FILTER CAKE FORMATION AND WATER LOSSES IN DEEP DRILLING MUDS

BY

WOLF VON ENGELHARDT

TRANSLATED BY

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PART I. PHYSICAL FOUNDATIONS OF THE FILTRATION OF CLAY SUSPENSIONS

The filtration processes that occur under the influence of pressure differences between the mud in the bore hole and the pore spaces of formation rocks are well known in both drilling operations and, of greater importance, in the subsequent production of oil. The usual precautions to improve muds lay stress upon keeping the resulting filter cake thin and reducing the velocity of filtration. The following is a report based on experimental investigations concerning the physical foundations of this filtration process. The behavior of muds in the well bore and the application of these investigations in practice will be treated in part II of this report.

A. General

In investigating the nature of mud filtration in our laboratories, the standard Baroid filter press was utilized. According to American procedures, one is accustomed to work at 7 atm. (approximately 100 psi) pressure and to indicate the quantity of filtrate collected and the thickness of filter cake for a 30-minute filtration time.

If \( F \) is the effective filter surface in square centimeters, \( P \) the filtration pressure in atmospheres, \( \eta \) the viscosity of the filtrate in centipoise, \( Q_t \) the quantity of filtrate in cubic centimeters separated after \( t \) seconds, and \( l_t \)
the filter cake thickness in centimeters for the same time, then the velocity of filtration in accordance with Darcy's law is:

\[
\frac{dQ_t}{dt} = k \cdot \frac{F}{\eta \cdot l_t}
\]  
(1)

where \( k \) is the permeability of the filter cake measured in darcys \((1 \text{ cm}^2 = 1.013 \times 10^8 \text{ darcys})\).

If \( b \) is the volume of filter cake which is formed per cubic centimeter of filtrate, one has:

\[
F \cdot l_t = b \cdot Q_t
\]  
(2)

If it is assumed that the permeability of the filter cake remains constant during the course of filtration, eq. (1), after introducing eq. (2), can be integrated to give the filtrate volume which is collected in \( t \) seconds:

\[
Q_t = \sqrt{\frac{2PF^2}{\eta}} \cdot \frac{k}{b} \cdot t
\]  
(3)

and for the thickness of the filter cake the following expression:

\[
l_t = \sqrt{\frac{2P}{\eta}} \cdot \frac{k \cdot b \cdot t}{Q_t}
\]  
(4)

Filtrate volumes and filter cake thicknesses should therefore increase in proportion to the square root of time. In fact, the measurement of the time relationships in the filtration of a very large number of field muds and different clay suspensions in our laboratories showed that for filtration times up to about one hour, very exact linear relationships were obtained. Accordingly, the filtration behavior of a mud is measured by two quantities: the permeability of the filter cake \( k \) and the specific volume \( b \) (which is filter cake volume divided by filtrate volume and is therefore dimensionless). From these two quantities, all filtration processes in the well bore should be predictable. As a consequence of the above equations, permeability and specific volume can be determined from experimental data according to the following formulae:

\[
k = \frac{Q_t \cdot l_t \cdot \eta}{2P \cdot F \cdot t}
\]  
(5)

\[
b = \frac{F \cdot l_t}{Q_t}
\]  
(6)

The specific volume \( b \) naturally depends in certain ways upon the porosity of the filter cake and the clay content of the mud, so that instead of the specific volume, the porosity of the filter cake can also be used as a characteristic quantity. Considering \( c \) as the clay concentration of the mud (grams clay dried at 150° C per gram of mud), \( \gamma_c \) the specific gravity of the dry clay, and \( \gamma_m \) that of the mud, the porosity of the filter cake \( \xi \) is obtained (von Engelhardt and Schindewolf, 1952) from:

\[
\xi = 1 - \frac{c \cdot \gamma_m}{\gamma_c} \left(1 + \frac{1}{b}\right)
\]  
(7)
Permeability, specific volume, and porosity of the filter cake and their relationships to the filtration time, pressure, temperature, electrolyte content, and clay concentration were investigated in our laboratories for various suspensions of Altwarmbuechen clays. This work has been reported in detail in another paper (von Engelhardt and Schindewolf, 1952), and only the results that are important in mud technology will be summarized here.

The composition of the Lower Cretaceous clay of Altwarmbuechen according to X-ray analysis is shown in Table 1.

| Illite     | 52  |
| Kaolinite | 21  |
| Quartz    | 15  |
| Montmorillonite | 9  |
| Calcite   | 2.5 |
| **Total** | 99.5|

B. The Formation of the Filter Cake

The derivation of the general filtration eqs. (3) and (4) assumes that permeability and porosity of the cake remain constant. Therefore, a linear relationship is expected between the volume of filtrate and the square root of time. However, investigations over longer periods (several hours) show that this assumption does not hold. In the case of prolonged experiments, somewhat more filtrate is continually collected than would be predicted by the parabolic function in eq. (3) (see figure 1). A detailed investigation of the filter cake that was formed in different periods of time showed that the cake is not built up homogeneously. The lower (oldest) layers are less porous and less permeable than the upper (youngest) layers; hence, the measured results for permeability and porosity of the entire cake reflect only average values. These mean values are changed with increasing filter cake thickness because of the following processes which occur simultaneously:

1. The thicker the filter cake is, the more porous and permeable are the newly formed layers.

2. In the course of time, the first formed layers decrease in porosity and permeability under the compressing influence of the filtrate flowing through them.

As is more completely discussed elsewhere (von Engelhardt and Schindewolf, 1952), both processes work against each other. In the initial period of about one hour the compression of the first formed layers predominates, whereas later the increasingly more porous structure of the newly formed layers influences the average values of permeability and porosity. Therefore, one observes first a slight decrease in the average porosity and permeability and after approximately one hour a gradual increase in these quanti-
ties. The initial decrease of average porosity and permeability is so insignificant that in this period eqs. (3) and (4) predict the course of filtration quite well. Later, the increase of the mean values signifies that the volume of filtrate and the filter cake thickness are increasing more quickly than before. In spite of this divergence from the simplified concept of filter cake formation, eqs. (3) and (4) retain their usefulness, because they are valid with good accuracy for both the initial course of filtration and for limited periods in the later filtration.

