Design and Implementation of a MATLAB/SIMULINK Based Simulator for Electric Machines and Drives With Loads

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1 Introduction

Motor analysis and design problems, particularly those involving complicated systems, require sophisticated simulation tools. Present tools tend to fall into one of two categories: detailed electromagnetic field simulators or high-level system simulators. Electromagnetic field simulators help with analysis of special materials, tooth structures, and other fine details of motor construction. They cannot address motor-load combinations, motors combined with realistic power supplies, or other system-oriented problems. High-level system simulators usually apply simple motor models, and concentrate on load operation and high-level system dynamics. With high-level simulation, it is difficult to address fast dynamic phenomena such as inverter-based motor controllers, frequency-dependent losses, or motor operating efficiency.

For dedicated drive applications, it would be very helpful to have a simulator that extends high-level system simulation down to the device level. At the device level, it is possible to model motor losses, pulse-width modulated (PWM) inverter action, and the fast dynamics needed to implement torque controls and nonlinear loops. This paper describes such a “mid-level” motor system simulation tool. The tool captures key phenomena based on a multi-state motor model combined with a magnetic saturation parameter. It includes models of inverter and rectifier systems that might be used for motor inputs or generator outputs. It addresses the interactions between a mechanical prime mover and a generator. The intent is a simulation system that can model a wide range of application problems. Typical applications would include an inverter-based ac induction motor drive with its mechanical load or a small auxiliary power system in which an engine drives an ac synchronous generator, which is then rectified and supplied to a battery bus.

This work was motivated originally by hybrid electric vehicle systems, in which an ac traction motor and inverter would be combined with an engine-generator auxiliary power unit (APU). Hybrid vehicle systems require considerable model detail to give an accurate analysis of energy budgets and accurate simulation of transient performance. The general needs extend to distributed power supply systems and to commercial and industrial power customers. The present work, as described here, supports analysis of machines and systems as needed for power-electronic building block (PEBB) applications. Distributed generation and power backup are representative examples of PEBB applications, for which mid-level motor simulation is needed.

The processing power of contemporary computers and the sophistication of the software running on these computers allows for simulation of very complex systems. This report deals with the simulation of electric machines and their drive systems, in a comprehensive context. The simulation uses the well-known SIMULINK and MATLAB packages. The report begins with modeling and control of induction motors, including inverter-based power input. It continues with synchronous machines, based on output rectifier loads.

2 Induction Motor Modeling and Simulation

Modeling and simulation of a three-phase induction motor and its electronic drive are considered in this section. The induction motor is modeled in stationary dq0 coordinates. The drive is a current regulated pulse-width modulated (PWM) inverter utilizing field oriented control (FOC). Simulation results in both motoring and generating modes are provided.
2.1 Induction Motor Model

This section details the conventional six-state induction motor model. The model is validated against a specific induction motor—a semi-custom unit manufactured for the University of Illinois Hybrid Electric Vehicle Program. The motor specifications are listed in Table 1 [1]. The per-phase equivalent circuit parameters are listed in Table 2 [2].

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<td>Stator leakage inductance</td>
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<tr>
<td>Referred rotor leakage inductance</td>
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The dynamic induction motor model used in this simulation is listed in eqs. (1)–(6) and is taken from [3]. The model is in stationary dq0 coordinates and contains six states: four flux linkage states, the motor speed and the motor shaft position. The primes indicate rotor quantities referred to the stator circuits.

\[
\frac{d\psi_{qs}}{dt} = \omega_b \left( v_{qs} + \frac{r_s}{X_{ls}} (\psi_{mq} - \psi_{qs}) \right) 
\]

\[
\frac{d\psi_{ds}}{dt} = \omega_b \left( v_{ds} + \frac{r_s}{X_{ls}} (\psi_{md} - \psi_{ds}) \right) 
\]

\[
\frac{d\psi'_{qr}}{dt} = \omega_b \left( v_{qr} + \frac{\omega_b}{\omega_r} \psi'_{dr} + \frac{r'_r}{X'_{tr}} (\psi_{mq} - \psi'_{qr}) \right) 
\]

\[
\frac{d\psi'_{dr}}{dt} = \omega_b \left( v_{dr} - \frac{\omega_b}{\omega_r} \psi'_{qr} + \frac{r'_r}{X'_{tr}} (\psi_{md} - \psi'_{dr}) \right) 
\]
\[
\frac{d\omega_{lm}}{dt} = \frac{1}{J_{lm}} \{T_{em} - T_{lm\text{ load}}\} 
\]

\[
\frac{d\theta_{lm}}{dt} = \omega_{lm} 
\]

Leakage reactances \(X_{ls}'\) and \(X_{lr}'\) are found by multiplying the base speed \(\omega_b\) (corrected for an equivalent two-pole motor) by the stator and referred rotor leakage inductances, respectively. The quantities \(r_s\) and \(r_r'\) are the stator and referred rotor winding resistances. The developed electric torque is denoted \(T_{em}\), \(T_{lm\text{ load}}\) is the load torque imposed on the motor, and \(J_{lm}\) is the total rotational inertia seen by the motor.

The quantities \(\psi_{mq}\) and \(\psi_{md}\) are defined as

\[
\psi_{mq} = X_M \left( \frac{\psi_{qs}}{X_{ls}} + \frac{\psi_{qr}'}{X_{lr}'} \right) - f_{sat}(\psi_m) 
\]

\[
\psi_{md} = X_M \left( \frac{\psi_{ds}}{X_{ls}} + \frac{\psi_{dr}'}{X_{lr}'} \right) - f_{sat}(\psi_m) 
\]

where \(X_M\) is defined as

\[
X_M = \left( \frac{1}{X_m} + \frac{1}{X_{ls}} + \frac{1}{X_{lr}'} \right)^{-1} 
\]

and \(f_{sat}(.)\) is a negative feedback term that simulates saturation of the induction motor magnetic material. Lastly, \(\psi_m\) is defined as

\[
\psi_m = \sqrt{\psi_{md}^2 + \psi_{mq}^2} 
\]

The magnetizing reactance of the motor \(X_m\) is the base speed multiplied by the magnetizing inductance. These equations are implemented into SIMULINK exactly as they are written [4]. Figure 1 shows the induction motor block diagram. The q-axis block implements eqs. (1) and (3), in addition to using eq. (7). The d-axis block implements eqs. (2) and (4) with eq. (8). These two blocks produce respective currents \(i_{qs}\) and \(i_{ds}\) given by

\[
i_{qs} = \frac{1}{X_{ls}} \{\psi_{qs} - \psi_{mq}\} 
\]

\[
i_{ds} = \frac{1}{X_{ls}} \{\psi_{ds} - \psi_{md}\} 
\]
The remaining zero-sequence current $i_{0s}$ is implemented separately as

$$\frac{di_{0s}}{dt} = \omega_b \frac{\psi_{0s} - r_s i_{0s}}{X_{ls}}$$

(13)

The zero-sequence quantities are included to facilitate the simulation of non-symmetrical situations such as a phase loss or fault. These currents are used as feedback for the electronic drive that will be explained later. The motor torque is computed using

$$T_{em} = \frac{3P}{4\omega_b} \left\{ \psi_{ds} i_{qs} - \psi_{qs} i_{ds} \right\}$$

(14)

where $P$ is the number of motor poles (four in this test case).

The motor model was tested in SIMULINK against the steady-state operation data [1] supplied by the motor manufacturer. The motor is supplied with the rated voltage of 67 V, at 60 Hz, and the motor shaft speed is varied slightly. The data points of shaft torque vs. speed are recorded. The simulated shaft torque and actual shaft torque provided by the manufacturer vs. shaft speed are shown in Figure 2. Note that as the motor is more heavily loaded, the simulation torque exceeds the actual motor torque. The torque mismatch shown in the figure is primarily
due to error in calculating the loss due to magnetic saturation. This data is not supplied by the manufacturer.

The simulated motor rms input current and the actual motor rms input current vs. shaft speed are shown in Figure 3. The simulated current is sinusoidal. The real motor has additional current harmonics due to inverter switching. In addition, the saturation function is uncertain, and accounts for a portion of the discrepancy. Because of additional losses, the actual motor current would always be expected to be higher than the simulated current.

The SIMULINK file used for the simulated motor steady-state load test is shown in Figure 4. The motor is modeled in $dq0$ coordinates and excited by

$$v_{qs} = \frac{67}{\sqrt{3}} \sin(120\pi t)$$

$$v_{ds} = \frac{67}{\sqrt{3}} \sin(120\pi t - \frac{\pi}{2})$$

$$v_{0s} = 0$$

These voltages represent a balanced three-phase set when transformed to stator abc coordinates. The transformations used will be discussed more in the section on FOC control. The “shaft speed ramp” block shown in Fig. 3 increases the motor shaft speed linearly with time. This represents loading the motor to the torque level needed to achieve this speed. The shaft speed and torque are then recorded at regular intervals to produce the plot in Figure 2. The simulation provides two ways to do this. The first is a continuous ramping in speed, the second is discrete changes in speed. The latter gives the motor time to reach steady state before the data point is recorded.

The rms phase current is found under the assumption of a purely sinusoidal current at the input frequency. The phase current is first rectified and then filtered to obtain the average. The average of the rectified current is given by

$$I_a = \frac{1}{\pi} \int_0^\pi I_0 \sin(\theta) d\theta = \frac{2I_0}{\pi}$$
Figure 2: Simulated and actual motor shaft torque vs. shaft speed.

Figure 3: Simulated and actual motor phase current vs. shaft speed.
This average current, assuming it is sinusoidal, can be related to the rms phase current by

\[ I_{rms} = \frac{\pi}{2\sqrt{2}} I_a \]

The plot in Figure 3 was generated this way.

2.2 Inverter Modeling

Models for the power structure of the PWM inverter and its interface to the induction motor are considered in this section and the FOC controller for the electronic drive is presented in the next section. The circuit diagram of an electronic drive with an induction motor is shown in Figure 5. The insulated gate bipolar transistors (IGBTs) used in this inverter switch at a frequency much higher than the fundamental of the desired motor terminal voltages. The duty ratio of the switches is modulated such that the moving average terminal voltages \( v_{ap}, v_{bp} \) and \( v_{cp} \) are sinusoidal and displaced by \( 2\pi/3 \) radians.

