Supplemental Data for Counting Moving Bodies Using Sparse Sensor Beams

Lawrence H. Erickson, Jingjin Yu, Yaonan Huang, Steven M. LaValle

This document contains supplemental simulation data for [1] and [2].

At each step, each body has an equal chance of being the next one to move (exponential random movement model). Bodies that are in a sink no longer move. No trial initially placed bodies into a sink, so it was always possible to reach a counting information state.

Two types of graphs were examined (directed cycles and directed paths), and two different methods of initially placing the bodies were examined (all in the same vertex to start and random initial locations). A short analysis of the simulation data follows.

1 Cycles - Same Initial Location

A set of m moving bodies navigated in a directed C_n until a counting information state was acquired. In the initial configuration, all bodies were placed in the same vertex. Results were averaged over 1000 trials.

The value of m ranged from 2 to 20. The value of n ranged from 2 to 10. Raw data is available in the associated text file (cycle-same.txt) posted with this document. The data is shown in Figure 1. Certain values of m are omitted from the graphs for readability purposes.

2 Cycles - Random Initial Location

A set of m moving bodies navigated in a directed C_n until a counting information state was acquired. An initial configuration was chosen uniformly at random from among all possible configurations. Results were averaged over 3000 trials.

The value of m ranged from 2 to 20. The value of n ranged from 2 to 10. Raw data is available in the associated text file (cycle-random.txt) posted with this document. The data is shown in Figure 2. Certain values of m are omitted from the graphs for readability purposes.

3 Paths - Same Initial Location

A set of m moving bodies navigated in a directed P_n until a counting information state was acquired. In the initial configuration, all bodies were placed in the same vertex (the vertex furthest from the sink). Results were averaged over 1000 trials. Bodies that entered the sink no longer moved.

The value of m ranged from 2 to 20. The value of n ranged from 2 to 10. Raw data is available in the associated text file (path-same.txt) posted with this document. The data is shown in Figure 3. Certain values of m are omitted from the graphs for readability purposes.

4 Paths - Random Initial Location

A set of m moving bodies navigated in a directed P_n until a counting information state was acquired. The starting configuration was randomized in each trial. A body has an equal probability of starting in any non-sink vertex. All bodies are placed independently. No bodies were initially placed in the sink (bodies that start in a sink never cause a sensor reading and are undetectable). Results were averaged over 3000 trials. Bodies that entered the sink no longer moved.

The value of m ranged from 2 to 20. The value of n ranged from 2 to 10. Raw data is available in the associated text file (path-random.txt) posted with this document. The data is shown in Figure 4. Certain values of m are omitted from the graphs for readability purposes.

5 Analysis

In a building evacuation most people would be going toward the exits of the building. Since people would presumably not be re-entering the building, the exterior can be considered a sink. Due to the lack of cyclic movement by any of the moving bodies (people), the amount of readings required to determine a counting information state are relatively small, almost linear. Note that due to the existence of a sink, the lower bound of Theorem 9 in[1] and [2] does note apply.

We have hypothesized that the length of the shortest directed cycle is crucial in determining the EE-convergence time. In both series of tests with directed cycles, the EE-convergence time dropped rapidly as the cycle increased in length.

The data in Section 1 was fit to equations of the form $y = a \times b^m + c$ (with y being the average number of sensor readings required to determine a counting information state). The best-fit parameters acquired are summarized in Table 1. All fits had a coefficient of determination of at least 0.99.

n	a	b	c
2	1.0228	2.0039	429.14
3	2.5202	1.4914	11.038
4	4.7563	1.3141	-4.0148
5	6.3305	1.2313	-4.5055
6	9.7911	1.1686	-9.0924
7	17.318	1.1139	-18.010
8	29.722	1.0773	-31.213
9	47.901	1.0537	-50.4791
10	76.3446	1.0372	-79.986

Table 1: Data from Section 1

The b parameter is the most important for the long-term behavior of the function. We have attempted to determine a function $b = f(n)$ that fits the data well, but have been unsuccessful. The function $b = 1/(n - 1.057) + .952$ has been the most successful so far, but it appears to correlate poorly with the measure values of b for higher values of n.

