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INFLUENCE OF POLYPHASE MOTORS ON THE VOLTAGE REGULATION OF CIRCUITS SUPPLYING SINGLE-PHASE WELDER LOADS

BY

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INFLUENCE OF POLYPHASE MOTORS ON THE VOLTAGE REGULATION OF CIRCUITS SUPPLYING SINGLE-PHASE WELDER LOADS

A REPORT OF AN INVESTIGATION conducted by
THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
in cooperation with
THE UTILITIES RESEARCH COMMISSION

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1. Preliminary Statement

During the past several years there has been a marked increase in the use of single-phase resistance welders. New applications of resistance welding are being developed, and it is expected that the applications will continue to increase in the future. This increased use of single-phase resistance welders has added to the voltage regulation problems which confront the utility engineer.

The resistance welding machine requires for its operation a high kva demand at a low power factor. The welder load is applied at frequent intervals and for relatively short periods. This changing load causes voltage changes on the power system adjacent to the welder. If these variations in voltage are severe enough, they can cause objectionable flicker in the lighting of other customers served from the same portion of the power system. There are two types of light flicker caused by a single-phase resistance welder: a non-cyclic flicker from single impulse machines such as spot, butt, flash and projection welders and a cyclic flicker from machines such as seam welders.

Light flicker is much more noticeable or objectionable at some frequencies of fluctuation than at others. Abrupt voltage changes recurring several times per minute (typical of the welding rate of numerous welding applications) must generally be limited to 1.0 - 1.5 v on a 120-v base if objectionable lighting flicker is to be avoided. Voltage changes at which cyclic flicker becomes objectionable are generally lower than for non-cyclic changes and depend on the duration and frequency of the disturbance. The most critical frequency is at about six fluctuations per second. At this frequency, the maximum allowable voltage variation is about 0.5 v.

Since such variations are objectionable, the power company engineer must be able to determine accurately the variation in voltage in order to provide service at the lowest cost. In general, these problems have been handled satisfactorily, with considerable ingenuity in devising schemes to keep the cost of service reasonable. However, in
some cases, voltage drops measured after installation of the welder were appreciably lower than had been calculated. Many power company engineers reasoned that this was caused by motor loads adjacent to the welder which had been neglected in the voltage drop calculations. Therefore, the study described in this bulletin was undertaken to determine what effect these motor loads have on the voltage drop and also to develop methods of calculating this effect.

This bulletin is divided into two main parts. The first presents analytical solutions of the voltage change produced by a single-phase welder, including the effect of the adjacent motor loads. The second part contains several charts from which the voltage drop may be determined for some of the most commonly encountered welder supply circuits. Included in this part is a description of how the charts were obtained together with a discussion of their limitations. A complete set of instructions for using the charts is also included, with an example to clarify these instructions. These charts offer a practical and time-saving method of determining the voltage drop when a large number of problems involving single-phase welders and various amounts of motor load are to be considered.

2. Acknowledgments

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R. O. Askey, Chairman, Public Service Company of Northern Illinois

J. F. Bracken, Commonwealth Edison Company

A. W. Brown, Illinois Northern Utilities Company

F. G. Mueller, Commonwealth Edison Company

H. E. Smith, Commonwealth Edison Company

R. E. Young, Public Service Company of Northern Illinois

The Advisory Committee was very active and made many valuable contributions during the progress of the work.

The investigation was carried on as a part of the work of the Engineering Experiment Station of the University of
Illinois under the general administrative direction of Dean W. L. Everitt, Director of the Station.

Acknowledgment is made to Professor C. A. Keener and Professor A. R. Knight of the Electrical Engineering Department for their suggestions, encouragement, and advice during the investigation.
II. Analytical Development of Voltage Change Equations

3. General Method of Solution

A single-phase welder is assumed to be connected to one phase of a symmetrical three-phase system. Three-phase induction and synchronous motors connected to the system are assumed to be symmetrical. Since the single-phase welder is the only unbalance on an otherwise symmetrical three-phase system, the performance of the system can be analyzed by the method of symmetrical components in a manner similar to that used for fault calculations. In this problem the welder will be considered as a line-to-line fault of impedance $Z_F$ as shown in Fig. 1. All currents and voltages are assumed sinusoidal.

![Symmetrical 3-Phase System Diagram](image)

**Fig. 1. Single-Phase Welder Considered as a Line-to-Line Fault on a Symmetrical Three-Phase System**

In Fig. 1, $I_a$, $I_b$, and $I_c$ represent the currents flowing into the fault from the respective phases; $V_a$, $V_b$, and $V_c$ represent the neutral-to-line voltage rises of the respective phases at the point of fault $F$ after the fault occurs.

4. Sequence Network Current and Voltage Relations

A symmetrical three-phase system which has become unbalanced due to a fault can be resolved into a positive-sequence system, a negative-sequence system, and a zero-
sequence system. In each of these systems the currents and voltages are symmetrical and the impedances in each phase are identical. Therefore each system can be replaced by a single-phase line-to-neutral network, and a single-line diagram can be used to represent the network.

Throughout this discussion ABC is taken as the positive sequence, phase A is considered as the reference phase, and subscripts 1, 2, and 0 are used to denote the positive, negative, and zero sequences respectively. Unless otherwise specified, all voltages are considered as rises and taken positive from neutral-to-line in the sequence networks.

Using the preceding definitions, the positive-sequence, negative-sequence, and zero-sequence networks can be represented as shown in Fig. 2. Here the heavy lines indicate the respective neutral buses and the letters F₁, F₂, and F₀ indicate the point of fault in the respective networks. Vₐ₁ is the neutral-to-line voltage rise at the point of fault in the positive-sequence network. Iₐ₁ is the current flowing from the positive-sequence network into the fault. Vₐ₂, Iₐ₂, Vₐ₀ and Iₐ₀ are similarly defined for the negative- and zero-sequence networks.

**Fig. 2. General Sequence Networks**

Very useful equations can be derived which apply to each network regardless of the type of fault. Let Vₐ₁, Vₐ₂, and Vₐ₀ be the neutral-to-line voltage rises of the respective phases at F before the fault occurs. The effect of the fault on the positive-sequence network is to change the positive-
Sequence voltage at the point of fault F from $V_{FA}$ to $V_{a_1}$ and the positive-sequence current flowing into the fault from zero to $I_{a_1}$. The relationship between $V_{FA}$, $V_{a_1}$, and $I_{a_1}$ can be found conveniently by the principle of superposition. If $Z_1$ is the impedance of the positive-sequence network between the fault and the neutral bus (generated emf's considered zero), the voltage drop caused by $I_{a_1}$ flowing from the neutral bus to the fault is $I_{a_1}Z_1$, or the voltage rise is $-I_{a_1}Z_1$. Therefore

$$\bar{V}_{a_1} = \bar{V}_{FA} - \bar{I}_{a_1} \bar{Z}_1$$

Equation (1) can also be developed by a superposition of currents instead of voltages as above.

In the usual case there are no negative-sequence voltages generated or induced in a system. Hence the negative-sequence voltage at F is zero before the fault occurs. It is $V_{a_2}$ after the fault and is due to the negative-sequence current $I_{a_2}$ flowing through $Z_2$, the impedance as viewed from the fault in the negative-sequence network. From Fig. 2 it can be seen that the negative-sequence voltage rise is

$$\bar{V}_{a_2} = -\bar{I}_{a_2} \bar{Z}_2$$

In similar manner it can be shown that

$$\bar{V}_{a_0} = -\bar{I}_{a_0} \bar{Z}_0$$

where $Z_0$ is the impedance as viewed from the fault in the zero-sequence network.

5. Determination of General Equivalent Circuit

Reference to Fig. 1 shows that the conditions at the point of fault F are:
\[ \bar{I}_a = 0 \]
\[ \bar{I}_b = - \bar{I}_c \]
\[ \bar{V}_b - \bar{I}_b Z_F = \bar{V}_c \] \hspace{1cm} (4)

The symmetrical components of the fault currents are:

\[ \bar{I}_{a0} = \frac{1}{3} (\bar{I}_a + \bar{I}_b + \bar{I}_c) = \frac{1}{3} (0 + \bar{I}_b - \bar{I}_b) = 0 \]

\[ \bar{I}_{a1} = \frac{1}{3} (\bar{I}_a + a\bar{I}_b + a^2\bar{I}_c) = \frac{1}{3} (0 + a\bar{I}_b - a^2\bar{I}_b) = \frac{a-a^2}{3} \bar{I}_b \] \hspace{1cm} (5)

\[ \bar{I}_{a2} = \frac{1}{3} (\bar{I}_a + a^2\bar{I}_b + a\bar{I}_c) = \frac{1}{3} (0 + a^2\bar{I}_b - a\bar{I}_b) = - \frac{a-a^2}{3} \bar{I}_b \]

\[ \bar{I}_{a2} = - \bar{I}_{a1} \]

The voltage rises at the point of fault in terms of their symmetrical components are:

\[ \bar{V}_a = \bar{V}_{a1} + \bar{V}_{a2} + \bar{V}_{a0} = \bar{V}_{a1} + \bar{V}_{a2} \]
\[ \bar{V}_b = \bar{V}_{b1} + \bar{V}_{b2} + \bar{V}_{b0} = a^2\bar{V}_{a1} + a\bar{V}_{a2} \] \hspace{1cm} (6)
\[ \bar{V}_c = \bar{V}_{c1} + \bar{V}_{c2} + \bar{V}_{c0} = a\bar{V}_{a1} + a^2\bar{V}_{a2} \]

Since \( I_{a0} = 0 \), \( V_{a0} \) must also be zero. Using the relations expressed in Equations (4) and (6), it can be seen that

\[ \bar{V}_b - \bar{V}_c = (a^2 - a) \bar{V}_{a1} - (a^2 - a) \bar{V}_{a2} = \bar{I}_b Z_F \]

Since \( \bar{I}_b = \bar{I}_{b1} + \bar{I}_{b2} + \bar{I}_{b0} = a^2\bar{I}_{a1} + a\bar{I}_{a2} = (a^2 - a) \bar{I}_{a1} \), the preceding equation becomes

\[ \bar{V}_b - \bar{V}_c = (a^2 - a) \bar{V}_{a1} - (a^2 - a) \bar{V}_{a2} = (a^2 - a) \bar{I}_{a1} Z_F \]

or

\[ \bar{V}_{a1} = \bar{V}_{a2} + \bar{I}_{a1} Z_F \] \hspace{1cm} (7)

In order to satisfy the conditions given in Equations (5) and (7), the sequence networks must be connected as shown in Fig. 3.
$I_{a_1}$ can be found in terms of the sequence network impedances and the voltage existing before the fault occurs. Solving Equation (1) for $V_{FA}$ and making use of Equations (2), (5), and (7)

$$\bar{V}_{FA} = \bar{V}_{a_1} + \bar{I}_{a_1} \bar{Z}_1 = \bar{V}_{a_2} + \bar{I}_{a_1} \bar{Z}_F + \bar{I}_{a_1} \bar{Z}_1 = \bar{I}_{a_1} \bar{Z}_2 + \bar{I}_{a_1} \bar{Z}_F + \bar{I}_{a_1} \bar{Z}_1$$

or

$$\bar{I}_{a_1} = \frac{\bar{V}_{FA}}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \quad (8)$$

$V_{a_1}$ and $V_{a_2}$ can also be determined in terms of the sequence network impedances and the voltage existing before the fault. From Equations (1) and (8)

$$\bar{V}_{a_1} = \bar{V}_{FA} - \bar{I}_{a_1} \bar{Z}_1 = \bar{V}_{FA} \left(1 - \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F}\right) \quad (9)$$

and from Equations (2), (5), and (8)

$$\bar{V}_{a_2} = -\bar{I}_{a_2} \bar{Z}_2 = \bar{V}_{FA} \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \quad (10)$$
6. Equations for Voltage Change at the Welder

Referring to Fig. 1, the line-to-line voltage rises at F after the fault are:

\[ V_{ab} = V_b - V_a \quad V_{bc} = V_c - V_b \quad V_{ca} = V_a - V_c \]

Substituting the relations given in Equations (6) into the above equations and using the definitions \( a = 1^\circ \leq 120^\circ \) and \( a^2 = 1 \leq 240^\circ \)

\[ V_{ab} = a^2 V_{a_1} + a V_{a_2} - V_{a_1} - V_{a_2} = (a^2 - 1) V_{a_1} + (a - 1) V_{a_2} \]
\[ = \sqrt{3} \left| 210^\circ V_{a_1} + \sqrt{3} \left| 150^\circ V_{a_2} \right. \right. \]

\[ V_{bc} = a V_{a_1} + a^2 V_{a_2} - a V_{a_1} - a V_{a_2} = (a - a^2) V_{a_1} + (a^2 - a) V_{a_2} \]
\[ = \sqrt{3} \left| 190^\circ V_{a_1} + \sqrt{3} \left| 270^\circ V_{a_2} \right. \right. \]

\[ V_{ca} = V_{a_1} + V_{a_2} - a V_{a_1} - a^2 V_{a_2} = (1 - a) V_{a_1} + (1 - a^2) V_{a_2} \]
\[ = \sqrt{3} \left| 330^\circ V_{a_1} + \sqrt{3} \left| 30^\circ V_{a_2} \right. \right. \]

The phase relations between the neutral-to-line voltage rises before the fault occurs are

\[ V_{F_{AB}} = \sqrt{3} \left| 210^\circ V_{F_A} - V_{F_{BC}} = \sqrt{3} \left| 90^\circ V_{F_A} - V_{F_{CA}} = \sqrt{3} \left| 330^\circ V_{F_A} \right. \right. \]

