TYPE-C WIND TURBINE MODEL ORDER REDUCTION AND PARAMETER IDENTIFICATION

BY

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THESIS

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ABSTRACT

Due to the increasing penetration of wind power, engineers are finding it useful to have models to investigate wind turbine performance. For different applications, an accurate modeling of aerodynamics, power electronics, electrical transients, or control systems are necessary. In other scenarios, these models may be unavailable or needlessly complex. Through use of singular perturbation analysis, this research shows that under normal system operating conditions the power injected in the network by a Type-C wind turbine generator (WTG) can be described by a first-order nonlinear dynamical model that relates the WTG power output to wind speed.

In this work the proposed model is validated through simulation and comparison to a published differential algebraic equation model as well as comparison to wind and power data measured from a real wind turbine. A parameter identification problem is also explored using the same data set. This analysis shows that turbine parameters can be estimated during normal operation of the machine. This technique may also be used in planning studies or to identify malfunctioning or underperforming turbines that require maintenance.
To my parents, for their love and support.
ACKNOWLEDGMENTS

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<th>Description</th>
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<tbody>
<tr>
<td>ARMAX</td>
<td>Autoregressive Moving Average with Exogenous Inputs</td>
</tr>
<tr>
<td>DAE</td>
<td>Differential Algebraic Equation</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input and Single-Output</td>
</tr>
<tr>
<td>WTG</td>
<td>Wind Turbine Generator</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Wind turbine blade pitch angle [degree]</td>
</tr>
<tr>
<td>$\beta_{\text{min}}, \beta_{\text{max}}$</td>
<td>Minimum and maximum blade pitch angle [degree]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>WTG tip speed ratio, dimensionless</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Intermediate variable used in calculating $C_p$, dimensionless</td>
</tr>
<tr>
<td>$\epsilon_1, \cdots, \epsilon_5$</td>
<td>Singular perturbation constants, dimensionless</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Voltage angle of infinite bus [rad]</td>
</tr>
<tr>
<td>$\phi_D$</td>
<td>DFIG terminal voltage angle [rad]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of air [kg/m$^3$]</td>
</tr>
<tr>
<td>$\omega_{\text{turbine}}$</td>
<td>Angular speed of wind turbine [rad/s]</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Electrical rotor speed [rad/s]</td>
</tr>
<tr>
<td>$\omega_{\text{ref}}$</td>
<td>Electrical rotor speed reference [rad/s]</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Synchronous electrical speed [rad/s]</td>
</tr>
<tr>
<td>$A$</td>
<td>Wind turbine swept area [m$^2$]</td>
</tr>
<tr>
<td>$B$</td>
<td>WTG torque parameter [p.u.$s^3$/m$^3$]</td>
</tr>
</tbody>
</table>
$C$ WTG power parameter [p.u. $s^3$]

$c_1, \cdots, c_9$ Performance coefficient parameters

$C_p$ Wind turbine power coefficient, dimensionless

$E'_d, E'_q$ d-axis and q-axis transient internal voltage [p.u.]

$H_D$ WTG inertia constant [s]

$I_{gc}$ complex current absorbed by grid side converter [p.u.]

$I_{qr}, I_{dr}$ DFIG rotor currents in d-axis and q-axis [p.u.]

$I_{qs}, I_{ds}$ DFIG stator currents in d-axis and q-axis [p.u.]

$K_P, K_I$ Proportional and integral gains of pitch angle controllers

$K_{I1}, \cdots, K_{I5}$ Integral gains of WTG power controllers

$K_{P1}, \cdots, K_{P4}$ Proportional gains of WTG power controllers

$M$ Scaled WTG inertia constant [$\text{frac{s}^2\text{rad}}$]

$P$ Active power injected by WTG [p.u.]

$P_R$ Rated output of WTG [W]

$P_{\text{ref}}$ WTG active power reference signal [p.u.]

$P_{\text{wind}}$ Theoretical amount of power in a steam of wind [W]

$Q$ Reactive power injected by WTG [p.u.]

$Q_{\text{ref}}$ WTG reactive power reference signal [p.u.]

$Q_{\text{set-point}}$ WTG reactive power set-point [p.u.]

$R$ Wind turbine blade radius [m]

$R_{\text{line}}$ Transmission line resistance [p.u.]

$T'_0$ DFIG transient open circuit time constant [s]

$T_m$ Mechanical torque of WTG [p.u.]

$V_C$ Wind turbine cut-in wind speed [$\frac{\text{m}}{\text{s}}$]

$V_D$ DFIG terminal voltage magnitude [p.u.]

$V_R$ Wind turbine rated wind speed [$\frac{\text{m}}{\text{s}}$]

$V_F$ Wind turbine furling wind speed [$\frac{\text{m}}{\text{s}}$]
\( V_{qr}, V_{dr} \)  DFIG rotor voltages in d-axis and q-axis [p.u.]
\( V_{qs}, V_{ds} \)  DFIG stator voltages in d-axis and q-axis [p.u.]
\( v \)  Wind speed \( [m/s] \)
\( x_1, x_2 \)  active power controller state variables [p.u.]
\( x_3, x_4 \)  reactive power controller state variables [p.u.]
\( x_5, x_6 \)  pitch angle controller state variables [p.u.]
\( X_{\text{line}} \)  Transmission line reactance [p.u.]
\( X_m \)  DFIG magnetizing reactance [p.u.]
\( X_r \)  DFIG rotor reactance [p.u.]
\( X_s \)  DFIG stator reactance [p.u.]
\( X'_s \)  DFIG stator transient reactance [p.u.]
CHAPTER 1
INTRODUCTION

1.1 Overview of Wind Energy

In a wind turbine, energy is extracted from the moving air by the blades and converted into rotational energy, which is transmitted via the drive unit to the rotor of the generator. There, the energy is converted from mechanical to electrical, and power flows from the stator out to the grid and finally to the consumer [1].

