INTELLIGENCE BY DESIGN IN AN ENTROPIC POWER GRID

BY

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DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical and Computer Engineering in the Graduate College of the University of Illinois at Urbana-Champaign, 2012

Urbana, Illinois

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In this work, the term *Entropic Grid* is coined to describe a power grid with increased levels of uncertainty and dynamics. These new features will require the reconsideration of well-established paradigms in the way of planning and operating the grid and its associated markets. New tools and models able to handle uncertainty and dynamics will form the required scaffolding to properly capture the behavior of the physical system, along with the value of new technologies and policies. The leverage of this knowledge will facilitate the design of new architectures to organize power and energy systems and their associated markets. This work presents several results, tools and models with the goal of contributing to that design objective. A central idea of this thesis is that the definition of products is critical in electricity markets. When markets are constructed with appropriate product definitions in mind, the interference between the physical and the market/financial systems seen in today’s markets can be reduced. A key element of evaluating market designs is understanding the impact that salient features of an entropic grid—uncertainty, dynamics, constraints—can have on the electricity markets. Dynamic electricity market models tailored to capture such features are developed in this work. Using a multi-settlement dynamic electricity market, the impact of volatility is investigated. The results show the need to implement policies and technologies able to cope with the volatility of renewable sources. Similarly, using a dynamic electricity market model in which ramping costs are considered, the impacts of those costs on electricity markets are investigated. The key conclusion is that those additional ramping costs, in average terms, are not reflected in electricity prices. These results reveal several difficulties with today’s real-time markets. Elements of an alternative architecture to organize these markets are also discussed.
To my parents: Violeta and Iván
ACKNOWLEDGMENTS

The whole problem with the world is that fools and fanatics are always so certain of themselves, but wiser people so full of doubts.

Bertrand Russell

People you interact with during this journey are among the most important components of the Ph.D. process. Many times, during those challenging and darker periods, I had people that put me again on track. My sincere thanks go out to those who touched my life and work during my years at Illinois.

I owe my deepest gratitude to my advisor, Prof. Sean P. Meyn. Working under his guidance has been a real joy both intellectually and personally. Bertrand’s quote is certainly appropriate to characterize Sean. Always full of doubts and new questions, always curious, always humble, always constructive and always open to new ideas. Since the first day we met, he has always been very willing to share all his deep expertise, insights and views on highly technical topics. In personal terms, the joy has not been less. His views on daily life issues, society, his drops of Buddhist wisdom along with many other unforgettable messages will always be in my memory. If I thought that I was lucky enough because I found an outstanding scholar and advisor to work with, I was certainly short on my assessment. I found also a great friend and an inspiring human being.

I have grateful words to the members of my Ph.D. committee: Thanks a lot for the participation and for the patience to find a date. In particular, I would like to thank Prof. Peter Sauer. He provided me all the support in transients periods of this trip. Certainly, without that support, achieving the steady state would have been much more uncertain. I also acknowledge Prof. George Gross for facilitating my transition to the ECE program, and for introducing me to this research area.
Many thanks to all the staff of the Power and Energy Systems group at Everitt Lab, and the staff of the Decision and Control group at CSL. In particular, I have special words to Karen Driscoll and Jana Lenz for all the help provided.

In terms of people, friends are a key component. In all these years living at Champaign-Urbana, I have had the honor of meeting amazing people. In particular, I want to thank Angel, Anupama, Beatriz, Cecilia, Dimitra, Elitsa, Eve, Hector, Gui, Marcos, Mehmet, Roberto, Patricio and Stan for being close to me on the happy and not so happy days.

I have special mentions to give to Marcos, Patricio, Hector and Roberto. Marcos thanks a lot for all your help in planning my moving to Urbana-Champaign. Patricio, my long-term roommate and neighbor: Thanks for all your help since day zero, when you picked me up at O'Hare with Marcos. Hector, my companion in many adventures even until today: Thanks a lot for your kindness during my first moments with the Power and Energy Systems group. Roberto, my roommate at Prairie Place who was always there with a smile: Many many thanks! I also have special words to Gui: Thanks for being such an amazing friend and scholar. Our many discussions and conversations in research topics were always a pleasure to me.

Also, special words to Mehmet, my dear friend the first years at Illinois: Without any doubts, you were the closest to a brother that I had in Champaign. All those moments lived at Castle apartments deserve a chapter rather than a paragraph. I will visit you soon in Istanbul, I promise! Many thanks to Dimitra for being close to me during the last period of this journey. Your kindness and sweetness were always a comfort during all these stressful months. Yes, it is jungle out there!

Special mentions to my friends outside Illinois: Jorge and Sergio. Your perseverance has always been an inspiration to me.

Last but not least, my family. Being here is just the result of their encouragement and support. Support since my first hobbies when I was a child to the days in which I realized I wanted to take a different career path. Violeta and Ivan this work is by and for you. I also dedicate this work to the loving memory of my Nona Maria, who left this world during the last period of this work.

Finally, a big *MIAU* to my sweet Olivia. Losing you so suddenly still hurts.
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CHAPTER 1

INTRODUCTION

Many changes are coming to the power industry due to new policies and new technologies. This is bringing new challenges in operating and planning the power grid. Some of these challenges take the form of increased or more exotic dynamics, and greater volatility and uncertainty. For example, renewable energy sources will inject volatile and uncertain patterns of energy into the grid, smart meters and appliances will increase demand uncertainty, new information technologies may increase cybersecurity risks, and new products and market participants may induce changes in dynamics and uncertainty.

The design and operation of the grid of the future will require a theoretical scaffolding able to deal with uncertainty and dynamics. These tools must complement the successful power and energy systems methodologies that are used today. It is necessary to understand and control complex dynamic systems subject to uncertainty, variability, and shared constraints.

In this chapter, we provide an overview about the motivation, vision, and topics developed in this work.

1.1 The Entropic Grid

The power grid has been defined as one of the biggest achievements of human engineering. This is evident after reviewing key characteristics of today’s grid. Maybe the most salient element of the grid is its complexity. Thousands of elements such as generating units, transmission lines, distribution systems and loads interact on a network, and are subject to strict constraints such as physical laws. An example of this complexity is clear by looking at the map of the European transmission system in Fig. 1.1.

In terms of technologies and operational paradigms, today’s grid heavily relies on big controllable generating plants. Nuclear, fossil-fueled and hydro
Interconnected network of ENTSO-E

Figure 1.1: Europe transmission system.

plants make up the bulk of installed capacity. The operation usually is based on centralized control schemes, making the role of Independent System Operators (ISOs) critical in this task. At the distribution level, end-users have historically had a well-defined role: being only consumers of energy with a passive role.

The power grid is key to the proper functioning of society, hence many mid- and long-term changes and initiatives are driven by political decisions and mandates. Politics is a key element of today’s grid. A clear example is the current trend of imposing renewable energy targets as a way to move towards a more sustainable grid.

Another characteristic of today’s grid is its relationship with markets structures. This is bringing many risks faced by both consumers and suppliers. Many of those risks are associated with the volatility seen on current electricity prices—from spikes lasting minutes to periodic bursts lasting weeks. Volatility that, as is going to be presented in later chapters, is inherent to current electricity markets designs. A picture with four emblematic cases of price volatility is shown in Fig. 1.2.

In this environment, consumers face the risk of being subject to market
Figure 1.2: Volatility of electricity prices around the world.

power by suppliers; the California crisis of 2001 and the Enron scandal together constitute one of the clearest examples. Similarly, suppliers are also subject to many risks. The volatility of prices is a big risk for suppliers, in terms of entry, as well as reliable day-to-day income. This has been particularly critical with the introduction of renewable sources. Suppliers associated with these technologies have additional risks due to the inherent uncertainty and volatility of such sources.

Summarizing, the current power grid is an already extremely complex system. What can we expect for the future? Many changes resulted from the Smart Grid initiatives [1]: the move from big generating plants at the transmission level to more distributed energy sources, many of them deployed at the distribution level; greater participation of renewable energy sources such as wind and solar; and the active role of end-users resulting from advanced metering infrastructure and the use of smart appliances. Deployment of more information technologies, which will improve the situational awareness of the grid, will facilitate its control, but might also bring new risks such as cybersecurity threats [2]. The list of expected changes and challenges is broad and diverse.
**Entropic Grid** The increased dynamics, volatility and uncertainty of tomorrow’s grid will increase the “disorder” or “uncertainty” of the system. In thermodynamics and in Shannon’s theory of coding and communication, the notion of entropy plays a central role. In a precise sense, entropy captures the number of attainable state trajectories in a system: The larger the entropy, the larger the number of possible state trajectories on a given time-horizon. In the proposed Smart Grid, with all the features described above, it is clear that the system will be much more “uncertain” and it is also clear that the range of possible system behaviors may increase greatly. For these reasons, until the appropriate level of intelligence can be assured, the term *Entropic Grid* is adopted to describe the new power grid, with all of its new complexity and uncertainty. The intention is to highlight the fact that although new technologies and policies bring many new opportunities for energy reduction and efficiency improvements, they also bring the potential for an extremely complex or “entropic” system. With proper design, this uncertainty can be reduced, and the term Smart Grid will be justified.

However, why is the need of a proper design so critical and necessary? On the planning and operation side, the expertise and intuition of system operators, along with the infrastructure of the power grid, were developed in an evolutionary way: over a long time horizon, with much trial and error. The new paradigm of new technology combined with new policy will bring uncertainty and complexity that rule out the reliance on evolutionary adaptation. Attempts of relying on trial and error can be a big waste of resources and a bridge to nowhere [3]. There is no time for evolution.

From a general perspective, the current designs are structured under two main paradigms: vertical integration or electricity markets. In the vertical integration design, a whole company owns and operates all the assets—generation, transmission and distribution. In the electricity market structure, markets and competitive forces are deployed at some levels of the industry, generally at the generation level. The motivations for the establishment of market structures are multifold and explained throughout this work. However, there is still no agreement about the real achievements of the deployment of markets in electricity. Issues such as system reliability [4], high tariffs trends [5], and the failure of spot prices as investment signals [6] are well-understood and recognized unsolved problems. On the other hand, innovation, increased efficiency in running companies, and the development of
new technologies are usually mentioned as the good outcomes [7, 8].

The main challenge is that many of those unsolved issues in the electricity market arena could be magnified in an entropic grid setting without proper design. New ways of thinking about operations and planning of power and energy systems and its associated markets emerge as critical requirements for the achievement of a reliable, sustainable and efficient system [9–11]. The overall goal is to obtain a better intuition regarding technological and system needs for the grid of the future, and how to ensure that these needs will be met via policy combined with markets.

1.2 Setting the Stage and Needs

It is recognized that the main challenge of electricity markets is the coexistence of two different coupled dynamical systems:

- A physical system driven by hard and soft engineering constraints in which reliability is the leading objective.

- A market/financial system driven by self-interests of players in which economic efficiency is the leading metric.

Additionally, each of these systems has its own uncertainties and complexities, not to mention the often orthogonal objectives among its players — energy companies, engineers, policy makers, and consumers [12, 13]. One of the ideas developed in this work is that many of the current issues of electricity markets can be explained as results of the interference between the physical and the market/financial systems. A common diagnosis of many electricity markets around the world is the fact that many design elements help to achieve financial/market objectives but need to be modified to attain the physical objectives, and conversely. The augmented level of complexity resulting from many Smart Grid initiatives could certainly increase such interference, reinforcing the need for careful and thoughtful design.

A key element to conciliate the physical and the market/financial systems is the definition of products. The product is the object that links the physical and market/financial systems. As discussed in Chapter 3, a bad product definition can certainly impact the market outcomes. Efficiency, risk allocation,
information aggregation and several other market issues are impacted by the product definition. However, if the product definition also does not match technological realities, then the physical system could also be impacted. For example, the lack of consideration of technological attributes in capacity markets might not facilitate the achievement of a reliable system [13].

In our view, one step required to define appropriate products is to start considering electricity not as a simple commodity, without qualitative distinctions, but as a product or service with attributes beyond energy. In other words, electricity should be seen as a more complicated product in which energy is just one of the dimensions or attributes to consider. Examples of other attributes might include flexibility, controllability and environmental impact of the resources.

In the electricity markets literature, there is almost no discussion of product definition issues. The explanation seems to be just another legacy of the idea of using spot markets and treating electricity as a commodity, in which the underlying product is 1 MWh. In a market for commodities the definitions are clear; e.g., a “product” is a barrel of oil or a pound of copper.

The elegant market formalism of Schweppes et al. [14] led many to believe that spot markets for energy would be sufficient to create appropriate economic signals for electricity markets. Empirical evidence collected over decades combined with recent theory demonstrates that energy-only markets are deficient on many grounds.

Recently, policy makers have started to recognize that services beyond energy are needed. In response, new federal policies have been implemented. One example of a “service” is the ramping of generation to respond to volatility in the grid. FERC order 755 is designed to ensure that ramping capabilities are properly compensated [15]. Appropriate market mechanisms to provide adequate compensation remain a subject of intense debate. These mechanisms are mainly grounded in the spot-pricing framework for electricity markets.

However, once the complicating factors and characteristics of the entropic grid are taken into account, the use of spot-markets, and in particular real-time markets, must be reconsidered. Several results presented in this thesis support this viewpoint. For example, in the context of volatile resources integration, the results presented in Chapter 4 show the need to reconsider market structure to stimulate greater penetration of volatile resources. The
consideration of dynamic costs, investigated in Chapter 5 for the case of ramping costs, illustrates how those additional costs are not captured by the electricity prices.

In order to evaluate market designs, it is crucial to understand the behavior of the physical system. This will require the use of models and tools tailored for the new entropic setting. A critical element is the consideration of the real costs and value of power. In the same line, it seems key to move beyond snap-shot-based modeling paradigms to the use of models in which dynamics and uncertainty are considered. All these elements are taken into account in the models presented throughout this work. An overview is provided in the next section.

Another need is to reconsider the economic framework in which current markets have been designed: competitive equilibrium analysis and spot pricing theory. It is recognized that competitive equilibrium analysis may give some valuable insights. However, it is a crude model of reality, and in particular too crude for long-term prediction of system behavior. In the competitive equilibrium setting, once dynamics and uncertainty are brought into play, the competitive outcomes of electricity markets are characterized by volatile and high prices [12]. Advocates of the current market designs may argue that there is no problem: If this is what the competitive equilibrium looks like, then we better accept volatile prices and price spikes because in the “long-term” society will be better. However, is it really so important that we achieve this “long-term” or “equilibrium” nirvana in the energy grid? How can we be so sure that we will converge?

Energy companies are seeking profits: Big profits, generally, may come from “disruptive technologies”, or from “market manipulation”. By the time we converge to the hypothesized optimal equilibrium, the entire system would have changed due to new policies and new technologies. Consequently, it is our belief that the notion of “long-term” convergence is meaningless in energy markets. Societal objectives such as efficiency, reliability and sustainability must be treated as drivers rather than the result of a “long-term” convergence process.

Based on all these considerations, an alternative architecture for organizing future power and energy systems is required. It should be constructed around the ideas of addressing entropy; achieving multifold objectives such as economic efficiency, reliability and sustainability; conciliating the physi-
cal and market/financial systems; and treating electricity as a service, not a commodity, that can be provided not only by conventional generators but, depending on the service, by many other technologies. Proper design is required to harness all that potential and achieve a reliable, efficient and sustainable grid. A visual representation of the type of resources, inputs and objectives of the future’s grid is pictured in Fig. 1.3.

The main way to address entropy is to reduce it. In that line, the role of a central authority setting rules is a key element of alternative architectures. It might be argued that this could create a loss of efficiency in the ideal sense of economics, but we see it as a necessary cost given the importance to society of reliable energy.

![Desires of Consumers & Suppliers, Energy & Volatility]

Desires of Consumers & Suppliers, Energy & Volatility

Power Consumption
Quality of Life

Capacitors, Batteries, Flywheels

Hydro, Wind, Solar, Flexible Manufacturing, Flexible HVAC

Nuclear, Coal, Gas Turbine Gen.

Power Grid

Power Consumption
Quality of Life

Figure 1.3: Multifold resources and attributes.

This idea of restricting behavior is not new and it is everywhere. A clear example is highway engineering: Nobody would claim that the highway system is efficient. On paper it might be possible to prove that efficiency can be improved by eliminating lanes or speed limits. In practice, it is well understood that the range of possible behavior of drivers must be reduced through lanes and speed limits to achieve a more predictable and safer transportation grid. It is required to accept that similar restrictions are necessary to improve the predictability and the achievement of societal objectives in the energy grid.

The best architecture for the energy highway of the future is not yet obvious and its realization will require a lot of research and effort, but it is likely
to include elements of today’s day-ahead markets combined with long-term contracts. However, it is certainly clear that lanes and speed limits, incentives and penalties, are going to be required in order to achieve a predictable and reliable system.

1.3 Scope and Contributions

The goal of this work is to contribute to the design and operation of power and energy systems able to cope with the many challenges associated with an Entropic Grid. As previously discussed, achieving this objective will require the deployment of new tools and models able to capture and handle uncertainty, dynamics and shared constraints. Such models and tools will form the scaffolding to understand the behavior of the physical system along with the systemic and private value of attributes associated with new technologies. The leverage of this knowledge will facilitate the design of new architectures to organize power and energy systems and their associated markets. These tools will also guide the formation of new government policies.

This thesis, for the purposes of control, market design and evaluation, focuses on

- Product definition in electricity markets
- Models that capture uncertainty and dynamics

The key role of the product definition in an electricity market is investigated in Chapter 3. Several analytical and numerical results, motivated by the analysis of real market designs, are used to unveil the importance of the product definition in the market outcomes. Based on the findings, elements and recommendations to define products in electricity markets are presented.

As mentioned above, another main topic developed in this thesis is to understand the impact that key features of the entropic grid can have on electricity markets. In order to achieve that goal, appropriate models able to handle uncertainty and dynamics are developed. Several dynamic electricity markets models, tailored to capture diverse elements of the entropic grid, are presented in Chapters 4, 5, and 6.

In Chapter 4, these models are used to investigate the value of volatile resources in electricity markets. The big research objective is to understand
how the volatility and uncertainty associated with these resources can impact their value. In addition, the incentives that current electricity markets provide to the deployment of volatile resources are investigated. The results show the need to implement policies and technologies able to cope with the volatility of renewable sources, and the need to update market structures.

In a similar vein, using a dynamic electricity market model in which ramping costs are considered, the impacts of those additional costs on electricity markets are investigated in Chapter 5. The key conclusion is the fact that at the competitive equilibrium those additional ramping costs, in average terms, are not reflected in average prices.

Finally, based on all the topics and findings, linking the definition of products ideas with the impacts on electricity markets once constraints, dynamics and uncertainty are considered, general elements to organize these markets around contractual agreements are discussed in Chapter 6. The viewpoints are illustrated via an investigation of the system-wide value of operational flexibility, along with the market implications.

1.4 Thesis Outline

This thesis is structured in several chapters. An overview of each chapter, with its topic and main contributions, is given here.

Chapter 2

Basic background material is presented. An overview of basic elements of competitive markets and electricity markets is provided. Basic notation and elements of Markov decision processes (MDP), a key element of the results of the rest of the chapters, are discussed. Specific models for an entropic grid setting are also reviewed. In particular, models for the dynamic economic dispatch of energy and reserves along with elements of dynamic electricity markets are reviewed. Those models are the starting point for many of the models developed throughout this work.
Chapter 3

The role of product definition in electricity markets is unveiled in this chapter. A product definition implemented in some U.S. electricity markets is used to illustrate several critical aspects that must be considered when electricity products are defined. Several analytical along with numerical results are presented. Based on these findings, elements of an alternative product definition which overcomes some of the investigated issues are discussed.

Chapter 4

The value of volatile resources in electricity markets is investigated in this chapter. A multisettlement electricity market model, considering a day-ahead and a real-time market, is the vehicle to perform the analysis. This is conducted in an idealized competitive equilibrium setting that incorporates both dynamics and uncertainty; closed form expressions are obtained for the supplier and consumer surpluses in this stochastic model. The results show the need to find resources and to create operational schemes that address volatility. Moreover, it is argued that current market structures must be updated to support greater renewable integration.

Chapter 5

The difficulties of incentivizing responsive generation in a real-time market setting are exposed. The analysis of a dynamic electricity market, that also includes also ramping costs, shows that average prices may coincide with average production marginal cost. Hence, the cost of ramping is not captured by those average prices. Several numerical illustrations are presented. The results underscore the need for finding alternative ways to structure electricity markets.

Chapter 6

Elements of an alternative view for organizing the markets in a Smart Grid scenario are discussed. Defining the products in these new markets will require a careful understanding of the systemic and private value of the differ-
ent attributes. The emphasis of the chapter is on investigating the resource allocation problem, getting resources appropriately sized in advance. The solution to this problem offers insights, and provides sensible guidelines for how many resources, and of which type, are required. Our viewpoints are illustrated via a detailed investigation of the system-wide value of operational flexibility, along with the market implications.

Chapter 7

A summary of the results and findings of this work is presented. In addition, several research avenues for future work are discussed.
CHAPTER 2

SCAFFOLDING FOR GRID AND MARKETS MODELING

In this chapter, basic background, terminology and references for the topics developed in the following chapters are provided. Elements of competitive markets and basic elements of current electricity market designs are presented. Similarly, an overview of Markov decision process concepts and models that capture uncertainty and dynamics are discussed. These models are used as a starting point for other ones presented throughout this work.

2.1 Electricity Markets

We provide a review and bibliography of key elements of electricity markets. Recognizing that each electricity market has its own peculiarities, there are common elements shared by many designs. We start providing basic concepts of competitive markets in general. A good overview is provided in [16].

2.1.1 Basics of Competitive Markets

The general idea behind competitive markets is to achieve a desirable society state in a decentralized way, without the need for a central planner. The optimal allocation of resources is achieved through an appropriate price system and individual optimization. The usual desirable society state is usually characterized in terms of the so-called Pareto efficiency. Informally, Pareto efficiency means that no agent can be better off without making another one worse off.

Closely related to the notion of Pareto efficiency in a decentralized way is the idea of Competitive equilibrium, the basic notion of equilibrium in economic systems. It resembles the notion of equilibrium in closed physical systems, in which, given a set of prices and allocations, each agent maximizes his
own welfare, subject to his physical constraints of production/consumption.

The welfare theorems of economics provide the link between these concepts [17]. The first welfare theorem states that any competitive equilibrium leads to Pareto efficiency, a formal characterization of Adam Smith’s invisible hand. The second welfare theorem states that any Pareto efficient allocation can be supported in a decentralized way by a competitive equilibrium. However, the conditions and assumptions behind these theorems are too idealized with respect to real world situations complicating their applicability [18]. This point is going to be evident in the case of electricity in which the physical reality of the process of generation, transmission and distribution challenges the deployment of market structures.

2.1.2 Basics of Electricity Market Design

The general structure of many electricity markets is based on the notion of a centralized market. A system operator receives offers and bids of consumers and suppliers respectively and runs a clearing mechanism determining the dispatch and prices of the system. The basic structure is pictured in Fig. 2.1.

![Figure 2.1: Centralized electricity market structure.](image)

A key element is a multisettlement structure of markets, in which the multiple markets are differentiated by the time at which the decisions are taken and the time at which the energy is delivered. The basic paradigm of this multisettlement structure is a day-ahead market (DAM) coupled with a real-time market (RTM).

The reasons for doing that are two-fold and are related to the economics and the physical operation of the system. From a physical viewpoint, in
centralized markets, there is a need to accommodate the commitment of the units in advance. From an economic viewpoint, the day-ahead market is a forward market which can improve the economic efficiency [19,20]. It is also pretty common to have a series of forward markets for different time scales, as the experiences of Chile and Brazil show [6]. The diverse time-frames are depicted in Fig. 2.2

![Figure 2.2: Time-frames of multi-settled markets.](image)

Another key element of many electricity market designs of interest for the current work is the lack of differentiation among technologies. This idea is explained by the underlying motivation of treating electricity as any other commodity.

### 2.2 Markov Decision Processes

Markov decision processes (MDPs) provide the theoretical framework for many of the models presented in this work. We provide a brief overview of some key concepts in the theory of MDPs that will facilitate the reading and introduce some notation of the present work. We follow [21,22].

In a colloquial way, an MDP aims to solve the problem of making decisions under uncertain and dynamic conditions. A system is evolving in time over a state space. Such evolution is the result of the dynamics of the system, uncertainty and actions taken in each state. Associated with the state and the actions, there is some reward or cost function. The overall goal is finding the best policy — which action to take given the current state — to achieve some objective.

In more formal terms, a Markov decision process consists of a state space
$X_\circ$, an action space $U_\circ$, and the controlled transition matrix,

$$P_u(x, y) := P\{X(t + 1) = y \mid X(t) = x, U(t) = u\} \quad x, y \in X_\circ, u \in U_\circ \quad (2.1)$$

For each state $x \in X_\circ$ there is a set $U_\circ(x) \subseteq U_\circ$ whose elements are admissible actions when the state process $X(t)$ takes the value $x$. A policy $\phi$ is a sequence of functions $\{\phi^t\}$ from $X_{t+1}$ to $U_\circ$ such that the allocation sequence defined by

$$U(t) = \phi^t(X(0), \ldots, X(t-1), X(t))$$

satisfies $U(t) \in U_\circ(X(t))$ for each $t$.

The Markov property for the controlled process is: For $x^0, x^1 \in X_\circ, u \in U_\circ, t \geq 0,$

$$P\{X(t + 1) = x^1 \mid (X(0), U(0)), \ldots, (X(t), U(t)); U(t) = u, X(t) = x^0\}$$

$$= P\{X(1) = x^1|U(0) = u, X(0) = x^0\}$$

$$= P_u(x^0, x^1) \quad (2.2)$$

Once a policy $\phi$ is adopted, the controlled transition matrix becomes a transition matrix for a Markov chain. All the stability theory and ergodic theory for Markov chains [21] provides the theoretical framework for analyzing and characterizing optimal policies for MDPs.

Usually a cost or penalty function is given. Based on these functions, performance criteria are considered such as

**Average cost:**

$$\phi := \limsup_{N \to \infty} \mathbb{E}_x \left[ \frac{1}{N} \sum_{k=0}^{N-1} c(X(k)) \right] \quad (2.3)$$

**Discounted cost:**

$$J(x) := \mathbb{E}_x \left[ \sum_{k=0}^{\infty} \beta^k c(X(k)) \right] \quad (2.4)$$

where the constant $\beta$ appearing in 2.4 is the discount parameter, assumed to lie in the open interval $(0, 1)$.

Usual methodologies for solving MDPs include linear programming approaches and algorithms such as Value Iteration (VIA) and Policy Iteration (PIA) [22]. However, solving any MDP for many power systems applications using those algorithms becomes really challenging due to the large-scale na-
ture of the system. For that reason, the use of approximation techniques such as approximate dynamical programming [23] and workload relaxations [22] or the use of reinforcement learning schemes such as TD- and Q-Learning becomes necessary.

The most basic approach to optimization via simulation is the stochastic approximation technique (SA) of Robins and Monro [24]. In one formulation, we have a real-valued function \( f: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R} \), which is interpreted as a cost that depends on a ‘parameter’ \( \theta \in \mathbb{R}^n \) and ‘chance’ \( w \in \mathbb{R}^m \). The goal is to minimize the average cost \( \bar{f}(\theta) = \mathbb{E}[f(\theta, W)] \), where the expectation is w.r.t. the random variable \( W \), which may or may not be known.

If \( \bar{f} \) were computable and differentiable, then we might apply the gradient algorithm to estimate the minimum,

\[
\theta_{k+1} = \theta_k - a_k \nabla \bar{f}(\theta_k), \quad k \geq 0, \tag{2.5}
\]

where \( \{a_k\} \) is a sequence of step-sizes. The SA method is simply this sequence without the averaging: It is assumed that a sequence of random variables \( W = \{W_k : k \geq 1\} \) is observed, and estimates of the optimal parameter \( \theta^* \) are generated by the “stochastic gradient” recursion,

\[
\theta_{k+1} = \theta_k - a_k \nabla f(\theta_k, W_{k+1}), \quad k \geq 0. \tag{2.6}
\]

Under general conditions it can be shown that (2.5) and (2.6) have the same limits, as \( k \) tends to infinity. There are also Newton-Raphson refinements to improve convergence and reduce variance. To generate the random variables \( W = \{W_k : k \geq 1\} \) with low computational costs one can apply recent techniques such as MCMC [25].

The most typical gain sequence is given by \( a_k = 1/(k+1) \), and in this case the SA algorithm is a generalization of the standard Monte Carlo method to estimate the mean of some function \( c(W) \). Denote \( \theta^* = \mathbb{E}[c(W)] \). It can be shown that \( \theta \) minimizes the mean-square error \( \mathbb{E}[(c(W) - \theta)^2] \). On writing \( f(\theta, W) = \frac{1}{2}(c(w) - \theta)^2 \), the SA recursion (2.6) becomes,

\[
\theta_{k+1} = \theta_k - a_k(\theta_k - c(W_{k+1})), \quad k \geq 0.
\]

Under the assumption that \( a_k = 1/(k+1) \), this recursion can be solved
exactly to give
\[ \theta_k = \frac{1}{k} \sum_{i=1}^{k} c(W_i), \quad k \geq 1. \tag{2.7} \]

All of the analytical tools that are known for the simple Monte Carlo recursion (2.7) have analogs in SA.

2.3 Models for the Entropic Grid

Conventional static optimization based models provide a valuable starting point for modeling and analysis. These models must be extended to take explicitly into account dynamics, uncertainty and volatility in an increasingly entropic energy network.

We begin with a description of network models suitable for operations and analysis. We introduce a dynamic dispatch model that captures some of the dynamic issues and uncertainty of the entropic grid.

2.3.1 Dynamic Dispatch for Operations

The models must include uncertainty in many forms. In the grid of the past, uncertainty arose from demand variability, generator outages, and weather. Volatility will increase substantially with greater integration of energy from solar and wind sources. The uncertainty and dynamics of these energy resources are very different from anything seen in the past. As the results of Chapter 3 show, the volatility that comes with renewable energy resources can be costly, even though these energy resources are free. Greater uncertainty could be achieved with the introduction of demand response, especially with dynamic, real-time pricing. While it is often argued that demand response has clear potential benefits for improving reliability and reducing overall volatility, a system with price-responsive demand may in fact be less predictable: The unconstrained behavior of consumers may reveal statistics as exotic as those of the wind!