The ratio of the line-to-line voltage after the fault to the line-to-line voltage before the fault for each of the phases is

\[ \frac{V_{ab}}{V_{F_{AB}}} = \frac{\sqrt{3} \left| 210^\circ V_{a_1} + \sqrt{3} \left| 150^\circ V_{a_2} \right. \right.}{\sqrt{3} \left| 210^\circ V_{F_A} \right.} = \frac{V_{a_1} - V_{a_2} \left| 120^\circ \right.}{\sqrt{3} \left| 210^\circ V_{F_A} \right.} \]
\[ \frac{V_{bc}}{V_{F_{BC}}} = \frac{\sqrt{3} \left| 90^\circ V_{a_1} + \sqrt{3} \left| 270^\circ V_{a_2} \right. \right.}{\sqrt{3} \left| 90^\circ V_{F_A} \right.} = \frac{V_{a_1} - V_{a_2}}{\sqrt{3} \left| 90^\circ V_{F_A} \right.} \]
\[ \frac{V_{ca}}{V_{F_{CA}}} = \frac{\sqrt{3} \left| 190^\circ V_{a_1} + \sqrt{3} \left| 270^\circ V_{a_2} \right. \right.}{\sqrt{3} \left| 90^\circ V_{F_A} \right.} = \frac{V_{a_1} - V_{a_2}}{\sqrt{3} \left| 90^\circ V_{F_A} \right.} \]
\[
\frac{V_{ca}}{V_{FCA}} = \frac{\sqrt{3} \left| 330^\circ V_{a_1} + \sqrt{3} \left| 30^\circ V_{a_2} \right. \right.}{\sqrt{3} \left| 330^\circ V_{F_A} \right.} = \frac{V_{a_1} - V_{a_2}}{V_{F_A}} \tag{11}
\]

Putting the values of \(V_{a_1}\) and \(V_{a_2}\) from Equations (9) and (10) into Equations (11) gives

\[
\frac{V_{ab}}{V_{F_{AB}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2 \left| 120^\circ \right.}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{12}
\]

\[
\frac{V_{bc}}{V_{F_{BC}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{12}
\]

\[
\frac{V_{ca}}{V_{F_{CA}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2 \left| 240^\circ \right.}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{12}
\]

In general only the magnitude of the voltage change is important. Thus

\[
\frac{V_{ab}}{V_{F_{AB}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2 \left| 120^\circ \right.}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{13}
\]

\[
\frac{V_{bc}}{V_{F_{BC}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{14}
\]

\[
\frac{V_{ca}}{V_{F_{CA}}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2 \left| 240^\circ \right.}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \tag{15}
\]

where the vertical lines indicate that only the magnitude of the resultant complex quantity is under consideration.
7. Change in Voltage at the Primary of the Welder Supply
Transformer Bank

In general, single-phase loads large enough to produce an appreciable voltage change are connected to a bus which is fed through a bank of transformers from a supply line. The change in voltage at the primaries of the transformers is usually more important to the utility engineer than the change in voltage at the bus since the primary voltage change directly affects all the other customers connected to the same supply line.

In calculating primary voltage changes, three specific circuits are encountered so often in practice that equations for each of these circuits are derived. Furthermore, the following analysis serves to show how other circuit configurations can be analyzed.

Circuit I. Transformer Bank. -- Consider a single-phase load connected at point F in the system shown in Fig. 4.

For the system of Fig. 4, let:

\[ E_{S_A} = \text{excitation voltage rise of source, neutral-to-line (phase A)} \]

\[ Z_{S_1} = \text{impedance of supply (source and line) to positive sequence} \]
\[ Z_{S2} = \text{impedance of supply to negative sequence} \]
\[ Z_{MP1} = \text{impedance of motor load and associated transformer bank at P to positive sequence (Equivalent Y-Y)} \]
\[ Z_{MP2} = \text{impedance of motor load and associated transformer bank at P to negative sequence (Equivalent Y-Y)} \]
\[ Z_{T1} = \text{impedance of plant-bus transformer to positive sequence (Equivalent Y-Y)} \]
\[ Z_{T2} = \text{impedance of plant-bus transformer to negative sequence (Equivalent Y-Y)} \]
\[ Z_{MF1} = \text{impedance of motor load at F to positive sequence (Y value)} \]
\[ Z_{MF2} = \text{impedance of motor load at F to negative sequence (Y value)} \]
\[ Z_F = \text{impedance of single-phase load.} \]

Using the preceding definitions, the equivalent circuit for the system of Fig. 4 is given in Fig. 5.

The motors in the equivalent circuit have been shown only as impedances, and therefore strictly speaking represent induction motors. Synchronous motors in such a circuit must be shown in the same manner as the source, that is, as an excitation voltage in series with an impedance. However, this does not affect the analysis of conditions after the fault, since the impedances \( Z_1 \) and \( Z_2 \) which appear in the solution are defined as the impedances viewed from the point of fault in the respective networks with all generated emf's considered as zero. These definitions are a result of the use of the principle of superposition.

The circuit of Fig. 5 is applicable after the fault occurs. In accordance with Equations (5) and (8) the currents \( I_{a1} \) and \( I_{a2} \) in Fig. 5 are

\[
\bar{I}_{a1} = \frac{\bar{V}_{FA}}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \quad (16)
\]

and
where

\[ \bar{Z}_1 = \left( \frac{\bar{Z}_{S_1} \bar{Z}_{MP_1}}{\bar{Z}_{S_1} + \bar{Z}_{MP_1}} + \bar{Z}_{T_1} \right) \bar{Z}_{MF_1} \]

(18)
and \( V_{FA} \) = voltage rise (neutral-to-line) at point F before the fault occurs.

The component of current in \( Z_S \) due to the fault is

\[
\bar{I}_{P_a_1} = \bar{I}_{a_1} \frac{\bar{Z}_{MF_1}}{\bar{Z}_S + \bar{Z}_{MP_1}} + \bar{Z}_T \frac{\bar{Z}_{MP_1}}{\bar{Z}_S + \bar{Z}_{MP_1}}
\]

\[
= \bar{I}_{a_1} \frac{\bar{Z}_{MF_1}}{\bar{Z}_S + \bar{Z}_{MP_1}} + \bar{Z}_T \frac{\bar{Z}_{MP_1}}{\bar{Z}_S + \bar{Z}_{MP_1}}
\]

The component of current in \( Z_S \) due to the fault is

\[
\bar{I}_{P_a_2} = \bar{I}_{a_2} \frac{\bar{Z}_{MF_2}}{\bar{Z}_S + \bar{Z}_{MP_2}} + \bar{Z}_T \frac{\bar{Z}_{MP_2}}{\bar{Z}_S + \bar{Z}_{MP_2}}
\]

\[
= \bar{I}_{a_2} \frac{\bar{Z}_{MF_2}}{\bar{Z}_S + \bar{Z}_{MP_2}} + \bar{Z}_T \frac{\bar{Z}_{MP_2}}{\bar{Z}_S + \bar{Z}_{MP_2}}
\]

Before the fault occurs, the equivalent circuit is the positive-sequence network shown in Fig. 6. In this figure \( I_A \)
is the load current before the fault, \( V_{P_A} \) is the voltage rise (neutral-to-line) at point P before the fault, and \( Z_{MF_1}^*, Z_{MP_1}^* \) are the respective impedances of the motor loads at F and P before the fault occurs. These impedances are determined from the impressed voltage, current, and power factor of the respective motor loads. In the case of an induction motor, such an impedance is the same as the impedance determined from the usual equivalent circuit. An impedance determined from the impressed voltage, current, and power factor of a synchronous motor is completely fictitious, since the equivalent circuit usually consists of an excitation voltage in series with the usual synchronous impedance. All other quantities in Fig. 6 are the same as previously defined.

From Fig. 6 it is seen that

\[
\bar{V}_{P_A} = \bar{V}_{FA} + \bar{I}_A \bar{Z}_{T_1} = \bar{V}_{FA} + \frac{\bar{V}_{FA}}{\bar{Z}_{MF_1}^*} \bar{Z}_{T_1} \tag{22}
\]

\( V_{P_A} \) is the positive-sequence voltage rise at P before the fault. \( V_{P_{a1}} \) is the positive-sequence voltage rise at P
after the fault. \( I_{pa_1} \) is the component current in \( Z_{S_1} \) due to the fault. Therefore by the principle of superposition

\[
\vec{V}_{pa_1} = \vec{V}_{PA} - \vec{I}_{pa_1} \vec{Z}_{S_1}
\]  

(23)

The negative-sequence voltage rise at \( P \) before the fault is zero; after the fault it is \( V_{pa_2} \). The component current in \( Z_{S_2} \) due to the fault is \( I_{pa_2} \). Therefore

\[
\vec{V}_{pa_2} = 0 - \vec{I}_{pa_2} \vec{Z}_{S_2} = - \vec{I}_{pa_2} \vec{Z}_{S_2}
\]  

(24)

Substituting \( V_{PA} \) from Equation (22) and \( I_{pa_1} \) from Equation (20) into Equation (23) gives

\[
\vec{V}_{pa_1} = \vec{V}_{FA} + \frac{\vec{V}_{FA}}{Z_{MF_1}} \vec{Z}_{T_1} - \vec{I}_{a_1} \left( \frac{\vec{Z}_{MF_1} \vec{Z}_{MP_1}}{\vec{Z}_{S_1} \vec{Z}_{MF_1} + \vec{Z}_{T_1} \vec{Z}_{S_1} + \vec{Z}_{MF_1} \vec{Z}_{MP_1}} \right) \vec{Z}_{S_1}
\]

Replacing \( \vec{I}_{a_1} \) by the value given in Equation (16), the above equation becomes

\[
\vec{V}_{pa_1} = \vec{V}_{FA} \left[ 1 + \frac{\vec{Z}_{T_1}}{\vec{Z}^*_{MF_1}} - \frac{1}{\vec{Z}_{1} + \vec{Z}_{2} + \vec{Z}_F} \vec{M} \right]
\]  

(25)

where

\[
\vec{M} = \frac{\vec{Z}_{MF_1} \vec{Z}_{MP_1} \vec{Z}_{S_1}}{\vec{Z}_{S_1} \vec{Z}_{MF_1} + \vec{Z}_{T_1} \vec{Z}_{S_1} + \vec{Z}_{MF_1} \vec{Z}_{S_1} + \vec{Z}_{MF_1} \vec{Z}_{MP_1}}
\]  

(26)

Similarly

\[
\vec{V}_{pa_2} = \vec{V}_{FA} \left[ 1 + \frac{\vec{Z}_{T_1}}{\vec{Z}_{1} + \vec{Z}_{2} + \vec{Z}_F} \frac{\vec{N} \vec{M}}{\vec{N}} \right]
\]  

(27)
where

\[
\bar{N} = \frac{\bar{Z}_{MF_1} \bar{Z}_{MF_2} \bar{Z}_{S_2}}{\bar{Z}_{S_2} \bar{Z}_{MF_2} + \bar{Z}_{T_2} \bar{Z}_{S_2} + \bar{Z}_{T_2} \bar{Z}_{MF_2} + \bar{Z}_{MF_2} \bar{Z}_{S_2} + \bar{Z}_{MF_2} \bar{Z}_{MF_2}} \quad (28)
\]

Equations similar to Equations (11), which apply to point F, can be written for point P. These equations are

\[
\frac{\bar{V}_{P_{ab}}}{\bar{V}_{P_{AB}}} = \frac{\bar{V}_{P_{a_1}} - \bar{V}_{P_{a_2}}}{\bar{V}_{P_{A}}} \quad |120^\circ|
\]

\[
\frac{\bar{V}_{P_{bc}}}{\bar{V}_{P_{BC}}} = \frac{\bar{V}_{P_{a_1}} - \bar{V}_{P_{a_2}}}{\bar{V}_{P_{A}}} \quad (29)
\]

\[
\frac{\bar{V}_{P_{ca}}}{\bar{V}_{P_{CA}}} = \frac{\bar{V}_{P_{a_1}} - \bar{V}_{P_{a_2}}}{\bar{V}_{P_{A}}} \quad |240^\circ|
\]

Putting the values of \(\bar{V}_{P_{a_1}}, \bar{V}_{P_{a_2}},\) and \(\bar{V}_{P_{a_2}}\) from Equations (22), (25), and (27) respectively into the preceding equations and considering only the magnitude of the voltage ratios gives

\[
\frac{V_{P_{ab}}}{V_{P_{AB}}} = \left| \frac{\bar{V}_{FA} \left[ 1 + \frac{\bar{Z}_{T_1}}{\bar{Z}_{MF_1}^*} \right]}{\bar{V}_{FA} \left[ 1 + \frac{\bar{Z}_{T_1}}{\bar{Z}_{MF_1}^*} \right]} \right| \quad \left| \frac{1 - \frac{\bar{Z}_{MF_1}^*}{(\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F) (\bar{Z}_{MF_1}^* + \bar{Z}_{T_1}^*)}}{\bar{M} + \bar{N} \quad |120^\circ|} \right|
\]

\[
\frac{\bar{V}_{P_{a_1}} - \bar{V}_{P_{a_2}}}{\bar{V}_{P_{A}}} \quad (30)
\]
\[
\frac{V_{P_{bc}}}{V_{P_B}} = \left[ 1 - \frac{Z_{M_1}^*}{(\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F)} \frac{Z_{M_1}^*}{(Z_{M_1}^* + \bar{Z}_1)} \right] \quad (31)
\]

and

\[
\frac{V_{P_{ca}}}{V_{P_C}} = \left[ 1 - \frac{Z_{M_1}^*}{(\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F)} \frac{Z_{M_1}^*}{(Z_{M_1}^* + \bar{Z}_1)} \right] \quad (32)
\]

The values of \( \bar{Z}_1, \bar{Z}_2, \bar{M} \) and \( \bar{N} \) in the foregoing equations are given respectively by Equations (18), (19), (26), and (28).

Circuit I. \( \text{Y}^\Delta \) Transformer Bank.-- Equations (30), (31) and (32) give the line-to-line voltage changes at the point P in Fig. 4, only when the transformer bank supplying the plant bus is connected \( \text{Y}^\Delta \). Equations are now derived for the case of a \( \text{Y}^\Delta \) transformer bank.

When a transformer bank is connected \( \text{Y} \) or \( \Delta \) and the lines are lettered as shown in Fig. 7, the positive-sequence line-to-neutral voltages and line currents are shifted \( 90^\circ \) in phase in the same direction in passing through the bank; the negative sequence line-to-neutral voltages and line currents are shifted \( 90^\circ \) in the opposite direction in passing through the bank.\(^4\) Whether the shift for the positive-sequence

---

voltages and currents is $90^\circ$ forward or $90^\circ$ backward depends upon the way the transformer windings are connected (for a given $Y$ there are two possible $\Delta$'s, and vice versa). The actual direction of the $90^\circ$ shift does not need to be known when determining the magnitudes of the voltages and currents caused by faults since the magnitudes will be the same for either shift. However, if the relative phase relations of the voltages and currents on the two sides of a $Y\Delta$ (or $\Delta Y$) bank must be known, as for example when two circuits each with a $Y\Delta$ bank are to be paralleled, then the actual direction of the shift must be determined from the transformer connections. Since only changes in voltage magnitudes are being determined in this investigation, either shift may be used in the analysis.

