COMMUNICATING ACCORDING TO THE STANDARDS: EXAMINING MATH TALK IN CHINESE AND U.S. MATHEMATICS CLASSROOMS

BY

MARC MCCONNEY

DISSELTATION

Submitted in partial fulfillment of the requirements for the degree of Doctorate of Philosophy in Educational Psychology in the Graduate College of the University of Illinois at Urbana-Champaign, 2013

Urbana, Illinois

Doctoral Committee:

Professor Michelle Perry, Chair
Professor Allison Ryan, University of Michigan
Professor Gloriana Gonzalez
Professor Richard Anderson
Abstract

This dissertation examines how some U.S. and Chinese teachers may employ some of the suggestions of the National Council of Teachers of Mathematics, *Standards*, during their discursive exchanges in the classroom. More importantly, this study attempts to operationalize one section of the *Standards*, Communications in the 3rd through 5th grades, by developing a coding scheme based on key ideas and principles from that section, and, then, by using those codes as a way to assess some teachers’ behaviors. In doing so, I found that Chinese teachers’ practices are more aligned with some of the suggestions of the *Standards*. However, the way in which teachers employ these suggestions varied greatly. A more in-depth look at segments of classroom discourse revealed that U.S. and Chinese teachers, even when employing some of the same the suggestions, have different intentions, strategies, and implementation. This study highlights the importance of specificity and consensus when evaluating teachers against the suggestions and principles of reform documents, reports, and studies. With these findings, we possibly can augment the professional development of U.S. teachers through the use of video, especially viewing teachers with contrasting approaches to our own.
To Mary and Eric.

For chapters that are finished and for chapters yet to be written
Acknowledgments

I would like to acknowledge the hard work, the patience, and the mentorship of my advisor, Michelle Perry, without whom no word in this text would have been written, read, and eventually placed in the tomes of the University’s dissertations. There is a quote from the movie *Fallen* “There are moments which mark your life. Moments when you realize nothing will ever be the same, and time is divided into two parts—before this, and after this.” Your guidance, support, and tutelage clearly demarcate my life in two – having chosen to study under you has clearly changed my life.

Through your compassion, your love, and your dedication, I found strength to refine my character and conjure the perseverance to complete this degree. Truly and literally, I could not have done this without you. You have been much more than an advisor. From your words and actions, I have learned as much from you as I have from anyone. I doubt that I could ever thank you enough or show you the true extent of my appreciation.

I must thank the Department of Educational Psychology and, in particular, the Child and Adolescent Development Division. During my time at the University of Illinois, I have encountered and called upon a number of staff, but none as encouraging and helpful as Beverly, Mary, Helen, and Julie. Similarly, without the assistance, formally and informally, of many professors, over the years, I am positive that my success would not have been the same.

I would like to thank Professor Dorothy Espelage for you advice and patience. I would like to thank Professor Jenny Singleton for helping me become a better graduate student and teacher. I would like to thank Professor Cynthia Carter-Ching for refining my guiding research questions, giving me more to read, and always being in my corner.
I would like to thank my committee for being gracious, flexible, and informative. To Professor Anderson, I have been constantly impressed by your scholarship and passion for education. Professor Gonzalez, thank you for thoughtful and challenging perspective to the dissertation.

I would like to thank Professor Allison Ryan – you have made an indelible mark on me as a student and teacher. Thank you for allowing me to teach, to hone my skills, and to take risks. Thank you for always answering my call. Thank you for being an inspiration. And thank you for always believing in me – I probably cannot say this enough. It meant the world to me.

I would like to acknowledge my colleagues in research: Lucia Flevares, thank you for being someone I could emulate, for the late night talks in the Armory, the countless critiques, and the support and praise for my particular and sometimes contentious ideas. I would like to thank Travis Wilson, Meg Schleppenbach, Sujai Kumar, Neesha Noronha, Leigh Mingle, Crystal Feil and Linda Sims. Without you all, my dissertation would have been a longer and more arduous process.

I would like to thank Kevin Miller for broadening my views as a researcher. Attempting to have one’s research be relevant to the very population studied is a daunting task. Thank you for your input and engaging research.

I would like to thank all of my friends during this long journey, but particularly, my childhood group, Brian, Reggie, Kim, and Kirk. Without you, most of these accomplishments would not have happened; and if they did, they would not have been as wonderful and fulfilling in your absence. Thank you for being there, at all times. Thank you for encouraging me to choose the right path, despite my stubbornness, my fears, and many mistakes. You all are truly a great family to have.
I would like to thank Ethan for all your counsel, large and small, during these times. Without the couch, Guinness (the dog), the lake, and the late night stories, my journey would have been a little less humorous, exciting, and rejuvenating. I would like to thank Wilson for all of the road trips, words of advice, respites of relaxation and merriment, the ventures into filming, and our overall exploration of leadership and mentorship.

I must thank Betty and Christina. My times with you were priceless - at U of I, Chicago, and D.C. Thank you for your support at all times, especially at moments when life, work, and my pursuits seemed unclear. Thank you for allowing me to be a part of your lives. Through you both, I have learned a great deal about myself.

During this time, there were several individuals to thank: All of the Scotts – for giving me a home when I needed, for support and constant love, and for sharing your lives with me; to Frances and King- parents are important, love is important, and the constant reminder to do the right thing is important, so thank you for all three of those things, but most importantly, thank you for sharing your sons with me; to Lisa and Giles – for giving me a chance and a second career; to Karla – for your kindness and making me grow even when I was resistant; to Tamara – for advice, friendship, and playing devil’s advocate most of the time; to Natalie – for support and encouragement; to Jimmie – for the good times and the better times; and to Cheri, Keith, Amanda, and Kathleen – for taking the time to understand me, for pushing me and believing in me.

I would like to thank Hales Franciscan High School (those who came before me, those with me, and those that I taught). Hales was the start and impetus for this journey into education. The Franciscan approach to teaching and learning has always guided my steps and my thinking. I was a young man when entered your doors, and was still a young man when you allowed me to
teach for the first time. Within those halls, I created my first guiding questions for the research that eventually followed.

Lastly, I would like to thank my wife, Robin McConney. Although your induction into this “finish your dissertation” group was at the end of the journey, the end was longer than we anticipated and more difficult than first imagined. You were a smile when I needed it; a shoulder when I wanted it; and a beautiful and caring wife at all times.

Like all doctoral students, I had to write these chapters alone. Yet, I am grateful and honored that my future chapters will be written with you – although, occasionally, we will disagree on the titles.
Table of Contents

List of Tables .................................................................................................................. ix

List of Figures .................................................................................................................. x

CHAPTER 1 Introduction ................................................................................................. 1
  U.S. Student Performance ............................................................................................ 1
  International and National Reasons ............................................................................ 4
  The NCTM Standards ................................................................................................. 7
  Rationale for this Study ............................................................................................... 10
  Narrowing the Focus: Communicative Standards
    Recommended by the NCTM ..................................................................................... 22
    Guiding Questions .................................................................................................. 23

CHAPTER 2 Conceptual Framework .............................................................................. 29
  Constructivism ........................................................................................................... 29
  Sociocultural Theory ................................................................................................. 32
  Importance of Discourse ........................................................................................... 35

CHAPTER 3 Methods ...................................................................................................... 39
  Data Source ................................................................................................................ 39
  Coding ......................................................................................................................... 41
  Development of this Coding Scheme ......................................................................... 41
  Coded Excerpts .......................................................................................................... 56
  The Coding Process .................................................................................................. 67

CHAPTER 4 Results ......................................................................................................... 69
  Descriptive Statistics ................................................................................................. 69
  Statistical and Exploratory Analyses ........................................................................ 76
  Quality of Behaviors ................................................................................................ 82

CHAPTER 5 Discussion and Conclusions .................................................................... 132
  Overview of the Findings: Alignment with the NCTM Standards ......................... 132
  A More Detailed Appraisal: Distinct and Dominant Features ............................. 140
  Limitations ................................................................................................................ 144
  Sociomathematical Norms ....................................................................................... 147
  Teachers’ Knowledge ............................................................................................... 149
  Conclusions ............................................................................................................... 150

REFERENCES ................................................................................................................ 157
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Copied Directly from the NCTM Standards</td>
<td>46</td>
</tr>
<tr>
<td>2</td>
<td>Listing of Codes</td>
<td>51</td>
</tr>
<tr>
<td>3</td>
<td>Coded Transcript from a Chinese Classroom</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>Coded Transcript from a U.S. Classroom</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>Other categories</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>T-Tests for whole-class time</td>
<td>71</td>
</tr>
<tr>
<td>7</td>
<td>Significant differences between groups</td>
<td>81</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>The number of teachers’ actions per group</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>for each category, except simple questions</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The average length (by number of words) of questions and explanations</td>
<td>74</td>
</tr>
<tr>
<td>3</td>
<td>The number of simple questions compared</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>to other codable behaviors, by country</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Correspondence analysis of teacher and behavior</td>
<td>79</td>
</tr>
</tbody>
</table>
CHAPTER 1

Introduction

U.S. Student Performance

U.S. researchers, agencies, and policy-makers have explicitly and implicitly measured U.S. students’ mathematical performance in two ways: they have compared U.S. students’ performance with their counterparts in other countries, or they have measured U.S. student achievement against state-wide or nationally devised standards. When compared to their international counterparts, U.S. students are described as average or consistently underperforming (American Institutes for Research, 2009; Dillon, 2010, December 7; PISA 2009; TIMSS 2003). Additionally, when assessing national performance, many students are at a basic or proficient level: having a small percentage performing at high or advanced levels (NAEP, 2005, 2007; NMAP, 2008). For some agencies, students’ national and international performances are combined to specify what concepts or procedures are strong or weak for U.S. students. The National Center for Education Statistics plans to conduct a new study to link national and international student assessments so that states can measure their performance against international benchmarks (National Center for Education Statistics, 2011).

When examining scores on international mathematics tests many European and Asian students consistently rank higher than U.S. students – with students from Finland, Japan, Korea, and China having enjoyed the highest rankings of all countries (TIMSS 1999, 2003, 2007; PISA, 2009; Zhou & Peverly, 2004). Recently, the U.S. media (Dillon, 2010, December 7; Nightly News, 2011) was surprised to find that Asian students, particular those in China, because they were taking the test for the first time, ranked the highest, mathematically, on the 2009 Program for International Student Assessment (PISA) test.
The PISA, started in 2000, tests 15 year olds from participating countries every three years. In the 2003 and 2006 cycles, U.S. students lagged behind their international counterparts. In each cycle, at least 12 to 15 countries scored higher, in mathematics, than the United States. Although U.S. students’ scores have increased from 2003 to 2009, they have been repeatedly outsourced by other countries; so much so, that in 2009, the U.S.’s rank has dropped to 24th. This outlook on mathematical performance is supported by other international studies (TIMSS 2003, 2007).

The 2003 Trends in International Math and Science Study (TIMSS) found that U.S. fourth grades ranked 12th in a group of 24 participating countries. In 2007 this trend continued, with the U.S. ranked 11th in a group of 37. Startlingly, the U.S. improvement from 1995 to 2007 is only 11 points for 4th graders and 8 points from 8th graders. The TIMSS study found that only 6 percent of 8th graders scored at an advanced level, whereas 10 percent of 4th graders did so. The study also found that at least 20 percent of students in Hong Kong, Korea, and Finland were similarly highly accomplished, and more than ten other countries had more than twice the percentage of advanced students as the United States. Although the percentage of U.S. students who performed at or above average was larger than the median, the results of the TIMSS study support the implications of the PISA tests: U.S. students’ math performance could be better.

Even when U.S. students are only tested against nationally devised standards, we are surprised. In 2005, according to the National Assessment of Educational Progress (NAEP) U.S. students perform mostly at a basic or proficient level. Only 6 percent of U.S. 8th graders who participated in the NAEP scored at an advanced level in math. In 2007, only 32 percent of 8th graders performed a proficiency standard, which is a considerably lower standard than advanced. Thus, more than two-thirds of these students are not performing well on a nationally designed
test. In 2009, the national average for 4\textsuperscript{th} and 8\textsuperscript{th} graders was slightly below the proficiency level. Interestingly, for four graders, this average scored was exactly the same in 2007. This finding caused David Driscoll (2011), the national chairman for the NAEP, to declare: “[T]he improvements in math clearly have not been enough. Over the past eight years progress has slowed, particularly at grade 4 where it had been very rapid for more than a decade. The percentage of students below \textit{Basic} has been reduced substantially, but it remains far too high” (D.P. Driscoll, letter to the governing board, November 1, 2011). Although some critics believe that these reports are not reliable because the standards set by the NAEP are stringent, the results are consistent with other studies and tests concerning students’ mathematical ability.

In an interesting report, \textit{Teaching Math to the Talented Tenth}, Hanushek, Peterson and Woessmann (2011) compared scores from the NAEP and the PISA to isolate possible educational advantages (e.g. both parents are college graduates) or disadvantages (e.g. no college educated parent) of groups. They examined whether the low PISA scores in the U.S. might be in part attributable in part to statistically significant groups with either predisposition. Even when they isolated students with at least one college-educated parent, they found only 10.3 percent reached the advanced level, a group still outperformed by all test-takers from 16 other nations.

The NAEP, over the years, has repeatedly found an average or below average performance of U.S. students – hinting that students’ processes are computationally and procedurally focused (NAEP, 2009, 2011). Sutton and Krueger (2002) state that “[m]any mathematics students spend much of their time on basic computational skills rather than engaging in mathematically rich problem-solving experiences” (p. 26). Such a focus could lead to rote memorization of formulas and the ability to solve problems previously practiced, but little else. In either instance, many have concluded that, overall, U.S. students’ achievement in and
understanding of mathematics could be considerably stronger. NCTM (2000) in its publication *Principles and Standards for School Mathematics* emphasized that “there is no question that the effectiveness of mathematics education in the United States...can be improved substantially” (p. 5).

U.S. students’ international and national performance have influenced the way we approach the training of pre-service teachers, designing of professional development programs, and assessing the depth of instruction in the classroom. This concentration on teachers’ practices eventually leads to comparing teachers of different countries and how teachers of varying levels of expertise approach new material and lessons. In fact, Andreas Schleicher, who oversees the analysis of education indicators for PISA, states “the common core effort is a great example about leveraging lessons from other countries.” The organizations that propelled the Common Core State Standards Initiative, which represent governors and state schools chiefs, aimed to make the standards comparable to those of high-performing countries. Whether they succeeded is still being debated. But the common standards represent a major U.S. endeavor to learn from abroad.

**International and National Reasons**

Many studies have focused exclusively on possible reasons for Asian students’ seemingly superior performance: examining the influence by cultural norms, familial support, nationally standardized curricula, language, and teacher’s preparation and instructional practice (Barth 2002; Corey, Peterson, Lewis, Bukarau, Jared, 2010; Fernandez, 2002; Gonzales, Guzman, & Jocelyn, 2004; Jacobs & Morita, 2002; Stigler & Hiebert, 1999; Stigler & Hiebert, 2004).

Naturally, and quite logically, we would assume that a combinatory effect is at work. Although any of these factors can account for superior performance on the part of any country, many of
these factors are overly broad for either a practical examination for or application in the U.S. No matter how much we understand about the cultural dynamics of Asian countries that possibly account for a deeper grasp of mathematics and motivation for academic success, we cannot implement those cultural norms. Similarly, familial approaches, supports, and beliefs about education are developed over time. Essentially, we have to concentrate on those possible influences that can be compared to and/or implemented in U.S. classrooms.

Some suggest that the U.S. should spend more money per student; others contend that smaller student-teacher ratios will make a difference. Some believe that student achievement can be, in part, be improved if teachers’ subject matter knowledge, beliefs about learning, and/or practice changes (Floden & Meniketti, 2005; Ma, 1999; Rosas & Campbell, 2010; Shulman, 1986). Thus, studies, interventions, and curricula have been designed to adjust or change one of these factors. In the end, many have attempted to change how teachers think about and approach the teaching of mathematics, assuming that what students witness daily becomes their own practice and determines how they come to understand mathematics. For many researchers and teachers, the most accessible of these influences is instructional practice.

To this end, some have examined the distinctive features of teaching mathematics in the U.S. and abroad, the feasibility of adopting techniques from other countries, strengthening the professional development of teachers, and/or various ways to substantively and substantially implement reform ideas in the school and classroom (Fernandez, 2002; Lim 2007; Stigler & Hiebert, 1999; Stigler & Hiebert, 2004). Sutton and Krueger (2002) stated, "The most direct route to improving mathematics achievement for all students is through better mathematics teaching" (p. 26). At the heart of these studies and efforts is whether teachers can change, and if so, how.
Teacher change, no matter the form, faces many challenges. Do we have to change teachers’ beliefs to change their practices? Do we have to spend more money to assist teacher change? Is it motivation, knowledge, or curriculum? Any one of these challenges can impede or facilitate teacher change. In some instances, changing any of these attributes would take considerable time and effort. However, other aspects of teaching can be refined by the teacher without extensive training, overhauling the general curriculum, or acquiring new technology. For example, the way teachers ask and model questions for the classroom can be implemented at any time and in any lesson. Thus, a powerful vehicle for change, and, possibly, student learning outcomes, is the way the teacher orchestrates communication in the classroom.

In addition to cross-cultural comparisons of teachers’ practices, many studies have focused heavily on teachers’ actions as they attempt to change their approaches in the classroom: namely, the structure of cooperative learning, the placement of tasks within a lesson, and establishing certain types of whole-class discussions according to recent reform efforts in math classes. These studies are often the result of a district or a school adopting a new curriculum or teachers undergoing a professional development program. Recently, throughout the nation, new curricula and programs have been based on the reform suggestions and initiatives.

Despite many city, state, and national efforts, U.S. student performance has not been consistent across school districts; nor have teachers’ practices been consistent with one another or aligned with any specific set of guidelines or suggestions (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; e.g. NAEP 2007, 2009, highlighting the inconsistency across states). To this end, states have sought to create and establish academic standards or adopt existing reform suggestions and efforts. Whether these states have created a state-wide standard or
adopted a national one, many are similar to or based, partially or wholly, on the National Council of Teachers in Mathematics (NCTM, 2000) ideas.

**The NCTM Standards**

While the push for major reform efforts in mathematics education has been ongoing for many years (Brownell, 1935; National Research Council, 1989), efforts have been renewed since 1989 when the National Council of Teachers of Mathematics (NCTM) released the original standards document, *Curriculum and Evaluation Standards for School Mathematics*. This document, along with a revised version and additional sections, were offered as a guide for mathematics education reform (NCTM, 1989, 2000; National Research Council, 1989). Teachers, for the last two decades, in the United States, were urged to change their approaches to teaching in both content and pedagogy (NCTM, 1989, 2000). They needed to change from an emphasis on procedural understanding, on the part of the student, to a more conceptual understanding, including the application of math to “real-world” situations. The documents also stressed process skills such as reasoning, problem-solving, communication, and connections, and moving away from focusing on computational skills to (Herrera & Owens, 2001). Classrooms should be transformed into student centered discussions and activities, which included students “discovering and constructing mathematical relationships, rather than merely memorizing procedures and following them by rote” (Herrera & Owens, 2001, p. 89).

This study examines how teachers, during whole class discussion, possibly align their questions and explanations with reform suggestions of the National Council for Teaching Mathematics (NCTM 2000), particularly the Communication section in the Principles and Standards for School Mathematics (the *Standards*). Such alignment should mean that teachers
do not rely heavily on achieving the right answers, rote memorization, and a computational dominance in the classroom.

In particular, I focus my efforts on NCTM for many reasons. First, the NCTM is a widely known and influential document. Second, the NCTM reflects the sweeping changes that are necessary for a deeper practice in math classes – teachers becoming more student-centered in their approaches, and students’ understandings becoming more conceptually based and applicable to real life situations. Additionally, many of the suggestions in the Standards can be found in other documents and reform efforts, like the Common Core Initiative (2010) or the Oregon Mathematics Leadership Institute (OMLI, 2005). Third, the NCTM suggestions and principles are based on practices in the U.S. and can be implemented without debate about culture, curriculum, or parental involvement.

One of the NCTM’s most prominent documents is the Standards, outlining a vision and roadmap for change in classroom approaches to learning, interactions, and expectations. This document calls for teachers and schools to concentrate on student-centered learning, emphasizing students’ conceptual understanding, an increase in the use of technology in classroom, and connections between and the application of mathematics to other subjects. Not only have states and agencies debated about U.S. teaching methods and curricula, many have sought new teaching guides, curricula and professional development programs, based in whole or part on reform principles.

Ultimately, these investigations, suggestions, and reform efforts have met with limited success. Either teachers have found it difficult to understand the purpose of reform curricula, implement reform practices in classrooms, or fully incorporate foreign techniques in U.S. classrooms (Charalambous & Philippou, 2004; Lim, 2007; Remillard & Bryans, 2004). At the
heart of this difficulty is a teacher’s ability to see what “good” practices look like. Chazan and Ball (1999) called for specific, yet realistic teachers’ behaviors, stating that “current math education discourse about the teacher’s role in discussion-intensive teaching…” leaves educators without a “framework for the kinds of specific, constructive pedagogical moves that teachers might make” (p. 2).

Initially, a teacher must possess the belief that reform efforts are beneficial and possible; but, then, teachers must be able to visualize and create those changes in their own classrooms. Studies also reveal how much teachers struggle in their new roles as facilitators within discourse-based learning environments (e.g., Nathan & Knuth, 2003; Rittenhouse, 1998). This research attempts to address this overall question: how can we describe teachers’ actions, according to the suggestions and principles of the Standards, in ways that are clearly identifiable and ultimately useful to the improving their practice?

Lastly, the NCTM purports that the PSTM has standards for the teaching of mathematics, the evaluation of the teaching of mathematics. Additionally, the Standards have guidelines for worthwhile mathematical tasks, a teacher’s role in discourse, and a student’s ability to participate in meaningful activities and discussions. These three aspects of a classroom and the Standards suggestions are the motivation for this dissertation.

To achieve this, I attempt to add more clarity and consistency to researchers’ definitions of some reform ideas for the purpose of measuring a teacher’s alignment with the Standards, and possibly providing teachers with more specificity about how the suggestions of these particular standards can be enacted. By first operationalizing the suggestion of the Standards and then cataloguing good practices from both China and the United States, I want to highlight any good practices that are and are not transparently captured by the suggestions of the Standards. I want
to map the use of certain types of teachers’ discursive actions in the classroom. Essentially, this process should add to the pool of video and transcript resources available to researchers, administrators and teachers who rely on professional development training as a way to affect teacher change.

Given this bold attempt, the heart of the study is exploratory, hopefully beginning a conversation that will seek to refine some of the assumptions, suggestions, and findings contained herein.

**Rationale for this Study**

**The NCTM’s reach.** The NCTM’s *Standards* reflect and have propelled the sweeping reform efforts we have witnessed in the United States for more than twenty years. States, cities, and school districts have either created standards or guidelines similar to those suggestions and principles contained in the *Standards* (for example OMLI, 2005; see also Common Core State Standards in Mathematics, 2010), or they have adopted, partially or in whole, *Standards*-based curricula and professional development materials (e.g. Children’s Math Worlds, Fusion 2000). Thus, many of the suggestions of the *Standards* represent a number of principles in other documents, approaches, and curricula.

Particularly, Principles and Standards for School Mathematics (NCTM 2000), which outlines the suggestions about communication in the classroom, represent very similar constructs to the Common Core. The Standards proposed content strands (called domains in CCSSM) and the importance of mathematical processes (called mathematical practices in CCSSM).

For example, one ubiquitous suggestion, either from the Common Core, the NCTM, or research articles, is the expectation for students to justify their answers. Justification seems to be a fundamental requirement for any teacher seeking to become more reform oriented. Other
similarities among reform approaches and documents include: having students challenge one another, think of multiple ways to solve a problem, apply the material to real-life events and phenomena, and posing a rich array of conceptual questions. By examining what the *Standards* suggest and attempting to implement them in a classroom, a teacher could shape his or her classroom into a student-centered, conceptually rich, but procedurally strong, mathematics class.

Furthermore, many teachers have knowledge of or are acquainted with reform ideas or the *Standards*. Reform ideas have become a staple for pre-service teachers, professional development programs, textbooks, or researchers attempting to assess in-class teacher practice (Jacobs et al., 2006; Tunks & Weller, 2009). At some level and to some extent, many teachers are affected by the suggestions of the *Standards* or reform efforts that call for teachers to change their practices in mathematics classrooms. Thus, teachers, wanting to change some of their practices, should be able to call upon the *Standards* as a guideline.

In fact, the *Standards* and studies examining the adoption and influence of this document assert that the suggestions and principles can be used to eliminate disparities in the quality of mathematics programs – across the nation (Delpit, 1995; Romberg, 1992); offer a clear focus for instruction, learning and assessment (Ravitch, 1995); provide demonstration of agreement and consensus (Labaree, 1984; O'Day & Smith, 1993); prepare teachers (Darling-Hammond, 2003; Labaree, 1984); and develop of a national curriculum in response to international comparison studies of countries whose students have achieved excellence in mathematics. The NCTM document asserts that its principles “describe particular features of high-quality mathematics education. The *Standards* describe the mathematical content and processes that students should learn. Together, the Principles and *Standards* constitute a vision to guide educators as they strive
for the continual improvement of mathematics education in classrooms, schools, and educational systems.”

In other words, the opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which students participate (NCTM 2000a). What students learn about particular concepts and procedures as well as thinking mathematically depends on how they engage in mathematical conversations and activities in their classrooms.

Since its inception, the NCTM’s principles and guidelines have had a noticeable impact on mathematics education in the United States, both directly and indirectly. Many mathematics methods courses, pre-service teacher training programs, methods of assessment, and professional development programs have been shaped to reflect some, if not all, of the suggestions of the Standards (Ferrini-Mundy, 1996; Findell, 1996; Research Advisory Committee, 1998). In addition, publishers have aligned mathematics textbooks with the Standards, claiming to either be based wholly on reform principles or at least influenced by recent trends in mathematics reform (e.g., Battista, 1999; Chandler & Brosnan, 1994).

Most importantly, some researchers (Fraivillig et al., 1999; Jacobs et al., 2006; Knuth & Peressini, 2001; Nathan & Knuth, 2003; Sherin, 2002) have focused exclusively on the instructional practices in U.S. classrooms and how teachers attempt to change. These studies provide crucial information on instructional practices in the United States, and have examined how U.S. teachers adopt new curricula, adhere to recent reform suggestions and principles, and participate in various forms of professional development (e.g. see Fraivillig, Murphy, & Fuson, 1999; Knuth & Peressini, 2001; Sherin, 2002; Wood, 1999). The reason for focusing on these investigations is that at the heart of these studies are teachers’ attempts to change: how they
examine their established approaches to mathematics and adopt new ways of believing and behaving in the classroom.

One central notion of the reform principles supporting the changes in instructional practices is that adherence to these reform principles will enhance U.S. students’ understanding of mathematical concepts and thereby increase their performance on mathematical tasks. As evidence, some mathematics programs have witnessed an increase in student achievement when adopting reform principles (Alternatives for Rebuilding Curricula, 2003; Hill, Rowan & Ball, 2005; Jacobs et al., 2007; NCTM, 2000; Fraivillig, Murphy, & Fuson, 1999; Fuson, William, & Drueck, 2000; Thompson & Senk, 2001).

