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RESTRAINT CHARACTERISTICS
OF FLEXIBLE RIVETED AND
BOLTED BEAM-TO-COLUMN
CONNECTIONS

by
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CONNECTIONS

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ABSTRACT

THE BEHAVIOR OF FLEXIBLE RIVETED AND BOLTED BEAM-TO-COLUMN CONNECTIONS IS DESCRIBED ALONG WITH THE FACTORS WHICH INFLUENCE THEIR MOMENT-ROTATION CHARACTERISTICS. THE LOCATION OF THE CENTER-OF-ROTATION OF THE CONNECTION ANGLES AND THE LOAD-DEFORMATION RELATIONSHIPS, AS DETERMINED FOR SHORT ANGLE SEGMENTS, ARE CONSIDERED TO BE THE MOST SIGNIFICANT FACTORS NECESSARY TO ESTABLISH THE RELATIONSHIP BETWEEN MOMENT AND ROTATION OF A CONNECTION. IT IS SHOWN THAT THE LOAD-DEFORMATION EXPRESSIONS FOR ANGLE SEGMENTS CAN BE DETERMINED ANALYTICALLY FOR DEFORMATIONS THAT ARE SUFFICIENTLY LARGE TO INSURE PLASTIC HINGES AT THREE CRITICAL LOCATIONS IN THE ANGLES. BASED ON THE DATA FROM TESTS CONDUCTED ON FULL SIZE FLEXIBLE CONNECTIONS THE LOCATION OF THE CENTER-OF-ROTATION CAN BE SHOWN TO BE A FUNCTION OF THE DEFORMATION OF THE CONNECTION ANGLES.

MOMENT-ROTATION INFORMATION IS READILY DETERMINED FOR CONNECTION ANGLES OF THICKNESSES UP TO AND INCLUDING THOSE 7/16 INCHES THICK, AND WITH FROM TWO TO 10 ROWS OF FASTENERS. IN THE DEVELOPMENT OF THESE RELATIONSHIPS THE INFLUENCE OF VARIABLES SUCH AS GAGE, FILLET RADIUS, YIELD POINT, FASTENER SIZE, AND ANGLE THICKNESS ARE ALSO CONSIDERED.

RECOMMENDATIONS ARE MADE FOR THE UTILIZATION OF THE RESTRAINT CHARACTERISTICS OF FLEXIBLE CONNECTIONS IN DESIGN. THE SAVINGS IN BEAM WEIGHT ARE SHOWN TO BE AS GREAT AS 6 OR 7 PER CENT AND THE REDUCTION IN DEPTH USUALLY 2 OR 4 INCHES, DEPENDING ON LOADING CONDITION AND FRAMING.
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I. INTRODUCTION

A. RIVETED AND BOLTED STRUCTURAL CONNECTIONS

Riveted and bolted structural steel connections which join beams to columns or girders have generally been divided into three broad groupings, flexible, semi-rigid, and rigid, depending upon the degree of rotation restraint that they provide (see Figure 1 for typical examples). Hechtman and Johnston have described these groups as follows:\(^1\)*

1. Flexible Connections are those which are capable of carrying the end shear, but which allow relatively free rotation between the end of the beam and the column. A flexible connection approaches the common assumption of pin end supports, in which case the beams are designed for full simple beam moment. This has been the general practice in the case of standard riveted building connections.

2. Semi-Rigid Connections are intermediate between rigid and flexible connections and transmit appreciable bending moment, with some rotation between the end of the beam and the column. Many connections assumed as 'flexible' are inherently 'semi-rigid,' thereby developing end moments which have not been considered in the design.

3. Rigid Connections are those in which the relative rotation between the end of the beam and the column is reduced to a minimum by the use of stiff connections, as in the case of continuous frames where full continuity is assumed in the analysis.

In practice, however, it has been simpler and more convenient to divide the connections into two groupings, completely flexible or completely rigid, and to ignore the condition which is actually more prevalent -- partially fixed.

Structures designed in accordance with the simplifying assumption that connections are either unable to resist moment or else are able to develop the full fixed end moment of a beam have performed successfully; as a result, designers have developed considerable confidence in this procedure. Unfortunately, the simplifications that are so desirable from the viewpoint of a designer stand in the way of a true appreciation of the actual behavior of structural connections.

The behavior of riveted beam-to-column connections has been the subject of considerable research and speculation for a number of years. (1-6) Experimental investigations have provided data concerning the moment-rotation

*Superscript numbers in parentheses refer to entries in References, Chapter VII.
characteristics of selected connections but there have been insufficient tests to provide an indication of the behavior of the many combinations of connections that are possible. Unfortunately, the complex behavior of riveted beam-to-column connections has made it difficult to develop analytical methods to predict their response to moment.

The failure-free performance of a structural element does not in itself indicate good design; good design also implies efficient and economical utilization of material. A further requirement for good design is that the structural components be able to support overloads up to but not exceeding those provided by the factor-of-safety. To accomplish this additional requirement the performance of members must be understood and design specifications must reflect actual behavior.

In the late 1940's interest developed in the possibility of using the ASTM A325 high-strength bolt as a structural fastener. By the early 1950's the high-strength bolt (properly tightened) had been accepted as a proven substitute for rivets. Whether the new high-strength bolt would behave similarly to the familiar rivet in beam-to-column structural connections remained to be investigated.

In the mid-1950's the Research Council on Riveted and Bolted Structural Joints authorized a test program at the University of Illinois to evaluate the effect of substituting high-strength bolts for rivets in beam-to-column connections. The specimens selected for this study were similar to the members used by Hechtman and Johnston in a test program of riveted connections.\(^{(1)}\)

To increase the usefulness of the Illinois research beyond a comparison of riveted and bolted joints, the study was broadened to include an investigation of the possibility of predicting analytically the behavior of flexible beam-to-column connections. Most of the specimens in the experimental program at Illinois were of the type commonly described as flexible. (A typical example is shown in Figure 1.)

B. OBJECT AND SCOPE OF INVESTIGATION

The flexible connection shown in Figure 1 was formerly designated by the American Institute of Steel Construction\(^{(7)}\) as a "standard beam connection"; however, the sixth edition of the manual designates this type of connection as a "framed beam connection." The connections discussed herein will be of the riveted and/or bolted type now known as "framed beam connections" and "heavy framed beam connections" having angles less than 1/2 in. thick.

Although the framed connections fall in the category of "flexible," tests have shown them to be capable of resisting some moment.\(^{(1,4,5,6)}\) Consideration of these beam end moments will reduce the computed value of the maximum positive beam bending moment caused by gravity loads and will make it possible for the designer to utilize a lighter beam. However, before concluding that consideration of end moment will prove economical, the increased design time required to include end moment calculations must be evaluated.
The end moment that a particular pair of connection angles will develop depends upon (1) the depth and length of the beam with which the angles are combined, (2) the gage or gages of the connection angles, (3) the type and size of fastener, (4) whether the connection is to a column flange, a column web, or a girder web, (5) the angle thickness, and (6) the physical properties of the angle material. Therefore, before the end moment of flexible connections can be utilized, a designer must have data that makes possible the rapid determination of end moments for all possible combinations of the variables just listed.

The moment-rotation behavior of a few full size flexible-type connections is available from the tests of other investigators. (1,4,5,6) Because of the many variables, however, it is not practical to require data from tests of full size specimens to provide the necessary design information. Therefore, a method of analysis capable of predicting the end restraint of a connection is required before the end moment capacity of flexible connections can be used in beam design. The development of such a method of analysis was one of the principal objectives of this study. An additional aim of this research was a better and more complete appreciation of flexible connection behavior.

The information produced by this study will make possible increased use of the provision in the AISC Specifications (7) which states:

Type 3 (semi-rigid) construction will be permitted only upon evidence that the connections to be used are capable of furnishing, as a minimum, a predictable proportion of full end restraint. The proportioning of main members joined by such connections shall be predicted upon no greater degree of end restraint than this minimum.

If "flexible"-type connections are found to have sufficient end restraint to warrant use of end moment in design, this type of connection should then be considered to be of the "semi-rigid" type. However, the connections studied in this report will be referred to as flexible, in spite of the moment resistance they possess, in order to distinguish them from the top and bottom angle-to-beam flange connections generally considered to be semi-rigid.

In order to keep the present study within reasonable limits, the scope has been restricted to connections with three to ten rows of fasteners in a vertical line, angles 5/16 in., 3/8 in., and 7/16 in. in thickness, and mild steel angle material.
II. THE BEHAVIOR OF FLEXIBLE CONNECTIONS

The behavior of flexible-type connections is complex. This complexity results from the fact that the connection angles may have yielded in certain locations, although the beam which is being supported may be carrying no more than its working load and thus may have an extreme fiber stress in flexure far below the yield point.

As the connection angles deform upon the application of moment, one part of the connection is pulled away from the supporting member to which it is attached and the other end of the same connection is pushed into the supporting member. The resultants of the forces which produce these deformations compose the couple which resists the applied moment. For descriptive purposes within this text the portion of the connection being pulled away from its support shall be said to be "in tension," and the portion being pushed towards its support shall be said to be "in compression." (See Figure 2.) Somewhere along the length of the connection angles there is a location where there will be no deformation attributable to the moment and, therefore, no tensile or compressive forces will be carried by the angles at that point. This location has been designated as "the center-of-rotation" and is illustrated in Figure 3.

As moment is applied to the connection angles, the strains which result are initially elastic. However, it has been observed by Rathbun, (5) and Hechtman and Johnston, (1) as well as in the tests reported herein (Appendix B), that the relationship between moment and rotation is non-linear, even at relatively low moments. (See Figure 4.) This lack of linearity in connection behavior exists while the angle strains are still elastic and results from the change in stiffness of the compression portion of the connection with respect to the tension portion as the moment increases.

To predict connection behavior the location of the "center-of-rotation" is required. The location depends upon the difference in the load-deformation behavior (or stiffness) along the length of the angle. If the behavior shown in Figure 2 is correct, the compression end would be many times stiffer than the tension end and the location of the center-of-rotation would be between mid-length and the compression end of the connection. During the initial stages of loading, however, the location was observed (Appendix B) to be near mid-length of the connection and would indicate that the tension and compressive ends of the connection were then of similar
stiffness. Generally the heel of the angle is not initially in firm contact with the column flange or web. Therefore, the backs of the angles at the compression end move a minute distance before coming into firm contact with the column over the full width of the angle leg. This clearance at the heel of the angle at the compression end allows both the tension and compression ends of the connection angles to resist initially the applied moment by flexure; this initial similarity in behavior of the tension and compression ends accounts for the observation that the center-of-rotation is close to the mid-length of the connection at early stages of loading.

As the outstanding leg of the angle at the compression end of the connection begins to bear against the column over more and more of its width, the stiffness of the compression end relative to the tension end increases. The increase in stiffness at the compression end has the effect of moving the center-of-rotation of the connection toward the compression end, which changes the slope of the moment-rotation curve. This results in a curve of the type shown in Figure 4.

Yielding initiates at the tension end of the connection angles at relatively low moments. The specific locations where yielding initiates (adjacent to fasteners or in angle fillets) are discussed in Appendix A. The distance along the length of the angles over which yielding has begun extends further toward the center-of-rotation with each additional increment of moment. As yielding propagates along the length of the angles their stiffness at the tension end decreases. This decrease in the stiffness reduces the relative stiffness of this region compared to the portion of the angles in compression and also has the effect of further moving the center-of-rotation towards the compression end. This decreasing stiffness accounts for the decreased increments of moment required to increase the rotation by a given amount. Figure 4, which is a typical moment-rotation curve for a pair of connection angles, clearly shows the decrease in connection stiffness with an increase in moment.

Tests reported herein (Appendix B) as well as tests by others have indicated that flexible connections can develop as much as 20 per cent of the full or theoretical fixed-end moment of a beam at working loads, depending on: the type of load, the end-rotation attributable to column deformation or the rotation of the supporting beam due to the beam connection, and the beam-span to beam-depth ratio. The simplifying assumption that flexible connections resist no moment has a conservative effect on the beam design; consideration of the end moment developed by flexible connections will reduce the required section modulus for a beam.

