COMBINED IRREGULAR ROUGHNESS AND FAVORABLE-PRESSURE-GRADIENT EFFECTS IN A TURBULENT BOUNDARY LAYER

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DISSEPTION

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Abstract

An experimental study of the combined impact of highly irregular surface roughness and moderate favorable-pressure-gradient (FPG) conditions on the structure of a turbulent boundary layer was assessed using two-dimensional particle image velocimetry (PIV) measurements in the streamwise–wall-normal plane and stereo PIV measurements in the wall-normal–spanwise plane. The roughness under consideration was replicated from a turbine blade damaged by deposition of foreign materials and contains a broad range of topographical scales. The two-dimensional PIV measurements were compared to measurements of smooth-wall flow under both identical FPG conditions as well as zero-pressure-gradient (ZPG) conditions in order to reveal the impact of roughness and FPG conditions on the underlying structure of the flow. The suppression of boundary layer thickness and Reynolds normal and shear stresses was observed in the smooth-wall FPG case. However, with the addition of surface roughness to the identical FPG condition, enhanced momentum deficit and Reynolds normal and shear stresses were found in the combined FPG and roughness condition. The result of quadrant analysis revealed the significant dominance of ejections over sweeps under FPG condition regardless of the surface conditions, while the comparable impact of sweeps and ejections was observed under ZPG conditions. Similar impacts of FPG and surface roughness were observed in the cross-plane stereo PIV measurements. Of interest, smooth-wall results displayed homogeneity in the spanwise direction, while strong inhomogeneity was observed in the FPG rough-wall case due to roughness protrusions along the spanwise direction.

In terms of flow structural modifications, inspection of instantaneous velocity fields in the 2D PIV measurements revealed vortex organization consistent with ZPG smooth-wall flow, though focused closer to the wall with a shallower inclination angle under smooth-wall FPG conditions. In contrast, the combined FPG and surface roughness effect promoted vortical structure penetration much further away from the wall and enhanced an momentum deficit, indicating that roughness
mitigates the FPG-induced focusing of these structural attributes toward the wall. Results from the two-point velocity correlations support these instantaneous observations. Instantaneous velocity fields in stereo PIV measurement revealed alternating, low- and high-momentum regions (LMRs and HMRs) in the spanwise direction that embody a significant fraction of the Reynolds shear stress. Consistent with the 2D PIV measurements, reduced wall-normal extent and less intense LMRs and HMRs were observed under FPG conditions, while these characteristics were mitigated due to the presence of surface roughness. To examine the average spatial structure of the flow, two-point velocity correlations were computed. While two-point correlations of velocity reflected the basic signature of spanwise-alternating LMRs and HMRs, correlations of velocity fields embodying only the largest spatial scales revealed an even higher degree of spanwise coherence of these patterns. However, the shortening of correlation, especially in spanwise direction was observed in FPG-Rough case.
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Chapter 1

Introduction

1.1 Motivation

Many practical flows of interest occur in the presence of complicating influences, such as non-zero pressure gradients that can accelerate/decelerate the flow as well as surface roughness. For example, the pressure surface of a turbine blade, wherein significant favorable-pressure-gradient (FPG) conditions exist, can also be subjected to surface damage due to a variety of damage mechanisms, including pitting, spallation and deposition of foreign materials (Bons, 2002). While surface roughness alone can cause notable modifications of the local flow behavior, the interaction between roughness and FPG conditions may further enhance, or possibly diminish, these effects.

The surface heat transfer in turbomachinery is tied to the non-zero pressure gradient turbulent boundary layer problem. It is well known that the surface heat transfer characteristics of turbine blades follow the Reynolds Analogy (relation of turbulent momentum and heat transfer) for the case of zero streamwise pressure gradient. However, under FPG conditions, significant discrepancy exists between the length of the transition zone deduced from the thermal boundary layer and from the momentum boundary layer (Blair, 1982; Sharma, 1987). Under FPG conditions, the transition zone length of the thermal boundary layer is longer than the momentum boundary layer, since the more inhibited heat transfer within a turbulent spot than that of momentum transfer under FPG conditions retards the development of a transitional thermal boundary layer (Chong and Zhong, 2006). Further complicating these processes, most engineering surfaces may be aerodynamically smooth at deployment, but can suffer cumulative surface damage due to numerous mechanisms. For example, fouling due to deposition of foreign materials on a heat transfer surface during the lifetime of the heat exchanger adds thermal resistance and hence reduces the heat transfer efficiency while increasing the pressure drop required to maintain the flow rate through the heat exchanger.
In marine systems, biological fouling induced by the attachment of barnacles, mussels, sponges, algae, slime and sea squirts on ship hulls and propellers leads to excessive energy losses and high fuel consumption (Bixler and Bhushan, 2012). Despite the practical importance of the combined impact of favorable pressure gradient and surface roughness in turbulent boundary layers, only a few studies have studied the statistical modifications imposed by such conditions. The flow structural modifications imposed by combined FPG and surface roughness conditions is highlighted in this research along with the statistical attributes. This research could serve as a benchmark for accurate flow simulations by either large-eddy simulation (LES) or Reynolds-averaged Navier–Stokes simulation (RANS).

1.2 Background of the turbulent boundary layer

Prandtl (1904) first suggested that there exists a thin layer close to a solid boundary within which vorticity varies rapidly as a result of the combined effects of viscous diffusion and convection, outside of which the vorticity is zero. Based on Prandtl’s initial efforts, numerous researchers have attempted to understand the behavior of turbulent boundary layers given their occurrence in a wide variety of practical applications, including flow over airborne and seaborne vessels, within oil pipelines and the atmospheric boundary layer. Recent efforts have found this flow to embody coherent structures that govern its dynamics and evolution, particularly Reynolds stresses that transport momentum and produce/dissipate turbulent kinetic energy via multi-scale interactions (Adrian, 2007).

The mean momentum equation for a canonical, steady turbulent boundary layer is given by

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'^2}}{\partial x},
\] (1.1)

where \(U\) and \(V\) are the mean streamwise and wall-normal velocities, \(P\) is the mean pressure, \(\overline{u'v'}\) is the Reynolds shear stress while \(\rho\) and \(\nu\) are the density and kinematic viscosity of the working fluid, respectively. Incompressibility is assumed, as is homogeneity in the spanwise (\(z\)) direction. Here, \(x\) and \(y\) are taken as the streamwise and wall-normal directions, respectively. Evaluating this
mean momentum equation outside the boundary layer (formally as $y \to \infty$) gives the relationship

$$-\frac{1}{\rho} \frac{dP_e}{dx} = U_e \frac{dU_e}{dx},$$

(1.2)

where $(\cdot)_e$ denotes quantities evaluated in the free stream. Thus, non-zero pressure gradient conditions are manifested by streamwise gradients in the free-stream velocity. Acceleration of the free stream (i.e., $dU_e/dx > 0$) yields $dP_e/dx < 0$ and is termed a favorable pressure gradient while deceleration (i.e., $dU_e/dx < 0$) yields $dP_e/dx > 0$ and is termed an adverse pressure gradient (APG). Deceleration is known to be a destabilizing effect owing to a higher probability of flow separation compared to ZPG flow, while acceleration provides enhanced stability in this regard. A constant free-stream velocity yields zero-pressure-gradient conditions.

Theodorsen (1952) proposed the horseshoe vortex to describe the coherent motions responsible for the transport processes in the ZPG turbulent boundary layer. This conceptual coherent structure model was later extended and refined by Head and Bandyopadhyay (1981) and Bandyopadhyay (1980) from flow visualizations of ZPG turbulent boundary layers. These visualizations revealed an abundance of inclined coherent structures near the edge of the boundary layer, though they appeared more as hairpin-like structures (i.e., long streamwise-oriented legs) compared to the horseshoe vortex of Theodorsen. Smith (1984) also visualized hairpin-like structures in low Reynolds number (Re) turbulent boundary layers. Later, Smith et al. (1991) postulated the hairpin vortex as the basic structure of wall turbulence and speculated the physical processes by which individual hairpin vortices could generate new structures during their evolution. Adrian et al. (2000b) visualized hairpin-like structures throughout a turbulent boundary layer on a smooth wall using the particle image velocimetry (PIV) technique. The inclination angle of the individual hairpin structures in the streamwise direction relative to the wall was approximately $45^\circ$, consistent with the observations of Head and Bandyopadhyay (1981). In addition, Adrian et al. (2000b) observed that these individual vortices tended to align with one-another in the streamwise direction to form larger-scale coherent motions that they termed hairpin vortex packets. The streamwise-aligned hairpin vortex heads were observed to form an inclined interface away from the wall at roughly $12^\circ$ beneath which a relatively uniform region of streamwise momentum deficit was apparent due to the collective induction of the vortices. They found that smaller, younger packets were born in
the near-wall region while the outer region of the flow was populated by packets that had grown to the boundary-layer edge during their evolution. Direct numerical simulations (DNS) of turbulent channel flow reported by Zhou et al. (1999) highlighted the capability of individual hairpin-like structures to spawn new vortices both upstream and downstream that, over time, evolved into trains of vortices consistent with hairpin packets. Recent studies highlight the persistence of these structural attributes even in the presence of surface roughness (Volino et al., 2007), including complex roughness with a broad range of topographical scales (Wu and Christensen, 2010). While all of the studies discussed above were performed under the simplified conditions of zero-pressure-gradient flow, many practical flows of interest occur in the presence of complicating influences like non-zero pressure gradients that can accelerate/decelerate the flow as well as surface roughness.

1.2.1 Favorable pressure gradient conditions

With respect to quantifying the strength of the streamwise pressure gradient, Clauser (1956) introduced the equilibrium parameter (Clauser pressure gradient parameter), $\beta$, given by

$$\beta = \frac{\delta^*}{\tau_w} \frac{dP_e}{dx} = -\frac{\Delta}{u_\tau} \frac{dU_e}{dx}; \Delta = \frac{\delta^* U_e}{u_\tau},$$

(1.3)

where $\delta^*$ is the displacement thickness, $P_e$ is the free-stream static pressure, $U_e$ is the free stream velocity, $\tau_w$ is the wall shear stress and $u_\tau$ is the friction velocity defined as $u_\tau = \sqrt{\tau_w/\rho}$. Alterna-
tively, Patel and Head (1968) proposed a non-dimensional pressure-gradient parameter

\[ \Delta p = \frac{\nu \, \frac{dP_e}{dx}}{\rho u_\tau^3}, \]  

(1.4)
since the velocity profile under the pressure gradient condition was observed to depart from the universal log-law related to the pressure gradient.

The acceleration parameter introduced by Launder (1964) has been widely used since it provides a useful indication of reversion in the turbulent boundary layer and also does not require evaluation of \( \tau_w \) for calculation of \( u_\tau \) (Launder, 1964; Moretti and Kays, 1965; Patel, 1965). The acceleration parameter \( K \) (also called the velocity gradient parameter), is given by

\[ K = \frac{\nu \, \frac{dU_e}{dx}}{U_e^2}, \]  

(1.5)
and characterizes the “strength” of the imposed pressure gradient. In contrast to \( \beta \) and \( \Delta p \), \( K \) is consisted of directly measurable quantities and is useful indicator regarding to the effect of pressure gradient on boundary layer transition. Favorable pressure gradients (FPG; \( K > 0 \)), wherein the free-stream flow in accelerated with downstream distance, are of particular interest due to their ability to actually revert a turbulent flow back to a laminar state if the imposed acceleration is strong enough (so-called “relaminarization” effect). Evidence suggests that \( K = 2.5 \sim 3.5 \times 10^{-6} \) is the critical value for the onset of relaminarization (Narayanan and Ramjee, 1969; Moretti and Kays, 1965; Jones and Launder, 1972). Taylor (1929) firstly observed a reversion of the flow from turbulent to laminar in his work on curved pipes and it has been extensively studied ever since (Launder, 1964; Moretti and Kays, 1965; Patel and Head, 1968; Blackwelder and Kovasznay, 1972; Narashimha and Sreenivasan, 1979). When relaminarization occurs, boundary-layer growth is suppressed as is the heat transfer rate, there exists a departure of the velocity profile from the universal log-law, the Reynolds number is decreased, as are the turbulence intensity and Reynolds stresses in the near-wall region.

Many studies have considered the impact of FPG conditions on smooth-wall turbulence (Narayanan and Ramjee, 1969; Launder, 1964; Schloemer, 1967; Herring and Norbury, 1967; McDonald, 1969; Coleman et al., 1977; Blair, 1992; Cardoso et al., 1991; Ichimiya et al., 1998; Piomelli et al., 2000;
Nagib et al., 2006; Dixit and Ramesh, 2010). Spalart (1986) conducted DNS of a sink-flow turbulent boundary layer with $K = 1.5$ and $3.0 \times 10^{-6}$. Sink flow refers to flow between two converging planes wherein $Re$, the skin friction coefficient, and $K$ remain constant at all streamwise positions owing to self-similarity (Sreenivasan, 1982). It was observed that as the intensity of the sink flow increased, the peak values of the Reynolds stresses shifted away from the wall. These results also revealed the suppression of vortex growth away from the wall and the development of much longer streaks with shallower angle compared to ZPG flow. Piomelli et al. (2000) used large-eddy simulation (LES) to study the impact of streamwise flow acceleration on the vortical structures that populate turbulent boundary layers at accelerations below and above the threshold for which relaminarization can occur. These simulations revealed the near-wall streaks to be more elongated in the presence of streamwise acceleration as well as a reduction in the number of coherent vortices along with an enhancement in their streamwise elongation and alignment. In addition, fewer intense ejection events were observed under FPG conditions. Similar observations were reported recently by Dixit and Ramesh (2010), including a reduced inclination angle of the large-scale structures as well as a significant increase in their streamwise extent with increasing FPG conditions. Jones et al. (2001) conducted both hot-wire and pitot-tube measurements on a sink flow with three mild FPG conditions ($K = 2.7, 3.59$ and $5.39 \times 10^{-7}$) at consecutive streamwise positions. The results showed the development of the log-law region as the velocity profiles evolve downstream due to decay of the wake strength. The streamwise turbulence intensity profiles $\langle u'u' \rangle / u_\tau^2$ for different $K$ were found to collapse using the outer flow scaling when $y/\delta > 0.8$ ($u_\tau$ is the friction velocity, $y$ is the wall-normal position and $\delta$ is the boundary-layer thickness).

While of significant practical importance, only a few studies have addressed the combined impact of FPG conditions and surface roughness. For example, Coleman et al. (1977) reported hot-wire measurements for flow over a porous surface (densely packed spheres) subjected to FPG conditions at $K = 2.9 \times 10^{-7}$. Comparing smooth- and rough-wall cases under FPG and ZPG conditions indicated decreased turbulent kinetic energy in the inner region. However, the values of the Reynolds shear stress correlation coefficients appeared insensitive to acceleration. Cal et al. (2008) reported measurements of these combined effects using heavy grit sandpaper for the rough surface and FPG conditions in the range $0.17 \times 10^{-7} < K < 2.6 \times 10^{-7}$. Due to the presence of
surface roughness, the boundary layer thickness was substantially increased compared to the smooth surface of the same FPG condition. However, the rate change of the boundary layer thickness for the combined effects was similar to the flow over a smooth surface under ZPG condition. It was concluded that the influence of roughness on the boundary-layer parameters dominated that of the FPG conditions for this range of acceleration. Agelinchaab and Tachie (2008) conducted PIV measurements in a turbulent boundary layer over two-dimensional square ribs with both FPG and APG conditions. The synergistic influences of APG and surface roughness were observed, while FPG conditions and roughness were found to compete with one-another. Finally, Tay et al. (2009) also reported results for low Reynolds number- FPG turbulent flows over smooth and rough surfaces. The surfaces consisted of sand grain and gravel for FPG conditions in the range $K = 0.1 - 3.9 \times 10^{-6}$ for flow in an asymmetric converging channel. The results also suggested that the effect of surface roughness was dominant over that of the FPG conditions. In addition, while the near-wall flow structure was modified significantly due to FPG and surface roughness effects, the distribution of Reynolds stresses was insensitive to such effects outside the roughness sublayer.

While the above studies highlight the statistical modifications imposed by combined FPG and roughness conditions, little is known about the underlying structural modifications imposed by such conditions. In addition, the roughness employed in these efforts was idealized, such as sand grain, regularly arrayed mesh, or uniform spheres, characterized by a dominant roughness scale arranged in an ordered manner despite the roughness present in most practical applications being highly irregular and containing a broad range of topographical scales. So it cannot be expected that the aforementioned idealized roughness models would reflect the full impact of practical roughness on wall turbulence. The discrepancies between Nikuradse’s (Nikuradse, 1950) friction-factor data from sandgrain experiments and those of Colebrook (1939) for more industrial-type roughness serve as excellent examples in this regard. Bons (2002) reported the heat transfer coefficient of the real roughness is overpredicted by 10% when the roughness is in the fully rough regime ($k^+ > 70$) , but the existing correlations of heat transfer and skin friction is severely underpredicted when the real roughness regime is $k^+ < 70$ with respect to the simulated (ordered cones or hemispheres) roughness indicating the flow characteristics of real roughness and simulated roughness are fundamentally different. An excellent review of the influence of actual surface roughness in gas turbines can be
found in Bons and McClain (2004). Recently Marchis and Napoli (2012) investigated the effects of irregular 2D and 3D roughness surface generated through the superimposition of sinusoidal functions with random amplitudes in turbulent channel flows via the LES technique. The results showed the 3D irregular roughness more effectively reduces the flow velocity inducing higher \( \text{rms} \) of the velocity fluctuations, Reynolds shear stress, and higher reduction of the anisotropy compared with those of 2D roughness despite both sharing the same mean roughness height. Taken together, these observations and discrepancies highlight the critical importance of understanding the flow over a highly irregular realistic roughness for proper modeling of practical rough-wall flows.

1.2.2 Challenges of defining the inner scales of the combined impact of FPG and surface roughness

The universal log law postulated by Prandtl (1925) and Karman (1931) has been widely used to define the wall shear stress, \( \tau_w \) and friction velocity \( u_\tau \) in a turbulent boundary layer with zero pressure gradient. In a turbulent boundary layer, viscous effects dominate in the near-wall (inner) region while inertial effects become important away from the wall (outer region). The overlap region develops between the inner and outer regions and is known to be the logarithmic layer where, for a canonical, ZPG, smooth-wall turbulent boundary layer the mean velocity can be represented as

\[
U^+ = \frac{1}{\kappa} \ln(y^+) + A, \tag{1.6}
\]

where \( \kappa \) is the von Karman coefficient, \( A \) is an additive constant (Townsend, 1976) and \((\cdot)^+\) represents normalization by inner scales \( u_\tau \) and \( y_\ast = \nu/u_\tau \), the viscous length scale. For ZPG flow, \( \kappa \approx 0.41 \) and \( A \approx 5.0 \) are generally used. Oftentimes, these constants are used with measurements of the mean velocity profile with \( y \) to determine \( u_\tau \) assuming a logarithmic profile without a direct measurement of \( \tau_w \) (termed the Clauser-chart method) owing to challenges in accurately measuring \( \tau_w \) in even the simplest, ZPG smooth-wall boundary layer (\( \tau_w \) in pipes and channels is much easier to document owing to its direct relationship to the constant streamwise pressure gradient that drives these internal flows). However, this curve-fit method for determination of \( u_\tau \) fails for the flow under strong FPG condition since its mean velocity profile departs from the universal log law with the increasing \( K \) (Launder, 1964; Patel and Head, 1968; Spalart, 1986). The von Karman
coefficient ($\kappa$) and the additive constant $A$ are found to be pressure-gradient-dependent variables (Nagib et al., 2006) that are not known \textit{a priori}. For this reason, several analytical and experimental methods have been proposed to define the skin friction coefficient, $C_f$, given by

$$C_f = \frac{\tau_w}{2\rho U_e^2}. \tag{1.7}$$

Dixit and Ramesh (2009) proposed the modified Clauser chart method for estimation of local skin friction in non-zero pressure-gradient flow except for the extreme cases of relaminarization or boundary-layer separation. The von Karman constant $\kappa$ and the intercept $A$ were estimated using an empirical polynomial curve fit of data from several sink-flow experiments (Jones et al., 2001; Dixit and Ramesh, 2008) and DNS results for the adverse pressure gradient flow (Spalart, 1986). The basic idea of this method is to apply initial (guessed) values of pressure-gradient-dependent parameters $\kappa$ and $A$ to estimate an initial $C_f$. Using the value of estimated $C_f$ and the known values of the acceleration parameter $K$, $\kappa$ and $A$ are recalculated. This procedure is repeated until the $\kappa$ and $A$ from the first guessed values and the estimation of the $C_f$ are closely matched. The percent difference of estimated $C_f$ with respect to the original reported values from other research is about 4.4%, however, relatively large differences (18.7%) were reported for the non-equilibrium flow data from oil-film interferometry.