In the drilling well, the filter cake is formed by a circulating stream of mud which continually removes the outermost loose parts while the first-formed layers are increasingly compressed under the influence of the filtrate flowing through them. Therefore, in the bore hole one will probably have to reckon with smaller permeabilities and porosities than are determined in the filter press. The values for permeability, porosity, and specific volume that are obtained in the filter press after approximately one hour will most nearly correspond to results in the well bore because they perhaps then reach their minimum. The numbers which are reported in the following paragraphs are therefore (unless otherwise indicated) always referred to a filtration time of one hour.

C. Effect of Pressure on Filtration

Since the filter cake is compressible, porosity and permeability are strongly influenced by the magnitude of the filtration pressure and, of course, eqs. (3) and (4) apply only for a constant pressure. The pressure dependence of filtration was investigated for the following mud suspensions:

1. Mud suspension of Altwarmbuechen clay, concentration = 0.30 gm. clay per gm. mud.
2. The same mud with an addition of 35 gm. NaCl per liter (in the filtrate, 24.4 gm. Cl per liter).
3. The same mud as No. 1. Before the addition of 35 gm. NaCl per liter, one percent sodium carboxymethyl cellulose (Tylose BS of the firm Kalle and Co., Wiesbaden, Germany) was added.

Filtrate quantity Q, filter cake thickness l, porosity $\xi$, specific volume b, and permeability k, are compiled for different pressures in table 2. All values are based on a filtration time of one hour. For muds 1 and 2, the most important properties of the filter cake are presented in figure 2 as a function of filtration pressure. In the range between 2 and 30 atm., permeability and porosity decrease with increasing pressure, and obviously, further pressure increases alter the cake only slightly. On the other hand, because the filtration velocity increases with pressure according to the Darcy law, the filtrate volume increases with pressure in spite of diminishing permeabilities. This increase is, to be sure, substantially less than would be expected at constant permeability and porosity.
Table 2. Effect of Pressure on Filtration of Muds 1, 2, and 3 (Filtration Time = 1 hour)

<table>
<thead>
<tr>
<th>Muds</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtrate Volume-cc (Q)</td>
<td>25.8</td>
<td>46.0</td>
<td>54.1</td>
</tr>
<tr>
<td>Filter Cake Thickness-cm (I)</td>
<td>15.6</td>
<td>17.4</td>
<td>20.8</td>
</tr>
<tr>
<td>Filter Cake Permeability Darcys x 10^6 (k)</td>
<td>3.7</td>
<td>4.5</td>
<td>5.1</td>
</tr>
<tr>
<td>Filter Cake Porosity</td>
<td>0.70</td>
<td>0.71</td>
<td>0.74</td>
</tr>
<tr>
<td>Spec. Volume (b)</td>
<td>0.69</td>
<td>0.66</td>
<td>0.72</td>
</tr>
<tr>
<td>Pres. atm. (P)</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>25</td>
<td>42</td>
</tr>
</tbody>
</table>

DEEP DRILLING MUDS
The effective pressure difference in the well bore is determined by the difference between the specific gravity of the mud and that of the fluids in the porous rock. If a mud specific gravity of 1.25 and an average specific gravity of 1.05 for the formation salt water are used, an increase in filtration pressure of about 2 atm. will result for an increase in depth of 100 meters. At a depth of 500 meters (1640 ft.) one therefore has to reckon with about 10 atm. of pressure difference, for 1000 meters (3280 ft.) about 20 atm., for 2000 m (6560 ft.) about 40 atm., and for 3000 m (9840 ft.) about 60 atm. Consequently the velocity of filtration and the formation of filter cake will be greater the deeper the drilling well. The effect to be expected can be estimated from the data in table 2. In the normal deep well, filtration pressures will generally be higher than the 7 atm. pressure at which muds are usually investigated in the Baroid filter press. Therefore, for the correct evaluation of the behavior of a specific mud in the drilling well, it may be more suitable to perform the filtration experiments at the pressure which corresponds to the depth in question. Indeed, a higher pressure is also recommended by this study because, as shown in figure 2, the permeability is changed very greatly with pressure in the lower pressure ranges and the measured results are more easily reproduced at higher pressures.

D. Effect of Temperature on Filtration

Provided the structure of the filter cake is not influenced by temperature, the velocity of filtration at different temperatures should be dependent only on the viscosity of the filtrate which, of course, changes with temperature. One infers from eq. (3) that for a given mud, the product of filtrate volume and the square root of viscosity should be constant for variations in temperature. Experiments on a mud suspension with 31 percent Altwarmbuechen clay gave the results which are listed in table 3. In the region from 20 to 50°C, the elevation of temperature seems to exert some minor influence on the structure of the filter cake, because the filtration velocity at 50°C is somewhat higher than

<table>
<thead>
<tr>
<th>Temperature, oC</th>
<th>Filtrate Volume, cc</th>
<th>Filtrate Viscosity, cp</th>
<th>Q : \sqrt{\eta_F}</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>17.6</td>
<td>1.032</td>
<td>17.9</td>
</tr>
<tr>
<td>20</td>
<td>18.1</td>
<td>1.032</td>
<td>18.4</td>
</tr>
<tr>
<td>20</td>
<td>18.1</td>
<td>1.032</td>
<td>18.4</td>
</tr>
<tr>
<td>30</td>
<td>19.1</td>
<td>0.862</td>
<td>17.7</td>
</tr>
<tr>
<td>30</td>
<td>19.0</td>
<td>0.862</td>
<td>17.7</td>
</tr>
<tr>
<td>30</td>
<td>20.6</td>
<td>0.862</td>
<td>19.1</td>
</tr>
<tr>
<td>50</td>
<td>25.0</td>
<td>0.617</td>
<td>19.6</td>
</tr>
<tr>
<td>50</td>
<td>25.1</td>
<td>0.617</td>
<td>19.7</td>
</tr>
</tbody>
</table>
that computed from the decrease in viscosity. However, the effect is not very great, so that for practical purposes, at least in the range up to 50°C, it can be assumed without introducing too much error that the temperature dependence of filtration is limited by the decrease in filtrate viscosity with increasing temperature. For temperatures above 50°C, one should expect higher filtration velocities than are calculated from the decrease in viscosity.