Figure 4: SIMULINK file used for motor tests.
In order to avoid component damage, the switches must operate such that the input source is never shorted and so the phase currents always have a path to take. Define the switching function \( q_{ij} \) as being equal to one when the switch \( q_{ij} \) is on and equal to zero when it is off. The switching constraints may now be written as

\[
q_{11} + q_{12} = 1 \tag{15}
\]

\[
q_{12} + q_{22} = 1 \tag{16}
\]

\[
q_{13} + q_{23} = 1 \tag{17}
\]

If the load is assumed to be balanced, a reference point may be placed at the center of the wye connected load. In addition, for analysis reasons, if the input source is split into two equal sources in series, the voltage at the node where the sources connect is equal to the center of the wye load. This allows the voltages \( v_{ap}, v_{bp} \) and \( v_{cp} \) to be written in terms of the switching functions as

\[
v_{ap} = \frac{1}{2} (q_{11} - q_{21}) V_{dc} = \frac{1}{2} (2q_{11} - 1)V_{dc} \tag{18}
\]

\[
v_{bp} = \frac{1}{2} (q_{12} - q_{22}) V_{dc} = \frac{1}{2} (2q_{12} - 1)V_{dc} \tag{19}
\]
\[ v_{cp} = \frac{1}{2} (q_{13} - q_{23}) V_{dc} = \frac{1}{2} (2q_{13} - 1)V_{dc} \quad (20) \]

where substitutions for \( q_{21}, q_{22} \) and \( q_{23} \) have been made using eqs. (15)-(17). The moving average of each phase voltage, denoted by an overbar, corresponds to replacing the switching functions with their corresponding duty ratios to obtain [5,6,7]

\[ \overline{v}_{cp} = \frac{1}{2} (2d_{11} - 1)V_{dc} \quad (21) \]
\[ \overline{v}_{kp} = \frac{1}{2} (2d_{12} - 1)V_{dc} \quad (22) \]
\[ \overline{v}_{ep} = \frac{1}{2} (2d_{13} - 1)V_{dc} \quad (23) \]

The duty ratios vary slowly in time compared to the switching period. The duty ratio function needed to produce sinusoidal phase voltages may be found by solving the following equation for the duty ratio

\[ V_0 \cos(\omega t) = \frac{1}{2} (2d - 1)V_{dc} \quad (24) \]

to obtain

\[ d(t) = \frac{V_0}{V_{dc}} \cos(\omega t) + \frac{1}{2} \quad (25) \]

where \( V_0 \) is the desired amplitude of the phase voltages. Therefore, in order to produce sinusoidal phase voltages at frequency \( \omega \) and amplitude \( V_0 \), the duty ratios of the switches \( q_{11}, q_{12} \) and \( q_{13} \) should be

\[ d_{11}(t) = \frac{V_0}{V_{dc}} \cos(\omega t) + \frac{1}{2} \quad (26) \]
\[ d_{12}(t) = \frac{V_0}{V_{dc}} \cos(\omega t - \frac{2\pi}{3}) + \frac{1}{2} \quad (27) \]
\[ d_{13}(t) = \frac{V_0}{V_{dc}} \cos(\omega t + \frac{2\pi}{3}) + \frac{1}{2} \quad (28) \]

The ratio \( k_d = 2V_0/V_{dc} \) is termed the depth of modulation and lies between zero and one. From eq. (24) it can be seen that the maximum value for \( V_0 \) is \( V_{dc}/2 \).
The simulation presented here assumes that the phase voltages produced by the inverter are perfectly sinusoidal and that all higher harmonics introduced by the PWM inverter are sufficiently filtered by the induction motor so as not to affect the torque. This assumption is validated by the fact that the harmonics produced by the switching are centered about the switching frequency and multiples of the switching frequency [5], and the switching frequency is deliberately set to a high value. The additional frequencies do produce extra losses, but these can be accounted for with separate correction functions, and do not affect motor dynamic performance.

The SIMULINK inverter model is shown in Figure 6. The inverter is placed in the stationary abc coordinate frame. The input variables to the drive are the intended reference motor phase currents $i_a^*$, $i_b^*$, and $i_c^*$ which are commanded by the FOC controller. The inverter supplies the phase voltages to the motor. It must control these voltages in such a way to produce the desired phase currents. The phase voltages are given by [4]

$$v_a = k_{CR} (i_a^* - i_a)$$
$$v_b = k_{CR} (i_b^* - i_b)$$
$$v_c = k_{CR} (i_c^* - i_c)$$

where $v_a$, $v_b$ and $v_c$ are the phase voltages produced by the electronic drive, $i_a$, $i_b$ and $i_c$ are the actual motor phase currents, and $k_{CR}$ is a high gain. If this gain is relatively high, or if a hysteresis-type current actuator is applied, the phase currents will track the reference values.

**Figure 6**: SIMULINK inverter block.
The IGBTs used are modeled as a forward voltage drop in series with an on-state resistance [2]. The circuit in Figure 5 indicates that the voltage drop suffered by each phase is equal to the forward voltage drop of one IGBT. This is because each phase is always connected in series with one IGBT. To provide a realistic IGBT model, this voltage drop includes both a fixed term and a resistive term. The resistive term is found by multiplying the phase currents by the IGBT on-state resistance, then directly subtracting this quantity from the phase voltages.

The voltage limit blocks in Figure 6 limit the phase voltages to the minimum of either the voltage commanded by the current regulator or one-half the dc bus voltage minus one IGBT forward drop. This mimics a standard PWM inverter, which, after reaching 100% depth of modulation, will go to alternative block modulation strategies, followed ultimately by square-wave operation. The voltage limit blocks simply clip the phase voltages when their amplitudes exceed one-half the dc bus voltage minus an IGBT forward drop.

The current drawn from the dc bus by the inverter must now be determined. In an electric vehicle or backup power application, the dc bus is likely to be supported by batteries, so bidirectional power flow is possible. The bus current can be found in any of several ways. Whether the current is into or out of the dc bus is determined by the current direction and the orientation of the switches. The switching action is determined by eqs. (26)-(28). The bus current can be found using the duty ratio information and the phase currents. Here, instead, the power into the electronic drive is found and then divided by the bus voltage to obtain the bus current. The power drawn by the drive is the phase currents multiplied by the phase voltages plus the inverter losses. The losses are due to the IGBT on-state resistance \( I^2R \) loss and forward drop \( VI \) loss. A given phase current is imposed on a single IGBT at any given time. For instance, in Fig. 5 both \( q_{11} \) and \( q_{21} \) cannot be on simultaneously. The current \( i_a \) flows in either \( q_n \) or \( q_{21} \), but not both. Therefore, the drive loss is produced by the phase currents through one IGBT on-state resistance and one forward drop. The drive dc bus current is given by

\[
i_{\text{drive}} = \frac{i_a v_a + i_b v_b + i_c v_c + (i_a^2 + i_b^2 + i_c^2) R_{\text{IGBT}} + (|i_a| + |i_b| + |i_c|) V_{\text{IGBT}}}{V_{\text{bus}}}
\]

where \( R_{\text{IGBT}} \) and \( V_{\text{IGBT}} \) are the on-state resistance and forward voltage drop, respectively, of the IGBTs. Note the absolute value of the currents was used to multiply the forward drop. No matter which way the current flows, it still flows through a forward voltage drop, be it either an IGBT or the corresponding reverse diode.

The drive current given by eq. (32) will become negative during regeneration, provided the regeneration is substantial enough to supply the losses. The transfer of power from the motor to the dc bus can be accomplished because the IGBTs have a reverse diode from collector to emitter. This diode is generally matched to the IGBT, and thus justifies a model matched to the IGBT itself.

The phase voltages to the motor may not be sinusoidal or balanced. This is especially true when the inverter produces block PWM because a voltage limit has been reached. Since the connection to the motor is 3-wire, the stator neutral is floating. This neutral voltage can be obtained by placing a small fictitious capacitor from the stator neutral to the dc bus neutral [4]. Remember, the dc bus neutral was defined as the midpoint of the dc bus voltage. The stator neutral voltage is given by
The phase voltages are then given by

\[
\begin{align*}
v_{as} &= v_a - v_g \\
v_{bs} &= v_b - v_g \\
v_{cs} &= v_c - v_g
\end{align*}
\]

where \(v_{as}, v_{bs},\) and \(v_{cs}\) are the actual stator winding voltages, and \(v_a, v_b,\) and \(v_c\) are the input phase voltages. In a stationary \(dq0\) coordinate system, the voltages are

\[
\begin{align*}
v_{qs} &= v_q - v_g \\
v_{ds} &= v_d \\
v_{0s} &= v_0 - v_g
\end{align*}
\]

So, given the desired voltages, the above equations are used to correct for the voltage imbalance in the stator.

### 2.3 Field Oriented Control

Field oriented control allows the induction motor torque to be controlled directly by changing the stator input currents. The results are very similar to the way the torque is controlled in a dc machine, but a complicated dynamic coordinate transformation is required for implementation. In an induction machine, the stator and rotor fields are not fixed electrically orthogonal to one another as they are in a dc machine. Another complication arises from the fact that the rotor field in an induction machine is induced by the stator field. Induction machine field oriented control controls the stator currents in such a way as to mimic the dc machine.

Induction machine field oriented control allows the following requirements for torque control to be met [6]

1. independent control of an equivalent armature current
2. independent control or constant value of (rotor) field flux
3. orthogonality between the field flux axis and the stator MMF axis to avoid interaction between the flux and MMF

In this section, the equations necessary for torque control of the induction motor will be developed. We start with the motor voltage equations in an arbitrary reference frame. For a more detailed discussion of reference frame transformation and elimination of the time varying inductances associated with the induction motor see [7]. The stator voltage equations for the induction machine in an arbitrary reference frame are
where

$$v_{ds} = r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega \lambda_{qs}$$  \hspace{1cm} (33)

$$v_{qs} = r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega \lambda_{ds}$$  \hspace{1cm} (34)

$$v_{0s} = r_s i_{0s} + \frac{d\lambda_{0s}}{dt}$$  \hspace{1cm} (35)

and for the rotor

$$v'_{dr} = r'_s i'_{dr} + \frac{d\lambda'_{dr}}{dt} - (\omega - \omega_r)\lambda'_{qr}$$  \hspace{1cm} (39)

$$v'_{qr} = r'_s i'_{qr} + \frac{d\lambda'_{qr}}{dt} + (\omega - \omega_r)\lambda'_{dr}$$  \hspace{1cm} (40)

$$v'_{0r} = r'_s i'_{0r} + \frac{d\lambda'_{0r}}{dt}$$  \hspace{1cm} (41)

where

$$\lambda'_{dr} = L'_{tr} i'_{dr} + L_m (i_{ds} + i'_{dr})$$  \hspace{1cm} (42)

$$\lambda'_{qr} = L'_{tr} i'_{qr} + L_m (i_{qs} + i'_{qr})$$  \hspace{1cm} (43)

$$\lambda'_{0r} = L'_{tr} i'_{0r}$$  \hspace{1cm} (44)

Here $\omega_r$ is the rotor speed, $\omega$ is the reference frame speed, and the primes indicate rotor referred quantities. The model described earlier in eqs. (1)-(6) can be obtained from these equations by setting the reference frame speed to zero, using eqs. (36)-(38) to substitute for the currents in eqs. (33)-(35), and using eqs. (42)-(44) to substitute for the currents in eqs. (39)-(41).