The current design practice of assuming flexible type connections to behave as pins neglects the moment in the column produced by the flexible-type connection. The magnitude of the moment carried to a column through a flexible connection is dependent on the type of load distribution (since for flexible connections the critical or maximum moment is not at the ends) and the stiffness of the beam. For a decrease in beam stiffness (load distribution
the same) there is an increase in the rotation of the end of the beam and associated with the increased end rotation there is an increase in connection moment. Therefore, if the moment resistance of flexible connections is considered in design, allowing the substitution of a lighter (and generally shallower) beam, the column moment will be increased. However, it will be shown later that this increase in column moment is small.
III. MOMENT-ROTATION PREDICTION BASED UPON LOAD DEFORMATION CHARACTERISTICS OF ANGLE SEGMENTS IN TENSION

A. METHOD OF PREDICTING RESTRAINT CHARACTERISTICS

It was suggested by Messrs. Beaufoy and Moharram that the relationship between moment and rotation for flexible-type connections can be derived from a consideration of the composite effect of short lengths of angles in tension or compression. Figure 5 illustrates how a connection angle is assumed to be subdivided into a number of short lengths or segments whose combined resistance to moment is considered equivalent to that of a single angle of the same total length.

The approach suggested above would make it possible to predict the moment-rotation characteristics of a flexible connection without dependence on tests of full size connections. What is required are the load-deformation characteristics of the angle segments and the location of the center-of-rotation.

Messrs. Beaufoy and Moharram utilized data obtained from tests on connections of the type indicated as semi-rigid in Figure 1 to provide the necessary relationship between load and deformation for an angle segment. The center-of-rotation was assumed to be at the mid-length of the connection for small moments and at the fastener row closest to the compression end for large moments. Where moment-rotation information existed for a flexible specimen whose connection angles had geometrical and physical properties similar to those of the angles of a semi-rigid connection, comparisons could be made between predicted and actual behavior of the flexible connection. Only two comparisons were possible in their study. Their predictions, when compared with actual tests results, provided a lower bound at small moments and an upper bound at large moments.

Since the available moment-rotation curves for flexible connections were few in number, additional verification that the load-deformation characteristics of angle segments can be used to predict moment-rotation characteristics was required. Experimental load-deformation relationships for connection angle segments and center-of-rotation data were obtained directly from the tests reported in Appendices A and B.

The primary objective of the work in this study was to replace the need for experimental data by the ability to predict analytically the behavior of flexible beam-to-column connections and to separate the effects of the many variables that affect this behavior.
B. LOCATION OF CENTER-OF-ROTATION

Since the location of the center-of-rotation of a pair of connection angles is dependent on the relative stiffnesses in the tension and compression regions of the connection angles, changes in connection angle thickness and geometry will influence the location of the center-of-rotation (See Appendix B). If the angle thickness is increased, the stiffness at the compression end increases linearly with the increase in thickness; at the tension end, the increase in stiffness is proportional to the cube of the ratio of the thicknesses. Therefore, while the relationship between stress and strain is still linear, increasing the angle thickness will decrease the ratio of the compression end stiffness to the tension end stiffness. This reduction in relative stiffness will be accompanied by a reduction of the distance to the center-of-rotation from the tension end, since the requirements of statical equilibrium dictate that the summation of the compressive forces along the face of the angles must equal the summation of the tensile forces. This was verified when the location of the center-of-rotation for Specimen FK-5 (7/16 in. thick) was compared with the location of the center-of-rotation of the other connections made up of 3/8-in.-thick angles. (Table B2, Appendix B)

After considerable yielding of the connection angles has taken place at the tension end, plastic hinges are formed, and the stiffness decreases; the stiffness is no longer related to the cube of the thickness of the angle. The deformation of the connection angles varies along their length. Consequently, while the extreme tension end may have developed plastic hinges, the material near the center-of-rotation behaves elastically. Therefore, no single value for stiffness will apply to the connection over its full length.

In spite of the many factors influencing the location of the center-of-rotation (magnitude of moment, angle thickness, fillet radius, gages for the angles, fastener size, etc.), experimental observations do not indicate a large range of locations when loads approach or exceed working loads. The data of Table B2 suggest that a center-of-rotation located 0.8 of the length of the angle from the tension end of the connection is representative of the test observations.

C. PREDICTION OF LOAD-DEFORMATION RELATIONSHIPS OF ANGLE SEGMENTS IN TENSION

If the load-deformation characteristics of the angle segments in tension (and thus the behavior of the angles) could be determined analytically, the entire procedure for determination of connection moment-rotation characteristics could be made independent of laboratory results. Therefore, considerable effort has been devoted to the development of such an analysis.

Consider the deformations of a typical flexible-type connection used with a beam of usual proportions (L/d between 12 and 24) and supporting a uniform load. If the load is increased until the extreme fibers of the beam
attain 36 ksi at the cross section resisting the greatest moment, the connection will have yielded over a considerable portion of its length as evidenced by the nonlinear moment-rotation behavior shown in Figure 4. An indication of the behavior of the connection angles can be obtained from a comparison of the deformation at the extreme tension end with the behavior of angle segments in tension (Appendix A). This comparison also indicates the extent to which yielding in the connection is necessary to accommodate the rotation introduced by a beam which is loaded to its yield moment.

To predict the load-deformation behavior of a particular pair of connection angle segments, recognition must be given to the various stages of behavior through which the angles go as they resist a load of increasing magnitude. These stages are (1) elastic, (2) a combination of elastic and inelastic, but without plastic hinges, and (3) a stage where plastic hinges have formed in critical locations in sufficient numbers to make it possible to calculate by plastic theory the load carrying capacity of the angles.

The initial portion of the load-deformation curve, the elastic portion, can be determined by using any one of several methods for determining elastic displacements. As an example consider the following: a connection angle with a thickness of 0.353 in., yield point of 40 ksi, 3/4-in. fastener size, and fastener locations as shown in Figure 6. When such angles are loaded, there are certain restraints placed on the specimen by geometry and end condition. It is assumed that the fasteners hold the angle legs fixed at points A and C. Point C (Figure 6c) moves downward only, because of symmetry, and the rotation at B is the same in both legs as a result of continuity.

Equations of Equilibrium dictate that $H_a = H_c$ and $V_a = V_c$ (Figure 6). If the angle is assumed fixed at A, and temporarily free to translate and rotate at C, then the horizontal movement at C caused by a horizontal load $H_c$ applied at C is: (See Figure 7a.)

$$\Delta_c = \frac{4.58H_c}{EI}.$$ 

Similarly, the horizontal movement at C caused by a vertical load $V_c$ applied at C is: (See Figure 7b.)

$$\Delta_c = \frac{2.1V_c}{EI}.$$ 

The horizontal movement at C caused by a moment $M_c$ applied at C is: (See Figure 7c.)

$$\Delta_c = \frac{3.51M_c}{EI}.$$ 

Since the connection angles are used in pairs, the displacements are symmetrical about the line of action of the load. For this reason there is no horizontal movement of the angle at point C perpendicular to the beam web. Therefore,

$$\Delta_c + \Delta_c + \Delta_c = 0,$$

$$(H_c + V_c + M_c)^{1/2} = 0.$$ (1)
The rotation of point C caused by a force $H_c$ for a fixed end condition at A is: (See Figure 7d.)

$$\text{Rot } C = \frac{3.51H_c}{E_1} ;$$

the rotation at C caused by a force $V_c$ is: (See Figure 7e.)

$$\text{Rot } C = \frac{1.45V_c}{E_1} ;$$

and the rotation at C caused by a moment $M_c$ is: (See Figure 7f.)

$$\text{Rot } C = \frac{3.51M_c}{E_1} .$$

If it is assumed that the fastener at C prevents rotation of the angle leg at point C, then the summation of the rotation at C must equal zero. Therefore,

$$\text{Rot } C + \text{Rot } C + \text{Rot } C = 0 , \hspace{2cm} \frac{3.51H_c}{E_1} + \frac{1.45V_c}{E_1} + \frac{3.51M_c}{E_1} = 0 . \hspace{2cm} (2)$$

Equations (1) and (2) make it possible to determine the relative magnitude of the two forces and the couple. $H_c$ and $M_c$ can be evaluated for any assumed value of $V_c$. Also, the movement in the direction of $V_c$ can be determined from the expression:

$$\Delta V_c = \frac{2.1H_c}{E_1} + \frac{1.64V_c}{E_1} + \frac{1.45M_c}{E_1} .$$

In Figure 8 can be seen the moment distribution in the angle segment if $V_c$ is taken equal to one kip. A one-kip load on a single angle segment one in. long produces a one-in.-kip moment at A.

Based on the above expressions it is determined that a one-kip load, on a pair of angle segments each one in. long, and with the properties previously stated, produces a movement in the direction of the applied load ($V_c$) equal to 0.00285 in. This can be interpreted as a resistance to deflection or as a "stiffness" of 350 kips per in.

The stage immediately following the elastic stage is complex because of the difficulty in determining a compatible strain distribution and is not treated here. However, after additional straining, plastic hinges will form at critical locations in the angle legs, and when the number of hinges reaches three (Figure 9) there will be a sufficient number of equations of condition to make it possible to solve for the unknown forces by methods of determinate analysis. In this stage the geometrical changes in the angles are large because the deformations in the angles are large compared to the restrained lengths of the angle legs. Therefore, before the load which causes a given deformation can be determined, it is necessary that the deformed shape of the angles be known or at least closely approximated so that the hinges can be located.

It is assumed that there is no difference in behavior between a length of the angle segment containing a fastener and a length of segment which does not.
The observation of photo-elastic coatings bonded to test specimens (Appendix A) has made it possible to observe strain distributions; and to determine when and in what order plastic hinges are formed in the angle segments and thus the connection angles. The hinges are formed in the following order: the first at the edge of the fastener in the outstanding leg, and the next two in the vicinity of the two intersections of the fillet and the angle legs.

After the three hinges have been formed, it is possible to determine the load associated with an assumed deformation if the position of the hinges is known. Movements of the hinges compatible with the movement of the web-connected leg were assumed in accordance with the construction shown in Figure 10. It was assumed that (1) the hinge at B moves on a circular path which has its center at A and a radius equal to the distance between points A and B, (2) the hinge at C moves on a circular path which has its center at D and has a radius equal to the distance between points C and D, and (3) the heel of the connection angle (reinforced by the fillet) does not deform; this assumption has the effect of fixing the distance between points B and C. These three relationships, combined with an appreciation of the angle behavior gained from photo-elastic strain determinations and photos made during the tests, are the basis for the construction.

The procedure to determine the forces associated with a particular deformation of a unit length of angle is as follows: (1) assume a movement of the web-connected leg in the direction of the beam web (away from the column), and (2) determine the position of the hinges consistent with this deformation as outlined above. The locations of the hinges consistent with deformations of 0.1, 0.2, and 0.3 in. are shown in Figure 11. The following equations express the behavior shown in Figure 11 mathematically, and the magnitude of $V$ can be determined for each deformation by solving the associated pair of simultaneous equations.

For an assumed $\Delta = 0.1$ in.,

\[
\sum M_{B1} = 0
\]

\[-2M_p - 0.095H + (g_1 - \frac{F}{2} - Th - R)V = 0\]

\[
\sum M_{C1} = 0
\]

\[-2M_p - (Th + R + 0.1)H + (g_1 - \frac{F}{2} - \frac{Th}{2} - 0.02)V = 0\]
For an assumed $\Delta = 0.2$ in.:

$$
\begin{align*}
\Sigma M_{B'} &= 0 \\
-2M_p - 0.185H + (g_1 - \frac{F}{2} - Th - R - 0.02)V &= 0 \\
\Sigma M_{C'} &= 0 \\
-2M_p - (\frac{Th}{2} + R + 0.2)H + (g_1 - \frac{F}{2} - \frac{Th}{2} - 0.03)V &= 0
\end{align*}
$$

For an assumed $\Delta = 0.3$ in.:

$$
\begin{align*}
\Sigma M_{B'} &= 0 \\
-2M_p - 0.26H + (g_1 - \frac{F}{2} - Th - R - 0.03)V &= 0 \\
\Sigma M_{C'} &= 0 \\
-2M_p - (\frac{Th}{2} + R + 0.3)H + (g_1 - \frac{F}{2} - \frac{Th}{2} - 0.07)V &= 0
\end{align*}
$$

$Th$ = angle thickness,  
$R$ = 0.8 fillet radius,  
$F$ = width across flats of nut or diameter of rivet head,  
$g_1$ = angle gage,  
$M_p$ = plastic moment.

It should be noted that these equations take into consideration all of the variables that contribute to the behavior of flexible connections; gage, angle thickness, physical properties of the material, fastener size, and fillet radius.

The assumed deformations and the corresponding computed loads provide data that can be used in the construction of a load-deformation curve. Calculated plastic load-deformation relationships are compared in Figure 12 with the results of tension tests conducted on pairs of angles whose geometry and properties are similar to those used in determining the calculated values. The correlation is good.

