PRODUCTION NOTE

University of Illinois at
Urbana-Champaign Library
THE STRENGTH OF CHAIN LINKS

BY

G. A. GOODENOUGH
AND
L. E. MOORE
THE Engineering Experiment Station was established by action of the Board of Trustees December 8, 1903. It is the purpose of the Station to carry on investigations along various lines of engineering, and to study problems of importance to professional engineers and to the manufacturing, railway, mining, constructional and industrial interests of the state.

The control of the Engineering Experiment Station is vested in the heads of the several departments of the College of Engineering. These constitute the Station Staff, and with the Director determine the character of the investigations to be undertaken. The work is carried on under the supervision of the Staff; sometimes by a research fellow as graduate work, sometimes by a member of the instructional force of the College of Engineering, but more frequently by an investigator belonging to the Station corps.

The results of these investigations will be published in the form of bulletins, and will record mostly the experiments of the Station's own staff of investigators. There will also be issued from time to time in the form of circulars, compilations giving the results of the experiments of engineers, industrial works, technical institutions and governmental testing departments.

The volume and number at the top of the title page of the cover are merely arbitrary numbers and refer to the general publications of the University of Illinois; above the title is given the number of the Engineering Experiment Station bulletin or circular, which should be used in referring to these publications.

For copies of bulletins, circulars or other information, address the Engineering Experiment Station, Urbana, Illinois.

Issued March, 1908
THE STRENGTH OF CHAIN LINKS

BY

G. A. GOODENOUGH

AND

L. E. MOORE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
PUBLISHED BY THE UNIVERSITY
This page is intentionally blank.
The chain is one of the most familiar as well as one of the most useful of mechanical devices. It is universally employed in hoisting and transmission, and for attaching and securing movable bodies, as, for example, in anchoring ships. As a rule, a chain is subjected to heavy loads and must transmit large forces, and upon its ability to withstand the stresses to which it is subjected by its loading may depend the success of a great mechanical operation, or even the safety of lives.

In view of these facts, it is surprising that the chain has received scant attention from investigators in the field of elasticity and strength of materials. Aside from two or three scattered memoirs, the theory of the stresses in chain links has been untouched. Experiments have been made, it is true, but these have been for the purpose of determining the ultimate strength of the chain, not for the purpose of testing a theory. Formulas for the loading of chains have been based upon the ultimate strength of the chain when tested to destruction and are thus purely empirical. No attempt seems to have been made to place such formulas on a rational basis supported by theory. It may be urged that the present empirical rules are satisfactory, inasmuch as they lead to satisfactory results. As a matter of fact, the results are not satisfactory; chains break, often with disastrous consequences, and the only reason that more do not
break is that a chain is seldom subjected to its rated load. Further arguments to show the importance of a rational analysis seem hardly necessary.

In undertaking the work described in this paper, three things were held in view.

1. The development of the theory of the stresses induced in chain links with given conditions as regards loading.
2. Experimental tests of the validity of the theory employed and also of the validity of the assumptions made as to the distribution of pressure between adjacent links.
3. The deduction from theoretical considerations alone of rational formulas for the loading of chains.

The beginning of this work dates back to 1900, when the analytical investigations were largely worked out. In 1906, Mr. R. M. Evans of the class of 1906 undertook the experimental verification of the theory, and presented in his graduating thesis certain of the results contained herein. The following year the experimental work was continued by Messrs. M. L. Millspaugh and R. L. Baker. The data obtained have been worked over carefully, all calculations have been repeatedly checked, and it is believed that the results derived are worthy of confidence, whatever may be the conclusions that are drawn from them.

**Method of Analysis**

The analytical investigation was first suggested by Bach's analysis of the stresses in a hollow cylindrical roller.\(^1\) It seemed evident that the general method there used could be employed to determine the stresses in links with circular or elliptical center lines. The fundamental equations may be found in Bach's work, but for the sake of completeness they are given in condensed form in this paper. (See Appendix A.) Grashof\(^2\) gives an analysis using the same fundamental equations, but owing to untenable assumptions, the analysis gives results wide of the truth. The only other analysis is that made by Winkler in a memoir published in Der Civilingenieur.\(^3\)

\(^1\) Bach, *Elasticitat und Festigkeit*, p. 458.


discussion of this memoir in which some of the results have been corrected is given by Professor Karl Pearson. To show Pearson's estimate of Winkler's work the following paragraphs from the introduction of the discussion are quoted:

"This is an important memoir both from the theoretical and practical standpoint; although many of its results require correction and modification. Some of these corrections have been made in Kapitel XL (Ringformige Körper) of the author's well known treatise: Die Lehre von der Elasticität und Festigkeit, Prag, 1867, but this treatise does not cover anything like the same area as the memoir. I propose therefore to indicate the correct analysis and compare its results with those of Winkler.

"The importance of the subject will be sufficiently grasped when I remind the reader that it is the only existing theory of the strength of the links of chains. To investigate the strength of such links by the complete theory of elasticity would involve even for the case of anchor rings an appalling investigation in toroidal and allied functions; while for the oval chain links with studs in ordinary use, any successful attempt at a general investigation seems inconceivable. We shall have the less hesitation, however, in applying the Bernoulli-Eulerian theory, if we remember how close an approximation Saint-Venant's researches on flexure have shown it to be in the case of straight bars. At the same time we are certainly going to put it to the very limit of its application, namely, to curved bars in which the dimensions of the cross sections are not very small as compared with either the length or the radius of curvature of the central axis." . . .

"Remembering that we need not assume adjacent cross sections of our link to remain undistorted, if we only suppose them to be approximately equally distorted, we can easily investigate an expression for the stretch at any point by a method akin to that which results from the Bernoulli-Eulerian theory."

The method here referred to is that given by Bach and Grashof for the analysis of bars with curved axes. An outline of it, as already stated, is given in Appendix A.

While the method employed in the investigations herein described

---

is essentially the same as that of Winkler and Pearson, there are one or two important points of difference in the assumptions made. Professor Pearson considers only two cases, the link with elliptical center line, and the link made up of two circular arcs and two straight lines. The analysis here given is extended to links of four and six circular arcs so as to approximate as closely as possible to the forms actually occurring; it is also extended to links with studs. Furthermore, it appears that in all cases Winkler assumed the pressure between adjacent links to be concentrated at a point at the end of the link. The present analysis assumes a distribution of pressure over a definite area. As will be shown later, this question of distribution has an important bearing upon the results obtained.

The complete analysis of the open link is given in Appendix B. The following is merely a brief outline of the method of attacking the problem. Consider one quadrant of the link as shown in Fig. 1. Denoting by 2Q the load on the link, the section at A lying along the minor axis will be subjected to a normal force Q. There will also be at this section a bending moment M, which can be determined from the conditions of the problem. Now assume any other normal section, as C, and consider the part of the link between sections A and C a free body. At C let two forces, each equal to Q but opposite in sense, be added to the system. One of these forces with the force Q at section A forms a couple whose moment is Qh; the other force is resolved into components, one Q cos \(\phi\) along the section, the other Q sin \(\phi\) normal to the section. The component Q cos \(\phi\) produces shearing stress and is neglected in the subsequent discussion. At the section C we have therefore:

- a normal force, \(P = Q \sin \phi\);
- a bending moment, \(M_b = Qh + M\).

The unknown moment M is now found from considerations explained in the analysis; and with P and \(M_b\) fully known, the intensity
of stress at any fiber is readily determined from the fundamental equation (C), Appendix A. It may be noted that instead of (C), the usual formula

\[ S = \frac{P}{f} + \frac{M}{I} \]

may be used, though the results may not be quite exact.

**Pressure Between Adjacent Links**

At the very beginning of the analysis arises a question as to the way in which the pressure between adjacent links is distributed. The analysis is somewhat simplified by assuming that two links have contact at one point only and that in consequence the pressure between them is concentrated at this point. [See Fig. 2(a)]. As a matter of fact, however, the links after a little wear have contact over a considerable surface and the pressure between them must be distributed in some way or other over this surface.

Referring to Fig. 2(b), suppose that contact exists over the arc EE, which subtends the angle \(2\alpha\) at the center \(O\). Though the parts of the link in contact are curved, the action of one link on another may be likened to that of a journal and bearing. We may assume (1) that the pressure is uniformly distributed along the arc EE, or if we make use of the more exact analysis of journal and bearing, we may assume (2) that the intensity is greatest at \(H\) and decreases towards \(E\), being at any point proportional to the cosine of the angle made with the axis \(XX\). Because angle \(\alpha\) is small, the second assumption changes but little the results obtained by using

![Fig. 2](image-url)
the first; hence we shall consider only the assumption of uniform
distribution.

A third possible distribution is represented in Fig. 2(c). Under
heavy load the link suffers a considerable distortion and the sides $e$
and $f$ approach each other. Now if the distributed pressure along
$EE$, Fig. 2(b), were in the nature of a fluid pressure so that the
points of application of the forces could move as the points $EE$ moved,
the law of distribution would be unchanged by the distortion of the
link. But the part $m$ of the adjacent link lying between the sides $e$
and $f$ is practically unyielding; hence when $e$ and $f$ approach each
other the part $m$ is pinched and there ensues a new distribution of
pressure. Evidently the result of this pinching action is to increase
the intensity of pressure near $E$, $E$ and to decrease it at $H$. We
cannot, of course, know the precise effect of the action just described.
For the sake of comparison with the other cases, we may assume,
however, that the effect is equivalent to concentrating the pressure
at the two points $E$, $E$.

In the subsequent analysis we shall make the three assumptions
just stated, namely:

1. Pressure concentrated at single point $H$, Fig. 2(a).
2. Pressure uniformly distributed over arc $EE$, Fig. 2(b).
3. Pressure concentrated at points $E$, $E$, Fig. 2(c).

As a matter of interest, we may in passing call attention to Gras-
hof's analysis. The links are supposed to be in contact along an
arc $EE$ subtending the angle $2\alpha$, as in Fig. 2(c). It is then assumed
that the part of the link lying between the sections $E$, $E$ takes no part
in the straining action, but acts as a rigid base or foundation to which
are attached the sides $e$ and $f$. As will be shown later, this neglected
part of the link plays a most important rôle, and Grashof's assump-
tion is anything but justified.

Experimental Verification of Analysis

Referring to Fig. 1, $OA$ and $OB$ denote respectively the semi-
minor and semi-major axes of the link. Under a load these axes
change, $OA$ becomes shorter and $OB$ longer, and these changes can
be measured with reasonable accuracy. Now the theoretical analysis
here employed furnishes a means of calculating the change of position of any point, as, for example, the point $A$, on the center line of the link. Thus for a given load, the new position $A'$ to which $A$ will move can be found. Evidently the component of $AA'$ in the direction of $AO$ is the change in $OA$, that is, one-half the change in the length of the minor axis; likewise, the component of $AA'$ in the direction of $OB$ is one-half the change in the major axis.

We have here a means of verifying theory by experiment. The changes of length of the axes of the link for given loads can be cal-

![Diagram of Dredge Chain](image)

**FIG. 3.—DREDGE CHAIN.**

Dimensions:

- $d = 1.000$ in.
- $b = 1.178$ in.
- $i = 1.161$ in.
- $r_2 = 1.173$ in.
- $a = 21^\circ$
- $a = 1.875$ in.
- $h = 1.240$ in.
- $c = 0.000$ in.
- $r_3 = 5.000$ in.
- $\beta = 79^915'$

culated from purely theoretical considerations. The actual changes for those loads can be measured. A comparison of the calculated and actual values of the changes of length affords therefore a delicate test of the theoretical analysis.

Because of the doubt regarding the distribution of pressure between adjacent links it was considered advisable to use circular rings of rectangular cross section. With these rings a true knife-edge bearing was possible, and the general theory (Appendix A) could be tested without danger of introducing unknown factors resulting from the
pressure distribution. The experiments on the rings are to be considered, therefore, as more reliable than the link tests in establishing the truth or falsity of the analysis. The tests of the actual chain links are, however, valuable in two ways: (1) They may be used to establish more firmly the analysis when applied to oval links; (2) Assuming that the ring experiments sufficiently establish the analysis, the link experiments may be used to test the assumptions made as to the distribution between adjacent links.

As already stated, the experiments were extended over a period of
two years and were performed as thesis work in the Laboratory of Applied Mechanics of the University of Illinois by senior students in the College of Engineering. The experiments of the first year (1906) made by R. M. Evans were wholly on chain links. Those of the second year (1907) made by Messrs. Baker and Millspaugh were partly on heavy chain links and partly on finished steel rings. These three men deserve great credit for the amount and the character of the work done and for their untiring efforts to do the work as well and accurately as possible.

It has not seemed necessary or desirable to differentiate in these pages between the tests made in the different years, as the objects of the tests and methods used were the same. The chain links tested were ordinary commercial links bought in the market. The test pieces for determining the modulus of elasticity of the material were ordered cut from the same bar from which the chains were made. The links with dimensions are shown in Fig. 3, 4, 5 and 6. In making the tests a short piece of chain, consisting of either three or five links,
five links being used whenever the dimensions of the machine would permit, was held in the jaws of the testing machine by a clevis at each end. These clevises were flattened to afford a better grip for the jaws of the machine. This is illustrated in Fig. 7. Deformations were measured by micrometers reading directly to .001 inch, and by interpolation to .0001 inch.

A micrometer having a "rachet contact" which insured practically the same pressure on the points in all measurements was used for measuring the deformations of the transverse or minor axis. To insure measuring between the same points each time small brass buttons were soldered to all the links except the 2-in. dredge chain.
THE STRENGTH OF CHAIN LINKS

FIG. 7.
link at the ends of the minor axis. The measurements were taken over these buttons. On the dredge chain a small spot was polished on the link itself at each end of the axis. The longitudinal deformations were taken between brass contact points screwed to the inside of two transverse bars of \( \frac{3}{4} \)-in. square iron. One of these bars was soldered to the link at each end of the long diameter or major axis. The distance between the contact points was then readily measured by means of an inside micrometer. An electric bell and battery were used in this connection, the bell ringing as soon as contact was established between the points. This device is shown in Fig. 8 and 9.

The dimensions of the rings were 12 in. outside and 9 in. inside diameter by 1 in. thick, as shown in Fig. 10. Two of these rings were cast steel and the third was wrought steel with a perfect weld. The rings were finished all over in a lathe. The method of holding the rings in the machine and applying the load is clearly shown in Fig. 11 and 12. The load was applied to the ring through knife edges. This was done to remove the uncertainty as to the distribution of pressure between the links. By referring to Fig. 11 and 12 it will be noted that the method of loading the rings gives flexibility in all directions and prevents any eccentricity of loading.