Type-C wind turbines are one of the most common types being installed today because they allow for variable speeds with less than fully rated power electronics. Variable speed is important because it allows for the most efficient extraction and reduced wear. This configuration is also known as a doubly fed induction generator (DFIG). A wound rotor induction generator is used, with rotor slip rings connected to a power frequency converter designed to inject rotor currents at the appropriate frequency. This converter is rated at approximately 30% of the nominal generator power. The stator is directly tied to the grid [2].

Depending on the type of analysis, different types of models are appropriate. For load flow studies, a static model is adequate. More detailed functional dynamic models can be used for transient stability, transient response, and control synthesis. The most complex physical models are necessary to analyze start-up transients, fault operation, and harmonics [3].

The vast majority of power system generation is composed of conventional synchronous generators. The swing equation is a well known model for this conventional generation that is simple, but also useful for transient stability and other types of power systems studies [4]. The purpose of this research is to develop a similar model for a type of renewable generation that is becoming a larger part of the power system.
1.2 Energy from the Wind

The power of a stream of air with density $\rho$ and cross-sectional area $A$ with a uniform, constant speed $v$ is

$$P_{wind} = \frac{1}{2} \rho A v^3.$$  

The fraction of this power converted into rotational energy at the hub (or electrical energy at the generator terminals, depending on the author) by the turbine is known as its power efficiency coefficient $C_p$. In 1919, Albert Betz used the conservation of mass and momentum to show that the theoretical upper limit of $C_p$ was $\frac{16}{27}$, or approximately 59%. This occurs when the turbine slows the wind passing through it to one third the upstream speed, $\frac{v_d}{v} = \frac{1}{3}$. The efficiency coefficient of modern wind turbines can approach 50% under ideal operating conditions [5].

The tip-speed ratio is the speed of the blade tip divided by the wind speed

$$\lambda = \frac{\omega_{turbine} R}{v},$$

where $\omega_{turb}$ is the rotational speed of the blades and $R$ is the distance from the center of the hub to the tip of the blade. Modern turbines can control the pitch of their blades using motors. We will denote this angle $\beta$. The performance coefficient can be approximated by a nonlinear function of $\lambda$ and $\beta$. Variable speed and pitch allow the turbines to track the maximum $C_p$ [2]:

$$C_p(\lambda, \beta) = c_1 \left( \frac{c_2}{\lambda_i} - c_3 \beta - c_4 \beta^3 + c_5 \right) e^{-\frac{c_7}{\lambda_i}},$$

$$\lambda_i = \left( \frac{1}{\lambda + c_8 \beta} - \frac{c_9}{\beta^3 + 1} \right)^{-1}.$$

The values for the constants $c_i$ were selected to fit manufacturer data by [2]. If the rotational speed data is not available or this complexity is not necessary or desired, a simpler model can be used that gives produced power as a function of wind speed and does not require knowledge of parameters that cannot be easily identified or any state variables. Figure 1.1 shows an idealized version of such a function in which generated power is zero up to some cut-in wind speed $V_C$, cubic in the wind speed past the cut-in speed.
until the rated speed $V_R$, constant from rated speed to the furling speed $V_F$, and finally zero after that. Mathematically this is written

$$P(v) = \begin{cases} \frac{P_R}{(V_R - V_C)^3}(v - V_C)^3 & V_C < v \leq V_R \\ P_R & V_R < v \leq V_F \\ 0 & \text{otherwise.} \end{cases}$$ (1.1)

Other smoother functions have also been proposed, but this is the most common model because of its simplicity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{idealized_power_curve}
\caption{Idealized wind turbine power curve}
\end{figure}

1.3 Review of Dynamic Models

For a more detailed description of the turbine, dynamic models must be used. These models are necessary to simulate the effects of the mechanical drive train, electric generator, power electronics, and other various control systems.

The mechanical drive train can be modeled in detail with the blades, hub, gear train, shafts, and generator all modeled separately, but it is often simplified to two rotating masses with a flexible, damped coupling between them. In [6], it is shown that this two mass model can offer improvements over a
single mass model transients that occur less than a second after a voltage drop.

For the generator, a fifth order model represents the dynamics of the flux linkages in both the stator and the rotor. A third order model neglects the stator dynamics, but is still used for transient stability analysis. Finally, a first order model would also neglect rotor dynamics, leaving just the swing equation. This model is still useful for longer term dynamic studies [7].

The power electronics for an indirect converter is usually a grid tied rectifier and a generator tied inverter connected with a DC link. If high frequency phenomena are being explored, such as during faults, a detailed model of these components is required. For normal power systems dynamic studies, however, it can be modeled as a simple fundamental frequency current source [2].

The many control systems on the WTG may have the most variation between different models. There is a controller to make sure the rotor is spinning at its optimal speed. The motors that adjust the pitch of the blades must be controlled. Another controller manages terminal voltage or reactive power injection. There are also controls to adjust yaw and possibly make other adjustments [1].

The full order dynamic model used for this research is described in [8]. There are ten differential equations, ten algebraic equations, one technical limit, and three reference inputs. There are three state variables which model the generator, two for the active power control, two for reactive power control, and three for pitch control. The network algebraic equations assume the machine is connected to an infinite bus through a line with a series impedance.

