DYNAMIC MODELING, SIMULATION, AND CONTROL OF
TRANSPORTATION HVAC SYSTEMS

BY
BIN LI

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Abstract

Air Conditioning and Refrigeration (AC&R) systems are ubiquitous in modern society since they perform the key engineering function of transporting thermal energy from one physical location to another. In doing so they are able to change the condition of a defined spatial environment to prescribe a particular temperature and humidity. Increased economic and environmental concerns have placed greater emphasis on the energy efficiency of these AC&R systems. These concerns necessitate better component and system design as well as better operation of existing systems using advanced control techniques. The use of advanced control techniques, particularly for transient system operation, is the focus of this dissertation.

Two significant challenges always exist in transient control of the AC&R systems: (i) developing control-oriented models that can capture the complex nonlinear thermodynamic behavior while balancing model simplicity with accuracy; and (ii) implementing control strategies that can achieve high performance and efficiency over a wide range of operating conditions. This dissertation makes contributions to these two fronts and is divided into two distinct parts. The first part of this dissertation introduces the development of a first-principles switched modeling framework, and presents simulation and experimental validation results in various AC&R system applications. These results show the validity of the modeling approaches to describe system transients under mode switching operations, such as cooling/heating mode switching and on/off cycling operation. An optimal operating strategy with on/off mode switching is illustrated as an example to demonstrate the effectiveness of the presented modeling tools in control design. To achieve high system performance under refrigerant phase transition conditions, a switching control strategy based on local models and local controller is introduced. This comprises the second part of this dissertation, where a first-principles invariant-order switched system with different operating models is formulated. Tools for designing controllers...
and analyzing the stability of the closed-loop switched system are presented. Simulation results demonstrate improved performance and efficiency with the presented control strategy compared to conventional control approaches in handling the nonlinear refrigerant phase transitions over a wide range of operating envelope.
To my family
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Nomenclature

Symbols

\[ A \] area \([m^2]\)
\[ c \] specific heat \([kJ/(kg\cdot K)]\)
\[ C \] flow coefficient \([\text{dimensionless}]\)
\[ f \] forcing function
\[ h \] refrigerant enthalpy \([kJ/kg]\)
\[ K \] gain in \textit{pseudo-state} equations; set to five \([s^{-1}]\)
\[ I \] solar radiation intensity \([kW]\)
\[ m \] mass flow rate \([kg/s]\)
\[ MC \] thermal capacitance \([kJ/K]\)
\[ NTU \] number of transfer units \([\text{dimensionless}]\)
\[ P \] refrigerant pressure \([kPa]\)
\[ \dot{Q} \] heat transfer rate \([kW]\)
\[ T \] temperature \([\degree C]\)
\[ u \] input
\[ V \] volume \([m^3]\)
\[ x \] state vector
\[ \dot{x} \] time derivative of state vector
\[ \tilde{x} \] refrigerant vapor quality \([\text{dimensionless}]\)
\[ Z \] coefficient matrix
\[ \alpha \] average refrigerant heat transfer coefficient \([kW/(m^2\cdot K)]\)
\( \bar{\gamma} \) mean void fraction [dimensionless]

\( \rho \) refrigerant density [kg/m³]

\( \eta \) efficiency [dimensionless]

\( \zeta \) fraction of heat exchanger length covered by zone, called normalized zone length [dimensionless]

**Subscripts**

- \( ac \) accumulator
- \( amb \) ambient
- \( c \) condenser
- \( c1,c2,c3 \) superheated, two-phase, sub-cooled zone in condenser
- \( dpr \) discharge pressure regulator valve
- \( e \) evaporator
- \( e0,e1,e2 \) superheated, two-phase, superheated zone in evaporator
- \( e01 \) boundary between superheated and two-phase zone in evaporator
- \( e12 \) boundary between two-phase and superheated zone in evaporator
- \( etot \) evaporation from given inlet two-phase refrigerant to saturated vapor
- \( f \) saturated liquid
- \( g \) saturated vapor
- \( hgl \) hot gas line
- \( i \) inlet
- \( j \) zone number; for evaporator, \( j \in \{0,1,2\} \) (0=superheated, 1=two-phase, 2=superheated)
- \( k \) compressor
- \( o \) outlet
- \( slhx \) suction line heat exchanger
- \( v \) expansion valve
- \( w \) heat exchanger structure (wall)
Chapter 1 Introduction

Air Conditioning and Refrigeration (AC&R) systems are ubiquitous in modern society since they perform the key engineering function of transporting thermal energy from one physical location to another. In doing so they are able to change the condition of a defined spatial environment to prescribe a particular temperature and humidity. While this may seem a simple enough task, it has had tremendous impact on the way we live and work [1]. Three examples, listed below, give more detail as to the impact level of these systems on our society: buildings, supermarket refrigeration and refrigerated transport.

Buildings are the dominant mode of energy usage today, which comprises over a third of the total energy use in the U.S. [2]. The heating, ventilation, and air-conditioning (HVAC) systems within these buildings are one of the major sources of the energy consumed [3], and AC&R systems remain the largest contributors to peak electrical demand [4]. Proper operation of these systems is essential to minimizing energy usage while maintaining occupant comfort. Occupant comfort is particularly important since people spend more than 90% of their lives inside of buildings whose interiors are conditioned by these systems and greater comfort leads to greater productivity [1].

Supermarkets are one of the most energy-intensive types of commercial buildings [5], and significant energy is used to maintain chilled and frozen food in both refrigerators and product display cases. According to the field monitored data released by Southern California Edison Company, refrigeration is the largest component of supermarket energy use, which accounts for 53% of the total energy consumption.

As an essential sector in the food supply cold chain, more and more attention is paid to food transport refrigeration because of increasing concerns on food safety and quality, as well as its impact on energy consumption and the environment, such as greenhouse gas CO₂ emissions
As shown in [9], food transport in the United Kingdom is responsible for 18,444 kton of CO$_2$ emissions of which 45% is attributed to heavy goods vehicles. Approximately a third of food transport in these vehicles is refrigerated, and the emissions from separate engine-driven refrigeration units and refrigerant leakage are not included. Not much data is available for energy consumption of transport refrigeration equipment during operation in the field, since the field fuel consumption depends on many factors; such as the types of vehicles in transport, types of operational mode (continuous modulation, on/off, pulldown and defrost), types of customer applications (urban distribution and long distance transport), types of transported products, climatic conditions, and fuel density [6, 10]. Repice and Stumpf [10] conducted a fuel consumption study for the long distance transport application, and the annual fuel use breakdown in terms of the types of operational mode is illustrated here in Figure 1.1. It is shown that the start/stop (on/off) frozen operation accounts for more than half of the annual fuel use, followed in importance by continuous perishable and pulldown mode.

**Figure 1.1 Annual fuel consumption for long distance transport** [10]

Increased economic and environmental concerns have placed greater emphasis on the efficiency of these AC&R systems. These concerns necessitate better component and system design as well as better operation of existing systems using advanced control techniques [11, 12]. As discussed in [11], two significant challenges always exist in transient control of AC&R systems: (i) development of control-oriented models that can balance simplicity with accuracy, and also capture the complex thermodynamic behavior; and (ii) design of control strategies that can achieve high performance over a wide range of operating conditions.
This dissertation makes contributions to these two fronts and is divided into two distinct parts. The first part of this dissertation introduces the development of a first-principles modeling framework, and presents simulation and experimental validation results in various AC&R system applications. The validation results demonstrate the capabilities of the modeling approaches to describe the transient behavior. An optimal operating strategy with operation mode switching is illustrated as an example to demonstrate the effectiveness of the modeling tools in control design. To achieve high system performance over a wide operating range, a switching control strategy is introduced. This comprises the second part of this dissertation which presents: a switched AC&R system framework with different operating models, tools for designing controllers, and analysis of the stability of the switched system. Simulation results demonstrate that the control strategy based on the switched system framework offers significant advantages compared to conventional control approaches over a wider operating envelope.

In summary, this dissertation addresses the challenging problem of transient modeling and control in the field of AC&R energy systems by presenting a validated modeling framework to describe the severe transients with system operating mode switching, and developing improved control techniques in performance and efficiency. The remainder of this chapter is organized as follows. Section 1 gives a background introduction of a vapor compression cycle (VCC), which is a representative operating mechanism in the AC&R systems. The control-oriented modeling framework is summarized in Section 2, where the modeling challenges for transient simulation are discussed. Section 3 overviews the control strategies for VCC systems, and discusses the formulation of the switched system and control problem. Finally, Section 4 presents an outline of the dissertation.

1.1 Vapor Compression Cycle Systems

A basic VCC system, as depicted in Figure 1.2(a), is a thermo-fluid system driven by the phase characteristics of the refrigerant that is flowing through it. It has four primary components: an evaporator, a compressor, a condenser and an expansion device. The refrigerant is used as the medium to absorb and remove heat from the space to be cooled and subsequently reject that heat elsewhere. The circulating refrigerant enters the compressor as a saturated or superheated vapor and is compressed to a higher pressure and higher temperature. The hot, compressed vapor is
routed through a condenser where it is cooled and condensed into a liquid by rejecting heat from the VCC system. The condensed liquid refrigerant is next routed through an expansion device where it undergoes an abrupt reduction in pressure and transitions to a liquid and vapor refrigerant mixture. The temperature of the mixture is colder than the temperature of the enclosed space to be refrigerated. The cold mixture is then routed through an evaporator where the refrigerant absorbs heat to lower the temperature of the enclosed space to the desired temperature. To complete the vapor compression cycle, the refrigerant vapor from the evaporator is routed back into the compressor. An ideal VCC system assumes an isentropic compression of the refrigerant across the compressor, an isenthalpic condition through the expansion device, and an isobaric process in the condenser and evaporator, as presented in the pressure-enthalpy (P-h) diagram in Figure 1.2(b). Note that the VCC system example is an air-cooled system, and in different applications, the secondary fluid could be water and controlled with water flow valves instead of the heat exchanger fans in Figure 1.2.

Figure 1.2  Ideal subcritical vapor compression cycle [11]: (a) system diagram (b) P-h diagram

There are also many optional but common components added into the VCC system, such as a liquid-line receiver tank, a suction-line accumulator, and a suction line heat exchanger. The
receiver tank is normally located at the condenser outlet and used to store excess refrigerant charge. The accumulator is connected with the evaporator outlet to ensure that only refrigerant vapor enters the compressor. Through the energy exchange between the cold refrigerant vapor leaving the evaporator and the warm refrigerant liquid exiting the condenser inside the suction line heat exchanger, the system performance can be improved. A more complex VCC example in the transport refrigeration systems is presented in Chapter 2. Although there are a number of system configurations, the basic operation principle is similar, and behaves as the four-component system does, shown in Figure 1.2.

1.2 Control-Oriented System Modeling Framework

For the sake of discussion, a specific system example, refrigerated transport, is considered here. An important characteristic of these refrigeration systems is temperature regulation so that the quality of perishable foods is preserved and the shelf life is extended during transport [6, 8, 13]. To satisfy customer needs for shipping a wide range of cargo under tight temperature control, the transport refrigeration industry has responded by improving temperature control techniques, providing greater cooling capacity and offering load flexibility, for instance, from single cargo space to multi-space systems as noted in [14]. Compared with stationary systems (i.e., building HVAC and supermarket refrigeration systems), transport refrigeration systems are required to perform reliably over a wider variety of operating conditions; these include broad temperature ranges of transported food products, wide variations in climatic conditions, and variations of operation mode (see Figure 1.1). Additionally, the refrigeration systems need to be designed to be energy efficient without compromising the temperature control of the products. To investigate and improve the refrigeration system performance, good knowledge of the system behavior is required and can be obtained either from modeling and simulation tools or through experimental studies. As presented in [15], the use of well verified numerical models can facilitate the understanding of system dynamic behavior, serve as a tool to evaluate alternative system designs and operating control strategies, and minimize the time and expense of test-cell experiments.

VCC modeling for the AC&R system applications has been an ongoing research in the Alleyne Research Group since 2000. Industrial interests in the modeling efforts have led to the
creation of the Thermosy toolbox in Matlab/Simulink for real-time simulation, control design and evaluation, and fault detection [11]. The mass flow devices, the expansion valve and compressor, are modeled using semi-empirical relationships, while the low-order dynamic heat exchanger models are derived with a moving-boundary lumped-parameter approach. The control-oriented system models are developed in Thermosys and validated using data obtained from the experimental systems at the Air Conditioning and Refrigeration Center (ACRC) at the University of Illinois at Urbana-Champaign. These nonlinear models are linearized around nominal operating conditions in Thermosys to enable the linear control implementation. Many different control techniques have been investigated to meet multiple performance objectives [11, 16, 17].

A key contribution of this research is the presentation of a first-principles switched modeling framework, which allows for simulating system transients under mode switch operations, such as continuous modulation with cooling/heating mode switch and on/off cycling operation. Figure 1.3 gives an example of the severe transients during the system off operation, which involves the highly nonlinear refrigerant phase transitions with the switching of thermodynamic states [18]. The switched modeling approaches to address this challenge are described in Chapter 2. The resulting models are developed in Thermosys, and model simulation and validation results are presented in Chapter 3 and 4.

![Condenser and Evaporator Diagram]

**Figure 1.3** One example of the refrigerant phase transitions inside the heat exchangers during the off operation [18]
1.3 Control of Vapor Compression Cycle Systems

The control objectives for VCC systems are typically defined as capacity control while maximizing the system efficiency. As discussed earlier, the amount of energy consumed by the VCC systems varies considerably and depends on the desired temperature of the enclosed conditioned space, the ambient conditions, the level of internal heat generation within the space, and so on. Therefore, these systems must be designed to meet varying cooling loads. A summary of various capacity control strategies, such as the simple on/off, compressor cylinder unloading, and variable speed compressor control, was conducted by Qureshi and Tassou [19] who found the variable speed compressor control provided the greatest flexibility to match varying loads, resulting in the best overall system efficiency (20% to 40% reductions in seasonal power consumption), albeit with a significant increase in cost.

To maximize the system efficiency for a given power input, the system should operate with the best heat transfer characteristics in the heat exchangers. Inside the evaporator, the liquid-vapor refrigerant mixture has a significantly greater heat capacity than the refrigerant solely in vapor form, and the two-phase portion of the evaporator provides virtually all of the system cooling capacity. Thus, the best energy transfer is obtained by maximizing the length of the two-phase zone within the evaporator. However, to prevent liquid refrigerant from entering the compressor to cause physical component damage, the VCC systems are normally designed to operate with a certain degree of superheat, defined as the temperature the exiting refrigerant is above the saturation temperature at the evaporator outlet. In the system configurations with an accumulator at the evaporator exit, the fluid entering the compressor is saturated vapor, which can ensure safe operation of the compressor while maximizing the evaporator’s performance. For the systems without an accumulator, in industrial practice, a generally accepted compromise is to regulate the outlet of the evaporator to the minimum value that retains vapor at the evaporator outlet throughout any anticipated system transients. This can also be termed as superheat regulation problem resulting in a tradeoff between efficiency and safety [20, 21].

The combination of the desire to match varying cooling loads with the evaporator superheat consideration requires that the control system should be designed to simultaneously meet multiple control objectives. Intuitively, utilizing all the actuation devices can fully exploit control authorities so as to maximize efficiency and performance; these control inputs include
the variable speed compressor, the variable orifice electronic expansion valve, and the variable speed heat exchanger fans in Figure 1.2. However, these all increase the cost and complexity of both the physical system and the control system. In the literature, only selected actuators were applied for control design while keeping others at constant actuation levels. In particular, the compressor speed $u_1$ and the expansion valve opening $u_2$ (see Figure 1.2) are the most frequently investigated as control inputs [11, 12, 17, 22-25]. There are limited studies exploring the modulation of the heat exchanger fan speeds during the control process [26]. The expansion valve opening and the evaporator fan speed were applied as the control inputs for superheat regulation in an automotive air conditioning example [16], while the compressor speed and the condenser fan speed were simulated as disturbances. In [27], the influence of the heat exchanger fan speeds on the system efficiency and performance was investigated. Moreover, additional valve control was introduced in [28] to use refrigerant charge as an input variable to control the condenser subcool.

One commonly used approach to operate the VCC system with variable inputs is to utilize single-input single-output (SISO) control (conventional PID loops). Typical input-output control loop pairings include expansion valve control of evaporator superheat and compressor control of cooling load. Since the cooling load is often difficult to measure directly, it can be estimated based on evaporator pressure or evaporator air inlet temperature. The drawback of applying multiple SISO control techniques has been well identified in [16, 17, 29], and the system performance is limited due to the inherent cross-coupling of the system dynamics. Different tools with the conventional control approach, such as decoupling techniques [17, 30, 31], were presented to demonstrate better performance over simple PID feedback loops.

Another popular control structure to compensate for the system coupling is the utilization of model-based multivariable control strategies. The multi-input multi-output (MIMO) control approach can be designed to coordinate multiple outputs and actuators, and meet multiple performance objectives. Additionally, it becomes more important for complicated systems where there may be multiple sets of heat exchangers, valves, and compressor racks that may be found in large distributed systems such as supermarket refrigeration systems [5]. The first model-based MIMO control approach to regulate evaporator pressure and superheat was proposed by He et al. [22], where a reduced-order linear model, developed from a first-principles nonlinear model, was
presented, and a Linear Quadratic Gaussian (LQG) approach was described to coordinate the inputs of the compressor speed and valve opening. The experimental control results demonstrated improved performance over the SISO control in terms of response speed and disturbance rejection. Since this initial effort, there have been many following studies related to the advanced control of VCC systems. A LQG controller based on a first-principles linear simulation model was developed in [24]. Qi and Deng [23, 32] introduced a multivariable control-oriented direct expansion air conditioning system model and applied the LQG MIMO technique to control the air temperature and humidity in an enclosed space served by the air conditioning system. A MIMO $H_\infty$ robust controller [33] was designed and implemented in an experimental HVAC system to demonstrate significantly superior performance over the conventional PI-based controllers. The utilization of Model Predictive Control (MPC) has emerged as a means to satisfy input and state constraints on the system while maintaining desired output tracking performance. The MPC framework is becoming increasingly prevalent due to its ability to handle the multivariable nature of the dynamics, constraints and optimality in an integrated fashion [12, 34]. In [26], a nonlinear predictive control algorithm based on a reduced-order first-principles nonlinear model was designed and tested on a water-to-water refrigeration system with the observation of dramatic performance improvement. Also, applications of various model-based control strategies have expanded to systems with multiple sets of heat exchangers, such as multi-evaporator compression cooling cycles [35-37].

1.3.1 Problem Formulation of Switched System and Control

The highly nonlinear behavior poses a challenging problem in the field of VCC system control, since the physical system exhibits dynamics that vary appreciably over the typical operating regime. In the model-based multivariable control framework reviewed above, most of the models applied for control design were either from system identification or linearization around nominal operating conditions from the nonlinear system models. One example in [38] showed that a linear controller failed to achieve acceptable performance for operating conditions more than 30% away from the nominal operating point. One of the popular solutions to this nonlinear control design problem is the use of gain scheduling [21, 22, 39], in which a nonlinear controller is constructed by interpolating a family of local controllers. In [39], an Youla
parameterization theoretical framework was presented for the generation of a local model and local controller network (LMN/LCN) and the effectiveness of the MIMO gain scheduling control strategies on performance regulation between design and off-design conditions were demonstrated.

![Diagram of VCC system operation with varying phase transitions]

**Figure 1.4  VCC system operation with varying phase transitions**

Although the system nonlinearity is appreciated for the VCC system control design in the literature, a particular heat exchanger formulation with a fixed number of fluid zones is normally assumed. There are few studies on the dynamic analysis and control implementation for the VCC system with varying phase transitions, as shown in Figure 1.4. The dynamic VCC system is switching between condenser subcooled and no condenser subcooled conditions, or between evaporator superheat and no evaporator superheat conditions. These types of refrigerant transitions cause nonlinear system behavior as discussed in [40, 41], and can occur due to varying system boundary conditions, such as the ambient conditions and the cooling loads.

This dissertation presents a first-principles dynamic switched system model with invariant order, which consists of a finite number of subsystems and logic rules that orchestrate the subsystem switching. A model-based MIMO control framework using Linear Matrix Inequalities (LMI) is described that is capable of handling the refrigerant transitions inside the fluid zones while achieving desirable system performance over a wide range of operating
conditions. This system model and control methodology is shown to guarantee stability and applicable to a broader class of multi-model switched systems.

1.4 Organization of Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 presents a control-oriented first-principles modeling framework that captures the VCC system transient behavior under mode switch operations, such as cooling/heating mode switch and on/off cycling. A refrigerated transport system example is introduced, and the modeling procedure of each of the components is presented. Chapter 3 describes the simulation environment for model simulation, validation, and control design. Experimental facilities representing different AC&R system applications are illustrated, along with the presentations of model validation results which demonstrate the modeling capabilities in predicting system dynamics under mode switch operations. Since refrigerant mass migration and redistribution are regarded as key factors affecting the cycling performance in AC&R system applications, an automotive air conditioning system is introduced in Chapter 4 to show the capabilities of the modeling approach in predicting the refrigerant mass migration in severe transients. Chapter 5 utilizes the validated system model and presents a hysteretic on-off control scheme with optimization algorithms for temperature regulation in the refrigerated transport application. Chapter 6 presents a first-principles nonlinear switched system framework with different subsystem models. A model-based MIMO switching control strategy is introduced to achieve desirable performance over a wide range of operating regime, followed by the stability analysis of the controlled VCC system with refrigerant phase transitions. This dissertation concludes in Chapter 7 with a summary of the results and recommendation for future research.
Chapter 2  Dynamic Modeling

This dissertation seeks to develop a control-oriented modeling framework employed to capture the dynamic characteristics of vapor compression cycles under system mode switch cycling operations. The modeling approach should be sufficiently accurate to describe the essential dynamic behavior, while remaining simple enough to provide insight into the dynamics. At the same time, the model should be compact enough to be useful as a control design tool. First, this chapter introduces common system mode switch operations in the target application of refrigerated transport systems. After presenting this introductory example, a switched modeling approach is examined to capture system transients under mode switch cycling operations, followed by the presentation of compressor, expansion devices, and other system component modeling. The validity of the presented modeling approach, as well as its generality in different system applications, is demonstrated in Chapter 3 and Chapter 4. Although the complete derivation for the heat exchangers is omitted in this chapter, sufficient detail is given for the reader to replicate the modeling process, and the interested reader is encouraged to refer to [18, 42-44] for more descriptions on equation derivations.

2.1 Refrigerated Transport System Example

As indicated in Figure 2.1, a typical refrigerated transport system consists of a refrigeration unit and a refrigerated cargo space. The refrigeration unit is interacting with the cargo space to meet its temperature requirements and satisfy the refrigeration demands over a wide range of operating conditions [6, 13]. The cargo space is coupled to the refrigeration unit such that the cargo space outputs, e.g. return air temperature (normally considered as the cargo space temperature), are the inputs to the refrigeration unit, e.g. evaporator air inlet temperature.
Simultaneously, the refrigeration unit outputs, e.g. evaporator air outlet temperature, are acting as the cargo space inputs, e.g. supply air temperature. There are two common methods of operation for temperature regulation in the refrigerated transport industry. One is compressor on/off cycling operation and the other is continuous operation by switching between system cooling and heating modes [10, 13, 45, 46]. The system operation mode (cooling, heating, or off) is driven by the difference between the measured cargo space temperature and the temperature set-point (see Figure 2.2). Other system actuators, such as condenser and evaporator fans, operate on the same schedule as the compressor for most transport systems.

Figure 2.1 A typical single cargo space refrigerated transport system
In contrast to cycling the refrigeration unit on and off for temperature regulation, the cooling/heating mode switch operation maintains the product temperature at a set-point below the ambient conditions by continuously running the system and cycling between cooling and heating modes. One reason to drive the refrigeration equipment to switch from cooling to heating mode is the need for defrosting the heat exchangers [47, 48]. Another reason is to maintain a continuous supply of air moving over the transported food product which is a requirement for fresh produce (i.e. strawberries). The cooling/heating mode switch allows for temperature regulation while maintaining continuous air circulation. This mode switch cycling operation is given particular attention in the following section due to the component function variations during transients.