Alternatively, the fully-integrated boundary-layer equation can be used to compute the skin friction coefficient as

$$\tau_w = \rho \left( \nu \frac{dU}{dy} - \langle u'v' \rangle - \int_0^1 \frac{\partial U^2}{\partial x} dy' + U \int_0^y \frac{\partial U}{\partial x} dy' - \int_0^U \frac{\partial \langle u^2 \rangle}{\partial x} dy' + \int_0^y \frac{\partial \langle v^2 \rangle}{\partial x} dy' + U_e \frac{dU_e}{dx} y \right). \tag{1.8}$$

Using the momentum balance between each term in the above equation, Cal et al. (2008) estimated the skin friction coefficients of LDV measurement data in a turbulent boundary layer subjected to combined FPG and roughness conditions.

Finally, Mehdi and White (2011) proposed an integral method to estimate $C_f$ suitable for the turbulent boundary layer. In contrast to the above integral momentum balance, here $C_f$ is
evaluated by fitting a Whittaker smoothing function to the total stress gradient, yielding

\[ C_f = \frac{4(1 - \delta^*)}{Re\delta} + 2 \int_0^1 2(1 - y)(-\langle u'v' \rangle)dy + 2 \int_0^1 (1 - y^2)(-\frac{\partial\tau}{\partial y})dy, \]  

(1.9)

where \( \delta^* \) and \( y \) have already been normalized by \( \delta \). The first and second terms represent the mean velocity profile and the Reynolds shear stress contribution. The last term illustrates the total stress gradient contribution that is fitted with a Whittaker smoothing function. Mehdi and White (2011) reported the percent difference between the estimated \( C_f \) from the integral method and previously reported data (turbulent boundary layer over a rough surface data from LDV and PIV) ranges from 0.2 to 6.6.

While the above-mentioned methods provide alternative approaches for determining \( \tau_w \) and hence \( u_\tau \) and \( y_* \), they still rely upon well-resolved measurements of the mean and turbulence statistics. Nevertheless, they are superior to the Clauser chart method given the pressure-gradient dependence of both \( \kappa \) and \( A \) and provide a means of independent determination of \( \tau_w \) under specific pressure-gradient conditions which can then be used to assess the validity of a logarithmic velocity profile and the corresponding \( \kappa \) and \( A \) under such conditions.

Direct measurements of \( C_f \) are also possible, at least for smooth walls, and oil film interferometry (OFI) is the commonly used method. This method is based on evaluation of the deformation of a thin (silicon) oil film (typically of an order of several wavelengths of visible light) when subjected to a shear stress on its top surface. Once the oil is sheared, a monochromatic light is used to illuminate the sheared pattern and the change in the interference pattern containing alternating patterns of light and dark fringes is recorded. The skin friction is determined as a function of the recorded fringe patterns, the wave length of the light source \( \lambda \), the kinematic viscosity of the oil \( \nu \), the index of refraction of the oil \( n \), and the refraction angle (e.g. camera angle relative to the wall-normal direction). Fernholz et al. (1996) reported mean skin friction values with an accuracy of \( \pm 4\% \) (Osterlund, 1999; Bourassa and Thomas, 2009). This OFI technique, however, is not suitable to define the skin friction on a rough surface. Recently Krogstad and Efros (2010) produced a friction balance for the measurement of skin friction of a rough surface. A segment of rough surface was attached on top of the micro force balance and positioned within the entire fetch of the rough surface panels with a minute gap of 0.6 mm. The horizontal movement of this floating
segment was converted to the drag force, yielding $\tau_w$. This direct measurement of the wall shear stress over a rough surface may provide a way to define the skin friction regardless of the surface condition; however, the complexity and calibration of the floating element (here, the micro force balance) must be carefully built in the system of the measurement and can be rather expensive and elaborate to construct.

1.3 Present study

The present effort explores the combined impact of highly-irregular surface roughness replicated from a damaged turbine blade and moderate FPG conditions ($K \approx 2.2 - 4.1 \times 10^{-7}$) on a turbulent boundary layer. For a low pressure turbine, the Re based on axial chord and inlet velocity is 60,000 and the acceleration parameter is $K \approx 3.8 \times 10^{-6}$ at the suction surface of the blade (Bons, 2012). Particle-image velocimetry (PIV) measurements in both the streamwise–wall-normal ($x-y$) plane and wall-normal-spanwise ($y-z$) planes were conducted to investigate the combined impact of FPG and roughness conditions by comparison with additional measurements of flow over a smooth wall under identical FPG conditions. The focus of the analysis is on both the statistical and structural modifications imposed by the combined impact of surface roughness and FPG conditions. The objectives of this work include:

- Understanding of statistical modifications imposed by the combined impact of FPG and surface roughness effects;
- Assessing the structural attributes of the flow over a smooth- and rough surface under identical FPG condition;
- Understanding the characteristics of the large-scale flow structures analyzed with proper orthogonal decomposition.

Chapter 2 summarizes the experimental methodology employed, while chapters 3 and 4 detail the results of these experiments. In particular, chapter 3 reports the statistical analysis of the experimental data while chapter 4 addresses the structural modifications imposed by the combined FPG and surface roughness effects. Finally, chapter 5 reports the conclusions of this effort as well as future work that could be conducted.
Chapter 2

Experiments

This chapter details the flow facility and particle image velocimetry (PIV) measurements undertaken in the streamwise-wall-normal ($x - y$) plane and wall-normal-spanwise ($y - z$) plane of turbulent boundary layers over a highly irregular rough surface under favorable-pressure gradient (FPG) conditions. Smooth-wall measurements in both image planes were conducted under the same FPG conditions for comparison with the rough surface data. In addition, a limited number of smooth-wall ZPG experiments were conducted to evaluate the effect of favorable pressure gradient conditions.

2.1 Experimental flow facility

All experiments were conducted in an open-circuit Eiffel-type, boundary-layer wind tunnel facility (Meinhart, 1994). The dimensions of the facility are 20 m long, 3.40 m wide, and 2.49 m tall. Figure 2.1 shows that the wind tunnel facility consists of three major sections: a conditioning section, including inlet and contraction area, a test section, and an exhausting section including the diffuser, fan and acoustic diffuser. Air is drawn to the conditioning section through an elliptical inlet to avoid flow separation and travels through a series of mesh screens and honeycomb in the settling chamber before it is accelerated through a contraction to obtain a nearly top hat inlet velocity profile with low turbulence levels (approximately 0.16%). The contraction section has a contraction area ratio $CR = 10$ and guides the flow into the test section. Following the test section, air enters the exhausting section through a long low-angle diffuser that transitions the cross sectional area from rectangular to circular shape to avoid flow separation while the flow expands. At the end of the exhausting section, the fan discharges the air into the room through an acoustic diffuser to damp aerodynamic noise.
Figure 2.1: (a) Schematic of the low-turbulence wind tunnel facility [Adapted from Meinhart (1994)]; (b) Detailed dimensions of the test section. (−−: ceiling profile for FPG set up; −−−: ceiling profile for ZPG set up)

The test section of the tunnel is 6 m long, 45.7 cm tall and 91.4 cm wide, and all boundary layers were formed on a smooth-wall boundary-layer plate suspended 100 mm above the bottom wall of the tunnel. This plate consists of two 3-m long and 91.4-cm wide smooth-wall sections smoothly joined at the streamwise center. It has transparent lateral and bottom windows with float glass insets 61 cm wide by 274.3 cm long to facilitate optical access from below. The test section of this facility is designed to ensure two-dimensionality of the flow along the tunnel’s spanwise centerline by allowing side-wall boundary layer growth less than 9% of the total width of the test section. The test section consists of four 152.4 cm long modules with eight plexiglass side windows that are 45.7 cm high and 111.8 cm long. There are two glass inserts in the plexiglass sides of the last module to transmit high-intensity laser light. The height of the test section is set by an adjustable false ceiling that smoothly joins to the exit of the contraction section and continues through the end of the test section into the diffuser section. There are static pressure taps positioned along the streamwise length of the boundary-layer plate located at 30.5 cm intervals to measure streamwise pressure gradient. Zero-pressure-gradient conditions were achieved by adjusting the ceiling height until all the pressure taps gave the same reading within the uncertainty of the pressure transducer.
Favorable-pressure-gradient conditions were achieved along the latter 3.25 m of the test section by linearly decreasing the height of the tunnel via the adjustable ceiling at an angle of $\alpha = 4^\circ$. At this angle, the acceleration parameter was $K \approx 4.2 - 5.3 \times 10^{-7}$ for a free-stream velocity of $U_e \approx 10$ m/s and $K \approx 2.3 - 4.0 \times 10^{-7}$ at $U_e \approx 16$ m/s. These velocities were chosen to achieve measurable FPG conditions within the wind tunnel capability. Due to the limit of the height of the test section, the first streamwise half of the test section ceiling (2.75 m) was raised to its maximum height to avoid interactions between the boundary layers that developed on the ceiling and the boundary-layer plate [see figures 2.1, 2.3(a)]. The boundary layers were tripped approximately 25 cm downstream of the elliptical leading edge of the boundary-layer plate with a 4.7-mm diameter circular rod. A PVC foam gasket was used to seal all the edges of boundary layer plate for minimization of cross flow. In addition, two 2.54 cm radius wooden fillets were attached between the side wall of test section and the boundary layer plate to avoid corner vortices.

Since the wind tunnel is open circuit, the room air was uniformly seeded by olive oil droplets with $\approx 1\mu$m diameter generated by nine Laskin nozzles with a working pressure of $\approx 60$ psi. Each nozzle had its own olive oil container and a vertical pipe with multiple exits to broadly disperse the seeding. This Laskin nozzle system was positioned upstream of the inlet to the wind tunnel distributed uniformly along the spanwise direction. Prior to each measurements, olive oil droplets were generated for a few minutes and seeded continuously during the experiment in the closed room to ensure nearly homogeneous mixture of air and seeding.

### 2.2 Highly irregular rough surface

As mentioned earlier, the goal of this research is to reveal the underlying structural modifications of the flow imposed by combined FPG and roughness conditions. The roughness considered here is based on the measured roughness patterns from an in-service turbine blade damaged by different mechanisms such as spallation, pitting, and deposition of foreign materials. Such roughness topographies and their characteristics were first documented by Bons et al. (2001). The topography associated with a turbine blade damaged by deposition of foreign materials was graciously provided to our research group by Professor Jeffrey P. Bons of Ohio State University. This rough surface is the same as that originally fabricated and studied by Wu and Christensen (2007) and Wu and
Figure 2.2: (a) Contour plot of a portion of the rough surface. The streamwise and spanwise positions of the measurement planes are demarcated by red and yellow lines. (b) Probability density function (pdf) of the roughness amplitude about the mean elevation (—) contrasted with a Gaussian distribution with an equivalent RMS (—) [Adapted from Wu and Christensen (2007)].

Christensen (2010) as well as Mejia-Alvarez and Christensen (2010) under ZPG conditions and is a scaled version of a profilometric surface scan of a turbine blade damaged by deposition of foreign materials. Since the original roughness heights of the damaged turbine blade surface were on the order of tens to hundred microns (Bons et al., 2001), it was scaled up in all three dimensions to generate fully-rough conditions for the relatively thick boundary layers generated by the flow facility employed (δ ≈ 50 mm for FPG, δ ≈ 100 mm for ZPG) at the Reynolds numbers considered herein. The average peak-to-valley roughness height of this surface is $k = 4.25 \text{ mm}$ while the root-mean square (RMS) roughness height, $k_{\text{rms}}$, is 1.0 mm. Figure 2.2(a) presents a topographical map of the rough surface, which is marked by a broad range of topographical scales arranged in an irregular manner. The roughness elements are elliptical in shape aligned in the streamwise direction and are attributable to cumulative deposition of foreign materials on the blade surface. As shown in figure 2.2(b), the probability density function (pdf) of surface elevation is close to a Gaussian distribution. More detailed information on fabrication of this scaled version of damaged turbine blade roughness can be found in Wu (2008) and Mejia-Alvarez (2010).

As described in Wu and Christensen (2007) and Mejia-Alvarez and Christensen (2010), a 3-m long replica of this topography was achieved by mirroring it in both the streamwise and spanwise directions and fabricated with a powder-deposition printer with a spatial resolution of 80 µm in the three directions. Over sixty individual tiles of each roughness case with a maximum footprint of
2525 × 30 mm² and a mean thickness 6 mm were fabricated and each contained two mirror images of the basic pattern. These roughness tiles were mounted on cast aluminum plates and placed along the downstream half of the boundary-layer plate by adjusting its height above the bottom wall of the tunnel such that the mean elevation of the roughness was coincident with the upstream smooth-wall conditions. Thus, the boundary layers under study were allowed to initially develop over the first 3 m of the smooth boundary-layer plate followed by an additional 3 m of development over the roughness. As noted earlier, in all cases the flow was tripped with a cylindrical rod near the upstream end of the boundary-layer plate and all measurements were conducted approximately 2.5 m downstream of the leading edge of the roughness.

2.3 Streamwise-wall-normal \((x − y)\) plane measurements

Two-dimensional PIV (2D PIV) was used to acquire over two thousand statistically independent, instantaneous velocity \((u, v)\) fields in the streamwise-wall-normal \((x − y)\) plane for both FPG-SM and FPG-Rough conditions. A dual-cavity Nd:YAG laser (200 mJ/pulse, 5 ns pulse duration, Quantel) served as the illumination source for the 1 \(\mu\)m tracer particles of olive oil generated by Laskin nozzles. The flow field was illuminated with a 500 \(\mu\)m thick laser lightsheet formed by the laser and a combination of cylindrical and spherical lenses. The laser was mounted on top of the wind tunnel and a high energy mirror was used to direct the light sheet into the wind tunnel normal to the boundary layer plate and parallel to the flow direction [see figure 2.3 (b)]. The scattered light from the tracer particles was imaged by a 4k × 2.8k, 12-bit frame-straddle CCD camera (TSI 11MP) viewing the 100 mm × 150 mm \((x × y)\) field of view. The roughness at the measurement location was painted black and sprayed with Rhodamine-B-doped paint to minimize reflections of the laser light. Rhodamine-B absorbed the green light (wavelength 520 – 570 nm) and fluoresces at a higher wavelength. A notch filter upstream of the camera suppressed the fluoresced light and therefore reduced the imaging of reflections of light from the complex roughness. Measurements were conducted under the aforementioned FPG conditions for both smooth and rough wall cases. For comparison, a smooth-wall case under ZPG conditions was also conducted in this measurement plane. All measurements were conducted 2.5 m downstream of the leading edge of the roughness.

All relevant parameters for ZPG-SM, FPG-SM, and FPG-Rough cases of \(x − y\) experiments at
Table 2.1: Relevant experimental parameters for streamwise–wall-normal plane 2D PIV measurements.

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface</th>
<th>$U_e$ [m/s]</th>
<th>Re</th>
<th>$\theta$ [deg]</th>
<th>$K \times 10^{-7}$</th>
<th>$\delta$ [mm]</th>
<th>$k$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG-SM 20 Hz</td>
<td>Smooth</td>
<td>10.33</td>
<td>8854</td>
<td>0</td>
<td>-</td>
<td>108.8</td>
<td>-</td>
</tr>
<tr>
<td>FPG-SM 20 Hz</td>
<td>Smooth</td>
<td>10.62</td>
<td>2164</td>
<td>4</td>
<td>3.89 - 4.11</td>
<td>48.29</td>
<td>-</td>
</tr>
<tr>
<td>FPG-SM 30 Hz</td>
<td>Smooth</td>
<td>15.89</td>
<td>2709</td>
<td>4</td>
<td>2.48 - 3.96</td>
<td>43.35</td>
<td>-</td>
</tr>
<tr>
<td>FPG-Rough 20 Hz</td>
<td>Rough</td>
<td>10.85</td>
<td>3125</td>
<td>4</td>
<td>3.54 - 3.74</td>
<td>56.93</td>
<td>4.25</td>
</tr>
<tr>
<td>FPG-Rough 30 Hz</td>
<td>Rough</td>
<td>16.37</td>
<td>4496</td>
<td>4</td>
<td>2.28 - 3.63</td>
<td>54.08</td>
<td>4.25</td>
</tr>
</tbody>
</table>

Table 2.2: Relevant experimental parameters for wall-normal–spanwise plane stereo PIV measurements.

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface</th>
<th>$U_e$ [m/s]</th>
<th>Re</th>
<th>$\theta$ [deg]</th>
<th>$K \times 10^{-7}$</th>
<th>$\delta$ [mm]</th>
<th>$k$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPG-SM 20 Hz</td>
<td>Smooth</td>
<td>10.25</td>
<td>2380</td>
<td>4</td>
<td>4.36</td>
<td>50.09</td>
<td>-</td>
</tr>
<tr>
<td>FPG-SM 30 Hz</td>
<td>Smooth</td>
<td>15.22</td>
<td>3476</td>
<td>4</td>
<td>3.12</td>
<td>51.41</td>
<td>-</td>
</tr>
<tr>
<td>FPG-Rough 20 Hz</td>
<td>Rough</td>
<td>10.69</td>
<td>2994</td>
<td>4</td>
<td>3.86</td>
<td>65.43</td>
<td>4.25</td>
</tr>
<tr>
<td>FPG-Rough 30 Hz</td>
<td>Rough</td>
<td>16.20</td>
<td>4466</td>
<td>4</td>
<td>2.56</td>
<td>65.96</td>
<td>4.25</td>
</tr>
</tbody>
</table>

20 Hz and 30 Hz wind-tunnel inverter frequencies are listed in table 2.1. Note that the boundary layer thickness, $\delta$, is taken at the wall-normal position where the mean streamwise velocity equals 99% of the free-stream velocity.

### 2.4 Wall-normal-spanwise ($y - z$) plane measurements

Stereoscopic PIV was used to acquire large ensembles of instantaneous, three-component velocity ($u, v, w$) fields in the wall-normal–spanwise ($y - z$) plane for both FPG-SM and FPG-Rough conditions. In this measurement arrangement, the primary velocity component (streamwise; $u$) was normal to the lightsheet while the in-plane velocity components represented the wall-normal ($v$) and spanwise ($w$) velocity components (primarily turbulent fluctuations).

The streamwise location of this cross-flow measurement plane was purposely positioned to sit within the streamwise field of view of the aforementioned 2D PIV measurements in the $x - y$ plane for the same flow conditions as a means of validating the present cross-flow measurements. Figure 2.3 (a) shows light-sheet locations for the $x - y$ and $y - z$ plane measurements. A 1-mm thick lightsheet perpendicular to the flow direction and the boundary-layer plate was formed and
propagated into the tunnel along the spanwise direction from a glass side wall. For stereoscopic PIV measurement, lightsheet thickness must be defined properly: it should be thick enough to capture the out-of-plane particulate motions, however, it should also be able to yield crisp, in-focus particle images as well. The laser was mounted on the side of the wind tunnel and a combination of spherical and cylindrical lenses was used to direct the light sheet into the wind tunnel normal to the boundary layer plate and the flow direction. The images of the scattered light was captured by two 4k × 2.8k, 12-bit frame-straddle CCD cameras (11MP TSI) viewing the 79 mm × 170 mm (y × z) field of view from upstream at an angle of θ = ±45° with respect to the x direction axis [see figure 2.3(c)]. The angle between the lens and the CCD array of each camera was rotated to satisfy the Scheimpflug condition to ensure uniform focus in both cameras across the entire field.
of view. The resulting image pairs from each camera were interrogated in an manner identical to that employed for the aforementioned 2D PIV measurements as described in §2.5. The relevant experimental parameters for the 20 Hz and 30 Hz cases are presented in table 2.2.

Following interrogation, recombination of these pairs of 2D velocity fields into a single, three-component velocity field in the measurement plane required calibration of the imaging system to properly map the image coordinate system to the object plane defined by the laser lightsheet. A target consisting of dots spaced at 5 mm intervals in both the horizontal and vertical directions was carefully aligned to be coincident with the laser lightsheet. Images of this target were then acquired by both cameras at this position as well as with the target translated ±250 μm upstream and downstream of lightsheet center. Using images of the target at multiple depths, a calibration mapping function was generated to map the two, 2-D image planes to the 3-D space defined by the laser lightsheet using the least-squares method of Soloff et al. (1997). Thus, the out-of-plane fluid motion was discerned from the distinct views of the tracer-particle motion within the laser lightsheet as imaged by the two cameras.

Figure 2.4 presents the target images acquired by each camera in the stereo imaging arrangement. The fiducial mark (diamond shape) is located at \( x = 0 \) mm and \( y = 73.15 \) mm. To minimize and regulate the gradient of light intensity shown in figure 2.4, Fiji J software was used to preprocess each pair of target images before processing the perspective calibration. This calibration was done using TSI Insight 8 software. A total of five calibration image pairs acquired by target
translation were used to determine the calibration of stereoscopic PIV measurements within the 1 mm lightsheet thickness. Figure 2.5 presents schematics of the calibration grids as viewed by the two cameras as well as the mapping of these grids onto a common reference frame using the derived mapping function.