1.4 WTG Data Set Analysis

The data set used in this research was provided by industry partners and contains time-stamped wind speed, active and reactive power, and substation voltage at 1 second intervals. Over five million samples (roughly 2 months) of data was obtained for two turbines in a period in the spring of 2010. Several periods of time were observed in which turbines had zero power output even at significant wind speeds. The data was taken from a site that was not commonly curtailed, so it is assumed this was due to a maintenance issue.
These periods of time were excluded from use in this research.

Figure 1.2 shows the active power output of a single turbine plotted against the wind speed when the recording was taken. It is clear that the plot follows the same trend as the idealized curve, but for any given wind speed there is a significant variance in power. This is a limit of a static model that will be addressed in this thesis.

![Experimental wind turbine power curve and best fit line](image)

**Figure 1.2: Experimental wind turbine power curve and best fit line**

### 1.5 Outline of the Thesis

The thesis is split into two main parts. Chapter 2 focuses on model order reduction and derives a 1st order WTG model from a 10th order model. Simulations of both models are compared. Chapter 3 develops a procedure for identifying parameters for the model using the field data. Experimental results will also be presented. Chapter 4 will summarize the results and suggest future topics of study.
2.1 Modeling Issues and Order Reduction Techniques

When modeling wind turbines or other physical systems, high-order differential equations are often necessary to create a realistic representation. Often small time constants or other parameters are the reason for the increased order of the system. If a differential equation involving a small parameter $\epsilon$ becomes lower order for $\epsilon = 0$ than $\epsilon \neq 0$, it is called a singular perturbation problem. These stiff, high order systems can be difficult to work with, so techniques have been developed to simplify the design with a reduced order model that captures the dominant phenomena. If the disregarded phenomena are also required, a second step can be taken to construct a boundary layer model [9], [10].

In this research, a model order reduction was performed on the 10th order model from [8]. If the stator losses are negligible and the turbine operates between cut-in and rated speed, the whole system of differential and algebraic equations reduces to

$$\frac{d\omega_r}{dt} = \frac{\omega_s}{M} \left( BC_p(\omega_r, v, 0) \frac{v^3}{\omega_r} - C\omega_r^2 \right)$$

(2.1)

$$P = C\omega_r^3,$$

(2.2)

where $\omega_r$ is the electrical rotor speed, $\omega_s$ is the synchronous speed, and $v$ is the wind speed. $M$ [s$^2$/rad] is the scaled inertia constant, and $B$ [s$^3$/m$^3$] and $C$ [s$^3$/rad$^3$] are constants related to turbine geometry and power settings, respectively.
2.2 Tenth-Order Nonlinear Dynamical Model

The differential equations that describe the 10th order model include a 3rd order model of induction machine, 2 equations for active power control, 2 equations for reactive power control, and 3 equations for blade pitch control. Algebraic equations relate rotor currents, rotor voltages, and generated power to the state variables. The network equations model the interaction of the turbine with the rest of the grid.

\[
\frac{1}{\omega_s} \frac{dE'_q}{dt} = - \frac{1}{\omega_s T_0} (E'_q + (X_s - X'_s)I_{ds}) + \left( \frac{X_m}{X_r} V_{dr} - \frac{\omega_s - \omega_r}{\omega_s} E'_d \right)
\]

\[
\frac{1}{\omega_s} \frac{dE'_d}{dt} = - \frac{1}{\omega_s T_0} (E'_d + (X_s - X'_s)I_{qs}) - \left( \frac{X_m}{X_r} V_{qr} - \frac{\omega_s - \omega_r}{\omega_s} E'_q \right)
\]

\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{2H_D} (B\omega_sC_p(\omega_r, v, \beta) \frac{v^3}{\omega_r} - E'_d I_{ds} - E'_q I_{qs})
\]

\[
\frac{dx_1}{dt} = K_{f1}(P_{ref} - P)
\]

\[
\frac{dx_2}{dt} = K_{f2}(K_{P1}(P_{ref} - P) + x_1 - I_{qr})
\]

\[
\frac{dx_3}{dt} = K_{f3}(Q_{ref} - Q)
\]

\[
\frac{dx_4}{dt} = K_{f4}(K_{P3}(Q_{ref} - Q) + x_3 - I_{dr})
\]

\[
\frac{dx_5}{dt} = K_{f5}(\omega_r - \omega_{ref})
\]

\[
\frac{dx_6}{dt} = x_5 - x_6 - \beta + K_P(\omega_r - \omega_{ref})
\]

\[
\frac{d\beta}{dt} = x_6
\]
\[ V_{qr} = K_P (P_{ref} - P) + x_1 - I_{qr} + x_2 \]  
(2.14)

\[ V_{dr} = K_P (Q_{ref} - Q) + x_3 - I_{dr} + x_4 \]  
(2.15)

\[ P = E_d' I_{ds} + E_q' I_{qs} - R_s(I_{ds}^2 + I_{qs}^2) \]
- \((V_{qr} I_{qr} + V_{dr} I_{dr})\)  
(2.16)

\[ Q = E_q' I_{ds} - E_d' I_{qs} - X_s'(I_{ds}^2 + I_{qs}^2) \]  
(2.17)

\[ I_{dr} = \frac{E_q'}{X_m} + \frac{X_m}{X_r} I_{ds} \]  
(2.18)

\[ I_{qr} = \frac{E_d'}{X_m} + \frac{X_m}{X_r} I_{qs} \]  
(2.19)

\[(2.20)\]