The moduli of elasticity of the different materials were determined from test specimens cut, except in the case of the cast steel rings, from the same bars from which the chains and ring were made. The modulus of elasticity of the cast steel was determined from test pieces poured from the same heat as the rings.
The plan followed in testing was to increase the load by such nearly equal increments that from fifteen to twenty readings would have been obtained when the estimated elastic limit was reached. In testing the links the load was increased to a point just beyond the elastic limit, which was indicated by the change in the increment of the deformation. The links were allowed to rest at least twenty-four hours and the second test was then run up to the same maximum load as in the first case, no attention being paid to the possible raising of the elastic limit by this treatment. It may be readily seen that the exact elastic limit is of little importance in this work compared to the modulus of elasticity. So long as the material was not injured to such an extent as to render values of the modulus of elasticity doubtful, the slight exceeding of the elastic limit was of no consequence. In testing the rings care was taken not to exceed the elastic limit.

In all cases save one, two tests of each specimen will be found recorded, and called the "first test" and the "second test" respec-
Fig. 12.
tively. These were not the only tests run. In some cases it was found desirable to run one or two preliminary tests to accustom the students to taking their observations and to get an idea of the behavior of the piece under load.

RESULTS

The results of the tests are given in tabular form on pages 37-44. Tables 6 to 11 inclusive apply to the tests of the circular rings; tables 12 to 18 to the tests of the four chain links shown in Fig. 3, 4, 5 and 6. The headings of the various columns render detailed explanation of the tables unnecessary.

The tabular values plotted to scale are shown in Fig. 13 to 25 inclusive. Fig. 13 to 18 show the results obtained from the circular rings; Fig. 19 to 25 those from the chain links. The first test in each case is denoted by a small circle o, the second test by a filled circle, ●. For the sake of convenience in comparison, the two tests are given in the same figure, but for distinctness, different origins have been used.

It is to be emphasized that the lines appearing in these figures are in all cases theoretical lines calculated from the known dimensions of the ring or link and from the modulus of elasticity experimentally determined. These lines in fact could have been drawn before the deflection tests were made.

DISCUSSION OF EXPERIMENTAL RESULTS

1. Circular Rings. Table 1 gives the calculated deformation for the three circular rings tested.

TABLE 1
DEFLECTION OF CIRCULAR RINGS

<table>
<thead>
<tr>
<th></th>
<th>Change of Length of Diameter per 1000-lb. Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical Diameter</td>
</tr>
<tr>
<td></td>
<td>Inches</td>
</tr>
<tr>
<td>No. 1</td>
<td>.00286</td>
</tr>
<tr>
<td>No. 2</td>
<td>.00294</td>
</tr>
<tr>
<td>No. 3</td>
<td>.00283</td>
</tr>
</tbody>
</table>
Fig. 13.

DEFLECTION IN INCHES - MINOR AXIS

Fig. 14.

DEFLECTION IN INCHES - MINOR AXIS
The values in this table were obtained from equations (H') Appendix C. The following is the calculation for ring No. 1:

Mean radius \( r = 5.25 \) in.

Area of cross section \( f = 1.5605 \) sq. in.

Modulus of elasticity (by experiment), \( 26,200,000 \).

\[ z = 0.006887; \text{ (See Appendix A).} \]

\[ \frac{1}{z} = 145.197; \]

\[ \frac{1}{1 + z} = 0.99315 \]

Substituting in formulas (H'), p. 65,

\[ \Delta x_a = - \left[ \frac{5.25 \times 145.197}{26,200,000 \times 1.5605} \left( \frac{2 \times 0.99315}{3.1416} - 0.7854 \right) \right] Q = -0.0000236 Q, \]

\[ \Delta y_a = - \left[ \frac{5.25 \times 145.197}{26,200,000 \times 1.5605} \left( \frac{2 \times 0.99315}{3.1416} - 0.5 \right) \right] Q = -0.0000247 Q. \]

As is evident from (H'), the curve giving the relation between the change of length of the axis and the applied load is a straight line through the origin. The value in the table gives the slope of this line. In Fig. 13 to 18 these lines have been drawn through the plotted points, and a comparison may be made between the line determined by calculation based on analysis and the points found by experiment.

The lines representing the mean of the experimental values will not, in general, pass through the origin, because of unavoidable errors at the beginning of the test. Hence the theoretical lines are not drawn through the origin, but are drawn with the proper slope in such a position as to permit the comparison to be made most easily. This course is entirely justified by the fact that the slope of the line, rather than its absolute position, is the important factor.

In ring No. 1, it will be seen that the agreement is remarkable; in fact, the calculated line is about as near the mean line of the points as could be drawn. It will be noticed that the points of the second test lie a little more regular in all cases than those of the first test.
Fig. 17.

Fig. 18.
In ring No. 2, the agreement is not quite so close as in No. 1, but still is fairly satisfactory. In the case of ring No. 3, vertical axis, the agreement is good, and it is also good in the second test for the horizontal diameter. In the first test the slope of the actual line seems slightly less than that of the calculated lines. It is possible that the modulus of elasticity as determined for rings 1 and 2 is a
little low. A higher value would make the theoretical lines slightly steeper.

The experiments on the rings, on the whole, seem to confirm in a satisfactory manner the theoretical analysis. We may therefore
THE STRENGTH OF CHAIN LINKS

conclude that the fundamental equations employed will give very closely the true stresses in rings, and that if proper assumptions regarding the distribution of pressure between links be made, the same equations will give the stresses in chain links. Assuming, therefore, the correctness of the analysis, we may use the results of the experiments on the links to throw some light on the question of distribution of pressure.

2. Chain Links. Fig. 19 and 20 show the experiments on the link shown in Fig. 3. In this, as in all the chain links, three calculations for the change of length of the axis were made. These correspond to the three assumptions as to the pressure noted in a previous section. See p. 6 and Fig. 2(a), (b), (c). The line of least inclination, indicated thus — — — — , corresponds to case (a), concentration at the end of the link; the intermediate full line corresponds to case (b), distributed pressure; while the line of greatest slope, indicated thus — — — — , corresponds to case (c), concentration at two points due to the possible wedging action. If we direct our attention to the second test, Fig. 19, we observe that the experimental points lie well within the region of these three lines; the same may be said of the test showing the change of length of the minor axis, Fig. 20.

In Fig. 21 and 22 are shown the experiments upon the long link of the conveyor chain, Fig. 4. The coincidence between the points of the second test and the theoretical line, Fig. 22, is striking. This test is perhaps of more weight than any other of the link tests because the length of the link caused large deflections. It will be observed that the points for the first test indicate in each case a line of smaller slope than the points for the second test. This fact may be explained possibly as follows: In the second test the links have become accommodated to each other, so to speak, and the action is more nearly that of a journal and bearing: hence condition (b) is approximated to rather than condition (a).

In Fig. 23 is shown the one test made on the link shown in Fig. 5. For some reason, a second test of this link was not made. The results are about as shown for the other links. Probably the points for the second test would have followed more closely the theoretical lines, as in the other cases.
The experiments upon the two-inch link, Fig. 6, are shown in Fig. 24 and 25. The results of the two tests are practically the same, and the agreement between the experimental points and theoretical lines is satisfactory.

The theoretical lines shown in Fig. 19-25 were obtained by calculation from formulas (J) and (K), Appendix C. The following table gives the results of the calculations thus made:

### TABLE 2

**BENDING MOMENTS AND DEFLECTIONS OF CHAIN LINKS**

<table>
<thead>
<tr>
<th>Case</th>
<th>Dredge Link, Curves, Fig. 3 and 20</th>
<th>Conveyor Link, Curves, Fig. 4 and 21 and 22</th>
<th>Proof Coil Link, Curves, Fig. 23</th>
<th>Two-Inch Link, Curves, Fig. 24 and 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Moment Mat End of Minor Axis</td>
<td>(a) -0.353 Qd</td>
<td>(b) -0.345 Qd</td>
<td>(c) -0.328 Qd</td>
<td>(d) -0.326 Qd</td>
</tr>
<tr>
<td>Increase of Length of Major Axis per 1000 lb. Load.</td>
<td>(a) 0.000324</td>
<td>(b) 0.000312</td>
<td>(c) 0.000289</td>
<td>(d) 0.000278</td>
</tr>
<tr>
<td>Decrease of Length of Minor Axis per 1000 lb. Load.</td>
<td>(a) 0.000353</td>
<td>(b) 0.000345</td>
<td>(c) 0.000328</td>
<td>(d) 0.000318</td>
</tr>
</tbody>
</table>

It is self-evident that the results obtained from the rough chain links would not be as concordant as those obtained from the finished rings. However, a comparison of the experimental values with the theoretical lines, Fig. 19 to 25, indicates that the theory is confirmed fairly well. The links tested exhibited some variety in form and size; and the results of the calculations show that the agreement of theory and experiment was equally good whether the link was long or short, of 1-in. or 2-in. iron. Tests of more links would have been desirable if sufficient time had been available. It may be stated that the computations are somewhat laborious and time-consuming. It is felt, however, that these four tests are sufficient to establish the validity of the analysis given in Appendix B.

In Fig. 22, additional lines have been drawn to give a comparison of the theory here developed with other theories. If we adopt the analysis usually given in our text-books for hooks and eccentrically
loaded bars, in other words, if we neglect the curvature of the link, the theoretical line for the deflection of the minor axis is the line marked "ordinary theory." On the other hand, if we adopt Grashof's assumption (see page 6) we get the steep line marked "Grashof's theory."

The question of the probable distribution of pressure between adjacent links is not definitely settled. In most cases the experimental points follow most closely the line corresponding to case (a), concentration at the end of the link, for the smaller loads. As the load is increased, however, the line through the points becomes steeper and its slope is about that of the theoretical line for case (b), distributed pressure. In a few of the experiments the points approached more closely the line for case (c). It is probable that the distribution depends somewhat upon the length of time a chain has been used. After the links have been fitted to each other and have worn slightly so as to make a bearing, the distribution will be that indicated by a line lying between the lines for cases (a) and (b). In this connection we may repeat the observation before made that in all cases the second test gave a line of greater slope than the first test.

By reference to Table 2 it will be seen that the assumed distribution influences in some measure the moment \( M \) at the end of the
minor axis, and through this the calculated stresses at various sections. The variation of \( M \) between cases (a) and (c) is about 7 per cent for the dredge link and 14 per cent for the conveyor link. If it is assumed that case (b) coincides most nearly with the actual distribution, the calculated stresses, taking the value of \( M \) from case (b), are not likely to vary more than 3 or 4 per cent from the actual
THE STRENGTH OF CHAIN LINKS

stresses even in the most extreme cases. Hence, in the subsequent calculations, we shall assume that the distribution is according to case (b).

DISTRIBUTION OF STRESSES IN LINKS

By means of the formulas developed in the Appendices, the intensity of stress can be calculated at any point of any cross section of the link. Thus for an open link, the moment $M$ at the end of the short axis is found from formula (F), and from this the moment at any other section is readily obtained. Now having $M_b$ and the normal force $P$ at the section in question, the stress at different points in the section is found by using different values of $y$ in formula (C). For the outer fiber $y = \frac{1}{2}d$, for the inner fiber $y = -\frac{1}{2}d$, at the axis $y = 0$, and so on.

These calculations have been made for the link shown in Fig. 5, and the results are exhibited in the following table:

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISTRIBUTION OF STRESS IN OPEN LINK</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Normal Force $P$</th>
<th>Moment $M_b$</th>
<th>Stress at Center of Section $y=0$</th>
<th>Stress in Outer Fiber $y=+\frac{1}{2}d$</th>
<th>Stress in Inner Fiber $y=-\frac{1}{2}d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>0.225 $Q$</td>
<td>+0.671 $Q$</td>
<td>+0.896 $Q/\ell$</td>
<td>+4.012 $Q/\ell$</td>
<td>-8.453 $Q/\ell$</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.257</td>
<td>+0.639</td>
<td>0.896</td>
<td>+3.863</td>
<td>-8.006</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>0.322</td>
<td>+0.544</td>
<td>0.896</td>
<td>+3.420</td>
<td>-6.677</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.500</td>
<td>+0.371</td>
<td>0.774</td>
<td>+2.785</td>
<td>-3.601</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>0.643</td>
<td>+0.178</td>
<td>0.774</td>
<td>+1.739</td>
<td>-1.325</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>0.766</td>
<td>+0.011</td>
<td>0.774</td>
<td>+0.836</td>
<td>+0.640</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>0.866</td>
<td>-0.124</td>
<td>0.774</td>
<td>+0.104</td>
<td>+2.234</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>0.910</td>
<td>-0.223</td>
<td>0.808</td>
<td>-0.504</td>
<td>+3.225</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>0.985</td>
<td>-0.284</td>
<td>0.843</td>
<td>-0.917</td>
<td>+3.777</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>1.000</td>
<td>-0.318</td>
<td>0.938</td>
<td>-1.546</td>
<td>+3.751</td>
</tr>
</tbody>
</table>

A better idea of the distribution of stress through the link is shown in Fig. 26. At section $a$, lying along the minor axis, the inner fiber is subjected to a tensile stress of $3.55 \frac{Q}{i}$, while the outer fiber is under a compression $1.55 \frac{Q}{i}$. At section $b$, the tensile stress at the inner fiber...
is a little greater, due entirely to the curvature at that section, and at section \( c \) this tensile stress is still greater because of the sharper curvature, notwithstanding the fact that the moment \( M_b \) is smaller. From here on, however, the tensile stress on the inside of the line rapidly decreases and reaches zero at the point \( L \). At section \( e \) the moment \( M_b \) changes sign by passing through the value zero; hence at this section the stress is uniformly distributed and equal to \( \frac{P}{j} \). From \( L \) to \( C \) the minor fiber of the link is in compression, the intensity of the compression reaching its maximum value \( 8.453 \frac{Q}{j} \) at the point \( C \). From \( A \) to \( K \) the outer fiber of the link is compressed, but from \( K \) to \( D \) it is in tension, the maximum intensity of the tension reaching the value \( 4.012 \frac{Q}{j} \) at the point \( D \). The lines \( HK \) and \( LM \) indicate the points of the link at which the stress is zero.

It will be observed that there are two points of maximum tensile stress; one at \( D \), the other at \( E \) on the inside of the link. The compressive stress in the outer fibers is small; but at the point \( C \) it is very large.

The following table gives the stresses in the same link when provided with a stud; and Fig. 27 shows the distribution of stress in such a link.