The cross-flow implementation of stereo PIV was validated by comparing the statistics of the present FPG-SM case with those calculated from the aforementioned 2D PIV measurements in the $x-y$ plane under identical conditions. Figure 2.6 presents wall-normal profiles of the Reynolds stress components $\overline{u'^2}$, $\overline{v'^2}$ and $-\overline{u'v'}$ from both measurements. Excellent consistency is noted in all three components between the $x-y$ 2D PIV measurements and the present stereo PIV measurements in the cross-flow plane ($y-z$), validating the methodology employed in the latter case. A similar comparison for the FPG-Rough case could not be made since the flow, particularly close to the wall, is quite sensitive to the local topographical conditions. Nevertheless, the consistency in the FPG-SM results is quite encouraging.
Figure 2.6: Comparison of Reynolds stress profiles from 2D PIV in the streamwise–wall-normal plane (△: $x - y$) and the present wall-normal–spanwise plane (□: $y - z$) measurements for FPG-SM conditions for FPG-SM 20Hz case.

2.5 PIV Interrogation details

The two thousand statistically-independent image pairs acquired in the $x - y$ plane for each flow condition (see table 2.1) were interrogated using a recursive, two-frame cross-correlation method with a final interrogation window size of $16^2$ pixels with an overlap of 50%. The resulting 2D velocity vector fields were then validated using standard-deviation and magnitude-difference comparisons to remove erroneous velocity vectors. A valid vector yield of 95–97% was achieved, minimizing the need for interpolation of holes where the erroneous vectors were removed. Finally, each velocity field was low-pass filtered with a narrow Gaussian filter to remove noise associated with frequencies larger than the sampling frequency of the interrogation. For the wall–normal-plane of measurements, the resulting image pairs from each camera were interrogated in an manner identical to that employed for the aforementioned 2D PIV measurements. Table 2.3 summarizes the specific interrogation and validation parameters utilized for all experiments. Table 2.4 shows corresponding magnifications and grid spacings for all cases.
Table 2.3: PIV Interrogation and validation parameters for \(x - y\) and \(y - z\) plane measurements.

<table>
<thead>
<tr>
<th>Interrogation parameters</th>
<th>1(^{st}) pass</th>
<th>2(^{nd}) pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-offset (pix)</td>
<td>12</td>
<td>determined from 1(^{st}) pass</td>
</tr>
<tr>
<td>y-offset</td>
<td>0</td>
<td>determined from 1(^{st}) pass</td>
</tr>
<tr>
<td>1(^{st}) window (pix(^2))</td>
<td>24 x 28</td>
<td>32 x 32</td>
</tr>
<tr>
<td>2(^{nd}) window</td>
<td>32 x 40</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Filters for removal of erroneous vectors</th>
<th>Absolute range</th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_{\text{min}}, U_{\text{max}}) (pix)</td>
<td>0, 20</td>
<td>Neighborhood size</td>
<td>3\times 3</td>
</tr>
<tr>
<td>(V_{\text{min}}, V_{\text{max}}) (pix)</td>
<td>-5, 5</td>
<td>Tolerance</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vector substitution parameters for 1(^{st}) validation pass</th>
<th>Mean</th>
<th>Gaussian Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neighborhood size (grid points)</td>
<td>3</td>
<td>Neighborhood size</td>
</tr>
<tr>
<td>Iteration</td>
<td>5</td>
<td>Gaussian radius (pix)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vector substitution parameters for 2(^{nd}) validation pass</th>
<th>Mean</th>
<th>Gaussian Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neighborhood size</td>
<td>3</td>
<td>Neighborhood size</td>
</tr>
<tr>
<td>Iteration</td>
<td>1</td>
<td>Gaussian radius</td>
</tr>
</tbody>
</table>

Table 2.4: Magnification and grid resolution of all experiments.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Case</th>
<th>Magnification ((\mu\text{m/pixel}))</th>
<th>(\Delta x) ((\mu\text{m}))</th>
<th>(\Delta y) ((\mu\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x - y)</td>
<td>ZPG-SM 20 Hz</td>
<td>35.31</td>
<td>282.4</td>
<td>282.4</td>
</tr>
<tr>
<td></td>
<td>FPG-SM 20 Hz</td>
<td>39.29</td>
<td>314.3</td>
<td>314.3</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 20 Hz</td>
<td>39.13</td>
<td>313.0</td>
<td>313.0</td>
</tr>
<tr>
<td></td>
<td>FPG-SM 30 Hz</td>
<td>39.31</td>
<td>314.5</td>
<td>314.5</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 30 Hz</td>
<td>39.07</td>
<td>312.5</td>
<td>312.5</td>
</tr>
<tr>
<td>(y - z)</td>
<td>FPG-SM 20 Hz</td>
<td>-</td>
<td>337.0</td>
<td>336.2</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 20 Hz</td>
<td>-</td>
<td>335.0</td>
<td>336.0</td>
</tr>
<tr>
<td></td>
<td>FPG-SM 30 Hz</td>
<td>-</td>
<td>337.0</td>
<td>336.2</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 30 Hz</td>
<td>-</td>
<td>335.0</td>
<td>336.0</td>
</tr>
</tbody>
</table>
2.6 Uncertainty

Uncertainty analysis is used to quantify an estimate of reasonable bounds on random error. Random error is attributed to the randomness of turbulent flow and the measurement error in PIV. The source of error in PIV originates from noise in the recorded image, pixel-locking bias error and the estimation of sub-pixel particle displacement. The recorded images are irregular in shape due to the noise contamination such as background speckles, aberration of the lenses, or noise in the recording medium (Adrian and Westerweel, 2011). When the particle size is less than two pixels, bias error that is the mean difference between measured and actual displacement, become significant. The bias error depends on the pixel resolution and the interpolation methods of sub-pixel displacement. The rms displacement error is minimum (approximately 2%) when particle diameter is 2-3 pixels (Westerweel, 2000).

Random error induced by the turbulent flow is also called sampling error since the turbulent statistics converge with the sufficient number of samples. Extensive explanation of the uncertainty analysis for 2D and stereoscopic PIV measurements are found in Mejia-Alvarez (2010). The sampling error can be expressed as

\[ \delta_s \left( \overline{\theta} \right) = \frac{S(\nu)}{\sqrt{n}}, \]  

(2.1)

where \( \theta \) is a given random variable, \( \delta(\overline{\theta}) \) represents the estimator of the standard error of the mean, \( n \) is the sample size and \( S(\theta) \) is the unbiased estimator of the standard deviation of the sample data defined as

\[ S(\theta) = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (\theta_j - \overline{\theta})^2}. \]  

(2.2)

If \( \theta \) is velocity, the parenthesis of the \( S(\theta) \) can be interpreted as a velocity fluctuation \( u' = u_j - U \) and is rewritten as

\[ S(u) = \sqrt{\frac{n}{n-1} \sum_{j=1}^{n} (u'_j)^2} = \sqrt{\frac{n}{n-1} \langle u'^2 \rangle^{1/2}}. \]  

(2.3)

Thus, the sampling error of a turbulent velocity is

\[ \delta_s(U) = \frac{1}{\sqrt{n}} \left( \sqrt{\frac{n}{n-1} \langle u'^2 \rangle^{1/2}} \right) = \frac{\langle u'^2 \rangle^{1/2}}{\sqrt{n-1}}. \]  

(2.4)
For simplicity, the maximum value of the turbulent intensity $\langle u'^2 \rangle$ is used to examine the upper bound of the sampling error, though the sampling error varies with respect to the wall-normal position. The sample size $n$ for the $x-y$ and $y-z$ plane measurements is the product of the total number of instantaneous realizations and the number of grid points in the streamwise (for $x-y$ plane) and spanwise (for $y-z$ plane) direction.

Moffat (1988) described the theory of error analysis. A result $R$ (random variable) from a set of measurements is represented by

$$R = R(X_1, X_2, X_3, \cdots, X_n), \quad (2.5)$$

where a variable $X_i$ represents the observation in a single-sample experiment or the mean of a set of $N$ observations in a multiple-sample experiment. $X_i$ has a known uncertainty $\delta X_i$. Uncertainty in a single measurement can be expressed as

$$\delta R_{X_i} = \frac{\partial R}{\partial X_i} \delta X_i, \quad (2.6)$$

where $\partial R/\partial X_i$ is the sensitivity coefficient for the $R$ with respect to the measurement $X_i$. The overall uncertainty in the result $R$ is

$$\delta R = \left\{ \sum_{i=1}^{N} \left( \frac{\partial R}{\partial X_i} \delta X_i \right)^2 \right\}^{1/2}, \quad (2.7)$$

By applying the root-sum square of the error method described above (Moffat, 1988), uncertainty of the product of two random variables can be expressed as (Mejia-Alvarez, 2010)

$$\delta(\langle \theta \gamma \rangle) = \left\{ \left[ \delta(\theta) \left( \frac{1}{n} \sum_{j=1}^{n} \gamma_j \right) \right]^2 + \left[ \delta(\gamma) \left( \frac{1}{n} \sum_{j=1}^{n} \theta_j \right) \right]^2 \right\}^{1/2}, \quad (2.8)$$

where $\gamma$ and $\theta$ can be regarded as components of the velocity fluctuation. Substituting the averages of each fluctuations in the parenthesis by their sampling error, the above equation is

$$\delta(\langle \theta \gamma \rangle) = \left\{ \left[ \delta(\theta) \delta(\gamma) \right]^2 + \left[ \delta(\gamma) \delta(\theta) \right]^2 \right\}^{1/2}. \quad (2.9)$$
Table 2.5: Relevant velocity scaling for all experiments.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Case</th>
<th>$U_e$ (m/s)</th>
<th>$\sigma_u$ (m/s)</th>
<th>$\sigma_v$ (m/s)</th>
<th>$\sigma_w$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG-SM</td>
<td>20Hz</td>
<td>10.33</td>
<td>0.98</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>FPG-SM</td>
<td>20Hz</td>
<td>10.62</td>
<td>0.97</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough</td>
<td>20Hz</td>
<td>10.85</td>
<td>1.10</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>FPG-SM</td>
<td>30Hz</td>
<td>15.89</td>
<td>1.30</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough</td>
<td>30Hz</td>
<td>16.53</td>
<td>1.77</td>
<td>0.88</td>
<td></td>
</tr>
</tbody>
</table>

$x - y$

<table>
<thead>
<tr>
<th>Plane</th>
<th>Case</th>
<th>$U_e$ (m/s)</th>
<th>$\sigma_u$ (m/s)</th>
<th>$\sigma_v$ (m/s)</th>
<th>$\sigma_w$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPG-SM</td>
<td>20Hz</td>
<td>10.25</td>
<td>0.84</td>
<td>0.32</td>
<td>0.52</td>
</tr>
<tr>
<td>FPG-Rough</td>
<td>20Hz</td>
<td>10.69</td>
<td>1.02</td>
<td>0.49</td>
<td>0.76</td>
</tr>
<tr>
<td>FPG-SM</td>
<td>30Hz</td>
<td>15.22</td>
<td>1.12</td>
<td>0.45</td>
<td>0.76</td>
</tr>
<tr>
<td>FPG-Rough</td>
<td>30Hz</td>
<td>16.20</td>
<td>1.55</td>
<td>0.70</td>
<td>1.10</td>
</tr>
</tbody>
</table>

$y - z$

The uncertainty of the normalized statistics for a single variable is

$$\delta \left( \frac{\langle \theta \rangle}{\rho} \right) = \frac{1}{\rho} \left\{ [\delta(\langle \theta \rangle)]^2 + \left[ \langle \theta \rangle \left( \frac{\delta(\rho)}{\rho} \right) \right]^2 \right\}^{1/2}$$  \hspace{1cm} (2.10)

and the uncertainty of the normalized product of two variables is calculated by

$$\delta \left( \frac{\langle \theta \gamma \rangle}{\rho \omega} \right) = \frac{1}{\rho \omega} \left\{ [\delta(\langle \theta \gamma \rangle)]^2 + \left[ \langle \theta \gamma \rangle \left( \frac{\delta(\rho)}{\rho} \right) \right]^2 + \left[ \langle \theta \gamma \rangle \left( \frac{\delta(\omega)}{\omega} \right) \right]^2 \right\}^{1/2}.$$  \hspace{1cm} (2.11)

For the normalization factor, the free stream velocity ($U_e$) is chosen for the uncertainty of the single point statistics and the RMS velocity fluctuations ($\sigma_u, \sigma_v, \sigma_w$) are used for the two-point statistics. The relative percentage of uncertainty normalized by a characteristic value of the random variable is

$$\epsilon(\theta) = \frac{\delta(\theta)}{\theta_c} \times 100.$$  \hspace{1cm} (2.12)

where the characteristic value ($\theta_c$) is the free stream velocity, $U_e$ for the percentage uncertainty of the ensemble velocity, while the standard deviations of velocity components ($\sigma_u, \sigma_v, \sigma_w$) are chosen for the two-point velocity correlations.

Apart from the above mentioned random error, there is a registration error in a stereo PIV calculation. When the calibration target and the light sheet are misaligned, the reconstructed displacements in three directions are different than the original data from the same position in the light sheet. Such difference is called registration error (van Doorne and Westerweel, 2007). The reconstruction of the three vector components based on the mismatched vector fields from two
Table 2.6: Random errors on an ensemble basis.

<table>
<thead>
<tr>
<th>Plane Case</th>
<th>n</th>
<th>$\delta_s(U)$ (m/s)</th>
<th>$\delta_s(V)$ (m/s)</th>
<th>$\delta_s(W)$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG-SM 20Hz</td>
<td>928 000</td>
<td>0.0010</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 20Hz</td>
<td>934 000</td>
<td>0.0010</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>930 000</td>
<td>0.0011</td>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>930 000</td>
<td>0.0013</td>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>930 000</td>
<td>0.0018</td>
<td>0.0009</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.7: Percentage of uncertainty in velocity ensembles.

<table>
<thead>
<tr>
<th>Plane Case</th>
<th>$\epsilon(U/\overline{U})$ (%)</th>
<th>$\epsilon(V/\overline{U})$ (%)</th>
<th>$\epsilon(W/\overline{U})$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG-SM 20Hz</td>
<td>0.010</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 20Hz</td>
<td>0.010</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>0.010</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>0.008</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>0.010</td>
<td>0.006</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.8: Percentage uncertainty of two point correlations.

<table>
<thead>
<tr>
<th>Plane Case</th>
<th>$\epsilon(u'u')/\sigma_u^2$ (%)</th>
<th>$\epsilon(v'v')/\sigma_v^2$ (%)</th>
<th>$\epsilon(u'v')/\sigma_u\sigma_v$ (%)</th>
<th>$\epsilon(w'w')/\sigma_w^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG-SM 20Hz</td>
<td>0.27</td>
<td>5.46</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 20Hz</td>
<td>0.28</td>
<td>1.92</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>0.19</td>
<td>1.35</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>0.12</td>
<td>1.52</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>0.05</td>
<td>0.37</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

26
Table 2.9: Residual errors on an ensemble basis.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Case</th>
<th>$0 \leq y/\delta \leq 0.3$</th>
<th>$0.3 &lt; y/\delta &lt; 0.7$</th>
<th>$0.7 \leq y/\delta \leq 1$</th>
<th>$0 \leq y/\delta \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y-z$</td>
<td>FPG-SM 20Hz</td>
<td>0.15</td>
<td>0.13</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 20Hz</td>
<td>0.22</td>
<td>0.12</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>FPG-SM 30Hz</td>
<td>0.15</td>
<td>0.17</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>FPG-Rough 30Hz</td>
<td>0.27</td>
<td>0.16</td>
<td>0.15</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Cameras also increases the registration error, but the mapping error is very small. Hutchins et al. (2005) showed the residual error from the mapping ranges from 0.2 pixel near the wall to 0.15 pixel towards the boundary layer. In present study, the ensemble averaged residual error in stereo PIV measurements was averaged along the spanwise direction ($y-z$). Table 2.9 shows the magnitudes of residual error averaged along the near wall region ($0 \leq y/\delta \leq 0.3$) and towards the boundary layer ($0.7 \leq y/\delta \leq 1$). Residual error for smooth wall cases is 0.15 - 0.2 pixel near the wall while it is 0.15 pixel away from the wall. The magnitude of residual error in the rough wall case ranges from 0.2 - 0.27 pixel near the wall to 0.18 - 0.23 pixel away from the wall. van Doorne and Westerweel (2007) quantified registration error in a laminar pipe flow. They found that misalignments as small as 0.1mm will lead to large registration errors. When the calibration planes and the light sheet are half the light sheet thickness, such misalignments lead to 5-8% error in the reconstructed in-plane displacement from the centerline of the pipe. The in-plane displacement is called the disparity map. Self-calibration proposed by Wienke (2005) can be used to correct the image disparity by adjusting the coefficients of the image mapping function. This method provides a possible way to minimize the registration error in a stereo PIV measurement.
Chapter 3

Results: Turbulence Statistics

This chapter presents characteristics of the single-point statistical analysis of the streamwise–wall-normal plane and wall-normal–spanwise plane PIV datasets for the flow over both smooth and rough surfaces under FPG conditions. Turbulent statistics from the two orthogonal plane measurements provide insight into the statistical modifications imposed by PFG conditions under these surface conditions. Flow over a smooth surface under ZPG conditions is also provided in the $x - y$ plane analysis.

3.1 Streamwise–wall-normal plane measurements

Ensemble averaging of the two thousand instantaneous velocity fields $(u, v)$ in streamwise–wall-normal plane $(x - y)$ for both smooth and rough surface conditions was used here to evaluate mean velocity, Reynolds stresses, and quadrant analysis. In some cases, line averaging in the streamwise direction is also employed to facilitate quantitative comparison of wall-normal profiles of statistics under the various flow conditions studied. The profiles computed from line averaging of the ensemble averaged statistics provide a measure of the bulk (mean) effect. The outer-layer variables, free-stream velocity, $U_e$ and boundary layer thickness, $\delta$ are used for normalization of these single-point statistics.

3.1.1 Mean velocity

Figure 3.1 presents the mean streamwise velocity profiles for the various flow and surface conditions at the two different Reynolds numbers studied. The free-stream velocity, $U_e$, was determined from the measured profiles as when the gradient of the line-averaged streamwise velocity $U$ plateaued with respect $y$. Following this, the corresponding boundary-layer thickness, $\delta$, was evaluated as
Figure 3.1: Mean streamwise velocity profile normalized by $U_e$ for (a) 20 Hz and (b) 30 Hz cases. Not all data points shown for clarity. ○: ZPG-SM, □: FPG-SM and △: FPG-Rough.

the $y$ location where the mean streamwise velocity $U$ reached 99% of $U_e$. Consistent with previous studies (Tay et al., 2009, for example), the velocity gradient near the wall in the FPG-SM case is much larger than that of the ZPG-SM case, indicating a substantial decrease of wall-normal growth of the boundary layer due to strong flow acceleration. The velocity gradient induced by pressure gradient can have a significant effect on the turbulent kinetic energy, Reynolds stresses, and turbulence production (Agelinaaab and Tachie, 2008). With the presence of surface roughness, however, the velocity gradient is retarded or flattened in the FPG-Rough case compared with the FPG-SM case, indicating opposing influence of FPG conditions and surface roughness. Similar trends are also observed at the higher Reynolds number as shown in figure 3.1 (b). Although $Re_\theta$ for the 30 Hz cases are 1.25–1.4 times higher than those of 20Hz cases, the mean velocity profiles appear $Re_\theta$ invariant. The previous study conducted by Tay et al. (2009) also reported the mean velocity profiles under mild FPG conditions to be independent of $Re_\theta$ based on more than a twofold increase in $Re_\theta$. Previous rough wall experiments under ZPG condition (Wu and Christensen, 2007; Mejia-Alvarez and Christensen, 2010) showed that the inner-scaled mean velocity profile of ZPG-Rough case was shifted downward relative to the smooth-wall profile due to the increased skin friction at the wall.

Figure 3.2 presents the ensemble-averaged streamwise velocity fields for the ZPG-SM, FPG-SM, and FPG-Rough cases. Compared to the ZPG-SM case, the tight near-wall contours of the FPG-
SM case highlight the intense velocity gradient in the near-wall region. In addition, the boundary layer thickness is clearly reduced due to the effect of FPG on a smooth surface. This boundary layer thinning effect under FPG implies the suppression of growth of vortical structures in the wall-normal direction. With the addition of surface roughness, however, these FPG induced effects become weaker via the opposing influence of FPG conditions and surface roughness. In particular, the FPG conditions act to suppress the vertical growth of the boundary layer while roughness tends to enhance it. Thus, the momentum deficit region close to the surface is reduced by FPG conditions, but increased by surface roughness. It is interesting to note that the broadening of the velocity contours due to the opposing influence of surface roughness and FPG condition renders these results similar to those of the ZPG-SM case. Cal et al. (2008) previously showed the similar trends based on LDV measurement of a turbulent boundary layer under FPG condition and sand grain roughness. Comparison of the smooth surface and the sand grid roughened surface under the identical FPG condition, they found significant enhancement of the boundary layer thickness and found its rate of change to be similar to the ZPG smooth surface case.

