Efficient Sharing of Resources in Distributed Systems

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Max/Min Fairness Using Efficient Sharing (ES): Method 1

<table>
<thead>
<tr>
<th>The rate</th>
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</thead>
<tbody>
<tr>
<td>( Rd, u(t) = \frac{C_{d,u} - \frac{q_{d,u}(t-\tau)}{\tau}}{\hat{N}_{d,u}(t-\tau)} )</td>
</tr>
</tbody>
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<table>
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<tr>
<th>Max/Min: Fractional flows</th>
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<tbody>
<tr>
<td>( \hat{N}<em>{d,u}(t-\tau) = \frac{S</em>{d,u}(t)}{R_{d,u}(t-\tau)} = \sum_j N_{d,u}(t-\tau) \frac{R_{d,u}^j(t)}{R_{d,u}(t-\tau)} )</td>
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<th>QoS, SLA check</th>
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<tr>
<td>( S_{d,u}(t) = \sum_j N_{d,u}(t-\tau) \cdot \varrho^{j}<em>{d,u} \cdot R</em>{d,u}^j(t) )</td>
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<th>Per flow weighted share at the link</th>
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<td>( \varrho^{j}<em>{d,u} \cdot R</em>{d,u}(t) )</td>
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Notations:
- \( C_{d,u} \) = Link capacity for down (d) and up (u) links
- \( q_{d,u}(t) \) = Queue size
- \( \tau \) = Control interval
- \( N_{d,u}(t) \) = Number of flows
- \( \varrho^{j}_{d,u} = \frac{R_{d,u}^j(t+\tau)}{R_{d,u}^j(t)} \) = Priority weight of flow \( j \)
- \( R_{d,u}^j(t) \) = Rate of flow \( j \)
MaxMin fairness example

- $C_u = 100\text{pkts/sec}$
- $q_u(t) = 0\text{pkts}$
- $\tau = 1\text{sec}$
- $N_u = 3$
- $R^1_u(t) = 10$, $R^2_u = 50$, $R^3_U = 50\text{ pkts/sec}$
- $\varphi^1_u = \varphi^2_u = \varphi^3_u = 1$
- $R_u(t - \tau) = 50\text{ pkts/sec}$
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- $R_u(t - \tau) = 50 \text{ pkts/sec}$

Then ES rate is

$$R(t) = \frac{100}{\frac{10}{50} + \frac{50}{50} + \frac{50}{50}} = 45.45 \text{ pkts/sec}.$$
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Then ES rate is

$$R(t) = \frac{100}{\frac{10}{50} + \frac{50}{50} + \frac{50}{50}} = 45.45 \text{ pkts/sec.}$$

Processor sharing (PS):

$$R(t) = \frac{100}{3} = 33.33 \text{ pkts/sec.}$$
Max/Min Fairness Using ES: Method 2

MaxMin fairness: Efficient Sharing

- Given a resource with capacity $X$ units/sec to be shared by $N$ sources,
- In this method we set $R(t - \tau) = \frac{X}{N}$, which is the processor sharing rate.
- Each source’s bottleneck fair (ES) share rate is denoted with $R_j(t)$.
- We also have $R_j(t) \leq R(t - \tau)$ as a source $j$ cannot send higher than its bottleneck fair share.
- Then the ES rate, $R(t)$ can be given as $R(t) = \frac{X}{\sum_j^N \left( \frac{R_j(t)}{R(t - \tau)} \right)} = \frac{X^2}{N \sum_j^N R_j(t)}$. 
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- Then the ES rate, $R(t)$ can be given as
  \[
  R(t) = \frac{X}{\sum_j^N \left( \frac{R_j(t)}{R(t - \tau)} \right)} = \frac{X^2}{N \sum_j^N R_j(t)}.
  \]

Then the rate for the example above, in the first iteration (round) becomes
\[
R(t) = \frac{100}{\frac{10}{33.33} + \frac{33.33}{33.33} + \frac{33.33}{33.33}} = 43.48 \text{ pkts/sec}.
\]

In the second iteration (round)
\[
R(t) = \frac{100}{\frac{43.48}{43.48} + \frac{43.48}{43.48} + \frac{43.48}{43.48}} = 44.84 \text{ pkts/sec}.
\]

Values of ES rate in next iterations (rounds or RTTs)
\[
44.9842555105713, \quad 44.9984250551231, \quad 44.9998425005512, \quad 44.9999842500055,
\]

Generalized Efficient Sharing (GES)

With priority weight, $\wp^j$ of flow $j$, the GES rate $R(t)$ can be given as
\[
R(t) = \frac{X}{\sum_j^N \left( \frac{\wp^j R_j(t)}{R(t - \tau)} \right)} = \frac{X^2}{N \sum_j^N \wp^j R_j(t)}.
\]

Then source $j$’s weighted share becomes $\wp^j R(t)$. 