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>Normal Force ( P )</th>
<th>Moment ( M_b )</th>
<th>Stress at Axis ( y=0 )</th>
<th>Stress in Outer Fiber ( y=+\frac{1}{8}d )</th>
<th>Stress in Inner Fiber ( y=-\frac{1}{8}d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>+0.555 ( Q )</td>
<td>+0.401 ( Q )</td>
<td>0.955 ( Q ) / ( l )</td>
<td>+2.814 ( Q ) / ( l )</td>
<td>-4.623 ( Q ) / ( l )</td>
</tr>
<tr>
<td>10°</td>
<td>+0.582</td>
<td>+0.373</td>
<td>0.955</td>
<td>+2.689</td>
<td>-4.246</td>
</tr>
<tr>
<td>20°</td>
<td>+0.602</td>
<td>+0.293</td>
<td>0.955</td>
<td>+2.314</td>
<td>-3.123</td>
</tr>
<tr>
<td>30°</td>
<td>+0.728</td>
<td>+0.152</td>
<td>0.895</td>
<td>+1.722</td>
<td>-0.805</td>
</tr>
<tr>
<td>40°</td>
<td>+0.892</td>
<td>+0.003</td>
<td>0.895</td>
<td>+0.913</td>
<td>+0.855</td>
</tr>
<tr>
<td>50°</td>
<td>+0.975</td>
<td>-0.109</td>
<td>0.895</td>
<td>+0.304</td>
<td>+2.180</td>
</tr>
<tr>
<td>60°</td>
<td>+1.029</td>
<td>-0.181</td>
<td>0.895</td>
<td>-0.089</td>
<td>+3.034</td>
</tr>
<tr>
<td>70°</td>
<td>+1.051</td>
<td>-0.211</td>
<td>0.895</td>
<td>-0.251</td>
<td>+3.216</td>
</tr>
<tr>
<td>80°</td>
<td>+1.041</td>
<td>-0.198</td>
<td>0.895</td>
<td>-0.180</td>
<td>+3.186</td>
</tr>
<tr>
<td>90°</td>
<td>+1.000</td>
<td>-0.056</td>
<td>0.989</td>
<td>+0.587</td>
<td>+1.480</td>
</tr>
<tr>
<td>End of short axis</td>
<td>+1.000</td>
<td>+0.107</td>
<td>1.000</td>
<td>+1.858</td>
<td>+1.424</td>
</tr>
</tbody>
</table>
It will be observed that in this case there are two sections at which the bending moment is zero. The tensile stress reaches a maximum for the outer fiber at $D$, and for the inner fiber at about the point $E$. The compression is greatest at point $C$, but is only a little over one-half that at $C$ in the case of the open link. The tensile stresses are also somewhat smaller than for the open link.

The following table gives the maximum tensile stresses (at points $D$ and $E$) and the maximum compressive stress (at point $C$) for each of the four links subjected to analysis.

<table>
<thead>
<tr>
<th>Link</th>
<th>Open Link</th>
<th></th>
<th>Stud Link</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tensile Stress</td>
<td>Compressive Stress at $C$</td>
<td>Tensile Stress</td>
<td>Compressive Stress at $C$</td>
</tr>
<tr>
<td></td>
<td>At $E$</td>
<td>At $D$</td>
<td></td>
<td>At $E$</td>
</tr>
<tr>
<td>Dredge, Fig. 3</td>
<td>3.98 Q/l</td>
<td>3.66 Q/l</td>
<td>8.38 Q/l</td>
<td>3.18 Q/l</td>
</tr>
<tr>
<td>Proof coil, Fig. 5</td>
<td>3.78</td>
<td>4.01</td>
<td>8.45</td>
<td>3.22</td>
</tr>
<tr>
<td>Two-inch, Fig. 6</td>
<td>3.72</td>
<td>3.47</td>
<td>7.94</td>
<td>3.20</td>
</tr>
<tr>
<td>Conveyor, Fig. 4</td>
<td>2.78</td>
<td>4.17</td>
<td>9.55</td>
<td>...</td>
</tr>
</tbody>
</table>

A study of the results presented in the preceding tables leads to some interesting conclusions:

In the first place, it may be observed that the maximum stresses for the different links are not widely different. The first three links may be regarded as typical of the forms ordinarily used in engineering practice, and in these the extreme variation in the maximum tensile stress is a little more than 7 per cent. It is also worthy of remark that the tensile stresses at the two points $D$ and $E$ are nearly the same. In some cases the greater stress will be at $D$, in others at $E$.

The conveyor link, on account of its relatively great length, presents an exception. As shown by the analysis, the increased length of the side makes the moment $M$ at the middle of the side small; consequently the moment at the end of the link is large, and the
stress at $D$ is considerably greater than that at $E$. It may be concluded, therefore, that so far as strength is concerned, the form of this link is not favorable.

The effect of the stud upon the distribution of stress is easily seen. The maximum tensile stresses are reduced about 20 per cent; but what is more essential, the heavy compressive stress at $C$ is reduced 50 per cent or more. We conclude, therefore, that provided the stresses are kept within the elastic limit of the material, the stud is of unquestioned value.

It has been the general opinion of engineers that the stud link chain is stronger than the open link chain; however, the experiments of committee D of the United States board appointed to test iron, steel and other metals (see Executive Document No. 98, House of Representatives, Forty-fifth Congress, Second Session), seem to indicate that the stud actually weakens the chain, causing it to rupture at a load lower than that required to break an open link chain. At first sight these experiments seem to disprove the results given in the preceding pages; however, in this case, fact and theory are easily reconciled. It is quite easy to understand that while the stud link is much stronger than the open link, provided the elastic limit is not reached, the former may rupture with a smaller load than the latter. In the first place, the collapse of the sides of the open link after the elastic limit is passed decreases the effective width of the link, and thus decreases the bending moments and stresses. If the iron of

![Fig. 28.](image-url)
which the link is constructed is ductile, the link may collapse until the sides become nearly parallel, and the stresses are lower than in the stud link, the sides of which are prevented from collapsing by the stud. The appearance of the two forms of link under heavy load will be somewhat as shown in Fig. 28. Thus the actual distortion of the open link gives it a form of greater strength, which is not the case with the stud link.

Near the load producing rupture it seems likely, therefore, that the stresses in the open link are less than those in the stud link subjected to the same load. Within the elastic limit, however, the reverse is true, and there can be no doubt that for ordinary working loads the chain made of stud links is materially stronger than the one made of open links.

**Formulas for the Loading of Chains**

Unwin, *Elements of Machine Design*, Part I, p. 438, gives the following formulas:

\[
P = 9 \, d^2, \text{ for studded link chain;}
\]

\[
= 6 \, d^2, \text{ for unstudded close link chain.}
\]

He says further: "For much used chain, subject frequently to the maximum load, it is better to limit the stress to \(3\frac{1}{2}\) tons per sq. in. Then

\[
P = 5 \, d^2."
\]

In these formulas, \(P\) denotes the load in tons, and \(d\) the diameter in inches of the iron from which the chain is made.

Unwin says that Towne limits the loads in ordinary crane chains to

\[
P = 3.3 \, d^2,
\]

but quotes the following table from Towne's *Treatise on Granes*.

<table>
<thead>
<tr>
<th>Diameter of Iron</th>
<th>Load on chain — tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{8})</td>
<td>0.06</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>0.25</td>
</tr>
<tr>
<td>(\frac{3}{16})</td>
<td>0.5</td>
</tr>
<tr>
<td>(\frac{1}{8})</td>
<td>0.75</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>1.5</td>
</tr>
<tr>
<td>(\frac{3}{8})</td>
<td>2</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>2.5</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>3</td>
</tr>
<tr>
<td>(\frac{7}{8})</td>
<td>4</td>
</tr>
<tr>
<td>(1)</td>
<td>5</td>
</tr>
</tbody>
</table>
This table seems to be obtained from the formula

\[ P = 8d^2. \]

Weisbach gives the formulas (Kent's *Pocket Book*, p. 339)

\[ P = 17800d^2, \text{ stud link}, \]
\[ P = 13350d^2, \text{ open link}, \]

where \( P \) denotes the load in pounds.

Bach, in his *Maschinenelemente*, p. 513, gives for chains with open links

\[ P = 1000d^2 \text{ for new chains, maximum load seldom applied.} \]
\[ P = 800d^2 \text{ for much used chain.} \]

\( P \) and \( d \) are taken in kilograms and centimeters, respectively. Using pounds and inches as the units, the formulas become

\[ P = 13750d^2; \]
\[ P = 11000d^2. \]

For a stud-link chain, Bach increases the safe load 20 per cent.

If we write the formula for the safe load

\[ P = kd^2, \]

the values of \( k \) given by the authorities quoted are as follows, \( P \) being taken in pounds:

<table>
<thead>
<tr>
<th></th>
<th>Open Link</th>
<th>Stud Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unwin</td>
<td>13,440</td>
<td>20,160</td>
</tr>
<tr>
<td></td>
<td>11,200</td>
<td>...</td>
</tr>
<tr>
<td>Weisbach</td>
<td>13,350</td>
<td>17,800</td>
</tr>
<tr>
<td></td>
<td>13,750</td>
<td>16,500</td>
</tr>
<tr>
<td></td>
<td>11,000</td>
<td>13,200</td>
</tr>
</tbody>
</table>

Referring to Table 5, we note that with links of the ordinary form the maximum tensile stresses are about as follows:

\[
\begin{align*}
\text{for open links, } 4 & \frac{Q}{j}; \\
\text{for stud links, } 3.2 & \frac{Q}{j}.
\end{align*}
\]

(1)
Of course these values will vary slightly with the form of the link; thus the conveyor link on account of its extreme length shows a maximum tensile stress of $4.17 \frac{Q}{f}$. In general, however, the value $4 \frac{Q}{f}$ cannot be very far from the truth for an open link of usual dimensions.

Denoting the load on the chain by $P$, and the maximum permissible unit stress by $S$, we have, since $P = 2Q$,

\[
\begin{align*}
\text{for open links,} & \quad S = \frac{4Q}{f} = \frac{2P}{f} ; \\
\text{for stud links,} & \quad S = \frac{3.2Q}{f} = \frac{1.6P}{f}.
\end{align*}
\]

Now taking $f = \frac{1}{4} \pi d^2$, we readily obtain

\[
\begin{align*}
\text{for open links,} & \quad P = \frac{1}{8} \pi d^2 S, \text{ say } 0.4 \ d^2 S ; \\
\text{for stud links,} & \quad P = \frac{1}{6.4} \pi d^2 S, \text{ say } 0.5 \ d^2 S.
\end{align*}
\]

Comparing these equations with the equation $P = kd^p$, we see that

\[
\begin{align*}
k = 0.4 \ S, & \quad \text{for open links,} \\
k = 0.5 \ S, & \quad \text{for stud links.}
\end{align*}
\]

Now using the values of $k$ just given, we obtain the following values of $S$ when we use the formulas ordinarily given.

<table>
<thead>
<tr>
<th></th>
<th>Open Link</th>
<th>Stud Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unwin</td>
<td>33,600</td>
<td>40,320</td>
</tr>
<tr>
<td></td>
<td>28,000</td>
<td>35,600</td>
</tr>
<tr>
<td>Weisbach</td>
<td>33,375</td>
<td>33,000</td>
</tr>
<tr>
<td></td>
<td>34,375</td>
<td></td>
</tr>
<tr>
<td>Bach</td>
<td>27,500</td>
<td>26,400</td>
</tr>
</tbody>
</table>

It will probably be agreed that these values are considerably in excess of the values usually regarded as permissible in machine construction.
THE STRENGTH OF CHAIN LINKS

So far, we have considered only the tensile stresses. Referring to Table 5 it is seen that the compression at the end of the link is more than double the maximum tensile stress; hence when a chain has its full load, if the maximum tensile stress is 30,000 lb., as indicated by the constants above, the compressive stress at the end is something over 60,000 lb. It is probable that when the maximum load is applied, the pinching action heretofore described reduces to a considerable degree this excessive compression. Furthermore, it will be noted that the part of the link subjected to this compression is restrained laterally by the sides of the adjacent link; and this lateral restraint offsets in some measure the compressive stress. In any case, however, this compression is a factor to be seriously considered.

Using the maximum tensile stress as a basis, the formulas

\[
\begin{align*}
P & = 0.4 d^2 S \text{ (open)} \\
P & = 0.5 d^2 S \text{ (stud)}
\end{align*}
\]

for open and stud links respectively, are proposed as substitutes for the formulas now in use. These formulas contain the safe maximum unit stress \( S \), and are in that respect more general than those quoted from Unwin, Bach and others. If desired, the usual form \( P = kd^2 \) is readily obtained by assuming a proper value of \( S \). Thus if \( S \) is taken at 15,000 lb. sq. in., we have

\[
\begin{align*}
P & = 6000 d^2 \text{ (open)}, \\
P & = 7500 d^2 \text{ (stud)},
\end{align*}
\]

respectively; if 20,000 lb. sq. in. is considered a permissible value of \( S \), the formulas become

\[
\begin{align*}
P & = 8000 d^2 \text{ (open)}, \\
P & = 10,000 d^2 \text{ (stud)},
\end{align*}
\]

respectively.

**SUMMARY OF RESULTS, AND CONCLUSIONS**

The following is a summary of the results obtained from the investigations herein described and the conclusions that may be drawn from them:

1. The experiments on the steel rings confirm the theoretical analysis employed in the calculation of stresses.
2. The experiments on the various chain links further confirm the analysis and show that the distribution of pressure between the links, in general, lies between the extremes (a), point contact and (c), pressure concentrated at opposite points, as in Fig. 2(c). For purposes of calculation case (b), uniform distribution of pressure over an arc $2\alpha$ may be assumed.

3. The load $2Q$ on the link produces an average intensity of stress \[ \frac{2Q}{2f} = \frac{Q}{f} \] in the cross section of the link containing the minor axis. With an open link of usual proportions the maximum tensile stress is approximately four times this value, i.e., $4\frac{Q}{f}$.

4. The introduction of a stud in the link equalizes the stresses throughout the link, reduces the maximum tensile stresses about 20 per cent, and reduces the excessive compressive stress at the end of the link about 50 per cent.

5. The stud-link chain of equal dimensions will, within the elastic limit, bear from 20 to 25 per cent more load than the open-link chain. The ultimate strength of the stud-link chain is, however, probably less than that of the open-link chain.

6. In the formulas for the safe loading of chains given by the leading authorities on machine design, the maximum stress to which the link is subjected seems to be underestimated and the constants are such as to give maximum stresses of from 30,000 to 40,000 lb. per sq. in. for full load.