3.1.2 Reynolds stresses

Figure 3.3 presents profiles of Reynolds normal and shear stresses for flow over the smooth and rough surfaces under FPG conditions. All Reynolds stresses are dampened by FPG conditions in the wall-normal direction except in the near-wall region. Furthermore, the near-wall peaks of the Reynolds stresses for FPG-SM are shifted closer to the wall than for the ZPG-SM and FPG-Rough cases. In addition, the peak values of Reynolds normal and shear stresses of the FPG-SM case are higher than the ZPG-SM case. The previous study of Fernholz and Warnack (1998) also reported a notable increase in Reynolds normal and shear stresses in the near wall region ($y/\delta \geq 0.1$) in a FPG smooth-wall turbulent boundary layer ($K = 1.18 - 1.69(10^{-8})$). In addition, they found the increment of turbulence production in the inner layer due to the intense velocity gradient, $dU/dy$. The FPG-Rough case shows enhanced Reynolds stresses due to the stronger effect of surface roughness than FPG condition near the wall as well as retarded near-wall peaks—a key characteristic of ZPG rough-wall turbulent boundary layers (Wu and Christensen, 2007). However, for $y/\delta > 0.4$, the Reynolds stresses are dampened by FPG condition indicating that
FPG conditions overwhelms the impact of surface roughness far from the wall.

Figures 3.4 and 3.5 present contour plots of normalized Reynolds stresses and corresponding turbulent kinetic energy (TKE) for the ZPG-SM, FPG-SM and FPG-Rough cases at 20 and 30 Hz, respectively. Consistent with previous studies of smooth-wall turbulence, FPG conditions weaken the Reynolds stresses and focus the peak values closer to the wall compared to ZPG conditions. This trend is evident when comparing $\langle u' u' \rangle$ and $\langle u' v' \rangle$ of the ZPG-SM and FPG-SM results.

The momentum equation of two dimensional, incompressible turbulent boundary layer shows the streamwise pressure gradient $dP/dx$ is balanced by $\nu \frac{\partial^2 U}{\partial y^2}$ in the near wall region and both the velocity gradient and turbulent stresses are influenced by changes in pressure gradient. The positive streamwise velocity gradient $dU/dx$ from FPG conditions is related to the production of negative wall-normal velocity gradient $dV/dy$ in a two dimensional incompressible flow (Smits and Wood, 1985). Townsend (1961) suggested the production of $\partial V / \partial y$ tends to flatten the large eddies and reduce their contributions to the Reynolds stresses. The reduced eddies also yield a substantial attenuation of turbulence intensity since the turbulent kinetic energy transfers to the mean motion of the flow (Tay et al., 2009). With the addition of roughness to the FPG conditions, both the Reynolds normal and shear stresses are enhanced relative to the FPG-SM case. In addition, the wall-normal position of peak activity in these stresses is displaced appreciably away from the wall. This combined impact of roughness and FPG conditions is particularly evident in $\langle u' u' \rangle$ and $-\langle u' v' \rangle$.

All things considered, the magnitude of TKE is reduced under the FPG condition, while the intense TKE is observed for the FPG-Rough case due to the presence of surface roughness. This opposing influence of FPG and surface roughness can be found in many studies. Agelinchaab and Tachie (2008) concluded the mean flow and turbulence quantities (Reynolds stresses) are independent of FPG conditions from their measurement of flow over 2D transverse ribs under FPG conditions ($0.2 \leq K(10^{-6}) \leq 1.32$). Cal et al. (2008) reported the stronger dominance of surface roughness over FPG conditions ($0.17 \leq K(10^{-6}) \leq 2.6$). Despite the dampening of turbulent fluctuations by FPG, surface roughness promotes enhancement of the turbulence level and intensifies the Reynolds stresses.
3.1.3 Quadrant analysis

Quadrant analysis was first proposed by Lu and Willmarth (1973) and is used herein to quantify quadrant contributions to the mean Reynolds shear stress (RSS), $\langle u'v' \rangle$, in the FPG-SM and FPG-Rough cases. In this regard, negative contributions to $\langle u'v' \rangle$ are attributed to ejection ($Q_2 : u' < 0, v' > 0$) and sweep ($Q_4 : u' > 0, v' < 0$) events, while positive contributions are due to the inward ($Q_3 : u' < 0, v' < 0$) and outward ($Q_1 : u' > 0, v' > 0$) interactions. In quadrant analysis, the mean RSS at each wall-normal position is decomposed into contributions from four quadrants excluding a hyperbolic hole of size $H$ as

$$\langle u'v' \rangle_Q(x,y;H) = \frac{1}{P} \sum_{j=1}^{P} u'(x,y)v'(x,y)I_Q(x,y;H),$$

(3.1)

where $P$ is the total number of velocity vector fields and $I_Q$ is the indicator function defined as

$$I_Q(x,y;H) = \begin{cases} 
1 & \text{when } |u'(x,y)v'(x,y)|Q \geq H\sigma_u(x,y)\sigma_v(x,y), \\
0 & \text{otherwise},
\end{cases}$$

(3.2)

where $\sigma_u \equiv \langle u'^2 \rangle^{1/2}$ and $\sigma_v \equiv \langle v'^2 \rangle^{1/2}$ are root-mean-square (rms) values of the streamwise and wall-normal velocities, respectively. Here, the hyperbolic hole of size $H$ represents a threshold on the strength of the RSS-producing events considered in the analysis. For $H = 0$, all contributions to the mean RSS are considered while only increasingly intense RSS-producing events are included with increasing values of $H$.

Lu and Willmarth (1973) conducted smooth-wall ZPG turbulent boundary layer experiments and found the largest contributions to the Reynolds stresses originated from the dominant features of low momentum fluid by ejection near the wall and sweeps of high-momentum fluid from the outer region toward the wall. More recent studies indicate that these ejections and sweeps are induced by the induction of hairpin vortices and associated vortex packets near the wall (Adrian et al., 2000b). Lu and Willmarth (1973) reported ejections and sweeps account for 77% and 55% of the RSS in the near-wall region. In this study of ZPG-SM 20 Hz case, the contributions of ejections and sweeps correspond to 69% and 63% in the near-wall region when all instantaneous $u'v'$ events are included
(\(H = 0\)). When only strong \(u'v'\) events are considered \((H = 4)\), clear dominance of ejection events over sweep events is observed (62% and 41% contributions to RSS, respectively). Both inward and outward interactions show negligible contributions (3%) to the RSS in the near wall region. Krogstad and Skare (1995) revealed the significant dominance of sweep events near the wall in the strong APG turbulent boundary layer flow while similar contributions of sweeps and ejections were observed in the outer region. Also consistent to the enhancement of Reynolds stresses under APG condition, the total contribution of both sweep and ejection events were found to be higher than that of ZPG case. Coupled with these enhanced magnitudes of sweep and ejection events, outward interactions were found to be enhanced in the presence of APG condition, especially when only strong \(u'v'\) events \((H = 4)\) were accounted. Similar results were also reported in the mild APG flow studied by Aubertine and Eaton (2005). Drozdz et al. (2011) concluded these enhanced sweep events and outward interactions of ascending and descending high speed flow motions in the near wall region were due to the effect of the delayed vortical structures induced by the mean velocity deceleration under APG conditions.

In contrast, previous studies showed the significant dominance of ejection events over sweep events under FPG conditions (Bourassa and Thomas, 2009; Drozdz and Elsner, 2011). Since the FPG condition tends to suppress the vortical structures and dampens the Reynolds stresses, one would expect the significant reduction in both magnitudes and numbers of each quadrants compared with ZPG-SM case. The remained fewer ejection events under strong FPG condition were found to have more intensified magnitudes compared with ZPG-SM case while sweep events were nearly eliminated (Bourassa and Thomas, 2009). Coupled with the elimination of sweep events and enhanced ejection events by the negative streamwise velocity fluctuation, the magnitude of inward interactions \(Q_3\) was found to be increased in the near wall region, though its contribution to the Reynolds stresses is still small compared to other contributions. Drozdz and Elsner (2011) reported weaker outward interactions and sweep events under FPG conditions while also identifying stronger ejection events and inward interactions.

Figures 3.6 and 3.7 present the contributions of the four quadrant events to the RSS for the ZPG-SM, FPG-SM and FPG-Rough cases at a threshold of \(H = 0\), wherein all instantaneous \(u'v'\) events are included in decomposition and a threshold of \(H = 4\) wherein only intense \(u'v'\) event

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are included. Previous zero-pressure-gradient studies (Wu and Christensen, 2007; Mejia-Alvarez and Christensen, 2010) reported ejections ($Q_2$) and sweeps ($Q_4$) dominate over inward ($Q_3$) and outward ($Q_1$) interactions under both smooth and rough ZPG conditions, with ejections and sweeps contributing to RSS-producing events at comparable levels. Schultz and Flack (2007) also reported the comparable dominance of $Q_2$ and $Q_4$, regardless of surface condition for the ZPG turbulent boundary layer. Consistent with previous studies, the current ZPG-SM case shows the dominant contribution to RSS-producing events of both $Q_2$ and $Q_4$. However, under FPG conditions, an enhanced dominance of $Q_2$ events over $Q_4$ events is observed in both smooth- and rough-wall cases. As a consequence of the peak shift in Reynolds stresses under the FPG condition, the peak locations for each quadrant are positioned closer to the wall compared with the ZPG-SM case (for FPG SM 20 Hz case, $y/\delta = 0.043$ while it is 0.089 for ZPG SM 20 Hz case). It is noticeable that the significant reduction in both sweep and ejection events through the entire boundary layer for the FPG SM case, except in the near wall region where the peak for $Q_2$ is higher than the ZPG-SM case. Although fewer ejection events occur under FPG condition by the dampening effect of FPG condition, a larger number of stronger $u'v'$ events persist near the wall. With the addition of surface roughness, the magnitudes of all quadrant events are increased, especially for $Q_2$ and $Q_4$. Due to the measurement challenges associated with strong reflections of laser light from the surface roughness, it is difficult to discern the peak value of each quadrant event in the FPG-Rough case. As observed in the FPG-SM case, significant reduction of $Q_2$ and $Q_4$ contributions throughout the boundary layer is observed due to the external acceleration as well as stronger dominance of ejection events $Q_2$ over sweep events $Q_4$. In particular, the dampening of sweeps under FPG condition is more pronounced when $H = 4$.

Figures 3.8–3.12 present contours of the quadrant contributions to the mean RSS when $H = 0$. As previously indicated with the profiles, significant dominance of $Q_2$ over $Q_4$ is observed under FPG condition regardless of the surface condition. This behavior is consistent with previous measurements conducted by Bourassa and Thomas (2009) and Drozdz and Elsner (2011) for an accelerating boundary layer on a smooth surface which revealed enhanced ejections in the near-wall region at the expense of reduced sweep events due to the external acceleration of the flow. For the case of adverse pressure gradient, however, Krogstad and Antonia (1994) showed the exactly
opposite behavior, with sweeps dominating ejections in the near-wall region. It is more clear to see the dampening of all $\langle u'v' \rangle$ quadrant events for the FPG-SM case for $y/\delta > 0.4$ due to the external acceleration. With the presence of surface roughness, the magnitudes of sweeps and ejections are enhanced for FPG-Rough case as shown in figures 3.11 and 3.12. In particular, slight enhancement of inward interactions $Q_3$ in the near-wall region is observed for the FPG cases regardless of surface condition compared to the ZPG-SM case. Such behavior is consistent with previous studies by Bourassa and Thomas (2009) and Drozdz and Elsner (2011).

When the threshold value $H = 4$ is employed, allowing only the most intense $u'v'$ events to be included in the quadrant decomposition, $Q_2$ ejections dominate over $Q_4$ sweeps in all ZPG-SM, FPG-SM, and FPG-Rough cases as shown in figures 3.8–3.16. Similarly, ZPG-Rough case from the previous research (Mejia-Alvarez and Christensen, 2010) also showed the dominance of $Q_2$ ejections over $Q_4$ when $H = 4$. The $Q_1$ outward interactions and $Q_3$ inward interactions are nearly zero for this threshold in all cases. Consistent with the $H = 0$ cases, the magnitude of ejections for the FPG-Rough case are much larger than the FPG-SM case due to the presence of roughness. The rough-wall flow, however, yields larger contributions from both ejections and sweeps compared to smooth-wall flow. As already discussed in the context of the mean velocity profile and Reynolds stresses, both the 20 Hz and 30 Hz cases have nearly the same profiles and contours of quadrant contributions, implying the quadrant contributions are less sensitive to the change of Reynolds number in the current range of mild FPG conditions.

### 3.2 Wall-normal–spanwise plane measurements

The stereoscopic PIV measurements in the cross-flow wall-normal–spanwise plane provides insight into the spanwise behavior of the flow under FPG smooth and rough surface conditions.

#### 3.2.1 Mean velocity

Figure 3.18 presents the ensemble- and spanwise-averaged streamwise velocity profiles in the $y - z$ measurement plane for both the smooth- and rough-wall FPG cases at 20 and 30 Hz tunnel frequency. Consistent with the $x - y$ plane measurements, intense velocity gradients close to the wall are observed in the FPG-SM case, while these gradients are attenuated in the FPG-Rough
case. It is noted that the intersection point between the profiles of FPG-SM and FPG-Rough cases occurs near $y/\delta \approx 0.5$.

Compared with the normalized streamwise velocity contours of the $x - y$ plane measurements (see figure 3.2), one can see the noticeable inhomogeneous velocity contours for the rough-wall case in the spanwise direction (figure 3.19), with regions of persistent low streamwise momentum apparent (i.e., bulges of lower-speed fluid penetrating further from the wall) bounded by regions of higher-speed fluid penetrating closer to the wall. These persistent low-momentum regions in the streamwise velocity are likely tied to large-scale roughness protrusions that generate low-speed wake regions. Such mean-velocity inhomogeneities in the rough-wall case induces the crossing point of the smooth- and rough-wall mean velocity profiles.

Previous research (Mejia-Alvarez, 2010) conducted on the identical rough surface used herein in the streamwise-spanwise ($x - z$) plane measurement also showed the inhomogeneous distribution of ensemble averaged velocity contours, while the smooth wall result was quite uniform. The spanwise-localized low momentum pathway bounded by the high momentum pathway apparent in a rough wall flow indicates the ‘channeling effect’ in the flow, or persistent wake regions due to the dominant roughness features. The preferential paths for the low- and high-momentum motions present in the ensemble averaged velocity of the rough-wall flow in the $x - z$ plane provide a clue to the inhomogeneous velocity contours of the rough-wall case in the current $y - z$ plane measurement.

### 3.2.2 Reynolds stresses

Figure 3.20 shows the profiles of Reynolds normal and shear stresses for the smooth- and rough-wall cases at different Reynolds number. Consistent with the $x - y$ plane measurements, all components of the Reynolds stresses are enhanced due to the presence of surface roughness. Also, all the peaks of the Reynolds stresses are shifted closer to the wall for the FPG-SM case compared with the FPG-Rough case. It is noted that $\langle w'w' \rangle$ induced by the presence of hairpin structures along the spanwise direction is higher than $\langle v'v' \rangle$. This is consistent with the result from Hutchins et al. (2005) for their cross-plane stereoscopic PIV measurements of flow on a smooth surface. The crossing point at $y/\delta \approx 0.7$ is visible for $-\langle u'v' \rangle$ profiles, indicative of the inhomogeneity of the Reynolds shear stress for the rough-wall case.
Figures 3.21 and 3.22 present contours of the ensemble-averaged Reynolds stresses, $\langle u'u' \rangle$, $\langle v'v' \rangle$, $-\langle u'v' \rangle$, and $\langle w'w' \rangle$, as well as the turbulent kinetic energy, TKE, in the $y-z$ measurement plane for the smooth- and rough-wall cases. While the smooth-wall results display the expected spanwise homogeneity, localized regions of intense Reynolds stresses and TKE are noted in the rough-wall case. (It should be noted that the turbulent effect seen in the $y-z$ measurement plane is not due to the rough-wall in the measurement plane. It is a projection of upstream turbulence, not a localized effects.) The peak RSS regions occur spatially-coincident with the persistent low-momentum regions in the mean velocity while the peak TKE regions occur at the spanwise boundaries of these low-momentum regions. One such instance of this behavior is apparent in the region $0 < z/\delta < 0.5$ where a region of low streamwise velocity is noted (see figure 3.19). Thus, the present roughness creates significant heterogeneities in the single-point statistics and the most intense turbulence is found to reside coincident with the low-momentum regions noted in the mean streamwise velocity. However, it is not clear whether these processes are due to unsteady shedding of small-scale vortices from roughness protuberances, for example, or larger-scale coherent motions within the flow that might be preferentially channelled along these low momentum paths.
Figure 3.2: Ensemble-averaged streamwise velocity normalized by $U_e$ for (a) ZPG-SM 20Hz, (b) FPG-SM 20Hz, (c) FPG-Rough 20Hz, (d) FPG-SM 30Hz and (e) FPG-Rough 30Hz cases in streamwise–wall-normal plane. The roughness topography beneath the field is also shown. (Flow is from left to right.)
Figure 3.3: (a), (c) Reynolds normal stress profiles, $\langle u'u' \rangle$ and $\langle v'v' \rangle$, and (b), (d) Reynolds shear stress, $-\langle u'v' \rangle$, profiles for the 20 Hz and 30 Hz cases, respectively. Not all data points shown for clarity.
Figure 3.4: Contour plots of ensemble-averaged (a–c) ⟨\(u'^2\)⟩, (d–f) ⟨\(v'^2\)⟩, (g–i) −⟨\(u'v'\)⟩, and (j–l) TKE normalized by \(U_e^2\) for (left) ZPG-SM, (middle) FPG-SM, and (right) FPG-Rough 20Hz cases in the streamwise–wall-normal plane.
Figure 3.5: As in figure 3.4, but for (left) FPG-SM and (right) FPG-Rough 30Hz cases.
Figure 3.6: Quadrant contributions to the mean RSS, $\langle u'v' \rangle_Q/U_e^2$, for (a–c) $H = 0$, and (d–f) $H = 4$ for the tunnel fan speed of 20 Hz. (Left : ZPG-SM, Middle: FPG-SM, Right : FPG-Rough)
Figure 3.7: Quadrant contributions to the mean RSS, $\langle u'v' \rangle_Q/U_e^2$, for (a–b) $H = 0$, and (c–d) $H = 4$, for the tunnel fan speed of 30 Hz. (Left : FPG-SM, Right : FPG-Rough)
Figure 3.8: Quadrant contributions to the mean RSS, $\langle u'v' \rangle_Q/U_e^2$, for the ZPG-SM 20 Hz case for $H = 0$. Contours of (a) $Q_1$, (b) $Q_2$, (c) $Q_3$ and (d) $Q_4$ normalized by $U_e^2$. 
Figure 3.9: As in figure 3.8 but for the FPG-SM 20Hz case.
Figure 3.10: As in figure 3.8 but for the FPG-SM 30 Hz case.
Figure 3.11: As in figure 3.8 but for the FPG-Rough 20 Hz case.
Figure 3.12: As in figure 3.8 but for the FPG-Rough 30Hz case.
Figure 3.13: Quadrant contributions to the mean RSS, $\langle u'v' \rangle_Q/U_\varepsilon^2$, for the ZPG-SM 20 Hz case for $H = 4$. 
Figure 3.14: As in figure 3.13 except for the FPG-SM 20Hz case.
Figure 3.15: As in figure 3.13 except for the FPG-SM 30 Hz case.
Figure 3.16: As in figure 3.13 except for the FPG-Rough 20 Hz case.
Figure 3.17: As in figure 3.13 except for the FPG-Rough 30 Hz case.