2.1.1 System Cooling/Heating Mode Switch Operation

As a transport system example, the dual-mode refrigeration unit studied in this dissertation is a commercially available TS-500 transport refrigeration unit manufactured by Thermo King Corporation. The refrigeration system is charged with refrigerant R404A. The primary mode of operation is the cooling cycle in which the unit extracts heat from the refrigerated cargo space and transfers it to the external ambient environment. In the second mode of operation, the heating cycle, the refrigeration unit delivers heat to the cargo space. Figure 2.3 shows a schematic of the system configuration where components are interconnected to form a vapor compression cycle (VCC) refrigeration system. The switch from cooling mode to heating mode is completed using a 3-way valve that directs the path of the refrigerant exiting the compressor at (1’) towards the evaporator through the discharge pressure regulator valve (2’) rather than to the condenser coil, as can be seen in Figure 2.3. In heating mode operation, the
refrigerant between the condenser inlet and the thermostatic expansion valve outlet (4) is trapped if the valve bleed port effect is not considered. The refrigerant flows through the hot gas line (3’) to the evaporator coil. The evaporator now functions as a condenser, taking superheated vapor in and condensing it into two-phase fluid while heating the cargo space. After separation in the accumulator (4’), the saturated refrigerant vapor passes through the throttle valve (5’) and finally returns to the compressor. Once the measured cargo space temperature exceeds the upper limit of the cargo space temperature set-point, the unit switches from heating to cooling mode operation where the superheated refrigerant vapor exiting the compressor at (1) is redirected by the 3-way valve to the condenser coil. A representative pressure-enthalpy (P-h) diagram for both cooling and heating modes is plotted in Figure 2.4, where refrigerant pressures along the heat exchanger coils are assumed to be uniform.

Figure 2.3  Schematic of the refrigeration unit in operation
2.2 Modeling Challenges

As illustrated above, the cooling/heating mode switch cycling operation is a complex and highly transient process involving component function variations as well as many indeterminate variables [49]. The compressor cycling with on and off operation produces very large transients which result in the destruction and creation of dynamic states, e.g. a loss of sub-cooled liquid in the condenser, the disappearance of the superheated vapor in the evaporator [50, 51]. Moreover, refrigerant migration and redistribution among the system components are always observed during the mode switch operations [52].

VCC system dynamic modeling is a challenging task in which the balance between complexity and accuracy must be considered. To investigate the dynamic behavior of VCC systems, heat exchangers are usually treated with transient models. Two common heat exchanger modeling approaches, finite-volume distributed-parameter and moving-boundary lumped-parameter methods, have been reported in the literature [29, 53-58]. Finite-volume approaches decompose the heat exchanger geometry to a finite set of small regions, allowing spatial effects to be captured by the model. Figure 2.5 illustrates the concept of discretizing the heat exchanger into many finite volume regions where the accuracy of the system model improves with an
increased number of regions. The complexity of the models is primarily used to capture the spatially varying fluid flow and heat transfer phenomena that occur in compact heat exchangers.

Many attempts have been made to model the system thermal dynamics (i.e., pressure, temperature, and refrigerant mass flow rate) with compressor on and off cycling operations using finite-volume approaches [15, 56, 59-63]. For example, a validated system model of a centrifugal chiller system using the finite-volume formulation was reported to predict transient performance including system starts-up in [56], while Kapadia et al. [61] applied the finite-volume approach to analyze the start-up performance of a split air-conditioning system. Additionally, from the open literature in heat pump applications, there have been extensive experimental investigations of system performance during the switch between the normal operating mode and defrosting mode. These include reverse-cycle defrosting processes [64-66] and hot-gas bypass defrosting methods [67, 68]. However, there are few studies on the development of simulation models to predict the system dynamics under normal/defrosting mode switch cycling operation. Krakow et al. [49] developed an analytical reverse-cycle defrosting model where the melting process on the coil surface was idealized by subdividing it into different stages. Based on the above modeling theory, Liu et al. [69] presented a validated reverse-cycle defrosting model for an air-source heat pump system where finite-volume distributed-parameter models for the condenser and evaporator during the defrosting cycle were developed.

**Figure 2.5  Schematic of the finite-volume modeling approach**

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Lumped-parameter methods have also been applied in various air conditioning and refrigeration applications to simulate the system transients [50, 70-74]. A system representation with three lumped-parameter condenser models (a superheated model, a two-phase model and a sub-cooled model) and one lumped-parameter evaporator model was introduced by Dhar and Soedel [50] to represent the start-up transients. Kumar et al. [73] chose to simulate an air-conditioning system with the compressor on and off cycling operation by using the moving-boundary lumped-parameter evaporator and condenser model representations in the system on and off steady-state conditions. Bendapudi et al. [75] presented a comparative study of centrifugal chiller system behaviors for start-up and load-change transients with flooded shell-and-tube heat exchanger models. In [75] both finite-volume and moving-boundary methods were quantitatively compared and the tradeoffs between the two formulations were carefully identified. While the moving-boundary methods were found to be computationally faster, they were not as computationally robust or accurate as the finite-volume approaches. Considering the control applications of vapor compression cycle systems with mode switch operations, the moving-boundary approach is selected to model heat exchanger dynamics due to its faster speed in real-time computer simulation and its compact nature (i.e., fewer states). Furthermore, moving-boundary models provide industrial practitioners and control engineers with physical insights [11] into the system’s dynamic behaviors that are useful for control design and embedded system uses.

2.2.1 Moving-boundary Lumped-parameter Formulation

In the moving-boundary lumped-parameter modeling approach [11, 42], as shown schematically in Figure 2.6 ((a) for condenser, and (b) for evaporator), heat exchangers are divided into control volumes or, zones based on the fluid phase, and the effective model parameters are lumped in each zone, resulting in a model of fairly low dynamic order. The location of the boundary between fluid phase regions is allowed to be a dynamic variable and vary throughout the length of the heat exchanger.
Recalling the cooling/heating mode switch operation in Figure 2.3, the component functions vary during transients. Specifically, when the system switches from cooling to heating mode operation, the evaporator acts like a condenser. Superheated vapor enters the evaporator coil and exits as two-phase fluid. When the system switches back to cooling mode operation, the evaporator starts to extract heat again from the cargo space. As described in [50], the compressor on/off cycling operation produces large transients which could result in the destruction and

Figure 2.6   Diagram of the moving-boundary heat exchangers: (a) condenser with three fluid zones; (b) evaporator with two fluid zones

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19
creation of dynamic states. Therefore, the fixed number of zones in the moving-boundary heat exchanger model representations severely limits their operating ranges, and the main challenge within this modeling framework becomes how to simulate the transitions of dynamic states under system mode switch cycling operations.

There have been quite a few studies investigating the heat exchanger model development with state transitions [76-81]. Pettit et al. [76] described useful techniques for controlling inactive states during evaporator model state transitions. In [77], the authors developed a generalized moving-boundary evaporator model to investigate its transients under larger disturbances. Two evaporator model structures were presented: a TP-V (two-phase and superheated regions) model, and a TP (two-phase region) model. The incorporation of the weighted wall temperature and time-dependent mean void fraction into the evaporator model was shown to guarantee the robust transition between the two model structures. A dynamic lumped-parameter model was developed in [80] to simulate the refrigeration system transients with variable capacity in both normal and abnormal working conditions, where two evaporator models: superheated evaporator and flooded evaporator were formulated. The evaporator outlet enthalpy was chosen as a dynamic state to determine which evaporator model is used in simulation. Continuously differentiable sigmoid functions were incorporated in the multi-zone condenser and evaporator modeling as presented in [78], and mode changes from one model structure to another could be avoided. McKinley and Alleyne [79] presented a new switching scheme for moving-boundary heat exchanger models to address the problems that the model matrices become singular and simulation fails under varying operating conditions. The condenser model was given as an example of their approach. Two model representations were described: a three-zone (superheated, two-phase and sub-cooled zones) condenser model, and a suitable two-zone condenser representation lacking a sub-cooled zone. Both models were comprised of governing equations for the state derivatives by applying the principles of mass and energy conservation. The pseudo-state variables were introduced to accommodate the transitions of dynamic states, and a novel switching criteria based on mean void fraction was presented. The switched condenser model also demonstrated robustness to varying conditions while enabling real-time simulation. Cecchinato and Mancini [81] presented a moving-boundary evaporator model consisting of two representations: two-zone (two-phase and superheated) and one-zone
(two-phase) models. Choices of state variables for model descriptions were discussed and refrigerant densities were used to provide an intrinsically mass conservative evaporator between these two models’ switching.

To summarize, most studies introduced above were about performance evaluation of heat exchangers with state transitions in the component level, such as the appearance and disappearance of the evaporator superheated zone and the condenser subcooled zone. In [76], the evaporator model was developed with the assumption that the entering refrigerant was two-phase or subcooled fluid, and it may not be the case when system switches to heating mode operation. So far, none of studies in the literature has presented the application of moving-boundary approaches to model and validate the system-level transients under mode switch cycling operations (cooling, heating, and off), which is therefore the focus of this dissertation.

2.3 Switched Modeling Approach

2.3.1 Moving-boundary Heat Exchangers

Using the switched modeling approach initiated by McKinley and Alleyne [79], a switched moving-boundary modeling framework is presented in this chapter to simulate heat exchanger behavior during mode switch operations. In this framework, the heat exchangers are developed with a constant structure that accommodates different model representations. The model’s dynamic state information, including both active and inactive states, is recorded in transients and the transitions between model representations are handled properly with different switching schemes.

To derive governing equations for heat exchangers, several assumptions about the fluid flow are required. These assumptions were commonly used in the past moving-boundary modeling efforts [11, 29, 79] and are as follows:

- The heat exchanger is assumed to be a long horizontal single-pass tube with a mass flow rate reduced by a factor of $1/n$, where $n$ is the number of heat exchanger circuits.
- The refrigerant flow is one-dimensional, compressible, and unsteady.
- The refrigerant pressure along the heat exchanger tube is assumed to be uniform, and the viscous friction is negligible. Therefore, the momentum conservation equation is not needed.
- Conduction along the heat exchanger axis is negligible.
- The two-phase slip flow can be modeled adequately through a void fraction correlation, such as the Zivi correlation [82].
- The air entering the heat exchanger has uniform velocity, temperature, and pressure.
- The air passing over the heat exchanger is assumed to be in dry conditions, and the effects of water vapor condensation on the system performance are not taken into account.

The heat exchanger model structure is illustrated in Figure 2.7. Air mass flow rate, air inlet temperature, refrigerant inlet and outlet mass flow rate, and refrigerant inlet enthalpy are used as the time varying model inputs, which can be provided by other component models, such as the compressor, and expansion devices. The choice of model outputs (i.e., condenser/evaporator pressure, refrigerant outlet temperature, air outlet temperature) will depend on the interfaces with other system component models.

![Figure 2.7 Heat exchanger model structure](image)

In the following sections, switched heat exchanger models (evaporator and condenser) are described in terms of different model representations. The heat exchanger tube wall energy governing equations involving the calculations of the wall-to-air and wall-to-refrigerant heat transfer rates were discussed in [18]. The refrigerant-side governing equations are derived by considering mass and energy conservation in each control volume (i.e., two-phase zone, and superheated zone in the evaporator). Only the final forms of the refrigerant-side governing
equations as well as switching criteria between different model representations are shown. The interested reader is referred to [18, 79] for the equation derivations and solution procedures to solve the ordinary differential equation sets.

### 2.3.1.1 Switched Evaporator

In the switched moving-boundary modeling framework, the evaporator is developed to accommodate three model representations (see Figure 2.8) to capture evaporator transients during mode switch operations. The two-zone (two-phase and superheated) or the one-zone (two-phase) evaporator model in Figure 2.8 can be used to describe the evaporator performance in cooling mode, while another two-zone (superheated and two-phase) model can represent the evaporator coil when the system is running in heating mode.

**Figure 2.8  Switched evaporator model representations**

The dynamic state vector in Equation 2.1 defines the evaporator conditions (i.e., pressure, enthalpy, temperature, and zone locations) at each instant in time. The *pseudo-state* technique [79] is applied to maintain a uniform state vector, independent of model representations. Each model representation is formulated in a nonlinear descriptor form in Equation 2.2 as in [11], with the uniform state vector $x_e$. The coefficient matrix $Z(x_e, u_e)$ contains thermodynamic variables, and $f(x_e, u_e)$ is a forcing function containing mass and energy balance terms. The NTU method is applied to represent the relationship between air inlet and outlet temperature in the evaporator,
as shown in Equations 2.3-2.4 and demonstrated in [79]. The reader is referred to [11] for a more
complete modeling definition of the matrix \( Z(x_e, u_e) \) and the function \( f(x_e, u_e) \).

It is worth mentioning here the choices of dynamic state variables. As discussed in [11],
redundant nature of thermodynamic properties results in freedom in choosing the physical
representation of the dynamic states. Different choices for heat exchanger models can be found
in the literature. In this dissertation, the mean void fraction \( \gamma_e \) is chosen as one of the state
variables to ensure refrigerant mass conservation during switching between different model
representations [79], although the mean void fraction is not an independent variable in certain
model presentations. For example, in the evaporator two-zone (two-phase and superheated)
model, the mean void fraction can be obtained from refrigerant pressure and refrigerant inlet
enthalpy, and hence auxiliary tracking equations are applied to maintain a constant number of
states \( x_e \).

\[
x_e = \begin{bmatrix} \xi_{e0} & \xi_{e1} & P_e & h_{e2} & T_{e0, w} & T_{e1, w} & T_{e2, w} & \gamma_e \end{bmatrix}^T \tag{2.1}
\]

\[
Z(x_e, u_e) \cdot \dot{x}_e = f(x_e, u_e) \tag{2.2}
\]

\[
T_{o, air, j} = T_{w, j} + (T_{i, air} - T_{w, j}) \exp(-NTU) \tag{2.3}
\]

\[
NTU = \frac{\alpha_{air} A_{air} \left( 1 - \frac{A_{fin}}{A_{air}} (1 - \eta_{fin, air}) \right)}{\bar{m}_{air} c_{air}} \tag{2.4}
\]

The refrigerant-side governing equations for the three evaporator model representations
(see Figure 2.8) as well as switching criteria are presented below.

1. **Two-phase and superheated two-zone evaporator model.** In this representation,
the inlet refrigerant to the evaporator is two-phase fluid. Equations 2.5-2.6 describe
the two-phase refrigerant dynamics, and the mass and energy conservation equations
for the superheated zone are given in Equations 2.7-2.8. The auxiliary mean void
fraction tracking, as presented in equation 2.9, is also included in this model.

\[
\frac{d\xi_{e1}}{dt} + \frac{\xi_{e1}}{\rho_{el}} \frac{dP_e}{dt} + \frac{\xi_{e1}}{\rho_{el}} \frac{d\gamma_e}{dt} + \frac{\xi_{e1}}{\rho_{el}} \frac{dV_e}{dt} + \frac{\bar{m}_{el2}}{\rho_{el} V_e} = \frac{\bar{m}_{el}}{\rho_{el} V_e} \tag{2.5}
\]
\[
\left[ \frac{\delta h_{e1}}{\delta P_e} - \frac{1}{\rho_{e1}} \right] \frac{dP_e}{dt} + \frac{\delta h_{e1}}{\delta \gamma_e} \frac{d\gamma_e}{dt} + \frac{h_{e,g} - h_{e1}}{\rho_{e1} V_e \delta h_{e1}} \dot{m}_{e12} = \frac{\dot{Q}_{e1} + \dot{m}_{e1} (h_{e,e} - h_{e1})}{\rho_{e1} V_e \delta h_{e1}} \]  
(2.6)

\[
\frac{d\xi_{e1}}{dt} - \frac{\xi_{e2}}{\rho_{e2} \delta P_e} \frac{dP_e}{dt} - \frac{\xi_{e2}}{\rho_{e2} \delta h_{e2}} \frac{dh_{e2}}{dt} + \frac{\dot{m}_{e12}}{\rho_{e2} V_e \delta h_{e2}} = \dot{m}_{e,e} \]  
(2.7)

\[
- \frac{1}{\rho_{e2}} \frac{dP_e}{dt} + \frac{dh_{e2}}{dt} - \frac{h_{e,g} - h_{e2}}{\rho_{e2} V_e \delta h_{e2}} \dot{m}_{e12} = \frac{\dot{Q}_{e2} - \dot{m}_{e,e} (h_{e,e} - h_{e2})}{\rho_{e2} V_e \delta h_{e2}} \]  
(2.8)

\[
\frac{\delta\overline{\gamma}_{e,tot}}{\delta P_e} \frac{dP_e}{dt} + \frac{\delta\overline{\gamma}_{e,tot}}{\delta h_{e,e}} \frac{dh_{e,e}}{dt} - \frac{d\gamma_e}{dt} = K (\overline{\gamma}_e - \overline{\gamma}_{e,tot}) \]  
(2.9)

Since the superheated zone at the evaporator inlet is made inactive in this model representation, Equation 2.10 is applied, and the pseudo-state equation 2.11 is used to govern the evaporator tube wall behavior for the inactive superheated zone. The dynamic wall temperature in the superheated zone tracks the wall temperature of the active two-phase zone.

\[
\frac{d\xi_{e,0}}{dt} = 0 \]  
(2.10)

\[
\frac{dT_{e,0,w}}{dt} = K (T_{e,1,w} - T_{e,0,w}) \]  
(2.11)

**2. Two-phase one-zone evaporator model.** Due to the disappearance of the superheated zone at the evaporator outlet, the pseudo-state equation 2.12 causes the superheated zone refrigerant enthalpy to track the saturated vapor enthalpy. Equation 2.13 is then used to govern the wall temperature in this inactive superheated zone by tracking the active two-phase zone state. The constant length of the only active two-phase zone is described in Equation 2.14, and Equations 2.10-2.11 still apply to represent the inactive superheated zone states at the evaporator inlet.

\[
\frac{dh_{e2}}{dt} = K (h_{e,g} - h_{e2}) \]  
(2.12)

\[
\frac{dT_{e,2,w}}{dt} = K (T_{e,1,w} - T_{e,2,w}) \]  
(2.13)
\[
\frac{d\varepsilon_{e1}}{dt} = 0 \quad (2.14)
\]

The refrigerant-side mass and energy conservation equations for this two-phase zone become:

\[
\frac{\varepsilon_{e1} \delta P_{e0} dP_e}{\rho_{e1} \delta P_e dt} + \frac{\varepsilon_{e1} \delta P_{e0} d\varepsilon_{e1}}{\rho_{e1} \delta \varepsilon_{e1} dt} = \frac{\dot{m}_{i,e} - \dot{m}_{o,e}}{\rho_{e1} V_{e}} \quad (2.15)
\]

\[
\left[ \frac{\delta h_{e1}}{\delta P_{e}} - \frac{1}{\rho_{e1}} \right] \frac{dP_e}{dt} + \frac{\delta h_{e1}}{\delta \varepsilon_{e1}} \frac{d\varepsilon_{e1}}{dt} = \frac{\dot{Q}_{e1} + \dot{m}_{i,e} (h_{i,e} - h_{e1}) - \dot{m}_{o,e} (h_{o,e} - h_{e1})}{\rho_{e1} V_{e} \varepsilon_{e1}} \quad (2.16)
\]

3. **Superheated and two-phase two-zone evaporator model.** When the system switches to the heating mode of operation, the evaporator behaves as a condenser, condensing the inlet superheated vapor into two-phase refrigerant. The mass and energy equations 2.17-2.18 are used to describe the refrigerant dynamics for the inlet superheated zone, and the refrigerant enthalpy in this zone is assumed to be average enthalpy computed in Equation 2.19. The conservation equations for the two-phase zone are shown in Equations 2.20-2.21.

\[
\frac{d\varepsilon_{e0}}{dt} + \left[ \frac{\varepsilon_{e0} \partial P_{e0}}{\rho_{e0} \delta P_e} + \frac{1}{2} \frac{\partial h_{e0}}{\rho_{e0} \delta P_{e0} \delta P_e} \right] \frac{dP_e}{dt} + \frac{\dot{m}_{e01}}{\rho_{e0} V_{e} h_{e0}} = \frac{\dot{Q}_{e0} + \dot{m}_{i,e} (h_{i,e} - h_{e0}) - \dot{m}_{o,e} (h_{o,e} - h_{e0})}{\rho_{e0} V_{e} h_{e0}} - \frac{1}{2} \frac{d h_{e0}}{dt} \quad (2.17)
\]

\[
\left[ \frac{1}{2} \frac{\partial h_{e0}}{\delta P_e} - \frac{1}{\rho_{e0}} \right] \frac{dP_e}{dt} + \frac{1}{\rho_{e0} V_{e} h_{e0}} \frac{\dot{m}_{e01}}{\delta \varepsilon_{e0} \delta h_{e0}} = \frac{\dot{Q}_{e0} + \dot{m}_{i,e} (h_{i,e} - h_{e0}) - \dot{m}_{o,e} (h_{o,e} - h_{e0})}{\rho_{e0} V_{e} h_{e0}} - \frac{1}{2} \frac{d h_{e0}}{dt} \quad (2.18)
\]

\[
h_{e0} = \frac{h_{i,e} + h_{e0}}{2} \quad (2.19)
\]

\[
- \frac{d\varepsilon_{e1}}{dt} + \frac{\varepsilon_{e1} \partial P_{e1}}{\rho_{e1} \delta P_e} dP_e + \frac{\dot{m}_{e01}}{\rho_{e1} V_{e} \varepsilon_{e1}} + \frac{\varepsilon_{e1} \partial P_{e1}}{\rho_{e1} \delta \varepsilon_{e1} \delta P_e} \frac{d\varepsilon_{e1}}{dt} = \frac{\dot{m}_{o,e}}{\rho_{e1} V_{e}} \quad (2.20)
\]

\[
\left[ \frac{\partial h_{e1}}{\delta P_e} - \frac{1}{\rho_{e1}} \right] \frac{dP_e}{dt} + \frac{\partial h_{e1}}{\rho_{e1} V_{e} \varepsilon_{e1}} \frac{\dot{m}_{e01}}{\delta \varepsilon_{e1} \delta h_{e1}} + \frac{\partial h_{e1}}{\delta \varepsilon_{e1} \delta P_e} \frac{d\varepsilon_{e1}}{dt} = \frac{\dot{Q}_{e1} - \dot{m}_{o,e} (h_{o,e} - h_{e1})}{\rho_{e1} V_{e} \varepsilon_{e1}} \quad (2.21)
\]
Since the refrigerant superheated zone at the evaporator outlet is inactive, Equation 2.22 is applied. With the pseudo-state equations 2.12-2.13, the dynamic states of this superheated zone are forced to track the corresponding states of the active zone.

\[
\frac{d\zeta_{e0}}{dt} + \frac{d\zeta_{el}}{dt} = 0
\]  (2.22)

4. **Switching criteria.** Refrigerant mass conservation is the major concern when choosing the switching criteria between different model representations [18, 79, 81]. Switching conditions between the two-zone (two-phase and superheated) and the one-zone (two-phase) evaporator model were presented in [18] with the assumption that the inlet refrigerant to the evaporator is two-phase fluid. Specifically, the conditions to trigger the switch from the one-zone to the two-zone (two-phase and superheated) evaporator model are given in Equations 2.23-2.24.

\[
\zeta_{el} (\overline{\gamma}_e - \overline{\gamma}_{etot}) > \zeta_{e\text{ min}}
\]  (2.23)

\[
\frac{d\overline{\gamma}_e}{dt} > 0
\]  (2.24)

If the mean void fraction \(\overline{\gamma}_e\) is above the equilibrium value \(\overline{\gamma}_{etot}\) for evaporation to saturated vapor, the term inside the parentheses in Equation 2.23 will be positive. This means there is excess vapor volume in the two-phase zone. The term on the left side of Equation 2.23 represents the normalized length of excess vapor volume, and \(\zeta_{e\text{ min}}\) on the right side is regarded as a tunable switching threshold indicating the minimum dimensionless length of the superheated zone within the total evaporator tube length. The value of this threshold is chosen to be 0.001 as a starting point. Therefore, these conditions can be stated as, ‘the existence and further increase of excess vapor volume in the two-phase zone indicate the occurrence of a superheated vapor zone at the evaporator outlet.’

