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Hydraulic Fracture Theory

Part I. – Mechanics of Materials

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ABSTRACT

This study takes up the problem of hydraulic fracture mechanics, orientation of fractures, and whether their control is possible.

In Part I, some theories on the mechanics of materials are adapted for use in dealing with problems of hydraulic fracture mechanics and also to help describe conditions of stress in porous sediments.

INTRODUCTION

Parts I and II of this report on hydraulic fracture theory are part of a project of the Oil and Gas Section of the Illinois State Geological Survey that has been carried on in consultation with the Department of Mining and Metallurgical Engineering, University of Illinois.

Part III will be a thesis in partial fulfillment of the requirements for a master's degree. It will deal with laboratory experiments suggested by the theoretical studies presented here.

The use of fracture treatment as a method for stimulating oil production has had a spectacular growth in the past several years. Results in general have been successful, but results for individual wells vary widely. An analysis of the mechanics of the pressure parting phenomenon is basic to the understanding of these variations in the results of fracture treatment.

One of the earliest references to pressure parting is included in a description of Dowell Incorporated acidizing services by Grebe and Stoesser (1935). They refer to "rock busting" as a common procedure accompanying acidizing of wells in order to secure greater penetration with the acid.

In a later article Grebe (1943) describes the intentional breakdown of a waste disposal well in 1930. Grebe assumed that parting took place along the bedding planes, and that the pressure at the sand face necessary to hold open the fracture was equal to the pressure caused by the overburden weight. From this assumption Grebe concluded that the average specific weight of the sediments could be calculated from the critical injection pressure. The critical injection pressure he defined as the pressure at which the injectivity of the well is sensitive to small pressure changes which indicate the opening or closing of a fracture.

Yuster and Calhoun (1945) describe pressure parting as observed in input wells of waterflood operations. They show that the injectivity increases suddenly when the pressure is increased above a certain value. They point out that the normal injectivity is restored, approximately, to its former value when the injection pressure is reduced, indicating that the fractures had closed.

Yuster and Calhoun stated that the opening of fissures is resisted by the tensile strength of the rock and the overburden pressure. Therefore, the fractures should follow planes of minimum tensile strength and paths of least overburden pressure. They suggest that the overburden pressure may have abnormally low values over limited areas due to the partial support and uneven distribution of the overburden load by overlying competent beds, and that an unequal distribution may also result from topographic features. This theory was used to explain the fact that some wells had critical pressures much lower than would be predicted by calculating the overburden pressure from the depth and average density of the sediments. They also suggested that cemented casing might act as a clamp, restraining the development of horizontal fractures in the vicinity of the well.

In October 1948, at the AIME meeting in Dallas, Texas, J. B. Clark presented a paper describing the "Hydrafrac Process" which had been developed by Stanolind Oil and Gas Company, now Pan American Petroleum. The following year the treatment was available to the oil industry. The treatment consisted essentially of the hydraulic breakdown of the producing section with a thickened sand-carrying fluid; the role of the sand was to prop open resulting fractures.

The purpose of fracture treatment is to increase the conductivity of fluid into or out of a well bore, or to increase the well's effective drainage area. The benefits derived from fracturing can be divided into three categories:

1) If one assumes a homogeneous reservoir rock, the effect of fractures is similar to increasing the size of the hole. After the fractures are produced, fluids, which formerly had to flow through the restricted section of rock surrounding the well, are able to move into the fracture at some distance from the well and flow within the fractures to the well bore with little opposition.

2) Production of fractures is one way to overcome the effect of a zone of abnormally low permeability surrounding the well bore. An impermeable sheath surrounding the well, sometimes called the skin factor, may result from several causes. Invasion of the drilling mud emulsions, deposition of paraffin or mineral matter, or swelling of clay in the pores may all contribute to the isolation of the well and thus reduce production markedly.

3) Fractures help connect systems of permeability and porosity that are otherwise isolated from the well. Any inhomogeneity of the reservoir rock may cause isolated permeability. Permeable sand lenses, solution cavities, reservoirs divided by impermeable shale laminations, and joint systems are all examples of situations where fractures radiating from the well might act as gathering lines, reaching from the well to isolated zones.

Thus, various beneficial effects might be obtained from the fracture treatment of a specific well. In all cases an increase in the mobility of fluids moving to or from the well is the result, but the fracture configuration which will best do the job differs for different wells. If bottom water or a gas cap is present, horizontal fractures seem to be in order. In other situations, where vertical permeability is interrupted by numerous shale streaks, vertical fractures might be best. This is pointed out by Clark and Reynolds (1954) who describe a method for obtaining vertical fractures.

The usefulness in fracturing a given well could be better decided if one could answer the following related questions: 1) For a given well what will be the fracture configurations, or, if control is possible, what fracture configurations are available? 2) What effects will various fracture configurations have on conductivity?

The first question has been treated by Clark and Reynolds (1954), McGuire et al. (1954), Scott et al. (1953), Zheltov and Kristianovich (1955), Hubbert and Willis (1957), and van Poollen (1957). This paper, also, deals with the first question, the problem of fracture orientation.

It is generally agreed that the compressive stress in the rock at the time of fracture will tend to control the orientation of the fracture. Although numerous other factors may enter into the problem, the compressive stresses are proably a dominant influence.

Part I of this paper presents several analytical tools that are useful in describing the state of stress in sediments around an oil well, then a general discussion of fracture orientation follows in Part II.

I am indebted to W. D. Rose, Professor of Petroleum Engineering at the University of Illinois, for numerous discussions of the substance of this paper and for criticism of the manuscript, to A. H. Bell, head of the Oil and Gas Section of the Illinois State Geological Survey, and to L. L. Whiting, Associate Geologist of the Survey, A. C. Bianchini, Assistant Professor of Theoretical and Applied Mechanics, H. L. Langhaar, Professor of Theoretical and Applied Mechanics of the University of Illinois, and L. R. Kern, Atlantic Refining Company, who assisted in various ways.

NOTATION

Symbols

α = Linear coefficient of thermal expansion, 1/T

β = Grain compressibility, in.^{2}/lb.

ϵ = Normal strain

E = Young's modulus, lb./in.^{2}

e = Unit volume dilatation

λ = Poisson's coefficient, lb./in.^{2}

G = Shear modulus, lb./in.^{2}

J = Linear coefficient of pore pressure expansion, in.^{2}/lb.

μ = Poisson's ratio

n = Fractional pore area in a random plane through a porous material, the porosity

N = Fractional area over which pore pressure effectively acts to produce tensile failure

ϕ = Angle of internal friction

r = Radial distance from the well bore, in.

σ = Total normal stress, lb./in.^{2}

σ' = Solid normal stress, lb./in.^{2}

$\bar{\sigma}$ = Effective normal stress, lb./in.^{2}

τ = Shear stress, lb./in.^{2}

T = Temperature

u, v, w = Components of displacement, in.

X, Y, Z = Body force per unit volume, lb./in.^{3}

$\bar{X}, \bar{Y}, \bar{Z}$ = Surface forces, lb./in.^{2}

γ = Shear strain

Superscripts

*, **, *** = Components of

$-$ = Effective component

' = Solid component

Subscripts

x, y, z, r, θ = Coordinate directions

h = Horizontal

xy = In the x plane parallel to the y axis

i = Internal, of or in the well bore

e = External or a specified external radius

o = Datum value

n = Normal to

cyclic = Two similar expressions are obtained by cyclic interchange of the subscripts

ELASTIC PROPERTIES OF POROUS MATERIAL

Compressibility

The elastic compressibilities of porous material are of special importance in this section and something will be said of these properties first.