Assuming the generator is connected to an infinite bus \(V \angle \phi\) through an impedance \(R + jX\), the network equations are

\[ E'_q - jE'_d = (R_s + jX'_s)(I_{qs} - jI_{ds}) + V_D \]  
(2.21)

\[ V_D e^{j\phi_D} = (R_{line} + jX_{line})(I_{qs} - jI_{ds} - I_{GC}) e^{j\phi} + V e^{j\phi} \]  
(2.22)

\[ I_{GC} = \frac{V_{qr} I_{qr} + V_{dr} I_{dr}}{V_D} \]  
(2.23)

The reference signals used in above are defined as

\[ P_{ref} = \begin{cases} 
C \omega^3_r & \text{if } \omega_r \leq \omega_{max} \\
P_{max} & \text{if } \omega_r > \omega_{max} \end{cases} \]  
(2.24)

\[ Q_{ref} = Q_{set-point} \]  
(2.25)

\[ \omega_{ref} = \frac{P_{ref}}{T_m} \]  
(2.26)

The limits on \(\beta\) are

\[ \beta_{min} - K_P(\omega_r - \omega_{ref}) \leq x_5 \leq \beta_{max} - K_P(\omega_r - \omega_{ref}). \]  
(2.27)
2.3 Seventh-Order Nonlinear Dynamical Model

If we assume operation in the optimal power tracking region, the pitch controller can be neglected, which gives us the 7th order model.

\[
\frac{1}{\omega_s} \frac{dE'_d}{dt} = -\frac{1}{\omega_s T'_0} \left( E'_d + (X_s - X'_s)I_{ds} \right) + \left( \frac{X_m}{X_r} V_{dr} - \frac{\omega_s - \omega_r}{\omega_s} E'_d \right)
\]

\[
\frac{1}{\omega_s} \frac{dE'_q}{dt} = -\frac{1}{\omega_s T'_0} \left( E'_q + (X_s - X'_s)I_{qs} \right) - \left( \frac{X_m}{X_r} V_{qr} - \frac{\omega_s - \omega_r}{\omega_s} E'_q \right)
\]

\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{2HD} \left( B\omega_s C_p(\omega_r, v, 0) \frac{v^3}{\omega_r} - E'_d I_{ds} - E'_q I_{qs} \right)
\]

\[
\frac{dx_1}{dt} = K_{I1}(P_{ref} - P)
\]

\[
\frac{dx_2}{dt} = K_{I2}(K_{P1}(P_{ref} - P) + x_1 - I_{qr})
\]

\[
\frac{dx_3}{dt} = K_{I3}(Q_{ref} - Q)
\]

\[
\frac{dx_4}{dt} = K_{I4}(K_{P3}(Q_{ref} - Q) + x_3 - I_{dr})
\]

The algebraic equations remain unchanged.

\[
V_{qr} = K_{P2}(K_{P1}(P_{ref} - P) + x_1 - I_{qr}) + x_2
\]

\[
V_{dr} = K_{P4}(K_{P3}(Q_{ref} - Q) + x_3 - I_{dr}) + x_4
\]

\[
P = E'_d I_{ds} + E'_q I_{qs} - R_s(I_{ds}^2 + I_{qs}^2)
- (V_{qr} I_{qr} + V_{dr} I_{dr})
\]

\[
Q = E'_d I_{ds} - E'_q I_{qs} - X'_s(I_{ds}^2 + I_{qs}^2)
\]

\[
I_{dr} = \frac{E'_d}{X_m} + \frac{X_m}{X_r} I_{ds}
\]

\[
I_{qr} = \frac{E'_q}{X_m} + \frac{X_m}{X_r} I_{qs}
\]
The network equations are also unchanged.

\[ E'_q - jE'_d = (R_s + jX'_s)(I_q - jI_d) + V_D \]  

(2.42)

\[ V_De^{j\phi_D} = (R_{\text{line}} + jX_{\text{line}})(I_q - jI_d - I_{GC})e^{j\phi_D} + Ve^{j\phi} \]  

(2.43)

\[ I_{GC} = \frac{V_qu + VdrI_{dr}}{V_D} \]  

(2.44)

The reference signals are simplified to

\[ P_{ref} = C\omega_r^3 \]  

(2.45)

\[ Q_{ref} = Q_{\text{set-point}}. \]  

(2.46)

The inequalities restricting \( \beta \) are now gone.

2.4 Singular Perturbation

A two time scale system can be written

\[ \frac{dx}{dt} = f(t, x, z) \]
\[ \epsilon \frac{dz}{dt} = g(t, x, z), \]

where \( x \in \mathbb{R}^n \) and \( z \in \mathbb{R}^m \). If \( \epsilon \ll 1 \), we think of the \( x \)-system as the slow system and the \( z \)-system as the fast system. If \( \epsilon = 0 \), the differential equations for \( z \) become algebraic constraints.

\[ g(t, x, z) = 0 \]

This equations can, perhaps implicitly, be solved for \( z \):

\[ z = h(t, x). \]

By replacing this function into the \( x \)-system we obtain the reduced system

\[ \frac{dx}{dt} = f(t, x, h(t, x)). \]
This reduced system is a good approximation to the original system if the fast system is stable.