Figure 3.18: Mean streamwise velocity profiles normalized by $U_e$ for the (a) 20 Hz cases and (b) 30 Hz cases. Not all data points shown for clarity. □: FPG-SM and △: FPG-Rough
Figure 3.19: Ensemble-averaged streamwise velocity normalized by $U_e$ for (a) FPG-SM 20 Hz, (b) FPG-Rough 20 Hz, (c) FPG-SM 30 Hz, and (d) FPG-Rough 30 Hz cases. The roughness topography beneath the field for the $y-z$ measurement is also shown. ($U$ is the out-of-plane velocity component while $V, W$ are the in-plane components)
Figure 3.20: Profiles of (a), (c) Reynolds normal stresses, $\langle u'^2 \rangle$, $\langle v'^2 \rangle$, and $\langle w'^2 \rangle$, and (b), (d) Reynolds shear stress, $-\langle u'v' \rangle$, for the 20 Hz and 30 Hz cases, respectively. Not all data points shown for clarity.
3.2.3 Quadrant analysis

The aforementioned $x - y$ plane measurements highlighted the dominant contribution of ejections ($Q_2$) over sweeps ($Q_4$) under FPG conditions regardless of the surface condition. Also, the stronger
magnitude of the ejections and sweeps was observed for the FPG-Rough case due to the presence of surface roughness. Consistent with the results for the $x-y$ plane measurements, a similar trend of the quadrant contributions is also observed in the $y-z$ plane measurements (profiles
in figures 3.23 and 3.24 and contour plots in figures 3.25–3.28). Consistent with the single-point
statistics where the flow over the rough surface intensifies the Reynolds stresses, the magnitude of
ejections ($Q_2$) for FPG-Rough case are much stronger than the FPG-SM case. It is interesting to
note that both ejections and sweeps show strong spatial inhomogeneity in the spanwise direction for
the FPG-Rough case, consistent with the inhomogeneities noted in all of the single-point statistics.
It should be noted that there exists an albeit weaker spanwise inhomogeneity in the results for
the FPG-SM cases, particularly the 20 Hz case. After careful consideration of all possible causes,
it was determined that this weak inhomogeneity is likely due to an error in the calibration of the
stereoscopic imaging system owing to a slight misalignment of the target translation along the light
sheet thickness.
Figures 3.29–3.32 present quadrant contributions to the RSS-producing events for a threshold defined by $H = 4$ which allows only the most intense $u'v'$ events to be included in the quadrant decomposition. It is noticeable that strongest contribution of $Q_2$ ejections to the RSS-producing events over $Q_4$ sweeps in both the FPG-SM and FPG-Rough cases. Consistent with the $H = 0$ results, the magnitudes of both ejections and sweeps are enhanced by the presence of surface roughness.

3.3 Summary

Two-dimensional PIV measurements in the streamwise–wall-normal plane of smooth- and rough-wall turbulent boundary layers under FPG conditions were conducted at different Reynolds numbers and compared to a smooth-wall ZPG case at similar free-stream velocity. In accelerating flows, an intense mean velocity gradient near the wall is observed as well as the thinning of the boundary layer thickness due to the external FPG conditions. However, with the presence of surface roughness under the same FPG conditions, this thinning effect is mitigated due to the increased
momentum deficit generated by the surface roughness. It is noted that the opposing influence of FPG conditions and surface roughness yields a mean flow more consistent with the ZPG-SM case.
It is known that FPG condition also dampens Reynolds stresses as well as the suppression of the boundary layer growth. Consistent with previous studies, Reynolds stresses were dampened by the imposed FPG conditions, with the peak values shifted closer to the wall for the FPG-SM case. However, the FPG-Rough case showed significant enhancement of Reynolds stresses relative to the FPG-SM case, again highlighting the opposing impact of FPG conditions and surface roughness.
In addition, the peak values of these stresses were displaced further away from the wall compared to the FPG-SM results. Similarly, turbulent kinetic energy was reduced under the FPG condition, while the enhancement of TKE was observed in the FPG-Rough results due to the competing influence of FPG condition and surface roughness. To study the modification of Reynolds shear stress by both FPG and the combined FPG and surface roughness conditions, quadrant analysis
was conducted. The quadrant analysis revealed the dominance of ejections $Q_2$ over the sweeps $Q_4$ under FPG conditions regardless of the surface condition, whereas both $Q_2$ and $Q_4$ contributions were dominant in the ZPG-SM case when all instantaneous $u'v'$ events are included ($H = 0$). For $H = 4$, wherein only intense $u'v'$ events are included, all cases showed ejections dominating over sweeps. With the addition of surface roughness, the magnitude of sweeps and ejections were much
larger than FPG-SM case for both $H = 0$ and $H = 4$. All of these trends were found to be insensitive to Reynolds number over the range studied.

Stereo PIV measurements in the wall-normal–spanwise cross-flow plane of smooth- and rough-wall turbulent boundary layers under FPG conditions were also conducted and the location was chosen within the streamwise region of the two-dimensional, $x$–$y$ plane PIV measurements. Consistent with the $x$–$y$ plane measurements, intense mean velocity gradients in the near-wall region as well as the suppression of the boundary layer vertical growth were observed in the FPG-SM case. In addition, the expected homogeneity in the spanwise direction for the FPG-SM case was observed while strong spanwise inhomogeneity was observed in the FPG-Rough case owing to the irregular protrusions of the surface roughness. The rough-wall result revealed the spatial imprints of low and high momentum regions in the mean streamwise velocity field where the lower speed fluid penetrates away from the wall while the higher speed fluid moves towards to the wall. Reynolds stresses and turbulent kinetic energy distribution for the FPG-SM case again showed spanwise homogeneity. However, the surface roughness promotes spanwise inhomogeneity in these turbulence quantities and thus presented localized regions of intense Reynolds stresses and TKE in the region $0 < \Delta z/\delta < 0.5$ where a region of low-momentum mean flow was observed. Consistent with the $x$–$y$ plane measurements, the decomposition of the mean Reynolds shear stress showed the
dominance of ejections $Q_2$ over sweeps $Q_4$ for $H = 0$ and an even more apparent dominance of $Q_2$ for $H = 4$. Interestingly, both ejections and sweeps show significant inhomogeneity in the spanwise direction for the FPG-Rough case with larger magnitudes of both ejection and sweep events.
Chapter 4

Results: Structural Attributes of the Flow

In the previous chapter, single-point mean and turbulence statistics were reported for the ZPG-SM, FPG-SM and FPG-Rough experiments. It was found that FPG-SM conditions produced a thinner boundary layer as well as a fuller velocity profile compared to the ZPG-SM case, yielding a more intense mean velocity gradient in the near-wall region under FPG conditions. The addition of surface roughness to the FPG conditions was found to broaden the tight near-wall contours that characterized the FPG-SM result, yielding mean streamwise velocity contours more reminiscent of the ZPG-SM case. In addition, the thinning of the boundary layer with increasing streamwise distance noted in the FPG-SM case was much weaker in the FPG-Rough case, indicative of competing influences wherein the FPG conditions acted to suppress boundary-layer growth while surface roughness enhanced the boundary-layer thickness. Both FPG conditions and roughness were also found to alter the behavior of the Reynolds normal and shear stresses. Consistent with previous studies of smooth-wall turbulence, FPG conditions weakened the Reynolds stresses and focused their peak values closer to the wall compared to ZPG conditions. With the addition of roughness to the FPG conditions, both the Reynolds normal and shear stresses were enhanced relative to the FPG-SM case. In addition, the wall-normal position of peak activity in these stresses was displaced appreciably away from the wall.

The focus of this chapter is on the structural characteristics of these flows, particularly the combined impact of irregular roughness and FPG conditions on the structural paradigm of the smooth-wall, ZPG turbulent boundary layer. Representative instantaneous velocity fields in the \( x - y \) plane and \( y - z \) measurement planes for the ZPG-SM, FPG-SM, and FPG-Rough cases at different Reynolds numbers are presented. Then, the average structural characteristics are assessed through two-point velocity correlations for each case. Finally, proper orthogonal decomposition (POD) is used with the \( y - z \) plane measurements to explore the characteristics of the larger and
smaller spatial scales of the flow under both FPG and roughness conditions.

4.1 Streamwise–wall-normal plane measurements

4.1.1 Instantaneous flow structures

Figure 4.1: Representative instantaneous velocity field in the $x - y$ plane for the ZPG-SM case. A constant advection velocity of $0.8U_e$ was removed to reveal embedded structure and vortex cores are highlighted with red circles. Counter-clockwise rotating vortex core (retrograde) is highlighted with dashed red circle. Background contours illustrate instantaneous streamwise velocity normalized by the free-stream velocity $U_e$. A Galilean decomposition has been applied to identify embedded vortices (hairpin vortices in larger-scale vortex packets, for example) moving at a uniform convection velocity (herein taken as $U_c = 0.8U_e$) in both the smooth- and rough-wall results from each field. Figures 4.1-4.3 present representative instantaneous velocity fields visualized in this fashion to highlight the impact of FPG conditions in the absence of and in the presence of roughness on the canonical structure of ZPG smooth-wall turbulence. It is well-established that the outer layer of ZPG smooth-wall turbulence is populated by hairpin-like structures that tend to coherently align in the streamwise direction to form larger-scale structural entities termed hairpin vortex packets (Adrian et al., 2000a). These packets appear in the $x - y$ plane as inclined interfaces formed by the heads of the streamwise-aligned hairpins (clockwise-rotating spanwise vortex cores) beneath which a region of streamwise
Figure 4.2: Representative instantaneous velocity fields in the $x - y$ plane for the (a) FPG-SM 20 Hz and (b) FPG-SM 30 Hz cases. A constant advection velocity of $0.8 \, U_e$ was removed to reveal embedded structure and vortex cores are highlighted with red circles. Background contours illustrate instantaneous streamwise velocity normalized by $U_e$. 
Figure 4.3: Representative instantaneous velocity fields in the $x - y$ plane for the (a) FPG-Rough 20 Hz and (b) FPG-Rough 30 Hz case. A constant advection velocity of $0.8 \, U_e$ was removed to reveal embedded structure and vortex cores are highlighted with red circles. Background contours illustrate instantaneous streamwise velocity normalized by $U_e$. 
momentum deficit is apparent due to the collectively-induced ejection events of these individual vortices (Adrian et al., 2000a; Christensen and Adrian, 2001). Recent measurements for the present roughness under ZPG conditions reveal this canonical outer-layer vortex organization to remain intact, though its characteristic spatial scales are modified, particularly within the roughness sublayer (Wu and Christensen, 2007, 2010).

Figure 4.1, which presents a representative velocity field from the ZPG-SM case, illustrating these defining characteristics of hairpin vortex packets. Several clockwise-rotating spanwise vortices, the imprint of hairpin heads, are indeed apparent, as highlighted by the red circles. These clockwise rotating vortices were termed prograde by Wu and Christensen (2006) because they rotate in a sense consistent with that of the mean shear ($\partial U / \partial y$). As also reported in Wu and Christensen (2006), counter-clockwise rotating vortices are also present (referred to as retrograde since their rotation is counter that of the mean shear) and are highlighted by the dashed red circle. The prograde vortices appear aligned in the streamwise direction and form an interface inclined at a shallow angle relative to the wall ($\sim 12^\circ$) moving at nearly the same speed in a manner consistent with that reported in previous smooth-wall ZPG studies (Adrian et al., 2000b; Christensen and Adrian, 2001). A region of slower-moving fluid ($\sim 0.5 U_e$) is apparent beneath this inclined interface as highlighted by the background contours of instantaneous streamwise velocity owing to the collective induction of the vortices in the packet. In this regard, each prograde vortex induces a strong ejection event just upstream and below its head which have been previously found to heavily contribute to the mean Reynolds shear stress, $\langle u'v' \rangle$ (Adrian et al., 2000b; Ganapathisubramani et al., 2003; Wu and Christensen, 2010). Near the wall, most of the vortices have a clockwise sense of rotation. This is expected, as the heads of hairpin vortices have negative fluctuating spanwise vorticity, $\omega'_z < 0$, which is consistent with the sense of the mean shear and hence their reference as prograde vortices. Wu and Christensen (2006) found the near-wall region to be densely populated by prograde vortices, with the population of retrograde vortices increasing near the outer edge of the log layer ($y \approx 0.2 \delta$).

When FPG conditions are imposed on smooth-wall turbulence, the characteristics of these vortex packets can change dramatically. As shown in figure 4.2 for the FPG-SM case, the inclination angle formed by the heads of the vortices is notably reduced and the magnitude of the streamwise
momentum deficit induced by the vortices that lies beneath the inclined interface is found to be much weaker compared to the ZPG-SM case. In addition, the strength of the ejections induced by the individual vortices appear weaker under FPG-SM conditions. All of these observations are consistent with the characteristics of the single-point statistics wherein boundary-layer growth was found to be suppressed by FPG-SM conditions as were the characteristic magnitudes of the velocity fluctuations as inferred from the Reynolds normal and shear stresses (see previous chapter). Furthermore, the notably reduced momentum deficit induced by vortex packets is consistent with the reduced mean momentum deficit noted in the near-wall region from the FPG-SM mean velocity profile. Thus, suppression of boundary-layer growth yields a commensurate suppression of the growth of vortex packets away from the wall compared to the ZPG-SM case while also weakening the turbulent motions induced by these structures. These structural trends are consistent with the results of Piomelli et al. (2000) and Spalart (1986) wherein fewer, more elongated and aligned vortical structures were observed in accelerating smooth-wall turbulent boundary layers compared to ZPG flow. Large-eddy simulation (LES) of smooth-wall ZPG and strongly accelerated turbulent boundary layers by Piomelli et al. (2000) revealed the more organized and elongated streaky vortical structure aligned in the streamwise direction of the FPG flow with fewer coherent eddies. In contrast, the ZPG flow eddies aligned in streamwise direction with a well-defined inclination angle from the wall and were found to extend for several hundred wall units into the outer layer. They conjectured the stretched vortices under FPG flow enhance the magnitude of coherent eddies and reduce their diameter owing to the conservation of angular momentum. Such smaller and intense eddies are more susceptible to viscous dissipation and this therefore explains the lower density of coherent structures in the FPG case. This yields fewer ejections, lower turbulence production, and therefore a decrease in the TKE. Spalart (1986) conducted direct numerical simulation (DNS) of the sink flow boundary layer for $K = 1.5, 3 \times 10^{-6}$ and $Re_\theta = 330$ and also reported reduced protrusion of vortex structures in the wall-normal direction and increased elongation in the streamwise direction at a shallower angle from the wall compared with ZPG flow.

When the surface roughness is added to FPG conditions, significant differences are noted in the underlying structure of the flow as is illustrated in figure 4.3 for the FPG-Rough cases at 20 and 30 Hz. Compared with the FPG-SM case where the streamwise momentum deficit is approximately
0.7U_e, the visualized vortex packet in the FPG-Rough case generates a momentum deficit of 0.5 – 0.6U_e that is more consistent with the ZPG-SM flow. In addition, the FPG-Rough packet is found to penetrate much further into the outer layer than is noted under FPG-SM conditions (This trend is consistently noted in visualizations of other velocity fields for the FPG-Rough case). In addition, the strength of the ejection events generated by the vortices in the visualized FPG-Rough packet are stronger than those visualized in the FPG-SM example. As such, the FPG-Rough packet in figure 4.3 is, at least qualitatively, more consistent with the ZPG-SM vortex packet in figure 4.1 than with the FPG-SM packet example in figure 4.2. As a baseline comparison, previous study conducted on an identical rough surface (Wu and Christensen, 2010) revealed the consistency in the characteristics of the instantaneous flow structures between ZPG-SM and ZPG-Rough cases by showing the aligned vortices in the streamwise direction forming an inclined interface where the significant momentum deficit region exists for ZPG-Rough case. Volino et al. (2007) also reported the instantaneous flow structure over a wire mesh surface to be qualitatively consistent to the smooth-wall flow.

Taken together, these instantaneous velocity fields indicate that the present roughness acts to mitigate the suppression of both the boundary-layer growth (and the commensurate wall-normal growth of vortex packets) as well as the suppression of the turbulent motions induced by these vortical structures imposed by FPG conditions.

### 4.1.2 Two-point velocity correlation coefficients

Two-point velocity correlation coefficients can be used to infer the average spatial characteristics of the underlying structure in the ZPG-SM, FPG-SM, and FPG-Rough cases. In the streamwise–wall-normal (x – y) plane, these correlation coefficients are computed as

\[ \rho_{i,j}(\Delta x, y; y_{ref}) = \frac{\langle u'_i(x, y_{ref})u'_j(x + \Delta x, y) \rangle}{\sigma_i(y_{ref})\sigma_j(y)}, \]  

(4.1)

where \( \Delta x \) is the spatial separation in the streamwise direction (statistical homogeneity is assumed across the limited streamwise field of view), \( y_{ref} \) is the wall-normal reference location at which the correlation maps are calculated and \( \sigma_i \) and \( \sigma_j \) are the root-mean-squares of the \( i^{th} \) and \( j^{th} \) velocity components.
Of particular interest, previous studies have found the spatial characteristics of the two-point correlation coefficient of streamwise velocity, $\rho_{uu}$, to be consistent with those of hairpin vortex packets, particularly its shallow inclination away from the wall and its relatively long streamwise extent (Wu and Christensen, 2006; Ganapathisubramani et al., 2005). The long streamwise extent of $\rho_{uu}$ can be interpreted as the imprint of the large scale, uniform streamwise momentum deficit induced by the collective induction of the vortices within hairpin packets. The length of a hairpin packet increases with increasing Reynolds number because there are more eddied per packet (Adrian et al., 2000b; Christensen and Adrian, 2001). In contrast, the spatial character of the two-point correlation coefficient of wall-normal velocity, $\rho_{vv}$, is more reminiscent with the spatial influence of a small-scale effect, i.e., the individual vortices in a packet. Since the induced flow within a packet is nearly parallel to the wall, the wall-normal velocity fluctuation $v$ has a small magnitude. As an individual eddy, both of $u$ and $v$ might have similar length scale, but owing to the collective and coherent induction of the individual vortices within packets, $u$ has much longer scale than $v$ (Liu et al., 2001). Finally, $\rho_{uv}$, the cross-correlation coefficient between the streamwise and wall-normal velocity fluctuations, embodies both the smaller-scale character of $\rho_{vv}$ that is likely due to the smaller-scale ejection events induced by the individual vortices as well as the larger-scale character of $\rho_{uu}$, indicative of the collective nature of this induction along the elongated streamwise extent of vortex packets (Wu and Christensen, 2006).

Figures 4.4–4.5 present two-point velocity correlation coefficients of streamwise velocity, $\rho_{uu}$ for the ZPG-SM, FPG-SM, and FPG-Rough cases at $y_{ref}/\delta = 0.15, 0.2$ and $0.3$. These locations were chosen since they sit near the logarithmic region of the flow. The inclination angle, $\beta$, for all cases is calculated by connecting a line through two points at the farthest regions where the contour level reaches at $\rho_{uu} = 0.5$.

The ZPG-SM results highlight the inclined nature of $\rho_{uu}$ at an angle consistent with the vortex packet noted in the instantaneous velocity fields shown in figure 4.1 for ZPG-SM flow. Furthermore, the $\delta$-scale streamwise extent of this correlation is again consistent with the streamwise extent of hairpin vortex packets. With increasing $y_{ref}$, a slight increase of the inclination angle, $\beta$, is noted from $10^\circ$ to $12^\circ$ as summarized in table 4.1. This trend of inclination angle matches well with the previous study (Wu and Christensen, 2010) for a smooth-wall turbulent boundary layer. In
Table 4.1: Streamwise extent $L_x(\Delta x/\delta)$ defined from the threshold $\rho_{uu} = 0.5$ and inclination angle of $\rho_{uu}$, $\beta(^\circ)$.

<table>
<thead>
<tr>
<th>$y/\delta$</th>
<th>Case</th>
<th>20Hz</th>
<th>30Hz</th>
<th>20Hz</th>
<th>30Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZPG</td>
<td>FPG SM</td>
<td>FPG Rough</td>
<td>FPG SM</td>
<td>FPG Rough</td>
</tr>
<tr>
<td>$y/\delta = 0.15$</td>
<td>$L_x$</td>
<td>0.56</td>
<td>0.62</td>
<td>0.51</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>9.98</td>
<td>8.44</td>
<td>9.19</td>
<td>7.67</td>
</tr>
<tr>
<td>$y/\delta = 0.20$</td>
<td>$L_x$</td>
<td>0.55</td>
<td>0.82</td>
<td>0.63</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>10.75</td>
<td>7.49</td>
<td>8.44</td>
<td>7.32</td>
</tr>
<tr>
<td>$y/\delta = 0.30$</td>
<td>$L_x$</td>
<td>0.57</td>
<td>1.10</td>
<td>0.82</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>12.58</td>
<td>7.23</td>
<td>8.39</td>
<td>6.35</td>
</tr>
</tbody>
</table>

addition, Adrian et al. (2000b) reported the inclination angle of hairpin packets to be approximately $15^\circ$.

Under FPG-SM conditions, $\rho_{uu}$ is found to undergo significant elongation in the streamwise direction as well as a reduced inclination angle compared to the ZPG-SM results (see table 4.1). These observations are consistent with previous studies of smooth-wall FPG turbulent boundary layers (Dixit and Ramesh, 2010). Dixit and Ramesh (2010) reported the average structural inclination angle under sink flow conditions decreases while its streamwise extent increases as the strength of the FPG conditions increases.