Compressibility is defined as the unit change in volume over the change in pressure, $\Delta V / V \Delta P$.

For porous materials two types of compressibility may be measured, depending on the surface to which the pressure is applied.

If a rock specimen is tested by applying a fluid pressure, not only to the external surface but also to the surfaces of the communicating pores, and $\Delta V / V \Delta P$ is measured, the property which is determined is the grain volume compressibility or the weighted average of the compressibilities of the mineral grains that make up the rock. The volume compressibility is commonly designated β, so the linear strain due to penetrating fluid pressure is

$$\epsilon = \frac{\beta P}{3} \qquad (1)$$

If, on the other hand, a rock is enclosed in an impermeable jacket of negligible strength and then tested under hydrostatic pressure, the property measured is the bulk compressibility. In this discussion the bulk volume compressibility, β', is expressed in terms of the other bulk elastic properties, Poisson's ratio, and Young's modulus according to the identity

$$\beta' = \frac{3(1 - 2\mu)}{E}$$

So the linear strain due to hydrostatic pressure applied to external boundaries is

$$\epsilon' = \left[\frac{1 - 2\mu}{E} \right] P \qquad (2)$$

The sign convention used here treats compressive stress and compressive strain as positive.

Hooke's Law

In general, sedimentary rock consists of a lattice work of mineral grains and connected channels occupied by fluid. The two interpenetrating phases interact with one another, the fluid delivering a pressure on its pore boundaries.

A form of Hooke's Law, taking into account the action of the fluid on the pore boundaries, was derived by Lubinski (1954) in order to solve problems in elasticity for porous sediments. A derivation of this expression follows.

Consider a unit cell of porous sediment, figure 1. We shall define as the solid stress in terms of gross area; n is the fractional area of a random slice through the material occupied by pores.

The force acting on an external boundary of the unit cell in the x direction is $\sigma'_x + nP$.

If the fluid pressure is zero, a porous elastic material may be treated as a conventional elastic material and Hooke's Law may be written

$$\epsilon_x = \frac{\sigma'_x}{E} - \frac{\mu}{E}[\sigma'_y + \sigma'_z] \qquad \text{(cyclic)} \qquad (3)$$

The presence of fluid pressure in the pores introduces an additional strain which may be visualized as follows. The fluid pressure, P, while acting on the total internal boundary of our unit cube in figure 1, is excluded from the solid portions of the external boundary. The effect of such a condition is the same as having the unit cube entirely immersed in a fluid at a pressure, P, and then having a superposed tension on the solid portions of the external surfaces of intensity (1 - n) P. This equivalence is indicated in figure 2.

The strain due to complete immersion in a fluid under pressure is the same in all directions and, according to (1), is $(\beta/3)P$ where β is compressibility of the grain material that makes up the rock.

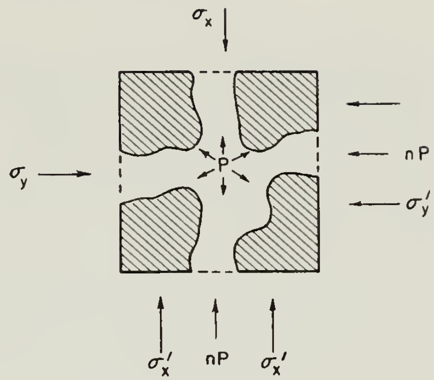

Fig. 1. - Forces acting on an element of porous material in two principal directions so as to define total normal stresses, σ, in terms of solid stresses σ'. Thus: $\sigma - \sigma' + nP$

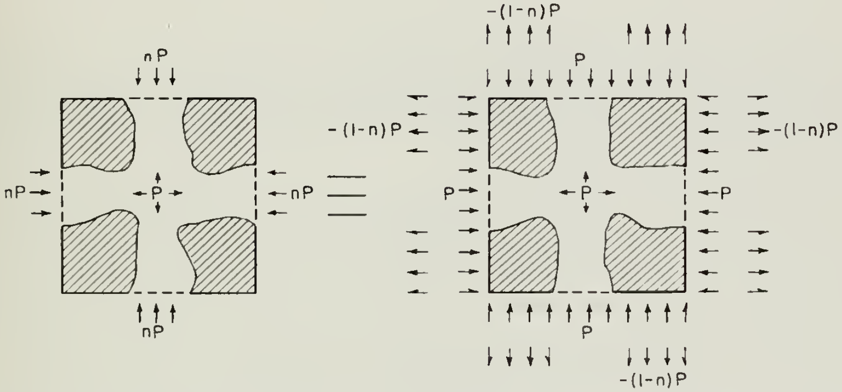

Fig. 2. - Diagram showing the equivalence of the fluid pressure distribution on an element of porous material, to the complete immersion of the element, together with a superposed tension on the external solid boundaries of intensity -(1-n)P.

The strain due to a tension of -(1-n)P on the solid portions of the external boundaries of the unit cube according to (2) is

$$-(1-n)\frac{(1-2\mu)}{E}P$$

So the strain due to the presence of fluid pressure is

$$\left[\frac{\beta}{3} - (1-n)\frac{(1-2\mu)}{E}\right]P \qquad (4)$$

and the expression for the total strain is the sum of (4) and the right hand term in (3)

$$\epsilon_x = \frac{\sigma_x'}{E} - \frac{\mu}{E}\left[\sigma_y' + \sigma_z'\right] + \left[\frac{\beta}{3} - (1-n)\frac{(1-2\mu)}{E}\right]P \quad \text{(cyclic)} \qquad (5)$$

Now we shall define the stress term differently and obtain an equivalent but simpler form of Hooke's Law. Again consider our unit cell of the porous material and the forces acting on it, indicated in figure 2, σ_x and σ_y, which will be called total normal stresses are defined as total forces per unit gross area acting on the faces of the cube. Note that this is the summation of the forces acting across the fluid portions of the boundary. The total stresses in terms of the solid stresses are then

$$\sigma_x = nP + \sigma'_x \qquad \text{(cyclic)} \tag{6}$$

Now substituting in (5) for $\sigma'_{x,y,z}$ we obtain

$$\epsilon_x = \frac{\sigma_x - nP}{E} - \frac{\mu}{E}\left[\sigma_x + \sigma_z - 2nP\right] - \left[(1-n)\frac{(1-2\mu)}{E} - \frac{\beta}{3}\right]P$$

Rearranging terms this becomes

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right] - \left[\frac{(1-2\mu)}{E} - \frac{\beta}{3}\right]P \qquad \text{(cyclic)} \tag{7}$$

The total stress, σ, does not exist as an entity within the porous system. It should be emphasized that the total stress is defined as the summation of the forces transmitted by the fluid and the solid per unit area. Although this definition makes the total normal stress seem a little obscure, actually it is the most useful form with which to work.

One advantage in using the total stress has already been demonstrated in the form of Hooke's Law that we have obtained in terms of the total stress, (7). These equations do not contain the pore area term, n. In Lubinski's equations (5), which give the strains in terms of the solid stresses, this additional property of the material must be included.

