Undisturbed Core Method for Determining and Evaluating the Hydraulic Conductivity of Unsaturated Sediments

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By

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ABSTRACT

This paper describes a new method developed to predict the transport of moisture and contaminants in soils. Study results indicate that this method could help simplify evaluation of municipal and industrial waste disposal sites for their potential environmental impact. Saturated and unsaturated hydraulic conductivities of several Illinois soils, calculated on the basis of pore size distribution, were shown to predict reliably the experimentally measured laboratory values. For coarse-textured soil materials and materials with a relatively narrow range of pore size, only one matching factor was required to calculate the hydraulic conductivity-water content relation accurately enough for many purposes; however, for fine-textured soil materials with a wide range of pore size distribution, two or more matching factors at a water content in the 0.3 to 0.4 bar range may be needed to obtain a useful evaluation for the unsaturated hydraulic conductivity.

KEY WORDS
unsaturated hydraulic conductivity, soil water, matching factor, permeability, groundwater, soil water potential, water retention

ACKNOWLEDGMENTS

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INTRODUCTION

At least two basic parameters of geologic sediments—hydraulic conductivity and soil water characteristic functions—must be evaluated before predictive analyses can be made of the transport of moisture and contaminants in the unsaturated-saturated zone of a soil. These parameters must be
measured accurately before determinations can be made of the potential environmental impact of municipal and industrial waste disposal sites. Reliable measurement of these parameters is also essential to planning and managing efficient schemes for irrigation water.

This research project was undertaken to determine the soil water parameters of some nonindurated geologic sediments in Illinois. Our specific objectives were:

- to determine the soil water characteristic functions in the laboratory, using undisturbed and disturbed samples.
- to determine the saturated-unsaturated hydraulic conductivity functions using the same undisturbed samples.
- to evaluate the hydraulic conductivity functions of all samples, using the capillary model and the pore size distribution data.

Before discussing the methodology and results of our study we will define several important soil water concepts that are critical to understanding and predicting soil water movement: soil water potential, soil water characteristic function, hydraulic conductivity function, and units for soil water potential.

**Soil Water Potential**

Soil water contains energy in different forms and quantities. The two principle forms of energy are kinetic energy, a function of velocity and potential energy that is a function of position of internal condition of the system. Since water moves very slowly in soil, kinetic energy can generally be ignored in the study of soil water systems; however, potential energy is of primary importance in determining the state and the movement of water in soil.

The spontaneous and universal tendency of all matter in nature is to move from a point of high potential energy to a point of low potential energy until an equilibrium condition is reached. Soil water systems obey the same universal pursuit of equilibrium.

A soil water system is subjected to a number of force fields, which causes its potential to differ from that of free water. The force fields commonly considered are gravitational potential, $\phi_g$, pressure potential,
\( \phi_p \), osmotic potential, \( \phi_o \), and gas potential, \( \phi_a \). The total potential, \( \phi_T \), of the soil water system can be considered as the sum of the individual potentials:

\[
\phi_T = \phi_g + \phi_p + \phi_o + \phi_a \tag{1}
\]

The gravitational potential, \( \phi_g \), and pressure potential, \( \phi_p \), are the primary force fields in soil water systems. The osmotic potential, \( \phi_o \), is dependent upon the presence of a solute in the soil water system. The gas potential, \( \phi_a \), is dependent upon external or internal gas pressure in the system. If the osmotic potential and gas potential are considered to have minor influence on the total potential, then Equation 1 can be simplified as follows:

\[
\phi_T = \phi_g + \phi_p \tag{2}
\]

At a height \( z \) above an arbitrary reference level, the gravitational energy of water in soil, \( E \), can be stated as follows:

\[
E = Mgz = \gamma_w gzv \tag{3}
\]

In equation 3, \( \gamma_w \) is the density of water, \( g \) is the acceleration of gravity and \( V \) is the volume of the mass, \( M \). From Equation 3, the gravitational potential energy, \( \phi_g \), can be expressed as follows:

\[
\phi_g = gz \text{ (per unit mass, } M) \tag{4}
\]

\[
\phi_g = \gamma_w gz \text{ (per unit volume, } V) \tag{5}
\]

\[
\phi_g = z \text{ (per unit weight, } W) \tag{6}
\]

In Equation 6, \( \phi_g \) depends only on \( z \) and is defined as the gravitational head in soil water systems.

Pressure potential, \( \phi_p \), is negative for unsaturated soil water systems and positive for saturated soil water systems. It can be shown that the pressure potential concept allows for the consideration of the entire moisture profile in the field in terms of a single continuous potential extending from the saturated region to the unsaturated region, below and above the water table.

The positive pressure potential for a saturated soil water system is fairly well understood. The negative pressure—less well understood—has often been termed capillary potential, soil suction, or (more accurately)
matrix potential. This potential results from the capillary and absorptive forces developed in the soil matrix.

In discussing pressure potential for unsaturated soil water systems, the capillary tube analogy is useful. Soil can be assumed to be a porous medium composed of capillary tubes of different sizes. In figure 1 the air water interfaces throughout the soil consists of menisci in which the curvature or radii indicate the state of tension in the soil water (much as a capillary tube does). As the moisture content of the soil is reduced, the air water interfaces recede into the smaller pores, the radii of curvature decrease, and the moisture tension increases.