The neutral-to-line voltage rises $\overline{V}_{pA}$, $\overline{V}_{pa1}$, and $\overline{V}_{pa2}$ given respectively in Equations (22), (25), and (27) are for the case of the $\Delta\Delta$ bank in Fig. 4 and were determined on an equivalent YY basis. In the case under consideration the plant-bus transformer bank is connected $Y\Delta$. Consequently, assuming a positive-sequence shift of $90^\circ$ forward, Equations (22), (25), and (27) become

$$\overline{V}_{pA} = j \overline{V}_{FA} \left[ 1 + \frac{\overline{Z}_{T1}}{\overline{Z}_{MF}^2} \right]$$

$$\overline{V}_{pa1} = j \overline{V}_{FA} \left[ 1 + \frac{\overline{Z}_{T1}}{\overline{Z}_{MF}^2} - \frac{1}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_F} \overline{M} \right]$$

$$\overline{V}_{pa2} = -j \overline{V}_{FA} \left[ \frac{1}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_F} \overline{N} \right]$$

Substituting these relations into Equations (29), the following equations are obtained for the ratios of the line-to-line voltages after the fault to the line-to-line voltages before the fault:
All symbols in the above equations have the same definitions as previously used.

A comparison of Equation (30) with (36), (31) with (37), and (32) with (38) shows that these equations differ only in the sign before $\bar{N}$, the plus sign applying when the plant-bus transformer bank is connected $\Delta\Delta$ and the minus sign when the bank is connected $\Delta$.

The ratios of the line-to-neutral voltages after the fault to the line-to-neutral voltages before the fault can be determined for point P by using Equations (33), (34) and (35). These ratios are

$$\frac{V_{pa}}{V_{PA}} = \left[ 1 - \frac{\bar{Z}_{MF1}^*}{(Z_1 + Z_2 + Z_F)(\bar{Z}_{MF1}^* + Z_{T1})} \right] (M + \bar{N})$$

$$\frac{V_{pb}}{V_{PB}} = \left[ 1 - \frac{\bar{Z}_{MF1}^*}{(Z_1 + Z_2 + Z_F)(\bar{Z}_{MF1}^* + Z_{T1})} \right] (M + \bar{N})$$

$$\frac{V_{pc}}{V_{PC}} = \left[ 1 - \frac{\bar{Z}_{MF1}^*}{(Z_1 + Z_2 + Z_F)(\bar{Z}_{MF1}^* + Z_{T1})} \right] (M + \bar{N})$$
The definitions of the symbols in these equations are the same as before, and the values of $Z_1$, $Z_2$, $M$, and $N$ are given respectively by Equations (18), (19), (26), and (28). The similarity between Equations (39), (40), and (41) and Equations (30), (31), and (32) should be noted.

Circuit II. ΔΔ or YΔ Transformer Bank. -- A second circuit often encountered in practice is shown in Fig. 8.

Again it is necessary to determine the change in the line-to-line voltages at point P caused by connecting the single-phase load at F. Voltage ratio equations for the circuit can be derived in a manner similar to that used for the circuit of Fig. 4. However, when there are no motors at P, the circuit of Fig. 4 reduces to that of Fig. 8 and $Z_{MP1} = Z_{MP2} = \infty$ may be substituted in the equations derived for Fig. 4. When these substitutions are made in Equations (26) and (28) the values of $M$ and $N$ become

$$
\bar{M} = \frac{\bar{Z}_{MF1} \bar{Z}_{S1}}{\bar{Z}_{S1} + \bar{Z}_{T1} + \bar{Z}_{MF1}}
$$
Substituting these values of \( M \) and \( \bar{N} \) into Equations (30), (31), (32), (36), (37), and (38) gives the following voltage ratio equations:

\[
\frac{V_{Pab}}{V_{PAB}} = \left[ 1 - \frac{Z_{MF1}^*}{(Z_1^*+Z_2^*+Z_F^*)} \left( \frac{Z_{MF1}^*Z_{S1}}{Z_{S1}^*+Z_{T1}^*+Z_{MF1}^*} + \frac{Z_{MF2}^*Z_{S2}}{Z_{S2}^*+Z_{T2}^*+Z_{MF2}^*} \right) \right]
\]

(42)

\[
\frac{V_{Pbc}}{V_{PBC}} = \left[ 1 - \frac{Z_{MF1}^*}{(Z_1^*+Z_2^*+Z_F^*)} \left( \frac{Z_{MF1}^*Z_{S1}}{Z_{S1}^*+Z_{T1}^*+Z_{MF1}^*} + \frac{Z_{MF2}^*Z_{S2}}{Z_{S2}^*+Z_{T2}^*+Z_{MF2}^*} \right) \right]
\]

(43)

\[
\frac{V_{Pca}}{V_{PCA}} = \left[ 1 - \frac{Z_{MF1}^*}{(Z_1^*+Z_2^*+Z_F^*)} \left( \frac{Z_{MF1}^*Z_{S1}}{Z_{S1}^*+Z_{T1}^*+Z_{MF1}^*} + \frac{Z_{MF2}^*Z_{S2}}{Z_{S2}^*+Z_{T2}^*+Z_{MF2}^*} \right) \right]
\]

(44)

where the plus sign before the last term is used when the plant-bus transformer bank is connected \( \Delta \Delta \) and the minus sign when the bank is connected \( Y \Delta \).

The line-to-neutral voltage ratio equations for point P and a \( Y \Delta \) connected transformer bank can be determined by substituting the preceding values of \( M \) and \( \bar{N} \) in Equations (39), (40), and (41). Thus

\[
\frac{V_{Pa}}{V_{PA}} = \left[ 1 - \frac{Z_{MF1}^*}{(Z_1^*+Z_2^*+Z_F^*)} \left( \frac{Z_{MF1}^*Z_{S1}}{Z_{S1}^*+Z_{T1}^*+Z_{MF1}^*} + \frac{Z_{MF2}^*Z_{S2}}{Z_{S2}^*+Z_{T2}^*+Z_{MF2}^*} \right) \right]
\]

(45)
Influence of Polyphase Motors

\[
\begin{align*}
\frac{V_{P_b}}{V_{P_B}} &= \left[ 1 - \frac{Z_{MF_1}^*}{(Z_1 + Z_2 + Z_F)(Z_{MF_1}^* + Z_{T_1})} \left( \frac{Z_{MF_1} Z_{S_1}}{Z_{S_1} + Z_{T_1} + Z_{MF_1}} + \frac{Z_{MF_2} Z_{S_2}}{Z_{S_2} + Z_{T_2} + Z_{MF_2}} \right) \right] \\
\frac{V_{P_c}}{V_{P_C}} &= \left[ 1 - \frac{Z_{MF_1}^*}{(Z_1 + Z_2 + Z_F)(Z_{MF_1}^* + Z_{T_1})} \left( \frac{Z_{MF_1} Z_{S_1}}{Z_{S_1} + Z_{T_1} + Z_{MF_1}} + \frac{Z_{MF_2} Z_{S_2}}{Z_{S_2} + Z_{T_2} + Z_{MF_2}} \right) \right] \\
\end{align*}
\]

(46)

The values of \( \overline{Z}_1 \) and \( \overline{Z}_2 \) in Equations (42), (43), (44), (45), (46) and (47) can be determined by letting \( Z_{MF_1} = Z_{MF_2} \) = \( \alpha \) in Equations (18) and (19).

This gives

\[
\overline{Z}_1 = \frac{(\overline{Z}_{T_1} + \overline{Z}_{S_1}) Z_{MF_1}}{\overline{Z}_{T_1} + \overline{Z}_{S_1} + Z_{MF_1}}
\]

(48)

\[
\overline{Z}_2 = \frac{(\overline{Z}_{T_2} + \overline{Z}_{S_2}) Z_{MF_2}}{\overline{Z}_{T_2} + \overline{Z}_{S_2} + Z_{MF_2}}
\]

(49)

Circuit III. \( \Delta \Delta \) or \( Y \Delta \) Transformer Bank.-- The third circuit to be considered is shown in Fig. 9. The circuit of Fig. 4 reduces to that of Fig. 9 when there are no motors at \( F \), that is, when \( Z_{MF_1} = Z_{MF_2} = \alpha \). Under this condition the equations derived for Fig. 4 reduce to the following:

\[
\begin{align*}
\frac{V_{P_{ab}}}{V_{P_{AB}}} &= \left[ 1 - \frac{(\overline{Z}_1 - \overline{Z}_{T_1}) \pm (\overline{Z}_2 - \overline{Z}_{T_2})}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_F} \right] 120^\circ
\end{align*}
\]

(50)
FIG. 9. CIRCUIT III, A THIRD TYPICAL SYSTEM FOR SUPPLYING A SINGLE-PHASE WELDER

\[
\frac{v_{PBc}}{v_{PB}} = \left[ 1 - \frac{(\bar{Z}_1 - \bar{Z}_{T1}) \pm (\bar{Z}_2 - \bar{Z}_{T2})}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right] 
\]

\[
\frac{v_{Pca}}{v_{PCA}} = \left[ 1 - \frac{(\bar{Z}_1 - \bar{Z}_{T1}) \pm (\bar{Z}_2 - \bar{Z}_{T2})}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right] \left[ 240^\circ \right] 
\]

\[
\frac{v_{Pa}}{v_{PA}} = \left[ 1 - \frac{(\bar{Z}_1 - \bar{Z}_{T1}) + (\bar{Z}_2 - \bar{Z}_{T2})}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right] 
\]

\[
\frac{v_{Pb}}{v_{PB}} = \left[ 1 - \frac{(\bar{Z}_1 - \bar{Z}_{T1}) + (\bar{Z}_2 - \bar{Z}_{T2})}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right] \left[ 240^\circ \right] 
\]

\[
\frac{v_{Pc}}{v_{PC}} = \left[ 1 - \frac{(\bar{Z}_1 - \bar{Z}_{T1}) + (\bar{Z}_2 - \bar{Z}_{T2})}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right] \left[ 120^\circ \right] 
\]
where

\[ \bar{Z}_1 = \frac{\bar{Z}_{S_1} \bar{Z}_{MP_1}}{\bar{Z}_{S_1} + \bar{Z}_{MP_1}} + \bar{Z}_{T_1} \]  \hspace{1cm} (56)  

\[ \bar{Z}_2 = \frac{\bar{Z}_{S_2} \bar{Z}_{MP_2}}{\bar{Z}_{S_2} + \bar{Z}_{MP_2}} + \bar{Z}_{T_2} \]  \hspace{1cm} (57)  

As in the preceding cases, the plus sign is used in Equations (50), (51), and (52) when the plant-bus transformer bank is connected \( \Delta \Delta \) and the minus sign is used when the bank is connected \( \nabla \nabla \).
III. MOTOR IMPEDANCES TO BE USED IN VOLTAGE CHANGE EQUATIONS

8. Introduction
The impedance $Z_{MF1}$ which appears in some of the voltage ratio equations is always determined from the voltage, current, and power factor of the motor load connected at $F$ before the fault occurs. The values to be used for the other motor impedances in the voltage ratio equations will be influenced by the type of single-phase load which is connected to the system. For some types of single-phase loads steady-state machine impedances can be used, while in other cases sub-transient or transient values must be used. To help in determining the proper motor impedances, single-phase loads are divided into the two following types:

1. Loads connected for an indefinitely long time or connected with an "on" period long enough for the system to reach a steady-state condition.
2. Loads connected with an "on" period so short that the system does not reach a steady-state condition before the "off" period occurs.

9. Synchronous Motors
For most practical problems involving voltage changes of synchronous machines it is sufficiently accurate to neglect the armature resistance and to use cylindrical rotor theory for both nonsalient-pole and salient-pole machines. In the latter case, the direct-axis reactance $X_d$ of the salient-pole machine is used as the synchronous reactance $X_s$ of the equivalent cylindrical-rotor machine.

For type 1 single-phase loads, the synchronous reactance $X_s = X_d$ is used for the positive-sequence motor impedance and the synchronous machine negative-sequence reactance $X_2$ is used for the negative-sequence motor impedance. For type 2 single-phase loads, the synchronous machine sub-transient reactance $X_d''$ should be used for the positive-sequence motor impedance with loads having "on" periods of 2 or 3 cycles or less, and the transient reactance $X_d'$ should be
used for the longer "on" periods. The negative-sequence reactance $X_2$ is used for the negative-sequence motor impedance for type 2 single-phase loads, since this value is inherently a transient value.

Synchronous machine impedances are affected some by saturation - in other words, by the load on the machine and/or the amount of field excitation. However, this refinement does not need to be considered in this investigation, since the negative-sequence motor impedance, which has the most effect on the voltage change, is nearly independent of saturation due to the damper windings used with synchronous motors. The negative-sequence motor impedance has more effect on the voltage change than the positive-sequence motor impedance, since it is the smaller of the two. This effect becomes apparent when the voltage ratio equations are used. The experimental data given in the following section also show that the magnitude of the field excitation has a minor effect on the voltage change.

Since it is usually impossible in practical cases to know the exact character of the motor loads, sufficient accuracy can be obtained by using values from the following table of typical constants for general purpose synchronous motors. The constants are for motors normally connected to utility lines; that is, 25-500 hp and 1800, 1200, or 900 rpm. The reactances are given in per unit with stator kva as the base, and the time constants are given in seconds.