The method just described, although it provides a significant portion of the load-deformation curve, does not provide information on the behavior prior to the formation of the third hinge. The load-deformation relationship for the initial portion of the curve (the elastic portion) can be determined readily as shown in Figures 7 and 8 and discussed earlier in this section. However, the portion of the load-deformation curve which remains undefined is the part which represents the transition from elastic behavior to a behavior governed by the presence of the three plastic hinges. This segment of the load-deformation curve is highly indeterminate but can be described by a smooth curve joining the elastic portion with the "three hinge portion." Based upon the above procedure it is possible to approximate, with sufficient accuracy for use in moment-rotation prediction, the load-deformation curves for a pair of angles (or angle segments) assembled back-to-back and being pulled in a direction parallel to the axes of the fasteners connecting the outstanding legs to their support.
D. THE LOAD-DEFORMATION RELATIONSHIP EXPRESSED AS AN EQUATION

It is shown in Section C that the load associated with a particular deformation of a pair of angle segments may be computed. The formulas are presented to facilitate the determination of this load for three values of deformation ($\Delta = 0.1$, $0.2$, and $0.3$ in.). To assist in the development of an expression describing the moment-rotation behavior of the corresponding connection, it is desirable to relate the load and deformation continuously over the range of application. To do this an equation of the following form is used.

$$ \text{Load} = C \times (\Delta)^n. $$

The $\Delta$ represents the deformation at the extreme tension end of the connection angles.

It is suggested in Section B, Chapter IV that because of the nonlinear behavior of the connection angles it is more convenient to apply a load factor and design for failure (initiation of yielding) than to use a working stress when considering end or connection moments. The tension end of the connection angles can be expected to provide a deformation between 0.1 in. and 0.3 in. when the supported beam begins to yield; therefore, the coefficient and the exponent in the equation above were evaluated so that the curve would pass through the origin and the two points which correspond to $\Delta = 0.1$ in. and $\Delta = 0.3$ in. The loads corresponding to these two $\Delta$'s are determined by the procedures of Section C. The values of load $P$ and $\Delta$ for a particular angle are substituted in the above equation for the conditions $\Delta = 0.1$ in. and $\Delta = 0.3$ in. The resulting equations are

$$ P(0.1) = C(0.1)^n, $$
$$ P(0.3) = C(0.3)^n. $$

$P$ may be determined from the relationships of Section C. The values of $C$ and $n$ can be determined using the above expressions. Values of $C$ and $n$ are tabulated in Table I for various common combinations of gage, fillet radius, yield point, fastener size, and angle thickness.

E. MOMENT-ROTATION RELATIONSHIPS OF CONNECTION ANGLES EXPRESSED IN EQUATION FORM

The moment-rotation characteristics of flexible type riveted or bolted beam-to-column connections can be closely approximated once the load-deformation relationships for angle segments and the location of the center-of-rotation are known. To take advantage of the connection restraint, an equation that will relate connection behavior to these two quantities is desired.

For deformations at the end of the tension portion of a flexible connection of 0.1 in. or greater, the distance from the center-of-rotation to the center-of-gravity of the force block, whose shape is determined by the load-deformation curve formulated in Section D, is approximately 0.53 of the distance from the center-of-rotation to the tension end. Also, the total force represented by the force block is approximately equal to 0.85
of the product of the length of the angle in tension times the load per unit length at the extreme tension end of the connection. If the deformation at the tension end is less than 0.1 in., the force block will approach a triangular shape as the assumed deformation is decreased. These relationships may be seen readily in Figure 12.

Although the center-of-rotation has been assumed to be at 0.8 of the length of the connection angles from the tension end, the location actually was observed (see Table 2, Appendix B) to vary with the magnitude of the deformation at the tension end of the connection. The varying distance of the center-of-rotation from the tension end can be conservatively approximated by the following expression if the tension deformation \( \Delta \) at the end of the connection is greater than 0.1 in.

\[
l_1 = (0.72 + 0.3 \Delta)k \text{ or } \frac{0.72k}{1 - 0.3\phi_1},
\]

where, \( l_1 \) = distance from the tension end to the center-of-rotation,
\( k \) = connection length,
\( \phi \) = connection rotation,
\( \phi l_1 = \Delta \) = the deformation at the tension end of the connection.

An expression for resisting moment has been developed utilizing the load-deformation relationships for angle segments in tension and the relationship above for the location of the center-of-rotation. The resisting moment can be seen to equal

\[
M = TR[0.53l_1 + 0.67(k - l_1)],
\]

where, \( TR \) = tension resultant = 0.85\((P \times l_1)\). The factor of 0.85 is related to the shape of the load-deformation curve of the connection angles. If substitutions are made for TR and \( l_1 \), the moment expression becomes

\[
M = 0.85P \left[ \frac{0.72k}{1 - 0.3\phi_1} \right] [0.565k],
\]

where, \( P = C(\Delta)^n \) or \( C(\phi l_1)^n \) and the small variation in \( l_1 \) makes it possible to replace \([0.53l_1 + 0.67(k - l_1)]\) by the approximation 0.565\(k\). The above expression can be further reduced to

\[
M = \frac{0.345 k^2 (0.72\phi_1)^n}{(1 - 0.3\phi_1)^{1+n}}
\]

for \( (\phi l_1) \geq 0.1 \text{ in.} \) \( (3) \)

The above expression adequately describes the connection behavior for maximum deformations \( (\Delta) \) of 0.1 in. and greater at the tension end of the connection, but the rapidly changing position for the center-of-rotation at smaller deformations makes predictions with Equation (3) less reliable for the smaller deformations. However, it has been found that for deformations less than 0.1 in., if \( n \) is taken equal to 0.4, good agreement with the test data is obtained. The expression for moment for maximum deformation at the tension end less than 0.1 in. is

\[
M = \frac{0.345 k^2 (0.72\phi_1)^{0.4}}{(1 - 0.3\phi_1)^{1.4}}
\]

for \( (\phi l_1) < 0.1 \text{ in.} \), \( (4) \)
FIGURE 1. COMMON TYPES OF BEAM-TO-COLUMN CONNECTIONS

FIGURE 2. DEFORMATIONS OF FLEXIBLE-TYPE CONNECTION ANGLES

Deformation at the top is the same as if a tensile load had been applied to angles through beam web.

Deformation at the bottom is the same as if a compressive load had been applied to angles through beam web.
FIGURE 3. LOCATION OF THE CENTER-OF-ROTATION

Rotation at failure 100 × 10^-3 radians

Rotation occurring at \( \sigma_y = 36 \text{ ksi} \) for range of common span lengths — 18 WF 50, uniform load

Deformation at tension end of 11-1/2 in. long connection

FIGURE 4. TYPICAL RELATIONSHIP BETWEEN MOMENT AND ROTATION FOR A FLEXIBLE CONNECTION
Flexible Connection

Equivalent Connection

FIGURE 5. FLEXIBLE CONNECTION COMPARED TO A SERIES OF ANGLE SEGMENTS IN TENSION AND COMPRESSION

Angle Th. = .353
Fillet radius = 1/2"

FIGURE 6. CONNECTION GEOMETRY ASSUMPTIONS CONCERNING ANGLE RESTRAINT
(a) Horizontal deflection at C due to a horizontal force at C
(b) Horizontal deflection at C due to a vertical force at C
(c) Horizontal deflection at C due to a moment at C
(d) Rotation at C due to a horizontal force at C
(e) Rotation at C due to a vertical force at C
(f) Rotation at C due to a moment at C

FIGURE 7. DETERMINATION OF LOAD DISPLACEMENT RELATIONSHIPS FOR ANGLE SEGMENT

(a) Reactions Corresponding To $V_C = 1$ Kip
(b) Moment Diagram

FIGURE 8. MOMENTS IN ANGLE DUE TO A ONE-KIP LOAD APPLIED VERTICALLY AT C
FIGURE 9. LOCATION OF PLASTIC HINGES FOR PREDICTION OF LOAD-DEFORMATION RELATIONSHIPS

FIGURE 10. CONSTRUCTION TO DETERMINE HINGE LOCATION FOR AN ASSUMED MOVEMENT OF BEAM CONNECTED LEG

Δ=0.1" (Assumed movement of vertical leg)
FIGURE 11. GEOMETRICAL RELATIONSHIPS USED TO DETERMINE VERTICAL FORCE ASSOCIATED WITH A VERTICAL MOVEMENT

FIGURE 12. COMPARISON OF TEXT RESULTS WITH PREDICTED LOAD-DEFORMATION DATA FOR ANGLE SEGMENTS IN TENSION
FIGURE 13. COMPARISON OF MOMENT-ROTATION CURVES DETERMINED BY THEORY AND BY TEST

* See Appendix B
FIGURE 14. INFLUENCE OF L/d ON END MOMENT
FIGURE 15. DETERMINATION OF END MOMENT TO MAINTAIN DESIRED SAFETY FACTOR
FIGURE 16. INCREASE IN COLUMN MOMENT AS A RESULT OF CONSIDERING END MOMENT
FIGURE 17. END MOMENT FOR CASE WHERE BEAMS OF EQUAL DEPTH AND THE SAME CONNECTION, BUT DIFFERENT L/d, FRAME OPPOSITE
FIGURE 18. END MOMENT FOR CASE WHERE BEAMS OF DIFFERENT DEPTH FRAME OPPOSITE, BUT L/d AND CONNECTION ARE THE SAME.
FIGURE 19. END MOMENT FOR CASE OF DIFFERENT LOADING CONDITIONS IN ADJACENT SPANS
FIGURE 20. END MOMENT DETERMINATION FOR DIFFERENT LENGTH CONNECTIONS
FRAMING OPPOSITE
FIGURE 21. END MOMENT PROVIDED BY FLEXIBLE CONNECTIONS

Note:
- $g_i = 2 - 9/16''$
- $\sigma_f = 36$ ksi
- Angle 4 $\times$ 3-1/2 $\times$ 3/8
- Fastener = 3/4' Dia.
- Uniform Load

End Moment, in.-kips

Rotation, Radians $\times 10^3$
FIGURE 22. RELATIONSHIP BETWEEN END MOMENT AND L/d
FIGURE 23. RELATIONSHIP BETWEEN SECTION MODULUS AND WEIGHT FOR THE MOST ECONOMICAL WF SECTIONS
FIGURE 24. APPROXIMATE SAVING IN WEIGHT WITH PERCENTS OF FIXITY FOR BEAM UNFORMLY LOADED
Load = \frac{M}{(d+g)} ; \Delta = d \times \phi

(a) Forces And Displacements In A Semi-Rigid Connection.

(b) Load-Displacement Curve Determined From M And \phi.

(c) Load-Displacement Curve For A Pair Of Angles Determined From Type Of Test Specimen Shown.

FIGURE A1. DETERMINATION OF LOAD-DISPLACEMENT RELATIONSHIP FOR CONNECTION ANGLES
FIGURE A2. TYPICAL TEST SETUP FOR TENSION AND COMPRESSION OF ANGLES

(a) Tension Specimen
(b) Compression Specimen

FIGURE A3. TYPICAL ANGLE SEGMENT TEST SPECIMENS

a. Tension Specimen
b. Compression Specimen
FIGURE A4. LOAD-DEFORMATION CURVE FOR SPECIMEN C-1

NOTE: Deformation is the movement of the vertical legs toward the bearing plate and does not include measured slip between vertical legs and loading plate.
FIGURE A5. COMPARISON OF BEHAVIOR OF SPECIMENS C-1 AND T-4

NOTE: Both Specimens Were 6" Long
FIGURE A6. SPECIMEN PS-1

(a) At failure
Deformation, inches

(b) Load-deformation curve

Load, kips/inch

ULTIMATE
Load 47.5
Deformation 1.43 in.

Gage 0.5 L - 2 1/2"

SPECIMEN
2 Angles 6 x 4 x 3/16

Deformation, inches

FIGURE A6. SPECIMEN PS-1
(a) Details of Specimen P.S.-2 and P.S.-3.

Steel blocks are used to insure similar behavior across width of specimen.