E. Effect of Electrolyte Content of Mud on Filtration

If one adds a soluble salt such as NaCl to a clay suspension, flocculation occurs, and the relatively small primary particles of clay minerals in the electrolyte-treated suspension aggregate in coarse flakes. Therefore, the filter cake that is formed from such flocculated muds has a greater permeability and also, at high salt contents, an increased porosity. Consequently, the velocity of filtration of such muds and the thickness of the filter cake that is formed in a given time are greater than in the case of electrolyte-poor suspensions.

The effect of NaCl additions to Altwarmbuechen clay suspensions was investigated at filtration pressures of 7, 20, and 60 atm, in three series of tests. The muds investigated are listed in table 4 and the results of the research in table 5. Filter cake permeabilities and porosities are presented as a function of salt content in figure 3.

Table 4. NaCl Content of Mud Suspensions of Altwarmbuechen Clay

<table>
<thead>
<tr>
<th>Mud No.</th>
<th>Clay Concentration c</th>
<th>Specific Gravity</th>
<th>NaCl added gm/1</th>
<th>Chloride Concentration in Filtrate gm/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.298</td>
<td>1.227</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>5</td>
<td>0.297</td>
<td>1.230</td>
<td>5.0</td>
<td>4.1</td>
</tr>
<tr>
<td>6</td>
<td>0.295</td>
<td>1.234</td>
<td>10.0</td>
<td>7.6</td>
</tr>
<tr>
<td>7</td>
<td>0.297</td>
<td>1.237</td>
<td>15.0</td>
<td>11.0</td>
</tr>
<tr>
<td>8</td>
<td>0.292</td>
<td>1.242</td>
<td>20.0</td>
<td>14.5</td>
</tr>
<tr>
<td>9</td>
<td>0.291</td>
<td>1.245</td>
<td>25.0</td>
<td>17.7</td>
</tr>
<tr>
<td>10</td>
<td>0.291</td>
<td>1.250</td>
<td>30.0</td>
<td>20.9</td>
</tr>
</tbody>
</table>

At all pressures investigated, the volume of filtrate produced in 60 minutes increases with the electrolyte content according to the degree of flocculation. While the permeability increases regularly with increasing salt content, the average porosity of the cake appears to exhibit a minimum at a chloride content in the filtrate of about 5 gm/l. This effect of electrolyte content on the properties of the filter cake is of great importance in the behavior of drilling muds containing salt. It is customary to counteract the severe formation of filter cake and the infiltration of formation rocks that follows from such flocculated muds through additions of floc-inhibiting colloids such as carboxymethyl cellulose (CMC). The action of CMC on the properties of the filter
Table 5. Filtration of Mud Suspensions Containing NaCl at 7, 20, and 60 atm.
Pressure (Filtration time = 1 hour)

<table>
<thead>
<tr>
<th>Mud No.</th>
<th>Filtrate Volume-cc (Q)</th>
<th>Filter Cake Thickness-cm (l)</th>
<th>Filter Cake Permeability Darcys x 10^6 (k)</th>
<th>Filter Cake Spec. Volume (b)</th>
<th>Filter Cake Porosity (ξ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pressure-atm. 7</td>
<td>20</td>
<td>60</td>
<td>Pressure-atm. 7</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>35.0</td>
<td>38.8</td>
<td>45.0</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>5</td>
<td>44.3</td>
<td>48.4</td>
<td></td>
<td>0.55</td>
<td>0.47</td>
</tr>
<tr>
<td>6</td>
<td>54.6</td>
<td></td>
<td></td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>49.3</td>
<td>54.7</td>
<td>64.0</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>8</td>
<td>51.2</td>
<td>60.3</td>
<td></td>
<td>0.71</td>
<td>0.74</td>
</tr>
<tr>
<td>9</td>
<td>51.1</td>
<td>60.0</td>
<td></td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>10</td>
<td>51.9</td>
<td>63.6</td>
<td>68.8</td>
<td>0.86</td>
<td>0.87</td>
</tr>
</tbody>
</table>
cake may be noted in the results previously described in part C, in which two muds with high NaCl content, one with and one without CMC added, were investigated. The filter cakes of the suspensions treated with CMC have a substantially lower permeability. This must be due on the one hand to the floc-inhibiting property of the colloid and on the other hand to the plugging of the filter cake pores by the large CMC anions.

F. Effect of Clay Concentration in Mud on Filtration

The results of several filtration experiments performed at 20 atm. pressure with suspensions of varying content of Altwarmbuechen clay are compiled in table 6.