Another advantage in working in terms of the total stresses arises from the continuity of the total normal stress across a boundary, whether solid (impermeable) or liquid. Whether the material outside the boundary is solid or liquid, the normal stresses on either side and normal to the boundary must be equal. In terms of the solid stresses, σ', we see from figure 3 that the surface pressure, P, delivered by a solid produces a solid normal stress on the porous side of the boundary equal to $(1 - n)P$. If the surface pressure is delivered by a liquid, the solid normal stress in the porous material will then be equal to P.

Yet another advantage to working in terms of the total stress will appear later with regard to the force field resulting from the motion of a fluid through the porous medium.

Readers interested in more detailed treatment of the elastic behavior of porous materials may wish to refer to Biot (1941), Gassmann (1951), and Geertsma (1957). Pore volume compressibility, which is not discussed in this paper, is treated by these authors. Lubinski (1954) outlines a method for the solution to problems of elastic stress in porous materials.

Thermal Stress Analogy

Lubinski (1954) points out the analogy between the problem of stresses resulting from thermal gradients in an isotropic material and the determination of stresses in a porous material that are caused by fluid pressure gradients.

He then outlines a method of solution following closely the method given by Timoshenko (1934, p. 204) for thermal stresses.

Hooke's Law for isotropic materials, which includes the strain due to changes in temperature, is:

$$\epsilon_x = \frac{\sigma'_x}{E} - \frac{\mu}{E}(\sigma'_y + \sigma'_z) - \alpha T \quad \text{(cyclic)} \tag{8}$$

where α is the linear coefficient of thermal expansion and T is the temperature above some datum at which the material is assumed to be unstressed.

The expression for the shear stresses in terms of the shear strains are unchanged, when expansion of the elements of the system due to temperature is considered. So,

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma = \frac{\tau_{zx}}{G} \tag{9}$$

G is the shear modulus, γ is the shear strain, and τ is the shear stress.

When the term αT in (8) is compared with

$$\left[\frac{(1-2\mu)}{E} - \frac{\beta}{3}\right] P$$

in (7) we see that their physical significance is nearly the same. The first represents a strain due to temperature change, the other to a change in pore pressure.

It will condense our formulation if we now define a new elastic constant,

$$J = \left[\frac{1-2\mu}{E} - \frac{\beta}{3}\right] \tag{10}$$

The physical meaning of J is analogous to α in (8) and we may call it the linear coefficient of pore pressure expansion.

Note that in Lubinski's form of Hooke's Law for porous materials (5) the term

$$\left[(1-n)\frac{(1-2\mu)}{E} - \frac{\beta}{3}\right]$$

has the same physical significance as α and J . So it can be seen that one effect of changes in pore pressure is a change in volume of the elements that make up the material, just as temperature produces a volume change.

Body Forces

There are no real body forces associated with a temperature gradient, but we might well suspect that a body force would be associated with a fluid moving through a porous medium, and, if this were the case, the analogy between thermal stresses and the stresses due to pore pressure gradients would be incomplete. As it turns out, the existence and nature of body forces due to fluid motion in a porous material is a matter of viewpoint. More specifically it depends on the definition of the normal stress term used.

For example, working in terms of the solid stress, σ', it is readily seen from figure 4 that

$$\sigma'_s + n(P + dP) = nP + \sigma'_s + d\sigma'_s$$

and

$$\frac{d\sigma'_s}{ds} = n\frac{dP}{ds} \tag{11}$$

This gives us a body force in terms of force per unit volume transmitted to the solid phase from the moving fluid.

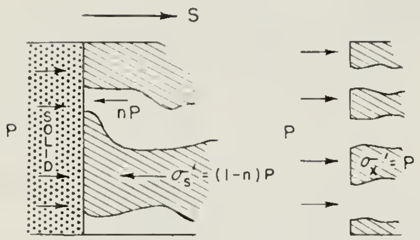

Fig. 3. - Diagram showing the normal stresses acting on two types of boundaries - solid and permeable.

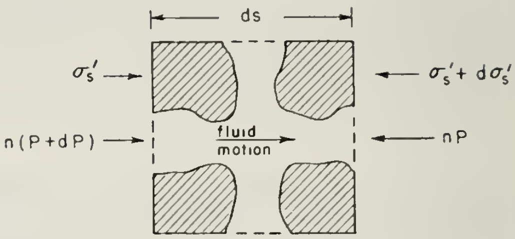

Fig. 4. - Diagram showing forces acting on the external surface of a porous element normal to direction of fluid motion.

Now converting this into terms of total stress using relation (6):

$$d\sigma_s' = d\sigma_s - n\,dP$$

$$\frac{d\sigma_s - n\,dP}{ds} = -n\frac{dP}{dn}$$

$$\frac{d\sigma_s}{ds} = 0 \qquad (12)$$

There is no body force due to the motion of the fluid in terms of the total stress. This seeming paradox is resolved when we realize that in working in terms of the total stress the fluid is treated as part of the body, so the force gradient in the liquid is cancelled by that in the solid. Still another view of the force field developed by the fluid may be taken when working in terms of the "effective" stress,which will be discussed in the next section.

As no body force exists due to the motion of the fluid when working in terms of the total stress, the effect of fluid pressure changes on the system may be assumed as entirely due to volume dilatation as is the case with thermal stresses. Thus the total stresses in porous systems with pressure gradients may be treated as completely analogous to thermal stresses.

Timoshenko's Thermal Stress Equations

We will proceed in the manner by which Timoshenko derives equations for the solution of problems in thermal stresses. Substituting in (7) using (10) we obtain

$$\epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] - JP \quad \text{(cyclic)} \qquad (13)$$

Defining the volume dilatation

$$e = \epsilon_x + \epsilon_y + \epsilon_z \qquad (14)$$

We can obtain by adding equations (13)

$$\sigma_x + \sigma_y + \sigma_z = \frac{Ee}{1-2\mu} + 3\frac{JEP}{1-2\mu} \qquad (15)$$

by rearranging the first of relations (13) we obtain

$$\sigma_y + \sigma_z = -\frac{E\epsilon_x}{\mu} + \frac{\sigma_x}{\mu} - \frac{JEP}{\mu} \tag{16}$$

subtracting (16) from (15) gives the normal stresses in terms of the strains.

$$\sigma_x = \frac{E\epsilon_x}{1+\mu} + \frac{\mu Ee}{(1+\mu)(1-2\mu)} + \frac{JEP}{1-2\mu} \quad \text{(cyclic)} \tag{17}$$

By making use of the identities (Timoshenko, 1934,p.11)

$$G = \frac{E}{2(1+\mu)} \tag{18}$$

and

$$\lambda = \frac{\mu E}{(1+\mu)(1-2\mu)} \tag{19}$$

relations (17) condense to

$$\sigma_x = 2G\epsilon_x + \lambda e + \frac{JEP}{1-2\mu} \quad \text{(cyclic)} \tag{20}$$

The shear stresses in terms of the strains are as given by equation (9)

$$\tau_{xy} = G\gamma_{xy}; \quad \tau_{yz} = G\gamma_{yz}; \quad \tau_{zx} = G\gamma_{zx}$$

The equation of equilibrium (Timoshenko, 1934, p. 195) is

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \quad \text{(cyclic)} \tag{21}$$