Once we have settled on network architecture, the specific policy for implementing demand response, and commitment decisions for wind, nuclear, and coal generation, we will have a significant variability and uncertainty.
To cope with these new dynamics combined with greater uncertainty, we will require better mechanisms for reserve management.

It is well known that the deployment of reserves is key to hedge against such uncertainty [26–29]. One way to address the implementation of reserves is by applying parallels between this task, and reserve management in inventory models [22, 30]. Optimal reserves will be zonal and dynamic, depending on current loads at various zones, the mix of available generation, and the ramping capabilities of generation units.

The reserve management approaches described in [22, 30] are based on a version of the models described next.

![Figure 2.3: Three bus system.](image)

**Network Topology** The basic topology is defined by a graph in which each node represents a bus, and each link represents a transmission line. A simple example is shown in Fig. 2.3. Located at each node are one or more of the following: Generation and exogenous demand. A lossless DC model is used to characterize the relationship between generation, demand, and power on the various links. There are \( \ell_n \) nodes, denoted \( \mathcal{N} = \{1, \ldots, \ell_n\} \), and \( L \) transmission lines, indexed by \{1, 2, ..., \( L \). The network is assumed to be connected.

**Demand-side** We denote by \( D_n(t) \) the demand at time \( t \), at bus \( n \), and by \( E_{dn}(t) \) the energy withdrawn by the consumer at that bus. We assume that there is no free disposal for energy, which requires that \( E_{dn}(t) \leq D_n(t) \) for all \( t \). If sufficient generation is available at bus \( n \) at time \( t \), then \( E_{dn}(t) = D_n(t) \).
In the event of insufficient generation, we have \( E_{Dn}(t) < D_n(t) \), i.e., the consumer experiences a blackout.

**Supply-side** We denote by \( E_{sn}(t) \) and \( R_{sn}(t) \) the energy and reserve produced by the supplier at bus \( n \). Generation capacity, \( G_s \), coincides with \( E_s + R_s \). The operational and physical constraints on the production of energy and reserve are expressed abstractly as

\[
(E_s, R_s) \in X_s
\] (2.8)

The usual DC power flow relations are included for the transmission system. In addition, rate constraints are imposed on generation that are a consequence of the physics of both generators and the grid.

**Constraints** The remaining assumptions on the dynamic network model are described as follows:

(i) Generation capacities are subject to strict bounds: For \( n \in \mathcal{N} \) and \( t \geq 0 \),

\[
G_n(t) \leq \overline{G}_n
\] (2.9)

where \( \overline{G} = (\overline{G}_1, \ldots, \overline{G}_{\ell_n})^T \) are fixed \( \ell_n \)-dimensional vectors.

(ii) Generation capacity is *rate constrained*: For all \( t_1 > t_0 \geq 0 \),

\[
\zeta^- \leq \frac{E_s(t_1) - E_s(t_0)}{t_1 - t_0} + \frac{R_s(t_1) - R_s(t_0)}{t_1 - t_0} \leq \zeta^+.
\] (2.10)

(iii) Lossless network, so it neither generates nor consumes energy. Consequently, the network is subject to the supply-demand balance constraint,

\[
1^T E_s(t) = 1^T E_D(t), \quad t \geq 0
\] (2.11)

(iv) Power flows over the network are consistent with the DC power flow model. Suppose bus 1 is selected as the reference bus, based on which the *injection shift factor matrix* \( H \in [-1, 1]^{N \times L} \) is defined, where \( H_{nl} \) denotes the power distributed on line \( l \) when 1 MW is injected into bus \( n \) and withdrawn at the reference bus [30].
Let $f_{l}^{\text{max}}$ denote the capacity of transmission line $l$. On letting $H_{l} \in \mathbb{R}^{N}$ denote the $l$-th column of $H$, the capacity constraint for line $l$ is expressed,

$$-f_{l}^{\text{max}} \leq (E_{s} - E_{D})^{T} H_{l} \leq f_{l}^{\text{max}} \quad (2.12)$$

The network shown in Fig. 2.3 provides an example of the general model in which there are sources of demand and supply at each of three nodes.

The vector of nodal power flows $P$ is given by

$$P = (E_{s1} - E_{D1}, E_{s2} - E_{D2}, E_{s3} - E_{D3})^{T},$$

where the directions of positive power flows are as indicated by arrows in the figure. If the impedances are identical in the three transmission lines, then with bus 1 chosen as the reference bus, the injection shift factor matrix is given by

$$H = \frac{1}{3} \begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & -1 \\ -1 & -1 & -2 \end{bmatrix} \quad (2.13)$$

The flow on the lines are given by $P^{T}H$.

### 2.3.2 The Social Planner’s Problem

Based on this model we can make precise the optimization goals posed by the social planner. This is the first step in assessing the efficiency of an outcome in a market setting.

**Objective Function** To formulate a control problem we introduce a cost function. The production cost for energy injected at bus $n$ at time $t$ is denoted $c_{n}^{E}(E_{Sn}(t))$, and for the reserve provided at that bus is $c_{n}^{R}(R_{Sn}(t))$. The marginal cost of black-out is denoted $c_{n}^{\text{bo}}$. For given generation, reserves, and demand $\{E_{Sn}, R_{Sn}, D_{n}, E_{Dn}\} \subset \mathbb{R}^{4}$, the cost function $c(E_{Sn}, R_{Sn}, D_{n}, E_{Dn})$ is defined by

$$\sum_{n \in \mathcal{N}} (c_{n}^{E}(E_{Sn}) + c_{n}^{R}(R_{Sn}) + c_{n}^{\text{bo}}(D_{n} - E_{Dn})). \quad (2.14)$$
Optimization Problem Putting all the above elements together, the basic optimal control problem becomes

\[
\min_{E_D, E_S, R_S} \int_0^\infty e^{-\eta s} E_x[c(s)] \, ds,
\]

subject to the operational/physical constraint (2.8), the network constraint (2.12), and energy-balance constraint (2.11). The optimization problem (6.5) is a sort of dynamic economic dispatch, in which the objective function corresponds to the expected discounted total cost.

Observe that the cost is a function of the \(4\ell_n\) variables contained in the \(\ell_n\)-dimensional vectors \(\{E_{sn}, R_{sn}, D_n, E_{Dn}\}\). For a network with 1000 generators in operation, such as the California system, this model is not suitable for mathematical analysis. Techniques from decision and control such as aggregate models and workload relaxations lead to a tractable model that can be approximately solved. For example, in [30] insights about effective reserve policies for a similar model are obtained.

This dynamic extension of the usual economic dispatch model is used as the starting point for analyzing the competitive equilibria and the impact of uncertainty, dynamics and constraints in electricity markets.

2.4 Dynamic Competitive Equilibrium for Electricity Markets

The competitive equilibrium is an idealization in which it is assumed that no market player is large enough to influence prices. In typical analyses of static market models, the prices in this equilibrium are equal to the marginal cost of production. In the literature it is found that this holds only on average [12, 31–33]. The sample path behavior of prices can look as erratic as the worst days during the crises in Illinois or California in the 1990s, or in Australia today.

To explain these conclusions, the Lagrangian decomposition analysis in the work [12] is recalled. The consumer and supplier’s objective function is the long-run discounted expected profit with discount rate \(\gamma\), represented by

\[
K_D := \mathbb{E}\left[\int e^{-\gamma t} \mathcal{W}_D(t) \, dt\right], \quad K_s := \mathbb{E}\left[\int e^{-\gamma t} \mathcal{W}_s(t) \, dt\right]
\]
The supplier and consumer each aim to maximize their respective mean discounted mean welfare $K_S, K_D$. We let $W(t) := W_S(t) + W_D(t)$ denote the welfare function of the social planner.

The form of the welfare functions $\{W_D(t), W_S(t)\}$ is not important for our purposes here. The only requirement is that the sum be equivalent to the negative of the cost $c(t)$ defined above (6.5): For some constant $\kappa$,

$$E[W(t)] = \kappa - E[c(t)], \quad t \geq 0.$$  

We now impose several idealistic assumptions for this dynamic model, each of which is an extension of what is assumed in the competitive equilibrium analysis of a static model.

(i) Consumers and suppliers share equal information. This is modeled as a filtration: An increasing family of $\sigma$-algebras, denoted $\mathcal{H} = \{\mathcal{H}_t : t \geq 0\}$. The demand process, and the decisions of the consumers and the suppliers are adapted to this filtration.

(ii) There is a price process $P^e$ that is adapted to $\{\mathcal{H}_t : t \geq 0\}$. However, prices are exogenous: For each $t_0 > 0$, the future prices $\{P^e(t) : t > t_0\}$ are conditionally independent of $\{E_D(t), E_S(t) : t \leq t_0\}$, given current and past prices $\{P^e(t) : t \leq t_0\}$.

Assumption (ii) is known as the price-taking assumption [34].

To Lagrangian decomposition of [12] is obtained on relaxing the constraint that $E_D(t) = E_S(t)$ for all $t$. Let $\{\lambda(t)\}$ denote a vector-valued stochastic process that is adapted to $\mathcal{H}$, of the same dimension as $E_D(t)$ or $E_S(t)$, and consider the Lagrangian relaxation: The dual functional $\Phi(\lambda)$ is defined as the supremum of

$$E\left[\int e^{-\gamma t} (W(t) + \lambda(t)^T(E_D(t) - E_S(t))) \, dt\right]$$

over all adapted processes $E_S, E_D$. Under the assumptions of our prior work, this optimization problem is decomposed into two problems — one for the
consumer and one for the supplier, as follows:

\[
\Phi(\lambda) = \max_{E_D} \mathbb{E}\left[ \int e^{-\gamma t} (W_D(t) + \lambda(t)^{T}E_D(t)) \, dt \right]
\]

\[
+ \max_{E_S} \mathbb{E}\left[ \int e^{-\gamma t} (W_S(t) - \lambda(t)^{T}E_S(t)) \, dt \right]
\]  

(2.16)

Under general conditions there is no duality gap: There is a process \( \lambda^* \) for which the solutions to (2.16) and the social planner’s problem (6.5) coincide. Moreover, by inspection it follows that this forms a solution to the competitive equilibrium, with \( P_e(t) = \lambda^*(t) \) for each \( t \).

Because of unique features of a power market, the equilibrium price process looks very different from the solution obtain for a static model:

(i) The price is marginal value to the consumer. In particular, when reserves are positive at a node in the network, the price is zero.

(ii) In the market models of [35] or [12], the average price for power never exceeds the average marginal cost: For each node \( n \),

\[
\mathbb{E}\left[ \int e^{-\gamma t} P_{n}(t) \, dt \right] \geq \mathbb{E}\left[ \int e^{-\gamma t}\left( \frac{d}{de} c_n^{e}(E_{sn}(t)) \right) \, dt \right]
\]

(iii) If the variance of demand or supply is large, then the variance of \( P_e \) will also be large.

Note that all of our discussion has focused on optimization in the real-time market (RTM). In practice, this is coupled with a day-ahead market (DAM) in which generation is scheduled to meet expected energy requirements, as well as reserves. A model designed to couple the two markets is proposed in [36], in which the welfare functions for both suppliers and consumers are extended to include “day-ahead” welfares,

\[
W_{D}^{\text{rti}}(t) = W_D(t) + W_{D}^{\text{da}}(t), \quad W_{S}^{\text{rti}}(t) = W_S(t) + W_{S}^{\text{da}}(t)
\]  

(2.17)

The details of this construction can be found in Chapter 2. As a final step, to illustrate the impact of dynamics on competitive prices, simulation results for a multiple technology case are surveyed. These results are taken from [12].
Primary and Ancillary Service Simulations

For a power market, the mean demand is often met by a primary service – the cheapest source of service capacity – through a prior contract. The deviations from mean demand can be met through primary service, as well as ancillary service. Typically, ancillary service is costly, but it can be ramped up/down at a much faster rate than the primary service. Nuclear and coal generators can be viewed as primary service providers while gas turbine generators are an example of more expensive, yet more responsive sources of ancillary service.

We consider the model of [35] in which \( G_p(t) \) and \( G_a(t) \) denote the instantaneous output of primary and ancillary generators at time \( t \).

Primary service takes on positive or negative values since it represents deviations from day-ahead schedules. Ancillary service is constrained to be non-negative. In addition, the two sources of generation are distinguished by their ramping capabilities: \( \zeta_{p+}, \zeta_{a+}, \zeta_{p-} \) and \( \zeta_{a-} \) represent the maximum rates for ramping up and down the primary and ancillary services, respectively.

\[
\begin{align*}
-\zeta_p^- (t_1 - t_0) &\leq G_p(t_1) - G_p(t_0) \leq \zeta_p^+ (t_1 - t_0), \\
-\zeta_a^- (t_1 - t_0) &\leq G_a(t_1) - G_a(t_0) \leq \zeta_a^+ (t_1 - t_0).
\end{align*}
\]

For simplicity, we focus on the case in which ramping down is unconstrained, i.e., \( \zeta_{p-}, \zeta_{a-} = \infty \). We assume that \( \zeta_{a+} > \zeta_{p+} \), to reflect the ability of ancillary service to ramp up faster than primary.

The demand at time \( t \) is given as \( D(t) \) and the reserve at that time is given by

\[
R(t) = G_p(t) + G_a(t) - D(t), \quad t \geq 0.
\]  

In our model, we consider some demand-response capabilities; that is certain loads can be turned off to maintain supply-demand balance in the event of reserves shortfall. When the demand exceeds the available supply capacity, demand response capacity is deployed first to balance the demand and supply. If the difference between the demand and available supply is less than the available demand response capacity – i.e., demand response capacity can sufficiently cover the reserve shortfall – the price is set by the cost of demand response. Otherwise, forced blackout will take place, in which case the price equals the choke-up price. Since the cost of demand response is typically lower than the cost of blackout, the price when the supply deficiency can be
covered by demand response would be lower than that if demand response was unavailable. Thus, demand response acts as a cushion between the normal security operations and the blackout. In our model, we use $\bar{r}_{\text{DR}}^{\text{max}}$ to denote total demand response capacity. That is, $\bar{r}_{\text{DR}}^{\text{max}}$ is the threshold up to which load can be shed. Hence, we have black-out situations only if $R(t) \leq -\bar{r}_{\text{DR}}^{\text{max}}$.

In this example the SPP can be converted to a cost minimization problem, in which the cost function on $(G^{p}, G^{a}, R)$ has the following form: For $t \geq 0$

$$C(t) := c^{p}G^{p}(t) + c^{a}G^{a}(t) + (c^{bo} - c^{dr})I(R(t) < -\bar{r}_{\text{DR}}^{\text{max}}) + c^{dr}I(R(t) < 0)$$

where $c^{p}$, $c^{a}$ represent the per unit production costs of primary and ancillary services respectively, $c^{dr}$ is the cost of demand response-based load shedding and $c^{bo}$ is the cost of blackout [35].

**Implications of these results for ancillary service providers** We illustrate the impacts using simulations based on a controlled random-walk (CRW) model. That is, the demand is modeled as a random walk of the form

$$D(k + 1) = D(k) + E(k + 1), \quad k \geq 0, \quad D(0) = 0,$$

in which the increment process $E$ is a bounded i.i.d. sequence. The reserve $R$ is modeled in discrete time using the expression (2.18).

The other model parameters used are as follows: $c^{p} = 1$, $c^{a} = 20$, $c^{dr} = 100$ and $c^{bo} = 400$. The ramp-up rates are taken as $\zeta^{p+} = 1/10$ and $\zeta^{a+} = 2/5$. The marginal distribution of the increment distribution was taken symmetric on $\{\pm 1\}$.

We perform experiments based on a family of threshold policies. Such policy is based on two thresholds $(\bar{r}^{p}, \bar{r}^{p})$. The policy considered, ramps up at maximum rate the primary service anytime $R(t) < \bar{r}^{p}$ subject to the constraint that $R(t)$ does not exceed the threshold

$$U^{p}(t) = \min(\zeta^{p+}, \max(0, \bar{r}^{p} - R(t))) \quad (2.19)$$

in which $U^{p}(t)$ represents the increment of primary service. If $R(t) \geq \bar{r}^{p}$, then primary service ramps and $U^{p}(t) = \bar{r}^{p} - R(t)$. For the ancillary service, if $R(t) + U^{p}(t) < \bar{r}^{a}$, then ancillary service ramps
\[ U^a(t) = \min(\zeta^a, \max(0, \bar{r}^a - R(t) - U^p(t))) \] (2.20)

in which \( U^a(t) \) represents the increment of ancillary service. If \( R(t) + U^p(t) \geq \bar{r}^a \) then ancillary service ramps down

\[ U^a(t) = \max(-G^a(t), \bar{r}^a - R(t) - U^p(t)) \] (2.21)

We perform experiments based on a family of threshold policies. Such policy is based on two thresholds \((\bar{r}^p, \bar{r}^a)\). Under the policy considered, primary service is ramped up whenever \( R(t) \leq \bar{r}^p \). Similarly, ancillary service is ramped up whenever \( R(t) \leq \bar{r}^a \). We refer the reader to [22, 30] for further details about the threshold policy.

In the numerics we considered discounting as in (6.5), but in discrete time, with discount factor \( \beta = 0.995 \). We find the “best-threshold” by estimating the discounted cost by the standard Monte Carlo estimate. We perform several experiments with varying levels of demand response. We use \( c^{dr} \) and \( \bar{r}_\text{DR}^{\text{max}} \) as simulation parameters to study sensitivity of average prices to demand response capabilities.

![Figure 2.4: Optimal thresholds for primary and ancillary service.](image)

Fig. 2.4 plots the optimal thresholds for primary and ancillary service with respect to the demand response capacity \( \bar{r}_\text{DR}^{\text{max}} \). The optimal threshold for primary service is much higher than that of ancillary service, which reflects the fact that the primary service ramps up slower than ancillary service, and primary service is less expensive. The low sensitivity is consistent with the conclusions of [35].

The average prices for primary service, and the conditional average of ancillary service are shown in Fig. 2.5, for various values of \( \bar{r}_\text{max}^{\text{DR}} \). The average price \( E[P^e] = c_p \) is consistent with the conclusions of [31]. The conditional
mean \( \mathbb{E}[P^e \mid G^a > 0] \) denotes

\[
\left[ \int_0^\infty e^{-\gamma t} P^e(t) \mathbb{I}(G^a(t) > 0) \, dt \right] \left[ \int_0^\infty e^{-\gamma t} \mathbb{I}(G^a(t) > 0) \, dt \right]^{-1}.
\]

Figure 2.5: Average prices and variance of \( P^e \).

Also shown in Fig. 2.5 is a plot of the variance of the equilibrium price with respect to \( \bar{r}_{\text{DR}_{\text{max}}} \). We see that the variance drops dramatically with an increase in the demand response capacity, even though the optimal thresholds are virtually unchanged.

Recall that the prices for primary and ancillary service are identical. However, the bulk of primary service is allocated in the day-ahead market. Consequently, ancillary service, such as provided by gas turbines, will be exposed to much greater variability in the efficient market outcome.

The results surveyed in this section can be interpreted in various ways. First, we have shown that prices are highly volatile even under the most idealistic assumptions of the competitive free market, so that volatile prices are usually seen in real world should not be surprising. On the other hand, under general conditions, in this equilibrium the average price of power is only the average marginal cost of generation [12, 32, 35]. Given the high variance of prices, we wonder how the suppliers can stay in business.

We can come to completely different conclusions on re-examining our assumptions: The results of this section are all based on the competitive equilibrium theory which, we believe, provides only a crude model of reality in power systems applications. Information is not symmetric: The consumers do not have access to the same information as the suppliers. Prices are not exogenous: The number of suppliers is finite, so there is ample opportunity
to exercise market power to influence prices in a real-world setting. Are the prices shown in the introduction the result of market power, or the natural price fluctuations in a competitive equilibrium? We do not know, because we do not know what the generator operators are thinking.

This brings us to a source of uncertainty that is potentially greater than the wind: The behavior of the consumers and suppliers of electricity. We cannot pretend to know exactly how the suppliers and consumers will behave to further their interests. This fact brings many research challenges.

### 2.5 Summary

Several basic results, terminology and references useful for the rest of this work were presented. These results underscore the main challenge, and promise, of this research: the need to understand complex economics and engineering models. This need becomes clear in an entropic grid setting in which it is mandatory to appropriately handle uncertainty and dynamics. Additional required background and literature are provided at the beginning of each chapter.
CHAPTER 3

PRODUCT DEFINITION IN ELECTRICITY MARKETS

In this chapter, the role of the product definition in electricity markets is investigated. Electricity auction research has been mainly focused on the auction structure such as pay-as-bid or uniform format. Recognizing the key role of this aspect, the definition of the product itself emerges also as a critical step. Appropriate products are fundamental for a constructive relationship between the market structures and the physical system. Poorly designed products may impact both the market performance—creating negative market conditions—and the physical operation of the system—inappropriate generation mix, for example. In this chapter, focusing on the market dimension, we investigate the impacts that the product definition can have on the market outcome. A product definition implemented in some U.S. electricity markets is used to unveil several critical aspects that must be considered when electricity products are defined. We illustrate those points by defining a product which overcomes some of the investigated issues. Our results demonstrate the importance of defining a proper product in electricity markets and provide guidelines for future research. This chapter is based on [37–40].

3.1 Introduction

There is an ongoing worldwide trend towards the deployment of market structures in the electricity industry. The idea of implementing electricity markets started a few decades ago and it was sustained by several dimensions. The causes of this trend are multifold—technological, academic and historical—and can be summarized as follows. On the technological side, economically efficient generating units of small- and mid-size capacity became reality [41]. Consequently, on the generation side emerged the possibility of having multiple suppliers of different sizes and the idea of implementing market in electrici-
ity, at least in the generation side, started to take shape. The idea was taken up in academia, and the framework of spot pricing for trading electricity emerged in the seminal work of [14]. Last but not least, there was the historical context of the late seventies and early eighties in which the deployment of market structures at many levels of society became popular [4,42,43]. These three dimensions paved the road to the deployment of market structures in electricity in Chile and the UK in the early eighties [6] with the hope that the harnessing of the competitive forces would stimulate innovation, facilitating the achievement of a more efficient system which eventually would result in affordable prices. Although the restructuring process has brought some benefits, in particular in terms of increasing the efficiency and management of utilities [7], many authors have questioned and criticized the real accomplishment of the original market hopes and objectives [4,43,44]. Moreover, some authors still believe that the salient characteristics of electricity make vertical integration essential for an efficient planning and operation of electrical systems [45]. An historical overview about the development of electricity markets along with discussion of future challenges is provided in [46].

A key design element of electricity markets is treating electricity as a commodity. Accordingly, MWhs should not be treated differently from other commodities such as copper or oil. In addition, the MWh commodity can be provided without apparent distinction by any generating technology. As a result of this electricity-as-a-commodity viewpoint, several market structures from other commodity markets such as financial derivatives or forward contracts started to be adopted in electricity. These structures are typically linked to the deployment of a spot market. Forward contracts are common instruments in commodity markets to hedge risk [47]. From the viewpoint of investments, a forward contract creates a long-term signal useful for investors who do not want to rely on the volatile spot markets. In addition, a forward contract market could also improve market efficiency. Using standard economic theory, the authors of [20] show how the implementation of a forward market can make a duopoly market competitive. However, for the particular case of electricity, and once some of its complexities are considered, there is no clear agreement about the market benefits of forward contracts [48,49].

From a physical perspective, however, the use of forward contracts may facilitate the achievement of other objectives such as resource adequacy or appropriate technology mix. The auction processes held in Chile and Brazil
are examples of the use of forward contracts for facilitating resource adequacy [6]. In addition, in the case of Brazil, the auction processes have facilitated the integration of new types of technologies. In terms of designing a market for electricity contracts, what and how to buy/sell are two natural questions that arise. Therefore, the essential issues are: (a) the product definition, the way in which the load is going to be categorized and what the basic unitary product is; and (b) the auction format, the way in which the sellers and the buyers are brought together and the method to clear the underlying product.

Several of the research efforts in electricity auctions have been primarily focused on the nature of the competitive bidding processes and on what auction formats and rules should be adopted, e.g., uniform or pay-as-bid formats [50], bypassing the discussion of the product definition. That discussion is important especially given the experience in other instances such as U.S. spectrum auctions, in which the results illustrate how the auction format and rules can impact the market outcomes [51].

In the literature we find little discussion about the characterization of the product in electricity markets. In the context of a public information game theory, [52] and [53] make an analysis about the impact of the demand packaging in the outcome efficiency, showing how vertical-type packaging does not have efficient equilibria. Similarly, [54] and [6] present some notions about the importance of the product definition.

This apparent lack of interest in the product definition might be also an effect of treating electricity as a standard commodity, a view that fails to capture many of the complexities associated with electricity production such as ramping rates. For example, due to technical limitations, a coal power plant has a maximum load ramping that prevents it from providing energy faster than a hydro power plant. In a similar way, nuclear units are usually used as base-load resource, due to their lack of ramping capabilities. Consequently, it is not only the energy that matters but also the instantaneous power and its trajectory. In addition, there are unique characteristics of electricity such as lack of massive storage capability, just-in-time manufacturing use and the several technical constraints of electricity generation that need somehow to be considered in the specification about what is being traded in these markets. Recognizing in the definition of products the multiple capabilities and services that different technologies can provide seems critical for having a constructive relationship between the physical systems and the
There are real market designs that help to illustrate the impact of a poorly defined product. A clear example is the auction process performed in Illinois during 2006 [38]. The level of prices attained in the process was so high that the auction was canceled after one year of its realization and a new scheme for the procurement of power was defined [55,56]. The final auction prices for a subset of the auction products and the spot market prices in Illinois during 2007 are illustrated in Figure 3.1. Note that the final auction prices of some products are above the market prices for about 90% of the time. In previous works the failure of the Illinois process has been attributed to the product definition based on the so-called *tranches* [38–40], definition that has been also used in auction process held in Maryland, Ohio, Pennsylvania and New Jersey. In addition to the Illinois experience, the aftermath of auctions using this type of products has been less than promising. For many years, electricity rates in New Jersey increased considerably after the implementation of auctions with these type of products. In Ohio the results of one auction realization were rejected by regulators. In Maryland, the implementation of the auction in 2007 resulted in a 72% increase of the electricity rates [40].

At a glance, the tranche-based product definition makes the winner of the contracts responsible for a fixed percentage of the auctioned load. The product is inherently random, resulting from the uncertainty in demand, in both the power and the energy associated. Due to the tranche definition, which seems to be very simple and convenient for the distribution companies, a series of problems during the auction implementation arise. In particular, as we explain in this chapter, tranche-based products may create economic inefficiencies and create conditions for market concentration and participant exclusion. In addition, these types of contracts fail to properly aggregate information in markets and provide insurance beyond the interest of end-consumers. Finally, the key structural characteristic—providing a fixed percentage of the load—creates conditions for high level of prices.

In this chapter, we discuss the impacts of product definition in electricity auctions. Although the implications of a poorly defined product are noticed in both the market behavior and the physical operation of the electricity system, our focus is mainly on the market performance. Through some cases and examples, we identify critical market aspects that should be considered in the product design. Our results demonstrate the importance of defining
properly the product in electricity markets and provides guidelines for future research. The structure of this chapter is as follows. The main elements of the 2006 Illinois Electricity Auction are discussed in Section 3.2. Section 3.3 is devoted to provide economic reasons along with illustrative examples to show the impact that the product definition can have in the market outcomes. Insights about competitive prices for tranche-based products are presented in Section 3.4. Based on our findings an alternative product definition is discussed in Section 3.5. Concluding remarks and future research directions are presented in Section 3.6.

3.2 The 2006 Illinois Electricity Auction

The 2006 Illinois Electricity Auction was created to procure supply for the Illinois distribution companies after the ending of the transition period of the Illinois electric industry. The basis of the Illinois electric industry is the enactment of the Electric Service Customer Choice and Rate Relief Law of December 1997. The restructuring of the Illinois electric industry led to the eventual establishment of the Ameren and Exelon holding companies with their respective generation assets removed from the regulated companies to form speculative market entities under the holding company structure.
During the transition period, which was legislatively extended and lasted nearly a decade, the tariffs of the residential and small commercial customers were artificially frozen and the Illinois utilities met their demands using long-term contracts. In 2004, the Illinois Commerce Commission (ICC) started a series of workshops and forums to study what steps to take following the end of the transition period. One of the recommendations was to introduce auctions in the procurement process for mid-term duration contracts, along the lines of other states, such as New Jersey and Maryland. In 2006 the ICC approved the use of the auction mechanism proposed by the Illinois utility subsidiaries of Exelon and Ameren. The auction was held in September 2006 and the impacts on the tariffs started to be felt from January 2007. The Auction outcomes were nearly universally disliked in light of the high prices that resulted.

The rationale for the Illinois Auction was to ensure reliable supply over the next 17-41 months, starting on January 2007, for the Illinois distribution companies owned by Ameren - CILCO, CIPS and IP - and the Exelon owned ComEd. Two auctions, one for fixed-priced customers and the other for hourly-priced customers, were run in parallel. Our analysis focuses only on the fixed price section because the outcomes of the hourly price section were immediately rejected by the ICC.

The 2006 Auction was designed and managed by NERA Economic Consulting, and used the format of the New Jersey electricity auction. The 2006 Illinois Electricity Auction is a multiple-product reverse simultaneous descendant “clock” auction. The reverse refers to the fact that the sellers, rather than the buyers, have the active role, and the descendant clock indicates the deployment of price decrements in the auction. The parties involved in the auction are the regulated distribution companies (the buyers) holding company buyers, twenty-one selling entities including the Exelon and Ameren generation subsidiaries and NERA, the Auction Manager. In addition, there are the observers the ICC and the Auction Monitor. The role of the Auction Monitor is to review the Auction results and to provide specific recommendations to the ICC. The ICC takes the final decision whether to accept or reject the Auction results. The structure of the Auction is summarized in Fig. 3.2.

The multiple products of the Auction are differentiated in terms of the distribution company, the customer class and the contract duration. The set
of products is showed in Fig. 3.3.

The unit of each product is specified in terms of the so-called tranche. The tranche of the chronological load over a given period is defined to be a specified fraction of the load at each point in time during that period. We illustrate the supply of a week-long load in terms of 10% tranches in Fig. 3.4. The total load is supplied by four sellers who provide one, two, three and four tranches, respectively.