The derivation of field oriented control is best done with the voltage equations placed in the synchronously rotating reference frame. The synchronously rotating frame is found by substituting the input radian frequency $\omega_r$ for $\omega$ in eqs. (33)-(44) as in eqs. (45)-(48). Note that
the rotor voltages $v_{qr}$ and $v_{dr}$ have been set equal to zero to represent a conventional cage-rotor machine.

$$v_{qs}^e = r_s i_{qs}^e + \frac{d\lambda_{qs}^e}{dt} + \omega_e \lambda_{ds}^e$$  (45)

$$v_{ds}^e = r_s i_{ds}^e + \frac{d\lambda_{ds}^e}{dt} - \omega_e \lambda_{qs}^e$$  (46)

$$0 = r_s i_{qr}^e + \frac{d\lambda_{qr}^e}{dt} + (\omega_e - \omega_r) \lambda_{dr}^e$$  (47)

$$0 = r_s i_{dr}^e + \frac{d\lambda_{dr}^e}{dt} - (\omega_e - \omega_r) \lambda_{qr}^e$$  (48)

with

$$\lambda_{ds}^e = L_{ls} i_{ds}^e + L_m (i_{ds}^e + i_{dr}^e)$$  (49)

$$\lambda_{qs}^e = L_{ls} i_{qs}^e + L_m (i_{qs}^e + i_{qr}^e)$$  (50)

$$\lambda_{dr}^e = L_{ls} i_{dr}^e + L_m (i_{dr}^e + i_{dr}^e)$$  (51)

$$\lambda_{qr}^e = L_{ls} i_{qr}^e + L_m (i_{qr}^e + i_{qr}^e)$$  (52)

The primes indicating referred rotor quantities have been left out for notational convenience. The superscripted $e$'s indicate the synchronously rotating reference frame. The motor shaft torque can be expressed as

$$T_{em} = \frac{3PL_e}{4L_r} (\lambda_{ds}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e)$$  (53)

An in depth discussion of induction motor torque control may be found in [9]. The equations necessary for torque control will be presented here, and it will be shown that they perform the desired the function. The rotor flux should be entirely along the d-axis in the synchronously rotating frame. This implies the following

$$\lambda_{qr}^e = 0$$  (54)

When substituted in the torque equation, the following is obtained
\[
T_{em} = \frac{3PL_m}{4L_{ir}} \lambda^e_{dr} i^e_{qs} \tag{55}
\]

where \( P \) is the number of motor poles. Now, if the rotor flux along the \( d \)-axis is held constant, the motor shaft torque is completely controlled by the current \( i^e_{qs} \). This is similar to a separately excited dc machine in which the field current is held constant and the shaft torque is controlled by the armature current. A constant rotor flux in the direction of the \( d \)-axis implies

\[
\frac{d\lambda^e_{dr}}{dt} = 0 \tag{56}
\]

This is needed to find the reference stator currents that produce a given shaft torque and flux level in the rotor. The \( q \)-axis reference current is found directly from eq. (55), and is

\[
i^e_{qs} = \frac{4L_r}{3PL_m} \frac{T_{em} \lambda^e_{dr}}{A_e} \tag{57}
\]

where \( T_{em} \) is the desired shaft torque, \( \lambda^e_{dr} \) is the desired rotor flux, and \( i^e_{qs} \) is the desired stator current. The second stator current is slightly more difficult to obtain. Start by substituting eq. (54) into the rotor voltage eq. (48) to obtain

\[
v^e_{dr} = 0 = r^e_{dr} + \frac{d\lambda^e_{dr}}{dt} \tag{58}
\]

where the externally applied rotor voltage \( v^e_{dr} \) is zero. Now, solve eq. (51) for the rotor current \( i^e_{dr} \) as follows

\[
\lambda^e_{dr} = (L_{ir} + L_m)i^e_{dr} + L_m i^e_{ds}
\]

\[
i^e_{dr} = \frac{2^e_{dr} - L_m i^e_{ds}}{L_{ir} + L_m} = \frac{\lambda^e_{dr} - L_m i^e_{ds}}{L_r} \tag{59}
\]

By substituting eq. (59) into eq. (58), the stator \( d \)-axis current is obtained as

\[
i^e_{ds} = \frac{1}{r^e_{dr}} \left( r^e_{dr} \lambda^e_{dr} + L_r \frac{d\lambda^e_{dr}}{dt} \right) \tag{60}
\]

Under this control concept, the derivative of the rotor flux is non-zero only when the field is changing, such as in the field weakening region of operation. The field oriented control block as implemented in SIMULINK is shown in Figure 7.
Figure 7: SIMULINK field oriented control block.

We now have the necessary stator currents in the synchronously rotating frame, but they need to be converted to the stationary frame, since that defines the actual time-dependent current values to be used as inputs by the inverter. Figure 8 shows graphically the transformation between two reference frames. Define the following vectors

\[
\begin{align*}
\mathbf{f}_{\text{do}} & = \begin{bmatrix} f_q^y & f_d^y & f_0^y \end{bmatrix}^T \\
\mathbf{f}_{\text{do}} & = \begin{bmatrix} f_q^x & f_d^x & f_0^x \end{bmatrix}^T
\end{align*}
\]

(61)

where these vectors represent circuit variables such as current or voltage. The $x$ and $y$ indicate the reference frames rotating at speeds $\omega_x$ and $\omega_y$, respectively. The two reference frames are related by the transformation [7]

\[
\mathbf{f}_{\text{do}}^y = {^x}K^y \mathbf{f}_{\text{do}}^x
\]

(62)

where the matrix $^xK^y$ is given by

\[
{^x}K^y = \begin{bmatrix}
\cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) & 0 \\
\sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(63)
The transformation from the synchronous reference frame to the stationary reference frame is found by substituting zero for $\theta_s$ and $\omega_s t$ for $\theta_s$ in transformation matrix of eq. (63) where $\omega_s$ is the angular velocity of the synchronously rotating reference frame. The SIMULINK block that implements this transformation is shown in Figure 9. The input $\theta_e$ is the position of the synchronous frame and given by

$$\theta_e = \int \omega_e dt$$

In order to go back to the stationary reference frame, $\theta_e$ must be found. Since we do not have access to the rotor field, it must be found indirectly from the stator currents and the rotor position. The rotor position can be determined by means of an angular resolver or an optical encoder attached to the rotor.

Substituting eq. (54) into eq. (47) the slip speed may be found as

$$\omega_e - \omega_r e = -\frac{r_r i_{qr}^e}{\lambda_{dr}^e}$$

where $\omega_r e$ is the rotor speed in electrical radians per second. The rotor current in eq. (65) is found by substituting eq. (54) into eq. (52), and is given by

$$i_{qr}^e = \frac{L_m}{L_r} i_{qs}^e$$
Figure 9: Transformation from the synchronous to the stationary reference frame.

The rotor flux $\lambda^{e}_{dr}$ is given by eq. (51). This equation also contains the rotor current $i^{e}_{dr}$. However, by substituting eq. (54) into eq. (48) and setting the time derivative of the rotor flux to zero, it can be shown that $i^{e}_{dr}$ is equal to zero. Equation (51) then becomes

$$\lambda^{e}_{dr} = L_n i^{e}_{ds}$$

(67)

The desired form for the slip frequency is found by substituting eqs. (66) and (67) into eq. (65) as

$$\omega_s - \omega_re = \frac{r_r i^{e}_{qs}}{L_r i^{e}_{ds}}$$

(68)

which, when rearranged and integrated gives the rotor synchronous position

$$\theta_e = \int \left( \frac{r_r i^{e}_{qs}}{L_m i^{e}_{ds}} + \omega_re \right) dt$$

(69)

The implementation of this equation is shown in Figure 7.

The SIMULINK block diagram containing the induction motor, electronic drive, and field oriented controller is shown in Figure 10. Note that since the motor is in $dq0$ coordinates, the motor input voltages and the currents must be transformed between the $dq0$ and the $abc$ coordinate systems. The relationship between the two coordinate frames is depicted graphically in Figure 11 and can be written as [7]

$$f_{q0} = K_s f_{abc}$$

(70)

where
This matrix represents the transformation from stationary \( abc \) coordinates to \( dq0 \) coordinates in an arbitrary reference frame. Here the transformation is from stationary \( dq0 \) coordinates to stationary \( abc \) coordinates. Therefore, the inverse of \( K_s \) must be used as in eq. (73).

\[
f_{\text{abc}} = K_s^{-1} f_{\text{dq0}}
\]

where

\[
K_s^{-1} = \begin{bmatrix}
\cos(\theta) & \sin(\theta) & 1 \\
\cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\
\cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1
\end{bmatrix}
\] (74)

Since both coordinate frames are stationary, \( \theta \) is equal to zero in eq. (74). The SIMULINK implementation of this transformation is shown in Figure 12. This transformation is used twice in the simulation. Once to convert the \( dq0 \) reference currents from the field oriented controller to \( abc \) coordinates for use by the drive, and once to convert the motor currents to \( abc \) coordinates to be used for feedback.

The remaining block in Figure 10 is the block labeled "field weakening." As the motor speed increases, so must the terminal voltage. This follows the behavior of conventional volts-per-hertz operation: the rotor flux is approximately proportional to the ratio of stator voltage to stator frequency. Frequency must rise to increase the speed, and the voltage must rise accordingly to maintain cost rotor flux linkage. At a certain speed, defined as the base speed, the inverter will hit a voltage limit imposed by the battery voltage or input dc bus. At this point, the rotor flux must be reduced in order to avoid derating the stator current. The usual scheme employed is to reduce the rotor flux inversely with rotor speed above the base speed, while the maximum allowed value of \( i_{qs}^* \) is held constant. Below base speed, motor operation constitutes a constant torque regime, in which the torque can have any positive or negative value up to the limit represented by the limiting value of \( i_{qs}^* \). Above base speed, the decreasing flux level results in motor operation limited to a certain maximum power, consistent with the rated value of \( i_{qs}^* \). This represents a constant power regime.

The motor model used so far has linear magnetic characteristics. This means, in essence, that the shaft torque increases with input voltage without limit. The actual motor will stall when
the shaft torque exceeds the **breakdown torque**. The breakdown torque limit exists because there is a limit on how intense the machine air gap flux can become due to saturation of the iron. This problem is addressed in linear models simply by limiting the torque reference used in the controller to that of the machine's breakdown torque. In practice, two torque limits are of interest. The motor's **rated torque**, which corresponds to the conventional steady-state rating, represents a value that can be produced continuously in most applications. The breakdown torque represents the maximum possible short-term value. In traction applications, high torque is rarely applied for more than a few seconds, so breakdown torque is representative of actual system limitations. In many other industrial systems, rated torque is a more appropriate limit, and breakdown torque provides short-term headroom, just as in conventional induction motor applications. For typical motor designs, breakdown torque is about 250% of rated torque.

Figure 13 shows the motor shaft torque, reference torque, and stator currents for a simulation example. The torque reference is left at zero until time has reached 1 s. This is done to allow the flux to build in the rotor. If the rotor flux is not established, the field oriented control will not function properly. At time $t = 1$ s, the torque reference is brought to 60 N·m. The current $i_{rp}$ climbs to approximately 150 A, and the shaft torque climbs to 60 N·m very nearly tracking the reference command. The reference then goes to $-60$ N·m at $t = 3$ s. Note that the current $i_{dq}$ stays constant because it is proportional to $\alpha_d$.

![Diagram of induction motor, electronic drive and FOC controller.](image)

**Figure 10:** Induction motor, electronic drive and FOC controller.

The next three figures illustrate an example of induction motor operation in the field weakening region. At time equal to one second, the torque reference is set to the motor rated torque of 41 N·m, and the motor is allowed to accelerate freely, based on a purely inertial load. Figure 14 shows the motor shaft speed and the shaft torque. The shaft torque falls off approximately as one over the shaft speed after reaching the base speed of about 5400 RPM. The motor currents in the synchronous reference frame are shown in Figure 15. Note that as the d-axis current begins to fall due to field weakening, the q-axis current increases. The controller is using the extra dc bus voltage in an attempt to keep the shaft torque at the reference value.
The q-axis current increases for a short time until the rated current of the motor is reached. This limit has been set equal to the steady state rated current of the motor for illustration of the field weakening effect. Once the current limit is reached, the torque begins to fall off.

The d-axis rotor flux and its time derivative are shown in Figure 16. The rotor flux decreases in proportion to one over the shaft speed above base speed. From the torque equation (55), it is immediately obvious why the shaft torque also decreases as one over the shaft speed when \( i_{qs} \) is constant. The shaft power in the field weakening region is approximately constant.