(b) Location of Photo-Elastic Material

FIGURE A7. SPECIMENS PS-2 AND PS-3
FIGURE A8. LOAD-DEFORMATION CURVES FOR SPECIMENS PS-2 AND PS-3
FIGURE A9. FRINGE LOCATIONS ON ANGLE SEGMENT WITH HIGH-SENSITIVITY PLASTIC -- SPECIMEN PS-3

Legend

- 0 Fringe
---- 1st Fringe
----- 2nd Fringe

Note: One fringe equal to approximately 900 micro-inches
Low Sensitivity Plastic
One fringe equal to approximately 5500 micro-inches

Legend
- Fringe
---- Ist. Fringe
......... 2nd. Fringe

High Sensitivity Plastic
One fringe equal to approximately 900 micro-inches

Fringes to close

FIGURE A10. SPECIMEN PS-3 ~ FRINGE LOCATION AT 8 KIPS

(a) Specimen T-3

(b) Specimen T-4

Note: Angles 4 x 4 x \(\frac{3}{8}\) Holes 13/16" dia.

FIGURE A11. DETAILS OF SPECIMENS T-3 AND T-4
FIGURE A12. SPECIMENS T-3 AND T-4 ~ LOAD-DEFORMATION CURVES
(a) Typical Specimen Shown Upside Down From Orientation In Testing Machine

(b) Typical Test Set-Up

FIGURE B1. TYPICAL SPECIMEN AND TEST SETUP
Figure B2. Typical Details of Specimens and Instrumentation

INSTRUMENTATION
Dials 1 & 2 Measure Rotation of Beam Relative to Column
Dials 3 & 4 Measure Rotation of Connection Angle Leg Relative to Beam Web

Figure B3. Details of Specimen FK-3
FIGURE B4. MOMENT-ROTATION CURVE FOR SPECIMEN FK-3
DETERMINATION OF CENTER-OF-ROTATION

CASE 1  Rotation Of Angle Leg Relative To Beam Web Is Insignificant (Measured By Dials 3 & 4).

Distance From Dial At Tension To Center-Of-Rotation.

\[ CR = \left( \frac{1}{1+2} \right) (\text{Beam Depth} + 2") \]

CASE 2  Rotation Of Angle Leg Relative To Beam Web To Be Considered.

Center-Of-Rotation Of Angle Leg Relative To Beam Web Obtained From Dial 3.

\[ A = \left( \frac{3}{3+4} \right) (\text{Connection Depth} - \frac{1}{2}) \]

Influence Of Rotation Indicated By Dials 3 & 4 On Dials 1 & 2.

\[ B = \text{Distance Between Dials 1 & 3} \]
\[ C = \text{Influence On Dial 1 Of Rotation Indicated By Dials 3 & 4} \]
\[ D = \text{Influence On Dial 2 Of Rotation Indicated By Dials 3 & 4. Calculate As For C} \]

\[ CR = \left( \frac{1-C}{1-C+2-D} \right) (\text{Beam Depth} + 2") \]

FIGURE B5. DETERMINATION OF LOCATION OF CENTER-OF-ROTATION
FIGURE B6. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FK-3
FIGURE B7. LOCATION OF STRAIN GAGES AND AVERAGE STRAINS -- SPECIMEN FK-3
NOTE: Specimen FK-4AB
All Fasteners Are Bolts - One Connection
With Washers The Other Without
Specimen FK-4AB-M
All Fasteners Are Bolts - No Washers Used
On Angle Leg Next To Column Flange
Specimen FK-4P
Angles Riveted To Beam Web, Bolted To Column Flange (No Paint)

Connection - 2 Angles 6 x 4 x 3/8 - 11 1/2"

Column 12 WF 65

FIGURE B8. DETAILS OF SPECIMENS FK-4AB, FK-4AB-M, AND FK-4P

FIGURE B9. MOMENT-ROTATION CURVES FOR SPECIMEN FK-4AB
FIGURE B10. STRAINS ON ANGLE LEG AGAINST COLUMN FLANGE -- SPECIMEN FK-4AB
FIGURE B11. AVERAGE STRAINS ALONG LENGTH OF ANGLE LEG NEXT TO BEAM WEB -- SPECIMEN FK-4AB
FIGURE B12. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FK-4AB

FIGURE B13. MOMENT-ROTATION CURVES FOR SPECIMEN FK-4AB-M
FIGURE B14. DEFORMATION THAT CONNECTION ANGLES CAN SUSTAIN PRIOR TO FAILURE -- SPECIMEN FK-4AB-M
FIGURE B15. AVERAGE STRAINS ALONG LENGTH OF ANGLE LEG NEXT TO BEAM WEB -- SPECIMEN FK-4AB-M

* Average of Near Side and Far Side Strain Values

FIGURE B16. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FK-4AB-M
FIGURE B17. LOCATION OF CENTER-OF-ROTATION DETERMINED BY STRAIN GAGES AND MECHANICAL DIALS -- SPECIMEN FK-4AB-M

FIGURE B18. SPECIMEN FK-4P AFTER TEST
FIGURE B19. MOMENT-ROTATION CURVES FOR SPECIMEN FK-4P

FIGURE B20. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL
DIALS -- SPECIMEN FK-4P
FIGURE B21. DETAILS OF SPECIMEN WK-4

NOTE: Angles Riveted to Beam Web, Bolted to Column Web (No Washers)

Column 12 WF 65

Connection - 2 Anges 6 x 4 x 3/8 - 11 1/2"

FIGURE B22. MOMENT-ROTATION CURVES FOR SPECIMEN WK-4
FIGURE B23. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN WK-4

NOTE: Specimen FB-4 (Shown)
Angles Riveted to Beam Web, Bolted to Column Flange (No Washers)
Specimen FB-4A (Not Shown)
One Angle Riveted to Column Flange, One Angle Bolted to Column Flange, Both Angles Bolted to Beam Web

FIGURE B24. DETAILS OF SPECIMENS FB-4 AND FB-A
FIGURE B25. MOMENT-ROTATION CURVES FOR SPECIMEN FB-4

FIGURE B26. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FB-4
a. STRAIN GAGE LOCATION

b. AVERAGE STRAIN AT VARIOUS MOMENTS

FIGURE B27. LOCATION OF STRAIN GAGES AND AVERAGE STRAIN -- SPECIMEN FB-4
FIGURE B28. MOMENT-ROTATION CURVES FOR SPECIMEN FB-A

Rotation, Radians x 10^3

Rotation due to movement of angle relative to beam web not included.

North
South

Moment, In-Kips
FIGURE B29. LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FB-4A

FIGURE B30. DETAILS OF SPECIMEN FK-5

NOTE: Angles Riveted to Beam Web, Bolted to Column Flange (No Washers)

Connection 2 Angles 6 x 4 x \( \frac{7}{16} \) 1'-2.5"

Column 12 WF 65-4'-0"

Moment, in.-kips

Distance From Tension End Of Connection Angles To Center-Of-Rotation, inches

Distance From Tension End Length Of Connection Angles

South

North

0 100 200 300 400 500

0 1 2 3 4 5 6 7 8 9 10

Compression End

Tension End

0 C C 0

M c

0 Eo

C 0 0 C

°I
c)

Moment, in.-kips

LOCATION OF CENTER-OF-ROTATION BASED ON READINGS FROM MECHANICAL DIALS -- SPECIMEN FB-4A

b, Bolted
FIGURE B31. MOMENT-ROTATION CURVES FOR SPECIMEN FK-5
a. STRAIN GAGE LOCATIONS

b. AVERAGE STRAIN AT VARIOUS MOMENTS

FIGURE B33. LOCATION OF STRAIN GAGES AND AVERAGE STRAIN -- SPECIMEN FK-5
FIGURE B34. DETAILS OF SPECIMEN WB-10AB

NOTE: All Fasteners Are Bolts (No Washers)
Rotation, radians x 10^{-3}

* Rotation Due To Movement of Angle Relative To Beam Web Not Included

FIGURE B35. MOMENT-ROTATION CURVES FOR SPECIMEN WB-10AB
FIGURE B36. LOCATION OF CENTER-OF-ROTATION BASED ON MECHANICAL DIALS -- SPECIMEN WB-10AB

FIGURE B37. SPECIMEN WB-10AB SHOWING ANGLE WITH PHOTO-ELASTIC MATERIAL
FIGURE B38. COMPARISON OF MOMENT-ROTATION CURVES FOR SPECIMEN WITH DIFFERENT FASTENERS AND DIFFERENT FASTENER ASSEMBLIES

FIGURE B39. SPECIMEN WITH ANGLES BOLTED TO THE BEAM WEB COMPARED WITH SPECIMEN WITH ANGLES RIVETED TO THE BEAM WEB
FIGURE B40. COMPARISON OF MOMENT-ROTATION CURVES FROM CONNECTIONS MADE TO COLUMN FLANGES AND WEB

FIGURE B41. EFFECT OF ANGLE THICKNESS ON MOMENT-ROTATION CURVES
<table>
<thead>
<tr>
<th>Connection Angles</th>
<th>$g_1$=2-1/4 in.</th>
<th>$g_1$=2-5/16 in.</th>
<th>$g_1$=2-3/8 in.</th>
<th>$g_1$=2-7/16 in.</th>
<th>$g_1$=2-1/2 in.</th>
<th>$g_1$=2-9/16 in.</th>
<th>$g_1$=2-5/8 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C$</td>
<td>$n$</td>
<td>$C$</td>
<td>$n$</td>
<td>$C$</td>
<td>$n$</td>
<td>$C$</td>
</tr>
<tr>
<td>2Ls 4x3-1/2x5/16</td>
<td>5.72</td>
<td>0.174</td>
<td>5.18</td>
<td>0.158</td>
<td>4.66</td>
<td>0.145</td>
<td>4.32</td>
</tr>
<tr>
<td>Fillet Radius = 3/8 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2Ls 4x3-1/2x3/8</td>
<td>8.95</td>
<td>0.181</td>
<td>8.12</td>
<td>0.167</td>
<td>7.39</td>
<td>0.156</td>
<td>6.77</td>
</tr>
<tr>
<td>Fillet Radius = 3/8 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2Ls 4x3-1/2x7/16</td>
<td>13.75</td>
<td>0.205</td>
<td>12.23</td>
<td>0.186</td>
<td>11.28</td>
<td>0.173</td>
<td>10.33</td>
</tr>
<tr>
<td>Fillet Radius = 3/8 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2Ls 6x4x3/8</td>
<td>10.82</td>
<td>0.215</td>
<td>9.67</td>
<td>0.200</td>
<td>8.81</td>
<td>0.185</td>
<td>8.05</td>
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<td>Fillet Radius = 1/2 in.</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2Ls 6x4x7/16</td>
<td>17.30</td>
<td>0.246</td>
<td>15.18</td>
<td>0.224</td>
<td>13.40</td>
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<td>12.0</td>
</tr>
<tr>
<td>Fillet Radius = 1/2 in.</td>
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<td></td>
<td></td>
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</tr>
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</table>

Note: Fasteners 3/4 in. diameter, plate and shape Y.P. = 36 ksi, $g_2$ = 2-1/4 in. (gage of leg connected to the beam web).
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>L₁</th>
<th>L₂</th>
<th>Fastener Diameter</th>
<th>g₁</th>
<th>g₂</th>
<th>Lower Yield Point, ksi</th>
<th>Angle Thickness, in.</th>
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<tr>
<td>C-1</td>
<td>4</td>
<td>6</td>
<td>3/4</td>
<td>2-1/4</td>
<td>-</td>
<td>40.4</td>
<td>0.382</td>
</tr>
<tr>
<td>PS-1</td>
<td>6</td>
<td>3</td>
<td>3/4</td>
<td>2-1/4</td>
<td>2-1/2</td>
<td>41.6</td>
<td>0.353</td>
</tr>
<tr>
<td>PS-2</td>
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<td>3/4</td>
<td>2-1/4</td>
<td>-</td>
<td>41.6</td>
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<td>-</td>
<td>41.6</td>
<td>0.347</td>
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<td>T-3</td>
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<td>3</td>
<td>3/4</td>
<td>2-1/4</td>
<td>-</td>
<td>40.4</td>
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<tr>
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<td>3/4</td>
<td>2-1/4</td>
<td>-</td>
<td>40.4</td>
<td>0.382</td>
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### TABLE B1

**CONNECTION ANGLE PROPERTIES**

<table>
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<tr>
<th>Specimen Number</th>
<th>Angle Size</th>
<th>Fillet Radius, in.</th>
<th>Leg, in.</th>
<th>Angle Thickness, in.</th>
<th>Elongation in 8 in., per cent</th>
<th>Reduction in Area, per cent</th>
<th>Lower Yield Point, ksi</th>
<th>Ultimate Strength, ksi</th>
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<tbody>
<tr>
<td>FK-3, FK-4AB, FK-4P, WK-4</td>
<td>6x4x3/8</td>
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<td>62.5</td>
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<td>-</td>
<td>-</td>
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<td>37.2</td>
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<td>0.30</td>
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<td>-------------------------------</td>
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<tr>
<td>3/8 in. th.</td>
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<td><strong>FK-4AB-M</strong></td>
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<td>75</td>
<td>80</td>
<td>84</td>
<td>85</td>
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<td>3/8 in. th.</td>
<td>S</td>
<td>70</td>
<td>77</td>
<td>82</td>
<td>85</td>
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<td><strong>FK-3</strong></td>
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<td>77</td>
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<tr>
<td>3/8 in. th.</td>
<td>S</td>
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<td>69</td>
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<td>69</td>
<td>73</td>
<td>77</td>
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<td>S</td>
<td>63</td>
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<td><strong>WK-4</strong></td>
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<tr>
<td>3/8 in. th.</td>
<td>S</td>
<td>58</td>
<td>69</td>
<td>77</td>
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<tr>
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<td>S</td>
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<td>7/16 in. th.</td>
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<td>84</td>
<td>94</td>
<td>96</td>
<td>97</td>
<td>96</td>
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<tr>
<td><strong>FK-4AB</strong></td>
<td>N**</td>
<td>51</td>
<td>62</td>
<td>68</td>
<td>75</td>
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<td>3/8 in. th.</td>
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<td><strong>FK-4R</strong>*</td>
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<td>3/8 in. th.</td>
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<td>50</td>
<td>60</td>
<td>72</td>
<td>76</td>
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</tr>
</tbody>
</table>

*Effect of Slip of Beam Web Relative to Angle Removed.