We now present the outcomes of the 2006 Illinois Electricity Auction. The Auction began on September 5, 2006, with the initial prices set by the Auc-
tion Manager. The Auction went through 39 rounds and terminated on the fourth day. The initial prices were 104 $/MWh for the large and 100 $/MWh for the small to medium customers of Ameren and ComEd.

The Auction was performed in rounds in which the Auction Manager set the prices for the 8 products for each round and each seller was allowed to offer one or more tranches of each of the 8 products. The only information released by the Auction Manager following each round was the price level for the next round and the range of oversupply for the total number of products. As long as there was an oversupply of any single product, a new round was launched by the Auction Manager with the prices in the new round modified from those in the previous round using non-negative decrements. The attainment of the supply-demand equilibrium in the 8 products signaled the end of the Auction. Each seller of a specific product received the identical price for each unit of the product sold. We provide a plot of the sequence of prices in Fig. 3.5.

3.2.1 Analysis of the Results

The dominant characteristic of the Auction results is the uniformly high prices for all the 8 products. The high prices are readily evident from the wholesale electricity market prices in 2007, the first year covered by the Auction outcomes. In fact, the prices set by the 2006 Illinois Electricity Auction were considerably higher than the average market prices in the Midwest ISO

Figure 3.4: Basic Auction product.
and PJM at the Illinois locations. It is interesting to note that, on the average, the ComEd purchase prices are lower than the Ameren ones. The high level of prices is evident from a comparison of the Auction prices with the daily locational marginal prices (LMP) on particular nodes of such system operators. We present the results of the comparison for the Ameren products of the Auction with the LMP at the so-called Illinois-Hub in the Midwest ISO.

The Midwest ISO evaluates LMPs on a five-minute basis and determines the hourly LMP as the time-weighted average of the five-minute values. We define the daily LMP to be the average of the 24 hourly LMP values. The Illinois-Hub is not a physical node but rather a fictitious node whose LMP is computed by the Midwest ISO using the LMP values from about 150 nodes located in the central, southern and southwestern parts of Illinois essentially covering the service of the three Ameren distribution companies. The Illinois-Hub LMP is a useful proxy for the wholesale electricity prices in the Ameren locations.

We use the daily Illinois-Hub LMPs nodal prices for the period from January 1 - December 31, 2007, to represent the market prices. We present the plot of these prices together with the price levels associated with the four Ameren products over the same period in Fig. 3.6.

We observe that the price of each product exceeds the average Illinois-Hub LMP for 2007. In fact, throughout the year, the daily LMP is below the
Auction prices expect for some short periods.

We can actually measure the fraction of time during which the exceptions occur. We do so by rearranging the average daily LMPs from the highest to the lowest to create a so-called price-duration curve. We present the Illinois-Hub LMPs duration curve in Fig. 3.9 together with the Auction prices for the four Ameren products. The LMPs exceed the Auction prices no more than 10% of the time. In other words, the market prices are below the Auction prices 90% of the days in 2007. Indeed, the Auction price of the BGS-LF17 product is above the market price about 97% of the days in 2007. These conclusions clearly point out that MWh of each of the four Ameren products may have been purchased at considerably lower prices in the market than those set by the Auction.

The final Auction prices show a clear decoupling between the prices of the products for small and medium size customers and those of the products for the large industrial and commercial customers for both distribution companies. Even though the initial Auction prices for the two sets of products started at nearly the same level - 100 $/MWh and 104 $/MWh, respectively - the final prices are at the level of 65 and 64 $/MWh for the small and medium customers of Ameren and ComEd, respectively, and 85 and 90 $/MWh for the large industrial and commercial customers of Ameren and ComEd, respectively. The higher prices of the latter products aimed at the large industrial and commercial customers of Ameren and ComEd reflect the uncertainty issues associated with this class of customers with the greater
flexibility to switch loads to alternative energy service providers. Since the latter customers are more likely to migrate their loads, they represent another source of uncertainty to the sellers. The difference between the price for these products and those for the small and medium customers is, in effect, the premium charged by the sellers to cover this additional uncertainty.

![Price duration curve](image)

Figure 3.7: Price duration curve.

There is also a counterintuitive outcome in the ComEd product prices, in that the longer contracts are priced lower than the shorter ones. Since, over the longer period, the uncertainty is higher, the expectation is that the prices associated with longer-term contracts tend to be higher than those covering shorter periods. The only explanation for this anomalous behavior is the desire of the large amount of nuclear generation in Illinois to find assured markets for longer period, even if the sales are at lower prices.

Our analysis indicates that the product definition is one of the key issues in the Auction design. An obligation that requires a fixed quantity of energy is vastly different from that requiring a fixed percentage of a given class load, as is the case of the tranche-based product in the 2006 Illinois Electricity Auction. The high prices attained in the 2006 Illinois Electricity Auction are therefore attributable, in part, to the problems entailed by the tranche-based product definition in use.
3.2.2 Product Definition Issues

Forward contracts have been used in many electricity markets around the world. The main objectives of such contracts are to

- Reduce the volatility of the price of electricity to the end users,
- Provide an assured market to generators at a specified price,
- Decrease the market power of certain companies in the electricity markets, and
- Provide effective price signals to stimulate investments in new generation assets.

The objective of reducing volatility of final prices serves the interest of risk-averse consumers, who wish to avoid the huge price fluctuations that are usual in the spot markets. Such risk-averse consumers are willing to pay a moderate risk premium for the security of less volatile prices. On the other hand, the economic’s literature suggests that the introduction of forward contracts may act as a pro-competitive device, with possible reduction in final prices, thus also benefiting consumers. Also, the markets provide good mechanisms to aggregate information and provide reliable signals for the necessity of further investments in generation.

Unfortunately, the contracts emanating from the tranche-based products used in the 2006 Illinois Electricity Auction fail to achieve the objectives above for a number of reasons, all of them entailed by the product definition problems. The main problems include the following:

- The tranche contracts provide full protection to the distribution companies, by transferring all the uncertainties to the sellers. However, this protection is beyond the interest of consumers.
- The uncertainty associated with the migration of distributors’ consumers has a component of information asymmetry that likely will drive prices up.
- The products delivered under the contracts are highly artificial since they involve the production of electricity that no single generating unit is capable of producing or doing so efficiently.
• The satisfaction of such contracts provides the impetus for consolidation of generation assets, leading to the concentration of the market into fewer entities. Such moves raise market power concerns and result eventually in reduced competition.

• The product definition does not allow comparison of demand provision among the sellers, because it refers to the unknown demand just in percentage terms. Thus, the information aggregation function of the market is undermined.

3.3 Analyzing a Product Definition

When a market for contracts is implemented, a natural question arises: How do the terms of the contract impact the market outcomes? This question has been overlooked in the electricity markets literature, mainly because in standard commodity markets the product definition is somehow natural—for example 1 barrel of oil or 1 pound of copper. However, electricity is radically different from any other commodity due to the technology involved, its link to a physical network that is highly complex, and its importance to society. Based on previous electricity auction processes, we claim that the product definition is a key element of any market for electricity contracts.

In this section, using a particular type of contract, we provide key elements that should be taken into account in the design of electricity contracts. Such elements are mainly related to economic and market performance. Although not discussed in this work, the definition of the contracts also impacts the achievement of other objectives beyond market and economic ones. In particular, the terms of the contracts will also play an important role in achieving objectives such as system reliability and environmental fulfillment. A non-interfering linkage between the market and the physical operation of the system can be achieved only by having products that capture the physical constraints and needs for achieving those objectives. Attributes such as location of the generating resources, volatility that different resources inject into the system, environmental impacts and flexibility should be also considered in the definition of appropriate products.
3.3.1 Tranche-based Products

We use the product definition used in the 2006 Illinois Auction to investigate the impacts of the product definition in market outcomes. We start defining the key terms of these type of contracts. Firstly, we introduce a load model. Assume that the load over a given period $H$ is a random variable $\tilde{l}(h)$. Moreover, assume that the load can be further decomposed as

$$\tilde{l}(h) = l_f(h) + \varepsilon(h) \tag{3.1}$$

where $l_f(h)$ is a deterministic part and $\varepsilon(h)$ is a random one. The deterministic part is forecasted. Consider an index set $\mathcal{I} = \{i : i = 1, ..., I\}$ of suppliers. The tranche-based contract ($\gamma_i(h)$) is defined to supply a fixed proportion ($\alpha_i$) of the total load,

$$\gamma_i(h) = \alpha_i \tilde{l}(h) : 0 \leq \alpha_i \leq 1 \land \sum_{i \in \mathcal{I}} \alpha_i = 1 \tag{3.2}$$

$$\Rightarrow \sum_{i \in \mathcal{I}} \gamma_i(h) = \tilde{l}(h) \tag{3.3}$$

Note that the contracts themselves are random variables as their associated power depends on the not-yet-realized load. Consequently, the contracts are associated not only with energy but to several other attributes such as risk insurance and other ancillary services. In forthcoming sections of the chapter, we focus on specific attributes associated with the tranche-based contract.

3.3.2 Model for Supplying Contracts

In order to illustrate economic issues emerging from the use of tranche-based products, a simple model for supplying contracts is presented. In this model, we focus only on the deterministic part of the load. Assume that the forecasted load ($l_f$) is decomposed in three components: base load ($l^b$), cycling load ($l^c$) and peak load ($l^p$), i.e., $l_f = l^b + l^c + l^p$. The load can be alternatively represented by the triplet $\mathbf{l} = (l^b, l^c, l^p)$. Assume an idealized set of generators ($\mathcal{I}$) to supply the load at the three load levels—base, cycling and peak levels. Considering a generator $i$, the total generated power is decomposed in base ($s^b_i$), cycling ($s^c_i$) and peak ($s^p_i$) power with their corresponding base
(\(c^b_i\)), cycling (\(c^c_i\)) and peak (\(c^p_i\)) costs. The consideration of different costs for each load segment allows one to capture some of the ramping capabilities of the resources. If a generator \(i\) cannot attend the peak demand then its cost is infinite, i.e., \(c^p_i = \infty\). The generated power is subject to the maximum power (\(P_{\text{max},i}\)), i.e., \(s_i = s^b_i + s^c_i + s^p_i \leq P_{\text{max},i}\). The generated power and cost can be alternatively represented by the triplet \(s_i = (s^b_i, s^c_i, s^p_i)\) and \(c_i = (c^b_i, c^c_i, c^p_i)\), respectively.

Let \((s_i)_{i \in \mathcal{I}} = (s^b_i, s^c_i, s^p_i)_{i \in \mathcal{I}}\) be the power allocation of all generators to supply the base, cycling and peak load segments. We say that the allocation \((s_i)_{i \in \mathcal{I}}\) is feasible if

\[
\sum_{i \in \mathcal{I}} s^b_i = l^b; \sum_{i \in \mathcal{I}} s^c_i = l^c; \sum_{i \in \mathcal{I}} s^p_i = l^p; \quad (3.4)
\]

\[s^b_i + s^c_i + s^p_i \leq P_{\text{max},i}, \forall i \in \mathcal{I} \quad (3.5)\]

The set of feasible allocations is denoted by \(\mathcal{F}\). We say that a feasible allocation \((s^*_i)_{i \in \mathcal{I}}\) is efficient if

\[
(s^*_i)_{i \in \mathcal{I}} \in \arg\min_{(s_i)_{i \in \mathcal{I}} \in \mathcal{F}} \sum_{i \in \mathcal{I}} s_i \cdot c_i \quad (3.6)
\]

where \(s_i \cdot c_i = s^b_i c^b_i + s^c_i c^c_i + s^p_i c^p_i\) is the standard inner product of vectors.

### 3.3.3 Market Outcome Analysis

By using several examples, we show that tranche-based contracts create problems such as economic inefficiency, competition reduction, market concentration, information aggregation, insurance distortion and information asymmetry.

**INEFFICIENCY** If tranche products are used to determine allocation, inefficiency occurs, provided that there are different generators. Consider two generators with the following costs: \(c_1 = (c^b_1, c^c_1, c^p_1) = (5, 15, 50)\) and \(c_2 = (10, 12, 15)\) and total capacity \(P_{\text{max},1} = P_{\text{max},2} = 10\). Assume that the demand is \(l = (l^b, l^c, l^p) = (4, 3, 3)\). By inspection, the efficient allocation is \(s_1 = (s^b_1, s^c_1, s^p_1) = (4, 0, 0)\) and \(s_2 = (s^b_2, s^c_2, s^p_2) = (0, 3, 3)\), with a total cost of \(4 \cdot 5 + 3 \cdot 12 + 3 \cdot 15 = 101\). However, any tranche allocation of \(\alpha \in [0, 1] \quad (44)\)
for generator 1 and \((1 - \alpha)\) for generator 2 will produce a total cost of

\[
\alpha(4 \cdot 5 + 3 \cdot 15 + 3 \cdot 50) + (1 - \alpha)(4 \cdot 10 + 3 \cdot 12 + 3 \cdot 15) = 215\alpha + (1 - \alpha)121,
\]

which is more expensive than the efficient one.

\[\square\]

**Participant exclusion** Consider the same system as before but \(c_1 = (c_{b1}, c_{c1}, c_{p1}) = (5, 15, +\infty)\). Being not able to provide power peak \((c_{p1} = \infty)\), generator 1 cannot supply a fixed proportion of the load \((\alpha_1)\). The unique tranche allocation is to assign the load to generator 2. Generator 1 is ruled out of the market. The cost of this allocation is 121.

\[\square\]

Note that in this scenario monopoly occurs and payments can be even higher than 121. In a general scenario with more generators, the tranche allocation may lead to the creation of bundling contracts—a third company could buy energy from different generators to meet the needs defined by the tranche. This bundling option works as a coordination device or as a mechanism of collusion. We examine it in the following example.

**Market concentration** Consider now three generators with the following costs: \(c_1 = (5, 15, +\infty)\), \(c_2 = (10, 11, 15)\) and \(c_3 = (12, 12, 13)\). Assume that the maximum capacities are \(P_{\text{max},1} = P_{\text{max},2} = 10\) and \(P_{\text{max},3} = 15\). The load is defined by \(l = (10, 7, 5)\). The optimal allocation is \(s_1 = (10, 0, 0)\), \(s_2 = (0, 7, 0)\) and \(s_3 = (0, 0, 5)\) with a total cost of 192. In the tranche allocation, generator 1 is excluded due to its inability to provide peak power. Thus, generators 2 and 3 have to supply a proportion of the load in its three levels subjected to their maximum powers, i.e., \(s_2 = \alpha l \leq P_{\text{max},2}\) and \(s_3 = (1 - \alpha)l \leq P_{\text{max},3}\). Therefore,

\[
s_2 = 10\alpha + 7\alpha + 5\alpha = 22\alpha \leq 10 \quad (3.7)
\]

\[
s_3 = 10(1 - \alpha) + 7(1 - \alpha) + 5(1 - \alpha) = 22(1 - \alpha) \leq 15 \quad (3.8)
\]
From Eqs. (3.7)-(3.8), the tranche allocation is restricted to $7/22 \leq \alpha \leq 10/22$. The corresponding generator costs and total cost ($c_T$) are

\[ c_2 = b_s b_s + c_s c_s + p_s p_s = 252\alpha \quad (3.9) \]
\[ c_3 = b_s b_s + c_s c_s + p_s p_s = 269(1 - \alpha) \quad (3.10) \]
\[ \Rightarrow c_T = c_2 + c_3 = 269 - 17\alpha \quad (3.11) \]

Consequently, the minimal total cost is $c_T \approx 261.3$ and occurs when $\alpha = 10/22$. Now, assume that a company buys (or makes a financial arrangement with) the first two generators. Then, the company can provide the allocation $(10\alpha, 7\alpha, 5\alpha)$ with a cost of $10\alpha \cdot 5 + 7\alpha \cdot 11 + 5\alpha \cdot 15 = 202\alpha < 252\alpha$. Depending on the auction’s rule, the firm will pocket the difference. Of course, the company can do even better by combining all three generators. In that case, the total cost for the company will be the efficient one, but now there is a monopolistic firm in the market, which may charge even higher prices. \(\square\)

The example above makes clear that the tranche-based product might promote the concentration of companies. This can have undesirable impacts in the competition and, consequently, on the final price.

But there are other elements and issues related to the tranche-based product definition. Unlike the previous points, these additional elements are related to uncertainty issues. First, the tranches market cannot work properly because it does not transmit or convey useful information. Consider the following example.

**Information Aggregation** Suppose that two companies have similar costs, but different beliefs about the demand. One thinks that the demand will be 110 MW while the other thinks it will be 120 MW. Let us assume that the price of 1% tranche contract is 1 $/MWh. Consequently, the first company is expecting $1.1 for the contract while the second expects $1.2. Note, however, that the price facing both companies is the same: 1 $/MWh. Hence, the same contract has different values for identical firms only because they expect different loads. Since the competition is in dollars per MWh, it is impossible to aggregate the generators’ beliefs in the electricity product. In contrast, a normal contract of 1 MWh during a period will pay exactly the same amount to both companies without being subjected to the demand
This phenomenon occurs because the tranche contract has an extra-dimension which is the load shape uncertainty. This additional dimension cannot be captured using a single dimensional price. This issue suggests that energy auctions with tranche-based contracts will reduce competition by favoring large generating companies that can take the risk of an uncertain product.

In terms of market considerations, there are two more elements that should also be taken into account in the design of contracts: first, understanding the type of insurance that the contract is providing, and second, how the contract terms can facilitate symmetry of information among the several parties involved.

**Uncertainty Protection** The tranche definition places all the uncertainty on the sellers’ shoulders. The role of the distribution companies is reduced to being simply the delivery channel to the end users, without carrying any uncertainty for the commodity delivered. The uncertainty in the tranche-based product is a function of the yet-to-be-realized loads in the period of interest since each seller must meet a specified fraction of the loads and thus carries a fraction of the energy delivered and the maximum capacity required to meet the peak. We illustrate in Fig. 3.8 the impacts of uncertainty using a four-seller case.

![Figure 3.8: The impacts on the tranche-based product sellers of the capacity and volumetric uncertainty in the load.](image-url)

For the weekly period, the actual load differs from that forecasted and the impacts on each of the sellers are indicated for this case. The loads and the
resulting load shape are inherently random, as they are dependent on various sources of uncertainty. Consequently, the power and energy associated with a tranche are uncertain. A tranche seller has a volumetric and a capacity uncertainty in what he sells. The volumetric uncertainty impacts the expected revenues, and the capacity uncertainty entails uncertainty in the utilization of the generation resources required to meet the tranche obligation. Such capacity and volumetric uncertainty was historically faced by the “utility” in the vertically integrated utility structure together with the uncertainty in generation such as forced outages and fuel price escalation.

The concentration of uncertainty into the sellers’ hands suggests that one of the objectives of the definition of contracts using tranches is to provide insurance to the distribution companies. Such an objective may be viewed as legitimate, given the goal of price volatility reduction. However, this mistaken view arises from a confusion between the distribution companies and the consumers. The distinction between the risks faced by the consumers and the distribution companies is clear. While the former care only about the price that they will pay, the latter face uncertainty about the total load that must be served. Also, while consumers are naturally assumed to be risk-averse, willing to pay a premium for less volatile prices, economists, in general, classify companies as risk neutral.

Now, risk-averse agents will buy full insurance if and only if the premium of the insurance is actuarially fair, that is, equal to the value that the insurance pays in the expected term. If the premium is higher than its actuarially fair value, the risk-averse consumers will underinsure. Since tranche products carry both the uncertainty of the load and the uncertainty of electricity prices, its actuarially fair value will be above an insurance just for the electricity price. Since the uncertainty faced by end users is restricted to the electricity price and does not include the uncertainty about the load, the tranche products provide more protection than the end users are interested in. In this situation, they would consider this premium above the fair value and would prefer to underinsure. However, in the Auction 100 % of the load was procured, with full insurance to the distribution companies. Of course, the distribution companies may be interested in having insurance for their total load risks. However, if they want to hedge these risks, they can do this through normal financial mechanisms.
Asymmetric information  The presence of migration risks in the contracts for the larger commercial and industrial consumers is an additional factor for the higher prices for the products for those customers. The Illinois restructuring legislation permits any customer to shift its load from the incumbent provider to an alternative. While such freedom has had no impact on the small residential customers, there have been major shifts of commercial and industrial loads over the past decade. The fact that such shifts may continue implies that the contracted energy consumed by the shifting customers will be supplied by other sources than the distribution company. Of course, while shifts into the distribution company may also occur, the integration of the new loads by the distribution entity entails changes in the total supply that must be delivered by the tranche sellers. Therefore, there are considerable uncertainties for the contract sellers that the loads in the future may change from the historical load shape. In particular, the uncertainty faced by small generation entities is far more marked than that faced by companies with many generation units since small entities have far less flexibility in dealing with such uncertainty. Such migration-risk impacts the valuation that each seller performs for the products associated with large customers. The sellers have no choice but to charge an additional insurance premium to cover against such additional uncertainty.

There are informational asymmetries in the market for the contracts to supply medium and large customer loads of the type economists classify as either adverse selection or moral hazard. The large consumers have the best information about their willingness to shift their loads from the distribution company. Given the direct contact between the distribution companies and the medium and large customers, the distribution companies are likely to be better informed that the sellers. As the seller of the contract is less informed than the buyer, it is possible that market problems occur that lead to high prices or even absence of trade.

A far more acute problem is that the adverse selection is the one with the moral hazard issues, with respect to the large consumers and the distribution companies. A generator requires a price that pays for the impacts of the expected migration, thereby leading to higher prices to the large consumers. Such prices provide an incentive for such medium and large customers to negotiate a direct deal with another generator for a period explicitly not allowing any migration. The existence of such contracts further adds to the
uncertainty of the contracts for the large customers products and consequent high prices that encourage large consumers to leave the distribution companies. There exists another, less serious, moral hazard problem for the distribution company, which will have less incentive to hold on to the large customers, since no direct losses are attributable to them.

The problems qualitatively discussed in this section are important and are likely to explain most of the problems that occurred in the 2006 Illinois Electricity Auction. However, this list of problems is not exhaustive. Although it seems almost impossible to provide quantitative figures for the relevance of these problems, it is clear that their existence calls for a careful consideration of alternative product definition. In the next section, a competitive equilibrium analysis of the equilibrium prices for tranche-based contracts is performed.

3.4 Market Clearance: A Comparison

We study the market clearing prices considering tranche-based products. We assume, following the treatment of bilateral contracts in several markets [57], that contracts and bilateral-transactions are explicitly considered into the dispatch of the system. The comparison focuses on the energy attribute of the tranche-based product. Other attributes associated with the tranche-based contract such as transmission capabilities, capacity, load-following and other ancillary services are not included. A comparison of the whole set of services associated with the tranche-based product would require the consideration of all those associated markets and it is beyond the scope of this work. However, as discussed in [40], the aggregate of all those additional services corresponded to a minimum part of the contract final prices. Hence, in terms of the value of the contracts, the most important service was energy.

In order to make the comparison of the energy attribute, the traditional economic dispatch problem is considered as a benchmark. We focus on the simplified case in which all the information is available, there is no uncertainty in the future load, and the only differences among suppliers are their costs and capacities.
3.4.1 Mathematical Formulation

Consider an index set \( I = \{i : i = 1, \ldots, I\} \) of suppliers, and an index set \( H = \{h : h = 1, \ldots, H\} \) of time horizon. The supplier \( i \) provides a power \( P_{i,h} \) at hour \( h \) subjected to a maximum of \( P_{\text{max},i} \) and a minimum of zero; its cost function is given by \( c_i(\cdot) \). The load for each hour is \( l(h) \) and its maximum during the period is \( l_{\text{max}} = \max\{l(1), \ldots, l(h), \ldots l(H)\} \). The centralized tranche allocation problem is defined as follows.

**Centralized tranche dispatch**

\[
\min_{\alpha_i} \sum_{i \in I, h \in H} c_i(\alpha_i l(h)) \quad \text{s.t.} \quad \sum_{i \in I} \alpha_i = 1 \quad \text{and} \quad 0 \leq \alpha_i \leq \min\left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}}\right\} \quad \forall i \in I
\]

(3.12)

In order to get insights about the optimal solution of the problem, consider its Lagrangian function

\[
\mathcal{L}(\alpha_i, \lambda, \mu_i^+, \mu_i^-) = \sum_{i \in I, h \in H} c_i(\alpha_i l(h)) + \lambda (1 - \sum_{i \in I} \alpha_i) + \sum_{i \in I} \mu_i^+ (\alpha_i - \min\left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}}\right\}) - \sum_{i \in I} \mu_i^- \alpha_i
\]

(3.13)

where the Lagrangian multipliers \( \mu_i^+ \) and \( \mu_i^- \) are nonnegative, and \( \lambda \) is unrestricted. The Karush-Kuhn-Tucker (KKT) optimality conditions \( \forall i \in I \) are

\[
\sum_{h \in H} l(h) \frac{\partial c_i(\alpha_i l(h))}{\partial P_i} - \lambda + \mu_i^+ - \mu_i^- = 0, \quad \text{(3.14)}
\]

\[
\mu_i^+ (\alpha_i - \min\left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}}\right\}) = 0 \quad \text{(3.15)}
\]

\[
\mu_i^- \alpha_i = 0 \quad \text{(3.16)}
\]

From the slackness conditions (3.15)-(3.16), if \( 0 < \alpha_i^* < \min\left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}}\right\} \) then \( \mu_i^* = \mu_i^- = 0 \). From Equation (3.14), the Lagrangian multiplier \( \lambda^* \) is equal to \( \sum_{h \in H} l(h) \frac{\partial c_i(\alpha_i l(h))}{\partial P_i} \). Given the convex structure of the problem, if the centralized dispatch has a solution it is possible to find a price that will support the efficient outcome. Let \( p_{\text{tranche}}^* \) be the competitive price associated
with the tranche-based contract. Assuming price-taking behavior, supplier $i$ faces the problem of maximizing his profits,

$$\max_{\alpha_i} \left[ \sum_{i \in I, h \in H} p_{\text{tranche}}^* \alpha_i l(h) - \sum_{i \in I, h \in H} c_i(\alpha_i l(h)) \right]$$

$$s.t \quad 0 \leq \alpha_i \leq \min \left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}} \right\}$$ (3.17)

The Lagrangian of this problem is given by

$$\mathcal{L}(\alpha_i, \lambda, \mu_i^+, \mu_i^-) = \sum_{h \in H} p_{\text{tranche}}^* \alpha_i l(h) - \sum_{h \in H} c_i(l(h)\alpha_i)$$

$$+ \mu_i^+ \left(\alpha_i - \min \left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}} \right\}\right) - \mu_i^- \alpha_i$$ (3.18)

Writing the KKT conditions,

$$- \sum_{h \in H} l(h) \frac{\partial c_i(\alpha_i l(h))}{\partial P_i} + p_{\text{tranche}}^* \sum_{h \in H} l(h) + \mu_i^+ - \mu_i^- = 0$$ (3.19)

$$\mu_i^+ \left(\alpha_i - \min \left\{1, \frac{P_{\text{max},i}}{l_{\text{max}}} \right\}\right) = 0$$ (3.20)

$$\mu_i^- \alpha_i = 0$$ (3.21)

Using Equation (3.19), the competitive equilibrium price is obtained and defined as

$$p_{\text{tranche}}^* = \frac{\sum_{h \in H} l(h) \frac{\partial c_i(\alpha_i^* l(h))}{\partial P_i}}{\sum_{h \in H} l(h)}$$ (3.22)

where $i$ is any generator such that $0 < P_i^*(h) = \alpha_i^* l(h) < P_{\text{max},i} \ \forall \ h$, i.e.,

$$\mu_i^+ = \mu_i^- = 0$$ as the constraints are non-binding. In addition, comparing Equations (3.14) and (3.19), the following relation between $\lambda^*$ and $p_{\text{tranche}}^*$ is obtained:

$$p_{\text{tranche}}^* = \frac{\lambda^*}{\sum_{h \in H} l(h)}$$ (3.23)

Our interest is to compare the tranche equilibrium price with the results of the economic dispatch problem. The prices associated with the economic dispatch are used as a proxy for electricity market prices—as in the case of implementing a spot-market dispatched at minimum cost. Certainly, in
this case, given that it is assumed that the load is supplied by the tranche-based products, there is no consideration of any type of interaction between a market for tranche-based contracts and a spot market for energy. The main objective of the comparison is to assess the effectiveness of tranche-based products to provide energy supply, and to assess the competitive prices in relation to those of a standard electricity market. A future research avenue certainly could be precisely to consider the interaction between those markets and study related strategic issues. In order to make a clear comparison, define $P_{i,h}^e$ as the power delivered by supplier $i$ at time $h$ in the economic dispatch context, which may not be equal to $P_{i,h}$—supplier allocation in the tranche dispatch context. The centralized economic dispatch is defined as follows.

**Centralized economic dispatch**

$$\begin{align*}
\min_{P_{i,h}^e} & \sum_{i \in I, h \in H} c_i(P_{i,h}^e) \\
\text{s.t.} & \sum_{i \in I} P_{i,h}^e = l(h), \quad \forall h \in H \\
& 0 \leq P_{i,h}^e \leq P_{\text{max},i}, \quad \forall i \in I, \forall h \in H
\end{align*}$$

(3.24)

In order to gain insight into the optimal solution of the problem, consider its Lagrangian function

$$\begin{align*}
\mathcal{L}(P_{i,h}^e, \lambda_h, \mu_{i,h}^+, \mu_{i,h}^-) &= \sum_{i \in I, h \in H} c_i(P_{i,h}^e) + \sum_{h \in H} \lambda_h \left(l(h) - \sum_{i \in I} P_{i,h}^e\right) \\
&\quad + \sum_{i \in I, h \in H} \mu_{i,h}^+ \left(P_{i,h}^e - P_{\text{max},i}\right) - \sum_{i \in I, h \in H} \mu_{i,h}^- P_{i,h}^e
\end{align*}$$

(3.25)

where the multipliers $\mu_{i,h}^+$ and $\mu_{i,h}^-$ are nonnegative, and $\lambda_h$ are unrestricted. The KKT optimality conditions $\forall i \in I, \forall h \in H$ are

$$\frac{\partial c_i(P_{i,h}^e)}{\partial P_{i,h}^e} - \lambda_h + \mu_{i,h}^+ - \mu_{i,h}^- = 0, \quad \text{(3.26)}$$

$$\mu_{i,h}^+ \left(P_{i,h}^e - P_{\text{max},i}\right) = 0 \quad \text{(3.27)}$$

$$\mu_{i,h}^- P_{i,h}^e = 0 \quad \text{(3.28)}$$
When the power limit constraints are non-binding, \( \mu_{i,h}^{+} = \mu_{i,h}^{-} = 0 \) \( \forall i \in I, \forall h \in H \). Therefore, the Lagrangian multipliers \( \lambda_{h}^{*} \) are equal to \( \frac{\partial c_{i}(P_{i,h}^{*})}{\partial P_{i,h}} \) \( \forall h \in H \). The solution of this problem provides, for each hour \( h \), the well-known marginal cost condition. The cheapest units will be loaded to their maximum power, while those units operating within their power limits will be loaded in such a way that they have the same marginal cost. The competitive price for each hour, \( p_{h}^{*} \), is given by the marginal cost of the last dispatched unit,

\[
p_{h}^{*} = \frac{\partial c_{i}(P_{i,h}^{*})}{\partial P_{i,h}}
\]

where \( i \) is any generator such that \( 0 < P_{i,h}^{e} < P_{\text{max},i} \).