When roughness is added to the FPG conditions, the streamwise extent of $\rho_{uu}$ is reduced compared to the FPG-SM case. In particular, its streamwise extent is found to be comparable to the ZPG-SM case close to the wall but grows with increasing wall-normal position to match the FPG-SM case in the outer layer. In addition, the inclination angle is larger in the near-wall region, consistent with the ZPG-SM case, but is reduced with increasing wall-normal position which is more consistent with the FPG-SM case. A reduction in the streamwise extent of $\rho_{uu}$ owing to roughness has been previously reported for ZPG conditions, including the present irregular roughness (Wu and Christensen, 2007, 2010; Mejia-Alvarez, 2010). This shortening of $\rho_{uu}$ induced by surface roughness has been also observed in other studies for turbulent flow over more idealized roughness.
such as woven mesh (Krogstad and Antonia, 1994; Volino et al., 2007) and sparsely distributed hemispheres (Tomkins, 2001). Krogstad and Antonia (1994) argued this shortening in streamwise extent of $\rho_{uu}$ in rough-wall flow is due to the higher inclination of the large-scale structures away from the wall based on their previous observation of the increase in the wall-normal Reynolds stress $\langle v'^+ \rangle$ over the rough wall due to the smaller damping influence of the vertical motions near the wall relative to the smooth wall case. Other studies have not reported a similar increase in the inclination angle of the large-scale motions for rough-wall flow.

Figure 4.6 shows one-dimensional profiles of $\rho_{uu}$ in the streamwise direction for all cases in the $x-y$ measurement plane. Under FPG-SM conditions, the profiles of $\rho_{uu}$ in the streamwise direction clearly show the extended characteristic length scale $L_x$–16% to 48% larger than the ZPG-SM case with increasing wall-normal position (see table 4.1). Here, $L_x$ is defined as the streamwise extent of the one-dimensional correlation profile associated with the $\rho_{uu} = 0.5$ level. Consistent with previous studies (Wu and Christensen, 2010; Mejia-Alvarez and Christensen, 2010), the shortening of $\rho_{uu}$ for the FPG-Rough case is observed compared with the ZPG-SM case. With increasing distance from the wall, the rough wall correlation under ZPG conditions collapses well with the ZPG-SM case, supporting Townsend’s wall similarity hypothesis (Mejia-Alvarez and Christensen, 2010; Volino et al., 2007). As reported in table 4.1, when $\rho_{uu} = 0.5$, FPG-Rough case is approximately 9% shorter than the ZPG-SM case. However, under FPG conditions, the characteristic length of $\rho_{uu}$ for the FPG-Rough becomes slightly longer than the ZPG-SM case with the increase of $y_{ref}$. Thus, near the wall, the FPG-Rough results are more consistent with the ZPG-SM results, while farther from the wall the FPG-Rough results are more consistent with the FPG-SM results. Therefore, roughness effects appear to dominate the structural characteristics in the near-wall region while FPG conditions appear to dominate these characteristics in the outer region of the flow.

Figure 4.7 shows one-dimensional profiles of $\rho_{uu}$ in the wall-normal direction for $\Delta x = 0$ and for different $y_{ref}$ values. As expected, the peak value of correlation occurs at each $y_{ref}$ position. Consistent with the profiles in streamwise direction, elongation of $\rho_{uu}$ in the wall-normal direction is observed for the FPG-SM case and such FPG induced elongation of $\rho_{uu}$ is diminished by the presence of surface roughness. However, with the increase of wall-normal reference position, the FPG-Rough case shows slight enhancement in its characteristic length scale at $y/\delta > y_{ref}$. Previous
study conducted for ZPG conditions of the flow over a smooth and rough surface (identical roughness used herein) by Mejia-Alvarez and Christensen (2010) found a shortening in the streamwise extent of $\rho_{uu}$ for the ZPG-Rough case compared to the smooth surface baseline within the roughness sublayer at $y/\delta = 0.1$, while the correlation for the ZPG-Rough case collapsed well with the smooth-wall result at the outside of the roughness sublayer at $y/\delta = 0.3$. Similar to that reported herein for FPG conditions, these trends indicate that roughness impacts the larger spatial scales of the flow within the roughness sublayer but such effects diminish away from the wall in a manner that supports Townsend’s wall similarity hypothesis.

Table 4.1 also shows the inclination angle and the characteristic length scale $L_x$ defined at the level of $\rho_{uu} = 0.5$ for all cases. Contrary to the ZPG-SM case, both FPG-SM and FPG-Rough cases become flatter with increasing wall-normal position. Taken together, the impact of roughness mitigate the stretching of $\rho_{uu}$ by FPG condition near the wall, while the influence of FPG overwhelmed that of roughness away from the wall.

Figures 4.8 and 4.9 present the two-point correlation coefficients of wall-normal velocity, $\rho_{vv}$, interpreted as the imprint of the individual hairpin-like vortices in a packet, for the 20 and 30 Hz cases, respectively. This correlation coefficient appears much less sensitive to both FPG and roughness effects, as only subtle differences are noted between the FPG-SM and FPG-Rough cases compared to the ZPG-SM results. The one-dimensional profiles of $\rho_{vv}$ in the streamwise direction shown in figure 4.10 clearly display the collapse among all cases consistent with previous studies (Wu and Christensen, 2010; Mejia-Alvarez and Christensen, 2010), indicating a strong similarity in the small-scale characteristics of these different flows up to $y/\delta = 0.2$. The profile of $\rho_{vv}$ for the FPG-SM case at $y/\delta = 0.3$, however, somewhat deviates from those of the ZPG-SM and FPG-Rough profiles which collapse. This deviation of $\rho_{vv}$ for the FPG-SM case is also noticeable in the contour plots of $\rho_{vv}$ at $y/\delta = 0.3$ as shown in figures 4.8-4.9. Figure 4.11 shows the one dimensional profiles of $\rho_{vv}$ in the wall-normal direction. As expected, little difference in the profiles is observed among the all cases, particularly compared to the strong FPG condition effects noted in $\rho_{uu}$.

In contrast to $\rho_{vv}$, the cross-correlation coefficient, $\rho_{uv}$, displays clear influences of both FPG and roughness effects as shown in figures 4.12 and 4.13. In particular, both the streamwise and wall-normal extents of $\rho_{uv}$ are significantly enhanced by FPG conditions but this enhancement is
somewhat mitigated by roughness as the FPG-Rough results appear qualitatively similar to the ZPG-SM results close to the wall. Thus, this analysis suggests that while the characteristics of the larger spatial scales are altered in the presence of both FPG and roughness, the characteristics of the smaller scales remain relatively immune to such effects. Previous study (Mejia-Alvarez and Christensen, 2010) revealed the enhancement of the streamwise extent of $\rho_{uu}$ in ZPG-Rough within the roughness sublayer compared to the ZPG-SM case, while the difference diminished with increasing wall-normal position. Quantitative comparison can be done by comparing one-dimensional profiles of $\rho_{uv}$ in the streamwise direction at different wall-normal positions.

Contrary to the profiles of $\rho_{uu}$ and $\rho_{vv}$, the profiles of $\rho_{uv}$ are asymmetric in the streamwise direction (Figure 4.14). Ganapathisubramani et al. (2005) reported similar observation of skewed streamwise profiles of $\rho_{uv}$ with a longer tail in the positive $\Delta x$ direction than the negative $\Delta x$ direction from their measurements of a ZPG turbulent boundary layer over a smooth surface. This asymmetric distribution of $\rho_{uv}$ reflects the signature of the hierarchy of hairpin packets growing from the surface with a characteristic angle (Adrian et al., 2000b). According to Adrian et al. (2000b), the younger, smaller packets lying close to the wall with lower inclination angle exist within the larger packets with high velocity and relatively higher inclination angle. From the visualizations of instantaneous structures of ZPG turbulent boundary layer, Adrian et al. (2000b) suggested this hairpin packets forms some type of linear ramp like patterns; hairpins in the downstream tend to be larger than those hairpins in the upstream yielding a strong positive value of wall normal velocity component $v$ at the downstream. This observation is also consistent with the present instantaneous structure results for the ZPG-SM case shown in figure 4.1. Thus, the $\rho_{uv}$ for the positive $\Delta x/\delta$ would be expected to be larger than that of negative $\Delta x/\delta$ (Ganapathisubramani et al., 2005).

It is noticeable that the difference in the characteristic length scale of $\rho_{uv}$ at $y/\delta = 0.3$ for the FPG-SM 20 Hz case is much longer than FPG-Rough 20 Hz case compared with those of 30Hz cases. This difference is also observed in the $\rho_{vv}$ profiles at $y/\delta = 0.3$. Also, the correlation for positive $\Delta x/\delta$ for the FPG-SM 20 Hz case shows little difference from the FPG-Rough 20 Hz case, while on the negative $\Delta x/\delta$ side, noticeable deviation between the two profiles exists. For the 30 Hz case, in contrast, the profiles of $\rho_{uv}$ for the FPG-SM and FPG-Rough cases show more deviation in positive side of $\Delta x/\delta$ so that the characteristic length scale for FPG-Rough case is longer than FPG-SM.
case. This can be explained by carefully observing the profiles of $\rho_{vv}$. Except for the $y/\delta = 0.3$ results, the profiles of $\rho_{vv}$ for the 20 Hz case shows a slightly longer correlation in the FPG-SM case when $\rho_{vv} \leq 0.2$. Opposite to the 20 Hz case, the profiles of $\rho_{vv}$ for the FPG-Rough 30 Hz case are slightly longer than those of FPG-SM case when $\rho_{vv} \leq 0.2$. These slight differences found in $\rho_{vv}$ seems to be amplified in $\rho_{uv}$. Besides these quantitative differences, it is clear that the cross-correlation $\rho_{uv}$ embodies both smaller and larger scale interactions, consistent with a reflection of the individual smaller-scale ejection events generated by individual hairpin-like vortices and the collective induction of such events across the vortices in a packet.

Figure 4.15 shows one-dimensional profiles of $\rho_{uv}$ in the wall-normal direction. For this direction, it is much clearer to observe the elongation of $\rho_{uv}$ under FPG-SM conditions compared to the ZPG-SM case. Also, the characteristic correlation length of the FPG-Rough case in the wall-normal direction is shorter than FPG-SM case, but it increases with increasing $y/\delta$. Taken together, the broadening of $\rho_{uv}$ in both the streamwise and wall-normal directions is observed under FPG condition, while this effect is reduced in the presence of surface roughness near the wall. However, with increasing $y$, yields elongation in both the FPG-SM and FPG-Rough cases.
Figure 4.4: Streamwise velocity correlation coefficients, $\rho_{uu}$, in the $x-y$ plane at $y_{ref} = 0.15, 0.2$ and $0.3\delta$ (from top to bottom rows) for the 20 Hz cases.
Figure 4.5: As in figure 4.4 but for the 30 Hz cases.
Figure 4.6: One-dimensional streamwise profiles of $\rho_{uu}$ from figures 4.4 and 4.5. Figure (a) illustrates how the characteristic length scale, $L_x$, in the streamwise direction was determined on the intercept with a threshold at $\rho_{uu} = 0.5$. 

Figure (b) to (f) show the variation of $\rho_{uu}$ at different $y^+$ values: 0.15, 0.2, and 0.3.
Figure 4.7: One-dimensional profiles of $\rho_{uu}$ from figures 4.4 and 4.5 in the wall-normal direction at $y_{ref} = 0.15, 0.2, 0.3$. 
Figure 4.8: Wall-normal velocity correlation coefficients, $\rho_{vv}$, in the $x-y$ plane at $y_{ref} = 0.15$, 0.2 and 0.30$\delta$ for the (a) ZPG-SM 20 Hz, (b) FPG-SM 20 Hz and (c) FPG-Rough 20 Hz cases, respectively.
Figure 4.9: As in figure 4.8, but for the 30 Hz cases.
Figure 4.10: One-dimensional profiles of $\rho_{vv}$ from figures 4.8 and 4.9 in the streamwise direction at $y_{ref} = 0.15, 0.2$ and 0.3.$
Figure 4.11: One-dimensional profiles of $\rho v$ from figures 4.8 and 4.9 in the wall-normal direction at $y_{ref} = 0.15, 0.2$ and 0.3.
Figure 4.12: Cross-correlation coefficients, $\rho_{uv}$, in the $x - y$ plane at $y_{ref} = 0.15, 0.2$ and $0.30\delta$ for the (a) ZPG-SM 20 Hz, (b) FPG-SM 20 Hz and (c) FPG-Rough 20 Hz cases, respectively. The contour levels range from -0.5 to 0 with increments of 0.05.
Figure 4.13: As in figure 4.12, but for the 30Hz cases.
Figure 4.14: One-dimensional profiles of $\rho_{uv}$ from figures 4.12 and 4.13 in the streamwise direction at $y_{ref} = 0.15, 0.2$ and $0.3\delta$. 
Figure 4.15: One-dimensional profiles of $\rho_{\text{uw}}$ from figures 4.12 and 4.13 in the wall-normal direction at $y^+=0.15$, 0.2 and 0.36.
Figure 4.16: Representative instantaneous velocity fields in the $y-z$ plane for the FPG-SM (a) 20 Hz and (b) 30 Hz cases. In-plane fluctuating velocity components ($v', w'$) are shown as vectors while background contours illustrate instantaneous streamwise velocity normalized by $U_e$. Contours range from -0.15 to 0.15 with the levels of 0.015. Not every vector is shown for clarity.

4.2 Wall-normal–spanwise plane measurements

4.2.1 Instantaneous flow structures

Figure 4.16 presents a representative instantaneous velocity field in the $y-z$ plane for the FPG-SM cases where the in-plane fluctuating velocity components ($v', w'$) (wall-normal and spanwise velocity fluctuations, respectively) are shown as vectors and the out-of-plane $u'$ component (streamwise velocity fluctuation) is presented as background contours. The FPG-SM case is marked by alternating, large-scale low- (LMR, where $u' < 0$, marked by 'blue' color) and high-momentum regions
Figure 4.17: Representative instantaneous velocity fields in the \( y - z \) plane for the FPG-Rough (a) 20 Hz and (b) 30 Hz cases. In-plane fluctuating velocity components \( (v', w') \) are shown as vectors while background contours illustrate instantaneous streamwise velocity normalized by \( U_e \). Not every vector is shown for clarity. The roughness topography beneath the field is also shown.

(HMR, where \( u' > 0 \), marked by ‘red’ color). While HMRs typically embody high-speed fluid moving towards the wall (termed ‘sweep’ events), LMRs are marked by low-speed fluid moving away from the wall (termed ‘ejection’ events). These LMRs and HMRs are bounded on either side by counter-rotating streamwise vortex cores that induce these ejection and sweep events. These HMRs and LMRs reflect the signature of hairpin packets in the turbulent boundary layer and are entirely consistent with those previously observed in cross-flow plane measurements in ZPG flow (Hutchins et al., 2004, 2005). The LMRs are likely due to the collectively induced momentum deficit region within the hairpin packets and are the cross-plane signature of the low-momentum regions noted
below the inclined interface of hairpin packets visualized in the \( x - y \) plane. It should be noted that the most intense \( u'v' \) event occur within LMRs and HMRs (Wu and Christensen, 2010). In contrast to ZPG conditions (Hutchins et al., 2004), the LMRs and HMRs observed under FPG-SM conditions do not extend as far into the outer region, likely due to the suppression of their growth away from the wall due to the FPG conditions. These observations are quite consistent with the character of these instantaneous structures in the \( x - y \) plane velocity fields presented earlier. As shown in figure 4.16, these HMRs and LMRs only extend to approximately \( y/\delta = 0.2 \) in average.

In this regard, one can conjecture that the vortical activity in the FPG-SM case as well as the strength of these motions in the \( y - z \) plane are notably reduced compared to ZPG flows by intuitively connecting to the results of the previous \( x - y \) plane measurements. In this regard, the imposed FPG conditions flatten the inclination of hairpin packets from the wall and reduces the intensity of the low-momentum regions. Thus, while the overall structural characteristics of the FPG-SM case are similar to its ZPG counterpart, FPG conditions certainly limit both the wall-normal extent as well as the intensity of these motions.

Figure 4.17 presents a representative instantaneous velocity field in the \( y - z \) plane for the FPG-Rough cases. Similar spatial imprints of LMRs and HMRs are observed in these fields; however, the intensity of the streamwise vortices bounding these large-scale motions and thus the intensity of the LMRs and HMRs themselves are notably stronger than the FPG-SM results. In addition, the LMRs and HMRs extend much farther away from the wall in the FPG-Rough case compared to that for FPG-SM flow. These differences are consistent with the previous measurements in the \( x - y \) plane. Taken together, these instantaneous velocity fields show that LMRs and HMRs exist under both FPG-SM and FPG-Rough conditions. However, roughness acts to mitigate the suppression of these motions due to FPG conditions, yielding flow behavior more reminiscent of ZPG conditions.
4.2.2 Two-point velocity correlation coefficients

In the wall-normal–spanwise ($y - z$) plane, the two-point velocity correlation coefficients are computed as

$$\rho_{ij}(\Delta z, y; y_{ref}) = \frac{\langle u_i'(z, y_{ref})u_j'(z + \Delta z, y) \rangle}{\sigma_i(y_{ref})\sigma_j(y)},$$  \hspace{1cm} (4.2)

where $\Delta z$ is the spatial separation in the spanwise direction, $y_{ref}$ is the wall-normal reference location at which the correlation maps are calculated and $\sigma_i$ and $\sigma_j$ are root-mean-squares of the $i^{th}$ and $j^{th}$ velocity components. Figures 4.18–4.23 present $\rho_{uu}, \rho_{vv}, \rho_{uw}$ and $\rho_{ww}$ in the $y - z$ plane at $y_{ref} = 0.15, 0.2$ and $0.3\delta$ for the FPG-SM and FPG-Rough cases, respectively. For $\rho_{uu}$, both smooth- and rough surface cases exhibit a strong positive correlation at $(\Delta z, y) = (0, y_{ref})$ bounded in the spanwise direction by large-scale regions of negative correlation. These characteristics are highly reminiscent of the spanwise ordering of LMRs and HMRs in the $y - z$ plane instantaneous velocity fields of figures 4.16–4.17, and therefore interpreted as the statistical imprint of these large-scale motions. However, despite these qualitative consistencies between the FPG-SM and FPG-Rough cases, a shortening of $\rho_{uu}$ is noted in the latter case, particularly in the spanwise direction. This reduction in spanwise and wall-normal coherence in the presence of roughness is also apparent in $\rho_{uw}$ and $\rho_{ww}$.

Hutchins et al. (2004) observed similar characteristics in inclined streamwise–spanwise plane PIV measurements for ZPG smooth-wall conditions. A positive streamwise velocity correlation $\rho_{uu}$ at the reference position $(\Delta z/\delta, y/\delta) = (0, 0.14)$ with a spanwise width of approximately $0.5\delta$ was bounded by negative correlations, separated by approximately $0.75\delta$. They found these two length scales seemed to match the width and separation between the low- and high-momentum regions from instantaneous velocity fields in the inclined plane. Marusic and Hutchins (2008) presented $\rho_{uu}$ with highly elongated regions of positive correlation, surrounded by anti-correlated regions in spanwise direction and argued this trend to be indicative of hairpins from their simultaneous measurements in $x - y$ and $y - z$ planes of a ZPG smooth-wall turbulent boundary layer.

In contrast, $\rho_{vv}$, which reflects the coherence of smaller-scale turbulent motions, shows little modification due to roughness, as was similarly observed for ZPG flow for smooth and rough walls (Wu and Christensen, 2010). Krogstad and Antonia (1994) also observed similar trend in $\rho_{vv}$.
contours slightly larger for the smooth surface than idealized rough surface (woven wire mesh), but considerably smaller than $\rho_{uu}$ since the wall-normal velocity fluctuations $v$ were dampened by the wall.

Figures 4.24–4.31 present one-dimensional profiles of the two-point correlations in the spanwise and wall-normal directions. Consistent with the observations from the 2D PIV measurements in the $x-y$ plane, the FPG-Rough case displays a noticeable shortening of the correlation for all cases, especially in $\rho_{uu}, \rho_{uw}$ and $\rho_{uv}$, compared to the FPG-SM case. The shortening of the correlation induced by the roughness indicates the loss of coherence of the flow and thus relates to the increase of randomly generated flow structures of the flow over a rough surface (Mejia-Alvarez, 2010).

Figure 4.24 illustrates one-dimensional profiles of $\rho_{uu}$ in the spanwise direction. The central region near $\Delta z/\delta = 0$ with positive correlation is bounded by negative correlation values on both sides, consistent with previous studies (Hutchins et al., 2005). Despite the similar trend in $\rho_{uu}$ between the FPG-SM and FPG-Rough cases, significant reduction of the coherence in the spanwise direction for the FPG-Rough case is observed which is consistent with the previous shortening of $\rho_{uu}$ in the streamwise direction. Table 4.2 summarizes the characteristic length scales of the FPG-SM and FPG-Rough cases at $y/\delta = 0.05$ determined from the one-dimensional profiles of $\rho_{uu}$ in the spanwise direction shown in figure 4.24. It is interesting to observe the spanwise coherence for both FPG-SM and FPG-Rough cases increases with the increase of wall-normal position of $y_{ref}$.

