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FLUID FLOW IN PETROLEUM RESERVOIRS

II. – Predicted Effects of Sand Consolidation


Walter Rose

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II. — PREDICTED EFFECTS OF SAND CONSOLIDATION

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ABSTRACT

There is much need for analytically developing valid descriptions of the phenomena of fluid flow in porous media. The Kozeny equation is frequently employed, and an important check on this application can be made by considering the question of sand consolidation.

Ordinarily it would be assumed that a consolidated sand is less permeable than the unconsolidated sand from which it was derived. To provide an analytic basis for this expectation, I have applied the Kozeny theory to the treatment of flow through various types of regular packings of spheres. Uniform size sphere packings are used frequently as models of unconsolidated sands, but a better representation considers packings made up of more than one sphere size.

Such models may be modified in three ways to represent the physical consequences of lithification: (1) cement may be located at points of particle contact as pendular rings; (2) cement may be deposited as an even layer on all pore surfaces; or (3) consolidation may be represented as a squeezing of all the particles together so that the point contacts become fused surfaces.

The Kozeny theory predicts an unexpected increase in permeability as pendular ring cement is deposited, but a sensible decrease in permeability for other types of consolidation. Three perplexing questions therefore arise: Is the Kozeny theory applicable to a treatment of what happens during consolidation? Have good models been chosen for study? Or ought we question the theory that consolidation always means a decrease in permeability? These questions are discussed, but none can be considered fully answered.

INTRODUCTION

In a previous paper (W. Rose, 1957a) the application of the Kozeny equation to describe homogeneous fluid flow through consolidated sandstones was questioned. An analytic check was made by taking a regular packing (cubic and rhombohedral) of uniform-sized spheres as the model of the unconsolidated sand, and cementing materials were represented as pendular rings deposited at sphere contacts. With such a representation, it was found that the specific surface area decreased with increased consolidation, which meant that the permeability predicted by the Kozeny equation would increase, according to:

$$K = f/tA^2 \quad (1)$$

where K is the permeability, f is the porosity, t is a textural constant, and A is the specific surface area per unit pore volume.

An increase in permeability with increase in cementation is contrary to expectations, but because the Kozeny equation has usually proved valid in applications of this sort, the result was reported as a paradox.

In an earlier work H. E. Rose (1950) also attempted to relate flow through consolidated media to fundamental considerations, and he concluded that the flow resistance was a statistical function of the quantity of cementing material present, and so was subject to considerable variation depending upon the uniformity in distribution of the bonding material. In consequence, he asserted that the permeability of consolidated sands cannot be uniquely calculated from the physical characteristics of the bed.

Thus, two separate analyses indicate that sand consolidation has a special effect on fluid flow phenomena. My own work, which has emphasized apparent departure from the indications of the Kozeny equation, still awaits experimental verification, but as H. E. Rose has stated, "It is also evident that the effect of the bonding material can only be to increase the resistance of the bed." Hence, the purpose of this paper is to extend the application of my previous analysis to consider a model composed of more than one sphere size. Although the influence of consolidation on tortuosity cannot be treated at this time, certain conceptual notions bring theoretical expectations and laboratory observations into closer harmony.

First, I see no necessity for the premise made by H. E. Rose that uneven distribution in bonding material is the proper explanation for the various anomalies he observed.

Cementation may be thought of as usually having causes locally independent of position, such as 1) flocculation and precipitation of clay minerals, micas, and inorganic salts; 2) solution and fusion at grain contacts due to overburden stress; and 3) entrapment of clay minerals and other suspended solids along pore surfaces and at grain contacts.

In other words, the post-depositional variation in environmental conditions throughout a large sedimentary sand body may give rise to variation in degree of cementation between widely separated locations, just as there are observed anisotropies in permeability, porosity, and other textural properties; but locally the environmental factors which cause consolidation must be regarded as virtually constant in space if not in time. Hence, the idea of uneven distribution of bonding material appears to be unnecessary, at least for lightly consolidated sands when small volume elements are being considered.

Actually the premise of uneven location of cement is not needed to explain the anomaly reported by H. E. Rose, for pore shapes and size distribution can not be represented by a single average particle diameter; moreover he admits that his data are of doubtful value.

PROPOSED MODEL OF A CONSOLIDATED SAND

The random packing of random-sized spheres appears to be a satisfactory model of an unconsolidated sand. True, quartz grains are double-domed elongated pyramids, but they acquire high sphericity through natural weathering; moreover, it can be shown that the sphere model is closely similar to a random packing of variously sized oblate and prolate spheroids if the particles have no preferential orientation.

