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INVESTIGATION OF THE PRESSURE CHARACTERISTICS AND AIR DISTRIBUTION IN BOX-TYPE PLENUMS FOR AIR CONDITIONING DUCT SYSTEMS

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THE ENGINEERING EXPERIMENT STATION,
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INVESTIGATION OF THE PRESSURE CHARACTERISTICS AND AIR DISTRIBUTION IN BOX-TYPE PLENUMS FOR AIR CONDITIONING DUCT SYSTEMS

A REPORT OF AN INVESTIGATION CONDUCTED BY THE ENGINEERING EXPERIMENT STATION UNIVERSITY OF ILLINOIS IN COOPERATION WITH THE AMERICAN GAS ASSOCIATION

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ABSTRACT

On October 1, 1947, the American Gas Association entered into an agreement for cooperative research with the Engineering Experiment Station of the University of Illinois, the overall objective being the improvement and simplification of air duct systems for residential all-year air conditioning installations. One phase of this research program has been a laboratory investigation of the pressure characteristics and air distribution in a type of plenum chamber designated as a box-plenum. This bulletin presents a report of this phase of the cooperative research project.

Two box-plenums having multiple branch ducts were investigated under a variety of test conditions to obtain design data applicable to box-plenums for air duct systems.

The initial laboratory apparatus included a 9-ft-long box-plenum with six branch ducts of 7-in. diam and a conventional abrupt expansion entrance section for introducing air into the end of the plenum. The pressure losses of this arrangement were found to be high; furthermore, a rotational flow effect occurred within the plenum and caused an unstable distribution of the air to the branch ducts. In an effort to improve the performance of the plenum, four additional entrance sections were developed and studied. The performance of the plenum with an entrance designated as Type 5 was found to be excellent, the pressure loss being about one half that with the abrupt expansion entrance section. This entrance section consisted of an internal expansion section with seven splitter vanes.

A similar arrangement incorporating a 3-ft-long box-plenum was also investigated. Again the performance of the plenum with the Type 5 entrance section was found to be much better than when the abrupt expansion section was used.

Studies were also made with the supply air entering the 3-ft box-plenum from the bottom. For this condition the pressure losses were found to be very high when the abrupt expansion entrance section was used; consequently, three additional bottom inlet entrance sections were developed and investigated. The performance of the system with a bottom entrance section designated as Type 9 was found to be good. This entrance section consisted of an external expansion section the sides of which had included angles of about 16 deg. An equation for approximating the total pressure loss of box-plenums with bottom entrance sections is given.
Another arrangement including the 9-ft-long box-plenum, the Type 5 end entrance section, and 7-, 8-, and 9-in.-diam branch ducts was investigated to determine the applicability of the previous results to practical supply air duct systems having branch ducts of various sizes and equivalent lengths. The results of this study indicate that a single equation can be used to express the total pressure loss of end inlet types of box-plenums of the sizes and shapes normally used in air duct systems for residences.
CONTENTS (CONCLUDED)

VII. RESULTS OF SERIES 3 STUDIES WITH THREE-FOOT PLENUM, 7-IN.-DIAMETER BRANCH DUCTS, BOTTOM ENTRANCE SECTIONS
24. Performance with Abrupt Expansion, Type 6, Entrance Section (Group 3B) 46
25. Performance with Cone Diffuser, Type 7, Entrance Section (Group 3C) 48
26. Performance with Internal Diffuser, Type 8, Entrance Section (Group 3D) 48
27. Performance with Divergent, Type 9, Entrance Section (Group 3E) 49

VIII. RESULTS OF SERIES 4 STUDIES WITH NINE-FOOT PLENUM; 7-IN., 8-IN., AND 9-IN.-DIAMETER BRANCH DUCTS; END ENTRANCE SECTIONS
28. Performance with Various Sizes of Branch Ducts Having Equal Equivalent Lengths (Groups 4B, 4C, and 4D) 51
29. Performance with Various Sizes of Branch Ducts Having Various Equivalent Lengths (Group 4E) 53
30. Total Loss Equation for Practical Box-Plenum Systems 54

IX. CONCLUSIONS AND RECOMMENDATIONS
31. General Conclusions 55
32. Box-Plenums with End Entrance Sections 55
33. Box-Plenums with Bottom Entrance Sections 56

APPENDIX A: Calibration of the Orifice Box 57
APPENDIX B: Calibration of the Branch Ducts 62
34. Branch Duct Resistance 62
35. Branch Duct Air Flow Rate by the Pressure-Drop Method 63
36. Branch Duct Air Flow Rate by the Dynamic-Tube Method 64

APPENDIX C: Evaluation of the Outlet Loss 66
APPENDIX D: Sample Data Sheet and Method of Compiling Data 67
APPENDIX E: Derivation of Equations and Computation of Results
37. Energy Leaving the Box-Plenum 70
38. Energy Entering the Box-Plenum 72
39. Evaluation of Losses 73
40. Derivation of Eq. (27), Section 20 76
FIGURES

NO.
1. Plenum, Branch Ducts, and Pressure Connections for Series 1 Studies 13
2. General Arrangement of Box-Plenum for Series 1 Studies 14
3. Details of End Entrance Sections 15
4. General Arrangement of Box-Plenum for Series 2 Studies 17
5. General Arrangement of Box-Plenum for Series 3 Studies 18
6. Details of Bottom Entrance Sections 20
7. General Arrangement of Box-Plenum for Series 4 Studies 21
8. Arrangement of Box-Plenum for Preliminary Analysis 23
9. Theoretical Total Loss in Terms of Branch Duct Velocity Heads 28
10. Theoretical Total Loss in Terms of Trunk Duct Velocity Heads 29
11. Definition of Losses Occurring in a Box-Plenum Arrangement 30
12. Plenum Losses with Five Types of End Entrance Sections (Series 1) 36
13. Relation Between Total Loss and Location of Branch Duct 1 38
14. Variation of Static Pressure and Air Flow Rate at Station 2 with Number of Branch Ducts in Use 39
15. Total Loss of 9-ft Box-Plenum with Type 5 End Entrance Section (Series 1) 40
16. Flow Pattern with Type 1 End Entrance Section for Series 2 Studies 43
17. Total Loss of 3-ft Box-Plenum with Type 5 End Entrance Section (Series 2) 44
18. Total Loss of 3-ft Box-Plenum with Type 6 Bottom Entrance Section (Series 3) 47
19. Flow Pattern with Type 6 Bottom Entrance Section for Series 3 Studies 48
20. Total Loss of 3-ft Box-Plenum with Type 9 Bottom Entrance Section (Series 3) 49
21. Total Loss of Box-Plenum for Unbalanced Conditions of Flow (Series 4) 51
22. Total Loss of Box-Plenum for Balanced Conditions of Flow (Series 4) 52
23. Arrangement for Calibration of Orifice Box 57
24. Calibration Curve for Air-Measuring Station 58
25. Calibration Curves for Individual Orifices in Orifice Box 59
26. Calibration Curves for Combinations of Orifices in Orifice Box 60
27. Representative Calibration Curve for Resistance of a Branch Duct 62
28. Representative Calibration Curve for Air Flow Rate in a Branch Duct 64
TABLES

1. Series 1 Studies — 9-ft Plenum, 7-in.-diam Branch Ducts, and End Entrance Sections 32
2. Series 2 Studies — 3-ft Plenum, 7-in.-diam Branch Ducts, and End Entrance Sections 33
3. Series 3 Studies — 3-ft Plenum, 7-in.-diam Branch Ducts, and Bottom Entrance Sections 33
4. Series 4 Studies — 9-ft Plenum, 7-in., 8-in., and 9-in.-diam Branch Ducts, and Type 5 End Entrance Section 33
5. Conditions and Results for Group 4E Studies 54
6. Values of $C$ and $n$ for Eq. (6), Appendix A 61
7. Sample Data Sheet 68
8. Corrected Gage Pressures, in. of Water 69
9. Corrected Air Flow Rates, cfm 69
I. INTRODUCTION

1. Preliminary Statement

The conventional type of air duct system for residential applications of winter, summer, or all-year air conditioning is the trunk duct system. As a result of research and experience, authoritative data are available for designing the trunk duct system. Since this system is characterized by a reduction in the cross-sectional area of the trunk duct at each branch takeoff, many intricate fittings are required. The fabrication and installation costs of trunk duct systems are usually high, generally from 20 to 50 percent of the cost of the entire air conditioning system. It is therefore desirable to develop a simpler and more inexpensive air duct system having an efficiency at least equal to that of the trunk duct system.

One such simplified system makes use of a box-plenum for distributing the air to the various supply branch ducts. "Box-plenum" refers to a type of plenum having a large cross-sectional area relative to the area of the supply air inlet. Box-plenums may be located in (a) attics, in conjunction with either "overhead supply" or ceiling panel systems; (b) ceilings of halls or closets in basementless homes; and (c) basements, when the conditioner is not centrally located or when the basement is too low to accommodate a bonnet type of plenum. In many cases the cost of a simplified air duct system using a box-plenum may be substantially less than that of a more complicated trunk duct system, since the box-plenum is rectangular without transition sections, and the branch takeoff fittings are simple butted connections.

The design of such plenums has been hampered by the lack of engineering data. Although abrupt expansion and contraction loss data are available which could conceivably apply to the respective entrance and outlet losses in a plenum, insufficient data are available for accurately evaluating (a) the effect of turbulent mixing of the air within a plenum; (b) the effect of the location of the air inlet to a plenum; (c) the effect of the number, size, and location of the branch ducts connected to a plenum; and (d) the effect of the physical dimensions of a plenum, such as length.

2. Objects and Scope of Investigation

To obtain data on the pressure characteristics and air distribution in box-type plenums, two box-plenums with multiple branch ducts were investigated under a variety of experimental conditions. The
main object was to develop data for use in the design of box-plenums for air conditioning duct systems. Specific objectives were as follows:

1. To study the air flow patterns within the plenums.
2. To evaluate the pressure losses occurring in plenums with conventional abrupt expansion entrance sections.
3. To develop types of entrance sections for improving the performance of the plenums and to evaluate the amount of improvement obtained.
4. To determine the effect of the fan discharging directly into the plenum.
5. To determine the distribution of air to the branch ducts.
6. To determine the effect of the location of the branch ducts on the plenum performance.
7. To determine the effect of adding equal resistance to each branch duct.
8. To determine the performance of plenums having multiple branch ducts of various diameters and equivalent lengths.
9. To develop design data for plenums having dimensions similar to those commonly used in residential air conditioning systems.

The scope of the investigation has been limited to the determination of performance data for box-plenums under conditions commonly encountered in residential air conditioning applications. Therefore the investigation has been confined to box-plenums 3 ft wide, 1 ft deep, and from 3 ft to 9 ft long, having a 12-in.-x-15-in. inlet and 7-in., 8-in., and 9-in.-diam branch ducts. However, the results are considered to be applicable to all box-plenums having approximately the same ratios of dimensions as those investigated.

3. Acknowledgments

This bulletin, a report of an investigation conducted in the Mechanical Engineering Laboratory, is the first to be published under the terms of a cooperative research agreement between the American Gas Association and the University of Illinois in connection with A.G.A. Research Project DGR-2-AC. The American Gas Association through its Committee on Domestic Gas Research (R. J. Rutherford, Chairman) has been represented by the Technical Advisory Group for Gas Summer Air Conditioning Research, the personnel of which changes somewhat from year to year. Since the project was initiated, the following persons have served on this Technical Advisory Group for at least one year during the period 1947-49.
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Glenn F. Zellhoefer, Eureka-Williams Corporation, Bloomington, Ill.
Eugene D. Milener, Secretary, American Gas Association, New York, N. Y.

Besides serving on this committee Messrs. Pierce and Zellhoefer have acted as Technical Advisors for the research work conducted at the University of Illinois in connection with A.G.A. Research Project DGR-2-AC.

The investigation, a part of the work of the Engineering Experiment Station and a project of the Department of Mechanical Engineering, was conducted under the general administrative direction of M. L. Enger, Dean Emeritus and former Director of the Engineering Experiment Station, and of Professor N. A. Parker, Head of the Department of Mechanical Engineering.

The Bulletin includes material from two theses written by A. G. Wilson and C. L. Brown under the supervision of Professor R. J. Martin and submitted in partial fulfilment of the requirements for the degree of Master of Science in Mechanical Engineering.

Acknowledgment is made to W. R. Hedrick, former Special Research Assistant, for aid in collecting and analyzing the data, and to W. K. Ender, graduate student, for aid in collecting the data.
II. DESCRIPTION OF APPARATUS

4. Series 1: Nine-Foot Plenum, 7-in.-diam Branch Ducts, End Entrance Section

The general arrangement of the apparatus is shown in Figs. 1 and 2. The four orifices in the orifice box were used to measure the total flow rate of the air entering the apparatus; since each could be closed off by a cover, they were used singly or in combination, depending upon the range of air flow rates used. Details of the calibration of the orifice box are given in Appendix A. The 12-in. centrifugal fan in the fan section discharged air from the orifice box into a 6-in.-long transition section and thence into the 12-in.-x-15-in. galvanized-iron trunk duct $5\frac{1}{2}$ ft long leading to the box-plenum. The fan was connected by a V-belt to a $1\frac{1}{2}$-hp, 220-v, d-c motor whose speed was varied over a wide range by two rheostats in the armature circuit. The resistance plate and egg-crate straightener provided uniform flow conditions at Station 2. The resistance plate consisted of a 24-gage galvanized-iron plate with numerous holes of $\frac{3}{4}$-in. diam; and the egg-crate straightener was made in accordance with specifications of the National Association of Fan Manufacturers. The top and bottom of the box-plenum were made of transparent material to permit visual observation of 480 directional indicators consisting of 2-in.-long silk threads uniformly spaced inside the plenum. The six 7-in.-diam branch ducts were made as exactly alike as possible, and were located in a symmetrical pattern about the plenum as shown in Fig. 2. Each branch duct consisted of four 12-in.-long flanged sections placed end to end; the section nearest the plenum was made of transparent material, the other three of 24-gage galvanized iron. The flow rate of the air leaving the plenum through any branch was measured by a flat-plate orifice installed in each branch duct. The calibration of the branch ducts is discussed in Appendix B. The adjustable cone-shaped dampers were used to control the air flow rates in the branch ducts. The sides of the box-plenum were made of 24-gage galvanized iron. The entire plenum assembly was supported by a steel frame and was designed to permit changing the length of the plenum. Asbestos tape was used over all joints to prevent air leakage.