7. The following formulas are applicable to chains of the usual form:

\[ P = 0.4 d^2 S, \] for open links,
\[ P = 0.5 d^2 S, \] for stud links,

where $P$ denotes the safe load, $d$ the diameter of the stock, and $S$ the maximum permissible tensile stress.
THE STRENGTH OF CHAIN LINKS

TABLE 6
FIRST TEST OF 12-INCH STEEL RING NO. 1

<table>
<thead>
<tr>
<th>Applied Load Pounds</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td></td>
<td>12.000 in.</td>
<td>9.000 in.</td>
</tr>
<tr>
<td>500</td>
<td>.2420</td>
<td>.2408</td>
</tr>
<tr>
<td>1,000</td>
<td>.2419</td>
<td>.2379</td>
</tr>
<tr>
<td>1,500</td>
<td>.2442</td>
<td>.2367</td>
</tr>
<tr>
<td>2,000</td>
<td>.2467</td>
<td>.2357</td>
</tr>
<tr>
<td>2,500</td>
<td>.2474</td>
<td>.2342</td>
</tr>
<tr>
<td>3,000</td>
<td>.2484</td>
<td>.2322</td>
</tr>
<tr>
<td>3,500</td>
<td>.2494</td>
<td>.2306</td>
</tr>
<tr>
<td>4,000</td>
<td>.2503</td>
<td>.2277</td>
</tr>
<tr>
<td>4,500</td>
<td>.2529</td>
<td>.2284</td>
</tr>
<tr>
<td>5,000</td>
<td>.2536</td>
<td>.2270</td>
</tr>
<tr>
<td>5,500</td>
<td>.2546</td>
<td>.2256</td>
</tr>
<tr>
<td>6,000</td>
<td>.2550</td>
<td>.2249</td>
</tr>
<tr>
<td>6,500</td>
<td>.2572</td>
<td>.2234</td>
</tr>
<tr>
<td>7,000</td>
<td>.2583</td>
<td>.2218</td>
</tr>
<tr>
<td>7,500</td>
<td>.2600</td>
<td>.2187</td>
</tr>
</tbody>
</table>

TABLE 7
SECOND TEST OF 12-INCH STEEL RING NO. 1

<table>
<thead>
<tr>
<th>Applied Load Pounds</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td>0</td>
<td>.9985</td>
<td>.0024</td>
</tr>
<tr>
<td>500</td>
<td>.9975</td>
<td>.0034</td>
</tr>
<tr>
<td>1,000</td>
<td>.9960</td>
<td>.0045</td>
</tr>
<tr>
<td>1,500</td>
<td>.9948</td>
<td>.0057</td>
</tr>
<tr>
<td>2,000</td>
<td>.9933</td>
<td>.0073</td>
</tr>
<tr>
<td>2,500</td>
<td>.9917</td>
<td>.0085</td>
</tr>
<tr>
<td>3,000</td>
<td>.9913</td>
<td>.0098</td>
</tr>
<tr>
<td>3,500</td>
<td>.9907</td>
<td>.0116</td>
</tr>
<tr>
<td>4,000</td>
<td>.9890</td>
<td>.0130</td>
</tr>
<tr>
<td>4,500</td>
<td>.9874</td>
<td>.0143</td>
</tr>
<tr>
<td>5,000</td>
<td>.9863</td>
<td>.0156</td>
</tr>
<tr>
<td>5,500</td>
<td>.9851</td>
<td>.0170</td>
</tr>
<tr>
<td>6,000</td>
<td>.9840</td>
<td>.0184</td>
</tr>
<tr>
<td>6,500</td>
<td>.9824</td>
<td>.0199</td>
</tr>
<tr>
<td>7,000</td>
<td>.9808</td>
<td>.0213</td>
</tr>
<tr>
<td>7,500</td>
<td>.9800</td>
<td>.0223</td>
</tr>
</tbody>
</table>
TABLE 8
FIRST TEST OF STEEL RING NO. 2

<table>
<thead>
<tr>
<th>Applied Load Pounds</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td>0</td>
<td>0.0076</td>
<td>0.0043</td>
</tr>
<tr>
<td>500</td>
<td>0.0054</td>
<td>0.0062</td>
</tr>
<tr>
<td>1,000</td>
<td>0.0047</td>
<td>0.0063</td>
</tr>
<tr>
<td>1,500</td>
<td>0.0036</td>
<td>0.0087</td>
</tr>
<tr>
<td>2,000</td>
<td>0.0018</td>
<td>0.0102</td>
</tr>
<tr>
<td>2,500</td>
<td>0.0007</td>
<td>0.0120</td>
</tr>
<tr>
<td>3,000</td>
<td>0.9994</td>
<td>0.0135</td>
</tr>
<tr>
<td>3,500</td>
<td>0.9975</td>
<td>0.0138</td>
</tr>
<tr>
<td>4,000</td>
<td>0.9958</td>
<td>0.0153</td>
</tr>
<tr>
<td>4,500</td>
<td>0.9953</td>
<td>0.0167</td>
</tr>
<tr>
<td>5,000</td>
<td>0.9932</td>
<td>0.0177</td>
</tr>
<tr>
<td>5,500</td>
<td>0.9920</td>
<td>0.0199</td>
</tr>
<tr>
<td>6,000</td>
<td>0.9917</td>
<td>0.0217</td>
</tr>
<tr>
<td>6,500</td>
<td>0.9905</td>
<td>0.0230</td>
</tr>
<tr>
<td>7,000</td>
<td>0.9887</td>
<td>0.0249</td>
</tr>
<tr>
<td>7,500</td>
<td>0.9870</td>
<td>0.0270</td>
</tr>
</tbody>
</table>

TABLE 9
SECOND TEST OF STEEL RING NO. 2

<table>
<thead>
<tr>
<th>Applied Load Pounds</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td>0</td>
<td>1.0050</td>
<td>0.0062</td>
</tr>
<tr>
<td>500</td>
<td>1.0028</td>
<td>0.0067</td>
</tr>
<tr>
<td>1,000</td>
<td>1.0023</td>
<td>0.0082</td>
</tr>
<tr>
<td>1,500</td>
<td>1.0018</td>
<td>0.0100</td>
</tr>
<tr>
<td>2,000</td>
<td>0.9964</td>
<td>0.0106</td>
</tr>
<tr>
<td>2,500</td>
<td>0.9961</td>
<td>0.0130</td>
</tr>
<tr>
<td>3,000</td>
<td>0.9964</td>
<td>0.0148</td>
</tr>
<tr>
<td>3,500</td>
<td>0.9960</td>
<td>0.0158</td>
</tr>
<tr>
<td>4,000</td>
<td>0.9943</td>
<td>0.0170</td>
</tr>
<tr>
<td>4,500</td>
<td>0.9932</td>
<td>0.0186</td>
</tr>
<tr>
<td>5,000</td>
<td>0.9919</td>
<td>0.0196</td>
</tr>
<tr>
<td>5,500</td>
<td>0.9908</td>
<td>0.0220</td>
</tr>
<tr>
<td>6,000</td>
<td>0.9899</td>
<td>0.0230</td>
</tr>
<tr>
<td>6,500</td>
<td>0.9884</td>
<td>0.0240</td>
</tr>
<tr>
<td>7,000</td>
<td>0.9871</td>
<td>0.0255</td>
</tr>
<tr>
<td>7,500</td>
<td>0.9860</td>
<td>0.0270</td>
</tr>
</tbody>
</table>
### THE STRENGTH OF CHAIN LINKS

#### TABLE 10
**FIRST TEST OF FORGED RING NO. 3**

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td>0</td>
<td>.0063</td>
<td>.0050</td>
</tr>
<tr>
<td>500</td>
<td>.0085</td>
<td>.0038</td>
</tr>
<tr>
<td>1,000</td>
<td>.0094</td>
<td>.0025</td>
</tr>
<tr>
<td>1,500</td>
<td>.0101</td>
<td>.9996</td>
</tr>
<tr>
<td>2,000</td>
<td>.0113</td>
<td>.9987</td>
</tr>
<tr>
<td>2,500</td>
<td>.0147</td>
<td>.9969</td>
</tr>
<tr>
<td>3,000</td>
<td>.0154</td>
<td>.9969</td>
</tr>
<tr>
<td>3,500</td>
<td>.0171</td>
<td>.9946</td>
</tr>
<tr>
<td>4,000</td>
<td>.0184</td>
<td>.9930</td>
</tr>
<tr>
<td>4,500</td>
<td>.0195</td>
<td>.9925</td>
</tr>
<tr>
<td>5,000</td>
<td>.0206</td>
<td>.9913</td>
</tr>
<tr>
<td>5,500</td>
<td>.0220</td>
<td>.9900</td>
</tr>
<tr>
<td>6,000</td>
<td>.0228</td>
<td>.9887</td>
</tr>
<tr>
<td>6,500</td>
<td>.0245</td>
<td>.9877</td>
</tr>
<tr>
<td>7,000</td>
<td>.0261</td>
<td>.9871</td>
</tr>
<tr>
<td>7,500</td>
<td>.0287</td>
<td>.9852</td>
</tr>
<tr>
<td>8,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### TABLE 11
**SECOND TEST OF FORGED STEEL RING NO. 3**

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Extensometer Readings</th>
<th>Deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal Axis</td>
<td>Vertical Axis</td>
</tr>
<tr>
<td>0</td>
<td>.0032</td>
<td>.0063</td>
</tr>
<tr>
<td>500</td>
<td>.0014</td>
<td>.0070</td>
</tr>
<tr>
<td>1,000</td>
<td>.0012</td>
<td>.0094</td>
</tr>
<tr>
<td>1,500</td>
<td>.0003</td>
<td>.0102</td>
</tr>
<tr>
<td>2,000</td>
<td>.9994</td>
<td>.0124</td>
</tr>
<tr>
<td>2,500</td>
<td>.9960</td>
<td>.0133</td>
</tr>
<tr>
<td>3,000</td>
<td>.9946</td>
<td>.0143</td>
</tr>
<tr>
<td>3,500</td>
<td>.9935</td>
<td>.0153</td>
</tr>
<tr>
<td>4,000</td>
<td>.9926</td>
<td>.0170</td>
</tr>
<tr>
<td>4,500</td>
<td>.9916</td>
<td>.0182</td>
</tr>
<tr>
<td>5,000</td>
<td>.9902</td>
<td>.0200</td>
</tr>
<tr>
<td>5,500</td>
<td>.9992</td>
<td>.0207</td>
</tr>
<tr>
<td>6,000</td>
<td>.9883</td>
<td>.0222</td>
</tr>
<tr>
<td>6,500</td>
<td>.9871</td>
<td>.0230</td>
</tr>
<tr>
<td>7,000</td>
<td>.9860</td>
<td>.0257</td>
</tr>
<tr>
<td>7,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 12
FIRST TEST OF DREDGE LINK

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Vertical Axis</th>
<th>Horizontal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Readings</td>
<td>Differences</td>
</tr>
<tr>
<td>1,000</td>
<td>.0098</td>
<td>.0125</td>
</tr>
<tr>
<td>2,000</td>
<td>.0101</td>
<td>.0129</td>
</tr>
<tr>
<td>3,000</td>
<td>.0103</td>
<td>.0136</td>
</tr>
<tr>
<td>4,000</td>
<td>.0106</td>
<td>.0136</td>
</tr>
<tr>
<td>5,000</td>
<td>.0109</td>
<td>.0138</td>
</tr>
<tr>
<td>6,000</td>
<td>.0109</td>
<td>.0138</td>
</tr>
<tr>
<td>7,000</td>
<td>.0116</td>
<td>.0140</td>
</tr>
<tr>
<td>8,000</td>
<td>.0112</td>
<td>.0149</td>
</tr>
<tr>
<td>9,000</td>
<td>.0117</td>
<td>.0146</td>
</tr>
<tr>
<td>10,000</td>
<td>.0119</td>
<td>.0152</td>
</tr>
<tr>
<td>11,000</td>
<td>.0121</td>
<td>.0149</td>
</tr>
<tr>
<td>12,000</td>
<td>.0124</td>
<td>.0158</td>
</tr>
<tr>
<td>13,000</td>
<td>.0126</td>
<td>.0160</td>
</tr>
<tr>
<td>14,000</td>
<td>.0129</td>
<td>.0163</td>
</tr>
<tr>
<td>15,000</td>
<td>.0131</td>
<td>.0170</td>
</tr>
<tr>
<td>16,000</td>
<td>.0132</td>
<td>.0171</td>
</tr>
<tr>
<td>17,000</td>
<td>.0138</td>
<td>.0180</td>
</tr>
<tr>
<td>18,000</td>
<td>.0142</td>
<td>.0188</td>
</tr>
<tr>
<td>19,000</td>
<td>.0155</td>
<td>.0192</td>
</tr>
<tr>
<td>20,000</td>
<td>.0161</td>
<td>.0199</td>
</tr>
<tr>
<td>21,000</td>
<td>.0176</td>
<td>.0221</td>
</tr>
<tr>
<td>22,000</td>
<td>.0216</td>
<td>.0267</td>
</tr>
</tbody>
</table>

### TABLE 13
SECOND TEST OF DREDGE LINK

[Modulus of Elasticity 24,600,000]

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Vertical Axis</th>
<th>Horizontal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Readings</td>
<td>Differences</td>
</tr>
<tr>
<td>1,000</td>
<td>.0033</td>
<td>.0035</td>
</tr>
<tr>
<td>2,000</td>
<td>.0035</td>
<td>.0038</td>
</tr>
<tr>
<td>4,000</td>
<td>.0038</td>
<td>.0045</td>
</tr>
<tr>
<td>6,000</td>
<td>.0042</td>
<td>.0054</td>
</tr>
<tr>
<td>8,000</td>
<td>.0048</td>
<td>.0062</td>
</tr>
<tr>
<td>10,000</td>
<td>.0051</td>
<td>.0078</td>
</tr>
<tr>
<td>12,000</td>
<td>.0059</td>
<td>.0088</td>
</tr>
<tr>
<td>14,000</td>
<td>.0061</td>
<td>.0101</td>
</tr>
<tr>
<td>16,000</td>
<td>.0068</td>
<td>.0111</td>
</tr>
<tr>
<td>18,000</td>
<td>.0069</td>
<td>.0120</td>
</tr>
<tr>
<td>20,000</td>
<td>.0099</td>
<td>.0120</td>
</tr>
<tr>
<td>22,000</td>
<td>.026</td>
<td>.0248</td>
</tr>
<tr>
<td>24,000</td>
<td>.0326</td>
<td>.0248</td>
</tr>
</tbody>
</table>
## TABLE 14
FIRST TEST OF CONVEYOR LINK

