Estimation of Time-Varying Hedge Ratios for Corn and Soybeans: BGARCH and Random Coefficient Approaches

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Abstract

This paper deals with the estimation of optimal hedge ratios. A number of recent papers have demonstrated that the ordinary least squares (OLS) method which gives constant hedge ratio is inappropriate and recommended the use of bivariate autoregressive conditional heteroskedastic (BGARCH) model. In this paper we introduce the use of a random coefficient autoregressive (RCAR) model to estimate time varying hedge ratios. Using daily data of spot and futures prices of corn and soybeans we find substantial presence of conditional heteroskedasticity, and also of random coefficients in the regressions of return from the spot market on the return from the futures markets. Hedging performance in terms of variance reduction of returns from alternative models are also conducted. For our data set diagonal vech presentation of BGARCH model provides the largest reduction in the variance of the return portfolio.

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1. INTRODUCTION

Numerous approaches are available to estimate hedge ratios. Traditionally, ordinary least squares (OLS) regression of the spot price on the futures price is run, with the slope coefficient being the hedge ratio [e.g. Ederington (1979); Anderson and Danthine (1980)]. However, this procedure is inappropriate because it ignores the heteroskedasticity often encountered in price series [Park and Bera (1987)] and is not based on conditional information [Myers and Thompson (1989)]. Recently, autoregressive conditional heteroskedastic (ARCH) [Engle (1982)] and generalized ARCH (GARCH) [Bollerslev (1986)] have been used to estimate time-varying hedge ratios (THR). The time-varying joint distribution of cash and future price changes has been examined for hedging financial instruments [Cecchetti, Cumby and Figlewski (1988)]. Bivariate GARCH (BGARCH) models also have been used to estimate THR in commodity futures [Baillie and Myers (1991); Myers (1991)], in foreign exchange futures [Kroner and Sultan (1990)], in interest rate futures [Gagnon and Lypny (1995)], and in stock index futures [Park and Switzer (1995) and Tong (1996)]. These more recent studies suggest that conventional hedging procedures can produce misleading results.

The initial findings obtained using GARCH models raise several questions. First, what is the sensitivity of THR to alternative specifications of the conditional covariance matrix, and what tests can be used to select the most appropriate model? Second, what is the degree to which the variance of returns is reduced with alternative procedures? Third, do alternative approaches that effectively capture the time-varying nature of hedge ratios exist. The sensitivity and degree to which time-varying hedge ratios reduce the variance of returns is important because of the difficulty in specifying and estimating GARCH models. Due to the complex structure of the covariance matrix, simplifying assumptions frequently are made to ensure non-negative variances and tractable solutions. Testing procedures have not been used to identify the most appropriate structure of the covariance matrix, and little empirical evidence exists regarding the effects of the simplifying assumptions on estimated hedge ratios. This is of particular importance, because while the limited evidence suggests that hedge ratios should be considered time-varying, the effective reduction of the variance of returns generated by these procedures over traditional constant hedge ratios has been small. Finally, alternative, more tractable approaches may need to be examined; here, we propose the use of a random coefficient (RC) model to estimate hedge ratios. The random coefficient approach has been successfully used to identify systematic risk in the market model literature [e.g. Bos and Newbold (1984)] and may prove useful in approximating effective hedge ratios.

In this article, we focus on model specification and empirical comparison of BGARCH and random coefficient models for hedge ratio estimation. In the next section, the appropriate hedging rule based on a mean-variance model is identified and alternative BGARCH models and their estimation results are presented. Diagnostic tests are used to identify the appropriate model specification. In Section 3, a random coefficient model for calculating THR is discussed and estimated. The results from examining the hedging effectiveness of the ARCH-type and random coefficient models, along with the conventional OLS hedging approach, are reported in Section 4. Finally, Section 5 offers some concluding remarks. Our analysis is similar to that of Baillie and Myers (1991); however, there are some important differences. The data sets are different, and we use nearby futures contracts. Our use of the random coefficient model in the context of hedge ratio estimation is new, and three alternative versions of the BGARCH model are considered. Finally, all our models are subjected to various diagnostic checks, specification tests and model selection procedures.

2. Time-Varying Hedge Ratios and Their Estimation Using BGARCH Models

2.1 The Time-Varying Hedging Rule

Using a mean-variance framework, hedge ratios have been estimated using OLS by regressing the returns from holding a spot contract on returns from holding a futures contract. In a similar context, assuming utility maximization and efficiency in future markets the conditional optimal one-period ahead hedge ratio, b^*_{t-1}, at time t can be derived as

$$b^*_{t-1} = \frac{-Cov(R_t^s,\ R_t^f|\psi_{t-1})}{Var(R_t^f|\psi_{t-1})}\ , \tag{1}$$

where R_t^s and R_t^f denote logarithmic differences of spot and futures prices from t-1 to t, respectively and Ψ_{t-1} is the information set at time t-1. This ratio is similar to the conventional hedge ratio except that the conditional variance and covariance replace their unconditional counterparts. Because conditional moments can change as the information set is updated, the hedge ratios also can change through time.

2.2 Specification of Selected BGARCH Models

The motivation behind using BGARCH models in the context of hedge ratio estimation is that daily commodity future and spot prices react to the same information, and hence, have non-zero covariances conditional on the available information set. We specify a general model as

$$\begin{aligned} R_t^s &= \mu_s + \epsilon_{st} \\ R_t^f &= \mu_f + \epsilon_{ft} \end{aligned} \tag{2}$$

$$\epsilon_t \mid \Psi_{t-1} \sim BN(0,\ H_t)\ , \tag{3}$$

where R_t^s, R_t^s are defined above, $\epsilon_t = (\epsilon_{st}, \epsilon_{ft})'$, BN denotes bivariate normal distribution, and H_t is a time-varying 2x2 positive definite conditional covariance matrix. A somewhat general form of H_t for a BGARCH (p,q) model can be written as,

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} \Gamma_i vech(\varepsilon_{t-1}\varepsilon'_{t-1}) + \sum_{i=1}^{p} D_i \; vech(H_{t-1}) \, , \tag{4}$$

where C is a 2x2 positive definite symmetric matrix and Γ_i and D_i are 3x3 matrices. The "vech" operation stacks the lower triangular elements of a symmetric matrix in a column.

The parameterization given in (4) is difficult to estimate since positive definiteness of H_t is not assured without imposing nonlinear parametric restrictions. Moreover, the model contains too many parameters, e.g., for p=q=1, H_t has 21 parameters. Here, we examine several simplifying specifications of H_t. One straightforward assumption that could be made is to specify that a conditional variance depends only on its own lagged squared residuals and lagged values. The assumption amounts to making Γ and D matrices diagonal. In this case, vech(H_t) of a BGARCH(1,1) model is given by

$$vech(H_t) = \begin{bmatrix} h^2_{ss,\,t} \\ h^2_{sf,\,t} \\ h^2_{ff,\,t} \end{bmatrix} = \begin{bmatrix} c_s \\ c_{sf} \\ c_f \end{bmatrix} + \begin{bmatrix} \gamma_{ss} & 0 & 0 \\ 0 & \gamma_{sf} & 0 \\ 0 & 0 & \gamma_{ff} \end{bmatrix} \begin{bmatrix} \epsilon^2_{ss,t-1} \\ \epsilon_{s,t-1}\epsilon_{f,t-1} \\ \epsilon^2_{ff,t-1} \end{bmatrix} + \begin{bmatrix} \delta_{ss} & 0 & 0 \\ 0 & \delta_{sf} & 0 \\ 0 & 0 & \delta_{ff} \end{bmatrix} \begin{bmatrix} h^2_{ss,t-1} \\ h^2_{sf,t-1} \\ h^2_{ff,t-1} \end{bmatrix} . \tag{5}$$