Cementation may be equivalent to the result of compaction, or of an even deposition of bonding material on all pore surfaces, or the cement may be deposited preferentially as pendular rings at points of grain contact. Hence, the model to be adopted for consolidated sands is equivalent to the model proposed for unconsolidated sands (that is, a random packing of random-sized spheres), as modified by the change in porosity, specific surface area, and tortuosity

which accompanies compaction and/or the location of bonding material at points of grain contact and/or as a layer on all pore surfaces. (In either case, the point contacts between the sand grains of the original unconsolidated matrix become altered as two- and perhaps three-dimensional surfaces of contact.)

The volumes and surface areas of pendular rings between spheres of unequal size have been derived and presented in another publication (Rose, 1957?). In regard to necessary accuracy for present purposes, these relationships can be expressed as:

$$\begin{aligned} \log (v/R_1^3) &\cong 3 \log \theta_1 - 6, \text{ and} \\ \log (a/R_1^2) &\cong 2.7 \log \theta_1 - 4 \end{aligned} \tag{2}$$

where v and a are the volume and area, respectively, of pendular rings regardless of the relative size of the sphere particles in contact, the angle θ_1 is defined by reference to figure 1, and R_1 is the radius of each larger sphere in contact with a smaller sphere.

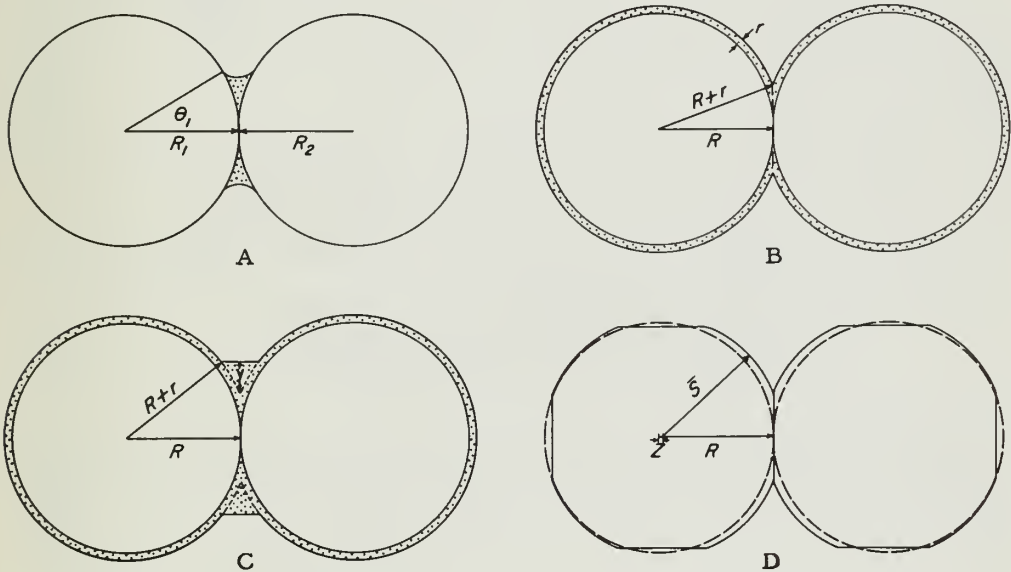


Fig. 1. - Sphere packing models of porous media showing various types of cementation.
 A - Pendular cement C - Uneven layered cement
 B - Even layered cement D - Cementation by compaction

Figure 1 represents the types of consolidated sand models which will be used, where diagram A depicts pendular ring bonding, diagram B shows cementation as layers on all pore surfaces, diagram C allows for the sometime probable tendency of layered cement to concentrate at points of grain contact, and diagram D refers to consolidation by compaction with fusion at the contact surfaces.

These physical models should suffice to represent the porous (lightly consolidated) sandstones which occur in nature, whether the cementation comes about through precipitation (or filtration) of dissolved or suspended minerals in the circulating groundwater, or is the result of fusion at grain contacts in response to the effects of overburden compaction.