Making use of (9) and (17) and substituting for the stresses in the equation of equilibrium, (16), assuming the body forces equal to zero,

$$\lambda \frac{\partial e}{\partial x} + G\left[2\frac{\partial \epsilon_x}{\partial x} + \frac{\partial \gamma_{xy}}{\partial y} + \frac{\partial \gamma_{xz}}{\partial z}\right] + \frac{JE}{1-2\mu}\frac{\partial P}{\partial x} = 0 \quad \text{(cyclic)} \tag{22}$$

By using the identities (Timoshenko, 1934, p. 7)

$$\epsilon_x = \frac{\partial \mu}{\partial x} \quad \text{(cyclic)} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \text{(cyclic)} \tag{23}$$

and the definition

$$\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(22) will simplify to

$$(\lambda + G)\frac{\partial e}{\partial x} + G\nabla^2 u + \frac{JE}{1-2\mu}\frac{\partial P}{\partial x} = 0 \quad \text{(cyclic)} \tag{24}$$

If the foregoing manipulations had been performed using the conventional form of Hooke's Law for isotropic materials,

$$\epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] \quad \text{(cyclic)} \tag{25}$$

rather than equations (7) and assuming the existence of body forces X, Y, Z, the result (Timoshenko, 1934, p. 200) would have been

$$(\lambda + G)\frac{\partial e}{\partial x} + G\nabla^2 u + X = 0 \quad \text{(cyclic)} \qquad (26)$$

When we compare (24) and (26) we see that the term

$$\frac{JE}{1-2\mu}\ \frac{\partial P}{\partial x}$$

in (22) takes the same form mathematically as a body force per unit volume.

The boundary equations (Timoshenko, 1934, p. 195) are

$$\overline{X} = \sigma_x \ell + \tau_{xy} m + \tau_{xz} n \quad \text{(cyclic)} \qquad (27)$$

where $\overline{X}$, $\overline{Y}$, $\overline{Z}$ are surface forces, and ℓ, m, and n are direction cosines.

After substituting for the stresses by making use of (9) and (17) and assuming that the surface forces are zero, we obtain

$$-\frac{JEP}{1-2\mu}\ell = \lambda e \ell + G\left[\frac{\partial u}{\partial x}\ell + \frac{\partial u}{\partial y}m + \frac{\partial u}{\partial z}n\right] + G\left[\frac{\partial u}{\partial x}\ell + \frac{\partial v}{\partial x}m + \frac{\partial w}{\partial x}n\right] \quad \text{(cyclic)} \qquad (28)$$

If the same operations had been performed on the boundary force equations (27) using the conventional form of Hooke's Law for isotropic materials and assuming the existence of surface forces the result would be

$$\overline{X} = \lambda e \ell + G\left[\frac{\partial u}{\partial x}\ell + \frac{\partial u}{\partial y}m + \frac{\partial u}{\partial z}n\right] + G\left[\frac{\partial u}{\partial x}\ell + \frac{\partial v}{\partial x}m + \frac{\partial w}{\partial x}n\right] \quad \text{(cyclic)} \qquad (29)$$

Comparing (23) and (24) we see that the term $-\left[\frac{JE}{1-2\mu}\right]P\ell$ is analogous to the surface force, $\overline{X}$.

We have now shown that the existence of fluid pressure and fluid pressure gradients within the porous system may be accounted for by assuming the existence of a fictitious body force per unit volume

$$\frac{JE}{(1-2\mu)}\ \frac{\partial P}{\partial x} \quad \text{(cyclic)}$$

and surface forces of intensity

$$-\frac{JEP}{(1-2\mu)}\ell \quad \text{(cyclic)}$$

thus satisfying the boundary and equilibrium equations.

If real body and surface forces exist, and the latter certainly will exist, they may be superposed on the components of stress obtained from the solution of equations (22) and (28).

In part II we will take advantage of the analogy between thermal stresses and the stresses resulting from pore pressure in obtaining the solution to a sample problem.

Effective Stress

In the field of soil mechanics a stress term known as "effective stress" has been defined which is useful because it is a measure of the pressure tending to compact soils and causing them to resist shear stresses. The effective stress, $\bar{\sigma}$, is defined, $\bar{\sigma} = \sigma - P$. (See Terzaghi and Peck, 1948, p. 52.)

The effective stress is the component of the total stress which tends to compact and distort the mineral grains.

The meaning of effective stress becomes more clear when we inspect relations (7). If the fluid pressure remains constant, a change in the total normal stresses produces strains that are functions of the bulk elastic properties of the material and that are accompanied by microscopic distortions in the porous framework.

Considering what happens when the fluid pressure changes and the total stresses remain constant, we see that the strain due to changes in fluid pressure may be conveniently divided into two components. One component

$$-\frac{(1-2\mu)}{E} P$$

is equivalent to the strain produced in a jacketed specimen of porous material by an external fluid pressure, P, except that it is opposite in sense. The nature of this type of strain is the same as that produced by changes in the total normal stresses and is accompanied by grain distortion.

The other component of strain caused by changes in fluid pressure, $(\beta/3)$ P, is equivalent to the strain produced in an unjacketed specimen when immersed in a fluid under pressure. The strain thus produced is hydrostatic microscopically as well as macroscopically and produces a uniform reduction in grain volume (we are assuming that the grains themselves are isotropic) with no grain distortion. Thus we have two types of strains, one accompanied by grain distortion and compaction, the other consisting of a uniform change in volume with no distortion. Separating these two types of strain we may put equations (7) in the form

$$\epsilon_x = \frac{(\sigma_x - P)}{E} - \frac{\mu}{E}[(\sigma_y - P) + (\sigma_z - P)] + \frac{\beta}{3} P \quad \text{(cyclic)}$$

We now define that component of the normal strain characterized by grain distortion and compaction as "effective strain", symbolized by $\bar{\epsilon}$. We see that this component of the normal strain is entirely due to normal stresses of intensity $(\sigma - P)$, symbolized by $\bar{\sigma}$, the effective stress. So we have the equations defining effective stress:

$$\bar{\sigma}_x = \sigma_x - P \quad \text{(cyclic)} \tag{30}$$

These two types of strain and the stresses that produce them are separated because they have very different effects on the properties of the material. The completely hydrostatic component of the normal strain, $(\beta/3)$ P, does not change the gross structure of the material, but it does alter slightly the distances between atoms in the crystal structure. This in turn changes resistance to shear along crystallographic planes and probably also changes the elastic constants. However, these effects are so small, as evidenced by the work of Griggs (1938) and others, as to be negligible within the range of pressures encountered in oil field work.

On the other hand, the properties of the porous material might well be expected to change with changes in the effective normal stresses. The properties of a porous material depend on its structure as well as on the properties of its constituents. Because a change in the effective stress changes slightly the structure of the porous substance, we might also expect a change in the elastic properties and strength of the material.

An increase in shear strength of granular materials due to an increase in the effective stress has been demonstrated experimentally many times. Measurement of strength and deformation of jacketed cylinders of sedimentary rock have been performed by Handin and Hager (1957) and others. Changes in bulk compressibility with effective confining pressure have also been measured by Zisman (1933).

The Coulomb-Mohr criterion of failure, which is developed in a later section, has been found to fit experimental data rather well. The Mohr theory of failure is based on Coulomb's equation,which states that resistance to shear is equal to some constant times the compressive stress normal to the plane of shear plus a constant which is the shear strength at zero stress normal to the plane of shear. Expressed analytically

$$\tau = \tau_0 + K\bar{\sigma}_n \tag{31}$$

where τ_0 is the resistance to shear at zero normal stress, $\bar{\sigma}_n$ is the effective stress normal to the plane of shear and K is a constant of proportionality, known as the coefficient of internal friction.