### Constants For Small Synchronous Motors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsaturated direct-axis reactance, $X_d$</td>
<td>0.60 - 1.45</td>
<td>1.15</td>
</tr>
<tr>
<td>Transient reactance, $X_d'$</td>
<td>0.20 - 0.50</td>
<td>0.37</td>
</tr>
<tr>
<td>Sub-transient reactance, $X_d''$</td>
<td>0.13 - 0.35</td>
<td>0.24</td>
</tr>
<tr>
<td>Negative-sequence reactance, $X_2$</td>
<td>0.13 - 0.35</td>
<td>0.24</td>
</tr>
<tr>
<td>Short-circuit transient time constant, $T_d'$</td>
<td>0.50 - 0.65</td>
<td>0.58</td>
</tr>
<tr>
<td>Short-circuit sub-transient time constant, $T_d''$</td>
<td>0.01 - 0.02</td>
<td>0.015</td>
</tr>
</tbody>
</table>
10. **Induction Motors**

The positive-sequence impedance of an induction motor is the impedance determined from the usual equivalent circuit. This impedance depends on the slip $S$; that is, on the load carried by the motor. The negative-sequence impedance can be determined by substituting a slip of $(2-S)$ for the $S$ of the usual equivalent circuit. This will show that the negative-sequence impedance is, for all practical purposes, equal to the blocked-rotor impedance. For transient studies involving induction motors, an impedance equal to the blocked-rotor impedance is used. This impedance might be compared to the sub-transient impedance of a synchronous machine. Since there is no field winding, there is no impedance of an induction motor which is comparable to the transient impedance of a synchronous machine.

For type 1 single-phase loads, the usual positive-sequence and negative-sequence impedances are used for the motor impedances in the voltage ratio equations. For type 2 single-phase loads, the blocked-rotor impedance should be used as the positive-sequence motor impedance with loads having "on" periods of 1 or 2 cycles; for "on" periods longer than this the usual positive-sequence impedance should be used. For all type 2 loads, the blocked-rotor impedance is used as the negative-sequence motor impedance.

The usual positive-sequence impedance of an induction motor varies with load. In general, this will not need to be taken into account, and the full-load value can be used. This follows from the fact that the negative-sequence impedance has much more effect on the voltage change than the positive-sequence impedance. The experimental data given in the next section confirm the above because they show that the load on an induction motor has little effect on the voltage change produced by connecting a single-phase load.

In practical cases where it is difficult, if not impossible, to determine the motor loads accurately, the following induction motor constants may be used. These constants are in per unit, the base kva being equal to the output kw (hp X 0.746). They are average values for 60-cycle, general purpose induction motors rated 10-500 hp and 1800, 1200, or 900 rpm.

<table>
<thead>
<tr>
<th>Impedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-load impedance</td>
<td>$0.8 \angle 27^\circ$</td>
</tr>
<tr>
<td>Blocked-rotor impedance</td>
<td>$0.135 \angle 70^\circ$</td>
</tr>
</tbody>
</table>
IV. EXPERIMENTAL DATA

11. Introduction

Many specific cases were investigated in the laboratory to verify the material of the preceding sections. The data and results of several typical cases are given in this section. As a matter of convenience the plant-bus transformer bank was considered to be connected $\Delta\Delta$ in all cases. In the laboratory this permitted the use of series impedances for the transformer bank. All motor impedances were measured by the usual test methods\(^2\), and only the results determined from such measurements are tabulated.

In each of the cases one or both of the ratios $V_{bc}/V_{FBC}$ and $V_{Pbc}/V_{PBC}$ were determined from measured voltages and from calculations using the previously developed equations and measured impedances. In most instances the difference between the experimental and calculated ratios is less than 0.5 percent, the maximum difference being less than 1 percent. Phase $bc$, the phase supplying the single-phase load, was used for the check between the experimental and calculated values. This phase is the most important in practical cases because it has the greatest voltage change when the transformer bank supplying the plant-bus is connected $\Delta\Delta$.

Cases 1 and 2 show that when an induction motor is connected to a three-phase system, there is definitely a reduction in the voltage change caused by connecting a single-phase load to the system. These cases also show that the amount of load on an induction motor has very little effect on the voltage change.

Cases 3 and 4 show that a synchronous motor also reduces the voltage change caused by connecting a single-phase load to a three-phase system. It is also shown that the load on a synchronous motor has little effect on the voltage change.

Case 5 shows that the excitation of a synchronous motor has only a small effect on the voltage change produced by a single-phase load.

Case 6 consists of several tests involving an induction motor and single-phase loads having various "on" and "off" periods. These tests show that the use of the blocked-rotor impedance for the positive-sequence motor impedance gives a voltage change which checks fairly well with that actually produced during the first cycle or two and that after this time the usual positive-sequence impedance gives a good check.

Case 7 includes several tests involving a synchronous motor and single-phase loads having various "on" and "off" periods. These tests indicate that the use of the sub-transient reactance for the positive-sequence motor impedance gives a result which checks reasonably well with the voltage change produced during the first one or two cycles and that the transient reactance gives a good check between this time and the time when steady-state conditions are reached.

12. Case 1. Induction Motor Connected to the Welder Bus

![Circuit Diagram]

**Fig. 10. Circuit for Case No. 1 of Experimental Data**

**Circuit and System Data**

\[
\begin{align*}
\bar{Z}_{S_1} &= \bar{Z}_{S_2} = 0.41 + j 0.25 \text{ ohm} \\
\bar{Z}_{T_1} &= \bar{Z}_{T_2} = 0.35 + j 0.20 \text{ ohm} \\
\bar{Z}_F &= R + j 0 \text{ ohms}
\end{align*}
\]
Induction Motor:
Rating - 7 ½ hp, 220 v, 1135 rpm, 3φ
No-load impedance = 1.62 + j 13.2 ohms
Approx. 3/4-load impedance = 5.63 + j 5.53 ohms
Blocked-rotor impedance = 0.875 + j 1.50 ohms
The single-phase load remained on long enough for the system to reach steady-state conditions.

Observed Data

<table>
<thead>
<tr>
<th></th>
<th>Voltages at F</th>
<th>Voltages at P</th>
<th>1-φ Current</th>
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<tbody>
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<td></td>
<td>$V_{ab}$</td>
<td>$V_{bc}$</td>
<td>$V_{ca}$</td>
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<tr>
<td>Motor Not</td>
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</tr>
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<td></td>
<td></td>
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<td>211.5</td>
</tr>
<tr>
<td>Motor Running</td>
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<td></td>
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</tr>
<tr>
<td>Approx. 3/4-Load</td>
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<td></td>
<td></td>
</tr>
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Calculated Data

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<tr>
<th>1-φ Current</th>
<th>$V_{bc}/V_{FBC}$</th>
<th>$V_{Pbc}/V_{PBC}$</th>
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</thead>
<tbody>
<tr>
<td>Motor Not Connected</td>
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<td></td>
<td>7.4</td>
<td>0.953</td>
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</tr>
<tr>
<td></td>
<td>15.35</td>
<td>0.901</td>
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</tbody>
</table>
### Sample Calculations

Consider the case where the motor is running at no-load and the single-phase load current is 5.4 amp. For this case:

\[ \bar{Z}_{S_1} = \bar{Z}_{S_2} = 0.41 + j 0.25 \]
\[ \bar{Z}_{T_1} = \bar{Z}_{T_2} = 0.35 + j 0.20 \]
\[ \bar{Z}_F = \frac{219}{5.4} = 40.6 + j 0 \]
\[ \bar{Z}_{MF_1} = 1.62 + j 13.2 \]
\[ \bar{Z}_{MF_2} = 0.875 + j 1.5 \]

From Equations (48) and (49):

\[ \bar{Z}_1 = \frac{(\bar{Z}_{T_1} + \bar{Z}_{S_1}) \bar{Z}_{MF_1}}{\bar{Z}_{T_1} + \bar{Z}_{S_1} + \bar{Z}_{MF_1}} = 0.709 + j 0.47 \]

\[ \bar{Z}_2 = \frac{(\bar{Z}_{T_2} + \bar{Z}_{S_2}) \bar{Z}_{MF_2}}{\bar{Z}_{T_2} + \bar{Z}_{S_2} + \bar{Z}_{MF_2}} = 0.46 + j 0.39 \]

Using Equation (14), the calculated voltage ratio for point F is:

\[ \frac{V_{bc}}{V_{FBC}} = \left| 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right| = 0.973 \]
The voltage ratio for point F determined from observed data is

\[ \frac{V_{bc}}{V_{Pbc}} = \frac{219}{224} = 0.977 \]

Using Equation (43), the calculated voltage ratio for point P is

\[ \frac{V_{Pbc}}{V_{PBC}} = 1 - \frac{Z_{MF1}}{(Z_{1} + Z_{2} + Z_{F})(Z_{S1} + Z_{T1})} \left( \frac{Z_{MF1}Z_{S1}}{Z_{MF1} + Z_{S1} + Z_{T1}} + \frac{Z_{MF2}Z_{S2}}{Z_{MF2} + Z_{S2} + Z_{T2}} \right) \]

\[ = 0.985 \]

The voltage ratio for point P determined from observed data is

\[ \frac{V_{Pbc}}{V_{PBC}} = \frac{226}{229} = 0.987 \]

13. Case 2. Induction Motor Connected to the Primary of the Welder Supply Transformer Bank

![Figure 11. Circuit for Case No. 2 of Experimental Data](image-url)
Circuit and System Data

\[ Z_s = Z = 0.41 + j 0.25 \text{ ohm} \]

\[ Z_{T1} = Z_{T2} = 0.35 + j 0.20 \text{ ohm} \]

\[ Z_F = R + j 0 \text{ ohms} \]

Induction Motor:
Rating - 7 ½ hp, 220 v, 1135 rpm, 3φ
No-load impedance = 1.62 + j 13.2 ohms
Approx. 3/4-load impedance = 6.1 + j 6.0 ohms
Blocked-rotor impedance = 0.875 + j 1.50 ohms

The single-phase load remained on long enough for the system to reach steady-state conditions.

Observed Data

<table>
<thead>
<tr>
<th></th>
<th>Voltages at F</th>
<th>Voltages at P</th>
<th>1-φ Current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( V_{ab} )</td>
<td>( V_{bc} )</td>
<td>( V_{ca} )</td>
</tr>
<tr>
<td>Motor Not Connected</td>
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<td>234.5</td>
<td>233.5</td>
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<tr>
<td></td>
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<tr>
<td>Motor Running</td>
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<tr>
<td>Motor Running</td>
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<td>Approx. 3/4-Load</td>
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Calculated Data

<table>
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<tr>
<th>1-φ Current</th>
<th>V_{bc}/V_{FBC}</th>
<th>V_{pbc}/V_{FBC}</th>
</tr>
</thead>
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<tr>
<td>Motor Not Connected</td>
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<td>7.4</td>
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<tr>
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<td>15.35</td>
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<td>Motor Running No-Load</td>
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<td>14.9</td>
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</tbody>
</table>

Sample Calculations

Consider the case where the motor is running under load and the single-phase load is 7.4 amp. For this case

\[ \bar{Z}_{S1} = \bar{Z}_{S2} = 0.41 + j 0.25 \]
\[ \bar{Z}_{T1} = \bar{Z}_{T2} = 0.35 + j 0.20 \]
\[ \bar{Z}_{M1} = 6.1 + j 6.0 \]
\[ \bar{Z}_{M2} = 0.875 + j 1.5 \]

\[ \bar{Z}_F = \frac{213}{7.4} = 28.8 + j 0 \]

From Equations (56) and (57)

\[ \bar{Z}_1 = \frac{\bar{Z}_{S1} \bar{Z}_{M1}}{\bar{Z}_{S1} + \bar{Z}_{M1}} + \bar{Z}_{T1} = 0.736 + j 0.442 \]

\[ \bar{Z}_2 = \frac{\bar{Z}_{S2} \bar{Z}_{M2}}{\bar{Z}_{S2} + \bar{Z}_{M2}} + \bar{Z}_{T2} = 0.655 + j 0.434 \]
Using Equation (14), the calculated voltage ratio for point F is

\[ \frac{V_{bc}}{V_{FBC}} = 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} = 0.954 \]

The voltage ratio for point F determined from observed data is

\[ \frac{V_{bc}}{V_{FBC}} = \frac{213}{223} = 0.955 \]

Using Equation (51), the calculated voltage ratio for point P is

\[ \frac{V_{Pbc}}{V_{PBC}} = 1 - \frac{(\bar{Z}_1 - \bar{Z}_T1) + (\bar{Z}_2 - \bar{Z}_T2)}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} = 0.977 \]

The voltage ratio for point P determined from observed data is

\[ \frac{V_{Pbc}}{V_{PBC}} = \frac{218}{223} = 0.978 \]


**Fig. 12. Circuit for Case No. 3 of Experimental Data**
Circuit and System Data

\[ Z_{S1} = Z_{S2} = 0.41 + j 0.25 \, \text{ohm} \]
\[ Z'_{T1} = Z'_{T2} = 0.35 + j 0.20 \, \text{ohm} \]
\[ Z_F = R + j 0 \, \text{ohms} \]

Synchronous Motor:

Rating - 5 kva, 220 v, 1200 rpm, 3\phi

Direct-axis reactance (unsat.) = 0 + j 8.7 ohms

Negative-sequence impedance = 1.33 + j 2.35 ohms

\[ Z^*_{MF1} \text{ (Full-load, } I_f = 3) = 7.95 + j 6.43 \, \text{ohms} \]

\[ Z^*_{MF1} \text{ (No-load, } I_f = 3) = 2.82 - j 15.9 \, \text{ohms} \]

The single-phase load remained on long enough for the system to reach steady-state conditions.