In the capillary tube shown in figure 2 the water above the water table will be in equilibrium when the upward component of the surface tension force is equal to the gravitational force acting on the suspended water. The height, h, to which the water will rise in the capillary tube is related mainly to the surface tension, σ, and radius, r, of the meniscus by the following equation:

$$h = \frac{2 \sigma \cos \theta}{\gamma_w g r}$$  \hspace{1cm} (7)

In figure 2 atmospheric pressure exists at points 1, 2, and 3. However, at point 4, just below the meniscus, the pressure is less than atmospheric pressure by an amount equal to \(h \gamma_w g\). Assuming that cosine \(\theta \approx 1\) for water in soil, and that the curvature of the water in the soil matrix is similar to that in a capillary tube of the same size (figure 2), the pressure potential per unit mass can be expressed from Equation 7 as follows:

$$\phi_p = -\frac{2 \sigma}{r \gamma_w} = gh$$  \hspace{1cm} (8)

The negative sign is used in Equation 8 because the pressure potential in an unsaturated soil water system is less than atmospheric pressure and because h would have a negative value in an unsaturated system.

From Equations 2, 4, and 8 the total potential per unit mass, excluding the osmotic potential and gas potential, can be stated as follows:

$$\phi_T = gz + gh$$  \hspace{1cm} (9)

On a unit weight basis, normally used in soil water studies, Equation 9 can be shown in the following form:
Figure 1. Capillary tubes showing configuration of the air water interfaces at different heights (after Dempsey and Elzefawy [22]).
In Equation 10, $H$ is the total soil water head, $z$ is the gravitational head, and $h$ is the pressure head. The pressure head is negative (suction) in unsaturated soil water systems and positive for saturated soil water systems.

To summarize, the criterion for the equilibrium in soil water systems is that the total water potential be equal throughout the system. To facilitate the analysis of particular systems, the total water potential is partitioned into various components that can be measured. Typically, the gravitational potential is determined by use of a measuring tape, the pressure potential by a piezometer for saturated systems and a tensiometer for unsaturated systems, and the gas potential by a pressure gauge.
Soil Water Characteristic Function

The relationship expressed in a soil water characteristic function is a soil property of fundamental importance in the analysis of water equilibrium and flow behavior in soil. Figure 3 shows relative soil water characteristic curves for two different soils. Physically, the curve tells (at any given moisture content) how much energy (per unit quantity of water removed) is required to remove a small quantity of water from the soil. It indicates how tightly water is held in the soil. Hillel (1), Taylor and Ashcroft (2), Kirkham and Powers (3), and Rose (4) have presented detailed explanations of how water is held in soil. Childs (5) has considered the mechanisms of water held in both swelling and non-swelling soils in great detail.

Cromey, Coleman, and Bridge (6) have described the methods used to determine the soil water characteristic curve—those used most frequently are the tensiometer, direct suction, pressure plate, and centrifuge methods. Because no single method can cover the entire moisture tension range, several measurement methods are generally used in determining these curves.

Figure 3. Relative soil-water characteristic curves for a clayey soil and sandy soil.
Figure 4 shows a simple type of tensiometer system that can be used for the low moisture-tension range (< 100 kN/m$^2$, or < 1 bar). The apparatus shown in figure 4 consists of a porous plate with its pores filled with water. The chamber beneath the porous plate is filled with water and connected to a flexible tube that is also filled with water. The negative head is equal to the distance, $h$, between the soil sample and the outflow end of the flexible tube in figure 4. The soil water characteristic curve is determined from the relationship between the water content of the soil sample and the magnitude of the negative pressure head of the water.

Hysteresis effects (figure 5) will often occur between soil water characteristic curves for drying and wetting. The hysteresis for the drying and wetting conditions arises from the influence of pore size distribution on water held in the soil. A complete moisture characteristic curve should consist of a drying (desorption) curve and a wetting (sorption) curve. The drying curve should start at saturated water content at close to zero suction and continue to a low water content at a high level of suction. The wetting curve should start at the high level of suction and low water content and proceed to saturation. This would characterize an envelope for water content and suction values in the given range. The influence of small moisture content changes on soil suction is shown by the smaller hysteretic curves inside the desorption and sorption curves in figure 5.

A useful simplification occurs when the soil suction is given in units of water head. A suction of 20 cm will lift a column of water 20 cm above a free water surface. Therefore, the suction on the moisture characteristics curve can be equated to the distance above a water table for equilibrium conditions. Also, by use of the soil water characteristic curve it is possible to estimate the equilibrium water content at various positions above the water table.

**Hydraulic Conductivity Function**

The flow of water through soils is often unsteady and unsaturated. Examples of such flows are the infiltration of water from the ground surface, the flow through the capillary fringe of an unconfined aquifer, the draining of soils, the evaporation from an aquifer close to the ground surface, the
Figure 4. Tensiometer system (after Dempsey and Elzeftawy [22]).

Figure 5. Hysteresis effects of drying and wetting conditions on matric suction.
fluctuations of groundwater level, the inflow of water from irrigation channels, and the land disposal of liquid wastes.

The general nonlinear partial differential equation that describes the transport of groundwater can be written as follows:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K \nabla \phi)$$  \hspace{1cm} (11)

where $\theta$ is the volumetric water content defined as the ratio of the volume of water, $V_w$, to the total volume of soil, $V$, $\nabla$ is the vector differential operator. $K$ is the hydraulic conductivity, and $\phi$ is the total potential.