Table 6. Filtration of Mud Suspensions of Different Clay Content (Filtration time = 1 hour)

<table>
<thead>
<tr>
<th>Clay Conc. C</th>
<th>Filtrate Volume cc</th>
<th>Filter Cake Thickness cm</th>
<th>Filter Cake Permeability Darcys × 10^6</th>
<th>Filter Cake Specific Volume</th>
<th>Filter Cake Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.153</td>
<td>54.4</td>
<td>0.27</td>
<td>2.2</td>
<td>0.22</td>
<td>0.64</td>
</tr>
<tr>
<td>0.190</td>
<td>47.5</td>
<td>0.29</td>
<td>2.1</td>
<td>0.27</td>
<td>0.61</td>
</tr>
<tr>
<td>0.229</td>
<td>41.7</td>
<td>0.31</td>
<td>2.0</td>
<td>0.33</td>
<td>0.58</td>
</tr>
<tr>
<td>0.273</td>
<td>36.8</td>
<td>0.35</td>
<td>1.9</td>
<td>0.42</td>
<td>0.57</td>
</tr>
<tr>
<td>0.307</td>
<td>33.6</td>
<td>0.39</td>
<td>2.0</td>
<td>0.52</td>
<td>0.57</td>
</tr>
<tr>
<td>0.346</td>
<td>30.5</td>
<td>0.40</td>
<td>1.8</td>
<td>0.58</td>
<td>0.53</td>
</tr>
<tr>
<td>0.383</td>
<td>27.9</td>
<td>0.40</td>
<td>1.6</td>
<td>0.63</td>
<td>0.49</td>
</tr>
</tbody>
</table>

It appears from these studies that the formation of the filter cake is dependent on the clay concentration of the mud. The porosity of the cake decreases with increasing clay content. The permeability is not so strongly dependent on clay content, but here too, a slight decrease is evident with increasing clay content. Corresponding results were obtained in investigations at 7 atm. pressure. Therefore, the velocity of filtration in drilling muds of varying clay content decreases with increasing clay concentration more rapidly than would be expected on the basis of the increasing ratio of clay to water and assuming constant filter cake properties.
Fig. 1. - Filtrate volume vs. time for the filtration of a clay suspension at 7 atm. pressure.

Fig. 2. - Permeability and porosity of filter cakes formed at different pressures with clay suspensions nos. 1 and 2.

Fig. 3. - Permeability and porosity of filter cakes formed from clay suspensions of different NaCl contents.
PART II. THE BEHAVIOR OF MUDS IN THE WELL BORE

In part I, the process of filtering a clay suspension was described on the basis of laboratory research with the Baroid filter press. Conditions in the drilling well, however, differ from the restrictions of the laboratory tests in several respects. In the well bore, the filtration does not occur over an even surface of constant thickness but through a hollow cylinder whose effective surface is diminished linearly with the deposition of filter cake. Furthermore, the mud cake is deposited in the bore hole on rock, that is, on a porous medium, while in the filter press such a support is practically absent. Usually, filtration occurs in a well under the influence of the circulating mud stream whereas any fluid movement in the filter press is relatively calm. The results which are obtained with the filter press are therefore not readily applicable to the effects in the drilling well. The following discussion will investigate the conclusions that can be drawn from both laboratory investigations and general considerations of filter cake formation in the well.

A. The Filtration Process in the Well Bore

With reference to the relation of filter cake and rock matrix, two limiting cases can be distinguished immediately. If the pore spaces of the rock are nearly equal to the size of the clay particles in the mud, no clay will penetrate into the formation. The filter cake will accumulate on the surface of the drill hole with a sharp boundary between rock and filter cake. On the other hand, if the pore spaces are large in comparison with the clay particles, so that they are not kept back and can form no bridges at the boundary of the bore hole, drilling mud will infiltrate into the rock. The depth of penetration of this clay will be limited, since every clay particle that is swept in will tend to get caught somewhere in a constriction of a rock poré. In this manner, a zone impregnated with clay will first be formed around the circumference of the bore hole, and only when this zone can absorb no more clay will the filter cake be deposited on the wall of the hole. Both of these limiting cases will be considered separately in the following discussion.

1. Microporous rocks (no impregnation by drilling mud)

   In the case of microporous rocks having pore spaces too small to be penetrated by drilling muds, the following zones can be distinguished during the filter cake formations:
   1. Filter cake
   2. Rock filled with filtrate (infiltrated zone)
   3. Rock with original pore contents
   The three zones are shown in figure 1 with the notations that are used in the following derivations.

   At the surface of the filter cake, the mud pressure \( P_w \) is effective; at the filter cake-rock boundary, the pressure \( P_o \); at the outer boundary of the infiltrated zone, the pressure \( P_i \); and at distance \( r_e \) from the center of the bore hole, the original rock pressure \( P_e \). In all cases of filter cake formation: \( P_w > P_o > P_i > P_e \).
Fig. 1. - Formation of filter cake in microporous rocks.

Fig. 2. - Formation of filter cake in macroporous rocks.
Consider the flow of filtrate through a hollow cylinder of thickness \( dr \) within the filter cake. The surface of this hollow cylinder is \( 2\pi rh \), where \( r \) is the radius and \( h \) the height of the cylinder. In accordance with the Darcy law one then obtains:

\[
- \frac{dQ}{dt} = -\dot{Q} = \frac{kK}{\eta_F} \cdot 2\pi rh \cdot \frac{dP}{dr}
\]

where \( kK \) is the permeability of the cake and \( \eta_F \) is the viscosity of the filtrate. Upon integration, the pressure obtained at this point is:

\[
P_r = \frac{\dot{Q} \cdot \eta_F}{2\pi h kK} \cdot \ln r + C
\]

The boundary condition for \( r = r_a \) is \( P_r = P_w \). Thus, one obtains:

\[
P_r = P_w - \frac{\dot{Q} \cdot \eta_F}{2\pi h kK} \cdot \ln \frac{r}{r_a}
\]

In particular, the equation for the pressures at the boundaries of the filter cake and rock is:

\[
P_o = P_w - \frac{\dot{Q} \cdot \eta_F}{2\pi h kK} \cdot \ln \frac{r_o}{r_a}
\]

By similar considerations of the hollow cylinders within the infiltered zone and within the original rock, corresponding expressions can be derived for the pressure \( P_1 \) at the outer limit of the infiltered zone and for the rock pressure \( P_e \). A combination of these equations leads to the following expression for the filtration velocity:

\[
Q = \frac{2\pi h (P_w - P_e)}{\left(\frac{\eta_F}{kK} \ln \frac{r_o}{r_a}\right) + \left(\frac{\eta_F}{k_i} \ln \frac{r_i}{r_o}\right) + \left(\frac{\eta_o}{k_o} \ln \frac{r_a}{r_i}\right)}
\]

where \( \eta_o \) is the viscosity of the original pore fluids.