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Vertical Axis</th>
<th>Horizontal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Readings</td>
<td>Differences</td>
</tr>
<tr>
<td>1,500</td>
<td>.1363</td>
<td>.0400</td>
</tr>
<tr>
<td>2,000</td>
<td>.1368</td>
<td>.0399</td>
</tr>
<tr>
<td>2,500</td>
<td>.1371</td>
<td>.0407</td>
</tr>
<tr>
<td>3,000</td>
<td>.1383</td>
<td>.0401</td>
</tr>
<tr>
<td>3,500</td>
<td>.1389</td>
<td>.0401</td>
</tr>
<tr>
<td>4,000</td>
<td>.1390</td>
<td>.0404</td>
</tr>
<tr>
<td>4,500</td>
<td>.1495</td>
<td>.0407</td>
</tr>
<tr>
<td>5,000</td>
<td>.1498</td>
<td>.0408</td>
</tr>
<tr>
<td>5,500</td>
<td>.1401</td>
<td>.0409</td>
</tr>
<tr>
<td>6,000</td>
<td>.1401</td>
<td>.0416</td>
</tr>
<tr>
<td>6,500</td>
<td>.1404</td>
<td>.0410</td>
</tr>
<tr>
<td>7,000</td>
<td>.1405</td>
<td>.0412</td>
</tr>
<tr>
<td>7,500</td>
<td>.1408</td>
<td>.0413</td>
</tr>
<tr>
<td>8,000</td>
<td>.1410</td>
<td>.0417</td>
</tr>
<tr>
<td>8,500</td>
<td>.1411</td>
<td>.0418</td>
</tr>
<tr>
<td>9,000</td>
<td>.1413</td>
<td>.0419</td>
</tr>
<tr>
<td>9,500</td>
<td>.1408</td>
<td>.0422</td>
</tr>
<tr>
<td>10,000</td>
<td>.1420</td>
<td>.0425</td>
</tr>
<tr>
<td>10,500</td>
<td>.1421</td>
<td>.0429</td>
</tr>
<tr>
<td>11,000</td>
<td>.1427</td>
<td>.0430</td>
</tr>
<tr>
<td>11,500</td>
<td>.1432</td>
<td>.0436</td>
</tr>
<tr>
<td>12,000</td>
<td>.1434</td>
<td>.0443</td>
</tr>
<tr>
<td>12,500</td>
<td>.1437</td>
<td>.0448</td>
</tr>
<tr>
<td>13,000</td>
<td>.1443</td>
<td>.0452</td>
</tr>
<tr>
<td>13,500</td>
<td>.1448</td>
<td>.0459</td>
</tr>
<tr>
<td>14,000</td>
<td>.1458</td>
<td>.0459</td>
</tr>
<tr>
<td>14,500</td>
<td>.1462</td>
<td>.0463</td>
</tr>
<tr>
<td>15,000</td>
<td>.1470</td>
<td>.0470</td>
</tr>
<tr>
<td>15,500</td>
<td>.1485</td>
<td>.0474</td>
</tr>
<tr>
<td>16,000</td>
<td>.1499</td>
<td>.0480</td>
</tr>
<tr>
<td>16,500</td>
<td>.1517</td>
<td>.0498</td>
</tr>
<tr>
<td>17,000</td>
<td>.1533</td>
<td>.0512</td>
</tr>
<tr>
<td>17,500</td>
<td>.1569</td>
<td>.0551</td>
</tr>
<tr>
<td>18,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Applied Load</td>
<td>Vertical Axis</td>
<td>Horizontal Axis</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------</td>
<td>-----------------</td>
</tr>
<tr>
<td></td>
<td>Readings</td>
<td>Differences</td>
</tr>
<tr>
<td>1,120</td>
<td>1.1835</td>
<td>0.292</td>
</tr>
<tr>
<td>2,040</td>
<td>1.1840</td>
<td>0.295</td>
</tr>
<tr>
<td>3,020</td>
<td>1.1843</td>
<td>0.300</td>
</tr>
<tr>
<td>4,000</td>
<td>1.1848</td>
<td>0.302</td>
</tr>
<tr>
<td>5,110</td>
<td>1.1852</td>
<td>0.308</td>
</tr>
<tr>
<td>6,040</td>
<td>1.1858</td>
<td>0.312</td>
</tr>
<tr>
<td>7,000</td>
<td>1.1860</td>
<td>0.315</td>
</tr>
<tr>
<td>8,050</td>
<td>1.1864</td>
<td>0.319</td>
</tr>
<tr>
<td>9,190</td>
<td>1.1867</td>
<td>0.323</td>
</tr>
<tr>
<td>10,000</td>
<td>1.1873</td>
<td>0.326</td>
</tr>
<tr>
<td>11,000</td>
<td>1.1880</td>
<td>0.329</td>
</tr>
<tr>
<td>12,020</td>
<td>1.1882</td>
<td>0.332</td>
</tr>
<tr>
<td>13,000</td>
<td>1.1886</td>
<td>0.336</td>
</tr>
<tr>
<td>14,090</td>
<td>1.1890</td>
<td>0.341</td>
</tr>
<tr>
<td>15,030</td>
<td>1.1892</td>
<td>0.345</td>
</tr>
<tr>
<td>16,090</td>
<td>1.1895</td>
<td>0.347</td>
</tr>
<tr>
<td>17,020</td>
<td>1.1897</td>
<td>0.350</td>
</tr>
<tr>
<td>18,000</td>
<td>1.1898</td>
<td>0.356</td>
</tr>
<tr>
<td>19,000</td>
<td>1.1907</td>
<td>0.355</td>
</tr>
<tr>
<td>20,000</td>
<td>1.1910</td>
<td>0.359</td>
</tr>
<tr>
<td>21,000</td>
<td>1.1918</td>
<td>0.365</td>
</tr>
<tr>
<td>22,000</td>
<td>1.1922</td>
<td>0.367</td>
</tr>
<tr>
<td>23,000</td>
<td>1.1923</td>
<td>0.372</td>
</tr>
<tr>
<td>24,000</td>
<td>1.1925</td>
<td>0.378</td>
</tr>
<tr>
<td>25,000</td>
<td>1.1933</td>
<td>0.380</td>
</tr>
<tr>
<td>26,000</td>
<td>1.1940</td>
<td>0.382</td>
</tr>
<tr>
<td>27,000</td>
<td>1.1944</td>
<td>0.389</td>
</tr>
<tr>
<td>28,000</td>
<td>1.2081</td>
<td>0.562</td>
</tr>
<tr>
<td>29,000</td>
<td>2.185</td>
<td>0.0673</td>
</tr>
</tbody>
</table>
### TABLE 16

**TEST OF PROOF COIL LINK**

[Modulus of Elasticity, 26,100,000]

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Vertical Axis</th>
<th>Average Deflections</th>
<th>Horizontal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Readings</td>
<td>Differences</td>
<td>Readings</td>
</tr>
<tr>
<td>1,000</td>
<td>2.006</td>
<td>2.002</td>
<td></td>
</tr>
<tr>
<td>2,000</td>
<td>2.010</td>
<td>2.006</td>
<td></td>
</tr>
<tr>
<td>3,000</td>
<td>2.017</td>
<td>2.008</td>
<td></td>
</tr>
<tr>
<td>4,000</td>
<td>2.021</td>
<td>2.010</td>
<td></td>
</tr>
<tr>
<td>5,000</td>
<td>2.022</td>
<td>2.016</td>
<td></td>
</tr>
<tr>
<td>6,000</td>
<td>2.026</td>
<td>2.019</td>
<td></td>
</tr>
<tr>
<td>7,000</td>
<td>2.030</td>
<td>2.021</td>
<td></td>
</tr>
<tr>
<td>8,000</td>
<td>2.034</td>
<td>2.027</td>
<td></td>
</tr>
<tr>
<td>9,000</td>
<td>2.038</td>
<td>2.029</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>2.041</td>
<td>2.032</td>
<td></td>
</tr>
<tr>
<td>11,000</td>
<td>2.045</td>
<td>2.034</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>2.049</td>
<td>2.038</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>2.054</td>
<td>2.043</td>
<td></td>
</tr>
<tr>
<td>14,000</td>
<td>2.060</td>
<td>2.047</td>
<td></td>
</tr>
<tr>
<td>15,000</td>
<td>2.066</td>
<td>2.052</td>
<td></td>
</tr>
<tr>
<td>16,000</td>
<td>2.071</td>
<td>2.051</td>
<td></td>
</tr>
<tr>
<td>17,000</td>
<td>2.078</td>
<td>2.061</td>
<td></td>
</tr>
<tr>
<td>18,000</td>
<td>2.087</td>
<td>2.068</td>
<td></td>
</tr>
<tr>
<td>19,000</td>
<td>2.097</td>
<td>2.070</td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td>2.108</td>
<td>2.076</td>
<td></td>
</tr>
<tr>
<td>21,000</td>
<td>2.119</td>
<td>2.088</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 17

**FIRST TEST OF TWO-INCH DREDGE LINK**

[Modulus of Elasticity 29,000,000 (Assumed)]

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>Vertical Axis</th>
<th>Deflections</th>
<th>Horizontal Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Readings</td>
<td></td>
<td>Readings</td>
</tr>
<tr>
<td>0</td>
<td>0.215</td>
<td>0.5488</td>
<td></td>
</tr>
<tr>
<td>3,000</td>
<td>0.223</td>
<td>0.5496</td>
<td></td>
</tr>
<tr>
<td>6,000</td>
<td>0.225</td>
<td>0.5501</td>
<td></td>
</tr>
<tr>
<td>9,000</td>
<td>0.230</td>
<td>0.5511</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>0.231</td>
<td>0.5518</td>
<td></td>
</tr>
<tr>
<td>15,000</td>
<td>0.238</td>
<td>0.5524</td>
<td></td>
</tr>
<tr>
<td>18,000</td>
<td>0.240</td>
<td>0.5528</td>
<td></td>
</tr>
<tr>
<td>21,000</td>
<td>0.244</td>
<td>0.5531</td>
<td></td>
</tr>
<tr>
<td>24,000</td>
<td>0.245</td>
<td>0.5535</td>
<td></td>
</tr>
<tr>
<td>27,000</td>
<td>0.249</td>
<td>0.5538</td>
<td></td>
</tr>
<tr>
<td>30,000</td>
<td>0.253</td>
<td>0.5540</td>
<td></td>
</tr>
<tr>
<td>33,000</td>
<td>0.254</td>
<td>0.5543</td>
<td></td>
</tr>
<tr>
<td>36,000</td>
<td>0.255</td>
<td>0.5549</td>
<td></td>
</tr>
<tr>
<td>39,000</td>
<td>0.263</td>
<td>0.5551</td>
<td></td>
</tr>
<tr>
<td>42,000</td>
<td>0.265</td>
<td>0.5555</td>
<td></td>
</tr>
<tr>
<td>45,000</td>
<td>0.269</td>
<td>0.5561</td>
<td></td>
</tr>
<tr>
<td>48,000</td>
<td>0.273</td>
<td>0.5563</td>
<td></td>
</tr>
<tr>
<td>51,000</td>
<td>0.277</td>
<td>0.5568</td>
<td></td>
</tr>
<tr>
<td>54,000</td>
<td>0.280</td>
<td>0.5570</td>
<td></td>
</tr>
<tr>
<td>57,000</td>
<td>0.283</td>
<td>0.5573</td>
<td></td>
</tr>
<tr>
<td>60,000</td>
<td>0.285</td>
<td>0.5577</td>
<td></td>
</tr>
<tr>
<td>Applied Load</td>
<td>0</td>
<td>3,000</td>
<td>6,000</td>
</tr>
<tr>
<td>--------------</td>
<td>----</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Readings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.0216</td>
<td>.5495</td>
<td>.0219</td>
</tr>
<tr>
<td>Differences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.0003</td>
<td>.0007</td>
<td>.0007</td>
</tr>
<tr>
<td>Deflections</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.0005</td>
<td>.0010</td>
<td>.00085</td>
</tr>
</tbody>
</table>

**Horizontal Axis**

| Applied Load | 0  | 3,000 | 6,000 | 9,000 | 12,000 | 15,000 | 18,000 | 21,000 | 24,000 | 27,000 | 30,000 | 33,000 | 36,000 | 39,000 | 42,000 | 45,000 | 48,000 | 51,000 | 54,000 | 60,000 | 75,000 |
|--------------|----|-------|-------|-------|--------|--------|--------|--------|-------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|               |    |       |       |       |        |        |        |        |       |        |        |       |       |       |       |       |       |       |       |       |       |       |
| Readings      |    |       |       |       |        |        |        |        |       |        |        |       |       |       |       |       |       |       |       |       |       |       |
|               | .5902 |       | .5890 |       | .5887 |       | .5880 |       | .5876 |       | .5870 |       | .5867 |       | .5865 |       | .5862 |       | .5860 |       | .5856 |       | .5852 |       |
| Deflections   |    |       |       |       |        |        |        |        |       |        |        |       |       |       |       |       |       |       |       |       |       |       |
|               | .5904 |       | .5900 |       | .5904 |       | .5902 |       | .5904 |       | .5904 |       | .5904 |       | .5904 |       | .5904 |       | .5904 |       | .5904 |       | .5904 |       |
TABLE OF SYMBOLS

\[ Q = \text{one-half of load applied to chain;} \]
\[ M_b = \text{bending moment at any chosen section;} \]
\[ M = \text{bending moment in side of link at end of minor axis;} \]
\[ P = \text{normal force on any section of link;} \]
\[ s = \text{intensity of stress at any point of a cross section;} \]
\[ \phi = \text{angle between any section and major axis of link;} \]
\[ \alpha = \text{one-half of assumed arc of contact between adjacent links;} \]
\[ f = \text{area of cross section of link;} \]
\[ E = \text{modulus of elasticity;} \]
\[ d = \text{diameter of iron in link;} \]
\[ r = \text{general symbol for radius of curvature;} \]
\[ r_2, r_3 = \text{radii of curvature of parts of link;} \]
\[ \varepsilon = \text{relative extension of any fiber;} \]
\[ \varepsilon_o = \text{relative extension of center line of link;} \]
\[ \omega = \text{ratio } \Delta d\phi : d\phi ; \]
\[ y = \text{distance of fiber from center line;} \]
\[ z = -\frac{1}{f} \int \frac{y}{r+y} df ; \]
\[ \Delta x, \Delta y = \text{deflections of major and minor axes of link, respectively;} \]
\[ S = \text{one-half of pressure between stud and side of link.} \]
APPENDIX A

THEORY OF STRESSES IN CURVED BARS *

Consider an element of the bar included between two cross sections $A_1A_2$ and $C_1C_2$, Fig. 29. The planes of these normal cross sections intersect in a line which pierces the plane of the paper at $M$; this line is the axis of curvature, that is, point $M$ is the center of curvature of the center line $AC$. For the sake of convenience, we shall make use of only one-half of the element, as shown in Fig. 30, and we shall consider the sides $B_1C_1$, $B_2C_2$ to be straight lines, since the sections $B_1B_2$ and $C_1C_2$ are taken indefinitely close to each other.

* The theory here given is substantially that laid down by Bach, Elasticität und Festigkeit, § 54.

Figs. 29, 30, 31.
Suppose now that an external force $P$ acts at right angles to the section $C_1C_2$. If this force is uniformly distributed over the cross section $C_1C_2$, each fiber will be elongated (or shortened) by an amount proportional to its original length. Thus, assuming that the stress is tensile, we shall have

$$\frac{C_1C_1'}{B_1C_1} = \frac{CC'}{BC'} = \frac{C_2C_2'}{B_2C_2} = \text{a constant.}$$

It follows that the plane of the cross section will in its new position $C_1'C_2'$ pass through the axis of curvature $M$.