Despite the pervasive reach of the Standards, their implementation has met with some difficulties. For some teachers, practices that are aligned with the Standards seem foreign, not resembling the ways in which they themselves were taught as students or instructed to teach. In these instances, teachers tend to fall back on traditional methods of teaching, especially when faced with confusion in the classroom or with impending standardized tests (Berry & Kim, 2008; Nathan & Knuth, 2003; Sherin, 2002; Williams & Baxter, 1996). Teachers have also remarked that it is difficult to understand exactly how to adopt many of the suggestions of the Standards (Hunter 2005; Nathan & Knuth, 2003; Sherin, 2002). One of the difficulties is actually understanding what good discursive practices look like in a variety of contexts (Blank, & Pechman, 1995; Lappan, 1997; Stigler, & Hiebert, 1999; Stigler & Perry, 2000). Teachers have difficulty implementing reform suggestions, especially when it comes to communication in the classroom.

Heaton (2000, p.47) compares this difficulty to Mark Twain’s description of wanting to pilot a riverboat. Thinking that the act of steering a boat was akin to what it looked like, Mark
Twain’s character soon found that once in the river, the dynamic between the captain and all the boat entails is more difficult than it looks. For Heaton, it was necessary to have a guideline as to how to operate the boat. Essentially, a guideline will have a great use if it supplied suggestions for what to do in many contexts.

Given these documented roadblocks to implementing a Standards-based curriculum or dynamic in a classroom, the time seems ripe for us, as educational researchers, to map out how a teacher can incorporate some of the principles of the reform movement. While the Standards have attempted to illustrate good teaching practices in the classroom, through examples in printed and online documents (McLeod et al., 1996) and various articles in *Teaching Children Mathematics*, I argue that more specific examples of good discursive practices should be studied and highlighted. Lampert, Rittenhouse, & Crumbaugh (1996) stated that “NCTM sidesteps the question of what exactly teachers need to teach and students need to learn for this kind of talk to be seen as an appropriate mode of public interactions among school children and their teacher” (p. 16).

In essence, instead of having examples that cover a broad range of NCTM suggestions simultaneously, I advocate studying those examples that might specifically illustrate informative, discursive, Standards-based practices in the classroom. Moreover, these examples should be taken directly from teachers in practice. Teachers need to witness how the ideas of the documents could be brought to life in classrooms (see, e.g., Stigler & Perry, 2000). Without these concrete examples, teachers will continue to work in the dark.

My dissertation offers a clear step in this direction. In particular, I document not only how U.S. but also how Chinese mathematics teachers may employ some of the suggestions of the Standards. Although the Chinese have not been in contact with the Standards, their superb
practice and performance in mathematics is well documented (Brenner, Herman, Ho, & Zimmer, 1999; Cai, 2000; Fan & Zhu, 2004; Miura, Chungsoon, Chang, & Okamoto, 1988; Stevenson & Stigler, 1992) and, because of this, their practice has the potential to offer us insight about specific ways in which the Standards can be implemented in U.S. classrooms.

**Cross-cultural investigations.** During the last two decades, when examining mathematical performance, there has been increasing interest in comparing schools across countries (Clarke et al., 2006; Leung et al., 2006; Schwippert, 2007). Reports and articles published from large-scale studies such as TIMSS, TIMSS-R, and PISA, suggests that studies can be a complex and nuanced, yet beneficial endeavor of examining teaching and learning in contrasting contexts. We are thus challenged by Clarke, Mesiti, Jablonka, and Shimizu’s question (2006), “…how best to learn from each other’s classroom practices” (p. 23). Large-scale studies provide massive data that can only be highlighted in slices in any particular study or article. So, how should teachers take advantage of these slices?

Schwippert (2007) suggested that researchers and teachers should develop a “feedback culture” (p.99). He stated that “[s]uch a culture allows practitioners to see feedback not as a threat but as a means of enhancing their practice and, in turn, ensuring sound educational outcomes for their students” (p. 99). Thus, the results of any particular study are welcomed, regardless of the findings, because those data are used to improve classroom practices. In this way, international studies can serve to evaluate and improve local practice.

When examining the results of international mathematical tests, a researcher will find an overwhelming display of how Asian students have outperformed U.S. students (e.g., Beaton, Mullins, et al., 1996; Hiebert et al., 2005; Stevenson & Stigler, 1992) and Chinese students have consistently ranked in the top-performing countries. Given these repeated findings, numerous
studies (e.g., House, 2006; Perry, 2000; Stigler & Hiebert, 1999) have sought to uncover why Asian students, particularly those from Japan, Korea, and China, seem to possess a better mathematical understanding and ability than their peers from other countries.

Although any one of these factors could account for the Asian students’ superior mathematical ability, one highly plausible reason for the superior performance of most Asian students is the types of instructional practices they encounter in the classroom (Linn, Lewis, Tsuchida, & Songer, 2000; Ma, 1999; Perry, 2000; Stevenson & Lee, 1995; Stigler & Hiebert, 1999). To cite one significant example, data from the TIMSS (1995, 1999) indicate that many Asian teachers do a better job than U.S. teachers of delivering a coherent and deep curriculum (e.g., Stigler & Hiebert, 1999). Furthermore, the fact that many U.S. teachers do not employ the suggestions and principles of the Standards likely adds to the performance differences between U.S. and Asian students.

We have known for decades that classroom practices in Asian countries seem to differ dramatically from those in U.S. classrooms (Stevenson, 1993; Stigler, Lee, & Stevenson, 1987; Stigler & Stevenson, 1991). For example, when compared to U.S. teachers, many Asian teachers spend more time presenting a problem, allowing their students to struggle with a problem, and challenging their students to find multiple solutions to the problem. Others (e.g., Hiebert et al., 2003) report that, in some Asian classrooms, teachers spend a considerable amount of time reviewing previous information and outlining the intended goals of the current lesson. Overall, the ways in which Asian teachers introduce new information and engage students with the materials differs from typical U.S. teachers’ practices.

Studying instructional practices of teachers is a viable avenue through which to examine possible influences on students’ learning and academic achievement (Stigler & Hiebert, 1999).
assume, like others (Jacobs et al., 2006; Perry, 2000; Stigler & Hiebert, 1999), that instructional practices are pivotal to students’ learning outcomes because students encounter most of their mathematics in school, which, in turn, determines how they learn, practice, and discuss mathematics. In this way, student mathematical performance is inextricably bound to teachers’ practices and the classroom environment.

Studying possible cross-cultural differences in teaching practices can be a useful vehicle with which we can possibly ask questions of our own practices. Additionally, the possible contrast between cultures begins the discussion of what types of question to ask when reviewing U.S. teachers’ practices. Such comparisons may disclose hidden practices and scripts that exist within a classroom. Lastly, contrasting the styles of two different cultural groups provide possible alternative ways of teaching, opening the possibility of transplanting positive activities, norms, or approaches (Stigler & Hiebert, 1997, 1999; Santagata & Stigler, 2000).

**Specificity with the Standards.** I want to make the suggestions of the *Standards* concerning communication more specific and more concrete, by outlining those key features of the NCTM *Standards* regarding communication in third through fifth grade. In essence, I want to codify key and pivotal words and phrases used in the *Standards*. Towards this end, I have created a coding scheme that includes ideas like: community acceptance of ideas, convincing peers, recognition and use of mistakes, etc. From this list, I define how each of these proposed actions take place. These key features are the foundation of my coding scheme.

The *Standards* suggest a complex orchestration of many actions: teachers’ use of tasks, curricula, problems and discourse. They recommend the types of behaviors that teachers should implement and the types of responses and participation students should employ in the class. They elaborate on the principles behind those suggestions; and attempt to give examples of what
good practices look like. Specifically, when addressing communication in the classroom, the Standards call for an overall, significant shift from a traditional approach to orchestrating meaningful discourse in the classroom (see for example Berry & Kim, 2008; Drake, 2006; Wood, 2001). The NCTM section on communication in general and in grades, 3rd, 4th, and 5th starts:

Instructional programs from pre-kindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely. (NCTM, 2000)

This foremost announcement can be interpreted in many ways. This flexibility could either be beneficial or frustrating to teachers and schools. On the one hand, having flexible suggestions allows schools and districts to mold their actions to the specific needs of teachers and students. On the other hand, when teachers are attempting to align themselves with reform ideas, particularly those suggestions of the Standards, or when they are faced with assessments based on these suggestions, more specificity might be helpful.

I do not suggest that the framers of the Standards intended to be more specific. Ideally, principles and suggestions would be broad, serviceable to a variety of groups. Yet, some studies show that despite teachers’ knowledge about and willingness to use the Standards, their practices do not reflect these reform ideas (Hiebert et al, 2003; Jacobs et al., 2006; Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999; Weiss, Pasley, Smith, Banolower & Heck, 2003). Given this disconnect, it is imperative that we are clear about how the Standards serve both as a
guideline for teachers who seek to change and an outline of assessment from which researchers can observe teachers. In other words, I am recommending that one of the next steps in understanding and improving practice in elementary mathematics classes is to specify the range of behaviors that instantiate the intentions of the NCTM Standards.

One particular study has used sections of the NCTM standards as a guideline to assess whether the principles have been incorporated into curricula, professional development programs, or classrooms (Jacobs et al., 2006). Jacobs et al. (2006) examined video portions of the 1995 and 1999 TIMMS study. They created codes to label and observe classroom practices aligned with Standards – ranging from problem solving to communication. How did they define communication? They quoted the NCTM: “Each student should be expected to not only to present and explain the strategy he or she used to solve a problem but also to analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies” (NCTM, 2000). Yet, the major aspect of how Jacobs et al. (2006) define communication in their published report is based on students offering alternative solutions. This practice is one of many suggestions of the Standards concerning communication in the classroom.

In the end, they found that teachers do not employ many of the Standards ideas, including reform ideas about communication in the classroom. They concluded that those teachers who are not adhering to any of the reform ideas are assumed to be teaching in a more traditional manner, possibly hindering students’ abilities to understand the material and perform well on tests. Although those teachers may have adhered to traditional ways to approach mathematics, they could not have implemented the range of ideas expressed in of the Standards, concerning communication, by simply encouraging students to vocalize alternative ways to solve
math problems. Thus, at minimum we should agree on what the Standards require, suggest, or illustrate as good practices.

For the Standards to have a lasting and common effect, teachers and districts should be able to interpret them broadly enough to be applicable to their unique systems and classrooms, but narrowly enough so that disparities in how we label reform practices do not lead to huge disparities in the quality of instruction across cities and states.

We know that when using reform centered materials or receiving instruction shaped by reform principles, students’ understanding and achievement has improved (Fraivillig, Murphy, & Fuson, 1999; Fuson, William, & Drueck, 2000). Researchers, whether conducting quantitative or qualitative studies, have consistently found that students who encounter, either through teachers practices or curriculum design, reform principles improve in mathematical performance (Fennema et al., 1996; Knapp & Peterson, 1995; Thompson & Sen, 2001; Villasenor & Kepner, 1993).

Despite these efforts, some researchers (e.g., Ball, 2001; Jacobs et al., 2006; William & Baxter, 1996) have suggested that many U.S. classrooms do not reflect the recommendations of reform efforts because teachers may find it difficult to implement change. More importantly, these studies have measured teachers’ alignment on two or three suggestions within the Standards. Studies also reveal how much teachers struggle in their new roles as facilitators within discourse-based learning environments (e.g., Nathan & Knuth, 2003; Rittenhouse, 1998). Hiebert (2003) and Van De Walle (2006) highlighted how traditional approaches for teaching mathematics continue to prevail in many classrooms across the United States. Whether teachers find it difficult to enact these principles or researchers find it difficult to define these principles
to either to examine teachers’ actions or to advise on teacher change, it is necessary for the Standards to be further examined in terms of intention and implementation.

**Teachers’ practices in other countries.** Some studies (e.g., Stigler, Fernandez, & Yoshida, 1996) suggest that Asian countries, specifically Japan and Korea, employ teaching practices that are aligned with the suggestions of the Standards, or more generally with reform suggestions. These studies range from examining how teachers use cooperative learning in classrooms to the techniques used to challenge students’ thinking. However, little research has focused on the ways in which Chinese teachers might naturally employ some of these reform ideas. In fact, some researchers have found that Chinese teachers’ approaches to assigning problems, questioning students, and providing explanations differ from teachers in other Asian countries (see Lewis & Tsuchida, 1998; Wang & Paine, 2003). For me, this suggests that Chinese teachers might implement practices like their other Asian counterparts or they might employ approaches far different from the suggestions of the Standards or U.S. teachers. Given this situation, and given their impressive performance in international comparisons, we need to explore how Chinese classrooms might reflect some of the practices of other countries, naturally align their approaches with those suggestions of the Standards, and, in many ways, employ practices far different than what we expect.

Some researchers have dealt specifically with discursive features when comparing Chinese and U.S classrooms. Perry and her colleagues (e.g., Perry, 2000; Perry, Vanderstoep, & Yu, 1993) have explored verbal exchanges in classrooms both across and within cultures (comparing traditional teaching approaches to those of classrooms that are attempting to employ reform ideas, e.g., McConney & Perry, 2011). One prominent difference between Asian and U.S.
classrooms is the types and amounts of verbal exchanges between the teacher and student (Sims et al., 2007; Stigler & Perry, 1988).

We know that instructional practices in high-achieving classroom influence learning outcomes for students. What we need to know, specifically, is whether the suggestions of recent reform efforts are being enacted in the classrooms in high-achieving countries. If so, this would, first, provide evidence that the proposed practices are successful and would, second, give us clear examples of exactly what these practices look like.

**Narrowing the Focus: Communicative Standards Recommended by the NCTM**

The *Standards* contain five major sections. One of these sections serves as a guideline for improving communicative features in the classroom. It contains descriptions of student-centered, discourse-rich classrooms. It suggests approaches and explains ways in which teachers can get and keep students engaged. Additionally, it provides some examples of good classroom exchanges. The NCTM does not explicitly state teachers should implement its suggestions in any particular way. Currently, *Teaching Children Mathematics* and *Mathematics Teaching in the Middle School* contain real-life examples of teachers attempting to apply some of the suggestions in the *Standards*.

However, taken collectively, from studies assessing teachers to compiling examples from these journals, many teachers will be left without a clear sense of what the *Standards* actually require. The possibility of a vague understanding and thus, a weak implementation of the *Standards* looms. My dissertation will add to our developing knowledge of what specific practices can be observed, which are aligned with the *Standards*, thus operationalizing some of the (oftentimes vague) principles recommended by the NCTM.
To aid in this effort, I developed a coding system based on the recommendations of the *Standards* and applied this to both Chinese and U.S. math lessons. My attempt to codify some of the suggestions of the *Standards* will aid researchers in analyzing discourse in the classroom and will benefit teachers by mapping the initial steps toward transforming their practices. Thus, I want to explore the data with four guiding questions:

**Guiding Questions**

1. Are U.S. teachers’ and Chinese teachers’ communicative practices aligned with NCTM *Standards*?

2. Are these observed practices an overlapping set or are they a distinct set within the two target cultures? If there are features unique to U.S. or Chinese teachers, in which ways are these practices supported by research on successful mathematics learning?

3. Are any other dominant practices or patterns of discourse, not specified by the *Standards*, prevalent in these classrooms? If so, do these repeated practices reflect some of the ideas of research concerning discursive features in classrooms, suggesting that these practices may facilitate mathematical learning?

4. More generally, what do meaningful discourse patterns and exchanges, according to the *Standards* or relevant research concerning discourse, look like in practice?

   *First question: Are U.S. teachers’ and Chinese teachers’ communicative practices aligned with NCTM Standards?*

   Some recent research suggests that U.S. teachers’ practices are not aligned with the *Standards* or reform suggestions (Jacobs et al., 2006; TIMSS 2003). Nevertheless, these researchers have not reached a consensus as to how to evaluate classroom practices. So far,
researchers, like many educators, have not been able to interpret what the NCTM actually means by way of some of its suggestions. For example, Jacobs et al. (2006) failed to define what the NCTM means by communication in the classroom. Instead, they selected a few suggestions from the Standards, and examined lessons based on very broad ideas on how teachers should align themselves with NCTM ideas. Thus, my first question, after codifying some of the ideas of the Standards, concerning what may naturally occur in these classrooms is justified.

Furthermore, asking in general what teachers normally do provides a background for witnessing what happens in a classroom and what could happen. Particularly, this first question applies directly to U.S. classrooms. Primarily, we will mostly study teachers in their natural environments, teaching in ways that are most comfortable to them. Thus, uncovering what already exists in the classroom and speculating about what could take place given the natural dynamics of a classroom is vital.

Discovering what happens in a Chinese classroom is also important. Although many practices in the Chinese educational system are culturally embedded, and any study’s ability to parcel what aspects of such a system is most influential on student performance is impossible, we nevertheless, can examine some practices that allow insight on improving any classroom discursive dynamic. Given that Chinese classrooms provide such a contrast to U.S. classrooms, we may be able to witness practices that make us reflect deeply on what we do well and what we fail to do in U.S. classrooms (Linn, Lewis, Tsuchida, & Songer, 2000). Lastly, any time a country consistently ranks the highest in a particular subject, other countries must pay attention. Moreover, researchers and educators are eager to dissect, understand, borrow, and reject norms and behaviors of that particular country.
Cross-cultural research is pivotal in contributing to the existing theories about learning, development, and behavior. When identifying groups of people who seem not to behave according to established theories, the range of predictor variables available for study in any target culture increases.

Second question: Are these observed practices an overlapping set or are they a distinct set within the two target cultures? If there are features unique to U.S. or Chinese teachers, in which ways are these practices supported by research on successful mathematics learning?

Although this manuscript is not devoted to comparing U.S. and Chinese teachers, at times, I find it necessary to juxtapose these classrooms to illustrate differences and possibilities. I assume that one would witness a difference between the two groups, given the drastic contrast noted by other researchers. Thus, if there are patterns that exist, within either group, it would be interesting to speculate about the assumptions, strategies, and beliefs that some of these teachers have about teaching and learning fractions.

Additionally, each group might display practices that are necessary and beneficial for their respective classroom practices and cultures. Although I do not offer these groups as representative in nature, I do believe that if they seem to align themselves with teachers (or groups) in other studies, certain assumptions can be advanced. For example, we may witness that many Chinese teachers frequently employ more conceptual questions and problem-solving techniques in their lessons. This observation would be congruent with previous research (Miller, Kelly, & Zhou, 2005; Perry 2000). We then could examine in what ways these practices are manifested and to what extent they possibly can or cannot happen in U.S. classrooms.
Third question: Are any other dominant practices or patterns of discourse, not specified by the Standards, prevalent in these classrooms? If so, do these repeated practices reflect some of the ideas of research concerning discursive features in classrooms, suggesting that these practices may facilitate mathematical learning?

I believe that the NCTM, with other reform documents, call for drastic changes in teachers’ behaviors, yet, their suggestions are not an exhaustive list of the dynamic a teacher should establish in the classroom. Given the dynamic nature of classrooms – the symbiotic relationship between teacher and students, teachers’ and students’ backgrounds, goals, and expectations, etc. – teachers will employ a host of strategies and behaviors not governed by reform principles. Yet behaviors may lessen, heighten, or modify their ability to exhibit those reform efforts. Significant to this research are those behaviors that are likely to promote greater comprehension on the part of the student. Thus, it is necessary to capture other behaviors that possibly reflect necessary or good practices.

Fourth question: More generally, what do meaningful discourse patterns and exchanges, according to the Standards or relevant research concerning discourse, look like in practice?

If future research will evaluate how closely educators follow, understand, or implement, in any way, suggestions found in reform documents, we (researchers and teachers) need to reach for, even in a basic form, a consensus on what is needed or wanted from teaching and learning mathematics, according to reform principles. Certainly, any set of guidelines should be amenable to the dictates of the classroom or school environment. Teachers need to teach according to their training, but also to the needs and dynamics of their current community of learners and any needs that arise from that culture in the classroom. More importantly, one of the major debates about reform efforts “is ambiguity about what reform teaching actually looks
like in practice and whether or not this kind of teaching can be measured and linked to pupil’s outcomes” (Jong, Pedulla, Reagan, Fernandez, & Smith, 2010). However, if we are to purport that any reform efforts will lead to improvements in student achievement, then we should understand what those efforts are and how they can be implemented in the classroom.

Answering this fourth question starts a conversation where none (at least publicly - in noted studies) existed. How do we, as researchers, align our assessment of teachers with their own understanding of their practice? When we reference a NCTM suggested practice, what does that behavior look like in practice? And what is the range of practices that satisfy that definition?

In sum, this study focuses on two aspects of research that does and could continue to influence the way we prepare U.S. teachers to approach mathematics in the classroom, and, indirectly, influence the way students understand, retain, and use mathematics. Given the wide range of studies comparing U.S. teachers to their international counterparts, or those simply investigating possible transferable educational behaviors from other countries, my current exploration seems supported. Second, the educational field is replete with studies concerning teachers’ alignment with the NCTM Standards or other reform efforts and studies that establish a platform for possible teacher development. This current study approaches both avenues.

I want to compare the discursive moves of Chinese and U.S. teachers, and how they might align their practices with some of the recommendations of the NCTM (2000) reform-centered document, the Standards. Specifically, I am interested in how teachers in these two countries, with notable differences in student achievement and teachers’ instructional practice, approach the teaching of their initial lessons in equivalent fractions.
Chinese students consistently outperform a majority of other students from different countries; thus, examining these classrooms may provide valuable insight into the types of instructional practices necessary for solid mathematical performance. Studying U.S. classrooms against the suggestions of the Standards is evident; the influence of these documents is pervasive, affecting professional development topics, curricula, and the implementation of reform efforts in school districts.

Despite the Standards’ prevalent reach in the U.S., I concede that it is not equally known in China. At first glance, this difference seems problematic for my study. However, this difference is crucial to the study. Chinese teachers are not well acquainted with the recent reform efforts in the United States or have not been trained in promoting the goals of the Standards, yet, consistently, many of their students are highly ranked as some of the top performers on international mathematics tests. The Standards are based on proven practice, grounded theory, and empirical research. Thus, although the Chinese teachers are unschooled in the NCTM Standards, they provide the perfect sample for asking whether teachers in high-achieving countries can naturally employ some of the suggestions of the Standards. And, if so, does this look different than when U.S. teachers, who have had exposure to NCTM Standards, try to implement best practice in teaching mathematics?
CHAPTER 2
Conceptual Framework

Constructivism

My dissertation borrows from two related perspectives: a constructivist view of mathematics learning and a sociocultural approach to understanding the use of discourse in the classroom. Both perspectives highlight how social interaction, participation in authentic tasks, and language are important to learning. For constructivists, improving students’ mathematics understanding involves an increase in their opportunity and ability to participate in meaningful mathematical activities, including discussions. Additionally, students must be able to approach and handle the material in a personally meaningful way. Thus interactions with the teacher and, more importantly, one’s classmates constitute a crucial source of opportunities to learn through cognitive conflict, reflection, and active cognitive reorganization (Piaget, 1970).

According to Kamii (1985), constructivism is “the theory according to which each child builds his own knowledge from the inside, through his own mental activity, in interaction with the environment” (p. 6). Essential to the classroom is the fulcrum on which students have to make a personal connection with material. The child will connect what is known to what is new, or they will apply a concept to a real-life event (Carpenter et al. 2004). The student starts with what they know or speculate and, then, confirm, refute, or reorganize their thoughts when presented with new or conflicting information. In this way, classrooms achieve what von Glaserfeld (1990) supports - that "knowledge is not passively received ... knowledge is actively built up by the cognizing subject" (p. 22). Thus, for a deeply engaging classroom, we should witness students who are handling the material as much as possible.
Such an approach leads to issues of authority and ownership in the classroom. Ultimately, constructivism asserts that mathematical learning is a process of meaning making where the student constructs knowledge - opposed to the idea that mathematics is a fixed body of facts to be received from the teacher or the textbook. Although each teacher establishes the direction of the class and, often, the text is fixed and used as a lesson plan, teachers can negotiate with students the types of discussions, participation, and scholarship will lead to success in the class. The climate and direction of a class can be inclusive of students’ needs and desires. Students, then, come to be responsible for a large part of their learning, ultimately giving them a stronger connection to the material.

Certainly, students cannot construct the entire body of mathematical knowledge, but they can personalize it (Cobb, 1990). For Cobb (1990), two major goals should define this personalization process: students should shed old ideas and schemas for new, complex structures, and, increasingly become autonomous learners during this process. Students should build more abstract, complex, and conceptually strong mental structures than the ones they currently possess. To build these structures students are introduced to contrasting ideas, given challenging tasks, and expected to engage in arguments concerning their perspectives. Combined with motivating students to become autonomous in their exploration of alternative explanations and initiating discussions, students should develop a responsibility for their own learning and realize the power of discovery. The NCTM’s approach supports these beliefs. Opportunities to learn mathematics are a function of the setting and kinds of tasks and discourse in which they participate (NCTM, 2000a).

In essence, from this perspective the transfer of knowledge from the teacher to the student is either not possible or does not result in substantive learning and understanding.
Therefore, we should expect teachers to *orchestrate* classroom discourse and activities – relinquishing some of the center stage of doing mathematics. Ideally, students will meet the challenge of being on “stage” and engage one another in collaboration as well as argumentation. Again, this notion is supported by the *Standards* and other reform efforts.

The focus is on how the student has a strong sense of control in her own learning process. Boomer (1986) stated that mathematical knowledge is “being personally constructed and applied according the principles internalized by the learner” (pp.4-5). The principles are derived from the dynamics of the classroom. The opportunities to engage mathematically are integral to how students come to understand the material. Teachers, then, are the orchestrators of the artifacts that students can use to make meaning. Thus, the lack of opportunities should produce less motivation in students, a more teacher-controlled lesson, and less overall input from students. If teachers view learning mathematics as an active process, considering students personal backgrounds, initiating discovery, and promoting intellectual conflict through debate, teachers can design instruction that goes beyond rote learning to meaningful learning that is more likely to lead to deeper, longer lasting understandings.

Constructivist learning includes a reliance on process, the exchange of differing points of view, and an emphasis on problem solving. Kamii and Lewis (1990) referred to the constructivist classroom as “a culture in which students are involved not only in discovery but in a social discourse involving explanation, negotiation, sharing, and evaluation” (p. 35). However, Cobb, Wood, and Yackel (1991) cautioned that the classroom environment must be risk-free so that students can question, exchange points of view, and be actively involved in discourse. A constructivist theory can provide teachers with a framework for teaching mathematics that encourages problem solving, reasoning, and communication in mathematics (Simon & Schifter,
Research studies have shown that students in constructivist classrooms have had a greater understanding of mathematics and experienced more success in the mathematics classroom than those in traditional classrooms (Cobb, Wood, Yackel, & Perlwitz, 1992).

The teacher’s role in modeling such behaviors is a sociocultural process. Not only is learning facts and formulas important in mathematical development, but also how students come to practice mathematics is essential. The classroom becomes an arena of artifacts to be used in acquiring how to participate in mathematics and social interactions.

**Sociocultural Theory**

Vygotsky (1978) asserted that social interaction is one of the major sources in the development of mental operations. Students’ very development is predicated on the type and quality of their social interactions while learning. Vygotsky (1978) stated: “Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological).” Thus, when studying how individuals eventually process information, we must examine those interactions that gave rise to the discovery and understanding of that information. Many of a child’s formal learning are first encountered in the classroom – in the United States, it is usually through one teacher. This is why we study teacher-student interactions; the classroom becomes the vehicle through which knowledge is mediated.