For a complete derivation of Equation 11 (5 or 6). An equation of this type applies to any nonreactive liquid in the porous medium; since we limit ourselves in this study to water, it is convenient to take the length of water column as the unit of potential. Potential gradients are then dimensionless, and if the time, $t$, is expressed in hours, the unit of hydraulic conductivity is centimeters per hour.

When the total potential is composed of only gravitational and negative pressure (capillary) components, Equation 11 may be written:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K \nabla h) + \frac{\partial \theta}{\partial z}$$  \hspace{1cm} (12)

where $h$ is the suction (negative pressure) potential, and $z$ is the vertical ordinate, positive upward.

When $h$ and $K$ are single-valued functions of $\theta$, Equation 12 becomes

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (D \nabla \theta) + \frac{\partial K}{\partial z}$$  \hspace{1cm} (13)

where

$$D(\theta) = K(\theta) \frac{\partial h}{\partial \theta}$$  \hspace{1cm} (14)

Childs and Collis-George (7) called $D$ the diffusivity of soil water and found it to be a function of $\theta$. Rogers and Klute (8) have shown that hydraulic conductivity, $K$, is uniquely related to soil moisture content, $\theta$. Two physical properties of the soil that enter into a saturated-unsaturated flow problem are hydraulic conductivity, $K(\theta)$, and soil moisture retention $h(\theta)$; these properties must be known if a solution of Equation 13 is to be obtained.
Childs and Collis-George (7), Millington and Quirk (9), Green and Corey (10), and others have explored the possibility of predicting the hydraulic conductivity of soils and other porous materials on the basis of pore size distribution. Such predictions are of interest because the hydraulic conductivity function, \( K(\theta) \), is relatively difficult to measure, whereas pore size distribution is easily obtainable by the standard measurement of moisture content versus suction (negative pressure).

The hydraulic conductivity is obtained by dividing the relation of moisture content and suction, \( h(\theta) \), into \( n \) equal water content increments, obtaining the suction, \( h \), at the midpoint of each increment, and calculating the conductivity by using the following equation (see ref. 7 for more details):

\[
K(\theta)_i = (30\gamma/\rho g n)(\varepsilon_p/n^2) \sum_{j=1}^{m} [(2j + 1 - 2i)h_j^2]
\]  

where

- \( K(\theta)_i \) = calculated conductivity for a specified moisture content corresponding to the \( i \)th increment, cm/min;
- \( \theta \) = moisture content, \( \text{cm}^3/\text{cm}^3 \);
- \( \gamma \) = surface tension of water, N/cm;
- \( \rho \) = density of water, \( g/\text{cm}^3 \);
- \( g \) = gravitational constant, \( \text{cm/s}^2 \);
- \( n \) = kinematic viscosity of water, \( \text{cm}^2/\text{s} \);
- \( \theta_s \) = saturated moisture content, \( \text{cm}^3/\text{cm}^3 \);
- \( \varepsilon \) = water-saturated porosity (\( \text{cm}^3/\text{cm}^3 \)), that is, \( \varepsilon = \theta_s \);
- \( p \) = constant whose value depends on the method of calculation 6, is equal to 2 in these calculations;
- \( \theta_o \) = lowest moisture content on the experimental \( h(\theta) \) curve;
- \( n \) = total number of pore classes between \( \theta = \theta_o \) and \( \theta_s \); \( n = m \theta_s / (\theta_s - \theta_o) \);
- \( i \) = last moisture-content increment on the wet end (for example, \( i - 1 \) identifies the pore class corresponding to \( \theta_0 \)).
h_j = suction (negative pressure) for a given class of moisture-filled pores (centimeters of water head); and

30 = the composite of the constant 1/8 from Poiseuille's equation, 4 from the square of r = 2γ/h, where r is the pore radius and 60 converts from seconds to minutes.

Green and Corey (10) concluded that Equation 15 yields reasonable values of the hydraulic conductivities for a range of soil types if a matching factor is used. Elzeftawy and Mansell (11) and Elzeftawy and Dempsey (12) stated that a matching factor at water saturation (the ratio of the measured to the calculated, saturated hydraulic conductivity) has a distinct advantage over match points because inaccuracies in calculated and experimentally evaluated K(θ) can be more easily tolerated at lower moisture content. Equation 15 can then be written by using the matching factor K_s/K_sc, in the following form:

\[ K(\theta)_i = \frac{K_s}{K_{sc}} \left( \frac{30\gamma^2/\rho g H}{(e\rho/n^2)} \right) \sum_{j=i}^{m} \left[ (2j + 1 - 2i)h_j^2 \right] \]  

where \( K_s \) is the measured saturated hydraulic conductivity, and \( K_{sc} \) is the calculated saturated conductivity.