Figure 4.25 shows the $\rho_{uu}$ profiles in the wall-normal direction for $\Delta z = 0$. The characteristic length scale of the FPG-Rough case in wall-normal direction is shorter than FPG-SM case. However, improved collapse between the FPG-SM and FPG-Rough cases is observed near $y/\delta > y_{ref}$ as $y_{ref}$ increases. Consistent with the previously-discussed $x-y$ plane measurements, the $\rho_{vv}$ profiles in the spanwise and wall-normal directions in figures 4.26 and 4.27 show similarity between FPG-SM and FPG-Rough cases, indicating negligible influence by both FPG and surface roughness conditions on the smaller scales of the flow.

Figures 4.28–4.31 present one-dimensional profiles of $\rho_{uw}$ and $\rho_{ww}$ in the spanwise and wall-normal directions. As observed for $\rho_{uu}$, shortening of the correlation due to surface roughness is observed for both $\rho_{uw}$ and $\rho_{ww}$ in both the spanwise and wall-normal directions. Similar to the previous study (Wu and Christensen, 2010), the spatial extent of $\rho_{ww}$ is situated between that of
Table 4.2: Streamwise extent $L_z(\Delta_z/\delta)$ of one-dimensional profiles of $\rho_{uu}$ at 0.05.

<table>
<thead>
<tr>
<th>$y/\delta$</th>
<th>20 Hz</th>
<th>30 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FPG SM</td>
<td>FPG Rough</td>
</tr>
<tr>
<td>0.15</td>
<td>0.48</td>
<td>0.31</td>
</tr>
<tr>
<td>0.20</td>
<td>0.59</td>
<td>0.37</td>
</tr>
<tr>
<td>0.30</td>
<td>0.75</td>
<td>0.49</td>
</tr>
</tbody>
</table>

$\rho_{vv}$ and $\rho_{uu}$, indicating the relatively smaller influence of both FPG and surface roughness on the $\rho_{ww}$ compared with $\rho_{uu}$.

Finally, in contrast to the instantaneous velocity fields in the $y-z$ plane that are clearly marked by alternating LMR’s and HMR’s across the entire spanwise extent of the field of view (both smooth and rough), the corresponding two-point correlations do not reflect this degree of periodicity in the large-scale motions. This is likely due to the broad range of scales present in these flows that masks alternating patterns and therefore reflected in the two-point correlations.

4.2.3 POD analysis

As noted, the two-point velocity correlations do not fully reflect the clear alternating patterns of high and low momentum regions. This is likely due to the masking effects caused by the multiple scale interactions of the flow. To extract these patterns, Hutchins et al. (2004) applied a de-jittering process that sorted PIV frames according to the dominant scales at a given wall-normal reference position. This analysis uncovered this alternating behavior in the statistics of the flow. Proper orthogonal decomposition (POD) is employed herein to yield scale separation so that the spatial characteristics of the larger and smaller scales of the flow can be analyzed in isolation.

This POD analysis is particularly well-suited for systems that are statistically inhomogeneous in one or more spatial directions, wherein complex infinite-dimensional processes can be represented using lower-dimensional approximate descriptions. In particular, POD generates a basis for the modal decomposition of the instantaneous fluctuating velocity fields and provides the most efficient way of identifying the motions which, on average, contain a majority of the turbulent kinetic energy (TKE) in the flow (Homes et al., 1996). Such a set of modes from the decomposition that is highly
related to the large-scale structures need not necessarily correspond to the coherent structures, but simply to the energy of the flow in a statistical sense (Kostas et al., 2005). In this regard, the lowest-order (most energetic) modes typically embody the larger spatial scales of the flow while the higher-order (less energetic) modes represent increasingly smaller spatial scales. The snapshot POD method is utilized herein to compute the POD modes and associated eigenvalues in the $y - z$ plane as it is more amenable to the discrete nature of PIV velocity fields.

POD has been widely used for identifying the dominant features of the flow as well as for constructing low-order reconstructions (low-pass-filtered versions) of the flow that represent the major flow dynamics with the least number of modes (Kostas et al., 2005). Gurka et al. (2006) conducted POD analysis on a vortical field of two dimensional PIV data of a turbulent boundary layer in a flume to extract large-scale structures of the flow. They observed the macro structure of elongated streamwise vortical structure with a small inclination angle of approximately $8^\circ$. Sen et al. (2007) applied snapshot POD analysis on the DNS of turbulent channel flow with the presence and absence of the three dimensional egg carton roughness elements. Their one-dimensional analysis revealed the slow convergence of the POD modes for the rough-wall case due to the broad range of spatial length scales due to the kinematics of the roughness elements. Using the first ten POD modes, they reconstructed Reynolds stresses that were well matched with their location and amplitude of the peak. It also revealed that adding additional POD modes did not significantly alter the inner-layer characteristics, implying the effect of roughness alters the lower-order, larger-scale, energy containing structures of the flow rather than the smaller-scale motions. Recently, Baltzer et al. (2010) applied POD to the DNS of a turbulent boundary layer to reveal the large-scale structures that can be observed in instantaneous snapshots of the flow. They revealed the POD modes can identify the statistically significant hairpin vortex packets.

The procedure of POD is well described by Cazemier et al. (1998). The goal of POD is to find the best approximation of a velocity field $u(x, t)$ where the spatial variable $x$, elements of a spatial domain $\Omega$ and the time $t$, elements of a time interval $T$ with $N$ deterministic spatial POD modes $\phi_i(x)$ and random temporal functions $a_i(t)$ as

$$
\min \int_{\Omega} \int_{T} \left( u'(x, t) - \sum_{i=1}^{N} \phi_i(x) \right)^2 \, dt \, dx.
\quad (4.3)
$$
Arbitrary variations of the unknown $\phi_i(x)$ and $a_i(t)$ lead to two eigenvalue problems. These eigenvalue problems are called the Fredholm integral equations of the second type, with positive definite Hermitian kernels. The Hilbert–Schmidt theorem ensures that the eigenfunctions are orthogonal and the eigenvalues, $\lambda_i$, are real and positive and form a decreasing, convergent series ($\lambda_i > \lambda_{i+1}$). The $i^{th}$ eigenvalue $\lambda_i$ represents the average turbulent kinetic energy in the $i^{th}$ POD mode since POD analysis is performed on the fluctuating velocity fields.

Classical POD: $\lambda_i \phi_i(x) = \int_{\Omega} \left( \int_T u(x,t)u(x',t)dt \right) \phi_i(x')dx$

Snapshot POD: $\lambda_i a_i(t) = \int_{T} \left( \int_{\Omega} u(x,t)u(x,t')dx \right) a_i(t)dt$. (4.4)

Both equations shown above are equivalent for solving for the POD modes $\phi(x)$ and coefficients $a_i(t)$. Lumley (1967) first proposed classical POD while the snapshot POD was introduced by Sirovich (1987). The snapshot POD is more efficient than the method of classical POD when the number of snapshots is smaller than the number of grid points. Thus, snapshot POD was applied in the present research and the singular value decomposition method was used to decompose corresponding eigenfunctions and eigenvalues of the flow. These were computed using built-in SVD functions within the Matlab software package.

The fractional contribution of the $i^{th}$ POD mode to the total turbulent kinetic energy $E$ can be expressed as

$$E_i = \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i},$$

where $E = \sum_{i=1}^{N} \lambda_i$ is twice the total turbulent kinetic energy of the flow and $N$ is the total number of basis functions.

Any instantaneous velocity field can be reconstructed by using first leading $K$ POD modes (low-order velocity field $u_L$) as

$$u(x, t_n) \approx u_L(x, t_n) = \sum_{k=1}^{K} a_k(t_n) \phi_k(x).$$

(4.6)
Table 4.3: Number of POD modes required to reconstruct 30% of the overall TKE for each $y - z$ measurement case.

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface</th>
<th>$Re_{\theta}$</th>
<th>$U_e$ [m/s]</th>
<th>$\delta$ [mm]</th>
<th>Number of modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPG-SM 20Hz</td>
<td>Smooth</td>
<td>2380</td>
<td>10.25</td>
<td>50.09</td>
<td>22</td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>Smooth</td>
<td>3476</td>
<td>15.22</td>
<td>51.41</td>
<td>25</td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>Rough</td>
<td>2994</td>
<td>10.69</td>
<td>65.43</td>
<td>18</td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>Rough</td>
<td>4466</td>
<td>16.20</td>
<td>65.96</td>
<td>15</td>
</tr>
</tbody>
</table>

This reconstruction is equivalent to a low-pass-filtered version of an instantaneous velocity field since the lowest-order modes typically select the largest scale motions of the flow. Thus, these low-order velocity fields reflect the largest scale motions of the flow while the smaller scale motions have been truncated away.

In this regard, of interest in the present effort was utilizing the lower-order POD modes as a low-pass filter to separate the large and small scales in the instantaneous fluctuating velocity fields. This filtering was accomplished by projecting each fluctuating velocity field onto the minimum number of modes in each case required to capture 30% of the TKE in the $y - z$ plane. Thus, the smaller spatial scales in the fields were truncated while the larger, more energetic scales were preserved Wu and Christensen (2010). Figures 4.33–4.34 illustrate the effect of this low-pass-filtering methodology applied to the instantaneous FPG-SM and FPG-Rough fields presented in figures 4.16–4.17 on page 91–92, respectively. Despite truncation of the smaller scales, there still exists strong visual similarities between the original, unfiltered velocity fields and the low-order reconstructions, particularly the spanwise-alternating regions of low and high momentum regions. Interestingly, the boundaries between the alternating LMRs and HMRs in both cases are marked by streamwise vortices that drive low-speed fluid away from the wall in the case of the LMRs and draw high-speed fluid toward the wall in the case of the HMRs. However, consistent with the original fields, the LMRs and HMRs for the FPG-Rough case extend much farther away from the wall than those in the FPG-SM case. This difference again highlights the growth of these motions away from the wall due to surface roughness. Apart from these differences, these representative low-order fields are marked by a distinct ordering of alternating LMRs and HMRs in the spanwise direction.
Table 4.4: Spanwise extent $L_z(\Delta_z/\delta)$ at $\rho_{uu} = 0.05$.

<table>
<thead>
<tr>
<th>$y/\delta$</th>
<th>20Hz</th>
<th>30Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FPG SM</td>
<td>FPG Rough</td>
</tr>
<tr>
<td>0.15</td>
<td>0.51</td>
<td>0.35</td>
</tr>
<tr>
<td>0.20</td>
<td>0.60</td>
<td>0.40</td>
</tr>
<tr>
<td>0.30</td>
<td>0.79</td>
<td>0.53</td>
</tr>
</tbody>
</table>

To explore the apparent spanwise ordering of LMRs and HMRs in both the FPG-SM and FPG-Rough cases further, all 2000 instantaneous velocity fields in each of these cases cases were projected onto the first few modes for the 20 Hz and 30 Hz cases derived from the snapshot POD analysis required to capture 30% of the overall TKE, respectively, yielding ensembles of instantaneous, low-pass-filtered velocity fields embodying only the larger-scale motions of these flows (see table 4.3). Figure 4.32 presents the cumulative energy distribution with respect to the number of POD modes for FPG-SM and FPG-Rough cases. Despite the qualitative similarity between the two cases in the cumulative energy distribution, fewer number of modes for the FPG-Rough case were required to reconstruct 30% of TKE than for the FPG-SM case, counter to the previous study (Sen et al., 2007).

These ensembles of low-pass-filtered fields were then used to compute two-point correlation coefficients as described by Eq. (4.2) at different $y_{rel}$ positions as shown in figures 4.35 and 4.40 for the FPG-SM and FPG-Rough cases. Since these correlations were computed from the low-order reconstructed velocity fields, they reflect the spatial coherence of only the larger-scale motions in the $y-z$ plane. Strikingly consistent with the overall character of the instantaneous low-pass-filtered velocity fields shown in figures 4.33–4.34, these two-point correlations reflect a spanwise-alternating ordering of large-scale motions well beyond that indicated by the original, unfiltered correlations shown in figures 4.18 and 4.23. In particular, $\rho_{vv}$ associated with the small scale interaction clearly display the alternating patterns of high and low momentum regions, reflecting the fact that they embody significant sweep ($v' < 0$) and ejection ($v' > 0$) events that contribute heavily to the overall Reynolds shear stress.

Consistent with the previous analysis of the two-point correlations derived from the original
velocity fields, these two-point correlations of the large-scale motions also show reduced coherence in the spanwise correlation due to the presence of surface roughness. Since the small-scale motions have been filtered, the magnitude of the correlations of the large-scale motions are enhanced. The one-dimensional profiles of these correlations in the spanwise and wall-normal directions shown in figures 4.41–4.48 reflect these characteristics. The maximum and minimum peaks of correlations at $y_{ref}$ have higher magnitude and wider regions near the peaks compared with the result of original correlations owing to the loss of the smaller scales of the flow.

4.2.4 Modal analysis

The spatial structure of the basis functions (POD modes) themselves in the FPG-SM and FPG-Rough cases were explored to identify any appreciable modifications of the characteristics of both the large and small scales of the flow. In particular, the competing influence of FPG and roughness conditions in enhancing/reducing the spatial extent of the larger spatial scales, respectively, as observed in the instantaneous velocity fields and two-point velocity correlations was explored. The basis function for each individual mode is presented by using the out of plane velocity vector component $u$ structure normalized by $U_e$ as shown in figures 4.49–4.50. It is interesting to observe the repeating alternating signs of the $u$ structure along the spanwise direction in both the smooth- and rough-wall conditions. The sum of these modal shapes reflect the LMRs and HMRs shown in the previous sections. It should be noted that the difference in contour levels among different surface conditions and Reynolds number is essentially meaningless since these magnitudes are scaled by the series sum of coefficients. It is also observed the gradual decrease of $u$ structure in both wall-normal and spanwise direction as the modal number increases. Since the energy ordering of POD modes generally results in a decreasing order of structural size as they follow the turbulent energy cascading from large scales containing high TKE to small scale, low-TKE structures. With the presence of surface roughness, the FPG-Rough case shows noticeable reduction of $u$ structure in the wall-normal and spanwise direction in each of the individual modes compared with the FPG-SM case as shown in figure 4.49. Sen et al. (2007) conducted POD modal analysis of the DNS data of a channel flow with a three-dimensional idealized (egg carton) roughness. Consistent with the present result, they revealed the size of structures decreases in the spanwise direction as the
number of modes increases and the size of the smooth- and rough-wall structures become similar. In other words, there is little structural difference between the rough- and smooth-wall flow at the smaller scales, indicating that roughness effects only the larger, energy-containing structures of the flow.
Figure 4.18: Two-point velocity correlation coefficients in the $y-z$ plane at $y_{ref} = 0.15 \delta$ for the FPG-SM and FPG-Rough 20 Hz cases. Contours range from -0.5 to 0.5 with the levels of 0.05. Dashed contours indicate negative levels.
Figure 4.19: As in figure 4.18, but for the 30 Hz cases.
Figure 4.20: Two-point velocity correlation coefficients in the $y - z$ plane at $\gamma_{\text{ref}} = 0.26$ for the FPG-SM and FPG-Rough 20 Hz cases.

(Figure continued on next page)
Figure 4.21: As in figure 4.18, but for the 30 Hz cases.
Figure 4.22: Two-point velocity correlation coefficients in the $y-z$ plane at $y_{ref} = 0.3\delta$ for the FPG-SM and FPG-Rough 20 Hz cases.
Figure 4.23: As in figure 4.18, but for the 30 Hz cases.
Figure 4.24: One-dimensional profiles of $\rho_{un}$ from figures 4.18 and 4.23 in the streamwise direction at $y_{ref} = 0.15, 0.2$ and $0.3\delta$. 

![Graphs showing one-dimensional profiles of $\rho_{un}$]
Figure 4.25: One-dimensional profiles of $\rho_{uu}$ from figures 4.18 and 4.23 in the spanwise direction at $y_{ref} = 0.15, 0.2$ and 0.35.
Figure 4.26: One-dimensional profiles of $\rho_{vv}$ from figures 4.18 and 4.23 in the streamwise direction at $y_{ref} = 0.15$, 0.2, and 0.3$\delta$. 
Figure 4.27: One-dimensional profiles of $\rho_{vv}$ from figures 4.18 and 4.23 in the spanwise direction at $y_c = 0.15, 0.2$ and $0.3\delta$. 

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Figure 4.28: One-dimensional profiles of $\rho_{uv}$ from figures 4.18 and 4.23 in the streamwise direction at $y_{ref} = 0.15, 0.2$ and $0.3\delta$. 
Figure 4.29: One-dimensional profiles of $\rho_{uv}$ from figures 4.18 and 4.23 in the spanwise direction at $y_{ref} = 0.15, 0.2$ and 0.3$\delta$. 
Figure 4.30: One-dimensional profiles of $\rho_{wall}$ from figures 4.18 and 4.23 in the streamwise direction at $y^+=0.15$, 0.2 and 0.3.$\delta$. 

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One-dimensional profiles of $\rho_{\text{aw}}$ from figures 4.18 and 4.23 in the spanwise direction at $y_{ref} = 0.15$, 0.2 and 0.3.δ.
Figure 4.32: Cumulative energy distribution for the FPG-SM and FPG-Rough cases. (○: FPG-SM, and \textit{wblue}△: FPG-Rough)
Figure 4.33: Low-order (large-scale) reconstruction of the smooth-wall instantaneous velocity realization in figure 4.16. Not every vector is shown for clarity.
Figure 4.34: Low-order (large-scale) reconstruction of the rough-wall instantaneous velocity realization in figure 4.17. Not every vector is shown for clarity.
Figure 4.35: Low-order reconstructed two-point velocity correlation of the 20 Hz case at $y/\delta = 0.15$ in figure 4.18.
Figure 4.36: Low-order reconstructed two-point velocity correlation of the 30 Hz case at $y/δ = 0.15$ in figure 4.19.
Figure 4.37: Low-order reconstructed two-point velocity correlation of the 20 Hz case at $y/\delta = 0.2$ in figure 4.20.
Figure 4.38: Low-order reconstructed two-point velocity correlation of the 30 Hz case at $y/\delta = 0.2$ in figure 4.21.
Figure 4.39: Low-order reconstructed two-point velocity correlation of the 20 Hz case at $y/\delta = 0.3$ in figure 4.22.
Figure 4.40: Low-order reconstructed two-point velocity correlation of the 30 Hz case at $y/\delta = 0.3$ in figure 4.23.
Figure 4.41: One-dimensional profiles of $\rho_{uu}$ from figures 4.35–4.40 in the spanwise direction.
Figure 4.42: One-dimensional profiles of $\rho_{uu}$ from figures 4.35-4.40 in the wall-normal direction.
Figure 4.43: One-dimensional profiles of $\rho_{\omega}$ from figures 4.35–4.40 in the spanwise direction.
Figure 4.44: One-dimensional profiles of $\rho_{cc}$ from figures 4.35-4.40 in the wall-normal direction.
Figure 4.45: One-dimensional profiles of $\rho_{uv}$ from figures 4.35–4.40 in the spanwise direction.
One-dimensional profiles of $\rho_{uv}$ from figures 4.35-4.40 in the wall-normal direction.
Figure 4.47: One-dimensional profiles of $\rho_{ww}$ from figures 4.35–4.40 in the spanwise direction.
Figure 4.48: One-dimensional profiles of $\rho_{ww}$ from figures 4.35–4.40 in the wall-normal direction.
Figure 4.49: POD modes in the $y - z$ plane for the FPG-SM case (left column) and the FPG-Rough case (right) at 20 Hz. (a,b) First mode (c,d) second (e,f) third and (g,h) fourth, (i,j) fifth mode streamwise velocity, $u$ normalized by $U_e$. Dashed contours indicate negative levels.
Figure 4.50: As in figure 4.49, but for the 30 Hz cases.
4.2.5 Low-order reconstructions of single-point turbulence statistics

Using the low-pass-filtered instantaneous velocity fields, the contributions of the larger-scale motions to the Reynolds stresses and turbulent kinetic energy were analyzed and compared with the results from the ensembles of the original, unfiltered instantaneous velocity fields. Figure 4.51 presents the profiles of the Reynolds normal and shear stresses observed in the ensembles of low-pass-filtered instantaneous velocity fields for FPG-SM and FPG-Rough with different Reynolds number.