This sociocultural perspective suggests that communities and their practices shape individuals thoughts and reasoning as they become more sophisticated learners. Given this framework, knowledge, skills, language, and understanding are developed, refined, and witnessed through social interactions. All social interactions either use or produce cultural tools, such as speech and writing, which, then mediate these social environments. Initially children
develop these tools to serve solely as social functions, ways to communicate needs. Later, they serve as building blocks for higher level thinking (Vygotsky 1978).

Because language is one of the primary tools children use during their social interactions and journey of learning, any discursive exchanges between a child and adult becomes the window through which we can witness the development of a child’s thinking. The internalization of ideas first occurs as external dialogue – speech modeled for or used with the child. Then, such speech becomes internal conversations within the individual – forming the basis for thought. Adults are responsible for teaching children how to label, define, identify, and represent objects, people, and events through language. Children learn what to say, when it is appropriate, and how to moderate what they say. Children essentially communicate with the world based on how they have been taught to do so. This practice of “imitation” is not limited to one’s general culture. Workplaces, churches, and classrooms are smaller cultures, with their own sets of rules and modes of communication.

Communication, according to sociocultural theory, is one of the main vehicles through which children understand their role, purpose, and power in a culture. This culture can be a society, a neighborhood, or a classroom. Through utilizing a sociocultural approach to understanding learning and development, insights may be gained into the use of talk with the mathematics classroom and into the ways that particular types of talk promote engagement in the practices of mathematics. We should then focus on talk in the classroom – analyzing what takes place between students and the teacher and among students themselves. What is of interest, therefore, are the constitutive features of classroom talk – features which signify that the teacher and students are participating more fully and critically in the sociocultural practices of mathematics.
More specifically, the language used in the classroom becomes the very resource from which students build meaning, connect ideas, and prune their existing mental schemas, if faulty. Vygotsky and others suggest that students gradually internalize the talk that occurs in groups, whether in dyads, cooperative groups, or whole-class discussions. They begin to challenge themselves and others, ask for reasons, and in general monitor their own mental work as others do their public speech. Initially, speech is used to label, identify, and define. Once, internalized, and understood as a tool, language begins to shape higher level thinking. These thoughts, from their inception through their refinement can be witnessed through various vehicles: choice of words. Essentially, communicating is thinking externalized.

Hence, teachers must create a link between mathematics and language (Rubenstein & Thompson, 2002; Stigler & Hiebert, 2004). This connection is best reflected when teachers help children communicate their ideas by elaborating upon what they already know (Kennedy, Tipps, & Johnson, 2004; Schmidt, 2004; Smith, 1999).

Particular to classrooms, where the teacher could be viewed as an instructor, and, sometimes, a more capable peer, researchers have attempted to study these settings as communities of learners (Brown, 1997; Lave & Wenger, 1991; Winbourne & Watson, 1998). Such communities involve central and peripheral participants (for classrooms, teachers and students), co-constructing knowledge, building on their practice, or negotiating meaning (Lave, 1991; Palinscar & Brown, 1985). These practices form the platform for learning, and, as they evolve, the learning culture of the classroom takes shape. These interactions are the basis for learning in the classroom. Thus, this perspective allows us to characterize a mathematics class as a complex human activity instead of an arena where a particular subject matter is dissected and investigated.
In classrooms, the teacher is seen as a more able practitioner, whereas students, who are understandably on the periphery of becoming better mathematicians, adopt the language and practices of their teachers. However, during this adoption, students and teachers negotiate rules, norms, and understandings (e.g., Cobb et al., 1992). Thus, a community practice and language is formed. Students combine the terms and phrases they acquire from the teacher or others with their own knowledge and emergent understandings. From this combination, students create meaning.

**Importance of Discourse**

As a conversational form, classroom talk differs from other discussions in a few, unique ways: there is an implied authority – the teacher (Schultz, Erickson, & Florio, 1982), the goal is to learn (as opposed to simply connect, affect mood, etc.): when a whole-class discussion is created, it is predictably controlled by a single conversant, the teacher (Nystrand, Gamoran, Kachur, & Prendergast, 1997), whose intention is to foster learning and manage behavior: and most important, to determine the types of questions asked and typically maintain the right to call on students and allocate turns, “in essence organizing and orchestrating the discussions” (Greenleaf & Freedman, 1993, p. 466; also see McHoul, 1978).

Given this structure, the teacher, no matter his or her approach, is the center of the classroom dynamic. The types of norms, especially verbal, that are established are paramount to the resulting success of the classroom. Language is not only a tool for the child, but for the teacher as well. The choice of tools (type of talk) determines the tasks, sophistication, and direction of the lesson.

Discursive features, theoretically and logically, provide the optimal perspective from which to make inferences about teaching approaches and to suggest change for the classroom.
from a constructivist and socio-cultural perspective of learning. Examining how students come to understand the appropriate ways to participate in a mathematical discussion or verbal exchange is equivalent to examining how students think. In this way, discourse is a practice. It is a cultural activity in which knowing mathematics is also doing mathematics (Schoenfeld, 1987; van Oers, 1996). Doing mathematics includes the repeated ways that teachers and students use discourse. Thus, how you speak mathematically and how you speak about mathematics becomes how you know and represent mathematics. Students’ beliefs about, approaches to, and understanding of mathematics start with discourse (Ball, Lubienski, & Mewborn, 2001; Shulman & Shulman, 2004; Sfard, 2001; Stein, 2001).

Carpenter, Franke, and Levi (2003) asserted that the very nature of mathematics presupposes that students cannot learn mathematics with understanding without engaging in discussion. This discussion starts with the teacher and could end with another student. More importantly, this discussion could be with oneself – reasoning internally or externally about how a problem is solved or simply how to understand the information that has been presented. How early teachers initially modeled, through talk, how to think about and solve problems becomes an internal conversation of sorts. We are not surprised that students use the same language as the teacher and the textbook. Students imitate the reasoning of their teachers or other students. Sfard believes that successful discourse in mathematics is essential to understanding, retention and success.

Students come to recognize and internalize particular ways of thinking and doing by participating in socio-mathematical practices with their teachers and others (Cobb & Bowers, 1999; Yackel & Cobb, 1996). Primarily, students learn these ways from the scaffolding and modeling by the teacher. Such activities become a repeated practice and students will gradually
make sense of these experiences in more complex and abstract ways. The types of discursive features a teacher employs signals to the students (and an observer) the way in which mathematics is approached, validated, and practiced in a given classroom and in the community of mathematicians in general.

Understanding cannot be separated from the discursive practices that guide and bound mathematical activity. Sfard’s (2001) notion of thinking as communicating, which implies that for one to engage in doing mathematics, one must be able to individually engage in the same kind of discursive activities that were modeled in outside discursive practices, is pivotal in justifying why observing discourse in a classroom is a viable way to uncover how teachers and students think about and eventually understand mathematics.

There is now a large body of empirical and theoretical evidence that demonstrates the positive effects of participating in mathematical dialogue within the classroom (e.g., Clarke, Keitel, & Shimizu, 2006; Fraivillig, Murphy, & Fuson, 1999; Goos, 2004; Kazemi & Franke, 2004; McClain & Cobb, 2001; Sfard & Kieran, 2001; White, 2003; Wood, Williams, & McNeal, 2006). However, simply participating is not enough. Learning how to justify one’s position, how to enter into an argument, and how to provide useful, but critical feedback to others’ ideas is an intentional practice on the part of the teacher. Therefore, providing comprehensible explanations about mathematical concepts is essentially a learned strategy. Sfard and Kieran (2001) asserted that mathematical communicating has to be taught (p. 70).

Without a teacher’s pedagogical support, students are, often, not able to understand others’ ideas and problem-solving procedures or elaborate on their own mathematical reasoning. Effective pedagogy focuses on support demands of and careful attention to students’ articulation of ideas. Franke and Kazemi (2001) claim that an effective teacher listens to students’
conjectures and anticipates mistakes and digressions. Yackel, Cobb, and Wood (1990) provide evidence to substantiate this claim. They report on the ways in which one teacher listened to, reflected upon, and learned from her students’ mathematical reasoning while they were involved in a discussion on relationships between numbers. They concluded that her mathematical subject knowledge and her focus on listening, observing, and questioning for understanding and clarification greatly enhanced her understanding of students’ thinking and anticipation of where discussions should be directed.

Lastly, teacher-student discourse is a key concern of the Standards and other reform-oriented projects, and has been the focus of many researchers analyzing reform curricula or student-centered talk in mathematics classrooms (e.g., Ball, 1992; D’Ambrosio, 1995; Fuson, William, & Drueck, 2000; Hufferd-Ackles, Fuson, & Sherin 2004; OMLI, 2005). The Standards and these studies recognize that one of the most critical avenues to facilitating a deeper understanding of mathematics is through good, student-centered discourse.
CHAPTER 3

Methods

Data Source

The data were collected as a part of a larger examination of U.S. and Chinese instructional practices in elementary mathematics classrooms. The eight participating Chinese schools were from Beijing, and the six U.S. schools were within a 50-mile radius surrounding a Midwest, university town, which included smaller, more rural areas. The socioeconomic status in the U.S. schools ranged from low to high. Additionally, there was ethnic and racial diversity among the U.S. students. Although the schools studied in China were all located in urban Beijing, most of the students in the Chinese classrooms were from either middle-class or working-class backgrounds, and they were ethnically and racially homogeneous.

The Chinese schools varied in terms of their perceived rigor and prestige. Two of the schools were viewed as top-level schools, while the others were viewed as middle-to-high level schools. The differences in prestige were virtually the same in the U.S. schools, with one viewed as top-level and the remainder viewed as middle-to-high level schools.

We asked teachers to inform us when they would be teaching an equivalent- or adding-fractions lesson, and we then made arrangements to capture these lessons on video. In the U.S. sample, these lessons occurred in the fourth grade, whereas in the Chinese sample, these lessons occurred in the fifth grade. Keeping the intended lesson content similar across sites was more important than keeping the grade levels the same. I anticipated that classroom discourse would differ more between lesson content (e.g., between equivalent fractions lessons and measurement lessons) than between fourth and fifth grades. For this reason, lessons that did not focus on this topic were omitted from analysis. Additionally, the NCTM suggests that teachers between the
third and fifth grades should have similar approaches in their mathematics classes. Thus, having teachers in either the fourth or fifth grade, especially when they are teaching similar content, should be acceptable for observation. Teachers’ behaviors were only coded during whole-class instruction. Primarily, whole-class discussion is accessible to all students. Within this context, teachers establish norms of behavior and create a platform for the primary vehicle of instruction.

Any initial lesson about fractions presents a chance for many complex verbal exchanges to occur. For one, while learning or explaining the operations of fractions, students produce many errors (Green, Flowers, & Peil, 2008; Niemi, 1996; Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989). These misconceptions call for more discourse because teachers are challenged with exposing and correcting students’ misunderstandings, and students, seeking clarification of ideas, either ask questions of the teacher or each other. Additionally, students’ misconceptions about the operation of fractions vary so greatly that teachers have difficulty deciphering the exact nature of individuals’ misunderstandings. Thus, teachers will find lessons ripe for scaffolding, peer collaboration, and guided instruction.

A second reason for choosing lessons about fractions as valuable observations is that it is centrally placed in the elementary curriculum of both countries. Both countries introduce fractions at almost the same age – between fourth and fifth grades. Other subjects are either introduced or formally covered at very different times in or across years. For example, algebraic concepts are introduced at disparate times in elementary school. Having teachers introduce the same topic, formally, to children who are relatively close in age (and possibly development) is invaluable to my research questions.
All of the lessons in both countries were videotaped during the spring semester. The average class size for the U.S. classes was 22 students, and in China, the average class size was 55 students.

All lessons were transcribed by native speakers. All of the Chinese lessons were translated to English and then back-translated for accuracy. For this study, I only examined the U.S. and Chinese videotaped classroom lessons where students either encountered their initial lessons in equivalent fractions or the addition of fractions with unlike denominators. The other lessons involved different content, and such differences may account for more or less the difference in direct instruction on the part of the teacher. I omitted eight lessons given this requirement. I assumed that teachers who had the same task – i.e. the initial lesson of the same content – would have to employ the same amount of review, direct instruction, challenges, and exchanges.

**Coding**

**Development of this coding scheme.** The coding scheme results from four iterative stages of development. First, I extracted significant concepts from the Standards section on communication, and then, second, excluded suggestions that fell beyond the scope of this study, like pausing and waiting for students responses. Third, I collapsed categories that were highly related, and, finally, added teacher behaviors that emerged from readings of the transcripts: behaviors that are not specifically highlighted in the Standards. Throughout these stages, I employed deductive and inductive processes. My deductive analyses started with à priori assumptions, questions, and theories, whereas my inductive analysis uncovered embedded information in the corpus itself. Because of the structure and purpose of this study, a combination of these methods was appropriate.
The majority of my coding scheme is predicated on established ideas and theories as outlined in the first two chapters of this dissertation. As a researcher, I approached the data with my own assumptions, informed by previous research, empirical studies, theoretical frameworks, and actual practice (as a teacher). More importantly, the foundation of this study is built on the principles and suggestions of the NCTM Standards, a document established over the past two and half decades by policy makers, researchers, and teachers.

I analyzed the data with a certain bias, only searching for those suggestions that explicitly deal with communication. Certainly, suggestions in other sections could influence how communication is used in the classroom. However, the Standards have devoted an entire section to communication, and educational research is replete with studies that also focus on discursive exchanges in the classroom (Knuth & Peressini, 2001; Nathan & Knuth, 2003; Sherin, 2002; Williams & Baxter, 1996). Therefore, isolating behaviors that particularly deal with teachers’ facilitation of whole-class discourse is supported.

Despite my predetermined questions, I could not allow them to eclipse important motifs that may have been embedded in the data: ones that I did not explicitly intend to examine. To note whether teachers, through their repeated actions, believed certain norms to be important to classroom communication, it was necessary for me to highlight teachers’ moves not specified by the Standards, as they may have been important to building a culture within the classroom and ultimately student understanding.

For example, I noted instances of student help. The Standards do not mention the role of student help in the classrooms. However, in both samples, a teacher’s call for one student to help another was notable. In some ways, this was a teacher’s move to arrive at the correct answer. In others, it may have been a teacher’s attempt to have one student talk directly to another student.
Thus, taking a general inductive approach (Bogdan & Biklen, 1998; Thomas, 2003; see in general, Lincoln & Guba, 1985; Patton, 1990), I attempted to notice themes and patterns first from the Standards and then from the transcripts. Subsequently, I then used these extractions as categories to reexamine the Standards and then the transcripts more closely, allowing other categories to emerge from several readings of the data.

**General inductive analysis.** General inductive analysis is associated with traditional approaches like grounded theory, discourse analysis, and narrative analysis. Such approaches call for the researcher to relinquish some of his or her external bias, and, instead, take note of patterns and themes in the corpus (Patton, 1987). As these patterns are discovered in the raw data, the researcher begins to develop categories, creating a model that captures key ideas, judged to be important by the researcher and implicitly disclosed by the participants through their emphasized or frequent behaviors and interactions (Thomas 2003).

Despite a researcher’s interests and questions, the participants in the study will have norms, shared understandings, and collective responses to any intervention, change, or even to the presence of the observer. These responses on the part of the participants are equally important as the researcher’s initial questions. Thus, through an inductive approach, researchers are able to capture important dynamics between participants – those that happen during or because of the research (Thomas 2003).

General inductive analysis differs from other approaches because it does not strictly adhere to traditional methodologies for qualitatively analyzing data. This approach compels the researcher to observe “frequent, dominant, or significant themes inherent in the raw data, without the restraints imposed by structured methodologies [and]…deductive data analysis” (Thomas, 2003, p.2). Thus, the researcher will perform an “open-coding,” allowing themes to arise from
the data (Strauss & Corbin, 1990). Nevertheless, this approach does have structure, and it calls for a systematic way to examine the data and develop codes.

The process is first guided by the researcher’s objectives and, then, by the multiple examinations and interpretations of the actual data (Thomas 2006). Inductive coding begins with close readings of text, searching for multiple meanings that are inherent in the text. The researcher then identifies segments that contain meaningful units, and creates a new category based on that segment. Additional segments are added to the category where they are relevant, and, where necessary, categories are split.

To support the initial researcher’s findings, trustworthiness can be assessed by a range of techniques such as (a) independent replication of the research, (b) comparison with findings from previous research, (c) triangulation within a project, (d) feedback from participants in the research, and (e) feedback from users of the research findings.

For this study, two independent coders were solicited to establish trustworthiness in the statements extracted from the *Standards*. The first coder was an elementary school teacher in a southern metropolitan area. At the time of the coding, she had been teaching fifth-grade for five years. She was a native of the United States and only fluent in English. She has knowledge of the NCTM *Standards*, and has participated in several professional development seminars where reform ideas were discussed or implemented. The second coder is a graduate student familiar with the *Standards*, reform ideas, and, specifically, the data set. She was born in China, and she is fluent in English and Chinese. Additionally, throughout this document, I compare my coding scheme to those from previous research, which is another form of establishing the trustworthiness of the coding scheme.
This inductive approach of allowing patterns to surface as the researcher is delving into the study was more prominent in two of the four stages. In the first stage, having read the *Standards* several times, I highlighted suggestions that were repeated, illustrated by examples, or supported by research concerning discourse. In the fourth stage, after using an established coding scheme on the transcripts, I recognized and categorized other prominent teachers’ actions that were not mentioned by the *Standards*.

**First stage.** In this first stage, I simply read the NCTM sections concerning communication and extracted phrases and statements that directly related to teachers’ behaviors that fostered classroom discourse. When searching for these ideas, I was informed by a plethora of research that supports some of the principles of the *Standards* (Berry & Kim, 2008; Hufferd-Ackles et al. 2004; Kazemi & Stipek, 2001).

I extracted phrases and statements verbatim from two sections in the NCTM document: (a) Communication and (b) Communication in the third through fifth grades, see Table 1. Although other sections imply what teachers might do when orchestrating discourse in the classroom, I excluded those sections from my investigation for two reasons: (1) the *Standards* have devoted space to addressing communicative change in the classroom, and (2) an attempt to comb through all of the sections that have some implications about discourse in the classroom would have been beyond the scope of this study. Extracting proposed teachers’ behaviors from these two sections give us an excellent platform from which to codify some of the suggestions of the *Standards*. 

45
Table 1

Copied Directly from the NCTM Standards

<table>
<thead>
<tr>
<th>Suggestion</th>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The communication process also helps build meaning and permanence for ideas and makes them public</td>
<td>Communication</td>
<td>59</td>
</tr>
<tr>
<td>Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections</td>
<td>Communication</td>
<td>59</td>
</tr>
<tr>
<td>Need for precision in that language</td>
<td>Communication</td>
<td>59</td>
</tr>
<tr>
<td>Especially in the face of disagreement—will gain better mathematical understanding as they work to convince their peers about differing points of view (Hatano &amp; Inagaki 1991).</td>
<td>Communication</td>
<td>59</td>
</tr>
<tr>
<td>Misconceptions can be identified and addressed</td>
<td>Communication</td>
<td>60</td>
</tr>
<tr>
<td>Thoughtful questions posed by a teacher or classmate can provoke them to reexamine their reasoning</td>
<td>Communication</td>
<td>60</td>
</tr>
<tr>
<td>Starting in grades 3–5, students should gradually take more responsibility for participating in whole-class discussions and responding to one another directly. They should become better at listening, paraphrasing, questioning, and interpreting others' ideas</td>
<td>Communication</td>
<td>61</td>
</tr>
<tr>
<td>They should express themselves increasingly clearly and coherently</td>
<td>Communication</td>
<td>61</td>
</tr>
<tr>
<td>Often, a student who has one way of seeing a problem can profit from another student's view, which may reveal a different aspect of the problem</td>
<td>Communication</td>
<td>61</td>
</tr>
<tr>
<td>Students must also learn to question and probe one another's thinking in order to clarify underdeveloped ideas</td>
<td>Communication</td>
<td>62</td>
</tr>
<tr>
<td>More-precise meanings. This observation is the foundation for understanding the concept of mathematical definitions</td>
<td>Communication</td>
<td>62</td>
</tr>
</tbody>
</table>
Table 1 (cont.)

| Must help students acquire mathematical language to describe objects and relationships | Grades 3-5 | 197 |
| Building a mathematical community of learners | Grades 3-5 | 197 |
| The focus is not on who is right or wrong but rather on whether an answer makes sense and can be justified | Grades 3-5 | 197 |
| Students listening to a number of peers and joining group discussions in order to clarify, question, and extend conjectures | Grades 3-5 | 193 |
| Speaking to one another in order to convince or question peers | Grades 3-5 | 193 |
| Include sharing thinking, asking questions, and explaining and justifying ideas | Grades 3-5 | 198 |
| Well-posed questions can simultaneously elicit, extend, and challenge students' thinking and at the same time give the teacher an opportunity to assess the students' understanding | Grades 3-5 | 197 |
| Periodically, teachers may need to explicitly discuss students' effective and ineffective communication strategies | Grades 3-5 | 197 |
| Refine their listening, questioning, and paraphrasing techniques | Grades 3-5 | 197 |
| Teachers must also routinely provide students with rich problems centered on the important mathematical ideas in the curriculum | Grades 3-5 | 197 |
| Students are working with situations worthy of their conversation and thought | Grades 3-5 | 197 |

*Any words or phrases that were repeated, either in the same section or a different section were only represented once.

**Second stage.** In the second stage, I excluded select phrases from this exhaustive list. These exclusions either could not be practically defined when attempting to code with these particular data, or were beyond the scope of this study. For example, the Standards call for teachers to have students listen to one another, but it is difficult to define what “listen to” actually means. We could interpret “listen to” in three different ways: it could be “wait-time,” a
check for understanding, or an attempt to disclose students’ possible confusion. Given the
design of this study, any of these interpretations would be hard to measure. Thus, some very
broad suggestions of the Standards would not allow me to accurately and adequately define
them. Next, I explain why I omitted each of the Standards suggestions that I chose to exclude.

Exceptions. I excluded five ideas of the Standards: teachers should have students listen
to one another; teachers should focus on precise mathematical language; they should use rich
mathematical tasks; they should use technology to promote sophisticated whole-class
discussions; and teachers should concentrate on improving students’ writing about mathematics.
The first exclusion was explained previously.

Unlike the possible category of “listening to,” the use of precise mathematical language
can be defined. However, examining such language is beyond the scope of this study. Although
every teacher should promote accuracy in using more formal mathematical language, the timing
(i.e. knowing your students’ readiness to advance and understanding the curriculum’s purposeful
use of informal language) dictates when precise mathematical terms should be employed.
Without controlling for curriculum, and, possibly, teacher expertise, how and when a teacher
should use precise mathematical language is difficult to ascertain. Even the Standards hesitate
on being specific about this suggestion. It states that “[f]or some purposes it will be appropriate
for students to describe their thinking informally, using ordinary language” (NCTM, p.61). It
also states that students should, at times, “learn to communicate in more-formal mathematical
ways…through the middle grades and into high school” (NCTM, p.61). In some sense, the
Standards suggest that this teacher action will be highly subjective.

The next two exclusions, although important, are not driven by a teacher’s verbal
exchanges. The types of technology and tasks, whether verbal or written, used in a classroom
are, in part, influenced by the school’s resources, policies, or curriculum. When comparing the U.S. and Chinese classrooms we did not control for the schools’ resources or policies concerning technology, manipulatives, or tools generally used in the classroom. The difficulty in observing, comparing, or making assumptions about the sophistication and benefit of technology and tasks in these classrooms compelled me to exclude the frequency and types of tasks and use of technology during these lessons. I recognize the importance of technology or written tasks in classrooms. However, they are a part of the larger “Discourse” (Gee, 1999) in the class. For this particular study, I have chosen only to concentrate on verbal exchanges (“discourse”).

**Third stage.** In the third stage, I logically collapsed categories based on theoretical and empirical studies that focus on discourse in mathematics classrooms (Berry & Kim, 2008; Hufferd-Ackles et al. 2004; Jacobs et al., 2006; Kazemi & Stipek, 2001; Knuth & Peressini, 2001; Nathan & Knuth, 2003; Sherin, 2002; Williams & Baxter, 1996). Many of these studies have developed coding schemes that resemble the suggestions of the *Standards*, or are based partially on the ideas found therein. To create a concise coding scheme, I relied on how these researchers conceptualized and described teachers’ discursive behaviors. For example, Kazemi and Stipek (2001) described a teacher’s behavior as “press for learning.” This category captures the NCTM’s suggestions about challenging students’ thinking and having students explain mathematics in conceptual ways. Under Kazemi’s definition a press for learning would include three NCTM suggested categories: to extend and challenge students, to require answers to make sense and be justified, and to convince others of your ideas.

In another study, Qiong and Yujing (2009) categorized teachers’ behaviors as “asking for a rule” versus “asking for a procedure” versus “asking for a justification;” yet, each category is a combination of a couple of suggestions from the *Standards*. Essentially, some teachers’ actions
either call for a combination of student responses, but one type of response was dominant, or teachers’ moves were so similar that they could be classified under a general group.

Additionally, I combined categories based on instances that arose from the transcripts themselves. Repeated readings of the transcripts and several examinations of the corpus allowed me to understand that some teachers’ behaviors, although couched in different words and phrases, essentially led to the same intended result. Instances from the transcripts illustrated how some codes were so closely related that they could be considered one category. For example, justifying one’s solution, reexamining one’s reasoning, and convincing one’s peers are so similar in how they are enacted in the classroom, they deserved to be combined.

Fourth stage. In this last stage, I was informed by a more qualitative approach to discovering codes within the data. I read through a third of the transcripts. From these readings, I observed repeated teacher actions governing the types of verbal exchanges in the classroom. These observations provided the platform for categories not specified by the Standards. From these initial categories, I examined the Standards again, solidifying the definition of these categories or modifying them until they accurately reflected these teachers’ behaviors. For this study, one category and two subcategories were not explicitly reflected in the Standards, but emerged from the data. The one category that was repeatedly used is a teacher’s elicitation for help. Often, in these classrooms, teachers would solicit the help of another student to assist a struggling classmate or answer a question for the teacher, who was feigning confusion. These instances were labeled “student help.”

The two subcategories were behaviors that were almost exclusively culturally specific. Teacher questions and explanations that ask for or give a rule or definition-based answer were derived from a close reading of the lessons. For example, in the Chinese classrooms, the
teachers often asked students to articulate the underlining rule of an answer or explanation. Lastly, within the category of Making Connections, I found it necessary to add a third subcategory – Everyday Reference. These are instances where a teacher refers to a common item or occurrence in the students’ daily life, but does not use that reference to explain a mathematical concept, nor is it used to solve a problem or clarify confusion. It seems that the reference was used to make a personal connection to the material. However, in these instances, students were left to make a connection to the material on their own. Thus, I felt compelled to include this last instance because it, at least, may possibly show teachers’ intentions on connecting to the material to something beyond the immediate classroom.

What follows, in Table 2, is the coding scheme used to examine the data. After the table, I follow with explanations and occasional examples to clarify each category.