Observed Data

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<thead>
<tr>
<th>V_\text{ab}</th>
<th>V_{bc}</th>
<th>V_{ca}</th>
<th>V_\text{ab}</th>
<th>V_{bc}</th>
<th>V_{ca}</th>
<th>1-\phi \text{ Current}</th>
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<td></td>
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Calculated Data

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<th>(V_{pbc}/V_{P_{IEC}})</th>
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</table>

Sample Calculations

Consider the case where the motor is running at no-load and the single-phase load current is 5.4 amp. For this case

\[
\bar{Z}_{S_1} = \bar{Z}_{S_2} = 0.41 + j 0.25 \\
\bar{Z}_{T_1} = \bar{Z}_{T_2} = 0.35 + j 0.20 \\
\bar{Z}_F = \frac{237}{5.4} = 43.9 + j 0 \]

From Equations (48) and (49)

\[
\bar{Z}_1 = \frac{\bar{Z}_{T_1} + \bar{Z}_{S_1}}{\bar{Z}_{T_1} + \bar{Z}_{S_1} + \bar{Z}_{MF_1}} = 0.672 + j 0.49 \\
\bar{Z}_2 = \frac{\bar{Z}_{T_2} + \bar{Z}_{S_2}}{\bar{Z}_{T_2} + \bar{Z}_{S_2} + \bar{Z}_{MF_2}} = 0.54 + j 0.419
\]
Using Equation (14), the calculated voltage ratio for point F is

\[
\frac{V_{bc}}{V_{FBC}} = \left| 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right| = 0.972
\]

The voltage ratio for point F determined from observed data is

\[
\frac{V_{bc}}{V_{FBC}} = \frac{237}{243.5} = 0.974
\]

Using Equation (43), the calculated voltage ratio for point P is

\[
\frac{V_{Pbc}}{V_{PBC}} = \left| 1 - \frac{\bar{Z}_{MF_1}}{(\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F)(\bar{Z}_{MF_1} + \bar{Z}_{T_1})} \left( \frac{\bar{Z}_{MF_1} \bar{Z}_{S_1}}{\bar{Z}_{S_1} + \bar{Z}_{T_1} + \bar{Z}_{MF_1}} + \frac{\bar{Z}_{MF_2} \bar{Z}_{S_2}}{\bar{Z}_{S_2} + \bar{Z}_{T_2} + \bar{Z}_{MF_2}} \right) \right| = 0.984
\]

The voltage ratio for point P determined from observed data is

\[
\frac{V_{Pbc}}{V_{PBC}} = \frac{238}{240.5} = 0.989
\]

15. Case 4. Synchronous Motor Connected to the Primary of Welder Supply Transformer Bank

![Fig. 13. Circuit for Case No. 4 of Experimental Data](image-url)
Circuit and System Data

\[ Z_{S1} = Z_{S2} = 0.41 + j 0.25 \text{ ohm} \]
\[ Z_{T1} = Z_{T2} = 0.35 + j 0.20 \text{ ohm} \]
\[ Z_F = R + j 0 \text{ ohms} \]

Synchronous Motor:
Rating - 5 kva, 220 v, 1200 rpm, 3φ
Direct-axis reactance (unsat.) = 0 + j 8.7 ohms
Negative-sequence impedance = 1.33 + j 2.35 ohms

The single-phase load remained on long enough for the system to reach steady-state conditions.

Observed Data

<table>
<thead>
<tr>
<th>Motor Conditions</th>
<th>Volts at F</th>
<th>Volts at P</th>
<th>1-φ Current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V_{ab}</td>
<td>V_{bc}</td>
<td>V_{ca}</td>
</tr>
<tr>
<td>Motor Not</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connected</td>
<td>238</td>
<td>238</td>
<td>237.5</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>230.5</td>
<td>233.5</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>227.5</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>223</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>215.5</td>
<td>225</td>
</tr>
<tr>
<td>Motor Running</td>
<td>238</td>
<td>238</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>231</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>228</td>
<td>231</td>
</tr>
<tr>
<td>(I_f = 3)</td>
<td>238</td>
<td>223.5</td>
<td>228.5</td>
</tr>
<tr>
<td></td>
<td>238</td>
<td>217</td>
<td>225</td>
</tr>
<tr>
<td>Motor Running</td>
<td>232</td>
<td>232.5</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>232</td>
<td>225.5</td>
<td>227.5</td>
</tr>
<tr>
<td>Approx.</td>
<td>232</td>
<td>223</td>
<td>226</td>
</tr>
<tr>
<td>Full-Load</td>
<td>232</td>
<td>219</td>
<td>223.5</td>
</tr>
<tr>
<td>(I_f = 3)</td>
<td>232</td>
<td>212</td>
<td>219.5</td>
</tr>
</tbody>
</table>
## Calculated Data

<table>
<thead>
<tr>
<th>1-Φ Current</th>
<th>(V_{bc}/V_{F_{BC}})</th>
<th>(V_{P_{bc}}/V_{P_{BC}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor Not Connected</td>
<td>5.3</td>
<td>0.965</td>
</tr>
<tr>
<td></td>
<td>7.4</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>10.4</td>
<td>0.933</td>
</tr>
<tr>
<td></td>
<td>15.5</td>
<td>0.900</td>
</tr>
<tr>
<td>Motor Running No-Load ((I_f = 3))</td>
<td>5.3</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>10.55</td>
<td>0.935</td>
</tr>
<tr>
<td></td>
<td>15.7</td>
<td>0.906</td>
</tr>
<tr>
<td>Motor Running Approx. ((I_f = 3))</td>
<td>5.2</td>
<td>0.968</td>
</tr>
<tr>
<td></td>
<td>7.3</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>10.3</td>
<td>0.936</td>
</tr>
<tr>
<td></td>
<td>15.3</td>
<td>0.905</td>
</tr>
</tbody>
</table>

### Sample Calculations

Consider the case where the motor is running under load and the single-phase load is 5.2 amp. For this case,

\[
\bar{Z}_S_1 = \bar{Z}_S_2 = 0.41 + j 0.25 \\
\bar{Z}_{MP_1} = 0 + j 8.7 \\
\bar{Z}_T_1 = \bar{Z}_T_2 = 0.35 + j 0.20 \\
\bar{Z}_{MP_2} = 1.33 + j 2.35
\]

\[
\bar{Z}_F = \frac{225.5}{5.2} = 43.4 + j 0
\]

From Equations (56) and (57)

\[
\bar{Z}_1 = \frac{\bar{Z}_{S_1} \bar{Z}_{MP_1}}{\bar{Z}_{S_1} + \bar{Z}_{MP_1}} + \bar{Z}_{T_1} = 0.738 + j 0.462
\]

\[
\bar{Z}_2 = \frac{\bar{Z}_{S_2} \bar{Z}_{MP_2}}{\bar{Z}_{S_2} + \bar{Z}_{MP_2}} + \bar{Z}_{T_2} = 0.687 + j 0.442
\]
Using Equation (14), the calculated voltage ratio for point F is

\[
\frac{V_{bc}}{V_{FBC}} = \left| 1 - \frac{Z_1 + Z_2}{Z_1 + Z_2 + Z_F} \right| = 0.968
\]

The voltage ratio for point F determined from observed data is

\[
\frac{V_{bc}}{V_{FBC}} = \frac{225.5}{232.5} = 0.971
\]

Using Equation (51), the calculated voltage ratio for point P is

\[
\frac{V_{Pbc}}{V_{PBC}} = \left| 1 - \frac{(Z_1 - Z_{T1}) + (Z_2 - Z_{T2})}{Z_1 + Z_2 + Z_F} \right| = 0.985
\]

The voltage ratio for point P determined from observed data is

\[
\frac{V_{Pbc}}{V_{PBC}} = \frac{229.5}{232.5} = 0.987
\]

16. Case 5. Effect of Synchronous Motor Excitation

![Circuit Diagram](image)

**FIG. 14. CIRCUIT FOR CASE NO. 5 OF EXPERIMENTAL DATA**
Circuit and System Data

\[ Z_{S1} = Z_{S2} = 0.41 + j 0.25 \text{ ohm} \]
\[ Z_{T1} = Z_{T2} = 0.35 + j 0.20 \text{ ohm} \]
\[ Z_F = R + j 0 \text{ ohms} \]

Synchronous Motor:
Rating - 5 kva, 220 v, 1200 rpm, 3φ
Direct-axis reactance (unsat.) = \( 0 + j 8.7 \) ohms
Negative-sequence impedance = \( 1.33 + j 2.35 \) ohms

The single-phase load remained on long enough for the system to reach steady-state conditions.

Observed Data

<table>
<thead>
<tr>
<th>Synchronous Motor Running Without Load</th>
<th>Motor Field Current</th>
<th>Voltages at F</th>
<th>1-φ Current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( V_{ab} )</td>
<td>( V_{bc} )</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>228</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td></td>
<td>228</td>
<td>218</td>
</tr>
<tr>
<td></td>
<td></td>
<td>227</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>234</td>
<td>234</td>
</tr>
<tr>
<td></td>
<td></td>
<td>233</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td></td>
<td>233</td>
<td>219</td>
</tr>
<tr>
<td></td>
<td>2.92</td>
<td>239</td>
<td>239</td>
</tr>
<tr>
<td></td>
<td></td>
<td>238</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td></td>
<td>238</td>
<td>223.5</td>
</tr>
<tr>
<td></td>
<td>3.72</td>
<td>244</td>
<td>244</td>
</tr>
<tr>
<td></td>
<td></td>
<td>242</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td></td>
<td>242</td>
<td>226.5</td>
</tr>
</tbody>
</table>
### Calculated Data

<table>
<thead>
<tr>
<th>Motor Field Current</th>
<th>1-φ Current</th>
<th>$V_{bc}/V_{FBC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calc.</td>
</tr>
<tr>
<td>1.05</td>
<td>9.4</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td>13.9</td>
<td>0.926</td>
</tr>
<tr>
<td>2.10</td>
<td>9.7</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td>14.2</td>
<td>0.926</td>
</tr>
<tr>
<td>2.92</td>
<td>9.9</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td>14.45</td>
<td>0.926</td>
</tr>
<tr>
<td>3.72</td>
<td>10.05</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td>14.65</td>
<td>0.929</td>
</tr>
</tbody>
</table>

### Sample Calculations

Consider the case where the motor field current is 3.72 amp and the single-phase load current is 10.05 amp. For this case:

\[
\begin{align*}
\bar{Z}_S_1 &= \bar{Z}_S_2 = 0.41 + j 0.25 \\
\bar{Z}_T_1 &= \bar{Z}_T_2 = 0.35 + j 0.20 \\
\bar{Z}_{MF_1} &= 0 + j 8.7 \\
\bar{Z}_{MF_2} &= 1.33 + j 2.35 \\
\bar{Z}_F &= \frac{233}{10.05} = 23.2 + j 0 \\
\end{align*}
\]

From Equations (48) and (49)

\[
\begin{align*}
\bar{Z}_1 &= \frac{(\bar{Z}_T_1 + \bar{Z}_S_1) \bar{Z}_{MF_1}}{\bar{Z}_T_1 + \bar{Z}_S_1 + \bar{Z}_{MF_1}} = 0.672 + j 0.49 \\
\bar{Z}_2 &= \frac{(\bar{Z}_T_2 + \bar{Z}_S_2) \bar{Z}_{MF_2}}{\bar{Z}_T_2 + \bar{Z}_S_2 + \bar{Z}_{MF_2}} = 0.54 + j 0.419 \\
\end{align*}
\]
Using Equation (14), the calculated voltage ratio for point \( F \) is

\[
\frac{V_{bc}}{V_{FBC}} = \left| 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right| = 0.950
\]

The voltage ratio for point \( F \) determined from observed data is

\[
\frac{V_{bc}}{V_{FBC}} = \frac{233}{244} = 0.955
\]

17. Case 6. Induction Motor Connected to the Welder Bus - Cyclic Welding Load

![Circuit Diagram]

**FIG. 15. CIRCUIT FOR CASE NO. 6 OF EXPERIMENTAL DATA**

**Circuit and System Data**

\[
\begin{align*}
\bar{Z}_{S1} &= \bar{Z}_{S2} = 0.41 + j 0.25 \text{ ohm} \\
\bar{Z}_{T1} &= \bar{Z}_{T2} = 0.35 + j 0.20 \text{ ohm} \\
\bar{Z}_F &= R + j 0 \text{ ohms}
\end{align*}
\]

**Induction Motor:**

- Rating - 7 ½ hp, 220 v, 1135 rpm, 3ϕ
- No-load impedance = 1.62 + j 13.2 ohms
- Approx. 1/2-load impedance = 4.27 + j 9.15 ohms
- Blocked-rotor impedance = 0.875 + j 1.50 ohms
The single-phase load "on" and "off" times are as shown on the oscillograms of Fig. 16.

### Observed Data

<table>
<thead>
<tr>
<th>Oscillogram Number</th>
<th>Motor Load</th>
<th>Voltages at F Before 1-φ Load</th>
<th>Voltages at F With 1-φ Load Steady-state</th>
<th>1-φ Current Phase bc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>V&lt;sub&gt;AB&lt;/sub&gt;</td>
<td>V&lt;sub&gt;BC&lt;/sub&gt;</td>
<td>V&lt;sub&gt;CA&lt;/sub&gt;</td>
</tr>
<tr>
<td>9-1</td>
<td>No-load</td>
<td>217.5</td>
<td>218</td>
<td>217.5</td>
</tr>
<tr>
<td>10-1</td>
<td>No-load</td>
<td>216.5</td>
<td>217</td>
<td>216.5</td>
</tr>
<tr>
<td>22-1</td>
<td>1/2-load</td>
<td>208</td>
<td>210</td>
<td>208.5</td>
</tr>
<tr>
<td>23-1</td>
<td>1/2-load</td>
<td>208</td>
<td>210</td>
<td>208.5</td>
</tr>
</tbody>
</table>

### Calculated Data

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-φ Load</th>
<th>Amplitude on Original Oscillogram</th>
<th>V&lt;sub&gt;bc&lt;/sub&gt;/V&lt;sub&gt;F&lt;/sub&gt;&lt;sub&gt;bc&lt;/sub&gt; From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillogram No. 9-1</td>
<td>Off</td>
<td>37.76</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>On</td>
<td>36.33</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>On</td>
<td>36.10</td>
<td>0.958</td>
</tr>
<tr>
<td></td>
<td>Off</td>
<td>37.44</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>Off</td>
<td>37.56</td>
<td>0.996</td>
</tr>
<tr>
<td></td>
<td>Off</td>
<td>37.72</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculated V<sub>bc</sub>/V<sub>F</sub><sub>bc</sub> = 0.961. Blocked-rotor impedance used for Z<sub>MF1</sub>. 
Fig. 16. Oscillograms for Case 6
Calculated Data (Cont.)