Tables 4.5, 4.6 summarize the peak locations for TKE and RSS for $y-z$ measurements computed
Figure 4.52: (a–b) $\langle u'^2 \rangle$, (c–d) $\langle v'^2 \rangle$, (e–f) $-\langle u'v' \rangle$, (g–h) $\langle w'w' \rangle$, (i–j) TKE normalized by $U_e$ for (left) FPG-SM 20 Hz, and (right) low-order FPG-SM 20 Hz cases.

from the original and the low-pass-filtered velocity fields. While only roughly 30% of the TKE is embodied in the larger-scale motions (consistent with the fact that this was the metric used to determine how many POD modes were used in this reconstruction for each case), these large-scale motions account for 40–50% of the RSS. For the FPG-SM cases (see figures 4.52 and 4.53), the contours of the filtered $\langle u'v' \rangle$ have similar magnitudes as the original, unfiltered data, except in the region near the wall where the dominant influence of the small scales would be expected. These observation are consistent with those of Liu et al. (2001) who conducted POD analysis on PIV data of channel flow in a streamwise-wall normal plane. The large-scale motions reconstructed using 50% of the total TKE were found to embody two-thirds to three-quarters of the total RSS in the outer region of the flow.

Of interest, the large-scale contributions to the single-point turbulence statistics in both the
smooth- and rough-wall cases show secondary peaks (or outer peaks) in contrast to the near-wall peaks from the original, unfiltered statistics (see figure 4.51). Hutchins and Marusic (2007) investigated the emergence of a second, distinct energy peak (inner and outer peaks) with increasing Reynolds number in smooth-wall profiles of $\langle u'^2 \rangle$ under ZPG conditions. They found the inner peak to be the strongest peak with a fixed location at $y^+ \approx 15$ and predominantly associated with streamwise scales of size $\lambda_x^+ \approx 1000$ regardless of Reynolds number and due primarily to the near-wall cycle of turbulence production. The outer peak was found to develop in the logarithmic region and its location is fixed approximately at $y/\delta \approx 0.06$ and is predominantly associated with structures of streamwise size $\lambda/\delta \approx 6$. In addition, the distance between the inner and outer peaks increased with Reynolds number, as did the magnitude of the outer peak relative to the inner one. Mathis et al. (2009) subsequently reported that the large scales residing in the log region amplitude modulate the small-scale structures in the near-wall region, with this modulation increasing with
Reynolds number. Consistent with these previous ZPG observations, the large-scale motions in the filtered FPG-SM and FPG-Rough cases develop peak locations further from the wall than the unfiltered single-point statistics. This effect is more dramatic in the FPG-SM case than the FPG-Rough case.

Quadrant analysis of the low-pass-filtered velocity fields was also conducted to assess the contributions of the large-scale motions to the overall Reynolds shear stress. Figures 4.58–4.61 show the
Figure 4.55: As in figure 4.52 but for the 30 Hz cases.

quadrant contributions of the large-scale motions to the mean RSS for FPG-SM and FPG-Rough cases for a threshold of $H = 0$ for the 20 and 30 Hz cases. In all cases, $Q_2$ and $Q_4$ events contribute in a comparable manner to the overall mean RSS in contrast to the previous observations of a dominance of $Q_2$ over $Q_4$ events when all spatial scales were evaluated. This indicates the dominance of $Q_2$ over $Q_4$ events in the previous observations to be predominantly associated with the smaller spatial scales of the flow. For $H = 4$, wherein only the intense $u'v'$ events are included, a clear dominance of $Q_2$ over $Q_4$ events is again apparent in both the smooth and rough cases as
Figure 4.56: Turbulent kinetic energy distribution of FPG-SM 20 Hz case (a) and 30 Hz case (c) with the corresponding low-order FPG-SM 20 Hz (b) and 30 Hz cases (d).

Figure 4.57: As in figure 4.56 but for the FPG-Full cases.

Finally, one-dimensional profiles of the quadrant contributions for the FPG-SM and FPG-Rough cases are shown in figures 4.66–4.69. Quadrant contributions of the low-pass-filtered velocity fields (right column) clearly show the dominant contributions of both $Q_2$ and $Q_4$ events for $H = 0$, while the stronger dominance of $Q_2$ over $Q_4$ for $H = 4$, consistent with the quadrant analysis of the original velocity fields. Of interest, the peaks of $Q_2$ and $Q_4$ events from the large-scale-motion contributions occur further away from the wall than the original, unfiltered results. This again implies the smaller spatial scales are responsible for the peak values close to the wall.
Table 4.5: Wall-normal locations of the peak values of TKE and $\langle u'v' \rangle$ from original ensemble averaged files.

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface</th>
<th>$y/\delta$</th>
<th>TKE ($10^{-3}$)</th>
<th>$y/\delta$</th>
<th>$\langle u'v' \rangle$ ($10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPG-SM 20Hz</td>
<td>Smooth</td>
<td>0.027</td>
<td>4.79</td>
<td>0.08</td>
<td>0.97</td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>Smooth</td>
<td>0.026</td>
<td>4.15</td>
<td>0.072</td>
<td>0.86</td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>Rough</td>
<td>0.057</td>
<td>7.92</td>
<td>0.088</td>
<td>1.85</td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>Rough</td>
<td>0.056</td>
<td>7.53</td>
<td>0.097</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Table 4.6: Wall-normal locations of the peak values of TKE and $\langle u'v' \rangle$ from the ensemble averaged files of the low-pass-filtered instantaneous velocity fields. The amount of reduced rates for TKE and $\langle u'v' \rangle$ are also specified in the parenthesis.

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface</th>
<th>$y/\delta$</th>
<th>TKE ($10^{-3}$)</th>
<th>$y/\delta$</th>
<th>$\langle u'v' \rangle$ ($10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPG-SM 20Hz</td>
<td>Smooth</td>
<td>0.14</td>
<td>1.05 (78.2%)</td>
<td>0.18</td>
<td>0.39 (59.3%)</td>
</tr>
<tr>
<td>FPG-SM 30Hz</td>
<td>Smooth</td>
<td>0.14</td>
<td>1.02 (75.4%)</td>
<td>0.17</td>
<td>0.37 (57.2%)</td>
</tr>
<tr>
<td>FPG-Rough 20Hz</td>
<td>Rough</td>
<td>0.13</td>
<td>2.15 (72.9%)</td>
<td>0.15</td>
<td>0.94 (49.1%)</td>
</tr>
<tr>
<td>FPG-Rough 30Hz</td>
<td>Rough</td>
<td>0.12</td>
<td>2.14 (71.5%)</td>
<td>0.14</td>
<td>0.91 (48.8%)</td>
</tr>
</tbody>
</table>

Figure 4.58: Low-order representation of quadrant contributions to the mean RSS, $\langle u'v' \rangle Q / U_c^2$, for the FPG-SM 20 Hz case and $H = 0$. (a) $Q_1$, (b) $Q_2$, (c) $Q_3$ and (d) $Q_4$.

4.3 Summary

Comparison of the results from the streamwise–wall-normal plane measurements with those of a ZPG smooth-wall turbulent boundary layer revealed the competing influence of FPG and roughness
effects. While vortex organization was found to persist under both FPG-SM and FPG-Rough conditions, its spatial characteristics were altered compared to ZPG-SM flow. In particular, inspection of instantaneous velocity fields revealed this organization to be focused closer to the wall in the FPG-SM case, with a shallower inclination angle noted as well as an elongated streamwise extent. In contrast, the FPG-Rough results revealed packet structures more consistent with the ZPG-SM
Figure 4.61: As in Figure 4.58 but for the FPG-Rough 30 Hz case.

Figure 4.62: Low-order representation of quadrant contributions to the mean RSS, $\langle u'v' \rangle Q/U_c^2$, for the FPG-SM 20 Hz case and $H = 4$. (a) $Q_1$, (b) $Q_2$, (c) $Q_3$ and (d) $Q_4$.

case, indicating that roughness mitigates the FPG-induced focusing of these structural attributes toward the wall. Two-point correlations coefficients of velocity support these instantaneous observations, as the inclination angle of $\rho_{uu}$ was decreased in the FPG-SM case but increased again in the FPG-Rough case. In addition, while FPG-SM conditions increased the streamwise extent of $\rho_{uu}$, FPG-Rough conditions yielded a shortening of $\rho_{uu}$.  

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Stereo PIV measurements in the wall-normal–spanwise plane of smooth- and rough-wall turbulent boundary layers under FPG conditions revealed the spatial imprints of LMRs and HMRs, particularly a reduction in their wall-normal extent and intensity for smooth-wall flow contrasted with enhancement of these characteristics by roughness. Nevertheless, the spanwise coherence of these motions, discerned from two-point velocity correlations, was notably reduced by the presence
of roughness, consistent with the previous observations of a reduction of streamwise coherence of such motions due to roughness. In contrast to the instantaneous velocity fields in the $y-z$ plane that are marked by alternating LMR’s and HMR’s across the entire spanwise extent of the field of view (both smooth and rough), the corresponding two-point correlations do not reflect this degree of periodicity in the large-scale motions. This is likely due to the broad range of scales present in these flows and therefore reflected in the two-point correlations. Hence, POD analysis as a scale decomposition was conducted to low-pass-filter the instantaneous velocity fields to effectively evaluate the characteristics of the large-scale motions of the flow in isolation (Wu and Christensen, 2010). Consistent spatial imprints of LMRs and HMRs were observed in the reconstructed instantaneous velocity fields and two-point velocity correlations, but enhanced spanwise alternating patterns of LMRs and HMRs were revealed compared to the unfiltered velocity fields and correlations. Modal analysis revealed the alternating patterns of mode shapes represented by $u'$. Using the low-pass-filtered instantaneous fields, Reynolds stress and TKE contributions of the large-scale motions were calculated and revealed a secondary peak in the log region. In contrast to the 30% TKE contributions of these motions, the larger scales were found to embody a majority of the Reynolds shear stress in both the FPG-SM and FPG-Rough cases. Finally, quadrant analysis was employed to study the RSS contributions of the large-scale motions. When all events were
Figure 4.66: Profiles of quadrant contributions for the original FPG-SM 20 Hz data and low-order reconstructed FPG-SM 20 Hz data. $H = 0$ and $H = 4$ cases are both shown.

considered, roughly equal contributions of ejections and sweeps to the mean RSS was observed, indicating that the dominance of ejections over sweeps in the original, unfiltered velocity fields is predominantly associated with smaller-scale motions.
Figure 4.67: As in figure 4.66 but for the FPG-SM 30 Hz case.
Figure 4.68: Profiles of quadrant contributions for the original FPG-Rough 20 Hz data and low-order reconstructed FPG-Rough 20 Hz data. $H = 0$ and $H = 4$ cases are both shown.
Figure 4.69: As in figure 4.68 but for the FPG-Rough 30 Hz case.
Chapter 5

Summary, Conclusions, and Future Work

The analysis of the flow characteristics of many practical flow systems is complicated due to the simultaneous occurrence of a non-zero pressure gradient and surface roughness conditions. While of significant practical importance, only a few studies have considered such conditions, typically employing simplified surface roughness, such as sand grain, mesh screens, or two-dimensional square bars. The present study explored the combined impact of highly-irregular surface roughness replicated from a damaged turbine blade and moderate FPG conditions on a turbulent boundary layer. The focus of analysis was on both the statistical and structural modifications imposed by the combined impacts of FPG and surface roughness effects compared to our present understanding of canonical zero-pressure gradient, smooth-wall flow.

The single-point statistical analysis of the streamwise–wall-normal \((x - y)\) plane 2D PIV and wall-normal–spanwise plane \((y - z)\) cross plane PIV measurements over smooth and rough surfaces under FPG conditions was conducted to explore the statistical characteristics under such conditions. The mean velocity profile of the accelerating flow shows an intense velocity gradient close to the wall that gives the mean profile a fuller shape compared to the ZPG-SM case. The presence of roughness mitigated these FPG effects, yielding a profile for the FPG-Rough that more resembles the ZPG-SM case. The ensemble-averaged \(U\) contours also show the tight contour ranges under FPG-SM case compared with ZPG-SM case indicating the boundary layer thinning effect under FPG conditions. Again, however, the presence of surface roughness broadened the tight contour range due to the increase of momentum deficit, while FPG conditions, in contrast, suppressed the boundary layer growth. It is interesting to note that such a competing influence of surface roughness and FPG conditions renders the FPG-Rough case quite similar to the ZPG-SM case. The opposing influence of FPG and surface roughness is also found in the Reynolds stresses. Consistent with previous studies (Piomelli et al., 2000; Ichimiya et al., 1998; Shah and Tachie, 2008; Agelinchaab...
and Tachie, 2008; Cal et al., 2008, 2009; Tay et al., 2009; Dixit and Ramesh, 2010), the Reynolds stresses were dampened by the imposed FPG conditions and the peaks were focused closer to the wall. It is well known that the surface roughness enhances Reynolds stresses regardless of pressure-gradient conditions. The present FPG-Rough cases highlight the impact of roughness in the near-wall region where the Reynolds stresses were found to be significantly enhanced by roughness. In contrast, the outer region of the flow appeared more strongly influenced by FPG conditions.

Consistent with the 2D PIV measurements in the $x - y$ plane, the wall-normal–spanwise plane measurements also revealed intense velocity gradients in the near-wall region as well as suppression of the vertical growth of the boundary layer in the FPG-SM case. Contrary to the homogeneous spanwise distribution of the FPG-SM mean and turbulence statistics, strong spanwise inhomogeneity was observed in the FPG-Rough case. The $U$ contours showed distinct waviness due to this inhomogeneity of the flow over the rough surface up to $y/\delta \approx 0.5$. This waviness resulted in spanwise alternating regions of low and high mean streamwise momentum. As previously observed in the $x - y$ plane, the Reynolds stresses and TKE in the FPG-SM case were dampened, while the irregular protrusion of surface roughness elements promoted spanwise inhomogeneity and thus presented localized regions of intense Reynolds stresses and TKE in the region $0 < \Delta z/\delta < 0.5$. This spanwise region is coincident with the location of reduced mean streamwise momentum. Quadrant analysis in the $x - y$ plane measurements revealed the dominance of ejections $Q_2$ over sweeps $Q_4$ under FPG conditions regardless of the surface condition, whereas comparable contributions of $Q_2$ and $Q_4$ were observed in ZPG-SM case when all the instantaneous $\langle u'v' \rangle$ events are included ($H = 0$). For all cases, a clearer dominance of $Q_2$ over $Q_4$ events was observed for $H = 4$, wherein only intense $u'v'$ events were included. The addition of surface roughness enhanced the magnitudes of sweeps and ejections. The same trend of dominance of $Q_2$ over $Q_4$ events was observed in the $y - z$ plane measurements, with significant inhomogeneity in the spanwise direction for the FPG-Rough case.

To explore the structural attributes imposed by FPG conditions and surface roughness, instantaneous velocity fields were inspected and two-point velocity correlations were computed. For the $x - y$ plane measurements, the streamwise alignment of hairpin-like vortices into larger-scale
packets was observed in all cases, though the characteristics of this organization were altered by FPG and surface roughness conditions. In particular, a reduction in the momentum deficit beneath the hairpin heads was observed, in addition to a focusing of this vortex organization closer to the wall in the FPG-SM case. However, with the presence of surface roughness, the momentum deficit beneath the hairpin heads was enhanced and the packets penetrated much further away from the wall compared to the FPG-SM case. The competing influence of FPG and surface roughness conditions mitigates the FPG-induced focusing of these structural attributes toward the wall. In accelerating flows, the two-point correlation coefficient $\rho_{uu}$ shows streamwise elongation and decreased inclination angle away from the wall. This suggests that the individual vortices within hairpin packets under FPG conditions tend to be elongated and flattened in streamwise direction. In contrast, due to the shortening effect of $\rho_{uu}$ owing to roughness, the characteristic streamwise length of $\rho_{uu}$ for the FPG-rough case is compensated. The wall-normal correlation coefficient $\rho_{vv}$, reflecting smaller-scale motions, shows negligible change by FPG and surface roughness conditions.

The stereo PIV measurements in the $y-z$ plane revealed the spatial imprints of low momentum regions bounded by high momentum regions (or vice versa), which is the signature of the hairpin vortex packet in the wall-normal–spanwise plane. Consistent with the $x-y$ plane measurements, the wall-normal extent and intensity of these motions were reduced in the FPG-SM case, while a significant enhancement of these characteristics was observed in the FPG-Rough case. Two-point velocity correlations of streamwise velocity ($\rho_{uu}$) revealed a strong positive correlation bounded by anti-correlation on either side, reflecting the spanwise-alternating nature of low- and high-momentum regions. For the FPG-Rough case, these correlations were significantly reduced in both wall-normal and spanwise extents, especially for the $\rho_{uu}, \rho_{uv}$, and $\rho_{uw}$. This reduction could reflect the generation of smaller-scale vortical structures at the rough wall that would necessarily reduce the average spatial coherence of the flow. Due to the broad range of scales present in all cases, the alternating patterns of low and high momentum regions across the spanwise direction observed in instantaneous fields was masked and thus the two-point correlations did not reflect this alternation. By conducting proper orthogonal decomposition, low-pass-filtered velocity fields embodying only the larger, more energetic scales while truncating smaller spatial scales were generated. Despite the truncation of the small scales, strong visual similarities between the original and the low-
order reconstructed instantaneous velocity fields was found, particularly the spanwise-alternating
low and high momentum regions. Two-point correlations computed from these low-pass-filtered
velocity fields revealed more clearly the spanwise-alternating nature of the low and high momentum
regions compared to the original, unfiltered velocity correlations. Modal analysis was conducted
to identify the patterns of the individual POD modes. The sum of first few modes corresponded
to the general patterns of the two-point velocity correlations. Alternating positive and negative
streamwise events were observed in both smooth- and rough- wall flow, with reduced patterns in
FPG-Rough case. Finally, while these low-pass-filtered fields embodied a minority of the TKE
(30%), the large-scale motions captured in these fields embodied a majority of the Reynolds shear
stress.

In summary, this effort has identified key aspects of turbulent boundary layer flow in the
simultaneous presence of FPG conditions and surface roughness that could be useful for those that
design and maintain engineering systems that embody such influences. For example, this effort
identified a clear dominance of roughness effects in the near-wall region while the outer region
appeared more strongly influenced by FPG effects. In this regard, an understanding of ZPG rough-
wall flow in the near-wall region would likely be sufficient for understanding and modeling the local
flow physics in this region dominated by roughness effects. In this regard, applications involving
heat transfer, for example, would likely be insensitive to FPG effects and instead would be driven
by surface-roughness effects. In contrast, flow away from the wall was found to be dominated by
FPG influences with little impact of roughness. Modeling and prediction of this region of the flow
would need to simply consider the impact of FPG conditions since roughness effects were found to
only penetrate to the log layer in the FPG-Rough case. Taken together, these observations provide
guidance on how one might model the physics of a FPG rough-wall turbulent boundary layer using
either Reynolds-averaged Navier–Stokes (RANS) methods or LES.

One of the main limitations of the effort presented herein was the lack of documentation of the
wall shear stress and hence viscous scales of the flow (the friction velocity, \( u_* = \sqrt{\tau_w/\rho} \) and the
viscous length scale, \( y_* = \nu/u_* \)). Unfortunately, measurements of \( \tau_w \) are difficult even in canonical
smooth-wall turbulent boundary layers and researchers typically rely on inferences based upon a
presumed logarithmic representation of the mean velocity profile (so-called Clauser chart method).
The occurrence of such a logarithmic representation of the mean profile cannot be presumed under FPG conditions and so determination of $\tau_w$ accurately in the present study was not possible. Indeed, even if a log profile is present, both the von Karman constant $\kappa$ and the intercept $C$ are pressure gradient-dependent variables (Nagib et al., 2006). Some studies have shown that the wall shear stress under non-zero pressure gradient conditions can be defined experimentally using oil-film interferometry (Osterlund, 1999; Ruedi et al., 2003; Nagib et al., 2006). However, this technique is limited to only smooth surfaces and is not adequate for a rough surface. Recently Krogstad and Efros (2010) suggested the micro-force balance as an alternative for defining the skin friction for flow over a rough surface directly by measuring the minute shift of an attached segment of a roughness element. Alternatively, integration of the mean momentum equation can also be used to assess the wall shear stress from flow-field measurements. Unfortunately, this methodology relies upon accurate evaluation of spatial derivatives in both the $x$ and $y$ directions. While the latter can be easily evaluated from PIV data, spatial derivatives in the streamwise direction are much smaller and therefore more susceptible to measurement uncertainties. Complicating this issue is the relatively narrow streamwise field of view of PIV measurements which provides a less than ideal signal from which to estimate streamwise derivatives. Thus, an obvious “next step” in this larger research effort would be the development of a robust method, either direct or indirect, for the evaluation for the wall shear stress so that the behavior of the flow in inner units can be fully evaluated.

Additional work is also needed in studying the impact of the upstream flow conditions on the flow within the developed FPG region where the present measurements were taken. The limitations of the wind-tunnel employed required the imposition of a mild adverse pressure gradient over the first quarter of the streamwise development length in order to raise the wind-tunnel ceiling to a height adequate enough to attain the moderate FPG conditions studied herein. Thus, the flow prior to the imposition of the FPG conditions was in a mild APG state rather than a “cleaner” ZPG state. Understanding what, if any, impact such a condition might have on the overall development of the smooth- and rough-wall flows studied would also be helpful.
References