We can apply this theory to our DFIG model. Introduce new variables for the inverse integral gain constants:

\[
\begin{align*}
\epsilon_1 &= \frac{1}{K_{I1}} & \epsilon_2 &= \frac{1}{K_{I2}} & \epsilon_3 &= \frac{1}{K_{I3}} & \epsilon_4 &= \frac{1}{K_{I4}}.
\end{align*}
\] (2.47)

Substituting these new variables into Equations (2.31), (2.32), (2.33), and (2.34) gives us

\[
\begin{align*}
\epsilon_1 \frac{dx_1}{dt} &= P_{ref} - P \\
\epsilon_2 \frac{dx_2}{dt} &= K_{P1}(P_{ref} - P) + x_1 - I_{qr} \\
\epsilon_3 \frac{dx_3}{dt} &= Q_{ref} - Q \\
\epsilon_4 \frac{dx_4}{dt} &= K_{P3}(Q_{ref} - Q) + x_3 - I_{dr}.
\end{align*}
\] (2.48) (2.49) (2.50) (2.51)

Rewrite \( T_0 = \frac{X_r}{\omega_s R_r} \) and define \( s = \frac{\omega_s - \omega_r}{\omega_s} \). Equations (2.28) and (2.29) become:

\[
\begin{align*}
\frac{1}{\omega_s} \frac{dE_q'}{dt} &= -\frac{R_r}{X_r} (E_q' + (X_s - X_s') I_{ds}) \\
&\quad + \left( \frac{X_m}{X_r} V_{dr} - s E_d' \right) \\
\frac{1}{\omega_s} \frac{dE_d'}{dt} &= -\frac{R_r}{X_r} (E_d' + (X_s - X_s') I_{qs}) \\
&\quad - \left( \frac{X_m}{X_r} V_{qr} - s E_q' \right).
\end{align*}
\] (2.52) (2.53)

Neglect rotor losses by setting \( R_r = 0 \) and define \( \epsilon_5 = \frac{1}{\omega_s} \) to give:

\[
\begin{align*}
\epsilon_5 \frac{dE_q'}{dt} &= \frac{X_m}{X_r} V_{dr} - s E_d' \\
\epsilon_5 \frac{dE_d'}{dt} &= \frac{X_m}{X_r} V_{qr} + s E_q' \\
\epsilon_5 \frac{dE_q'}{dt} &= - \frac{X_m}{X_r} V_{qr} + s E_q'.
\end{align*}
\] (2.54) (2.55)

By approximating all \( \epsilon_i \) variables by 0 we get algebraic equations from (2.48),
\begin{align}
P &= P_{ref} \\
x_1 &= I_{qr} \\
Q &= Q_{ref} \\
x_3 &= I_{dr} \\
V_{dr} &= s \frac{X_r}{X_m} E'_d \\
V_{qr} &= s \frac{X_r}{X_m} E'_q.
\end{align}

Equation (2.30) is the only remaining differential equation, but it still depends on a number of the algebraic equations. The following section will work towards removing those algebraic equations.

\section{2.5 First-Order Nonlinear Dynamical Model}

Neglect stator losses by setting $R_s = 0$. Equation (2.37) then becomes

\begin{equation}
P = E'_d I_{ds} + E'_q I_{qs} - (V_{qr} I_{qr} + V_{dr} I_{dr}).
\end{equation}

From (2.60) and (2.61), we can rewrite the last term of the previous equation

\begin{equation}
V_{qr} I_{qr} + V_{dr} I_{dr} = s \frac{X_r}{X_m} (E'_q I_{qr} + E'_d I_{dr}).
\end{equation}

Replacing the currents with their values from Equations (2.39) and (2.40) gives

\begin{align}
s \frac{X_r}{X_m} (E'_q I_{qr} + E'_d I_{dr}) &= s \frac{X_r}{X_m} (E'_q (-\frac{E'_d}{X_m} + \frac{X_m}{X_r} I_{qs})) \\
&\quad + E'_d (\frac{E'_q}{X_m} + \frac{X_m}{X_r} I_{ds})) \\
&= s (E'_d I_{ds} + E'_q I_{qs}).
\end{align}

Plugging this into equation (2.62) results in

\begin{equation}
P = (1 - s) (E'_d I_{ds} + E'_q I_{qs}).
\end{equation}
From (2.56) and (2.45) it follows that \( P = C \omega_r^3 \), thus
\[
E_d' I_{ds} + E_q' I_{qs} = \frac{C \omega_r^3}{1 - s} = C \omega_s \omega_r^2.
\] (2.66)

Then, from (2.30), it follows that
\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{2H_D} \left( B \omega_s C_p(\omega_r, v, 0) \frac{v^3}{\omega_r} - C \omega_s \omega_r^2 \right).
\] (2.67)

Define \( M = \frac{2H_D}{\omega_s} \) to arrive at the final result
\[
\frac{d\omega_r}{dt} = \frac{\omega_s}{M} \left( BC_p(\omega_r, v, 0) \frac{v^3}{\omega_r} - C \omega_r^2 \right)
\] (2.68)

and
\[
P = C \omega_r^3.
\] (2.69)

2.6 Comparison of Nonlinear Dynamic Models

2.6.1 Model Initialization and Simulation

The 7th order model is of the form
\[
\frac{dx}{dt} = f(x, z)
\] (2.70)
\[
0 = g(x, z).
\] (2.71)

To simulate the system we must first solve consistent initial conditions. We add the additional constraint that the system must be in equilibrium, that is:
\[
x(0) = x_0
\] (2.72)
\[
z(0) = z_0
\] (2.73)
such that

\[ 0 = f(x_0, z_0) \]  \hspace{1cm} (2.74)  
\[ 0 = g(x_0, z_0). \]  \hspace{1cm} (2.75)

These equations can be solved analytically or with the help of a numerical solver. Once the initial conditions are calculated, we use numerical integration to solve for the time evolution. Because of the wide range of time constants we must use a method which can handle stiff, differential-algebraic equations. The ode15s solver in MATLAB is designed to solve just such problems. A complete description of the solver is available in [11]. For consistency, the same technique was used to solve the reduced order model.