Table 2

Listing of Codes

<table>
<thead>
<tr>
<th>Community acceptance of ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple perspectives or alternative solutions</td>
</tr>
<tr>
<td>Challenging and extending</td>
</tr>
<tr>
<td>Misconceptions/Mistakes/Errors</td>
</tr>
<tr>
<td>Paraphrasing/Restating/Revoicing</td>
</tr>
<tr>
<td>Making connections</td>
</tr>
<tr>
<td>Previous material</td>
</tr>
<tr>
<td>Other subjects</td>
</tr>
<tr>
<td>Real-life application</td>
</tr>
<tr>
<td>Everyday connections</td>
</tr>
<tr>
<td>Types of questions and explanations</td>
</tr>
<tr>
<td>Procedural</td>
</tr>
<tr>
<td>Conceptual</td>
</tr>
</tbody>
</table>
Explicit dialogue concerning norms

**What supports a mathematical justification**

**How to disagree**

**Other**

Acceptance of spontaneous contributions

Student help

Simple questions

**Community acceptance of ideas.** This category encompasses instances where a teacher asks a student or the class if another student is correct or incorrect. Often, in the transcripts, teachers would invite students to pass judgment on the validity of students’ answers or explanations; and this was done in variety of ways. For example, the teacher could ask the entire class, by a show of hands, who disagrees, and, then, select students to challenge any particular point of an answer or argument. Thus, a teacher could ask the class if they accept or reject an answer. Either action is coded under this category.

**Multiple perspectives or alternative solutions.** This category captures the instances when teachers request a student to either repeat a strategy in their own words or simply invite students to give a view or answer different from the initial answer. The *Standards* are clear that students profit from seeing views that are different from their own. Teachers who encourage students to explain their way of solving a problem greatly contribute to the various ways in which students gain skills to view and solve a problem through different avenues. Thus, involving multiple perspectives within a class discussion promotes comparison and contrast with other views.
**Types of questions and explanations.** This category treats questions and explanations on the part of the teacher similarly. Either can be a procedurally, conceptually, or rule-based action. Under this code, I first identify whether a teacher asked a question or gave a mathematically relevant explanation. Either could then be labeled as conceptual, procedural, or rule/definition based.

Conceptual and procedural questions also arise in challenges from the teacher. However, in that category, such questions are only coded after an initial question was asked by the teacher and some type of response given by the student.

The last type of question in this category, those that are either rule or definition based, are ones that arose from reading the transcripts. Evident in the examination of the lessons, both in the U.S. and Chinese classrooms, are teachers’ questions that call for students to recite a mathematical rule or the give the definition of a term. I collapsed these two types of explanations.

**Challenging and extending.** Within this category, we witness several different moves: instances where a teacher challenges a student’s initial answer by asking further questions or attempting scaffolding. This category is the largest, containing several related notions embedded in the *Standards*: to convince one’s peers, justify one’s solutions, re-examine one’s thinking, defend challenges by other students, and scaffolding by the teacher. This category is distinguished from the codes in *Questions and Explanations* by the timing of a teacher’s move. We are coding teachers’ questions after an initial question was asked and an answer was given.

**Misconceptions/Mistakes/Errors.** For this category, I note times when a teacher acknowledges a mistake on the part of a student. A teacher can explore the mistake, note the mistake and give an answer, or offer an explanation that leads to other questions that check for
understanding. In contrast, a teacher can simply note the mistake as an error, offer the correct answer, or ignore the mistake. Although I note both instances, only the former was derived from the suggestion of the Standards.

For this study, I do not attempt to define what a mistake is or suggest how a teacher should handle such. Simply, I want to record when teachers acknowledge and spend time correcting or investigating a mistake. Thus, this study notes if extended discourse was initiated around any given mistake. If a teacher were simply to give a correct answer, it is coded as a correction.

Paraphrasing/Restating. This category captures all teacher requests for students to interpret others’ ideas, paraphrase what they just heard, either from the teacher or another student, or revoicing (on part of the teacher). There are explicit and implicit teacher moves in this category. For example, a teacher could ask one student to “repeat” what another student stated, or a teacher could “restructure” what a student just said. For this study, I assume that both moves serve the same purpose, which will be explored later.

Making connections. Often, the Standards state that teachers should have students make connections to previous mathematical concepts, other subjects, or to real-life events. I highlight instances where teachers apply (or ask students to apply) their mathematical concepts to real-life activities, connect them with other subjects, or compare them to previous lessons.

Explicit dialogue concerning norms. This umbrella category captures a couple of teacher’s behaviors. Although these behaviors are not related conceptually, they serve an overall purpose – to establish explicit social and sociomathematical norms in the classroom. Usually, teachers in this study are explicit about how students offer and support mathematical arguments, disagreements, and general participatory behaviors. Although the Standards do not emphasize
such a broad category, they do mention these subcategories. Thus, teachers’ behaviors in this category include instances where a teacher was explicit about establishing a norm. The three subcategories are teachers’ instructions about what supports a mathematical justification, how to offer a disagreement, and other.

**Acceptance of spontaneous contributions.** Spontaneous contributions, especially when they are mathematical in content, could be as important as students’ mistakes. In this study, teachers’ responses to spontaneous contributions, where students offer an answer or suggestion without having been asked, are either labeled as accepted or denied. Sometimes, these contributions are students’ attempts either to enter a conversation or ask a question. Thus, for the teacher to acknowledge these contributions in some form is decisive to how the culture of the class flows. If there is extended discourse following such a contribution, it will be captured by another category.

**Simple questions.** Lastly, in contrast to behaviors emphasized in the *Standards*, I note instances where teachers ask simple questions. A simple question involves mathematics where only a simple answer is sufficient, and does not challenge the student to engage the material in any complex or thoughtful manner. Particularly, these questions do not pertain to the present subject matter. For example, teachers’ repeated requests to solve simple multiplication problems during a lesson concerning the equivalence of fractions would be considered simple.

I include these questions because they serve as a baseline of the type of mathematical discourse in a classroom, and, as such, are a staple of most mathematics classes. They are necessary in many exchanges. However, in this study, they give a drastic contrast to those types of teacher behaviors requested by the *Standards.*
Coded Excerpts

Given these codes and their definitions, I find it useful to provide examples of how I actually coded two episodes, which can be found in Tables 3 and 4. In both excerpts, I provide 10 coded minutes of the lesson. The first example is taken from a Chinese classroom and the second from a U.S. lesson. I offer a third example to capture the remaining set of categories.

Table 3

Coded Transcript from a Chinese Classroom

<table>
<thead>
<tr>
<th>T</th>
<th>Please look at the screen together with me and complete the equation. Who will try? Hao Jiaqi.</th>
<th>Simple question</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>50 divided by equals 2. En, 50 divided by, 50 multiplied by…</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Multiplied.</td>
<td></td>
</tr>
<tr>
<td>Ss</td>
<td>50x3 divided by 25x3 equals 2.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Right?</td>
<td>Community acceptance</td>
</tr>
<tr>
<td>Ss</td>
<td>Yes.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Ok. Second one?</td>
<td>Simple question</td>
</tr>
<tr>
<td>S</td>
<td>The quotient of 50 divided by 5 divided by the quotient of 25 divided by 5 equals 2.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Good. Sit down. Do you agree?</td>
<td>Community acceptance</td>
</tr>
<tr>
<td>S</td>
<td>Agree.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Then what's your reason? Zhang Yuan.</td>
<td>Conceptual question</td>
</tr>
<tr>
<td>S</td>
<td>The rationale of consistent quotient.</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 (cont.)

<table>
<thead>
<tr>
<th>T</th>
<th>Oh, Sit down, Very good. Then <strong>who will talk about the content of the rationale of consistent quotient?</strong> Tang Xiaoyun.</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Two such numbers divided or multiplied…</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>Who will help her,</strong> Wang Jin?</td>
<td>Student help</td>
</tr>
<tr>
<td>S</td>
<td>If the two numbers are multiplied or divided by same times, the quotient keeps unchanged.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>What are the two numbers?</strong></td>
<td>Simple question</td>
</tr>
<tr>
<td>S</td>
<td>Dividing number and the divided number.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>If the dividing number and the divided number are multiplied or divided by same times, the quotient keeps unchanged.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>Right?</strong></td>
<td>Community acceptance</td>
</tr>
<tr>
<td>S</td>
<td>Yes.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Say it again.</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>If the dividing number and the divided number are multiplied or divided by same times, the quotient keeps unchanged.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Sit down. Now we know that there's the rationale of consistent quotient in division, right? <strong>Then what kind of number did we learned before?</strong></td>
<td>Simple question</td>
</tr>
<tr>
<td>S</td>
<td>Fraction.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Fraction. Let's look at fraction. <strong>Who will read the question?</strong> Yan Fenwei.</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 (cont.)

<table>
<thead>
<tr>
<th>S</th>
<th>Represent the following colored areas. <strong>Are these colored areas equivalent?</strong></th>
<th>Simple question</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Good. Students from the first group please. Sit down, Su Le?</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>1/2.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>1/2. <strong>Who will tell why you fill in 1/2? Cao Zhe?</strong></td>
<td>Conceptual question</td>
</tr>
<tr>
<td>S</td>
<td>Because the paper is divided into 2 parts.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>What did you get, Yu Sujuan?</strong></td>
<td>Alternative answers</td>
</tr>
<tr>
<td>S</td>
<td>Divided unit one into 2 parts, and one part of them is 1/2.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>1/2. Good. <strong>Ok, second group. Shao Ming?</strong></td>
<td>Alternative answers</td>
</tr>
<tr>
<td>S</td>
<td>1/4.</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>2/4.</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>2/3.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>Why 2/4?</strong></td>
<td>Conceptual question</td>
</tr>
<tr>
<td>S</td>
<td>Because unit one is divided into 4 parts, 2 parts of them is 2/4.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Very good. <strong>Third group, Shi Shenqi?</strong></td>
<td>Alternative answer</td>
</tr>
<tr>
<td>S</td>
<td>4/8.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td><strong>Why?</strong></td>
<td>Conceptual question</td>
</tr>
</tbody>
</table>
Table 3 (cont.)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S</strong></td>
<td>Because unit one is divided into 8 parts, that is 4 parts of them.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Very good. Then you see are the colored areas equivalent?</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Equivalent.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Equivalent, good. Just now I said there's rationale of consistent quotient in division. Then in fraction there is also some basic rationale. In today's class we will investigate the basic rationale of fraction together. Now please look at the question. Who will say what's the first question? Hou Ruoling.</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Compare 3/4, 6/8 and 9/12.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Ok. Now let's investigate the relationship between these three fractions. Please take out of the three pieces of rectangle paper you prepared before class. Be quick. Please represent 3/4, 6/8 and 9/12 in the rectangles with shaded areas. Understand?</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Yes.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Ok, shaded area. Start. Hurry up. Zhu Yu is almost done, Gao Xing is not bad too. Other students please hurry up. <strong>Ok, who will talk about how did you get 3/4? Zhao Yi.</strong> Procedural question</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Divided the paper into four equal parts, three parts of them is 3/4.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td><strong>Put up your hands if you did the same as her.</strong> Ok, sit down. <strong>Who will talk about how did you do 6/8? LiuXinggoa.</strong> Community acceptance Procedural question</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Divide the paper into 8 equal parts and 6 parts ...</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td><strong>What did you get?</strong> Simple question</td>
</tr>
</tbody>
</table>
Table 3 (cont.)

S 6/8.

T Very good. Next one, Zhang Hong. Simple question

S Divided the paper into 12 equal parts, 9 parts of them is 9/12.

T Right? Then please compare the shaded areas in these three pieces of paper if you finish. What can you find? Simple question

S The same.

T They are the same. Right? That is to say the shaded areas are in the same size. It shows that the three fractions are.... Restating

Table 4

Coded transcript from a U.S. Classroom

T Today we're going to talk about how to write common and decimal fractions as percents, and percents are what I usually give you in your grades. 99 percent out of 100, and then we're going to learn how to write percents as common decimal fractions. Community acceptance

T Okay? Time - it's evening, and the clock was set for eight o'clock at what time will a 125 minute video end? And our person of the day said 10:10pm. Thumbs up or thumbs down? Procedural question

T Okay, I see some thumbs down. How would we go about figuring this out? Kirsten? Restating

S Um, 125 minus 60.

T Okay, because there's 60 minutes in an hour. Okay, so that's 1 hour.
Table 4 (cont.)

S 60 is another hour, (teacher interjects... 60 is another hour) and you just put those 2 together.

T And then what do I have left over? Simple question

S 5 minutes.

T 5 minutes. So what do I do with all this information? (student interjects... you enter...) It was 8 o'clock to begin with, pm. Kirsten? Procedural question

S You add the, you add those 2 hours.

T So that'd be 9 o'clock, 10 o'clock.

S It's 10, 10:05.

T Thumbs up or thumbs down now? Okay so we all agree on that. Community acceptance

T Okay, estimate and round 368 dollars and 27 cents to the nearest tenth. That would be a dime. So you do 368 to the nearest dime. Thumbs up or thumbs down on this answer? (student says out loud... thumbs down) Teacher explanation Community acceptance

T He has 360 dollars and 27 cents. Now if we're doing this to the nearest dime, can someone help us out here? Student help

S Um, if you're going to the nearest dime then you want to go over to the 7 and see if it's lower than 5 or 5 or higher.

T And it happens to be...

S Higher than 5.

T So what do I do to the 2? Procedural

S Make it a 3 and do 7 plus 2.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 4 (cont.)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Very good! 368 dollars and 30 cents to the nearest tenth. <strong>Thumbs up or thumbs down on this one?</strong></td>
<td>Community acceptance</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Okay, it says write these decimal fractions as common fractions using words and common fractions. <strong>So the first one, thumbs up or thumbs down? Is this written correctly?</strong> Okay, we need some help here. Vanessa can you help us?</td>
<td>Community acceptance</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>It's supposed to be 4 and 6 tenths.</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>Okay 4 and 6 over tenths. <strong>Thumbs up or thumbs down if you agree.</strong> Good job. <strong>Now how would I write that as words?</strong> Vanessa go ahead.</td>
<td>Community acceptance</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Um, 4 and 6 tenths.</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>And you need to write the word and 6 tenths. And make sure you end it in the t-h-s. <strong>Thumbs up or thumbs down for this one?</strong> Good Job.</td>
<td>Community acceptance</td>
</tr>
<tr>
<td><strong>T&amp;S</strong></td>
<td>How about this next one? Um, he didn't do it. Where is it? Is it up there? (student interjects...huh uh (meaning no)) Okay, so someone help me out on this one. Ashley. How would I write it in the common fraction?</td>
<td>Procedural question</td>
</tr>
<tr>
<td><strong>SS</strong></td>
<td>5 over 100.</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>5 over 100. So we say, how do we say that?</td>
<td></td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>5 hundredths.</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>5 hundredths. Okay, so in words I would say 5 hundredths. And I need to make sure I end it in the t-h-s. Yes?</td>
<td></td>
</tr>
<tr>
<td><strong>S</strong></td>
<td><strong>Um, do you have to put a line between the 5 and the hundredths?</strong></td>
<td>Spontaneous Contribution</td>
</tr>
</tbody>
</table>
Table 4 (cont.)

<table>
<thead>
<tr>
<th>T</th>
<th>Ah, ha (meaning yes). You mean like here? Or here?</th>
<th>Accepted</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>There.</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>No. The only time you use the hyphen is when 21, 22, 23, 33, all of those numbers. Okay? Thumbs up on this, do we feel comfortable with this now? Great.</td>
<td>Teacher explanation</td>
</tr>
<tr>
<td>T</td>
<td>Okay, Mr. Ross is ordering boxes of crayons for the children in a school. He estimates that each teacher will have about 25 children in their class. If there are 21 classrooms in the school, how many boxes of crayons will he need to order? If each box</td>
<td>Simple question</td>
</tr>
<tr>
<td>T</td>
<td>Okay, so what our person has done 25 times 21 equals 535 is that what we came up with so far? Do we agree? Sums stand? Okay, let's recheck some math here.</td>
<td></td>
</tr>
<tr>
<td>T&amp;S</td>
<td>Help me out. 1 times 5 is? (students all say...5). 1 times 2 is? (students all say...2). Bring the 0 down. 2 times 5 is? (...10). 2 times 2 is? (.4) Plus the 5, should be 525, right? (.Yeah) Okay, now I want to take that times what? (.69 cents)</td>
<td>Simple questions</td>
</tr>
<tr>
<td>T</td>
<td>Help me add this up. (she's doing it all on the board)</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Notice I... (mumbles- kind of talking to self)</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>2. 12.</td>
<td></td>
</tr>
</tbody>
</table>
Table 4 (cont.)

<table>
<thead>
<tr>
<th>T</th>
<th>Carry the one.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>6. 3.</td>
</tr>
<tr>
<td>T</td>
<td>Does that look better?</td>
</tr>
<tr>
<td>S</td>
<td>Yes.</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td><strong>How many of you agree with that?</strong> Okay. I think there were just some multiplication errors in there. He had the right process, just not the right steps. So he was on the right road.</td>
</tr>
<tr>
<td>Community acceptance</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Okay, what would be the best way to estimate 6287 and 3772? And our person said 6000 and 3000. <strong>What do you think, thumbs up or thumbs down?</strong> Well, I see some thumbs down. Terry?</td>
</tr>
<tr>
<td>Community acceptance</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Okay, I had 6000 on this one and 4000.</td>
</tr>
<tr>
<td>T</td>
<td>Okay, <strong>why - why did you choose the 4000?</strong></td>
</tr>
<tr>
<td>Conceptual question</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Because I looked at the number and it's the 3 next to the 7 and so you round up.</td>
</tr>
<tr>
<td>T</td>
<td>Okay, and <strong>we looked down here all the numbers had been rounded to the thousand so we knew it was going to be rounded to the thousand because that's what our answer showed.</strong> And 6, this would be rounded to 6000 and then that would be rounded to 4000. Okay?</td>
</tr>
<tr>
<td>Teacher explanation procedural</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>You may put those away please and we will do our timed test. Please get your pencil, your testing folder, and your marking instruments out.</td>
</tr>
<tr>
<td>T</td>
<td>Get your marking instrument out now so you're not looking for it. (she says to a specific student) (to everyone...) And you need to fold it so that CDF3 is showing.</td>
</tr>
</tbody>
</table>
Table 4 (cont.)

T And you fold it so you only have the one side showing, please.

T Want to give them back here so she can get going please. (to a specific student) (to everyone...) Fold it so you only have the one side showing. Fold it down with the one side showing. What time are you leaving today? (to a specific student)

T Okay. Are you ready now? (student answers yes) Okay, you've got 2 minutes. Start.

Table 5

Other categories

<table>
<thead>
<tr>
<th>Category</th>
<th>Teacher</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Making Connections</em></td>
<td>U.S., Teacher 5</td>
<td>You have one candy bar in front of you. How would you write it as a fraction?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Do you want words?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nope, I want you to write it as a fraction, using numbers, on the board.</td>
</tr>
<tr>
<td><em>Rule/definition based – question</em></td>
<td>U.S. Teacher 5</td>
<td>So first of all, what is the definition of a fraction? Write it on your board. What is a fraction? Talk about it with your partner. What is a fraction?</td>
</tr>
<tr>
<td><em>Spontaneous Contribution</em></td>
<td>U.S., Teacher 4</td>
<td>Four.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><em>(inaudible) – this was the spontaneous question</em></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Four out of eight. Yes, you have a question?</td>
</tr>
</tbody>
</table>
No, but ahh like on the one-half it ends with a two and then there’s the two fourths and four and that fraction ends with a four and there's four pieces of eighths and...

And you're beginning to see the pattern that we're going to be looking for. Good thinking.

Oh, I see it

You see it too? As we go along let's see if we can pick up more people seeing that pattern. Good observation.

**Challenging/Extending**  China, Teacher 3


From 3/4 to 6/8 the numerator and denominator are all multiplied 1 time.

How many times?

All multiplied 2 times.

That is the numerator and denominator are all multiplied by?

2

2, then you get?

6/8.

How about the results?

The results are equivalent

Who are equivalent?

The Coding Process

After completing the coding scheme, I recruited a graduate student to help establish reliability in coding the transcripts and in identifying any behaviors that were relevant in the corpus. The second coder was familiar with the corpus, and, additionally, was fluent in Chinese. Her assistance was invaluable in understanding whether behaviors and episodes in the Chinese lessons could be classified accurately by the coding scheme. Moreover, her translations and insights were necessary in analyzing the transcripts in a more qualitative way.

Initially, we analyzed classroom discourse from another data set. I used another data set for two reasons: (1) I wanted to preserve as much of my corpus as possible for establishing reliability, and I believed that training on lessons similar to my corpus would prove effective, and, (2) Wanting to establish my coding scheme as a tool for assessment, I wanted to know if it could be applied to classroom lessons that were not specific to this current study. We found this precursory training helpful in then analyzing the current U.S. and Chinese lessons.

We analyzed four U.S. classroom lessons, one from each of four participating teachers. These lessons were from fourth-grade teachers, who were using the reform curriculum *Math Trailblazers* (Wagreich et al., 1997). At times, when we disagreed, we either refined our definition or we searched the current corpus to find similar behaviors and used those to settle the dispute. We spent most of our time attempting to understand and simply refine the current coding scheme. Once we believed that we had agreement about the categories, we selected 25 percent of the corpus of the data to establish reliability.

We reached reliability in three separate ways. First, we established what a codable behavior was. Not all teachers’ behaviors, whether mathematically relevant or not, were codable. For example, a teacher could ask a student to read from a textbook. Although the
student may be reading about mathematics – i.e. how to solve a question, the teacher’s request to read was not coded. Second, we found that some of the teachers’ behaviors created episodes of verbal exchanges. These episodes were primarily scaffolding events where a mistake was examined, a challenged issued, or a clarification was necessary. The coders had to agree on what started and concluded an episode. These episodes were then used to examine to qualitative differences of teachers’ behaviors. Third, we established reliability for correctly labeling codable behaviors. For agreement on what qualified as a codable behavior, we achieved a Cohen’s Kappa of .93. For exchanges that composed episodes, our Cohen Kappa was .84. Lastly, our Cohen’s Kappa for labeling teachers’ behaviors was .82. After establishing reliability at these levels, I coded the remaining lessons.
CHAPTER 4

Results

I present the results in three parts: (a) descriptive statistics, (b) exploratory analysis and significance testing, and (c) a qualitative look at episodes within the data. The descriptive statistics illustrate the environment in which these teachers employed some of these behaviors, highlighting the frequency of their actions and comparing some differences that appear between groups. The exploratory and significance analyses compare differences or similarities between the teachers. Lastly, to illustrate how these practices might be defined and how they are enacted, I focus on some qualitative aspects of discursive practices within selected episodes.

Descriptive Statistics

One of the initial questions in juxtaposing these two groups is to ask: in what ways do teachers already employ some of these suggestions? More interestingly, how often do teachers, who have some knowledge of the Standards, and others, who are unaware of the NCTM, practice strategies that are aligned with reform ideas? To answer these questions, I wanted to look at the overall and average number of codable events in these classrooms. In this way, I attempted to show the type of talk and frequency of who’s talking, in general: the teacher or student.

**Total amount of codable behaviors.** The average length of a U.S. class period was 40.5 minutes, whereas the average class period for a Chinese lesson was 42.4 minutes, which was not a significant difference. With the exception of one U.S. teacher, most teachers used the entire class period for whole-class instruction. One U.S. teacher used a significant portion of class to lead the students, sometimes in groups and other times as individuals, through tasks. Thus, her instruction was not available to all of the students at the same time.
From Figure 1, we see that, during the whole-class instruction, Chinese teachers employed more codable behaviors, as defined in this study, than their U.S. counterparts, except in four categories: attending to mistakes (although the quality of this type of response is important), restating students’ answers, offering students problems that focus on real-life applications, and referencing everyday events, which is not a suggestion of the Standards.

![Figure 1. The number of teachers’ actions per group for each category, except simple questions.](image)

Although the average difference in whole-class instruction time between the two groups was minimal, the difference in the sheer number of codable questions and explanations was large: 653 for the Chinese teachers and 363 for U.S. teachers. This finding suggests that Chinese teachers, more than their U.S. counterparts, seemed to align more of their class time with some of the principles advanced by the Standards, as I have defined them.

**Adjusted whole-class discussion.** Not all whole-class time was devoted to mathematics. Some teachers attended to discipline, reminding students of homework, or physically arranging
the class for group work. Thus, I wanted to examine whole-class time that was used for mathematics, specifically. This more restrictive view of whole-class time is permissible because no coded events existed within those non-mathematics periods.

To account for this issue, I recalibrated whole-class time to omit classroom management, the explanation of non-mathematical issues, like discussions about students’ absences or past behaviors, and the arrangement or preparation of students for a task that lasted longer than a minute. Given this adjustment and using a two-way $t$-test, the amount of time devoted to these whole-class mathematical conversations was significantly longer in Chinese classrooms, see Table 6.

Table 6

<table>
<thead>
<tr>
<th>T-Test for whole-class time</th>
<th>U.S Teacher</th>
<th>Chinese Teacher</th>
<th>t</th>
<th>Df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted Whole-Class Time</td>
<td>36.31</td>
<td>40.97</td>
<td>-2.09 *</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>(40.17)</td>
<td>(9.11)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $p < .02$

The standard deviations appear in parentheses below the means.

The results depicted in Table 6 indicate that Chinese teachers not only employed more of these behaviors that were captured in the coding system; they possibly also took up more whole-class time to do so. At the very least, more whole-class time was used for mathematical talk. Thus, in this study, they spent more time on mathematically relevant material than the U.S. teachers.

A considerable amount of the U.S. teachers’ whole-class, but non-mathematical, time was spent arranging students into groups for a particular task. Interestingly, Chinese teachers also instructed students to work in groups; however, the time for the arrangement of and instruction for these groups and tasks was very small – only a minute or two. More importantly, the purpose of these groups in the Chinese classrooms was rarely to practice individual work
while the teacher circled the classroom, as was typically the practice in the U.S. classrooms; instead, they were used to collaborate on problem solving and then to report to the entire class.

These differences imply that Chinese teachers have more mathematical talk in their classrooms. Although the total time for the class is about the same, Chinese teachers employed about double the amount of mathematical, codable events in the class; they used more whole-class time for such talk; and they used more time (whole-class discussions and group work) to have students contribute to the mathematics of the lesson.

**Teacher talk versus student talk.** The NCTM Standards call for a student-centered approach to engendering mathematical discussions in classrooms. At the basic level, the more teachers contribute compared to their students, the less time and space students have to possibly initiate the topics for discussions, pose arguments to other students, or offer their own thoughts and procedures. Although a teacher-dominated discussion does not preclude a student-centered approach, the finite amount of time in a lesson dictates that the more a teacher talks, the less the students can contribute. Examining the proportion of student talk versus teacher talk can give us an idea of the opportunities afforded to the students (also see Sims, Perry, Schleppenbach, McConney, & Wilson, 2008).

To highlight teacher versus student talk, I used the number of words in the entire turn that concluded within a codable event. The most common occurrence was a teacher simply asking a mathematical question. For example:

Teacher: That’s one-half your candy bar. But we also have what here? Elizabeth?

Student: Two-fourths.

However, there were times when a teacher explained a concept, reiterated an instruction and, then, asked a question. In most cases, the question alone did not convey the teacher’s intent.
Thus, in all such cases, I needed to count what preceded the question. The following example illustrates how the teacher’s question is only understood with the explanation that preceded it. In this next example the teacher referenced another student’s answer:

Teacher: Right? Ok, now through our observation from left to right as well as from right to left, we found such two patterns. Right? Who can try to summarize in your own words, what change happening to the denominator and the numerator can keep the fraction unchanged? Who will try? You please.

Student: The denominator and the numerator, the denominator and the numerator of fraction are multiplied or divided at same time, the result keeps unchanged.