**Units for Soil Water Potential**

The normal methods of expressing potential in soil water systems are shown in table 1 for the various measurement systems. Relationships for the measurement systems are shown in table 2. For potentials expressed on a per unit weight basis or on a per unit volume basis, the dimensions are those of length (centimeter, meter, or foot) or of pressure (dyne/square centimeter, newton/square meter, or pound/square foot), respectively. Equations for converting between the three forms of potential are stated as follows:

\[ \frac{\text{energy}}{\text{mass}} = g \frac{\text{energy}}{\text{weight}} \]  

(17)

\[ \frac{\text{energy}}{\text{volume}} = \gamma_w \frac{\text{energy}}{\text{mass}} \]  

(18)

\[ \frac{\text{energy}}{\text{volume}} = g\gamma_w \frac{\text{energy}}{\text{weight}} \]  

(19)
Table 1. Potential expressed in the major measurement systems.

<table>
<thead>
<tr>
<th>Potential</th>
<th>cgs system</th>
<th>mks system</th>
<th>English system</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy mass</td>
<td>dyne cm = erg/gm</td>
<td>Newton meter = Joule/kg</td>
<td>ft lb</td>
</tr>
<tr>
<td>energy weight</td>
<td>dyne cm = cm/dyne</td>
<td>Newton meter = meter</td>
<td>ft lb = ft</td>
</tr>
<tr>
<td>energy volume</td>
<td>dyne cm³ = dyne/cm²</td>
<td>Newton meter = Newton/meter²</td>
<td>ft lb = lb/ft²</td>
</tr>
</tbody>
</table>

\[
100 \text{ kM/m}^2 = 1 \text{ bar} = 14.5 \text{ psi} = 29.5 \text{ in Hg} = 75.1 \text{ cm Hg} = 33.4 \text{ ft water} = 1020 \text{ cm water}
\]

In making analyses of soil water systems, it is convenient to use one of these methods consistently for expressing potential rather than to use more than one method in the same analyses. Of the three methods, potentials expressed as energy per unit weight appear to be utilized most in the literature and are used in this paper.

MATERIALS AND METHODS

Determinations of soil water characteristic and hydraulic conductivity functions were made of triplicate undisturbed and disturbed soil samples. The undisturbed core samples (5.4 cm in diameter and 3 cm in height) were collected from sites in Illinois. The soil cores used in this study were obtained during previous studies and taken from storage. All the samples had been allowed to dry, but otherwise were undisturbed. The locations and some physical properties of the undisturbed core samples are shown in table 3. The disturbed soil samples (table 4) were passed through a 2-mm sieve, oven dried, and hand packed in the "Tempe" test cells.

All samples were placed in "Tempe" pressure cells and saturated with water. The Tempe" pressure cell operates under the same physical principles as does the porous plate apparatus of the ASTM Test for Capillary-
Table 2. Conversions for potential units.

<table>
<thead>
<tr>
<th>cm</th>
<th>meter</th>
<th>ft</th>
<th>erg gm</th>
<th>Joule kg</th>
<th>ft lb slug</th>
<th>dyne cm²</th>
<th>Newton meter²</th>
<th>lb ft²</th>
<th>bar</th>
<th>atmosphere</th>
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<td>.0980</td>
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<td>1.013</td>
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Table 3. Location, type and particle size data of undisturbed samples used in study.

<table>
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<th>Sample No.</th>
<th>Material</th>
<th>Depth (cm)</th>
<th>Soil Horizon</th>
<th>Particle Size (%)</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sand</td>
</tr>
<tr>
<td>RU-43</td>
<td>Equality Fm.</td>
<td>390-395</td>
<td>B2</td>
<td>9</td>
</tr>
<tr>
<td>EL-53</td>
<td>Equality Fm.</td>
<td>740-750</td>
<td>B2</td>
<td>8</td>
</tr>
<tr>
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<td>Peoria Loess</td>
<td>0-30</td>
<td>A</td>
<td>5</td>
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<tr>
<td>12-30&quot; Piatt</td>
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<td>30-76</td>
<td>B2</td>
<td>10</td>
</tr>
<tr>
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</tr>
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<td>107-122</td>
<td>B2</td>
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<td>2A</td>
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<td>2B1</td>
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<td>Ogle</td>
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<td>no. 20</td>
<td>Ogle</td>
<td>185-195</td>
<td></td>
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<td>61-150</td>
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<td>Cg</td>
<td>19</td>
</tr>
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<td>Alexander</td>
<td>215-175</td>
<td>Cg</td>
<td>14</td>
</tr>
<tr>
<td>no. 8</td>
<td>Alexander</td>
<td>215-275</td>
<td>Cg</td>
<td>13</td>
</tr>
<tr>
<td>no. 9</td>
<td>Alexander</td>
<td>215-275</td>
<td>Cg</td>
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[1] Follmer [24]: all Clark County locations
Table 4. Properties of disturbed samples.

<table>
<thead>
<tr>
<th>soil sample</th>
<th>particle size (%)</th>
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<tr>
<td></td>
<td>sand</td>
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<tr>
<td>Fine Ottawa Sand</td>
<td>100</td>
</tr>
<tr>
<td>Coarse Ottawa Sand</td>
<td>100</td>
</tr>
<tr>
<td>Ground Ottawa Sand</td>
<td>100</td>
</tr>
<tr>
<td>Richland Loess</td>
<td>4</td>
</tr>
<tr>
<td>Roxana Silt</td>
<td>11</td>
</tr>
</tbody>
</table>

Moisture Relationships for Coarse and Medium-Textured Soils by Porous-Plate Apparatus D 2325-68 (1974), with maximum 1 atm pressure. Figures 6 and 7 show the laboratory setup and a schematic of the pressure cell. A constant temperature was maintained at all times in the laboratory.