This form is called the "diagonal vech" representation of H_t. The necessary conditions for this H_t to be positive definite are

$$\begin{gathered} c_s > 0,\; c_f > 0,\; c_s c_f - c^2_{sf} > 0, \\ and\;\; \gamma_{ss} > 0, \gamma_{ff} > 0,\; \gamma_{ss}\gamma_{ff} - \gamma^2_{sf} > 0 \;. \end{gathered} \tag{6}$$

Engle and Kroner (1995) suggested another parameterization which is almost guaranteed to be positive definite. This form, the "positive definite" parameterization is written as

$$H_t = \begin{bmatrix} c_{ss} & c_{sf} \\ c_{fs} & c_{ff} \end{bmatrix} + \begin{bmatrix} \gamma_{ss} & \gamma_{sf} \\ \gamma_{fs} & \gamma_{ff} \end{bmatrix}' \begin{bmatrix} \epsilon^2_{ss,\ t-1} & \epsilon_{f,\ t-1}\epsilon_{f,\ t-1} \\ \epsilon_{f,\ t-1}\epsilon_{s,\ t-1} & \epsilon^2_{ff,\ t-1} \end{bmatrix} \begin{bmatrix} \gamma_{ss} & \gamma_{sf} \\ \gamma_{fs} & \gamma_{ff} \end{bmatrix} + \begin{bmatrix} \delta_{ss} & \delta_{sf} \\ \delta_{fs} & \delta_{ff} \end{bmatrix}' H_{t-1} \begin{bmatrix} \delta_{ss} & \delta_{sf} \\ \delta_{fs} & \delta_{ff} \end{bmatrix}. \tag{7}$$

Note that the number of parameters to be estimated for this specification is 11. When the γ and δ parameters in the variance covariance function are zero, H_t becomes a constant conditional covariance proposed by Myers and Thompson (1989),

$$H_t = \begin{bmatrix} c_{ss} & c_{sf} \\ c_{fs} & c_{ff} \end{bmatrix}. \tag{8}$$

Bollerslev (1990) introduced another attractive way to simplify H_t. He assumed that the conditional correlation between ϵ_{st} and ϵ_{ft} is constant over time and expressed H_t as

$$H_t = \begin{bmatrix} h^2_{ss,\ t} & h^2_{sf,\ t} \\ h^2_{fs,\ t} & h^2_{ff,\ t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho_{sf} \\ \rho_{sf} & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix}, \tag{9}$$

where ρ_{sf} (< 1) is the time-invariant correlation coefficient, and the individual variances $h^2_{s,\ t}$ and $h^2_{f,\ t}$ are assumed to be a standard univariate GARCH process, for example,

$$h^2_{s,\ t} = c_s + \gamma_{ss}\epsilon^2_{s,\ t-1} + \delta_{ss}h^2_{s,\ t-1}.$$

It is clear that the "constant correlation" representation involves only 7 parameters. Also, positive definiteness of the specification is assured if $h_{s,\ t} > 0$ $h_{f,\ t} > 0$. Because of these attractive features, most of the applications of the BGARCH model use this representation [see for example, Bollerslev (1990), Baillie and Bollerslev (1990), and Kroner and Sultan (1993)].

However, constancy of correlation is a very strong assumption and validity of model (9) remains an empirical question.

2.3 Data and Selected Sample Statistics

Daily data of cash and futures prices for corn and soybeans are used for the period from October 1988 to December 1989. The cash prices are central Illinois elevator bids. March, July, November nearby futures contracts for soybeans, and March, July, December nearby futures contracts for corn are used. The return on the futures and cash markets are defined as a percentage change of each price, i.e.

$$R^{is}_t = 1000 \; log\; (S^i_t / S^i_{t-1})$$

$$R^{if}_t = 1000 \; log\; (F^i_t / F^i_{t-1}),$$

where S^i_t and F^i_t refer to i^{th} cash and futures prices respectively, and i is either corn or soybeans. Switching to a nearby contract takes place on first day of the expiration month. The use of nearby contracts is motivated by hedging practices in the grain industry. Day-to-day purchasing/selling hedging activities, which our data and model best reflects, are most frequently based on nearby contracts as opposed to more distant maturities. To reduce the possible effects of discontinuities in the data series from using price differences between futures prices from contracts with different maturities, changes in the futures prices for the nearby contract are substituted into the data series on the day of the switch. While this may cause some problems as the conditional variance for the new contract is specified in terms of past squared innovations and conditional variances of the old contract, no appreciable differences were encountered in examining the futures price series around the switch points. Similarly, examination of the optimal hedging ratios around the switch points revealed little evidence of dramatic change in their magnitudes. More importantly, since we use the same relevant data set in examining alternative models, our comparative findings, which are the primary focus of the research, should remain valid.

Several preliminary diagnostic tests on the return series of the data are conducted. Using the Phillips-Perron unit root test, nonstationarity of the return series is examined. The unit root hypotheses for the all return series of cash and futures of corn and soybeans are rejected (Table 1). The sample characteristics of univariate unconditional distribution of the return series of corn and soybeans are summarized in Table 2. Not surprisingly, cash and futures returns for the corn and soybeans show typical fat-tail non-normal distributions. In Table 3, test results on autocorrelation of the series and bivariate normality are reported. The Ljung and Box Q statistic is calculated to examine the autocorrelation of univariate return series. There is no evidence of serial correlation. However, the Q^2 statistics, based on squared return series, provide strong evidence of conditional heteroskedasticity.

Bivariate normality is examined using the Bera-John (1983) omnibus test statistic

$$C_3 = N\left(\sum_{i=1}^{2} T_i^2/6 + \sum_{i=1}^{2} (T_{ii}-3)^2/24\right) , \tag{10}$$

where N is a sample size and $T_i = \Sigma_t[\hat{y}_i^3]_t /N$, $T_{ii} = \Sigma_t[\hat{y}_i^4]_t /N$, where $\hat{y} = R(X - m)$, X is the vector of individual observation of a given return series, m is the mean of X, R is the matrix whose eigen vectors are the same as the eigen vectors of the sample covariance matrix of X's, but with the eigen values raised to -1/2. Note that in the univariate case, T_i is the measure of skewness, and T_{ii} is the measure of kurtosis. Under the null hypothesis of bivariate normality, C_3 is asymptotically distributed as χ^2 with 4 degrees of freedom. Given the high values of the test statistics bivariate normality is rejected soundly for both corn and soybeans return series.

2.4 Estimation Results

All parameters in the various BGARCH models are estimated using the maximum likelihood method. The log likelihood function of BGARCH model is given by

$$1(\theta) = \frac{1}{T} \sum_{t=1}^{T} 1_t(\theta)$$
$$where \quad 1_t(\theta) = -½\log(2\pi) - ½\log|H_t| - ½\epsilon'_t \, H_t^{-1}\epsilon_t \tag{11}$$

where θ is the underlying parameter vector. The form of H_t depends on which specification of the conditional variance is used. As discussed earlier, three specifications of H_t, namely positive definite, diagonal vech, and constant correlation forms of BGARCH are estimated. As in most of the applied work in ARCH, we use Berndt, Hall, Hall and Hausman (1974) algorithm (BHHH) for maximizing $1(\theta)$.