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**Figure 11:** Transformation between \( abc \) and \( dq0 \) coordinate frames.

![Transformation between abc and dq0 coordinate frames](image1.png)

**Figure 12:** Transformation from stationary \( dq0 \) coordinates to stationary \( abc \) coordinates.

![Transformation from stationary dq0 to abc](image2.png)
Figure 13: Induction motor shaft torque, torque reference, and stator currents.

Figure 14: Induction motor shaft speed and torque.
Figure 15: Induction motor stator currents in synchronous reference frame.

Figure 16: The d-axis rotor flux and time derivative of d-axis rotor flux.
2.4 Simulation of Induction Motoring and Generation

This section considers induction motoring and generation based on the induction machine model given in the previous section. The examples use a FOC electronic drive interfaced to a battery pack. The SIMULINK block diagram for this example is shown in Figure 17. The induction motor and FOC controller are contained within a single block. In addition, there is a proportional-integral speed controller coupled to the field controller. In this example, we consider a comprehensive system in which an internal combustion engine is coupled to the induction machine. The induction machine can be used as a "combined starter/alternator" in this case—an application of current interest in automotive systems. It could also represent any number of induction generator applications.

Figure 17: Induction generator setup.

Figure 18 shows the power drawn from the battery pack by the electronic drive. There is an initial spike in power during startup. This is due to the electronic drive bringing the induction motor up to a speed of 325 rad/s. Once the motor is up to speed, the power drops close to zero because the load is inertial in this example. At 1.5 s the combustion engine is brought to 40% throttle and attempts to overspeed the motor. The field oriented controller commands a negative torque to hold the speed at 325 rad/s resulting in a net power flow to the battery pack. At 3.5 s the combustion engine is brought to 80% throttle resulting in an increased power flow to the battery pack. In this example, the system runs open loop, but it is a simple matter to implement a closed loop power control. In fact, the best option would be to incorporate speed and torque control to maintain the combustion engine at its most efficient operating point for a given load.

Figure 19 shows the induction motor shaft speed in rad/s and the battery pack state-of-charge. Note there is an initial drop in the battery pack state-of-charge during the engine startup transient. When generation commences, the battery pack state-of-charge begins to rise.
3 Synchronous Generator and Rectifier Modeling and Simulation

Modeling and simulation of a permanent magnet synchronous generator and its corresponding output delivered to an SCR-based rectifier are considered in this section. First, operation of a simplified three-phase bridge rectifier will be presented. Next, the synchronous generator model will be presented in stationary $abc$ coordinates. Finally, interfacing of the generator and rectifier will be considered and simulation results given.
3.1 SCR-based Rectifier

The section details the operation and simulation of a typical phase-controlled rectifier with variable input frequency capability. A mid-level rectifier simulation must include both the average dynamics (the usual high-level simulation case) and the switch timing. In a rectifier used at conventional power frequencies, ripple is high and average dynamics are not adequate for modeling. Furthermore, the control is fundamentally nonlinear. Switches act to alter device timing. The switch action is coupled with circuit currents and voltages. Because of the interaction between switch timing control and the circuit, a rectifier can serve as an example of a mixed-mode simulation problem. A useful simulator must address the analog dynamic behavior of the circuit state variables, the digital processing and behavior of switch controls, and the variable switch timing action. It is common for a phase-controlled rectifier to be coupled to a driven wound field or permanent magnet synchronous generator when dc output is desired. The rectifier considered here is a three-phase bridge consisting of six silicon controlled rectifiers (SCRs), a firing circuit, and a current controller. The rectifier is used to convert the ac current from the generator’s three-phase voltage source into a dc current for the load. A simplified rectifier circuit diagram is shown in Figure 20.

![Simplified rectifier model.](image)

Figure 20: Simplified rectifier model.

This circuit has three ac input voltage sources, six SCRs and a dc current source load. The voltages of the three input sources form a balanced three-phase set as in eq. (75)

\[
\begin{align*}
    v_a &= V_o \sin(\alpha t) \\
    v_b &= V_o \sin(\alpha t - \frac{2\pi}{3}) \\
    v_c &= V_o \sin(\alpha t + \frac{2\pi}{3})
\end{align*}
\]

(75)

The basic operation of the rectifier is to cyclically connect the input sources to the load in a manner that transfers power from the input to the load. The load is a dc current source, which means that power transfer can only take place if the voltage imposed across the load by the rectifier has a dc component. To understand how this is done, we must first look at the operation of a single SCR.
The SCR is very much like a diode; it is capable of conducting current in the forward direction and is reverse blocking. The circuit symbol for the SCR is shown in Figure 21. The main difference between the two is the SCR can also forward block. In fact, if just placed in a circuit, it will block in both directions. Forward conduction of the SCR is initiated by applying a current pulse to the third gate terminal. Forward conduction will then commence, provided the device was initially forward biased from anode to cathode. After this point, the SCR acts much like a diode [8]. The SCR will continue to conduct until the current flowing through the device goes to zero. This is a very important aspect of the SCR; its turn-on time is directly controllable by the gate signal timing, but its turn-off time is determined by the current flow through the device. Turn-off is determined by the circuit in which the SCR resides.

![Figure 21: Silicon controlled rectifier.](image)

As shown in Figure 20, the SCRs $S_1$, $S_3$ and $S_5$ conduct current to the load. These will be termed the high side SCRs. The devices $S_2$, $S_4$ and $S_6$ conduct current from the load, and will be termed the low side SCRs. The average load voltage is controlled by when these SCRs are turned on and how long they stay on. In the case of a current source load as we have here, a particular SCR will stay on until commutated off by another SCR on the same side. For instance, if $S_1$ is conducting, it can be turned off by sending $S_3$ a gate pulse when $v_b>v_a$. Since $v_b$ is greater than $v_a$, when $S_3$ is turned on, $S_1$ will be reverse biased, and the load current will be shifted to $v_b$. In a real rectifier which has some line inductance associated with each input source phase, there will be a short period of time called the commutation time, during which both $S_1$ and $S_3$ carry the load current. This case will be discussed later when the full rectifier and generator models are considered together.

Figure 22 shows the phase voltages input to the rectifier. The amplitude of the phase voltages is taken as one volt for simplicity. For ease of analysis, the time axis has been multiplied by the frequency of the sinusoids to transform it to radians. The delay angle $\alpha$ defines the turn-on of the SCRs. The delay angle is defined to be zero for $S_1$ at $\alpha=\pi/6+2k\pi$ and zero for $S_2$ at $\alpha=-\pi/6+(2k+1)\pi$ for $k=0,1,2,\ldots$. It may be similarly defined for the remaining SCRs, only shifted by $2\pi/3$ radians for the phase b SCRs, and $4\pi/3$ for the phase c SCRs. The reason $\alpha=0$ is defined this way is because of the assumption of continuous current conduction due to the current sourced load. Consider trying to turn on $S_3$ before the point $\alpha=0$ as defined above. In this situation, as we attempt to turn on $S_3$, $S_1$ is conducting and $v_a>v_b$. Therefore, $S_3$ is reverse biased and will not conduct. However, if an attempt is made to turn on $S_2$ after the point $\alpha=0$, forward bias exists and $S_3$ will turn-on, taking the current away from $S_1$. 

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As mentioned above, it is necessary for the load voltage to have a dc component in order to transfer power to the dc current source. For this simplified rectifier, it is easy to demonstrate that this dc component exists. The average load voltage may be calculated by realizing that each SCR comes on at $\alpha$ and stays on for $2\pi/3$ radians. The average voltage on the high side of the load is

$$\langle v_{\text{high}} \rangle = \frac{3V_o}{2\pi} \int_{\alpha}^{\pi/3 + \alpha} \sin(\theta + \frac{\pi}{6}) d\theta = \frac{3\sqrt{3}V_o}{2\pi} \cos(\alpha)$$

where $\theta = \alpha$ and $V_o$ is the amplitude of the phase voltages, in this case one. The average voltage of the low side is evaluated in a similar manner.

$$\langle v_{\text{low}} \rangle = \frac{3V_o}{2\pi} \int_{\alpha}^{\pi/3} \sin(\theta + \frac{\pi}{6} + \pi) d\theta = -\frac{3\sqrt{3}V_o}{2\pi} \cos(\alpha)$$

The dc component of the load voltage is

$$V_L = \langle v_{\text{high}} \rangle - \langle v_{\text{low}} \rangle = \frac{3\sqrt{3}V_o}{\pi} \cos(\alpha)$$

for $0 \leq \alpha \leq \pi$. This shows that the load voltage has a dc component whose amplitude is dependent on the amplitude of the input phase voltages and on the delay angle $\alpha$. Equation (76)
even implies that this voltage may be made negative by making the delay angle greater than 90°. For this example, it is possible to make $V_r$ negative for as long as desired because of the current source load. An inductor in the load can emulate the characteristics of a current source, and this is done in many cases. However, in contrast to the example above, when the voltage across the inductor is reversed, the current will begin to fall, and will eventually become zero. Once this happens, the SCRs will turn-off and the load voltage will fall to zero.

Figure 23 shows the phase $a$ voltage, the switching functions for $S_1$ and $S_2$ for $\alpha=0$, and the voltage imposed on the load by phase $a$. The product $S_1v_a$ is the voltage provided to the high side of the load and $S_2v_a$ is provided to the low side. Figure 24 shows the high and low side load voltages for $\alpha$ equal to 0, $\pi/6$, $\pi/3$ and $\pi/2$ radians. The voltage across the load for these delay angles is shown in Figure 25. It is clear from these plots that the average load voltage decreases as the delay angle increases. The average load voltage for $\alpha=\pi/2$ is zero with this ideal current-source load.

Consider a real rectifier example in which the rectifier load is a battery pack with a series RL filter. The resistance is parasitic, representing the inductor, connection, and battery losses. It would be made as small as possible in practice. The inductance is provided for current filtering to reduce losses. If the load current is perfectly filtered, the system losses are $I_d^2R_1$, where $I_d$ is the dc load current and $R_1$ is the total series resistance of the RL combination and the battery pack. The largest unwanted frequency component of the load voltage for this rectifier subsystem is six times the input frequency. There are higher frequency components as well, at multiples of $6f_{in}$. These voltage components create current components of the same frequency in the load. If they are not filtered, the losses become

$$P_{loss} = \left( I_d^2 + \sum I_{dj} \right) R_1,$$

where the $I_{dj}$ terms are the additional current components. This loss is always greater than $I_d^2R_1$. Since the only power transferred to load is due to the zero component, it is desirable to eliminate the $I_{dj}$ components. The largest and hardest to filter component is $I_{d6}$, and the filter is chosen with the reduction of this component in mind.

In a real world application, the delay angle must be calculated and translated into a real time quantity that may be used to trigger the SCRs. Consider the expression for the load voltage in eq. (76). This equation is nonlinear in the control variable $\alpha$, which makes the control issue difficult. However, if we define the control variable to be $\kappa = \cos(\alpha)$, the problem becomes linear [5]. A possible control strategy is to sense the average load voltage, compare it to the desired load voltage, and use the error, possibly with a PI controller, to adjust $\kappa$.