In addition to the force $P$ normal to the section, let there be a couple of moment $M_b$ acting at the section in question. It is assumed that the sense of the couple is such as to increase the curvature of the bar. The section $C_1C_2$ of the unloaded bar is brought to $C_1'C_2'$ by the normal force $P$, as just explained. The couple causes it to assume a new position $C_1''C_2''$, Fig. 31. The plane of the section in this position intersects the plane of the section $B_1B_2$ in the line $M'$. The angle between the cross sections is increased from $d\phi$ to $d\phi + \Delta d\phi$, and the radius of curvature is shortened from $r$ to $\rho$.

Let $ds$ denote the length $BC$ and $\Delta ds$ the elongation $CC''$ due to the force $P$ and moment $M_b$. The ratio $\frac{\Delta ds}{ds}$ we shall denote by $\varepsilon_0$;

hence

$$\varepsilon_0 = \frac{CC''}{BC}.$$ 

Consider now a fiber lying along $PP_1$ at a distance $y$ from the center line $BC$. The extension of the fiber is $P_1P''$; hence we have

$$\varepsilon = \frac{P_1P''}{PP_1}.$$ 

To determine $\varepsilon$, let $C''D$ be drawn parallel to $C_1C_2$. Then

$$P_1P'' = P_1D + DP'' = CC'' + DP'' = \varepsilon_0 ds + y \cdot \text{angle } DC''P'' = \varepsilon_0 ds + y \cdot \Delta d\phi,$$

and

$$PP_1 = (y + r)d\phi.$$ 

Therefore

$$\varepsilon = \frac{\varepsilon_0 ds + y \cdot \Delta d\phi}{(r + y)d\phi} = \frac{\varepsilon_0 \frac{ds}{d\phi} + y \frac{\Delta d\phi}{d\phi}}{r + y}. $$
Let \( \omega \) denote the ratio \( \frac{\Delta d\phi}{d\phi} \); then since \( ds = r d\phi \),

\[
\varepsilon = \frac{\varepsilon_0^2 + \omega y}{r + y} = \varepsilon_0 + (\omega - \varepsilon_0) \frac{y}{r + y}.
\]  

(1)

If \( E \) is the modulus of elasticity, the stress corresponding to the elongation \( \varepsilon \) is

\[
s = E\varepsilon = E \left[ \varepsilon_0 + (\omega - \varepsilon_0) \frac{y}{r + y} \right].
\]  

(2)

The stresses developed over the section must hold in equilibrium the external forces and couples; hence denoting an element of the area of the section by \( df \), we have from ordinary static conditions,

\[
P = \int s \, df = \int E \left[ \varepsilon_0 + (\omega - \varepsilon_0) \frac{y}{r + y} \right] df.
\]  

(3)

\[
M_b = \int y \cdot s \, df = \int Ey \left[ \varepsilon_0 + (\omega - \varepsilon_0) \frac{y}{r + y} \right] df.
\]  

(4)

The integrals involved, considering \( E \) a constant, are

\[
\int df, \quad \int y \, df, \quad \int \frac{y}{r + y} \, df, \quad \text{and} \quad \int \frac{y^2}{r + y} \, df.
\]

Evidently \( \int df = i \), and since \( y \) is measured from a gravity axis, \( \int ydf = 0 \). For the sake of convenience, let

\[
\int \frac{y}{r + y} \, df = -zfr;
\]  

(5)

then

\[
\int \frac{y^2}{r + y} \, df = \int \left( y - r \frac{y}{r + y} \right) df = -r \int \frac{y}{r + y} \, df = zfr.
\]  

(6)

Inserting these values of the integrals in (3) and (4), we get

\[
P = Ef \left[ \varepsilon_0 - (\omega - \varepsilon_0) z \right],
\]

\[
M_b = Ef (\omega - \varepsilon_0) zr.
\]
By slight reduction the following important formulas are now obtained:

\[ \omega = \frac{1}{E f} \frac{M_b}{z r} \],

\[ \epsilon_0 = \frac{1}{E f} \left( P + \frac{M_b}{r} \right) \],

(B) \[ \omega = \frac{1}{E f} \left( P + \frac{M_b}{r} + \frac{M_b}{z r} \right) \].

Inserting the expressions for \( \epsilon_0 \) and \( \omega \) given by (A) and (B) in (2) we finally obtain for the intensity of stress at any point of the section

\[ s = \frac{P}{f} + \frac{M_b}{f r} + \frac{M_b}{z f r} \frac{y}{r + y} \].

(C)

Formula (C) gives the stress in terms of the force \( P \), couple \( M_b \), and other terms which depend solely upon the geometry of the system under consideration. In applying this formula care must be taken to give the quantities their proper signs. Thus:

\( P \) is positive when it tends to produce tension, negative when it tends to produce compression;

\( M_b \) is positive when it tends to increase the curvature of the bar, negative when it tends to decrease the curvature;

\( y \) is positive when measured towards the convex side of the bar, negative when measured towards the concave side, that is, towards the center of curvature.

When the value of \( s \) as determined from formula (C) is positive, the stress is tensile; if \( s \) is negative, the stress is compressive.

The function \( z \) as defined by (5), that is,

\[ z = -\frac{1}{f} \int \frac{y}{r + y} df \],

may be obtained by integration in the case of regular sections, circles, rectangles, etc. The following expressions for \( z \) are all that are required for present purposes.
For a circular cross section of radius $a$,

$$z = \frac{1}{4} \left( \frac{a}{r} \right)^2 + \frac{1}{8} \left( \frac{a}{r} \right)^4 + \frac{5}{64} \left( \frac{a}{r} \right)^6 + \frac{7}{128} \left( \frac{a}{r} \right)^8 + \ldots$$

$$\frac{1}{z} = 4 \left( \frac{r}{e} \right)^2 - 2 - \frac{1}{4} \left( \frac{e}{r} \right)^2 - \frac{1}{128} \left( \frac{e}{r} \right)^4 - \ldots$$

For a rectangular cross section of width $b$ and depth $2a$,

$$z = \frac{1}{3} \left( \frac{a}{r} \right)^2 + \frac{1}{5} \left( \frac{a}{r} \right)^4 + \frac{1}{7} \left( \frac{a}{r} \right)^6 + \ldots$$
APPENDIX B

ANALYSIS OF OPEN LINK

To give an idea of the general method employed, a simple case is taken first, the more complicated general case later.

It is assumed that a quadrant of the center line of the link is made up of two circular arcs, Fig. 32, one, $BE$, having a radius equal to the diameter $d$ of the iron of the link, the other arc, $EA$, having a radius $AC = r$. Denoting by $a$ and $b$ the major and minor semi-axes, $BO$ and $AO$, respectively, the following geometrical relations are easily deduced:

$$r = \frac{a^2 + b^2 - 2ad}{2(b - d)},$$

$$\sin \alpha = \frac{r - b}{r - d}, \quad \tan \alpha = \frac{r - b}{a - d}.$$

Let it be assumed first that the pressure between two links is concentrated at a point. Denoting this pressure by $2Q$, the normal...
force at section $A$ is $Q$. The unknown bending moment at this same section may be denoted by $M$. At any point $H$ between $E$ and $A$ on the center line introduce two equal and opposite forces each equal to $Q$. One of these forces may be combined with $Q$ giving a moment at $H$ of magnitude

$$Qr (1 - \sin \phi).$$

Adding to this moment the moment $M$ at section $A$, we have for the bending moment at section $H$,

$$M_b = M + Qr (1 - \sin \phi). \quad (1)$$

The other force at $H$ may be resolved into two components, one normal to the section and thus producing tension, the other lying in the plane of the section. The latter component is neglected. The former has the value,

$$P = Q \sin \phi. \quad (2)$$

For sections lying between $B$ and $E$, that is, for values of $\phi$ between 0 and $\alpha$, we have likewise,

$$M_b = M + Q (b - d \sin \phi), \quad (3)$$

$$P = Q \sin \phi. \quad (4)$$

The unknown moment $M$ is determined from the following considerations. As shown in Appendix A, the distortion of the link under load changes the angle $d\phi$ between two adjacent cross sections by the amount $\Delta d\phi$, this change being positive at some sections, negative at others. Because of the symmetry of the link, sections $A$ and $B$ originally at right angles remain at right angles; that is, the summation of the changes of angle $\Delta d\phi$ between $B$ and $A$ must be zero.

Hence

$$\sum_{B}^{A} \Delta d\phi = 0,$$

or since $\Delta d\phi = \omega \cdot d\phi$,

$$\int_{0}^{\alpha} \omega \, d\phi + \int_{\alpha}^{\pi} \omega \, d\phi = 0. \quad (5)$$
The general expression for $\omega$ [Eq. (B), Appendix A] is

$$\omega = \frac{1}{E_f} \left( P + \frac{M_b}{r} + \frac{M_b}{z r} \right). \quad (6)$$

For sections between $\phi = 0$ and $\phi = a$, $r = d$; hence,

$$\omega_1 = \frac{1}{E_f} \left( Q \sin \phi + \frac{M_b}{d} + \frac{M_b}{z_1 d} \right), \quad (7)$$

the subscript 1 being used to distinguish the $\omega$ and $z$ of this part of the link from those of the other part. For sections lying between $\phi = a$ and $\phi = \frac{\pi}{2}$,

$$\omega_2 = \frac{1}{E_f} \left( Q \sin \phi + \frac{M_b}{r} + \frac{M_b}{z_2 r} \right). \quad (8)$$

Inserting the proper values of the moment $M_b$ from (1) and (3) we have,

$$E_f \omega_1 = \frac{M + Q b}{d} \left( 1 + \frac{1}{z_1} \right) - \frac{Q}{z_1} \sin \phi, \quad (9)$$

$$E_f \omega_2 = \frac{M + Q r}{r} \left( 1 + \frac{1}{z_2} \right) - \frac{Q}{z_2} \sin \phi.$$  

Inserting these values of $\omega_1$ and $\omega_2$ in (5), integrating and reducing, we get finally,

$$M =$$

$$Q d \left[ \frac{1}{z_1} \left( 1 - \cos \alpha \right) + \frac{1}{z_2} \cos \alpha - \alpha \frac{b}{d} \left( 1 + \frac{1}{z_1} \right) - \left( 1 + \frac{1}{z_2} \right) \frac{\pi}{2} - \alpha \right] \alpha \left( 1 + \frac{1}{z_1} \right) + \frac{d}{r} \left( 1 + \frac{1}{z_2} \right) \left( \frac{\pi}{2} - \alpha \right)$$

$$= \frac{D}{(D)}$$

The value of $M$ being found by means of formula (D) the bending moment at any section is readily obtained; and with the bending
moment and normal force given, the intensity of stress at any fiber is readily determined by means of formula (C).

For a circular ring we have,

\[
\begin{align*}
P &= Q \sin \phi, \\
M_b &= M + Qr (1 - \sin \phi),
\end{align*}
\]

whence

\[
\int_{0}^{\pi/2} \omega \, d\phi = \int_{0}^{\pi/2} \left[ \left( \frac{M}{r} + Q \right) \left( 1 + \frac{1}{z} \right) - \frac{Q \sin \phi}{z} \right] d\phi = 0.
\]

Integrating and reducing,

\[
M = Qr \left( \frac{2}{\pi (1 + z)} - 1 \right).
\]

We shall now take up a more complicated example which agrees more closely with conditions met in practice. In the first place, the quadrant of the center line of the link cannot usually be represented closely by two circular arcs. Links actually measured show the form shown in Fig. 33. The center line BA is made up of four parts:
(1) the arc $BE$ of radius $d$ struck from $C_1$ as a center; (2) the arc $EF$ with radius $r_2$ and $C_2$ as center; (3) the arc $FG$ with radius $r_3$ and $C_3$ as center; (4) in some cases, a straight part of length $e$.

Secondly, as explained previously, the assumption that the pressure between adjacent links is concentrated at a point is not justified. We shall make the assumption that the pressure is distributed along an arc, as shown in Fig. 2 (b); afterwards the resulting equations will be modified to suit the assumption represented by Fig. 2 (c), namely that the pressure is regarded as concentrated at two points $E$ and $E$.

![Diagram of chain links](image)

The distributed pressure along the angle of contact $2\alpha$ (Fig. 34) gives rise to a normal force and a bending moment independent of the force $Q$ and moment $M$ at section $A$. We have now to derive expressions for the force $P$ and moment $M_b$ at any section included within the angle $\alpha$.

The length of an element of arc of the circumference in contact is $\frac{d}{2}d\phi$; hence the pressure over the element of arc with the assumption of uniform distribution is

$$p\frac{d}{2}d\phi.$$

The vertical component of this force is

$$p\frac{d}{2}\cos\phi d\phi,$$
and the sum of such vertical components must be in equilibrium with the external force $2Q$. That is,

$$\frac{pd}{2} \int_{-\alpha}^{\alpha} \cos \phi \, d\phi = 2Q,$$

whence

$$pd \sin \alpha = 2Q,$$

or

$$p = \frac{2Q}{d \sin \alpha}.$$

Take any section $OS$ making an angle $\theta$ with $OX$ and for the present consider the angle $\phi$ constant. We are to find the moment and normal force at section $T$ due to the distributed pressure between sections $T$ and $S$. The intensity of pressure in the direction $OS$ is $p$ and the pressure along an element of arc of the circumference is therefore

$$dF = p \frac{d}{2} d\theta.$$

Now at $T$ introduce two equal and opposite forces parallel to $OS$ and of magnitude $dF$. One of these combines with $dF$ acting through $S$ to form a couple whose moment is,

$$d \sin (\theta - \phi) dF = p \frac{d^2}{2} \sin (\theta - \phi) d\theta.$$

The other force $dF$ is resolved into components, the one perpendicular to section $T$ being

$$p \frac{d}{2} \sin (\theta - \phi) d\theta.$$

If now we vary $\theta$ from $\phi$ to $\alpha$ and take the sum of the forces and moments for each element $d\theta$, we get:

Normal force at $T = \frac{pd}{2} \int_{\phi}^{\alpha} \sin (\theta - \phi) \, d\theta$;

Moment at $T = \frac{pd^3}{2} \int_{\phi}^{\alpha} \sin (\theta - \phi) \, d\theta$. 
These are the force and moment at section $T$ arising from the distributed pressure between sections $T$ and $U$.