The static pressures at Stations 1, 2, 61, and 62 were measured by piezometer rings; those at all other stations, by single static-pressure

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taps. Rubber tubing connected all pressure measuring stations to inclined draft gages mounted on the instrument panel. These gages, which read directly to 0.01 in. of water, were calibrated with a hook-gage type of micromanometer. The fan speed was measured by a speed indicator that read directly in revolutions per minute. The power input to the motor was measured with a wattmeter that read directly to 10 watts.

In addition to the arrangement shown in Figs. 1 and 2, which was designated “remote” to signify that the fan was not discharging directly into the box-plenum, several other arrangements were utilized. A “close-coupled” fan position was obtained by removing the 5½ ft of trunk duct (including the egg-crate straightener and the resistance plate) and connecting the fan section directly to the plenum. To facilitate this change in arrangement the orifice box was mounted on casters. Several other arrangements were provided by changing the

![Figure 1. Plenum, Branch Ducts, and Pressure Connections for Series 1 Studies](image)
locations of branch ducts 1 and 5 along the sides of the plenum. Further variation of conditions was made possible by completely closing off any branch duct or combination of branch ducts, so that studies could be made with from one to six branch ducts in use.

When any of the foregoing arrangements was used, one of the five galvanized-iron end entrance sections shown in Fig. 3 was located at the plenum entrance just downstream from Station 2. The principal details of these five entrance sections are as follows.
Type 1—Abrupt expansion section. This is the conventional method of introducing air into the end of a box-plenum. The performance of this section was used as the basis for evaluating the effectiveness of Types 2, 3, 4, and 5. The arrangement shown in Fig. 2 has this section installed.

Type 2—Divergent section. The length of 18 in. was selected as a maximum distance external to the plenum that would be available for external entrance sections in most practical applications.

Type 3—Divergent section with splitter vanes. This is the Type 2 section with 7 full-depth splitter vanes installed.

Type 4—Internal section with expansion ring. This section has 5 vanes and is located entirely inside the plenum.

Type 5—Internal expansion section. This section was obtained by inscribing a semicircle about the upstream end of the Type 3 section and placing it inside the plenum. Layout details for fabricating Type 5 entrance sections are shown in Fig. 3b.

5. Series 2: Three-Foot Plenum, 7-in.-diam Branch Ducts, End Entrance Section

The general arrangement of the apparatus (Fig. 4) is similar to the Series 1 arrangement shown in Fig. 2 and described previously, except that the length of the box-plenum was reduced to 3 ft and a microdynamic tube was installed near each branch duct orifice. The function of the microdynamic tubes is discussed in Appendix B.

This box-plenum was studied with only the Type 1 and Type 5 end entrance sections installed at the plenum entrance (see Fig. 3). During some of the studies made with the Type 5 entrance section installed, branch ducts 2 and 4 were located at the end of the plenum on each side of branch 3.

6. Series 3: Three-Foot Plenum, 7-in.-diam Branch Ducts, Bottom Entrance Section

The general arrangement of the apparatus is shown in Fig. 5. Except for the trunk duct between the resistance plate and the bottom of the box-plenum, all the apparatus was the same as that used in Series 2. The plenum and branch ducts were the same as shown in Fig. 4 except that branch 6 was located at one end of the plenum instead of on the top and that the air was introduced through the bottom of the plenum rather than through the end. The air leaving the fan section (Fig. 5) flowed through the trunk duct and into an auxiliary plenum 36 in. wide, 18 in. deep, and 57 in. long. The air
All Branch Ducts 7 in Dia.  
Unlabeled numerals designate pressure measuring stations.

Figure 4. General Arrangement of Box-Plenum for Series 2 Studies
then flowed into a rectangular convergent section of 6-in. radius and 12-in.-x-15-in. throat dimensions, through an egg-crate straightener, and finally into the bottom of the box-plenum. The function of the convergent section and straightener was to provide uniform flow conditions at Station 2, which was located between the egg-crate straightener and the bottom entrance section.

A second arrangement included two additional 7-in.-diam branch ducts, making a total of eight identical branch ducts connected to the box-plenum. They were located in a symmetrical pattern about the plenum, so that all four sides of the plenum were similar to the one having branch ducts No. 1 and No. 2 attached to it (see Fig. 5).

Studies were made with each of four types of bottom entrance sections—Types 6, 7, 8 and 9 as shown in Fig. 6—and with the plenum arrangement as shown in Fig. 5, which included six branch ducts. In addition, studies were made with the Type 9 entrance section when the plenum arrangement included eight branch ducts. The principal details of these bottom entrance sections are as follows.

Type 6—Abrupt expansion section. This is the conventional method of introducing air into the bottom of a box-plenum. The performance of this section was used as the basis for evaluating the effectiveness of Types 7, 8, and 9. The arrangement shown in Fig. 5 has this section installed.

Type 7—Abrupt expansion section with an inverted cone. The functions of this cone were to reduce the energy loss as the supply air was turned through an angle of 90 deg and to direct air more uniformly toward the branch ducts located in the sides of the plenum.

Type 8—Internal diffuser section. This diffuser section was designed on the same principle as the Type 5 end entrance section; namely, that of dividing the entering air into several small streams.

Type 9—Divergent section. The 18-in. length of this section was selected as a maximum distance external to the plenum that would be available for external entrance sections in most practical applications. When this section was used, the box-plenum was raised 18 in. higher than shown in Fig. 5, and the entrance section was installed between Station 2 and the bottom of the box-plenum.

7. Series 4: Nine-Foot Plenum; 7-in., 8-in., and 9-in.-diam Branch Ducts; End Entrance Section

The arrangement of the apparatus (Fig. 7) includes four 7-in., two 8-in., and two 9-in.-diam branch ducts. Since this arrangement was used to investigate the applicability of the results obtained with the
FIGURE 6. DETAILS OF THE BOTTOM ENTRANCE SECTIONS
Type 5 end entrance section in systems similar to those encountered in practice, the branch ducts were installed at arbitrarily selected locations along the sides and downstream end of the plenum, and the Type 5 entrance section was installed at Station 2. The centers of all the branch ducts were located on the same level, midway between the top and the bottom of the plenum.
III. Preliminary Analysis of Losses

8. Statement of the Losses

In order to predict the relative magnitudes of the various pressure losses that occur in a box-plenum and to provide a basis for comparison with laboratory data, the expected losses for an arrangement similar to that used for the initial studies were calculated and analyzed. The plan view of the box-plenum arrangement used for this theoretical analysis is shown in Fig. 8.

Four vertical cross-sections of this arrangement are designated as o, p, m, and d. Since the plenum and all the branch ducts are supplied with air from a common source, Section o, and since all the air is delivered to the atmosphere through a parallel system made up of the six branch ducts, one branch was used to calculate the total loss. For this purpose, branch duct 3 was arbitrarily selected. The total-pressure loss of the arrangement between Section o and the atmosphere, Section a, is composed of the following losses:

1. Abrupt expansion loss from o to p as the air enters the plenum.
2. Turbulent mixing loss within the plenum from p to m.
3. Friction-pressure loss within the plenum from p to m.
4. Abrupt contraction loss from m to d as the air flows from the plenum into the branch duct.
5. Total-pressure loss in the branch duct from d to a.

It was assumed that the air flowing through the system was incompressible and at the standard density of 0.075 lb per cu ft and that a uniform flow pattern existed throughout the arrangement. The several pressure losses for these idealized flow conditions were then calculated as follows.

9. Abrupt Expansion Loss

The Borda formula for the loss of head during the abrupt expansion occurring from o to p is

\[ h_e = \frac{(v_o - v_p)^2}{2g} \]  

in which

- \( h_e \) = head lost during expansion, ft of fluid flowing
- \( v_o \) and \( v_p \) = mean velocities at Sections o and p, respectively, ft per sec
- \( g \) = acceleration due to gravity, ft per (sec)\(^2\)

This equation may be expressed in terms of the areas and one velocity as
in which

\[ h_e = \left(1 - \frac{A_o}{A_p}\right)^2 \frac{v_o^2}{2g} \]  (2)

\[ A_o = \text{area of the cross-section designated as } o, \text{ ft}^2 \]

\[ A_p = \text{area of the cross-section designated as } p, \text{ ft}^2 \]

or, in the more convenient form for air flow,

\[ H_e = \left(1 - \frac{A_o}{A_p}\right)^2 (VP)_o \]  (3)

in which

\[ H_e = \text{head lost during expansion, in. of water} \]

\[ (VP)_o = \left(\frac{V_o}{4005}\right)^2 = \text{velocity head, in. of water, at Section } o \]

\[ \text{for air at standard density and } V_o \text{ in fpm} \]

Substituting the values of \( A_o \) and \( A_p \) shown in Fig. 8 into Eq. (3) yields

\[ H_e = 0.34 (VP)_o \]  (4)

which states that the abrupt expansion loss from \( o \) to \( p \) as the air enters the box-plenum equals 0.34 of the velocity head existing in the trunk duct at Section \( o \).
10. **Turbulent Mixing Loss**

With respect to the turbulent mixing loss within the plenum, the assumption of a uniform flow pattern existing at every point throughout the arrangement demands that the turbulent mixing loss be zero. Therefore for this analysis the assumption must be that

\[ H_t = 0 \]  

in which

\( H_t \) = turbulent mixing loss within the box-plenum, in. of water.

11. **Friction-Pressure Loss Within the Box-Plenum**

The friction-pressure loss from \( p \) to \( m \) can be estimated by means of the friction chart. The circular equivalent of the 12-in.-x-36-in. plenum for equal friction and capacity is approximately 22 in. in diameter. Since the friction-pressure loss in a given duct is not a function of the velocity head alone, an air flow rate must be specified. Selecting 1000 cfm and referring to the friction chart, the friction-pressure loss in a 22-in.-diam duct is 0.01 in. of water per 100 ft of length, or 0.0009 in. of water for a 9-ft length. However, since the rate of air flow in the plenum is not constant but decreases as air is delivered to each of the branch ducts, the estimated friction-pressure loss of the 9-ft plenum is less than 0.0009 in. of water. This loss is negligibly small—less than 7 percent of the abrupt expansion loss of 0.0136 in. of water for 1000 cfm flowing as calculated by Eq. (4). Therefore it can be concluded that for all practical purposes

\[ H_p = 0 \]  

in which

\( H_p \) = friction-pressure loss of the plenum, in. of water.

12. **Abrupt Contraction Loss**

The abrupt contraction loss from \( m \) to \( d \) as the air flows from the plenum into branch duct 3 can be calculated by employing the Borda formula under the proper conditions. Thus

\[ h_c = \frac{(v_c - v_d)^2}{2g} \]  

in which

\( h_c \) = head lost during contraction, ft of fluid flowing

\( v_c \) = mean velocity at the vena contracta formed in the branch duct, ft per sec

\( v_d \) = mean velocity at Section \( d \), ft per sec.

This equation may be expressed in terms of the area and one velocity as

\[ h_c = \left( \frac{A_d}{A_c} - 1 \right)^2 \frac{v_d^2}{2g} \]  

(8)
or, in the more convenient form for air flow,

\[ H_c = \left( \frac{A_d}{A_c} - 1 \right)^2 (VP)_d \]  

(9)
in which

- \( H_c \) = head lost during contraction, in. of water
- \((VP)_d\) = \((V_d/4005)^2\) = velocity head, in. of water, at section \(d\)
  for air at standard density and \(V_d\) in fpm
- \(A_d\) = area of cross-section designated as \(d\), ft\(^2\)
- \(A_c\) = area of vena contracta of the air stream in the branch duct, ft\(^2\)

The ratio \(A_c/A_d\) is commonly known as the coefficient of contraction. Designating this coefficient by \(C\), Eq. (9) becomes

\[ H_c = \left( \frac{1}{C} - 1 \right)^2 (VP)_d \]  

(10)

McElroy \(^1\) gives the following equation for \(C\):

\[ C = \sqrt{\frac{1}{z-z - \left( \frac{A_d}{A_m} \right)^2 - \left( \frac{A_d}{A_m} \right)^2}} \]  

(11)
in which \(z\) is an empirical factor which was designated as the contraction factor. For abrupt contraction the value of \(z\) was given as 2.50. Substituting the values of \(A_m\) and \(A_d\) given in Fig. 8 into Eq. (11), the value of \(C\) is found to be 0.634. Equation (10) then becomes

\[ H_c = 0.34 (VP)_d \]  

(12)
which states that the abrupt contraction loss from \(m\) to \(d\) as the air leaves the box-plenum equals 0.34 of the velocity head existing in the duct at section \(d\).