If the filtration takes place in a rock whose pores are filled with water, the viscosities \( \eta_F \) and \( \eta_o \) are of the same order of magnitude. The same is also true for the permeabilities \( k_i \) and \( k_o \) of the infiltered zone and the unaffected rock, respectively. However, according to the results presented in part I, the permeability of the filter cake \( kK \) is of the order of \( 10^{-6} \) darcys. Rock permeabilities, of course, vary between wide limits. Clays have permeabilities that are even less than \( 10^{-6} \) darcys while coarse-grained rocks can have permeabilities above 10 darcys. However, permeable rocks in the usual sense, that is, the sandstones and limestones that are normally encountered in drilling operations, generally have permeabilities above one millidarcy (\( 10^{-3} \) darcys). In such cases, the last two parts in the denominator of eq. (5) may be ignored. This is not altered by the fact that \( r_e/r_i \) can in certain cases be substantially greater than \( r_o/r_a \) and \( r_i/r_o \), because these ratios only enter into eq. (5) logarithmically. Thus, for rocks with permeabilities above one millidarcy, the processes in the interior of the rock can be neglected. The velocity of fluid movement is governed only by the flow of the filtrate through the mud cake, and the pressure \( P_o \) is practically equivalent to the original formation pressure \( P_e \).
It follows that the thickness of filter cake that is formed after a definite time is independent of the permeability of the rock and is determined only by the properties of the mud, assuming the rock permeability is greater than one millidarcy. Therefore, in general, no conclusions as to rock permeability can be drawn from the thickness of the filter cake, which, of course, can be deduced from microlog data (Tunn, 1952).

For water-filled rocks with permeabilities above one millidarcy, eq. (5) may be simplified to the following expression:

\[
\dot{Q} = \frac{2\pi h k K}{\eta_F \ln \frac{r_o}{r_a}} (P_w - P_e)
\]

(6)

This equation can be integrated because \( r_a \) can be expressed either as a function of the filtrate volume \( Q \) or \( Q \) as a function of \( r_a \). In the first case, an equation for the increase of filter cake thickness with time is obtained, and in the second case, the result is an equation for the volume of filtrate which has penetrated into the rock. Restricting ourselves to the former case, the introduction of the specific volume of the filter cake leads to the following expressions:

\[
b \cdot Q = \pi h r_o^2 - \pi h r_a^2 \\
dQ = - \frac{2\pi h r_a}{b} \cdot dr_a
\]

(7)

After substituting for \( dQ \) in eq. (6) and recognizing that \( r_a = r_o \) when \( t = 0 \), integration leads to the following relation between time and filter cake thickness:

\[
t = \frac{\eta_F}{2bkK (P_w - P_e)} \left[ r_a^2 \ln \frac{r_a}{r_o} + \frac{1}{2} (r_o^2 - r_a^2) \right]
\]

(8)

If the filtration takes place opposite an oil-bearing formation, the assumptions introduced in deriving eq. (6) are no longer completely valid. Since the invading mud filtrate is not able to displace all of the oil, the infiltered zone contains residual oil. As a result, \( k_i \), or the permeability of the infiltered zone relative to the filtrate, is decreased and the second part of the denominator in eq. (5) becomes somewhat larger. Nevertheless, as a first approximation, it is permissible to disregard the effect of this second part as compared to that of the first.

2. Macroporous rocks (impregnation by drilling mud)

If the pore spaces in the rock are larger than the clay particles, drilling mud will be deposited in the rock at the beginning of the filtration process. Four concentric zones must then be considered: the filter cake, an impregnated zone, an infiltered zone, and the rock with its original pore content (see figure 2). The first clay particles that enter a specific rock will reach on the average the same maximum depth of penetration \( r_s \) (as measured from the center of the well bore) before becoming stuck. The following particles will
fill up the pore spaces between \( r_0 \) and \( r_s \). Only when this impregnated zone is completely developed will the filter cake begin to form. Since only 20 to 30 percent of the rock volume is free pore space, relatively small volumes of penetrating mud will suffice to build up impregnated zones of significant thickness.

The depth of penetration of the clay \( r_s \) will depend both on the kind of rock and also on the nature of the drilling mud. The impregnated zones will be thicker in the case of coarse-grained, highly permeable rocks than in the case of low permeabilities. On the other hand, well-dispersed drilling muds will produce larger impregnated zones than will flocculated muds with coarse clay particles. The permeability of the impregnated zone \( k_s \) will, in any case, be smaller than that of the rock. If the clay particles that fill up the rock become as thickly stratified as in the filter cake, \( k_s \) may even be smaller than the permeability of the filter cake. Probably such an impervious layer will not develop, but it must be recognized that the permeability of the impregnated zone is of the same order of magnitude as that of the filter cake.

The uncertainty as to the thickness of the impregnated zone, the quantity of clay that is deposited in it, and its resulting permeability complicate quantitative calculations of the filtration process in macroporous rocks. Experimental investigations on the penetration of drilling muds in a specific rock are presently being conducted in our laboratories. However, general conclusions concerning this particular filtration process are already possible.