The general model was estimated for all BGARCH specifications. In all cases, the hypothesis that $\mu_s = \mu_f = 0$ could not be rejected. The results presented below reflect the restricted model with no drift terms in (2). Restricting the model does not alter appreciably the results from the BGARCH models. Other specifications, such as including a dummy variable for daily effect and seasonal dummies were tried. However, estimates for dummy variables were insignificant.

Tables 4 and 5 present the estimate results of the constant conditional covariance model (equation 8) and the various BGARCH models for corn and soybeans, respectively. Likelihood ratio (LR) statistics also are presented. Under the null hypothesis of conditional homoskedasticity, the LR statistics are asymptotically distributed as $\chi^2(p)$ where p is the number of extra parameters in the different BGARCH models. From the test results, it is clear that conditional homoskedasticity is rejected overwhelmingly when tested against any of the conditional heteroskedastic models.

Now, consider the estimates of three heteroskedastic models. Most of the variance parameters of constant correlation and diagonal vech models are significant and the values of the estimated parameters seem to be quite reasonable. For the diagonal vech model, the conditions for positive definiteness of H_t as stated in (6) are satisfied. Also, we note that for this model, the estimates of $\gamma_{ss} + \delta_{ss}$ and $\gamma_{ff} + \delta_{ff}$ are all close to, but less than, one. On the other hand, in the positive definite parameterization model, many of the parameter estimates are not significant. Of course, it is not possible to choose a model just on the basis of significance and implications of parameter estimates. For model selection, we subject the three BGARCH models to more rigorous model testing. In the next section, we perform a number of diagnostic checks, such as, conditional bivariate normality, autocorrelation and conditional heteroskedasticity tests, using the standardized residuals obtained from the estimated models. Model selection criteria such as Akiake's Information Criteria (AIC) and Bayesian Information Criteria (BIC) are also used.

2.5 Model Specification Tests

Economic theory does not provide specific guidelines for the appropriate parameterization of the BGARCH model. Therefore, the selection of an appropriate model is essentially an empirical question. Based on the standardized residuals, $\hat{H}_t^{-1/2}\hat{\epsilon}_t$ from the constant correlation, diagonal vech and positive definite parameterizations, we conduct the Ljung-Box Q, Q^2, and conditional normality tests. The results are reported in Table 6. For the constant correlation model, as in the original data, we note from the Q statistic values that there is no evidence of serial correlation in the residuals. The Q^2 statistic, based on squared residuals, provides an indirect test of conditional heteroskedasticity. None of the Q^2 statistics are significant, which contrasts sharply with the Q^2 values for the return series reported in Table 3. This indicates that the constant correlation model does take account of the conditional heteroskedasticity present in the data. Finally, the Bera-John test statistics for bivariate normality are much lower than those for the original series; for instance, for the original soybean series, the test statistic was 1047.75. This was reduced to 243.21 for the standardized residuals. However, all the statistics are still significant indicating that constant correlation BGARCH does not take into account all the non-normality present in the data. Again, the primary source of non-normality is the high kurtosis values.

The diagnostic test results for the diagonal vech and positive definite BGARCH models also provide somewhat mixed results. All the skewness and kurtosis coefficients indicate that the conditional distributions are still fat-tailed non-normal. The calculated Bera-John test, rejects the null hypothesis of bivariate conditional normality for both the corn and soybean data. However, the use of these BGARCH filters results in a significant reduction in the non-normality. For example, the value of the Bera-John statistic, 563.16, for the unconditional distribution of return series of corn is reduced to 87.61 and 34.44, respectively, for the diagonal vech and positive definite representations. The $Q(12)$ statistics indicate the absence of autocorrelation for all the specifications. The Q^2 (12) suggest that most of the heteroskedasticity has been eliminated under the diagonal vech model, with only the soybean futures series showing signs of a significant non-constant variance. However, the test results are less attractive under the positive definite BGARCH representation; all series demonstrate significant non-constant variances.

To assist in selecting an appropriate BGARCH specification, AIC and BIC are also calculated, and are presented in Tables 4 and 5. For both corn and soybeans, the positive definite parameterization has the smallest AIC and BIC values, followed rather closely by the diagonal vech and the constant correlation models.

The overall selection of the most appropriate model, because of the mixed testing results, must be made with care to balance the test findings. Based on the specifcation tests which favor the diagonal vech and the constant correlation models, the model selection criteria which support the positive definite and the diagonal vech models, and our previous concerns about the parameter estimates of the positive definite form, the diagonal vech model appears to be appropriate for the time-varying hedge ratio estimation for both corn and soybeans.

The BGARCH models provide the time-varying conditional variances and covariances of R^s_t and R^f_t. Thus, the time-varying hedge ratio at time t-1 can be obtained from,

$$b^*_{t-1} = \frac{h^2_{sf,t}}{h^2_{ff,t}} = HR_t \; . \qquad (12)$$

The time-varying hedge ratios of corn and soybeans calculated from the above equation are plotted in Figures 1-3. To save space we do not present the other THR graphs, but they are quite similar. In Figures 1 and 2, the hedge ratios based on diagonal vech and constant correlation BGARCH, and the constant hedge ratio estimates of 0.931 based on the OLS method are illustrated for corn. The means and standard deviations of the time-varying hedge ratios of constant correlation and diagonal vech BGARCH strategies for corn are 0.970 and 0.931, and 0.101 and 0.125 over the sample period. The estimated time-varying hedge ratios from the two specifications demonstrate similar movements around its mean. In the case of soybeans (Figure 3), the constant hedge ratio from OLS is 0.897. The mean and standard deviation of time-varying hedge ratios of the diagonal vech BGARCH model are 0.962 and 0.199 over sample period. Unlike the case of corn, for soybeans, this sample mean is quite different from the OLS hedge ratio. In Figure 3, the THR has one substantial drop at the beginning of the period (October - November 1988) and high peak around July 1989. These large changes in the hedge ratios occur near maturity of the harvest contracts (October - November) and during the summer when the crops are most susceptible to the effects of weather.

3. Comparison with Random Coefficient Model

The conventional hedge ratio model is given by

$$\begin{aligned} R_t^s &= \alpha + \beta R_t^f + \epsilon_t \\ \epsilon_t &\sim i.i.d.\ N(0,\sigma^2). \end{aligned} \tag{13}$$

The primary objection to this model is the time invariance of the coefficient β. Previously, we discussed the use of ARCH models to capture the dynamic nature of hedge ratios. The random coefficient (RC) model is another approach to take account of the time-varying nature of hedge ratios and here we explore the possibility of using a RC model as an alternative to ARCH models. In a different context, this type of model has been used before to estimate time-varying coefficients. For example, Bos and Newbold (1984) used a RC model to estimate time-varying systematic risk in the market model framework. A random coefficient autoregressive (RCAR) model is written as

$$\begin{aligned} R_t^s &= \alpha + \beta_t R_t^f + \nu_t \\ (\beta_t - \bar{\beta}) &= \phi(\beta_{t-1} - \bar{\beta}) + \mu_t , \end{aligned} \tag{14}$$

where β_t is a time-varying coefficient, and ν_t and μ_t are independent and identically distributed random variables with means zero and variances σ^2_ν and σ^2_μ respectively, and $|\phi| < 1$. When ϕ = 0, the RCAR model is identical to the standard RC model of Hildreth and Houck (1968). The parameters of the model can be estimated using the nonlinear maximum likelihood procedures described in Pagan (1980). The estimated results are shown in Table 7 for corn and soybeans. All parameter estimates for corn and soybeans are significant except the constant term α.