2.6.2 Numerical Results

The 7th and 1st order models were initialized with the following parameters: 

\[ X_m = 3.5092, \quad X_s = 3.5547, \quad X_r = 3.5859, \quad \omega_s = 120\pi, \quad R_s = .01015, \quad R_r = .0088, \quad H_D = 4, \quad \rho = 1.225, \quad R = 15, \quad S_b = 10^6, \quad C = 3.2397 \times 10^{-9}, \quad K_{P1} = 1, \quad K_{P2} = 1, \quad K_{P3} = 1, \quad K_{P4} = 1, \quad K_{I1} = 5, \quad K_{I2} = 5, \quad K_{I3} = 5, \quad K_{I4} = 5, \quad Q_{set-point} = 0, \quad R_{line} = .1, \quad X_{line} = .03, \quad V = 1, \quad \phi = 0. \]

The wind speed \( v \) was 10 m/s from \( t = 0 \) s to 100 s, 15 m/s from \( t = 100 \) s to 250 s, and 13 m/s for all following times.

Figure 2.1 compares the responses of the full and reduced order models. Clearly very little accuracy was lost when reducing the order of the model. When the models are in steady state near half their rated power, the reduced order model has 0.6% more generation than the full order model because the losses have been neglected.
Figure 2.1: Comparison of full and reduced order model step responses with identical parameters
CHAPTER 3
PARAMETER IDENTIFICATION

System identification is defined by [12] to be “the process of determining a difference or differential equation (or the coefficient parameters of such an equation) such that it describes a physical process in accordance with some predetermined criterion.” If nothing is known about the physical system other than the inputs and outputs, this is known as a black box problem. Some common black-box models include ARMAX and Box-Jenkins, which are special cases of the general SISO model. For a linear state space system, the Kalman filter can be used.

Prior attempts by the author to develop a black box model from measured wind turbine data were largely unsuccessful. As noted in [13], these models are unfeasible in most cases for nonlinear systems. Some knowledge of the nonlinearities should be built into the models. In the previous section a structure for the model has been developed using a priori knowledge of the system, so only the coefficient parameters need to be identified. This is known as a gray box problem.

3.1 Gray Box Identification Procedures

To evaluate the performance of a model, some metric is needed to quantify its performance. As suggested in [13], we choose to minimize a norm of the prediction error vector. This problem can be solved using techniques from the literature on nonlinear optimization. If $\theta$ is the vector of unknown parameters, the set of reference data is $y$, and $\hat{y}(\theta, v)$ is the solution to the
initial value problem for the dynamic model, we can write the problem

\[
\begin{aligned}
\text{minimize} \quad & ||\hat{y}(t, \theta) - y(t)|| \\
\text{subject to} \quad & \hat{x}(0) = h(\theta) \\
& \hat{x}(t, \theta) = f(\hat{x}(t, \theta), \theta) \\
& \hat{y}(t, \theta) = g(\hat{x}(t, \theta), \theta).
\end{aligned}
\]

There are two issues with this formulation. In practice, the real data \( y(t) \) will be sampled at discrete points in time. Additionally, it is more computationally feasible to fulfill the constraints by solving an initial value problem to calculate \( \hat{y}(t, \theta) \), then treat the problem as an unconstrained optimization. This new problem is

\[
\begin{aligned}
\text{minimize} \quad & F(\theta) \\
F(\theta) = \sum_{i=1}^{n} (\hat{y}_i(\theta) - y_i)^2.
\end{aligned}
\]

Since the evaluation of the objective function requires the solution of an initial value problem, it cannot be analytically differentiated. This means a number of common optimization algorithms such as Newton’s method cannot be used. Additionally, computing the objective function is computationally expensive, so a good solution technique will attempt to minimize the number of function evaluations.

There are several ways to solve the problem even without analytic derivatives. In this work a quasi-Newton [14] method which computes numerical derivatives for the Jacobian and does not need to explicitly calculate the Hessian is used. An alternative method is to use a derivative-free method such as Nelder-Mead [15]. In practice this method was found to have slower convergence than the chosen method, but for problems whose first derivative is not smooth it may be more suitable.

In our study it was assumed there was an initial guess of the unknown parameters that was roughly within an order of magnitude of the true value. It was found that the objective function was mostly convex in this region. If the initial guess was worse or if a different system was being studied, the procedure may converge to a non-global minimum, meaning the optimal set of parameters would not be found. In this case the problem is much more
difficult and a global optimization technique must be used. Additional constraints could be added to the problem formulation. In our study the parameters must be positive to be physically meaningful. The BFGS-B algorithm is a straightforward variant of the chosen method that could handle this type of constraint, but it was not found to be necessary because the search naturally avoids such solutions.