Mud filtration in rocks with large pore spaces breaks down into two phases. First, the "internal filter cake" forms in the impregnated zone, and second, the normal filter cake is deposited on the rock wall. The mathematical treatment of the formation of the impregnated zone can follow a form analogous to that utilized in the previous paragraphs in considering filter cake formation. The permeability of the filter cake \( k_K \) is replaced by the permeability of the impregnated zone \( k_s \). An intermediate radius \( r_b \) is introduced to designate the increasing thickness of the impregnated zone between \( r_s \) and \( r_0 \) \((r_s \geq r_b \geq r_0)\). Further, it should be borne in mind that only a fraction of the total available pore space in the impregnated zone is filled by the volume of "internal filter cake." This fraction shall be designated as \( p' \). If the penetrating clay in the impregnated zone fills all pores of the rock, then \( p' = p \), the porosity of the rock. In general, \( p' \) will be smaller than \( p \). The progress of the formation of the impregnated zone may be derived in a manner analogous to the development of eq. (8), and the expression is:

\[
t = \frac{p' \cdot \eta F}{2b k_s (P_w - P_e)} \left[ r_b^2 \ln \frac{r_b}{r_s} + \frac{1}{2} (r_s^2 - r_b^2) \right] \quad (9)
\]

This process is terminated when \( r_b = r_0 \); consequently, for the entire impregnation time:

\[
T_s = \frac{p' \cdot \eta F}{2b k_s (P_w - P_e)} \left[ r_0^2 \ln \frac{r_0}{r_s} + \frac{1}{2} (r_s^2 - r_0^2) \right] \quad (10)
\]
In the subsequent period of normal filter cake formation, the impregnated zone must also be considered, because its permeability is of the same order of magnitude as that of the filter cake. According to eq. (5), the velocity of filtration during the formation of the external filter cake is:

\[
\dot{Q} = \frac{2\pi h (P_w - P_e)}{\left(\frac{\eta F}{kK} \ln \frac{r_o}{r_a} \right) + \left(\frac{\eta F}{k_s} \ln \frac{r_s}{r_o}\right)} \tag{11}
\]

Integration of this equation leads to the following expression for the formation time of a filter cake of thickness \(r_o - r_a\):

\[
t = \frac{\pi F}{2bkK (P_w - P_e)} \left[r_o^2 \ln \frac{r_a}{r_o} + \frac{1}{2} (r_o^2 - r_a^2)\right] + \frac{\pi F}{2bk_s (P_w - P_e)} \left[\ln \frac{r_s}{r_o} (r_o^2 - r_a^2)\right] \tag{12}
\]

As a first approximation, if the permeability of the impregnated zone is set equal to that of the filter cake \((k_s = k_K)\), eq. (12) may be simplified to:

\[
t = \frac{\pi F}{2bkK (P_w - P_e)} \left[r_o^2 \ln \frac{r_a}{r_o} + \left(\frac{1}{2} + \ln \frac{r_s}{r_o}\right) (r_o^2 - r_a^2)\right] \tag{13}
\]

B. Examples of Calculations on the Filtration Process

Assuming a static mud column and rock permeabilities above one millidarcy, the time required for the formation of filter cake may be represented by the following equation:

\[
t = R \cdot A \tag{14}
\]

where:

\[
R = \frac{\pi F}{2bkK (P_w - P_e)}
\]

and

\[
A = r_o^2 \ln \frac{r_a}{r_o} + \left(\frac{1}{2} + \ln \frac{r_s}{r_o}\right) (r_o^2 - r_a^2)
\]

The factor \(R\) depends only on the depth of the bore hole and the character of the mud and shall be called the mud function. The factor \(A\) contains the bore hole radius, the thickness of the impregnated zone, and the thickness of the filter cake and shall be called the geometric function.

Eq. (14) applies to the microporous rocks and also to macroporous rocks with impregnation (assuming the permeabilities of the impregnated zone and the filter cake are equal). In computing total time for the latter case, the interval required for impregnation must, of course, be added to the formation time of the external filter cake. The impregnation time may be computed from:
\[ T_s = R \cdot B \]

where \[ B = p' \left[ r_o^2 \ln \frac{r_o}{r_s} + \frac{1}{2} (r_s^2 - r_o^2) \right] \] (15)

The factor B contains the fractional pore volume \( p' \), the bore hole radius, and the radius of the impregnated zone. The mud function \( R \) is the same as above.

In figures 3 and 4, the geometric functions A and B are presented as plots of the thickness of the filter cake and of the impregnated zone, respectively. A well bore radius of 10 cm and \( p' = 0.15 \) were assumed. The mud function \( R \) may be calculated from values of the permeability and specific volume that have been experimentally determined for a given mud under conditions that are appropriate for the depth of the well. The dimensions of \( R \) are sec. cm\(^{-2} \), provided pressures are given in atmospheres, the viscosity in centipoises, and the permeability in darcys. Since the dimensions of A and B are cm\(^2 \), multiplication by \( R \) gives time in seconds.

Using eq. (14) and (15), the effect on the filtration process of (1) pore size, (2) well depth, and (3) the flocculated condition of the mud, can now be investigated. As mentioned previously, these calculations apply only to the formation of filter cake in a static mud column. With regard to the depth of the impregnated zone, its permeability and its clay content, the lack of experimental data makes it necessary to assume certain values for these factors. Therefore, no claim can be made as to the quantitative exactness of these calculations in predicting the filtration process in the well bore.

1. Effect of pore size of the rock

As a basis for calculations, the following data for depth, pressure, and temperature are assumed:

<table>
<thead>
<tr>
<th>Depth</th>
<th>Temp.</th>
<th>( P_{e} ) Pressure in Pore Spaces of Rock</th>
<th>( P_{w} ) Mud Pressure</th>
<th>( P_{w} - P_{e} ) Filtration Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 m.</td>
<td>40°C</td>
<td>105 atm.</td>
<td>123 atm.</td>
<td>18 atm.</td>
</tr>
</tbody>
</table>

The mud is assumed to have the same properties as the 30 percent Altwarmbuechen clay suspension that was described in part I. Under the above conditions, the following values apply:

<table>
<thead>
<tr>
<th>Spec. gr.</th>
<th>( \eta ) F Viscosity of Filtrate at 40°C</th>
<th>( k ) Permeability of Filter Cake at 18 atm. darcys</th>
<th>( b ) Specific Volume of Filter Cake at 18 atm.</th>
<th>( R ) Mud Function day \cdot cm(^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.234</td>
<td>0.653 cp</td>
<td>2.0 \cdot 10^{-6}</td>
<td>0.46</td>
<td>0.228</td>
</tr>
</tbody>
</table>
Fig. 3. - The dependence of function A on the filter cake thickness for different zones of impregnation ($r_o = 10 \text{ cm}$).