In order to understand how the tranche-based product prices \( p_{\text{tranche}}^{e} \) are related to the benchmark prices \( p_{h}^{*} \), we focus on the structure of these problems. In particular, the economic-dispatch problem for the peak-hour has a very similar structure to the centralized tranche dispatch problem. The economic-dispatch problem for the peak-hour is given by,

\[
\begin{align*}
\min_{P_{i,h}^{e}} & \quad \sum_{i \in I} c_{i}(P_{i,h}^{e}) \\
\text{s.t.} & \quad \sum_{i \in I} P_{i,h}^{e} = l_{\text{max}}, \\
& \quad 0 \leq P_{i,h}^{e} \leq P_{\text{max},i}, \quad \forall i \in I
\end{align*}
\]

By writing \( P_{i,h}^{e} = \kappa_{i}l_{\text{max}} \), the problem for the peak-hour can be written as

\[
\begin{align*}
\min_{\kappa_{i}} & \quad \sum_{i \in I} c_{i}(\kappa_{i}l_{\text{max}}) \\
\text{s.t.} & \quad \sum_{i \in I} \kappa_{i} = 1, \\
& \quad 0 \leq \kappa_{i} \leq \frac{P_{\text{max},i}}{l_{\text{max}}}, \quad \forall i \in I
\end{align*}
\]

which reads similar to the centralized tranche problem, with the only difference that in the former problem the objective function spans over all the hours. When cost functions \( c_{i}(\cdot) \) are monotonically increasing and its derivatives are non-decreasing, it is straightforward to prove that:

a. Centralized tranche and economic dispatch optimal solutions are related...
by
\[ \alpha_i^* = \frac{P_{i,h_{\text{peak}}}^e}{l_{\text{max}}} \quad \forall i \in \mathcal{I} \] (3.32)

b. Centralized tranche and economic dispatch total costs are bounded by
\[ \sum_{i \in \mathcal{I}, h \in \mathcal{H}} c_i(P_{e,i,h}^*) \leq \sum_{i \in \mathcal{I}, h \in \mathcal{H}} c_i(\alpha_i^*l(h)) \] (3.33)
c. Competitive prices for tranche and economic dispatch are bounded by
\[ \frac{\sum_{h \in \mathcal{H}} p_{h}^*}{H} \leq p_{\text{tranche}}^* \leq p_{h_{\text{peak}}}^* \] (3.34)

Stricter bounds will depend on the specific form of the cost functions, e.g., linear or quadratic, and the level of similarity among suppliers.

**Proposition 1** The optimal tranche-based allocations \( \alpha_i^* \) are related to the dispatch of the peak hour \( \kappa_i^* = \frac{P_{i,h_{\text{peak}}}^e}{l_{\text{max}}} \) by
\[ \alpha_i^* = \kappa_i^* = \frac{P_{i,h_{\text{peak}}}^e}{l_{\text{max}}} \quad \forall i \] (3.35)

**Proof** It is straightforward to prove that the set of \( \kappa_i \) that solves (3.31) also is the optimal solution of the centralized tranche problem (3.12). Using the assumption that the cost functions \( c_i(x) \) are monotonically increasing, we obtain
\[ \sum_{i \in \mathcal{I}} c_i(\kappa_i^*l_{\text{max}}) \leq \sum_{i \in \mathcal{I}} c_i(\kappa_i^*l_{\text{max}}) \quad \forall \kappa_i \]
\[ \Rightarrow \sum_{i \in \mathcal{I}} \kappa_i^*l_{\text{max}} \leq \sum_{i \in \mathcal{I}} \kappa_i^*l_{\text{max}} \quad \forall \kappa_i \]
\[ \Rightarrow \kappa_i^* \leq \kappa_i \quad \forall i, \kappa_i \] (3.36)
\[ \Rightarrow \kappa_i^*l(h) \leq \kappa_i l(h) \quad \forall h, i, \kappa_i \]
\[ \Rightarrow \sum_{i \in \mathcal{I}, h \in \mathcal{H}} c_i(\kappa_i^*l(h)) \leq \sum_{i \in \mathcal{I}, h \in \mathcal{H}} c_i(\kappa_i l(h)) \quad \forall \kappa_i \]
Consequently, \( \kappa_i^* \) is also a solution of (3.12). Hence, the centralized tranche allocation is just settled by the dispatch of the peak-hour. In other words,
\[ \alpha_i^* = \kappa_i^* = \frac{P_{i,h_{\text{peak}}}^e}{l_{\text{max}}} \quad \forall i \] (3.37)
Proposition 2 The total cost associated to the tranche-based products is lower-bounded by the economic dispatch cost,

$$\sum_{i \in I, h \in H} c_i(P_{e, i, h}^*) \leq \sum_{i \in I, h \in H} c_i(\alpha_i^* l(h)) \quad (3.38)$$

**Proof** Note that in the centralized tranche dispatch, the optimal power supplied by generator $i$ at time $h$, $P_{i,h}^*$, is a fraction of its optimal power offered at the peak hour, $P_{i,h,peak}^*$. In other words,

$$P_{i,h}^* = \alpha_i^* l(h) = P_{i,h,peak}^* l(h)_{max} \quad (3.39)$$

Consider an inexpensive unit $j$ which has to be used to its maximum power at any hour under an economic criterion. As shown before, the centralized tranche problem will assign $P_{j,h,peak}^* = P_{j,peak}^* l(h)_{max}$ at the peak hour but it will also assign $P_{j,h}^* = P_{j,peak}^* l(h)_{max}$ at any other hour. Therefore, as $P_{j,h} \neq P_{j,peak}^*$ for the non-peak hours, we move away from the optimal solution of the dispatch problem. Therefore,

$$\sum_{i \in I, h \in H} c_i(\alpha_i^* l(h)) = \sum_{i \in I, h \in H} c_i (P_{i,h}^*) \geq \sum_{i \in I, h \in H} c_i (P_{i,h,peak}^*) \quad (3.40)$$

Proposition 3 The tranche price is upper-bounded by the marginal price of the economic dispatch at the peak hour,

$$p_{\text{tranche}}^* \leq p_{h,peak}^* \quad (3.41)$$

**Proof** Consider that

$$\frac{\partial c_i(\alpha_i^* l(h))}{\partial P_i} \leq p_{h,peak}^*, \quad \forall \ h \quad (3.42)$$

and therefore

$$\frac{\sum_{h \in H} l(h) \frac{\partial c_i(\alpha_i^* l(h))}{\partial P_i}}{\sum_{h \in H} l(h)} = p_{\text{tranche}}^* \leq \frac{\sum_{h \in H} l(h)p_{h,peak}^*}{\sum_{h \in H} l(h)} = p_{h,peak}^* \quad (3.43)$$
Proposition 4 The tranche price is lower-bounded by the average of the hourly marginal price of the economic dispatch

\[
\frac{\sum_{h \in \mathcal{H}} p^*_h}{H} < p^*_\text{tranche}
\] (3.44)

Proof First of all, we establish a lower bound relating tranche prices with the economic dispatch ones,

\[
\frac{\sum_{h \in \mathcal{H}} l(h)p^*_h}{\sum_{h \in \mathcal{H}} l(h)} < \frac{\sum_{h \in \mathcal{H}} l(h)\frac{\partial c_i(\alpha_i l(h))}{\partial P_i}}{\sum_{h \in \mathcal{H}} l(h)}
\] (3.45)

Consider that the marginal supplier of the tranche problem is the supplier \(i\). For all \(h\) where \(l(h) < l_{\text{max}}\) the supplier \(i\) will reduce its power by \(\alpha_i[l_{\text{max}} - l(h)]\). Consequently, the power in hour \(h\) will be given, as expected, by

\[
P^*_{i,h} = P^*_{i,h_{\text{peak}}} - \alpha_i^*[l_{\text{max}} - l(h)] = \alpha_i^*l(h)
\] (3.46)

In the economic dispatch context, the supplier \(i\) is the marginal supplier at the peak hour generating a power equal to \(P^*_{i,h_{\text{peak}}} = P^*_{i,h_{\text{peak}}} = \alpha_i^*l_{\text{max}}\). For any other hour \(h\), two cases are possible:

- The supplier is still the marginal supplier. In this case, supplier \(i\) reduces its power by \(l_{\text{max}} - l(h)\) at hour \(h\) and therefore \(P^*_{i,h} = P^*_{i,h_{\text{peak}}} - (l_{\text{max}} - l(h)) \prec P^*_{i,h_{\text{peak}}} - \alpha_i^*[l_{\text{max}} - l(h)] = P^*_{i,h} = \alpha_i^*l(h) \forall h\). Considering that marginal cost functions are non-decreasing, then \(p^*_h = \frac{\partial c_i(P^*_{i,h})}{\partial P_i} < \frac{\partial c_i(\alpha_i* l(h))}{\partial P_i}\) holds.

- The supplier is no longer providing power. In this case, as supplier \(i\) is ruled out, there must be a supplier \(j\) with a lower marginal cost than supplier \(i\) such that \(p^*_h = \frac{\partial c_j(P^*_{j,h})}{\partial P_j} < \frac{\partial c_i(\alpha_i* l(h))}{\partial P_i}\) holds.

By using the previous result for all hours, it is clear that Equation (3.45) is satisfied. In order to prove the original proposition, we focus on the following bound for the economic dispatch problem:

\[
\frac{\sum_{h \in \mathcal{H}} p^*_h}{H} < \frac{\sum_{h \in \mathcal{H}} l(h)p^*_h}{\sum_{h \in \mathcal{H}} l(h)}
\] (3.47)
This relationship is easily proved by considering that marginal cost functions are non-decreasing with respect to the load levels. Both the left and right sides of expression (3.47) are of the form

\[ \sum_{h \in H} \eta_h p^*_h \]  

(3.48)

with \( \sum_{h \in H} \eta_h = 1 \). On the left side the coefficients are given by \( \kappa_h = \frac{1}{H} \), and on the right side by \( \tau_h = \frac{l(h)}{\sum_{h \in H} l(h)} \). If for a particular hour \( h = i \), \( \kappa_i \geq \tau_i \), then necessarily in order to respect the constraint \( \sum_{h \in H} \tau_h = 1 \), \( \kappa_j \leq \tau_j \) for any other hour \( h = j \neq i \). Given that \( \tau_j \geq \tau_i \), it is clear that \( l(j) \geq l(i) \). By using the monotonicity of the marginal costs, it is obtained that \( p^*_j \geq p^*_i \). Hence, hours with higher loads and higher prices are weighted more and so expression (3.47) follows. By combining (3.47) and (3.45) the original proposition is proved.

\[ \square \]

3.4.2 Linear Cost Function Case

In the particular case of linear cost functions given by \( c_i(x) = \beta_i x \), a clear tranche-price bound can be found. In this case the solution of the centralized tranche problem is characterized by a marginal supplier, \( i = m \), for which \( \mu^+_m = \mu^-_m = 0 \), \( \alpha_m \in [0, \min\{1, \frac{P_{\max,i}}{l_{\max}}\}] \). By the complementary slackness conditions we also know that

\[ \beta_i < \beta_m \Rightarrow \alpha_i = \min \left\{ 1, \frac{P_{\max,i}}{l_{\max}} \right\}, \quad (3.49) \]

\[ \beta_i > \beta_m \Rightarrow \alpha_i = 0 \]

In this case, the tranche equilibrium price is given by

\[ p^*_\text{tranche} = \beta_m \]  

(3.50)

It is interesting to note that, in the linear case, the economic-dispatch problem of the ‘peak-hour’ has the same form, except for a scaling factor given by \( \sum_{h \in H} l(h) \), as the tranche-dispatch problem. Hence, it is clear that the competitive tranche-base price will be equal to the maximum hourly price.
of the economic dispatch problem,

\[ p^*_{\text{tranche}} = \max\{p_h\} \]  

(3.51)

In this simple linear case, the bound (3.51) is telling us that the tranche competitive price is always the maximum of the benchmark prices over the period. Under mild conditions, similar bounds are illustrated in the numerical exercise of the next section for a more general cost function structure. This is just a consequence of the structural feature of the tranche-based product of providing a fixed-percentage of the load.

Certainly, these high prices are even more likely to happen once uncertainty is considered and additional risk premiums are expected. For example, as Figure 3.1 illustrates, in the Illinois process all the products were above market prices more than 85% of the time. However, the riskier products were above market prices 90% or 97% of the time.

### 3.4.3 Quadratic Cost Function Case

We illustrate the type of bounds that could emerge in tranche-based markets using a simple 5-generator test system. Consider a time horizon of 168 hours. Cost functions have the quadratic form \( c_i(x) = a_i x + b_i x^2 \) and the system data is shown in Table 3.1. A typical load pattern is considered over the time horizon of study. Using a price duration curve, a comparison of the optimal tranche-contract price and prices associated with the economic dispatch is shown in Figure 3.9.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Suppliers</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i ) [$/MWh]</td>
<td>0.03</td>
<td>0.66</td>
<td>1.66</td>
<td>5</td>
<td>3.33</td>
<td></td>
</tr>
<tr>
<td>( b_i ) [$/(\text{MWh}^2)]</td>
<td>0.03</td>
<td>0.16</td>
<td>0.66</td>
<td>1.33</td>
<td>3.33</td>
<td></td>
</tr>
<tr>
<td>( P_{\text{max},i} ) [MW]</td>
<td>400</td>
<td>300</td>
<td>200</td>
<td>100</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>( P_{\text{min},i} ) [MW]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: 5-generator system data

In this particular case, it is interesting to note that the benchmark prices are below the tranche competitive price about 87% of the time. These results
show that tranche product prices, even in the most idealized situation, could be above benchmark market prices for long time periods. Certainly, these bounds depend on the technology mix of generators and the load patterns. In terms of technology mix, the more homogeneous the mix, the closer the tranche product prices to the average benchmark prices. This is clearly visualized by thinking of the extreme case of having all suppliers with similar costs and a flat demand. With respect to the load pattern, higher tranche-based product prices are expected for more variable loads due to the structural flaw of providing a fixed percentage of the load. A product required to supply a fixed percentage of the load could result in situations in which the set of most expensive suppliers are required to provide energy even in the periods of extremely low load levels, e.g., base-load periods.

3.5 An Alternative Definition: Energy-Ramping-Blocks Products

The results presented in this work clearly illustrate the importance of defining appropriate products. We should highlight that this is just a starting point and our hope is that these results could increase the interest in this exciting and important topic. Finding a proper product is a highly challenging task.
for the many reasons explained in this work. Moreover, the assessment of the ultimate level of appropriateness will only be possible once the market is implemented. However, it is clear that careful analysis and research, aiming to make the market and the physical systems coexist by defining appropriate products, will increase the chances of positive outcomes—or at least will avoid potential bad outcomes.

As an alternative product, we propose one defined by the so-called energy-ramping-blocks (ERB). This type of product might facilitate the contracting of energy and the provision of investment signals to achieve an appropriate technology mix. This product naturally captures some of the features of different controllable technologies associated with electricity generation.

ERB products focus on three load segments—the base, the cycling and the peaking segments. For each segment, unitary products are defined in terms of horizontal blocks. The time and capacity of each block are key parameters in this product definition. In addition to the delivery time, the three load segments are differentiated by ramping capabilities of the different technologies. In other words, in order to be able to provide cycling and peaking segments, some minimum ramping requirements must be satisfied. In this way, there is no problem about arbitraging between different segments because the product for each load segment captures some unique features of different technologies. For example, nuclear units certainly can provide base-block products, but they cannot participate in the cycling and peaking segments. A basic set of ERB products is illustrated in Figure 3.10.

ERB products do not carry any capacity and volumetric uncertainty. Hence, they provide a suitable definition for contracting bulk-power. In addition, the use of markets based on these products can provide appropriate long-term signals for the different types of technological attributes required for each load segment. In addition, the efficiency of the physical operation—thinking of physical contracts—could be closer to an optimal dispatch.

We do not presume that ERB products are the panacea as they may also have their limitations. For example, ERB products might not be proper to deal with generating units with volatile output. Appropriate products for those technologies must consider some uncertainty in their supply; hence, interruptible contracts as defined by [58] seem a good starting point to think about products for uncontrollable units.
3.6 Summary

In this chapter, the impact of product definition in electricity markets is studied. Using a product definition implemented in some U.S. electricity markets, we reveal several consequences that an improper product definition can have in the market outcomes in terms of market efficiency, concentration, uncertainty allocation and clearing prices. We provide several economic reasons along with illustrative examples. Our findings provide guidelines about the desired attributes of appropriate products. The key challenge is defining products that can effectively link the markets and their associated physical systems. To emphasize our findings, we present an alternative product based on energy-ramping blocks that overcomes some of the critical issues discussed in this chapter. Our results demonstrate the importance of properly defining products in electricity markets and provide guidelines for future research.

Figure 3.10: ERB product illustration.
CHAPTER 4

THE VALUE OF VOLATILE RESOURCES IN ELECTRICITY MARKETS

In this chapter, the impact of volatility on renewable energy sources is investigated. While renewable resources most certainly provide environmental benefits, and also help to meet aggressive renewable energy targets, their deployment may have pronounced impacts on system operations. There is an acute need to understand these impacts in order to fully harness the benefits of renewable resource integration. This chapter addresses the integration of wind energy resources in a multi-settlement electricity market structure. Analysis is conducted in an idealized competitive equilibrium setting that incorporates both dynamics and uncertainty; closed form expressions are obtained for the supplier and consumer surpluses in this stochastic model. These formulae reveal that the value of wind generation, under the current operational and market practices, falls dramatically with volatility. These findings motivate the need to find resources and to create operational schemes that address volatility. Moreover, it is argued that current market structures must be updated to support greater renewable integration. This chapter is based on [33,36].

4.1 Introduction

Energy security combined with environmental concerns has led regions of the United States and other countries to adopt renewable portfolio standards which mandate that a certain percentage of electricity production must come from renewable resources [59,60]. Among the many renewable energy sources, wind power is the most attractive for massive deployment due to rapid installation, low investment costs, and low operational/maintenance costs [61]. These factors coupled with legislative stimulus available for wind installation projects have resulted in explosive growth in installed wind generation ca-
Figure 4.1: ISO-NE day-ahead and real-time demand and supply (www.iso-ne.com).

Pac"cy over the last decade [60]. However, its extensive deployment presents major challenges in system operations due to the inherent characteristics of wind generation: limited control capabilities, forecasting uncertainty, and intermittency in the generation outputs [62]. To overcome these challenges and fully harness the benefits associated with wind resource integration and deployment, we need to understand the unique characteristics of wind generation and the overall impact on the power system and market operations.

The operation of a power system and its electricity markets is a challenging task because of its complexity, the variability and uncertainty in electricity supply and demand, and a range of physical constraints.

Such complexities are further compounded in a market environment driven by private interests as well as regulatory policies [63]. These factors coupled with the interconnectedness of the power system require a coordination process to dispatch the supply resources and meet the demand requirements. In most jurisdictions, an independent entity known as the independent system operator (ISO) is in charge of this centralized coordination process. The ISO is responsible for operating the system and its electricity markets in a reliable and economic manner. The ISO typically maintains extra generation capacity online at all times as reserves to ensure that electricity supply is reliable in spite of uncertainty as well as variability in both demand and supply. Fig. 4.1 illustrates the reserve policy of ISO New England: Scheduled generation is roughly 4 GWs greater than the day-ahead forecast throughout the day.

With low penetration of wind resources, demand and supply exhibit well-understood patterns, so that the evolution of system and market conditions are largely predictable. As the deployment of wind resources increases, the resulting increase in uncertainty will force the ISOs to adjust their operating
policies to ensure reliability of electricity supply.

Figure 4.2: Generation and load data from BPA balancing authority – two weeks in Spring, 2012.

Shown in Fig. 4.2 is the total load as well as the available wind and thermal generation in the balancing area of Bonneville Power Administration (BPA) for several days during Spring, 2012.

Demand exhibits variability, but is highly predictable. Thermal generation capacity also exhibits some variability. Note however that the jumps in thermal capacity arise from start-up or shut-down of units as a result of the day-ahead commitment decisions. Hence this variability is also predictable. The variability in wind generation is much more pronounced, on time-scales ranging from several minutes to several days.

Deep penetration of wind resources will increase the variability as well as uncertainty of energy supply. Reliability requirements will then necessitate the procurement of additional reserves. These impacts of wind generation are well recognized, and attempts to quantify them have been pursued [64–67]. It is strongly implied in [68–70] that the introduction of wind resources will require new ways of thinking about market operations. It is argued in [69] that the allocation of risk and uncertainty associated with wind generation, regardless of whether it is used as a supply-side resource or a demand-side resource, is a challenging problem. Analysis of dynamic stochastic models to address these questions is presented in [30,31,71–73].

The impact of wind generation on the grid and its markets is investigated in this chapter, focusing on two parameters: penetration and volatility. The goal is to understand how the low cost of wind generation may be offset
by the impacts associated with its volatility. The analytical framework is
based on [12, 30, 31], but the model is extended to capture the inter-related
day-ahead and real-time electricity markets.

In the prior work [31], a stochastic control model is introduced that is
intended to model a real-time electricity market under the ideal assumptions
of a competitive equilibrium, but in a dynamic setting. An analogy with
manufacturing systems is exploited to quantify optimal reserves. An exact
expression for the optimal reserves is obtained, whose value grows linearly
with the variance of demand. In this chapter similar conclusions are obtained
when supply is subject to volatility. Consequently, if the currently adopted
‘use all the wind’ policy is utilized, then the volatility introduced into the
grid will create the need for higher reserves.¹

Volatility also has negative impact on the market. The competitive equi-
librium for the coupled markets introduced in this chapter admits an exact
analysis. The expected surpluses for the consumers and suppliers are ob-
tained in two scenarios: one in which consumers own the wind generation
resources and one in which suppliers own the resources.

In the economist’s competitive equilibrium model, prices are not impacted
by the decisions of players. This is of course an extreme idealization of reality.
The competitive equilibrium is naive, but it is the standard benchmark in
economic studies. The conclusions of the chapter demonstrate that even
under these ideal conditions, without attention to the physics of the grid,
current market structures will not support high levels of renewable energy.

The numerical experiments presented in Section 4.3 illustrate the critical
role of volatility in determining the value of wind generation under current
operational paradigms. The model introduced in this chapter can be used to
establish thresholds for the coefficient of variation beyond which there is no
value of additional wind generation for consumers and/or suppliers.

However, these results must be understood in a limited context: Nothing
in this chapter implies that we should reduce efforts to harness more renew-
able energy. The framework of this chapter is based on standard economic
analysis, considering the welfare of consumers and suppliers with limited
time-horizons (days or weeks). When longer time-scales are considered (one
or two generations), then the cost of renewable integration is more palatable

¹Note that if the ideal competitive equilibrium framework is abandoned, then other
difficulties can emerge - market dynamics can even cause instability [74, 75].
since sustainability and a clean environment are essential for the welfare of our grandchildren.

Finally, just as in other markets such as computer memory, the costs associated with renewable generation will drop quickly once there are incentives and markets to create resources that mitigate volatility. Such resources include both storage and demand response such as batteries, flexible manufacturing and demand response in commercial and residential buildings (e.g. responsive HVAC, water heaters, and pool pumps) which have been explored in many recent chapters [76, 77].

The remainder of the chapter contains three additional sections. Section 4.2 provides a description of the multi-settlement electricity market model and a survey of some of its features. A refinement of this model that incorporates wind generation is introduced in Section 4.3. This section contains the derivation of mean surpluses in the competitive equilibrium. Concluding remarks and suggestions for future research are given in Section 4.4.

4.2 Electricity Market Models

In many jurisdictions around the world, electricity is traded using two or more interrelated markets that constitute the so-called multi-settlement structure. Although all markets trade in the MWh commodity, they are differentiated by the time intervals between the trading decisions and the energy delivery, which range from year(s), month(s), one day, hour(s), to minutes. The market model introduced in this chapter is based on a multi-settlement structure, consisting of a day-ahead market (DAM) and a real-time market (RTM); this is based on the structure that is commonly adopted by ISOs in the U.S. today. There are economic and operational reasons for adopting such a multi-settlement structure.

From an economic point of view, the multi-settlement structure allows sellers and buyers to hedge against the risk associated with the uncertain real-time conditions. In addition, some forward markets, which trade energy years and months in advance, create signals for resource investments, facilitating the achievement of resource adequacy.

From an operational point of view, the physical constraints on the gen-
eration units – ramping limitations and start-up constraints – make it impossible to support the “just-in-time” electricity production within a market structure which uses only real-time trading. The clearing of the DAM one day prior to the actual production and delivery of energy allows the ISO to schedule generation in such a way that physical constraints are satisfied with high probability. As supply and demand are not perfectly predictable in the DAM, the RTM – which is operated minutes ahead of the actual “real time” – allows fine-tuning of the resource allocation decisions made in the DAM. The RTM is typically cleared every 5 to 15 minutes, so as to maintain a continuous balance between supply and demand.

In what follows, the main design elements used in our analysis to represent the coupling of the DAM and the RTM are presented. While real-world DAM and RTM clearing operates at discrete decision epochs, we construct continuous time models to approximate the coupled markets. The models are constructed in a stochastic setting to capture uncertainty in supply and demand.

4.2.1 Elements of the Multi-Settlement Model

The multi-settlement market model introduced in this chapter is defined by a coupling of two markets — day-ahead and real-time.

In today’s markets, some aspects of reliability concerns are addressed through the use of reserves. Part of the policy to manage reserves is considered in the clearing of the different markets. That is, each supplier offers a bundle of energy and capacity which contributes towards meeting demand and reserve requirements. A reserve process will be critical in the multi-settlement market model described here.

For each \( t \geq 0 \), the total demand is denoted \( D^{\text{rt}}(t) = d^{\text{da}}(t) + D(t) \) and the total on-line capacity is \( G^{\text{rt}}(t) = g^{\text{da}}(t) + G(t) \), where \( d^{\text{da}}(t) \) and \( g^{\text{da}}(t) \) denote the forecasted demand and supply at time \( t \). Hence, \( D(t) \) and \( G(t) \) are the deviations from the forecasted demand and supply, respectively.

The demand and generation capacity are assumed to be aggregates from many consumers and suppliers, where consumers may be utility companies, and suppliers generation companies. It is assumed that the transmission grid has ample capacity — we disregard these potential constraints and other
The reserve process is defined as the difference of generation capacity and demand

\[ R(t) \triangleq G^{ui}(t) - D^{ui}(t) = G(t) - D(t) + r^{da}(t), \quad t \geq 0, \quad (4.1) \]

in which \( r^{da}(t) = g^{da}(t) - d^{da}(t) \) represents the forecasted reserves cleared in the day-ahead market. Throughout this chapter a fixed reserve policy is adopted for the day-ahead market, similar to what is used by many ISOs,

\[ r^{da}(t) \equiv r^{da}_0 \text{ is constant.} \quad (4.2) \]

Under the assumptions to be imposed below, the reserves process \( R \) forms a state-process for the optimization problems considered by the consumer and supplier in the RTM, and the threshold level \( r^{da}_0 \) is a decision variable for consumers and suppliers in the DAM.

The operational models used in this chapter are an adaptation of the dynamic models of [31]. The stochastic model considered in [30,31] consists of the following components:

(i) **Volatility:** The deviation in demand \( D \) is modeled as Brownian motion: \( D \) is a driftless Brownian motion with instantaneous variance \( \sigma^2 \).

(ii) **Ramp Constraints:** Since generation cannot increase instantaneously, there exists \( \zeta \in (0, \infty) \) such that

\[ \frac{G(t') - G(t)}{t' - t} \leq \zeta, \quad \text{for all } t \geq 0, \text{ and } t' > t. \quad (4.3) \]

No corresponding lower bound is imposed.

Under these assumptions, the reserve process can be viewed as a controlled stochastic system, which is written as the SDE,

\[ dR(t) = \zeta dt - dI(t) - dD(t) \quad (4.4) \]

where \( I \) is non-decreasing. That is, \( dG(t) = \zeta dt - dI(t) \) models the upper bound (4.3) on the rate of increase in generation. The Brownian motion assumption allows for closed-form expressions for mean welfare of consumers
and suppliers in a competitive equilibrium. The *existence* of a competitive equilibrium, and some structural properties similar to what is found here, can be established under much more general assumptions on statistics, dynamics, and costs [12,78].

### 4.2.2 Competitive Equilibrium Framework

The market analysis in this chapter is immersed in the competitive equilibrium framework. We perform a backwards analysis, starting from the characterization of the real-time market model of [31] and then the coupling with the day-ahead market. In this way, the multi-settlement case is reduced to a variant of the RTM model of [31].

**Real-time Market Model**  The market analysis in [31] is based on the physical model described above, along with the following assumptions on costs, utility, and prices:

(i) **Cost:** The production technology of the supplier is subject to a production cost \( c(G(t)) \) for the production capacity \( G(t) \) made available at time \( t \geq 0 \). The cost is a linear function of \( G(t) \), of the form \( cG(t) \) for some constant \( c > 0 \).