The derivative of a function is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}. \quad (3.3)$$

We can approximate the derivative as a finite difference by setting $h$ to a small but finite value as in Equation (3.4). Assuming $f(x)$ has already been calculated this requires only one additional function evaluation. Equation (3.5) gives a more accurate approximation, but requires two function evaluations. In this work the first approach was used for the calculation of gradients.

$$f'(x) \approx \frac{f(x + h) - f(x)}{h}. \quad (3.4)$$

$$f'(x) \approx \frac{f(x + \frac{h}{2}) - f(x - \frac{h}{2})}{h}. \quad (3.5)$$

To approximate a Hessian matrix with finite differences, the diagonal elements would look like

$$f''(x) \approx \frac{f(x + h) - 2f(x) + f(x - h)}{2h}, \quad (3.6)$$

and off-diagonal elements would be calculated as

$$\frac{\partial^2 f(x_1, x_2)}{\partial x_1 \partial x_2} \approx \frac{f(x_1 + h, x_2 + h) - f(x_1 + h, x_2 - h)}{4h^2} + \frac{-f(x_1 - h, x_2 + h) + f(x_1 - h, x_2 - h)}{4h^2}. \quad (3.7)$$

The symmetry of the Hessian means not all the elements need to be calculated, but the remaining terms would still clearly require many function evaluations. For this reason we will use an approach that can approximate the Hessian without using the direct finite difference approach.
3.2 BFGS Optimization

The Taylor series expansion of $f$ gives us the following approximation of $f(x_{k+1})$ from the point $x_k$:

$$f(x_{k+1}) \approx f(x_k) + \nabla f(x_k)^T(x_{k+1} - x_k) + \frac{1}{2}(x_{k+1} - x_k)^T H_k (x_{k+1} - x_k),$$

(3.8)

where $H_k$ is the Hessian of $f$. Taking the gradient with respect to $(x_{k+1} - x_k)$ gives us

$$\nabla f(x_{k+1}) \approx \nabla f(x_k) + H_k (x_{k+1} - x_k).$$

(3.9)

For Newton’s method, set $\nabla f(x_{k+1}) = 0$ and derive

$$\nabla f(x_k) = H_k (x_{k+1} - x_k)$$

(3.10)

$$H_k^{-1} \nabla f(x_k) = (x_{k+1} - x_k)$$

(3.11)

$$x_{k+1} = x_k - H_k^{-1} \nabla f(x_k).$$

(3.12)

Quasi-Newton methods are of the form

$$x_{k+1} = x_k + \alpha_k d_k$$

(3.13)

$$d_k = -D_k \nabla f(x_k).$$

(3.14)

We will assume the step size $\alpha_k = 1$. Setting $D = H^{-1}$ gives the aforementioned Newton’s method. Setting $D = I$ gives the gradient descent method. This technique is very simple and requires the fewest operations per iteration, but the slow convergence of this method means it will take many iterations and thus a very long time to converge.

Rather than set $\nabla f(x_{k+1}) = 0$, we choose $D_k$ to be an $H_k$ that fulfills (3.9). In case of scalar $x$, this means

$$H_k = \frac{\nabla f(x_{k+1}) - \nabla f(x_k)}{x_{k+1} - x_k}.$$  

(3.15)

Because we need $H_k$ to calculate $x_{k+1}$, we cannot use this formula directly. Instead, we use the approximation calculated using values only from the
previous steps:

\[ H_k = \frac{\nabla f(x_k) - \nabla f(x_{k-1})}{x_k - x_{k-1}}. \]  

(3.16)

The vector case is more complicated. Define \( p = x_k - x_{k-1} \) and \( q = \nabla f(x_k) - \nabla f(x_{k-1}) \). Then the BFGS method is

\[ H_k = H_{k-1} + \frac{qq^T}{q^T p} - \frac{H_{k-1}pp^T}{p^T H_{k-1} p} H_{k-1}. \]  

(3.17)

In the case of scalar \( x \), this reduces to the secant method. It is derived by adding the constraints that the matrix must be symmetric and have the minimum Frobenius norm of the difference between iterations. This choice has the property that if \( H_k \) is positive definite, \( H_{k+1} \) will also be positive definite [16].

In practice, the inverse Hessian is approximated directly by

\[ H_k^{-1} = \left( I - \frac{qq^T}{q^T p} \right)^T H_{k-1}^{-1} \left( I - \frac{qq^T}{q^T p} \right) + \frac{pp^T}{q^T p}. \]  

(3.18)

This method saves the computation of computing the inverse at each step. This equation is derived using the Sherman-Morrison formula, which is a special case of the matrix inversion lemma.

### 3.3 Application to Reduced Order Model

To apply this method to the reduced order model, we use the preceding equations, making substitutions for the appropriate variables. In this case the time series functions we want to match are \( P \) and \( \hat{P} \), the function we are trying to minimize is \( F \), and the variables are \( \theta \).

In our application the unknown parameters are \( B, C \), and \( M \). Additionally the initial condition \( \omega_r(0) \) can be treated as an unknown parameter, or it can be fixed to \( \sqrt{\frac{P(0)}{C}} \). Allowing \( \omega_r(0) \) to vary freely allows for a solution in which \( \hat{P}(0) \neq P(0) \), but is more accurate in the following period of time. Fixing the initial condition forces the initial outputs to agree, but could cause higher total error. This approach makes the optimization simpler because
there are fewer variables. The effect of the initial condition eventually decays, so especially over a long time period it may be advantageous to fix $\omega_r(0)$.