An important issue here is to test for the constancy of the beta coefficient. When ϕ in the time-varying coefficient model (14) is not equal to zero, then testing $H_0 : \sigma_\mu^2 = 0$ is complicated.

If the beta coefficient is not constant over time, under (14), Var (β_t) = $\sigma_\mu^2/1-\phi^2$. Under the null, $H_0 : \sigma_\mu^2 = 0$, the parameter ϕ is not identified. Therefore, a test can not be conducted using the conventional large sample tests such as the likelihood ratio, Rao's score (RS), or Wald tests because the nuisance parameter, ϕ, is identified only under alternative hypothesis. This phenomenon results in a violation of regularity conditions for the standard testing procedures. For example, in the formulation of the RS test, the information matrix derived from the likelihood function with the restrictions of $\sigma_\mu^2 = 0$ becomes singular. Here, we follow the Davies (1977) approach for testing $\sigma_\mu^2 = 0$. This procedure involves computing the standard RS statistic for a given value of ϕ, say RS(ϕ). Then the test is based on a critical region of the form

$$\{\sup_{\phi\in(-1,1)} RS(\phi) > c\} \ , \qquad (15)$$

where c is a suitably chosen constant. Davies (1987) provided an approximation to the p-value of the test [for a description of the computation of the p-value, see Bera and Higgins (1992)]. The supremum of the RS (ϕ) for corn and soybeans are reached at ϕ = - 0.76 and 0.28, respectively. It is interesting to note that the corresponding maximum likelihood estimate of ϕ in Table 7 are -0.758 and 0.280. The p-values of the Davies test are very close to zero both for corn and soybeans. Therefore, the null hypothesis of constant coefficient model is rejected for both corn and soybeans.

The test of a RC model against the alternative of RCAR model is carried out with a LR test under the following testable hypothesis,

$$\begin{aligned} H_0 &: \phi = 0 \\ H_a &: \phi \neq 0 \ . \end{aligned} \qquad (16)$$

The calculated LR statistics are 15.699 and 4.169 for corn and soybeans respectively. Thus, the null hypothesis, RC model, is rejected, although for soybeans the rejection is only at 5% significance level.

These test results indicate that the RCAR model provides a good representation of the data set. In order to trace time-variant hedge ratios, i.e., β_t, we employ a fixed interval smoothing process using the following conditional means and variances of β_t over time period, t = 1,2,...,T [see Newbold and Bos (1985, p. 39)]

$$\begin{aligned} &= E[\beta_t | R_{s1}...R_{sT};\ R_{f1}...R_{fT}] \\ &= \beta\ (t|T) \\ &= \beta(t|t) + A_t[\beta(t+1|T) - \beta(t+1|t)] \end{aligned} \quad (17)$$

$$\begin{aligned} &= Var\ [\beta_t | R_{s1}...R_{sT};\ R_{f1}...R_{fT}] \\ &= P(t|T) \\ &= P(t|t) - A_t[P(t+1|t) - P(t+1|T)]A_t \\ \textit{where}\ A_t &= P(t|t)\phi P(t+1|t)^{-1}\ . \end{aligned} \quad (18)$$

The smoothing process begins by setting t = T-1 in (17) and (18), thus obtaining the mean and covariance of the conditional distribution of β_{T-1}. The maximum likelihood estimates, β, ϕ, σ_μ, σ^2, presented in Table 7, are used for the fixed coefficient in the estimation of stochastic parameter over sample period. The plots of β_t, time-varying hedge ratios for corn and soybeans, are given in Figures 4 and 5, respectively. For corn, there is some similarity between the plots of THR using RCAR model and those reported in Figures 2 and 3 using the BGARCH model; although in Figure 4 we observe more fluctuation. For soybeans, quite interestingly, the behavior THR obtained from BGARCH and RCAR models, as can be seen from Figures 3 and 5, are very close.

4. Hedging Performance of BGARCH and RCAR Models

The results of the diagnostic tests for that BGARCH and RCAR models led us to identify alternative specifications for estimating the time-varying hedge ratios. However, the hedging performance of the models still remains a practical question. We now investigate the in- and out-of-sample hedging performance of the BGARCH and RCAR models. Under the assumption that the time-varying conditional variance and covariance estimated from the diagonal vech model is the generating process, the percentage reduction in the conditional variance of the returns portfolio relative to the no hedging scenario is used to evaluate hedging performance. In-sample results are based on the 311 observations used in the estimation process, while the out-of-sample hedging performance is assessed over the next 60 observations.

The results of the in- and out-of-sample hedging performance are reported in Table 8. For the corn, using the BGARCH models, the in-sample hedge performance is very similar, with an average variance reduction of 79 percent. For soybeans, the BGARCH representations also produce similar reductions in the variance of returns, except for the positive definitive parameterization. The diagonal vech specification, which is somewhat a better model for our data, provides the largest variance reduction, 79.42 and 77.00 percent reductions for corn and soybeans, respectively. For both corn and soybeans, the RCAR model performs rather poorly, reducing the corn and soybean variances by 73.70 and 73.23 percent, respectively, which is inferior even to the constant hedging (OLS) procedure.

Out-of-sample results verify the attractiveness of the diagonal vech parameterization; its use leads to the largest variance reductions for both commodities. The other BGARCH models and the RCAR model perform very poorly; particularly the poor out-of-sample performance of the positive definite model is quite unexpected.

Overall, the results indicate the usefulness of appropriately specified BGARCH models for establishing hedge ratios. These findings are consistent with Baillie and Myers (1991) but somewhat different from Myers (1991) who encountered little difference in hedging performance between time-varying and other procedures. Clearly, the disparate findings may be a result of differences in the commodities and data, as well as the specification procedures employed here. The results associated with the RCAR model framework caution against the use of a procedure that does not directly specify and estimate the changing nature of variance matrix which is fundamental to the measurement of time-varying hedge ratios.

While the BGARCH model shows potential for improved risk management, further study is needed to assess the costs of its specification and implementation relative to the gains in variance reduction. The implementation of the BGARCH framework can require frequent and costly position changes in the futures market. In addition, the estimation and continual updating of models for practical use can be time-consuming and costly. Assessment of the hedging performance of these models in framework which explicitly incorporates these costs will provide a more complete understanding of the usefulness of time-varying procedures for managing price risk.

5. Conclusion and Further Research

Various BGARCH and RCAR models were applied to estimate time-varying hedge ratios. This remedies the constancy of hedge ratios based on conventional OLS estimation and permits the hedge ratios to be based on conditional information. After rigorous tests of model specification, the diagonal vech parameterization was found to be appropriate for both corn and soybeans. The BGARCH hedge ratios performed better at reducing the variance of the portfolio return for corn and soybeans. A diagonal vech parameterization provided the largest reduction in the variance of the portfolio return. This is consistent with the results of diagnostic and specification test for the representation of conditional second moment. For both corn and soybeans, the constancy hypothesis of the hedge ratio from OLS was rejected against random coefficient autoregressive (RCAR) model. However, the RCAR model performed poorly at reducing the variance of returns, perhaps, suggesting that this procedure may not be appropriate in the presence of conditional heteroskedasticity. For our data Q^2 statistics provide a strong indication of conditional heteroskedasticity.