The saturated hydraulic conductivities of all samples were determined utilizing an apparatus (figure 8) similar to that used in the ASTM Test for Permeability of Granular Soils (Constant Head) D 2434-68 (1974). The samples were then subjected to air pressure.

After their saturated hydraulic conductivity was determined, the samples were allowed to drain following sequential subjection to air pressures of 100, 200, 300, 500, 800, and 1000 cm of water. They were then placed in a 15-bar porous-plate apparatus to determine the equilibrium moisture content retained in the soil samples for air pressures of 1, 3, 5, and 15 atm (using procedures similar to ASTM D 2325-68 or ASTM Test for Capillary-Moisture Relationships for Fine-Textured Soils by Pressure-Membrane Apparatus D 3152-72 (1977)). The water content (by volume) was determined from the weight of the pressure cell corresponding to each state equilibrium pressure and the oven-dry weight of the soil samples.

The measured hydraulic conductivity function, \( K(\theta) \), of all samples was evaluated by the instantaneous profile method suggested by Watson (13) and described by Rogers and Klute (8). Another method, described by Elzeftawy and Mansell (11), was also used to determine \( K(\theta) \) of each sample. This method is based on the utilization of a unit hydraulic gradient to provide a steady-state, downward, unsaturated flow of water across the soil core. The computer program developed by Elzeftawy and Dempsey (12) was used to
Figure 6. Tempe pressure cells set-up in the laboratory.
Figure 7. Detailed cross section of Tempe cell (after Soilmoisture Equipment Corp. [23]).

Figure 8. Permeameter used for determining saturated hydraulic conductivity.
calculate the hydraulic conductivities, using Equation 16 and the soil water-retention curves.

The Lakeland fine sand samples were taken at three depth intervals from the Agricultural Experiment Station farm of the University of Florida at Quincy, Florida (Elzeftawy and Mansell {11}).

RESULTS

Amerman (14) and Philip (15) have pointed out the importance of including information about the unsaturated soil properties in large-scale hydrogeologic investigations. For example, to incorporate principles of soil physics into a rainfall-runoff model, it is possible to use either a numerical solution of the unsaturated flow equation or a simple infiltration equation such as that given by Green and Ampt (16) or derived by Philip (17). In the first approach, the soil water characteristic (the relationship between soil suction head, h, and volumetric water content, ϑ) and the conductivity function (the relationship between the unsaturated hydraulic conductivity K and ϑ) must be known. In the second approach, composite hydraulic parameters, specifically the Green-Ampt (16) wetting front suction, hf, and Philip (17) sorptivity, S, must be estimated or computed directly from specified functions of h, K, and ϑ.

The need to specify relationships among h, K, and ϑ presents a significant problem in hydrology because of the difficulty of obtaining measurements of these parameters and of presenting the collected data. Gardner, et al. (18), Campbell (19), and Clapp and Hornberger (20) have attempted to use power curves to describe the soil moisture characteristic of soils and have had only limited success in estimating the hydraulic conductivities from these power curves; however, Elzeftawy and Mansell (11) have shown that the calculated hydraulic conductivity using Equation 16 provided a good estimation of the K(ϑ) function of Lakeland fine sand.

The measured and calculated values of hydraulic conductivity of Lakeland fine sand are presented in Figure 9 for three different profile depths. The measured hydraulic conductivity values at water saturation, Ks, were used as the only matching factor to determine the calculated curves of the K(ϑ) function. It is well known that it is quicker and simpler to determine Ks experimentally then it is to measure unsaturated hydraulic conductivities
Figure 9. Experimental and calculated hydraulic conductivity of Lakeland fine sand.

at any moisture content below the saturation value. But the pronounced deviation between the calculated and measured $K(\theta)$ values for volumetric water contents less than 10 percent suggests that a second matching point somewhat within the "field capacity" range of water content may be needed.

The average densities and water-saturated hydraulic conductivities of the undisturbed samples of Lakeland sand are shown in table 5. The variation of bulk density with depth of the soil profile is almost negligible; however, the hydraulic conductivity of the bottom layer (60 to 90 cm) is much higher than that for the surface layer (0 to 15 cm).

Selected physical properties of the undisturbed (Drummer and Dana) and disturbed (Ottawa sand and Fayette C horizon) samples used are presented in table 6. The saturated hydraulic conductivities of the Fayette C and Dana soils are larger than expected, which might be attributable to the low bulk
Table 5. Bulk density and saturated hydraulic conductivity of Lakeland fine sand.

<table>
<thead>
<tr>
<th>soil depth, cm</th>
<th>bulk density, $\rho_s$, $\rho_s + t^a$, g/cm$^3$</th>
<th>saturated hydraulic conductivity $K_s$, $K_s + t^a$, cm/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 15</td>
<td>1.56 ± 0.06</td>
<td>14.80 ± 1.12</td>
</tr>
<tr>
<td>30 to 45</td>
<td>1.57 ± 0.03</td>
<td>13.00 ± 0.93</td>
</tr>
<tr>
<td>60 to 90</td>
<td>1.57 ± 0.05</td>
<td>17.10 ± 1.09</td>
</tr>
</tbody>
</table>

$^a$t-distribution at 95 percent confidence level.