Fig. 4. - The dependence of function B on the thickness of the impregnated zone ($r_o = 10 \text{ cm}$, $p^t = 0.15$).
The results of filter cake formation on four different rocks with the above mud are presented in figure 5. One rock shows no impregnation, and the other three have impregnation zones of 1, 4, and 8 cm thickness. It will be noted that the development of mud cake is significantly different from that in the filter press because of the particular geometrical relationships in the well bore. The velocity with which cake is formed decreases somewhat only at the beginning, then follows more or less a linear relationship and accelerates at the end. In microporous rocks where the small pore size prevents any mud penetration \((r_s - r_o = 0)\), filter cake formation begins immediately, while in the macroporous rocks, the formation of the external filter cake does not begin until the accumulation of clay in the interior of the rock is completed. In this latter case, the velocity with which the external filter cake is formed diminishes as the impregnated zone extends further away from the well bore.

2. Effect of well depth

With increasing depth, temperature and filtration pressures increase. The decrease of filtrate viscosity with increasing temperatures and the increase in pressure accelerate the formation of filter cake but, on the other hand, arrest the normal decrease in the permeability of the filter cake with increasing pressure.

The net effect of these individual factors may be computed for a specific mud that was investigated at several different pressures in the laboratory. The following temperatures and pressures are assumed for a microporous rock without impregnation.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>25</td>
<td>52</td>
<td>62</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>40</td>
<td>105</td>
<td>123</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>70</td>
<td>210</td>
<td>246</td>
<td>36</td>
</tr>
</tbody>
</table>

The following data for a 30 percent Altwarmbuechener drilling mud are taken from laboratory research (see section C of part I).

<table>
<thead>
<tr>
<th>No.</th>
<th>Pw−P̅e atm.</th>
<th>Temp. °C</th>
<th>k darcys</th>
<th>b</th>
<th>F cp.</th>
<th>R day · cm⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>25</td>
<td>3.7 ⋅ 10⁻⁶</td>
<td>0.50</td>
<td>0.894</td>
<td>0.280</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>40</td>
<td>2.0 ⋅ 10⁻⁶</td>
<td>0.46</td>
<td>0.653</td>
<td>0.228</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>70</td>
<td>1.3 ⋅ 10⁻⁶</td>
<td>0.45</td>
<td>0.406</td>
<td>0.0886</td>
</tr>
</tbody>
</table>

Filter cake thicknesses computed from these values are presented as a function of time in figure 6. With greater depth, the velocity of filter cake formation increases quite considerably. For example, the velocity at 2000 meters (6,560 feet) is about three times as great as at 500 meters (1640 feet).
Fig. 5. - Rate of formation of filter cake for different impregnated zone thicknesses ($r_0 = 10$ cm.).

Fig. 6. - Rate of formation of filter cake at different depths assuming no impregnated zone ($r_0 = r_0 = 10$ cm.).

Fig. 7. - Rate of formation of filter cake for different drilling muds assuming no impregnated zone. I - Normal mud ($c = 0.30$); II - Mud with 35 gm. NaCl per liter; III - Mud with 35 gm. NaCl per liter plus 1% carboxymethyl cellulose.
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3. Effect of salt content in the mud

To demonstrate the effect of salt content on the formation of filter cake in the well bore, the following assumptions of depth, pressure, and temperature were taken as a basis for calculations:

<table>
<thead>
<tr>
<th>Depth</th>
<th>Temp.</th>
<th>Pe Pressure in Pore Spaces of Rock</th>
<th>Pw Mud Pressure</th>
<th>Pw - Pe Filtration Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 m.</td>
<td>40°C</td>
<td>105 atm.</td>
<td>123 atm.</td>
<td>18 atm.</td>
</tr>
</tbody>
</table>

For muds No. 1 (salt free), No. 2 (35 gm/liter NaCl), and No. 3 (35 gm/liter NaCl, 1% CMC) that were discussed in section C of part I, the following values apply for the above assumed conditions:

<table>
<thead>
<tr>
<th>Mud No.</th>
<th>k darcys</th>
<th>b</th>
<th>7F cp</th>
<th>R day · cm⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0 × 10⁻⁶</td>
<td>0.46</td>
<td>0.653</td>
<td>0.228</td>
</tr>
<tr>
<td>2</td>
<td>9.5 × 10⁻⁶</td>
<td>0.62</td>
<td>0.653</td>
<td>0.0356</td>
</tr>
<tr>
<td>3</td>
<td>1.0 × 10⁻⁶</td>
<td>0.60</td>
<td>0.653</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Filter cake thicknesses calculated for a microporous rock without impregnation are presented as a function of time in figure 7. The rapid formation of filter cake in the flocculated mud and the beneficial effect of carboxymethyl cellulose are easily discernible.

C. Applications in Practice

A series of simplifying conditions as well as arbitrary assumptions as to the thickness and condition of the zone impregnated with mud underlies the previous calculations. Accordingly, the numerical values that have been presented cannot be applied directly to actual situations in the field. Nevertheless, these laboratory investigations and the calculated curves of filter cake formation facilitate a general understanding of the process of filtration and the different conditions that are important in drilling technique and practical mud control.