Switching from the one-zone (two-phase) to another two-zone (superheated and two-phase) model representation could occur when the system operation transitions from cooling to heating mode, and the conditions are defined as follows:
\[ h_{i,e} > h_{v,g} \]  \hspace{1cm} (2.25)
\[ \frac{dh_{i,e}}{dt} > 0 \]  \hspace{1cm} (2.26)

If the inlet refrigerant enthalpy is above the saturated vapor enthalpy value, Equation 2.25 will be satisfied. This means the superheated refrigerant vapor enters the evaporator. So, these conditions can be explained as, ‘the inlet refrigerant in the evaporator becomes superheated vapor and the inlet enthalpy is continuing to increase.’

Similarly, switching back to cooling mode operation could drive the evaporator to switch from the two-zone (superheated and two-phase) to the one-zone model when the switching conditions given in Equations 2.27-2.28 are satisfied.

\[ h_{i,e} < h_{v,g} \]  \hspace{1cm} (2.27)
\[ \frac{dh_{i,e}}{dt} < 0 \]  \hspace{1cm} (2.28)

2.3.1.2 Switched Condenser

A switched moving-boundary condenser model was developed in [79] to accommodate the presence or absence of a subcooled region. However, additional capability is needed to simulate condenser dynamics under system mode switch operations. Recall the cooling/heating mode switch operation in the transport system example. When the system shifts from cooling to heating mode operation, the refrigerant flow is redirected by the 3-way valve (see Figure 2.3) to the hot gas line rather than to the condenser coil. In this scenario, the condenser acts as if the system were ‘shutting down’ where the remaining refrigerant inside the coil continues to flow out of the condenser and enters the receiver tank due to the inertia and pressure differential. Therefore, similar to the switched evaporator model development, a switched condenser model is presented to consist of five different model representations (see Figure 2.9) to handle system cycling transients.
The dynamic state vector in Equation 2.29 represents the condenser conditions at each time instant. The descriptor form state derivative equation 2.30 is applied to describe the condenser model with different coefficient matrices $Z(x_i,u_i)$ and forcing functions $f(x_i,u_i)$ for different model representations. Again, the *pseudo-state* variables, which represent absent states in inactive zones [79], are implemented to maintain the uniform state vector $x_i$, and provide reasonable initial conditions when the model representations switch from one another. The equation derivations of the switched condenser model and its capability to simulate system on and off cycling operation were discussed in [18]. In this chapter, the refrigerant-side governing equations for two condenser model representations (two-zone (superheated and two-phase), and one-zone (two-phase) models) are presented for the condenser operating during
cooling/heating mode switch in the transport application. Detailed descriptions of all the five model representations and switching criteria can be found in [18, 79].

\[
x_c = \begin{bmatrix} \xi_{c1} & \xi_{c2} & P_c & h_{c3} & T_{c1,w} & T_{c2,w} & T_{c3,w} & \tau_c \end{bmatrix}^T \tag{2.29}
\]

\[
Z(x_c, u_c) \cdot \dot{x}_c = f(x_c, u_c) \tag{2.30}
\]

1. **Superheated and two-phase two-zone condenser model.** In this representation, the condenser model is described as a two-zone component without subcooled region at the condenser outlet. This is the case in cooling mode operation when there exists a receiver tank connected to the condenser outlet as shown in Figure 2.3. The refrigerant flowing out of the condenser is normally saturated liquid or two-phase fluid at steady-state conditions [43, 83]. The mass and energy conservation equations for the superheated zone are given in Equations 2.31-2.32, where a similar assumption as Equation 2.19 for the refrigerant enthalpy in this zone is made. The refrigerant-side governing equations for the two-phase zone are presented in Equations 2.33-2.34.

\[
\frac{d\xi_{c1}}{dt} + \left[ \frac{\xi_{c1}}{\rho_{c1}} \frac{\partial \rho_{c1}}{\partial P_c} + \frac{1}{2} \frac{\xi_{c1}}{\rho_{c1}} \frac{\partial \rho_{c1}}{\partial h_{c1}} \frac{\partial h_{c1}}{\partial P_c} \right] \frac{dP_c}{dt} + \frac{\dot{m}_{c12}}{\rho_{c1} V_c} \frac{\xi_{c1}}{\rho_{c1} V_c} \left( \frac{\rho_{c1}}{\rho_{c2}} \right) = \frac{\dot{m}_{i,c}}{\rho_{c1} V_c} \frac{1}{\rho_{c1}} \frac{\partial P_{c1}}{\partial h_{c1}} \frac{dh_{c1}}{dt} \tag{2.31}
\]

\[
\left[ \frac{1}{2} \frac{\delta h_{c1}}{P_c} \right] \frac{dP_c}{dt} + \frac{(h_{c1} - h_{c1})}{\rho_{c1} V_c} \frac{\dot{m}_{c12}}{\rho_{c1} V_c} \frac{\xi_{c1}}{\rho_{c1} V_c} = \frac{\dot{Q}_{c1}}{\rho_{c1} V_c} \frac{\xi_{c1}}{\rho_{c1} V_c} \frac{1}{\rho_{c1}} \frac{dh_{c1}}{dt} \tag{2.32}
\]

\[
\frac{d\xi_{c2}}{dt} + \frac{\xi_{c2}}{\rho_{c2}} \frac{\partial \rho_{c2}}{\partial P_c} \frac{dP_c}{dt} - \frac{\dot{m}_{c12}}{\rho_{c2} V_c} \frac{\xi_{c2}}{\rho_{c2} V_c} \frac{\partial \rho_{c2}}{\partial \tau_c} \frac{d\tau_c}{dt} = - \frac{\dot{m}_{o,c}}{\rho_{c2} V_c} \tag{2.33}
\]

\[
\left[ \frac{\delta h_{c2}}{P_c} \right] \frac{dP_c}{dt} - \frac{(h_{c2} - h_{c2})}{\rho_{c2} V_c} \frac{\dot{m}_{c12}}{\rho_{c2} V_c} \frac{\xi_{c2}}{\rho_{c2} V_c} = \frac{\dot{Q}_{c2}}{\rho_{c2} V_c} \frac{\xi_{c2}}{\rho_{c2} V_c} \frac{1}{\rho_{c2}} \frac{dh_{c2}}{dt} \tag{2.34}
\]

Since the subcooled zone is inactive, Equation 2.35 is satisfied, and the pseudo-state equations 2.36-2.37 are applied [79].

\[
\frac{d\xi_{c1}}{dt} + \frac{d\xi_{c2}}{dt} = 0 \tag{2.35}
\]
\[
\frac{dh_{c3}}{dt} = K(h_{c,f} - h_{c3}) \quad (2.36)
\]

\[
\frac{dT_{c3,w}}{dt} = K(T_{c2,w} - T_{c3,w}) \quad (2.37)
\]

2. **Two-phase one-zone condenser model.** When the length of the superheated zone \(\zeta_{c1}\) in the condenser two-zone (superheated and two-phase) model becomes smaller than the given minimum threshold \(\zeta_{c,\text{min}}\) in transients, the condenser could switch to the two-phase one-zone model representation (see Figure 2.9). The governing equations for the refrigerant-side of the active two-phase zone become:

\[
\frac{\delta h_{c2}}{\delta P_c} \frac{dP_c}{dt} + \frac{\delta h_{c2}}{\delta \gamma_c} \frac{d\gamma_c}{dt} = \frac{\dot{m}_{c2} - \dot{m}_{o,c}}{\zeta_{c2} V_c} \quad (2.38)
\]

\[
\left[ \frac{\delta h_{c2}}{\delta P_c} \frac{1}{\rho_{c2}} \right] \frac{dP_c}{dt} + \frac{\delta h_{c2}}{\delta \gamma_c} \frac{d\gamma_c}{dt} = \frac{\dot{Q}_{c2} + \dot{m}_{c2} (h_{c2} - h_{c2}) - \dot{m}_{o,c} (h_{o,c} - h_{c2})}{\rho_{c2} V_c \zeta_{c2}} \quad (2.39)
\]

The constant length of the two-phase zone is described by:

\[
\frac{d\zeta_{c2}}{dt} = 0 \quad (2.40)
\]

Equations 2.41-2.42 are used for the inactive superheated zone and the *pseudo-state* equations 2.36-2.37 are still applied for the subcooled zone.

\[
\frac{d\zeta_{c1}}{dt} = 0 \quad (2.41)
\]

\[
\frac{dT_{c1,w}}{dt} = K(T_{c2,w} - T_{c1,w}) \quad (2.42)
\]

Similar switching conditions based on the refrigerant inlet enthalpy as Equations 2.25-2.26 can be used to trigger the switch from the two-phase one-zone model to the superheated and two-phase two-zone condenser model.
2.3.1.3 Model Equivalence

A PDE method was presented in [11] to model the moving-boundary heat exchangers. In detail, the governing ordinary differential equations were obtained by integrating the governing partial differential equations (PDEs) (Equations 2.43-2.45) along the length of the heat exchanger tube to remove the spatial dependence and assuming lumped parameters associated with each fluid zone. The heat exchanger modeling approach, discussed in this dissertation, uses the unsteady state form of the mass and energy conservation equations, and expands these equations at each fluid control volume to yield a set of governing ordinary differential equations [79]. The equivalence of these two modeling approaches in describing heat exchangers with the same dynamic state variables is demonstrated here.

\[
\frac{\partial (\rho A)}{\partial t} + \frac{\partial (\dot{m})}{\partial z} = 0 \quad (2.43)
\]

\[
\frac{\partial (\rho A h - AP)}{\partial t} + \frac{\partial (\dot{m} h)}{\partial z} = \rho_i \alpha_i (T_w - T_r) \quad (2.44)
\]

\[
(C_p \rho A_w) \frac{\partial (T_w)}{\partial t} = \rho_i \alpha_i (T_r - T_w) + \rho_o \alpha_o (T_u - T_w) \quad (2.45)
\]

The two-phase and superheated two-zone evaporator model is presented as an example, and the mass and energy conservation equations for both zones are shown in Equations 2.5-2.8. Combine Equations 2.5 and 2.7 to remove the intermediate variable \( \dot{m}_{e12} \), and the mass conservation equation along the evaporator tube can be obtained in Equation 2.46. Substitute the intermediate variable \( \dot{m}_{e12} \) in Equations 2.6 and 2.8, and the energy conservation equations for the superheated and two-phase zone can be rewritten in Equations 2.47 and 2.48 respectively.

\[
\left( \rho_{e1} - \rho_{e2} \right) \frac{d \zeta_{e1}}{dt} + \left( \zeta_{e1} \frac{\delta \rho_{e1}}{\delta P_e} + \zeta_{e2} \frac{\delta \rho_{e2}}{\delta P_e} \right) \frac{d P_e}{dt} + \zeta_{e1} \frac{\delta \rho_{e1}}{\delta h_{e1}} \frac{d h_{e1}}{dt} + \zeta_{e2} \frac{\delta \rho_{e2}}{\delta h_{e2}} \frac{d h_{e2}}{dt} + \zeta_{e1} \frac{\delta \rho_{e1}}{\delta \theta_e} \frac{d \theta_e}{dt}
\]

\[
= \frac{\dot{m}_{e,c} - \dot{m}_{n,e}}{V_e} \quad (2.46)
\]
Using the PDE method and choosing the same dynamic state variables \( P_e \), \( T_e \), and \( h_e \), the governing ordinary differential equations for both evaporator zones (superheated and two-phase) are presented in Equations 2.49-2.52.

\[
\begin{align*}
\left( \rho_{e,2} h_{e,2} - \rho_{e,2} h_{e,2} \right) \frac{d \xi_{e,2}}{dt} - \xi_{e,2} \left( h_{e,2} - h_{e,2} \right) \frac{\delta P_{e}}{\delta P_{e}} + \left( h_{e,2} - h_{e,2} \right) \frac{d P_{e}}{dt} + \
\left( \rho_{e,2} \frac{\delta h_{e,2}}{\delta P_{e}} \left( h_{e,2} - h_{e,2} \right) \frac{d h_{e,2}}{dt} \right) = V_e \left( \dot{Q}_{e} - \dot{m}_{i,e} \left( h_{e,2} - h_{e,2} \right) \right) \\
\end{align*}
\]  
(2.47)

\[
\begin{align*}
\left( \rho_{e,2} h_{e,2} - \rho_{e,2} h_{e,2} \right) \frac{d \xi_{e,2}}{dt} - \xi_{e,2} \left( h_{e,2} - h_{e,2} \right) \frac{\delta P_{e}}{\delta P_{e}} + \left( h_{e,2} - h_{e,2} \right) \frac{d P_{e}}{dt} + \
\left( \rho_{e,1} \frac{\delta h_{e,1}}{\delta P_{e}} \left( h_{e,1} - h_{e,1} \right) \frac{d h_{e,1}}{dt} \right) = V_e \left( \dot{Q}_{e} - \dot{m}_{i,e} \left( h_{e,1} - h_{e,1} \right) \right) \\
\end{align*}
\]  
(2.48)

Combine Equations 2.49 and 2.51 to remove the intermediate variable \( \dot{m}_{e,12} \), and the same mass conservation equation along the evaporator tube as Equation 2.46 is obtained. Substitute the variable \( \dot{m}_{e,12} \) in Equation 2.50 with the expression in Equation 2.49, and the energy
conservation equation for the superheated zone can be rewritten and shown identically as Equation 2.47. By applying a similar technique to replace the variable $\dot{m}_{e_{12}}$ in Equation 2.52 with the expression in Equation 2.51, the identical two-phase zone energy equation as Equation 2.48 can be achieved with the following equations:

$$\rho_{e_{1}} h_{e_{1}} = \rho_{e_{1}, f_{1}} h_{e_{1}, f_{1}} (1 - \gamma_{e_{1}}) + \rho_{e_{1}, g_{1}} h_{e_{1}, g_{1}} \gamma_{e_{1}}$$  \hspace{1cm} (2.53)

$$\frac{\delta(\rho_{e_{1}} h_{e_{1}})}{\delta P_{e}} = \frac{d(\rho_{e_{1}, f_{1}} h_{e_{1}, f_{1}})}{dP_{e}} (1 - \gamma_{e_{1}}) + \frac{d(\rho_{e_{1}, g_{1}} h_{e_{1}, g_{1}})}{dP_{e}} \gamma_{e_{1}} = h_{e_{1}} \frac{\delta P_{e}}{\delta P_{e}} + \rho_{e_{1}} \frac{\delta h_{e_{1}}}{\delta P_{e}}$$  \hspace{1cm} (2.54)

$$\frac{\delta(\rho_{e_{1}} h_{e_{1}})}{\delta \gamma_{e_{1}}} = \rho_{e_{1}, g_{1}} h_{e_{1}, g_{1}} - \rho_{e_{1}, f_{1}} h_{e_{1}, f_{1}} = h_{e_{1}} \frac{\delta P_{e}}{\delta \gamma_{e_{1}}} + \rho_{e_{1}} \frac{\delta h_{e_{1}}}{\delta \gamma_{e_{1}}}$$  \hspace{1cm} (2.55)

### 2.3.2 Accumulator/Receiver Tank

The accumulator is a device normally used to separate vapor from liquid in two-phase flow, thus preventing damage to the compressor. The primary function of the receiver tank in the refrigeration system is to store excess refrigerant mass to ensure system capacity over a large range of operating conditions, and the existing refrigerant is assumed to be saturated liquid. Since the structure of the refrigerant-side mass and energy governing equations for both accumulator and receiver tank components is similar, the accumulator model development is introduced here to capture system behavior in mode switch transients.

In the refrigerated transport example (see Figure 2.3), when the system is in cooling mode of operation, due to the presence of the suction line heat exchanger in the refrigerant circuit, the refrigerant entering the accumulator is superheated vapor and the accumulator acts as a superheated vapor tank with heat transfer characteristics. However, since the evaporator behaves like a condenser in heating mode, refrigerant liquid accumulates at the bottom of the accumulator and saturated refrigerant vapor flows out of the accumulator tank. Therefore, a switched accumulator is developed with two model representations as shown in Figure 2.10.

Two dynamic states, refrigerant pressure $P_{ac}$, and average refrigerant enthalpy $h_{ac}$, are defined to describe the refrigerant dynamics inside the accumulator for both representations. The structure of refrigerant-side mass and energy governing equations in each model representation is the same and is given below:
\[
\frac{\delta \rho_{ac}}{\delta P_{ac}} \frac{dP_{ac}}{dt} + \frac{\delta \rho_{ac}}{\delta h_{ac}} \frac{dh_{ac}}{dt} = \frac{\dot{m}_{ac} - \dot{m}_{o,ac}}{V_{ac}} \tag{2.56}
\]

\[-\frac{1}{\rho_{ac}} \frac{dP_{ac}}{dt} + \frac{dh_{ac}}{dt} = \frac{(UA)_{ac} (T_{amb} - T_{ac}) + \dot{m}_{ac} (h_{v,ac} - with_{ac}) - \dot{m}_{o,ac} (h_{v,ac} - h_{ac})}{\rho_{ac} V_{ac}} \tag{2.57}\]

where the term \((UA)_{ac} (T_{amb} - T_{ac})\) represents the heat transfer rate from the ambient air to the refrigerant inside the accumulator. Examples of the calculations of the heat transfer coefficients \((UA)_{ac}\) in cooling and heating mode are available in [44]. The difference in the governing equations between the two representations is the exiting refrigerant enthalpy \(h_{o,ac}\) in Equation 2.57. In the two-phase mixture model, the exiting refrigerant is saturated vapor, while the exiting enthalpy is assumed to be the average enthalpy \(h_{ac}\) in the superheated vapor model representation [84].

**Figure 2.10  Switched accumulator model representations**

The accumulator model switching criteria are determined based on the mean void fraction \(\bar{\gamma}_{ac}\), which is defined as the ratio of refrigerant vapor volume to total volume, and can be represented as a function of the refrigerant pressure \(P_{ac}\) and the refrigerant enthalpy \(h_{ac}\). For example, the switch occurs from the two-phase mixture model to the superheated vapor model when the switching conditions, as given in Equations 2.58 and 2.59, are satisfied.
If the mean void fraction value is above one as shown in Equation 2.58, this means the accumulator is filled with vapor and the average refrigerant enthalpy is above the saturated vapor enthalpy. So, these switching conditions can be stated as, ‘the refrigerant enthalpy inside the accumulator becomes larger than the saturated vapor enthalpy and continues to increase.’

When the system operation switches from cooling to heating mode, the conditions to trigger the switch from the superheated vapor accumulator to the two-phase mixture model are explained in Equations 2.60 and 2.61.

\[
\overline{\gamma}_{ac} > 1 \quad (2.58)
\]
\[
\frac{dh_{ac}}{dt} > 0 \quad (2.59)
\]

The mean void fraction \(\overline{\gamma}_{ac,calc}\) in the superheated vapor model is calculated in Equation 2.62. If this value is below one, the average refrigerant quality \(\tilde{x}_{ac}\) inside the accumulator will be below one, and Equation 2.60 will be satisfied. This means the refrigerant enthalpy is below the saturated vapor enthalpy, and there is excess liquid volume accumulating at the bottom. Therefore, these conditions can be described as, ‘the mean void fraction indicates there is noticeable excess liquid volume inside the accumulator superheated vapor model and it continues to accumulate.’

\[
\overline{\gamma}_{ac,calc} = \frac{\tilde{x}_{ac}\rho_{w,f}}{\tilde{x}_{ac}\rho_{w,f} + (1-\tilde{x}_{ac})\rho_{w,g}} \quad (2.62)
\]

### 2.4 Static Component Modeling

As discussed in [11], the issue of time scale is of critical importance for dynamic modeling of vapor compression cycle systems. Under the assumption that the dynamics of the components (i.e., compressor, expansion and regulation valves, and suction line heat exchanger) are generally an order of magnitude faster than those of the dynamic components (i.e., heat
exchangers, receiver tank, and accumulator), these components are modeled using steady-state equations. In this section, the modeling approaches of each “static” component are briefly introduced by taking the refrigeration unit system depicted in Figure 2.3 for example. Details about the semi-empirical approach to modeling the compressor and expansion devices (electronic expansion valve and thermostatic expansion valve) are available in [11, 18, 83], and more descriptions about the individual “static” component models of the refrigeration unit operating in cooling and heating mode can be found in [44].

2.4.1 Suction Line Heat Exchanger

The suction line heat exchanger (SLHX) is a refrigerant liquid to refrigerant vapor heat exchanger. The thermal capacitance of the SLHX is assumed to be small in comparison to that of the evaporator and condenser; therefore, the associated dynamics can be neglected and its governing equations are described using steady-state equations. In cooling mode operation of the refrigeration unit (see Figure 2.3), hot refrigerant liquid exiting the receiver tank flows through the inner coil of the SLHX where it loses heat to the cold refrigerant vapor flowing through the outer shell of the SLHX. During heating mode, the two-phase refrigerant exiting the evaporator passes through the outer shell of the SLHX without undergoing significant heat transfer due to the trapped refrigerant in the inner coil. The reader is referred to [44] for details on this component model.

2.4.2 Throttle Valve

The throttle valve, also known as a suction pressure regulator valve, is a mechanical control valve to regulate the pressure of the refrigerant vapor at the compressor inlet. In this refrigerated transport system example, this valve model is developed using an empirical map that was presented in [44].

2.4.3 Scroll Compressor

The TS-500 refrigeration unit contains an open-drive scroll compressor. The compressor is operated using a diesel engine and can run at three speeds, denoted as high speed, low speed and null (approximately 4000rpm, 2500rpm and 0rpm, respectively). The compressor mass flow rate is computed using:
\[ \dot{m}_v = V_k \omega_k \rho_k \eta_{vol} \]  \hspace{1cm} (2.63)

where \( V_k \) is the cylinder volume, \( \omega_k \) is the compressor speed, and \( \rho_k \) is the refrigerant inlet density. The volumetric efficiency \( \eta_{vol} \) is calculated using a performance mapping approach provided in [44].

### 2.4.4 Thermostatic Expansion Valve

As another mass flow device in the system, in cooling mode operation, the thermostatic expansion valve (TXV) controls the refrigerant mass flow entering the evaporator by maintaining a certain level of refrigerant superheat at the evaporator outlet. Equation 2.64 is used to calculate the mass flow rate, where the flow coefficient \( C_f \) is determined via a semi-empirical mapping approach [18].

\[ \dot{m}_v = C_f \sqrt{\rho_v \left( P_{o,alk} - P_e \right)} \]  \hspace{1cm} (2.64)

In heating mode operation, the disappearance of superheat at the evaporator outlet results in the closed position of the expansion valve; however, the bleed port effect of the TXV can be modeled by assuming the valve as a fixed orifice device with a constant flow coefficient value under these conditions.

### 2.4.5 Discharge Pressure Regulator Valve

Since the compressor in the TS-500 refrigeration unit example is not operated using a variable speed drive, the pressure ratio across the compressor cannot be varied arbitrarily using the compressor speed. Instead, the compressor is fixed at high or low speed to meet the desired capacity during heating mode, and the discharge pressure regulator (DPR) valve is used to regulate the pressure ratio. By adjusting the valve opening, a backpressure is created which drives the compressor to produce more work and provide more generated heat to the cargo space. Once the discharge pressure is settled, an iterative process is applied to compute the flow rate across the DPR valve, where the flow coefficient is a mapping function of the calculated mass flow rate. In this way, the pressure regulating performance is achieved as a result of valve opening changes. The development of the flow coefficient map can be found in [44].
2.4.6 Hot Gas Line

The hot gas line is a series of pipes that are engaged only in heating mode to connect the DPR valve outlet to the evaporator inlet. This component is modeled as a pressure drop element with heat transfer characteristics, and more modeling details are given in [44].