Table 6. Selected physical properties of soils used at indicated depths.

<table>
<thead>
<tr>
<th>property</th>
<th>Drummer 0 to 30 cm</th>
<th>Drummer 30 to 75 cm</th>
<th>Drummer 75 to 90 cm</th>
<th>Dana 0 to 10 cm</th>
<th>Ottawa sand 0.85 to 2 mm</th>
<th>Fayette C horizon 120 to 150 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>sand, %</td>
<td>6.00</td>
<td>6.00</td>
<td>6.00</td>
<td>6.1</td>
<td>100.00</td>
<td>7.00</td>
</tr>
<tr>
<td>silt, %</td>
<td>77.20</td>
<td>80.50</td>
<td>82.60</td>
<td>74.9</td>
<td>0.00</td>
<td>75.00</td>
</tr>
<tr>
<td>clay, %</td>
<td>16.80</td>
<td>13.20</td>
<td>10.50</td>
<td>19.00</td>
<td>0.00</td>
<td>18.00</td>
</tr>
<tr>
<td>bulk density, g/cm$^3$</td>
<td>1.52</td>
<td>1.30</td>
<td>1.43</td>
<td>1.22</td>
<td>1.65$^a$</td>
<td>1.25$^a$</td>
</tr>
<tr>
<td>saturated hydraulic conductivity, $K_s$, cm/h</td>
<td>$2.11 \times 10^{-2}$</td>
<td>$3.6 \times 10^{-2}$</td>
<td>$2.5 \times 10^{-2}$</td>
<td>$1.92 \times 10^{-0}$</td>
<td>$1.34 \times 10^{+1}$</td>
<td>$2.15 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

$^a$hand packed.

densities and, in the Dana soil, the high organic matter content. The grain size distribution of the natural soils material (Drummer, Dana, and Fayette) is similar; however, these soils differ widely in their bulk densities and hydraulic conductivities. For instance, the bulk densities of the Drummer surface layer (0 to 30 cm) and Fayette C horizon are 1.52 and 1.25 gm/cm$^3$, respectively; the difference between their corresponding $K_s$ values is 1 order of magnitude. The Dana and Fayette soils material are similar in grain size analysis and bulk density; however, their saturated hydraulic conductivities are 1 order of magnitude apart, which probably indicates the effect of different natural soil structures.

Soil moisture content suction characteristic curves obtained by sequential drainage are shown in figure 10 for the three profile depths of Drummer soil and the surface layer (0 to 15 cm) of Lakeland fine sand. It is significant that the amount of water retained at relatively low values of suction (for example, between 0 and 1000 cm of suction) depends upon the
capillary effect and the pore size distribution and therefore is strongly affected by the soil structure. On the other hand, water retention in the higher suction range is due increasingly to adsorption and is thus influenced less by the structure and more by the texture and specific surface of the soil material. Figure 10 indicates that, in general, the greater the clay content, the greater the water content, at any particular suction (compare Lakeland sand and Drummer silty loam) and the more gradual the slope of the curve.

The effect of compaction upon a soil is to decrease its total porosity, and especially to decrease the volume of the large interaggregate pores; this means that water content at saturation and the initial decrease of water content with the application of low suction are reduced. The data presented in table 6 and figure 10 for the 30 to 75-cm and 75 to 90-cm depth of Drummer samples support the previous statement: note the similarity in their particle-size analysis and the differences in their bulk densities and the saturated water contents.

The calculated and experimental hydraulic conductivities of three layers of Drummer soil profiles are shown in figure 11. The experimental data were obtained by the unit gradient method as published by Elzeftawy and Mansell (12). The hydraulic conductivity of this soil at saturation is generally about 4 orders of magnitude larger than at 50 percent of saturation. The calculated results were consistent with the experimental data; however, the calculated numerical values below 0.32 cm$^3$/cm$^3$ water content were less than the experimentally hydraulic conductivities obtained (not shown in figure 11).

Hydraulic conductivities as a function of moisture content of Ottawa sand and Fayette C horizon are shown in figures 12 and 13. The lines represent the calculated values of $K(\theta)$ obtained by Equation 16 and the soil-moisture retention curve. The circles are the experimental data points. These soil materials represent a wide range of pore size distributions over which the calculations of hydraulic conductivities are based. Figure 13 shows that a change in water content of Fayette soil from 0.47 to 0.30 cm$^3$/cm$^3$ has reduced the hydraulic conductivity from 2.2 X $10^{-1}$ cm/h to 4.0 X $10^{-3}$ cm/h, respectively.
Green and Corey (10) and others have stated that using a matching factor at water saturation has a distinct advantage, since inaccuracies in calculated values of $K(\theta)$ can be more easily tolerated at lower water contents; however, in studying phenomena such as evaporation, the early stages of water infiltration, and the movement of solutes such as contaminants in the unsaturated zone, more accurate methods are needed to determine the unsaturated hydraulic conductivities of soils at lower values of water content. Bruce (21) suggested that matching factors somewhat below the bubbling pressure are sufficiently accurate for calculating the unsaturated hydraulic conductivities of coarse-grained soils; however, he also stated that the indiscriminate use of such methods for calculating the hydraulic conductivities of fine-grained soils is inadvisable. In our study good results have

Figure 10. Soil moisture-suction relationships of Lakeland fine sand and Drummer soil.
been obtained for the fine-grained soils (Drummer and Fayette) using saturated hydraulic conductivity as the only matching factor. However, we noticed that the calculated values deviated from the experimental results, especially within the range of low water content (less than 0.35 cm$^3$/cm$^3$ moisture content). For this reason, the Dana loam samples were chosen to investigate the possibility of using two or more matching factors to calculate the $K(\theta)$ function. Some of the physical properties of Dana soil are presented in table 6.