There are a number of issues that require further attention. In particular, the relationship between the BGARCH and random coefficient models is an interesting issue. For our data set, the performance of RCAR model is not very encouraging. However, from a theoretical point of view, it is a viable alternative to the BGARCH model for estimating time-varying hedge ratios, and perhaps would work better for other data sets. The fundamental difference between these two models is that the BGARCH models consider the joint conditional distribution of the spot and future prices while RC model specifies a conditional mean model for the spot price given the futures price. In the RCAR model the time varying nature of the model is incorporated through the mean equation and in BGARCH models, this is achieved through the variance specification. It would be interesting to test BGARCH and RCAR models against each other directly using Cox's non-nested hypotheses testing procedures. Finally, from the behavior of the standardized residuals, we noted that the BGARCH models under the assumption of conditional normality does take account of most of the high degree of kurtosis in the data. However, some kurtosis still remains unexplained. Therefore, use of a bivariate conditional t-distribution would seem to be worthwhile.

Table 1. Phillips-Perron Test for Unit Roots of the Return Series

	Corn		Soybeans	
	Cash	Futures	Cash	Futures
$Z(t^*_\alpha)$	-18.30	-17.39	-19.60	-15.92
$Z(\Phi_1)$	167.98	151.82	244.14	91.85
$Z(t_{\tilde{\alpha}})$	-18.77	-17.94	-19.37	-17.22
$Z(\Phi_2)$	131.70	129.86	216.40	115.92
$Z(\Phi_3)$	111.64	101.16	210.76	43.07

Note: Critical Values	$Z(t^*_\alpha)$	$Z(\Phi_1)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_2)$	$Z(\Phi_3)$
1%	-3.43	6.43	-3.96	8.27	6.09
5%	-2.86	4.59	-3.66	6.25	4.68

The tests are based on the following regressions,

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t$$

$$y_t = \tilde{\mu} + \tilde{\beta}(t - T/2) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$Z(t^*_\alpha)$ test for H_0: $\alpha^* = 1$
$Z(\Phi_1)$ test for H_0: $\mu^* = 0, \alpha^* = 1$
$Z(t_{\tilde{\alpha}})$ test for H_0: $\tilde{\alpha} = 1$
$Z(\Phi_2)$ test for H_0: $\tilde{\mu} = 0, \tilde{\alpha} = 1, \tilde{\beta} = 0$
$Z(\Phi_3)$ test for H_0: $\tilde{\alpha} = 1, \tilde{\beta} = 0$

Table 2. Sample Statistics of the Return Series -- January 1989 - December 1990

Variable	Mean	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
Corn						
Spot	-.56	13.23	-.51	4.41	-56.05	37.18
Futures	-.73	12.50	-.25	4.84	-50.06	43.34
Soybeans						
Spot	-1.14	14.39	-.61	4.22	-61.70	36.29
Futures	-1.04	13.41	-.37	4.04	-46.80	43.68

Futures Contracts: Corn - May, July, December
Soybean - March, July, November

Table 3. Test Results of Autocorrelation and Bivariate Normality for the Return Series

	Corn		Soybean	
	Spot	Futures	Spot	Futures
$Q(12)$	10.84	14.91	19.83	10.25
$Q^2(12)$	35.04	52.41	39.30	35.45
Bivariate Normality				
T_1	-0.38	-0.23	-0.41	0.83
T_{11}	4.40	9.38	3.87	11.75
C_3	563.16		1047.75	

T_1 : Skewness
T_{11} : Kurtosis
C_3 : Bera-John statistic,
Critical Values of $\chi^2(p)$ are:

	1%	5%
p = 4	13.277	9.488
p = 12	26.217	21.026

Table 4. Estimation Results of Alternative Models for Corn

		BGARCH		
	Constant Conditional Covariance	Constant Correlation	Diagonal Vech	Positive Definite
C_{ss}	176.257 (10.985)	23.936 (6.808)	68.418 (16.299)	29.921 (13.936)
C_{ff}	168.182 (11.106)	14.245 (3.323)	37.956 (10.575)	11.129 (4.206)
C_{fs}	153.328 (3.001)		47.854 (12.531)	12.091 (6.705)
γ_{ss}		0.136 (0.044)	0.197 (0.053)	0.770 (0.125)
γ_{sf}			0.145 (0.430)	0.160 (0.076)
γ_{fs}				-1.027 (0.151)
γ_{ff}		0.121 (0.019)	0.132 (0.039)	-0.479 (0.101)
δ_{ss}		0.738 (0.064)	0.454 (0.102)	0.341 (0.212)
δ_{sf}			0.561 (0.095)	-0.012 (0.165)
δ_{fs}				0.507 (0.206)
δ_{ff}		0.790 (0.022)	0.641 (0.079)	0.918 (0.153)
ρ_{sf}		0.887 (0.011)		
Log Likelihood	-2233.55	-2209.02	-2193.17	-2175.02
AIC		4432.04	4404.34	4372.04
BIC		4458.22	4437.99	4413.06
LR Statistics		49.08 [4]	80.76 [6]	117.06 [8]

The numbers in parentheses are standard errors. The AIC and BIC are the Akaike Information and the Bayesian Information Criteria. The LR Statistics test the constant conditional covariance model (8) against the respective BGARCH formulation. The numbers in brackets are the degrees of freedom, p. At the 1% level, the critical χ^2 [p] are: 13.24 [4], 15.81 [6], and 20.09 [8].

Table 5. Estimation Results of Alternative Models for Soybeans

		BGARCH		
	Constant Conditional Covariance	Constant Correlation	Diagonal Vech	Positive Definite
C_{ss}	206.418 (13.691)	43.937 (15.341)	58.103 (10.183)	50.222 (20.064)
C_{ff}	180.204 (11.547)	31.477 (11.664)	37.862 (7.079)	47.140 (25.029)
C_{fs}	161.755 (5.073)		45.811 (7.836)	48.176 (4.778)
γ_{ss}		0.200 (0.037)	0.345 (0.043)	0.741 (0.119)
γ_{sf}			0.314 (0.043)	-0.001 (0.119)
γ_{fs}				-0.591 (0.128)
γ_{ff}		0.133 (0.039)	0.304 (0.048)	0.239 (0.128)
δ_{ss}		0.605 (0.090)	0.485 (0.048)	0.595 (0.933)
δ_{sf}			0.543 (0.040)	-0.142 (0.099)
δ_{fs}				0.239 (0.079)
δ_{ff}		0.703 (0.079)	0.589 (0.043)	0.951 (0.686)
ρ_{sf}		0.857 (0.010)		
Log Likelihood	-2322.59	-2294.14	-2181.97	-2166.10
AIC		4602.28	4381.94	4354.20
BIC		4628.46	4415.60	4395.34
LR Statistics		56.90 [4]	281.24 [6]	312.98 [8]

The numbers in parentheses are standard errors. The AIC and BIC are the Akaike Information and the Bayesian Information Criteria. The LR Statistics test the constant conditional covariance model (8) against the respective BGARCH formulation. The numbers in brackets are the degrees of freedom, p. At the 1% level, the critical χ^2 [p] are: 13.28 [4], 16.81 [6], and 20.09 [8].