13. Total-Pressure Loss in the Branch Duct

The friction-pressure loss in the branch duct from \(d\) to \(b\) can be evaluated by means of the friction chart when the flow rate and equivalent length are known. The velocity in the branch duct can be determined from the area and the known volume of air flowing per

\(^1\) "Pressure Losses Due to Bends and Area Changes in Mine Airways." U. S. Bureau of Mines Information Circular No. 6663, November 1932.
unit of time; consequently the velocity pressure existing at b can be evaluated. This velocity pressure plus the friction-pressure loss from d to b represents the total-pressure loss of the branch duct.

Since the six branch ducts shown in Fig. 8 are of identical size and length and since the turbulent mixing and friction-pressure losses of the plenum as given by Eqs. (5) and (6) are assumed to be zero, each branch duct will deliver the same volume of air, in cfm, and each will have the same total-pressure loss. Therefore the total pressures in all branch ducts at points just outside the plenum will be equal in magnitude when the ducts have the same diameters and equivalent lengths. These total pressures depend on the actual equivalent lengths of the branch ducts but not on the losses occurring within the box-plenum. Consequently, for the purposes of this analysis it is only necessary to consider the losses from Section o to Section d.

14. Analysis of the Total Loss of the Box-Plenum

The total loss \( L_t \), in. of water, of the box-plenum between Section o and Section d is

\[
L_t = H_o + H_t + H_b + H_e
\]  

which upon substituting Eqs. (4), (5), (6), and (12) becomes

\[
L_t = 0.34(VP)_o + 0.34(VP)_n
\]  

Since the air flow rates in the six identical branch ducts are equal, \((VP)_o\) and \((VP)_n\) can be related as follows:

\[
Q_o = 6Q_n
\]  

in which

- \(Q_o\) = flow rate at Section o, cfm
- \(Q_n\) = flow rate in any branch duct, cfm

Moreover,

\[
Q_o = A_oV_o
\]

and

\[
Q_n = A_nV_n
\]

A relationship in terms of areas and velocities is

\[
A_oV_o = 6A_nV_n
\]

or in terms of velocity heads

\[
A_o^2(VP)_o = (6A_n)^2(VP)_n
\]

which when rearranged becomes

\[
(VP)_o = \left(\frac{6A_n}{A_o}\right)^2(VP)_n
\]
Equation (20) can be generalized to any number of identical branch ducts by expressing Eq. (15) as

\[ Q_o = N Q_n \]  \hspace{1cm} (21)

in which \( N \) = number of branch ducts. Equation (20) then becomes

\[ (VP)_o = \left( \frac{N A_n}{A_o} \right)^2 (VP)_n \]  \hspace{1cm} (22)

Since the term \( (N A_n) \) represents the summation of the areas of the branch ducts, it can be expressed, by replacing \( N \) with sigma, as \( \Sigma A_n \).

Equation (22) then becomes

\[ VP_o = \left( \frac{A_1 + A_2 + A_3 + \ldots}{A_o} \right)^2 (VP)_n = \left( \frac{\Sigma A_n}{A_o} \right)^2 (VP)_n \]  \hspace{1cm} (23)

Substituting the right hand member of Eq. (23) into Eq. (14),

\[ L_t = 0.34 \left( \frac{\Sigma A_n}{A_o} \right)^2 (VP)_n + 0.34 (VP)_n \]  \hspace{1cm} (24)

which expresses the total loss with any given number of identical branch ducts as a function of the velocity head existing in any one of the branch ducts. The value of any total loss, \( L_t \), given by Eq. (24) is necessarily associated with a single air flow rate or velocity in a branch. However, this loss can be more conveniently expressed as the number of branch duct velocity heads lost by dividing the total loss, \( L_t \), in inches of water, by the branch duct velocity head, also in inches of water. This number, being dimensionless, is then independent of any flow rate or velocity. The desired dimensionless expression is therefore

\[ \frac{L_t}{(VP)_n} = 0.34 \left( \frac{\Sigma A_n}{A_o} \right)^2 + 0.34 \]  \hspace{1cm} (25)

Equation (25) gives a means of predicting the total loss of the plenum arrangement shown in Fig. 8 for any number of identical branch ducts in terms of the velocity head existing in any one of the branch ducts.

An expression equivalent to Eq. (25) is

\[ \frac{L_t}{(VP)_o} = 0.34 + 0.34 \left( \frac{A_o}{\Sigma A_n} \right)^2 \]  \hspace{1cm} (26)

in which the total loss is expressed in terms of the velocity head existing in the trunk duct. Equations (25) and (26), plotted on Figs. 9 and 10 respectively, can be used for comparison with the actual results obtained from the arrangement shown in Fig. 2.

Comparison of Eqs. (4) and (12) shows that the predicted expansion and contraction losses will be equal when the velocities at the inlet (Section o) and outlet (Section d) are equal. However, since it is
common practice to design air duct systems so that the velocities decrease progressively from the conditioner to the register, the velocity at \( o \) will normally exceed that at \( n \). For practical applications, therefore, the expansion loss at the plenum entrance will be larger than the contraction loss at the outlets; hence, initial efforts to reduce the loss of this box-plenum should be directed toward reducing the loss occurring at the plenum entrance.

In this theoretical analysis of the losses, it was assumed that no turbulent mixing loss occurred within the plenum, an assumption which was known to be invalid. Consequently, the results of the investigation were expected to show actual losses of larger magnitude than those predicted by Eqs. (25) and (26).

The several losses in the box-plenums studied have been designated by the symbols and nomenclature shown in Fig. 11. Since later studies indicated that it was not feasible to evaluate separately the entrance and turbulent mixing losses, which are mutually dependent, the sum of these two losses has been designated as the plenum loss, \( L_p \). The contraction loss occurring as the air flows from the plenum into each branch duct has been designated as the outlet loss, \( L_n \). The sum of the plenum loss, \( L_p \), and the outlet loss, \( L_n \), has been designated as

\[
\frac{L_f}{(VP)_p} = 0.34 \left( \frac{\Sigma A_n}{A_o} \right)^2 + 0.34
\]

Figure 9. Theoretical Total Loss in Terms of Branch Duct Velocity Heads
Figure 10. Theoretical Loss in Terms of Trunk Duct Velocity Heads
the total loss, $L_t$. Station 2 represents the initial reference point of the box-plenum being investigated. Since duct losses in general can be determined by methods in common use, the losses of the branch ducts were considered to be incidental to the main objectives of the investigation. The total loss $L_t$, plenum loss $L_p$, and outlet loss $L_n$, being of primary importance, were evaluated for numerous arrangements of the laboratory apparatus.
IV. PROCEDURE

15. General Procedure

For any given box-plenum arrangement, the total air flow rate was varied in about six equal increments to provide a range of from 100 to 350 cfm per branch duct. Before each experiment, calculations were made of the gage readings corresponding to the respective total flow rates desired. Each of these flow rates was then obtained by adjusting the fan speed until the desired value of the static pressure at Station 0 was established. The static pressures and velocity pressures at the several stations, fan speed, and power input to the motor were recorded. A representative data sheet is shown in Appendix D. When the number of branch ducts in use was varied, those not in use were sealed at the ends.

During many of the experiments in Series 1, 2, and 3, the box-plenum was first "balanced" so that equal flow rates existed in all 7-in.-diam branch ducts. This balanced condition was obtained by adjusting the dampers at the ends of the branch ducts until all ducts discharged 175 cfm, a value selected because it was the mean flow rate normally used in practice with 7-in.-diam branch ducts. The total flow rate was then varied from 100 to 350 cfm-per branch in about six equal increments without further adjustment of the dampers.

The four basic variations of experimental conditions for Series 1, 2, and 3 are defined as follows:

Basic 1—All dampers removed. The equivalent length of each 7-in.-diam branch duct was then 54 ft at a flow rate of 175 cfm. If the flow rates of the branch ducts were unequal, the box-plenum was considered to be "unbalanced."

Basic 2—Dampers adjusted to obtain equal or "balanced" flow rates in all branch ducts. This is a modification of Basic 1 with minimum dampering in which at least one branch duct is not dampered.

Basic 3—All dampers adjusted to increase the equivalent length of each branch duct by 46 ft. The equivalent length of each 7-in.-diam duct was then 100 ft at a flow rate of 175 cfm. This again represents an unbalanced condition.

Basic 4—Dampers adjusted to obtain equal flow rates in all branch ducts. This is a modification of Basic 3 with minimum dampering in which at least one branch duct is maintained as in Basic 3.

16. Description of Studies

The studies made with the 9-ft box-plenum having 7-in.-diam branch ducts and the end position of the entrance section (Series 1)
### Table 1

**SERIES 1 STUDIES—9-FT PLENUM, 7-IN.-DIAM BRANCH DUCTS, AND END ENTRANCE SECTIONS**

<table>
<thead>
<tr>
<th>Group</th>
<th>Fan Position</th>
<th>Entrance Section*</th>
<th>Experimental Conditions</th>
<th>Number of Branch Ducts in Use</th>
<th>Remarks†</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>Special</td>
<td>Special</td>
<td>Calibration</td>
<td>One</td>
<td>Calibration of orifice box and branch duct orifices</td>
</tr>
<tr>
<td>IB</td>
<td>Remote</td>
<td>Type 1</td>
<td>Basic 1, 2, 3, 4</td>
<td>Six</td>
<td>Rotation occurred. Duplicate studies required for C and CC flow</td>
</tr>
<tr>
<td>IC</td>
<td>Close-Coupled</td>
<td>Type 1</td>
<td>Basic 1, 2</td>
<td>Six</td>
<td>Rotation occurred. Duplicate studies required for C and CC flow</td>
</tr>
<tr>
<td>ID</td>
<td>Remote</td>
<td>Type 2</td>
<td>Basic 1, 2, 3, 4</td>
<td>Six</td>
<td>Rotation occurred. Duplicate studies required for C and CC flow</td>
</tr>
<tr>
<td>IE</td>
<td>Remote</td>
<td>Type 3</td>
<td>Basic 1, 2, 3, 4</td>
<td>Six</td>
<td>No rotation</td>
</tr>
<tr>
<td>IF</td>
<td>Remote</td>
<td>Type 4</td>
<td>Basic 1, 2, 3, 4</td>
<td>Six</td>
<td>Rotation occurred. Duplicate studies required for C and CC flow</td>
</tr>
<tr>
<td>IG</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2, 3, 4</td>
<td>Six</td>
<td>No rotation</td>
</tr>
<tr>
<td>IH</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Six</td>
<td>No rotation. Branch duct 1 located 7&quot; from upstream corner of plenum</td>
</tr>
<tr>
<td>IJ</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Six</td>
<td>No rotation. Branch duct 1 located 10.5&quot; from upstream corner of plenum</td>
</tr>
<tr>
<td>IK</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Six</td>
<td>No rotation. Branch duct 1 located 11&quot; from upstream corner of plenum</td>
</tr>
<tr>
<td>IL</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Six</td>
<td>No rotation. Branch duct 1 located 21&quot; from upstream corner of plenum</td>
</tr>
<tr>
<td>IM</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>No rotation. Three separate piezometer rings formed by Sta. 3, 4, 11, and 12; Sta. 5 and 10; and Sta. 6, 7, 8, and 9</td>
</tr>
<tr>
<td>IN</td>
<td>Remote</td>
<td>Type 5</td>
<td>Special</td>
<td>Six</td>
<td>No rotation. Piezometer ring formed by Sta. 6, 7, 8, and 9</td>
</tr>
<tr>
<td>IO</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1</td>
<td>Varied</td>
<td>No rotation</td>
</tr>
<tr>
<td>IP</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1</td>
<td>Varied</td>
<td>No rotation</td>
</tr>
<tr>
<td>IQ</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1 and Special</td>
<td>Varied</td>
<td>No rotation. Piezometer ring formed by Sta. 6, 7, 8, and 9</td>
</tr>
</tbody>
</table>

*See Fig. 3 for description.

†Rotation—During several studies, the entering air flowed down one side and back the other side of the plenum instead of expanding and completely filling the plenum section near the entrance. This condition was designated as Clockwise (C) or Counterclockwise (CC) Rotation, depending upon the direction of the air flow around the plenum as viewed from above. For example, when CC rotation occurred, the air stream from the entering duct supplied air to branch ducts 1, 2, 3, 4, and 5 in order.

The studies made with the 3-ft box-plenum are shown in Table 2 for the end entrance section (Series 2) and in Table 3 for the bottom entrance section (Series 3). The studies made with the 9-ft box-plenum having 7-, 8- and 9-in.-diam branch ducts (Series 4) are in Table 4.
### Table 2

**Series 2 Studies—3-ft Plenum, 7-in.-diam Branch Ducts, and End Entrance Sections**

<table>
<thead>
<tr>
<th>Group</th>
<th>Fan Position</th>
<th>Entrance Section*</th>
<th>Experimental Conditions</th>
<th>Number of Branch Ducts in Use</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>Remote</td>
<td>Type 1</td>
<td>Calibration</td>
<td>One</td>
<td>Calibration of branch duct orifices</td>
</tr>
<tr>
<td>2B</td>
<td>Remote</td>
<td>Type 1</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Apparatus arranged as shown in Fig. 4</td>
</tr>
<tr>
<td>2C</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Branch ducts located as shown in Fig. 4</td>
</tr>
<tr>
<td>2D</td>
<td>Remote</td>
<td>Type 5</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Branch ducts 1, 3, and 5 located as shown in Fig. 4. Branch ducts Nos. 2, and 4 located at end of plenum on either side of branch duct 3</td>
</tr>
</tbody>
</table>

*See Fig. 3 for description.