(ii) **Value of power:** For each unit of energy delivered, the consumer obtains \( v \) units of utility. Thus, the utility of power to the consumer is \( v \min(D(t), G(t)) \).

(iii) **Disutility from power loss:** If the demand is not met \( (R(t) < 0) \), the consumer suffers utility loss \( c^o |R(t)| \) for some \( c^o > 0 \).

(iv) **Perfect competition:** The price of power \( P(t) \) in the RTM is assumed to be *exogenous* – it is independent of the decisions of the market players. The consumers and suppliers each observe the price process.

(v) **Myopic consumer:** The consumer is *myopic* in the sense that their internal consumption preferences are not subject to the ramping constraints (4.3).
The objectives of the consumer and the supplier are specified by the respective welfare functions,

\[ W_S(t) := P(t)G(t) - cG(t) \]
\[ W_D(t) := v \min(D(t), G(t)) - c^{bo} \max(0, -R(t)) - P(t)G(t) \] (4.5)

It is assumed that the consumer and supplier each optimize the discounted mean-welfare,

\[ K_S := \mathbb{E}\left[ \int e^{-\gamma t}W_S(t)\,dt \right] \]
\[ K_D := \mathbb{E}\left[ \int e^{-\gamma t}W_D(t)\,dt \right] \] (4.6)

where \( \gamma > 0 \) is the discount rate.

In order to characterize the competitive equilibrium, we first focus on the solution of the social planner’s problem (SPP) as defined in standard economics texts. The SPP is described as the maximization of the discounted mean of the total welfare,

\[ K = \mathbb{E}\left[ \int e^{-\gamma t}(W_S(t) + W_D(t))\,dt \right] \]

The optimal solution is obtained in [31], following [79], defined so that the resulting reserve process is a reflected Brownian motion (RBM) on the half-line \( (-\infty, \bar{r}^*) \), with

\[ \bar{r}^* = \frac{1}{\theta_+} \log \left( \frac{v + c^{bo}}{c} \right) \] (4.7)

where \( \theta_+ \) is the positive solution to the quadratic equation \( \frac{1}{2}\sigma^2\theta^2 - \zeta\theta - \gamma = 0. \)

For \( \gamma \approx 0 \) we have \( \theta_+ \approx 2\zeta/\sigma^2 + \gamma/\zeta \). It is also shown in [31] that the solution to the average cost case is obtained as the limit of the discount-cost solution, as \( \gamma \downarrow 0 \), which results in \( \theta_+ = 2\zeta/\sigma^2 \).

From (4.7), it can be appreciated that in a system with volatile demand and ramping-constrained supply, the optimal reserve threshold is directly proportional to the demand variance \( \sigma^2 \), and inversely proportional to the ramping rate \( \zeta \).

Fig. 4.3 shows plots of the optimal total welfare \( K^* \) for \( \zeta = 0.1, 100, 200, 300, 500 \) and various values of \( \sigma \). The discount fact was \( \gamma = 1/12 \), the model was ini-
The model of Cho and Meyn [31] does not take into account volatility from supply, and there is no consideration of the coupled markets. Fig. 4.2 makes clear the need for considering the volatility from both supply and demand if resources such as wind are included. The corresponding extensions to the RTM model to capture these features are discussed in Section 4.3.

A closed form solution for the unique real-time market equilibrium is obtained in [31]. It is shown that the price of power can be expressed as a static function of the optimal reserves $R^e(t)$ obtained in the SPP. The equilibrium price functional is a piecewise constant function of the equilibrium reserve process,

$$p^e(r^e) = (v + c^{bo})I\{r^e < 0\}$$

(4.8)

The sum $v + c^{bo}$ is in fact the maximum price the consumer is willing to pay, often called the *choke-up price*.

**MULTI-SETTLEMENT MARKET MODEL**  The market model is based on the welfare functions for consumers and suppliers.
Definition 1  The total welfare for consumers are given by, respectively, 

\[
W^\text{ttl}_D(t) := v\min(D(t), G(t)) - c^\text{bo} \max(0, -R(t)) - P^e(t)G(t) - p^\text{da}(t)g^\text{da}(t) 
\]

\[
W^\text{ttl}_S(t) := P(t)G(t) - cG(t) + p^\text{da}(t)g^\text{da}(t) - c^\text{da}g^\text{da}(t) 
\]

where \( P^e(t) \) is the market clearing price in the RTM and \( p^\text{da}(t) \) is the DAM price.

The two welfare functions can be decomposed in terms of real-time and day-ahead welfare components.

Proposition 5  The total consumer welfare can be expressed as

\[
W^\text{ttl}_D(t) = W_D(t) + \{ (v - p^\text{da}(t))d^\text{da}(t) + (P^e(t) - p^\text{da}(t))r^\text{da}_0 \} 
\]

where \( W_D(t) \) is given in (6.1).

Proof  The total consumer welfare expression can be transformed as

\[
W^\text{ttl}_D(t) := v\min(D(t), G(t)) - c^\text{bo} \max(0, -R(t)) \\
- P^e(t)G(t) - p^\text{da}(t)g^\text{da}(t), \\
= v\min(D(t) + d^\text{da}(t), G(t) + g^\text{da}(t)) - c^\text{bo} \max(0, -R(t)) \\
- P^e(t)(R(t) + D(t) - r^\text{da}_0) - p^\text{da}(t)g^\text{da}(t), \\
= v\min(D(t) + d^\text{da}(t), G(t) + g^\text{da}(t) - d^\text{da}(t) + d^\text{da}(t)) \\
- c^\text{bo} \max(0, -R(t)) - P^e(t)(R(t) + D(t)) \\
+ P^e(t)r^\text{da}_0 - p^\text{da}(t)g^\text{da}(t), \\
= v\min(D(t), R(t) + D(t)) + vd^\text{da}(t) \\
- c^\text{bo} \max(0, -R(t)) - P^e(t)(R(t) + D(t)) \\
+ P^e(t)r^\text{da}_0 - p^\text{da}(t)(r^\text{da}(t) + d^\text{da}(t)), \\
\]

Recalling the definition of \( W_D(t) \) in (6.1), routine calculations give (4.11).

The terms in brackets in (4.11) are beyond the control of the consumers or suppliers in the RTM. We now focus on finding the expectation of welfare for consumers, based on the following average prices:
Definition 2 (i) The average price in the DAM is denoted
\[
\overline{p}^{da} = \gamma \int_{0}^{\infty} p^{da}(t) e^{-\gamma t} dt
\]

(ii) The average price in the RTM is the function of the initial reserves,
\[
\overline{p}^{e}(r) := \gamma \int_{0}^{\infty} \mathbb{E}[P^{e}(t) \mid R^{e}(0) = r] e^{-\gamma t} dt
\]

In [31], \( \overline{p}^{e}(r) \) is shown to be a non-increasing function of \( r \), satisfying \( c^{rt} \leq \overline{p}^{e}(r) \leq v + c^{bo} \) for each \( r \in (-\infty, \bar{r}^{*}] \), and \( \overline{p}^{e}(\bar{r}^{*}) = \min_r \overline{p}^{e}(r) = c^{rt} \).

Proposition 6 The total discounted mean welfare for consumers is given by
\[
K_{\text{D}}^{\text{util}} = K_{\text{D}}(r) + \gamma^{-1} r(\overline{p}^{e}(r) - \overline{p}^{da}) + \int_{0}^{\infty} (v - p^{da}(t)) d^{da}(t) e^{-\gamma t} dt \tag{4.12}
\]

Proof The following is obtained by taking expectations of each side of (4.11),
\[
K_{\text{D}}^{\text{util}} = K_{\text{D}} + \int_{0}^{\infty} \mathbb{E}[(v - p^{da}(t)) d^{da}(t) + (P^{e}(t) - p^{da}(t)) r^{da}] e^{-\gamma t} dt
\]

The convention that the reserves in the DAM are constant will help to simplify this expression.

Using Def. 2, the total discounted mean welfare is simplified to
\[
K_{\text{D}}^{\text{util}} = K_{\text{D}}^{\text{D}} + \int_{0}^{\infty} \left\{ (v - p^{da}(t)) d^{da}(t) + (\overline{p}^{e}(r) - p^{da}(t)) r^{da} \right\} e^{-\gamma t} dt
\]

Substituting the expression \( r^{da} = R(0) = r \) and using Def. 2 gives (4.12).

The three terms on the right-hand side of (4.12) are interpreted as follows: \( K_{\text{D}}(r) \) is the real-time market welfare. The second term, \( \gamma^{-1} r(\overline{p}^{e}(r) - \overline{p}^{da}) \), couples the DAM and RTM, avoiding double-payments for reserves already settled in the day-ahead market. The third term is the day-ahead welfare due to energy settlements.
For the case of the supplier, similar steps are followed. Recall that the supplier welfare was introduced in Def. 1.

**Proposition 7** The total supplier welfare can be decomposed in terms of real-time and day-ahead welfare expressions,

\[
W_{\text{ttl}}^S(t) = W_s(t) + r_0^0 (p^\text{da}(t) - P(t)) + d^\text{la}(t)(p^\text{da}(t) - c)
\]  

(4.13)

where \( W_s(t) \) is given in (6.1).

**Proof** Replacing \( r^\text{da}(t) = r_0^0 \) in (4.1), and recalling the welfare definition (4.10), gives

\[
W_{\text{ttl}}^S(t) := P(t)(R(t) + D(t) - r_0^0) - c(R(t) + D(t) - r_0^0) + p^\text{da}(t)g^\text{da}(t) - c^\text{da}g^\text{da}(t),
\]

\[
= P(t)(R(t) + D(t)) - c(R(t) + D(t)) - r_0^0(P(t) - c) + p^\text{da}(t)g^\text{da}(t) - c^\text{da}g^\text{da}(t)
\]

\[
= W_s(t) - r_0^0(P(t) - c) + r_0^0(p^\text{da}(t) - c^\text{da})
\]

\[
+ d^\text{la}(t)(p^\text{da}(t) - c^\text{da})
\]

(4.14)

As a single technology is assumed, \( c^\text{da} = c \). Hence, the supplier’s welfare simplifies to

\[
W_{\text{ttl}}^S(t) = W_s(t) + r_0^0(p^\text{da}(t) - P(t)) + d^\text{la}(t)(p^\text{da}(t) - c)
\]  

(4.15)

Following similar arguments as in the consumer’s case, it is possible to find the total expected supplier welfare. The proof is omitted, since it follows the same steps used in the proof of Prop 6.

**Proposition 8** The total discounted mean welfare for suppliers is given by

\[
K_{\text{ttl}}^S = K_s(r) + (\overline{p}^\text{la} - \overline{p}(r))\gamma^{-1}r + \int_0^\infty (p^\text{da}(t) - c) d^\text{la}(t) e^{-\gamma t} dt.
\]

(4.16)

The terms in the welfare expression (4.16) are interpreted as follows: The first is the real-time welfare. As in the supplier’s case, \( (\overline{p}^\text{la} - \overline{p}(r))\gamma^{-1}r \) avoids
the double-payments of the reserves settled in the day-ahead market. The last term is the day-ahead welfare due to energy settlements.

**Competitive Equilibrium of the Coupled Market** We begin by constructing the solution of the SPP problem, and from this the equilibrium prices are obtained in the coupled market.

Expressions for the welfare of the suppliers and consumers in the coupled market are obtained in Props. 6 and 8, and based on these results the total welfare is obtained,

\[ K_{\text{ttl}} = K + \int_{0}^{\infty} (v - c) \, d^{\text{aw}}(t) \, e^{-\gamma t} \, dt \]

(4.17)

in which \( K = K_D + K_s \). Hence, the total welfare for the coupled market includes the total welfare for the RTM of [31] plus the day-ahead welfare. Given this expression for the total welfare function, it is possible to solve the SPP.

**Proposition 9** In the coupled market, the optimal day-ahead reserves correspond to the optimal initial reserves of [31], given by:

\[ \bar{r}^* = \frac{1}{\theta^+} \log \left( \frac{v + c^{bo}}{c} \right) \]

(4.18)

**Proof** Applying the first order optimality conditions applied to the total welfare (4.17), it is clear that,

\[ \frac{dK_{\text{ttl}}}{dr} = 0 \Leftrightarrow \frac{dK}{dr} = 0 \]  

(4.19)

**Proposition 10** In the coupled market, the expected RTM price is equal to the marginal cost,

\[ \overline{p}^e = c \]

(4.20)

**Proof** This is a consequence of having, in the coupled market, the same optimal reserves process as obtained in the Cho and Meyn model [31], which is defined by the threshold (4.18) (see § 4 of [31] for details).

Based on the foregoing, we can compute the equilibrium day-ahead price for the coupled market.
Proposition 11 The expected DAM price is equal to the expected RTM price,

$$\bar{p}^{\text{da}} = \bar{p}^e = c$$  \hspace{1cm} (4.21)

Proof The first order optimality conditions to the optimal consumer and supplier welfare give

$$0 = \frac{dK^\text{ttl}}{dr} = \frac{dK_D}{dr} + \gamma^{-1}(\bar{p}^e - \bar{p}^{\text{da}})$$
$$0 = \frac{dK^\text{ttl}}{dr} = \frac{dK_S}{dr} + \gamma^{-1}(\bar{p}^{\text{da}} - \bar{p}^e)$$

With $r = \bar{r}^*$ it is known that

$$\frac{dK_S}{dr} = \frac{dK_D}{dr} = 0$$

from which it follows that

$$\bar{p}^{\text{da}} = \bar{p}^e = c$$  \hspace{1cm} (4.22)

The settlement of the expected energy demand, $d^{\text{da}}(t)$, in the DAM is an independent optimization problem. It follows that the equilibrium price for the linear-cost and single-technology case is given by $p^{\text{da}}(t) = c$. Hence, in the competitive equilibrium, the average prices of both reserves and energy in the day-ahead market are equal to the marginal cost of production.

4.3 Who Commands the Wind?

This section explores an extension of the DAM/RTM that differentiates between the generation of wind and conventional resources. Recall that it is assumed in this chapter that all the wind generation available is dispatched and injected into the system. Conventional generators serve the residual demand. Volatility of the wind will result in volatility of this residual demand, which will result in higher reserves in the dynamic competitive equilibrium. With low penetration of wind resources, the increase in demand volatility will be negligible, and hence the impact on the market outcome will not be significant. Potential negative market outcomes are possible with a combination of high penetration and high volatility of wind generation.
To quantify these claims, expressions are derived for the total welfare, differentiated by who commands the wind resources: the consumer or the supplier. Closed form expressions for the discounted mean welfare of the consumer and supplier in each of the two settings are quantified.

Perhaps surprisingly, in the numerical results that follow it is found that the supplier can achieve significant gains when the consumer commands the wind. The explanation is that the higher volatility forces the consumer to pay for higher reserves in the DAM.

This section is concluded with a discussion of some of the key issues, from a market perspective, for integrating wind power into electricity markets.

4.3.1 Consumers Command the Wind

We first consider a setting in which the wind resources are commanded by the demand-side. The total wind capacity is denoted \( G_{\text{w}}^{\text{ttl}}(t) = g_{\text{w}}^a(t) + G_w(t) \). The consumer surplus at time \( t \) is thus given by

\[
W_{D,w}^{\text{ttl}}(t) = v \min(D^{\text{rtl}}(t), G_{\text{w}}^{\text{rtl}}(t) + G_w^{\text{rtl}}(t)) - c^{bo} \max(0, -R(t)) - P(t)G(t) - p^{da}(t)g^{da}(t)
\]

As in [69], the resulting market is modeled by interpreting wind generation as a negative load. The resulting residual demand is denoted by \( D^{\text{net}}(t) = D(t) - G_w(t) \). Expressions for consumer and supplier welfare with respect to residual demand are obtained as follows:

\[
W_{D,w}^{\text{rtl}}(t) = v \min(D^{\text{rtl}}(t) - G_w^{\text{rtl}}(t), D^{\text{rtl}}(t) + R(t) - G_w^{\text{rtl}}(t)) \\
- c^{bo} \max(0, -R(t)) \\
- P(t)(R(t) + D(t) - G_w(t) - r_0^{da}) \\
- p^{da}(t)(d^{da}(t) - g_w^{da}(t) + r_0^{da}) + vG_w^{\text{rtl}}(t)
\]

\[
= v \min(D^{\text{net}}(t), R(t) + D^{\text{net}}(t)) \\
- c^{bo} \max(0, -R(t)) - P(t)(R(t) + D^{\text{net}}(t)) \\
+ (v - p^{da}(t))d^{da}(t) + (P(t) - p^{da}(t))r_0^{da} \\
+ p^{da}(t)g_w^{da}(t) + vG_w(t)
\]
Hence the total welfare can be expressed as the sum

\[
W_{D,W}^{rtl}(t) = W_{D,W}^{rt}(t) \\
+ \left\{ vd\Delta(t) - p\Delta(t)(d\Delta(t) - g\Delta_W(t)) \right\} \\
+ \left\{ (P(t) - p\Delta(t))r_0\Delta + vG_W(t) \right\}
\]

(4.23)

The first term is the real-time welfare expression from [31] for the equivalent load \(D_{net}\); the next term in brackets corresponds to the DAM welfare in which the welfare gain \(p\Delta(t)g\Delta_W(t)\) is due to the ownership of wind generation by consumers. The final term in brackets contains elements that are beyond the control of the consumer under the assumptions of this chapter.

For the supplier surplus, the expression is similar to the case without wind:

\[
W_{S,W}^{rtl}(t) = (P(t) - c^{\text{fi}})G(t) + (p^{\text{fi}}(t) - c^{\text{da}})g^{\text{da}}(t) \\
= W_{S,W}^{rt}(t) + (p^{\text{da}}(t) - c^{\text{da}})g^{\text{da}}(t) + (p^{\text{da}}(t) - P(t))r_0^{\text{da}}
\]

in which \(W_{S,W}^{rt}(t)\) is the supplier surplus obtained in the RTM for serving the residual demand \(D_{net}(t)\).

4.3.2 Suppliers Command the Wind

The alternative in which wind resources are a part of the supply side is considered now. As in the previous case, it is assumed that all the available wind generation is dispatched. The total social welfare will be unchanged, but the distribution between suppliers and consumers will be different. In particular, there is a shifting of the benefits in the DAM of dispatching wind from consumers to suppliers. However, in the RTM suppliers face the liability of injecting volatility into the system, quantified by a term associated with wind power deviations. From the consumer’s viewpoint, following familiar
calculations,

\[ W^{\text{rt}}_{D,W}(t) = v \min(D^{\text{rt}}(t), G^{\text{rt}}(t) + G_w^{\text{rt}}(t)) \]
\[ - c^b \max(0, -R(t)) \]
\[ - P(t)(G(t) + G_w(t)) - p^{\text{da}}(t)(g^{\text{da}}(t) + g_w^{\text{da}}(t)) \]
\[ = W^{\text{rt}}_{D,W}(t) \]
\[ + \left\{ (v - p^{\text{da}}(t))d^{\text{da}} + (P(t) - p^{\text{da}}(t))r_0^{\text{da}} \right\} \]
\[ + (v - P(t))G_w(t) \]

In this case, the impact of the wind is captured in the last term. The term 
\( vG_w(t) \), as before, has no impact on the discounted mean welfare expression. The term 
\( -P(t)G_w(t) \) quantifies payments/compensations in the real-time market related to wind power deviations.

The supplier welfare now includes terms due to wind generation:

\[ W^{\text{rt}}_{S,W}(t) = (P(t) - c^r)G(t) + P(t)G_w(t) \]
\[ + \left( p^{\text{da}}(t) - c^{\text{da}} \right)(g^{\text{da}}(t) - g_w^{\text{da}}(t)) + p^{\text{da}}(t)g_w^{\text{da}}(t) \]
\[ = W^{\text{rt}}_{S,W}(t) + \left( p^{\text{da}}(t) - c^{\text{da}} \right)(g^{\text{da}}(t) - g_w^{\text{da}}(t)) \]
\[ + \left( p^{\text{da}}(t) - P(t) \right)r_0^{\text{da}} + P(t)G_w(t) + p^{\text{da}}(t)g_w^{\text{da}}(t) \]  

(4.24)

Based on this expression it is appreciated that the welfare gain \( p^{\text{da}}(t)g_w^{\text{da}}(t) \), which in the previous setting was taken by the consumers, is now taken by the suppliers. In addition, suppliers face the impacts of the wind power deviations \( P(t)G_w(t) \).

Each of these expressions is based on the assumption that all available wind generation will be utilized. Under this assumption, regardless of who commands the wind, there is always a **systemic** impact of wind volatility reflected by the more volatile residual demand. All the terms that can be impacted by wind volatility, such as real-time prices and optimal reserve levels, are exactly the same no matter who commands the wind. When the wind resources are commanded by suppliers, the benefits quantified by \( p^{\text{da}}(t)g_w^{\text{da}}(t) \) go into the suppliers’ pocket. However, in the case in which suppliers command the wind, the term \( P(t)G_w(t) \) can create losses for suppliers, and gains for consumers.
4.3.3 Computing Welfares

It is possible to find closed form expressions for the mean welfare for the suppliers and consumers. These expressions are used in the numerical experiments described in the next section.

The real-time mean welfare \( K_s^* \) and \( K_d^* \) are obtained in closed form by substituting the price \( P^e(t) \) into the respective welfare functions defined in (6.1): For any time \( t \),

\[
E[W^*_s(t)] = (v + c^{bo})(E[D(t)I\{R^e(t) \leq 0\}] + E[R^e(t)I\{R^e(t) \leq 0\}]) \tag{4.25}
\]

\[
E[W^*_d(t)] = -(v + c^{bo})(E[D(t)I\{R^e(t) \leq 0\}]) \tag{4.26}
\]

Computations of these value functions are performed based on these representations, and on the following expression for the expectation of the welfare associated with wind deviation:

\[
E[P(t)G_W(t)] = (v + c^{bo})(E[G_W(t)I\{R^e(t) \leq 0\}]) \tag{4.27}
\]

CLOSE-FORM EXPRESSIONS In this section the formulae for discounted mean social welfare are derived. We begin with some generalities: \( X \) is a Markov process on a general state space \( X \), with semigroup \( \{P^t : t \geq 0\} \). For a given \( \gamma > 0 \), the resolvent is the Laplace transform

\[
U_\gamma := \int_0^\infty e^{-\gamma t} P^t dt \tag{4.28}
\]

We let \( c: X \rightarrow \mathbb{R} \) denote a generic function satisfying \( U_\gamma |c| (x) < \infty \) for each \( x \). In this case the function \( h = U_\gamma c \) has the representation

\[
h(x) = E\left[\int_0^\infty c(X(t)) e^{-\gamma t} dt\right] \tag{4.29}
\]

This is part of our motivation for considering the resolvent. The other motivation comes from its relationship with the generator.

A function \( h \) is in the domain of the extended generator if there exits a function \( g \) such that the process below is a local martingale for each initial
condition of $\Phi$

$$M_T := h(\Phi_T) - h(\Phi_0) - \int_0^T g(\Phi_s) ds, \quad T \geq 0.$$  \hspace{1cm} (4.30)

We let $\mathcal{A}$ denote the extended generator, and denote $\mathcal{A}f = g$ when $M$ is a local martingale (see [80,81]).

Under our assumption that $U_\gamma|c|$ is finite-valued, the function $h = U_\gamma c$ is in the domain of the extended generator with

$$\mathcal{A} h = \gamma h - c.$$  \hspace{1cm} (4.31)

Consequently, the domain of the extended generator includes the range of the resolvent.

Our goal is to compute solutions to dynamic programming equations of the form (4.31), when the function $c$ is given. The models of interest are limited to three special cases:

(i) $\Phi = R$: The reflected Brownian motion (4.4) on this domain, so that $X = (-\infty, \bar{r}]$.

(ii) $\Phi = (R, D)$: where $R$ is in (i), and the demand process $D$ also appears in (4.4). In this case $X = (-\infty, \bar{r}] \times \mathbb{R}$.

(iii) $\Phi = (R, G_W)$: where $R$ is in (i), and the wind deviation process $G_W$ also appears in (4.4) once the demand is replaced by the residual demand. In this case $X = (-\infty, \bar{r}] \times \mathbb{R}$.

The following result identifies a large class of functions in the domain of $\mathcal{A}$. The result is an interpretation of Itô’s formula for reflected diffusions [82].

**Proposition 12** Suppose that $h: (-\infty, \bar{r}] \rightarrow \mathbb{R}$ is $C^2$ and satisfies $h'(\bar{r}) = 0$. Then $h$ is in the domain of the extended generator, and $\mathcal{A}h = Dh$, where

$$Dh = \zeta h' + \frac{1}{2} \sigma^2 h''.$$  \hspace{1cm} (4.32)

It is often easy to solve the DP equation for $D$,

$$\zeta h'_0(r) + \frac{1}{2} \sigma^2 h''_0(r) = \gamma h_0(r) - c(r).$$  \hspace{1cm} (4.33)
where the function \( h_0 \) is piecewise \( C^2 \). In particular, the functions defined below satisfy (4.33) with \( c \equiv 0 \):

\[
\varphi_+(r) = e^{-\theta_+ r}, \quad \varphi_-(r) = e^{-\theta_- r}
\]  

(4.34)

where \( \theta_- < 0 \) and \( \theta_+ > 0 \) denote the two roots of the quadratic equation,

\[
\frac{1}{2} \sigma^2 \theta^2 - \zeta \theta - \gamma = 0.
\]  

(4.35)

These functions are building blocks for the solution of (4.31):

**Lemma 1** Suppose that \( c \) is a piecewise continuous function, and that \( h_0 \) is a piecewise continuous function that is \( C^2 \) on each of the intervals \((-\infty, 0]\) and \((0, \infty)\), satisfying (4.33) for \( r \neq 0 \). Then the function \( h \) defined below is in the domain of the extended generator for \( R \), and satisfies (4.31):

\[
h(r) = h_0(r) + \begin{cases} a_- \varphi_-(r) & r \leq 0 \\ b_- \varphi_-(r) + b_+ \varphi_+(r) & 0 < r \leq \bar{r} \end{cases}
\]  

(4.36)

where the constants \( \{a_-, b_-, b_+\} \) solve the system of linear equations,

\[
\begin{bmatrix}
1 & -1 & 0 \\
\theta_- & -\theta_- & \theta_+ \\
0 & \theta_- e^{-\theta_- r} & \theta_+ e^{-\theta_+ r}
\end{bmatrix} \begin{bmatrix}
a_- \\
b_- \\
b_+
\end{bmatrix} = \begin{bmatrix}
h_0(0+) - h_0(0-) \\
h_0'(0-) - h_0'(0+) \\
h_0'(\bar{r})
\end{bmatrix}
\]  

(4.37)

**Proof** The matrix equation (4.37) represents three constraints for the function \( h \): Continuity at the origin, differentiability at the origin, and finally the constraint \( h'(\bar{r}) = 0 \). While this function is \( C^1 \) and not \( C^2 \), it can be approximated by \( C^2 \) functions to establish the local martingale property.

Lemma 1 is the idea behind the proof of [22, Proposition 3.4.13], which considers the special case in which \( c \) is a continuous piecewise linear function. A special case required in the analysis that follows uses

\[
h_0(r) = \gamma^{-1} c(r) \quad \text{when} \ c(r) = \mathbb{I}\{r \leq 0\}.
\]  

(4.38)
Solving Dynamic Programming Equations

In view of (4.26), to compute the discounted mean welfare functions it is sufficient to solve (4.31) for the four functions of \((r,d)\) and one function of \((r,w)\),

\[
\begin{align*}
    c_{W}(r,w) &= w \mathbb{I}\{r \leq 0\} \\
    c_{A}(r,d) &= d \mathbb{I}\{r \leq 0\}, \quad c_{B}(r) = r \mathbb{I}\{r \leq 0\} \\
    c_{C}(r) &= r \mathbb{I}\{r \leq 0\}, \quad c_{R}(r) = r
\end{align*}
\]

We denote by \(h_{W}, h_{A}, h_{B}, h_{C}, h_{R}\) the respective solutions to (4.31). We then apply (4.25–4.26–4.27) to obtain

\[
\begin{align*}
    K_{s}^{*} &= (v + c_{bo}) (h_{A} + h_{B}) - ch_{R}, K_{D}^{*} = -(v + c_{bo}) h_{A}, \\
    \mathbb{E}[P(t)G_{W}(t)] &= (v + c_{bo}) h_{W}
\end{align*}
\]

The functions \(\{h_{B}, h_{C}\}\) can be computed by a direct application of Lemma 1:

**Proposition 13** The function \(h_{B0}(r) = (\gamma^{-1}r + \zeta\gamma^{-2})\mathbb{I}\{r \leq 0\}\) solves (4.33) with \(c = c_{B}\). The function \(h_{C0}(r) = \gamma^{-1}\mathbb{I}\{r \leq 0\}\) solves (4.33) with \(c = c_{C}\).

An application of Lemma 1 then gives \(h_{B} = h_{B0} + \varphi_{B}\) and \(h_{C} = h_{C0} + \varphi_{C}\), where \(\{\varphi_{B}, \varphi_{C}\}\) are piecewise continuous, and are linear combinations of \(\{\varphi_{+}, \varphi_{-}\}\) on the two line segments \((-\infty, 0]\) and \((0, \bar{r}^{*}]\).

The function \(h_{R}\) is computed in a similar fashion. The proof of Prop 14 follows from Prop 12. The constant \(k\) is chosen so that \(h_{R}^{\prime}(\bar{r}) = 0\).

**Proposition 14** \(h_{R}(r) = \gamma^{-1}r + \zeta\gamma^{-2} + k\varphi_{-}(r), r \leq \bar{r}, \) with \(k = (\theta_{-}\gamma\varphi_{-}(\bar{r}))^{-1}\)

Computation of \(h_{A}\) and \(h_{W}\) is a bit more complex: Let \(c_{D} = h_{C}^{\prime}\) (the derivative of \(h_{C}\)), and suppose that \(h_{D}\) is in the domain of the extended generator and solves (4.31) with \(c = c_{D}\). We then have

**Proposition 15** \(h_{A}(r,d) = dh_{C}(r) - \sigma^{2}h_{D}(r), h_{W}(r,w) = wh_{C}(r) + \sigma^{2}h_{D}(r)\)

The proof of Prop 15 follows from the following lemmas.