The calculation of the arccosine can be done in a simple way. Let $\kappa$ be the control input which varies between zero and the amplitude of the phase voltages, scaled to 1 V. Let $v_{ar}$ be a cosine function of the same frequency as the phase voltages that is equal to one at $\alpha=0$ for phase $a$ as illustrated in Figure 26. The SCR $S_1$ is to be turned on at $\alpha=\omega t_{on}$. The time $t_{on}$ is exactly the point where $\kappa$ and $v_{ar}$ meet. The trigger for $S_1$ is derived by comparing $\kappa$ and $v_{ar}$. As $v_{ar}$ crosses and falls below $\kappa$, the comparator triggers $S_1$. The remaining high side SCRs $S_3$ and $S_5$ are triggered in a similar fashion. The low side SCR $S_2$ can be triggered by comparing $-\kappa$ to $v_{ar}$. The only difference is that the comparator must detect when $v_{ar}$ cross and goes above $-\kappa$. The remaining low side SCRs are triggered similarly.
Figure 23: Phase $a$ voltage, switching functions and voltage imposed on load.
Figure 24: High and low side load voltages for various values of $\alpha$. 

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Figure 25: Voltage across the load for various values of $\alpha$. 
Figure 26: Waveforms for SCR phase control.

The signals $v_{ar}$, $v_{br}$ and $v_{cr}$ are not difficult to obtain. Consider the phase $a$ voltage

$$v_a = V_0 \sin(\omega t)$$

Integrate this voltage to obtain

$$v_{ar} = -\frac{V_0}{\omega} \cos(\omega t)$$  \hspace{1cm} (77)

and we now have a set of orthogonal signals from which $v_{ar}$ may be synthesized by using, for example, operational amplifiers. The signal we need to create is

$$v_{ar} = \cos(\omega t - \frac{\pi}{6})$$  \hspace{1cm} (78)

The construction of this signal is shown graphically in Figure 27. From the figure, $v_{ar}$ is given by

$$v_{ar} = \sin(\frac{\pi}{6})\sin(\omega t) + \cos(\frac{\pi}{6})\cos(\omega t)$$  \hspace{1cm} (79)

The signals $v_{br}$ and $v_{cr}$ are constructed in a similar manner.
Synthesis of the reference signals $v_{ar}$, $v_{br}$ and $v_{cr}$ can be done this way if the source impedance is not too high. If the source impedance is too great, the terminal voltages will become very much current dependent, and there will be a significant amount of switching noise present. The reference signals may become distorted and their amplitudes or phases changed. This will make the average load voltage differ from the form given in eq. (76).

The effect of scaling the amplitude of $v_{ar}$ on the phase delay angle is shown in Figure 28. The actual phase delay angle is reduced by reducing the amplitude of $v_{ar}$. The change in average load voltage can be written as

$$\Delta V_L = \frac{3\sqrt{3}V_0}{\pi} (\cos(\alpha_1) - \cos(\alpha_2))$$

Figure 27: Construction of the signal $v_{ar}$.

Figure 28: Effect of scaling the amplitude of $v_{ar}$ on the delay angle.
The cosine function can be eliminated from the previous equation substituting the following expressions

\[
\cos(\alpha_1) = \frac{\kappa}{V_1} \\
\cos(\alpha_2) = \frac{\kappa}{V_2}
\]

The change in the average load voltage is then

\[
\Delta V_L = \frac{3\sqrt{3}V_0}{\pi} \left( \frac{V_1 - V_2}{V_1V_2} \right) \kappa
\]

Now, let \(V_1 = 1\), which is the desired value, and let \(V_2 < 1\). The expression for the average load voltage is

\[
\Delta V_L = \frac{3\sqrt{3}V_0}{\pi} \left( 1 - \frac{V_2}{V_2} \right) \kappa
\]

which shows that the average load voltage increases as the amplitude of the reference signal \(v_{\varphi}\) decreases.

The second problem of \(v_{\varphi}\) having phase error is simpler. The delay angle varies linearly with changes in the phase of \(v_{\varphi}\). The last difficulty is that of obtaining the line frequency needed to construct the reference signals. There is a division by input frequency in eq. (77). If the line frequency is constant, this is not a problem. However, in this case the rectifier may be coupled to a generator that varies in speed over a wide range. Therefore, there is a need to sense the input frequency in some way. This can be done in several ways, including sensing the generator shaft speed. Most conventional variable-frequency rectifiers use phase-locked-loops (PLLs) to synthesize signals that are in phase with the input voltages, and of magnitude one. The PLL does a good job of maintaining the reference signal phase and amplitude in the presence of rectifier input distortion. There is a slight disadvantage to using PLLs. If the input frequency changes too fast, they will lose their lock condition. It is also possible to use an automatic gain control (AGC) amplifier to convert eq. (77) to fixed-amplitude output, but such an approach offers no special advantages over a PLL.

The PLL is shown in block diagram form in Figure 29. It consists of a mixer, lowpass filter, integrator, and a voltage controlled oscillator (VCO). The mixer used is a multiplier that has inputs \(v_{in}(t)\) and the VCO output signal \(v_{vco}(t)\). The output of the mixer is given by

\[
\begin{align*}
\nu_m(t) &= v_{in}(t)v_{vco}(t) = V_0 \cos(\omega_{in}t + \theta_{in})\cos(\omega_{vco}t + \theta_{vco}) \\
&= V_0[\cos(2\omega_{in}t + \theta_{in} + \theta_{vco}) + \cos(\theta_{in} + \theta_{vco})]
\end{align*}
\]

where
\[ \begin{align*}
    v_{in}(t) &= V_0 \cos(\omega_{in}t + \theta_{in}) \\
    v_{vco}(t) &= \cos(\omega_t t + \theta_{vco})
\end{align*} \]

\[ \begin{align*}
    v_{in}(t) \rightarrow \text{Lowpass Filter} \rightarrow V_{low} \rightarrow k_{PLL} \rightarrow 1/s \rightarrow \theta_{vco}
\end{align*} \]

\[ \cos(\omega t + \theta_{vco}) \]

\[ \text{VCO} \]

**Figure 29:** Block diagram of the Phase-Locked-Loop.

The mixer output frequency components are the sum and difference of the input voltage and the VCO frequencies. This signal is then passed through a low pass filter to attenuate the higher frequency component. The output of the low pass filter is approximately

\[ v_{low} \approx k \cos(\theta_{in} - \theta_{vco}) \]

This signal is then scaled by \( k_{PLL} \) and integrated, and the result used to control the VCO. The loop attempts to minimize this value. At the minimum, the VCO voltage is

\[ v_{vco}(t) = \cos(\omega_{in}t - \frac{\pi}{2}) = \sin(\omega_{in}t) \quad (82) \]

Even though the VCO was shown here to have the same frequency as the input source, this process will still work even if it does not. If there is a frequency difference between \( v_{in} \) and \( v_{vco} \), then there is a corresponding increase in phase error with time. As the phase is corrected, the frequencies will become equal.

The input to the rectifier is provided by the synchronous generator. Therefore, the input frequency is dependent directly on the generator shaft speed. The fact that this speed is not constant requires the filter in the PLL to be at least second order, with two free integrators. This will allow the PLL to track a ramp in frequency [10]. The PLL delivers a signal with amplitude of one that lags the input signal by 90°. There is a separate PLL for each phase to give a balanced three-phase set of signals from which \( v_{0r}, v_{br} \) and \( v_{cr} \) can be constructed.

The SIMULINK block diagram of a PLL is shown in Figure 30. Saturation has been added to the input and VCO signals that effectively squares the sinusoids. This eliminates distortion present in the input signal. The PLL process works in much the same way, however. The \( t_{out} \) signal is the width of the gate pulse. In a real rectifier, the SCR may be initially reverse biased when the gate pulse arrives. This prevents the SCR from coming on. This problem is alleviated by extending the gate pulse to an angular duration of \( \pi/2 \) radians. If the SCR is initially reverse biased, it will come on at the instant it becomes forward biased.

The process of construction of the timing signals \( v_{0r}, v_{br} \) and \( v_c \) is shown in Figure 31. These signals are constructed directly from the three-phase balanced set delivered by the PLLs. If unbalanced operation, such as the loss of a generator phase, is required, the signals may be
constructed as described above by using integration to create an orthogonal set of signal vectors for each phase.

Figure 30: Block diagram of Phase-Locked-Loop.

Figure 31: Construction of the reference signals \( v_{ar}, v_{br}, \) and \( v_c \).

The SIMULINK block diagram for generation of the trigger pulses used to turn on the individual SCRs is shown Figure 32. Most elements of this block – but not all – are digital, which emphasizes the mixed-mode nature of the complete rectifier simulation. The reference signals \( v_{ar}, v_{br}, \) and \( v_c \) are compared with \( \kappa \) to detect when the reference signals fall below \( \kappa \) and trigger the high side SCRs. The low side SCRs are triggered by comparing the reference signals to \(-\kappa\). The signals SCR_\( a \), SCR_\( b \), and SCR_\( c \) are used to control the high and low side phase \( a \), \( b \), and \( c \) SCR pairs, respectively.

These trigger signals activate the gate drivers for each SCR as shown in Figure 33. The gate drivers are positive edge-triggered D flip-flops. The Q-output of each flip-flop is used to trigger its respective SCR. This output is held high for the length of time specified by \( T_{\text{gate}} \). Ideally, the trigger should be held high for \( \pi/2 \) radians to assure correct SCR firing.
Figure 32: Generation of SCR gate trigger pulses.

Figure 34 shows the block diagram model for each SCR. The state of the SCR, either on or off, is modeled by a positive edge-triggered D flip-flop. The flip-flop output $Q$ is equal to one when the SCR is in the on-state. The output $\bar{Q}$ is equal to $1-Q$. The clock signal for the flip-flop is state with Boolean operations as follows

$$CLK = V_{bias}V_{gate}\bar{Q} + V_{gate}\bar{Q}_d$$

where $\bar{Q}_d$ is a delayed version of $\bar{Q}$, $V_{bias}$ is equal to one when the SCR is forward biased, and $V_{gate}$ is equal to one when the gate signal is active. If the SCR is initially off and the gate signal is made active, the clock input will become high until the flip-flop changes state, and the clock will then go low. The gate signal remains high for $\pi/2$ radians, but the clock signal will stay low. If the SCR is reset, the $\bar{Q}$ output will become high again. If the gate signal is still active, the second term in the clock equation above will cause the flip-flop to be clocked again. In other words, an attempt will be made to again turn the SCR on. This mimics the action of a real rectifier which provides a continuous gate pulse train for $\pi/2$ radians. If the SCR terminal conditions allow it, the SCR will go into the on-state again.

The SCR may be reset (turned off) in several ways. The usual reset is the phase current crossing zero. This action provides a reset signal given that the SCR was in the on-state initially. The second way to cause a reset is for the SCR current to be less than a value $i_{eps}$ after the gate signal has become inactive, given that the SCR was initially in the on-state. This prevents the SCR from remaining on indefinitely with zero phase current. The last way to reset the SCR is
provided by the Reset Logic block in Figure 37. This reset prevents two SCRs on the same side, high or low, from being active when there are no active SCRs on the other side. This situation can occur when one of the SCR gates has been disabled to simulate a faulty SCR. The reset for phase \( a \) is given by

\[
\overline{R}_{AL} = (\overline{S}_{LA} + \overline{S}_{LB}) + S_{HA} + S_{HB} + S_{HC}
\]

where this reset is active low as indicated by the over-bar. This equation says reset the low-side phase \( a \) SCR if no high-side SCRs are on, and the low-side phase \( a \) and phase \( b \) SCRs are on. This applies for an \( abc \) sequence.