### Table 3

**Series 3 Studies—3-ft Plenum, 7-in.-diam Branch Ducts, and Bottom Entrance Sections**

<table>
<thead>
<tr>
<th>Group</th>
<th>Fan Position</th>
<th>Entrance Section*</th>
<th>Experimental Conditions</th>
<th>Number of Branch Ducts in Use</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A</td>
<td>Remote</td>
<td>Type 6</td>
<td>Calibration</td>
<td>One</td>
<td>Calibration of branch duct orifices</td>
</tr>
<tr>
<td>3B</td>
<td>Remote</td>
<td>Type 6</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Apparatus arranged as shown in Fig. 5</td>
</tr>
<tr>
<td>3C</td>
<td>Remote</td>
<td>Type 7</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Branch ducts located as shown in Fig. 5</td>
</tr>
<tr>
<td>3D</td>
<td>Remote</td>
<td>Type 8</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Branch ducts located as shown in Fig. 5</td>
</tr>
<tr>
<td>3E</td>
<td>Remote</td>
<td>Type 9</td>
<td>Basic 1, 2</td>
<td>Varied</td>
<td>Eight branch ducts in a symmetrical pattern of two branch ducts per plenum side</td>
</tr>
</tbody>
</table>

*See Fig. 6 for description.

### Table 4

**Series 4 Studies—9-ft Plenum, 7-in., 8-in., and 9-in.-diam Branch Ducts, and Type 5 End Entrance Section**

<table>
<thead>
<tr>
<th>Group</th>
<th>Experimental Conditions</th>
<th>Number of Branch Ducts in Use</th>
<th>Remarks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>4A</td>
<td>Calibration</td>
<td>One</td>
<td>Calibration of branch duct orifices</td>
</tr>
<tr>
<td>4B</td>
<td>Basic 1</td>
<td>Varied</td>
<td>Equivalent length of all branch ducts adjusted to 62 ft</td>
</tr>
<tr>
<td>4C</td>
<td>Basic 2</td>
<td>Varied</td>
<td>Balanced to obtain equal velocities in all branch ducts</td>
</tr>
<tr>
<td>4D</td>
<td>Basic 2</td>
<td>Varied</td>
<td>Balanced to obtain equal total pressures in the upstream ends of all branch ducts</td>
</tr>
<tr>
<td>4E</td>
<td>Basic 1</td>
<td>Eight</td>
<td>Variable equivalent lengths of branch ducts</td>
</tr>
</tbody>
</table>

*See Fig. 7 for arrangement of apparatus.
V. RESULTS OF SERIES 1 STUDIES WITH NINE-FOOT PLENUM, 7-IN.-DIAMETER BRANCH DUCTS, END ENTRANCE SECTIONS

17. Performance with Abrupt Expansion, Type 1, Entrance Section (Groups 1B and 1C)

Since no published data are available for evaluating the turbulent mixing loss with a plenum, the assumption is ordinarily made that the turbulent mixing loss is negligible and that the entire plenum loss is represented by the entrance loss alone. The error introduced by this assumption may lead to gross undersizing of the air duct system.

The results of this investigation show that, with the remote position of the fan, the plenum loss was 1.17 velocity heads based on the velocity in the trunk duct for the unbalanced flow conditions (Basic 1) and that it was increased to 1.77 velocity heads based on the trunk duct velocity for the balanced flow condition (Basic 2). This plenum loss under balanced conditions, which includes both the entrance and the turbulent mixing losses, is more than five times the normally accepted value for the abrupt expansion entrance loss of 0.34 velocity heads shown in Eq. (4). It should be noted that the 0.34 value represents an abrupt expansion loss when both the upstream and downstream flow patterns are uniform. However, in the box-plenum, the downstream flow pattern was not uniform and in some regions indicated a reverse flow. Therefore the 1.77 value for the combined entrance and turbulent mixing losses in the plenum could not be evaluated separately without tedious and complicated pressure traverses whose accuracy would be highly questionable. Although the difference between 1.77 and 0.34 velocity heads does not truly represent the turbulent mixing loss, the magnitude of this difference indicates that the mixing loss is appreciable.

The performance of the box-plenum with this abrupt-expansion entrance section was adversely affected by a flow pattern phenomenon that occurred with the fan in both the remote and the close-coupled position. Instead of expanding and uniformly filling the plenum section near the entrance, the entering air flowed down one side of the plenum and back the other side in a rotational motion. The particular direction of rotation which would occur might be either clockwise or counterclockwise; it could not be predicted in advance. The effect of this rotation on the distribution of air to the branch ducts was of considerable importance, because the rate of flow in a given duct depended on the location of this duct with respect to the rotating stream. Moreover, the rotation could be readily changed from clock-
wise to counterclockwise, or vice versa, thus upsetting the former distribution of air to the branch ducts. For example, branch duct 1 delivered more air than duct 5 when counterclockwise rotation occurred. However, when branch 1 was closed off for 5-20 sec, the direction of rotation shifted to clockwise and was stable in that direction, even when No. 1 was reopened. Branch duct 1 then delivered only 80 percent of its former volume.

Because of the unequal distribution of the air to the branch ducts, high resistances had to be imposed upon some of the branch ducts with the dampers in order to obtain equal flow rates which represented “balanced” conditions; consequently the system had a high balancing loss. Moreover, when the system was balanced for a given direction of rotation, it would become unbalanced if the direction of rotation changed.

Plenum losses, the studies indicated, were the same for either direction of rotation. Furthermore, the distribution of air to the branch ducts was “symmetrical”; for example, if branch ducts 1 and 5 (see Fig. 2) delivered 180 and 150 cfm respectively during counterclockwise rotation, they would deliver 150 and 180 cfm respectively during clockwise rotation. The impossibility of developing reliable design data for such unstable conditions is apparent. Furthermore, an abrupt expansion entrance section should not be used in practical applications, since the balance of an air duct system could conceivably be upset either by stopping and restarting the fan or by temporarily closing off a register.

The purpose of the Basic 3 and 4 experimental conditions was to determine the effect upon the pressure loss and air flow characteristics of the box-plenum when equal resistance is added to each branch duct. Comparison of the results with those obtained from the Basic 1 and 2 experimental conditions showed that the total pressure at Station 2 was higher by an amount equivalent to the resistance added. However, the calculated total losses of the box-plenum were the same for the Basic 1 and 3 conditions, and also for the Basic 2 and 4 conditions. It was therefore concluded that the total loss of the plenum was unaffected by the addition of equal resistances to each branch duct. Also, the added resistance had no perceptible effect upon the air flow characteristics, since the flow rates in the individual branch duct were comparable, and rotational flow occurred in the plenum.

To investigate the effect of the fan discharging directly into the plenum, studies were made with the fan close-coupled to the box-plenum. The experimental conditions used previously with the remote
fan position were duplicated to provide direct comparison of results. The total loss of the box plenum with the fan in the close-coupled position was slightly greater than the corresponding loss with the fan in the remote position. For all practical purposes, however, the losses can be considered the same for either arrangement. Furthermore, the distribution of the air to the outlets was the same, and rotational flow occurred during both experimental conditions.

18. Comparative Performance of Five End Entrance Sections
   (Groups 1B, 1D, 1E, 1F, and 1G)

   Since the use of the abrupt expansion entrance section (Type 1) resulted in high losses and unstable flow conditions, four additional entrance sections (Fig. 3) were developed and studied. The plenum
losses, shown in Fig. 12, were obtained for the Basic 1 and Basic 2 experimental conditions with all six branch ducts in use. Rotation occurred with Types 1, 2, and 4. As previously discussed, the use of any entrance section which results in rotational flow should be avoided.

The effect of adding equal resistances to each branch duct was also investigated with Types 2, 3, 4, and 5. Results indicated that the plenum loss was not affected, a conclusion which agreed with that obtained with the Type 1 section.

The following factors were selected for evaluating the performance of any entrance section:

1. The plenum loss under balanced conditions of flow.
2. The additional pressure required to establish balanced conditions of flow.
3. The stability of the box-plenum, or the ability to maintain fixed flow rates.
4. The space requirements of the entrance section external to the plenum.

The type 5 entrance section was superior to the other types in all these respects. Its favorable characteristics are as follows:

1. The plenum loss under balanced conditions of flow was only 51 percent of that for the Type 1 entrance section.
2. The additional pressure required to obtain balanced conditions of flow was negligible.
3. The stability of the box-plenum was excellent, since no rotational flow occurred or could be made to occur.
4. The space requirement was a minimum, since the entrance section occupied no space external to the plenum.
5. The entrance section was easily fabricated and installed.

Because of these favorable characteristics, further studies were made of this type of entrance section under a wide variety of experimental conditions.

19. Effect of Branch Duct Location (Groups 1H, 1I, 1J, 1K, and 1L)

Studies were made with the Type 5 entrance section to investigate the effect of changing the location of one or more of the branch ducts. Results are presented in Fig. 13. The total loss of the box-plenum increased considerably when branch duct 1 was located closer than approximately 12 in. from the upstream corner of the box-plenum. Therefore, if excessive pressure losses are to be avoided, the centerline of any branch duct at the point of takeoff from the plenum should
be 12 in. or more from the upstream corners of the plenum. An additional group of experiments was conducted with both branch ducts 1 and 5 located 7 in. from the corner of the plenum. As shown by point A in Fig. 13, the total loss was less under these conditions. This indicates that a symmetrical location of branch ducts is desirable when a distance \( D \) less than 12 in. must be used. For a distance \( D \) greater than 12 in., the performance of the plenum was not affected by the relative locations of the branch ducts.

20. Equations for Total Loss (Groups 1M, 1O, 1P, and 1Q)

To determine the plenum performance under more general conditions of operation, the effect of varying the number of branch ducts in use was investigated. The relationships of the static pressures and air flow rates at Station 2 are shown in Fig. 14. Since, for any given number of branch ducts in use, numerous combinations were possible, several representative combinations were studied for each condition. The static pressure requirements were independent of the particular branch ducts in use in any one combination; for example, in Fig. 14 the curve for three branch ducts in use applies to all combinations of three ducts,
such as branch ducts 1, 2, and 3 and branch ducts 4, 5, and 6. From the curves of Fig. 14 and the equations developed in Appendix E, the curve of Fig. 15 was derived. The plotted points, as compared with the curve, represent deviations of less than three percent in the data.

![Figure 14: Variation of Static Pressure and Air Flow Rate at Station 2 with Number of Branch Ducts in Use](image)

The equation of this curve, which represents the unbalanced flow condition, can also be expressed as

\[ L_t = 0.80(VP)_o + 0.45(VP)_n \]  

in which

- \( L_t \) = total loss of the box-plenum, in. of water.
- \((VP)_o\) = velocity head based on the velocity at Station 2, in. of water.
The total loss, therefore, can be expressed as the sum of a function of the velocity head in the trunk duct and a function of the velocity head in the branch duct. The term \(0.80(VP)_n\) represents the plenum loss, \(L_p\); the term \(0.45(VP)_n\) represents the outlet loss, \(L_n\). The plenum loss includes the entrance and turbulence losses; however, as previously stated, these two losses depend on each other and it is not feasible to separate them.

For the balanced flow condition an equation similar in form to Eq. (27) was derived. In this connection, the balancing loss with the Type 5 entrance section (see Fig. 12) was extremely small when all six branch ducts were in use. Moreover, as the number of branch ducts in use was reduced, the balancing loss became progressively less; in fact, it was necessarily zero when only one branch duct was in use.
Therefore, with all six in use, the data obtained were used to derive the following equation of total loss for the balanced flow condition:

\[ L_t = 0.90(VP)_o + 0.45(VP)_n \]  \hspace{1cm} (28)

Equation (28) applies specifically to the case in which all branch ducts are located more than 12 in. from the upstream corners of the box-plenum.

The relations expressed in Eq. (27) and Eq. (28) can be used to develop a general equation for the total loss of a box-plenum:

\[ L_t = K_o(VP)_o + K_n(VP)_n \]  \hspace{1cm} (29)

in which

\( L_t \) = total loss of any box-plenum, in. of water.

\( K_o \) = a constant, the value of which depends on factors such as the design of the entrance section, amount of turbulence within the plenum, and the physical dimensions of the plenum.

\( K_n \) = a constant, the value of which depends upon the performance of the plenum takeoff fitting.

\( (VP)_o \) = velocity head based on the velocity in the trunk duct, in. of water.

\( (VP)_n \) = velocity head based on the average branch duct velocity, in. of water.