**North Connection Assembled With Washers.

***See Reference 6.
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where, $K$ is determined by solving the equation for $K$ and then evaluating $K$ for values of $M$ and $\phi$ associated with $A = 0.1 \text{ in.}$ as determined in Equation (3).

To establish the applicability of the above expressions to the prediction of $M-\phi$ relationships, comparisons are made in Figure 13 with the behavior of specimens reported in Appendix B. The predicted curves are based on physical and geometrical properties identical with those of the test specimens. The predicted curves compare favorably with the actual test results except in the case of Specimen FK-3 (a very flexible connection) whose predicted stiffness is approximately 20 per cent less than the stiffness observed in tests. However, this was a very flexible connection and consequently provided very little restraint.

It is possible, utilizing the equations just described, and values of $C$ and $n$ derived from the predicted load-deformation relationships discussed in Section D, to predict the moment rotation characteristics of flexible connection angles within the limitations of geometry and physical properties that have been considered.

The procedures developed herein to predict the $M-\phi$ characteristics of flexible connections should not be extrapolated to include angle thicknesses and gages outside the limits of Table 1. This limitation is imposed because it is expected that if the angle thickness is increased beyond these limits factors not significant in this development, such as the contribution of fastener deformation, may become major sources of deformation. Additional research is needed to more clearly establish the limits of this theory.
IV. DESIGN CONSIDERATIONS

A. CONSTANT-LOAD AND CONSTANT-STRESS BEAM LINES

If it is desired to determine the actual end moment of a flexible connection, a method is required that will take into consideration the nonlinear relationship between moment and rotation of the connection. Such a method was first introduced by Batho and consisted of combining on a single plot the connection moment-rotation curve and curves of the end moment of the beams versus end rotation (Figure 14). The intersection of the curves gives the equilibrium position of the connection and the beam for the angles, beam size, load, and span length shown. Because the beam load is held constant, for any end moment, the line representing the beam behavior is called a constant-load beam line. It follows, then, that for constant load and an increase in end moment, the maximum positive moment (and maximum stress) of the beam will decrease.

A designer is interested in maintaining a limiting value of maximum moment (and, therefore, stress) at a critical section on the beam rather than on maintaining a constant load. In order to keep the maximum stress constant, Hechtman and Johnston modified the Batho procedure. In their method Hechtman and Johnston constructed a line describing the relationship between end moment and end rotation if the stress at some critical section were held constant (critical section is the location along the length of the beam where the moment is a maximum). Such a line is called a constant-stress beam line. To maintain a constant stress at the critical section the load on the beam must be increased as the end moment increases. Since flexible type connections rarely develop more than 20 per cent of the fixed end moment of the beam, the critical section will never be at the end when flexural stresses govern the beam design and the beam size will be determined by the maximum positive moment.

The equation for the constant-maximum-stress beam line derived for a uniform load, maximum stress at the center, and a given working stress is

\[ M = 2\sigma_w S - \frac{3ES\phi}{L/d}, \]

where, \( M \) = the moment at the center,
\( \sigma_w \) = the allowable working stress,
\( S \) = the section modulus,
\( E \) = the modulus of elasticity,
\( \phi \) = the end rotation,
\( L \) = the span length,
\( d \) = the beam depth.
For a concentrated load in the middle of a span with identical end conditions the constant-maximum-stress beam line equation is

\[ M = \frac{c \phi S - 2ES\phi}{L/d}. \]

Where the load is maintained constant, the constant-load beam line equation is

\[ M = M_r - \frac{2EI\phi}{L}, \]

where, \( M_r \) is the fixed end moment for the load the beam is supporting. The beam must have identical moment resisting connections at each end and the load must be symmetrical for these equations to apply.

In Figure 14 three constant-stress beam lines are constructed for a beam carrying a uniform load but with varying span lengths. Note the increase in end moment with increasing span length. This increase is related to the decrease in the stiffness of the beam relative to the connection as the length of the beam is increased.

B. SAFETY FACTOR

Consider the 18WF50 beam supported by a flexible connection as shown in Figure 15. The moment-rotation characteristics for the connection, and the constant-stress beam lines for a working stress of 24 ksi and a maximum stress of 36 ksi are also shown. In elastic design, failure in flexure is associated with the extreme fiber reaching the yield stress at the critical section and the safety factor is the ratio of the failure (or yield)

stress to the working stress. Since stress is proportional to load in the elastic design of beams, the beam shown in Figure 15 can be expected to carry an overload of \( (36/24) \) or 1.5 times the load required to produce the working stress. This load-stress relationship assumes that the behavior of the connection and the beam are linear up to the failure (initiation of yielding) of the beam. As has been observed in tests, the relationship between moment and rotation for flexible-type connections is actually nonlinear under usual loading conditions. In Figure 15 it can be seen that because of the nonlinearity of the connection behavior, moment capacity of the connection increases only slightly between the working stress condition and yield stress condition on the beam. Therefore an increase in load in the ratio of the yield stress to the working stress will cause more than a proportional increase in the extreme fiber stress. The end moment to be utilized in working stress design should be the end moment developed when the extreme fiber has just reached the yield point multiplied by the ratio of the working stress to the yield stress as indicated by Hechtman and Johnston.\(^1\) This will insure that when the working load is multiplied by the safety factor the extreme fiber will just reach the yield point (see Figure 15).

To illustrate the preceding discussion consider the following example. The 18WF50 beam shown in Figure 15 can support a uniformly distributed working load of 2.47 kips per ft.
(\omega = 24 \text{ ksi}^{*}) \text{ if there is no end moment. The intersection of the moment-rotation and the constant-stress curves for the connection and the beam indicates that the end moment actually will be 209 \text{ in.-kips} (17.4 \text{ ft-kips}) when the beam working stress is 24 \text{ ksi}; therefore the beam can carry an additional working load of 0.24 \text{ kips per ft} because of the resistance of the connection to moment. Thus the total working load when end moment is considered is 2.71 \text{ kips per ft}. Since the yield stress is taken to be 36 \text{ ksi}, the safety factor against yield ordinarily would be expected to be 36/24 or 1.5 and the failure load should be (2.71 \times 1.5) 4.06 \text{ kips per ft}. When the beam is stressed to 36 \text{ ksi} the end moment is 241 \text{ in.-kips} (20.1 \text{ ft-kips}) (Figure 15) and the corresponding load is 3.99 \text{ kips per ft}. Therefore the yield point is reached at a lower load than the 4.06 \text{ kips per ft} which would be the case if the connection behavior had been linear. To insure the proper working load, the end moment used must be the end moment developed when the extreme fiber of the beam reaches the yield stress multiplied by the ratio of the working stress to the yield stress [for this example, 241 \times (24/36) = 160 \text{ in.-kips}]. For the example, the permissible working load that corresponds to a 1.5 overload is 2.65 \text{ kips per ft} (an increase of 7 per cent in the distributed load as a result of the end restraint).

Because of the nonlinearity of connection behavior, beam design based on the working load times the safety factor (to achieve yield stress at the extreme fiber) rather than working load will be more direct.

C. ALLOWABLE STRESS DESIGN

The AISC Specifications permit the design of beams with end moments intermediate between the pinned end condition and full fixity (Section 1.2, Type 3). Little use is made of this provision, however, because of limited dependable information available concerning moment capacities of semi-rigid connections. Even when the connection behavior is known, the actual end moment that connections will provide is influenced by framing conditions and the load in the adjacent spans. Also, there is the possibility that deformations of the column may influence the beam end moment.

Since the connections discussed herein are of the type currently described as flexible, (relatively little resistance to moment) their previous use in design has not included consideration of their moment resisting capacity. However, methods will be discussed for taking advantage of the resistance to rotation that these connections possess and use will be made of the ability to predict connection behavior developed in this study. Connections of the type currently described as semi-rigid can be treated similarly.

The simplification of assuming zero end restraint for flexible connections has meant that columns actually have been subjected to moments which have not been taken into consideration...
in their design. Neglecting these moments, as tests have shown (Appendix B), is unconservative especially at corners or other areas lacking complete symmetry of framing and loading. There is no evidence, however, that this simplification has resulted in any real concern over the adequacy of columns in actual structures joined by flexible-type connections. However, the following question might be raised: Will the additional moment delivered to the columns through the connections be significant enough to require consideration in the determination of the total connection moment in column design?

Figure 16 shows a constant-load beam line for an 18WF50. This is the beam section required for the load and span shown in the figure if no consideration is given to the end moments. Also shown in the figure is the constant-stress beam line for a 16WF45 which would be the section required if the resisting moment of the connections is considered. It should be noted that a reduction in beam depth with no change in the connection size or load has the effect of increasing the end moment from 230 in.-kips to 255 in.-kips. The increase is approximately 10 per cent over the already existing (but currently ignored) moment of 230 in.-kips. To properly design columns would warrant consideration of this moment in the column design.

The problem of unbalanced moments at columns and girders also requires consideration, but because of the non-linear connection behavior the actual end moment must be determined by trial and error.

Where a beam frames into a girder and there is no member framing opposite, or within a short distance, it would seem reasonable to assume that there will be no end moment because of the small torsional restraint of the girder.

Figure 17 shows the conditions that would exist if two beams of the same size but different span-length to beam-depth ratio framed into the web of a girder. If it is assumed that the web of the girder remains vertical, there will be an unbalanced moment. If the web of the girder is allowed to rotate in the direction of the unbalanced moment, the M-$ curves of the connections will be shifted along the abscissa until the joint is in equilibrium (see Figure 17). The actual end moment in this case is approximately equal to the average of the two unbalanced moments.

Consider next the case where beams of different depth, but the same size connection, frame opposite each other at a girder as is shown in Figure 18. If the span-length to beam-depth ratios in this case are equal and the load approximately uniformly distributed, the unbalanced moment will be small enough to be neglected.

If in design it is desired to entertain the possibility that an adjacent span will be carrying none or only a portion of its live load capacity at the time the beam being considered is loaded to failure (beam has reached the yield stress), a construction as shown in Figure 19 will describe the behavior. Because of the rotation of the girder as a result of the unbalanced moment, the available end
moment again can be considered to be equal approximately to the average of the unbalanced end moments. The procedure for the less conservative case of adjacent spans assumed to carry only a portion of their live load capacity when the beam being sized is loaded to failure would be similar to Figure 19 or to the case of different depths (Figure 18). The largest load capacity and the greatest economies will occur when failure is assumed to take place in all spans simultaneously.

The condition that will exist if connections of different lengths frame opposite one another is shown in Figure 20. In this case the smallest of the two end moments would be the moment to use in design.

In the case where a beam frames into a column and there is no beam framing opposite it is recommended that no end moment be considered. If beams frame into a column opposite one another and are of comparable span length-to-beam ratio and connection size, the end moment should be that associated with zero rotation of the column. Whenever the beam connection is made to the column flange it is suggested that the column flange be as thick as the connection angles or greater in thickness before considering that the column flange deformation can be neglected. If the column flange thickness is less than that of the angles, it is suggested that the connection be treated as pinned.