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-ϕ Load</th>
<th>Amplitude on Original Oscillogram mm</th>
<th>(V_{bc}/V_{FBC}) From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Off</td>
<td>33.03</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>On</td>
<td>31.75</td>
<td>0.960</td>
</tr>
<tr>
<td>3</td>
<td>On</td>
<td>31.66</td>
<td>0.958</td>
</tr>
<tr>
<td>4</td>
<td>On</td>
<td>31.52</td>
<td>0.954</td>
</tr>
<tr>
<td>5</td>
<td>On</td>
<td>31.52</td>
<td>0.954</td>
</tr>
<tr>
<td>6</td>
<td>On</td>
<td>31.53</td>
<td>0.954</td>
</tr>
<tr>
<td>7</td>
<td>On</td>
<td>31.49</td>
<td>0.953</td>
</tr>
<tr>
<td>8</td>
<td>On</td>
<td>31.47</td>
<td>0.953</td>
</tr>
<tr>
<td>9</td>
<td>On</td>
<td>31.47</td>
<td>0.953</td>
</tr>
<tr>
<td>10</td>
<td>On</td>
<td>31.49</td>
<td>0.953</td>
</tr>
<tr>
<td>11</td>
<td>Off</td>
<td>32.81</td>
<td>0.994</td>
</tr>
<tr>
<td>12</td>
<td>Off</td>
<td>32.94</td>
<td>0.997</td>
</tr>
<tr>
<td>13</td>
<td>Off</td>
<td>33.02</td>
<td>1.0</td>
</tr>
<tr>
<td>14</td>
<td>Off</td>
<td>32.96</td>
<td>0.997</td>
</tr>
<tr>
<td>15</td>
<td>Off</td>
<td>33.10</td>
<td>1.0</td>
</tr>
<tr>
<td>16</td>
<td>Off</td>
<td>33.06</td>
<td>1.0</td>
</tr>
<tr>
<td>17</td>
<td>Off</td>
<td>33.12</td>
<td>1.0</td>
</tr>
<tr>
<td>18</td>
<td>Off</td>
<td>33.06</td>
<td>1.0</td>
</tr>
<tr>
<td>19</td>
<td>Off</td>
<td>33.03</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>Off</td>
<td>33.03</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculated \(V_{bc}/V_{FBC} = 0.959\). Blocked-rotor impedance used for \(Z_{MF_1}\).

Calculated \(V_{bc}/V_{FBC} = 0.949\). Approx. 1/2-load impedance used for \(Z_{MF_1}\).
Calculated Data (Cont.)

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-Φ Load</th>
<th>Amplitude on Original Oscillogram (mm)</th>
<th>$V_{bc}/V_{FBC}$ From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillogram No. 10-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Off</td>
<td>37.67</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>2 On</td>
<td>36.45</td>
<td>0.968</td>
<td></td>
</tr>
<tr>
<td>3 On</td>
<td>36.20</td>
<td>0.962</td>
<td></td>
</tr>
<tr>
<td>4 On</td>
<td>36.08</td>
<td>0.959</td>
<td></td>
</tr>
<tr>
<td>5 On</td>
<td>36.12</td>
<td>0.959</td>
<td></td>
</tr>
<tr>
<td>6 On</td>
<td>36.12</td>
<td>0.959</td>
<td></td>
</tr>
<tr>
<td>7 Off</td>
<td>37.37</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td>8 Off</td>
<td>37.58</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>9 Off</td>
<td>37.61</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>11 Off</td>
<td>37.62</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Calculated $V_{bc}/V_{FBC} = 0.962$. Blocked-rotor impedance used for $Z_{MF_1}$.

Calculated $V_{bc}/V_{FBC} = 0.953$. No-load impedance used for $Z_{MF_1}$.

| Oscillogram No. 23-1                |          |                                        |                                               |
| 1 Off                                | 33.10    | 1.0                                    |                                               |
| 2 On                                 | 31.96    | 0.965                                  |                                               |
| 3 On                                 | 31.71    | 0.958                                  |                                               |
| 4 On                                 | 31.62    | 0.955                                  |                                               |
| 5 On                                 | 31.56    | 0.954                                  |                                               |
| 6 On                                 | 31.61    | 0.955                                  |                                               |
| 7 On                                 | 31.55    | 0.954                                  |                                               |
| 8 Off                                | 32.90    | 0.995                                  |                                               |
| 9 Off                                | 33.08    | 1.0                                    |                                               |
| 10 Off                               | 33.14    | 1.0                                    |                                               |
| 11 Off                               | 33.17    | 1.0                                    |                                               |
| 12 Off                               | 33.14    | 1.0                                    |                                               |

Calculated $V_{bc}/V_{FBC} = 0.959$. Blocked-rotor impedance used for $Z_{MF_1}$.

Calculated $V_{bc}/V_{FBC} = 0.949$. Approx. 1/2-load impedance used for $Z_{MF_1}$. 
Sample Calculations
Consider the case of oscillogram No. 22-1. For this case

\[ \bar{Z}_S_1 = \bar{Z}_S_2 = 0.41 + j 0.25 \quad \bar{Z}_{MF_1} = 0.875 + j 1.5 \]
\[ \bar{Z}_T_1 = \bar{Z}_T_2 = 0.35 + j 0.20 \quad \bar{Z}_{MF_2} = 0.875 + j 1.5 \]

\[ \bar{Z}_F = \frac{199}{9.38} = 21.2 + j 0 \]

From Equations (48) and (49)

\[ \bar{Z}_1 = \frac{(\bar{Z}_{T_1} + \bar{Z}_S_1) \bar{Z}_{MF_1}}{\bar{Z}_{T_1} + \bar{Z}_S_1 + \bar{Z}_{MF_1}} = 0.46 + j 0.39 \]
\[ \bar{Z}_2 = \frac{(\bar{Z}_{T_2} + \bar{Z}_S_2) \bar{Z}_{MF_2}}{\bar{Z}_{T_2} + \bar{Z}_S_2 + \bar{Z}_{MF_2}} = 0.46 + j 0.39 \]

Using Equation (14), the calculated voltage ratio for point F is

\[ \frac{V_{bc}}{V_{FBC}} = \left| 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right| = 0.959 \]

The voltage ratio for point F determined for the second cycle on the oscillogram is

\[ \frac{V_{bc}}{V_{FBC}} = \frac{31.75}{33.03} = 0.960 \]
18. Case 7. Synchronous Motor Connected to the Welder Bus - Cyclic Welding Load

![Diagram of circuit and system data]

Fig. 17. Circuit for Case No. 7 of Experimental Data

Circuit and System Data
\[ \bar{Z}_{S_1} = \bar{Z}_{S_2} = 0.41 + j 0.25 \text{ ohm} \]
\[ \bar{Z}_{T_1} = \bar{Z}_{T_2} = 0.35 + j 0.20 \text{ ohm} \]
\[ \bar{Z}_F = R + j 0 \text{ ohms} \]

Synchronous Motor:
- Rating - 5 kva, 220 v, 1200 rpm, 3φ
- Direct-axis reactance (unsat.) = 0 + j 12.9 ohms
- Negative-sequence impedance = 1.06 + j 1.41 ohms
- Transient reactance = 0 + j 3.08 ohms
- Sub-transient reactance = 0 + j 1.56 ohms

The single-phase load "on" and "off" times are as shown on the oscillograms of Fig. 18.
FIG. 18. OSCILLOGRAMS FOR CASE 7
Observed Data

<table>
<thead>
<tr>
<th>Oscillogram Number</th>
<th>Motor Load</th>
<th>Field Current</th>
<th>Voltages at F Before 1-Φ Load</th>
<th>Steady-state Voltages at F With 1-Φ Load</th>
<th>1-Φ Current Phase bc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>V&lt;sub&gt;AB&lt;/sub&gt;</td>
<td>V&lt;sub&gt;BC&lt;/sub&gt;</td>
<td>V&lt;sub&gt;CA&lt;/sub&gt;</td>
</tr>
<tr>
<td>5-2</td>
<td>No-load</td>
<td>1.75</td>
<td>226</td>
<td>227</td>
<td>227</td>
</tr>
<tr>
<td>6-2</td>
<td>No-load</td>
<td>1.75</td>
<td>226</td>
<td>227</td>
<td>227</td>
</tr>
<tr>
<td>8-2</td>
<td>No-load</td>
<td>1.75</td>
<td>226</td>
<td>227</td>
<td>227</td>
</tr>
<tr>
<td>24-2</td>
<td>1/2-load</td>
<td>3.30</td>
<td>224</td>
<td>225</td>
<td>225</td>
</tr>
</tbody>
</table>

Calculated Data

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-Φ Load</th>
<th>Amplitude on Original Oscillogram mm</th>
<th>V&lt;sub&gt;bc&lt;/sub&gt;/V&lt;sub&gt;F&lt;sub&gt;BC&lt;/sub&gt;&lt;/sub&gt; From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillogram No. 5-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Off</td>
<td>35.22</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>On</td>
<td>33.87</td>
<td>0.961</td>
</tr>
<tr>
<td>3</td>
<td>On</td>
<td>33.80</td>
<td>0.959</td>
</tr>
<tr>
<td>4</td>
<td>Off</td>
<td>35.19</td>
<td>0.998</td>
</tr>
<tr>
<td>5</td>
<td>Off</td>
<td>35.25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculated \( V_{bc}/V_{F_{BC}} = 0.965 \). Sub-transient reactance used for \( Z_{MF1} \)
Calculated Data (Cont.)

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-φ Load</th>
<th>Oscillogram No. 8-2</th>
<th>Amplitude on Original Oscillogram</th>
<th>( V_{bc}/V_{FBC} ) From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Oscillogram No. 8-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>35.19</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>Off</td>
<td>35.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>On</td>
<td>33.89</td>
<td></td>
<td>0.963</td>
</tr>
<tr>
<td>3</td>
<td>On</td>
<td>33.81</td>
<td></td>
<td>0.961</td>
</tr>
<tr>
<td>4</td>
<td>On</td>
<td>33.68</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>5</td>
<td>On</td>
<td>33.66</td>
<td></td>
<td>0.957</td>
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<tr>
<td>6</td>
<td>On</td>
<td>33.72</td>
<td></td>
<td>0.958</td>
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<tr>
<td>7</td>
<td>On</td>
<td>33.64</td>
<td></td>
<td>0.957</td>
</tr>
<tr>
<td>8</td>
<td>On</td>
<td>33.70</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>9</td>
<td>On</td>
<td>33.70</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>10</td>
<td>On</td>
<td>33.69</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>11</td>
<td>On</td>
<td>33.69</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>12</td>
<td>On</td>
<td>33.69</td>
<td></td>
<td>0.958</td>
</tr>
<tr>
<td>13</td>
<td>Off</td>
<td>35.17</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>14</td>
<td>Off</td>
<td>35.20</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>15</td>
<td>Off</td>
<td>35.20</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>16</td>
<td>Off</td>
<td>35.14</td>
<td></td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculated \( V_{bc}/V_{FBC} = 0.965 \). Sub-transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.957 \). Transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.953 \). Direct-axis reactance used for \( Z_{MF_1} \).

| Oscillogram No. 6-2 |
|----------------------|-----------------|-----------------|
| 1                    | Off             | 35.17           | 1.0             |
| 2                    | On              | 33.73           | 0.960           |
| 3                    | On              | 33.71           | 0.959           |
| 4                    | On              | 33.62           | 0.957           |
| 5                    | On              | 33.60           | 0.956           |
| 6                    | Off             | 34.92           | 0.994           |
| 7                    | Off             | 35.12           | 1.0             |
| 8                    | Off             | 35.15           | 1.0             |
| 9                    | Off             | 35.12           | 1.0             |
| 10                   | Off             | 35.16           | 1.0             |

Calculated \( V_{bc}/V_{FBC} = 0.965 \). Sub-transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.957 \). Transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.953 \). Direct-axis reactance used for \( Z_{MF_1} \).
### Calculated Data (Cont.)

<table>
<thead>
<tr>
<th>Cycle Number Indicated on Oscillogram</th>
<th>1-Φ Load</th>
<th>Oscillogram No. 24-2</th>
<th>Amplitude on Original Oscillogram (nm)</th>
<th>( V_{bc}/V_{FBC} ) From Oscillogram Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Off</td>
<td></td>
<td></td>
<td>35.21</td>
<td>1.0</td>
</tr>
<tr>
<td>2 On</td>
<td></td>
<td></td>
<td>33.58</td>
<td>0.955</td>
</tr>
<tr>
<td>3 On</td>
<td></td>
<td></td>
<td>33.46</td>
<td>0.950</td>
</tr>
<tr>
<td>4 On</td>
<td></td>
<td></td>
<td>33.51</td>
<td>0.951</td>
</tr>
<tr>
<td>5 On</td>
<td></td>
<td></td>
<td>33.43</td>
<td>0.950</td>
</tr>
<tr>
<td>6 On</td>
<td></td>
<td></td>
<td>33.41</td>
<td>0.950</td>
</tr>
<tr>
<td>7 On</td>
<td></td>
<td></td>
<td>33.51</td>
<td>0.951</td>
</tr>
<tr>
<td>8 On</td>
<td></td>
<td></td>
<td>33.43</td>
<td>0.950</td>
</tr>
<tr>
<td>9 On</td>
<td></td>
<td></td>
<td>33.44</td>
<td>0.950</td>
</tr>
<tr>
<td>10 On</td>
<td></td>
<td></td>
<td>33.46</td>
<td>0.951</td>
</tr>
<tr>
<td>11 On</td>
<td></td>
<td></td>
<td>33.42</td>
<td>0.950</td>
</tr>
<tr>
<td>12 On</td>
<td></td>
<td></td>
<td>33.50</td>
<td>0.951</td>
</tr>
<tr>
<td>13 On</td>
<td></td>
<td></td>
<td>33.48</td>
<td>0.951</td>
</tr>
<tr>
<td>14 Off</td>
<td></td>
<td></td>
<td>35.08</td>
<td>0.998</td>
</tr>
<tr>
<td>15 Off</td>
<td></td>
<td></td>
<td>35.15</td>
<td>1.0</td>
</tr>
<tr>
<td>16 Off</td>
<td></td>
<td></td>
<td>35.10</td>
<td>0.998</td>
</tr>
<tr>
<td>17 Off</td>
<td></td>
<td></td>
<td>35.19</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculated \( V_{bc}/V_{FBC} = 0.965 \). Sub-transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.957 \). Transient reactance used for \( Z_{MF_1} \).