When examining the length of students’ responses, I used the response that directly answered or attempted to answer the teacher’s question. Sometimes, students would answer spontaneously. If the teacher called on a specific student, but another student answered, I combined the length of both students’ responses. I calculated the average length, using the number of words, of teachers’ codable questions and explanations compared to their students’ responses to these codable actions. Figure 2 shows this relationship.
Compared to their U.S. counterparts, Chinese students have more opportunities to talk, and Chinese teachers talked less. On average, Chinese students’ contributions are equal to those of U.S. teachers’ mathematical codable events. When examining who is talking in the classroom, and possibly directing the mathematics used in the lesson, Chinese students seem to have a notable contribution to that dialogue.

In general, Chinese teachers seem to employ more suggestions of the Standards during whole-class discussions than U.S. teachers in this study. Additionally, they seemed to do so without sacrificing the opportunities for students to contribute to the mathematical discourse.

**Simple talk vs. coded talk.** Another possible measure for students’ substantive contribution to meaning-making in the classroom is the frequency to which they can elaborate on their ideas. Often, a teacher’s use of a simple question, as defined in this study, did not promote substantive thought on the part of the student. Usually, such questions were leading, focusing on the right answer. Nevertheless, simple questions are necessary for classroom discussion, but do little to advance deep procedural or conceptual understanding. Thus, by comparing simple
questions to the other coded behaviors, I can provide at least a rough idea of how in-depth or superficial the lesson may be.

When comparing the simple questions to the total number of other categories, U.S. teachers asked more simple questions than all other codable practices combined (Figure 3). Chinese teachers demonstrated the reverse pattern. Although Chinese teachers used more simple questions than their U.S. counterparts, their other codable practices outnumbered their use of simple questions. These other practices are the ones supported by the *Standards*.

![Figure 3. The number of simple questions compared to other codable behaviors, by country.](image)

Although it would appear that Chinese teachers simply talk more than their U.S. counterparts in, roughly, the same amount of class time, this is not the case. Some mathematical statements are not coded behaviors, according to this investigation. For example, U.S. teachers often give instructions that last longer than a minute or two. Within these instructions are explanations about mathematics. These explanations are not questions directed at the students, either as simple questions or coded behaviors. Thus, when U.S. teachers are explaining
mathematics, but not requiring students to participate, their Chinese counterparts are continual using the suggestions of the Standards to engage multiple students.

After reading these transcripts repeatedly and watching the videos of these classrooms, these descriptive statistics are only a glimpse of how these events unfolded in these settings. Although we have an idea of raw numbers and proportions, I now turn to ask what do these behaviors look like? The first step in highlighting how behaviors operate in a classroom, I wanted to visually depict a teacher’s performance. Thus the next sets of analyses illuminate these practices more fully.

**Statistical and Exploratory Analyses**

Correspondence analysis can highlight how the U.S. and Chinese classrooms are different in how they employ some of these practices. To note differences in each category, I conducted simple t-tests. I advance reasons why I selected these analyses. Then I give an interpretation of the findings.

**Why correspondence analysis?** Correspondence analysis (CA) is a tool for solving classification problems. For this study, I want to associate the types and frequency of behaviors with particular teachers because examining how teachers use behaviors in conjunction with one another is pivotal to this study. Specifically, I wanted to examine whether teachers, by country, seemed to approach the teaching of equivalent fractions similarly. Although comparing the means of the groups along each category would show a significant difference between the groups, I still needed to ask what did this difference look like? Thus, one difficulty of the study was to illustrate how these different behaviors operated together within a group of teachers.

The first step in illustrating the dynamics of these classrooms was to use an analysis that begins to highlight how these practices are balanced in the classroom. Do they appear together
having a strong or weak association or are some so dominant that the opportunities for others is low? I chose correspondence analysis because it will possibly highlight how similar or dissimilar groups of teachers are based on their use of these behaviors. Additionally, it may possibly show the strength of association some of these behaviors have.

Correspondence analysis places categorical data in a Euclidean space, which makes it suitable for visual analysis. This analysis is often used where the data, arranged in a two-way contingency table, has many rows and columns and is not easy to interpret by visual inspection. The map produced by a correspondence analysis helps reveal relationships within the table. It produces a visual representation of the relationships between the row categories and the column categories in the same space.

Despite representing the data, visually, in a Euclidean space, correspondence analysis uses a form of chi-square distance rather than Euclidean distance between points. Correspondence analysis seeks to represent the interrelationships of categories of row and column variables on a two-dimensional map. To use an analogy, it is an attempt to plot a cloud of data points (the cloud having height, width, thickness) on a single plane to give a reasonable summary of the relationships and variation within them. The data or profile points are said to have mass. (This is the analogy that is used.) The point distance matrix is the input to principal components analysis, yielding the dimensions which correspondence analysis uses to map points.

Correspondence analysis uses another analogy, taken from mechanics: inertia or the moment of inertia. Technically, it is the integral of mass times the squared distance to the centroid (e.g., Greenacre, 1984, p. 35). Inertia is defined as the total chi-square for a two-way frequency table divided by the total sum of all observations in the table. Thus, inertia shows us the strength of association between categorical variables.
Another way of looking at correspondence analysis is to consider it as a method for decomposing the overall inertia by identifying a small number of dimensions in which the deviations from the expected values can be represented. This is similar to the goal of factor analysis, where the total variance is decomposed, so as to arrive at a lower – dimensional representation of variables that allows one to reconstruct most of the variance/covariance matrix of variables.

For this study, it sorts teachers and their actions into groups or clusters, so that the degree of association is strong between members of the same cluster and weak between members of different clusters. Each cluster describes, in terms of the data collected, the class to which its members might belong. Additionally, clusters allow us to view the strength of association between behaviors. Thus, some actions are more likely to co-exist with other types of actions.

Correspondence analysis is a tool of discovery, and, I hope to discover if teachers, individually or within cultures, are more similar or dissimilar to each other in how they employ some of these practices. It may reveal associations and structure in the data that, although not previously evident, nevertheless are sensible and useful once found. Additionally, this analysis might suggest what further statistical analyses are necessary to properly examine the data.

Figure 4 shows two visually important aspects: Teachers, when examined according to this coding scheme, cluster similarly by country, and behaviors can be grouped; that is some behaviors have a stronger association with one another than others when employed by these particular teachers. For the Chinese teachers, some behaviors, like challenging, conceptual questioning, asking for alternative solutions, and community acceptance, cluster together. This clustering illustrates how teachers in their respective groups are more similar to one another than they are to any subset of the teachers combined.
This similarity is also supported by the fact that two dimensions explain 94.86% of associations we witness in the figure. In this type of analysis, the total possible number of dimensions that could account for 100% of the analysis (all information contained in the table) would be the number of rows minus one and the number of columns minus one. In short, having two dimensions explain a majority of the associations in the model is considered excellent (Benzecri, 1992).

The model suggests that the Chinese teachers employ some of the same behaviors while the U.S. teachers seem to employ other behaviors. Although each group exhibited almost all of
the behaviors at some point during their lessons, each group seemed to rely on certain groups of
questions and explanations to dominate their classrooms.

In Figure 4, one can see that challenges, community acceptance of ideas, conceptual
questions, and alternative solutions seem to dominate the types of codable behaviors that exist in
the Chinese classrooms. On the other hand, U.S. classrooms seem to deal with mistakes, use
simple questions, restate students’ answers and employ procedural questions more than other
codable practices.

Explicit instructions about norms, using previous lessons as connectors to current
material, employing rule-based questions, and applying the material to real life situations did not
have the strongest associations to the other behaviors that any of the teachers were employing.
All teachers seem to have a staple of procedural and conceptual questions, some challenges,
restating students’ ideas, handling mistakes in some way, asking for community acceptance, and,
in some way, eliciting alternative views among students.

We do see that U.S. teachers, as group, are more associated with spontaneous
contributions and real-life applications, but this association, overall, is weak. Visually, we note
that these categories are not used in conjunction with the other codable practices.

**T-tests.** Using correspondence analysis we can visualize how these teachers are
associated with certain behaviors, we have to ask if, in fact, there are differences in the frequency
of use of these codable practices. After establishing a graphical association, I asked: to what
degree are U.S. teachers employing real-life applications in their lessons versus the extent to
which their Chinese counterparts are doing so. Thus, a post hoc analysis using a two tailed t-test
for each category would highlight if either group is incorporating more of a particular behavior
in their lessons. Lastly, for some of these categories with smaller n’s, t tests are viable.
Visually, a correspondence analysis map could possibly show the strength of association between the variables in the rows (teachers) and columns (categories), and t-tests could possibly show the significant degree of difference (or the lack of) between groups and along these categories.

**Individual categories.** Only certain categories were significantly different between the two groups. U.S. and Chinese teachers in this study seem to differ in the type of actions they employ in the classroom. Although Chinese teachers employed more of these behaviors on a whole, they did not perform significantly different in all categories (see Table 7). U.S. teachers handled mistakes and spontaneous contributions more than their Chinese counterparts. Additionally, U.S. teachers restated their students answers significantly more. In other categories, however, Chinese teachers perform significantly different than the U.S. teachers.

Table 7

<table>
<thead>
<tr>
<th>Category</th>
<th>China</th>
<th>U.S.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistakes</td>
<td>1.3</td>
<td>2.3</td>
<td>2.13**</td>
</tr>
<tr>
<td>Challenges</td>
<td>8.6</td>
<td>2.0</td>
<td>2.17**</td>
</tr>
<tr>
<td>Spontaneous Contributions</td>
<td>0.0</td>
<td>1.5</td>
<td>2.26*</td>
</tr>
<tr>
<td>Procedural Questions</td>
<td>6.2</td>
<td>3.0</td>
<td>2.17*</td>
</tr>
<tr>
<td>Conceptual Questions</td>
<td>11.9</td>
<td>3.2</td>
<td>2.11**</td>
</tr>
<tr>
<td>Alternative Perspectives</td>
<td>10.2</td>
<td>4.2</td>
<td>2.13**</td>
</tr>
<tr>
<td>Community Acceptance</td>
<td>12.0</td>
<td>6.4</td>
<td>3.56**</td>
</tr>
</tbody>
</table>

Table 7 (cont.)
This analysis suggests that for a number of codable practices, Chinese teachers significantly perform differently than U.S. teachers. Many of the Chinese lessons revolve around asking procedural and conceptual questions to a number of students. Additionally, there seems to be a focus on having students understanding the mathematical rule that governs an operation. U.S. teachers handle mistakes and spontaneous contributions more than the Chinese teachers do. However, the average number of events in both categories (mistakes and spontaneous contributions) is low.

Quality of Behaviors

When defining each category, I could only list them as a range of possible actions. For example, the NCTM Standards suggest that a teacher should have students justify their answers. This attempt by the teacher could take many forms. Reasonably, this statement could be satisfied in several different ways. A teacher could directly ask a student to prove their answer. A teacher could remind a student of a norm in the classroom that encourages the student to justify her answer. A teacher could prompt another student to challenge the student who answered. Lastly, a teacher could feign confusion. Without much clarification from the Standards, teachers and researchers could interpret the Standards in various ways. By examining episodes that highlight how different teachers can employ these behaviors in various ways, we could clarify a range of possible behaviors that satisfy the spirit of these reform efforts. In this section, I approach the data more qualitatively. I use vignettes to show how researchers can approach interpreting reform suggestions.
I take this more in-depth approach because my quantitative analyses could not adequately capture whether these teachers were employing some of the principles of the *Standards* or using behaviors that personally work for them. Strauss and Corbin (1990) advance that qualitative methods can be used to highlight any phenomenon that is not widely known or understood. Others (e.g., Heath, 1982; Hihebert & Wearne, 1993) have argued that highlighting vignettes from classroom conversations adds to our understanding of why we might witness differences between one group and another. Thus, qualitative methods are appropriate in situations where the researcher has determined that quantitative measures cannot adequately describe or interpret a situation.

In this following section, I will first analyze the categories that are notably different between the two groups. The categories with these sorts of notable differences are: mistakes, conceptual questions and challenges, procedural questions, community acceptance of ideas, alternative perspectives, simple questions, and other codable behaviors not noted in the *Standards*. I believe these practices within these categories, more than others, provide new ideas about how the NCTM suggestions can be actualized in the classroom. Then, I will discuss how similar or typical behaviors are within a group. To some extent, members of each group show similarities within the group – the Chinese teachers having the greater similarity relative to the U.S. teachers. For many of these examples, Chinese teachers’ practices seemed interchangeable. Any one of the teachers appeared to be very similar to the others, whereas the U.S. teachers had more variability in their approaches to teaching.

**Mistakes**

Defining mistakes in educational literature has not been consistent, allowing researchers to examine a range of behaviors. Some have defined mistakes as factual errors. Others have
included incomplete answers or answers that have not satisfied what a teacher was looking for (Schleppenbach, Flevares, Sims, & Perry, 2007). Mistakes can also be right answers with faulty or obscure procedures. I simply labeled answers as mistakes when the student was, in fact, incorrect, or where the teacher explicitly treated the answer as factually incorrect.

**Chinese teachers’ handling of mistakes.** Often, in the Chinese classrooms, teachers would signal that an answer was incomplete. Such responses *could* be mistakes. However, within these classrooms, Chinese teachers rarely stated that a student was wrong. Instead, they would assert that there was more to be said. These occurrences were either mistakes, depending on the explicit response of the teacher, or they were challenges on the part of the teacher. In short, for the Chinese lessons, a teacher could explicitly state that the student made a mistake (mistake) or the teacher could ask the student (or another) for a clearer or more complete answer (challenge or student help). In the second instance a student would give the correct answer, yet the teacher wanted to hear another student voice the answer, usually in a way that everyone could understand, or the teacher was searching for the rule or reason in addition to any exceptions.

For example, the following episode is treated as a mistake because the teacher suggests that first student “missed some points” and then directs the question to any other student. The teacher did not challenge or scaffold the first student.

**Teacher:** In division.

**Student:** In division, if the divided number and the dividing number are all multiplied or divided [the] same time, the quotient keeps unchanged.

**Teacher:** Right?

**Students:** Right.
Teacher: **He still missed some points. Who can make it complete? You please say it again. See who can find it.**

Student: In division, if the divided number and the dividing number are all multiplied or divided same times, the quotient keeps unchanged.

Teacher: Any problems?

Student: Except 0.

Teacher; Except 0, right? Multiplied or divided by a same number, but except 0. Ok, this is the rationale of consistent quotient we've learned before. Next let's see some questions. Ok, Lang Kun.

Although the teacher sought the correct answer from another student (signaling to the first student that he was wrong), this teacher requested a rule-based explanation to accompany the right answer. Interestingly, Chinese teachers surrounded mistakes with requests for rule-based explanations or student help. The following vignette is typical in how student help is achieved:

Student: If we apply the rationale of consistent quotient to fractions, to multiply or divide both of the numerator and the denominator will be the same thing.

Teacher: Oh, He said it's the same thing. Do you understand?

Student: Yes.

Teacher: Who can explain it again to me? I don't understand it very well. Please make it more clearly. Zhou Chenguang.

Student: In fractions, if the denominator and the numerator are multiplied or divided by same times, the fraction keeps unchanged.

Teacher: That is to say, he changed all the numbers related to division into the names about fraction. Right? Zhou Chengyun, is that what you mean?

Student: Yes.

Teacher: In fractions, if the denominator and the numerator are multiplied or divided by same times (except 0), the fraction keeps unchanged. Is that true?
Student: Yes.

The first student gives a right answer, but apparently a mistake in phrasing versus providing the wrong answer. Similar to the first vignette, this teacher seeks a clearer answer. The teacher seeks another student’s help, and through the exchange with the second student, restates and clarifies the first student’s answer. The teacher’s restatement signals that the right answer has been achieved.

Seeking another student’s help in clarifying or correcting the mistaken student seemed to be a staple for Chinese teachers in this study. Although they employed other behaviors, like simple questions and community acceptance, they relied heavily on having another student intervene. In the following example, the teacher passes the responsibility of answering to another student, twice, until the desired pattern was achieved. The students are seeking equivalent fractions, based on an example in the book. Within the episode, the teacher used simple questions and the community’s acceptance to signal the right or wrong answer.

Teacher: We studied the characteristics of when an answer stays the same in the fourth-grade classes. Now let's do some completion according to it. Look at the first question.

Teacher: Ok, who would like to do it first? Fu Rong.

Student: 4/5 equals to 8/10.

Teacher: The second. Look carefully. Read the question silently first and think it over.

Teacher: OK, who wants to do it? Liu Ji.

Student: 30/6 equals to 15/3, equals to 10/2, equals to 5/1.

Teacher: Is that right? It is right! … Read it silently first.
Teacher: Who wants to answer it? Liu Ruying.

Student: 24/12 equals to 48/24, equals to 4/2, equals to 6/3.

Teacher: Good. Well done! You have grasped this part pretty good. Let’s look at the blackboard, now.

For Chinese teachers, mistakes ranged from factual errors on the part of students to not receiving a complete answer. These teachers did not signal what a right answer would be. They eventually accepted that the student in question, and often one or two other students, recited the answer, an exception, and the rule governing the process. In this way, mistakes served as a way of eliciting alternative perspectives and highlighting the rule that governs a lesson.

**U.S. teachers’ handling of mistakes.** Although U.S. students seemed to have made more factual mistakes than their Chinese counterparts, U.S. teachers did not use these opportunities to arrive at a “clearer” explanation. In fact, in no instances did the teachers, after a mistake was made, signal that the answer was partially true – that is, that the first student was correct, yet the teacher wanted to uncover the mathematical rule for the concept or operation, or that the teacher wanted to involve other students. For the U.S. classrooms, teachers sought the correct numerical answer, asking several simple questions to lead the class or providing the answer and explanation themselves. In the next episode, I illustrate a typical way the U.S. teachers in this study handled mistakes.

Teacher: What color would they have to be if they are equivalent to one-half?

Student: That's what he said.

Student: Yellow.

Teacher: Yellow, so tell me all of the yellow numbers you see up here.

Student: One-fourth.

Teacher: **Not one-fourth.** How many fourths are yellow?
Student: Two.
Teacher: So say the fraction.
Student: Two-fourths.
Teacher: Two-fourths.
Student: Four-eighths.
Teacher: Four-eighths.
Student: Eight-sixteenths.
Teacher: Right.

In this example the teacher simply pointed to the correct numbers that she wanted to hear from the student. For each answer, the teacher restated the answer to signal that it was correct. The teacher’s restating of a student’s answer, without an immediate question, signals that the class had arrived at the right answer. Usually, teachers used “right” in the form of a question, attempting to request the community’s acceptance of an idea, but here, the teacher simply ended the episode.

There are times when the teacher asked if the community agreed, but often, it came at the end of the teacher partially or wholly supplying the correct answer. Thus, it was not a way to involve other students in helping solve the problem or reciting the relevant rule.

Teacher: Eights. Now we're going to color in this one, and this one. So what's our fraction there? The part that's colored in, what is our fraction?
Student: (inaudible)
Teacher: It is? I see why you say that though. You're just counting the parts, one two, three, four five. But it's not really, it's more than that.
Teacher: Is it 2/4? One, two, three, four. So what we've got colored in right now is 2/4? Does everybody agree with that?
At times, the teacher would supply the answer, and, then, explain the student’s confusion or the proper procedure. The following example shows a teacher, after noting a student’s mistake and allowing contributions to the discussion, ultimately providing the relevant mathematics to solve the problem.

Teacher: Okay, so you added eight plus eight, eight plus eight and got sixteen then you added?

Student: Four and four… four and four and got eight.

Teacher: Then?

Student: Umm, I added them both up.

Teacher: Add them both, so sixteen and eight is?

Student: Umm...inaudible...)

Teacher: I'm sorry?

Student: An inch and two-fourths.

Teacher: An inch and two-fourths.

Teacher: Now, I'm having a little bit of problem with this.

Student: I think, I think it would be eight times four would be thirty two.

Ss Yeah, yeah.

Student: Because you do length times width....

Student: Thirty-two area squared.

Different Student: He mixed me up.

Student: Yeah.

Teacher: Okay first of all, let's get back to Scott, because Scott's has an error here. And you know what Scott, it's okay cause
that's what we come to schools to have errors as long as we fix it up. He does have an error here. What we are after is area length times width. Let me see if I can help out Scott. Scott, when you walk around you're taking a tape measure and you're walking it here. You just lost the idea of putting down the area rugs.

Student: Okay.

Teacher: Okay, because now you're dealing with one dimension. And you're going this way and you're going this way and you're just making a fence. So when we add this up, you are here and now I'm not sure where you're getting. Now if I have sixteen inches and I have eight inches, I'm thinking one and two-fourth inches doesn't show me anything and I'm not sure where you're getting.

Student: He got the perimeter. He got the perimeter and not the area.

Teacher: Right, but to get this kind to get a fraction of a number and such a low one inch out of twenty-four inches, the answer would be twenty-four and we're dealing with inches. So, that is walking around making a fence, okay? We're talking about…

Student: But eight times four is thirty-two.

Another student: Yeah, eight times four is thirty-two. That's right.

Teacher: So that's why it's so important when you deal with area. If you make the mistake about adding you aren't going to get the right answer. Because it is - it is thirty-two inches squared is the right answer. The key is that there are two dimensions going on. There are two dimensions and that is the part is the hard part and we just have to practice it. Do you get that now, Scott?

Student: I get it now.

This episode lasted two and a half minutes, with the teacher supplying 70% of the talk.

The Standards state that “instead of doing virtually all the talking, modeling, and explaining themselves, teachers must encourage and expect students to do so. Teachers must do more listening [and] students more reasoning” (NCTM 2010). In the above episode, we witnessed the teacher leading the student to an apparent understanding, given the student’s responses. The
NCTM calls for students’ involvement and supplying the mathematical content where possible. This teacher (and others), by offering most of the mathematical content, offers little opportunity for students to handle their own mistakes or engage with other students on solving problems.

At the end of the episode the teacher stated the correct answer is: “Because it is - it is thirty-two inches squared is the right answer. The key is that there are two dimensions going on.” However, before the teacher made this statement, different students had the right mathematics in mind and knew the answer. One student commented on the correct mathematics: “Thirty-two area squared.” Another student commenting on why one student confused the others: “He got the perimeter. He got the perimeter and not the area.” These statements show that when the mistake was made the other students possessed enough knowledge and reason to supply most of the mathematics, to correct each other, and reach the correct answer. At these moments, the teacher could have asked for student help or could have requested a conceptual explanation for either Scott’s confusion or a solution to the problem. Moreover, the teacher could have simply asked Scott why he believed his was right or wrong, using his response to have other students challenge him or explain their own ideas. In fact, the teacher announced that she would help Scott understand, amidst the spontaneous contributions from her students.

Despite the teacher’s dominance in this exchange, we have witnessed two overall behaviors that are aligned with the suggestion of the Standards: many students are involved in the conversation and the teacher takes time to offer a conceptual (visual) way of understanding the problem and Scott’s confusion.

The preceding example shows how a mistake started an episode. Additionally, it is one of the rare times that a U.S. teacher, after a mistake had been noted, employed another
suggestion of the *Standards*—a conceptual explanation—but in this episode it was given by the teacher and not by a student.

Interestingly the same teacher dealt with another mistake in a slightly different way—she simply explained why students would make a mistake. Although she may possess sufficient pedagogical content knowledge to know why students commit errors, at the time, she could have allowed the students to disclose their ideas or confusion. Essentially, she supplied the answer and the explanation.

Teacher: I want to know why someone got twelve-thirds.

Student: We're not even in the thirds.

Teacher: But someone got that answer. I heard twelve-thirds going on too.

Student: He's got three-thirds.

Teacher: Let's talk about twelve-thirds real quickly cause we're running out of time. Twelve-thirds? *Why isn't this a good answer?* That's more than one. Do we have more than one going on? You know what? I think someone said it because there's three left.

Student: That's what Taylor said.

Teacher: And that's why I think that they said it. Yeah and the problem is that's a real common mistake is that you think that now this is a whole because you know part of it is gone. But you have to consider what the whole was. Now here's the next part, I want you to eat another… I want you to eat think of the part that was there first when I say go. Because some of you are going to make some big mistakes here. We're thinking about the whole. You've already eaten one-fourth of it or a quarter of it. I want to have half of this candy bar gone. Do that.

Although mistakes were acknowledged in these U.S. classrooms, they did not seem to reach the level of complexity that the *Standards* require. Instead of using the confusion of the student as a basis for discussion, these teachers either typically supplied the answer or led the students through computations to arrive at the correct destination.
Chinese teachers took a different approach to mistakes. The first difference that stands out in the way Chinese teachers signal that a mistake has been made is that they routinely treated incomplete answers as mistakes. The second contrast to their U.S. counterparts is the specificity in the type of answer they are seeking: either they want the answer to be rule-based or an overall accurate use of terminology when dealing with the material. In the following example, the teacher signaled a mistake by passing the responsibility of giving a correct answer to another student.

Teacher: It is because the denominator changed, it required the numerator change, too. Right? Then what is the rationale for the change?

Teacher: What are we based upon to do this? Yaowangyuan?

Student: We are based upon the requirements.

Teacher: Sit down please. Who can make it clearer? Qianqing.

Student: The basic rationale of fraction.

Teacher: Yes, We're not only based upon the requirements, but also the basic rationale of fraction. Right?

The requirements (the text) do call for the denominator and numerator to be changed in this instance. Maybe the student did not understand that the teacher was calling for a mathematical rule. Nevertheless, she wanted the rule that governed the procedure. Thus, the first student’s was answer was not clear enough.

Teacher: Then it comes to?

Student: It comes to 4/16.

Teacher: Go ahead.

Student: 1/4 is multiplied 4 times from left to right.

Teacher: Who can help her? You have four people in your group. Jia Yixin please?
Student: The numerator and denominator of 1/4 are all multiplied 4 times, then it comes to 4/16.

Teacher: Ok. Then you said the numerator and denominator of 1/4?

Student: Are all multiplied 4 times.

Teacher: That is to say 4 times 4, equals what?

Student: Equals 4/16.

Teacher: Right? Go ahead.

Student: Then I'll multiply the numerator and denominator of 1/4 by 2 times, it becomes 2/8. That is to say 1/4 equals one times two divided by four times 2 equals 2/8.

Teacher: Good. He found the pattern. Is it the same as yours?

The teacher wanted the class to understand that one could not simply multiply from left to right. Visually, one could multiple 4/4 from left to right, but the teacher sought to highlight the rule *that you multiple the numerator and denominator by the same number*. Again, the student’s answer was not clear enough. Thus, the teacher sought help from another student.

Chinese teachers typically sought help from other students when confronted with an individual’s mistakes. Additionally, Chinese teachers were specific about the language used to give an answer. There is a sense of completeness to answers. This completeness seems to include using correct mathematical language, some conceptual understanding, and rules that govern mathematics operations. The next example is typical of episodes in Chinese classrooms, whether the teacher is dealing with mistakes or challenging student’s responses.

Teacher: What is the relationship between fraction and division? Song Yiping?

Student: The divided number equals the numerator in the fraction, while the dividing number equals the denominator in the fraction.

Teacher: *Stop! Let's correct it for him.*
Student: Correspond to….

Teacher: Good. Repeat it again.

Student: The divided number corresponds to the numerator in the fraction, the dividing number corresponds to the denominator in the fraction, and the division symbol corresponds to the fraction bar.

Teacher: Do you agree?

Again, we witnessed and incomplete answer, which was signaled as a mistake. The first student clearly mentioned the division that is inherent in a fraction – “the divided number…” and “the dividing number….” Yet, the first answer was not conceptually accurate because “equals” is not precisely the same as “corresponds.” Additionally, we see that the teacher asked other students to judge whether the second answer offered was correct. In short, the teacher did not want others to infer the relationship between fractional notation and division, she wanted this stated explicitly.