Taking $\phi$ constant, we obtain

$$\int_\phi^\alpha \sin(\theta - \phi)d\theta = 1 - \cos(\alpha - \phi),$$

and making use of the relation $p = \frac{2\theta}{d \sin \alpha}$ we get:

Normal force at $T = \frac{Q}{\sin \alpha}[1 - \cos(\alpha - \phi)]; \quad (12)$

Moment at $T = \frac{Qd}{\sin \alpha}[1 - \cos(\alpha - \phi)]. \quad (13)$

The normal force and moment at section $T$ due to the force $Q$ and moment $M$ at section $A$ have been shown to be respectively,

$$Q \sin \phi,$$

and

$$M + Q(b - d \sin \phi).$$

The total normal force at section $T$ is therefore,

$$P = Q \sin \phi + \frac{Q}{\sin \alpha}[1 - \cos(\alpha - \phi)] = \frac{Q}{\sin \alpha} - Q \cot \alpha \cos \phi. \quad (14)$$

The moment at $T$ due to the distributed pressure is opposite in sense to the moment due to $Q$ and $M$ at section $A$; hence the net moment at $T$ is

$$M_b = M + Qb - Qd \sin \phi - \frac{Qd}{\sin \alpha}[1 - \cos(\alpha - \phi)]$$

$$= M + Qb - \frac{Qd}{\sin \alpha} + Qd \cot \alpha \cos \phi. \quad (15)$$

Referring to Fig. 33, we see that (14) and (15) give the values of
For the arc \( EF \) with radius \( r_2 \), the values of \( M_b \) and \( P \) are found to be,

\[
\begin{align*}
M_b &= M + Qh - Qr_2 \sin \phi, \\
P &= Q \sin \phi,
\end{align*}
\]

whence for this arc, \( \omega_2 \) is given by the expression

\[
\omega_2 = \frac{1}{E}\left[ \frac{M + Qh}{r_2} \left(1 + \frac{1}{z_2}\right) - \frac{Q}{z_2} \sin \phi \right].
\] (18)

For the arc \( FG \), likewise

\[
\begin{align*}
M_b &= M + Qr_3 - Qr_2 \sin \phi, \\
P &= Q \sin \phi,
\end{align*}
\]

whence

\[
\omega_3 = \frac{1}{E}\left[ \frac{M + Qr_3}{r_3} \left(1 + \frac{1}{z_3}\right) - \frac{Q}{z_3} \sin \phi \right].
\] (20)

For the straight part \( GA \),

\[
\begin{align*}
M_b &= M, \\
P &= Q,
\end{align*}
\]

whence

\[
\omega_4 = \frac{1}{E}\left[ Q + \frac{M}{r_4} \left(1 + \frac{1}{z_4}\right) \right].
\] (21)

Since the normal sections at \( B \) and \( A \) must remain at right angles, the summation of \( \omega \cdot d\phi \) from \( B \) to \( A \) must be zero; that is

\[
\int_0^\alpha \omega_1 d\phi + \int_\alpha^\beta \omega_2 d\phi + \int_\beta^\gamma \omega_3 d\phi + \int_\gamma^A \omega_4 d\phi = 0.
\] (22)

The first three integrals present no difficulties. Care must be taken in evaluating the last integral, however, because of the infinite factors that it contains. The expression for \( \frac{1}{z_4} \) is

\[
\frac{1}{z_4} = 16 \left( \frac{r_4}{d} \right) - 2 - \frac{1}{16 \left( \frac{d}{r_4} \right)} \ldots
\]
Now since for the straight part $GA$, $r_4$ is infinite, it appears that $\frac{1}{z_4}$ is also infinite; and neglecting the finite terms, we may write (21) in the form

$$\omega_4 = \frac{1}{E_f} \cdot \frac{M}{r_4} \frac{16r_4^2}{d^2} = \frac{16M}{r_4} \cdot \frac{16}{E_f} \cdot \frac{d^2}{r_4}.$$ 

Therefore

$$\omega_4 \cdot d\phi = \frac{16M}{E_fd^2} r_4 \cdot d\phi.$$ 

But $r_4 d\phi = ds$; hence

$$\int_0^\pi \omega_4 d\phi = \frac{16M}{E_fd^2} \int_0^\pi ds = \frac{1}{E_f} \cdot \frac{16M}{d^2}.$$ 

(23)

If now we substitute in (22) the values of $\omega_1$, $\omega_2$, and $\omega_3$ given by (16), (18) and (20) and for the fourth integral the value just obtained we get the following (neglecting the constant factor $E_f$):

$$\begin{align*}
M + \frac{Qb}{d} \left(1 + \frac{1}{z_1}\right) \int_0^\alpha d\phi - \frac{Q}{z_1} \sin \alpha \int_0^\alpha d\phi + \frac{Q}{z_1} \cot \alpha \int_0^\alpha \cos \phi d\phi \\
+ \frac{M+Qh}{r_2} \left(1 + \frac{1}{z_2}\right) \int_0^\alpha d\phi - \frac{Q}{z_2} \int_0^\alpha \sin \phi d\phi + \frac{M+Qr_3}{r_3} \left(1 + \frac{1}{z_3}\right) \int_0^\alpha d\phi \\
- \frac{Q}{z_3} \int_0^\alpha \sin \phi d\phi + 16 \frac{M_e}{d^2} &= 0.
\end{align*}$$

Integrating and reducing, the following is obtained:

$$M = -Qd$$

$$\begin{align*}
\int \left[ \frac{a}{d^2} \left(1 + \frac{1}{z_1}\right) - \frac{1}{z_1} \left(\frac{\alpha}{\sin \alpha} - \cos \alpha\right) + \frac{b}{z_2} \left(1 + \frac{1}{z_2}\right) \left(\beta - \alpha\right) \\
- \frac{1}{z_2} \left(\cos \alpha - \cos \beta\right) + \left(1 + \frac{1}{z_2}\right) \frac{\pi}{2} - \beta - \frac{\cos \beta}{z_3} \right]
\end{align*}$$

(F')

If the assumption is made that the pressure may be considered as concentrated at two points subtending the angle $2\alpha$ (see Fig. 2 (c)), we readily obtain instead of (12) and (13):

Normal force at $T = Q \sec \alpha \sin (\alpha - \phi)$. 

(12')

Moment at $T = Qd \sec \alpha \sin (\alpha - \phi)$. 

(13')
Equations (14) and (15) then become

\[ P = Q \tan \alpha \cos \phi, \quad (14') \]

\[ M_b = M + Qb - Qd \tan \alpha \cos \phi, \quad (15') \]

whence

\[ \omega_i = \frac{1}{E} \left[ \frac{M + Qb}{\alpha} \left( 1 + \frac{1}{z_i} \right) - \frac{Q}{z_i} \tan \alpha \cos \phi \right]. \quad (16') \]

Using this value of \( \omega_i \) in (22), we get finally

\[ M = -Qd \left[ \frac{b}{d} \left( 1 + \frac{1}{z_i} \right) - \frac{1}{z_i} \tan \alpha \sin \alpha + \frac{h}{\alpha} \left( 1 + \frac{1}{z_i} \right) \left( \beta - \alpha \right) \right] \]

\[ \frac{1}{z_i} \left( \cos \alpha - \cos \beta \right) + \left( 1 + \frac{1}{z_i} \right) \left( \frac{\pi}{2} - \beta \right) - \frac{\cos \beta}{z_i} \]

\[ \alpha \left( 1 + \frac{1}{z_i} \right) + \frac{d}{r_1} \left( 1 + \frac{1}{z_i} \right) \left( \beta - \alpha \right) + \frac{d}{r_2} \left( 1 + \frac{1}{z_i} \right) \left( \frac{\pi}{2} - \beta \right) + 16 \frac{d}{d} \] \quad (F')

It will be observed that the change in the assumed law of distribution changes the second term in the numerator of (F) from

\[ \frac{1}{z_i} \left( \frac{\alpha}{\sin \alpha} - \cos \alpha \right) \]

to

\[ \frac{1}{z_i} \tan \alpha \sin \alpha. \]

If we assume concentration at the end of the link this term is

\[ \frac{1}{z_i} (1 - \cos \alpha). \]

From the value of \( M \) as determined from (F') or (F'), the bending moment \( M_b \) at any section is obtained, and then the stress at any fiber is found by means of (C).
APPENDIX C

DERIVATION OF THEORETICAL FORMULAS FOR THE CHANGE OF LENGTH IN THE AXES OF THE LINK

The analysis given in Appendices A and B may be employed to calculate the change in length of either axis of the link due to a given load. As has been stated, the comparison of this calculated change of length with the change actually measured is the basis of the experimental verification of the theory.

The following discussion is substantially that given by Bach: Let $O P C D$, Fig. 35, be the center line of a curved bar before it is subjected to external forces. The point $O$ is chosen as the origin, and the tangent and normal at $O$ are taken as the $Y$- and $X$-axes respectively. When the bar is subjected to external forces, it is distorted and the center line changes its form. Any point $C$ is thereby moved to a new position, and if $x_c$, $y_c$ are the original coordinates...
of C, these receive increments $\Delta x_e$ and $\Delta y_e$ respectively. These
increments we now propose to determine by the principles hitherto
developed.

Choose any point P whose coordinates are $x$, $y$, and let $r$ be the
radius of curvature at this point. An element of arc at P has the
direction $PT$ and its length is $ds = r d\phi$, where $\phi$ as usual denotes the
angle between $r$ and the $x$-axis. Because of the action of the external
forces and couples, this element of arc will turn about P — which
we for the present consider fixed — through the angle $\Delta d\phi$. This
rotation causes the point C to move to $C_1$ on the circular arc
$CC_1 = PC \cdot \Delta d\phi$. The components of the displacement $CC_1$ along
the $X$- and $Y$-axes are respectively

$$PC \cdot \Delta d\phi \sin PCF = PC \cdot \sin PCF \cdot \Delta d\phi = (y_e - y) \cdot \Delta d\phi$$
and

$$-PC \cdot \Delta d\phi \cos PCF = -PC \cdot \cos PCF \cdot \Delta d\phi = -(x_e - x) \cdot \Delta d\phi \quad (1)$$

In addition to the coordinate increments due to the change in
inclination of the section at $P$ there are increments due to the
lengthening (or shortening) of the arc element $ds$ at $P$. The exten-
sion of the element is $\varepsilon_0 ds$; hence because of this extension the point
$C$ is moved in the direction of the $X$-axis a distance,

$$\varepsilon_0 ds \sin \phi = \varepsilon_0 dx$$
and in the direction of the $Y$-axis a distance

$$\varepsilon_0 ds \cos \phi = \varepsilon_0 dy.$$

Adding together the changes just deduced (and replacing $\Delta d\phi$ by
$\omega \cdot df$), we have:

$$\begin{align*}
\text{change along } X\text{-axis} &= (y_e - y) \omega d\phi + \varepsilon_0 dx, \\
\text{change along } Y\text{-axis} &= -(x_e - x) \omega d\phi + \varepsilon_0 dy.
\end{align*} \quad (2)$$

The total increments of the coordinates $x_e$ and $y_e$, made up of the
changes for all the arc elements lying between $O$ and $C$, are found
by summation. Thus,

$$\begin{align*}
\Delta x_e &= y_e \int_0^{\phi_e} \omega d\phi - \int_0^{\phi_e} y \alpha d\phi + \int_0^{\phi_e} \varepsilon_0 dx, \\
\Delta y_e &= \int_0^{\phi_e} x \omega d\phi - x_e \int_0^{\phi_e} \omega d\phi + \int_0^{\phi_e} \varepsilon_0 dy.
\end{align*} \quad (G)$$
We have now to apply these fundamental formulas (a) to the circular ring and (b) to the chain link.

I. Distortion of Circular Ring

Referring to Fig. 36, it is evident that the point $A$ at the extremity of the transverse diameter is the point whose coordinate increments are desired. Denoting by $x, y$ the coordinates of any point $H$ on the center line between $A$ and $B$, we have

$$
\begin{align*}
  x_a &= y_a = r, \\
  x_a - x &= r \cos \phi, \\
  y &= r \sin \phi.
\end{align*}
$$

At $H$, the normal force and bending moment are

$$
\begin{align*}
  P &= Q \sin \phi, \\
  M_b &= M + Qr (1 - \sin \phi),
\end{align*}
$$

whence

$$
Ef \cdot \varepsilon_0 = \frac{M + Qr}{r},
$$

and

$$
Ef \cdot \omega = \frac{M + Qr}{r} \left(1 + \frac{1}{z}\right) - \frac{Q \sin \phi}{z}.
$$
Since for the quadrant $BA, \int_0^\pi \omega d\phi = 0$, the first of equations (G) reduces to the simpler form

$$\Delta x_a = - \int_0^{\pi/2} y \omega \, d\phi + \int_0^r \xi_0 \, dx. \quad (7)$$

Inserting in (7) proper values from (3), (5) and (6), we get,

$$Ef \cdot \Delta x_a = -(M + Qr) \left(1 + \frac{1}{z}\right) \int_0^{\pi/2} \sin \phi \, d\phi + \frac{Qr}{z} \int_0^{\pi/2} \sin^2 \phi \, d\phi$$

$$+ \frac{M + Qr}{r} \int_0^r \, dx.$$  

$$= -(M + Qr) \left(1 + \frac{1}{z}\right) + \frac{Qr\pi}{4z} + M + Qr$$

$$= -\frac{1}{z} (M + Qr) + \frac{\pi}{4z} Qr.$$

In a similar way,

$$\Delta y_a = - \int_0^{\pi/2} (x_a - x) \omega \cdot d\phi + \int_0^r \xi_0 \, dy = - \int_0^{\pi/2} r \cos \phi \cdot \omega \, d\phi + \int_0^r \xi_0 \, dy.$$  

$$Ef \cdot \Delta y_a = -(M + Qr) \left(1 + \frac{1}{z}\right) \int_0^{\pi/2} \cos \phi \, d\phi + \frac{Qr}{z} \int_0^{\pi/2} \sin \phi \cos \phi \, d\phi$$

$$+ \frac{M + Qr}{r} \int_0^r \, dy.$$  

$$= -\frac{1}{z} (M + Qr) + \frac{Qr}{2z}.$$

These results may be written as follows:

$$\Delta x_a = - \frac{1}{Ef} \cdot \frac{1}{z} \left[ M + Qr \left(1 - \frac{\pi}{4}\right) \right], \quad (H)$$

$$\Delta y_a = - \frac{1}{Ef} \cdot \frac{1}{z} \left[ M + \frac{1}{2} Qr \right].$$
THE STRENGTH OF CHAIN LINKS

If finally the expression for $M$ given by equation (E) be introduced, the equations take the form

$$
\begin{align*}
\Delta x_a &= -\frac{1}{Ef} \cdot \frac{QR}{z} \left[ \frac{2}{\pi (1 + z)} - \frac{\pi}{4} \right], \\
\Delta y_a &= -\frac{1}{Ef} \cdot \frac{QR}{z} \left[ \frac{2}{\pi (1 + z)} - 0.5 \right].
\end{align*}
$$

(H')

II. Distortion of Chain Link

Referring to Fig. 33, the point $A$ at the end of the minor axis is the point whose movement under load is desired. Since for the quadrant $BA$, $\int \omega \, d\phi = 0$, the equations (G) when applied to the coordinates of point $A$ become simply,

$$
\begin{align*}
\Delta x_a &= -\int_0^{\phi_a} y\omega \, d\phi + \int_0^{x_a} \varepsilon_0 \, dx, \\
\Delta y_a &= \int_0^{\phi_a} x\omega \, d\phi + \int_0^{y_a} \varepsilon_0 \, dy.
\end{align*}
$$

(G')

There are four parts in the quadrant $BA$ of the center line; hence separate expressions for $x$, $y$, $\omega$ and $\varepsilon_0$ must be derived for each of these parts and furthermore the integrations must be separated.