There is one additional constraint, but is not really a reset. A control line from each low side SCR’s \( \bar{Q} \) output is logically anded with the enable for the respective high-side SCR. The same is true for each high-side SCR. This prevents both SCRs belonging to one phase from coming on simultaneously. Again, this problem arises when simulating fault conditions.

Figure 35 shows the SCR pair for phase \( a \), the gate driver block, and the check for forward bias. The check for forward bias is done by comparing the phase voltage with either \( V_h \), the high side load voltage, or \( V_l \), the low side load voltage. Figure 36 shows how the high and low side load voltages, and the load current are determined. Define the following vectors

\[
S_H = [S_1 S_3 S_5]^T
\]

\[
S_L = [S_2 S_4 S_6]^T
\]

\[
v_{abc} = [v_a v_b v_c]^T
\]

\[
i_{abc} = [i_a i_b i_c]^T
\]

The high and low side load voltages are given by

\[
v_H = \frac{S_H^Tv_{abc}}{S_H^TS_H + \overline{Z}_H} + \overline{Z}_H (v_b + v_L) (Z_H + Z_L) \quad (83)
\]

\[
v_L = \frac{S_L^Tv_{abc}}{S_L^TS_L + \overline{Z}_L} + \overline{Z}_L (v_H - v_b) (Z_H + Z_L) \quad (84)
\]

where

\[
Z_H = \begin{cases} 1 & S_H^TS_H > 0 \\ 0 & S_H^TS_H = 0 \end{cases}
\]

\[
Z_L = \begin{cases} 1 & S_L^TS_L > 0 \\ 0 & S_L^TS_L = 0 \end{cases}
\]

and \( \overline{Z}_H = 1 - Z_H \), \( \overline{Z}_L = 1 - Z_L \), and \( V_b \) is a dc voltage source in series with the load. \( Z_H \) and \( Z_L \) are logical variables, therefore the sum in \( Z_H + Z_L \) in eqs. (83) and (84) is equal to zero or one.
The expressions for the high and low side load voltages do not define exactly the load voltage. They are used to determine if the SCRs are forward biased or not. For instance, if no SCRs are on, \( S_{HF} = 0 \) and \( S_{L} = 0 \), then by eqs. (83) and (84), \( v_{HF} = 0 \) and \( v_{L} = 0 \). This does not mean that the voltage across the load is zero. It means that the load is not physically connected to the rectifier by any of the SCRs. Either a high or low side SCR may be turned on at this point. However, as soon as any of the SCRs are turned, the load voltage is then well defined with respect to the rectifier.

Consider the case where \( S_{I} \) is in the on-state. The high side load voltage, as given by eq. (83) is \( v_{a} \). This is correct because the high side of the load is connected directly to phase \( a \). The low side load voltage, as given by eq. (84), is \( v_{a} - v_{b} \). This is confirmed by summing the voltages from phase \( a \) to the low side of the load. In order for a low side SCR to be turned on, its respective phase voltage must be lower than \( v_{L} \).

When two high side SCRs are on, the high side load voltage, from eq. (83), is equal to the average of the two phases connected to the high side. This represents the commutation process, in which one SCR is turning off and another is turning on, both on the same side. The fact that the load voltage is equal to the average of the connected phase voltages will be shown later.
Figure 34: SIMULINK SCR model.

Figure 35: Phase a SCR pair with gate driver.
**Figure 36:** Determination of the high and low side load voltages and the load current.

The SIMULINK block diagram of the complete SCR bridge rectifier is shown in Figure 37. The memory blocks are used to prevent an algebraic loop from forming due to the feedback of the load voltage for determination of forward bias. The Brdg_en input is the bridge enable. It directly enables or disables the SCRs. In this case, it affects the entire bridge. However, each SCR has its own enable that can be used to simulate a single- or multiple-device fault.

**Figure 37:** SIMULINK block diagram of SCR bridge.
3.2 Generator Modeling

The generator considered here is a permanent magnet synchronous generator. The model is presented in stationary $abc$ coordinates to be compatible with the rectifier model. The model contains no damper windings, the permanent magnets are simulated as a constant current applied to a fictitious field winding, and the machine is assumed to be of the round rotor type. The model for a conventional synchronous generator with no damper windings is

$$
\begin{bmatrix}
\mathbf{v}_{abc} \\
\mathbf{v}_{fd}
\end{bmatrix} = \frac{d}{dt} \begin{bmatrix} 
\mathbf{L}_s & \mathbf{L}'_{sr} \\
\mathbf{L}'_{sr}^T & L_r 
\end{bmatrix} \begin{bmatrix}
-\mathbf{i}_{abc} \\
\mathbf{i}_{fd}
\end{bmatrix} + \begin{bmatrix}
-\mathbf{R}_s & 0 \\
0 & R_{fd}
\end{bmatrix} \begin{bmatrix}
\mathbf{i}_{abc} \\
\mathbf{i}_{fd}
\end{bmatrix}
$$

(85)

where

$$
\mathbf{v}_{abc} = [v_a, v_b, v_c]^T
$$

are the stator voltages, and

$$
\mathbf{i}_{abc} = [i_a, i_b, i_c]^T
$$

are the stator currents. The matrices $\mathbf{L}_s$ and $\mathbf{R}_s$ are

$$
\mathbf{L}_s = 
\begin{bmatrix}
L_{aa} & -L_{ab} & -L_{ac} \\
-L_{ba} & L_{bb} & -L_{bc} \\
-L_{ca} & -L_{cb} & L_{cc}
\end{bmatrix}
$$

and

$$
\mathbf{R}_s = 
\begin{bmatrix}
r_s & 0 & 0 \\
0 & r_s & 0 \\
0 & 0 & r_s
\end{bmatrix}
$$

where $L_{aa}$, $L_{bb}$, and $L_{cc}$ are the magnetizing inductances of each phase, and the others are coupling inductances between the phases. The resistance $r_s$ is the series resistance of each phase. The vector $\mathbf{L}'_{sr}$ is the referred rotor inductance and is given by

$$
\mathbf{L}'_{sr} = L_m \begin{bmatrix}
\sin\left(\frac{\theta_r}{2}\right) \\
\sin\left(\frac{\theta_r}{2}, -\frac{2\pi}{3}\right) \\
\sin\left(\frac{\theta_r}{2}, +\frac{2\pi}{3}\right)
\end{bmatrix}^T
$$

(86)

where

$$
\theta_r = \int_0^t \omega_r dt
$$
is the machine rotor position. In this case, the machine contains permanent magnets, so \( i_{fd} = i_{fl} \) is constant, and the rotor equation involves no dynamics. The stator equation can be written as

\[
v_{abc} = -R_s i_{abc} - L_s \frac{d i_{abc}}{dt} + \frac{L_m}{2} \omega_r i_{fd} L_m \begin{bmatrix}
\cos\left(\frac{\ell}{2} \theta_r\right) \\
\cos\left(\frac{\ell}{2} \theta_r - \frac{2\ell}{3}\right) \\
\cos\left(\frac{\ell}{2} \theta_r + \frac{2\ell}{3}\right)
\end{bmatrix}
\]  

(87)

The constant \( L_m \) can be evaluated from open circuit data. The terminal voltage of the unloaded generator is related to the speed by the constant \( k_g \) in units of V·s/rad. The rms phase voltage of the unloaded generator is given by

\[
V_{phase} = k_g \omega_r
\]

(88)

The quantity \( I_{fd} L_m \) is then

\[
I_{fd} L_m = \frac{2\sqrt{2}}{P} k_g
\]

(89)

Since the machine is of the round rotor type, the shaft torque is given by

\[
T_{em} = L_m i_{fd} \frac{P}{2} \left[ \left( i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \cos\left(\frac{\ell}{2} \theta_r\right) + \frac{\sqrt{3}}{2} \left( i_b - i_c \right) \sin\left(\frac{\ell}{2} \theta_r\right) \right]
\]

(90)

The rotor speed equation is

\[
J \frac{d \omega_r}{dt} = \frac{N_{sm}}{N_{ICE}} T_{mech} - T_{em}
\]

(91)

where the driver torque \( T_{mech} \) has been scaled by the drive gearing ratio. The inertia \( J \) is the total inertia of the generator, driver and transmission.

The SIMULINK block diagram of the generator is shown in Figure 38. The block labeled SM solves the machine equations for the various terminal constraints imposed by the rectifier. The block \( T_{generator} \) determines the generator shaft torque and the block \( V_{abc} \) gives the terminal voltages of the motor. The shaft torque is determined directly from eq. (90) and the terminal voltages are provided by eq. (87). The rotor speed block is not shown here, but is implemented directly from eq. (91).
3.3 Generator and Rectifier Interface

The problem of interfacing a permanent magnet generator with a phase-controlled bridge rectifier is considered in this section. The schematic of the generator and rectifier is shown in Figure 39. The load comprises an inductance $L_L$, a resistance $R_L$, and a dc source $V_b$. The voltage $V_b$ could represent a battery pack or other constant load voltage, the inductance is used as a current filter, and $R_L$ is the inductance and connection resistance. The six SCRs model the rectifier. The generator is modeled by three ac sources with a series resistance and inductance. The phase currents and voltages are labeled $i_a$, $i_b$ and $i_c$, and $v_a$, $v_b$ and $v_c$, respectively. The generator internal sources and cross coupling terms are represented by $v_{aa}$, $v_{bb}$, and $v_{cc}$.

Figure 38: Block diagram of the synchronous generator.

Figure 39: Generator and rectifier diagram.
There are three basic configurations which are possible within the bridge structure:

1. All SCRs in the bridge are off.
2. Two of the SCRs are on – one on the high side and one on the low side.
3. Three SCRs are on -- two high side SCRs and one low side SCR, or two low side SCRs and one high side SCR are on.

The first case occurs when the bridge is disabled, during start-up or when the load voltage is high enough to reverse bias the SCRs. It can also occur in a situation called discontinuous current mode when the load current falls to zero for a short interval during each rectifier output cycle. The second case is the most common. Current is flowing to the load through a high side SCR and from the load through a low side SCR. The third case happens during switch commutation when the load current is shifted from one generator phase to another. During this time, two SCRs on the same side share the load current. This commutation time is dependent on the generator series phase inductance.

Thirteen bridge configurations can be constructed this way. They are listed in Table 3, in which $V_H$ and $V_L$ refer to the high and low side load voltages, respectively. Equations describing each of these bridge configurations are created and used by the simulator. The bridge configuration is determined from external controls and the generator and load voltages. The configuration is then used to select the proper set of equations to describe the system.