Experimental data for sedimentary rocks has been found to fit this simple relation reasonably well. However, Handin and Hager (1957) point out that resistance to shear seems to depend on stresses other than those normal to the plane of shear. Although the Coulomb - Mohr theory thus leaves something to be desired, we shall use it here as the best machinery presently available. Probably a great deal of experimental work will be required in order to take better account of shear strength as a function of the stress condition of the material.

Before the Coulomb-Mohr criterion of failure is discussed, something should be said of the influence of the effective stress on the elastic constants of the material.

Experimental Values

An experimental study was undertaken (Cleary, 1958) to test the applicability of the elastic theory of porous materials developed in the previous section for sandstones and to learn typical values of the elastic constants. These experiments demonstrated that the bulk elastic constants are strongly a function of the effective stresses in the material, as suggested in the above paragraphs. E increases markedly with mean effective confining pressure. As the elastic theory presented here was developed under the classical assumption of constant elastic properties, it is somewhat inadequate, but may be used for order-of-magnitude calculations if "average" values of the "constants" are selected within the range of stress conditions that exist in the problem under consideration. Table 1 shows reasonable values of the elastic constants for two sandstones, one of high and one of low porosity, obtained from these tests.

In summary, the effective stress produces approximately the same changes in properties of a porous material, such as shear strength and elasticity, in the presence of fluid pressure as would a total stress in the absence of fluid pressure.

Table 1. - Elasticity Values Used in Sample Problems

	E lb./in.^{2}	μ	$\frac{JE}{1-\mu}$
Porous sand	2×10^6	.1	.7
Hard sand	6×10^6	.2	0

For the sake of completeness we will obtain the force field developed by the moving fluid in terms of the effective stress, $(\sigma - P)$. In the previous section we showed that this body force in terms of the total stress was zero, or

$$\frac{d\sigma_s}{ds} = 0$$

where s is the direction of fluid motion. From the definition of effective stress given by (30):

$$d\sigma_s = d\bar{\sigma}_s + dP$$

Therefore

$$\frac{d\sigma_s}{ds} = -\frac{dP}{ds}$$

Hence, the effective stress gradient equals the negative gradient of the fluid pressure. This view of the body force caused by the motion of a fluid is particularly useful in soil mechanics where the shear strength of soils as a function of hydraulic gradients is of concern. (See Terzaghi and Peck, 1948, p. 53.)

Coulomb-Mohr Criterion of Shear Failure

A method for determining the stress condition of impending shear failure for granular materials is used in the field of soil mechanics (Terzaghi and Peck, 1948, p. 95-97). The same method was used by Hubbert (1951) in a discussion of the mechanics of faulting, and by Hubbert and Willis (1957) in estimating the stress condition in a faulted region from which the orientation of hydraulic fractures might be inferred.

This method combines the graphical presentation of normal and shear stresses, known as the Mohr's circle, with a plot of Coulomb's equation for the internal friction of granular materials.

The Mohr's circle construction is developed as follows: A two-dimensional stress block (fig. 5) is oriented so that the normal stresses σ_x and σ_z are perpendicular to planes of zero shear. An arbitrary shear plane AA' passes through the block making an angle θ with σ_x, the plane of least principal stress. An elementary wedge of material, ds, dx, and dz, is indicated in the block and expanded below it. The normal stress on the shear plane is σ_n, and τ is the shear stress.

The equilibrium equation in direction AA' is

$$-\tau\, ds + \sigma_z\, dx \sin\theta - \sigma_x\, dz \cos\theta = 0$$

and

$$\sin\theta = \frac{dz}{ds} \qquad \cos\theta = \frac{dx}{ds}$$

So

$$\tau = \frac{1}{2}(\sigma_z - \sigma_x) \sin 2\theta \qquad (32)$$

This gives us the relation between the shear stress, τ , and the principal stresses, and the angle Θ, which the shear plane makes with the least principal stress.

The equilibrium equation for the direction normal to the shear plane is

$$-\sigma_n\, ds + \sigma_z\, dx \cos\theta + \sigma_x\, dz \sin\theta = 0$$

$$\sigma_n = \sigma_z \cos^2\theta + \sigma_x \sin^2\theta$$

$$\sigma_n = \frac{(\sigma_x + \sigma_z)}{2} + \frac{(\sigma_z - \sigma_x)}{2} \cos 2\theta \qquad (33)$$

Now making use of equations (31) and (32), if τ is plotted against σ_n for all values of Θ while holding σ_x and σ_z constant, we obtain a circle (fig. 6). The circle intersects the σ axis at σ_x and σ_z. Its center lies on the axis at point

$$\frac{(\sigma_z + \sigma_x)}{2}$$

An arbitrary point, A, on the circle, satisfies equations (32) and (33) for a particular angle Θ such that 2Θ is the angle which the radius vector of A makes with the positive σ axis. Once the properties of the circle are recognized it is possible to construct it directly from σ_x and σ_z. After the circle is constructed, the normal and shear stresses for any plane making an angle Θ with the lesser of the two principal stresses can be read directly from the construction.

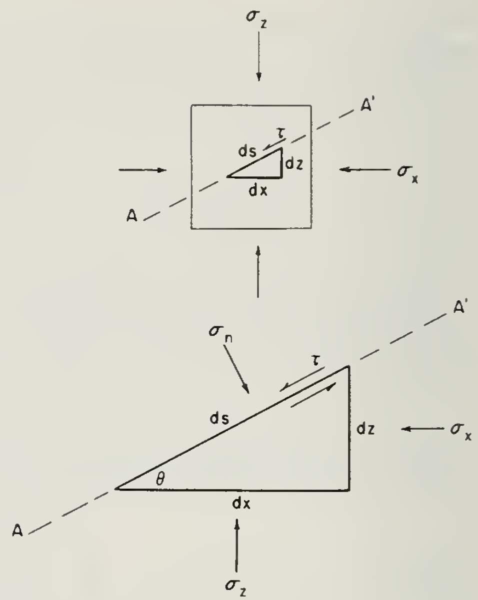

Fig. 5. - Stress block showing normal stresses in two principal directions. The shear stress is in a plane that makes an angle θ with σ_x . The shear stress is indicated on the wedge ds, dx, dz.

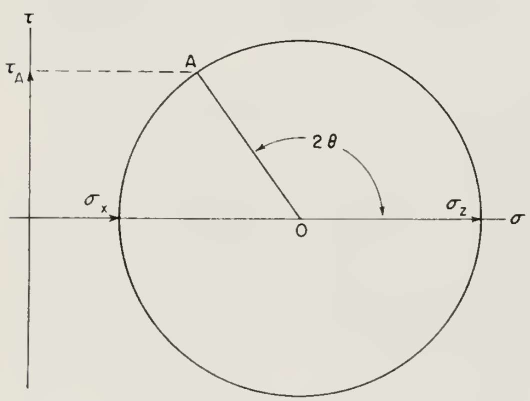

Fig. 6. - Mohr's circle showing the shear stress, τ_A, in a plane making an angle θ with the least principal stress.