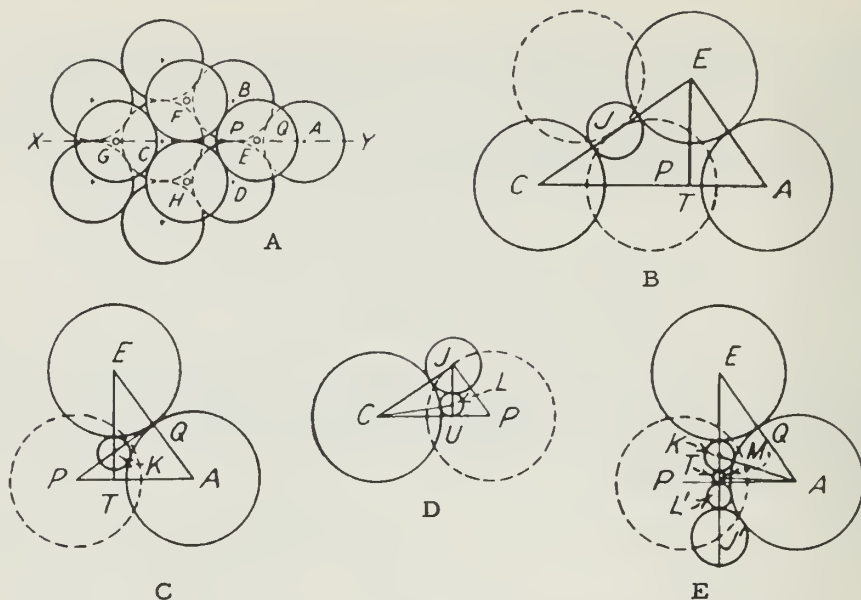


Fig. 2. - Plan view of rhombohedral packing and the geometry of the filling process (after White and Walton, 1937).

- A - The unfilled rhombohedral packing with center of spheres in lower layer indicated by crosses and center of spheres in upper layer indicated by circles.
- B - The secondary sphere is shown in section X-Y.
- C - The tertiary sphere is shown in section X-Y.
- D - The quaternary sphere is shown in section X-Y.
- E - The quinary sphere is shown in section X-Y.

That is, the environmental factors which give rise to cementation determine just where the silicon dioxide, iron oxide and hydroxides, clays, and the calcium and magnesium carbonates will be located, and in what quantity. Nevertheless, the local effect will be fairly uniform, and contiguous sand grains of the original matrix can be expected to show similar lithification as depicted schematically in figure 1 and also as shown by Spock (1953, fig. 22).

Figure 2 shows the specific sphere packing model which will be considered, and it may be described as a "filled" rhombohedral packing. That is, secondary, tertiary, quaternary, and quinary spheres of the sizes specified in table 1 are inserted within (but without enlarging) the various void spaces of the rhombohedral matrix to achieve a maximum sphere particle density (that is, minimum porosity). This arrangement (White and Walton, 1937) is used in the analyses below in a way similar to the use previously made of the "filled" cubic packing (Rose, 1957?).

As will be evident, the "filled" rhombohedral packing is characterized by a high occurrence of large-to-small sphere contacts, more so than in the case of the "filled" cubic packing, so that an advantage is gained of having a model of the following description:

The model is regular, which simplifies the analytics; but it also possesses wide particle size variation and sphere-to-sphere intermixing.

These features give the model a virtual character of randomness which, although not real, provides excellent similitude with the prototype unconsolidated sand sediments of nature. The latter is taken to be a locally isotropic assemblage of random-sized particles (of high sphericity) distributed randomly in space.

INFLUENCE OF PENDULAR RING CEMENT

Table 1 specifies the way the secondary J sphere of radius $0.414R_1$ fits in the largest void space left by the primary E spheres, which then leaves space to insert two tertiary K spheres, eight quaternary L spheres, and eight quinary M spheres. During the course of this filling, the fractional porosity is reduced from 0.2595 to about 0.149. Table 1 also gives the resulting number of sphere-to-sphere contacts.

As in my previous work (Rose, 1957?), it is next assumed that an angle, θ_1 equals 10° , defines a reasonable size for the pendular ring cement at sphere contacts between the largest (primary E) spheres; hence the defining angles and surface areas of all pendular rings are calculable from geometry as is discussed in the cited references. The values given in table 1, in fact, have been obtained by direct methods, rather than by making use of various available approximations such as given by Equations 2.

Table 2 and figure 3 show the results obtained. If the rhombohedral packing containing only E size spheres is filled with pendular ring cement occupying a fractional pore space volume of only 4.09×10^{-3} , the specific surface area of the resultant system (A_C) is reduced enough so the $(A/A_C)^2$ is increased to 1.15. Adding the smaller J, K, L, and M spheres continues this trend in a way comparable to what was observed for the filled cubic packing.