To facilitate design, curves can be constructed which give the value of end moment that a particular flexible connection will provide when combined with a uniformly loaded beam stressed to the yield point. Figure 21 is an example of such a plot. In this figure constant-stress beam lines were constructed for a number of the more efficient WF sections and three span-length to beam-depth ratios. The beam-lines intersect only those moment curves of connections with a length recommended by the AISC Specifications. For a particular connection length and span-length to beam-depth ratio, the beam size has very little effect on the end moment. Because of the small influence of beam size, beam size can be neglected and the end moment determined on the basis of the span-length to beam-depth ratio (see Figure 22). The span-length to beam-depth ratios along the abscissa correspond to the end-rotations in Figure 22 which results when there is no end moment, the beam is stressed to the yield point, and the span-length to beam-depth ratios are those shown.

Plots similar to Figure 22 can be constructed using the methods proposed in this paper for variations in gage, angle thickness, fillet radius, fastener size, and yield point which are the factors which affect the moment-rotation behavior of the connection angles. With such an assortment of plots the end moment may be quickly determined.

D. BEAM WEIGHT SAVINGS

No complication of a design procedure can be justified unless there are advantages, either economic or behavioral. Since flexible connections as now used behave satisfactorily, it is necessary to anticipate some weight
savings in order to justify the additional work required to take end moment into consideration. To facilitate an investigation of this question a relationship between section modulus and weight may be used. Figure 23 shows a plot of section modulus versus weight for the most efficient wide flange beams available. These sections are from the AISC Steel Construction Manual. The relationship between section modulus and beam weight can be described quite closely by two straight lines and a transition curve. The equations for these lines and the curve are shown on the figure. A similar curve involving British beam sections is shown in the book The Plastic Methods of Structural Analysis, by B. G. Neal. Utilizing a curve of this type it is possible to determine the weight reduction corresponding to a reduction in section modulus resulting from the consideration of end moment in design.

Also included in Figure 23 is a curve showing the relationship between section modulus and weight for a group of 27 WF's with 14-in. flanges (not among the most economical sections). It will be noticed that a line connecting these points has a different slope than the curve representing the most economical sections. Therefore, if the beam chosen for a particular design is selected on the basis of considerations other than having the most efficient section available, for example, the section is chosen because of depth limitations or because of the necessity for a wider flange width to insure stability, for these sections a greater reduction in weight will be possible for a given reduction in section modulus.

Weight savings mentioned herein are based on the utilization of the most efficient sections in any particular case. In order to determine the percentage of weight savings that will result if end restraint is taken into consideration, Figure 24 has been prepared. The odd shape of the curve between a section modulus of about 65 and 155 is the result of the transition curve shown in Figure 23. In most cases the percentage of weight savings can be approximated by taking 40 per cent of the percentage of end-fixity furnished by the connection.

The following is an example illustrating the design procedure if the connection moment is considered; the simple beam that would have been required is included for comparison. The data governing the design are:

- Span Length - 24 ft
- Working Load - 2.6 kips per ft
- Working Stress - 24 ksi
- Yield Stress - 36 ksi
- Safety Factor - 1.5
- Connection - 2 angles
  - 4 x 3 1/2 x 3/8 x 11 1/2 in.

In the case of simple beam design the maximum beam moment for the failure load (2.61 x 1.5 = 3.9 kips per ft) is 3370 in.-kips and the required section modulus is 93.6 in.³ (3370/36). The simple beam design requires an 18WF55 beam. If the end moment is considered it is necessary to determine the moment that will be developed; see Figure 22 (for L/d = 16 and 4 rows of fasteners) to determine the moment developed by the connection. The end moment is 210 in.-kips; hence the maximum beam moment is reduced to 3160 in.-kips (3370 - 210). A moment of 3160 in.-kips requires a section modulus of at least
88 in.\(^3\) if the yield stress of 36 ksi is not to be exceeded. An 18WF50 (Section Modulus = 89 in.\(^3\)) meets this requirement. The weight saved in this case is 9.1 per cent (5/55).

E. CONCLUSIONS

1. For connection angle thicknesses of 5/16 to 7/16 in., the moment-rotation characteristics of flexible-type connections can be predicted with sufficient accuracy to utilize their restraint in design.

2. The additional design time required to take the end restraint of flexible connections into consideration, with the aid of prepared charts, will be slight and result in beam weight savings of approximately 10 per cent, depending on loading conditions and framing.

3. The predicted connection behavior can be utilized to provide stiffness factors required in methods of analysis of frames with semi-rigid connections. (10)
APPENDIX A: TENSION AND COMPRESSION TESTS OF ANGLE SEGMENTS

A. INTRODUCTION

Messrs. Beaufoy and Moharram in their paper entitled, "Derived Moment Angle Curves for Web Cleat Connections," draw attention to the comparative behavior of semi-rigid and flexible connections. The authors used the moment-rotation relationship and geometry of a particular flange-cleat connection to determine the load-deformation relationship for a pair of angles of the same size and loaded as shown in Figure A1. They assumed that flexible connection behavior is equivalent to short lengths of angles acting as shown in Figure 5; it can be shown that the load-deformation characteristics of the angles can be used to determine the moment resistance of the flexible connection. Correlations between predicted moment-rotation curves and experimental curves was good. This has made it possible to use the results of tests of flange-cleat or semi-rigid types of connections in the prediction of the moment-rotation characteristics of flexible-type connections.

The correlation established by the work mentioned above has shown the comparative behavior of the two different types of connections. Since the behavior of a flexible connection is related to the behavior of strips or segments of angles loaded as shown in Figure 5, it should be possible to obtain the required load-deformation relationship of the angles by testing a segment of similar geometry and loading. The tests reported in this appendix were conducted to provide this information.

The scope of this work is limited to angles with dimensions close to those used in most flexible connections. Also, such variables as angle thickness, gage, fastener size, fillet radius, and yield point have been kept within limits normally expected in conventional building framing of mild structural grade steel.

B. TEST RESULTS

Six tests were conducted on small two-angle specimens as part of a study of the behavior of connection angles. There were five tension specimens and one compression specimen. Figure A2 shows the geometry and manner of loading of the specimens which were included in the program.

1. Specimen C-1

It has been observed that the center-of-rotation in flexible connections (Appendix B) initially is near
mid-height of the connection: this finding indicates that (for typical construction) initially portions of the connection in tension and in compression have similar stiffnesses. Specimen C-1 was included in the program to determine the behavior of the compression end of the connection so that the behavior could be compared to that at the tension end (Figure A3a), especially in the early stages of loading. The test setup for Specimen C-1 is shown in Figure A2 and a photograph of the specimen is shown in Figure A3b. It can be seen from the load-deformation curve in Figure A4 that the behavior is as expected; there is an abrupt increase in stiffness when the heel of the angle attains firm contact with the bearing plate. The deformation which is recorded in the load-deformation plot is the movement of the heel of the angle towards the bearing plate plus the difference between the elastic strains in the vertical legs of the angles and the elastic strains in the loading plate and was obtained by subtracting the movement of the loading plate relative to the vertical angle legs from the movement of the loading plate towards the bearing plate. A comparison between the results from the test on Specimen C-1 and those from T-4 (See Figure A5) shows the tension and compression specimens to have similar stiffnesses in the early stages of loading. However, they differ markedly in the latter stages of loading.

2. Specimen PS-1

The development of an analytical procedure for predicting load-deformation relationships for angle segments in tension involves assumptions as to how the connection angles are restrained at the fasteners, the locations of regions of inelastic behavior, and the order in which inelastic action progresses throughout the specimen. Specimen PS-1 was tested to provide a basis for more realistic assumptions concerning angle behavior. The typical test setup for angles in tension is shown in Figure A2.

With Specimen PS-1 (Figure A6), not only were load deformation data accumulated but a photo-elastic material was bonded to the surfaces of the specimen to provide a detailed history of the response of the angle to load. Because considerable inelastic action takes place in flexible connections, even at working loads, a low-sensitivity, photo-elastic material was used on the angles so that the material would be effective at the large strains. Calibration of the photo-elastic material showed that the first fringe was formed at 5500 micro in. per in. of strain. A drawback to the use of a low-sensitivity material was that it could not be used to accurately detect the strains associated with the beginning of yielding; however, the photo-elastic results did show the progression of inelastic behavior.

As would be expected, the horizontal leg (Figure A6) of the test specimen was the most highly strained element. In the horizontal leg of the specimen, where the bolt is located, large strains develop first at the edge of the bolt head or nut attaching the horizontal leg to the tee section.
Following this initial yielding adjacent to the fastener in the horizontal leg, yielding began in the fillets of the angles in both the horizontal and vertical legs at approximately the same load. Large strains were indicated adjacent to the fastener in the vertical leg much later in the tests.

3. Specimen PS-2

Specimen PS-2 was added to the program to determine the strains throughout the thickness of the angle leg rather than just on the surface as was the condition in Specimen PS-1. To accomplish this, a special specimen was utilized, (see Figure A7). This specimen was designed to duplicate the behavior of that portion of the angles directly restrained by the fasteners since this is the only location along the length of the angles that the end conditions are easily reproduced. The specimen was restrained across the full width by the use of the steel blocks. With the specimen restrained in this manner, it was assumed that the strain indications of photo-elastic material mounted on the edges of Specimen PS-2 would be typical of the strain distribution cut parallel to the end of the special specimen.

The plastic used on Specimen PS-1 was of low sensitivity to accommodate large strains. It was decided in this specimen to use two different plastics. On the edge of one angle a high-sensitivity plastic was bonded to show strains in the early part of the test and on the other angle the low-sensitivity plastic was used to record the large inelastic strains. In this manner both the behavior at the beginning and the end of the tests could be observed. Other instrumentation, as in previous tests, included slip dials to measure the relative movement of the component parts.

The specimen was loaded in 500-lb. increments through the early stages of loading and then in one-kip increments. A photograph of the isochromatics was taken at each increment of loading, and at 0, 3, 6, and 9 kips iso-clinics were also taken. The dial indicators were read at all increments. The load deforma-tion curve for the angles is shown in Figure A8b. Unfortunately, as a result of poor photographic technique, it was not possible to interpret properly the pictures of the iso-chromatics and iso-clinics. The test was rerun on Specimen PS-3.

4. Specimen PS-3

Specimen PS-3 was the same as PS-2 in every detail. The load-deformation curve obtained in the test of PS-3 is shown in Figure A8b. From the photo-elastic studies, it was possible to determine the maximum strain location and points of contra-flexure. In Figures A9 and A10 the fringe locations can be seen as the load on the specimen was increased.

Upon installation of the specimen in the testing machine, before any load was applied, pictures were taken of the iso-chromatics to obtain an indication of the strains in the angles resulting from the tightening of the bolts. These are shown in Figure A9a. Compressive strains can be observed at the fasteners and bending strains in
the fillets of the angles.

The initial shape of the specimen as fabricated can have a considerable influence on strains in the early stages of loading. It can be noticed at the 2 1/2-kip load that the top surface of the horizontal leg was in tension from the fillet to the block which was used as a nut. Considering the restraint provided by the fillet and the nut one should find a point of contra-flexure situated between them. The point of contra-flexure was not there because the tightening of the bolts caused flexural straining of the angle legs as a result of the angle legs not being at right angles to each other initially; the angle between the two legs was actually less than 90°.

The point of contra-flexure will not be associated with zero strain in this specimen, as would be the case in a continuous beam with roller supports (where no force parallel to the longitudinal axis will be developed as a result of deflections alone). High initial tension in the fasteners makes possible high friction forces between the horizontal leg and the T-section at the fastener and this friction force will keep the horizontal legs from moving towards each other as the vertical load is applied. This horizontal force which is necessary to keep the specimen in a shape forced on it by the external restraints will produce tensile strains in the horizontal leg. These tensile strains are superimposed on the strains resulting from the bending of the angle leg. Therefore the point of contra-flexure is not a point of zero strain but a section through the angle where there are tensile strains of uniform magnitude. It can be seen that at a 2 1/2-kip load the point of uniform strain in the horizontal leg was about 1.65 in. from the heel of the angle (approximately 1/4 in. from the steel block).