Table 6. Autocorrelation, Heteroskedasticity, and Conditional Normality Test Results for the Corn and Soybean BGARCH Models

	Corn			Soybean		
	Constant Correlation	Diagonal Vech	Positive Definite	Constant Correlation	Diagonal Vech	Positive Definite
Q(12)						
ϵ_{st}	9.15	19.08	10.84	17.92	12.31	20.33
ϵ_{ft}	14.19	11.28	14.91	10.72	8.44	11.32
$Q^2(12)$						
ϵ^2_{st}	11.57	7.19	35.04	6.59	12.37	39.60
ϵ^2_{ft}	11.14	19.44	52.41	11.30	29.73	33.32
Conditional Bivariate Normality						
T_s	0.17	0.05	0.24	-0.06	0.06	-0.03
T_f	-0.63	-0.24	-0.28	-0.60	-0.45	-0.13
T_{ss}	6.23	5.54	4.35	6.99	5.11	3.94
T_{ff}	5.82	3.19	3.54	4.21	3.67	3.71
C_3	261.16	87.61	34.44	243.21	74.37	18.88

T_s and T_f measure skewness of the standardized residuals, respectively for spot and future BGARCH models. T_{ss} and T_{ff} measure kurtosis. C3 is the Bera-John statistic, χ^2 [4], where the critical values at the 1% and 5% levels are 13.28 and 9.49, respectively.

Table 7. Estimation Results of the Ordinary Least Squares and Random Coefficient Autoregressive Corn and Soybean Models

	Corn	Soybeans
OLS	$R_{st} = 0.126 + 0.931\ R_{ft}$	$R_{st} = -0.128 + 0.897\ R_{ft}$
	(0.357) (0.029)	(0.446) (0.029)
	$R^2 = 0.774$, D.W. = 2.26	$R^2 = 0.703$, D.W. = 1.63
	$\sigma^2 = 39.413$	$\sigma^2 = 61.018$
	log likelihood = -1012.61	log likelihood = -1080.58
RCAR model	$R_{st} = 0.205 + \beta_t\ R_{ft}$	$R_{st} = -0.057 + \beta_t\ R_{ft}$
	(0.319)	(0.381)
	$\beta_t - 0.954 = -0.758\ (\beta_{t-1} - 0.954)$	$\beta_t - 0.934 = 0.280\ (\beta_{t-1} - 0.934)$
	(0.029) (0.123) (0.029)	(0.050) (0.134) (0.050)
	$\sigma^2 = 27.283$	$\sigma^2 = 30.070$
	(2.705)	(3.860)
	$\sigma_\mu = 0.025$	$\sigma_\mu = 0.197$
	(0.015)	(0.044)
	log likelihood = -985.393	log likelihood = -1058.99

The standard errors are given in parentheses.

Table 8. Performance Alternative Hedging Models Compared to No Hedging

	Average Percentage Variance Reduction from the No Hedge Position			
	Corn		Soybean	
	In-sample	Out-of-Sample	In-Sample	Out-of-Sample
Constant Hedging (Bivariate Conditional)	-77.42	-66.47	-73.97	-80.84
Constant Hedging (OLS)	-78.35	-67.70	-74.23	-80.80
BGARCH (Constant Correlation)	-78.75	-43.13	-73.90	-42.91
BGARCH (Diagonal Vech)	-79.42	-69.61	-77.00	-85.69
BGARCH (Positive Definite)	-78.92	-0.87	-65.17	2.76
RCAR model	-73.70	-68.51	-73.23	-17.07

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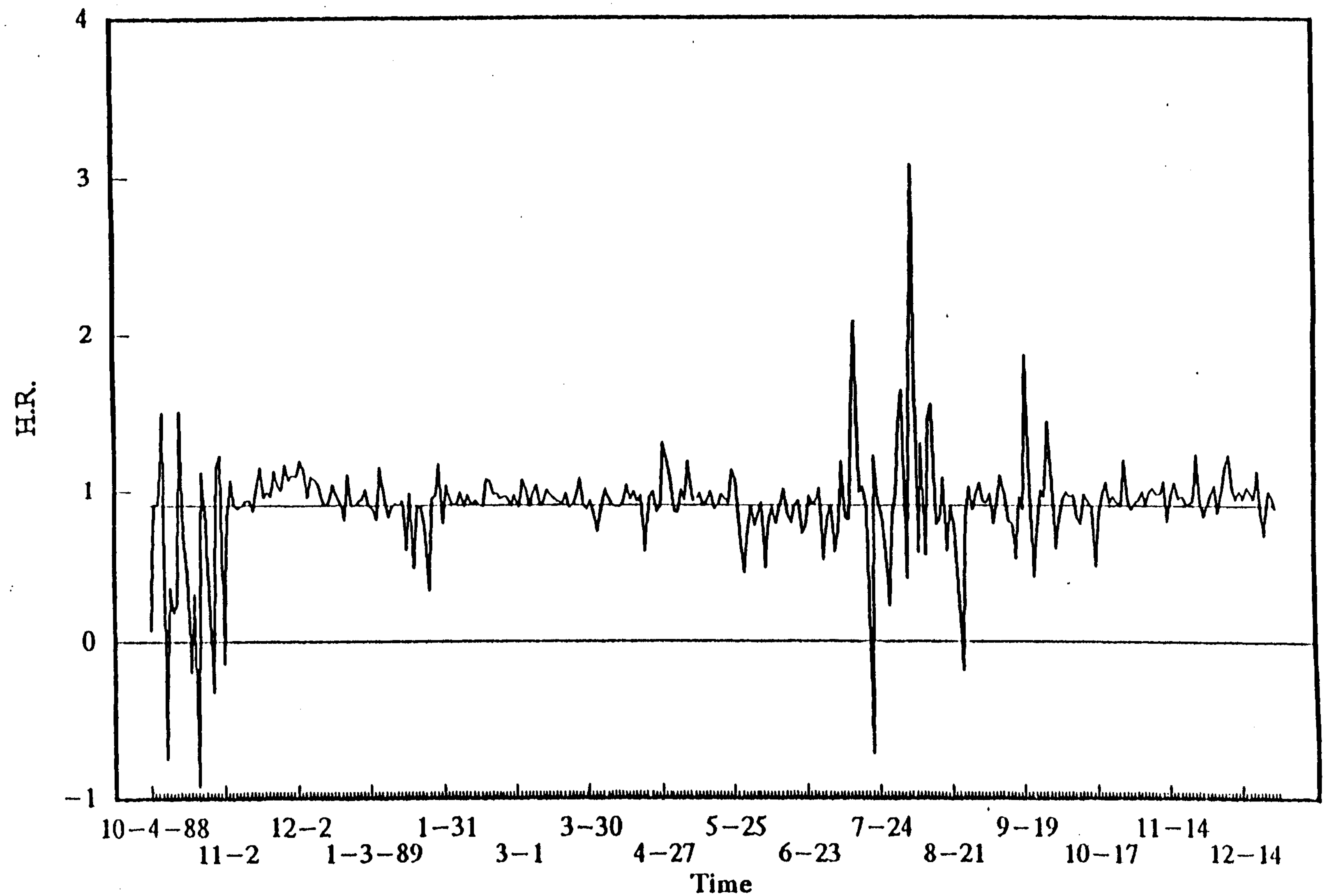