Coulomb's equation (31) for the shear resistance of granular materials under compression has already been given. It was also pointed out that the significant stress normal to the plane of shear is the effective stress, $\bar{\sigma}$, defined by equation (30) if pore fluid pressure is present. Coulomb's equation (31) may be written

$$\tau = \tau_o + \tan\phi\, \bar{\sigma}_n \qquad (34)$$

in which $\tan\phi = K$ and ϕ is defined as the angle of internal friction.

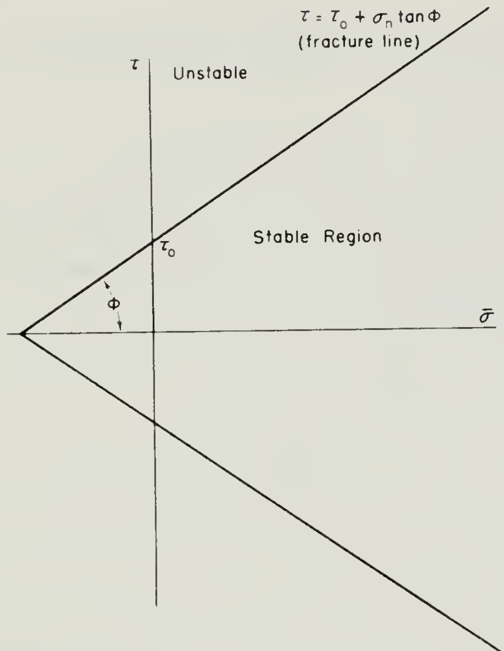

Fig. 7. - Fracture lines showing regions of stability and instability.

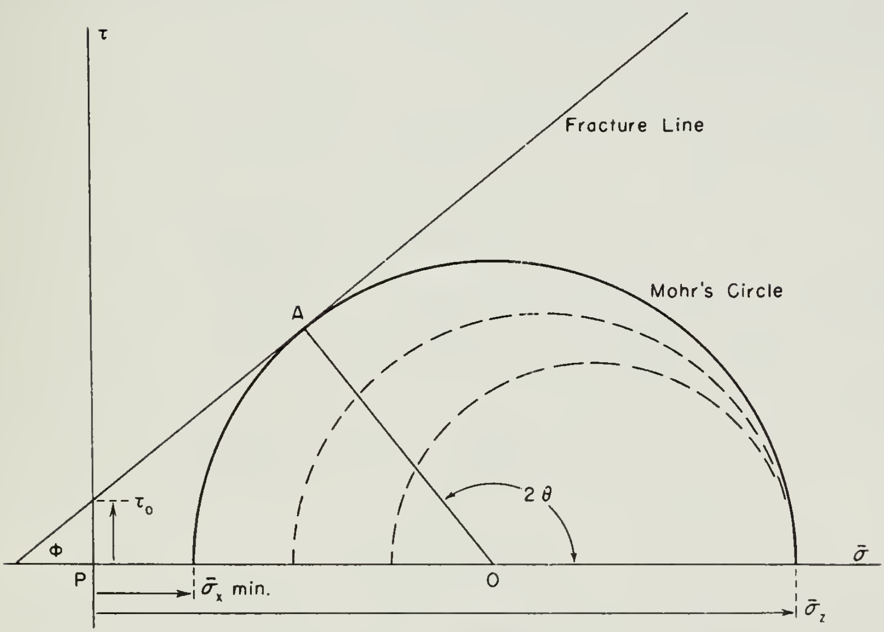

Fig. 8. - Mohr's circle tangent to the fracture line, yielding minimum value of $\bar{\sigma}_x$ for the given value of $\bar{\sigma}_z$, the minimum and maximum principal stresses.

Now, if we plot the shear resistance τ against the effective normal stress, $\bar{\sigma}_n$, we obtain two straight lines defining regions of stability and instability on the graph (fig. 7).

Combining Mohr's Circle with Coulomb's Equation

Figure 8 shows Mohr's circle diagram and fracture lines plotted from Coulomb's equation. The diagram has been plotted in terms of the effective stress. It was therefore necessary to substitute for the total normal stresses in (32) and (33) their expressions in terms of the effective stress. However, this does not change the form of (32) and (33).

The dotted circles on the diagram indicate the states of stress for decreasing values of $\bar{\sigma}_x$. The solid line circle shows the minimum value of $\bar{\sigma}_x$ for the given set of conditions. If $\bar{\sigma}_x$ were smaller the circle would fall partly in the unstable region. If this were the case the shear stress would exceed the shear strength along certain planes, an impossible condition. So, given the angle of internal friction, ϕ, for the rock, and the effective vertical compressive stress, one can construct a Mohr's circle tangent to the fracture line and read directly $\bar{\sigma}_x$ min, the minimum value of the effective horizontal principal stress.

Algebraic Expression for σ_x Minimum in Terms of Pore Pressure

It is convenient to have an algebraic expression for σ_x min. This we can obtain directly from figure 8, which shows the Mohr's circle tangent to the fracture line. This is the condition of impending failure. In other words, this is the condition such that the shear strength is equal to the shear stress along a plane making an angle θ with the horizontal. Thus, a minimum value of $\bar{\sigma}_x$ is defined for given values of ϕ, τ_o, σ_z, and P.

From the figure we note that

$$PO - AO = \bar{\sigma}_x \min$$

and

$$PO = \frac{\bar{\sigma}_x \min + \bar{\sigma}_z}{2}$$

and

$$AO = \sin\phi\left[\frac{\bar{\sigma}_x \min + \bar{\sigma}_z}{2} + \frac{\tau_o}{\tan\phi}\right]$$

Therefore

$$\bar{\sigma}_x \min = \frac{(1 - \sin\phi)}{(1 + \sin\phi)}\,\sigma_z - \frac{2\cos\phi\,\tau_o}{1 + \sin\phi} \tag{35}$$

Making use of (35) and (30) we can obtain this expression in terms of total stresses.

$$\sigma_x \min - P = \frac{(1 - \sin\theta)}{(1 + \sin\theta)}\,(\sigma_z - P) - \frac{2\cos\theta\,\tau_o}{1 + \sin\phi} \tag{36}$$

Tensile Strength

There is one further restriction to be placed on the allowed values of σ_x based on the assumption that the rocks will not sustain an appreciable tensile stress over any wide area.

The following analogy is offered to justify this assumption. If a brick from an old brick wall were placed in a testing machine and pulled apart, the measured value of the breaking stress might be about 600 pounds per square inch. But if by some means a short section of the wall were tested, the value of the breaking stress would be much lower because of cracks and other imperfections which would exist in such a section. Finally, if a long section of the old wall were tested, in all likelihood the tensile strength would be either 0 or negligible, for probably somewhere in a long section of the wall there would be a crack that would almost completely sever the wall.

Similarly, a sedimentary bed will ordinarily contain joints or planes of weakness along which tensile stresses would be relieved. From this assumption we make the following restriction on the value of the normal stresses

$$\sigma \geq P$$

or

$$\sigma_x \min = P \quad \text{(cyclic)} \tag{37}$$

Tensile Failure Due to Pore Fluid Pressure

If forces are being applied over a very limited region within the rock, the tensile strength of the rock may have to be considered. As noted previously, the fluid does not act over the total area of a section of cemented porous material. However, as we are now considering the condition of failure, it is not valid to use the pore area term used in (6) above as the effective unit area.