As the load was increased, this point of uniform strain gradually moved towards the fillet. At the 6-kip load, which was about the last load for which it was still possible to identify iso-chromatics clearly, the point of uniform strain had moved to about 1 3/8 in. from the heel of the angle. This agrees with the location of the point of contra-flexure for moments of equal magnitude located at a critical section in the fillet and at the edge of the steel block. The critical section in the fillet is not where the fillet is tangent to the angle leg, but it is within the fillet a short distance. This point was located by determining where the moment and the section modulus of the angle are increasing at the same rate. The location of this critical point was approximately 0.2 of the radius of curvature of the fillet beyond the point of tangency. For the particular specimen being tested this places the critical section at approximately 0.82 in. from the heel of the angle. The edge of the steel block is about 1.88 in. from the heel of the angle. This would put the point of contra-flexure, based on equal moments at these two locations, at 1.35 in. from the heel of the angle and agrees with the 1 3/8 in. observed in the tests.

In the vertical leg, the location of maximum straining is near the juncture of the fillet and angle leg.
as in the case of the horizontal leg. The point of contra-flexure in the vertical leg is located near the steel block. It can be determined from these observations that the moment in the vertical leg adjacent to the steel block is much less than the moment at the fillet. This same observation may be obtained from a theoretical analysis.

An examination of the load-deformation curve, Figure A8b, shows that at approximately 4 1/2 kips there was a marked change in the behavior. The photo-elastic material also indicated that at this load the iso-chromatics in the region of the fillets and the fasteners were becoming too numerous to separate on the photographs. It would appear therefore that the abrupt change in slope of the load-deformation curve is associated with the formation of plastic hinges at the critical locations. Based on this test it can be concluded that plastic hinges may form in angles having geometrical and physical properties similar to those of PS-3, and that this will occur when the deformation is approximately 0.020 in. This deformation is well below that which can be expected (for gravity loads) near the top of flexible-type connections supporting beams in which the extreme beam fiber at the critical section is stressed to $36\text{ ksi}$. (See Figure 4.)

At approximately 8 kips (Figure A10) the low-sensitivity plastic started to show fringes. The location of the point of contra-flexure in the horizontal leg appeared to remain at 1 3/8 in. and the high-sensitivity plastic on the vertical leg showed that there was no plastic hinge formed adjacent to the steel block on the vertical leg of the angle. The point of contra-flexure remained in the same approximate location through the 12-kip loading (the fringes could not be separated beyond this point).

5. Specimens T-3 and T-4

One question that comes to mind when considering the applicability of the results of a simple tension test to the prediction of the load-deformation curves for angles in tension is whether the stiffness of the tension specimen is comparable to stiffness of an equal length of angle which is actually part of a connection angle. If a pair of angles is tested in tension, of a length comparable to the 3-in. fastener spacing generally used, there will be unrestrained edges at each end. If a flexible connection is divided up over its length into 3-in. increments, the only segment to have a free edge, of those which must resist tension, is the one at the end. All other segments have a common boundary with another segment which produces continuity between the two. This continuity affects the stiffness of the angles. It was therefore considered desirable to obtain some indication of the difference in stiffness between a specimen with its boundaries free and another specimen with the boundaries fixed. Specimens T-3 and T-4 were included to provide this information. Figure All shows the geometry of the specimens.

The load-deformation curves for Specimens T-3 and T-4 are shown in Figure A12. The two ends of Specimen
T-4 between the center line of the fastener and the free boundary when added together form a specimen equivalent to Specimen T-3. If, in Specimen T-4, the load carried by the end portions was known, the load carried by an interior portion could be determined.

The separation of the effects of the end of the connections is possible because of their similarity with Specimen T-3. If the load-deformation curve for Specimen T-3 is subtracted from that of Specimen T-4, the result should be a third curve which would describe behavior of the interior portion of Specimen T-4 (Figure A12). From a comparison of the 3-in. center strip with the 3-in. specimen with ends unstrained, it can be seen that the interior strip was about 7 1/2 per cent stiffer than an exterior strip. The stiffness condition which prevails over most of the length of the angle at the tension end is similar to that of the interior portion of the Specimen T-4. Thus, if specimens are to be tested similar to Specimen T-3, to provide load-deformation information, it appears reasonable that wherever this data is applied to the interior portion of the flexible connection the load should be increased approximately 7 per cent for the same deformation.

C. DISCUSSION OF TEST RESULTS

The similarity in the load-deformation relationship of the specimen loaded in compression (Specimen C-1) with those loaded in tension (Specimen T-4) for low loads explains why the center-of-rotation (Appendix B) of a flexible connection is located near the mid-length of the connection for low moments. Since the load-deformation relationship is the same at both ends of the connection, there must be equal lengths in tension and compression; therefore, the center-of-rotation must initially be at mid-length.

Plastic hinges adjacent to the fasteners and at the angle fillets will form in the connection angles when the displacement is approximately 0.02 in.

Specimens PS-2 and PS-3 were 1 3/8 in. long and were restrained over their entire width by steel blocks. Specimen PS-1, which was 3 in. long, was restrained only by the fasteners (about 1/3 of the length) and therefore would be expected to be less stiff than Specimens PS-2 and PS-3. A comparison of the load-deformation curves for the two types of specimens (Figures A6 and A8) shows the completely restrained specimens to be approximately 30 per cent stiffer than Specimen PS-1. This would indicate that stiffness is affected by changing the fastener spacing, but the small variations in spacing associated with conventional design requirements will not make this a significant factor.
VI. APPENDIX B: BEAM-TO-COLUMN TESTS

A. INTRODUCTION

The adoption and rapid acceptance of the high-strength bolt as a structural fastener prompted the initiation of a research program in the 1950's to study the behavior of this type of fastener and various types of connections assembled with them. One such connection, the beam-to-column connection, was evaluated in the studies reported herein.

B. DESCRIPTION OF SPECIMENS AND TESTS

The specimens were designed for comparison with those used in several prior investigations, that is, with a column stub and beams projecting from either the web or flanges of the column. This arrangement makes it possible to conduct two tests simultaneously (Figure Bla).

The beams and columns were cut from rolled sections of material meeting the specifications of ASTM designation A7; rivets were furnished from material meeting ASTM designation A141 requirements; and the high-strength bolts met the requirements of ASTM designation A325-55T. The specimens were fabricated, except for bolting, in the shop of a large steel fabricator. Bolting was done in the research laboratory.

Between the time of the initial series of tests and the present series, changes have been adopted in the high-strength bolt assembly requirements. In order that the present series of tests might more closely agree with the current trends in bolting, most of the high-strength bolts were assembled without washers under the nut or bolt head, whenever this element was the one in contact with the angle material. Also, a turn-of-nut method of tightening equivalent to the "snug plus 1/2 turn" method was used.

Coupon tests were run on the structural material used in the specimens and the results are listed in Table B1.

The beam-to-column specimens were tested in a 600,000-lb Universal testing machine with a setup shown in Figure Blb. Instrumentation used in the tests consisted of mechanical dials, SR-4 electric resistance strain gages, white-wash, and photo-elastic materials. The locations of the dials and an indication of their functions are shown in Figure B2. All specimens had 40-in. moment arms. The ends of the beams were supported on rollers to reduce the restraining effect of frictional forces to a minimum.

The test specimens are identified by combinations of letters and numbers...
that have the following meanings: the first character in the specimen designation is the letter F or W. The F indicates that the beams are connected to the column flange, while W indicates a connection to the column web. The second character is either a B or a K. This is a designation taken from earlier versions of the AISC Specifications and indicates whether the connection angle leg is attached to the beam web with one or two lines of fasteners, respectively. The initial two characters are followed by a number which indicates the rows of fasteners in the connection. For example, in Specimen FK-3 the beams are connected to the column flange, it is a K-type connection, and there are three rows of fasteners.

To gain an understanding of connection behavior, the following information was obtained in the tests: moment-rotation relationships for the beams relative to the columns, the change in location of the center-of-rotation with increase in moment, and the contribution to rotation caused by slip of the angle leg relative to the beam web.

1. Specimen FK-3

Figure B3 shows the details of Specimen FK-3; the moment-rotation curves are shown in Figure B4. The bolt assembly consisted of 3/4 in. diameter A325 regular semi-finished hex bolt and a heavy semi-finished hex nut without washers, installed so that the nuts were against the connection angles. The angles were riveted to the beam web. The center-of-rotation was determined from the rotation dials by the procedure shown in Figure B5. The change in location of the center-of-rotation with increasing moment is shown in Figure B6.

A number of SR-4 electric strain gages were mounted on the web of the column, 1 1/2 in. from the face of the column, to provide an additional means of locating the center-of-rotation. The transition from tension to compression in the beam web will not correspond to the center-of-rotation as previously defined because the straining in the compression zone has a much steeper gradient than the straining in the tension zone. However, it is possible to determine the approximate difference in the location of the point of zero load at the surface of the column flange (center-of-rotation) and the point of zero strain in the column web 1 1/2 in. from the column flange. Consider the web to be a large plate with an edge loading of the same distribution as the column flange delivers to the column web. Using the theory of elasticity it can be shown that the point of zero strain in the web, 1 1/2 in. from the exterior surface of the flange, is approximately one inch closer to the mid-length of the connection than the zero load point on the column flange surface. Figure B7 shows the strain in the column web as the moment delivered to the column through the connection was increased.

2. Specimen FK-4AB

Specimen FK-4AB was included in the program to provide a specimen with all bolted connections. Figure B8 gives the details of the specimen and in Figure B1b the specimen can be seen in
place in the testing machine. To examine the influence of washers on the connection stiffness, the south connection was assembled without washers while washers were placed on the north connection.

In addition to measurements of relative movement by the usual mechanical dials, measurements were also taken of bolt elongations. Strain gages were added to one of the connection angles to provide additional information concerning the behavior of the angles, particularly with regard to the center-of-rotation. Because of the complex behavior of connection angles of the flexible type, the location of a strain gage on one side of the angle would be very difficult to interpret without knowledge of the strain on the opposite side of the angle leg. For this reason, it was considered important to mount strain gages on both sides of the angle leg, as well as on both legs of the connection angle. To provide clearance between the strain gage and the column flange a 1/2-in. wide by 1/8-in. deep slot was machined in the column flange at each strain gage with the slot extending beyond the toe of the angle to provide access for the lead wires. On the angle leg against the beam web notches were cut in the beam web at each strain gage location to provide clearance and lead wire access.

The moment-rotation curves for the two connections are shown in Figure B9. The effect of the washers was to increase the stiffness approximately 10 per cent.

The strain gages placed on the angles have been used to obtain an indication of the location of the point of contra-flexure in the leg of the angle against the column flange and also to help locate the center-of-rotation. Figure B10 shows how the strains varied at a section 2 3/4 in. from the end of the angle, as the loads were increased on the specimen. The point of contra-flexure as indicated by the strain gage readings was approximately 1 3/8 in. from the heel of the angle which agrees with photo-elastic evidence in Appendix A.

Figure B11 presents the average strains along the length of the angle leg against the beam web. The forces in the leg necessary to produce these strains can be determined when the loads are low and the behavior is still elastic. The forces represented by the strains in the beam-connected leg produce a couple; and to satisfy the conditions of equilibrium, the area of the tension zone should equal the area of the compression zone and the force represented by this area multiplied by the distance between the center of gravities of the two different areas should equal the moment applied to that particular connection. When the applied moment was 40 in.-kips the tension and compression resultants determined from the strain gage readings were each 5.5 kips and the calculated moment resisted by the connection was 42 in.-kips, assuming both angles to be resisting the same moment. At 80 in.-kips applied moment the tension resultant was 10.1 kips and the compression resultant 13.5 kips; if these two values are averaged the resisting moment is 97 in.-kips. Since only one angle was instrumented, it is possible that
the other angle may resist a different moment and discrepancies may exist between the actual applied moment and the moment indicated by the strain gages. Figure B12 shows the location of the center-of-rotation with increasing moment as determined from the mechanical dials.

3. Specimen FK-4AB-M

Specimen FK-4AB-M was added to the program upon completion of the test on Specimen FK-4AB. The specimen angles were machined in the laboratory to a 3/8-in. thickness from a 1/2-in. thick angle. It was felt that the strain gages would provide more reliable information if mounted on machined angles rather than being bonded to angles subject to the tolerances and deformations associated with standard rolling mill methods and shop fabricating procedures, as was the case in Specimen FK-4AB. Mechanical dials were mounted in the same manner as on the other specimens to measure relative movements, and strain gages were mounted on one of the connection angles to obtain a quantitative measure of the strain distribution in the angles.

As noted in Figure B8, no washers were used under the nuts but they were used under the finished-hex bolt heads in an effort to provide about as much restraint as would be provided by the heavy semi-finished nut. Moment-rotation curves for the two connections are shown in Figure B13.

The tremendous ductility of structural steel connections of this type is exhibited in Figure B14. Even at this extreme deformation, the ultimate or maximum moment capacity of the connection had not been reached. The test was concluded because the loading frame would not permit any more deflection of the specimen.