**Lemma 2** The function \(H_{1}\) defined by \(H_{1}(r,d) = dh_{C}(r)\) is in the domain of the extended generator for the bivariate process \((R, D)\), and satisfies

\[
\mathcal{A}H_{1}(r,d) = \gamma H_{1}(r,d) - c_{A}(r,d) - \sigma^{2}c_{D}(r)
\]
Lemma 3 The function $H_2$ defined by $H_2(r, w) = wH_C(r)$ is in the domain of the extended generator for the bivariate process $(R, G_W)$, and satisfies,

$$AH_2(r, w) = \gamma H_2(r, w) - c_W(r, w) + \sigma_W^2 c_D(r)$$  \hspace{1cm} (4.42)$$

Computation of each of the welfare functions is possible once we can compute $h_D$. For this we note that $c_D$ can be expressed,

$$c_D(r) = \begin{cases} A_- e^{-\theta_- r} & r \leq 0 \\ B_- e^{-\theta_- r} + B_+ e^{-\theta_+ r} & 0 < r \leq \bar{r}^* \end{cases}$$  \hspace{1cm} (4.43)$$

where $A_-, B_-, B_+$ are constants. Computation of $h_D$ then follows from Lemma 1 combined with the following result:

Lemma 4 Writing $c(r) = c_D(r)$, the function $h_0$ given below solves (4.33):

$$h_0(r) = \begin{cases} (\theta_- \sigma^2 - \zeta)^{-1} A_- r e^{-\theta_- r} & r \leq 0 \\ (\theta_- \sigma^2 - \zeta)^{-1} B_- r e^{-\theta_- r} + (\theta_+ \sigma^2 - \zeta)^{-1} B_+ r e^{-\theta_+ r} & 0 < r \leq \bar{r}^* \end{cases}$$

Proof We apply the product rule $Dfg = f'Dg + g'Df + \sigma^2 f'g'$, using $f(r) = r$ and $g(r) = e^{-\theta r}$, where $\theta$ is any solution to (4.35). This gives

$$Dfg(r) = r\gamma e^{-\theta r} + \zeta e^{-\theta r} - \theta \sigma^2 e^{-\theta r}$$

Consequently, considering the two possibilities for $\theta$, on defining $h_+ = r\varphi_+/(\theta_+ \sigma^2 - \zeta)$, and $h_- = r\varphi_-/(\theta_+ \sigma^2 - \zeta)$, we obtain

$$Dh_+ = \gamma h_+ - \varphi_+ \quad \text{and} \quad Dh_- = \gamma h_- - \varphi_-$$

From the formula (4.43) we conclude that $Dh_0 = \gamma h_0 - c_D$, with $h_0$ defined in the lemma.

4.3.4 Numerical Examples

Using the results of the previous section, closed-form expressions for the discounted mean welfare of the consumer and the supplier are obtained. We
now provide illustrative examples and discuss our results.

The following set of parameters – expressed in $/MWh – is used in all our experiments:

\[ c_{bo} = 200,000, \quad v = 50 \]
\[ c_{da} = c_{rt} = 30. \]  

(4.44)

Based on the previous analysis, \( p_{da} = 30. \)

The discount factor is \( \gamma = 1/12 \) (corresponding to a 12 hour time horizon), the ramp rate \( \zeta = 200 \), the mean demand was taken to be \( \bar{D} = 50,000 \) MW, and its standard deviation \( \sigma = 500. \)

The standard deviation of wind \( \sigma_w \) and wind resource penetration are treated as variables, but are scaled as follows: The coefficient of variation of wind and the percentage of wind resource penetration are denoted, respectively, by

\[ c_v := \frac{\sigma_w}{\mathbb{E}[G_{W}(t)]} \quad \text{and} \quad k = 100 \frac{\mathbb{E}[G_{W}(t)]}{\bar{D}} \]  

(4.45)

Figure 4.4: Shown on the left is the optimal reserve level as percentage of DAM demand for different coefficient of variation \( c_v \). The plot on the right shows the optimal total welfare for different values of \( c_v \).

Fig. 4.4 shows the optimal reserves as a percentage of \( \bar{D} \), calculated using (4.7), with respect to \( c_v \) and \( k \), and the other parameters held constant. The value of \( \bar{r}^* \) in the model without wind generation – indicated as \( k = 0 \) in the figure – is approximately 10% of \( \bar{D} \). This value rises quickly with increased wind penetration or increased variability.

In the following subsections, the impact of wind generation volatility on the total consumer and supplier welfare under the two different commanding
schemes is illustrated.

![Graph showing consumer and supplier welfare](image)

**Figure 4.5:** Consumer and supplier welfare when consumer owns wind for different coefficient of variation $c_v$.

**Consumers Command the Wind** Shown in Fig. 4.5 is the consumer and supplier welfare when the consumers command the wind generation resources. It is seen that the negative impact of wind volatility results in a decay in welfare with increasing volatility, and the consumer breaks even at $c_v \sim 0.1$. Below this threshold value, the consumer sees benefit with additional wind generation; beyond it, the consumer welfare decreases rapidly. With high volatility, the consumer is better served by reducing the wind generation injected into the system.

Remarkably, for the given set of model parameters, the threshold at $c_v = 0.1$ is largely invariant to the level of wind resource penetration.

**Suppliers Command the Wind** If the suppliers command wind generation, the welfare expressions are impacted by the wind-deviation term $E[P(t)G_w(t)]$. In real time, this term is potentially harmful for suppliers. If $G_w(t) \ll 0$, then it is likely that reserves are negative, so that $P(t) = (v + c^{bo})$. This will result in high penalties to the supplier, who is charged the real-time price for not delivering the power promised in the DAM. This is seen in the plots of consumer and supplier welfare shown in Fig. 4.6. For low volatility levels, suppliers see benefit from owning and dispatching power from wind. Supplier welfare drops quickly for $c_v > 0.1$. 

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Figure 4.6: Consumer and supplier welfare when suppliers own wind for different coefficient of variation $c_v$.

4.3.5 Updating Markets Structures

The results of this chapter show that the value of renewable energy can be low if it is volatile. Some might argue that this means we should abandon efforts to increase our renewable energy portfolio, but this is a misreading of our research.

- The framework of this chapter is based on standard economic analysis. There are of course many issues to consider beyond the value of consumption, the cost of blackout, and the cost of coal. Moreover, while the notion of utility used here is standard, it is based on just two parties—an abstract collection of consumers and of generators. The interests of society are far broader.

- Traditional sources of energy will be gone in just a few generations. Optimistic estimates conclude that the natural gas supply in North America will run out in just 100 years, at its current 20% of total energy consumption [83]. Moreover, the price of natural gas has dropped dramatically in recent years. In response, natural gas consumption is likely to climb to at least 40%, which would drop the time horizon to 50 years.

- Finally, the results of this chapter imply that high levels of renewable generation can be injected into the grid provided that we also install technologies that can offset volatility (reduce $\sigma^2$), or reduce the cost of
load shedding ($c^{\text{sh}}$ in this model). Resources such as storage and demand response, along with today’s regulation technologies, will expand in the future to meet these goals [76,77,84–86].

Hence it is possible to create a more sustainable energy infrastructure, and it is necessary if we want energy for our grandchildren. Unfortunately, current markets for electricity do little to help us move forward.

There are several reasons why innovation has been slow in energy markets. First, there has been little consideration of the physics of the grid. In particular, energy cannot be treated as a “commodity” without recognizing the different values of different forms of generation. Remarkably, current markets are based on the assumption that one megawatt is as good as another, imposing uniform prices on generation from all sources. Consequently, in many of today’s markets there are no incentives for responsive generation that can ramp quickly, and few incentives for reliability.

A second deficiency in today’s market is that the true costs of generation are not always considered. There are short-term costs, such as those associated with ramping, and long-term costs associated with pollution. Fortunately these gaps are beginning to receive attention in the systems and control literature [70,72,87–89].

4.4 Summary

When dynamics and uncertainty are taken into account, social welfare may fall dramatically as more wind generation is dispatched. The environmental and sustainability benefits of wind power could be eclipsed due to the cost of additional reserves, and the adverse impact of volatile prices. These conclusions hold under the most ideal circumstances in which the consumers own all wind generation resources and, more importantly, the perfect competition setting in which price manipulation is excluded.

Closed-form formulae show that under the current scheme of dispatch all the wind, key features of current electricity markets create adverse incentives to the deployment of volatile resources. When consumers control the wind, consumer welfare falls and supplier welfare will eventually rise with increases in either wind penetration or its volatility or both. In the case in which suppliers control the wind, the penalties in real-time associated with wind
deviation can have negative impacts on supplier welfare. In either case, the entity that commands wind resources may suffer.

These facts mean that new technologies and new policies must be created to mitigate the impact of volatility. There are control challenges, and also policy challenges: Appropriate incentives and penalties must be introduced to help improve reliability of the power grid, stimulate innovation in new technologies, and encourage the participation of new actors in these markets.
CHAPTER 5

TOWARD UNDERSTANDING THE IMPACT OF RAMPING COSTS IN POWER MARKETS

The purpose of this chapter is to expose the difficulties of incentivizing responsive generation in a real-time market setting. In an analysis of a dynamic electricity market, that includes cost of capacity and ramping of generation, it is found that

- In a competitive equilibrium, average price may coincide with average marginal production cost. Hence, the cost of ramping is not captured by the average prices.

- If a price for ramping is included in the market, then the competitive equilibrium price is no longer unique. A complete characterization of possible prices is obtained.

The analysis is based on the solution to what economists call the social planner’s problem (SPP), which is a stochastic control problem in this dynamic setting. Numerical experiments expose the structure of the solution to the SPP, and illustrate the main issues regarding the competitive equilibrium. These findings underscore the need to find alternative ways to structure electricity markets.

5.1 Introduction

The Smart Grid vision is expected to change many paradigms in the power and energy industry. In particular, the implementation of new technologies and policies will bring new means of interaction between participants—suppliers, consumers and system operators. Increased deployment of renewables and participation of new players (both consumers and new generation service providers) will bring many challenges to both engineers and policy makers.
The new technological environment, along with the empirical results of several decades of implementing electricity markets, are forcing some to reconsider established paradigms in electricity markets. The elegant market formalism of Schweppe et al. [14] led many to believe that real-time markets for energy would be sufficient to create appropriate economic signals for electricity markets. Empirical evidence over decades combined with recent theory demonstrate that real-time markets have deficiencies on many grounds. One explanation is that the 30-year-old real-time market paradigm treats electricity as a commodity, much like bread or natural gas.

This view of electricity ignores engineering realities. The commodity perspective does not recognize the many different attributes and services, beyond energy, that different technologies can provide. Policy makers have started to recognize these facts, and in response new federal policies have been implemented. One example of a “service” is the ramping of generation to respond to volatility in the grid. FERC order 755 is designed to ensure that ramping capabilities are properly compensated [15]. Appropriate market mechanisms to provide adequate compensation remain a subject of intense debate.

Market design is non-trivial in part because the “product” is not easily defined. A generator or a battery can provide ancillary service to the grid. These resources have very different attributes in terms of cost, capacity, reliability, and bandwidth of service. There are hundreds of other resources that can provide ancillary service. Do we have a single real-time market for all? If so, how is the price determined as a function of cost, reliability, ramping rate, and capacity?

It is a thesis of this work that another difficulty with market design in the electricity sector is the focus on real-time markets. Distinguishing the dynamic attributes of generation is difficult in a real-time setting. Moreover, even in simple markets with just one or two generation classes, it is known that real-time markets can lead to uncertainty, uncertain dynamics, and high price volatility [12,74,78]. These theoretical results reflect what is seen today in electricity markets all over the world. There is no theory to predict better behavior if we add real-time markets for ancillary services.

Markets for electricity and ancillary service must be based on the costs and attributes of service providers. From the point of view of the service provider, important attributes include the following:
1. Cost and capacity of energy
2. Cost and capacity of ramping
3. Reliability of income

From the point of view of the grid (ISO or utility company):
1. Value of energy
2. Value of ramping
3. Reliability of service

An electricity product is thus a multi-attribute concept, and markets must take this into account. The precise definition of a product will require abstractions since it is impossible to define exactly “the cost of ramping” or the “reliability of ramping”.

A starting point is to construct models to obtain insight regarding these costs and values. Following [89], it is argued that we must first reconsider the social planner’s problem (SPP) of economics in a dynamic setting, taking into account the dynamic costs and benefits of the various services. The SPP is a centralized stochastic control problem, and hence a realistic yet tractable representation of the physical system is key. The use of stochastic differential equation models and approximate dynamic programming approaches is a promising avenue [30, 36, 86, 90]. The next step is defining appropriate products based on the value that different technologies can offer. The last point is the implementation of market structures and pricing schemes in which the use of contractual arrangements emerges as a natural alternative [89].

In order to capture the value of different technologies, it is mandatory to consider the real cost and benefits of producing and consuming power. This will require moving from the standard use of cost and utility functions towards the use of cost and utility functionals.

So, instead of having cost and utility functions that are a function of quantities at a given time, the model will be constructed around cost functionals \( c[f(.)] \) or utility functionals \( u[f(.)] \) in which the argument is a function in time, \( f: [0, T] \rightarrow \mathbb{R}^m \), where \( T \) is the time horizon, and \( m \) indicates the number of technologies in the system; it is assumed that \( m = 1 \) in this chapter.
In addition, functionals are assumed to be of integral form, such as

\[ C[f(t)] = \int_0^t \{ \alpha f(s) + \beta |\dot{f}(s)| \} ds \]  

(5.1)

where \( \alpha \) and \( \beta \) are non-negative constants.

From the suppliers’ viewpoint, a nuclear unit might have a very small \( \alpha \) and a very large \( \beta \) reflecting the fact that producing energy is cheap, but ramping it up and down can be costly. Similarly, a peaking unit might have large \( \alpha \) and small \( \beta \) reflecting the fact that once in operation the cost of ramping up and down is not so high. Many other technological costs can be represented by appropriate functionals.

From the consumers’ viewpoint, a utility functional can be constructed to represent many attributes beyond energy. For example, some consumers might care about the environmental impact of the energy they consume over a period. In that line, as the results in [36] show, having more volatile resources might require having an increased number of fast units, or the deployment of more reserves. So “green” consumers might put some value on faster units that will make up for the inherent volatility of a “green” system. Utility functionals can also capture the utility or disutility of flexibility to consumers. Using the same functional form (5.21), a consumer that requires little flexibility in consumption might have very small \( \beta \) reflecting the small value of changing consumption patterns over time. On the other hand, a consumer that requires more flexibility in consumption might have larger values of \( \beta \).

Given functionals that define cost and utility over some time horizon, the market problem is examined within the context of dynamic competitive equilibrium theory as in [12, 31]. One conclusion in this prior work is that the volatility of prices is not necessarily a sign of market failure. Even in the most idealized competitive equilibrium setting, volatility in prices is a natural outcome. These results are extended in the present chapter to explicitly model the costs of both energy and ramping. The main objectives of this chapter are to

- Understand how prices, in a competitive market setting considering dynamics and uncertainty, are impacted by ramping costs.
- Investigate the impact of having differentiated prices for capacity and
ramping.

- Quantify mean values of economics metrics, in particular, prices and welfares.

In the literature, there is little discussion about the impact of ramping costs on market structures. In [91], Tanaka studies the impact of ramping costs on electricity markets, restricted to a deterministic and single-price setting. In the present chapter, such impacts are investigated from a general perspective and under broader settings. In particular, following our previous work, we consider a dynamic electricity market with uncertainty. Additionally, we investigate multi-pricing schemes—having a single price or differentiated prices for capacity and ramping. We focus special attention on quantifying mean values that can provide insights about the appropriateness of real-time markets. Based on our analytical and numerical findings, the main conclusions of this chapter include,

- In a competitive equilibrium, price is never marginal cost, but average price may coincide with average marginal cost. Hence, the cost of ramping is not captured by the average prices.

- If a price for ramping is included in the market, then the competitive equilibrium price is no longer unique. A complete characterization of possible prices is obtained in the chapter.

These results expose the difficulties of incentivizing responsive generation in a real-time market setting, reinforcing the need to develop alternative market designs. Other results in this chapter can facilitate the design of alternative market structures. In particular, the quantification of mean costs and utilities is valuable information in the design of appropriate contractual arrangements [89].

This chapter has five additional sections. In Section 5.2, the basic concepts of competitive equilibrium analysis are presented. Section 5.3 is devoted to the characterization of competitive prices in a dynamic and deterministic setting. In Section 5.4, the analysis is extended to a dynamic and stochastic setting and the main analytical results of the chapter are presented. In Section 5.5, numerical experiments illustrating the analytical findings are discussed. Finally, concluding remarks and future avenues for research are presented in Section 5.6.
5.2 Competitive Equilibrium Analysis

In this chapter, we focus on integral functionals to capture costs or utility. For the finite-horizon market model, an example is,

\[ F[f(t)] = \int_0^t \mathcal{F}(f(s), \dot{f}(s)) \, ds \]  

(5.2)

where \( \mathcal{F} \) is a real-valued function of two variables. For the case of costs, this representation captures the costs of capacity and ramping. In order to perform a competitive equilibrium analysis, we need to construct an appropriate social planner’s problem (SPP). Players and basic modeling assumptions are reviewed in this section.

5.2.1 Models and Players

In order to capture the impact of ramping costs, we extend the queuing model introduced in [30] for power generation. In this model, there are two players — consumers and suppliers — and transmission capacity is assumed to be enough to allocate all the transactions. Given that the impact of ramping costs is expected to be key in coping with the uncertainty associated with volatile resources, we focus mainly on the real time operation of the grid. Consequently, we focus on supplying the deviation of the expected demand. \( D = \{ D(t) : t \geq 0 \} \) denotes the residual demand process.

The capacity of a generator is the maximum amount of power that can be extracted from the generator at a given time. The respective capacities of the aggregated of generators at time \( t \) are denoted by \( G(t) \). The reserve process at time \( t \) is defined by

\[ R(t) = G(t) - D(t), \quad t \geq 0. \]  

(5.3)

We say that loss-of-load occurs at time \( t \) if \( R(t) < 0 \).

The objectives of the consumer and the supplier are specified by their respective welfare functions,

\[ W_s(t) := P(t)G(t) - c(G(t)) - c_R(\dot{G}(t)) \]

\[ W_d(t) := B(G(t)) - P(t)G(t) \]  

(5.4)
in which $c(G(t))$ and $c_R(G(t))$ represent the costs for capacity and ramping, respectively, and $B(G(t))$ represents the utility function for consumers.

It is assumed that the consumer and supplier each optimize either discounted or total mean-welfare,

$$W_s := \mathbb{E} \left[ \int e^{-\gamma t} W_s(t) \, dt \right],$$

$$W_d := \mathbb{E} \left[ \int e^{-\gamma t} W_d(t) \, dt \right]$$

(5.5)

where $\gamma > 0$ is the discount rate. In the total welfare case $\gamma = 0$.

In order to characterize the competitive equilibrium, we first focus on the solution of the SPP as defined in standard economics texts. The SPP is described as the maximization of the social welfare,

$$W = \mathbb{E} \left[ \int e^{-\gamma t} (W_s(t) + W_d(t)) \, dt \right].$$

For many of the results presented in this chapter, in terms of competitive prices, the explicit solution of the SPP is not required. This situation changes on the numerical experiments in which the solution of the SPP is quantified.

5.2.2 Revisiting Simple Cases

Here we introduce simple cases in order to review terminology and concepts of competitive equilibrium (CE) theory. These results and examples will facilitate the transition from standard economic results in a static setting to the case in which dynamics and uncertainty are considered.

The underlying idea of the CE theory is using prices to achieve a state, the so-called competitive equilibrium, in which consumers and suppliers agree. In this analysis two assumptions are crucial:

- Price taking assumption: prices are not impacted by players’ decisions.
- Information symmetry: Information is available to both sellers and consumers without distinctions.

The equilibrium is assumed to arise from the separate optimization problems performed by consumers and suppliers. The price $P$ is exogenous and independent of the decisions of consumers or suppliers.
In its simplest form, the problem faced by the consumer is

\[
\max_{G_D} \quad U(G_D) - PG_D \\
\text{s.t.} \quad G_D \in X_D
\]

in which \(U(G_D)\) is an abstract utility function for consumers, and \(X_D\) represents constraints faced by the consumer.

The supplier maximizes income minus cost,

\[
\max_{G_S} \quad PG_S - C(G_S) \\
\text{s.t.} \quad G_S \in X_S
\]

in which \(C(G_S)\) is a cost function, and \(X_S\) represent supplier constraints.

A competitive equilibrium exists if there exists a price \(P\) such that suppliers and consumers agree, i.e., \(G_D = G_S\). An equilibrium is called efficient if the equilibrium solves the following SPP:

\[
\max_{G_D, G_S} \quad U(G_D) - C(G_S) \\
\text{s.t.} \quad G_S = G_D \\
G_S \in X_S, \ G_D \in X_D
\]

The usual pictographic representation of equilibrium points, represented by the intersection of two curves, is shown in Fig. 5.1. For a model without state constraints, this is explained as follows: from the consumer’s viewpoint, for given prices, it is possible to find a demand function \(G_D(P)\). This function, as its name suggests, will represent how many units of \(G_D\) the consumer demands for a particular price \(P\). It is clear that the price is the independent variable. However, it is standard in economics to use the inverse demand function \(P(G_D)\). In this representation, price would seem to be the dependent variable and the quantity the independent variable. However, this is just a graphical representation. Price is still the independent variable.

**Proposition 16** The inverse demand function for consumers is given by the marginal value,

\[
P_D(G_D) = \frac{\partial U}{\partial G_D}
\]

Following a similar reasoning, the supply function \(G_s(P)\) will describe how
many units of $G_s$ the supplier can provide for a particular price $P$. Its inverse is denoted $P(G_s)$, and it is identified in

**Proposition 17** The inverse demand function for supplier is given by the marginal cost,

$$P_s(G_s) = \frac{\partial C}{\partial G_s}$$

(5.10)

The two propositions follow directly from the first-order conditions of optimality. Consequently, they hold only for unconstrained markets. In a competitive equilibrium, the optimizers agree, $G_s^* = G_D^*$, and the price must satisfy $P^* = P_s(G_s^*) = P_D(G_D^*)$; that is, marginal cost coincides with marginal utility.

![Figure 5.1: Pictographic representation of the competitive equilibrium.](image)

The graphical representation represented by the intersection of the suppliers and consumers inverse demand functions is only possible in simple cases. Once dynamic, temporal and other type of constraints are considered, it is not straightforward to write explicit inverse demand and supply functions. However, even in those cases, it is possible to find competitive prices. The impact of hard constraints is examined in the following example.

**Impact of ramping constraints** In the book chapter [12], the conditions for the existence of the competitive equilibrium are thoroughly studied. Under those conditions, it is possible to decouple the process of finding the
competitive prices and the efficient allocation. The existence of the competitive equilibrium depends on the properties of the centralized problem. As explained in Theorem 3 of [12], if the centralized problem has a solution and there is no duality gap, then a competitive equilibrium exists.

In many situations, the construction of inverse demand functions is straightforward. This is the result of the fact that usually consumers are not subject to hard constraints. In these cases, the competitive price will be equal to the marginal value for consumers evaluated at the centralized problem solution. Using these ideas, we investigate the impact of ramping constraints on prices.

Consider the competitive equilibrium model in which the utility function for the consumer is piecewise linear,

\[ U(G_D) = v \min(D, G_D) - c^{bo}(G_D - D) \]  

in which \( c^{bo} \) represents the cost of load shedding. In this example the consumer is not subject to constraints. Hence the inverse demand function is given by

\[ P_D(G_D) = \frac{\partial U}{\partial G_D} = (v + c^{bo})\mathbb{I}((G_D - D) < 0), \quad G_D \neq D \]  

If \( G_D = D \), then the consumer will accept any price between zero and \((c^{bo}+v)\).

![Figure 5.2: Inverse demand function, simple case.](image)

It is assumed that the supplier’s cost function is linear.

\[ C(G_S) = c G_S \]  

Two cases are considered – the standard setting in which the supplier is
unconstrained, and a more realistic setting with constraints.

**Unconstrained model**  From the preceding we know that price coincides with marginal cost,

\[ P_s(G_s) = \frac{\partial C}{\partial G_s} = c \]  

(5.14)

The equilibrium is obtained by computing the intersection of supply and demand curves. This gives \( G_s = G_d = D \), provided \( c < (v + c^b) \). The competitive equilibrium is efficient.

![Figure 5.3: Inverse demand and supply functions unconstrained.](image)

![Figure 5.4: Inverse demand function.](image)

**Ramping constraints**  Given that the supplier is subject to ramping constraints, it is not possible to find a simple inverse supply function. Hence, in order to find an expression for the equilibrium prices, we use the existence results from [12]. Assuming the solution of the SPP exists, the competitive price will be given by the inverse demand function of consumers evaluated at the SPP solution,
\[ P(G^{spp}) = (v + c^bo) \mathbb{I}((G^{spp} - D) < 0) + c\mathbb{I}(G^{spp} = D) \quad (5.15) \]

Note that price will be equal to marginal cost only if suppliers can effectively follow demand. In general, either because ramping constraints are active or demand is uncertain, price will never be equal to marginal cost and it will show high volatility patterns, taking either 0 or \((v + c^bo)\) values.

The message of these examples can be summarized as follows. Once more reality—constraints and dynamics—is brought into consideration, competitive prices are no longer the intersection of textbook’s inverse supply and demand curves. Prices still can be calculated, but interesting and many times undesirable behaviors might emerge, such as extreme price volatility. In the next section, a deep investigation about the impact of ramping costs on prices is provided.

5.3 Competitive Prices — Deterministic Case

The remainder of the chapter concerns dynamic models, beginning with a finite time-horizon setting and deterministic dynamics. Suppliers and consumers are not subject to any physical and temporal constraints. The two welfare functions are integrals over the time horizon of length \( T \),

\[
W_s := \int_0^T W_s(t) \, dt, \quad W_D := \int_0^T W_D(t) \, dt \quad (5.16)
\]

where the integrand will be specified in two different settings, differentiated by the pricing framework.

In the first case there is a single price for capacity, and in the second there are two prices: one for capacity, and one for ramping. In the first stage of analysis it is assumed that the initial condition is specified. This section concludes with an analysis of the case where initial capacity is a decision variable for consumers and suppliers. This case leads to an elegant solution in which average price is precisely average marginal cost.
5.3.1 One Price

Assume that there is a single price $\hat{P}(t)$ associated with the capacity $G_S(t), G_D(t)$ at time $t$. It is assumed in this case that consumer and supplier welfares are given by

\begin{align*}
W_D(t) &:= B(G_D(t)) - \hat{P}(t)G_D(t) \\
W_S(t) &:= \hat{P}(t)G_S(t) - c(G_S(t)) - c_R(\dot{G}_S(t))
\end{align*}

(5.17)

(5.18)

The social welfare function is the sum

$$W(t) = B(G_D(t)) - c(G_S(t)) - c_R(\dot{G}_S(t))$$

(5.19)

The solution of the SPP, denoted $G^{\text{spp}}(t)$, is the optimizer of the following functional optimization problem:

$$\max_{G_S, G_D} W = \int \left[ B(G_D(t)) - c(G_S(t)) - c_R(\dot{G}_S(t)) \right] dt$$

s.t. $G_D(t) = G_S(t)$

(5.20)

This functional optimization problem is similar to the classical variational problem of finding a function $f$ over a time interval with the aim of minimizing an integral functional,

$$F[f(t)] = \int_0^t F(f(s), \dot{f}(s)) \, ds$$

(5.21)

An elegant study of the optimality condition for this type of problem is provided in [92].

**Lemma 5** The optimality conditions for an integral functional on $f$ and $\dot{f}$, over a time interval, are given by the Euler-Lagrange equation

$$\frac{\partial F}{\partial f} - \frac{d}{dt} \left( \frac{\partial F}{\partial \dot{f}} \right) = 0$$

Along with transversality conditions depending on the boundary conditions for the end points:

- For fixed initial and final points, no additional conditions are required.
• For fixed initial but free final point, a transversality condition is re-
quired,
\[ \frac{\partial F}{\partial \dot{f}} \bigg|_{t=T} = 0 \]

• For free initial and final points, transversality conditions are required,
\[ \frac{\partial F}{\partial \dot{f}} \bigg|_{t=0} = \frac{\partial F}{\partial \dot{f}} \bigg|_{t=T} = 0 \]

**Proof** These results can be straightforwardly shown by performing admis-
sible variations and setting the differential of \( F \) equal to zero,
\[ \frac{\delta F}{\delta f(t)} = 0 \]

Details can be found in Chapter 7 of [92].

Applying Lemma 5, it is possible to find the optimality conditions for the
SPP (5.20),
\[ \frac{\delta W}{\delta G(t)} = 0 \Rightarrow \frac{\partial W}{\partial G} - \frac{d}{dt} \left( \frac{\partial W}{\partial \dot{G}} \right) = 0 \]
from which the following is obtained as a condition for optimality,
\[ \frac{\partial B}{\partial G} = \frac{\partial c}{\partial G} - \frac{\partial^2 c_R}{\partial^2 \dot{G}} \dot{G}(t) \]

In addition, boundary conditions are required. If the initial condition is fixed,
\( G(0) = g \), a transversality condition is required for optimality at the ending
point,
\[ \frac{\partial c_R}{\partial G} \bigg|_{t=T} = 0 \]

If end-points are free, transversality conditions at both initial and final points
are required,
\[ \frac{\partial c_R}{\partial \dot{G}} \bigg|_{t=0} = \frac{\partial c_R}{\partial \dot{G}} \bigg|_{t=T} = 0 \]

As in the static case, computation of equilibrium prices is based on an
analysis of the optimization problems faced by the players. We begin with
the consumer’s optimization problem:

**Proposition 18** The competitive price is given by the marginal utility func-
tion of the consumer evaluated at the SPP solution,

\[ \hat{P}(t) = \frac{\partial B}{\partial G} \bigg|_{G(t)=G^{pp}} \]  

(5.22)

**Proof** Recall that the consumer is not subject to temporal or spatial constraints. The consumer’s optimization problem reduces to a static one for each time. On differentiating the welfare \( W_D(t) := B(G_D(t)) - \hat{P}(t)G_D(t) \) introduced in (5.17) with respect to \( G_D(t) \), the first order optimality conditions imply that \( \hat{P}(t) = \frac{\partial B}{\partial G} \). At the equilibrium \( G_D = G^{pp} \), and the formula is proved. \( \square \)

Ramping costs in the supplier’s optimization complicate analysis, and in particular rule out the simple marginal-cost representation.