For the data in Section 2, the best fit lines are summarized in Table 2

n	a	b	c
2	1.4867	2.0083	180.47
3	5.1204	1.4837	-6.8949
4	8.9579	1.3211	-15.039
5	12.870	1.2424	-19.684
6	16.375	1.1967	-22.781
7	18.468	1.1703	-23.631
8	20.250	1.1524	-24.986
9	24.715	1.1324	-30.494
10	26.660	1.1220	-32.405

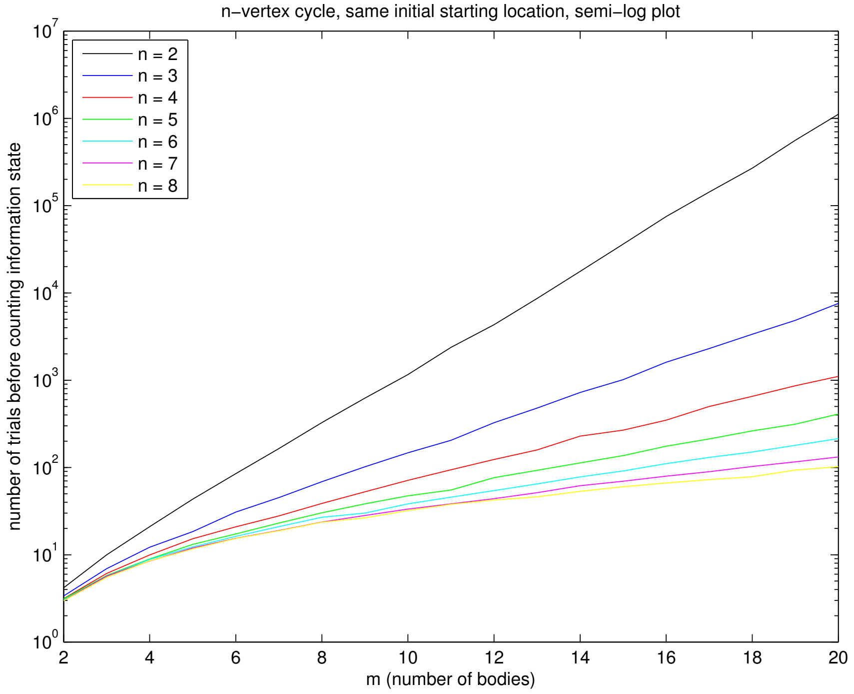
Table 2: Data from Section 2

While the b parameter in this data set looks similar for small values of n, it diverges from the data in Section 1 once n becomes larger than 7. In this data, the function $b = 1/(n - 1) + 1$ fits the existing data almost perfectly, though further examinations with higher values of n are required to verify this. This would suggest that the EE-convergence time for m bodies in an environment whose minimum directed cycle is of size n would be upper-bounded by a function of order $O((1/(n-1)+1)^m)$.

References

[1] L. H. Erickson, J. Yu, Y. Huang, and S. M. LaValle, “Counting moving bodies using sparse sensor beams,” in *Algorithmic Foundations of Robotics X*. Springer, 2013, pp. 427–442.

[2] ——, “Counting moving bodies using sparse sensor beams,” *IEEE Transactions on Automation Science and Engineering*, 2013, Accepted for publication.