The pore area term, n, was defined as the area of pore in any random plane passed through the material. Tensile failure, whether due to an external tension or internal fluid pressure, will not occur in a plane. The surface of fracture, while tending to remain normal to the least compressive stress, will follow a tortuous path that takes advantage of statistical variations in the strength of the material and larger scale cracks and weaknesses if present. In spite of these considerations, the fluid pressure in a consolidated material must act over something less than the total area in the plane of fracture. Let us designate this fraction as N. It would be difficult to obtain a value for N but we know that it must be greater than n. Note that for an unconsolidated sand n would equal about .35 whereas N would equal 1.

An idealized one-dimensional model of a consolidated porous material subjected to an internal fluid pressure and an external compressive stress is depicted in figure 9. The model consists of a plug with a reduced middle section fitted in a cylinder. The cross section of the annulus represents N as described above. The bore of the cylinder is unity, so the area of the reduced section of the plug is (1 - N). This represents the solid area which must be severed along the plane of fracture.

Suppose that we have obtained the tensile strength, σ_t, in terms of the gross area by conventional laboratory methods. In terms of the solid area along the potential surface of fracture the tensile strength is $\sigma_x/(1 - N)$.

The question is, what "pore" pressure, P, is required to cause the model to fail in tension? If additional details are not known, there are several possible answers to this question.

(1) Assume, for example, that the material is ductile so that failure will occur when the shear stress reaches a certain critical value, τ_s, per unit solid area. Then the pressure causing tensile failure will be determined by equating

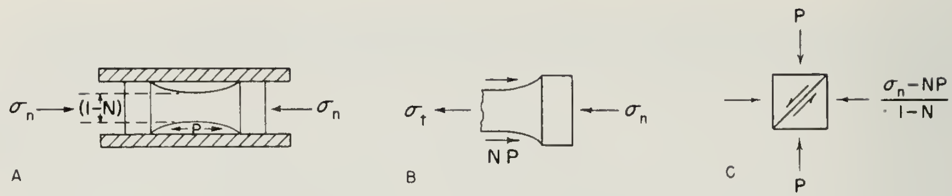

Fig. 9A. - One-dimensional model of porous material loaded by an external stress, σ_n, normal to the plane of potential fracture, and a pore fluid pressure P. The solid area in the plane of failure is (1 - N).

B. - Free-body diagram showing the forces acting to produce tension normal to the plane of failure in a one-dimensional model of porous material.

C. - A cubic element in the section of failure with the forces acting on it.

the expression for the shear strength in terms of the tensile strength and solid area of failure

$$\tau_s = -\frac{\sigma_t/(1 - N)}{2}$$

to the shear stress in terms of an external axial load, σ_n , and pore fluid pressure necessary to cause failure (fig. 9A),

$$\tau_s = \frac{1}{2}\left[\frac{(\sigma_n - NP)}{(1 - N)} - P\right]$$

This reduces to

$$P = \sigma_t + \sigma_n \qquad (38)$$

This tells us that, regardless of the value of N, the material will fail when the pore pressure, P, exceeds the external pressure, σ_n, by an amount equal to the tensile strength of the material.

(2) Now suppose that the model is composed of a brittle material. The material will fail in pure tension when the axial stress exceeds a certain value, σ_t, per unit gross area or actually $\sigma_t/(1 - N)$ in terms of the solid section of failure.

The pressure necessary for failure is then determined by $\sigma_n + \sigma_t = NP$ (fig. 9B).

$$P = \frac{1}{N}(\sigma_n + \sigma_t) \qquad (39)$$

This indicates that the pressure required to rupture a brittle porous material in tension exceeds the external stress needed to rupture it and is inversely proportional to N.

As it turns out, actual experiments on brittle materials do not indicate that the above argument will hold.

(3) P. W. Bridgman (1947) performed tensile tests on brittle materials under confining pressure. The essential features of these tests could be represented by our figure 10.

The values of pressure necessary to cause failure in these experiments ranged from what would be predicted by

$$P = \sigma_t + \sigma_n$$

on up to much higher values. It was also found that the type of fluid in contact with the specimen greatly affected the results of the experiment. If the specimen was jacketed, much higher values for the "strength" were obtained.

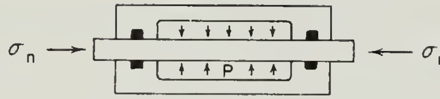

Fig. 10. - "Pinch-off" apparatus might be used to measure tensile strength of rock loaded by pore fluid pressure.

In one set of experiments a uniform glass rod was allowed to protrude through packed glands at either end of a pressure chamber. Pressure within the chamber caused failure of the specimen in tension and the pressure causing failure was approximately

$$P \approx \sigma_t + \sigma_n \tag{40}$$

where σ_n is a small unknown external stress due to friction in the packing. Here we note that although the specimen failed in tension it was actually slightly in compression due to friction in the packing.

Bridgman's explanation for this paradox is roughly as follows. The surface of the brittle material contains microscopic fractures and imperfections which act as stress raisers. If the fluid can enter these it can act locally in the axial direction, but in a gross diagram of the experiment it appears to act only radially. Thus the fluid may propagate a fracture across the section while it is in compression. What will really happen depends on the detailed properties of the material and must be determined by actual experiment. So it would seem that the pore pressure necessary to rupture sandstone and other porous rock materials should lie in the region

$$\sigma_n + \sigma_t \leq P \leq \frac{1}{N}(\sigma_t + \sigma_n)$$

One is tempted to speculate further than this. Observing that the internal surface of a porous rock contains, to the nth degree, the microscopic stress raisers and microscopic fractures encountered by Bridgman for polished specimens (1947), we may well suspect that P, the pore pressure necessary for tensile failure, is simply $\sigma_n + \sigma_t$. A straightforward experiment, using an apparatus modeled after figure 10, might settle this question. The pressure necessary to rupture successive identical rock specimens would be plotted against successive values of σ_n , an external stress applied to the ends of the specimen. One could then determine whether $\frac{dP}{d\sigma_n} = 1$ or $\frac{dP}{d\sigma_n} = \frac{1}{N}$ where $N < 1$

One difficulty would be that of obtaining identical specimens if natural material were used.

Summary

As a review of this section we will now plot the change in stress condition of two buried strata, considering stress a function of the pore fluid pressure, and assuming that it has changed uniformly with time throughout the strata.

The example in which the vertical stress, σ_z, is maximum and one of the horizontal stresses, σ_x, is minimum is considered in this and all succeeding examples.

Let the vertical stress, σ_z, equal 5,000 psi. The least horizontal stress when the pore fluid pressure is zero, will be denoted by σ_{x_0}.

The assumed elastic properties of a porous sandstone are listed in table 1.

A restriction on the possible conditions of stress is given by (37). This is the requirement that for large masses of rock the tensile strength is negligible and consequently the total stress, σ_x, cannot be less than the fluid pressure, P. In figure 11 this requirement is plotted as a straight line through the origin with a slope of one. This line defines a set of minimum values of σ_x for any given pore fluid pressure, P.