The value of \( K_n \) for the branch ducts used in this investigation was 0.45 (see Appendix C). For the five entrance sections studied, the values of the plenum losses for balanced flow conditions shown in Fig. 12 are also values of \( K_o \). Therefore the equations of the total loss of the plenum for the five types of entrance sections and for balanced flow conditions are as follows:

Type 1 \[ L_t = 1.77(VP)_o + 0.45(VP)_n \]  \hspace{1cm} (30)
Type 2 \[ L_t = 1.42(VP)_o + 0.45(VP)_n \]  \hspace{1cm} (31)
Type 3 \[ L_t = 1.18(VP)_o + 0.45(VP)_n \]  \hspace{1cm} (32)
Type 4 \[ L_t = 1.47(VP)_o + 0.45(VP)_n \]  \hspace{1cm} (33)
Type 5 \[ L_t = 0.90(VP)_n + 0.45(VP)_n \]  \hspace{1cm} (28)
VI. RESULTS OF SERIES 2 STUDIES WITH THREE-FOOT PLENUM, 7-IN.-DIAMETER BRANCH DUCTS, END ENTRANCE SECTIONS

21. Performance with Abrupt Expansion, Type 1, Entrance Section (Group 2B)

Box-plenums installed in residential air conditioning systems are normally about 3 ft wide and 1 ft deep, and vary in length from a minimum of 3 ft to a maximum of 9 ft. Since the length is subject to considerable variation, the box-plenum studied in the laboratory was so designed that it could be made 3, 6, or 9 ft long. After the Series 1 studies had been conducted with the 9-ft-long box-plenum, the Series 2 studies then were made with the 3-ft-long box-plenum and either the Type 1 or Type 5 entrance section.

The results of the studies made with the Type 1 entrance section installed as shown in Fig. 4 indicated that the plenum loss was 1.10 velocity heads based on the velocity in the trunk duct for the unbalanced flow condition and that it was increased to 1.60 velocity heads when the system was balanced for equal flow rates in all branch ducts. Since these values of the plenum loss are values of $K_o$ in Eq. (29), the equations of total loss for these experimental conditions are as follows:

\[
L_u = 1.10(VP)_o + 0.45(VP)_n \quad (34)
\]

\[
L_t = 1.60(VP)_o + 0.45(VP)_n \quad (35)
\]

During these studies the silk directional indicators showed a definite flow pattern within the plenum. The supply air stream retained its shape until it reached the end of the plenum, and then split into two streams that traveled back along the sides of the plenum toward the entrance, as shown in Fig. 16. During the unbalanced flow conditions, this flow pattern resulted in increased air flow from the branch ducts which were connected to the plenum at locations farthest away from the entrance. For example, with all six branch ducts in use, each of branch ducts 1, 5, and 6 delivered 14.8 percent of the total air quantity flowing, each of outlets 2 and 4 delivered 17.5 percent, and outlet 3 delivered 20.6 percent.

To investigate the effect of branch duct location a number of experiments were conducted with various combinations of five branch ducts in use. The results of the study conducted with branch duct 5 in use and branch duct 1 sealed off at the damper end indicated that high resistances had to be imposed on several of the other branch ducts to obtain equal air flow rates. The plenum loss under balanced conditions of flow was much higher than the corresponding loss as determined
from the studies made with both branch ducts 1 and 5 in use, and either branch duct 2, 3, 4, or 6 sealed off at the damper. Therefore the branch ducts on both sides of the plenum closest to the entrance should be located directly across from each other—for example, the relative locations of branch ducts 1 and 5 (Fig. 4). It was also observed that

a slight instability of the air flow in the box-plenum occurred when branch duct 1 was not in use; the flow pattern tended to shift and upset the balance of flow, a condition that would be undesirable in practice. Equation (35) applies specifically to the case in which both branch ducts 1 and 5 were in use.

22. Performance with Internal Expansion, Type 5, Entrance Section (Groups 2C and 2D)

The results of the studies made with the Type 5 entrance section installed at Station 2 in the arrangement shown in Fig. 4 indicated that the plenum loss was 0.85 velocity heads based on the velocity
in the trunk duct for the unbalanced flow condition and that it was increased to 1.10 velocity heads when the plenum was balanced. As with the 9-ft box-plenum, the use of the Type 5 entrance section in the 3-ft box-plenum resulted in a significant decrease in the plenum loss. The equations of total loss are as follows:

Unbalanced Flow— \[ L_t = 0.85(VP)_u + 0.45(VP)_u \]  
Balanced Flow— \[ L_t = 1.10(VP)_u + 0.45(VP)_u \]

The results with balanced flow conditions are shown graphically in Fig. 17. Points A and B represent the loss with four and five branch
ducts in use, respectively, when the branch ducts in use did not include branch duct 3. The open-circle points and the curve were obtained with branch duct 3 in use. Thus the loss is increased somewhat if the branch duct located at the end of the box-plenum is not in use. In order to investigate this effect further, branch ducts 2 and 4 were moved to the end of the plenum, one on each side of branch duct 3, and studies were made with six, five, and four branch ducts in use. Though results indicated that minimum loss was obtained when one branch duct was located in the center of the end of the box-plenum, no appreciable difference was observed with three branch ducts located at the end of the box-plenum. Equation (37) applies specifically to the case in which branch duct 3 is in use.

23. *Correlation with Results of Series 1*

Though the losses of the 9-ft and 3-ft box-plenums with the Type 1 entrance section installed are not exactly comparable because two distinct types of flow patterns occurred, comparison of Eq. (30) for the 9-ft plenum with Eq. (35) for the 3-ft plenum shows good agreement for the balanced flow condition. The higher loss for the 9-ft plenum might be attributable to the increased frictional resistance of the longer plenum.

With the Type 5 entrance section installed at Station 2, the total loss under balanced conditions of flow was slightly higher for the 3-ft plenum than for the 9-ft plenum. As indicated by Eqs. (37) and (28), the plenum loss of the 3-ft plenum is \(0.20(VP)\) higher than the plenum loss of the 9-ft plenum. This higher loss is probably due to the fact that the centerlines of branch ducts 1 and 5 were located only 9 in. from the upstream corners of the 3-ft plenum. Since, as shown in Fig 13, studies made with the 9-ft plenum indicated that an increased loss should be expected if the branch ducts are located less than 12 in. from the upstream corners of the plenum, Eq. (37) represents good correlation with the previous results. For all practical purposes the plenum loss can be considered as one velocity head based on the velocity in the trunk duct, \(1.0(VP)\), provided that the centerline of any branch duct at the point of takeoff is not located within about 12 in. of the upstream corners of the box-plenum. This plenum loss is also applicable to any box-plenum length between 3 ft and 9 ft. Since Eqs. (28) and (37) were in good agreement, it was considered unnecessary to investigate the 6-ft box-plenum.
VII. RESULTS OF SERIES 3 STUDIES WITH THREE-FOOT PLENUM, 7-IN.-DIAMETER BRANCH DUCTS, BOTTOM ENTRANCE SECTIONS

24. Performance with Abrupt Expansion, Type 6, Entrance Section (Group 3B)

In conventional forced warm-air systems, the warm-air bonnet is directly attached to the outlet from the furnace and is relatively the same size. The pressure losses that occur as the air enters the bonnet are therefore small, and are normally assigned to the pressure loss of the unit. However, a number of other applications are encountered in which the trunk duct from the conditioner is connected to the bottom of a box-plenum and in which the velocities, and usually the losses, are high. Consequently, to obtain performance data for this type of application, the Series 3 studies were made with the supply air entering the bottom of the 3-ft box-plenum. The initial studies were made with the arrangement shown in Fig. 5; the results obtained under balanced conditions of flow are shown in Fig. 18. The multiplicity of points for a given number of branch ducts in use shows that the loss depended on the particular branch ducts in use. For example, when different combinations of four branch ducts in use were studied, the total loss varied from 2.44 to 3.15 velocity heads based on the velocity in the branch ducts. A study was made to determine whether the loss for a given number of branch ducts in use was a function of the number of sides of the box-plenum "in use." For example (Fig. 5), when branch ducts 1, 3, 4, and 6 were in use, all four sides of the plenum were considered to be in use; in contrast, when branch ducts 1, 2, 4, and 5 were in use, only two sides of the plenum were considered to be in use. This study indicated that correlation of the results with the number of sides in use was not possible. The curve shown in Fig. 18 is therefore an approximation of the loss that would occur in a practical installation of a box-plenum having a bottom entrance section.

The equation of this curve can be expressed as

\[ L_t = 2.50(VP)_o + 0.45(VP)_n \]  

(38)

With respect to the flow pattern, the arrows of Fig. 19 show the general direction of air movement inside the plenum as viewed from the plenum side. Although this flow pattern was essentially the same for all of the Series 3 studies, some instability was evidenced by the erratic fluctuations of the gage fluid in the inclined draft gages.

Because the losses depend on the particular branch ducts in use and unstable flow conditions were experienced, it was not possible to
Series 3—Basic 2 Studies
Type 6 Bottom Entrance Section

\[ \frac{L_I}{(VP)_{kr}} = 2.50 \left( \frac{\sum A_0}{A_0} \right) + 0.45 \]

**Figure 18. Total Loss of 3-ft Box-Plenum with Type 6 Bottom Entrance Section (Series 3)**
develop acceptable design information for a box-plenum system incorporating this abrupt expansion entrance; furthermore, Eq. (38) indicates that the plenum loss is excessively large. Therefore this type of entrance should be avoided in practice.

25. Performance with Cone Diffuser, Type 7, Entrance Section (Group 3C)

In an attempt to reduce the high losses experienced with the abrupt expansion entrance section, the Type 7 entrance section shown in Fig. 6b was installed. Studies made with four, five, and six branch ducts in use indicated that no improvement was obtained; in fact, the plenum loss was increased from 2.50\((VP)_o\) to about 3.00\((VP)_o\). Therefore no further studies were made of this entrance section.

26. Performance with Internal Diffuser, Type 8, Entrance Section (Group 3D)

In a second attempt to reduce the losses obtained with the abrupt expansion entrance section, the internal diffuser entrance section shown in Fig. 6c was installed. The results of studies conducted with this entrance section showed that the plenum loss was increased from 2.50\((VP)_o\) to about 5.00\((VP)_o\). Therefore this entrance section was dismissed from further consideration.
27. Performance with Divergent, Type 9, Entrance Section (Group 3E)

Since the losses of the box-plenum were excessively high with the first three types of entrance sections, the divergent entrance section shown in Fig. 6d was installed and a study conducted. This section occupied 18 in. of space external to the box-plenum, whereas the other

![Figure 20. Total Loss of 3-ft Box-Plenum with Type 9 Bottom Entrance Section (Series 3)](image-url)
three types were located entirely within the plenum. The results for the balanced flow conditions are shown in Fig. 20. The equation of the curve can be expressed as

$$L_t = 1.20(VP)_o + 0.45(VP)_n$$

(39)

It may be noted that the loss still depends on the particular branch ducts in use. However, the deviations of the plotted points from the curve are much less than those for the abrupt expansion section illustrated in Fig. 18. Moreover, comparison of Eqs. (38) and (39) shows that the divergent entrance section has significantly decreased the total loss of the box-plenum.

Comparison of Eq. (39) with Eqs. (28) and (37) shows that the total loss of the box-plenum with this type of bottom entrance section is only slightly greater than the total losses obtained with the best type of end entrance section.
VIII. RESULTS OF SERIES 4 STUDIES WITH NINE-FOOT PLENUM; 7-IN., 8-IN., AND 9-IN.-DIAMETER BRANCH DUCTS; END ENTRANCE SECTIONS

28. Performance with Various Sizes of Branch Ducts Having Equal Equivalent Lengths (Groups 4B, 4C, and 4D)

To develop design data for installations of box-plenum systems it was necessary to investigate the validity of Eqs. (28) and (37) when applied to practical systems having branch ducts of various diameters and resistances. Consequently, a box-plenum arrangement was assembled as shown in Fig. 7 using 7-in., 8-in., and 9-in.-diam branch ducts so located as to conform with installations normally used in practice.

Preliminary studies were conducted with the resistance of each branch duct adjusted to represent 62 ft of equivalent length—an unbalanced condition of flow. Results are shown as plotted points in

![Figure 21. Total Loss of Box-Pleum for Unbalanced Conditions of Flow (Series 4)](image-url)
Fig. 21. Superimposed upon this graph is the curve of Eq. (27) obtained from the Series 1 studies conducted with 7-in.-diam branch ducts only. The plotted points show that Eq. (27) is a conservative expression for the total loss under unbalanced conditions of flow.

In order to obtain balanced conditions of flow, it was necessary to decide what constituted a "balanced" condition for a box-plenum having different sizes of branch ducts. One method of "balancing" the box-plenum was to establish equal velocities in all branch ducts. For this purpose the velocity selected was 655 fpm, which corresponds to an air flow rate of 175 cfm in a 7-in.-diam branch duct. On this basis, the air flow rates required were 175, 228, and 290 cfm for the 7-in., 8-in., and 9-in.-diam branch ducts respectively. The results of the studies made under these conditions are shown as open circles and

Figure 22. Total Loss of Box-Plenum for Balanced Conditions of Flow (Series 4)
the solid curve in Fig. 22. Superimposed upon this graph is the curve of Eq. (28) obtained from the Series 1 studies conducted with 7-in.-diam branch ducts and balanced conditions of flow. A comparison of the two curves indicates that for values of $\Sigma A_n/A_o$ larger than about 1.4 the loss of this plenum is less than the loss calculated from Eq. (28). Moreover, in the range of $\Sigma A_n/A_o$ between 1.4 and 1.2 the actual loss of this plenum is only slightly larger than the loss calculated from Eq. (28). Since in air duct systems the branch duct velocities are normally less than the velocities leaving the conditioner or in the trunk duct, box-plenum systems will ordinarily be designed with the values of $\Sigma A_n/A_o$ greater than about 1.2.

Another method of "balancing" the box plenum was to assume that the air flow rate in each branch duct should be of such magnitude that, for the particular equivalent lengths involved, the total-pressure loss from the plenum to the atmosphere would be equal for all branch ducts. On this basis, the calculated air flow rates required were 175, 241, and 318 cfm for the 7-in., 8-in., and 9-in.-diam branch ducts respectively. The results of the studies made under these conditions are shown as crosses in Fig. 22. Comparison of these points with the open circles indicates that the total loss is essentially the same for both methods of balancing the system.