If the Kozeny equation (Equation 1) is to be regarded as a valid relationship between permeability, porosity, and specific surface area, and if cementation is to be considered only as the deposition of pendular ring material, it is to be expected that permeability increases with cementation to a degree somewhat equivalent to the $(A/A_C)^2$ values as given in table 2. This is because the porosity term is affected insignificantly by the addition of the cement material; and apparently the same thing can be said about the effect of such minor cementation on the tortuosity term.

From my work to date on this subject (Rose, 1957a, 1957?, and herein), the rule appears to be established for a large variety of sphere packings, that the specific surface area bounding the flowing fluid (A_C) is always smaller when pendular ring material is present than when it is absent (A). That is, the dome area, A_D (the influence of which is removed by the addition of pendular ring material), is always greater than the added pendular ring surface area, A_p ; moreover, this is true regardless of the sphere size ratio of the particles in contact.

My original work (Rose, 1957a) showed how the $(A/A_C)^2$ ratio could be caused to increase as more and more pendular ring material was added. This effect was demonstrated only for cases of regular packings of regular-sized spheres, but it may be presumed to apply to filled as well as unfilled packings.

Table 1. - Sphere Components of a Filled Rhombohedral Packing with Unit Cell Volume Equal to $4\sqrt{2}R_1^3$

<i>i, j</i> spheres	Radius of sphere	Number of spheres in unit cell	Fraction of unit cell volume filled by each size sphere	Number of contacts per sphere between spheres of two (<i>i, j</i>) sizes	Sphere size ratio (R_i/R_j)	Number of pendular rings of <i>i, j</i> -th type in a unit cell	Number of dome sur- faces per unit cell located	The angle θ_1 measur- ing the pendular ring size (compare fig. 1)	The surface area of each pendu- lar ring of each type				
							E J K L M						
E	R_1	1	0.740	E-E	6	1.000	6	12	-	10.0°	0.045 R_1^2		
				J	6	0.414	6	6	-	-	8.8	0.044	
				K	8	0.225	8	8	-	-	8.2	0.043	
				L	24	0.177	24	24	-	-	8.0	0.040	
				M	8	0.116	8	8	-	-	7.8	0.039	
J	0.414 R_1	1	0.053	J-E	6	-	-	6	6	-	21.3	-	
				J	0	1.000	-	-	-	-	-	-	-
				K	0	0.543	-	-	-	-	-	-	-
				L	8	0.427	8	8	-	-	19.0	0.057	
				M	0	0.280	-	-	-	-	-	-	-
K	0.225 R_1	2	0.017	K-E	8	-	-	8	8	-	36.4	-	
				J	0	-	-	-	-	-	-	-	
				K	0	1.000	-	-	-	-	-	-	
				L	0	0.788	-	-	-	-	-	-	
				M	8	0.516	8	8	-	-	33.0	0.061	
L	0.177 R_1	8	0.032	L-E	24	-	-	24	24	-	45.2	-	
				J	8	-	-	8	8	-	44.5	-	
				K	0	-	-	-	-	-	-	-	
				L	0	1.000	-	-	-	-	-	-	
				M	0	0.645	-	-	-	-	-	-	
M	0.116 R_1	8	0.009	M-E	8	-	-	8	8	-	67.2	-	
				J	0	-	-	-	-	-	-	-	
				K	8	-	-	-	8	-	64.0	-	
				L	0	-	-	-	-	-	-	-	
				M	0	1.000	-	-	-	-	-	-	

Table 2. - Surface Area Characteristics of Certain Filled Rhombohedral Packings

Sphere sizes which packing contains	Resulting fractional porosity f	Unit cell pore volume (bulk volume = $4\sqrt{2} R_1^3$)	Fraction of pore space occupied by pendular ring material ($\theta_1 = 10^\circ$)	A/R_1	A_p/R_1	A_D/R_1	A_c/R_1	$(A/A_c)^2$	\bar{K}
E	0.2595	$1.47 R_1^3$	4.09×10^{-3}	8.55	0.183	0.768	7.96	1.15	1.145
E, J	0.207	1.17	1.03×10^{-2}	12.60	0.458	1.737	11.32	1.24	1.227
E, J, K	0.190	1.076	1.86×10^{-2}	14.88	0.816	2.813	12.89	1.34	1.315
E, J, K, L	0.158	0.894	5.82×10^{-2}	21.36	2.562	7.715	16.20	1.74	1.639
E, J, K, L, M	0.149	0.843	8.08×10^{-2}	24.26	3.678	10.125	17.81	1.86	1.710