Calculated \( V_{bc}/V_{FBC} = 0.953 \). Direct-axis reactance used for \( Z_{MF_1} \).
Sample Calculations

Consider the case of oscillogram No. 8-2. For this case

\[
\begin{align*}
\bar{Z}_S_1 &= \bar{Z}_S_2 = 0.41 + j 0.25 \\
\bar{Z}_T_1 &= \bar{Z}_T_2 = 0.35 + j 0.20
\end{align*}
\]

\[
\begin{align*}
\bar{Z}_{M_F_1} &= 0 + j 1.56 \\
\bar{Z}_{M_F_2} &= 1.06 + j 1.41
\end{align*}
\]

\[
\bar{Z}_F = \frac{218}{9.0} = 24.1 + j 0
\]

From Equations (48) and (49)

\[
\bar{Z}_1 = \frac{(\bar{Z}_{T_1} + \bar{Z}_S_1) \bar{Z}_{M_F_1}}{\bar{Z}_{T_1} + \bar{Z}_S_1 + \bar{Z}_{M_F_1}} = 0.401 + j 0.501
\]

\[
\bar{Z}_2 = \frac{(\bar{Z}_{T_2} + \bar{Z}_S_2) \bar{Z}_{M_F_2}}{\bar{Z}_{T_2} + \bar{Z}_S_2 + \bar{Z}_{M_F_2}} = 0.472 + j 0.370
\]

Using Equation (14), the calculated voltage ratio for point F is

\[
\frac{V_{bc}}{V_{F_{BC}}} = \left| 1 - \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F} \right| = 0.965
\]

The voltage ratio for point F determined for the second cycle on the oscillogram is

\[
\frac{V_{bc}}{V_{F_{BC}}} = \frac{33.89}{35.19} = 0.963
\]
V. MAGNITUDE OF THE EFFECT OF THE MOTORS IN PRACTICAL CASES

19. Introduction

In order to give some conception of the magnitude of the effect of the motors in practical cases, calculations were made using data furnished by the Commonwealth Edison Company of Chicago, Illinois, for six typical single-phase welder installations supplied by their system in the Chicago area. In each of these cases the plant-bus transformer bank was connected YΔ and the motor loads were for all practical purposes composed only of induction motors. A value of 4 percent (on the motor base) was used for the reactance of the transformers supplying the motor load at point P. This value is considered to be a reasonable average for these transformers. An exact value is difficult to determine but is really not necessary, as this value can be changed appreciably without producing much effect on the magnitude of the voltage change at P.

In the table of calculated results, only the drop in the line-to-neutral voltage of phase A is shown, since the drops in the line-to-neutral voltages of the other two phases are considerably less. In the case of the line-to-line voltage drops, the voltage drop between lines B and C is not shown, since it is much smaller than the other two.

It should be noted in Case 4 that the effect of the motors was to increase the voltage drop between lines C and A. However, between lines A and B, where there is much more drop, there is a considerable improvement due to the motors. Since the maximum voltage drop is of most importance, increasing the voltage drop between lines C and A does not appear to be objectionable. Although the voltage drop may be increased by the motors under some conditions, it always occurs on one of the phases which does not have the maximum drop in voltage.
20. Circuit and System Data

![Diagram showing a circuit with a source, supply system impedance, transformer bank, and motor load.]  

**Fig. 19. Welder Supply System Used in the Determination of the Voltage Changes Produced in Practical Cases**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Welder Load kva</th>
<th>Welder Load volts</th>
<th>Welder Power Factor percent</th>
<th>Duration of Weld cycles</th>
<th>Plant-Bus Transformer Bank kva</th>
<th>Impedance of Plant-Bus Transformer percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72</td>
<td>232</td>
<td>67</td>
<td>30</td>
<td>3-75</td>
<td>2.40</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>230</td>
<td>40</td>
<td>10</td>
<td>3-50</td>
<td>3.25</td>
</tr>
<tr>
<td>3</td>
<td>147</td>
<td>223</td>
<td>89</td>
<td>10</td>
<td>3-25</td>
<td>3.25</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>226</td>
<td>98</td>
<td>12</td>
<td>3-50</td>
<td>2.40</td>
</tr>
<tr>
<td>5</td>
<td>58</td>
<td>221</td>
<td>60</td>
<td>15</td>
<td>3-25</td>
<td>2.40</td>
</tr>
<tr>
<td>6</td>
<td>94</td>
<td>218</td>
<td>47</td>
<td>24</td>
<td>3-37-1/2</td>
<td>2.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Resistance of Supply System ohms</th>
<th>Reactance of Supply System ohms</th>
<th>Motors at P hp</th>
<th>Motors at F hp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.538</td>
<td>0.484</td>
<td>2200</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.424</td>
<td>0.451</td>
<td>1900</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>0.310</td>
<td>0.481</td>
<td>3300</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>0.361</td>
<td>0.493</td>
<td>750</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>1.678</td>
<td>1.151</td>
<td>650</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>0.380</td>
<td>0.571</td>
<td>2000</td>
<td>120</td>
</tr>
</tbody>
</table>
21. **Calculated Results**

The following results are based on the lettering convention for the primary and secondary lines shown in Fig. 7.

**DROP IN LINE-TO-NEUTRAL VOLTAGE OF PHASE A AT POINT P**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Percent Drop, No Motors Connected</th>
<th>Percent Drop, All Motors Connected</th>
<th>Percent Improvement Due to Motors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.77</td>
<td>0.62</td>
<td>19.5</td>
</tr>
<tr>
<td>2</td>
<td>0.81</td>
<td>0.61</td>
<td>24.6</td>
</tr>
<tr>
<td>3</td>
<td>1.17</td>
<td>0.86</td>
<td>26.5</td>
</tr>
<tr>
<td>4</td>
<td>1.04</td>
<td>0.91</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>1.76</td>
<td>1.44</td>
<td>18.2</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>0.78</td>
<td>25.7</td>
</tr>
</tbody>
</table>

**DROP IN VOLTAGE BETWEEN LINES A AND B AT POINT P**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Voltage A</th>
<th>Voltage B</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.54</td>
<td>0.50</td>
<td>7.4</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.42</td>
<td>12.5</td>
</tr>
<tr>
<td>3</td>
<td>1.09</td>
<td>0.83</td>
<td>23.9</td>
</tr>
<tr>
<td>4</td>
<td>1.14</td>
<td>0.98</td>
<td>14.0</td>
</tr>
<tr>
<td>5</td>
<td>1.04</td>
<td>1.02</td>
<td>2.0</td>
</tr>
<tr>
<td>6</td>
<td>0.73</td>
<td>0.59</td>
<td>19.2</td>
</tr>
</tbody>
</table>

**DROP IN VOLTAGE BETWEEN LINES C AND A AT POINT P**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Voltage C</th>
<th>Voltage A</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.61</td>
<td>0.46</td>
<td>24.5</td>
</tr>
<tr>
<td>2</td>
<td>0.74</td>
<td>0.54</td>
<td>27.0</td>
</tr>
<tr>
<td>3</td>
<td>0.66</td>
<td>0.55</td>
<td>16.7</td>
</tr>
<tr>
<td>4</td>
<td>0.41</td>
<td>0.44</td>
<td>-7.3</td>
</tr>
<tr>
<td>5</td>
<td>1.60</td>
<td>1.21</td>
<td>24.4</td>
</tr>
<tr>
<td>6</td>
<td>0.84</td>
<td>0.64</td>
<td>23.8</td>
</tr>
</tbody>
</table>

22. **Sample Calculations**

In all calculations the following base quantities were used:

Base $3\phi \text{kva} = 1000$
For the high voltage part of the circuit:

Base line-to-line voltage = 3760 v

\[
\text{Base line current} = \frac{1,000,000}{\sqrt{3} \times 3760} = 153.6 \text{ amp}
\]

Base impedance = \( \frac{3760}{\sqrt{3}} \) = 14.13 ohms

For the low voltage part of the circuit:

Base line-to-line voltage = 240 v

\[
\text{Base line current} = \frac{1,000,000}{\sqrt{3} \times 240} = 2405 \text{ amp}
\]

Base impedance = \( \frac{240}{\sqrt{3}} \) = 0.0576 ohm

Consider the drop in the line-to-neutral voltage of phase A for case 6.

The welder kva at 240 v = \( \left( \frac{240}{218} \right)^2 \times 94 = 114 \text{ kva} \)

\[
\bar{Z}_F = \frac{1000}{114} \angle 62^\circ = 8.77 \angle 62^\circ = 4.12 + j 7.74 \text{ per unit}
\]

\[
\bar{Z}_{T_1} = \bar{Z}_{T_2} = (0 + j 0.024) \frac{1000}{112.5} = 0 + j 0.214 \text{ per unit}
\]

\[
\bar{Z}_{S_1} = \bar{Z}_{S_2} = \frac{0.38 + j 0.571}{14.13} = 0.0269 + j 0.0404 \text{ per unit}
\]

\[
\bar{Z}_{MF_1} = \bar{Z}_{MF_1}^* = \frac{1000 \times 0.8 \angle 27^\circ}{120 \times 0.746} = 8.94 \angle 27^\circ
\]

\[
= 7.96 + j 4.06 \text{ per unit}
\]
\[ \bar{Z}_{MF_2} = \frac{1000 \times 0.135 \angle 70^\circ}{120 \times 0.746} = 1.51 \angle 70^\circ \]

\[ = 0.515 + j \, 1.415 \, \text{per unit} \]

\[ \bar{Z}_{MP_1} = \frac{1000}{2000 \times 0.746} \left( 0.8 \angle 27^\circ + 0.04 \angle 90^\circ \right) \]

\[ = 0.549 \angle 29.5^\circ = 0.478 + j \, 0.270 \, \text{per unit} \]

\[ \bar{Z}_{MP_2} = \frac{1000}{2000 \times 0.746} \left( 0.135 \angle 70^\circ + 0.04 \angle 90^\circ \right) \]

\[ = 0.116 \angle 74.5^\circ = 0.031 + j \, 0.112 \, \text{per unit} \]

From Equations (18) and (19)

\[ \bar{Z}_1 = \frac{(\frac{\bar{Z}_{S_1}}{\bar{Z}_{S_1} + \bar{Z}_{MP_1}} + \bar{Z}_{T_1})}{\bar{Z}_{S_1} + \bar{Z}_{MP_1}} \bar{Z}_{MF_1} \]

\[ = 0.0324 + j \, 0.246 \, \text{per unit} \]

\[ \bar{Z}_2 = \frac{(\frac{\bar{Z}_{S_2}}{\bar{Z}_{S_2} + \bar{Z}_{MP_2}} + \bar{Z}_{T_2})}{\bar{Z}_{S_2} + \bar{Z}_{MP_2}} \bar{Z}_{MF_2} \]

\[ = 0.0229 + j \, 0.211 \, \text{per unit} \]
From Equations (26) and (28)

\[
\bar{M} = \frac{\bar{Z}_{MF_1} \bar{Z}_{MP_1} \bar{Z}_{S_1}}{\bar{Z}_{S_1} \bar{Z}_{MP_1} + \bar{Z}_{T_1} \bar{Z}_{S_1} + \bar{Z}_{T_1} \bar{Z}_{MP_1} + \bar{Z}_{MF_1} \bar{Z}_{S_1} + \bar{Z}_{MF_1} \bar{Z}_{MP_1}}
\]

\[= 0.0266 + j 0.0352 \text{ per unit} \]

\[
\bar{N} = \frac{\bar{Z}_{MF_2} \bar{Z}_{MP_2} \bar{Z}_{S_2}}{\bar{Z}_{S_2} \bar{Z}_{MP_2} + \bar{Z}_{T_2} \bar{Z}_{S_2} + \bar{Z}_{T_2} \bar{Z}_{MP_2} + \bar{Z}_{MF_2} \bar{Z}_{S_2} + \bar{Z}_{MF_2} \bar{Z}_{MP_2}}
\]

\[= 0.0153 + j 0.0258 \text{ per unit} \]

Using Equation (39), the voltage ratio for point P is

\[
\frac{V_{P_a}}{V_{P_A}} = \left| \frac{\bar{V}_{P_a_1} + \bar{V}_{P_a_2}}{\bar{V}_{P_A}} \right| = \left| 1 - \frac{\bar{Z}_{MF_1}}{(\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_F)(\bar{Z}_{MF_1} + \bar{Z}_{T_1})} \right| (\bar{M} + \bar{N})
\]

\[= | 1 - 0.00785 \angle 9.2^\circ | = 0.9922 \]

This is the same as a drop in the line-to-neutral voltage of phase A at point P of

\[
\frac{V_{P_A} - V_{P_a}}{V_{P_A}} \times 100 = (1 - 0.9922) 100 = 0.78\%
\]

When there are no motors connected to the system (all motor impedances = \(\omega\)), a similar calculation shows that the drop in the line-to-neutral voltage of phase A at point P is 1.05 percent. Therefore the improvement due to the motors is

\[
\frac{1.05 - 0.78}{1.05} \times 100 = 25.7\%.
\]
VI. CHART SOLUTION OF VOLTAGE DROPS

23. Chart Development

The equations developed in the preceding sections can be used to calculate the voltage drop caused by a single-phase welder provided all the impedances are known. However, a great amount of labor is involved in making these calculations, as is evident from an examination of certain equations such as (26), (28) and (31). For use by the power company engineer, who would need to make many such calculations, it would be desirable to obtain the voltage drops by a less time-consuming method. For this reason a set of charts was prepared, from which it is possible to obtain graphically the voltage drop caused by a single-phase welder.

In order to make possible the construction of these charts it was necessary to make some assumptions concerning the system. They are all met fairly well in practice and are entirely reasonable. These assumptions are:

a. The system can be represented by the circuit of Fig. 20.

b. The impedance of the transformers supplying the welder is composed entirely of reactance; that is, the resistance is negligible.

c. The impedance of the transformers supplying the motors at P is assumed to be 0.04 per unit on the motor base (hp X 0.746 = base kva).

d. The motor loads at both P and F are entirely induction motors.

e. The positive- and negative-sequence impedances of the motors are assumed to be 0.8 \angle 27^\circ \text{ per unit and } 0.135 \angle 70^\circ \text{ per unit respectively (hp X 0.746 = base kva).}

f. The voltage-drop range of the charts is limited to small voltage changes that would normally be permitted on a power system.

In those cases which do not satisfy the above conditions, it will be necessary to calculate the voltage drop by the methods of the preceding sections.