From the geometry of Fig. 33, and from the results obtained previously (Appendix B) we have the following:

$$
\begin{align*}
x &= d(1 - \cos \phi), \quad y = d \sin \phi, \\
P &= \frac{Q}{\sin \alpha} - Q \cot \alpha \cos \phi, \\
M_b &= M + Qb - \frac{Qd}{\sin \alpha} + Qd \cot \alpha \cos \phi, \\
Ef \cdot \varepsilon_0 &= \frac{M + Qb}{d}, \\
Ef \cdot \omega &= \frac{M + Qb}{d} \left(1 + \frac{1}{z_1}\right) - \frac{Q}{z_1} \left(\frac{1}{\sin \alpha} + \cot \alpha \cos \phi\right).
\end{align*}
$$

For arc $BE$, subtending angle $\alpha$
\[
\begin{align*}
\text{For arc } EF, \\
\text{subtending angle } \beta - \alpha
\end{align*}
\]
\[
\left\{ \begin{array}{l}
x = i - r_2 \cos \phi, \quad y = r_2 \sin \phi + b - h. \\
P = Q \sin \phi. \\
M_b = M + Qh - Qr_2 \sin \phi. \\
E_f \cdot \varepsilon_0 = \frac{M + Qh}{r_2}. \\
E_f \cdot \omega = \frac{M + Qh}{r_2} \left(1 + \frac{1}{z_2} \right) - \frac{Q}{z_2} \sin \phi.
\end{array} \right.
\]
\[
\text{For arc } FG, \\
\text{subtending angle } \frac{\pi}{2} - \beta
\]
\[
\left\{ \begin{array}{l}
x = a - r_3 \cos \phi, \quad y = b - r_3 + r_3 \sin \phi. \\
P = Q \sin \phi. \\
M_b = M + Qr_3 - Qr_3 \sin \phi. \\
E_f \cdot \varepsilon_0 = \frac{M + Qr_3}{r_3}. \\
E_f \cdot \omega = \frac{M + Qr_3}{r_3} \left(1 + \frac{1}{z_3} \right) - \frac{Q}{z_3} \sin \alpha.
\end{array} \right.
\]
\[
\text{For straight part } GA
\]
\[
\left\{ \begin{array}{l}
x = a + u, \quad y = b. \\
P = Q. \\
M_b = M. \\
E_f \cdot \varepsilon_0 = Q. \\
E_f \cdot \omega = 16 M \frac{r_4}{d^2}.
\end{array} \right.
\]

The above expressions for \( x, y, \varepsilon_0 \) and \( \omega \) are substituted in the equations (G') and the resulting integrals are evaluated for each arc separately. Then the results are combined.

For the straight part \( GA \), the radius \( r_4 \) is infinite, and care must be exercised to get correct results. Letting \( u \) denote the distance of a point in \( GA \) from \( G \), we have
\[
x = a + u,
\]
whence
\[
x \omega \, d\phi = 16 M \frac{r_4}{d^2} (a + u) \, r_4 \, d\phi.
\]
But
\[
r_4 \, d\phi = du,
\]
THE STRENGTH OF CHAIN LINKS

and therefore
\[ \int xw \, d\phi = \frac{16}{d^2} \int_0^1 (a + u) \, du = 16 \frac{M}{d^2} \left( a + \frac{e^2}{2} \right). \]

Likewise
\[ \int yw \, d\phi = \frac{16}{d^2} \int_0^1 du = 16 \frac{Mb}{d^2}. \]

It is readily seen that for the part \( GA \),
\[ \int \varepsilon_y \, dx = Qe, \text{ and } \int \varepsilon_y \, dy = 0. \]

The results of the substitution and integration are the following equations:
\[ E_1 \cdot \Delta x_\alpha = - \frac{1}{z_1} \left( 1 - \cos \alpha \right) (M + Qb) - \frac{Qd}{2 \varepsilon_1} \left[ \frac{2 \left( 1 - \cos \alpha \right)}{\sin \alpha} - \sin \alpha \cos \alpha \right] \]
\[ - \frac{1}{z_2} (\cos \alpha - \cos \beta) (M + Qh) \]
\[ + \frac{Qr_2}{2z_2} \left[ \beta - \alpha + \sin \alpha \cos \alpha - \sin \beta \cos \beta \right] \]
\[ + \frac{h - b}{z_3} \left( 1 + \frac{1}{z_2} \right) (\beta - \alpha) (M + Qh) \]
\[ - \frac{1}{z_2} (\cos \alpha - \cos \beta) (h - b) Q - \frac{\cos \beta}{\varepsilon_3} (M + Qr_3) \]
\[ + \frac{Qr_3}{2z_3} \left( \frac{\pi}{2} - \beta + \sin \beta \cos \beta \right) \]
\[ + \left( 1 + \frac{1}{z_2} \right) \left( \frac{r_3 - b}{z_3} \right) \left( \frac{\pi}{2} - \beta \right) (M + Qr_2) \]
\[- (r_3 - b) \frac{\cos \beta}{z_2} Q + Qe - \frac{16 be}{d^2} M. \quad (J) \]
\[ E_1 \cdot \Delta y_\alpha = \alpha \left( 1 + \frac{1}{z_1} \right) (M + Qb) + \frac{Qd}{z_1} (1 + \cos \alpha) \]
\[ - \frac{Qd}{2z_1} \left( \frac{2 \alpha - \alpha \cos \alpha}{\sin \alpha} + \cos^2 \alpha \right) \]
\[ - \frac{1}{z_1} \sin \alpha (M + Qb) \]
\[ + \frac{i}{z_2} \left( 1 + \frac{1}{z_2} \right) (\beta - \alpha) (M + Qh) \]
\[ - \frac{1}{z_2} (\sin \beta - \sin \alpha) (M + Qh) \]
It will be observed that both (J) and (K) have the general form,

\[ Ef' = c_1 M + c_2 Q \]  

(8)

where \( c_1 \) and \( c_2 \) are constants depending entirely upon the dimensions and configuration of the link. Since however,

\[ M = kQ \]

we have

\[ \Delta x_a = c'Q, \quad \Delta y_a = c''Q \]  

(9)

where \( c' \) and \( c'' \) are other constants. Equation (9) shows that the change in the length of either axis is directly proportional to the load.
APPENDIX D

ANALYSIS OF STUD-LINK.

When the link has a transverse stud, as shown in Fig. 37, the analysis of the stresses is much more complicated than in the case of the open link. Hence we shall give here only the general method of attack and the final equations.

Referring to Fig. 37, let \( 2S \) denote the pressure between the end of the stud and the side of the link. Then one-half of this pressure may be considered as acting on the quadrant \( AB \) of the link; and to simplify the work, we shall assume the line of action of this force \( S \) to lie in the minor axis of the link. The quadrant \( AB \) is therefore subjected to the external force \( Q \) and moment \( M \), as before, and in addition to the transverse force \( S \).

The introduction of this force \( S \) gives rise to new terms in the expressions for the normal force and bending moment at any section.
Assuming the link quadrant to be made up of three circular arcs and a straight part, as shown in Fig. 33, we readily obtain the following results:

For arc $BE$,

\[
\begin{align*}
P &= \frac{Q}{\sin \alpha} - Q \cot \alpha \cos \phi + S \cos \phi, \\
M_b &= M + Qb - \frac{Qd}{\sin \alpha} + Qd \cot \alpha \cos \phi - S (a + e - d + d \cos \phi).
\end{align*}
\]

For arc $EF$,

\[
\begin{align*}
P &= Q \sin \phi + S \cos \phi, \\
M_b &= M + Qh - Qr \sin \phi - S (a + e - i + r \cos \phi).
\end{align*}
\]

For arc $FG$,

\[
\begin{align*}
P &= Q \sin \phi + S \cos \phi, \\
M_b &= M + Qr \sin \phi - S (e + r \cos \phi).
\end{align*}
\]

For straight part $GA$,

\[
\begin{align*}
P &= Q, \\
M_b &= M - S (e - u).
\end{align*}
\]

From these expressions for $P$ and $M_b$ we may derive expressions for $\varepsilon_a$ and $\omega$ as in Appendix C for open links.

We have in the case of the stud link two unknown quantities to determine, the force $S$ and the moment $M$ at the section $A$; hence we must obtain two relations between $M, S,$ and $Q$. As in the analysis of the open link, one equation is found from the relation

\[
\int_{\text{Sec. } B}^{\text{Sec. } A} \omega \cdot d\phi = 0. \tag{1}
\]

To obtain a second relation, we make use of the fact that the decrease in the length of the minor axis must be equal to the decrease in the length of the stud. Now an expression for the change of the minor axis may be found by the method of Appendix C, using equations (G'). The length of the stud may be taken as $2b - d$; hence if $E'$ and $f'$ denote respectively the modulus of elasticity of the material of the stud and the average area of cross-section, we have

\[
\text{decrease of length} = \frac{2S (2b - d)}{E'f'}. \tag{2}
\]
One-half of this decrease of length is equal to $\Delta y_a$, the change of the \textit{Y-} coordinate of the point $A$ of the link. We have therefore,

$$- \Delta y_a = \frac{S (2b - d)}{E'f'}.$$  

(3)

If now we denote by $k$ the ratio $\frac{E_f}{E'}$ we obtain finally

$$- E_f \cdot \Delta y_a = kS (2b - d).$$

(4)

Equation (4) gives a second relation between $M$, $S$ and $Q$.

The following simultaneous equations are finally obtained:

$$A_1 M = B_1 S d - C_1 Q d,$$
$$A_2 M = B_2 S d - C_2 Q d,$$

(5)

in which the coefficients have the following values:

$$A_1 = \alpha \left( 1 + \frac{1}{z_1} \right) + \frac{d}{r_2} \left( 1 + \frac{1}{z_2} \right) \left( \beta - \alpha \right) + \frac{d}{r_3} \left( 1 + \frac{1}{z_3} \right) \left( \frac{\pi}{2} - \beta \right) + 16 \frac{e}{d}.$$

$$B_1 = \alpha \left( 1 + \frac{1}{z_1} \right) \frac{a + e - d}{d} + \frac{1}{z_2} \sin \alpha + \left( 1 + \frac{1}{z_2} \right) \frac{a + e - i}{r_2} \left( \beta - \alpha \right)$$

$$+ \frac{1}{z_2} \left( \sin \beta - \sin \alpha \right)$$

$$+ \frac{e}{r_3} \left( 1 + \frac{1}{z_3} \right) \left( \frac{\pi}{2} - \beta \right) + \frac{1}{z_3} \left( 1 - \sin \beta \right) + 8 \frac{e^2}{d^2}.$$
\[ + \frac{i}{r_z} \left( \frac{a + e - i}{d} \right) \left( 1 + \frac{1}{z_2} \right) (\beta - d) - \frac{1}{z_2} \left( \frac{a + e - 2i}{d} \right) \left( \sin \beta - \sin \alpha \right) \]

\[ - \frac{1}{2z_2} \left( \frac{\beta - \alpha + \sin \beta \cos \beta - \sin \alpha \cos \alpha}{d} \right) \]

\[ + \frac{1}{z_3} \left( \frac{a - e}{d} \right) (1 - \sin \beta) + \frac{ae}{r_3d} \left( 1 + \frac{1}{z_3} \right) \left( \frac{\pi}{2} - \beta \right) \]

\[ - \frac{1}{2z_3} \left( \frac{\pi}{2} - \beta - \sin \beta \cos \beta \right) + \frac{8}{3} \frac{e^2}{d^3} (3a + e) - k \left( \frac{2b - d}{d} \right). \]

\[ C_z = \frac{a}{d} \left( 1 + \frac{1}{z_1} \right) - \frac{1}{z_1} \left( \frac{\alpha}{\sin \alpha - \cos \alpha} \right) - \frac{1}{z_1} \left( \frac{b}{d} \sin \alpha - 1 \right) \]

\[ - \frac{1}{2z_2} \frac{\alpha}{d} \left( \alpha \cot \alpha + \cos^2 \alpha \right) + \frac{i\hbar}{r_2d} \left( 1 + \frac{1}{z_2} \right) (\beta - \alpha) \]

\[ - \frac{1}{z_2} \frac{i}{d} \left( \cos \alpha - \cos \beta \right) - \frac{1}{z_2} \frac{\hbar}{d} \left( \sin \beta - \sin \alpha \right) \]

\[ + \frac{1}{2z_2} \frac{r_2}{d} \left( \sin^2 \beta - \sin^2 \alpha \right) + \frac{a}{d} \left( 1 + \frac{1}{z_3} \right) \left( \frac{\pi}{2} - \beta \right) - \frac{1}{z_3} \frac{\alpha}{d} \cos \beta \]

\[ - \frac{1}{z_3} \frac{r_3}{d} (1 - \sin \beta) + \frac{1}{2z_3} \frac{r_3}{d} (1 - \sin^2 \beta). \]

These coefficients are first determined from the known constants \( z_1, z_2, \) and \( z_3, \) and the known dimensions of the link. The solution of Eqs. (5) then gives the values of \( M \) and \( S, \) and from these, values of the normal force \( P \) and moment \( M_b \) for any section are readily found. Having \( P \) and \( M_b, \) the stress at any point of the cross section is found from the general equation (C).

With the open link the greatest tensile stress is either at the end or at the side of the link, that is, at sections on the major or minor axis. See Fig. 26. In the case of the stud link, the greatest tensile stress is usually at a point on the inside of the link at some distance from the end of the minor axis. See Fig. 27. To determine the exact position of the section of maximum tension, we insert the expressions for \( P \) and \( M_b \) in (C) and thus obtain

\[ S = c + m (Q \sin \phi + S \cos \phi), \]

in which \( c \) and \( m \) are constant for all sections.
Taking the first derivative, we get,
\[ \frac{dS}{d\phi} = m (Q \cos \phi - S \sin \phi), \]
and equating this to zero, we find
\[ \frac{Q}{S} = \tan \phi. \]
Hence at the section for which
\[ \phi = \tan^{-1} \frac{Q}{S}, \]
the tensile stress will be a maximum.
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION


Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. (Out of print).