Figure 5. RCAR and Constant Hedge Ratios for Soybeans

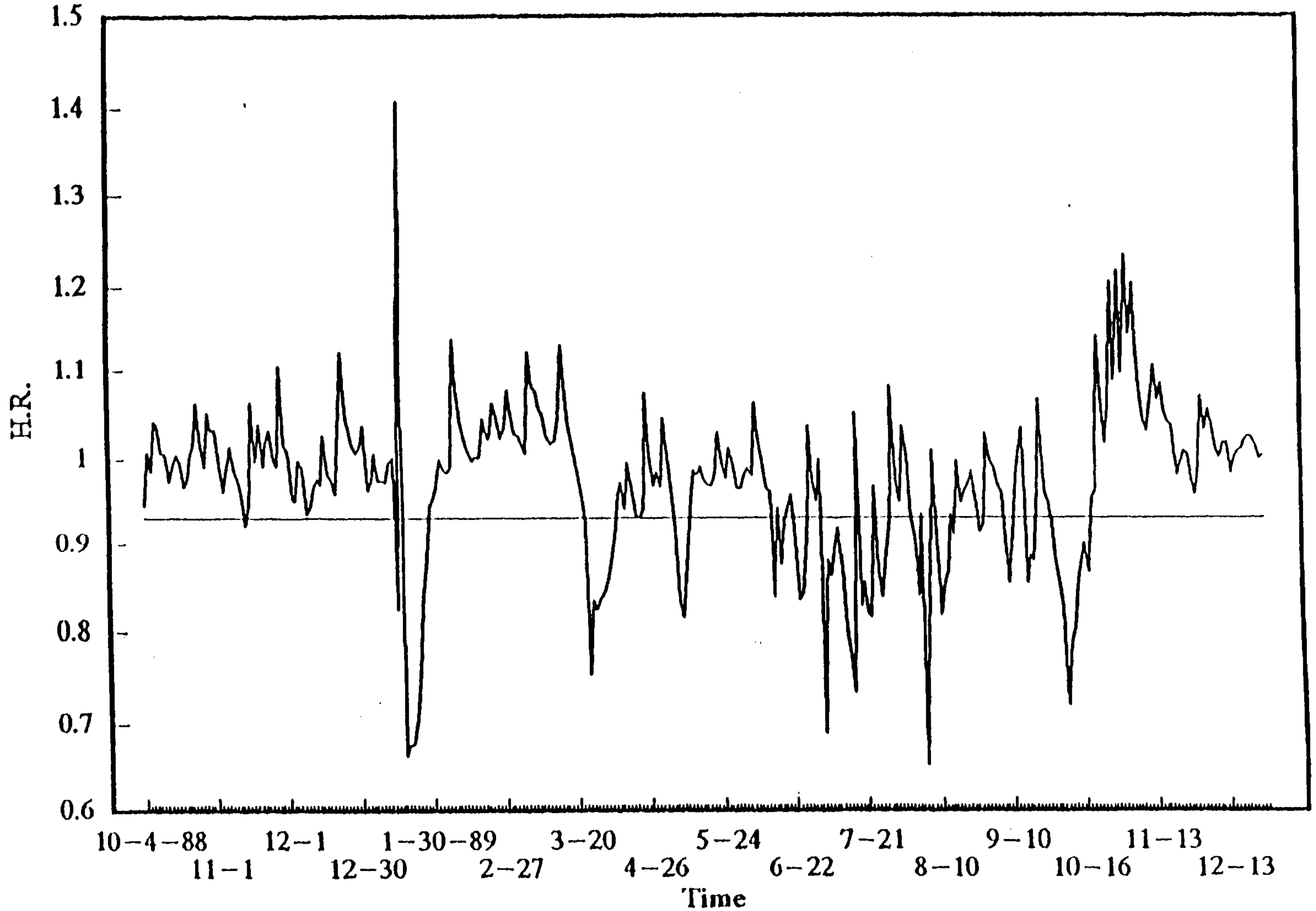

Figure 1. Constant Correlation BGARCH and Constant Hedge Ratios for Corn

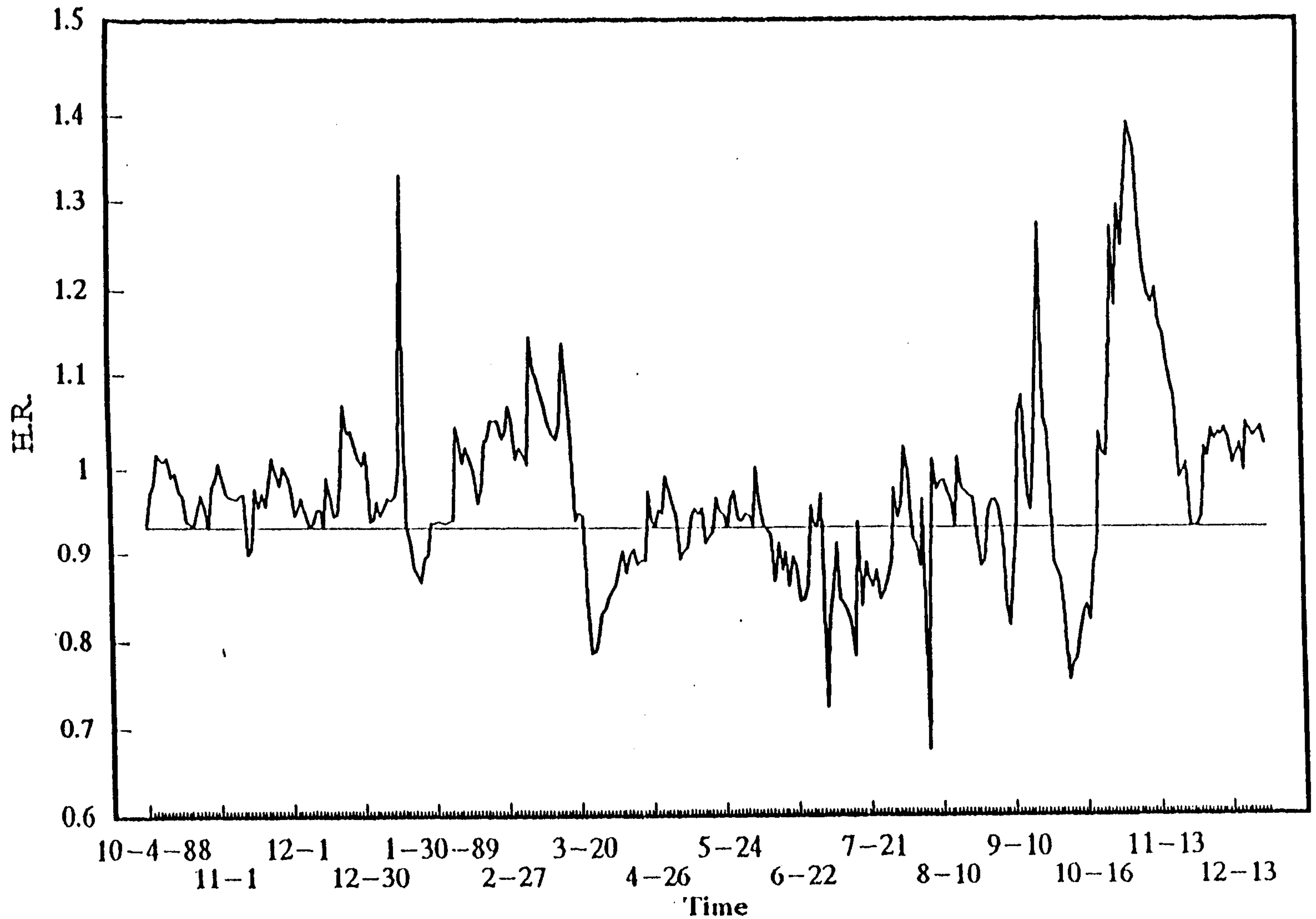