**Proposition 19** The competitive price is given by the marginal cost of capacity minus the time derivative of the marginal cost of ramping, evaluated at the SPP solution,

\[ \hat{P}(t) = \left[ \frac{\partial c}{\partial G} - \frac{\partial^2 c_R}{\partial G^2} \right] \bigg|_{G(t)=G^{pp}} \]  

(5.23)

**Proof** The optimization problem faced by the supplier is given by

\[ \max_{G_S} W_S = \int \left[ \hat{P}(t)G_S(t) - c(G_S(t)) - c_R(\dot{G}_S(t)) \right] dt \]  

(5.24)

The optimality condition for this problem can also be found by using results from Lemma 5:

\[ \frac{\delta W_S}{\delta G_S(t)} = 0 \Rightarrow \frac{\partial W_S}{\partial G_S} - \frac{d}{dt} \left( \frac{\partial W_S}{\partial \dot{G}_S} \right) = 0 \]  

(5.25)

from which it is obtained that

\[ \hat{P}(t) = \frac{\partial c}{\partial G_S} - \frac{\partial^2 c_R}{\partial G^2_S} \dot{G}_S(t) \]  

(5.26)

At the equilibrium \( G_S = G^{pp} \) and the formula is proved. \( \square \)

Once ramping costs are considered, the equilibrium price is no longer equal to marginal cost. The price can go above marginal production cost when \( \ddot{G}(t) < 0 \), and the price can be negative for some periods of time.
However, similar to what is seen in dynamic-stochastic models [12, 31, 78], the average of prices over the period $T$ are equal to the average marginal cost of capacity over the period, plus some boundary terms related to the ramping costs:

**Proposition 20** In the deterministic case, average price is equal to the average marginal cost of capacity plus boundary terms related to ramping cost.

$$
\frac{1}{T} \int_0^T \hat{P}(t) dt = \frac{1}{T} \int_0^T \frac{\partial c}{\partial G} dt + \frac{1}{T} \left( \frac{\partial c_R}{\partial G} \bigg|_{t=0} - \frac{\partial c_R}{\partial G} \bigg|_{t=T} \right) \quad (5.27)
$$

**Proof** Direct application of the average to price expression (5.23). □

The boundary terms vanish if the constraint $G(0) = g$ is relaxed. That is, the average price is exactly the average marginal cost of capacity:

**Proposition 21** If the initial condition is free, then the average price is equal to the average marginal cost of capacity,

$$
\frac{1}{T} \int_0^T \hat{P}(t) dt = \frac{1}{T} \int_0^T \frac{\partial c}{\partial G} dt \quad (5.28)
$$

**Proof** In the free end-points, in addition to the Euler-Lagrange equation, the transversality condition becomes

$$
\frac{\partial c_R}{\partial G} \bigg|_{t=0} = \frac{\partial c_R}{\partial G} \bigg|_{t=T} = 0 \quad (5.29)
$$

As in the previous proof, this implies that the average price is average marginal cost of capacity. □

### 5.3.2 Two Prices

We study now the case in which different prices for both capacity and ramping are considered. The welfare functions for consumers and suppliers are given by

$$
W_D(t) := B(G(t)) - P_1(t)G(t) - P_2(t)\dot{G}(t) \quad (5.30)
$$

$$
W_S(t) := P_1(t)G(t) + P_2(t)\dot{G}(t) - c(G(t)) - c_R(\dot{G}(t)) \quad (5.31)
$$
Note that the previous section is just a particular case in which \( P_2 \equiv 0 \). Following the assumptions of the previous case, suppliers and consumers are not subject to physical or temporal constraints. Hence, the SPP is unchanged,

\[
\max_{G_D, G_S} W = \int \left[ B(G_D(t)) - c(G_S(t)) - c_R(\dot{G}_S(t)) \right] dt
\]

\[
\text{s.t. } G_D(t) = G_S(t)
\]

(5.32)

In order to find expressions for the competitive prices, as before, the problems faced by consumers and suppliers are analyzed.

**Proposition 22** The competitive prices of capacity and ramping are related to the value for consumers by the following ordinary differential equation (ODE):

\[
P_1(t) - \dot{P}_2(t) = \frac{\partial B}{\partial G} \bigg|_{G(t) = \text{SPP}}
\]

(5.33)

**Proof** The result follows directly by writing the optimality conditions of the problem faced by the consumer. Unlike the previous case in which the consumer’s optimization problem was reduced to a static one for each time. In this case, the term \( P_2(t)\dot{G}_D(t) \) creates a time coupling. Hence, the optimality condition must be obtained by using Lemma 5,

\[
\frac{\delta W_D}{\delta G(t)} = 0 \Rightarrow \frac{\partial W_D}{\partial \dot{G}} - \frac{d}{dt} \left( \frac{\partial W_D}{\partial G} \right) = 0
\]

(5.34)

The proposition is proved by doing the explicit calculations,

\[
P_1(t) - \dot{P}_2(t) = \frac{\partial B}{\partial G}
\]

(5.35)

along with the fact that at equilibrium \( G_D(t) = G^{\text{SPP}} \). □

A similar result is found by looking at the optimization problem faced by the supplier problem.

**Proposition 23** The competitive prices of capacity and ramping are related to the supplier’s costs by the ODE,

\[
P_1(t) - \dot{P}_2(t) = \left[ \frac{\partial c}{\partial G} - \frac{\partial^2 c_R}{\partial G^2} \dot{G}(t) \right]_{G_S(t) = \text{SPP}(t)}
\]

(5.36)
**Proof** The result also follows by writing the optimality conditions of the problem faced by the supplier,

\[
\frac{\delta W_s}{\delta G(t)} = 0 \Rightarrow \frac{\partial W_s}{\partial G} - \frac{d}{dt} \left( \frac{\partial W_s}{\partial G} \right) = 0 \tag{5.37}
\]

Hence,

\[
P_1(t) - \dot{P}_2(t) = \frac{\partial C}{\partial G} - \frac{\partial^2 C_R}{\partial G^2} \dot{G}(t) \tag{5.38}
\]

along with the fact that at equilibrium \( G_s(t) = G^{sup} \).

An interesting point is the fact that there is a lack of uniqueness on \( P_1(t) \) and \( P_2(t) \). In other words, there are multiples pairs of \( P_1(t) \) and \( P_2(t) \) that provide a competitive equilibrium. This fact might seem strange, but it is not. Both prices are dependent and a link with the single price of the previous section can be established.

**Proposition 24** \( P_1(t) \) and \( P_2(t) \) are related to a single price for capacity, \( \hat{P}(t) \), by

\[
P_1(t) - \dot{P}_2(t) = \hat{P}(t) \tag{5.39}
\]

**Proof** The relationship can be found by working the welfare expressions. We will focus only on the consumer welfare. Supplier is analogous. The total consumer welfare is given by

\[
W_D = \int \left( B(G(t)) - P_1(t)G(t) - P_2(t)\dot{G}(t) \right) dt \tag{5.40}
\]

integrating by parts the term \( P_2(t)\dot{G}(t) \), we obtain

\[
W_D = \int \left( B(G(t)) - P_1(t)G(t) + \dot{P}_2(t)G(t) \right) dt - P_2(t)G(t) \big|_0^T \tag{5.41}
\]

Consequently, the welfare functions for suppliers and consumers are then given by

\[
W_D(t) = B(G(t)) - (P_1(t) - \dot{P}_2(t))G(t) \tag{5.42}
\]

\[
W_s(t) := (P_1(t) - \dot{P}_2(t))G(t) - c(G(t)) - c_R(\dot{G}(t)) \tag{5.43}
\]

with the condition that \( P_2(T)G(T) = P_2(0)G(0) \).
Hence, it is clear that prices for capacity and ramping are coupled, and related to the single price of capacity by

\[
P_1(t) - \dot{P}_2(t) = \dot{P}(t) \tag{5.44}
\]

An intuitive way to understand the lack of uniqueness for \(P_1(t)\) and \(P_2(t)\) is the following. The price of ramping \(P_2(t)\) could be associated to the Lagrange multiplier of an additional constraint in the SPP, given by \(\dot{G}_D(t) = \dot{G}_S(t)\). However this constraint is just a redundant one. The solution for the problem considering only the \(G_D(t) = G_S(t)\) constraint is also a solution of the problem with the new constraint \(\dot{G}_D(t) = \dot{G}_S(t)\). Hence, a relationship between the Lagrange multipliers of the original problem and the new one can be found.

This lack of uniqueness implies that any pair of prices satisfying (5.36), (5.33) would provide the same welfares for consumers and suppliers. An illustration of this situation is developed for the case in which \(c(x) = cx\), \(c_R(\dot{x}) = \frac{1}{2}c_R\dot{x}^2\) and the demand is inelastic. First, the welfare for the single price case is evaluated,

- \(\dot{P}(t) = c - c_R\dot{G}\)

Hence, supplier welfare in this case is given by,

\[
W_s = \int \left[ \dot{P}(t)G(t) - cG(t) - \frac{1}{2}c_R\dot{G}^2 \right] dt
\]

\[
W_s = \int \left[ (c - c_R\dot{G}(t))G(t) - cG(t) - \frac{1}{2}c_R\dot{G}^2 \right] dt \tag{5.45}
\]

\[
W_s = \int \left[ -c_R\dot{G}(t)G(t) - \frac{1}{2}c_R\dot{G}^2 \right] dt
\]

Integrating by parts the term \(\int -c_R\dot{G}(t)G(t)dt = -c_R[\dot{G}G - \int \dot{G}^2 dt]\),

\[
W_s = \frac{1}{2} \int c_R\dot{G}^2 dt - c_R\dot{G}G \bigg|_0^T \tag{5.46}
\]

Using the fact that \(\dot{G}(0) = \dot{G}(T) = 0\), we obtain that the supplier
welfare is
\[ W_s = \frac{1}{2} \int c_R \dot{G}^2 dt \] (5.47)

Now, it is possible to find several pairs of appropriate \( P_1 \) and \( P_2 \) that will give the same welfare as with \( \hat{P}(t) \).

- \( P_1 = c \) and \( P_2 = c_R \dot{G} + K \)
  
  Hence, the supplier welfare will be given by,
  \[ W_s = \int \left[ P(t)G(t) + P_2(t)\dot{G} - cG(t) - \frac{1}{2}c_R \dot{G}^2 \right] dt = \frac{1}{2} \int c_R \dot{G}^2 dt + K \int \dot{G} dt \] (5.48)

By comparing with (5.46), it is clear that \( K \) is
\[ K = \frac{-c_R \dot{G}G}{\int G dt} \] (5.49)

- \( P_1 = 0 \) and \( P_2 = -ct + c_R \dot{G} + K \)
  
  \[ W_s = \int \left[ (-ct + c_R \dot{G})\dot{G} - cG(t) - \frac{1}{2}c_R \dot{G}^2 \right] dt = \frac{1}{2} \int c_R \dot{G}^2 dt - cT G(T) + K \int \dot{G} dt \] (5.50)

the result is clear by integrating by parts the term \( \int -ct \dot{G} = -c[tG - \int G dt] \). By comparing with 5.46, it is clear that in this case \( K \) is
\[ K = \frac{cT G(T) - c_R \dot{G}G}{\int G dt} \] (5.51)

5.4 Competitive Prices — Stochastic Case

In this section, we study the competitive equilibrium prices in the case in which uncertainty in demand is considered. We focus on the case in which there is a single price. This setting would be appropriate for analyzing a real-time market in which ramping costs are included. The following welfare functions are considered:

\[ W_b(t) := v(G_D(t)) - c^{bo}(D(t) - G_D(t)) - P^e(t)G_D(t) \] (5.52)
in which $v(.)$ and $c^{bo}$ represent the utility and disutility function for consumers, respectively, and $c(.)$ and $c_R(.)$ represent capacity and ramping costs for suppliers. It is assumed that the consumer and supplier each optimize the total discounted cost,

$$W_S := E \left[ \int e^{-\gamma t} W_S(t) \, dt \right],$$

$$W_D := E \left[ \int e^{-\gamma t} W_D(t) \, dt \right]$$

where $\gamma > 0$ is the discount rate.

We first establish formulas for $P^e$.

**Proposition 25** Suppose that $(G^*)$ is a solution to the SPP that defines a competitive equilibrium with price process $P^e$. Then,

$$P^e(t) = \nabla v(G^*(t)) + \nabla c^{bo}(D(t) - G^*(t)), \quad t \geq 0.$$  

**Proof** In this case we have that $W_D(t) := v(D(t)) - c^{bo}(D(t) - D^*_D(t)) - P^e(t)G^*_D(t)$. The formula follows by writing the optimality conditions for consumers and using the fact that consumer is not subject to any type of constraints. In addition, at the competitive equilibrium $G^* = G^*_D$.  

For the case of piece-wise linear utility functions, similar to the case presented in [31], the price is a function of the social planner’s reserve process.

**Proposition 26** For the particular case of piece-wise linear benefit function, given by $v \min(D, G) - c^{bo}R_*$, the competitive price is given by,

$$P^e(t) = (v + c^{bo})I(G^*(t) - D(t) < 0)$$

**Proof** Direct application of Prop. 25.  

Unlike [31], in this case, the supplier is not subject to ramping constraints. Hence, an expression for the equilibrium prices can also be obtained by looking at the supplier’s problem.
Proposition 27 Suppose that \((G^*)\) is a solution to the SPP that defines a competitive equilibrium with price process \(P^e\). Then,
\[
P^e(t) = \nabla c(G^*(t)) - e^{-\gamma t} \frac{d}{dt} \left[ e^{-\gamma t} \frac{\partial c_R}{\partial G^*} \right] \geq 0. 
\] (5.57)

Proof The welfare function of the supplier is given by \(W_S(t) := P^e(t)G_s(t) - c(G_s(t)) - c_R(\dot{G}_s(t))\). In this case, the supplier is not subject to ramping constraints. Hence, the optimality conditions for maximizing the supplier welfare
\[
W_S = E\left[ \int e^{-\gamma t} W_S dt \right] 
\] (5.58)
are given by
\[
\frac{\delta W_S}{\delta G_s(t)} = 0 \Rightarrow \frac{\partial W_S}{\partial G_s} - \frac{d}{dt} \left( \frac{\partial W_S}{\partial \dot{G}_s} \right) = 0 
\] (5.59)
along with appropriate boundary conditions depending on whether the initial condition is fixed. By using (5.59) the expression for the price is obtained.

\(\square\)

The expression for equilibrium prices in terms of the cost functions allows one to quantify explicitly the expected price related to the expected costs.

Proposition 28 In the stochastic case, expected price is equal to the expected marginal cost of capacity plus boundary terms related to expected ramping costs.
\[
E\left[ \int e^{-\gamma t} P^e(t) dt \right] = E\left[ \int e^{-\gamma t} \nabla c(G^*(t)) dt \right] + E\left[ \frac{\partial c_R}{\partial G_s} |_{t=0} \right] 
\] (5.60)

Proof
\[
W_S(G_s(t), \dot{G}_s(t), t) = E\left[ \int e^{-\gamma t} W_S(G_s(t), \dot{G}_s(t), t) dt \right] 
\] (5.61)
We perform a perturbation on \(G_s(t)\) of the functional associated with suppliers welfare by using
\[
G^\alpha_s(t) = G_s(t) + \alpha h(t) \\
\dot{G}^\alpha_s(t) = \dot{G}_s(t) + \alpha \dot{h}(t) 
\] (5.62)
in which \(h(t)\) is an admissible variation.
Using the fundamental result for optimality conditions,

$$
\frac{d}{d\alpha} W_s(G_s^*(t) + \alpha h(t), \dot{G}_s^*(t) + \alpha \dot{h}(t), t) \bigg|_{\alpha=0} = 0
$$

(5.63)

By (5.61), (5.62) and (5.63) the following is obtained:

$$
\frac{d}{d\alpha} W_s = E \left[ \int e^{-\gamma t} \frac{d}{d\alpha} \left( P^e(t)(G_s(t) + \alpha h(t)) - c(G_s(t) + \alpha h(t)) - c_R(\dot{G}_s(t) + \alpha \dot{h}(t)) \right) dt \right]
$$

$$
\frac{d}{d\alpha} W_s = E \left[ \int e^{-\gamma t} \left( P^e(t)h(t) - \nabla c h(t) - \frac{\partial c_R}{\partial \dot{G}} \dot{h}(t) \right) dt \right]
$$

$$
\frac{d}{d\alpha} W_s = E \left[ \int e^{-\gamma t} \left( P^e(t) - \nabla c + e^{\gamma t} \frac{d}{dt} \left( e^{-\gamma t} \frac{\partial c_R}{\partial \dot{G}} \right) \right) h(t) dt \right] - b.t.
$$

(5.64)

Hence,

$$
\frac{d}{d\alpha} W_s \bigg|_{\alpha=0} = 0 \Rightarrow P^e(t) = \nabla c \left( G^*(t) \right) - e^{\gamma t} \frac{d}{dt} \left( e^{-\gamma t} \frac{\partial c_R}{\partial \dot{G}} \right)
$$

(5.65)

plus additional boundary conditions depending on the end-points.

Taking expectations in (5.65), we obtain

$$
E \left[ \int e^{-\gamma t} P^e(t) dt \right] = E \left[ \int e^{-\gamma t} \nabla c \left( G^*(t) \right) dt \right] - E \left[ \int \frac{d}{dt} \left( e^{-\gamma t} \frac{\partial c_R}{\partial \dot{G}} \right) dt \right]
$$

$$
E \left[ \int e^{-\gamma t} P^e(t) dt \right] = E \left[ \int e^{-\gamma t} \nabla c \left( G^*(t) \right) dt \right] + E \left[ \frac{\partial c_R}{\partial \dot{G}} \bigg|_{t=0} \right]
$$

(5.66)

As in the deterministic case, for the free-end points case the expected price is exactly the expected marginal cost of capacity,

**Proposition 29** If initial condition is free, then expected price is equal to the expected marginal capacity cost,

$$
E \left[ \int e^{-\gamma t} P^e(t) dt \right] = E \left[ \int e^{-\gamma t} \nabla c \left( G^*(t) \right) dt \right]
$$

(5.67)

**Proof** The formula is proved by considering the additional transversality condition required for optimality,

$$
E \left[ \frac{\partial c_R}{\partial \dot{G}} \bigg|_{t=0} \right] = 0
$$
In the case in which the initial condition is fixed, it is possible to find an expression for the expected price in terms of the sensitivity to the initial condition.

**Proposition 30** If the initial condition is fixed, then expected price is equal to expected marginal cost plus a sensitivity term to the initial condition,

\[
E\left[\int e^{-\gamma t} P^e(t) \, dt\right] = E\left[\int e^{-\gamma t} \nabla c \left(G^*(t)\right) \, dt\right] + \nu^* \tag{5.68}
\]

In which the initial condition \(G_S(0) = g_0\) is captured in the Lagrange multiplier \(\nu^*\).

**Proof** Consider a Lagrangian relaxation of the suppliers problem, in which the constraint associated with the initial condition \(G_S(0) = g_0\) is captured in the Lagrange multiplier \(\nu\). For this we define the Lagrangian,

\[
\mathcal{L}_s(G_s, \nu) = E\left[\int e^{-\gamma t} W_S(t) \, dt\right] - \nu[G_S(0) - g_0] \tag{5.69}
\]

We perform a feasible perturbation on \(G_s(t)\) of the Lagrangian by using

\[
G^\alpha_s(t) = G_s(t) + \alpha
\]

\[
\dot{G}^\alpha_s(t) = \dot{G}_s(t) \tag{5.70}
\]

and we apply the local Lagrange multiplier theorem of [92]. If \((G^*)\) is a solution of the constrained problem, then

\[
\left.\frac{d}{d\alpha} \mathcal{L}_s(G_s^\alpha(t) + \alpha, \dot{G}_s^\alpha(t), t)\right|_{\alpha=0} = 0 \tag{5.71}
\]

Calculating \(\frac{d}{d\alpha} \mathcal{L}_s\), we obtain,

\[
\frac{d}{d\alpha} \mathcal{L}_s = E\left[\int e^{-\gamma t} \frac{d}{d\alpha} \left[P^e(t)(G_s(t) + \alpha) - c(G_s(t) + \alpha) - c_R(\dot{G}_s(t))\right] dt\right]
\]

\[
- \frac{d}{d\alpha} \nu^* [G_S(0) + \alpha - g_0]
\]

\[
\frac{d}{d\alpha} \mathcal{L}_s = E\left[\int e^{-\gamma t} \left[P^e(t) - \nabla c\right] dt\right] - \nu^* \tag{5.72}
\]
5.5 Numerical Illustrations

Following the same structure of Section 5.4, numerical illustrations for the deterministic and stochastic cases are presented. These numerical illustrations will help to visualize the impact of ramping costs on competitive prices. In the deterministic case, the impact of the ramping cost on the prices is the deviation from the marginal cost of capacity. The level of deviation will depend on both the ramping costs and the trajectory of supply. In the stochastic case, prices are going to be equal to the marginal value for consumers.

One of the most interesting features of these prices is that, in average terms, the impacts of ramping costs are mild or even negligible. Consequently, the validity of these prices to provide appropriate compensations to suppliers and to work as signals for investment becomes questionable.

5.5.1 Deterministic Case

The first experiment is in a deterministic context. We focus on the case in which demand is inelastic, and no reserves are deployed. It is also assumed that the end points are fixed. The family of demand trajectories is given by the quadratic expression,

\[ D(t) = -at^2 + aTt \]  \hspace{1cm} (5.74)

in which \( a \) is just a parameter used to modify the quadratic shape, and \( T \) is the time length. With this family of demand the price of electricity is given by

\[ P^e(t) = c + 2acR \]  \hspace{1cm} (5.75)

Two demand trajectories are compared to illustrate the impact of the shape on the prices, \( a = 1 \) and \( 6 \). The additional parameters used are \( c = 200 \) and
\( c_R = 50 \).

Figure 5.5: Demand and prices for the inelastic demand case, \( c = 200 \), \( c_R = 50 \).

The equilibrium prices illustrate the impacts of the additional ramping costs. Prices are no longer equal to the marginal cost of capacity since they are impacted by the trajectory of supply. The average price is the marginal cost of capacity plus some additional terms related to the boundary terms.

5.5.2 Stochastic Case

We now move into a stochastic setting in which uncertainty from demand is considered. For the purposes of these experiments, a simple cost structure is imposed:

**Constant marginal benefits for consumption of energy:** The benefit of consumption of energy is given by \( v \min(D, G) \).

**Constant marginal production cost:** \( c^g \) represents the marginal costs for any additional unit of capacity.

**Constant marginal loss-of-load value:** If there is excess demand for power, then loss-of-load occurs. The cost of excess demand is given by \( c^{bo} |r| \) when \( r < 0 \), where \( c^{bo} > 0 \).

In addition, a structure for ramping cost is imposed.

**Quadratic ramping cost:** The cost of ramping is modeled as a quadratic function of the instantaneous rate of change of the available capacity \( \frac{c_R}{2} \dot{G}^2(t) \)
Hence, the cost function has the following form:

\[ C(t) := c^g G(t) + (v + c^bo)R_-(t) + \frac{c_R}{2} \dot{G}^2(t), \quad t \geq 0. \]  

(5.76)

Using the definition of the reserve process (6.3), it is possible to rewrite the cost function (6.4) in terms of the reserve process.

\[
C(t) := c^g (R(t) + D(t)) + (v + c^bo)R_-(t) + \frac{c_R}{2} \dot{G}^2(t) \\
\quad c^g R(t) + (v + c^bo)R_-(t) + \frac{c_R}{2} \dot{G}^2(t) + c^g D(t) \\
(\dot{c}^bo + v - c^g)R_-(t) + c^g R_+(t) + \frac{c_R}{2} \dot{G}^2(t) + c^g D(t)
\]

Given the fact that \(D(t)\) is exogenous, for terms of optimization the terms \(c^g D(t)\) can be dropped off. Hence, the cost in terms of the reserve process can be written as

\[
C(t) := (\dot{c}^bo + v - c^g)R_-(t) + c^g R_+(t) + \frac{c_R}{2} \dot{G}^2(t), \quad t \geq 0
\]

(5.77)

in which \(R_-(t) = \max(-R(t), 0)\) and \(R_+(t) = \max(R(t), 0)\).

The reserve process (6.3) can be interpreted as a special case of a controlled random walk of the form:

\[ Q(t + 1) = Q(t) - U(t) + A(t + 1), \quad t \geq 0 \]  

(5.78)

Assuming that \(D(t + 1) = D(t) - A(t + 1)\), it is clear that the dynamics of the reserve process is given by

\[
R(t + 1) = R(t) + (G(t + 1) - G(t)) - (D(t + 1) - D(t)) \\
R(t + 1) = R(t) + U(t) + A(t + 1)
\]

in which \(U(t) := G(t + 1) - G(t)\). Using this definition for the control \(U(t)\), it is possible to write the cost (5.79) as a function of the state \(R(t)\) and the control \(U(t)\),

\[
C(x, u) := c(x) + \frac{c_R}{2} u^2, \quad t \geq 0
\]

(5.79)

in which \(c(x) = (\dot{c}^bo + v - c^g)x_- + c^g x_+\). This cost function has similarities to the cost function of speed scaling applications [90], in which the cost
considers the trade-offs between delay of service and the power consumption associated with the control $u$.

Several expected values are computed using standard Monte Carlo estimators. First, we focus on the quantification of expected prices. The state and action spaces are discretized. The optimal policy is numerically obtained using a standard value iteration algorithm (VIA) [22]. The price functional for a particular sample path is given by

$$P^e(t) = (v + c^{bo})I(R(t) < 0) + (c - c_R\tilde{G})I(R(t) = 0)$$

In these experiments, the parameters are given by $c^{bo} = 3000$, $c = v = 50$ and $c_R = 500$. The state space is given by $\{−480\Delta, ..., 0, ..., 480\Delta\}$, in which $\Delta = 1/24$. The demand process is given by a symmetric distribution around zero. Such a distribution is constructed by replicating a standard geometric distribution with parameter 0.93 on the negative demand side. Sample paths for generation, demand, reserves and prices are presented in Figs. 5.6, 5.7 and 5.8.

In order to find an estimator of the expected price, $10^3$ sample paths of $10^4$ time steps were considered. The expected price in this case is given by

$$E[P^e(t)] = 45.3618$$

The next experiment aims to verify that the expected price is mildly or not impacted at all by ramping costs. Several simulations are performed, quan-
identifying the expected prices for different values of the ramping cost parameter \( c_R = [0, \ldots, 1000] \), and two values of the capacity marginal cost \( c = 50, 100 \). The results are illustrated in Fig. 5.9.

Figure 5.7: Sample paths for reserves.

Figure 5.8: Sample paths for prices.

Figure 5.9: Prices and ramping costs for different values of \( c \).
As expected from the analytical results of the previous section, the expected prices do not reflect the impact of ramping costs: expected prices are usually close to the marginal cost of capacity \( c \).

Finally, quantification of the impact of ramping costs in suppliers welfare is investigated. Mean values of the supplier welfare,

\[
W_s(t) := P(t)G(t) - c(G(t)) - c_R(\dot{G}(t))
\]  

are numerically evaluated for different values of the ramping cost parameter \( c_R = [0, ..., 5000] \). The capacity marginal cost is set at \( c = 50 \). The results are shown in Fig. 5.10. In general, for higher ramping costs the supplier welfare decreases. This result is coherent with the fact that prices, do not properly capture the additional ramping costs.

![Figure 5.10: Supplier welfare and ramping costs.](image)

The main message of these results is the fact that under spot pricing of electricity, even if ramping costs are considered, suppliers with high ramping costs might not be properly compensated. This is clear when the relative welfare respect to the case in which \( c_R = 0 \) is analyzed. The numerical results show that the expected welfare for higher values of \( c_R \) is lower than the welfare when \( c_R \) is zero. Hence, a natural question is: Why would suppliers with ramping costs want to participate in these markets? Suppliers participating only in the real-time markets, as usually peaking units do, will have little incentive to stay. These results demonstrate the need to investigate alternative compensation schemes once the real costs of generation along with dynamics, uncertainty and shared constraints are considered.
5.6 Summary

The use of spot markets, even with the explicit consideration of ramping costs, does not guarantee the creation of incentives for investments on appropriate technologies. This conclusion is apparent from the current state of the power grid, and is reflected in the analysis of the competitive equilibrium model introduced in this chapter: First, in average terms, competitive prices do not properly capture the impact of ramping costs. Second, once prices for power and ramping are considered, there is a lack of uniqueness on the competitive equilibrium. These analytical results were illustrated through simulation. Under several scenarios, it is shown that expected prices are mainly related to marginal capacity costs. Similarly, once the supplier welfare is quantified, taking into account the cost of ramping, it is possible to appreciate the negative impact of ramping costs.

More realistic consideration of the costs and benefits associated with power generation may prove to be the appropriate way to incentivize needed resources. However, we challenge the widely adopted paradigm of using spot market schemes, and in particular, real-time prices as a foundation for market design. The grid needs reliable resources for energy and for ramping service, and innovation that will bring better services that will allow greater adoption of renewable energy. Generators need a reliable source of revenue, and signals that inform them how to best invest in new technology to increase their revenue. Current research focuses on contract design as a means to satisfy these goals.
CHAPTER 6

DIFFERENTIATED ELECTRIC POWER PRODUCTS

In this chapter, elements of an alternative view for organizing the markets in a Smart Grid scenario are discussed. The key element is the consideration of electricity as a multi-attribute product associated to contractual arrangements. The specification of these attributes will first require a clear understanding of the social planner’s problem (SPP), as defined in standard economics. Such understanding will facilitate the valuation of the resource attributes with respect to the fulfillment of societal objectives. The emphasis of the chapter is investigating on the SPP for resource allocation, getting resources appropriately sized in advance. The solution of this problem offers insight, and provides sensible guidelines for how many resources, and of which type, are required. These viewpoints are illustrated via a detailed investigation of the system-wide value of operational flexibility, along with the market implications. A general discussion about the role of contracts to support multi-attribute products is also included. This chapter is based on [89].