* A is the surface area of the unconsolidated packing per unit pore volume
 A_p is the same function referring to the pendular ring surface areas added
 A_D is the same function referring to the dome areas subtracted

$$A_c = A + A_p - A_D$$

$$\bar{K} = S(A/A_c)^2$$

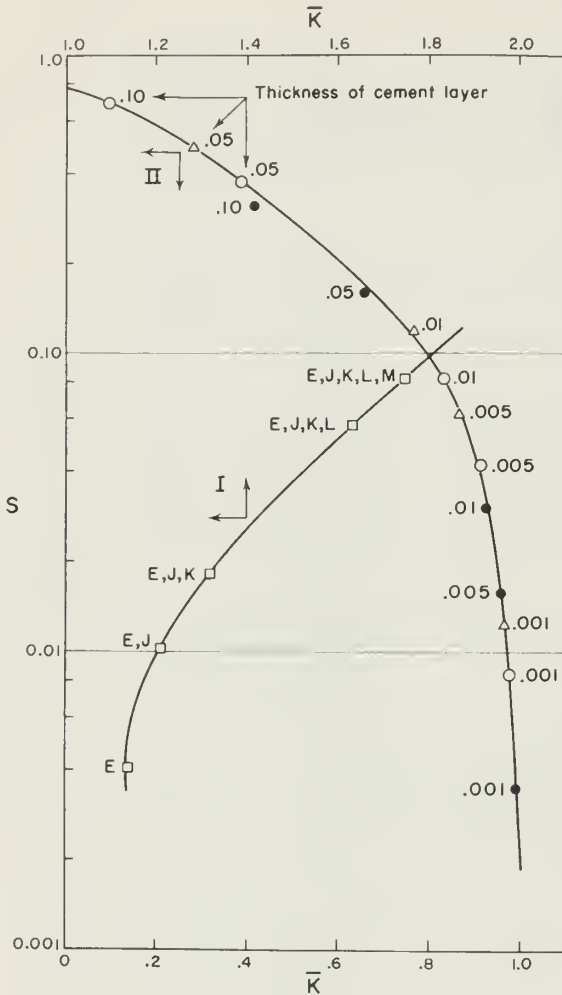


Fig. 3. - Fractional change in permeability, \bar{K} , as a function of the ratio of the volume of cement added to the pore volume, S , for various sphere packing examples.

Curve I shows the increase in permeability resulting from the location of pendular cement at the particle contacts of various filled-rhombohedral systems. Letters indicate the particle size distribution of the packing.

Curve II shows the effect of layered cement in decreasing permeability for unfilled rhombohedral packing (circles), unfilled cubic packing (solid dots), and E, J-filled rhombohedral packing (triangles). Numbers indicate cement layer thickness.

In the current work (compare also Rose, 1957?), the $(A/A_c)^2$ ratio is seen to be increased as more pendular rings are added when proceeding with the sequence of adding secondary and smaller spheres to "fill" the basic regular patterns. Figure 3 illustrates this latter effect.

It may be concluded, therefore, that the indicated anomaly of pendular ring cements causing an increase in permeability is minimized for the cases of well sorted sands (minimum number of grain contacts), and of low degree of consolidation (minimum amount of cement deposition).

INFLUENCE OF LAYERED CEMENTS

If a single sphere particle is considered, it is apparent that adding a layered cement would increase the tortuosity and surface area but would decrease the porosity by increasing the grain volume. All three effects would lead to a decrease in permeability according to Equation 1, and according to expectations.

With modification, this simple argument can also be applied to a treatment of complex sphere packings such as the filled cubic and rhombohedral packings discussed above.

Let \bar{K} be the fractional change in permeability due to consolidation, and neglecting the influence of deposited cements on tortuosity, then from Equation 1 we have:

$$\bar{K} = (f_c/f)^3 (a/a_c)^2 \quad (3)$$

where f and a refer to the fractional porosity and the surface area per unit bulk volume, respectively, and where the subscript "c" designates the system after cement has been added.

Then:

$$A/A_c = \frac{\sum 4\pi R^2 N}{N [4\pi (R+r)^2 - 2\pi (R+r)(r)(n)]} \quad (4)$$

and:

$$f_c/f = \frac{V_B - \{ \sum (N [1.33\pi (R+r)^3 - \sum 0.33\pi (n)(r^2)(3R-r)]) \}}{V_B - \sum 1.33\pi R^3 N} \quad (5)$$

where N is the number of R-sized spheres per unit cell volume; (n/2) is the number of contacts between each R-sized sphere; V_B is the bulk volume of the unit cell; and r is the thickness of the layered cement.