DISCUSSION

A method of predicting the saturated-unsaturated hydraulic conductivities of some Illinois soils utilize the soil moisture content-suction head
relation, \( h(\theta) \), to calculate the unsaturated hydraulic conductivity of soil. The value of the hydraulic conductivity at saturation, \( K_s \) (the soil permeability), was used as a matching factor during the calculations. The \( h(\theta) \) relations and the saturated conductivities, \( K_s \), of soils were determined in the laboratory using the commercially available "Tempe" cell. Undisturbed samples of Drummer and Dana soils and disturbed samples of Fayette C soil, Ottawa sand, and other soils were used in this study. Published data on some agricultural soils were also used to validate results of our investigations. On the basis of our study results, the following conclusions can be made:

1. The model successfully predicts the hydraulic conductivity of a wide range of soils.
2. The proposed simplified laboratory procedure is reliable and can be used to determine easily the soil moisture-suction relationships of disturbed or undisturbed soil samples.
3. Evaluation of the unsaturated hydraulic conductivities of soils using the proposed "Tempe" cell method is quick and economical.
Figure 13. Experimental and calculated hydraulic conductivities of Fayette C horizon.

The experimental and calculated hydraulic conductivities of the Dana sample are shown in figure 14; the circles represent the experimental data and the lines represent the calculated values. The calculated hydraulic conductivity function using $K_s$ as the only matching point is shown in the figure by the solid line; note the deviation between the calculated and experimental data below a water content of 0.45 cm$^3$/cm$^3$. Better results were obtained when two matching factors were used, particularly when the saturated hydraulic conductivity, $K_s$, and another experimental value somewhat below the bubbling pressure of the soil were used. (The value $K(\theta) = 1.01 \times 10^{-3}$ cm/min was arbitrarily chosen where $\theta = 0.40$ cm$^3$/cm$^3$.) In this case, the dashed line represents the $K(\theta)$ function calculated with two matching factors in Equation 16. Using two matching factors in the calcu-
lations of \( K(\theta) \) functions of many fine-grained soils has reduced the error in predicting the hydraulic conductivity values at low soil water content.

Results of our study presented in graphical form in Appendix B, indicate that the method described in this report for calculating the hydraulic conductivity of soil materials can be used with confidence for many practical applications describing the pore transport system.
REFERENCES


LIST OF PUBLICATIONS


Appendix A
Soil-Moisture Characteristics of Samples Used in This Study.
Soil moisture content (%) vs. Soil suction -h (cm) for:

- Sample 21
- Sample 22
- Sample 25
- Sample 26
Soil moisture content $\theta$ (%) vs. Soil suction $h$ (cm) for different samples:

- **Sample 2**
- **Sample 3**
- **Sample 4**
- **Sample 5**
Soil moisture content \( \theta \) (%)

Sample 6

Sample 7

Sample 8

Sample 9
Appendix B

The hydraulic conductivity soil-moisture relationships for samples used in this study. Note, the rewetting of dry samples may have increased the saturated hydraulic conductivity in some samples.
Fine Ottawa Sand

$K_s = 1.03 \times 10^1 \text{ cm/hr}$
$\theta_s = 0.363 \text{ cm}^3/\text{cm}^3$
$\rho = 1.70 \text{ gm/cm}^3$

Coarse Ottawa Sand

$K_s = 1.34 \times 10^1 \text{ cm/hr}$
$\theta_s = 0.382 \text{ cm}^3/\text{cm}^3$
$\rho = 1.65 \text{ gm/cm}^3$

Ground Ottawa Sand

$K_s = 1.81 \times 10^0 \text{ cm/hr}$
$\theta_s = 0.420 \text{ cm}^3/\text{cm}^3$
$\rho = 1.50 \text{ gm/cm}^3$

Richland Loess

$K_s = 5.05 \times 10^0 \text{ cm/hr}$
$\theta_s = 0.520 \text{ cm}^3/\text{cm}^3$
$\rho = 1.30 \text{ gm/cm}^3$
Roxana Silt

\[ K_s = 8.76 \times 10^{-1} \text{ cm/hr} \]
\[ \theta_s = 0.49 \text{ cm}^3/\text{cm}^3 \]
\[ \rho = 1.19 \text{ gm/cm}^3 \]

RU-43

\[ K_s = 2.87 \times 10^{-1} \text{ cm/hr} \]
\[ \theta_s = 0.480 \text{ cm}^3/\text{cm}^3 \]
\[ \rho = 1.23 \text{ gm/cm}^3 \]

EL-53

\[ K_s = 1.37 \times 10^{0} \text{ cm/hr} \]
\[ \theta_s = 4.80 \text{ cm}^3/\text{cm}^3 \]
\[ \rho = 1.21 \text{ gm/cm}^3 \]