2.5 Refrigerated Cargo Space Modeling

As described in Figure 2.1, a single cargo space transport refrigeration system consists of a refrigeration unit and a refrigerated cargo space. The refrigeration unit system uses energy to extract heat from the refrigerated cargo space and transfer it to the external ambient environment to maintain the food temperature within allowable ranges for food safety and shelf life considerations [13].

The refrigerated cargo space model introduced here accommodates the following important effects in food transportation: (i) varying ambient conditions, including ambient temperature, solar radiation intensity, and wind speed; and (ii) air infiltration. The major modeling assumptions are as follows:

- Air inside the cargo space is well-mixed and the air temperature is uniform.
- Outside and inside wall surface temperatures of the cargo space are assumed to be uniform respectively.
- Heat conduction through the walls is one dimensional.

Three dynamic states, cargo space temperature $T_{\text{space}}$, interior surface temperature $T_{\text{is}}$ and exterior surface temperature $T_{\text{es}}$, are defined to describe the model dynamics, as shown in Equation 2.65. The cargo space air, interior, and exterior surface heat balance governing equations are presented in Equations 2.66-2.68 by applying energy conservation principles [18].

$$x = \begin{bmatrix} T_{\text{space}} & T_{\text{is}} & T_{\text{es}} \end{bmatrix}^T$$

$$\frac{dT_{\text{space}}}{dt} = \frac{\dot{Q}_{\text{cond}} - \dot{Q}_{\text{inf}} - \dot{Q}_{\text{occ}}}{(MC)_{\text{air}}}$$

$$\frac{dT_{\text{is}}}{dt} = \frac{\dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}}}{(MC)_{\text{space, w}}}$$
\[
\frac{dT_{ex}}{dt} = \frac{\dot{Q}_{solar} - \dot{Q}_{outconv} - \dot{Q}_{cond}}{(MC)_{space,w}}
\] (2.68)

The solar load \(\dot{Q}_{solar}\) and air infiltration load \(\dot{Q}_{inf}\) are computed from Equations 2.69 and 2.70. \(\dot{Q}_{inconv}\) and \(\dot{Q}_{outconv}\) represent the convective heat transfer from the surface to the air, and the heat conduction from the exterior to interior surface is given by \(\dot{Q}_{cond}\). The VCC refrigeration system capacity provided to the cargo space is denoted as \(\dot{Q}_{vcc}\). More information on the surface heat transfer coefficients and heat balance processing modeling approach can be found in [18].

\[
\dot{Q}_{solar} = \beta I_{solar\_radiation}
\] (2.69)

\[
\dot{Q}_{inf} = V_{space} \rho_{space} c_{space} (T_{amb} - T_{space})(ACH)
\] (2.70)

With the development of the refrigeration unit and cargo space models described in this chapter, the transport refrigeration system can then be represented to simulate system mode switch operations and evaluate system performance under different temperature regulation approaches (i.e., on/off cycling, and continuous cooling/heating cycling) discussed in the following chapters.
Chapter 3  Model Simulation and Validation

A set of simulation tools, the Thermosys toolbox, was developed in [11] for the purpose of model validation, dynamic analysis, and control design in various air conditioning and refrigeration system applications. In this chapter, this software toolbox is briefly introduced, and the modeling approaches discussed in Chapter 2 are applied to develop component and system models for simulation studies. Two experimental test beds representing different applications are illustrated, followed by the presentations of model validation results which demonstrate the modeling validity in predicting system behavior under mode switch operations. A simulation case study for temperature regulation in a transport refrigeration example involving door-opening events is given at the end of the chapter to demonstrate modeling capabilities.

3.1 Simulation Platform

As described in [11], the Thermosys toolbox is a library of models and tools for simulating dynamic thermal systems, such as vapor compression cycle systems. This library is developed for use in Matlab, and these models are created using the visual programming package Simulink. The open architecture style of programming allows users to customize the models for their own applications. Also, the availability of additional toolboxes in Matlab, such as the Control Design and System Identification toolboxes, increases the flexibility of the Thermosys toolbox since these other Matlab options can be used to gain flexibility in system analysis.

Figure 3.1 presents an overview of the Thermosys toolbox from the Simulink browser. The fluid property functions are implemented as lookup tables in Thermosys, and the data contained in those tables are generated from the fluid property functions provided with the
software package Engineering Equation Solver (EES) [85]. The auxiliary tools include the mean void fraction calculation based on a slip ratio correlation [11].

Figure 3.1 Overview of Thermosys library structure

3.1.1 Component Models

The component models developed in this dissertation are added into the components block as shown in Figure 3.1. The nonlinear governing differential equations presented in Chapter 2 are applied to develop the dynamic models, such as the moving-boundary heat exchangers, accumulator, receiver tank, and refrigerated cargo space. The semi-empirical modeling approaches are used to derive steady-state algebraic equations for the “static” models, such as compressor, suction line heat exchanger, and several types of expansion and regulating valves.

Each of the component models is organized as a masked subsystem, and the model consists of two essential parts: a graphical user interface (GUI) and a component function. The image of the component type is displayed with the component mask. By double clicking the component, the interactive GUI is activated which then allows the user to specify the
component’s physical parameters, operating conditions, and other necessary information, such as heat transfer coefficient correlations. Underneath the mask, the component inputs, physical parameters, and calculated thermodynamic properties are passed to the component function for calculation. The outputs of the component function are then converted to the component outputs. The component function for simulation is a user-defined function, and both Simulink block diagrams and component S-functions are applied in the Thermosys toolbox. More information about the component features in Thermosys is presented in [11], and examples of the switched component model implementation in Thermosys can be found in [18].

3.1.2 System Models

To build a system model for analysis, the inputs and outputs of each component need to be connected in a logical manner with a basic understanding of the system ‘information flow’. Take the single cargo space transport refrigeration system given in Chapter 2 for example. The refrigeration unit in Figure 2.3 is constructed with component models (cooling and heating mode components separately), along with pipe models and hydraulic resistance elements [86] connecting each component. The pipe models calculate pressure drops between any two components, and the hot gas line component can be regarded as a variation of the pipe model. The refrigeration unit model is coupled to the cargo space through the air flow interactions and the refrigerated transport system, as represented in Thermosys, is shown in Figure 3.2. Various external inputs can be applied to the system for simulation studies, such as compressor speed, air volumetric flow rates to the heat exchangers, and ambient conditions.

As mentioned earlier, one advantage of this Thermosys toolbox is its ability to interface with other toolboxes in Matlab for system analysis and control design. The Stateflow toolbox in Matlab is an interactive design environment for modeling and simulating event-driven systems based on finite-state machine theory. Stateflow allows the user to combine graphical and tabular representations, including state transition diagrams and flow charts, to model system response to events, such as time-based conditions [87]. The state transition diagrams in the Stateflow toolbox are applied here as an example to describe the cooling/heating mode switch temperature control algorithm in the transport applications, as illustrated in Figure 3.3. Figure 3.4 shows the
schematic of closed-loop system by combining the refrigeration system model in Thermosys and Stateflow temperature control diagrams.

Figure 3.2 Refrigerated transport system example in Thermosys
3.2 Experimental System

The experimental systems used for dynamic model validation and control research are introduced in this section.

3.2.1 Experimental Test Bed at ACRC

Much of the model validation research, including switched heat exchanger modeling framework, and system transients with on and off cycling operation in this dissertation, was
conducted on experimental facilities available as part of the Air Conditioning and Refrigeration Center (ACRC) at the University of Illinois at Urbana-Champaign.

The experimental system is a dual-evaporator “trainer” system containing the following components: a tube-and-fin condenser, two tube-and-fin evaporators, an internal heat exchanger, a suction-line accumulator, a liquid-line receiver, a semi-hermetic compressor, and a variety of expansion devices. A full suite of sensors for pressures, temperatures and mass flow rates in the system allow for detailed observations of system transient behaviors, while numerous manual valves allow the system to be configured in many different ways. A photograph of the experimental system is shown in Figure 3.5. Detailed descriptions about this system, including sensors, actuators, system components along with physical parameters, data acquisition and control, can be found in [11, 17, 83].

![Figure 3.5 Photograph of the experimental system at ACRC](image)

**3.2.2 Industrial Experimental Facility**

All experimental data about the system cooling/heating mode switch operation presented in this dissertation were collected at the Thermo King Corporation test facility located in Minneapolis, Minnesota. The experimental refrigeration system as shown in Figure 3.6 is
instrumented with type-T thermocouples and pressure transducers. Air temperatures entering and leaving the heat exchangers are measured with thermocouple grids placed near the coils. Immersion thermocouples are used to monitor the refrigerant temperature at different locations in the refrigeration unit. Four thermocouple stands (see Figure 3.6(b)) are used for measuring the air temperature profile inside the cargo space. Table 3.1 presents accuracy of sensors in measurements. Along with the status of each solenoid valve in the refrigeration system, each temperature sensor and pressure transducer is connected to an Agilent 34970A data acquisition system to observe the system behavior during testing. For proprietary reasons, the physical parameters of the refrigeration unit components cannot be provided; however, the reader is encouraged to refer to [44] for more information regarding cooling and heating mode operation of the refrigeration unit as well as its instrumentation.

![Figure 3.6](image)

**Figure 3.6**  (a) Photograph of the industrial experimental facility  (b) Instrumentation inside the refrigerated cargo space

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-T thermocouples for temperature</td>
<td>±0.5°C</td>
</tr>
<tr>
<td>Pressure transducer at compressor discharge side</td>
<td>±10kPa</td>
</tr>
<tr>
<td>Pressure transducer at compressor suction side</td>
<td>±10kPa</td>
</tr>
<tr>
<td>Pressure transducer at receiver tank outlet</td>
<td>±10kPa</td>
</tr>
<tr>
<td>Pressure transducer at evaporator outlet</td>
<td>±10kPa</td>
</tr>
</tbody>
</table>
3.3 Model Validation

One of the key challenges in any modeling effort is the model validation. Given the lack of experimental model validation in the literature about the applications of the switched moving-boundary framework in predicting system performance under mode switch operations, this research attempts to increase the confidence in the model’s fidelity through various validation scenarios. This section is divided into three parts. The switching schematic between different model representations of the switched heat exchangers developed in Chapter 2 is illustrated with varying external system inputs, and comparisons between data collected from the experimental system (see Figure 3.5) and model outputs are presented. The second model validation scenario includes system on and off cycling operation, where both compressor speed and electronic expansion valve (EEV) opening inputs operate on the same on and off schedule while maintaining constant air flow rates across the heat exchangers. Finally, the developed refrigeration unit model as depicted in Figure 3.2 is compared against the test data collected on the experimental platform in Figure 3.6 during cooling/heating mode switch cycling, and the temperature pull-down and continuous regulation validation results are presented for a transport refrigeration temperature control system (see Figure 3.4). The reader is encouraged to refer to [11, 44, 83] for the validation of the individual component models in the experimental system described in Figures 3.5 and 3.6.

Particular attention is paid to the calculations of the refrigerant-side and air-side heat transfer coefficients in the heat exchangers under mode switch operations. The heat exchanger tube wall-to-air heat transfer coefficients are calculated using $j$-factor correlations [83]. The two-phase refrigerant-side heat transfer coefficients are defined by empirical equations. In the model validation scenarios presented in this section, the two-phase refrigerant heat transfer correlation developed by Dobson and Chato [88] is chosen for the condenser, and the evaporator model uses the correlation from Wattelet et al. [89]. A Gnielinski correlation [90] is applied for the single-phase refrigerant-side heat transfer coefficient calculation. When the system switches from cooling to heating mode, the two-phase heat transfer correlation shifts from the Wattelet correlation [89] for evaporating flows to the Dobson-Chato correlation [88] for condensing flows. When the system turns off with zero refrigerant mass flow, the refrigerant heat transfer coefficients are computed with Equation 3.1, where the thermal conductivity $k$ is a function of
refrigerant states, pressure and enthalpy. Free convection correlations in [91] can be used to calculate air-side heat transfer coefficients with no air flow conditions.

\[
\alpha = \frac{k}{D}
\]  

(3.1)

3.3.1 Scenario I: VCC System with Switched Heat Exchangers

As discussed in [79], when the vapor compression refrigeration system is in operation, the refrigerant entering the condenser is a superheated vapor due to the compression process. The refrigerant outlet conditions range from a two-phase mixture to a subcooled liquid, which results in the condenser model switching between three-zone (superheated, two-phase and subcooled) and two-zone (superheated and two-phase) models under different operating conditions. Meanwhile, there also exist two different refrigerant distribution configurations in the evaporator model of the refrigeration system, which are two-zone (two-phase and superheated) and one-zone (two-phase) model representations. Therefore, four possible models (see Figure 3.7), named the ‘3-2 model’, ‘3-1 model’, ‘2-1 model’, and ‘2-2 model’, respectively, can be developed to describe the VCC system operation.

Figure 3.7  Four possible models to represent the VCC system operation
The purpose of this scenario study is to verify the system performance by applying different model representations to capture the heat exchangers’ transients. Test data for validation are obtained from the experimental system introduced in Figure 3.5, where a single evaporator (evaporator #2) system configuration with an EEV is used. Figure 3.8(a) depicts the schematic of the experimental system studied here, and the related system model is developed in Thermosys as given in Figure 3.8 (b).

Figure 3.8  (a) Schematic of the VCC experimental system  (b) VCC system model

The applied external system inputs are the compressor speed, and the valve opening command as shown in Figures 3.9 and 3.10, while the air volumetric flow rates across the heat exchangers are kept fixed, and the air inlet temperatures are assumed to be constant in experiments.
The plots in Figure 3.11 through Figure 3.14 compare experimental data with various system model outputs. The switching schematic among the VCC system operation models (see Figure 3.7) can be obtained from the condenser subcooled and the evaporator superheated temperature transients as shown in Figures 3.13 and 3.14, and this diagram is presented in Figure 3.15 to demonstrate the effectiveness of the developed switched heat exchanger modeling framework.
Figure 3.12  Scenario I: evaporator pressure

Figure 3.13  Scenario I: condenser subcooled temperature

Figure 3.14  Scenario I: evaporator superheated temperature

Figure 3.15  Scenario I: switching schematic of the VCC operation model with switched heat exchangers
3.3.2 Scenario II: System On and Off Cycling Operation

Most of the model validation efforts on the VCC system on and off cycling operation are presented in [18], and one example is introduced in this chapter. Multiple on and off duty cycling step changes in both compressor speed and valve opening inputs are applied in the experimental system (see Figures 3.5 and 3.8(a)), which serves three main purposes: (i) the validation of the switched moving-boundary modeling approach; (ii) the validation of the refrigerant heat transfer coefficient calculations in transients [18]; and (iii) the demonstration of the compressor duty cycling capabilities for control applications. The system step inputs are plotted in Figures 3.16 and 3.17. The interested reader is referred to [18] for system operating conditions in this model validation scenario.

![Compressor speed input](image)

**Figure 3.16** Scenario II: compressor speed input

![Valve opening command input](image)

**Figure 3.17** Scenario II: valve opening command input

Due to the closed position of the expansion valve, the condenser and the evaporator become isolated from each other when the system turns off. Figure 3.18 describes the dynamic switching of different model representations in the condenser model structure during the on and off cycling operation. The comparisons between experimental data and model outputs are presented in Figure 3.19 to Figure 3.23.
Figure 3.18  Scenario II: Switching schemes of different condenser model representations

Figure 3.19  Scenario II: condenser pressure

Figure 3.20  Scenario II: evaporator pressure

Figure 3.21  Scenario II: condenser air outlet temperature
3.3.3 Scenario III: System Cooling/Heating Mode Switch Operation

To validate the model performance of the cooling/heating mode switch operation in a refrigerated transport system, comparisons with experimental data sets are described in this section. The experimental scenario involves a temperature pull-down and control test for the enclosed cargo space, as seen in Figure 3.6. The test procedure can be described as, ‘high speed pull-down of the cargo space temperature from ambient temperature to a given set-point (fresh or frozen) using the refrigeration unit; then continuously run cooling/heating mode switch cycle operation with low speed to maintain the space temperature.’ The temperature and pressure measurements are collected every 10 seconds.

Two parts are included in this model validation scenario. The refrigeration unit model developed for simulating cooling/heating mode switch operation is compared with the test data, where the refrigerant mass distribution among the components during transients is illustrated. Secondly, the temperature pull-down and regulation validation results are presented for the temperature control transport system.
Specific attention is paid to the modeling of the evaporator inlet refrigerant flow rate and enthalpy (see Figure 2.3) due to their key effects on the system performance during the mode switching [48]. The evaporator inlet flow is provided by the TXV in cooling mode, and then it is determined by the refrigerant flowing through the DPR valve and the hot gas line after the system switches to heating mode operation. Equations 3.2 and 3.3 are used to describe the transients of the refrigerant inlet conditions during the mode switch by assuming the uniform mixing of two refrigerant flows where $n$ is the number of parallel-passes in the evaporator (in this example, $n$ is equal to 11).

\[
\dot{m}_{r,e} = \frac{\dot{m}_v + \dot{m}_{dpr}}{n} \quad (3.2)
\]

\[
\dot{h}_{r,e} = \frac{\dot{m}_v \dot{h}_{v,e} + \dot{m}_{dpr} \dot{h}_{v,hgl}}{\dot{m}_v + \dot{m}_{dpr}} \quad (3.3)
\]

During heating mode operation, the mass flow rate across the TXV bleed port is considered. In cooling mode steady-state operation, the mass flow rate across the DPR valve $\dot{m}_{dpr}$ is assumed to be zero.

**3.3.3.1 Refrigeration Unit Model Validation**

The experimental data for refrigeration unit model validation is chosen from the low compressor speed temperature regulation test, and the cooling/heating mode switch cycle as shown in Figure 3.24 is regarded as the unit model input. Other model inputs for simulation include the evaporator and condenser air inlet temperatures as given in Figures 3.25 and 3.26. The initial operating conditions of the refrigeration unit are listed in Table 3.2.

![Figure 3.24](image-url)  
**Figure 3.24**  Scenario III: cooling/heating sequence as model inputs
Figure 3.25  Scenario III: evaporator air inlet temperature as model inputs

Figure 3.26  Scenario III: condenser air inlet temperature as model inputs

Table 3.2  Scenario III: refrigeration unit initial conditions

<table>
<thead>
<tr>
<th>Evaporator</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>395</td>
</tr>
<tr>
<td>Refrigerant inlet enthalpy</td>
<td>kJ/kg</td>
<td>89.04</td>
</tr>
<tr>
<td>Air inlet temperature</td>
<td>°C</td>
<td>0.16</td>
</tr>
<tr>
<td>Air mass flow rate</td>
<td>kg/s</td>
<td>0.7</td>
</tr>
<tr>
<td>Refrigerant outlet superheat</td>
<td>°C</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accumulator</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>382.36</td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>Kg</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compressor</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet pressure</td>
<td>kPa</td>
<td>287.47</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>kPa</td>
<td>1726.5</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>°C</td>
<td>11.23</td>
</tr>
<tr>
<td>Outlet temperature</td>
<td>°C</td>
<td>82.7</td>
</tr>
</tbody>
</table>
The plots in Figures 3.27-3.29 compare the experimental results with various model outputs. When the system operation switches from cooling to heating mode, superheated vapor exiting the compressor is redirected to enter the evaporator instead of the condenser coil, which results in heat exchanger pressure changes as seen in Figures 3.27 and 3.28. Due to the cycling input of the evaporator air inlet temperature (also the cargo space temperature) in Figure 3.25, the evaporator air outlet temperature increases to exceed the inlet temperature during heating mode and drops back to the initial condition after the system cools down, as shown in Figure 3.29.
The condenser and evaporator pressures are two important variables to determine the refrigeration system performance during cooling/heating mode switch cycles, and the model tracks the dynamics of the pressures quite well considering the large cycling transients and the complexity of the system. The predicted evaporator pressure responds slightly faster than the experimental curve in Figure 3.28, especially when the system operation switches from heating to cooling mode. This is due to the sudden decrease of the evaporator inlet refrigerant flow rate and the less accurate predictions of the refrigerant-side heat transfer rates during such large transients. As presented in Figure 3.29, there are some differences between the model output and the experimental measurement due to the thermal time constants of the temperature sensors and steady-state model deviations. However, their shapes and trends are similar.

![Figure 3.27 Scenario III: condenser pressure](image)

**Figure 3.27** Scenario III: condenser pressure

![Figure 3.28 Scenario III: evaporator pressure](image)

**Figure 3.28** Scenario III: evaporator pressure

Figure 3.30 describes the switching transitions between different model representations in the evaporator model structure (see Figure 2.8) for this validation scenario, where relevant switching criteria are discussed in Chapter 2. In cooling mode, the evaporator is the two-zone (two-phase and superheated) model. When the system mode switch occurs, the normalized length of the superheated vapor zone is tracked until it becomes less than the switching threshold (0.5% of the total evaporator tube length), and the evaporator switches to the one-zone two-phase
model, and then moves to the superheated and two-phase two-zone model for heating mode operation. When the system operation shifts back to cooling mode, a switch is triggered from the two-zone (superheated and two-phase) to the one-zone (two-phase) evaporator model, since the switching conditions described in Equations 2.27 and 2.28 are satisfied, and then the pseudo-states are applied to represent the dynamics of the inactive superheated inlet zone. Due to superheat regulation by the TXV in cooling mode, the superheated zone at the evaporator outlet becomes active and the evaporator model switches back to the two-zone (two-phase and superheated) representation. Three repeated cooling/heating mode switch cycles are presented in Figure 3.31 to highlight the dynamic switching in terms of the length traces of three different zones (superheated, two-phase and superheated) in the evaporator during transients. The zone lengths \( \zeta_e^0, \zeta_e^1 \) and \( \zeta_e^1 \) are dimensionless and normalized based on the total evaporator tube length to add up to one.

![Figure 3.29 Scenario III: evaporator air outlet temperature](image.png)

**Figure 3.29** Scenario III: evaporator air outlet temperature

![Figure 3.30 Scenario III: switching transitions in the evaporator](image.png)

**Figure 3.30** Scenario III: switching transitions in the evaporator
During cooling mode operation, the condenser is a two-zone (superheated and two-phase) model configuration due to the existence of the receiver tank, and maintains its model representation during the mode switch transients as shown in Figure 3.32. There is no refrigerant flow entering the condenser coil in heating mode, which leads to the decrease in length of the superheated vapor zone at the condenser inlet. Given longer durations for heating mode operation, possible switches to different condenser model representations (see Figure 2.9) could occur. Figure 3.33 presents the dynamic switching in terms of the mean void fraction transients in the switched accumulator model (see Figure 2.10). The accumulator switches from the two-phase mixture in heating mode to the superheated vapor model in cooling mode as the switching conditions given in Equations 2.58 and 2.59 are satisfied.

Figure 3.31  Scenario III: evaporator superheated/two-phase/superheated zone length trace in transients
As mentioned in Chapter 2, an advantage of the switched modeling techniques is the tracking of vapor and liquid refrigerant transients in the heat exchangers and the accumulator, which enables the calculation of refrigerant mass variations. The refrigerant mass distribution among the system components during the mode switch is explored based on the developed unit model, and three consecutive cycles are chosen again (see Figure 3.34) to illustrate the refrigerant mass migration in the high-side (condenser and receiver tank) and the low-side (evaporator and accumulator) components. When the system switches from cooling to heating mode operation, two-phase refrigerant flows through the suction line heat exchanger and enters the accumulator, which results in the sudden increase of refrigerant mass inside the accumulator as shown in Figure 3.34. During heating mode operation, around 15% of the refrigerant mass in
the high-side components migrates to the low-side due to the bleed port effect of the TXV. When the system switches back to cooling mode operation, the refrigerant mass is redistributed among different system components.