29. Performance with Various Sizes of Branch Ducts Having Various Equivalent Lengths (Group 4E)

The studies discussed in the preceding section were conducted with each branch duct having an equivalent length of 62 ft. To confirm the validity of Eqs. (28) and (37) for the design of practical box-plenum systems, studies were made with the arrangement shown in Fig. 7 and with the dampers adjusted to obtain the arbitrarily selected equivalent lengths listed in column C of Table 5. Assuming a total pressure in the box-plenum of 0.15 in. of water and utilizing the second method discussed above for determining balanced conditions of flow, the required static pressures and predicted volumes were calculated from the equation

$$ (TP)_{\text{plenum}} = (SP)_{n1} + (VP)_{n1} + 0.45(VP)_{n1} \quad (40) $$

For a given size of branch duct, the velocity pressure at Station $n1$ is dependent upon the air flow rate, while the static pressure depends on both the air flow rate and the equivalent length. By trial, a flow rate was found such that the static pressure as determined from the friction chart, and the corresponding velocity pressure, satisfied Eq. (40) when $(TP)_{\text{plenum}}$ equalled 0.15 in. of water. Values of the static
pressure and flow rates for the several branch ducts are given in columns D and E of Table 5. The calculated flow rates in column E represent balanced conditions of flow. Column F shows the actual flow rate as measured for each branch duct when the dampers were adjusted for the required static pressures at Station n1.

<table>
<thead>
<tr>
<th>Branch Duct No.</th>
<th>Branch Duct Diam, in.</th>
<th>Selected Equivalent Length, ft</th>
<th>Required Static Pressure at Station n1, in. of Water</th>
<th>Calculated Air Flow Rate, cfm</th>
<th>Measured Air Flow Rate, cfm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>62</td>
<td>0.0830</td>
<td>375</td>
<td>369</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>125</td>
<td>0.1125</td>
<td>225</td>
<td>228</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>150</td>
<td>0.1138</td>
<td>280</td>
<td>285</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>125</td>
<td>0.1125</td>
<td>225</td>
<td>232</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>125</td>
<td>0.1180</td>
<td>150</td>
<td>163</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>150</td>
<td>0.1223</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>100</td>
<td>0.1146</td>
<td>167</td>
<td>165</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>62</td>
<td>0.0950</td>
<td>205</td>
<td>202</td>
</tr>
</tbody>
</table>

Since the calculated flow rates of column E and the measured flow rates of column F are in good agreement, the results of this study represent both balanced and unbalanced conditions of flow. The total loss for this arbitrary box-plenum arrangement is plotted as point A in Figs. 21 and 22. Comparison of this total loss with the corresponding total loss from Eq. (28) indicates that box-plenum systems having branch ducts of various sizes and equivalent lengths can be conservatively designed in practice by the use of Eq. (28).

30. Total Loss Equation for Practical Box-Plenum Systems

The results of the Series 4 studies showed that Eq. (28) was applicable to practical systems utilizing a box-plenum about 9 ft long. Since this equation was in good agreement with Eq. (37), which applied to the 3-ft-long box-plenum, the mean value of the two can be used to obtain the following equation:

\[ L_t = 1.00 (VP)_o + 0.45 (VP)_n \]  

This equation is applicable to practical box-plenum duct systems of about the same range of proportionate dimensions as those studied, provided that the supply air is introduced into the box-plenum at one end and through a Type 5 entrance section and that the centerlines of all branch ducts at the point of takeoff from the plenum are at least 12 in. from the upstream corners of the box-plenum. Layout details for fabricating Type 5 entrance sections were shown in Fig. 3b.
31. **General Conclusions**

The following general conclusions may be drawn from the results of this investigation.

1. The total-pressure loss associated with a box-plenum is composed of an entrance, a turbulence, and an outlet loss. A separate evaluation of entrance and turbulence losses is not feasible.

2. The loss due to turbulent mixing within a box-plenum having several branch ducts attached is seemingly large, for the plenum loss, which includes the entrance and turbulence losses, is much higher than the theoretical entrance loss alone.

3. Unless the entrance section of a box-plenum is carefully designed, large pressure losses and unstable distribution of air to the branch ducts will occur.

4. The pressure loss and air flow pattern within a box-plenum are not affected by the addition of equal resistance to each branch duct.

5. For all practical purposes the total-pressure losses are the same for the remote and close-coupled positions of the fan; therefore the results will apply to practical applications in which the fan is located very close to the box-plenum.

6. In general, the most “symmetrical” arrangement of a box-plenum duct system will provide the best performance. Examples of a “symmetrical” arrangement are: (a) the inlet duct located at the center of the end of the box-plenum; and (b) the same number and spacing of branch ducts on each side of the box-plenum.

7. Lower pressure losses and better overall performance are obtained when the supply air is introduced into the box-plenum at the end instead of at the bottom; therefore the end inlet condition should be used whenever practicable.

32. **Box-Plenums with End Entrance Sections**

The following conclusions are applicable to box-plenum duct systems in which the supply air enters through the end of the plenum.

1. When the Types 1, 2, and 4 entrance sections were used, rotational flow occurred and the air distribution was unstable; therefore these three types should not be used in practice.

2. If excessive pressure losses are to be avoided, the centerline of any branch duct at the point of takeoff should be at least 12 in. from the upstream corners of the box-plenum.

3. The Type 5 entrance section was superior to the other types from the standpoint of total-pressure loss, pressure required to obtain
balanced conditions of flow, stability of the air flow in the box-plenum, and space requirements; therefore this type of entrance section is recommended for use.

(4) The total loss of box-plenums of about the same range of proportionate dimensions as those studied and having the Type 5 entrance section installed at one end can be obtained from the equation

\[ L_t = 1.00(VP)_o + 0.45(VP)_n \]

in which

- \( L_t \) = total loss, in. of water.
- \( (VP)_o \) = velocity head based on the average velocity in the trunk duct, in. of water.
- \( (VP)_n \) = velocity head based on the average velocity in the branch ducts, in. of water.

This equation applies to all cases in which the centerlines of all branch ducts are located at least 12 in. from the upstream corners of the plenum. When branch ducts must be located closer than 12 in., a correction to the total loss should be applied as indicated in Fig. 13.

33. Box-Plenums with Bottom Entrance Sections

The following conclusions are applicable to box-plenum duct systems in which the supply air enters through the bottom of the plenum.

(1) When the conventional abrupt expansion, Type 6, entrance section was used, the plenum loss was very high and the distribution of the air to the branch ducts was unstable; moreover, the pressure loss varied considerably with the positions of the branch ducts about the plenum. Therefore this type of entrance section should not be used.

(2) The performance of the plenum was not improved by the use of the inverted cone, Type 7, entrance section and was very poor with the internal diffuser, Type 8, entrance section; therefore these two sections also should be avoided.

(3) Whenever a bottom entrance section is to be used, a divergent entrance section similar to the Type 9 is recommended as a means of reducing the pressure loss and improving the performance. An included angle between sides of 7–20 deg and a length of section as long as practicable are recommended.

(4) The total loss of box-plenums of about the same range of proportionate dimensions as that studied and having an entrance section similar to the Type 9 installed in the bottom can be approximated from the equation

\[ L_t = 1.20(VP)_o + 0.45(VP)_n \]
APPENDIX A

CALIBRATION OF THE ORIFICE BOX

The arrangement used for calibrating the orifice box is shown in Fig. 23. The special equipment required for calibration purposes consists of the apparatus downstream of Station 2. After the calibration, the special equipment was removed and replaced by the box-plenum shown in Fig. 2.

To facilitate calibration of the orifice box the air measuring station was first calibrated so that a single reading of the velocity pressure in the center of the duct could be used to determine the mean velocity of flow. For six different air flow rates, the average of the square roots of the velocity pressures obtained by a 20-point traverse was plotted against the corresponding velocity pressure existing at the center of the duct, as shown in Fig. 24. The equation of the curve can be expressed as

$$\sqrt{(VP)_a} = 0.862\sqrt{(VP)_c}$$

in which

$$\sqrt{(VP)_a} = \text{average of the square roots of the velocity pressures in inches of water obtained during the 20-point traverse.}$$

\((VP)_c\) = velocity pressure at the center of the calibration duct, in. of water.

Since Eq. (1) eliminated the need for additional 20-point traverses, the pitot-tube was fixed at the center of the duct during all subsequent calibrations.

The initial calibration for the orifice box was conducted with the 11-in.-diam orifice in use and the other three sealed. The fan speed was maintained approximately constant and the air flow rate was varied in several steps by adjusting the damper located downstream of the air-measuring station. The orifice differential pressure, \(h_o\), was then plotted against the air flow rate in cfm of air at standard density, and a calibration curve for the 11-in.-diam orifice was obtained. The following equations were used to determine the air flow rate referred to standard density:

\[
d_a = \frac{1.327(b - 0.378p_w)}{460 + t}
\]

\[
V_a = 1096.5 \sqrt{\frac{(VP)_a}{d_a}}
\]

**Figure 24. Calibration Curve for Air-Measuring Station**
Air Flow Rate at Orifice Box, $Q_o$, in c.f.m. At Standard Density of 0.075 lb. per cu. ft.

**Figure 25. Calibration Curves for Individual Orifices in Orifice Box**

\[
Q_a = V_a A
\]  
\[
Q_o = Q_a \sqrt{\frac{d_a}{d_s}}
\]

in which

- $d_a = $ density of air at pressure, temperature, and moisture content existing during calibration, lb per cu ft.
- $b = $ barometric pressure corrected for temperature, in. of mercury.
- $p_w = $ partial pressure of water vapor, in. of mercury. This is equal to the product of the relative humidity and the saturation pressure of water vapor at the temperature $t$.
- $t = $ dry-bulb temperature of air, deg F.
- $V_a = $ velocity of air under actual conditions, fpm.
- $\sqrt{(VP)_a} = $ average of the square roots of the velocity pressures in inches of water as determined from Eq. (1).
- $Q_o = $ air flow rate under actual conditions, cfm.
Air Flow Rate at Orifice Box, $Q_o$, in c.f.m. At Standard Density of 0.075 lb per cu ft.

$A =$ area of duct, sq ft. At the measuring station, the internal area was 0.545 sq ft.

$Q_o =$ air flow rate referred to standard density, cfm.

$d_o =$ density of standard air, or air having a density of 0.075 lb per cu ft.

In the same manner, the 8-in., 7-in., and 5-in.-diam orifices were separately calibrated. The calibration curves for each of the four orifices when used individually are shown in Fig. 25.

Calibrations were also conducted with combinations of two, three, and four orifices open; representative calibration curves are shown in Fig. 26.

The equations of all the calibration curves were computed and used, rather than the actual plots, to determine the total air flow rate, $Q_o$, entering the experimental arrangement through the orifice box. These equations are of the form

$$Q_o = C(h_o)^n$$

in which the values of $C$ and $n$ are given in Table 6.
### Table 6

VALUES OF $C$ AND $n$ FOR Eq. (6), APPENDIX A*

<table>
<thead>
<tr>
<th>Diameters of Orifices in Use, in.</th>
<th>$C$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>344</td>
<td>0.500</td>
</tr>
<tr>
<td>7</td>
<td>672</td>
<td>0.500</td>
</tr>
<tr>
<td>8</td>
<td>882</td>
<td>0.500</td>
</tr>
<tr>
<td>11</td>
<td>1018</td>
<td>0.500</td>
</tr>
<tr>
<td>5, 7</td>
<td>992</td>
<td>0.494</td>
</tr>
<tr>
<td>5, 8</td>
<td>1170</td>
<td>0.489</td>
</tr>
<tr>
<td>5, 11</td>
<td>1875</td>
<td>0.487</td>
</tr>
<tr>
<td>7, 8</td>
<td>1480</td>
<td>0.487</td>
</tr>
<tr>
<td>7, 11</td>
<td>2305</td>
<td>0.487</td>
</tr>
<tr>
<td>8, 11</td>
<td>2340</td>
<td>0.481</td>
</tr>
<tr>
<td>5, 7, 8</td>
<td>1780</td>
<td>0.479</td>
</tr>
<tr>
<td>5, 7, 11</td>
<td>2465</td>
<td>0.477</td>
</tr>
<tr>
<td>5, 8, 11</td>
<td>2620</td>
<td>0.477</td>
</tr>
<tr>
<td>7, 8, 11</td>
<td>2930</td>
<td>0.477</td>
</tr>
<tr>
<td>5, 7, 8, 11</td>
<td>3225</td>
<td>0.469</td>
</tr>
</tbody>
</table>

* $Q_o = C(h_o)^n$
34. Branch Duct Resistance

For the resistance of each branch duct with its damper removed, a calibration curve was obtained by plotting the static pressure at the upstream pressure tap, Station n1, against the air flow rate in the branch duct. All calibrations were conducted with the branch ducts in their normal positions about the box-plenum, so that Fig. 2 also represents the calibration arrangement for Series 1. During each calibration, only one branch duct was in use; therefore the orifice box

---

**Figure 27. Representative Calibration Curve for Resistance of a Branch Duct**
flow rate, \( Q_o \), was equal to the branch duct flow rate, \( Q_n \). For example, when branch duct 5 was calibrated, branch ducts 1, 2, 3, 4, and 6 were sealed; the orifice box differential pressure, \( h_o \), and the static pressure at Station 51 were recorded; and the flow rate, \( Q_5 \), was plotted against the static pressure, \( (SP)_{51} \), to obtain the calibration curve shown in Fig. 27. In the same manner, resistance calibration curves were established for each of the other branch ducts.