The complete formulation is given below. Here we assume $\omega_r(0)$ is unknown.

\[
\begin{align*}
\text{minimize } & F(\theta) \\
F(\theta) = & \sum_{i=1}^{n} \left( \hat{P}_i(\theta) - P_i \right)^2 \\
\theta = & [B\ C\ M\ \omega_r(0)]^T
\end{align*}
\] (3.19)

And the solution procedure is

\[
\begin{align*}
\theta_{k+1} = & \theta_k + d_k \\
d_k = & -D_k \nabla F(x_k) \\
D_k = & \left( I - \frac{qp^T}{q^T p} \right)^T D_{k-1} \left( I - \frac{qp^T}{q^T p} \right) + \frac{pp^T}{q^T p} \\
D_0 = & I.
\end{align*}
\] (3.22, 3.23, 3.24, 3.25)

After using numerical integration to find $\hat{P}(t)$, linear interpolation is used to find the values of $\hat{P}_i$ at the time measurements of $P_i$ were taken.

3.4 Numerical Identification Results

The first identification study presented here uses the 7th order model data as the reference data. Figure 3.1 shows the 7th order model plotted against the 1st order model with identified parameters. Table 3.1 lists the parameters used in the 7th order model and the identified model for comparison. The difference in the parameters identified was on the order of 1%. Though small, the difference is not an error. The way the problem has been formulated, the objective is to minimize the difference in the outputs between the two systems. These identified parameters actually produce a better fit than the exact parameters from the 7th order model. The new parameters are adjusted slightly to account for the simplifying assumptions made in Chapter 2. The RMS error is reduced from $2.75 \times 10^{-3}$ with the 7th order parameters in Figure 2.1 to $3.14 \times 10^{-4}$ with the identified parameters in Figure 3.1.
Figure 3.1: Comparison of step responses of full model and reduced order model with identified parameters

Table 3.1: Comparison of actual and identified parameters

<table>
<thead>
<tr>
<th></th>
<th>$B$</th>
<th>$C$</th>
<th>$M$</th>
<th>$\omega_r(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Values</td>
<td>4.33e-004</td>
<td>3.24e-009</td>
<td>2.12e-002</td>
<td>3.87e+002</td>
</tr>
<tr>
<td>Identified Values</td>
<td>4.30e-004</td>
<td>3.30e-009</td>
<td>2.10e-002</td>
<td>3.86e+002</td>
</tr>
</tbody>
</table>

The next study uses the field data as a reference. Unlike the previous problem, the values of the parameters (and even the structure) of the reference system are unknown. Figure 3.2 shows the wind speed used as an input to the models as well as the observed responses. The sharp changes in the wind speed are translated directly into power fluctuations by the static model. The dynamic model and the measured data show a smoother output. From these observations we conclude that the parameters can be effectively identified and a large improvement has been made over the static model. The objective function is not, however, reduced to zero. The remaining error can be attributed to the simplifications that were made to reduce the order of the models as well as errors in the measured data. One of the significant limitations of the measured wind speed data is placement of the anemometer on the nacelle. Because it is located behind the blades, there will be deviations due to turbulence that do not realistically represent what the overall turbine experiences [2].

When performing an identification study it is important to avoid using
the same data to train a model as well as to verify the fit quality. If this is done, there is a danger that the identified model will be “over-fit” to the training data. For this reason we divide the reference data into a training set and a verification set. Figure 3.3 shows the RMS error between the different models and the verification data and the amount of training data that was used to identify the parameters. This plot shows that even a few data points is enough to train a basic static model. Additional training data only offers a marginal improvement to the fit quality. On the other hand, the dynamic model performs very poorly when given a small amount of training data, but after learning from enough data it outperforms the static model. From this plot we estimate at least 15 minutes of training data would be adequate. In practice the actual amount of data depends on the “richness” of the signals, but this is hard to quantify for a nonlinear system.

3.5 Sensitivity Studies

In this section the sensitivity of the parameters is investigated. In this procedure the identified parameters from 3.1 were perturbed and the RMS difference between the new simulation output and the 7th order model was recorded. Parameters were increased or decreased by up to 50% of their optimal value. Figure 3.4 shows the result of perturbing each parameter individually. Figures 3.5, 3.6, and 3.7 show pairwise sensitivities.

Figure 3.8 is a sensitivity plot showing how variations in the 1st order model parameters affect the difference from the real wind turbine data. In this case we see there are two values of $C$ that cause the error to have a local minimum. In this case the solver may not have converged to the global minimum.
Figure 3.2: Comparison of static and dynamic models with identified parameters
Figure 3.3: Comparison of training data required for static and dynamic models

Figure 3.4: Sensitivity of RMS error to changes in each parameter
Figure 3.5: Sensitivity of RMS error to changes in $B$ and $C$ with $M$ held constant

Figure 3.6: Sensitivity of RMS error to changes in $B$ and $M$ with $C$ held constant
Figure 3.7: Sensitivity of RMS error to changes in $C$ and $M$ with $B$ held constant

Figure 3.8: Sensitivity of RMS error to changes in each parameter
CHAPTER 4

CONCLUDING REMARKS

This thesis has presented a general background in wind turbine modeling, outlined the model order reduction work performed, and discussed the application of parameter identification techniques that can be used to fit the model to realistically simulate existing turbines.

The presented reduced order model is shown to significantly improve upon static power curve methods for modeling the normal operation of the turbine. Unfortunately, our set of measured data did not include any faults or other abnormal scenarios, but the 1st order model is expected to perform as well as the swing equation for synchronous machines in these conditions.

Future work could include modifying the identification technique to improve performance when run in real time for identification of changing parameters that could indicate a maintenance problem. A recursive identification method such as that discussed in [17] may significantly lower the computational burden required.
REFERENCES