Although Chinese teachers seem to deal with mistakes in ways that differ from their U.S. counterparts, their approaches may not bring them closer to the ideals of the Standards. The Standards encourages teachers to help students understand the reason for their mistakes. We do not necessarily see this practice in the Chinese classrooms. We see the correction of mistakes by involving multiple students in the conversation, but we do not witness students struggling through their confusion or debating with other students about right answers.

Conceptual questions and challenges

Although conceptual questions and challenges were treated separately in the quantitative portion of the dissertation, they will be examined together in this section. Challenges and conceptual questions take the same form when simply looking at the question posed by the teachers. Conceptual questions usually were formed by asking why questions (if they were not
prompted by a student’s answer), were formed as a result of having students recognize patterns, or having students explain a procedure shown to the class. Challenges occurred after a student provided an answer to a different type of question. They differ in their timing – whether the teacher was responding to a particular student’s response or if she was posing an open question to the class, but focused on one student. Sometimes, challenges created an exchange of dialogue between the teacher and the students, whereas conceptual questions could have ended after the student’s response or extended into an episode – like challenges. However, follow up questions and student responses are very similar for both behaviors. Thus, the differences I witnessed across groups for challenges, I have also witnessed with many of the conceptual questions.

The following excerpts highlight the similarity between conceptual questions and challenges. In the first, we witness a conceptual question followed by an exchange; in the second example we witness a classic challenge dialogue.

Teacher: Length times width, here we are… why is it that we only we have four sides how come we only do this two-sided thing? Jonathan, do you know why?

Another Student: I know.

Teacher: Can I have two lengths and two widths cause that is what I'm dealing with four sides here?

Student: No.

Teacher: Chris?

Student: You know you have four inches there so, you put it sorta like you umm have, you have a four inches both on both sides and then…

Teacher: Like this side up here?

Student: Yeah and then you have eight on the other side umm where it's at and then you add them all together and you get the answer. Because you have both
of them on the top both of them on the side so you can add them up...inaudible...

Another Student: No you got to times it.

Teacher: And that's a good point, I'm hearing you add and I'm not liking that that word add, multiply.

This episode highlights a typical conceptual question. This particular exchanged involved multiple students. Thus, like challenges, it produced a conversation about how different students think and solve problems. This next example of a challenge shows a typical exchange after the initial question; thus, showing how qualitatively, we can treat the two situations the same.

Student: Because, because the numerator of the fraction and, because to keep it equal to 2/5, the numerator and the denominator of the fraction should all multiply or divide a same number.

Teacher: Divided.

Student: Divided by a same number. The fraction keeps unchanged. So they should all be divided by 5.

Teacher: Then why do you choose 5? Will 6 be ok?

Student: No.

Teacher: Why not?

Student: If so the fraction doesn't equal 2/5.

Teacher: Zhao Ande?

Student: Because the resulting fraction is 2/5, to let the numerator be 2, it must be 10 divided by 5.

In the preceding episode we witnessed a teacher challenge a student, and with the resulting exchange, advance a second challenge, seeking help from another student, and used a community acceptance of the answer given. Taken together, these episodes show that conceptual or challenge questions can, and often, lead to similar follow up exchanges. Because each behavior can start from the same types of questions and result in an exchange with students where other codable practices are used, I look at these behaviors together.

**Chinese teachers’ use of conceptual questions and challenges.** Additionally, for Chinese teachers some challenges and mistakes were very similar – these teachers sought a specific rule-based answer or a conceptual justification for an answer. An episode containing a challenge could be a response to an incomplete answer or to a correct answer. Usually, the teacher asked a student to defend his position – and did so by asking why. Such a request aligns with the expectations of the *Standards*. In fact, the *Standards* state that why questions are excellent prompts for a discussion about a student’s thinking.

Typical in a Chinese classroom was the use of why or why not. Often students could not simply give the correct answer. The following are examples of the types of conceptual questioning found in these Chinese classrooms.

Teacher: Second one.

Student: False.


Student: Because they are not divided by a same number.

Teacher: They are not divided by a same number, so the fraction?

Student: Changes.
Teacher: It's not the previous one. Right? Ok, next one.

Student: False.

Teacher: Why false? Li Guangxu.

Student: Because one multiplied while the other divided.

The following example shows that Chinese teachers challenged students even when the answer was correct. Often, the students were led to give not only their personal reasons why they came to a conclusion but also to supply the rule that governed the operation at hand, as in this example:

Teacher: Who will try the first question? The first question?

Student: If the numerator and denominator are all multiplied or divided by a same number, the fraction keeps unchanged.

Teacher: Who agrees with him?

Student: It's false.

Teacher: Why false?

Student: Except 0.

Teacher: Yes, he missed a limitation, except 0. The second question, LiYang?

Student: If the numerator is divided by 4 times, and the denominator is multiplied by 4 times, the fraction is unchanged. This is false.

Teacher: Why false?

Student: To keep the fraction unchanged, the numerator and denominator should be either multiplied or divided at the same time.

In this episode we find two challenges – both asking why the student chose his answer ("Why false?") Such why questions seemed to be commonplace for most of the Chinese teachers. Additionally, to arrive at the first challenge, the teacher asked for the class to accept
the answer of classmate. Lastly, we discover another press for specificity – the zero exception to the rule. As a researcher and teacher, I have heard many students and teachers repeat: “If the numerator and denominator are multiplied by the same number, the resulting fraction would be equivalent.” Yet, multiplying the fraction by zero does not produce an equivalent fraction. This exception becomes conceptually important in understanding the rule.

Other episodes were equally challenging – calling for students to justify their answers and requiring specificity in their justifications. The next episode shows the use of why questions, but it also highlights how the Chinese teachers incorporated other codable behaviors within the episode.

Student: 3/8 equals 3/8 multiplied with 3 at same time
Teacher: Who multiplied with 3?
Student: 8 multiplied with 3.
Teacher: Oh, why?
Student: Because according to the basic rationale of fraction, the denominator and numerator all multiplied with 3, so 8 should be multiplied with 3.
Teacher: How about the result?
Student: The result? The resulting fraction keeps unchanged.
Teacher: Who will say it again? I won't say it. Qiao Deshen please say it again.
Student: According to the basic rationale of fraction, the denominator and numerator all should be multiplied or divided by a same number. Because the numerator is multiplied with 3, the denominator should also be multiplied with 3.
Teacher: Yes, in this way the fraction keeps?
Student: Unchanged.
Teacher: Is he right? Very good. Second question. Who will try? Hao Yu?
The preceding two episodes shows how a challenge, starting with a simple question, is surrounded by the teacher soliciting multiple students, requiring a rule-based answer, and asking the community to accept or challenge the target student’s answer. Logically, a challenge should produce a scaffolding event or a series of conceptually rich questions. Alternatively, a challenge can reasonably be followed by a procedural question. In essence, a challenge should be supported by other codable questions, constructing the type of dialogue envisioned by the Standards.

The most notable aspect of a conceptual question posed by a Chinese teacher is the full answer given by the student. The students’ answers were usually based on a mathematical rule or they were structured as a justification for how or why a student performed a task.

A second, prominent feature of conceptual questions in Chinese classrooms is how they are surrounded by other actions that are supported by the Standards. Easily, a conceptual question can be surrounded by simple questions: the teacher ask the question, receives the right answer, and uses a simple question to follow. These questions would review the material, check for understanding, or assure that the correct computation has been used. However, when conceptual questions lead to other Standards supported actions, the exchanges become in depth and sophisticated. The following is an example of a typical exchange between teacher and student in a Chinese classroom, for this study.

Teacher: Let's compare 3/4, 6/8, and 9/12 by looking at these pictures. What do you think, from the pictures, is their relationship? Dong Fang?

Student: I found by comparing the pictures that 3/4, 6/8, and 9/12 are equal to each other.

Teacher: Good. Does anyone have different opinion? Do you agree with her?

Students: Agree.
Teacher: *So, it means, 3/4..... let another student say it again. From looking at the pictures, I find? Liu Ruying.*

Student: From looking at the pictures, I find that 3/4 is equal to 6/8, is equal to 9/12.

Teacher: *Is she right?*

Students: Right.

Twice the teacher asks for a community acceptance of an idea and for alternative answers. Twice, a student returns a full answer (instead of a phrase like “equivalent or equal”). Sometimes these answers are based on mathematical rules or known material.

Teacher: Is the value of the fraction still the same after the change?

Students: It is still the same.

Teacher: The same...Li Moru.

Student: The numerator and denominator of 3/4 are both enlarged by 2 times - that is 6/8. The value of the fraction does not change.

Teacher: The value of the fraction does not change.

Student: The numerator and denominator of 3/4 are enlarged by 3 times - that is 9/12. The value of the fraction does not change.

Teacher: Could anyone use one sentence to express the meaning of the two sentences she just said? Summarize them. Liu Ruying.

Student: *The numerator and denominator are enlarged by some times...... If the numerator and denominator are enlarged by the same times, the value of the fraction keeps the same.*

Often, students in the Chinese classes responded to a conceptual question with a reference to a rule or implied that their answer was justified by some earlier accepted understanding. This norm is seen throughout the transcripts. Students were expected to offer full answers to conceptual questions and witnessed follow-up questions that elicited alternative answers.

Student: 15/20 is equal to the fraction with the denominator of 20 divided by 5…

Teacher: Wait a moment. Why do you divide 20 by 5?

Student: Because the denominator in the final fraction should be 4. So in order to reduce the numerator in that fraction, we should divide it by 5. 20 is divided by 5 and the result is 4.

Teacher: What about its denominator?

Student: The denominator is also reduced by 5 times. The final result is 3 over 4.

Teacher: Is it right?

The follow-up question was procedurally based. One codable behavior was followed by another. Additionally, the student was expected to give a full answer, and, sometimes, they are required to answer follow-up questions. Lastly, the answer (or exchange) is offered to the class for approval. We see this norm clearly in the next episode:

Teacher: Ok, now observe the three groups, can you find any common pattern? Li Zhenhui, please.

Student: All divided by same times.

Teacher: Make it clear.

Student: The numerators of the three groups of fraction.

Teacher: What of the fraction? Who can help him to make it complete? Ling Feng.

Student: The numerators and the denominators of these three groups of fraction are all divided by the same times, the fractions keep unchanged.

Teacher: Agree?

Student: Agree.
For these Chinese students, teachers accepted full, clear answers. If it was necessary to arrive at the right and complete answer by asking another student to add to or refine a previous response, they did.

**U.S. teachers’ use of conceptual questions and challenges.** U.S. teachers made use of conceptual questions. Usually, teachers would ask why questions if they were seeking information on how a student understood a pattern, performed a procedure, or when a student made an error. At times, teachers would relate the information to an everyday event, but were not trying to apply the material to a practical, real-life scenario.

In this first episode, we witness how many of these teachers restricted conceptual questions to disclosing the reasoning behind a mistake.

**Student:** Twelve-thirds?

**Student:** No.

**Teacher:** Why isn't this a good answer?

**Student:** Inaudible.

**Teacher:** I've been hearing bottom smaller than the top. Yeah, that's more than one. Do we have more than one going on? You know what? I think someone said it because there's three left.

**Student:** That's what Taylor said.

**Teacher:** And that's why I think that they said it.

**Student:** Inaudible.

**Teacher:** Yeah and the problem is that's a real common mistake is that you think that now this is a whole because you know part of it is gone. But you have to consider what the whole was. Now here's the next part, I want you to eat another I want you to eat think of the part that was there first when I say go. Because some of you are going to make some big mistakes here. We're thinking about the whole. You've already eaten a quar one-fourth of it or a quarter of it. I want to have half of this candy bar gone.
Student: Okay, that's easy. Do that.

In the United States, challenges, on average, happened twice in a lesson. When they did occur, many were in response to a student disagreeing with the answer of another. These disagreements were not spontaneous. All of them happened at the prompting of the teacher. Another prominent feature of the episodes surrounding a challenge in the U.S. lessons was the lack of other codable events or any significant extended discourse concerning the student’s justification. Thus, these episodes were significantly shorter than those in the Chinese classrooms. On average, these episodes only lasted about four turns – enough for the teacher to challenge, the student to answer, the teacher to have a follow up explanation or question, and, maybe another student’s response. The following episodes highlight this type of exchange.

Teacher: You can either have one half of my candy bar or you can have two-fourths of my candy bar. Which one do you want?

Student: It doesn't matter.

Teacher: Why not?

Student: Cause they are both the same.

Teacher: They are both the same. Because one half is everybody, equivalent to two-fourths. That's the whole idea that we're going to work with today.

Different teacher, similar exchange:

Teacher: Is it 2/4? One, two, three, four. So what we've got colored in right now is 2/4? Does everybody agree with that?

Teacher: *Who doesn't agree with 2/4? Why Jared?*

Student: If you divide the wall again you have half and then…

Teacher: OK. Well, this is a half right here. How many fourths in a half? If we were able to divide those again and we want an equivalent fraction to 2/4, what would the denominator be? We're going to divide these into halves again. What would the denominator be here?
Even in episodes where there are five or more turns, the exchanges are still typical of the teacher either supplying the answer or stopping with the correct answer. In short, episodes involving challenges in the U.S. classrooms typically contained simple questions. Either those simple questions led to the challenge or they followed a challenge question.

Teacher: Three pieces. So one third is one of the three pieces. OK. Now, can somebody give me an example of an equivalent fraction for one third?

Student: One sixth?

Teacher: OK. He says one-sixth.

Student: Can I change it?

Teacher: Sure, and explain to me why you want to change it.

Student: Three fifth, I mean three-sixths.

Teacher: OK. He wants to change it to three-sixths.

Student: Because it's half of six.

Teacher: Three is half of six. So you multiply this by...

Student: (inaudible)

Teacher: But, remember whatever you do to the denominator, you must do to the numerator. So, if that's multiplied by two, and that's multiplied by two...

Student: It's going to be two-sixths.

Teacher: Two-sixths. And that's great, and that's OK. You knew what you were doing. So, is one third an equivalent fraction to two-sixths?

In this exchange, the teacher led the students to the correct answer by simple questions.

The following vignettes were the typical settings where conceptual questions occurred.

For the U.S. classes, when a conceptual question was asked, students responded in a few ways: they refrained from answering, offered a phrase or two, or were led to the correct answer.

Essentially, when they did answer, these correct answers usually were computational.
**Procedural Questions**

Mathematics classrooms are typically replete with procedural questions. They lead to strong computational understanding, and if used frequently, they lead to students being able to supply correct answers, quickly. How students solve a problem is usually the center of a whole class discussion, especially if the procedure was incorrect or very inventive. Additionally, teachers will call on students to present to the class – “come to the board and show your work” or “tell us how you solved that.” At times, these calls for a demonstration can lead to a challenge of a student’s ideas, request for clarification, checks for understanding, or disclosures of confusion. Just as why-questions are common examples of a conceptually based request, how questions anchor procedural requests.

Although, we have examples of procedural questions in both countries, I witnessed a difference in how a procedural question was handled by the student, and how that student’s answer was accepted by the teacher. In both countries, asking students how they arrived at a procedure seemed fairly the same. However, the answers given by students were far different. Students in the U.S. could give an adequate answer without much challenge or probing on the part of the teacher. In contrast, students in China gave more detailed answers. I contend that explanations are expected in Chinese classrooms, and thus, students usually offer full answers.

In U.S. classrooms, the recurrence of short and sometimes vague answers leaves teachers supplying the correct mathematics for the discussion. They restate and substantively add to the student’s answer or they supply the necessary procedural explanation, which was frequent and prominent (to the researcher). When teachers provided procedural explanations, they attempted to introduce new material, review old material, or correct some mistake on the part of a student. Such explanations add clarity to the lesson, at times. However, when used too often, those
explanations seem to prohibit students from dealing with the material in a personally meaningful way. The teacher controls the content, the procedure, and the discussion. Although the procedure to a problem is being discussed, the discussion is unilateral. This dynamic is antithetical to the principles of the *Standards* (NCTM, 2000).

**U.S. teachers’ use of procedural questions.** Below is an example of how a U.S. teacher poses a procedural question and the typical student response. We should note the length of the student’s answer and its complexity, and the teacher’s acceptance of such.

**Teacher:** So somebody said that two eighths is the same as one fourth. So how did we, how did this person get two eighths to one fourth? What math did you have to do in order to do that? Erica?

**Student:** Well, you divided it.

**Teacher:** Right. You have to divide both the numerator and the denominator by the same number.

**Teacher:** So what number did they divide by? What did she divide by?

**Student:** Two.

**Teacher:** Two divided by two is one and eight divided by two is four. And then that gives us two names for that fraction. Still the same amount.

Here is another episode that highlights a common exchange when a procedural question is posed.

**Teacher:** Two halves equal a whole. Put one half on your blue sheet. Just leave the other one over here to the side. Now, we want to find an equivalent fraction to one half, which we've already done up here, but let's do it with the tile squares. **How could we do that? Shamali?**

**Student:** Take two fourth.

**Teacher:** Two fourth and lay it down.

**Student:** One, on the other side..

**Teacher:** Lay them up on the side with the half?
Student: Yeah.

Teacher: So, we know we have a half and you said we put two-fourths, one fourth plus one fourth, that equals two-fourths correct? So, those are our equivalent fractions, everybody agree with that?

These examples illustrate students offering a phrase or short sentence to stay in the conversation. The teacher then restated the answer, which served to validate the student’s contribution, and then added more accurate terms or filled any voids in the original answer. In some ways, restating is a necessary and good practice. The Standards recognize that all students will not be able to articulate their thoughts. Some students’ personal procedures will confuse other students. Thus, teachers are often translators. However, teachers have to achieve a balance between adding clarity and seizing authority of the mathematical talk in the classroom. Essentially, teachers can allow students to provide content and reasoning when students possess enough knowledge to do so. If the students could reasonably arrive at the right answer for the right reasons, teachers should simply orchestrate the discussion. We do not witness this dynamic in the first or second episode.

In the first episode, the student simply said “you divided it”. Although the class probably understood the references made in the sentence – “you” the teacher, and “it” the number – the teacher seemed compelled to add clarity: “You have to divide both the numerator and the denominator by the same number.” However, this statement could have been provided by the first student or by other students. Equally, in the second episode, the teacher offered more of the answer (clarity) than the student, yet it seemed as if the student could have provided the same information. Essentially, in the second episode the student said “take two fourths, on one side, and yeah.” I would argue that many other students in the class did not follow the reasoning of the answer, if they understood the answer at all. The teacher stated “you said put two-fourths”
and then explained that further. Students have access to the teacher’s methods, thoughts and interpretations, but not other students.

Some of these episodes, in U.S. classrooms, often led to a student giving an answer that never disclosed what the student was thinking. Either the student relied on the textbook, an earlier answer, or gave an answer that did not reflect a complete thought. In the following episode, the student gives an answer that makes good use of the textbook, but did require the student to think about possible answers.

Teacher: Now, you're going to be having assignments where they're going to ask you what fraction is equivalent to like one-half. What can you do to find out?

Student: Look in your green notebook.

Teacher: Look in your green notebook. If I wanted you to tell me all of the fractions on this page that's equivalent to one-half what would you tell me? All of the fractions on this page they're equivalent to one-half. What would you say, Jordan?

Jordan: It will be….

Teacher: What color would they have to be if they are equivalent to one-half?

Student: Yellow.

Teacher: Yellow. So tell me all of the yellow numbers you see up here.

None of the students’ answers dealt with any mathematics. Additionally, the first student’s answer was vague – we, and the students in the classroom, cannot know to what the student’s referred. The student never mentions the page, an example, a rule from the text, a fraction – nothing. In fact, from the answer given, the teacher could not know either. Although, we conclude that the teacher assumed correctly, we, in fact, cannot know. In the end, the teacher tells the class the answer.
In this next example, the student gave a numerical answer instead of explaining what they would do if they were in the teacher’s role. Although the teacher may have intended on the student giving a simple numerical answer (given the lack of follow-up questions), procedural questions inherently ask for an individual to explain how they performed an act or how they might solve a problem, but this student appeared to have no expectation to explain and was not required to do so:

Teacher:          Gerald, what is the pattern? What do we need to do to the top to get to the bottom?

Student: Add… I mean multiply.

Teacher: Multiply by what?

Student: By three.

Teacher: Is that what I'm multiplying by?

Student: No by two.

Teacher: By two's.

Equally conceivable in this episode is a line of questioning that would have compelled the student to explain to the class how to multiply the numerator and denominator by two, achieving equivalent fractions. However, we are left to infer what the student meant because he said “multiply”, “three” (mistake), and “two”. The student was not required to give a complete thought, and mathematical terms were not required in this request, and were barely used by the student.

The preceding examples show us attempts to have the students think about how they solve problems. However, the students’ ability to fully disclose what they are thinking does not take place. Instead we see what the teacher knows about the mathematics, and by explaining it to the class, implies that this will allow students to learn the concepts.
**Chinese teachers’ use of procedural questions.** In contrast to the examples from U.S. procedural episodes, Chinese students’ answers are longer, the teacher attempts more scaffolding and clarification, and the exchanges lead to other codable events. In this latter dynamic, we see the power of an initial codable event – the opportunity to employ other suggestions from the Standards. Thus, procedural requests (or conceptual ones) can be opportunities to create an exchange where many suggestions (from the Standards) are used. In the following episode, we see an exchange where the teacher asks for a public presentation of a problem-solving method, a complete answer by a student, then the teacher’s attempt to involve other students.

Teacher:  **See how does 3/4 change into 9/12? Liu Chi?** You please.

Student:  The denominator and numerator are all divided 3 times.

Teacher:  Divided 3 times. That is 12 divided by 3, then 9 divided by 3.

Student:  Then we got ¾.

Teacher:  Then we got ¾.

Teacher:  Is it right? **Summarize a pattern from what you observed. Can you?** Ok, start.

Teacher:  **Who will try to summarize the pattern?** LiHong please.

Student:  I think when we extract some numbers are equivalent or not, we should divide the numerator of bigger one by the numerator of the smallest, and divide the bigger one's denominator by the smaller one's denominator, if the multiple is equivalent, the two fractions are equivalent.

Teacher:  Just like 12 divided by 4, and 9 divided by 3. How many times are they divided actually?

Student:  3 times.

Similar to the last U.S. episode, this teacher wants the students to understand the pattern of multiplying the numerator and denominator by the same number and producing an equivalent fraction. Yet, we witness Chinese students using mathematical terms; we see the teacher ask for
a community acceptance of an answer, restate a response, and at least call on two different students to comment on the same problem. The next example continues with procedural questions surrounded by other codable behaviors. Particularly interesting in this exchange is that the teacher also presses for justifications that are rule based.

Teacher: 6/12. Look at the next one. How to change 10/24 to make its denominator 12? Luojiaying.

Student: The numerator and denominator of 10/24 all divided by 2.

Teacher: If they are all divided by 2, the denominator becomes 12 and the fraction keeps unchanged. Right? What is the numerator?

Student: 5

Teacher: 5/12. In this question the numerator changed as who changed?

Student: The denominator.

Teacher: It is because the denominator changed, it required the numerator change, too. Right? Then what is the rationale for the change?

Teacher: What are we based upon to do this? Yaowangyuan.

Student: We are based upon the requirements.

Teacher: Sit down please. Who can make it clearer? Qianqing.

Student: The basic rationale of fraction.

Teacher: Yes, We're not only based upon the requirements, but also the basic rationale of fraction. Right?

The requirements are the curriculum – the textbook. Unlike a previous U.S. episode where the student relied on the textbook, this Chinese teacher needed to hear a full answer. The teacher wanted them to know that there is an underlying mathematical concept here – the basic rationale of a fraction – if you multiply or divided the numerator and denominator by the same number, the fraction remained unchanged. They repeated this principle constantly throughout all
of the classrooms. Sometimes, this adherence to repeating the principle and incorporating precise mathematical terms seems subtle, yet redundant.

Teacher: Think carefully, if I want to change 1/2 into a fraction having 12 as its denominator, how would the denominator change? Liu Ji

Student: The fraction should be changed into 6/12.

Teacher: How could you change it? What change should be made to the denominator?

Student: The denominator should be enlarged by 6 times.

Teacher: Well. What happened to the denominator 2?

Student: Multiplied by 6.

Teacher: Multiplied by 6. According to the basic characteristic of fractions, what will happen to the numerator if the denominator is multiplied by 6?

Student: The numerator should be multiplied by 6, too.

The student used the phrase “enlarged by 6 times”. The class, similar to what we see in U.S. classrooms, could assume that meant “multiplied by”. However, the teacher did not accept possibly synonymous words, and students were required to say exactly what they did so that others could follow along. This episode stands in contrast to many of the U.S. classrooms because it shows how complete of an answer the teacher required.

Community acceptance of ideas

The Standards support students learning to challenge one another and justify their answers to the class. When teachers request that a given answer be accepted by the general community, they signal to students that disagreements are welcomed and expected. They also communicate implicitly that students’ ideas are important in determining the veracity and practice of mathematics in the classrooms. In traditional settings, teachers could simply label answers correct or incorrect, retaining authority over the mathematics. Having the community
decide on the mathematics encourages participation and the possibility of students talking and challenging one another. The *Standards* state: “Students need opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom (*teachers should encourage students to talk in math class and to evaluate each others’ thinking*) to see whether they can be understood and if they are sufficiently convincing” (NCTM, 2000 p.60).

This category was difficult to define because of the wide range of actions that could qualify as a community acceptance of an answer (as the *Standards* suggest); thus, community acceptance looked very different between the groups and among the U.S. teachers. The difference lies mostly in the phrasing and timing of the request. I captured the instances when a teacher explicitly asked if students agreed or disagreed. Both groups used community acceptance as a way to check for confusion and understanding and to encourage disagreements.

For example, there were times when a request for acceptance seemed to be a check for understanding, first in a U.S. class, then in a Chinese class:

**U.S. class**

Teacher: OK, six would be our denominator. What would our equivalent fraction be to one third? Have the same value as one third. You need help? What do we multiply the three by to get six? Elizabeth?

Student: Two.

Teacher: Two. Multiply that by two. So, what do we do to the denominator Ashley?

Student: You do to the top.

Teacher: To the numerator. So what's our numerator?

Student: Four.

Teacher: **Everybody agree with that?** Now we're going to divide it again. So what's our denominator now? We just added the six again. What's our denominator now?
Student: (inaudible response)
Teacher: Don't think so.
Student: Are we dividing the top?
Teacher: No, we're just dividing the six, we're in the six, now we divide the six again in half. Morgan?

In this U.S. episode, which is typical of many others, the teacher, who does not pause, was simply checking to see if students understand and are following along. During her questions and explanation, she signaled that there was a wrong answer, but she makes no apparent statement that students are actually attempting to “divide” the six in the denominator, instead of multiplying by two. Certainly, switching from the word “multiplying” the denominator to the word “dividing” is confusing. Yet, read in its entirety, the teacher wanted the kids to realize that 1/3, 2/6, and 4/12 are equivalent. In essence, she wanted to know if the students are following the logic of multiplying the denominator and numerator by the same number. In short, her statement asking “if everybody agrees” felt, to this researcher more like a check for understanding (what she was attempting to do) than an opportunity to disagree.

The Chinese example is similar, in the sense that the teacher wanted to know if students were following along, but arises from a different task; the teacher wants to know if many students can produce equivalent fractions.

Chinese class

Teacher: Students from each group please tell us what are the fractions you take? Tell about how the numerators and denominators change first, and then see if the fractions are equivalent. Who will try? Wang Guanting, you please.