![Figure 3.34](image)

**Figure 3.34** Scenario III: refrigerant mass migration among system components in transients

### 3.3.3.2 Refrigerated Transport System Validation with Temperature Regulation

The model validation presented here, which is identical to the experimental study, includes a temperature pull-down and regulation test through cycling the system operation between cooling and heating mode. The temperature control objective is to maintain the cargo space temperature at the fresh set-point of -1.11°C with an allowable variation of +2.5/-1.0°C. The schematic of the refrigeration unit temperature control system is presented in Figure 3.4, and the ambient temperature as the system input is shown in Figure 3.35. The main cargo space model parameters used in this study are given in Table 3.3.
With the cooling/heating temperature control algorithm, the resulting actuator performance in terms of compressor speed and cooling/heating mode switch cycles is given in Figures 3.36 and 3.37 respectively. The plot in Figure 3.38(a) compares the cargo space temperature with the measured experimental data, and a closer comparison between time 10000 and 12000 seconds is given in Figure 3.38(b). As can be seen, the measured space temperature is drifting slightly below the set-point due to the varying cargo space thermal capacitance, which is not captured by the developed system model. However, overall the model predictions match the experimental results quite well in the time domain. The validation results further demonstrate the capability of the proposed dynamic modeling approach.
Figure 3.36  Scenario III: compressor speed for temperature pull down and regulation

Figure 3.37  Scenario III: cooling/heating mode switch cycles for temperature pull down and regulation

(a)
3.4 Simulation Case Study

The validation results described above indicate that the developed transport refrigeration model in Figure 3.2 is suitable for simulating the system operating characteristics during cooling/heating mode switch cycles. This section presents a case study on urban transport delivery to demonstrate the simulation capability in evaluating various models and scenarios. In the urban delivery application, customers typically have multiple stops with frequent door-openings over the course of the day, and cooling/heating mode switching is preferably chosen for temperature regulation [10]. This case study introduces two door-opening events during operation, and the duration times are 5 and 10 minutes, respectively, as shown in Figure 3.39. The refrigeration system is assumed to be in the ‘OFF’ condition during the door-opening event, and the cargo space temperature set-point is 0°C. Additionally, a constant ambient temperature (28°C) is applied here. The interested reader is referred to [92] for the validated refrigeration load calculations caused by the door-openings.
Figure 3.39  Case study: door opening events for simulation inputs

After each door-opening event, the refrigeration system restarts with high compressor speed to cool down the cargo space temperature to the set-point and then the system regulates the temperature with low speed, cycling between the cooling and heating mode of operation. The refrigeration unit inputs determined by the temperature control algorithm are shown in Figures 3.40 and 3.41, and the cargo space temperature performance is plotted in Figure 3.42. As illustrated, the presented modeling capability provides a tool for evaluating different scenarios, systems, and operation strategies prior to extensive experimental testing.

Figure 3.40  Case study: compressor speed input to refrigeration unit

Figure 3.41  Case study: cooling/heating mode switch cycles to refrigeration unit
Figure 3.42  Case study: cargo space temperature performance with effects of door-opening events
Chapter 4 Refrigerant Mass Migration

Refrigerant mass migration and redistribution are regarded as key factors affecting the cycling performance of air conditioning and refrigeration systems. In this chapter, an R134a automotive air conditioning experimental system is presented as an example to investigate the refrigerant migration during compressor shut-down (off) and start-up (on) operations. Using the switched modeling framework discussed in Chapter 2, a dynamic system model to simulate the refrigerant mass distribution behavior in on and off cycling transients is developed in Thermosys. Model validation against experimental data is presented to demonstrate the capabilities of the modeling approach in predicting the refrigerant mass migration among the components during shut-down, and the resulting refrigerant redistribution behavior in start-up. To further investigate the potential of the dynamic modeling tools, simulation case studies are described: (i) refrigerant mass migration prediction with multiple system on and off duty cycling operation; (ii) system configuration’s impacts on the refrigerant mass migration; and (iii) system start-up performance investigation.

4.1 Introduction

As discussed in Chapter 2, one common way for capacity control of air conditioning and refrigeration systems is by on-off cycling operation. These systems operate in a cycling mode in which they stop and start the refrigerant flow to modulate the amount of cooling or heating capacity provided to the enclosed environmental spaces. The correct prediction of refrigerant mass migration during the dynamic operation is paramount, since the refrigerant mass migration directly affects the system transient performance [52, 93].

The variations of refrigerant mass distribution in the on-off cycling transient conditions have been experimentally studied in different system applications, such as residential heat pump
systems [94-97], household refrigerator-freezers [98, 99], and automotive air conditioning systems [100]. All experimental results present the observation of refrigerant migration from the high-pressure components (i.e., condenser) to the low-pressure components (i.e., evaporator) during the compressor off period, and refrigerant redistribution from non-equilibrium state to steady-state refrigerant flow conditions across the mass flow components during the start-up. Cycling losses (defined as the energy consumption difference between a system with a continuously running compressor and a system with a cycling compressor, both having the same operating temperatures and the same cooling load [59]) caused by the on-off operation have been as well identified and quantified experimentally and analytically [59, 93, 95, 96, 98-105]. Among these studies, the refrigerant migration is recognized as the major contributor to affect the system transient and cycling performance. The off-cycle migration necessitates the refrigerant redistribution during the on-cycle, which reduces the system start-up performance, including a reduction in evaporator capacity and an increase in the system power requirements. Several possible designs to improve system cycling performance by controlling the refrigerant migration have been discussed in the literature [59, 96, 98, 100, 101, 105, 106]. Peuker and Hrnjak [100] reported a 28% reduction in compressor energy during the first 25 seconds of the start-up without cooling capacity losses when the refrigerant mass migration is prevented during the shut-down period.

Attempts have been made to model the system thermal dynamics (i.e., pressure, temperature, and refrigerant mass flow rate) with compressor start-up and shut-down operations [15, 18, 50, 59, 60, 62, 70-72, 75], but few results are available concerning the modeling of refrigerant migration behaviors. Murphy and Goldschmidt [70, 71] presented simulated refrigerant migration results in some partial system components during start-up or shut-down transients. The refrigerant mass transients after start-up have been simulated by Koury et al. [15], and Ozyurt and Egrican [107] compared the steady-state refrigerant mass distribution of the system model with experimental measurements. Hermes and Melo [60] developed a first-principles household refrigerator model to simulate the system transients and point out the model potential in refrigerant migration analysis during the cycling operation. Additionally, Cheng et al. [108] presented a validated observer design approach to estimate the immeasurable two-phase zone length in the evaporator during the start-up process, which can be used to track the
refrigerant mass distribution in transients. The objective of the study in this chapter is to simulate and validate the refrigerant mass distribution transients among the system components during the on-off cycling operation, and to provide industrial practitioners with dynamic modeling tools to predict system performance (i.e., refrigerant mass migration) at varying conditions. In the following sections, a dynamic model of an R134a automotive air conditioning system [100] is presented as an example to capture the refrigerant migration in compressor on and off cycling operation.

4.2 Experimental Facility and System

The facility used for experimental studies mentioned in this chapter, consists of two environmental chambers as shown in Figure 4.1. The compressor is installed between the two chambers and is driven by a speed controlled electrical motor. A torque transducer in combination with a tachometer is used to determine the compressor power. The condenser is installed in an open-loop wind tunnel inside the condenser chamber. In the evaporator chamber, the HVAC module containing the evaporator is attached to the wind tunnel and can therefore be considered part of the open-loop wind tunnel. The HVAC module flaps are set to intake fresh air for these experiments.

Before and after each heat exchanger, thermocouple grids consisting of welded Type-T thermocouples are used to determine the dry-bulb air temperatures. Air flow rates are determined by flow nozzles. The indoor and outdoor blowers exhaust the air directly into the chambers. An external R404A chiller system with the evaporator mounted on the ceiling inside the condenser chamber compensates for the heat rejected by the condenser. PID-controlled electric heaters are installed in both chambers to heat the room to the specified test conditions.
The system used for transient modeling and validation is a state of the art R134a automotive air conditioning system consisting of the following components: compressor, condenser, fixed orifice tube, evaporator, and accumulator. The components are installed into the experimental facility in Figure 4.1 with the same difference in vertical height as in the vehicle. The schematic of the experimental system is shown in Figure 4.2. The pipe lengths are comparable to the original vehicle system with the exception of the liquid tube section. Due to the arrangement of the calorimetric chambers, the liquid tube for the breadboard system is 3.7 times longer than the liquid tube of the real vehicle system. As seen in Figure 4.2, ball valves are installed on either side of each component and, by closing these ball valves simultaneously, the refrigerant mass can be trapped in each section. The ball valves are chosen because of their very low flow resistance, and they are installed as close as possible around the components. Table 4.1

Figure 4.1  Schematic of the experimental facility including breadboard system [52]
contains physical parameters of the system components, and complete descriptions about the experimental system set-up and component specifications are given in [52].

![Image of the experimental system]

**Figure 4.2 Schematic of the experimental system**

**Table 4.1 Component physical parameters**

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Condenser</strong></td>
<td>Inlet header volume</td>
<td>0.0001332</td>
<td>m³</td>
</tr>
<tr>
<td></td>
<td>Internal volume</td>
<td>0.0004271</td>
<td>m³</td>
</tr>
<tr>
<td></td>
<td>Outlet header volume</td>
<td>0.0001332</td>
<td>m³</td>
</tr>
<tr>
<td><strong>Evaporator</strong></td>
<td>Inlet header volume</td>
<td>0.0001926</td>
<td>m³</td>
</tr>
<tr>
<td></td>
<td>Internal volume</td>
<td>0.0005372</td>
<td>m³</td>
</tr>
<tr>
<td></td>
<td>Outlet header volume</td>
<td>0.0001486</td>
<td>m³</td>
</tr>
<tr>
<td><strong>Accumulator</strong></td>
<td>Volume</td>
<td>0.001331</td>
<td>m³</td>
</tr>
<tr>
<td><strong>Liquid tube</strong></td>
<td>Fluid flow length</td>
<td>4.91</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Hydraulic diameter</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td><strong>Fixed orifice</strong></td>
<td>Internal diameter</td>
<td>0.001823</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Tube Length</td>
<td>0.0762</td>
<td>m</td>
</tr>
<tr>
<td><strong>Compressor</strong></td>
<td>Volume</td>
<td>0.00022</td>
<td>m³</td>
</tr>
<tr>
<td></td>
<td>Fixed displacement</td>
<td>0.0002147</td>
<td>m³/rev</td>
</tr>
<tr>
<td><strong>Pipes</strong></td>
<td>Compressor-to-condenser length</td>
<td>2.96</td>
<td>m</td>
</tr>
</tbody>
</table>
Peuker and Hrnjak [100] experimentally investigated the refrigerant mass distribution among the system components during the compressor shut-down and start-up period using the quick-closing valve method and refrigerant recovery techniques. The accuracy and repeatability of the refrigerant mass measurement approach were verified. Moreover, a transparent accumulator, along with transparent inlet and outlet tubes, was used to visualize the refrigerant flow during the transient. The published experimental data are used for the validation of the dynamic model presented in this chapter. The reader is encouraged to refer to [52, 100] for further details regarding the refrigerant mass measurement method, experimental results and analysis.

4.3 System Modeling

This section is divided into two parts. Using the switched modeling framework discussed in Chapter 2, the component models are introduced for simulating the system cycling transients in the simulation platform Thermosys presented in Chapter 3, and the liquid tube model is given particular attention. The second part presents methods to calculate the refrigerant mass in each component during transients.

4.3.1 Component Modeling

The automotive air conditioning system as shown in Figure 4.2 is subdivided into five components, each of which is briefly discussed below.

*Heat exchangers.* The condenser and evaporator components are developed with different model representations to accommodate the transitions of dynamic states, and keep track of the vapor and liquid refrigerant phase changes during on and off cycling transients. Specifically,
Four model representations are applied here to describe the condenser dynamics: the three-zone model (superheated, two-phase and subcooled), the two-zone model (superheated and two-phase), the one-zone model (two-phase), and another one-zone model (superheated) in Figure 2.9. The evaporator model consists of two model presentations as can be seen in Figure 2.8: the one-zone model (two-phase), and the two-zone model (two-phase and superheated).

**Accumulator.** The accumulator is a key component in the automotive air conditioning system. Its primary function is to store excess refrigerant mass to ensure system capacity over a large range of operating conditions. The two-phase mixture accumulator model in Figure 2.10 is used with governing equations 2.56 and 2.57.

**Liquid tube.** The liquid tube is treated as a separate component, since it involves dramatic refrigerant flow dynamics in system shut-down and start-up transients as discussed in [70, 71]. The authors in [70, 71] chose two different approaches to model the liquid tube: a discretized volume approach in shut-down and a tank model method at start-up transients. In this section, a simplified switched liquid tube model with three different representations, shown in Figure 4.3, is developed to represent the refrigerant migration, and the refrigerant pressure drop along the liquid tube is neglected. The refrigerant-side mass and energy governing equations in each model representation can be described in Equations 4.1 and 4.2, which are similar to those in the accumulator model development (see Equations 2.56 and 2.57). The difference in the governing equations for each representation is the exiting refrigerant enthalpy $h_{o,lt}$ in Equation 4.2. In the two-phase liquid-vapor mixture model, the exiting refrigerant enthalpy is calculated based on the entering refrigerant enthalpy $h_{i,lt}$ and the mean void fraction $\bar{\gamma}_{lt}$, while the exiting enthalpy is assumed to be the average enthalpy $h_{lt}$ in the superheated vapor and subcooled liquid model representations.

\[
\frac{\delta \rho_{lt}}{\delta P_{lt}} \frac{dP_{lt}}{dt} + \frac{\delta \rho_{lt}}{\delta h_{lt}} \frac{dh_{lt}}{dt} = \frac{\dot{m}_{i,lt} - \dot{m}_{o,lt}}{V_{lt}} \tag{4.1}
\]

\[
-\frac{1}{\rho_{lt}} \frac{dP_{lt}}{dt} + \frac{d\rho_{lt}}{dt} = \frac{(UA)_{lt}(T_{amb} - T_{lt}) + \dot{m}_{i,lt}(h_{i,lt} - h_{lt}) - \dot{m}_{o,lt}(h_{o,lt} - h_{lt})}{\rho_{lt}V_{lt}} \tag{4.2}
\]
Figure 4.3 Switched liquid tube model structure

Similar to the accumulator model switching conditions discussed in Chapter 2, the mean void fraction $\overline{\gamma}_h$, which is a function of refrigerant pressure $P_h$ and enthalpy $h_h$, determines the model switching criteria between liquid tube model representations. For example, the switch occurs from the two-phase mixture to the superheated vapor model (see Figure 4.3) when the switching conditions, as given in Equations 4.3 and 4.4, are satisfied.

$$\overline{\gamma}_h > 1$$  \hspace{1cm} (4.3)

$$\left. \frac{dh_h}{dt} \right| > 0$$  \hspace{1cm} (4.4)

*Fixed orifice tube and compressor.* These two mass flow devices are considered to be “static” component models. As presented in [70, 71], the mass flow rate across the fixed orifice tube influences the refrigerant pressure in both heat exchangers as well as the refrigerant mass distribution during the cycling transients. Therefore, an accurate valve model is necessary. Equation 4.5 is used to calculate the orifice mass flow rate, where the flow coefficient $C_f$ is determined via a semi-empirical mapping approach [109], and the refrigerant density $\rho_v$ is assumed to be a function of the refrigerant inlet conditions (i.e., liquid-vapor mixture, or vapor) to the orifice tube, as given in Equation 4.6. Equation 2.63 is applied for the compressor mass
flow rate computation, where the volumetric efficiency $\eta_{vol}$ is found using a performance mapping approach similar to the one given in [109].

$$\dot{m}_v = C_f \sqrt{P_v (P_{in} - P_e)}$$  \hspace{1cm} (4.5)

$$\rho_v = \rho \left( P_{in}, h_{i,v} \right)$$  \hspace{1cm} (4.6)

### 4.3.2 Refrigerant Mass Calculation

Approaches are explored here to evaluate the refrigerant mass distribution behaviors in the on-off cycling operation. Poggi et al. [110] presented a summary of available refrigerant mass calculation methods in the literature. The authors in [110] also pointed out that the major challenge is the mass evaluation in two-phase components, such as heat exchangers. For general numerical simulation, the refrigerant mass migration in each system component can be obtained by Equation 4.7 with known refrigerant inlet and outlet mass flow rate conditions. An alternative way to evaluate the refrigerant mass distribution in heat exchangers during transients is using the mean void fraction $\bar{\gamma}_e$ and $\bar{\gamma}_t$, which is an integral form of a local void fraction [79, 111]. The Zivi void fraction correlation [82] is applied in this study, since it shows a good agreement with measurements as reported in [112, 113]. The two-phase and single-phase (vapor or liquid) refrigerant mass calculations in the heat exchangers are given in Equations 4.8 and 4.9.

$$M_{\text{component}} = M_{\text{steady-state}} + \int (\dot{m}_i - \dot{m}_o) \, dt$$  \hspace{1cm} (4.7)

$$M_{\text{two-phase.exchangers}} = \left( \bar{\gamma}_g (1 - \bar{\gamma}) \rho_f \right) \zeta_{\text{two-phase exchangers}}$$  \hspace{1cm} (4.8)

$$M_{\text{single-phase.exchangers}} = \rho_{\text{single-phase}} \zeta_{\text{single-phase exchangers}}$$  \hspace{1cm} (4.9)

The total refrigerant mass in the automotive air conditioning system (see Figure 4.2) is described below:

$$M_{\text{total}} = M_{\text{exchangers}} + M_{\text{accumulator}} + M_{\text{liquid tube}} + M_{\text{compressor}} + M_{\text{pipes}}$$  \hspace{1cm} (4.10)

where the calculation of the single-phase refrigerant mass in components, such as superheated vapor in pipes, is based on the refrigerant density and component volume information.
4.4 Model Validation

As mentioned earlier, the refrigerant mass migration experimental data presented in [100] are used for transient validation in this section. The model validation scenario, which is identical to the experimental scenario in [100], includes shut-down and start-up step changes in compressor speed (see Figure 4.4) while maintaining the dry air flow rates across the heat exchangers. The interested reader is referred to [52] for the experimental study regarding the effect of water vapor on the refrigerant migration during the cycling operations. The steps in system inputs for the model validation here are summarized in Table 4.2 along with the condenser and evaporator air inlet temperature conditions. The compressor shut-down time period is 3 minutes. The operating conditions before system input changes, including the refrigerant mass distribution among the components, are listed in Table 4.3.

![Figure 4.4 Model validation: compressor speed input](image)

**Table 4.2 Model validation: system inputs**

<table>
<thead>
<tr>
<th>Input</th>
<th>Step time for shut-down</th>
<th>Before shut-down</th>
<th>Step time for start-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor speed</td>
<td>0s</td>
<td>900 rpm</td>
<td>180s</td>
</tr>
<tr>
<td>Cond. air mass flow rate</td>
<td></td>
<td>0.525 kg/s</td>
<td></td>
</tr>
<tr>
<td>Evap. air mass flow rate</td>
<td></td>
<td>0.156 kg/s</td>
<td></td>
</tr>
<tr>
<td>Cond. air inlet temperature</td>
<td></td>
<td>35°C</td>
<td></td>
</tr>
<tr>
<td>Evap. air inlet temperature</td>
<td></td>
<td>35°C</td>
<td></td>
</tr>
</tbody>
</table>
### Table 4.3 Model validation: operating conditions

<table>
<thead>
<tr>
<th>Section</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Evaporator</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>340.15</td>
</tr>
<tr>
<td>Inlet enthalpy</td>
<td>kJ/kg</td>
<td>118.92</td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>kg</td>
<td>0.16852</td>
</tr>
<tr>
<td><strong>Accumulator</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>334.24</td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>kg</td>
<td>0.18272</td>
</tr>
<tr>
<td><strong>Compressor</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inlet pressure</td>
<td>kPa</td>
<td>328.33</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>kPa</td>
<td>1345.12</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>°C</td>
<td>4.92</td>
</tr>
<tr>
<td>Outlet temperature</td>
<td>°C</td>
<td>62.34</td>
</tr>
<tr>
<td>Refrigerant mass flow rate</td>
<td>kg/s</td>
<td>0.034</td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>kg</td>
<td>0.05974</td>
</tr>
<tr>
<td><strong>Condenser</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>1341.38</td>
</tr>
<tr>
<td>Refrigerant inlet temperature</td>
<td>°C</td>
<td>63</td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>kg</td>
<td>0.2196</td>
</tr>
<tr>
<td><strong>Liquid tube</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refrigerant mass</td>
<td>kg</td>
<td>0.36396</td>
</tr>
<tr>
<td><strong>Fixed orifice tube</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inlet pressure</td>
<td>kPa</td>
<td>1312</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>kPa</td>
<td>340.15</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>°C</td>
<td>47.07</td>
</tr>
<tr>
<td>Refrigerant mass flow rate</td>
<td>kg/s</td>
<td>0.034</td>
</tr>
</tbody>
</table>

By closing the ball valves shown in Figure 4.2 simultaneously, the refrigerant masses are kept in five sections: compressor, condenser, liquid tube, evaporator and accumulator. The refrigerant mass distribution in each section is measured at the following times during the transient scenario: 0 (compressor turns off), 5, 10, 20, 60, 180 (compressor turns on), 185, 190,
200, 220, 240 and 480. All times are in seconds. A total refrigerant mass of 1000g is used for the experimental study, and the pressure and temperature measurements are taken every 1.5 seconds.

The plots in Figures 4.5-4.9 compare the experimental results with various system model outputs. During the off cycle, the refrigerant migrates from the high-pressure to the low-pressure components through the fixed orifice tube, which results in the heat exchanger pressure changes as seen in Figures 4.5 and 4.6. The evaporator air outlet temperature, as shown in Figure 4.7, increases towards the ambient temperature (35°C) as the thermal mass of the heat exchanger and refrigerant mass inside approach an equilibrium condition. In addition, the system model predicts the transients of the refrigerant mass flow rate across the compressor at start-up shown in Figure 4.9.

Figure 4.5  Model validation: condenser pressure

Figure 4.6  Model validation: evaporator pressure

Figure 4.7  Model validation: evaporator air outlet temperature
Figure 4.8  Model validation: evaporator refrigerant outlet temperature

Figure 4.9  Model validation: refrigerant mass flow rate across the compressor

The total refrigerant mass comparison between experimental measurement and system model prediction is given shown in Figure 4.10. The total refrigerant mass in the experimental system differs up to 2.7% from the target refrigerant mass of 1000g as a result of the measurement technique used. The amount of refrigerant mass is maintained in the transient simulation, which further proves the validity of the switched modeling approach developed in Chapter 2. There are three main reasons accounting for the approximately 10% mass prediction deviation:

- The volume of heat exchanger headers is neglected in the modeling process.
- The refrigerant mass in the compressor $M_{\text{compressor}}$ (see Equation 4.10) is neglected in the total refrigerant mass calculation, since there is little change to the refrigerant amount in the compressor [97, 100].
- The impact of choices of void fraction correlations on the refrigerant mass evaluation is discussed in [110, 113], and the Zivi correlation [82] used here underestimates the refrigerant mass against the experimental results with a maximal deviation of 10%.
The refrigerant migration during the transient scenario is described in Figures 4.11 and 4.12. The experimental results show that before the system turns off, 58% of the total refrigerant mass is located in the high-pressure components (condenser and liquid tube). Three minutes after the compressor is stopped, only 11% of the total refrigerant mass is found in these components (see Figure 4.11). It can also be seen that the majority of the mass migration takes place in the first minute after system shut-down, and the refrigerant redistribution during start-up is almost completely achieved after one minute [100]. To quantify the model performance, a mass prediction error method, as computed in Equation 4.11 with the root mean square (RMS) value, is applied here. N=12 is the number of data samples available. Approximately 8% error is found in predicting the refrigerant mass migration in the high-pressure components, and the prediction error is 4% for the migration in the low-pressure components as shown in Figure 4.12.

$$E_{\text{error}} = \frac{1}{M_{\text{total}}} \sqrt{\frac{1}{12} \sum_{k=1}^{12} (M_{\text{model},k} - M_{\text{experiment},k})^2} \quad (4.11)$$

**Figure 4.10** Model validation: total refrigerant mass in the system

**Figure 4.11** Model validation: refrigerant mass migration in the high-pressure components
Figure 4.12  Model validation: refrigerant mass migration in the low-pressure components

Figure 4.13  Model validation: switching schemes in the condenser

Figure 4.13 describes the dynamic switching of different model representations inside the condenser model for this validation scenario, where relevant switching criteria are discussed in Chapter 2 in this dissertation and in [18]. The condenser model switches from the initial three-zone (superheated, two-phase and sub-cooled) model to the final one-zone (superheated) model during the off cycle. The vapor and liquid refrigerant transients are tracked through the model switching techniques, which enables the calculation of refrigerant mass variations using Equations 4.8 and 4.9. The refrigerant mass migration in the condenser component is given in Figure 4.14, while Figure 4.15 presents the refrigerant migration inside the liquid tube component. Two minutes after system shut-down, the liquid tube is filled with superheated...
vapor, which agrees with the experimental observation [100]. The initial high compressor mass flow rate (see Figure 4.9) and low orifice tube mass flow rate explain the over-shoot of the refrigerant mass migration in the condenser in the first 20 seconds of the start-up.