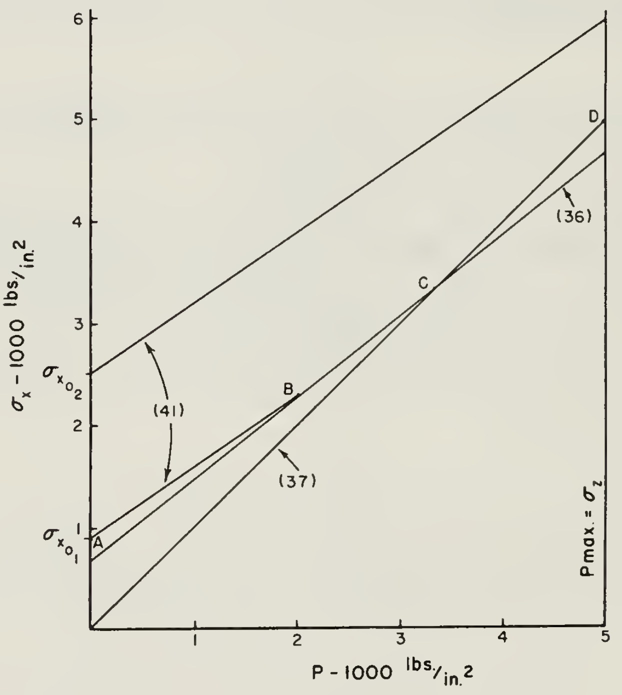

Fig. 11. - The least horizontal stress, σ_x, as a function of a pore fluid pressure changing with time for the case $\sigma_x < \sigma_z$

Another restriction on possible values of σ_x is imposed by the limited shear strength of the material. This restriction is imposed by plotting (36) on the diagram. Let ϕ equal 45° and τ_0 equal 200 psi.

The value of ϕ selected is not to be taken too seriously. Published values cannot be relied upon in this application. In a sense, the value of ϕ which we seek applies to the yield strength of the material because if any yielding does occur the least stress will go immediately to a higher value and yielding will then cease. Doubtless if the rock is near enough the condition of failure for shear strength to be considered, shear joints would already exist in the rock and therefore the coefficient of friction that we seek would be for slippage along pre-existing fractures. For this reason a modest value is assumed for τ_o. It is hoped that future studies will make possible a more confident application of this theory.

We see in figure 11 that for the interval $0 < P < 3300$ psi, the shear strength is the limiting factor whereas the "negligible tensile strength" requirement decides the minimum value of σ_x for higher values of P.

It now remains to plot the change in σ_x due to changes in pore fluid pressure using equations (7). The conditions of the problem require that horizontal expansion be prevented while free expansion is allowed in the vertical direction, and that there is no change in σ_z. Letting $\epsilon_x{}^*$ and $\epsilon_y{}^*$ be the strain components and letting $\sigma_x{}^*$, $\sigma_y{}^*$, and $\sigma_z{}^*$ be the components of the total stress due to the change in fluid pressure, these conditions may be expressed $\sigma_z{}^* = 0$; $\epsilon_x{}^* = \epsilon_y{}^* = 0$ and from symmetry $\sigma_x{}^* = \sigma_y{}^*$. Substituting in (7) gives

$$\sigma_x{}^* = \left[\frac{JE}{1 - \mu}\right] P$$

and

$$\sigma_x = \sigma_{x_o} + \left[\frac{JE}{1 - \mu}\right] P \qquad (41)$$

The experimental value of $\frac{JE}{1-\mu}$ in table 1 (p. 13) is o.7. This value has been substituted in (41) and the curves plotted for two assumed values of σ_{x_o} on the diagram in figure 11.

Figure 11 gives us the range of possible values of P and σ_x, the least horizontal compressive stress, and it also shows how σ_x will change with fluid pressure.

Assuming that the virgin pore fluid pressure was 2,000 psi it is evident from figure 11 that $\sigma_{x_{o_1}}$ is a minimum inasmuch as its curve AB cannot intersect the yield line below the point B. Therefore, possible values of σ_x must fall within the region A B C D.

Figure 12 shows a diagram similar to figure 11. Here we have plotted values for a hard sandstone. The assumed properties of the material are listed in table 1.

It is not meaningless to consider the change in fluid pressure in a hard nearly impervious formation if it has some sort of joint system and lies adjacent to a permeable formation in which the fluid pressure is changing. Suppose that the rock we are now considering is entirely impervious save for a few widely spaced closed vertical joints, and that it forms an impermeable cap on a porous sand.

For an impervious rock, J must be equal to zero. Therefore (40) becomes $\sigma_x = \sigma_o =$ constant. The horizontal stress does not change with fluid pressure. This is indicated in figure 12 by horizontal lines $\sigma_{x_{o_1}}$ and $\sigma_{x_{o_2}}$. But, when the fluid pressure communicating with the joints reaches the value of σ_{x_o}, any further increase in P must be accompanied by an equal increase in σ_x due to requirement (37).

Let the stress condition be that indicated by point A in figure 12. Then let P be increased from 2,000 psi to 3,000 psi. What will be the horizontal strains in the rock? Assume that σ_y is greater than 3,000 psi then it will not be influenced directly by the change in P.

The conditions of the problem are then $\epsilon_y{}^* = 0$; $\sigma_z{}^* = 0$; $\sigma_x{}^* = 1{,}000$ psi

For impervious rock Hooke's Law is simply

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z) \qquad \text{(cyclic)}$$

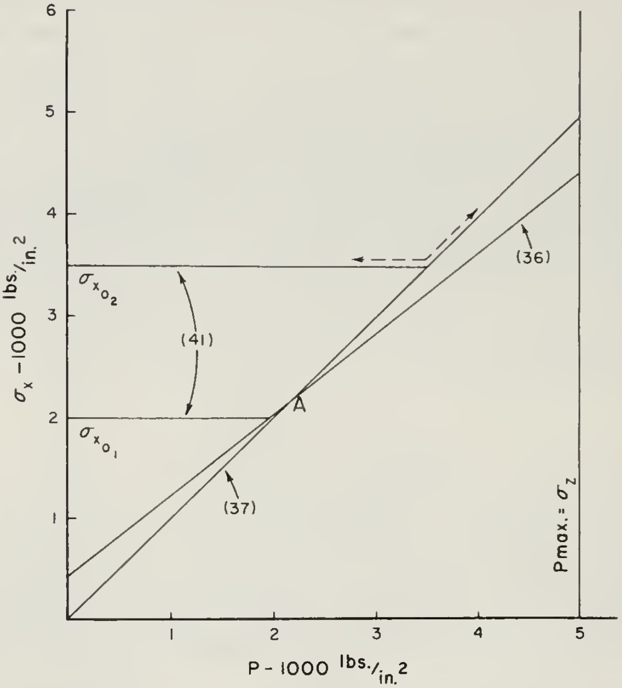

Fig. 12. - The least horizontal stress, σ_x, as a function of pore pressure for a stratum of very low porosity. The path of changing σ_x is indicated by the dotted arrow.

This gives us, when we substitute the boundary conditions,

$$\epsilon_x{}^* = \frac{1,000}{E} - \frac{\mu}{E} \sigma_y$$

$$0 = \sigma_x - \mu[1,000]$$

$$\epsilon_x{}^* = \frac{[1 - \mu^2]}{E} [1,000]$$

Letting μ equal .2 and E equal 6 x 10^6 gives $\sigma_x{}^*$ = .001 in./in.

A joint system perpendicular to the X axis and an average spacing of 100 inches would then have opened about .01 inch per joint.

Some of the basic theory presented here will be used in the solution of sample problems in Part II.

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