The charts were constructed by plotting the results of a large number of calculations. The number of calculations that were made was sufficient to cover the range of each
variable that affects the voltage drop. The manner in which each variable is taken into account is the result of an experimental process of determining a workable chart form, and not the result of any mathematical procedure. The charts give only the maximum line-to-line voltage change for a ΔΔ transformer bank and the maximum line-to-neutral voltage change for a ΥΔ transformer bank. These two voltage changes are given since they are the most important in a practical case.

The representation of several of the variables by a group of radial straight lines is not an exact representation but is extremely close to being exact. The error involved in using these straight lines is less than the error that is inherently present in any graphical solution. In most cases the error of the system data is greater than any error caused by the non-linearity of the charts. For these reasons it is thought that the charts represent a very practical solution to the time-consuming problem of making voltage drop calculations.

24. Instructions for Using the Charts

The circuit diagram to which these charts apply is shown in Fig. 20. All voltage drops computed or read from the charts will be for the point P (the primary of the welder supply transformer) and for the phases indicated in Fig. 21.

![Diagram of circuit](image-url)
The following procedure is suggested when it is desirable to change the usual method of welder calculations as little as possible. It is, of course, apparent that the voltage drop including the effect of all the motors could be obtained immediately by the use of Step C.

Step A. Calculate the voltage change at the point P by the usual method, neglecting the effect of the motors at either the point P or the point F. If this calculation gives a result which is definitely acceptable or one so large that the motors could not possibly bring the voltage drop within acceptable limits the procedure may be stopped here.

Step B. If Step A indicates the advisability of a further calculation, then the chart with the correction factor K should be used. With this chart, correction is made for the motors at point P. It is necessary to determine first the number M, which is a function of the motors at P and also of the magnitude of the system supply impedance to the point P. The product of the hp at P and the system supply impedance $Z_s$ in per unit on a 10,000 kva base gives the number M. The value of K, corresponding to this value of M, multiplied by the value of the voltage drop computed in Step A will give the voltage drop at P corrected for the effect of the motors at P. The procedure may
be stopped at this point if satisfactory results are obtained.

Step C. If a further refinement seems necessary, the correction of the motors at the point F may be taken into account. This involves the calculation of the values which are necessary for the use of the chart that takes into account the effect of motors at both F and P. These values are:

a. The supply system impedance in per unit on a 10,000 kva base; this was calculated in Step B above.

b. The welder kva at the rated secondary voltage of the welder supply transformer. This is the kva that the welder would take if rated secondary voltage were applied to its terminals. The welder power factor must also be known.

c. The value of the supply-system reactance divided by the supply-system resistance. This is denoted by the X/R ratio on the charts.

d. The welder supply transformer impedance in per unit with the welder kva (as found in (b) above) as the per unit base.

e. The constant M. This was found in Step B.

f. The ratio of the motor hp load at F to the welder kva at F.

After these quantities have been determined, the charts may be used by the following procedure to find the voltage drop at P directly.

g. Select the chart corresponding to the ratio of the motor hp load at F to the welder kva at F -- calculated in part (f) above. In general, there will be no chart corresponding to this ratio. In such a case, it will be necessary to use two charts and interpolate between the two values of the voltage drop obtained from these two charts -- one above and one below the actual ratio. It will also be necessary to select from two charts the chart that has values of welder kva in the range of the particular problem.

h. Enter the charts at the lower left hand corner. In this region it will be necessary to select the group of curves corresponding to the per
FIG. 22. SAMPLE CHART ILLUSTRATING THE USE OF THE CHARTS
unit impedance of the welder transformer (welder kva as base). This was calculated in (d).

i. From the power factor scale on the left hand margin select the welder power factor. Then move horizontally to the right to an intersection with the curve corresponding to the value of X/R for the supply system impedance.

j. From this intersection move vertically upward until an intersection is made with the line corresponding to the magnitude of the welder kva.

k. From this point move horizontally to the right until an intersection is made with the line corresponding to the value of the number M.

l. From this point move vertically downward to an intersection with the line corresponding to the per unit value of the supply system impedance.

m. Then move horizontally to the left and read the voltage drop in percent on the vertical scale in the middle of the chart.

25. Numerical Example

![Diagram of circuit]

**FIG. 23. CIRCUIT FOR THE EXAMPLE ILLUSTRATING THE USE OF THE CHARTS**
Data as Given:

Motor load at P - 600 hp
Motor load at F - 40 hp
Welder takes 73 kva at 228 v and 65% power factor (at the welder terminals)
Impedance of YΔ transformer bank is 3.75% on its own rating of 100 kva
Transformer voltage ratio: 2170/240

Impedance from the source to the point P may be given in either of two ways:

1. Three-phase short circuit capacity at P is 14,300 kva at an impedance angle of 56.3° or X/R is 1.5.
2. Impedance per phase in ohms is \(0.548 + j 0.822\) (2170/3760 volt base).
   Note that these are equivalent forms.

Conversion of Data to a Usable Form:

a. The supply impedance must be expressed in per unit on a 10,000 kva base as indicated on the charts.
   1. If the impedance is expressed as a short-circuit capacity, it can be converted to a per unit value directly. For this case the short-circuit capacity is 14,300 kva. The value on a 10,000 kva base is then
      \[
      \frac{10,000}{14,300} = 0.70 \text{ per unit}
      \]
   2. If the impedance is expressed in ohms, then the base impedance for 10,000 kva at the particular voltage must first be found.

      Base voltage = \(\frac{3760}{\sqrt{3}} = 2170\)
      Base kva per phase = \(\frac{10,000}{3} = 3,333\)
      Base current = \(\frac{3,333,000}{2170} = 1536\)
Base impedance $= \frac{2170}{1536} = 1.413$

The impedance $0.548 + j 0.822$ becomes

$$Z_S = \frac{0.548 + j 0.822}{1.413 + j 1.413} = 0.388 + j 0.582 \text{ per unit}$$

$$Z_S = \sqrt{(0.388)^2 + (0.582)^2} = 0.70 \text{ per unit}$$

b. The rated transformer voltage is 240 v. Therefore the welder kva must be corrected to 240 v.

$$240 \times 73 = 80 \text{ kva at 65% power factor lagging.}$$

Note: The kva on the charts is always the kva the welder would take if rated secondary voltage were applied to its terminals.

c. $X/R$ ratio

1. This was given directly as 1.5
2. For this method of expressing the impedance

$$\frac{X}{R} = \frac{0.582}{0.388} = 1.5$$

d. The $\Delta$ transformer has an impedance of 3.75\% on 100 kva. This must be converted to 80 kva since the charts have this impedance expressed with the welder kva as base.

$$\frac{80}{100} \times (0.0375) = 0.03 \text{ per unit on the 80 kva base}$$

e. Number $M$ for the effect of the motor load at P.

$$M = (Z_s) X (\text{Motor hp at P}) = 0.70 \times 600 = 420$$

f. Motor hp at $F$ $= \frac{40}{80} = \frac{1}{2}$

Welder kva

Since there is no chart for the ratio of motor load at $F$ to welder kva equal to the value of $\frac{1}{2}$ obtained in (f) above, voltage drops for ratios of 1 and 0 must be found from the charts and the desired value interpolated from the two voltage drops.

- Ratio = 0; Voltage drop = 0.96\% (Chart #2a)
- Ratio = 1; Voltage drop = 0.90\% (Chart #3a)
- Ratio = $\frac{1}{2}$; Voltage drop = 0.93\% (Interpolated)
The following conclusions can be drawn as a result of the preceding investigation of the effect of polyphase motors on the voltage change caused by single-phase welders connected to a three-phase system.

a. The effect of the motors is to reduce the voltage change on the phase or phases having the maximum drop in voltage. In a practical case this effect can be appreciable and hence should be considered in any accurate analysis of the effect of a single-phase load.

b. The voltage change with the motor effect included can be calculated on the basis of the method and equations developed in this investigation.

c. The amount of load on the motors has very little effect on the voltage change.

d. The magnitude of the field excitation of a synchronous motor has little effect on the voltage change.

e. In practical cases it is found that the exact character of the motor loads adjacent to the single-phase welder cannot usually be determined. It is also found that these loads are largely composed of induction motors. Because of this and of the statement made in (c), most practical cases which involve single-phase welders having "on" periods longer than two cycles can be handled with sufficient accuracy by using the average full-load and blocked-rotor impedances given in Section 10 for the positive- and negative-sequence motor impedances respectively. For the small number of cases involving either "on" periods shorter than two cycles or appreciable synchronous-motor load, the motor constants discussed in Sections 9 and 10 should be used.

f. Balanced three-phase loads other than motors have not been considered up to this point for two reasons. The first is that the usual industrial load is largely motor load. The second reason is that the positive- and negative-sequence impedances of the other balanced three-phase loads are equal, and therefore they do not affect the voltage change.
caused by the welder nearly as much as the same kva of three-phase motor load. In cases where there are appreciable balanced three-phase loads other than motors connected to the system, they can be taken into account by including their impedances when setting up the positive- and negative-sequence networks.

g. The charts included in this bulletin can be used as a graphical solution of the equations developed in the first part of the bulletin. When a number of single-phase welder loads are to be considered these charts offer a valuable time-saving method of solution.

h. All the equations and charts given in this bulletin are based upon sinusoidal currents and voltages. However, there are single-phase welders which have phase-shift heat control and do not draw sinusoidal currents except at the 100-percent heat setting. There are two types of phase-controlled welders that need consideration. The first is the type which does not use series capacitors for power factor correction. For this type it is recommended that the voltage drop be determined for the 100-percent current setting, since the maximum drop will occur under this condition.3

For the second type, which uses series capacitors to correct the power factor to approximately unity, the maximum drop in the line-to-line voltage for a ΔΔ transformer bank or the line-to-neutral voltage for a YA bank does not occur at 100 percent current but at a current reduced by phase control.3 Therefore the maximum voltage drop cannot be determined by the methods of this bulletin since the current would be non-sinusoidal. The solution of this problem was not included in the preceding investigation and remains a problem for further study.

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"K"x (Voltage Drop Without Motors) = Actual Voltage Drop

Chart 1 Correction Factor, K, for Motors at P Only
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Chart 2a  Chart for Determining Voltage Drops Caused by Single-Phase Welders

0 - 520 Welder kva  Motor hp at F = 0
Welder kva  Welder kva

System Impedance, M

Transformer Impedance, 0.05 p.u.
(On Welder kva as a Base)

Transformer Impedance, 0.03 p.u.
(On Welder kva as a Base)

Transformer Impedance, 0.01 p.u.
(On Welder kva as a Base)

Welder kva  At Rated Secondary Voltage

Source
System Impedance
Primary Voltage Drop GC
Primary Voltage Drop AN

Constant M

M = (System Impedance in pu) x (Motor hp at F)
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Chart 2b  Chart for Determining Voltage Drops Caused by Single-Phase Welders

0 - 2600 Welder kVA  Motor hp at F = 0
Welder kVA

System Impedance, $J$

Transformer Impedance, 0.05 p.u.
(On Welder kVA as a Base)

System Impedance, $J$

Transformer Impedance, 0.03 p.u.
(On Welder kVA as a Base)

System Impedance, $J$

Transformer Impedance, 0.01 p.u.
(On Welder kVA as a Base)

System Impedance
In Per Unit on 10,000 kVA Base

Motor hr at F = 0
Welder kVA
Chart 3a Chart for Determining Voltage Drops Caused by Single-Phase Welders

System Impedance, $I$

Transformer Impedance, 0.05 p.u.
(On Welder kVA as a Base)

Transformer Impedance, 0.03 p.u.
(On Welder kVA as a Base)

Transformer Impedance, 0.01 p.u.
(On Welder kVA as a Base)

Motor hr at $F$

Welder kVA = 1

Constant $M$

$M = (\text{System Impedance in pu}) \times (\text{Motor hr at P})$

Welder kVA

Motor hr at $F = 1$

System Impedance

In Per Unit on 10,000 kVA Base

0 - 520 Welder kVA

Chart for Determining Voltage Drops Caused by Single-Phase Welders
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Chart 3b Chart for Determining Voltage Drops Caused by Single-Phase Welders

Constant M

\[ M = (\text{System impedance in pu}) \times (\text{Motor hp at } P) \]

- Welder kva
- At Rated Secondary Voltage
- System Impedance
- Transformer Impedance
  - 0.05 pu (On Welder kva as a Base)
  - 0.03 pu (On Welder kva as a Base)
  - 0.01 pu (On Welder kva as a Base)

- System Impedance
  - In per unit on 10,000 kva Base

- Motor hp at P = 1
- Welder kva

0 - 2600 Welder kva
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Chart 4a Chart for Determining Voltage Drops Caused by Single-Phase Welders

0 - 520 Welder kva
Motor hp at F = 2
Welder kva

Welder kva.
At Rated Secondary Voltage

System Impedance, I

Transformer Impedance, 0.02 pu
(On Welder kva as a Base)

System Impedance, I

Transformer Impedance, 0.01 pu
(On Welder kva as a Base)

System Impedance
In Per Unit on 10,000 kva Base

Constant M
M = (System Impedance in pu)(Motor hp at F)
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Chart 4b Chart for Determining Voltage Drops Caused by Single-Phase Welders

- Welder kVA
- At Rated Secondary Voltage
- Transformer Impedance 0.8 pu
  (On Welder kVA as a Base)
- System Impedance

- Motor hp at F = 2
- Welder kVA

Constant M

\[ M = \text{System Impedance in pu} \times \text{(Motor hp at P)} \]
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Chart 5a  Chart for Determining Voltage Drops Caused by Single-Phase Welders

- Welder kva
  At Rated Secondary Voltage

- System Impedance
  Transformer Impedance 0.02 pu
  (On Welder kva as a Base)

- System Impedance
  Transformer Impedance 0.05 pu
  (On Welder kva as a Base)

- System Impedance
  Transformer Impedance 0.01 pu
  (On Welder kva as a Base)

- System Impedance
  Transformer Impedance 0.00 pu
  (On Welder kva as a Base)

- Welder Power Factor

- Motor hp at P = 3
  Welder kva

System Impedance
In Per Unit on 10,000 kva Base

Constant M
M = (System Impedance in pu) x (Motor hp at P)
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Chart 5b Chart for Determining Voltage Drops Caused by Single-Phase Welders

0 - 2600 Welder kva                      Motor hp at F = 3
Welder kva
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