![Figure 4.14 Model validation: refrigerant mass migration in the condenser](image1)

![Figure 4.15 Model validation: refrigerant mass migration in the liquid tube](image2)

As shown in Figure 4.12, prior to system start-up, over 80% of the total refrigerant mass is located in the low-pressure components (evaporator and accumulator). The experimental study [100] shows that the evaporator is starved 5 seconds after the start-up operation, which coincides with the occurrence of superheated vapor at the evaporator outlet, since the relatively warm thermal mass of the evaporator leads to the evaporation of the refrigerant inside, and the refrigerant mass is migrated towards the condenser before returning to the steady-state distribution as described in Figure 4.14. Similar phenomena are observed in [98, 102], and Figure 4.16 highlights the dynamic switching in terms of the length traces of two different zones in the evaporator model. The evaporator model stays in the one-zone (two-phase) model representation (see Figure 2.8) during the shut-down transients, where the normalized length of the superheated vapor zone is set to be 0.5% of the total evaporator tube length, and the pseudo-states discussed in Chapter 2 are applied to represent the dynamics of this inactive zone.
switch occurs from the one-zone to the two-zone (two-phase and superheated) model after start-up, since the switching conditions described in Equations 2.23 and 2.24 are satisfied and the superheated zone becomes active. The normalized superheated zone length is tracked until it becomes less than the switching threshold (0.5%), and then the two-zone evaporator model representation switches back to the one-zone model for steady-state conditions, as presented in Figure 4.16. The zone lengths are dimensionless and normalized based on the total evaporator tube length to add up to one.

The accumulator holds 56% of the total refrigerant mass before start-up, and Figure 4.17 presents the refrigerant mass migration inside the accumulator. During the off period, the pressure gradient drives the refrigerant migration from the evaporator to the accumulator component in the first 60 seconds, and the migration continues as a result of the temperature gradient [95, 96, 100, 102]. During the first 20 seconds of the system start-up, 35% of the total refrigerant mass leaves the accumulator. Real time videos were taken in the experimental study [52], and snapshots of the accumulator during transients are shown in Figure 4.18.

![Figure 4.16 Model validation: evaporator two-phase and superheated zone length trace in transients](image)
4.5 Simulation Case Studies

The potential of the presented dynamic modeling tools to simulate the refrigerant mass migration can be investigated in the following aspects:

- **Performance evaluation.** With the developed modeling approach, it is possible to evaluate the refrigerant mass migration for different operating conditions (i.e., ambient temperature, different refrigerant fluids) and therefore predict the cycling performance of current air conditioning and refrigeration systems. Other objectives, such as different component designs, can be evaluated for transient scenarios.

- **Control implementation.** As discussed in this chapter, the refrigerant migration and redistribution have been identified as a major reason for the cycling losses. However, it is not always feasible to improve the system performance by completely avoiding the migration during the off period [105]. Therefore, it is important to investigate various means of operating the on and off cycles to both meet performance requirements and simultaneously manage refrigerant distribution. This balance can be achieved by adjusting the timing or duration of the on and off cycles. Another
interesting option is the variation in the start-up and shut-down profiles to include ramping up and down the compressor speed.

In this section, three simulation scenarios are presented: (i) refrigerant mass migration prediction with multiple system on and off duty cycling operation; (ii) component parameter variation’s impacts on the refrigerant mass migration; and (iii) system start-up performance investigation.

4.5.1 System On and Off Duty Cycling Operation

The scenario consists of three compressor on and off cycles (see Figure 4.19) while maintaining the air flow conditions across the heat exchangers. The first compressor on and off cycle operation is identical to the model validation scenario presented above, and the system initial operating conditions for simulation are the same as those in Table 4.3. The three compressor off time periods are 3 minutes, 2 minutes and 1 minute respectively.

![Figure 4.19 Simulation case study I: compressor speed input](image)

The plots in Figures 4.20-4.22 show the refrigerant mass migration in the system components during transients. The experimental data for the first on and off cycle is given for comparisons, since this is the only cycle data collected from the experimental study. The total refrigerant mass calculated from the system model is maintained constant under the compressor duty cycling operation, as described in Figure 4.23.
Figure 4.20  Simulation case study I: refrigerant mass migration in the high-pressure components

Figure 4.21  Simulation case study I: refrigerant mass migration in the low-pressure components

Figure 4.22  Simulation case study I: refrigerant mass migration in the accumulator.

Figure 4.23  Simulation case study I: total refrigerant mass in the system.
4.5.2 Liquid Tube Length Variation

Due to the arrangement of the experimental facility, the liquid tube for the experimental study is 3.7 times longer than the liquid tube of the real vehicle system, as shown in Figure 4.1. The dynamic characteristics of the real system can be represented using the dynamic model described above using the length of the liquid tube as in an actual vehicle system. The compressor speed cycle in Figure 4.4 is applied here. The refrigerant mass migration in the liquid tube in the experimental system (called ‘experimental system’) is compared to the system model prediction with the shorter liquid tube (called ‘predicted system’), as shown in Figure 4.24. Figure 4.25 presents the refrigerant mass migration in the low-pressure components during the compressor off period. As presented in Figure 4.24, it takes two minutes after the system turns off until the liquid tube in the experimental system is completely filled with superheated vapor. By comparison, it takes 20 seconds with the shorter liquid tube based on the predicted system model results. As a result, the refrigeration mass migration equilibrium is achieved more quickly for the shorter liquid tube (see Figure 4.25).

![Figure 4.24 Simulation case study II: comparison of the refrigerant mass migration in the liquid tube during the off period](image-url)

![Figure 4.25 Simulation case study II: comparison of the refrigerant mass migration in the low-pressure components during the off period](image-url)
4.5.3 Ramping Compressor Speed at Start-Up

In this simulation example, the performance variations are investigated if the compressor is speed is increased linearly during the start-up. The baseline case has a step change of the compressor speed as shown in Figure 4.4. Three other cases are presented for comparisons: ramping the compressor speed from 0 to 900 rpm in 10, 30 and 90 seconds. The start-up compressor speeds are shown in Figure 4.26. The resulting refrigerant mass migration in the condenser is presented in Figure 4.27. In the baseline case, the initially high compressor mass flow rate (see Figure 4.9) and low orifice tube mass flow rate explain the overshoot of the refrigerant mass migration in the condenser in the first 20 seconds of the start-up. As can be seen in Figure 4.27, the overshoot phenomenon becomes less obvious with an increase in compressor speed ramping time.

![Figure 4.26 Simulation case study III: compressor speed input for ramping up cases](image)

**Figure 4.26** Simulation case study III: compressor speed input for ramping up cases

![Figure 4.27 Simulation case study III: refrigerant mass migration in the condenser for ramping up cases](image)

**Figure 4.27** Simulation case study III: refrigerant mass migration in the condenser for ramping up cases

Figure 4.28 presents the non-dimensional air-side cooling capacity \( \dot{Q}_{air} / \dot{Q}_{ss} \) during the system start-up, where \( \dot{Q}_{air} \) is the air-side cooling capacity as a function of time and \( \dot{Q}_{ss} \) represents the air-side cooling capacity at a steady-state operating condition. It can be seen that
ramping the compressor speed results in slower cool-down times. The bar graph in Figure 4.29 describes the tradeoff between energy consumption and energy efficiency for the different start-up cases. The compressor energy consumption $W_{\text{comp}}$ and cooling energy $Q_{\text{air}}$ are calculated using Equations 4.12 and 4.13, and the ratio $Q_{\text{air}}/W_{\text{comp}}$ represents the energy efficiency during the system start-up. The integration time applied here is the first 100 seconds of the start-up. As shown in Figure 4.29, an increase in ramping time results in a more energy efficient start-up performance (the ratio increases from 2.08 in the baseline case to 2.25 in the ramp-30s case and 2.78 in the ramp-90s case), at the expense of the cooling energy reduction provided to the passenger compartment (the cooling energy drops from 285.47 kJ in the baseline case to 246 kJ in the ramp-30s case and 177.65 kJ in the ramp-90s case). The model prediction results here are in good agreement with the experimental ramping study presented in [52] which determines a ratio of 2.04 for the baseline case and 2.33 for the 30 second compressor speed ramping case.

\[
W_{\text{comp}} = \frac{\int_{0}^{100} m_k (h_{\theta,k} - h_{i,k}) dt}{\eta_{\text{isentropic}}}
\]

\[
Q_{\text{air}} = \int_{0}^{100} \dot{Q}_{\text{air}} dt = \int_{0}^{100} m_{\text{air},c} c_{\text{air}} (T_{\text{i,air,c}} - T_{\text{o,air,c}}) dt
\]

**Figure 4.28** Simulation case study III: air-side cooling capacity during start-up for ramping up cases
Figure 4.29  Simulation case study III: tradeoff between energy consumption and efficiency during start-up for ramping up cases
Chapter 5  Optimal On-Off Control

As previously discussed in this dissertation, one common temperature control approach in refrigerated transport systems is a simple on-off feedback controller with hysteresis [13, 46]. Current practice usually involves the experimental tuning of the hysteresis settings to maintain some desired thresholds for temperature regulation. This tuning is done either in the field on test systems or in test cells at dedicated R&D facilities. This chapter presents a hysteretic on-off control scheme with optimization algorithms for temperature regulation in the refrigerated transport system applications.

A nonlinear simplified refrigerated transport system model, which consists of a vapor compression cycle and a cargo space, is introduced first to serve as an analytical tool for control design. An optimal on-off control strategy is developed based on the time domain analysis of the temperature oscillations. A cost function involving temperature variations from the set-point, energy consumption and average compressor on-off cycling frequency is described for minimization. Choices of weighting values in the cost function give the flexibility of the control approach to meet different requirements. Simulation examples are shown that demonstrate the ability and robustness of the presented method in achieving temperature regulation and system efficiency control in the transport system applications. Finally, the simulations are augmented by experimental validation via a novel hardware-in-the-loop load emulation approach.

5.1 Introduction

In the hysteretic on-off control scheme [114], the compressor’s on or off control action is driven by the difference between a measured cargo space temperature and a given temperature set-point. Therefore the cargo space temperature oscillates around the set-point. The amplitude
and frequency characteristics of the temperature oscillation cycles influence the system performance variables including: temperature variations for food preservation, mechanical component wear and energy consumption. The limitations of the on-off control with regard to system variability are well explained in Leva et al. [115] and Deng et al. [116]. Leva et al. [115] presented an adaptive on-off control scheme for a household freezer system by adding a linear filter with adaptive gains in the loop to modify the oscillation characteristics to meet specific requirements. The adaptive control method is shown to be sufficiently simple to be implemented on available microcontrollers and is able to improve the performance of the dynamic system. Deng et al. [116] compared different on-off switching control methods associated with costs, and presented an on-off control method with low complexity and computational requirements. Through minimizing the cost function involving the temperature variations and the compressor wear, the optimal temperature cycle period and duty ratio are obtained to control the energy input to a first-order room model. Predictive control is also involved in the on-off switching actuation studies. For example, Ricker [117] formulated a linear time-invariant (LTI) model-based predictive control structure for the on-off actuators to maintain the display case temperature within constraints while simultaneously minimizing the compressor cycling frequency in a supermarket refrigeration process. Additionally, an optimal on-off control strategy was implemented in a chilled water cooling system application by Jian and Zaheeruddin [118], where the chilled water temperature and the chiller energy input were utilized in the cost function.

The study in this chapter develops an optimal on-off control scheme based on a nonlinear system model to provide optimal temperature hysteresis settings. Furthermore, this chapter explicitly illustrates the effectiveness of the scheme on temperature regulation as well as its impact on reducing energy consumption and component wear. With the developed control framework, the practicing control designers can rapidly narrow down potential choices of hysteresis set-points in simulation before moving to expensive test-cell experiments, which will reduce the commissioning time and operation costs. Additionally, controllers can be co-developed for new system configurations prior to committing to hardware. This affords upstream design flexibility that is very useful to advanced product development groups.
5.2 System Model

A complete refrigerated transport system model is described as seen in Figure 2.1 in Chapter 2. The system transients under compressor on and off cycling operation are validated against experimental measurements as presented in Chapter 3, which proved the validity of the modeling framework. The objective of this study is to demonstrate the potential of the developed on-off control scheme on the refrigerated transport applications; therefore, the control design is carried out on the simplified transport system model as depicted in Figure 5.1. The system consists of a basic vapor compression cycle (VCC) and a refrigerated cargo space. The cargo space is coupled to the VCC system, and the return air temperature, also considered to be the cargo space temperature, is used to drive the compressor in an on-off cycling fashion. As mentioned previously, the heat exchanger fans are considered to cycle with the compressor.

![Figure 5.1 A simplified refrigerated transport system](image)

Using the validated modeling approaches presented in Chapter 2, the transport system model is implemented in Thermosys, and the nonlinear VCC model along with the dynamic state information is summarized in Figure 5.2. The physical parameters from the experimental system in Figure 3.5 are used to develop the VCC model, and the major parameters of the refrigerated cargo space model applied in this study are listed in Table 5.1.
5.3 On-Off Control Algorithm

5.3.1 Basic Hysteretic On-Off Control

Due to the prohibitive cost constraints associated with continuously variable compressor speed control, the compressor on-off speed control approach is still by far the most common paradigm in the refrigerated transport systems. The cooling capacity provided to the cargo space is modulated by turning on or off the subsystem driving the compressor; this may either be an electric motor, an engine, or a clutched belt system driven off the vehicle’s main engine. The control principle is usually a simple hysteresis loop. The block diagram of such a hysteretic feedback control system is given in Figure 5.3, and the typical cargo space temperature performance based on this approach is presented in Figure 5.4. The hysteresis parameters
determine the size of the temperature oscillation ($T_{\text{high}}$ and $T_{\text{low}}$) around the given set-point $T_0$. The limitation of this basic on-off control is its inability to regulate the temperature oscillation amplitudes upon changing conditions, such as ambient temperature and varying food temperature requirements. Should tighter temperature control or higher system efficiency be desired, it may be necessary to consider other control approaches.

![Figure 5.3 Basic hysteretic on-off feedback control system](image)

**Figure 5.3** Basic hysteretic on-off feedback control system

![Figure 5.4 Basic hysteretic on-off control system performance](image)

**Figure 5.4** Basic hysteretic on-off control system performance

### 5.3.2 Optimal Hysteretic On-Off Control

#### 5.3.2.1 Time Period Function

As shown in Figure 5.4, one temperature oscillation cycle around the temperature set-point $T_0$ can be divided into four time periods, $t_1$, $t_2$, $t_3$, and $t_4$. The time $t_0$ is the starting time for
one oscillation cycle. According to the hysteresis loop principle, the time periods \( t_1 \) and \( t_4 \) represent the compressor off time, while the compressor turns on during the time periods \( t_2 \) and \( t_3 \) to cool down the enclosed cargo space. For a given transport system (see Figure 5.1), the time periods are functions of the hysteresis parameters and the ambient external conditions, as presented in Equations 5.1-5.4 where the ambient temperature \( T_{amb} \) is taken into account. Particularly, for a prescribed ambient condition, the time period \( t_2 \) depends on the temperature upper bound \( T_{high} \) and the time period \( t_4 \) is calculated from the temperature lower bound \( T_{low} \). The reader is encouraged to examine Appendix A for the derivations of the time period functions for the transport system example in Figure 5.1.

\[
\begin{align*}
    t_1 &= t_1(T_{high}, T_{low}, T_{amb}) \\ \\
    t_2 &= t_2(T_{high}, T_{amb}) \\ \\
    t_3 &= t_3(T_{high}, T_{low}, T_{amb}) \\ \\
    t_4 &= t_4(T_{low}, T_{amb})
\end{align*}
\]

### 5.3.2.2 Cost Function

A key contribution of the study in this chapter is the formulation of the hysteretic feedback as an optimal on-off control problem via the definition of a cost function for minimization. The cost function, developed based on the time-domain results presented in Figure 5.4, is composed of three parts: temperature performance, energy consumption, and average compressor on-off cycling times. The last element is indicative of wear on the VCC equipment. The approach of calculating the accumulated food quality loss [5] is applied here to evaluate the cargo space temperature performance. Specifically, the areas above and below the given temperature set-point \( T_0 \), noted as

\[
\int_{t_0}^{t_0+t_1+t_2} \left( T_{space}(t) - T_0 \right) dt \quad \text{and} \quad \int_{t_0}^{t_0+t_1+t_2+t_3+t_4} \left( T_0 - T_{space}(t) \right) dt
\]

(see Figure 5.4), are defined to describe the temperature performance around the desired set-point in one temperature oscillation cycle. Recalling that the time periods \( t_2 \) and \( t_3 \) represent the compressor on time, the energy consumed by the compressor for one temperature cycle is

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defined as \[ \int_{t_0}^{t_0+t_1+t_2+t_3} (V_{rpm} N_{torque}(t)) \, dt \] [119], where \( V_{rpm} \) is the compressor on speed and \( N_{torque} \) is the compressor shaft torque. Additionally, the average compressor on-off cycling times in a given time period \( H \) is described as \( \frac{H}{t_1+t_2+t_3+t_4} \). Therefore, the cost function \( J \) can be written in Equation 5.5 by combining these three parts. Three weights, \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \), are introduced in the cost function and described further in Table 5.2.

\[
J = \sum_{1}^{n} \left[ \begin{array}{c}
\alpha_1 \left( \int_{t_0}^{t_0+t_1+t_2} (T_{space}(t) - T_0) \, dt \right) + \\
\alpha_2 \left( \int_{t_0+t_1+t_2+t_3}^{t_0+t_1+t_2+t_4} (T_0 - T_{space}(t)) \, dt \right) + \\
\alpha_3 \left( \int_{t_0+t_1}^{t_0+t_1+t_2+t_3} (V_{rpm} N_{torque}(t)) \, dt \right) + \\
\frac{H}{t_1+t_2+t_3+t_4} 
\end{array} \right]
\]

(5.5)

<table>
<thead>
<tr>
<th>Weight</th>
<th>Function</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>to influence the temperature performance above the given set-point ( T_0 )</td>
<td>( \text{kJ}^{-1}\cdot\text{K}^{-1} )</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>to influence the temperature performance below the given set-point ( T_0 )</td>
<td>( \text{kJ}^{-1}\cdot\text{K}^{-1} )</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>to penalize the average compressor on-off cycling times</td>
<td>N/A</td>
</tr>
</tbody>
</table>

In order to compute the integrals of the temperature performance terms in Equation 5.5, a piecewise linear function is applied to approximate the temperature oscillation trajectory, and the integrals can be estimated as triangular areas, as shown in Equations 5.6 and 5.7. The effects of such approximations on the system performance are discussed in the following sections.
\[
\int_{t_0}^{t_0 + t_1 + t_2} (T_{space}(t) - T_0) \, dt \approx \left( \frac{T_{high} - T_0}{2} \right) (t_1 + t_2) \quad (5.6)
\]

\[
\int_{t_0 + t_1 + t_2}^{t_0 + t_1 + t_2 + t_3 + t_4} (T_0 - T_{space}(t)) \, dt \approx \left( \frac{T_0 - T_{low}}{2} \right) (t_3 + t_4) \quad (5.7)
\]

Since the energy term in the cost function is the integration of power consumption over the total compressor on time, not just the peak energy when the compressor turns on, Equation 5.8 can be regarded as an appropriate candidate to represent the compressor energy consumption in one temperature oscillation cycle (see Figure 5.4).

\[
\int_{t_0 + t_1}^{t_0 + t_1 + t_2 + t_3} (V_{rpm}N_{torque}(t)) \, dt \approx V_{rpm}N_{torque,ss} (t_2 + t_3) = K (t_2 + t_3) \quad (5.8)
\]

where \(N_{torque,ss}\) is the steady state torque value and \(K\) is a constant.

Substituting Equations 5.6-5.8 into Equation 5.5 and rearranging terms yield the cost function \(J\) in Equation 5.9. The time periods, \(t_1, t_2, t_3,\) and \(t_4,\) are functions of the temperature upper bound \(T_{high}\) and the temperature lower bound \(T_{low}\) in polynomial forms (see Equations A.1-A.4 in Appendix A). Consequently, for the given ambient condition \(T_{amb}\) and weight values, \(\alpha_1, \alpha_2,\) and \(\alpha_3,\) the optimal values \(\hat{T}_{high}\) and \(\hat{T}_{low}\) of the temperature bounds can be found by minimizing the cost function \(J,\) as described in Equation 5.10.

\[
J = \frac{t_2 + t_3 + \alpha_1 \left( \frac{T_{high} - T_0}{2} \right) (t_1 + t_2) + \alpha_2 \left( \frac{T_0 - T_{low}}{2} \right) (t_3 + t_4) + \alpha_3}{t_1 + t_2 + t_3 + t_4} \quad (5.9)
\]

\[
\left[ \hat{T}_{high}, \hat{T}_{low} \right] = \arg \min_{T_{high,low}} J \quad (5.10)
\]

Furthermore, the optimization framework associated with the cost function discussed above can be revised to accommodate practical temperature requirements and constraints. Given the recommended temperature ranges around the set-point \(T_0\) (see \(T_{highrec}\) and \(T_{lowrec}\) in Figure 5.4) for perishable food in transportation [120], the temperature performance and food quality loss can be evaluated by defining the area above the recommended temperature upper bound.
\(T_{\text{high rec}}\) and the area below the temperature lower bound \(T_{\text{low rec}}\), illustrated in Figure 5.4. Using similar triangular approximations as Equations 5.6 and 5.7, the areas to describe the temperature performance in one oscillation cycle become:

\[
\int (T_{\text{space}}(t) - T_{\text{high rec}}) \, dt \approx \frac{(T_{\text{high}} - T_{\text{high rec}})^2}{2(T_{\text{high}} - T_0)} (t_1 + t_2) \quad (5.11)
\]

\[
\int (T_{\text{low rec}} - T_{\text{space}}(t)) \, dt \approx \frac{(T_{\text{low rec}} - T_{\text{low}})^2}{2(T_0 - T_{\text{low}})} (t_3 + t_4) \quad (5.12)
\]

The cost function is re-written in Equation 5.14 by replacing the temperature performance terms in Equation 5.9 with Equations 5.11 and 5.12. Choices of weighting values \(\alpha_1\) and \(\alpha_2\) will determine the closeness of optimal values \(\hat{T}_{\text{high}}\) and \(\hat{T}_{\text{low}}\) to the recommended temperature upper and lower bounds. Additionally, there may exist a prescribed maximum compressor on-off switching frequency \(\omega_{\text{max}}\) during operations. Therefore, this optimization scheme with constraints becomes:

\[
\left[\hat{T}_{\text{high}}, \hat{T}_{\text{low}}\right] = \arg \min_{\hat{T}_{\text{high}}, \hat{T}_{\text{low}}} J_{\text{constraint}} \quad (5.13)
\]

where

\[
J_{\text{constraint}} = \frac{t_2 + t_3 + \alpha_1 (T_{\text{high}} - T_{\text{high rec}})^2}{2(T_{\text{high}} - T_0)} (t_1 + t_2) + \alpha_2 (T_{\text{low rec}} - T_{\text{low}})^2}{2(T_0 - T_{\text{low}})} (t_3 + t_4) + \alpha_3 H
\]

\[
\frac{1}{t_1 + t_2 + t_3 + t_4} \leq \omega_{\text{max}} \quad (5.15)
\]

5.3.2.3 Optimization Algorithm

The nature of the hysteretic on-off control system in Figure 5.3, in terms of interactions between temperature oscillation amplitude and frequency [115], ensures the existence of optimal values, \(\hat{T}_{\text{high}}\) and \(\hat{T}_{\text{low}}\), in Equation 5.10 by choosing appropriate values of the weights, \(\alpha_1\), \(\alpha_2\), and \(\alpha_3\). Iterative algorithms for optimization such as Newton’s method [121] can be used to find
the local minimum of the defined cost function in Equation 5.9. Similar methods can be applied in the constrained optimization to search for the optimal temperature bounds where the local minimum of the cost function in Equation 5.14 is achieved and the constraint in Equation 5.15 is satisfied.