Student: We take 2/4 and 4/8.
Teacher: 4/8
Student: and 8/16.
Teacher: Oh, 8/16?
Student: For these three fractions, their denominators are all multiplied.
Teacher: Yeah, the three fractions. You can choose any 2 from them.
Student: 2/4 and 4/8. Their numerators and denominators are all multiplied 2 times. The fraction keeps unchanged.
Teacher: Agree?
Student: Agree.

For both situations, the teacher wanted to know if the class is following the example presented. The multiplication seems simple, and the teacher assumed that many, if not all students, recognized the pattern. Most of the instances in both groups took on various forms, but conformed to this general assumption.

There were times when community acceptance led to longer exchanges. In those times, the teacher prompted a student to disagree or the teacher disagreed. However, many of the requests seemed to be a check for understanding.

**Alternative perspectives**

Most of the requests for alternative perspectives took the same form, in both countries: “Does anyone have anything else?” All teachers used this basic question at some time or another.

**U.S. teachers’ use of alternative perspectives.** In the U.S. classroom, this request seemed to serve two purposes: a) teachers wanted to know if students had any different ideas or solutions to add to the conversation, and b) they were checking for understanding. The following is a good example of how a teacher solicited different ideas:
Teacher: What is a fraction and why do we need fractions? Why do we need to be able to write numbers in fraction form? Alex?

Alex: Because we need, if there's more than the number and less than the other, we need something in between.

Teacher: Ok, all right. What do you think Jennifer?

Jennifer: If there's less than a number then you'll need to write like a half.

Teacher: Ok, less than which number?

Jennifer: One.

Teacher: Less than one, ok? If we want to talk about less than one, we need some way to express it.

The teacher starts with a conceptual question, uses an open-ended question to solicit any answer the student might give, and finishes by restating what both students contributed.

Although both students seem to offer answers that reflect a lesson or statement learned previously, they could have offered any thought, especially the second student, Jennifer.

In other instances, like many teachers in this study, U.S. teachers used this request to see if many students understood a concept.

Teacher: Three-fourths. Nick says three fourth. Well if, in order to reduce it we need to be able to divide it by the same number, correct? Will five-sixths divided equally to come up with three-fourths?

Student: inaudible response

Teacher: Well I like your effort, and that's OK.

Student: Ten-twelfths.

Teacher: Anybody else think they know another one?

Student: Twenty-twenty-fourths.

Teacher: Twenty-twenty-fourths. How about another one. Jared?

Student: Thirty-thirty-sixths.
Teacher: Thirty-thirty-sixths. Now, here I'm going to show you the easy way to figure this out. All you have to do is if you look, right here you start with five-sixths, you use your multiplication table because everything is set up perfectly, everyone of these is an equivalent fraction to five-sixths. Everybody see that? So if I asked you to come up with one for seven-eighths, see how simple it is to find equivalent fractions to seven-eighths using your multiplication table. Now let me show you a different way.

The teacher used the basic question: “Does anyone have anything different/more?” Yet, we can see from the exchange that the goal is to see if students can produce equivalent fractions.

**Chinese teachers’ use of alternative perspectives.** Chinese teachers’ requests for alternative perspectives covered a wider range: they called for student help to a) solve a problem, b) offer a more complete answer, c) give a different perspective, or d) check for understanding. This range of actions seem to be consistent throughout the Chinese lessons. Any one teacher could use all of these types of actions, or just a couple. Seeking alternative answers that support, extend, or clarify a previous answer is aligned with the *Standards*. Additionally, it is the very pillar on which Jacobs et al. (2006) based their assessment.

I will give an example of each practice, in order.

**Solve a problem**

Teacher: Yes, I saw some of you had a very warm discussion just now. Now let's listen to some of your ideas. What pattern do you find from the equation? Li Xuelian please.

Student: From left to right, the numerator and denominator of 1/4 are all multiplied 4 times to 4/16.

Teacher: Then it comes to?

Student: It comes to 4/16.

Teacher: Go ahead.

Student: 1/4 is multiplied 4 times from left to right.
Teacher: Who can help her? You have four people in your group. Jia Yixin please?

Student: The numerator and denominator of 1/4 are all multiplied 4 times, then it comes to 4/16.

Teacher: Ok. Then you said the numerator and denominator of 1/4 ?

Student: Are all multiplied 4 times.

Teacher: That is to say 4 times 4, equals what?

Student: Equals 4/16.

Teacher: Right? Go ahead.

Complete answer

Student: If numerator and denominator are multiplied or divided by same times, the fraction keeps unchanged.

Teacher: Anything else? Who has more to say? Yang Zhichao.

Student: Except 0.

Teacher: Any more? Chen Guojiao.

Student: Multiplied the same times at same time.

Teacher: Say it completely.

Student: In fraction, if numerator and denominator are all multiplied or divided same times, the fraction keeps unchanged.

Teacher: Is it complete now?

Student: Yes.

We see that the teacher wanted the complete rule, but used the form “…have anything different/more?”

Different perspective
Teacher: Let's look at this sentence again. Is his expression very accurate? Do you have different opinions?

Teacher: Please, another student read this sentence again? Qiao Ran?

Student: The numerator and denominator of a fraction are multiplied or divided by the same number - the value of the fraction keeps the same.

Teacher: Some are raising their hands. Qu Cheng?

Student: I think it should use the term “times” instead of simply using numbers.

Teacher: Times? We use *times* with the term "enlarge" or "reduce". When it comes to "multiply" or "divide", we can use a number. We say enlarge or reduce by how many times.

Teacher: Anyone else have different idea?

Student: We say multiply or divide by a number, but it will be wrong if the number is zero.

Teacher: Why couldn't it be zero?

Student: Because when any number is multiplied by zero, the result will be zero.

Teacher: So what? What will become zero?

Student: The numerator and denominator will all become zero.

The teacher started the episode with requesting different opinions about a statement read from the text. When the first student introduced an idea, she clarified the use of terms, then asked again if there is a different idea. These open ended questions allowed for different responses – students giving their opinions.

*Check for understanding*

Teacher: What do you think, from the pictures, is their relationship? Dong Fang?

Student: I found by comparing the pictures that 3/4, 6/8, and 9/12 are equal to each other.

Teacher: Good. Does anyone have different opinion? Do you agree with her?
Students: Agree.

Teacher: So, it means, 3/4…… let another student say it again. From looking at the pictures, I find? Liu Ruying?

Student: From looking at the pictures, I find that 3/4 is equal to 6/8, is equal to 9/12.

Teacher: Is she right?

Students: Right.

Although the teacher asked if there is anything different, her efforts led to having more than three students understand the concept. This action occurred frequently in these Chinese lessons.

**Simple Questions**

Simple questions are a staple of any classroom. In fact, you could not conduct a coherent lesson without them. For basic concepts, to lecture, or to provide a review of material a teacher may use simple questions. However, if simple questions dominate the classroom, a teacher and students have less time to employ and handle other types of questions and explanations – like those suggested by the Standards. More importantly, simple questions often lead to simple answers, allowing very little intellectual autonomy on the part of the student.

Additionally, noting the use of simple questions in this study is striking. Because all teachers used simple questions more than any other type of question, highlighting these types of questions would allow us to see how some norms are established in the classroom. These norms would be behaviors that are literally performed every day and many times in a lesson.

I note that all teachers eventually use the same types of simple questions. All teachers will elicit addition and subtraction answers within a fraction lesson. All teachers will ask students to give a simple answer. All teachers at some point simply have one student reiterate
what another student said (to check for understanding) or repeat what the teacher said. Lastly, at some point teachers will ask students to recite what is in the textbook.

**U.S. teachers’ use of simple questions.** In many of the U.S. classrooms, such questions were used to check for understanding or to lead students to the correct answer. Once the correct answer was achieved, teachers would move to another problem or explain how or why a student gave a particular answer. The following episode epitomizes a teacher’s search and satisfaction with the right answer.

Teacher: Okay, Jennifer you have one candy bar and you want to be fair about it so how many pieces do you divide that into?

Student: One-sixteenth.

Teacher: Not...okay, each person is going to get one-sixteenth but you take the candy bar and break it into how many pieces?

Teacher: Sixteen pieces and each piece is going to be called one-sixteenth. And Jennifer if she doesn't get hoggy about it and only eats her piece - how much are you going to eat of that candy bar?

Student: One-sixteenth.

Teacher: One out of the sixteen pieces or one-sixteenth, okay? Now, this is basically what we're doing today is that we are only working with these fractions. Now, there are other fractions that we can do this with. But to make this easy for you to see what's going on, we're going to do some coloring now.

The students start to color different fractions strips, but they do not further discuss the equivalency of fractions during this time in class. In these ways, simple questions are used to supply whole numbers. Although simple questions are necessary in classrooms, without the use of more probing questions they can, sometimes, lead nowhere. The next episode supports this notion, and this was common in many of the U.S. classrooms.

Teacher: You can either have one half of my candy bar or you can have two-fourths of my candy bar. Which one do you want?
Student: It doesn't matter.

Teacher: Why not?

Student: Cause they are both the same.

Teacher: They are both the same. Because one half is ..everybody?

Ss & T: Equivalent to two-fourths.

Teacher: That's the whole idea that we're going to work with today. We're been working with that in measurements. You know when we measure things on the ruler. You can either say that a line is one and one-half inches long or what else could you say? It's one and what? And it's the same length?

Student: Eight-sixteenths.

Teacher: You can say it's one and eight-sixteenths. What else could you say? Come on think. Our brains are rusty today.

Student: I'm rusty.

Teacher: Okay, if you're measuring on your ruler. You could say this line is one and one half inches long or you can say it's one and eight-sixteenths inches long. What else could you say? Madison?

Student: It's one and one-eighth long?

Teacher: Not one-eighth, how many eighths will it take to make a half an inch?

Student: Two.

Teacher: Eighths..?

Students: Four.

Teacher: Four what? Say it.

Student: Four-eighths.

Teacher: Four-eighths. Right Melanie?

Teacher: Four-eighths. Okay, now I got one for you. How many fours would it take?
Student: Oh, umm...to make one with eight.

Teacher: To make a half. How many...

Student: Oh, two!

Teacher: Two-fourths to make a half.

Teacher: Now, do you see how long it took you to think your way through that? And some of you are still looking at me like where are these numbers coming from?

Even when using only simple questions, although conceptual or procedural questions or explanations were necessary, the exchange became incoherent. The teacher talked about one and one-half inches, using a ruler, to be equivalent to one and eight-sixteenth inches. Although mathematically true, this was not visually available to the students because they did not have rulers available and thus could not see eight-sixteenths on the ruler. Thus, the confusion starts. The first student stated that one-eighth was equivalent to one-half. Given this mistake, the teacher asked a question that could either be about multiplication or addition. The student then attempted to multiply the eighth by two – maybe then arriving at sixteen. One student realized that if one wants to keep eight as the denominator and create an equivalent fraction with one-half, one should use four as the numerator. Yet, the exchange became more confusing by the teacher’s simple question of “how many fours?” This should, at least, be “how many fourths” if not “how many one-fourths…?” The episode ended with the teacher admitting that such a simple exchange landed in confusion. In the end, we are not sure how to create equivalent fractions or why any particular process of achieving such fractions work.

**Chinese teachers’ use of simple questions.** In Chinese classrooms, some of the simple questions looked the same as they did in American classrooms. However, when there is a difference, it is striking. Some simple questions disclosed either the procedural or conceptual
underpinnings of the mathematics being discussed. At other times, Chinese teachers use simple questions to lead to or follow from procedural or conceptual questions - leading to a coherent exchange between teacher and student. In the next example, the simple questions follow:

Teacher: 9/12 is equal to 3/4. Is there anyone else that is equal to 9/12?
Student: 6/8 is also equal to 3/4.
Teacher: Right. We can also write 6/8 is equal to 3/4. And you said 9/12 is equal to 6/8.
Teacher: Let us study these two fractions first. Let's look at this equation. 9/12 is equal to 3/4. What change has happened to the numerator and denominator of 9/12? Wang Peng?
Student: Its numerator and denominator are both enlarged by 3 times.
Teacher: Multiply by 3 times?
Student: Reduced by 3 times.
Teacher: What’s… that?
Student: Reduced.
Teacher: Right, so that means 12 is divided by how many?
Student: 3
Teacher: How about the numerator?
Teacher: 9 is divided by 3. Do the values of these two fractions change?
Student(s): No.

In this episode, the teacher highlighted how a person would arrive at equivalent fractions by either multiplying or dividing the numerator and the denominator, resulting in understanding that the value (representation) of equivalent fractions is the same. The students could possibly understand how to create an equivalent fraction and why the process works, either by multiplying or dividing - with only the use of simple questions.
At other times, Chinese teachers’ use simple questions to lead to conceptual or procedural ones. In the next example, the teacher uses simple questions to create equivalent fractions by dividing numerators and denominators; then, asks why this procedure is possible. The episode concludes with noting and handling a mistake and asking for alternative summaries of the mathematical rule.

Teacher: So what should we fill in? 8 divided by what? Come on, Liu Xianzhe.

Student: Equals 8 divided by 10.


Student: Equals 16 divided by 20.


Student: 30 divided by 6 equals 15 divided by 3.

Teacher: Go on.

Student: Also equals to 10 divided by 2, and 6 divided by 1.

Teacher: What divided by 1?

Student: 5 divided by 1.

Teacher: 24 divided by 12, Liu Yunfeng?

Student: 24 divided by 12 equals 49 divided by 96. Oh, 24. Also equals 4 divided by 2, and 8 divided by 4.

Teacher: **What is your basis?** Wang YiYang.

Student: The rationale of consistent quotient.

Teacher: The rationale of consistent quotient. What's the content of it? Wang Yuqin?

Student: If the numerator and denominator of a fraction multiplied by a same number, or divided by a same number, the quotient keeps unchanged.

Teacher: Kind of wordy. Who would like to supplement? She missed something. Hong Ren.
Student: If the numerator and denominator of a fraction multiplied or divided by a same number, the quotient keeps unchanged.

Teacher: You miss it, too.

Student: Except 0.

Teacher: Yeah, except 0. In division if the numerator and denominator of a fraction multiplied or divided by a same number except 0, the quotient keeps unchanged. This is the rationale of consistent quotient, which we learned in fourth grade. Look at the following exercise, tell about the relationship between fraction and division first, and then fill in the blank. LuoXiao.

Or simple questions followed and supported the use of procedural and conceptual questions, as this next example shows.

Teacher: Ok, then what is the difference between what is said in book and the pattern you summarized? Liu Shan.

Student: The book says the numerator and denominator are multiplied or divided by a same number. While what we summarized is to multiply or divide the same times.

Teacher: So that's basically the same thing. Right? Then, anything else different? You please.

Student: The book says "except zero".

Teacher: Did we say it just now?

Student: No.

Teacher: Then who can tell us why the pattern says "except zero"? You please.

Student: Because any numeral multiplied by zero is zero.

Teacher: Let's take an example, ok?

Teacher: 3/4 equals. What is it when 3 multiplied by 0?

Student: The numerator is 0.

Teacher: What's the result?
Student: 0
Teacher: Can it be all right?
Student: No.
Teacher: If 3/4 divided, Can 0 be the dividing number?
Student: No.
Teacher: Then does it make sense to divide 3 by 0?
Student: No.
Teacher: So there must have a stipulation of the pattern. Must except…
Student: Except 0.
Teacher: Because 0 cannot be dividing number in division, 0 cannot be denominator in fraction, either.

The simple questions in this episode bring to life the conceptual idea offered by the students. The teacher concluded with an example. With an exchange of questions and answers, the teacher, to some degree, proves the rationale of the consistent quotient correct. The use of simple questions to highlight an example or to prove a mathematical idea stands in stark contrast to using simple questions mainly for computational tasks.

Other codable behaviors

Spontaneous contributions. Although U.S. teachers significantly handled more spontaneous contributions than their Chinese counterparts, these episodes were not qualitatively notable. A U.S. teacher would accept a spontaneous contribution – that is, acknowledge it, but did not adequately address it. For example:

Kelly: I have an example of two numbers. Like three sixths can be one half because it is half of something.
Teacher: Good. OK. And that’s exactly what we are going to be learning today. Kelly said that three sixths and one half equal the same thing, they represent the same amount. And that's what we mean by an equivalent fraction.

The student’s outburst was not ignored, but it was not explained. The Standards suggest that a teacher restate the student’s position when it is not complete – adding precise mathematical language and clarity, bringing attention to how the student shared her thinking and uncovering mathematics involved in the contribution (NCTM 2000).

In fact, for this episode, if other students did not have a firm grasp on equivalence, they would be confused. They would have to automatically know that a calculation was implied by the teacher and they would have to figure out which numbers to use. Almost all of the spontaneous contributions were handled this way. For other instances, students were simply led to the right answer. This does not support the vision of the Standards or other reform documents, like the Common Core State Standards (2010). Thus, U.S. teachers had spontaneous contributions in their classrooms, whereas Chinese teachers did not. These unsolicited contributions were not ignored but they were not used as the Standards suggest.

**Rule-based episodes.** The U.S. teachers’ use of rule-based requests was rare, and we were liberal with the instances that we did code. The very few times that U.S. teachers suggested that a rule should be used to support a student’s procedure appeared in the following form:

Teachers: Now guys if we use division to get lowest terms, what could we have used to get a bigger equivalent fractions. Multiplication. Six times what gives you twelve?

Students: Two.

Teachers: Two. Whatever I do to the denominator, I have to do...?

Students: To the numerator.
Teachers: To the numerator. So, five times two is ten. Your equivalent fraction was ten-twelfths.

The other two examples are similar to this.

Chinese teachers heavily incorporated rule-based questions in their lessons. First, teachers ensured that students justified their answers by stating a mathematical rule. Second, Chinese teachers, again, seemed to have students repeat the same rule to make sure that many students understood the same concept. The following is a typical exchange in a Chinese classroom involving a rule-based question:

Teacher: Oh, why?

Student: Because according to the basic rationale of the fraction, the denominator and numerator all multiplied with 3, so 8 should be multiplied with 3.

Teacher: How about the result?

Student: The result? The resulting fraction keeps unchanged.

Teacher: Who will say it again? I won't say it. Qiao Deshen please say it again.

Student: According to the basic rationale of fraction, the denominator and numerator all should be multiplied or divided by a same number. Because the numerator is multiplied with 3, the denominator should also be multiplied with 3.

Teacher: Yeah, in this way the fraction keeps…?

Student: Unchanged.

There were very few rule-based episodes in U.S. lessons. Additionally, the vignette presented was typified the others. In glaring contrast, Chinese teachers used rule-based questions often, and the requests were very similar across the Chinese lessons.
CHAPTER 5
Discussion and Conclusions

Overview of Findings: Alignment with the NCTM Standards

Despite the differences between the U.S. and Chinese classrooms, I cannot claim that either group exemplifies the ideal Standards classroom. Both groups employed simple, procedural, and conceptual questions, and at times involved many students in providing or refining an answer. Rarely, in the U.S. classrooms, did these practices rise to the level of highlighting instances where reform suggestions were realized. In the Chinese classes, the use of these types of questions were more prevalent and to a greater extent. However, the Standards call for more than a focus of procedural, challenging, and conceptual questions and the expectations of having alternative perspectives offered in the classroom (NCTM, 2000). Given the numerous examples in the NCTM over the years, ideal classrooms would center on problem solving, justification, and creating opportunities for students to compare and contrast their ideas. Other initiatives, like OMLI and the Common Core, are aligned with the Standards.

Both groups employed simple, procedural, and conceptual, challenging questions more than any other codable practice. In both groups, the teacher retained authority in classroom. Despite the Chinese teachers’ ability to get students to talk a lot, similar to exchanges in the U.S. classrooms, the talk is directed toward the teacher, whereas the Standards call for students to talk to and challenge one another (NCTM, 2000).

When teachers, in both groups, used other codable practices they did so inconsistently – maybe a practice or two were distinctive for the group, but others were rare. For example, U.S. teachers attended to spontaneous contributions, whereas Chinese teachers did so once or twice. However, when any U.S. teacher accepted and addressed a spontaneous contribution, she or he
simply provided the right answer. These opportunities were not used to realize the ideas of the Standards. The Standards recognize that some “students in the lower grades need help from teachers in order to share mathematical ideas with one another in ways that are clear enough for other students to understand” (NCTM, 2000, p. 61). Thus, teachers would need to take the time to a) paraphrase the student’s contribution, b) encourage other students to attempt to answer, and c) model good questions and explanations if necessary.

I documented major differences between the groups. Strikingly, Chinese classrooms highlighted how procedural and conceptual questions can be used to involve many students in solving one problem. Along these lines, Chinese teachers, in more quantitative and qualitative ways, are aligned with some of the suggestions of the Standards.

**U.S. classrooms.** Like others’ research (Jacobs 2006; Johnson, 2003; Hiebert et al., 2005; National Research Council, 2000; Stigler & Hiebert, 1999), U.S. teachers, in this study, did not adhere to the principles of reform ideas. These classes were more traditional – focusing on whether the correct answer and explanation have been heard, either from a student or the teacher. Rarely were students challenged by the teacher, beyond a correct or semi-correct answer, nor were they encouraged to challenge one another. Given the dominant use of simple questions, the evidence suggests that students were not often given the opportunity to comment on procedures publicly presented to the class. Mostly, students responded to teachers’ questions with short phrases. Such responses did not indicate that the speaker fully understood the mathematics involved. Additionally, given the brevity of the answer, the rest of the class would have to assume what the speaker meant. Sometimes, U.S. teachers would restate the student’s answer, but these restatements apparently were also assumptions, on the part of the teacher.
Overall, the U.S. classroom could be characterized as a standard, teacher-dominated classroom, where simple questions were used to work through most of the mathematics. Particularly, U.S. teachers used many simple questions to solicit calculations from the students. In fact, when mathematical explanations were given, most often the teachers supplied them. Moreover, none of the students’ explanations alone or collectively, within any given exchange, rivaled the sophistication or amount of words of their teachers’ explanations. For example, the following two episodes represent the depth of typical U.S. students’ responses:

Teacher: Ok, I ate one half of it and how much did you eat? You ate half. Ok, now what does the denominator of a fraction tell us? Why do we have a two in the denominator? Pat?"

Pat: To show how many parts there are.

This is a typical student response to a conceptual question from the teacher. The next episode is a response to a procedural question:

Teacher: So three-fifths, another name for three-fifths, if we want a decimal name is sixty-one hundredths. And then how do you make that into a percent? Sam?

Sam: Sixty percent.

Teacher: Sixty percent. You move the decimal two places - move the decimal point two places and you have sixty percent. And those are all the same name, different names for the same amount.

U.S. teachers offered more procedural and conceptual explanations than their students. A typical U.S. teacher’s explanation usually followed a question that was posed to a student. Sometimes, the student answered in short phrases, like above, but other times teachers simply answered their own questions:
Teacher: So somebody said that two-eighths is the same as one-fourth. So how did we, how did this person get two-eighths to one-fourth? What math did you have to do in order to do that? Tyler?

Tyler: Well, you divided it.

Teacher: Right. You have to divide both the numerator and the denominator by the same number. So what number did they divide by? What did she divide by? Two. Two divided by two is one and eight divided by two is four. And then that gives us two names for that fraction. Still the same amount.

But I ate one fourth of the candy bars and I also ate two eighths of the candy bars. Ok, now someone also said that six-eighths can be changed to three-fourths.

Ok, so we had to divide both the numerator and denominator by the same number in order to write a new name for this fraction.

What was that number? Two. Again we divided by two. Six divided by two is three; eight divided by two is four, ok? So you need to know this, that a fraction can represent a part of a whole thing, it can represent a part of a set of things."

U.S. teachers often offered explanations when their students could have been requested to do so. Their explanations were longer and more sophisticated than their students.

In fact, U.S. teachers, overall, talked more than their students. This dominance allowed them to retain control of the mathematics and left little opportunity for students to handle the material. The dynamic allowed the teacher to lecture – to attempt to simply transfer knowledge. Thus, for some teachers being the source of the mathematics may have felt like a coherent, linear class – the teacher lectured and explained, students learned and showed their understanding through answering questions correctly.

Yet, this dynamic has been the source of attention and argument for researchers and educators. Yu (2005) stated that having a teacher pose a question while students search for the right answer was a major problem in traditional classrooms. Many teachers believe that the
vertical (teacher to student) exchange is “synonymous with dialogue” (Yu, 2005). “When the teacher talks most, the flow of ideas and knowledge is primarily from teacher to student. When students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community” (NCTM, 2000). Yet, I found that much of the teacher talk is the kind that realistically could have been provided by the students. Instead, students gave simple, short answers that did not compare in length or sophistication to their teachers’.

Not surprisingly, these students were responding to simple questions. As noted in Chapter 3, U.S. teachers used more simple questions than other codable questions combined. Thus, the opportunities for students to provide more depth explanations were few.

**Chinese classrooms.** One reason for the Chinese teachers’ use of more codable practices is that there was simply more mathematical talk (questions and explanations) in the Chinese classrooms. Interestingly, the teachers talk (they question and explain) more than U.S. teachers do. However, this talk covers more mathematics, and it requires the students to talk more as well. This seems counterintuitive. Yet, it is possible for a teacher to talk, and therefore to direct or orchestrate the discussion, in the classroom, and have his or her talk generate more talk from the students. I argue that the types of questions make a difference. The *Standards* and others would agree (NCTM, 2000; Hiebert & Stigler, 2000; Perry, Vanderstoep, & Yu, 1993).

For Chinese teachers, they used at least two other codable actions effectively – community acceptance of ideas and requesting alternative perspectives. They did so in a way that seems unique to their group. They were successful in compelling students to provide multiple answers to one questions or problem. They sought alternative explanations; they
challenged student answers; and they asked for rule-based explanations. In this way, aspects of the Standards were met.

In other ways, some of these same practices seemed repetitive and superfluous. During these episodes, teachers were assessing whether many students understood a particular operation, concept, or rule. The Standards do suggest that student thinking become public, but this principle supported students’ disclosure of confusion, disagreement, or unique problem solving approaches, and it is not clear that these goals were met by these practices.

I believe that some of the procedural and conceptual questions highlighted in this study were aligned with the Standards (NCTM, 2000) and other reform objectives, like the Common Core State Standards (2010) and the Oregon Mathematics Leadership Institute (2005). Although the Common Core standards do not specifically call for types of procedural or conceptual questions, it suggests that students be able to “reason abstractly and quantitatively” and “construct viable arguments and critique the reasoning of others” (Common Core, 2010). These requirements are satisfied through the procedural and conceptual exchanges between teacher and student. Using such questions, teachers are able to promote “reasoning [that] entails … a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects” (Common Core, 2010). Although we did not witness teachers achieve all of these practices in any particular vignette, we did see students pressed beyond computational performances and challenged to offer alternative solutions and ideas.

The Common Core, OMLI, and the Standards suggest that having students justify answers and recognize what is an acceptable argument for others should be a norm in math classes (Common Core, 2010; OMLI, 2005; NCTM, 2000). Sometimes, procedural questions
can lead to justifying, but, mostly, teachers’ use of why questions produce justifications from students. Additionally, in many of these exchanges are the opportunities for other students to ask questions, disclose their own confusion, or agree. Therefore, for a few of the categories, I contend that, in this study, Chinese teachers practice more of what the Standards suggest than their American counterparts. Particularly, Chinese teachers employ more procedural and conceptual questions that challenge their students in the midst of whole-class discussions. More importantly, because they frequently ask if other students agree with a given answer, they provide the opportunities for possible disagreement, exceptions, different ways to solve a problem, or the disclosure of confusion. For those suggestions that we see clearly enacted in Chinese classrooms, they give us ample material to fashion exchanges that begin to realize reform ideas.