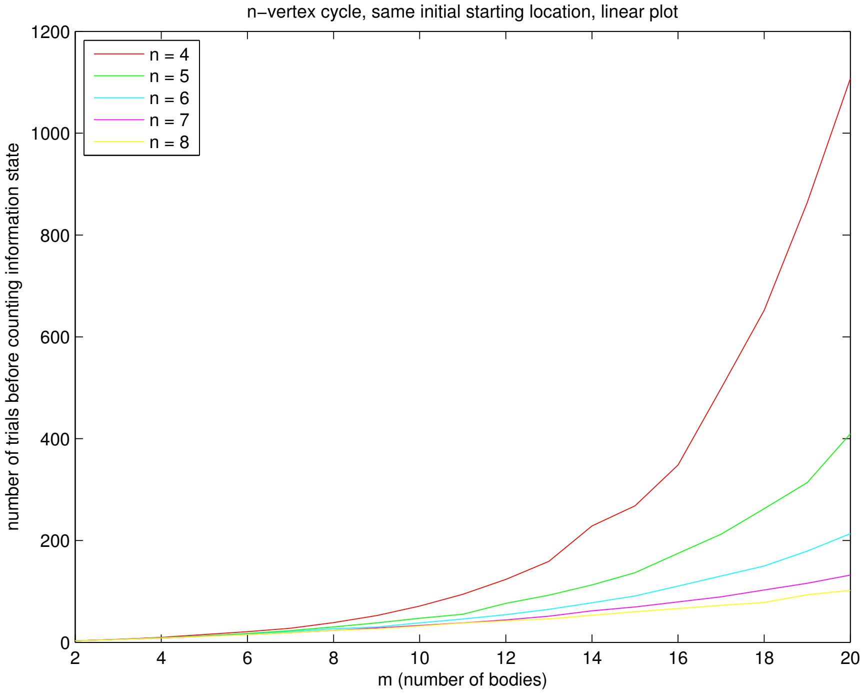
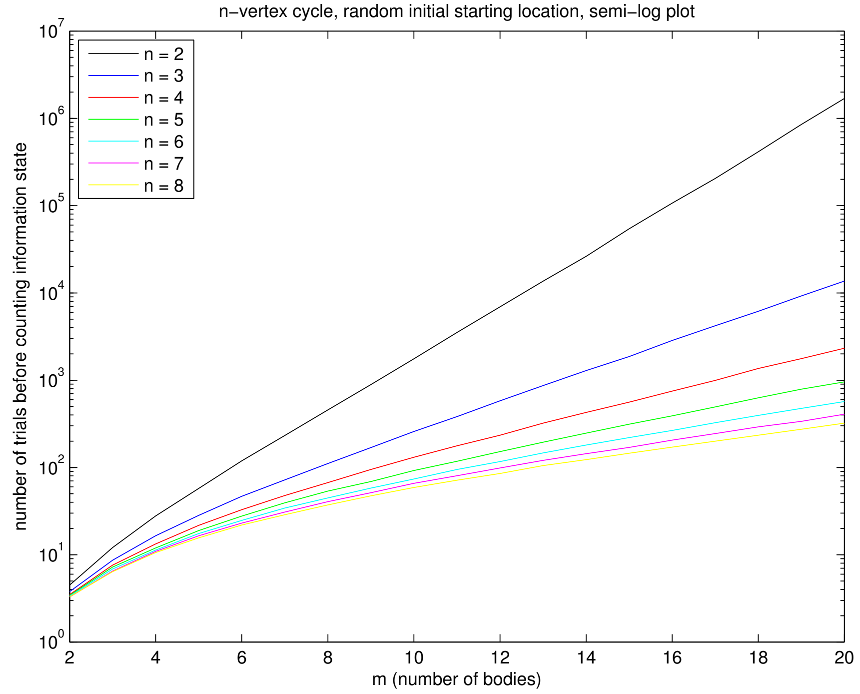


Figure 1: Simulation in cycle C_n with m bodies detailing the average number of readings required to obtain a counting information state. All bodies start in the same vertex.



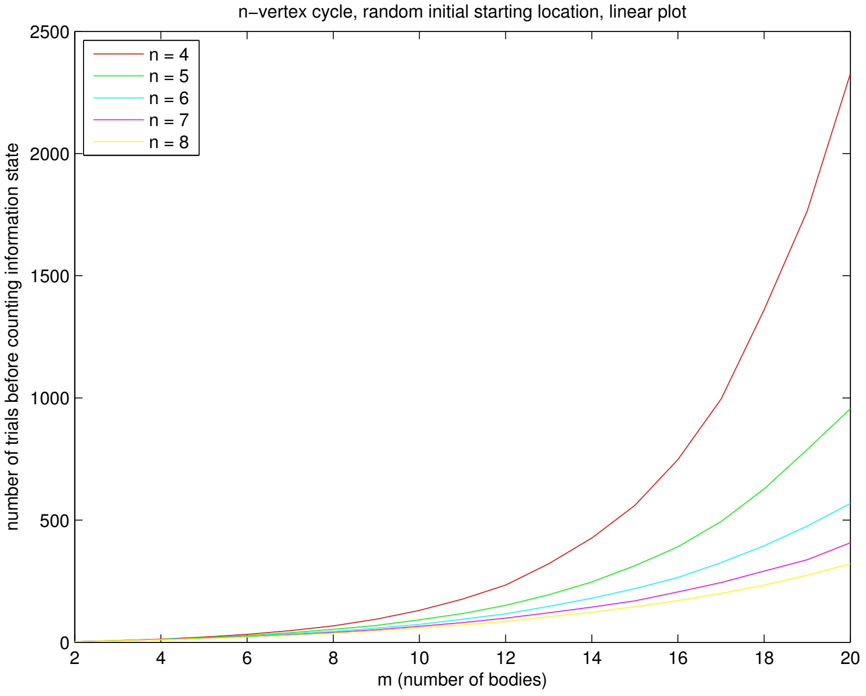


Figure 2: Simulation in cycle C_n with m bodies detailing the average number of readings required to obtain a counting information state. The initial location of the bodies is randomized.