For the special case of a rhombohedral packing (E spheres only), where R = 1, we may write:

$$\bar{K} = 0.317 \left[\frac{(1.467 - 12.566r + 25.133r^2 - 16.755r^3)^3}{(1 - 4r - 5r^2)^2} \right] \quad (6)$$

If J-spheres of radius 0.414 are added, then we have:

$$\bar{K} = 0.854 \left[\frac{(1.171 - 14.717r + 64.084r^2 - 34.046r^3)^3}{(1.171 - 6.685r - 10r^2)^2} \right] \quad (7)$$

Calculations with equations 6 and 7 give the results tabulated in table 3 and shown in figure 3. Also shown is the influence on permeability where layered cement is added to a cubic packing; these results also have been obtained from calculations based on equations 3, 4, and 5.

As in the case of adding pendular cement, layered cements also decrease the surface area of the resulting system. This is because of the large number of contacts between adjacent spheres which therefore limits the growth of any one sphere. The effect is not large, however, and is more than offset by the decrease in porosity which results when the cement is deposited, hence, \bar{K} is seen to decrease markedly in a way proportional to the degree of consolidation.

Table 3 and figure 3 show essentially the same trend, whether the packing be cubic, rhombohedral, or filled-rhombohedral. It is true that adding a layer of given thickness results in more pore space being filled in the rhombohedral than in the cubic packing, but the decrease in permeability is also greater, as would be expected, because of the greater packing density in the former case.

Regarding the latter, it should be apparent from equations 4 and 5 that still further decreases in permeability result when the packing density is held constant but the number of contacts is diminished. For example, calculation shows that when n is decreased from 12 to 6 for the unfilled-rhombohedral packing, and when r is taken as 0.1, then \bar{K} is decreased from 0.092 to 0.008.

Of passing interest is the fact that all of the points in figure 3 showing the decrease in permeability due to layered cements fall on approximately the same curve regardless of the packing form. That is, for the systems studied, \bar{K} seems to be a unique function of the amount of layered cement added, whereas the same is not true for the various reported cases of pendular cement which have so far been investigated.

Table 3. - Effect of Layered Cement on Permeability

	Thickness of layer r	$(A/A_c)^2$	$(f_c/f)^3$	Fraction of pore space occupied by cement S	\bar{K}
Unfilled rhombohedral packing	0.001 R	1.0040	0.9744	0.0086	0.9823
	0.005	1.0206	0.8781	0.0424	0.9145
	0.010	1.0422	0.7688	0.0839	0.8351
	0.050	1.2698	0.2319	0.3856	0.3739
	0.100	1.8182	0.0279	0.6967	0.0922
E, J-filled rhombohedral packing	0.001 R	1.0116	0.9630	0.0125	0.9742
	0.005	1.0595	0.8266	0.0615	0.8758
	0.010	1.1255	0.6812	0.1201	0.7667
	0.050	2.0790	0.1314	0.4916	0.2732
Unfilled cubic packing	0.001 R	1.0020	0.9901	0.0033	0.9921
	0.005	1.0101	0.9515	0.0164	0.9611
	0.010	1.0207	0.9066	0.0328	0.9254
	0.050	1.1198	0.5906	0.1610	0.6614
	0.100	1.2913	0.3202	0.3168	0.4135

Further calculations for examples of filled porosity, where tertiary, quaternary, and quinary spheres are present, will show the same trend. This leads to the conclusion that the Kozeny equation always predicts a decrease in permeability when layered cements are added to the porous matrix. Evidently the effect is related to the amount of cement added, and to the nature of the packing arrangement (that is, the number of contacts between adjacent spheres and the particle size distribution).

Of some importance nevertheless is the similarity between the model which has been used above to represent sand consolidated with layered cements, and the model which can be selected to represent the consolidation of porous media by sintering or compaction. For the simplest example, consider a cubic packing of uniform size lead balls of radius R , and then apply a triaxial load so that the original six point-contacts on the surface of each sphere become flattened to circular contacts.