Since the 7-in.-diam branch ducts were made alike, they had nearly identical calibration curves. A general equation for all the calibration curves is

\[
(SP)_{n1} = \left( \frac{Q_n}{S} \right)^2
\]

in which

- \((SP)_{n1}\) = static pressure at the upstream pressure tap of any branch duct, Station \( n1 \), in. of water.
- \( Q_n \) = flow rate in any branch duct referred to air at standard density, cfm.
- \( S \) = a constant of proportionality evaluated by calibration.
- \( n \) = subscript denoting the number of the branch duct concerned. For example, for branch duct 2, \( n = 2 \).

Whenever major changes were made in the apparatus, such as converting from one arrangement to another, the branch ducts were recalibrated to obtain new calibration curves. In general, the value of \( S \) varied slightly with the particular arrangement being calibrated; however, an average value of \( S \) can be applied to all 7-in.-diam branch ducts for any one series of studies. For example, the average value of \( S \) was 710 for Series 1.

Equation (1) also expressed the resistance of the 8-in. and 9-in.-diam branch ducts, provided the proper value of \( S \) was used.

35. Branch Duct Air Flow Rate by the Pressure-Drop Method

In order to measure the flow rates in individual branch ducts, a flat-plate orifice was installed in each branch duct 2 ft from the upstream end. For Series 1, the pressure-drop method for calibrating the orifices was used. Specifically, the difference in the static pressures existing at the two pressure taps in each branch duct (see branch duct 5, Fig. 2) was plotted against the flow rate, \( Q_n' \), to establish a flow rate calibration curve for each branch duct. A representative calibration curve is shown in Fig. 28.

In the actual studies when more than one branch duct was in use, the sum of the flow rates of the individual branch ducts did not quite agree with the total flow rate as determined by the orifice box.
36. Branch Duct Air Flow Rate by the Dynamic-Tube Method

For the purpose of obtaining better agreement between the air flow rates determined at the orifice box and at the branch ducts, the dynamic-tube method of calibration was used to obtain calibration curves for Series 2, 3, and 4. A microdynamic tube was located along the axis of each branch duct on the downstream side of the orifice and at approximately the vena contracta of the stream passing through the orifice. Also, a static-pressure tap was installed on the branch duct in the same plane as the end of the microdynamic tube; consequently, when the tap and tube were connected differentially to an inclined draft gage, the velocity pressure at the vena contracta was measured. (See branch duct 5 in Fig. 4.) Though the dynamic-tube method did not completely eliminate differences in flow rates, the
deviations obtained by this method were less than those obtained by the previous method. Studies indicated that the orifice box volume was correct and that the branch duct orifices were affected by the flow conditions within the box-plenums. Therefore the branch duct flow rates as measured by the branch duct orifices were corrected to the total air flow rate as measured at the orifice box, by the equation

$$Q_n = \left( \frac{Q_o}{\sum Q_n'} \right) Q_n'$$

(2)

in which

- $Q_n =$ corrected flow rate in any branch duct referred to air at standard density, cfm.
- $Q_o =$ total flow rate referred to air at standard density, as determined from the orifice box calibration curves, cfm.
- $Q_n' =$ flow rate in any branch duct referred to air at standard density as determined from its calibration curve, cfm.
- $\sum Q_n' =$ sum of the flow rates of the branch ducts in use, cfm.

Equation (2) was used to correct the air flow rates in the branch ducts as determined by either the pressure-drop or the dynamic-tube method of calibration.
APPENDIX C

EVALUATION OF THE OUTLET LOSS

In order to compare the various types of entrance sections, it was desirable to evaluate separately the abrupt contraction loss that occurred as the air flowed from the plenum into the branch ducts. Two methods were used to evaluate this pressure loss.

First, if the total pressure in the plenum just ahead of an outlet to a branch duct and the total pressure at Station \( n1 \) were determined, the difference between these total pressures would represent the outlet loss. Some of the Group 1Q studies were conducted with Stations 6, 7, 8, and 9 (see Fig. 2) connected with a piezometer ring. With this piezometer, the total pressure close to the outlet to branch duct 3 was obtained. The difference between this total pressure and the total pressure at Station 31 represented the contraction loss occurring at the outlet to branch duct 3.

A second method used to evaluate the outlet loss was to neglect the entrance and turbulent mixing losses when only one branch duct was in use—that is, to assume the outlet loss equal to the difference in total pressures existing at Station 2 and Station \( n1 \). The Group 1A studies with one branch duct in use and with the Type 5 entrance section installed were used to evaluate the outlet loss by this method. Later studies proved this assumption logical, for the sum of the entrance and turbulent mixing losses was only 10 percent of the outlet loss when only one branch duct was in use.

As the results obtained by these two methods were in good agreement, an average loss was selected—0.45 of the velocity head existing in the branch duct, or in equation form,

\[
\frac{L_n}{(VP)_n} = 0.45 \tag{1}
\]

Equation (1) was in good agreement with the commonly accepted value of abrupt contraction loss for the area ratio involved. Therefore this value of outlet loss was used in all equations which expressed the total loss of the various box-plenum arrangements studied.
APPENDIX D

SAMPLE DATA SHEET AND METHOD OF COMPILING DATA

A sample data sheet is shown in Table 7. The data it gives were obtained during the first of the Group 1G studies, and are representative of those obtained when using the general procedure described in Section 15. The pressure differentials at the orifice box, $h_o$, are given in column C; the static pressures at Station 2, $(SP)_o$, are given in column D. The values of the differential pressures across the orifices in branch ducts 1 to 6 are given in columns E to K. These pressures, as corrected by means of the gage calibration curves, are shown in Table 8. The total air flow rate entering the plenum, $Q_o$, and the corrected air flow rates leaving the system through the branch ducts are shown in Table 9. Comparison of the branch duct flow rates shows that they are nearly equal. Therefore only a small amount of dampering would be required in order to balance the distribution from the plenum during the Basic 2 experimental condition.

The next step in the procedure was to plot the six values of $(SP)_o$, given in column C of Table 8 against the corresponding values of $Q_o$, given in column B of Table 9. The plotted points, the curve relating them, and the equation of the curve are shown in Fig. 14 for the condition of six branch ducts in use. The equation is

\[(SP)_o = \left( \frac{Q_o}{3500} \right)^2 \]

The derivation and use of this equation are given in Appendix E.

The same procedure was followed for all but a few special studies.
**Table 7**

**SAMPLE DATA SHEET**

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Orifice Box Orifices in Use</th>
<th>$h_a$</th>
<th>$(SP)_o$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
<th>$h_5$</th>
<th>$h_6$</th>
<th>Wattmeter, kw</th>
<th>Fan Speed, rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$8^\circ$</td>
<td>0.314</td>
<td>0.020</td>
<td>0.020</td>
<td>0.022</td>
<td>0.024</td>
<td>0.020</td>
<td>0.020</td>
<td>0.021</td>
<td>0.140</td>
<td>588</td>
</tr>
<tr>
<td>2</td>
<td>$11^\circ$</td>
<td>0.218</td>
<td>0.045</td>
<td>0.054</td>
<td>0.054</td>
<td>0.061</td>
<td>0.050</td>
<td>0.052</td>
<td>0.057</td>
<td>0.170</td>
<td>611</td>
</tr>
<tr>
<td>3</td>
<td>$11^\circ$</td>
<td>0.416</td>
<td>0.085</td>
<td>0.102</td>
<td>0.105</td>
<td>0.117</td>
<td>0.095</td>
<td>0.096</td>
<td>0.105</td>
<td>0.325</td>
<td>828</td>
</tr>
<tr>
<td>4</td>
<td>$7^\circ$, $11^\circ$</td>
<td>0.371</td>
<td>0.148</td>
<td>0.187</td>
<td>0.191</td>
<td>0.211</td>
<td>0.184</td>
<td>0.178</td>
<td>0.188</td>
<td>0.400</td>
<td>937</td>
</tr>
<tr>
<td>5</td>
<td>$5^\circ$, $7^\circ$, $8^\circ$, $11^\circ$</td>
<td>0.222</td>
<td>0.216</td>
<td>0.271</td>
<td>0.276</td>
<td>0.310</td>
<td>0.284</td>
<td>0.265</td>
<td>0.274</td>
<td>0.670</td>
<td>1028</td>
</tr>
<tr>
<td>6</td>
<td>$5^\circ$, $7^\circ$, $8^\circ$, $11^\circ$</td>
<td>0.331</td>
<td>0.300</td>
<td>0.305</td>
<td>0.302</td>
<td>0.440</td>
<td>0.385</td>
<td>0.307</td>
<td>0.381</td>
<td>0.140</td>
<td>1208</td>
</tr>
</tbody>
</table>

**Remarks:** Basic 1 experimental conditions (all dampers removed).
**TABLE 8**

**CORRECTED GAGE PRESSURES, IN. OF WATER**

<table>
<thead>
<tr>
<th>Run No.</th>
<th>( \ell_0 )</th>
<th>((SP)_0)</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( h_3 )</th>
<th>( h_4 )</th>
<th>( h_5 )</th>
<th>( h_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>J</td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>0.021</td>
<td>0.020</td>
<td>0.023</td>
<td>0.024</td>
<td>0.023</td>
<td>0.020</td>
<td>0.021</td>
</tr>
<tr>
<td>2</td>
<td>0.219</td>
<td>0.046</td>
<td>0.054</td>
<td>0.056</td>
<td>0.062</td>
<td>0.057</td>
<td>0.055</td>
<td>0.057</td>
</tr>
<tr>
<td>3</td>
<td>0.419</td>
<td>0.088</td>
<td>0.102</td>
<td>0.108</td>
<td>0.118</td>
<td>0.105</td>
<td>0.102</td>
<td>0.105</td>
</tr>
<tr>
<td>4</td>
<td>0.373</td>
<td>0.152</td>
<td>0.184</td>
<td>0.194</td>
<td>0.210</td>
<td>0.197</td>
<td>0.186</td>
<td>0.189</td>
</tr>
<tr>
<td>5</td>
<td>0.253</td>
<td>0.222</td>
<td>0.273</td>
<td>0.279</td>
<td>0.308</td>
<td>0.278</td>
<td>0.273</td>
<td>0.276</td>
</tr>
<tr>
<td>6</td>
<td>0.333</td>
<td>0.306</td>
<td>0.363</td>
<td>0.394</td>
<td>0.440</td>
<td>0.395</td>
<td>0.403</td>
<td>0.383</td>
</tr>
</tbody>
</table>

*Pressures in Table 7 corrected by means of gage calibration curves.*

**TABLE 9**

**CORRECTED AIR FLOW RATES, CFM**

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Orifice Box Flow Rate</th>
<th>Branch Duct Flow Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Q_0 )</td>
<td>( Q_1 )</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>496</td>
<td>78</td>
</tr>
<tr>
<td>2</td>
<td>755</td>
<td>121</td>
</tr>
<tr>
<td>3</td>
<td>1049</td>
<td>171</td>
</tr>
<tr>
<td>4</td>
<td>1365</td>
<td>225</td>
</tr>
<tr>
<td>5</td>
<td>1628</td>
<td>271</td>
</tr>
<tr>
<td>6</td>
<td>1928</td>
<td>326</td>
</tr>
</tbody>
</table>

*Air flow rates determined from pressures in Table 8 and calibration equations (see Appendix A and B); then corrected by Eq. (2), Appendix B.*
37. Energy Leaving the Box-Plenum

To determine the total loss, $L_t$, defined in Fig. 11, it is necessary to evaluate the energy entering the plenum at Station 2 and that leaving the plenum at Station $n1$. The energy leaving the plenum is represented by the total pressure $(TP)$ at Station $n1$, which is the sum of the static pressure $(SP)$ and the velocity pressure $(VP)$. Therefore, at the upstream pressure tap of any branch duct

$$(TP)_{n1} = (SP)_{n1} + (VP)_{n1}$$  \hspace{1cm} (1)

Since Eq. (1), Appendix B, is

$$(SP)_{n1} = \left(\frac{Q_n}{S}\right)^2$$

it can be substituted into the first equation to obtain

$$(TP)_{n1} = \left(\frac{Q_n}{S}\right)^2 + (VP)_{n1}$$  \hspace{1cm} (2)

In order to express the term $(VP)_{n1}$ in equation form,

$$Q_n = A_nv_n$$  \hspace{1cm} (3)

in which

- $Q =$ air flow rate, cfm
- $A =$ area, sq ft
- $v =$ mean velocity, fpm
- $n =$ subscript denoting the number of the branch duct concerned. For example, for branch duct 5, $n = 5$.

Since, for air at standard density,

$$v_n = 4005\sqrt{(VP)_{n1}}$$  \hspace{1cm} (4)

substitution of Eq. (4) into Eq. (3) yields

$$(VP)_{n1} = \left(\frac{Q_n}{4005A_n}\right)^2$$  \hspace{1cm} (5)

Defining a symbol $V$ as

$$V = 4005A_n$$  \hspace{1cm} (6)

Eq. (5) can be expressed

$$(VP)_{n1} = \left(\frac{Q_n}{V}\right)^2$$  \hspace{1cm} (7)

and substituted in Eq. (2) to obtain

$$(TP)_{n1} = \left(\frac{Q_n}{S}\right)^2 + \left(\frac{Q_n}{V}\right)^2$$  \hspace{1cm} (8)
The right-hand member of this equation can be combined into one term by the following procedure.