Max/Min Fairness Using ES: Method 3

MaxMin fairness: Efficient Sharing

- Given a resource with capacity $X$ units/sec to be shared by $N$ sources,
- Each source’s bottleneck fair (ES) share rate is denoted with $R^j(t)$.
- We also have $R^j(t) \leq \frac{C}{N(t-\tau)}$ as a source $j$ cannot send higher than its bottleneck fair share.
- In this method, first
  
  $N(t) \leftarrow 0; \hat{N}(t) \leftarrow 0; \hat{X}(t) \leftarrow 0;$
  
  for each flow $j$ do
    
    $N(t) \leftarrow N(t) + 1;$
    
    if ($R^j(t) < \frac{X}{N(t-\tau)}$) then
      
      $\hat{X}(t) \leftarrow \hat{X}(t) + R^j; \hat{N}(t) \leftarrow \hat{N}(t) + 1;$
      
      end if
  
  end for

- The ES rate $R(t)$ is then given by
  
  $R(t) = \frac{X - \hat{X}(t)}{\tilde{N}(t)}; \tilde{N}(t) = N(t) - \hat{N}(t).$
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  if $(R^j(t) < \frac{X}{N(t-\tau)})$ then
  
  $\hat{X}(t) \leftarrow \hat{X}(t) + R^j$; $\hat{N}(t) \leftarrow \hat{N}(t) + 1$

  end if

  end for

- The ES rate $R(t)$ is then given by

  $R(t) = \frac{X - \hat{X}(t)}{\tilde{N}(t)}$; $\tilde{N}(t) = N(t) - \hat{N}(t)$.

$\hat{N}(t)$ and $\hat{X}(t)$ are number of flows and sum of rates of the flows bottlenecked at other resources.
Max/Min Fairness Using ES: Method 3

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- Each source’s bottleneck fair (ES) share rate is denoted with $R^j(t)$.
- We also have $R^j(t) \leq \frac{C}{N(t-\tau)}$ as a source $j$ cannot send higher than its bottleneck fair share.
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  \[ \text{for each flow } j \text{ do} \]
  \[ N(t) \leftarrow N(t) + 1; \]
  \[ \text{if } (R^j(t) < \frac{X}{N(t-\tau)}) \text{ then} \]
  \[ \hat{X}(t) \leftarrow \hat{X}(t) + R^j; \hat{N}(t) \leftarrow \hat{N}(t) + 1; \]
  \[ \text{end if} \]
  \[ \text{end for} \]
- The ES rate $R(t)$ is then given by
  \[ R(t) = \frac{X - \hat{X}(t)}{\tilde{N}(t)}; \tilde{N}(t) = N(t) - \hat{N}(t). \]
Max/Min Fairness Using ES: Method 3

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- We also have $R_j(t) \leq \frac{C}{N(t-\tau)}$ as a source $j$ cannot send higher than its bottleneck fair share.
- In this method, first $N(t) \leftarrow 0; \hat{N}(t) \leftarrow 0; \hat{X}(t) \leftarrow 0$; for each flow $j$ do $N(t) \leftarrow N(t) + 1$; if $(R_j(t) \leq \frac{X}{N(t-\tau)}$ then $\hat{X}(t) \leftarrow \hat{X}(t) + R_j$; $\hat{N}(t) \leftarrow \hat{N}(t) + 1$; end if end for
- The ES rate $R(t)$ is then given by $R(t) = \frac{X - \hat{X}(t)}{\hat{N}(t)}$; $\tilde{N}(t) = N(t) - \hat{N}(t)$.

$\hat{N}(t)$ and $\hat{X}(t)$ are number of flows and sum of rates of the flows bottlenecked at other resources.

Then rate for the example above

$$R(t) = \frac{100 - 10}{2} = 45.0 \text{ pkts/sec}.$$ 

Generalized Efficient Sharing (GES)

With priority weight, $\varphi^j$ of flow $j$, the GES rate $R(t)$ can be given as

$$R(t) = \frac{X - \hat{X}(t)}{\sum_{j} \tilde{N}(t) \varphi^j}$$

$\tilde{N}(t)$ flows bottlenecked at the current resource with capacity $X$ units/sec. Then source $j$’s weighted share at the current resource becomes $\varphi^j R(t)$. 

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Efficient Sharing