Figure 5.5 shows the schematic of the optimal on-off control system. The benefit of this control approach is its freedom to choose the size and frequency of the temperature oscillation to meet system requirements. The quantitative relationship between desired time-domain characteristics and optimal control weights allows more flexibility in control design for the system performance tradeoff. The same set of weights can be used for several different simulated system models, thereby automatically choosing the set-points for new system designs without retuning. Moreover, the developed scheme is simple enough to be implemented on the existing feedback control systems by just employing an additional optimization algorithm.

![Figure 5.5 Optimal hysteretic on-off feedback control system](image)

### 5.4 Simulation Examples

The feedback control system with the optimization algorithm, as illustrated in Figure 5.5, is built in Thermosys for simulation. The simplified model in Figure 5.1 is used to represent the refrigerated transport system. The measured cargo space temperature and the optimal results of the temperature upper and lower bounds, \( \hat{T}_{\text{high}} \) and \( \hat{T}_{\text{low}} \), will determine the actions of the actuator; that is, compressor on or off.

Given that three performance terms are involved in the cost function for optimization, choices of weighting values provide the flexibility of the control approach to achieve desirable system performance. It is an open question on how to select the weighting values, since they depend on various performance requirements. Normally, a group of baseline weights can be
chosen at the starting point, and options of weighting values can be found for different performance control purposes. Therefore, maps involving weight values, ambient conditions and system performance specifications, such as temperature requirements for different foods in transport, could be generated to help find the appropriate weights. In this study, more focus is put on the system performance analysis with the optimal on-off control scheme. However, the mapping studies are in line with current industrial practices whereby manufacturers have different settings commanded from the driver cab for different foods.

In this section, examples of temperature regulation and compressor on-off cycling frequency control are presented first to demonstrate the ability of the developed algorithm. A case study with varying ambient temperature conditions is described in the second scenario to demonstrate the potential of the optimal control scheme in the refrigerated transport applications. To this end, the robustness of the approach to modeling uncertainties and disturbances is discussed. The nominal temperature set-point $T_0$ is set to be 5°C here, and the optimization time period $H$ is 2 hours.

### 5.4.1 Constant Ambient Temperature (32°C) Scenario

Since the ambient temperature is unchanged in this scenario, the time periods, $t_1$, $t_2$, $t_3$, and $t_4$ become functions of the temperature upper $T_{\text{high}}$ and lower bound $T_{\text{low}}$ (see Equations 5.1-5.4). The baseline weights are chosen in Equation 5.16 to ensure that each term in the cost function $J$ in Equation 5.9 has the same order of magnitude. The minimum of the cost function can be found through the iterative Newton’s method, and the optimal results are given in Equation 5.17. Figure 5.6 shows the contour plot of the cost function. It can be seen that the sequence $(T_{\text{high}}, T_{\text{low}})_n$ converges towards the optimal point to seek the local minimum of the cost function after four iterations from its initial guess. Simulation results with the optimal temperature bounds in Equation 5.17 show that the actual system cost $J$ based on the temperature performance integral calculations is 1.1006, which gives a 2.2% difference from the minimum cost $J_{\text{min}}=1.0764$ with the piecewise linear function approximation from 5.6 and 5.7. Moreover, through extensive simulations (see Figure 5.7), the average ratios of the integrals to the triangular areas are 1.04:1 and 1.17:1 for the temperature performance above and below the set-point respectively. The same optimal results as seen in Equation 5.17 are obtained with the ratios
considered for the baseline optimization, and the minimum cost $J_{\text{min}}$ becomes 1.0957, even closer to the actual system cost. In this study, the piecewise linear function approximation is applied in rewriting the cost functions in Equations 5.9 and 5.14, since it is simple and sufficient to achieve the optimal temperature bounds within allowable prediction errors. Starting with the baseline weights, two case studies are included in the following subsections.

$$\alpha_1 = 0.4, \alpha_2 = 0.2, \alpha_3 = 100 \quad (5.16)$$

$$\hat{T}_{\text{high}} = 7.1, \hat{T}_{\text{low}} = 3.8 \quad (5.17)$$

![Figure 5.6 Constant ambient case: contour plot of the cost function with the baseline weights (cost = $J$-1)](image)

**Figure 5.6** Constant ambient case: contour plot of the cost function with the baseline weights (cost = $J$-1)
5.4.1.1 Tighter Temperature Control Case

In transportation, different kinds of food may require different levels of temperature regulation around the desired set-point, which could be challenging for the basic control scheme in Figure 5.3. The weights $\alpha_1$ and $\alpha_2$ in the cost function of Equations 5.9 can be used to tune the temperature performance above and below the given set-point, respectively. In this case, compared to the baseline optimal results in Equation 5.17, tighter performance below the set-point is required. This results in an increase of the weight $\alpha_2$ as shown in Equation 5.18. The resulting optimal values in Equation 5.19 show that the lower temperature bound $T_{\text{low}}$ is closer to the set-point 5°C than that in the baseline case. The contour plot with the weights in Equation 5.18, showing the sequence convergence to the optimal point, is presented in Figure 5.8.

$$\alpha_1 = 0.4, \alpha_2 = 0.5, \alpha_3 = 100$$

(5.18)

$$\hat{T}_{\text{high}} = 7.4, \hat{T}_{\text{low}} = 4.4$$

(5.19)
5.4.1.2 Constrained Temperature Control Case

As discussed earlier in this chapter, there normally exist recommended temperature ranges for the transported food, and prescribed maximum compressor on-off cycling frequencies for the component wear minimization [116]. Assume that the recommended temperature upper bound \( T_{\text{high rec}} \) is 5.5°C, the lower bound \( T_{\text{low rec}} \) is 4.5°C, and the maximum frequency \( \omega_{\text{max}} \) is \( 1/720\text{s}^{-1} \) in this case. The inequality constrained optimization problem described in Equations 5.13-5.15 is formulated as an unconstrained problem by incorporating a logarithmic barrier function [121] to which the Newton’s method can be applied.

Four simulation tests involving choices of the temperature upper and lower bounds are shown here. In Test 1, without optimization, the recommended temperature bounds (4.5-5.5°C) are used to control the compressor on or off operations. Through implementing the constrained optimization algorithm with the weights shown in Equation 5.18, the obtained optimal temperature bounds \( \hat{T}_{\text{high}} \) and \( \hat{T}_{\text{low}} \) in Equation 5.20 are included in Test 2. Since the weight \( \alpha_i \) can be used to tune the temperature performance above the recommended upper bound (see Equation 5.14 and Table 5.2), the weights in Equation 5.21 are chosen in Test 3 and 4 for tighter temperature control purposes. The constraint requirement in Equation 5.15 is applied for
optimization in Test 4; while no compressor on-off cycling frequency constraints are considered in Test 3 for comparisons. Table 5.3 summarizes the resulting optimal temperature upper and lower bounds for each test along with the average compressor cycling frequency information. The plot in Figure 5.9 compares the cargo space temperature performance for the four simulation tests, and the actuator on-off actions in terms of compressor speed are presented in Figure 5.10. It can be clearly seen that the compressor on-off switching frequency is higher than the maximum allowable for the recommended temperature ranges in Test 1. By increasing the weight $\alpha_i$ value from 0.4 to 2.5 (see Equation 5.18 and 5.21), better temperature performance above the recommended upper bound (5.5°C here) can be achieved in Test 3 and 4 compared to Test 2 as shown in Figure 5.9. The comparison results between Test 3 and 4 presented in Table 5.3 show the tradeoff between temperature performance and on-off cycling frequency where the frequency requirement is met in Test 4 at the cost of poorer temperature performance below the recommended lower set-point.

![Figure 5.9 Constrained temperature control case: cargo space temperature performance](image)

**Figure 5.9 Constrained temperature control case: cargo space temperature performance**

$$\hat{T}_{\text{high}} = 7.4, \hat{T}_{\text{low}} = 4.2$$ (5.20)
\[ \alpha_1 = 2.5, \alpha_2 = 0.5, \alpha_3 = 100 \]  

Figure 5.10  Constrained temperature control case: on-off actions in compressor speed

Table 5.3  Constrained temperature control case: performance comparisons

<table>
<thead>
<tr>
<th>Test study</th>
<th>Optimal upper bound $T_{\text{high}}$ (°C)</th>
<th>Optimal lower bound $T_{\text{low}}$ (°C)</th>
<th>Average compressor on-off frequency $\omega$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained temperature control Test 1</td>
<td>5.5</td>
<td>4.5</td>
<td>4.8/720</td>
</tr>
<tr>
<td>Constrained temperature control Test 2</td>
<td>7.4</td>
<td>4.2</td>
<td>0.84/720</td>
</tr>
<tr>
<td>Tighter temperature control without constraints Test 3</td>
<td>6</td>
<td>3.7</td>
<td>1.38/720</td>
</tr>
<tr>
<td>Tighter temperature control with constraints Test 4</td>
<td>6</td>
<td>3.4</td>
<td>0.98/720</td>
</tr>
</tbody>
</table>

5.4.2 Varying Ambient Temperature Scenario

The case studied in this scenario involves varying external conditions, which could become important factors influencing the system performance for long-time or long-distance transportation. Two 8-hour simulation examples with varying ambient temperature are given
here. Compared to the existing transport system with fixed hysteretic on-off control parameters, the simulation results for the system with the optimization algorithm embedded demonstrate the possible benefits of the developed on-off control approach.

A typical August temperature profile in Minneapolis is discretized for simulation as shown in Figure 5.11. The chosen control objective is to maintain the cargo space temperature at the nominal set-point of 5°C with an allowable variation of +2.5/-1.5°C. For the system with a basic on-off control scheme, the upper and lower temperature bounds are then set to be 7.5°C and 3.5°C respectively, and the cargo space temperature performance is presented in Figure 5.12. The transport system with the optimal on-off control algorithm is constructed (see Figure 5.5), and the weights $\alpha_1, \alpha_2, \text{ and } \alpha_3$ are chosen in Equation 5.22. Recall that the optimization time period $H$ is assumed to be 2 hours in this scenario, and therefore the optimal temperature upper and lower bounds $\hat{T}_{\text{high}}$ and $\hat{T}_{\text{low}}$ can be updated through minimizing the cost function in Equations 5.9 every 2 hours given a varying ambient temperature. Note that this optimization can be implemented online with known or predicted ambient conditions. The optimal temperature bounds along with the cost $J$ information are summarized in Table 5.4, and Figure 5.13 presents the corresponding cargo space temperature performance in simulation. The compressor on or off actuations in terms of compressor speed based on the optimal bounds are shown in Figure 5.14.

$$\alpha_1 = 0.4, \alpha_2 = 0.5, \alpha_3 = 60 \quad (5.22)$$

![Figure 5.11 Varying ambient case: ambient temperature](image-url)
Figure 5.12  Varying ambient case: cargo space temperature performance for the system with the fixed hysteresis

Figure 5.13  Varying ambient case: cargo space temperature performance for the system with the optimization algorithm

Figure 5.14  Varying ambient case: on-off actions in compressor speed for the system with the optimization algorithm

Note that the temperature requirements are met for both system examples (see Figures 5.12 and 5.13), while the system performance comparisons in average power consumption $P$ and average compressor on-off cycling frequency $\omega$ are given in Table 5.5. The average power consumption for a certain amount of time $H$ is defined below:
Here \( h_{in} \) and \( h_{out} \) are the refrigerant inlet and outlet enthalpy across the compressor, and \( \dot{m} \) represents the refrigerant mass flow rate, which can be calculated via simulation.

\[
P = \frac{\int_{t_0}^{t_f} \dot{m}(t)(h_{out}(t) - h_{in}(t))}{H}
\]

(5.23)

The impact of varying ambient temperature on the system performance is presented in Table 5.5. As the ambient temperature drops, the system needs less energy for temperature regulation since less cooling capacity provided to the cargo space is required. Meanwhile, the system needs to work much longer to cool down the cargo space at higher ambient temperature.
due to its low cooling capacity, which explains the increase of the average compressor on-off cycling frequency. For reference, this phenomenon is experimentally documented in Björk and Palm [122]. The results in Table 5.5 also show that the average power consumption in the transport system with fixed hysteresis is 0.192kW and the value drops to 0.1802kW by implementing the optimal on-off control scheme, which leads to an around 6.2% improvement in power savings. However, it should be pointed out that less energy is consumed in the system with optimization at the expense of higher compressor on-off cycling frequency (see Table 5.5; $\omega$ increases from 0.57/720 to 1.28/720s$^{-1}$), which demonstrates the system performance tradeoff between energy consumption and on-off cycling frequency.

### 5.4.3 Robustness Analysis

In the optimal control framework, the cost function for minimization is developed based on the polynomial time period functions, $t_1$, $t_2$, $t_3$, and $t_4$, which are identified from the nominal system model (see Equations A.1-A.4 in Appendix A). The robustness of this approach to modeling uncertainties and disturbances is investigated in this section, and the presented control scheme is used for analysis here. From the nominal cargo space model parameters given in Table 5.1, the space internal volume is perturbed with a 10% increase; additionally, so are the space surface area and thermal capacity values. The actual system performance cost $J$ (e.g., 1.0717 at ambient 30°C) with the perturbed model is compared with the nominal performance cost (e.g., 1.0872 at ambient 30°C) as seen in Figure 5.15, and the prediction error of the performance cost is within 1.5%.

As mentioned earlier, solar radiation conditions could be another important factor to influence the transport system performance. A constant solar radiation intensity value (170W-m$^{-2}$) is used as a nominal operating condition for control design in this study, and the varying solar radiation conditions are regarded as disturbances for robustness analysis. A typical solar radiation profile is applied in the nominal system model where the intensity value drops from 600 to 250W/m$^2$ as the ambient temperature drops from 33 to 30°C. The actual system performance cost $J$ is also shown in the bar graph in Figure 5.15. It is observed in simulation that more power is consumed by the system with disturbances (0.2455kW at ambient 33°C) due to
the higher heating load put on the system. Compared to the predicted performance cost (1.0915 at ambient 33°C), the actual cost increased to 1.1117, which is a less than 2% difference.

![Figure 5.15 Robustness analysis: performance cost comparisons](image)

**Figure 5.15** Robustness analysis: performance cost comparisons

### 5.5 Hardware-in-the-loop Load Emulation Experimentation

To precisely replicate many of the experimental conditions that may be experienced in transport, without needing the extensive test-cell facility hardware, a novel hardware-in-the-loop load emulation technique [21] is applied here to determine the system performance with the developed control approach. In this section, the load emulation approach will be briefly introduced, and experimental studies corresponding to the varying ambient temperature simulation scenario discussed above are presented to verify the validity of the optimal on-off control scheme.

#### 5.5.1 Load Emulation Framework

The load emulation approach was invented to experimentally simulate different ambient conditions and space load parameters on a given VCC system without the need for replicating such conditions in a test chamber thereby providing a more flexible testing environment. The schematic of the hardware-in-the-loop emulation experimental system is presented in Figure 5.16, where the software-in-the-loop system for simulation studies is given in Figure 5.16(a) for
comparisons. The refrigerated cargo space model in Figure 5.16(b), with the physical parameters (see Table 5.1 for example) and ambient conditions, is implemented on-line to take in measurements from the experimental VCC system and determine the virtual cargo space temperature. The emulation unit, attached to the evaporator side, tracks the virtual temperature and then places an equivalent load on the VCC system. The interested reader is referred to [21] for more details on the load emulation augmentation and implementation on the experimental system as shown in Figure 3.5.

![Diagram](image)

**Figure 5.16** (a) Software-in-the-loop system (b) hardware-in-the-loop load emulation system

### 5.5.2 Experimental Studies

The purpose of experimental testing is to determine the actual system performance by implementing the optimal on-off control algorithm. The varying ambient temperature scenario is chosen here for experimental studies. The ambient temperature conditions for the load emulation testing are presented in Figure 5.17 and the temperature regulation requirements applied in the simulation studies (3.5-7.5°C) are still in effect. Figure 5.18 shows the load emulation results for the system with the basic on-off control scheme, where the system load of evaporator air inlet temperature is tracking the virtual cargo space temperature. With the developed on-off control
scheme on the varying ambient temperature, the temperature upper and lower bounds are updated every 2500 seconds through the optimization algorithm, and the optimal values can be found in Table 5.4. The resulting load emulation results are presented in Figure 5.19.

![Figure 5.17 Load emulation testing: varying ambient temperature](image1)

**Figure 5.17** Load emulation testing: varying ambient temperature

![Figure 5.18 Load emulation results: system with fixed hysteresis](image2)

**Figure 5.18** Load emulation results: system with fixed hysteresis

![Figure 5.19 Load emulation results: system with optimization algorithm](image3)

**Figure 5.19** Load emulation results: system with optimization algorithm

It can be seen from Figures 5.18 and 5.19 that the average compressor on-off cycling frequency $\omega$ experimentally increases from 0.65/720 to 1.7/720s-1 by applying the optimization
algorithm, which is in agreement with the simulation results shown in Table 5.5. In experiments, the power consumed by the compressor are measured using an AC watt-transducer [11], and the results show that there is approximately a 6.8% improvement in power savings with the optimal control compared to the basic on-off control scheme, which is also well predicted in simulation studies.
Chapter 6    LMI Control Design

To effectively control vapor compression cycle (VCC) systems whose dynamics are highly nonlinear, it is necessary to develop plant models and control laws for different operating regions. Using the first-principles modeling framework discussed in Chapter 2, this chapter presents an invariant-order switched system that captures four operating models over a broad range of conditions. To synthesize a multi-input multi-output (MIMO) control system, the Linear Quadratic Regulator (LQR) technique is framed as a control optimization problem with Linear Matrix Inequality (LMI) constraints which can be simultaneously solved for the set of considered linear systems. Stability and performance characteristics of the controlled switched system are guaranteed. Simulation results demonstrate improved performance and efficiency with the presented control strategy compared to conventional control approaches in handling the nonlinear refrigerant phase transitions over a wide operating envelope.

6.1 Introduction

An ideal VCC system, as shown in Figure 1.2, is a thermodynamic system driven by the phase characteristics of the refrigerant that is flowing through it. VCC systems transfer heat by means of repeating phase transitions in the refrigerant which can lead to the changing of the fluid characteristics in each of the heat exchangers (condenser and evaporator). The mixture of liquid and vapor refrigerant distributed in the heat exchangers makes the phase transition process highly nonlinear and complex [11, 40, 41]. Therefore, it is necessary to develop control strategies which account for system nonlinearities and maintain desirable performance over a wide operating envelope.
Advanced control of VCC systems in various air conditioning and refrigeration (AC&R) applications has been widely studied in the literature [22-24, 33, 39]. In particular, multi-input multi-output (MIMO) control design is attractive due to the strong coupling in VCC systems. He et al. [22] proposed a Linear Quadratic Gaussian (LQG) MIMO controller with gain scheduling to adapt to changing operating conditions. Rasmussen and Alleyne [39] presented a Youla parameterization framework for the generation of a local model and local controller network (LMN/LCN) and demonstrated the effectiveness of MIMO gain scheduling control strategies on performance regulation of the nonlinear system. A LQG controller based on a first-principles simulation model was developed in [24], and the controller performance was evaluated for operating conditions 30% away from the nominal operating point [38].

![Schematic of closed-loop switching control system](image)

**Figure 6.1 Schematic of closed-loop switching control system**

Although it has been shown that VCC systems exhibit nonlinear behavior across their wide range of operating conditions, a particular heat exchanger formulation with a fixed number of fluid zones is normally assumed for VCC system control design in the open literature. It has also been identified that highly nonlinear dynamics evolve when the system loses its condenser subcooled zone or evaporator superheated zone [41]. These types of refrigerant transitions can occur due to varying system boundary conditions. This dissertation goes beyond previous efforts to develop a switching control framework (see Figure 6.1) that is capable of handling the transitions inside the fluid zones. Based on the validated switched modeling approaches presented in Chapters 2 and 3 to capture refrigerant phase transitions, this chapter presents a switched system model, thereby enabling model-based MIMO control design for the nonlinear VCC system. The VCC system can be represented by a set of four possible operating models as
illustrated in Figure 3.7. The key contribution of this work is the formulation of an invariant-order first-principles switched model system amenable to switching controller design.

### 6.2 System Operation

As discussed in Chapter 3, when the VCC system is in operation, the refrigerant entering the condenser is a superheated vapor due to the compression process. The refrigerant outlet conditions range from a two-phase mixture to a subcooled liquid, which results in the condenser model switching between three-zone (superheated, two-phase and subcooled) and two-zone (superheated and two-phase) models under different operating conditions. Meanwhile, there also exist two different refrigerant distribution configurations in the evaporator model of the refrigeration system, which are two-zone (two-phase and superheated) and one-zone (two-phase) model representations. Therefore, four possible models (see Figure 3.7), named the ‘3-2 model’, ‘3-1 model’, ‘2-1 model’, and ‘2-2 model’, respectively, can be developed to describe the VCC system operation. Exogenous signals, such as a desired reference trajectory or system disturbances, and/or endogenous scheduling variables, such as measured pressure or temperature signals [39] can drive the VCC system to switch between different operating models.

The switched model framework is described in Figure 6.2, and the switched VCC system performance is validated with test data as presented in Chapter 3. Note that the conditions to trigger the switching between different operating models are state-dependent, and they are discussed in detail in Chapter 2. Another example to describe the switched system operation is

---

**Figure 6.2 State-dependent switched model diagram**

The switched model framework is described in Figure 6.2, and the switched VCC system performance is validated with test data as presented in Chapter 3. Note that the conditions to trigger the switching between different operating models are state-dependent, and they are discussed in detail in Chapter 2. Another example to describe the switched system operation is
shown in Figure 6.3. The evaporator cooling capacity, acting as the exogenous signals, drives the switching of the operating models from one to another.

![Figure 6.3 Example of model switching with variation of cooling capacity](image)

### 6.3 System Modeling and Linearization

As mentioned earlier, the VCC system dynamics are highly nonlinear. In order to apply linear control techniques on the VCC system, a linearized model of the system dynamics is needed. In the switched system model development, the nonlinear dynamic model for each individual operating model is linearized. The interested reader is referred to [42] for the linearization procedure.