If now it is assumed that the density of the material is not changed, and that, except for the flattened areas, the remaining surfaces of the compressed lead pellets assume an approximate spherical shape of radius \bar{S} , then:

$$a/a_c = \frac{R^2}{\bar{S}^2 - 3\bar{S}(\bar{S} + Z - R)} \quad (8)$$

$$f_c/f = \frac{8(R - Z)^3 - 1.33\pi\bar{S}^3 + 2\pi(Z + \bar{S} - R)^2(4R - Z - \bar{S})}{3.81 R^3} \quad (9)$$

where Z/R is the fractional flattening of each sphere, and where Z , R , and \bar{S} are related by:

$$4.19 \bar{S}^3 - 6.26 (Z + \bar{S} - R)^2 (4R - Z - \bar{S}) - 4.19 = 0$$

Physically, this model and the layered cement model have an equivalent appearance (fig. 1), which is that of spheres grown bigger during consolidation, and which now have flat surfaces of contact with adjacent spheres. There is, however, a basic difference in the two lithification processes. When layered cements are deposited, the grain volume increases, decreasing pore volume (and porosity) since bulk volume remains constant. In the sintering process, on the other hand, the bulk volume decreases and the grain volume remains constant, and consequently the pore volume (and porosity) decrease in what proves to be a significantly different way.

Equations 8 and 9 show that permeability is decreased when a sediment is squeezed together and compacted, a result which follows expectations. For example, in the extreme case of compacting so that the sphere centers are brought 10 percent closer in all directions (i.e., $Z/R = 0.1$), the Kozeny equation predicts that the reduced permeability will be only 7 percent of the original permeability (i.e. $\bar{K} = 0.0714$). And if there is preferential orientation in the compaction, the resultant permeability values will have a directional variation not unlike that found in nature where the overburden loads a more or less laterally unconfined sedimentary stratum.

DISCUSSION

As it occurs in nature, cementation must result in uniform distributions (i.e. layered cements) and/or localized depositions (i.e. pendular cements); or, perhaps, compression and/or fusion occurs at points of grain contact due to the compaction of the overburden sediments.

If various types of sphere packings are taken as the model of the originally unconsolidated sand, the Kozeny equation predicts that permeability is decreased as layered cements are added. This is because of the large decrease in free volume (porosity), which more than offsets the effect of a slight decrease in the pore surface area.

With these same models, the Kozeny equation predicts an increase in permeability when the cements are located at points of grain contacts as pendular rings. This is because a large decrease in pore surface area attends the location of pendular cement, which offsets the rather negligible opposing effect of decrease in porosity.

Sintering and compaction, it has been shown, produce a result similar to the effect of adding layered cements; and since it is ordinarily supposed that permeability should decrease with consolidation, one is left only with the need to rationalize the paradox of finding intuition contradicted in the case of sands cemented with pendular ring deposits.

At least three explanations are possible. It has already been proposed (Rose, 1957a and 1957?) that the Kozeny equation itself might not accurately express the phenomena being considered. Such a view is given much support by the recent analysis of Scheidegger (1957), who points to various defects of the Kozeny theory. On the other hand, we must admit that the Kozeny theory has been applied with success to the treatment of fluid flow in both unconsolidated and consolidated porous media (Carman, 1956).

One expects, therefore, to meet some success in transiting from an unconsolidated sand (where Kozeny theory certainly applies) to a consolidated sand (where Kozeny theory sometimes applies).

For example, Carman (1956) notes that the "two extreme cases of cubic and close packing gave fairly close agreement. . . with the Kozeny equation." Elsewhere Carman implies "the failure of the Kozeny theory for most consolidated media," when this is observed, is related more to the difficulty of independently measuring tortuosity and specific surface area, rather than to any fundamental defect in the theory. And, as will be noted, the work discussed in this paper is free from any uncertainty about tortuosity and specific surface area.

The second explanation of the anomaly has to do with the question as to whether or not pendular ring cements actually are found in nature. If such configurations do not exist, and if natural cementation is always of the layered and/or sintered and/or compacted type, then the Kozeny theory conforms with the expectation that consolidated media should have a lower permeability than the original unconsolidated matrix.

The question of the prevalence of pendular ring cements in nature is involved. Little information is to be found in the geologic literature. In essence, one must know from the standpoint of the petrophysicist the structure of the original sand, and then trace the history of diagenetic processes which have occurred over geologic time to produce the resultant consolidated sand. And to find an explicit solution, all of these factors must be known with some quantitative detail.

Perhaps as a compromise one might be tempted to postulate that natural cementation is principally of the layered type, with only a minor tendency for additional pendular cement to be deposited. Such a configuration is illustrated in figure 1c, which is a simplified depiction of the probable deposition of extra cementing material at the junction point of adjacent sand particles as the latter grow in size with layered cement.