Strain gage locations are shown in Figure B15. A somewhat different technique was used on this specimen to provide access to the strain gages and to provide clearance between the strain gages and the face of the column. While in Specimen FK-4AB, bolts were machined in the face of the column to provide clearance for the strain gages, in this specimen cold-rolled strips 1/8 in. thick were placed between the angle leg and the column flange. A 2 1/2-in.-wide strip was centered on each fastener and this provided a 1/2-in.-wide access slot to the gages midway between the fasteners. The cold-rolled strip at the compression end of the specimen was wider and extended beyond the end of the connection angles so that the heels of the angles at the compression end would come into bearing on the cold-rolled strip. If the strain gage readings on the two faces of the angle leg attached to the beam web are averaged, then effects due to flexing of the leg will be removed and the result will be a value of strain which can be attributed to the tensile or compressive forces acting in the angle leg. Plotted in Figure B15 are values of average strain along the length of the angle leg next to the beam web for various moments.

The location of center-of-rotation based on data from mechanical dials is shown in Figure B16.

The relationship between the point of zero strain along the line of the
strain gages on the web-connected leg and the center-of-rotation is similar to that of Specimen FK-3 where the strain gages were mounted on the column web. Specimen FK-4AB-M had the strain gages located one inch from the face of the column, which was less than the 1 1/2-in. distance for Specimen FK-3. Since the strain gages were 1/2 in. closer to the column flange, the point of zero strain should be about 3/4 in. closer to the mid-length of the connection than the center-of-rotation. See Figure B17 for a comparison of the center-of-rotation as determined by mechanical dials and by strain gages.

4. Specimen FK-4P

In this specimen the angles were riveted to the beam web and bolted to the column flange; the specimen configuration is shown in Figure B8. A photograph of the specimen after the test is shown in Figure 18. There were no washers underneath the nuts. However, washers were used underneath the bolt heads to approximate the stiffness of the heavier bolt head which were being considered for adoption at the time the tests were run.

The moment-rotation curves are plotted in Figure B19 and, as can be seen, there was little difference in behavior between the two connections on either sides of the specimen.

The location of the center-of-rotation for the specimen is shown in Figure B20.

5. Specimen WK-4

Specimen WK-4 was included in the program to provide an indication of the contribution of column flange deformation to the overall connection rotation. In the drawing of the Specimen (Figure B21) it may be noted that the beam connections were made to the column web. The moment-rotation curve is plotted in Figure B22 and the center-of-rotation location in Figure B23.

To obtain an indication of the change in stiffness resulting from different size bolt heads and nuts, the bolts were assembled without washers under either the bolt head or nut. The nut was a heavy semi-finished type and the bolt was the regular semi-finished hex type.

6. Specimen FB-4

The connection designated previously as AISC Type-B are the type most commonly used in practice. Specimen FB-4 was one of that group. It was expected that this type of connection would be less stiff than the K-type connection because of the larger rotation possible between the beam web and the angle leg connected to the beam web by a single line of fasteners. Therefore, this connection type should afford the minimum degree of restraint possible with flexible type connections. The details of the specimen are shown in Figure B24.

The results of the connection tests are shown in Figure B25 and the location of the center-of-rotation in Figure B26. Since there was no important difference in behavior between the moment-rotation curves for the specimen with one or two lines of fasteners, the number of lines of fasteners did not
to have any significant effect.

This specimen, like some of the others, had strain gages mounted on the web of the column to provide an indication of the load distribution along the length of the column in the area adjacent to the connection angles. Figure B27 shows the distribution of strain in the column web 1 1/2 in. from the face of the column.

7. Specimen FB-4A

Specimen FB-4A was included to provide an opportunity to study a specimen which uses an erection procedure calling for one of the angles to be shop riveted to the column flange while the other angle is to be shipped loose and to be bolted in place in the field.

Figure B24 shows the specimen in detail, Figure B28 the moment-rotation curves, and Figure B29 the location of the center-of-rotation.

8. Specimen FK-5

To determine the effect of depth on the behavior of a connection, a specimen with five rows of fasteners was tested. The angles of K-type connections are 7/16 in. thick in this size. This specimen had the connection angles riveted to the beam web and shop bolted to the column. Figure B30 shows the specimen details, Figure B31 the moment-rotation curves, and Figure B32 the location of the center-of-rotation.

This specimen had strain gages mounted on the column web to provide an additional measure of the location of the center-of-rotation. The data from the strain gage readings are shown in Figure B33.

9. Specimen WB-10AB

The largest specimen included in the original program was FK-5. When the original objectives of the program were enlarged to include an attempt to find an analytical procedure for predicting the moment-rotation characteristics of flexible connections, it was decided to add to the program a specimen with a connection comparable to the largest standard connections. This would make it possible to check the analytical procedure developed, not only in the middle range of specimen sizes, but at the upper limit on a connection with 10 rows of fasteners. A drawing of the specimen is shown in Figure B34. The shape of the beam used to apply the moment to the connection angles has no particular significance. It was developed solely to utilize existing material. The end of the beam was an 18WF50 beam cut from a specimen previously tested; to this were welded plates to form a web and flanges as shown. Figure B35 shows the moment-rotation curves for the north and south connections, and Figure B36 the location of the center-of-rotation based on the mechanical dial readings.

Although a number of small angle-segment specimens had been tested with photo-elastic materials bonded to them, this material had not been utilized on the beam-to-column connection specimens. Since the Specimen WB-10AB was the last to be tested in this particular program, it was decided to put photo-elastic
materials on one of the angles, in addition to strain gages, in the hope that this would give further information as to the location of the center-of-rotation and a better understanding of the behavior. It was not possible to mount photo-elastic material on both sides of the angles so the strain information is for only one side of each leg. Figure B37 shows the location of the photo-elastic material that was bonded to the angles.

C. DISCUSSION OF TEST RESULTS

The results of the tests conducted in this program can be used to make comparisons between the behavior of riveted and bolted joints and to investigate the behavior of specimens with one and two lines of fasteners. In making comparisons, the angle properties and specimen geometry must be considered. (See Table B1.)

The various fasteners and fastener assemblies used in the tests have introduced several different restraint conditions. There was an all-riveted specimen (FK-4R, reference number 6), an all-bolted specimen with washers under all-bolt heads and nuts (FK-4AB, North), an all-bolted specimen with no washers at all (FK-4AB, South), and a specimen with the connection angle riveted to the beam web and bolted to the column flange (FK-4A, bolts assembled with washers). In Figure B38, these tests are compared. It appears that bolted specimens assembled without washers have approximately the same restraint characteristics as riveted specimens. The use of hardened washers increased joint stiffness sufficiently to displace the moment-rotation curve somewhat from the results of the test conducted on a connection without washers. There seems also to be a difference in behavior between specimens in which the angle leg is fastened to the beam web is bolted or riveted. Whether this can be attributed to normal scatter or is the result of some variable is not apparent, although it would be expected that the bolted specimen would be stiffer than the riveted specimen since washers were used.

Specimen FB-4 connections were assembled with rivets in the angle leg connected to the beam web and bolts in the column-connected leg, while the connections of specimen FB-4A had one angle leg riveted to the column flange and the remainder of the connection bolted. Since rivets and bolts without washers have been shown to provide similar moment-rotation characteristics in the connections (Figure B38), the only difference in behavior that would be expected would result from the clearance provided for the insertion of the web bolts. Examination of Figure B39 shows that this slip was considerable and had a significant effect on the restraint characteristics of the bolted connection. This slip occurred at a rotation which would be reached or exceeded in most structures at working loads. If in Specimen FB-4A the rotation of the angle leg connected to the beam web relative to the beam web is subtracted from the total rotation, then a behavior comparable to that of Specimen FB-4, which had the angles riveted to the beam web, is obtained. Specimens which have the
connection angles riveted to the beam web show little effect from slip of the angles relative to the beam web because the holes are filled by the rivet shanks. This suggests that where high-strength bolts are to be used, the moment-resistance of the connection might be increased considerably by using interference-fit bolts.

Connections assembled with two lines of bolts in the web connected leg (FK-4AB) also exhibit very little slip of the angle leg relative to the beam web, but their greater cost would not warrant their use for the small additional moment resistance they possess. Slip could also be prevented by an increase in bolt clamping force, possibly by the higher clamping force of the higher-strength bolts (ASTM A490) that have been adopted for use since these tests were completed.

In order to determine whether column flange deformations are large enough, in comparison to the angle deformations, to affect the connection rotation, a comparison of connections FK-4P and WK-4 is presented in Figure B40. No significant difference is apparent between the specimens that could be attributed to large column flange deformations. It would appear then that the column flange deformations can be neglected in most normal designs, but it would seem advisable to restrict the columns used to those with flanges somewhat greater in thickness than the angles until more is known of the behavior of the thinner flanges.

There was little difference in angle thickness in the specimens included in the program and therefore, the tests do not afford an opportunity to study fully the effect of angle thickness on behavior. Specimens FK-4AB and FK-4AB-M had connection angles of slightly different thicknesses, but the south connection of FK-4AB was identical in other respects to both connections of FK-4AB-M. A comparison of the test results shows FK-4AB-M, the specimen with thicker angles, was somewhat stiffer than FK-4AB. The behavior of flexible connections, after inelastic straining is present over a considerable portion of their length, was shown in the body of this report to be related to the plastic moment capacity of the angle material. The plastic moment for a unit length is equal to:

\[ M_p = \frac{(\text{angle thickness})^2 \sigma_y}{4} \]

From this expression it can be seen that the plastic moments will be proportional to the square of the angle thickness and directly proportional to the yield point. Based on this relationship, the moment of FK-4AB-M should be approximately 1.14 times that of FK-4AB at the same rotation. This is borne out by the test results in Figure B41.

In Figure 42, specimens with one and two lines of rivets attaching the angles to the beam web are compared. There is little difference in their behaviors although there was a difference in the geometry of the specimens; FB-4 had a 3/8-in. fillet compared with a 1/2-in. fillet in FK-4P. This would tend to make FK-4P stiffer. On the other hand, the angles of FB-4 were thicker than the angles of FK-4P and would increase the stiffness of FB-4 relative to FK-4P. The effects tend to
cancel each other, a condition evident
in Figure 42.

Of considerable importance in the
prediction of the moment-rotation rela-
tionship for flexible connections is
the location of the center-of-rotation.
Table B2 has been prepared to show the
variations in the location of the
center-of-rotation as measured by the
mechanical dials. Comparison of the
location of the center-of-rotation for
the different specimens is made at
equal values of deformation at the
tension end.

If the connection angles of one
specimen are thicker than those of
another, but physical properties and
geometry are similar, the ratio of
stiffness of the tension end to the
stiffness of the compression end will
be greater for the thicker angle.
This is because the stiffness at the
tension end is proportional to the
square of the thickness, while at the
compression end the increase in stiff-
ess is directly proportional to the
thickness. As a result, an increase
in the thickness affects the stiffness
at the tension end more than the stiff-
ess at the compression end. Since
the location of the center-of-rotation
is a function of the relative stiff-
ess of the two ends of the connection,
the center-of-rotation should be
nearer the center or mid-height for
the thicker angle. Specimen FK-5 had
7/16-in. angles and its center-of-
rotation was closer to the center of
the connection than most of the others.

Washers also stiffened the bolted
connections and this increased stiff-
ness, as evidenced by a shift of the
center-of-rotation to a point nearer
the connection center for Specimens
FK-4AB(M) and FK-4A.

Specimen WB-10AB appears to have
had a center-of-rotation much further
from the connection center based on
mechanical dial readings, than all the
others in spite of the fact that the
angles were 7/16 in. thick. An exami-
nation of the specimen after the test
showed the angle leg next to the beam
web to be bent noticeably. This bend-
ing would have the effect of moving the
center-of-rotation away from the con-
nection center. The photo-elastic
material bonded on WB-10AB shows the
center-of-rotation to be about 80 per-
cent of depth.

Increasing the length of a connec-
tion increases the moment that can be
resisted by the square of the ratio of
the length, if the maximum deformation
at the tension end of the connection is
the same in each case. When allowance
is made for the difference in fillet
radius, length, and thickness, the mo-
ment resisted by WB-10AB should be
approximately 3.5 times that of FK-5
when its rotation is one-half of that
of FK-5. This is what the data show.

Included in the program of tests
also were specimens which were painted
on their faying surfaces before assem-
by. No difference was observed in
their behavior from the behavior of
those not painted.
VII. REFERENCES


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