Particularly, the differences between these two groups centered on multiple students becoming involved in procedural and conceptual (including challenging) exchanges. They appear to be the hallmark of the Chinese lessons. Multiple students were involved in two ways – teachers asked if the community accepted the answer given or the teacher called for different students to restate a previous answer (making sure many students understood a concept) or to add to or offer a different perspective. These practices appeared in U.S. classrooms but not consistently so, and they, on average, were not the result of an extended, sophisticated exchange as we witnessed in many of the Chinese episodes. Furthermore, Chinese teachers used other types of questions to lead to or follow from a procedural or conceptual explanation. Thus, for Chinese teachers, leading to a procedural or conceptual understanding seemed to be paramount to any other practice in the classroom.
An advantage in questioning. This priority on the types of questions produces a stark contrast between these two groups. Question type determines the response from, and, therefore, the cognitive demand of the student (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956). Although simple questions are necessary for any lesson, they are considered low level questions (Hiebert et al., 2003; OMLI, 2005). The typical response comes from rote memory or cursory calculations. Many researchers have suggested that higher level questions, which are procedurally and/or conceptually rich, produce deeper levels of learning (Gall, 1984; Marzano, Pickering, & Pollock, 2001; Perry et al., 1993; Redfield & Rousseau, 1981). Higher level questions require the student to analyze, contrast, apply, and justify. Various types of questions have a role in the classroom, but researchers are concerned about lower level questions dominating whole-class discussion.

Thus, a teachers’ use of higher level questions should promote a deeper understanding and longer retention on the part of the student. For the Standards, teachers should “ask questions that force the students to consider the context or implications of their ideas” (NCTM, 2000). For others, pressing or challenging the student is at the heart of a reform-oriented classroom (Stigler & Hiebert, 1999; Kazemi & Stipek, 2001), and one in which the student personally connects to the material, reexamines their own thinking, and becomes more proficient in crafting a solid justification of her position, using increasingly precise mathematical language.

In Chapter 3, we witnessed how Chinese teachers used an abundance of questions that required students to articulate the procedural and conceptual underpinnings of math problems. However, the abundance of higher level questions in the Chinese classrooms may be a result of students possessing more knowledge. Verplaetse (2000) claimed that when students succeed in acknowledging or recalling knowledge at the factual level, teachers would tend to move to a
higher level of questioning. The students’ abilities provide the platform from which the direction of a lesson can be built. This has the potential to deepen the interaction with students and further expand their thinking.

If Chinese students possess more mathematical knowledge than their U.S. counterparts, Chinese teachers may have an advantage over U.S. teachers, because students are ready for a deeper analysis of the material and more challenging questions. Whether teachers are motivated solely from their own experience to generate such questions or if the types of questions used are a result of the readiness on the part of the students is a question yet unanswered. Yet, we do know that teachers’ use of challenging questions that prompt students to reflect on mathematical rules, the application of mathematics, or how and why mathematics works, can happen at any grade level (NCTM, 2000).

A More Detailed Appraisal: Distinct and Dominant Features

I did witness two distinct features in these two groups – U.S. teachers occasionally made reference to common items or events in the students’ life, and Chinese teachers employed the help of other students when solving a problem. I do not believe that either distinct feature highlights ways in which teachers can be assessed with any reform suggestions, from NCTM to Common Core. Nevertheless, I believe that a discussion of both features is pertinent, because teachers’ repeated use suggest that they are important.

In the U.S. classrooms, using everyday references was not dominant - it did not occur more than other practices nor did it appear to be a staple for the U.S. teachers. The Standards (2000, p. 198) state: “Real-world contexts provide opportunities for students to connect what they are learning to their own environment. Students' experiences at home, at school, and in their community provide contexts for worthwhile mathematical tasks.” However, including common
events into a lesson do not, in and of themselves, satisfy reform ideas. This practice presents an opportunity for a rich discussion or application of mathematic to real life scenarios. Whether the teacher capitalizes on the teachable moment is the question.

Realistically, there are attempts (from neutral actions to failures) and successes in using everyday experiences when explaining mathematical concepts. Attempts are opportunities. In some of the U.S. classrooms, these opportunities were failures because they ended in confusion. At other times, they were not sufficiently applied to the mathematics; thus, they were neutral. The next two episodes, neutral and failing respectively, exemplify the more common ways in which everyday references were used:

Teacher: Tell this first problem again, Hunter.

Student: 27 children divided into 3...3 groups.

Teacher: Okay, I'm thinking, when you go to P.E., you can probably group. You happen to be a class of 20.

Teacher: So if we said, going to P.E., and Mrs. Fluster says, hey, I need you guys to get into 3 groups. Would you end up having 3 groups and then you'd stick 3 kids in them?

Student: Nooo...

Teacher: That's where you were heading.

Student: No. 3 in each group. Each group. 3 in each group!

Teacher: It was each three in each group? Okay, ummm... if we were going to do 3 in each group, then what do you think we need to get to this? Hunter?

Student: Ummm... 9?

Referring to the gym class where groups are established frequently, the teacher presented an example that students could easily relate to. The students achieved the correct calculation, but were not expected to understand the concept of equivalence. Clearly, the students understood the
multiplying 3 x 9 equals 27. However, in a lesson about fractions, no fraction was mentioned by teacher or student. Although the exchange produced equal groups, no one talked about equivalence – the main point of the lesson. Thus, students may not have been confused, but the example of a real life situation did not highlight how fractions represent parts of wholes.

The next example clearly resulted in some confusion:

Teacher: So we've had a nice unit on decimals so you already know about decimals. And fractions and decimals are very much related to each other. So a fraction is another way to express something that is less than one whole thing.

So if you buy a gallon of milk at the grocery store and you only drink one cup for breakfast, you didn't drink the whole gallon did you? And you wouldn't probably be able to drink the whole gallon without kind of running over at the seams. And so you need some way to talk about less that that whole gallon of milk. If you order a pizza, if it's a large pizza, probably you cannot eat the whole thing. And you need a way to talk about what part of that pizza you ate.

Teacher: OK, now you also need to know that we have special names for the numbers in a fraction. And what do we call the top number? Gordon?”

From the teacher’s perspective, she wanted to highlight that wholes can have parts. However, we are left to assume that all of the kids understood this example. From the episode it is not clear if the part we consumed was equal to the parts that remained. Students did not label any parts. In essence, from this exchange, it would be perfectly reasonable for a student to believe a fraction can represent a smaller part of a whole, and that part does not have to be equal to any other parts. Moreover, the connection to the “special names” to the part or to the whole was omitted, which may have left students further confused.

Having an opportunity to connect mathematics to real experiences is better than not having a chance to engage in such a discussion or having an expectation of applying what you know to the real world. Although some of the references did not explain the underlying math in
full, they may have kept the students engaged. We can assume, for some of the references, students thought about mathematics outside of the classroom. Such engagement is a desired outcome for all teachers. For this reason, teachers may continue to use these references.

The danger, however, lies in those times when students are left to assume how the mathematics operate in a given scenario. For example, U.S. teachers often used food sharing as a way to explain fractions and equivalence. For some students sharing food equally is obvious – take a whole pizza and divide it equally among six students. Each student’s piece represents \( \frac{1}{6} \) of the pizza. Additionally, if we took a pizza, cut it into 12 pieces, and gave each student two pieces, they would still have \( \frac{1}{6} \) of the whole pizza. This was a typical scenario in U.S. classrooms involving the explanation of fractions, equivalence, and applying it to food.

Some students would understand this reference immediately. If all pieces were equal and each student received a piece, we have achieved equality. Additionally, some students understand this idea because the pizza is drawn in two dimensions and looks like a circle, with each slice appearing the same. Nevertheless, some children will become distracted with the real-life aspects of a pizza – rarely is it cut equally, it is often cut in squares, and pizzas have ingredients which can be distributed unevenly which makes each piece qualitatively unequal.

Thus, simply making reference to a real life item carries a certain danger for distraction (see e.g. Brown, McNeil, & Glenberg, 2009). Such distractions may leave certain students wondering how two qualitatively different items are mathematically equivalent. Without the opportunity to explore this phenomenon, by challenging students, having students challenge one another, or using many examples to show equivalence, teachers may leave students confused.

Yet, U.S. teachers used these examples often. We may be witnessing habit. Many teachers may have been taught this way as students themselves, or some may repeatedly
encounter these examples in traditional textbooks. Without post-observation interviews that chronicle each question and its motivation, I am speculating. However, other researchers have suggested that U.S. teachers may teach according to how they learned mathematics when they were younger (e.g., Clift & Brady, 2005). Others suggest that more veteran teachers might be wedded to tradition (e.g., Remillard & Bryans, 2004). Whether teachers rely on their personal history as a student learning mathematics, a traditional text book, or approaches that have been successful for them as an experienced teacher, commenting on a common item or event without unpacking the mathematics in the situation does not meet the suggestions of the Standards, nor does it allow the students to analyze how math operates in everyday life.

Although this action was notable, it was not dominant. Not this practice, nor any other distinctive feature, was a widely used behavior for the U.S. teachers. The standard use of simple, procedural, and, sometimes, conceptual questions were the staples of the U.S. classrooms. Although U.S. teachers attended to spontaneous contributions and mistakes more than their Chinese counterparts, they did not do so in some unique way.

Limitations

This dissertation has several limitations. First, the NCTM does not purport to be amenable to codification, like I have attempted. One may argue that my treatment of the Standards is an oversimplification. Given the examples that appear in the Standards and, over the years, in the publications Teaching Children Mathematics, Mathematics Teaching in the Middle School, and Mathematics Teacher, along with conference papers and studies, the guidelines from the Standards are meant to be broad, applicable to various situations. Thus, several lessons, despite the contrast in cultures and approaches, cannot fully capture the intent of the Standards. I do argue that a conversation about the specificity of practices should take place.
Often, teachers may believe, because reform suggestions are so broad, that they are employing reform suggestions in the classroom. For example, the Standards call for the application of mathematical concepts to real life situations. Some teachers, however, simply make a reference to an everyday activity. In the moment of teaching, where students are engaged and agreeing with what is said, this reference could feel successful, especially if the analogy is novel and personally meaningful. In real classes, this practice happens.

Despite the intention of remaining broad and applicable to many classrooms and styles, some researchers and agencies found it necessary to be more specific about reform ideas and standards (Common Core, 2010; OMLI, 2005). Although the NCTM (2000) offer examples of teachers practices, these examples are not exhaustive and do not identify the range of possible practices a teacher can employ to be accurately assessed as implementing the Standards. The Common Core standards focus on student proficiency in content areas. By codifying what students are expected to learn at each grade level, the Common Core standards, like the nationalized Chinese curriculum, give the teachers the opportunity to know what the student should have learned and what the student encounter later. Specificity is key.

The Oregon Project not only addresses students’ performance, it concentrates heavily on teacher development and assessment. In terms of assessment, OMLI researchers (Weaver & Dick, 2006) have developed an evaluation (of a lesson) protocol. Concerning discourse, the evaluation is specific about what an observer should witness – five possible types of discourse that take place in episodes during the lesson. The types of discourse range from low to high, and they are specific in terms of how to code teacher questions and students’ responses. I believe that level of specificity was lacking in parts of the Standards concerning communication in the classroom; therefore, this dissertation addressed that issue.
Second, these studies say much about these particular groups of teachers. One could argue that they are not a representative sample of many U.S. and Chinese classrooms. They are not. Although these groups were compared, I did so to distinguish patterns, if they were present, to elicit good practices if they were there, and possibly give a range to how we define good, Standards-based practices despite their various instantiations in the classroom. Certainly, we can find many classrooms where reform ideas are priority, given that many districts have adopted reform-oriented curricula and teachers have completed professional development seminars. Yet, I believe that these highlighted differences and practices are real. I believe that we can witness any of these actions in many schools that we would enter today.

Third, given the few number of transcripts and the lack of diversity in the teaching practices, I cannot say that these teachers display the range of practices that could satisfy the ideals of reform principles. In essence, my hope to adequately outline a range of practices, from this study alone, cannot be realized. The results presented contain good examples of procedural and conceptual exchanges. Some show the ability of the teacher to have students verbally contribute more of the mathematics during a lesson. Thus, a more specific understanding about how some these of behaviors can be achieved is present in this study. Yet, more classes, with varying levels of experience on the part of the teachers, are needed to fully highlight how converting parts of the Standards into a guideline could be truly beneficial.

Nevertheless, these attempts commence a conversation about what qualifies as a reform practice – particularly, how teachers might define reform practices. In Chapter 1, I asked if we could witness good practices in these classrooms so that we could begin to define what counts as reform actions. To some extent, I believe we have. More importantly, I believe that we have
started a conversation about how to assess teachers according to any type of standards – i.e.
dissecting overall topics, like discourse, into identifiable chunks, like OMLI (2005).

Fourth, I implied that students’ responses reflected the sophistication of the teachers’
questions. This may be true. If so, then my concentration on teachers’ types of questions is
justified. Nonetheless, I also understand that students acquire academic norms from earlier
grades. This may be particularly true for Chinese students, where a national curriculum and
culture is shared by many. Teachers, in turn, relying on students having certain previous skills
and knowledge and embedded beliefs about education and teachers, can simply build on those
societal and educational norms.

In essence, Chinese teachers’ depth of questions could be the result of recognizing and
relying on a solid platform from which to engage and challenge students ability to handle more
sophisticated math conversations. The benefit may come from established norms. A culture of
giving full, complete answers, or, at least, an attempt to do so, could be the product of that
particular teacher or it could be the product of the educational system.

**Sociomathematical Norms**

One limitation to my examination of these classrooms is the types of norms that are
established before this particular grade level. To some extent teaching happens exactly where
the student is. Introducing new content involves reviewing previous concepts, using analogies,
and having the teacher speak more about the mathematics than her students (more than you
would witness in a regular lesson). Thus, all teachers will perform the same actions at particular
times during a new lesson. However, in other instances, teaching happens because of what came
before a particular moment – how parents or previous teachers have taught mathematics. What
counts as a justification? Why give a complete answer? Is mathematics a product of skill or
talent? These questions are sometimes answered earlier in a child’s life. How to communicate in a math class and to what extent are processes that are established over time.

The motivation to provide clearer responses or to supply a full answer, no matter the question, maybe established in earlier grades. Full, complete answer may be requirement from all math teachers. If taught from the beginning of one’s academic career, witnessing full answers from these Chinese students is simply normal. It, then, is not the result of the type of question the teacher asked. The teacher’s choice of question simply reinforces what is already expected and previously learned.

Nevertheless, we can be and should be concerned with those norms that can be established with an academic year especially in a mathematics classroom. According to Yackel & Cobb (1996), teachers and students can negotiate what counts as a good mathematical answer or justification. Within the classroom, the teacher can address what role do students play in determining the type and depth of mathematics in the classroom. For example, given the teacher’s initiation, students can accept mistakes as the beginning of whole-class discussion and analysis, rather than possibly believing that wrong answers are quickly corrected. In the end, students should come to understand that the basis for explanation is mathematical rather than status-based (Yackel & Cobb, 1996).

Students’ understanding of mathematics as a practice and not a set of rules given by a teacher is a goal of the Standards and achievable in the classroom (NCTM, 2000). The Standards consistently highlight the expectations for student talk and thinking in the classroom. For example, the sociomathematical norms in a reform-oriented classroom should include the expectation that the students are to present their methods of finding solutions by describing their actions fully instead of simply accounting for computational methods (Pang, 2003).
Teachers cannot change the past, and students’ previous knowledge or skills may be difficult to alter once acquired. Students will undoubtedly come to each class with expectations and beliefs about education in general and mathematics, specifically. Nevertheless, teachers have the ability to shape how their particular students understand the practice of mathematics. Teachers who pay attention to their students’ thinking, challenge these students to explain in full, reflect on their own thinking, and justify their answers, will produce students who are comfortable communicating about mathematics in increasingly sophisticated ways.

**Teachers’ Knowledge**

Lastly, can the differences between Chinese and U.S. teachers’ practices in a mathematics class be a result of how much they know about the subject? Some research suggests that teacher knowledge is key (Ball, 1991; Ball et al., 2001; Hill, Row, & Ball, 2005; Ma, 1999; Sherin, 1996; Zhou, Peverly, & Xin, 2006). Zhou, Peverly, and Xin in a comparison of third-grade mathematics teachers in the U.S. and China, contend that American teachers were more knowledgeable about general educational and psychological theories, whereas Chinese teachers concentrated their efforts on possessing stronger subject matter knowledge as well as a strong grasp of the mathematics trajectory that their students would follow – in the current and coming years. This is not the first study to assert this notion (Ma, 1999).

One reason that Chinese teachers have an advantage in knowing a student’s trajectory: a national curriculum gives the direction. Thus, math teachers in China not only know what they should teach, but they know where students should land in each grade level. Many Chinese teachers are responsible for more than one grade, teaching fourth and fifth, for example. Lastly, the variability between Chinese teachers in the same school or district is low. They teach from the same curriculum, which is organized according to grade level.
Second, an overwhelming number of Chinese teachers only teach math, whereas many U.S. teachers teach more than one subject at the fourth and fifth-grade levels. Additionally, the Chinese teachers convene and agree on the best approaches to the material and discuss best practices. What is a good or typical practice in one school or district or state in the U.S. may be different once a student transfers schools.

This preparation is in stark contrast to many U.S. teachers’ experiences. Many U.S. teacher preparation programs focus on how to teach mathematics rather than on mathematics itself, and once U.S. teachers become certified they do not often have the opportunity to improve their knowledge of the subject, unless it is on their own. Although some teachers are offered professional development during their careers, this development is not teacher driven – i.e. it does not usually arise from teachers realizing deficits in either subject matter knowledge or their pedagogical content knowledge (An, Kulum & Wu, 2004).

Zhou, Peverly, and Xin (2006) also found that more experienced American teachers were better able to identify important points for teaching fraction concepts. Inexperienced American teachers did not possess this skill. For Chinese teachers, however, mastery of this skill did not depend on experience. In fact, many Chinese teachers hold a certificate in teaching math. This certificate is granted after three years of post-secondary study. The researchers found that the less experienced Chinese teachers demonstrated the same proficiency as their more experienced Chinese counterparts. In conclusion, how Chinese teachers are prepared to teach mathematics may account for their approaches to their initial lessons in equivalent fractions.

Conclusions

This was an exploratory analysis. I wanted to highlight how videos of classrooms can be used to lend specificity to the Principles and Standards for School Mathematics (NCTM, 2000).
I believe that my attempt at codifying the suggestions and principles was justified. Additionally, I believe that teachers either employ some suggestions of the Standards or they use distinctive practices that are not specified in the Standards, but these behaviors are useful in their particular classroom. For some, they learned reform approaches from professional development seminars; and, for others, experience and expertise bring knowledge about student learning and a solid understanding of the curriculum.

I believe we can learn from hypothetical situations crafted by the framers of reform documents, like the Standards or Common Core. They are replete with examples of how teachers can engage in exchanges and scaffolding scenarios that keep students motivated while modeling best practices in questioning, arguing, and justifying. Equally important, we should examine exchanges from real classrooms. These scenarios not only highlight reform ideas in practice, they illustrate teachable moments – those opportunities to realize reform suggestions.

Essentially, we are searching for instances where teachers’ actions can fall into the range of what we accept as student-centered, conceptually rich discussions and how teachers can recognize immediate opportunities for change (Mok, Cai, & Fung, 2008).

The NCTM Standards stipulate that students need opportunities to communicate math ideas and solve problems with others, that they should engage in mathematical activities with confidence and enthusiasm, and that teachers should use assessment strategies that focus on understanding rather than on right answers. Although many of the episodes from this study would not qualify as the ideal Standards-suggested approach, they do represent the opportunities for reform ideas to be realized.

Assessment. In terms of specificity and an approach to classroom observation, the Oregon Leadership Institute (2005) offers an evaluation process that can be a viable path for
assessing teachers according to the *Standards*. First, the OMLI process is *specific* about what is being observed, especially outlining the desire outcomes for discourse in math classes. Second, an interview with the teacher precedes classroom observation. Lastly, a post-observation discussion happens. I suggest, through implications in this dissertation, that being more specific about what the *Standards* look like in real classrooms and how researchers define alignment would allow teachers to conform to reform-oriented ideas. In essence, if teachers know how they will be assessed and what assessment looks like in real classrooms, they could state in pre-observation interviews or questionnaires if their lesson include reform-oriented ideas or not and why.

In this way, assessment becomes a reliable and powerful tool for the researcher and the teacher. Educators, researchers, and other stakeholders would have a consensus from which to build and design future studies. For teachers, specifically, reform ideas would be clear, understandable, and, in many instances, viewable. Teachers could decide what practices could be readily implemented in their classrooms, and what actions need a gradual process. Researchers could be more confident about defining and observing certain behaviors, knowing that a) teachers either agree or disagree with the actualization of a reform-oriented suggestion, b) researchers are not left to define practices in various ways, and c) possibly, there exist a library detailing the range of practices that define any given reform idea. As it stands, researchers and educators could disagree about the look, the impact, and the range of reform-oriented practices in the classroom.

**Implications.** What’s possible? Teachers can learn much from video, especially when the classrooms are real and similar to their own. Viewing their own classrooms, teachers’ depth of analysis is greater, knowing the motivations behind their own actions and the history of the
students’ abilities. Although teacher preparation and improvement happens on many levels, like increasing a teachers’ content knowledge, teachers’ critical reviews of themselves remain a viable avenue toward teacher change. Equally important, teachers can learn from cross-cultural studies of teachers facing similar content. In the end, research has continually suggested that an integral part of teacher preparation programs should be a systematic analysis of teaching (Hiebert, Morris, Berk & Jansen, 2007; Schaefer, 1967; Sherin, 2002).

Additionally, using video for examining professional development has been accepted over the years. Video gives many teachers insight in a convenient, non-embarrassing way. Many teachers can view themselves, and improve upon their performance as the year progress. Actions that seemed coherent and cogent during the class may, after viewing, appear less so (Jacobs & Morita, 2002; Sherin, 2004). Earlier in this chapter, I illustrate how some teachers’ use of everyday references may feel like applying math to real-life situations, when, in that it is merely a reference. Additionally, teachers can view teachers from other countries – avoiding criticism of or comment on a peer. More importantly, teachers from different cultures may approach math in ways that could awaken possibilities and ideas not yet implemented in U.S. classrooms.

Particularly, the coding scheme developed for this dissertation allows teachers to think of reform ideas in very concrete ways. Understanding that practices can range from one implementation to another, teachers will have the option of determining the dynamic of their class and the ability of their students and choose an approach that is comfortable for them or their environment. Additionally, having vignettes from real classrooms, where actual confusion and limited understanding on the part of the student occurs and where a teacher struggles with
decisions about challenging, leading, and scaffolding occur, provide teachers in training or teachers seeking to improve examples that are believable.

More importantly, and particular to this investigation, teachers can view opportunities where the suggestion of the *Standards* can be realized. In many of these examples, teachers began to employ some of these behaviors, but fell short of actualizing the benefits of certain teacher practices. For example, to make a personal connection to the material, some teachers mentioned an everyday event, but did not connect that event to a mathematical concept. The students did not have to apply the concept to the real life issue, which is suggested by the *Principle and Standards*. Teachers, viewing such shortfalls, can become more aware of their own behaviors, or, they will plan lessons to involve real life application, so that this possible mistake would happen.

From this coding scheme, teachers can plan several lessons, insuring that students routinely encounter a variety of questions, ranging from procedural to challenging. Additionally, teachers can either anticipate the result of a teacher to student or student to student exchanges or become more proficient in recognizing such. Thus, teachers will plan what students will encounter and be aware of what answers, explanations, justifications, and types of talk, in general should fill the classroom.

Teachers have opportunities to realize reform ideas in the classroom despite the level of mathematical sophistication on the part of the students. Particularly, discourse is the one aspect of teaching that is the direct control of teacher. Textbooks, student skills, policies, and resources are given to many teachers, without their input. When teachers are given the opportunity to change these more external aspects, the change is slow, occurring in the next academic year or longer. What teachers can affect immediately is the delivery of the content to students. We have
long established that some teachers enact a different curriculum than what was intended. The most accessible aspect of the classroom is discourse. What the teacher says and how he or she says it, the types of questions used, and the kinds of explanations expected are changes that can take place any day.

**Future studies.** Future studies should, overall, concentrate on studies that expand our knowledge about possible culture differences. Although China and the United States present a contrast of approaches to mathematics in the elementary school grades, other countries, like Finland, Italy, and Japan, have structured classrooms and deliver content in ways slightly different than either of the two countries studied in this dissertation. From these studies we could refine the ways in which we understand and enact suggestions from the NCTM and the Common Core Standards.

Additionally, future studies should examine the approaches that high achieving states and districts have; those practices that depend more on the interaction between teacher and pupil than it does on the economic and material resources of a particular school.

Lastly, future studies may be designed to establish and revise professional development programs. This dissertation and other research highlight how video of classrooms practices can be used to raise questions about one’s own teaching and offer suggestions about improvements in the classrooms. Video of classrooms provide a record of teaching that can be used repeatedly – analyzed, praised, and critiqued often, allowing the viewer to capture nuances and themes in classrooms and lessons.

**Changing culture.** We cannot change American culture as mathematics researchers. Changing how children view education in general, on a national level, would be a monumental task. If the Chinese culture is the root cause for the difference we see between these two groups,
then all comparisons between these two countries do not add to our understanding of how teachers’ change in discursive practices may affect student learning outcomes.

However, how students view mathematics is malleable in every math class they take. Classrooms are cultures, and mathematics instruction is affected by the beliefs, histories, curricula, and trends of a particular culture. Each U.S. teacher, in the course of an academic year, can establish norms and expectations in his or her classrooms. Within this micro-culture, students learn to think about mathematics and how to think mathematically. We desire for teachers to become student-centered, having students participate in procedurally and conceptually rich discussions. They learn what it means to understand and do mathematics.

Stigler & Hiebert (2004) state that “teaching is cultural… We must find a way to improve the standard operating procedures in U.S. mathematics classrooms—to make incremental and continuous improvements in the quality of the instruction that most students experience. A focus on teaching must avoid the temptation to consider only the superficial aspects of teaching: the organization, tools, curriculum content, and textbooks. The cultural activity of teaching—the ways in which the teacher and students interact about the subject—can be more powerful than the curriculum materials that teachers use” (p.16). This interaction can propel mathematics classrooms toward reform-oriented lessons – the uncomfortable move to a class where a teacher shares more authority with students, has students talk more, devoting as much time to mistakes as they do right answers, ensuring that students become procedurally strong and conceptually rich. Changing the culture of a mathematics classroom through discourse is possible, desirable, and most beneficial.
References


problems, potential solutions. *Child Development Perspectives, 3*, 160-164.


students use their heads? Principles of high-quality Japanese mathematics instruction.


*Educational Psychologist, 27*, 243-261.


*Tutorials in Quantitative Methods for Psychology, 7*(1), 5-14.


Exploring the link between reformed teaching practices and pupil learning in elementary 


Kvale, 1996. InterViews: An introduction to qualitative research interviewing. 


sociable mathematical discourse in school. In D. Olson & N. Torrance (Eds.), Handbook of 


Teasley (Eds.) Perspectives on Socially Shared Cognition (pp.63-82). Washington, DC: 
American Psychological Association.


*Educational Studies in Mathematics, 72,* 161-183.