From geometry, one can state that the ratio of the surface area added to that subtracted at each contact can be approximated by:

$$\frac{(2Rr + r^2)^{\frac{1}{2}} + y}{(R + r)} \quad (10)$$

where R is the sphere size, r is the thickness of the cement layer, and y is a measure of the amount of pendular cement deposited on top of the layered cement. It is easily seen that for all reasonable values of r and y , the subtracted area greatly exceeds the added area, which means that the (A/A_C) ratio thereby is markedly decreased; hence \bar{K} tends to become unreasonably large.

A third idea to be considered is an interesting possible explanation as to why the Kozeny theory predicts an increase in permeability for cases of pendular ring deposition. The fact of the matter is that no one can say at this time what the relationship is between the permeability of consolidated porous media and the permeability of the parent matrix prior to diagenesis. Evidently no experiments of this sort have been done except trivial ones which demonstrate that ultimately a loose sand can be plugged (partially or completely) if enough cementation occurs. Available data, however, do not show whether permeability is increased or decreased during the period of initial consolidation, and this is the question being considered here.

Among other factors, it is clear that the Kozeny theoretical inclusion of the influence of porosity and specific surface area on permeability is reasonable, whether or not quantitative accuracy has been obtained. That is, porosity is a measure of the total free volume, and hence for isotropic media, porosity is a measure of the total cross-sectional area available for flow. The specific surface area term bears a relationship to the mean hydraulic radius (cf. Scheidegger's treatment of this point, 1957), and as such establishes the pore size equivalence between the porous rock and a bundle-of-tubes model.

The surface area concept is also important in another way. Thus, in the final analysis, the conductivity which porous media afford to the flow of fluids must be related to the extent to which the pore wall boundaries exert a "drag" on the moving fluids. Clearly, the greater the surface area (for a given porosity condition), the greater will be the influence of the zero-velocity boundary surfaces in resisting fluid flow.

A simple case will illustrate the point. If we choose two capillary tubes of unit length so that together they have the same internal volume as a single larger tube of unit length, then the smaller tubes, when arranged in parallel, afford a flow conductivity which is only one-half that afforded by the larger tube alone. The total cross-sectional area (porosity) has not been changed, but the zero-velocity boundary surfaces have increased by a factor of the square root of two. Obviously, the decrease in flow conductivity which is indicated by Poiseuille's law is directly attributable to the fact that the fluid must drag over more pore wall surface area with no compensating increase in hydraulic radius or total cross-sectional area.

In the case of packings of sphere particles, the first addition of pendular cement at points of grain contact also does not change the porosity significantly; but the surface area of the pore boundaries is greatly decreased, as has been shown from geometry. Hence, perhaps it is not unreasonable that the permeability should increase even though the surface area removed by the deposition of the pendular cement is that portion most distant from the central pore spaces.

In order to study the magnitude of the effect, one needs experimental data now nonexistent; for otherwise we have only the indications of the Kozeny theory which are by no means conclusive. Presumably, there is little hope of applying the "drag" theoretical approach to the permeability problem (Scheidegger, 1957). because of the formidable obstacles of analysis which will be encountered.

CONCLUSIONS

1) Kozeny theory predicts an increase in permeability when various types of sphere packings have cement deposited as pendular rings at points of grain contact.

2) Kozeny theory predicts a decrease in permeability if the cement is deposited as an even layer, or if the particles are pressed together by compaction. This trend is diminished, and perhaps reversed, if pendular cement is subsequently deposited over the layered cement at certain preferential locations.

3) Since it is usual to assume that permeability should always decrease during natural consolidation, one might question the validity of the Kozeny theory in this application.

4) One also might question the reasonableness of the models which have been selected, suggesting the need for further geologic work to classify the physical characteristics of sedimentary rock before and after diagenesis.

5) Finally, one might go to the laboratory and see if permeability can be increased in certain cases, and thus reconcile what initially have been assumed to be conflicting ideas.

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- * Compare with Scheidegger's personal communication to the author, dated September 18, 1957: "I think the difficulties with the Kozeny equation originate in the fact that it assumes in essence that the resistance to flow results entirely from the sticking of the liquid to the walls of the pores and thus automatically arrives at an equation where permeability is universally proportional to the specific internal surface of the medium. All other effects that might contribute to the flow resistance are summarily neglected."

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CIRCULAR 242

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