Figure 2. Diagonal Vech BGARCH and Constant Hedge Ratios for Corn

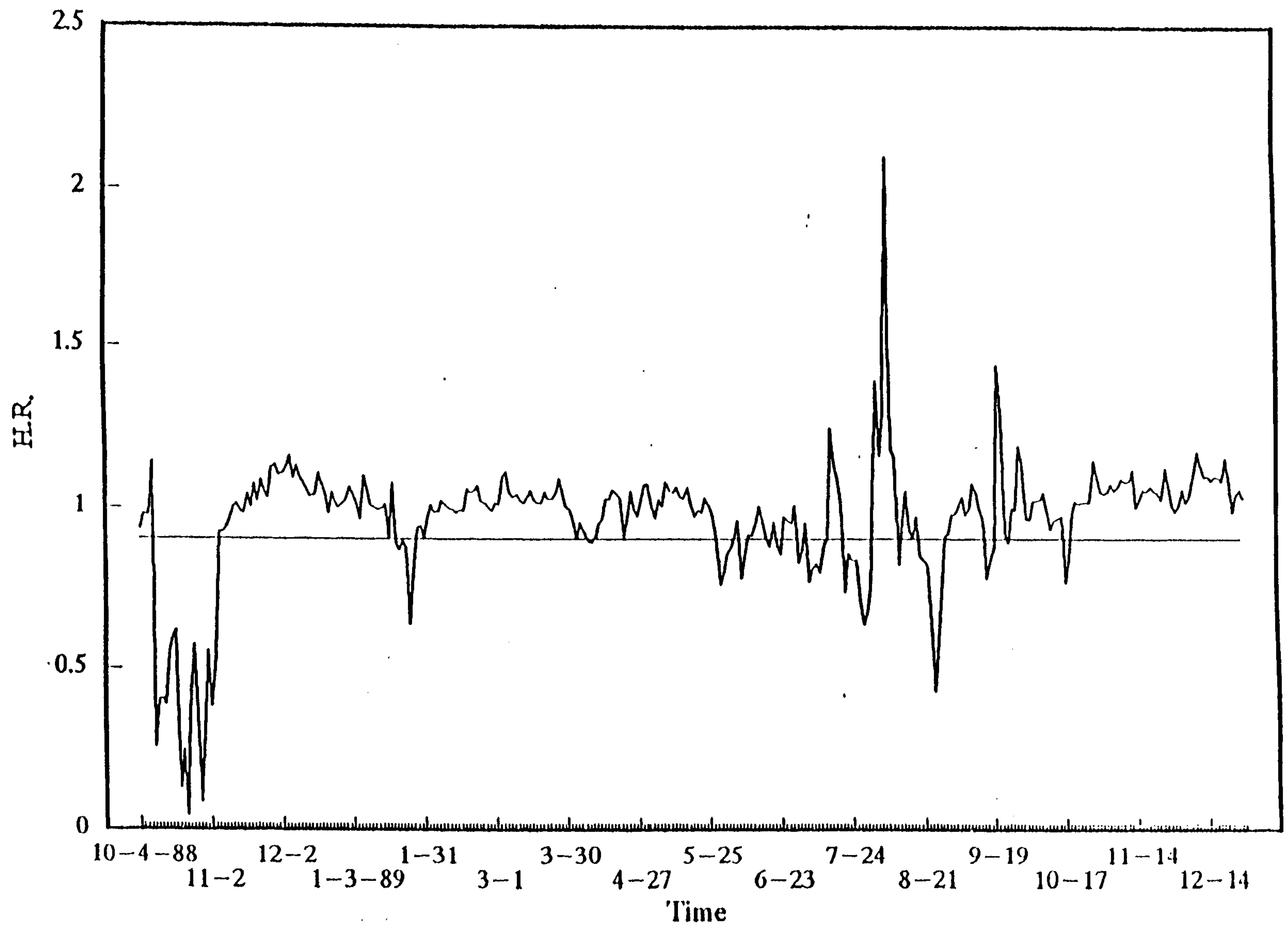

Figure 3. Diagonal Vech BGARCH and Constant Hedge Ratios for Soybeans

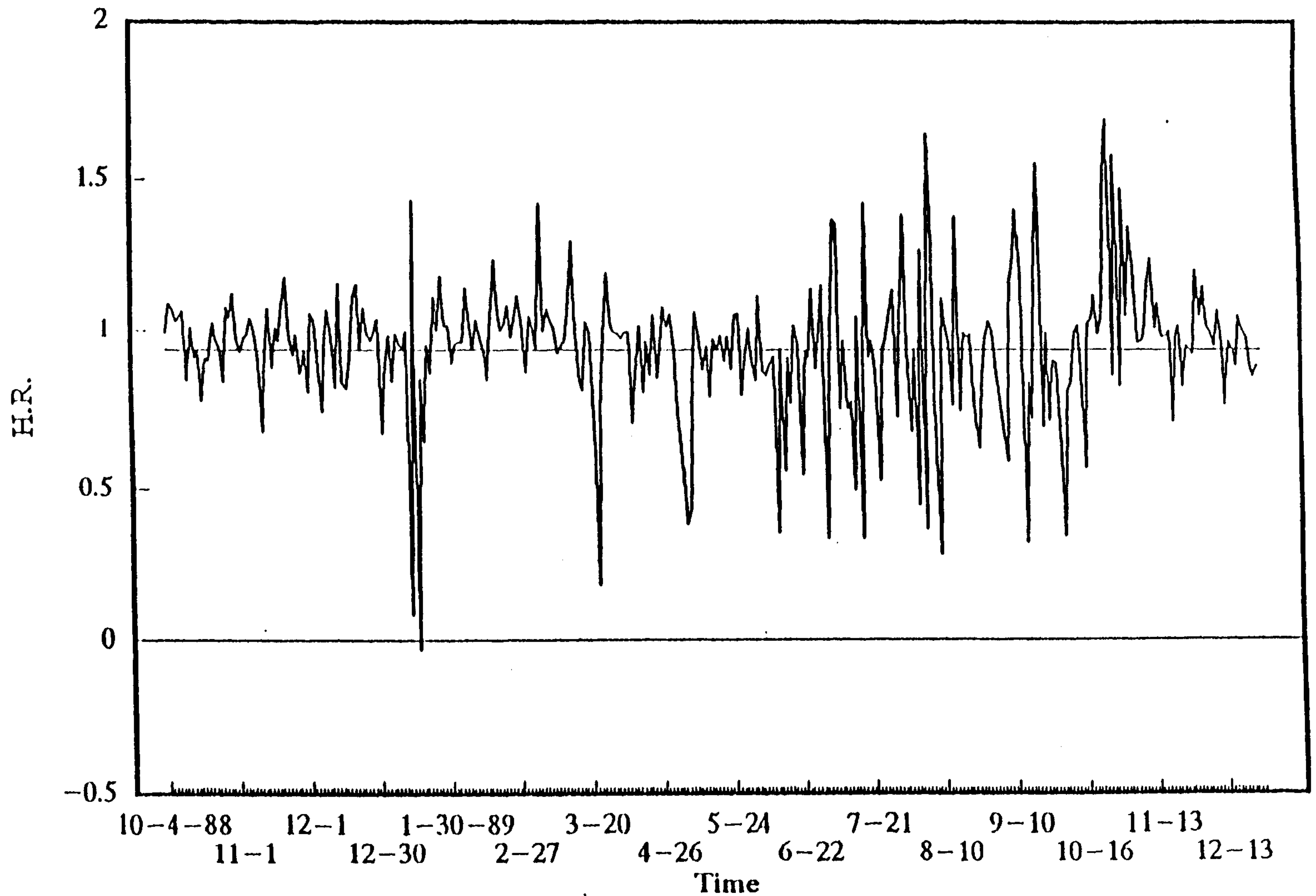

Figure 4. RCAR and Constant Hedge Ratios for Corn