First of all, a knowledge of the structure of filter cake and the process of its formation under static conditions is important. Except for a short initial period, it can be seen in figure 5 that the formation of mud cake proceeds at practically constant velocity. In the filter press, on the other hand, mud cake thickness increases in a parabolic function; that is, the rate of deposition continually decreases and after a given time for a specific mud, the amount of filtration is so small as to be negligible. Such a condition can never be realized in the well bore. Thus, in order to reduce the possibility of clogging the well with mud cake, its permeability can be decreased by chemical treatment. This may be partially accomplished by the addition of bentonite or carboxymethyl cellulose. However, the filtration will never quite cease in the case of a static mud column. Other than this, the above results apply to flowing mud.
It may be deduced from the investigations with the filter press that the filter cake in the bore hole is formed inhomogeneously. The layers nearer the rock are more consolidated than the more porous outer layers. Consequently, the circulating mud stream will repeatedly remove the more porous particles of the cake while the first formed layers become thicker and more impermeable because of the filtrate flowing through them. Therefore, the filter cake will develop a certain equilibrium thickness depending on particular conditions in the well bore, especially the ascending mud velocity. Of course in each case the thickness of the cake will be less as the mud circulation increases. Where high filter cake permeabilities are present, static conditions in the well must be avoided as much as possible since mud cake can grow to appreciable thicknesses very quickly.

The harmful action of mud flocculation has previously been discussed in part I. Flocculation can be caused in a drilling mud either by the presence of cement or by salt which may be encountered in the drilling operations. Wherever permeable formations are exposed to such flocculated muds, heavy filter cake formation will occur. The addition of salt-stabilized bentonite, carboxymethyl cellulose, or the starch products can be used to counteract this condition. A heavy filter cake formation will also develop if the drilling mud contains too little good clay and has become enriched with other minerals such as limestone or dolomite.

Although the permeability of the filter cake is reduced by increased pressure, investigations with the filter press show that at greater depths, there is also an increased velocity of filtration and filter cake formation. The acceleration of the filtration process is clearly shown on the curves of figure 6. This is very important and indicates the necessity for mud control as depth increases in order that the velocity of filtration will remain small.

The kind of permeable rock is of particular importance in the formation of filter cake. In the case of microporous rocks, an especially quick growth of filter cake can be expected. This is confirmed by observations in the field. For example, in drilling the Schreib chalk, which is encountered below diluvial and Tertiary beds in Holstein (northern Germany), experience has shown that the danger exists of the well bore becoming completely filled with mud cake in a very short time so that the drilling tools become stuck.

A core of the Schreib chalk near Eisendorf in Holstein gave a porosity of 36 percent and a permeability of 0.5 millidarcys. The rock is very porous but its pore spaces have an average diameter of only about 1 micron. Such a rock can no doubt absorb filtrate, but the formation of an impregnated zone is out of the question. Consequently, the formation of filter cake takes place relatively quickly. Such formations cannot be drilled without danger unless the mud is controlled so as to form a filter cake with as little permeability as possible. This is accomplished by the addition of bentonitic clays and carboxymethyl cellulose.

As previously discussed, rocks with higher permeabilities and larger pore spaces tend to form impregnated zones. The penetrating clay then retards the velocity of the filter cake formation. An example of this is the Bentheimer sand (Valendis) of Emsland (western Germany). In this formation, the velocity of filter cake formation is slowed down so much by the
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impregnated zone that drilling difficulties never occur. The fact that the filtration process in the Bentheimer sand is retarded and actually proceeds very slowly is apparent on electric logs. The thickness of the infiltered zone can, of course, be computed from resistivity measurements. Such calculations in the Bentheimer sand of the Georgsdorf, Scheerhorn, and Ruhlermoor fields (western Germany) show that the infiltered zone is generally less than 10 centimeters in depth. This limited invasion by the filtrate is quite understandable if one assumes plugging of the pores by clay particles.

Correspondingly, the filter cake thicknesses are also relatively small in these sands. As an example of this, mud cake thicknesses calculated from microlog data for a well in the Bentheimer sand of the Apeldorn field are presented in figure 8. It is interesting to note on this profile that one can discern the influence of the nature of the rock on the thickness of the cake. The filter cake is thinnest opposite the coarsely grained rocks between the depths of 800 and 806 meters. This is to be expected since the impregnation of the strata with mud will be greatest in the largest pores. Of course, the formation of the impregnated zone also depends upon the size of the clay particles, since the ease of penetration in a given rock will increase as particle size decreases.

Figure 9 shows the thickness of the infiltered zone in the Bentheimer sand of a well in the Ruhlermoor field, as calculated from electrical resistivity data. The formation in this well down to a depth of 884 meters is an oil sand with about 18 percent capillary water (Haftwasser). Below is a water zone (Randwasser) with 10 percent NaCl. The thickness of the infiltered zone is obviously greater opposite the water zone. It is conjectured that the drilling mud became flocculated by the salt water and the resulting coarse clay particles were no longer able to penetrate into the rock and form an impregnated zone.

While the formation of impregnated zones is convenient in normal drilling operations because the formation of filter cake is slowed down, these zones can be extremely harmful to oil production. The impregnation of oil sands by clays will substantially reduce the effective permeability for the flow of oil into the well. Part of the alluvial clay will probably be removed by the flowing oil, but it is also possible that, in certain cases, considerable quantities of clay remain in the rock and obstruct fluid movement. Thus, in drilling through oil-bearing formations, if the reservoir rock contains large pores, it will be necessary to utilize muds that do not form impregnated zones. Such muds are, for example, clay-free and oil-base or flocculated drilling muds that contain, as much as possible, large clay particles that cannot penetrate into the rock.

REFERENCES


Fig. 8 - Principal grain size of the Bentheimer sandstone in Apeldorn well no. 4 and filter cake thicknesses as calculated from microlog data.

Fig. 9 - Depth of the infiltered zone in the Bentheimer sandstone of Ruhlermoor well no. 80 at the transition between the oil and water zones.