0-12 in. Piatt

\[ K_s = 2.11 \times 10^{-2} \text{ cm/hr} \]
\[ \theta_s = 0.432 \text{ cm}^3/\text{cm}^3 \]
\[ \rho = 1.52 \text{ gm/cm}^3 \]
12-30 in. Platt

$K_s = 3.6 \times 10^{-2} \text{ cm/hr}$

$\theta_s = 0.512 \text{ cm}^3/\text{cm}^3$

$\rho = 1.30 \text{ gm/cm}^3$

30-60 in. Platt

$K_s = 2.5 \times 10^{-2} \text{ cm/hr}$

$\theta_s = 0.488 \text{ cm}^3/\text{cm}^3$

$\rho = 1.43 \text{ gm/cm}^3$

Sample 11

$K_s = 1.15 \times 10^{0} \text{ cm/hr}$

$\theta_s = 0.700 \text{ cm}^3/\text{cm}^3$

$\rho = 1.32 \text{ gm/cm}^3$

Sample 12

$K_s = 7.04 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.740 \text{ cm}^3/\text{cm}^3$

$\rho = 1.15 \text{ gm/cm}^3$
Sample 21

$K_s = 2.61 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.450 \text{ cm}^3 / \text{cm}^3$

$\rho = 1.34 \text{ gm/cm}^3$

Sample 22

$K_s = 1.04 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.480 \text{ cm}^3 / \text{cm}^3$

$\rho = 1.33 \text{ gm/cm}^3$

Sample 25

$K_s = 4.18 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.500 \text{ cm}^3 / \text{cm}^3$

$\rho = 1.34 \text{ gm/cm}^3$

Sample 26

$K_s = 3.70 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.580 \text{ cm}^3 / \text{cm}^3$

$\rho = 1.31 \text{ gm/cm}^3$
Sample 27

$K_s = 1.24 \times 10^{-2}$ cm/hr

$\theta_s = 0.570$ cm$^3$/cm$^3$

$\rho = 1.20$ gm/cm$^3$

Sample 28

$K_s = 4.19 \times 10^{-1}$ cm/hr

$\theta_s = 0.540$ cm$^3$/cm$^3$

$\rho = 1.21$ gm/cm$^3$

Sample 29

$K_s = 1.13 \times 10^0$ cm/hr

$\theta_s = 0.480$ cm$^3$/cm$^3$

$\rho = 1.35$ gm/cm$^3$

Sample 30

$K_s = 3.24 \times 10^{-2}$ cm/hr

$\theta_s = 0.520$ cm$^3$/cm$^3$

$\rho = 1.37$ gm/cm$^3$
Sample 17

Hydraulic conductivity (cm/hr)

Soil moisture content $\theta$ (%)

$K_s = 2.40 \times 10^6$ cm/hr

$\theta_s = 0.450$ cm$^3$/cm$^3$

$\rho = 1.13$ gm/cm$^3$

Sample 19

Hydraulic conductivity (cm/hr)

Soil moisture content $\theta$ (%)

$K_s = 5.13 \times 10^6$ cm/hr

$\theta_s = 0.590$ cm$^3$/cm$^3$

$\rho = 1.03$ gm/cm$^3$

Sample 20

Hydraulic conductivity (cm/hr)

Soil moisture content $\theta$ (%)

$K_s = 4.81 \times 10^6$ cm/hr

$\theta_s = 0.590$ cm$^3$/cm$^3$

$\rho = 1.12$ gm/cm$^3$

Sample 1

Hydraulic conductivity (cm/hr)

Soil moisture content $\theta$ (%)

$K_s = 1.51 \times 10^6$ cm/hr

$\theta_s = 0.610$ cm$^3$/cm$^3$

$\rho = 0.88$ gm/cm$^3$
Sample 2

$K_s = 2.80 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.580 \text{ cm}^3/\text{cm}^3$

$\rho = 0.86 \text{ gm/cm}^3$

Sample 3

$K_s = 5.60 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.690 \text{ cm}^3/\text{cm}^3$

$\rho = 0.90 \text{ gm/cm}^3$

Sample 4

$K_s = 1.64 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.710 \text{ cm}^3/\text{cm}^3$

$\rho = 0.82 \text{ gm/cm}^3$

Sample 5

$K_s = 4.53 \times 10^{-1} \text{ cm/hr}$

$\theta_s = 0.630 \text{ cm}^3/\text{cm}^3$

$\rho = 1.21 \text{ gm/cm}^3$
Sample 6

$K_s = 7.61 \times 10^{-2} \text{ cm/hr}$
$
\theta_s = 0.720 \text{ cm}^3/\text{cm}^3$
$
\rho = 0.89 \text{ gm/cm}^3$

Sample 7

$K_s = 4.05 \times 10^0 \text{ cm/hr}$
$
\theta_s = 0.540 \text{ cm}^3/\text{cm}^3$
$
\rho = 1.30 \text{ gm/cm}^3$

Sample 8

$K_s = 3.18 \times 10^0 \text{ cm/hr}$
$
\theta_s = 4.90 \text{ cm}^3/\text{cm}^3$
$
\rho = 1.36 \text{ gm/cm}^3$

Sample 9

$K_s = 4.39 \times 10^0 \text{ cm/hr}$
$
\theta_s = 0.400 \text{ cm}^3/\text{cm}^3$
$
\rho = 1.31 \text{ gm/cm}^3$