The desired form is

$$(TP)_{n1} = \left(\frac{Q_n}{T}\right)^2$$  \hspace{1cm} (9)

in which

$T$ = a "total-pressure" constant which includes the "static-pressure" constant $S$ and the "velocity-pressure" constant $V$.

Equating Eqs. (8) and (9),

$$\left(\frac{Q_n}{T}\right)^2 = \left(\frac{Q_n}{S}\right)^2 + \left(\frac{Q_n}{V}\right)^2$$  \hspace{1cm} (10)

which, by inspection, can be written

$$\left(\frac{1}{T}\right)^2 = \left(\frac{1}{S}\right)^2 + \left(\frac{1}{V}\right)^2$$  \hspace{1cm} (11)

and solved for $T$ to obtain

$$T = \frac{VS}{\sqrt{S^2 + V^2}}$$  \hspace{1cm} (12)

Values of $V$ in Eq. (6) for the branch duct sizes used in this investigation are as follows.

For any 7-in.-diam branch duct:

$$V = 1070$$  \hspace{1cm} (6a)

For any 8-in.-diam branch duct:

$$V = 1400$$  \hspace{1cm} (6b)

For any 9-in.-diam branch duct:

$$V = 1770$$  \hspace{1cm} (6c)

Thus, after evaluating $S$ by calibration, an equation expressing the total pressure at Station $n1$ can be determined analytically. For example, for the arrangement shown in Fig. 2 the resistance of any branch duct may be expressed in the form of Eq. (1), Appendix B, as

$$(SP)_{n1} = \left(\frac{Q_n}{710}\right)^2$$

in which 710 = the constant of proportionality, $S$.

Since the branch ducts shown in Fig. 2 are of 7-in. diameter, Eq. (6a) provides

$$V = 1070$$

Equation (12) is then

$$T = \frac{(1070)(710)}{\sqrt{(710)^2 + (1070)^2}} = 592$$
By Eq. (9) the final result is

$$(TP)_{n1} = \left( \frac{Q_n}{592} \right)^2$$

which expresses the total pressure, in inches of water, at Station $n1$ as a function of the air flow rate in the branch duct concerned, provided only that the branch duct is not dampered. In the same manner, a total-pressure equation was obtained for the branch ducts installed in each of the several arrangements.

38. **Energy Entering the Box-Plenum**

To evaluate the energy entering the plenum, it is necessary to determine the total pressure at Station 2, a pressure-measuring station located very close to the plenum entrance. An expression for the total pressure at Station 2 can be obtained by an analysis similar in every way to the foregoing. An expression similar to Eq. (1) is

$$(TP)_{o} = (SP)_{o} + (VP)_{o}$$

where the subscript refers to Station 2.

In compiling the data, the air flow rates at Station 2 were plotted against the static pressure at Station 2 to determine curves such as those shown in Fig. 14. The equations of all such curves could be expressed in the form

$$(SP)_{o} = \left( \frac{Q_{o}}{S_{o}} \right)^2$$

in which

$S_{o} = \text{a "static-pressure" constant of proportionality, whose numerical value depends on the experimental conditions. For example, in Fig. 14 the value of } S \text{ is 3500 for six branch ducts in use, 2830 for five branch ducts in use, etc.}$

An expression similar to Eq. (3) is

$$Q_{o} = A_{o}v_{o}$$

and, following the analysis in Section 34 above,

$$(VP)_{o} = \left( \frac{Q_{o}}{4005A_{o}} \right)^2$$

or, for the 12-in.-x-15-in. trunk duct at Station 2,

$$(VP)_{o} = \left( \frac{Q_{o}}{5006} \right)^2$$

in which 5006 is the numerical value of $V_{o}$, a "velocity-pressure" constant at Station 2.
Thus, Eq. (13) is

\[(TP)_o = \left(\frac{Q_o}{S_o}\right)^2 + \left(\frac{Q_o}{5006}\right)^2\]  \hspace{1cm} (18)

or, noting the form of Eqs. (9) and (12),

\[(TP)_o = \left(\frac{Q_o}{T_o}\right)^2\]  \hspace{1cm} (19)

in which

\[T_o = \frac{(5006)S_o}{\sqrt{S_o^2 + (5006)^2}}\]  \hspace{1cm} (20)

As an example of the procedure, the equation of the curve for six branch ducts in use as shown in Fig. 14 is

\[(SP)_o = \left(\frac{Q_o}{3500}\right)^2\]

The value of \(S_o\) is therefore 3500. Using Eq. (20),

\[T_o = \frac{(5006)(3500)}{\sqrt{(3500)^2 + (5006)^2}} = 2870\]

and Eq. (19) becomes

\[(TP)_o = \left(\frac{Q_o}{2870}\right)^2\]

which expresses the total pressure, in inches of water, at Station 2 as a function of the air flow rate at the entrance of the box-plenum for this particular condition. In the same manner, an equation for the total energy entering the plenum was obtained for each experimental condition.

39. Evaluation of Losses

The difference in total pressure between Station 2 and Station \(n1\) was defined in Fig. 11 as the total loss, \(L_t\), of the box-plenum. This definition states that

\[L_t = (TP)_o - (TP)_{n1}\]  \hspace{1cm} (21)

Substituting Eqs. (9) and (19) into this equation yields

\[L_t = \left(\frac{Q_o}{T_o}\right)^2 - \left(\frac{Q_{n1}}{T}\right)^2\]  \hspace{1cm} (22)

Equation (22) applies to each branch duct for the studies made when all dampers were removed, that is, the unbalanced conditions of air distribution maintained during the Basic 1 studies. However, since the branch air duct flow rates were not equal during the Basic 1 conditions, the value of the total loss computed from Eq. (22) depended on the particular branch duct flow rate selected. Therefore the average
total loss was desired. In order to eliminate the need for a separate calculation for each branch duct, and also to permit the losses to be generalized to any air flow rate, the simplifying assumption was made that

\[ Q_n = \bar{Q}_n \]  

(23)

in which \( \bar{Q}_n \) = average branch duct flow rate referred to standard density, cfm. Upon investigation it was found that this assumption introduced a maximum deviation of less than 1.0 percent for all studies conducted with 7-in.-diam branch ducts exclusively. Since this deviation was well within the limits of accuracy of this investigation, Eq. (22) was written

\[ L_t = \left( \frac{Q_o}{T_o} \right)^2 - \left( \frac{Q_n}{T} \right)^2 \]  

(24)

In order to generalize this equation, it is possible to write

\[ Q_o = NQ_n. \]  

(25)

in which \( N \) = number of branch ducts in use. For example, for any combination of three branch ducts in use, \( N = 3 \).

Substitution of Eq. (25) into Eq. (24) provides the general equation for the total loss with any number of branch ducts in use:

\[ L_t = \left( \frac{NQ_n}{T_o} \right)^2 - \left( \frac{Q_n}{T} \right)^2 \]  

(26)

This equation can be expressed in a dimensionless form by dividing it by Eq. (7) to provide, with the aid of Eqs. (6a) and (23),

\[ \frac{L_t}{(VP)_{n1}} = \left( \frac{NQ_n}{T_o} \right)^2 - \left( \frac{Q_n}{1070} \right)^2 \]  

(27)

which reduces to

\[ \frac{L_t}{(VP)_{n1}} = \left( \frac{1070N}{T_o} \right)^2 - \left( \frac{1070}{T} \right)^2 \]  

(28)

Equation (28) expresses the total loss of the box-plenum in terms of the average velocity heads in the branch ducts; it was used to calculate the total loss of all studies made with Basic 1 (unbalanced) experimental conditions in which 7-in.-diam branch ducts were used exclusively. For the Series 4 studies conducted with 7-in., 8-in., and 9-in.-diam branch ducts it was necessary to use Eq. (22) instead of Eq. (28), obtain the total loss based on each branch duct, and calculate the average value.
As an example of the application of Eq. (28), the total loss will be determined from the data given in Fig. 14 for six branch ducts in use. The value of $T$ was found to be 592 in the example cited in Section 34, and the value of $T_o$ was found to be 2870 in Section 34. Therefore Eq. (28) becomes

$$\frac{L_t}{(VP)_{n_1}} = \left[ \frac{(1070) (6)}{2870} \right]^2 - \left( \frac{1070}{592} \right)^2 = 1.74$$

which states that, for this particular experimental condition, the total loss is 1.74 velocity heads based on the average velocity in the branch ducts. Note that this value is plotted in Fig. 15 at a value of $\Sigma A_n/A_o$ of about 1.3.

During the Basic 2 (balanced flow) experimental conditions the dampers were adjusted until the air flow rates of the branch ducts in use were equal. Therefore, any branch duct flow rate, $Q_n$, was also the average branch duct flow rate, $\bar{Q}_n$, when only 7-in.-diam branch ducts were used. Moreover, at least one branch duct was not dampered during the Basic 2 conditions. Since these were precisely the conditions used in the derivation of Eq. (28) for the Basic 1 conditions, Eq. (28) was also applicable to the one or more branch ducts not dampered during the Basic 2 conditions. Furthermore, since whatever resistance was added to balance the box-plenum was charged against its performance, the total loss computed from Eq. (28) for any one undampered branch duct represented the total loss of the plenum under balanced conditions of flow. Therefore, Eq. (28) was also used to calculate the total loss of the plenum for all Basic 2 experimental conditions during the Series 1, 2, and 3 studies.

A relationship between the velocity head existing in the trunk duct and that in the branch ducts is obtained as follows. Equating Eqs. (15) and (25),

$$A_o v_o = N \bar{Q}_n$$

or, with the aid of Eqs. (3) and (23),

$$A_o v_o = N A_n v_n$$

For air at standard density and the respective areas, Eq. (30) becomes

$$(1.25) (4005) \sqrt{(VP)_o} = N (0.267) (4005) \sqrt{(VP)_{n_1}}$$

which provides the relationship for expressing any loss in terms of either the velocity head in the trunk duct or the average velocity head in the branch ducts:

$$(VP)_{n_1} = \left( \frac{21.8}{N^2} \right) (VP)_o$$

(32)
With respect to the other losses involved, the plenum loss, $L_p$, which is the sum of the entrance and turbulent mixing losses, is defined in Fig. 11 as the difference between the total loss, $L_t$, and the outlet loss, $L_n$. In equation form, this states that

$$L_p = L_t - L_n$$ (33)

or, in a dimensionless form in terms of the velocity head in any branch duct,

$$\frac{L_p}{(VP)_{n_1}} = \frac{L_t}{(VP)_{n_1}} - \frac{L_n}{(VP)_{n_1}}$$ (34)

The outlet loss, $L_n$, was evaluated separately; it is given by Eq. (1), Appendix C. Therefore Eq. (34) can be written

$$\frac{L_n}{(VP)_{n_1}} = \frac{L_t}{(VP)_{n_1}} - 0.45$$ (35)

Equation (35) was used to evaluate the plenum loss after the term $L_t/(VP)_{n_1}$ was evaluated by either Eq. (22) or Eq. (28).

40. Derivation of Eq. (27), Section 20

In analyzing the initial data obtained when the number of branch ducts in use was varied from one to six, the total losses as determined by Eq. (28) were plotted against the number of branch ducts in use. However, since a completely dimensionless plot was desirable, the number of branch ducts in use was expressed in a dimensionless form as the ratio of the trunk duct area to the sum of the areas of the branch ducts in use. This ratio, which was designated as $A_o/\Sigma A_n$, is used as the abscissa in Fig. 15.

The problem was then the relating of the plotted points in Fig. 15 to a single curve. In this connection, an attempt was made to fit the plotted points with the simplest relation which might hold. This was

$$L_t = K_o (VP)_o + K_n (VP)_n$$ (36)

where $K_o$ and $K_n$ are constants. Using the value of the outlet loss determined by the methods discussed in Appendix C, Eq. (36) became

$$L_t = K_o (VP)_o + 0.45 (VP)_n$$ (37)

Next, Eq. (32) was used to eliminate $(VP)_o$ and obtain

$$L_t = K_o \left( \frac{N^2}{21.8} \right) (VP)_n + 0.45 (VP)_n$$ (38)

which, when divided by $(VP)_n$, became

$$\frac{L_t}{(VP)_n} = K_o \left( \frac{N^2}{21.8} \right) + 0.45$$ (39)
However, since the identity could be written ¹
\[ \frac{N^2}{21.8} = \left( \frac{\sum A_n}{A_o} \right)^2 \]  
(40)

Eq. (39) was expressed as
\[ \frac{L_t}{(VP)_n} = K_o \left( \frac{\sum A_n}{A_o} \right)^2 + 0.45 \]  
(41)

The optimum value of $K_o$ was found to be 0.80; hence the final result was
\[ \frac{L_t}{(VP)_n} = 0.80 \left( \frac{\sum A_n}{A_o} \right)^2 + 0.45 \]  
(42)

which is the equation of the curve shown in Fig. 15. The plotted points, as compared with the curve, represent deviations of less than 3 percent in the data. Since 3 percent was within the limits of accuracy expected, it was concluded that the relations expressed by Eqs. (36) and (42) provided a satisfactory correlation with the plotted points. Equation (29) in Section 20, which is identical to Eq. (36) above, therefore served as the basis for analyzing all subsequent data.

¹ See also Chap. III, Section 14.
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