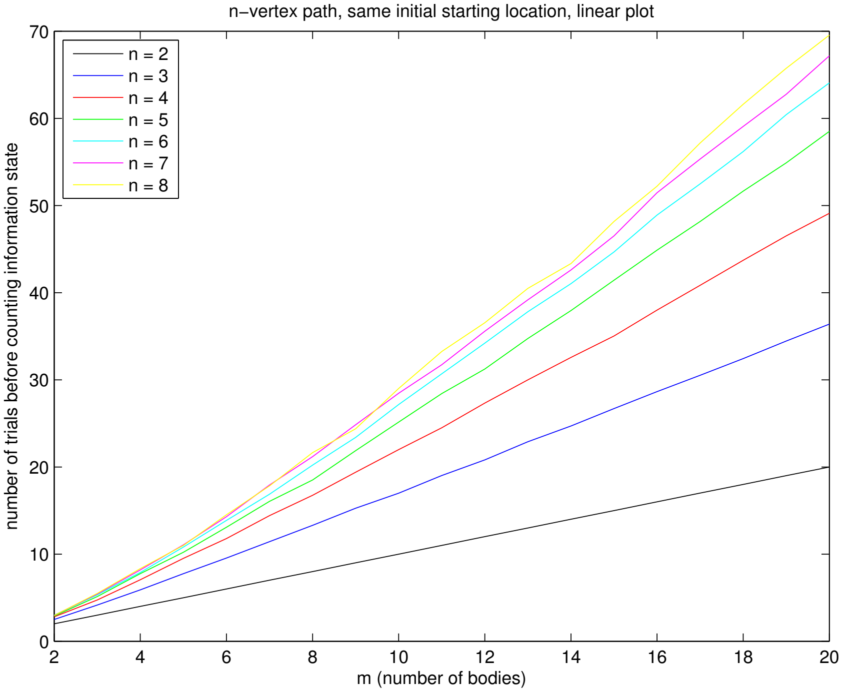


Figure 3: Simulation in path P_n with m bodies detailing the average number of readings required to obtain a counting information state. All bodies start in the same vertex (furthest from the sink).

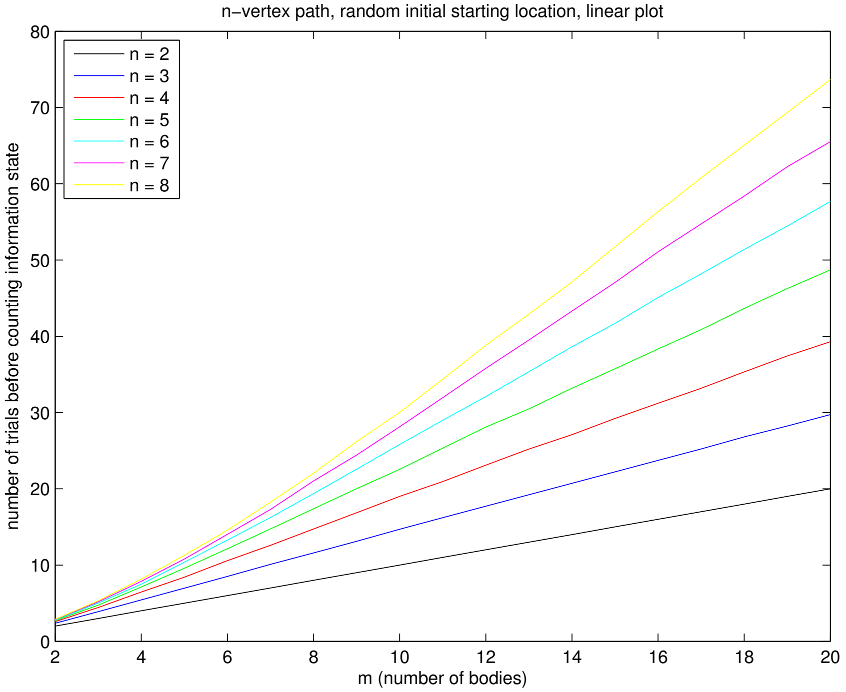


Figure 4: Simulation in path P_n with m bodies detailing the average number of readings required to obtain a counting information state. The initial location of the bodies are random (no bodies are initially placed in a sink).