The dynamic states of the switched model are described in Equation 6.1. First-order pseudo-state equations discussed in Chapter 2 are added in each operating model to ensure the continuity of states and maintain a 13th-order switched model of the system.

\[
x = \begin{bmatrix} \xi_c & P_e & h_c & T_{c1,w} & T_{c2,w} & T_{c3,w} & P_e & h_c & T_{c1,w} & T_{c2,w} & \overline{P}_c & \overline{P}_c \\
\end{bmatrix} \tag{6.1}
\]

The state-space representation is given in Equation 6.2. The vectors \(\delta x\), \(\delta u\), \(\delta w\) and \(\delta y\) are the dynamic deviations from the steady state operating point about which the system is linearized, as described as \(\delta x = x - x^{eq}\), \(\delta u = u - u^{eq}\), \(\delta w = w - w^{eq}\) and \(\delta y = y - y^{eq}\). The input variables, \(u = [u_1 \ u_2 \ u_3 \ u_4]^T\), represent compressor speed, valve opening, condenser air flow rate, and evaporator air flow rate (see Figure 1.2). The output variables are condenser pressure, condenser heat rejection, evaporator pressure, and evaporator cooling capacity, denoted as \(y = [P_c \ \dot{Q}_c \ P_e \ \dot{Q}_e]^T\). The condenser and evaporator air inlet temperatures are regarded as disturbances, \(w = [T_{air,in,c} \ T_{air,in,e}]^T\). The matrices \(A_i\), \(B_i\), \(C_i\), \(D_i\), \(F_i\) and \(H_i\) are obtained from the model linearization, and \(i=1,2,3,4\) denotes the model number. Note that one linear model is
derived for each operating model, and this chapter focuses on the switching operation between
different operating models. However, multiple linearized models can also be developed for a
single operating model for gain scheduling control implementation [22, 39] because the
dynamics within each model are themselves nonlinear.

\[
\delta \dot{x} = A_r \delta x + B_r \delta u + F_r \delta w \\
\delta y = C_r \delta x + D_r \delta u + H_r \delta w
\] (6.2)

The dynamics of the linearized model are compared against those of the nonlinear model
for each operating model. One validation of the linearized model for ‘3-2 model’ operation (see
Figure 3.7) is given here. Figure 6.4 shows the system input signals, and Figure 6.5 compares the
dynamic response of condenser pressure, evaporator pressure, and evaporator air outlet
temperature, of the linearized and nonlinear models. The dynamic responses match well, thereby
validating the linearization.

![Figure 6.4](image)

**Figure 6.4** System input signals for linearized model validation; dashed line corresponds
to the right Y-axis
6.4 LMI-based Control Design

This section presents an LMI representation of LQR control design to develop a switching control framework as shown in Figure 6.1. In this study, the condenser and evaporator air inlet temperatures are assumed to be invariant, therefore, the linearized state-space representation becomes:

\[
\delta \dot{x} = A_\sigma \delta x + B_\sigma \delta u \\
\delta y = C_\sigma \delta x + D_\sigma \delta u
\]  \hspace{1cm} (6.3)

where the \( D \) matrix is a zero matrix. Since the switching of the system operation is state-dependent, the switching signal \( \sigma(t) \) is defined in Equation 6.4, denoting the operating model number \( \{1, 2, 3, 4\} \).

\[
\sigma(t^+) = \phi(x(t), \sigma(t))
\]  \hspace{1cm} (6.4)
Using the standard LQR technique, the state feedback control \( \delta u = -K_\delta \delta x \) for each operating model can be designed to minimize the performance objective function given in Equation 6.5. The weighting matrices \( Q_y \) and \( R \) are applied to balance output errors and control efforts. The state feedback gain \( K_\delta \) can be obtained by solving the corresponding algebraic Riccati equation.

\[
J = \int_0^\infty \left( \delta y^T Q_y \delta y + \delta u^T R \delta u \right) dt
\]

\[
= \int_0^\infty \left( \delta x^T C^T Q_y C \delta x + \delta u^T R \delta u \right) dt
\]

(6.5)

### 6.4.1 Stability of Switched Systems

The authors in [123] presented that there are roughly two kinds of problems in the stability study of switched systems, one is the stability analysis of switched systems under given switching signals (arbitrary and constrained), and the other is the synthesis of stabilizing switching signals for a given switched system. Compared to the arbitrary switching, the constrained switching normally involves restrictions on the switching signals, such as time-dependent and state-dependent switching [124]. The stability analysis of the switched systems is extensively studied in the literature, and a great number of analysis tools, such as common Lyapunov functions, multiple Lyapunov functions, and dwell-time for slow switching, were presented in [123-127].

It is known [123, 124] that the existence of a common quadratic Lyapunov function is sufficient to guarantee stable state trajectories in the switched system with all switching sequences provided that the system states remain continuous (no state jump in switching). The conditions for the existence in the continuous-time switched linear system \( \dot{x}(t) = A_i x(t) \) can be expressed as LMIs in Equation 6.6. The Lyapunov function is constructed as shown in Equation 6.7 and \( \dot{V}(x) < 0 \).

\[
A_i^T P + PA_i < 0, \ \forall i \in \{1, \ldots, N\}
\]

(6.6)

\[
V(x) = x^T P x
\]

(6.7)
In the switched VCC system control study discussed in this chapter, there exist state
jumps at the switching instant due to the change of the equilibrium conditions applied in
linearization for different operating models, thereby causing the discontinuities of the defined
Lyapunov function in Equation 6.7. The stability analysis tools using multiple Lyapunov
functions were discussed in [125, 127] to handle the discontinuous problems, therefore, the
concept of multiple Lyapunov functions approach is brought in to meet the system performance
and stability requirements. In this framework, multiple Lyapunov matrices $P_1, P_2, \ldots, P_N$ are
applied, and the resulting Lyapunov function of the switched system is defined in Equation 6.8.
The indicator function $\xi_i$ is given in Equation 6.9.

$$V(x) = x^T \left( \sum_{i=1}^{N} \xi_i P_i \right) x$$

$$\xi_i = \begin{cases} 1, \text{ when the operating model } i \text{ is active} \\ 0, \text{ otherwise} \end{cases}$$

6.4.2 LMI Formulation of LQR Control

Using the LMI techniques discussed in [128], the LQR problem can be reformulated as a
convex optimization problem with LMI constraints. In the switching control study, the
optimization becomes as a suboptimal multi-objective control problem described as follows:

$$\text{min}_{\{S_{\sigma=1} \ldots S_{\sigma=N}\},\{Y_{\sigma=1} \ldots Y_{\sigma=N}\},\{\tilde{X}_{\sigma=1} \ldots \tilde{X}_{\sigma=N}\}} \sum_{\sigma=1}^{N} \left( \text{Tr}(Q S_{\sigma}) + \text{Tr}(\tilde{X}_{\sigma}) \right)$$

subject to

$$(\tilde{A}_\sigma - \tilde{B}_\sigma K_\sigma)^T S^{-1}_\sigma + S^{-1}_\sigma \left( \tilde{A}_\sigma - \tilde{B}_\sigma K_\sigma \right) + Q + K_\sigma^T R K_\sigma < 0$$

and

$$\tilde{X}_\sigma - R^{1/2} Y_\sigma S^{-1}_\sigma Y_\sigma^T R^{1/2} > 0$$
A stronger switching condition, that is the Lyapunov functions’ values at the switching instant are non-decreasing, is added to find multiple Lyapunov matrices $P_1 = S_1^{-1}$, $P_2 = S_2^{-1}$, ..., $P_N = S_N^{-1}$ for stability. The condition can be expressed by:

$$
\delta x^T P_j \delta x \leq \delta x^T P_i \delta x
$$

(6.13)

at those states $\delta x$ where the state trajectories pass from operating model $i$ to operation model $j$. As presented in [123, 127], this constrained matrix inequality condition in Equation 6.13 can be replaced with a LMI condition without constraints using $S$-procedure, and the multiple Lyapunov function matrices can be found by solving LMIs problems.

Using the Schur complement, the LMIs in Equations 6.11-6.12 can be rewritten as Equations 6.14-6.15. Hence, the LMI-based LQR design problem can be summarized as the minimization of the multi-objective function in Equation 6.10 under the constraints given by Equations 6.13-6.15. The suboptimal state feedback gain for each operating model involved is presented in Equation 6.16. The main advantage of this formulation is that the solution provides multiple Lyapunov function matrices $P_1, P_2, ..., P_N$ to ensure system stability during model switching and achieve desired performance of the closed-loop system. The interested reader is referred to [129, 130] for a detailed derivation of LQR design with LMI formulation.

$$
\begin{bmatrix}
\dot{A}_\sigma S_\sigma + S_\sigma \dot{A}_\sigma^T - \dot{B}_\sigma Y_\sigma - Y_\sigma^T \dot{B}_\sigma^T & Y_\sigma^T & S_\sigma \\
Y_\sigma & -R^{-1} & 0 \\
S_\sigma & 0 & -Q^{-1}
\end{bmatrix} < 0
$$

(6.14)

$$
\begin{bmatrix}
\dot{X}_\sigma & R^{1/2} Y_\sigma \\
Y_\sigma^T R^{1/2} & S_\sigma
\end{bmatrix} > 0
$$

(6.15)

$$
K_\sigma = Y_\sigma S_\sigma^{-1}
$$

(6.16)

6.4.3 State Observer Design

Since some state variables in Equation 6.1 cannot be measured directly (e.g., normalized two-phase zone length in the condenser and evaporator), a standard Kalman filter [131] is
developed based on the linearized plant model in Equation 6.3 for each operating model, as
given below where $\delta\hat{x}$ is an estimate of the state variable $\delta x$.

$$
\delta\dot{x} = A_\sigma \delta\hat{x} + B_\sigma \delta u + L_\sigma (\delta y - \delta y_\sigma)
$$

(6.17)

The observer gain matrix $L_i$ is computed from:

$$
L_\sigma = H_\sigma C_\sigma^T V^{-1}
$$

(6.18)

where $H_\sigma \in \mathbb{R}^{13 \times 13}$ is the solution of the Riccati equation:

$$
H_\sigma A_\sigma^T + A_\sigma H_\sigma - H_\sigma C_\sigma^T V^{-1} C_\sigma H_\sigma + W = 0
$$

(6.19)

The noise covariance matrices $W$ and $V$ in Equation 6.19 are selected as the identity
matrices, $I^{13 \times 13}$ and $I^{4 \times 4}$ respectively to reach a proper balance between fast convergence and
stable performance. With the state observer, the MIMO feedback control diagram for each
operating model is presented in Figure 6.6.

![Figure 6.6 Block diagram of MIMO feedback control for operating model $i$](image)

### 6.5 Stability Verification Study

The stability of the controlled switched system is always a concern, and the concept of
the multiple Lyapunov function approach is applied in the LQR control study with LMI
formulation as discussed above. In this section, the values changes of the Lyapunov function in
given switching sequences are examined in simulation for the stability verification.

Two cases are considered in the verification scenario by specifying different amplitudes
of the reference signals at time $t=0s$. Figure 6.7 presents the condenser pressure reference signals
in simulation, and two corresponding switching sequences are described in Figures 6.8 and 6.9,
respectively. In the case study 1, the reference signals trigger the switching from the ‘3-2 model’ to the ‘3-1 model’, while a switching from the ‘3-1 model’ to the ‘2-1 model’ occurs in the case study 2.

![Chart showing condenser pressure reference inputs in both cases](image1)

**Figure 6.7** Condenser pressure reference inputs in both cases

![Diagram showing switching sequence for stability verification in case study 1](image2)

**Figure 6.8** Case study 1: switching sequence for stability verification

![Diagram showing switching sequence for stability verification in case study 2](image3)

**Figure 6.9** Case study 2: switching sequence for stability verification

The Lyapunov function of the switched system is given in Equation 6.8 with multiple Lyapunov matrices $P_1$, $P_2$, $P_3$ for each operating model. The corresponding values of the Lyapunov function in the case study 1 (see Figure 6.8) are plotted in Figure 6.10, and Figure 6.11 presents the values of the constructed Lyapunov function in the case study 2. It can be seen from Figures 6.10-6.11 that using the multiple Lyapunov approach, the stability of the controlled switched system under constrained state-dependent switching is verified.
Figure 6.10  Corresponding values of Lyapunov function in case study 1

Figure 6.11  Corresponding values of Lyapunov function in case study 2
6.6 **Performance Comparison Simulation Study**

Building upon the control framework discussed above, a model-based switching control case study is presented to show the advantages of the developed approach compared to conventional control approaches. Figure 6.12 describes an appropriate control architecture for VCC system operation. As an outer loop, a higher level planning algorithm, such as set-point optimization, can be placed to achieve an optimal balance between desired capacity and efficiency [31]. This is a shift from the current practice where the actuators are used to meet specific control objectives. The inner loop regulation is studied in this dissertation, where the local control network is composed of four individual controllers, and the switched system model is used to represent the dynamics of the nonlinear VCC system.

This section is divided into two parts. First, a conventional control approach in VCC system applications is described. Second, for a particular VCC system, one case study in simulation demonstrates the effectiveness of the switching control in achieving desired performance.

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**Figure 6.12** Model-based switching control block diagram

### 6.6.1 Conventional PI Controller Implementation

Conventional proportional-integral (PI) controllers are constructed with four individual single-input single-output control loops. The SISO feedback configuration used for the controller design of the particular VCC system is shown in Figure 6.13. The controllers $K_1(s)$, $K_2(s)$, $K_3(s)$, and $K_4(s)$ are independent PI controllers, since some types of PI algorithms are prevalent in the majority of industrial controllers for these kinds of systems [31]. The actuator inputs are the compressor speed, $u_1$, the expansion valve opening, $u_2$, the condenser air flow rate, $u_3$, and the
evaporator air flow rate, $u_4$. The corresponding output variables are the difference between condenser and evaporator pressure, $P_c - P_e$, the evaporator superheat, $T_{esh}$, the condenser pressure, $P_c$, and the evaporator air outlet temperature, $T_{air, out}$. 

![Diagram](image)

**Figure 6.13** Conventional PI control for VCC system

### Table 6.1 Tuned gains for four PI controllers

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Proportional Gain</th>
<th>Integral Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor speed</td>
<td>3</td>
<td>0.4</td>
</tr>
<tr>
<td>Expansion valve opening</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>Condenser air flow rate</td>
<td>0.001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Evaporator air flow rate</td>
<td>0.005</td>
<td>0.009</td>
</tr>
</tbody>
</table>

### 6.6.2 Model-based Switching Controller Implementation

A case study involving two models’ switching (‘3-2 model’ and ‘2-2 model’) is described here to present the reference tracking performance of the controlled system using the developed control design approach.

The LMI-based control problem is to find the suboptimal state feedback gain matrix $K$ and observer gain matrix $L$ for each operating model, $(K_1, L_1)$ and $(K_2, L_2)$. To track the references, integrators are included in the switching control framework. In the LQR design, the weighting matrices $Q_y$ and $R$ are chosen in Equation 6.20. For comparisons, four PI controllers are tuned to evaluate the reference tracking performance of the closed-loop system, and the controller gains are listed in Table 6.1.
\[ Q_y = \begin{bmatrix} 0.01 \\ 0.1 \\ 0.01 \\ 1 \end{bmatrix} \quad R = \begin{bmatrix} 10^3 \\ 2 \times 10^6 \\ 2 \times 10^6 \end{bmatrix} \]  \hspace{1cm} (6.20)

6.6.2.1 Case study 1

Figure 6.14 presents the operating model switching sequence between the ‘3-2 model’ and the ‘2-2 model’ with the step changes of references. The system performance in terms of tracking the references (pressure difference, evaporator superheat, condenser pressure, and evaporator cooling capacity) are shown in Figures 6.15-6.18. It can be seen that the designed model-based switching controllers result in effective reference tracking for each output with less oscillation compared to the conventional PI control.

![Case study 1: corresponding switching between operating models](image)

**Figure 6.14**  Case study 1: corresponding switching between operating models

![Case study 1: pressure difference between condenser and evaporator](image)

**Figure 6.15**  Case study 1: pressure difference between condenser and evaporator

![Case study 1: evaporator superheat](image)

**Figure 6.16**  Case study 1: evaporator superheat

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A zoomed in plot is highlighted in Figure 6.19, which compares the cooling capacity performance with these two control approaches. Figures 6.20-6.23 present the corresponding actuator input signals during the operating models’ switching from the ‘3-2 model’ to the ‘2-2 model’.
Figure 6.21  Case study 1: expansion valve opening input from time 1600s to 2400s

Figure 6.22  Case study 1: condenser air flow rate input from time 1600s to 2400s

Figure 6.23  Case study 1: evaporator air flow rate input from time 1600s to 2400s
Chapter 7  Conclusions and Future Work

7.1 Summary of Research Contributions

This dissertation focuses on dynamic modeling and control of vapor compression cycles that are common cooling mechanisms in heating, ventilation and air-conditioning (HVAC) systems. The primary contributions of this dissertation fall into two distinct areas. First, with respect to the vapor compression cycle modeling, this dissertation presents a validated dynamic modeling framework to capture severe transient system behavior under mode switch cycling operations. Second, with respect to the vapor compression cycle control, this dissertation presents a first-principles nonlinear switched system framework amenable to switching control design over a wide range of operating envelope.

7.1.1 Dynamic Modeling of Vapor Compression Cycles

The first contribution of this research is the presentation of a control-oriented switched modeling framework to describe system transients under mode switching operations, such as cooling/heating mode switch and on/off cycling operation. The heat exchanger components are derived using the moving-boundary lumped-parameter modeling approach, and developed with multiple model representations to accommodate the refrigerant phase changes occurring in mode switch transients. One example in refrigerated transport system applications is illustrated to demonstrate the validity of the presented dynamic modeling approach in the simulation of the cooling/heating mode switch temperature regulation. Another example in an automotive air conditioning system is presented to demonstrate the capabilities of the modeling approach in predicting refrigerant mass migration in on/off cycling operation. Building upon a validated system model, a hysteretic on-off control scheme with optimization algorithms for temperature
regulation is developed and implemented on an experiment test bed, demonstrating optimization tradeoffs in VCC system on/off capacity control operation. This optimization approach illustrates of the potential usage of the dynamic modeling tools presented in the first section of this dissertation.

7.1.2 Control of Vapor Compression Cycles

Nonlinear control techniques, such as gain scheduling, have been widely used in the control of vapor compression cycle systems which exhibit nonlinear behavior over a typical operating regime. However, there has been a lack of studies on the system control with refrigerant phase transitions where highly nonlinear dynamics evolve with the switching of thermodynamic representations. The second section of this dissertation presents a first-principles dynamic switched system model with invariant order, which consists of four subsystems and logic rules that orchestrate the subsystem switching. A stable model-based MIMO switching control framework using a Linear Matrix Inequalities (LMI) technique is described that is capable of handling the refrigerant transitions inside the fluid zones over a wide operating envelope. The simulation evaluation results demonstrate improved performance and efficiency with the presented control strategy compared to conventional control approaches.

7.2 Future Work

HVAC system research is expanding beyond the traditional focus on performance to encompass other quantitative and qualitative criteria of increased economic and environmental concerns. These quantitative challenges include efficient energy consumption while realizing a comfortable and safe environment for the multi-domain (mechanical, thermal, chemical, and electrical) energy systems. The qualitative problems include reliability and fault detection. Dynamic modeling and control of vapor compression cycle systems are discussed in this dissertation and this research has many aspects that have yet to be explored. A few of these are mentioned here, including modeling and simulation, as well as control and optimization research.
7.2.1 Modeling and Simulation

Experimental validation results in this dissertation have demonstrated the capabilities of the developed modular modeling approach in predicting system transient behavior. Future research in this aspect can involve the extension of this switched modeling framework to complex refrigeration systems, such as multi-zone variable refrigerant flow systems, since the variable refrigerant system technology has been utilized in more and more applications due to its high energy efficiency and control flexibility [132]. The multi-zone system operation is subject to varying environmental inputs and disturbances; therefore, steady state simulations are not sufficient to describe and regulate the actual system performance. These systems may need to run simultaneous heating and cooling operations, switch from cooling to heating mode and vice versa, or turn on and off the indoor units in any of the individual temperature-controlled zones. A prototype system modeling example can be found in [133, 134]. Furthermore, this modular modeling concept can be utilized to simulate system dynamics in more complex distributed system applications with multi-domain and multi-time-scale subsystems. This will not only enhance the simulation capabilities presented in this research, but also provide valuable tools for advanced control and optimization implementation.

7.2.2 Control and Optimization

In this dissertation, the vapor compression cycle system is formulated as a switched system, and a model-based switching control framework is presented to account for high system nonlinearity with refrigerant phase transitions over a wide operating regime. It is possible to expand the switched system with more than four subsystems to describe the system behavior over a larger set of operating conditions than considered here. Consequently, would be an interesting avenue to seek optimal switching sequences [135] in the presence of disturbances and other exogenous signals. From the system operation perspective, the choices of the switching sequences would be related to the refrigerant mass redistribution among the system components, which would provide great physical meaning. Furthermore, with the switched system models available, the nonlinear system dynamics will provide endless opportunities for implementation of control and optimization strategies in a multitude of system configurations, such as complex multi-zone refrigeration systems, and hybrid multi-domain HVAC system applications [136].
Appendix A Time Period Function Generation

In this appendix, more information is provided on the derivation procedure of the governing equations for the time periods, \( t_1, t_2, t_3, \) and \( t_4 \), discussed in Chapter 5. The temperature set-point \( T_0 \) is chosen to be 5°C in this study. Given certain values of \( T_{\text{amb}}, T_{\text{high}} \) and \( T_{\text{low}} \), the time period data is collected after simulation of the transport system model (see Figure 5.1). Available identification software, such as Matlab System Identification Toolbox [137], is used to identify the time-temperature correlations. The resulting \( t_1, t_2, t_3, \) and \( t_4 \) functions are presented in Equations A.1-A.4 and can then be used in the system cost functions (see Equations 5.9 and 5.14). As an example, the \( t_1 \) function with the ambient temperature \( T_{\text{amb}} \) fixed at 30°C is shown in Figure A.1.

\[
t_1 = 487.16T_{\text{high}}^2 + 7.85T_{\text{amb}}^2 - 2778.43T_{\text{high}} + 3325.76T_{\text{low}} - 280.63T_{\text{amb}} - 636.6T_{\text{high}}T_{\text{low}} + 238.98T_{\text{high}}T_{\text{amb}} - 203.84T_{\text{low}}T_{\text{amb}} - 5.04T_{\text{high}}^2 + 3.36T_{\text{low}}^2 - 35.26T_{\text{high}}^2 + 0.67T_{\text{high}}^2 + 38.75T_{\text{high}}T_{\text{low}}T_{\text{amb}} - 0.63T_{\text{high}}T_{\text{low}}T_{\text{amb}}^2 + 1230.13
\]  

\[\text{(A.1)}\]

\[
t_2 = 16.38T_{\text{high}}^3 - 329.98T_{\text{high}}^2 + 2180.73T_{\text{high}} + 50.73T_{\text{amb}} - 18.13T_{\text{high}}T_{\text{amber}} + 1.62T_{\text{high}}^2T_{\text{amb}} - 4713.44
\]  

\[\text{(A.2)}\]

\[
t_3 = 1.23T_{\text{amb}}^3 - 448.8T_{\text{low}}^2 - 50.7T_{\text{amb}}^2 + 1258T_{\text{high}} + 6436T_{\text{low}} + 1252T_{\text{amb}} - 687T_{\text{high}}T_{\text{low}} + 40.6T_{\text{high}}T_{\text{amb}} - 392.8T_{\text{low}}T_{\text{amb}} + 16.9T_{\text{low}}^2T_{\text{amb}} - 0.57T_{\text{low}}^2T_{\text{amb}} + 63.11T_{\text{high}}T_{\text{low}}^2 - 4.79T_{\text{high}}T_{\text{amb}} + 12.18T_{\text{high}}T_{\text{low}}T_{\text{amb}} - 16790
\]  

\[\text{(A.3)}\]

\[
t_4 = 12.96T_{\text{low}}^2 - 163.91T_{\text{low}} + 0.016T_{\text{amb}}^2 - 3.26T_{\text{amb}} + 0.51T_{\text{low}}T_{\text{amb}} + 502.4
\]  

\[\text{(A.4)}\]
Figure A.1 Example map of the time period $t_1$ function
List of References