<table>
<thead>
<tr>
<th>Bridge Configuration</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $v_H=v_a$, $v_L=v_b$</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>2 $v_H=v_a$, $v_L=v_b=v_c$ (commutation)</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>3 $v_H=v_a$, $v_L=v_c$</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>4 $v_H=v_b$, $v_L=v_c$ (commutation)</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>5 $v_H=v_b$, $v_L=v_c$</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>6 $v_H=v_b$, $v_L=v_a=v_c$ (commutation)</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>7 $v_H=v_b$, $v_L=v_a$</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>8 $v_H=v_b$, $v_L=v_a$ (commutation)</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>9 $v_H=v_a$, $v_L=v_a$</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>10 $v_H=v_a$, $v_L=v_b$ (commutation)</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>11 $v_H=v_a$, $v_L=v_b$</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>12 $v_H=v_c$, $v_L=v_b$ (commutation)</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>13 $v_H=0$, $v_L=0$</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>off</td>
</tr>
</tbody>
</table>

The system equations can be derived using eq. (87) and the circuit diagram in Figure 39. First, partition $\mathbf{L}_s$ as

$$
\mathbf{L}_s = \begin{bmatrix} \mathbf{L}_a \\ \mathbf{L}_b \\ \mathbf{L}_c \end{bmatrix}
$$
where \( \mathbf{L}_a, \mathbf{L}_b, \text{ and } \mathbf{L}_c \) are row vectors. Now, assume that SCRs \( S_2 \) and \( S_4 \) are in the on-state and using eq. (87), write the loop equation as follows

\[
v_a - v_b = R_L i_a + L_L \frac{di_a}{dt} + V_b
\]

\[-r_s i_a - \frac{P}{2} \omega_L L_m d i_{f1} \cos(f_2 \theta_r) + r_s i_b + L_b \frac{d\alpha_{f2}}{dt} - \frac{P}{2} \omega_L L_m d i_{f2} \cos(f_2 \theta_r - \frac{2\pi}{3})
\]

\[= R_L i_a + L_L \frac{di_a}{dt} + V_b
\]

\[-(r_s + R_L) i_a + r_s i_b + (L_b - L_a) \frac{di_{ab}}{dt} - L_L \frac{di_a}{dt} + \frac{P}{2} \omega_L L_m d i_{f1} \left[ \cos(f_2 \theta_r) - \cos(f_2 \theta_r - \frac{2\pi}{3}) \right] = V_b
\]

Define \( \mathbf{L}_x = \mathbf{L}_b - \mathbf{L}_a \) and use a subscripted numeral to indicate a particular component of a vector. The above equation can now be written as

\[(L_{x1} - L_{L}) \frac{di_a}{dt} + L_{x2} \frac{di_b}{dt} + L_{x3} \frac{di_c}{dt} = (r_s + R_L) i_a - r_s i_b + \frac{P}{2} \omega_L L_m d i_{f1} \left[ \cos(f_2 \theta_r) - \cos(f_2 \theta_r - \frac{2\pi}{3}) \right] + V_b
\]

This is one state equation for the system. Since there are three state variables, three equations are needed. The second equation is found by applying KCL at the load

\[i_a + i_b = 0
\]

After taking derivatives, the second state equation is

\[\frac{di_a}{dt} + \frac{di_b}{dt} = 0
\]

The final equation is found by noting that the phase \( c \) current is zero since it is open circuited. Hence, the third equation is

\[\frac{di_c}{dt} = 0
\]

Placing these state equations in matrix form yields

\[
\begin{bmatrix}
L_{x1} - L_{L} & L_{x2} & 0 \\
1 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{di_{ab}}{dt} \\
\frac{di_{ab}}{dt} \\
\frac{di_{ab}}{dt}
\end{bmatrix}
= \begin{bmatrix}
r_s + R_L & -r_s & 0 \\
0 & 0 & 0 \\
0 & 0 & L_m L_{f1}
\end{bmatrix}
\begin{bmatrix}
\cos(f_2 \theta_r) - \cos(f_2 \theta_r - \frac{2\pi}{3}) \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
V_b \\
0 \\
0
\end{bmatrix}
\]
This equation may now be put into standard form by inverting the matrix on the left and multiplying. The solution for the commutation configuration is similar.

Assume that SCRs $S_1$, $S_4$ and $S_6$ are in the on-state. Here, the load current is being shifted from phase $b$ to phase $c$. Equation (92) also holds in this case and doesn’t need to be modified. The second equation is KCL at the load and in this case is

$$i_a + i_b + i_c = 0 \Rightarrow \frac{di_a}{dt} + \frac{di_b}{dt} + \frac{di_c}{dt} = 0$$

(96)

The phase $c$ current is no longer zero. The last equation is found by noting that $v_b = v_c$.

$$-r_s i_b - L_{b} \frac{d\theta_{abc}}{dt} + \frac{p}{2} \omega_L L_{md} I_{fd} \cos(\frac{p}{2} \theta_r - \frac{2\pi}{3}) + r_s i_c + L_c \frac{d\theta_{abc}}{dt} - \frac{p}{2} \omega_L L_{md} I_{fd} \cos(\frac{p}{2} \theta_r + \frac{2\pi}{3}) = 0$$

(97)

Defining $L_y = L_c - L_b$, these equations are put in matrix form as follows

$$\begin{bmatrix} L_{x1} - L_L & L_{x2} & L_{x3} \\ L_{y1} & L_{y2} & L_{y3} \\ 1 & 1 & 1 \end{bmatrix} \frac{di_{abc}}{dt} = \begin{bmatrix} r_s + R_L & -r_s & 0 \\ 0 & r_s & -r_s \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\frac{p}{2} \theta_r) - \cos(\frac{p}{2} \theta_r - \frac{2\pi}{3}) \\ \cos(\frac{p}{2} \theta_r - \frac{2\pi}{3}) - \cos(\frac{p}{2} \theta_r + \frac{2\pi}{3}) \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{p}{2} \omega_L L_{md} I_{fd} \cos(\frac{p}{2} \theta_r - \frac{2\pi}{3}) - \cos(\frac{p}{2} \theta_r + \frac{2\pi}{3}) \\ 0 \end{bmatrix}$$

(98)

As with the previous equation, this equation can be placed in standard form by inverting the matrix on the left and multiplying through. The other configurations are found in a similar fashion and all thirteen are listed in the Appendix.

Each of the configurations above are of the form

$$A_j \frac{d\theta_{abc}}{dt} = B_j i_{abc} - \frac{p}{2} \omega_L L_{md} I_{fd} U_j(\theta_r) + \begin{bmatrix} V_b \\ 0 \\ 0 \end{bmatrix}$$

where the index $j$ indicates that these matrices are for a specific configuration. These equations may be put into standard state space form by multiplying by $A^{-1}$. Multiplying yields

$$\frac{d\theta_{abc}}{dt} = A_j^{-1} B_j i_{abc} - \frac{p}{2} \omega_L L_{md} I_{fd} A_j^{-1} U_j(\theta_r) + A_j^{-1} \begin{bmatrix} V_b \\ 0 \\ 0 \end{bmatrix}$$

(99)
This is the form of the state equations used by the simulation program. This is only one of thirteen equations, however. Define the row vector of switching functions $Q$ as

$$Q = [q_1 \ q_2 \ \cdots \ q_{12} \ q_{13}]$$

where the switching functions $q_j$ take on values of one or zero, and are constrained in the following manner

$$\sum_{j=1}^{13} q_j = 1 \quad (101)$$

These switching functions are also a function of the state of the SCRs contained within the rectifier bridge.

$$q_j = q_j(S_1, S_2, S_3, S_4, S_5, S_6) \quad (102)$$

Let $U_j(\theta_i) = M_j W(\theta_i)$ where $M_j$ is matrix of scalars and $W(\theta_i)$ is defined as

$$W(\theta_i) = \begin{bmatrix} \cos(\frac{\pi}{2} \theta_i) \\ \cos(\frac{\pi}{2} \theta_i - \frac{2\pi}{3}) \\ \cos(\frac{\pi}{2} \theta_i + \frac{2\pi}{3}) \end{bmatrix}, \text{ and } V_b = \begin{bmatrix} V_b \\ 0 \\ 0 \end{bmatrix}.$$  

The system state equations can now be written as

$$\frac{di_{abc}}{dt} = q_1 A_1^{-1} [B_1 i_{abc} - \frac{\pi}{2} \omega_t L_{md} I_{fa} M_1 W(\theta_r) + V_b] + q_2 A_2^{-1} [B_2 i_{abc} - \frac{\pi}{2} \omega_t L_{md} I_{fa} M_2 W(\theta_r) + V_b] + \cdots + q_{13} A_{13}^{-1} [B_{13} i_{abc} - \frac{\pi}{2} \omega_t L_{md} I_{fa} M_{13} W(\theta_r) + V_b]$$

and simplified to give

$$\frac{di_{abc}}{dt} = QA^{-1} [B_i i_{abc} - \frac{\pi}{2} \omega_t L_{md} I_{fa} MW(\theta_r) + V_b] \quad (103)$$

where $Q$ is defined as in eq. (100) and the matrices $A$, $B$, and $M$ are defined as

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{13} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{13} \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_{13} \end{bmatrix}$$

49
The state of a single SCR is determined by the external control and the terminal conditions of that SCR. The collection of the six SCR states is used to determine the system configuration used. The SIMULINK block used to determine the system configuration is shown in Figure 40. The two inputs SCR_H and SCR_L are vectors whose components are $S_1$, $S_3$, and $S_5$ and $S_2$, $S_4$, and $S_6$, respectively. These values are converted to a unique index by the following transformation

\[
Index = 1 + S_6 + 2S_5 + 4S_4 + 8S_3 + 16S_2 + 32S_1
\]  

(104)

**Figure 40:** SIMULINK configuration block.

The MATLAB function Config_sm translates the index of eq. (104) to an index ranging from 1 to 13. This index is supplied to generator model to select the proper configuration.

The generator block that calculates the phase currents and current derivatives is shown in Figure 41. The three integrators provide the phase currents by integrating the right-hand-side of eq. (103). The current derivatives are simply the inputs to these integrators. The MATLAB function A_smfl is used to calculate the first term of eq. (103) $QA^{-1}B_{abc}$. The function takes as inputs the configuration number and the three phase currents.

**Figure 41:** Determination of synchronous machine currents and current derivatives.

The Eint block is used to calculate the remaining terms in eq. (103), the motor shaft speed, and the generator internal voltages. The SIMULINK block diagram of Eint is shown in Figure 42. The integrator labeled theta_r integrates the shaft speed and provides $\theta_r$ on the
interval \([0,2\pi]\). The MATLAB function \(\text{phi}_\text{sm}\) implements the matrix function \(W(\theta)\). The MATLAB function \(\text{A}_\text{smf2}\) computes the terms \(QA^{-1}\left(\frac{\omega}{2}, l_{md} I_{fd} M W(\theta) + V_b\right)\) in eq. (103).

The block labeled \(E_{abc}\) in Figure 42 computes the internal phase voltages of the generator. The SIMULINK block diagram for \(E_{abc}\) is shown in Figure 43. Computation of the internal voltages is done using eq. (103) with the currents and current derivatives set equal to zero.

\[ u(2)^* \cos(\text{Poles}_\text{sm}/2*u(1)) \]  
\[ u(2)^* \cos(\text{Poles}_\text{sm}/2*u(1)-2*\pi/3) \]  
\[ u(2)^* \cos(\text{Poles}_\text{sm}/2*u(1)+2*\pi/3) \]

**Figure 42:** SIMULINK block \(E_{int}\).

**Figure 43:** Computation of generator no-load phase voltages.

The block used to calculate the generator terminal voltages is labeled \(V_{abc}\) and is shown in Figure 38. The blocks contained in this subsystem block are shown in Figure 44. The terminal voltages are calculated using the internal generator internal voltages and eq. (87).
Figure 44: Calculation of generator terminal voltages.