6.1 Introduction

It is recognized that one of the main challenges of designing electricity markets is the coexistence of two coupled dynamical systems: a physical system driven by hard and soft engineering constraints, and a market/financial system driven mainly by the self-interests of players. The differences are also evident in the views and metrics of the two main professional communities: engineers usually care about reliability, while economists care about market efficiency.

After three decades of deploying markets in electricity and recognizing some positive outcomes, there are still deep unsolved issues and open ques-
tions. Well known unresolved issues surround basic questions such as system reliability [4], high tariffs [5], and the failure of spot prices as investment signals [6]. While innovation, increased efficiency in running companies, and the development of new technologies are usually proclaimed as positive consequences of the restructuring of the electricity industry [8], many authors still argue about the real achievements of the process [5, 43, 44]. Without proper design, many of the current unsolved issues in the market arena could be magnified by the deployment of Smart Grid technologies [3, 93].

Many of the current issues in electricity markets are a consequence of the interference between the physical and the financial/market systems of electricity markets. A common diagnosis of many electricity markets around the world is the assertion that many design elements are intended to achieve financial/market objectives but need to be modified to attain physical objectives [12, 13].

The seminal work on spot pricing of electricity by Schweppe et al. [14] provided the theoretical framework to develop and implement the idea of markets for electricity. The underlying starting point is the idea of treating electricity as a commodity, in terms of MWhs, without qualitative distinctions. As a result of this commoditization of electricity, several market structures from other commodity markets —financial derivatives, forward contracts — started to be replicated in electricity. These structures typically depend on the implementation of a spot market in which “delivered power” is the essential product for sale.

In our view, one step required for conciliating the physical and the market systems is to start considering electricity not as a simple commodity such as oil or copper, but as a product offering more services than plain energy. In other words, electricity should be seen as a multi-attribute product in which energy is just one dimension. Nuclear power, for example, provides steady power with little ramping capability; wind and solar power provide environmentally friendly power but little control; gas turbines provide a lot of flexibility and controllability that is required to compensate for the inherent volatility of power supply and demand. The Smart Grid will provide the technological conditions for even more service differentiation. The success of the whole new paradigm will depend critically on the design of appropriate market structures that take into account this reality.

It is often claimed that real-time pricing schemes are a necessary compo-
nent for the new market structures in a Smart Grid setting. There is an over-riding belief in both industry and academia that an intelligent grid can only be realized with real-time pricing: fluctuations in energy prices for consumers based on the current strain on the grid. The vision held by many economists [94, 95] is that customers will be exposed to high prices when there is a shortage of electricity. If prices are too high, the theory says, they will postpone their laundry or dim their lights. The market and consumers are expected to adapt and evolve towards an economic ideal.

In reality, the notion that residential consumers would accept having this volatility passed on to them is very implausible: exposing consumers to the price fluctuations of current energy markets could cause public rejection of the exciting current trend towards a sustainable energy future. Indeed, recent research has exposed the dangers of real-time pricing: Real-time pricing can lead to uncertainty along with high volatility of prices [12, 31, 74]. Additionally, the review of a few small-scale tests reported in [96] shows that the use of prices to control the grid has not been successful. We will avoid costly mistakes and public backlash only by looking more closely at the needs of the grid, the public, and the planet.

In this chapter, we provide elements of an alternative view for organizing the markets in a Smart Grid scenario. Our starting point is to treat electricity not as a simple commodity. In that line, we discuss the importance of taking into account the true cost of power generation, which depends not just on the total instantaneous power delivered but also on the shape of the generation trajectory. In our view, a natural way to accommodate multi-attribute products is through the use of contracts for different products and services. The use of contracts will lead to a more predictable system. Contracts will also simplify the decision making by ISOs, generators, and utility companies. The central authority will have greater certainty about what resources will be available in the future to satisfy their needs, and suppliers will have long-term signals that will enhance investment in new services and technologies. In addition, we believe that prices will be less volatile when carefully constructed contracts replace today’s real-time markets.

The specification of multi-attribute products will first require a clear understanding of the social planner’s problem (SPP), as defined in standard economics theory [93]. Such understanding will facilitate the valuation of the resource attributes with respect to the fulfillment of societal objectives.
such as reliability, sustainability and efficiency.

We set out to illustrate this approach to design an SPP that is tractable and captures key dynamical issues. In this chapter we focus on a single pair of generator attributes: the cost of generation and the ramping capability of generation. We investigate the value that investing in operational flexibility, in terms of ramping rates, has for the system. Once we understand the SPP and the system-wide impact of ramping rates, the value of securing contracts for various resources will be clearer. This chapter illustrates the construction of supply and demand functions for ramping rates that can be used to construct simple market structures for these services.

The rest of the chapter is organized as follows. Section 6.2 contains a survey of recent cases of chaotic prices around the world which illustrate important issues of spot markets for electricity. In addition, an overview of the use of contracts in current electricity markets is presented. In Section 6.3, the importance of having appropriate models representing the physical reality to capture the different attributes provided by electricity suppliers is discussed. In addition, the evaluation of flexibility, in terms of ramping rates, is discussed. Several numerical illustrations are presented. Our findings illustrate the need for having products and markets associated with ramping rates. The rationale and challenges for a new architecture are discussed in Section 6.4. The chapter concludes with a summary in Section 6.5.

6.2 The Grid Today

6.2.1 The Lens of Perfect Competition

The current way of evaluating markets is through the lens of perfect competition. This presumes symmetric information and lack of market power. A few examples from the past year will serve to assess if these assumptions hold true. Additionally, these examples will illustrate the amount of volatility that the current market system is exposed to and why it is likely that volatility and dysfunction will not go away without new ways of thinking about markets.

During periods of calm, prices in both real-time and day-ahead markets hover between $10 and $60 per megawatt hour (MWh) in the United States,
New Zealand, Australia, and Canada. But it is not unusual to see huge deviations, or even prices that are negative. A clear example happened in New Zealand is shown in Fig. 6.1.

Figure 6.1: New Zealand prices.

In a typical hour from the spring of 2011, a price map of energy in the midwestern United States shows that prices spanned from well over $200/MWh in parts of Missouri to below negative $20 /MWh in parts of Minnesota. This happens in large part because of the unpredictable intermittency of renewable energy. Imagine that strong winds hit the turbines in Minnesota, flooding the system with effectively free energy. A coal or nuclear generator in the region cannot afford to shut off during a transient period of low demand and turn back on later; the process is too slow and expensive. Instead many generators stay on, hoping that the windy weather will end or that demand will surge from the cold. They are willing to pay what is essentially a fine to the utilities for the resulting strain they place on the grid — injecting excess power when it is not wanted.

Equally bizarre are extreme price spikes. In Texas, real-time prices for power exceeded $2,000 for several hours one day in February 2011 during a cold snap that actually froze some power-generating equipment; and during an August heat wave, day-ahead prices were over ten times the norm for half of the days of the month. It is arguably in the best interest of generators to not worry about 10% of their equipment failing (or being scheduled for repairs) during periods of high demand, since the resulting spike in energy price more than makes up for the losses incurred by the malfunctioning ge-
generators — often by a factor of ten. A 2011 study by the non-profit Texas Coalition for Affordable Power claims that electric deregulation — allowing the generators to operate under the rules of a free market — has cost Texas residential consumers more than billions in higher rates since deregulation began there in 1999. In New Zealand on 26 March 2011, power prices reached $20,000 per MWh for several hours, caused by repairs on several generators in the middle of the day. The result was a NZ$25 million dollar windfall for generation companies over a six hour trading period.

All these chaotic behaviors are the result of the interference between the physical and the market systems associated with electricity markets [12, 31, 74]. Power markets have been designed through the lens of perfect competition, where efficiency is the goal. In addition, the fundamental assumptions include lack of market power and symmetry of information. Based on the previous examples, consider these two questions:

- Is there market power? Yes - generators in Texas and New Zealand know that withholding power can cause an enormous jump in prices.

- Is information symmetric? Of course not! Do utility companies know the intent behind maintenance schedules in Texas? Does the New Zealand electricity authority know if the generators deliberately withheld power?

Recall the two fundamental assumptions of competitive equilibria theory: market power does not exist, and information is completely symmetric among the market participants. In the case of electricity markets operating in real time, these assumptions are absolutely false, and there is no reason to see significant improvements by market evolution alone.

In addition, immersed in the analysis and design of electricity markets is the concept of marginal cost: When static market models are considered through the lens of competitive equilibria theory, economists conclude that prices should coincide with marginal cost.

Consider how you might define this concept – you might ask an engineer. For example, the operator of a coal-fired generator would respond, *this depends on how the generator is operated.* A steady stream of power will be cheap. If the generator is forced to ramp up and down, then the cost will be far greater. Concepts surrounding the term “marginal cost” are the founda-
tion of every economist’s education, yet this term is virtually undefinable in the context of power generation.

We strongly believe that many of the current unsolved issues in the market arena along with the crazy prices observed today could be magnified by many of the smart grid initiatives, such as incentives for introduction of renewables without commensurate incentives for operating reserves; FERC mandates for pricing demand response (DR) resources, without careful rules on how the DR is delivered; and smart meters to homes, without a science to support their use.

6.2.2 Contracts Today

Contracts have been a subject of extensive research in commodity markets. In particular, some results show that under certain settings, contracts might reduce market power in oligopolistic spot markets [20]. Considerable research has been performed on the role of contracts to hedge against uncertainty in spot markets [47]. In addition, some authors argue that contracts are needed as the result of transactions costs [97].

In the particular case of electricity, research on the value of contracts is staged in the context of market efficiency, and even in this limited scope there is no clear agreement about the benefits of contracts. For example, some authors have found that long-term contracts might reduce market power [48]. Others argue that contracts could enhance market power [49]. Both viewpoints have merit, since no market model can capture all the complexities and underlying issues.

In terms of research frontiers, major goals are to broaden the analysis to include metrics beyond efficiency, to understand the role of contracts to achieve other objectives such as incentivizing and securing reliable, responsive and even green energy supply. In the case of resource adequacy, there exists empirical evidence of the use of contracts to achieve some of these objectives. For example, the auction processes held in Brazil, Chile and Illinois illustrate the use of forward contracts with the aim of facilitating resource adequacy [6, 38–40].

The design of contracts for attributes beyond energy will require the understanding of the value that such attributes can have for the system. Achieving
that will require the use of appropriate models, in particular models able to handle uncertainty and dynamics, and provide insight on market outcomes. Realistic models of cost/value are also required – long-term and short, including the value of reliability.

6.3 The Value of Operational Flexibility

Understanding the value of attributes is the first step towards the creation of appropriate products (or services) and contracts associated with them. We describe the role of the social planner’s problem in designing and evaluating markets and capturing attributes value. We illustrate our viewpoints by considering the value of operational flexibility in terms of ramping rates.

6.3.1 Social Planner’s Problem — Capturing System Needs

In order to capture the value of attributes for the system, consumers, and suppliers, it is necessary to construct an appropriate social planner’s problem (SPP) as understood in standard economic theory [16]. An effective SPP must describe appropriately the physical reality of the energy system, especially when the available resources is much more diverse than only standard generating units, as illustrated in Fig. 6.2. In that line, the consideration of operational issues into the planner’s problem becomes key.

However, the big challenge is how to capture key issues and constraints without constructing intractable and unsolvable models. In that line, the use of stochastic differential equation (SDE) models as in [30,98] is our starting point. In addition to tractable models for representing the physical reality, key components of the SPP are the welfare functions of the players. Capturing the value of different attributes is the key step to design proper contracts and market structures.

In previous work [31,33] the welfare functions of the supplier and consumer were taken to be of the simple piecewise linear form,

\[
W_D(t) := v \min(D(t), G(t)) - c^R_\text{bo} R_\text{bo}(t) - P(t)G(t) \\
W_S(t) := P(t)G(t) - cG(t) \tag{6.1}
\]
For the consumer, there is unit value $v$ for consumption, $c_{bo}$ for loss-of-load, and $P(t)G(t)$ is the price paid for power. The supplier receives this payment, but the cost of production is $cG(t)$. In this model, the attribute MWh has an important, explicit and instantaneous value for the system. This model ignores the dynamical considerations surveyed in Section 6.2. In particular, the cost of ramping is ignored in this previous work.

In this section the cost of ramping is modeled in a long-term setting: We consider a continuum model in which more responsive generation can be procured, but this comes with a cost for more sophisticated technology. To understand the value of more responsive generation sensitivities are computed, such as

$$
\frac{d E[W_s(t)]}{d \zeta}
$$

(6.2)

where $\zeta$ is the system ramp-rate limit. In this analysis the fine-details of the cost of generation are not considered.

6.3.2 Model

In order to capture the value of operational flexibility, we start with a queuing model introduced in [30] for power generation. For the sake of simplicity, we consider the case without transmission constraints and with only 2 technologies — primary and ancillary services — illustrated in Fig. 6.3. Given
our aim of finding the value of flexibility, we focus mainly on the real-time operation of the grid. Our main interest is supplying of the deviation of the expected demand. The residual demand process, in which the mean demand is subtracted, is denoted $D = \{D(t) : t \geq 0\}$. In power systems terms, $D(t)$ corresponds to the difference between the day-ahead expected demand and the actual demand at time $t$ over a 24-hour period.

Primary generators correspond to generating units providing the bulk of the required power. Examples are large nuclear units or coal units, with large investment costs and small operational costs. In contrast, ancillary generators correspond to more flexible generators or peaking units such as gas turbines, with lower investment cost but often higher operational cost.

The capacity of a generator is the maximum amount of power that can be extracted from the generator at a given time. The respective capacities of the primary and ancillary generators at time $t$ are denoted by $G^p(t)$ and $G^a(t)$. The reserve process at time $t$ is defined by

$$R(t) = G^p(t) + G^a(t) - D(t), \quad t \geq 0.$$  

We say that loss-of-load occurs at time $t$ if $R(t) < 0$.

Capacity is subject to strict ramp-rate limits: For constants $\zeta^p, \zeta^a$, and for each $0 \leq t_0 < t_1 < \infty$,

$$\frac{G^p(t_1) - G^p(t_0)}{t_1 - t_0} \leq \zeta^p, \quad \frac{G^a(t_1) - G^a(t_0)}{t_1 - t_0} \leq \zeta^a.$$

The lower rate constraints on the generators are relaxed.

We model deviation of the demand process $D$ as a zero-mean Brownian motion with instantaneous variance $\sigma_D^2 > 0$. In this way we can apply techniques from [22] to compute quantities such as average cost. Both analytical and simulation results contained in [22] suggest that our conclusions do not
change dramatically with the precise distribution of demand.

For the purposes of computation, a simple cost structure is imposed:

**Constant marginal benefits for consumption of energy:** The benefit of consumption of energy is given by \( v \min(D, G^p(t) + G^a(t)) \).

**Constant marginal production cost for primary and ancillary service:** \((c^p, c^a)\) represents the marginal costs for the primary and the ancillary services.

**Constant marginal loss-of-load value:** If there is excess demand for power, then loss-of-load occurs. The cost of excess demand is given by \( c^{bo}|r| \) when \( r < 0 \), where \( c^{bo} > 0 \).

The cost function on the state-space \((G^p, G^a, R)\) has the form

\[
C(t) := c^p G^p(t) + c^a G^a(t) + (v + c^{bo}) R(t), \quad t \geq 0.
\]  

(6.4)

In addition, it is assumed that \( c^{bo} \gg c^a > c^p \), and \( \zeta^a > \zeta^p \). The higher cost of ancillary service is justified by its more flexible ramping capability.

### 6.3.3 Optimization Problem

The basic optimal control problem, assuming that resources become available at the beginning of the time horizon and considering average cost metrics, becomes

\[
\min_{m, E_D, E_s, R_s} \left\{ i(m) + \limsup_{T \to \infty} \mathbb{E}_{\mathcal{Z}} \left[ \frac{1}{T} \int_0^T C(t) \, dt \right] \right\},
\]

(6.5)

The decision variables include \( m \) representing investments decisions and \( E_D, E_s, R_s \) the scheduling of energy and reserves. In the general case, the problem is subject to operational/physical constraints, network constraints, and energy-balance constraints. Transmission constraints are not considered here.

### 6.3.4 Operational Costs

The operational costs are obtained using results from [30]. The optimal policy is affine under either the discounted or average-cost criterion. A sample path is shown in Fig. 6.4. In particular, the average-cost optimal policy is affine
Figure 6.4: Sample path of the supply capacity.

The steady state cost is given by

\[ \phi(\bar{r}) = \gamma_0^{-1} \left( \frac{\zeta^{p+}}{\bar{r}^{p+}} c^a + e^{-\gamma_0 \bar{r}^{p+}} (v + c^{bo}) \right) e^{-\gamma_1 (p^{p+} - p^a)} + (\bar{r}^{p+} - \gamma_1^{-1}) c^p \]  

(6.7)

6.3.5 Investment Costs

Piecewise linear investment costs are assumed for the ramping capabilities of resources. These are constructed based on the assumption that primary/ancillary technologies have high/low investment costs, and low/high operational costs.

A convex relaxation of the inherent integer constraints associated to investment decisions is used for calculations: The investment costs for \(\zeta^+_{p,a}\) ‘units’ of ramping capabilities of primary/ancillary service are given by

\[ i(\zeta^{p+}_{p,a}) = \alpha_1^{p,a} \zeta^{p+}_{p,a} \mathcal{H}(0, b^p) + \left( \alpha_1^{p,a} b^p + \alpha_2^{p,a} (\zeta^{p+}_{p,a} - b^p) \right) \mathcal{H}(b^p, b^p) + \left( \alpha_1^{p,a} b^p + \alpha_2^{p,a} (b^p - b^p) + \alpha_3^{p,a} (\zeta^{p+}_{p,a} - b^p) \right) \mathcal{H}(b^p, b^p) \]

in which \(\mathcal{H}(u, v) = [H(\zeta^{p+}_{p,a} - u) - H(\zeta^{p+}_{p,a} - v)],\) where \(H(\cdot)\) is the Heaviside
function and $\alpha_{1,2,3}^{p,a}$ and $b_{1,2,3}^{p,a}$ are constants.

### 6.3.6 Economic Value of Flexibility

In order to quantify the economic value of flexibility, we follow a approach similar to [99] for the case of valuing reserves. This involves the construction and analysis of an equivalent cost $C$ in terms of the expected costs and the risk preferences. Here we take

$$C_T = \mu_{C_T} + k\sigma_{C_T}$$

(6.8)

in which $\mu_{C_T}$ and $\sigma_{C_T}$ correspond to the mean and standard deviation of the total cost, and $k$ is a constant that capture the risk preference of the firm. The system-wide value of ramping rates is defined as the sensitivity,

$$V_T(\zeta_{p,a}^+) := -\frac{dC_T}{d\zeta_{p,a}^+}$$

(6.9)

The results here are restricted to the risk-neutral case in which $k = 0$.

Given these elements, supply and demand functions can be constructed that define a clearing price for a hypothetical market associated with ramping rates. Such supply and demand functions are quantified by evaluating marginal cost and marginal utility. Cost/utility functions for suppliers/consumers are given by,

$$C_S(t) := c^p G^p(t) + c^a G^a(t) + i(m_p) + i(m_a)$$

$$U_D(t) := v \min(D(t), G^p(t) + G^a(t)) - c^{bo} R_-(t)$$

(6.10)

Hence, the marginal utility function is

$$V_D(\zeta_{p,a}^+) := \frac{dE[U_D(t)]}{d\zeta_{p,a}^+}$$

(6.11)

and the marginal cost is

$$V_S(\zeta_{p,a}^+) := \frac{dE[C_S(t)]}{d\zeta_{p,a}^+}$$

(6.12)

The equilibrium price associated with a particular ramping rates mix will be
given by the intersection of these demand and supply functions. A numerical example is given in the next section.

6.3.7 Numerical Experiments

The optimal value of ramping rates in the SPP problem, and the supply and demand functions, are computed here for these parameters:

\[
\begin{align*}
    c^{bo} &= 10^5, \quad c_a = 500, \quad c_p = 20, \quad \sigma_D = 250, \\
    \alpha^p_1 &= 2 \times 10^5, \quad \alpha^p_2 = 1.2\alpha^p_1, \quad \alpha^p_3 = 1.5\alpha^p_1, \\
    \alpha^a_1 &= 2 \times 10^4, \quad \alpha^a_2 = 1.2\alpha^a_1, \quad \alpha^a_3 = 1.5\alpha^a_1, \\
    b^p_{1,2,3} &= 2, 5, 12, \quad b^a_{1,2,3} = 3, 15, 40
\end{align*}
\]

![Figure 6.5: System operational cost and ancillary ramping rate sensitivity.](image)

The total cost (6.8) can be expressed as

\[
C_T = \phi(\bar{r}) + i(\zeta^+_p) + i(\zeta^+_a)
\]

For these parameters, its minimum cost is given by

\[
\zeta^+_p = 3.1 \quad \zeta^+_a = 8.6
\]

It is possible to appreciate in Fig. 6.5 that system operational costs decreased with more system flexibility, reflecting the fact that the more flexible the system, the lesser loss-of-load situations. In Fig. 6.6 the total cost is
shown. Beyond $\zeta^{+} = 8.6$, the system operational benefits are eclipsed by the investment costs.

\[ C_T = 1.83 \times 10^8 \]

Figure 6.6: Total system cost as a function of total ramp-rate $\zeta^{+}$.

The demand and supply functions are shown in Fig. 6.7. The two curves intersect, as expected, at the optimal value $\zeta^{+} = 8.6$. The clearing price of this hypothetical market associated to ramping rates is approximately 5000 $/MW.

\[ C_S(t) := c_p G_p(t) + c_a G_a(t) + i(p) + i(a) \]
\[ U_D(t) := v_{\min}(D(t),G_p(t) + G_a(t)) \]

\[ V_S(\zeta^{+}_{p,a}) := \frac{dE[C_S(t)]}{d\zeta^{+}_{p,a}} \]
\[ V_D(\zeta^{+}_{p,a}) := \frac{dE[U_D(t)]}{d\zeta^{+}_{p,a}} \]

$\zeta^{+}_{p,a} = 8.6$

Figure 6.7: Clearing the ramping market.

6.4 Toward a New Architecture

The solution of the SPP problem provides insights with respect to system needs and attribute values. The next task is to design appropriate market structures to procure resources to meet these needs. The success will depend mainly on the creation of appropriate incentives for both suppliers and consumers.
There are several reasons to make greater use of contracts in electricity markets:

1. Contracts can easily incorporate the multi-attribute nature of both power generation and demand response

2. Suppliers will have long-term signals that will enhance investment in new services and technologies

3. The decision-making processes of ISOs, generators, and utility companies will be simplified. Complexity of resource allocation on short time-scales is simplified when resources are secured in advance through contracts. In particular, the on-going unit commitment problem solved every five minutes in today’s real-time market can be eliminated or simplified.

4. Contracts can lead to a more predictable system — similar to the use of lanes and speed limits in highways that reduce the range of possible behavior by market participants.

5. For similarly reasons, prices will be less volatile when carefully constructed markets for contracts replace today’s real-time markets.

Certainly, the extensive use of contracts could create a loss of efficiency, since this amounts to regulation of the behavior of market participants. This is similar to the “inefficiencies” seen in today’s highways, that we all know is necessary to enhance reliability. Following the highway analogy, some of these contracts will be handed down by government, and will be enforced through regulatory frameworks.

Extensive research is required in this exciting topic. In particular, changing the commodity paradigm will require questioning the need for real-time markets, as well as significant changes to current day-ahead markets. The design of contracts must take into account potential monopolization and market power, and built-in mechanisms for evolution of contract terms in response to a rapidly evolving grid. These are just a few critical avenues for future research.
6.5 Summary

The main message of this chapter is the importance of organizing electricity markets around differentiated electric power products; the treatment of electricity as a commodity does not incentivize the kinds of services needed in the power grid. New products defined in terms of contracts are an alternative to organize these markets. To be successful, it is critical to appropriately capture the value of attributes such as instantaneous energy, ramping capabilities, and environmental impacts. Therefore, design and analysis of markets cannot be based on a snapshot model; it must include the dynamics, uncertainty and complexity of the power grid, just as these features are considered in any engineering analysis. To obtain meaningful estimates of the value of attributes, we require a better understanding of system costs, utility to consumers, and the long-term needs of society.
This final chapter contains a summary of the main results and ideas developed in this work. Moreover, future research avenues are discussed.

7.1 Summary

The term Entropic Grid is coined in this thesis to characterize a grid with increased levels of uncertainty and dynamics. Those changes are being driven by several Smart Grid initiatives. In this work, it is argued that proper design is mandatory to fully achieve the potential that the Smart Grid could provide. Without proper design, the interference between the physical and the market/financial systems, which is recognized in today’s electricity markets, could be magnified.

Achieving this objective will require the reconsideration of well-established paradigms in the way of planning and operating the grid and its associated markets. A proper scaffolding of models and tools, complementing successful power and energy methodologies, will be the vehicle to make the transition from the Entropic to the Smart Grid.

A central contribution of this thesis is to demonstrate the key role of product definition in electricity markets. Market designs based on appropriate definitions of the various products will not create interference between the physical and the market/financial systems. There was a lack of research in this area, mainly because in standard commodity markets there is no need to define products. In this thesis, concrete contributions to start filling the gaps are discussed. Several recommendations to define appropriate products in electricity markets are presented in Chapter 3. This point of view is illustrated with in-depth analyses of several prototype models.

A key requirement to evaluate market designs is to understand the behav-
ior of the physical system. Such understanding requires the developing of appropriate models able to capture and handle the salient characteristics of the entropic grid—uncertainty and dynamics. In order to achieve this, dynamic electricity market models tailored to the physical reality are presented in Chapters 3, 4 and 5.

The investigation of the impact of volatility and uncertainty of renewable sources of energy is one of the areas in which dynamic market models for the purpose of market analysis are constructed. With this purpose, in Chapter 4 a multi-settlement dynamic electricity market model—considering day-ahead and real-time markets—is introduced. Analysis is conducted in an idealized competitive equilibrium setting that incorporates both dynamics and uncertainty. Closed form expressions are obtained for the supplier and consumer surpluses in this stochastic model. These expressions allow one to quantify the impact that volatility and uncertainty can have on market outcomes. The results show the need to implement policies and technologies able to cope with the volatility of renewable sources. In addition, the results also show the need to update the market structures to properly incentivize the deployment of renewable sources.

Motivated by the need to consider the real costs and benefits of power, the idea of using functionals rather than functions is presented in Chapter 5. A dynamic market model with the consideration of ramping costs is implemented. A complete characterization of the competitive prices, under a broad range of settings, is presented and several conclusions are obtained. First, in average terms, competitive prices do not properly capture the impact of ramping costs. Second, once prices for power and ramping are considered, there is a lack of uniqueness on the competitive equilibrium. These analytical results are illustrated through several numerical simulations. More realistic consideration of the costs and benefits associated with power generation may prove to be the appropriate way to incentivize needed resources. However, based on these results, the widely adopted idea of using spot market schemes, and in particular, real-time prices as a foundation for market design, must be reconsidered.

General elements of an alternative architecture to organize markets are discussed in Chapter 6. These steps can be understood as a natural consequence of the results developed in this thesis. Hence, the key ideas are summarized in understanding the social planner’s problem in a dynamic and
uncertain setting, the definition of products based on the value that different technologies can offer and the implementation of contractual agreements. These steps are illustrated via an investigation of the system-wide value of operational flexibility.

Exciting and challenging times are being faced in power and energy systems. The ultimate realization of many of the Smart Grid expectations will depend on intelligent design. It is hoped that the contributions of this dissertation will help to engineer a more reliable power grid.

7.2 Future Work

The results of this dissertation reveal many exciting open research frontiers.

7.2.1 Market Design

In terms of electricity markets, the notion of treating electricity as a service rather than a commodity opens many possibilities for research in the development of new market designs. In particular, designing markets at the distribution level due to the fact that many Smart Grid initiatives are expected to be deployed precisely at that level. Also in terms of market design, a natural extension to the market models presented in this work is the consideration of strategic issues. The economic models in this work assume a competitive equilibrium setting, disregarding strategic issues. Even in that idealized setting, as the results of this work show, the outcomes are not the best ones: volatile prices, few incentives for deploying renewable sources and inappropriate capture of key costs. New market designs will have to deal with strategic and market power issues. This is of particular interest if market designs constructed around the notion of contractual arrangements are implemented.

7.2.2 Reliability in Power and Energy Systems

In an entropic grid setting the well-known notions and ways to quantify reliability should be complemented with new ideas and methodologies. The difficulty in achieving reliability guarantees is due in part to the breadth
of the meaning of the term “reliability”. It is recognized that reliability is dependent on *time scales*: The usual time scale dependence in reliability analysis of power systems is in terms of adequacy and operational reliability, defined respectively on long- and short-time horizons. Similarly, reliability has a *locational* dimension: A black-out in the Boston area has more impact than a black-out in New Salem. In addition, reliability depends upon both the frequency and the magnitude of disruption. For these reasons, system reliability cannot be summarized in a single scalar quantity, and for example, the usual procedure of planning energy systems for achieving specific reliability targets, i.e., LOLP of 1 day in 10 years, becomes hard to justify. The several dimensions of reliability can be separated into two broad classes: The first is a measure of the frequency of *rare-events*, such as the LOLP emphasized by today’s practitioners. The second is *average-cost* metrics, where the ‘cost’ arises from various sources, and these costs are *persistent* rather than rare. Understanding and creating appropriate metrics for reliability in this new setting is an exciting future research area.

### 7.2.3 Models for Planning

In terms of planning models, an extensive area of research is the development of long-term resource planning models using ideas, tools and methodologies from decision & control, and simulation & learning disciplines [21–23]. The proposed models will be able to capture operational issues and the underlying dynamics and uncertainty of the grid of the future, and convey this information to longer time scales. Techniques from decision & control such as aggregate models and workload relaxations lead to tractable models that can be approximately solved [22], in particular, using similar approaches to [22,30,98,100] in which a fluid-model based on first-order statistics is constructed and effective policies are constructed based on a lower dimensional relaxation. The impact of variability on the optimal control solution is investigated by introducing variability in the relaxation. These models could be used to complement the models developed in this work and get a better understanding of the impact of dynamics and uncertainty along with the value of flexibility of the generating resources and other technological attributes.
CHAPTER 8

REFERENCES


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Control; held jointly with the 2009 28th Chinese Control Conference, 2009, pp. 3575–3580.