The complete rectifier and generator SIMULINK block diagram is shown in Figure 45. The block labeled SCR Bridge contains all of the components used to construct the rectifier. The block labeled PM Machine models the permanent magnet synchronous generator. The block labeled Mech dynamics contains the generator rotor speed equation. The input port $v_c$ is the battery pack capacitor voltage. The output $i_{\text{load}}$ is the rectifier load current supplied to the load, in this case a battery pack. In this particular setup, the driver is a constant and the phase delay angle is controlled directly by a rate-limited step function. The rectifier enable input is made active only after the PLLs have gained a lock on the generator frequency.

Figure 45: Generator and rectifier block diagram.

In the remainder of this section, several generator and rectifier simulations are examined. The first is a simulation of free acceleration of the generator. This is done by directly disabling the rectifier via the Bridge en input. Figure 46 shows the generator phase $a$ voltage and shaft speed during free acceleration. The phase voltage magnitude and frequency vary linearly with the shaft speed of the generator.

Figure 47 shows the rectifier load current for various values of $\kappa$. The steady-state load current is close to being a linear function of $\kappa$. Since the load is not a perfect current source, this
relation is not exactly linear. These plots display the dynamic response of the rectifier load current to a step in $\kappa$.

![Figure 46: Generator phase $a$ voltage and shaft speed.](image)

![Figure 47: Rectifier load current for various values of $\kappa$.](image)
Figure 48 shows the rectifier phase $a$ voltage current. Switch commutation is very apparent in this figure. Figure 49 shows the rectifier phase $a,b,$ and $c$ voltages and the bridge configuration number. It can be seen from this plot that the commutating phase voltages are equal. During each cycle of operation, the bridge steps through each configuration in sequence. Figure 50 illustrates the loss of the high-side phase $a$ SCR gate drive at time $t=0.3$ s. Note that the bridge can still function, but at a reduced capacity. The remaining SCRs still function properly, including the low-side phase $a$ SCR. The load current falls off dramatically. This can be corrected to some extent by decreasing the bridge delay angle. This setup can be modified to simulate faults as well. For instance, a situation with a shorted SCR can be simulated by forcing the corresponding flip-flop output to remain high during the fault condition. The real modification to the program is to change the rectifier commutation resistance to simulate opening of the SCR after the fault is cleared.

Figure 48: Phase $a$ voltage and current demonstrating switch commutation.
Figure 49: Rectifier phase $a$, $b$ and $c$ voltages and the bridge configuration.
Figure 50: Illustration of the loss of the high-side phase a SCR.

The rectifier model is versatile enough to be used in many applications. For instance, varying gating methods can be applied directly. Voltage and current feedback control are readily applied. The rectifier is capable of accepting varying input voltage and frequency. A constant frequency input source can be simulated by simply holding the generator speed constant and removing the coupling terms from the generator model. Partial operation of the bridge is simulated by disabling individual SCRs. Faulted conditions can be simulated by forcing the individual SCRs to stay active. However, precautions must be taken to ensure the phase current returns to zero during the opening of a fault. The SCR can be modeled as a forward drop with a series resistance, like the IGBT, although the present version of the simulator treats the device as ideal. Alternatively, the forward drop and on-state resistance can be added directly in series with the input of the rectifier.

4 Summary

This report describes in detail mid-level dynamic modeling of the induction machine, FOC controller, the synchronous machine, and a three-phase SCR-based bridge rectifier. In addition, a simulator is developed within the MATLAB/SIMULINK environment capable of simulating all of the above listed systems in various setups. The components are graphically implemented in SIMULINK in a modular format. This makes the construction, as well as any modifications, of a proposed design very easy. The graphical layout is also intuitive and very easy to follow.
The attributes listed above make the simulator both a useful tool for design evaluation and a sophisticated instructional aid. The level of complexity implemented within the simulator makes it accurate enough to be of use for design evaluation. The ease with which a given system can be modified makes rapid design refinement possible. This includes not only rapid changes in component parameters, but also changes in the component structures. Each of the component blocks is implemented directly into SIMULINK, which makes changes and additions easy and direct. As an educational tool, the simulator allows a student to simulate a real world system without the overhead involved in constructing a physical system.
Appendix: Switch configuration matrices for the SCR-based rectifier coupled with the PM synchronous machine

The state equations for the synchronous machine variables take the form of [9]

$$A \frac{di_{abc}}{dt} = Bi_{abc} - \frac{r}{2} \omega_r L_{md} I_{fd} U(\theta_r)$$

where $A$ is an inductance matrix, $B$ is a resistance matrix, and $U(\theta_r)$ is the driving input vector and is a function of the shaft position. The matrices for each switch configuration are listed below. The row vectors $L_a$, $L_b$, and $L_c$ are the rows of $L_s$, and $\phi_a=0$, $\phi_b=-2\pi/3$, and $\phi_c=2\pi/3$. The subscripted numerals indicate a particular vector component.

Configuration 1: switches $S_1$ and $S_4$ in the on-state.

$$A = \begin{bmatrix}
(L_b - L_a) & L_a & 0 \\
1 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
r_s - R_L & -r_s & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$U = \begin{bmatrix}
\cos(\theta_r + \phi_a) - \cos(\theta_r + \phi_b) \\
0 \\
0
\end{bmatrix}$$

Configuration 2: switches $S_1$, $S_4$, and $S_6$ in the on-state.

$$A = \begin{bmatrix}
(L_b - L_a) & (L_b - L_a) & (L_b - L_a) \\
(L_b - L_c) & (L_b - L_c) & (L_b - L_c) \\
1 & 1 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
r_s - R_L & -r_s & 0 \\
0 & -r_s & r_s \\
0 & 0 & 0
\end{bmatrix}$$

$$U = \begin{bmatrix}
\cos(\theta_r + \phi_a) - \cos(\theta_r + \phi_b) \\
\cos(\theta_r + \phi_c) - \cos(\theta_r + \phi_b) \\
0
\end{bmatrix}$$
Configuration 3: switches $S_1$ and $S_6$ in the on-state.

$$A = \begin{bmatrix} (L_c - L_a) - L \ 1 \ 0 \\ 1 \ 0 \ 1 \\ 0 \ 1 \ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} r_s + R_L \ 0 \ -r_s \\ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos(\theta + \phi_a) - \cos(\theta + \phi_e) \\ 0 \\ 0 \end{bmatrix}$$

Configuration 4: switches $S_1$, $S_3$, and $S_6$ in the on-state.

$$A = \begin{bmatrix} (L_c - L_a) - L \ (L_c - L_a) - L \ (L_c - L_a) - L \\ (L_a - L_b) - L \ (L_a - L_b) - L \ (L_a - L_b) - L \\ 1 \ 1 \ 1 \end{bmatrix}$$

$$B = \begin{bmatrix} r_s \ R_L - r_s \ 0 \\ -r_s \ 0 \ r_s \\ 0 \ 0 \ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos(\theta + \phi_a) - \cos(\theta + \phi_e) \\ \cos(\theta + \phi_e) - \cos(\theta + \phi_a) \\ 0 \end{bmatrix}$$

Configuration 5: switches $S_3$ and $S_6$ in the on-state.

$$A = \begin{bmatrix} 0 \ (L_c - L_b) - L \ (L_c - L_b) - L \\ 0 \ 1 \ 1 \\ 1 \ 0 \ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \ r_s + R_L \ -r_s \\ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos(\theta + \phi_b) - \cos(\theta + \phi_e) \\ 0 \\ 0 \end{bmatrix}$$
Configuration 6: switches $S_2$, $S_3$, and $S_6$ in the on-state.

$$A = \begin{bmatrix}
(L_e - L_b)_1 & (L_e - L_b)_2 & (L_e - L_b)_3 \\
(L_e - L_a)_1 & (L_e - L_a)_2 & (L_e - L_a)_3 \\
1 & 1 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & R_s + r_s & -r_s \\
r_s & 0 & -r_s \\
0 & 0 & 0
\end{bmatrix}$$

$$U = \begin{bmatrix}
\cos(\theta_r + \phi_b) - \cos(\theta_r + \phi_c) \\
\cos(\theta_r + \phi_a) - \cos(\theta_r + \phi_c) \\
0
\end{bmatrix}$$

Configuration 7: switches $S_2$ and $S_3$ in the on-state.

$$A = \begin{bmatrix}
(L_a - L_b)_1 & (L_a - L_b)_2 & -L_L & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
-r_s & r_s + R_L & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$U = \begin{bmatrix}
\cos(\theta_r + \phi_b) - \cos(\theta_r + \phi_a) \\
0 \\
0
\end{bmatrix}$$

Configuration 8: switches $S_2$, $S_3$, and $S_5$ in the on-state.

$$A = \begin{bmatrix}
(L_e - L_b)_1 - L_L & (L_e - L_b)_2 & (L_e - L_b)_3 \\
(L_e - L_a)_1 & (L_e - L_a)_2 & (L_e - L_a)_3 \\
1 & 1 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
R_L - r_s & r_s & 0 \\
r_s & 0 & -r_s \\
0 & 0 & 0
\end{bmatrix}$$

$$U = \begin{bmatrix}
\cos(\theta_r + \phi_b) - \cos(\theta_r + \phi_a) \\
\cos(\theta_r + \phi_c) - \cos(\theta_r + \phi_b) \\
0
\end{bmatrix}$$
Configuration 9: switches $S_2$ and $S_3$ in the on-state.

\[
A = \begin{bmatrix}
(L_a - L_e)_1 & (L_a - L_e)_2 - L_L \\
1 & 0 & 1 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-r_s & 0 & r_s + R_L \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
\cos(\theta_r + \phi_c) - \cos(\theta_r + \phi_a) \\
0 \\
0 \\
\end{bmatrix}
\]

Configuration 10: switches $S_2$, $S_4$, and $S_5$ in the on-state.

\[
A = \begin{bmatrix}
(L_a - L_e)_1 & (L_a - L_e)_2 & (L_a - L_e)_3 - L_L \\
(L_a - L_b)_1 & (L_a - L_b)_2 & (L_a - L_b)_3 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-r_s & 0 & r_s + R_L \\
-r_s & r_s & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
\cos(\theta_r + \phi_c) - \cos(\theta_r + \phi_a) \\
\cos(\theta_r + \phi_b) - \cos(\theta_r + \phi_a) \\
0 \\
\end{bmatrix}
\]

Configuration 11: switches $S_4$ and $S_5$ in the on-state.

\[
A = \begin{bmatrix}
0 & (L_b - L_e)_1 & (L_b - L_e)_2 - L_L \\
0 & 1 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & r_s & r_s + R_L \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
\cos(\theta_r + \phi_c) - \cos(\theta_r + \phi_b) \\
0 \\
0 \\
\end{bmatrix}
\]
Configuration 12: switches \( S_1, S_4, \) and \( S_5 \) in the on-state.

\[
A = \begin{bmatrix}
(L_b - L_a)_1 & (L_b - L_a)_2 - L_L & (L_b - L_a)_3 \\
(L_a - L_c)_1 & (L_a - L_c)_2 & (L_a - L_c)_3 \\
1 & 1 & 1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
r_s & R_L - r_s & 0 \\
-r_s & 0 & r_s \\
0 & 0 & 0
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
\cos(\theta_r + \phi_a) - \cos(\theta_r + \phi_b) \\
\cos(\theta_r + \phi_a) - \cos(\theta_r + \phi_a) \\
0
\end{bmatrix}
\]

Configuration 13: all switches in the off-state.

\( B = 0 \)

\( U = 0 \)

The state derivatives are zero. The value for \( A \) is irrelevant.
6 References


