A Design for Non-Sticking Plug and Ring Gages and Locators

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I. THE NATURE OF THE PROBLEM

It is well known that if a plug is inserted in a close-fitting hole, the plug has a tendency to stick or jam if it is not truly in line with the hole when it enters. When the plug or ring is manipulated by hand, it is difficult to align the two closely enough to prevent initial sticking. The plug and ring must then be realigned, and often the attempt at realignment results in sticking in another position. The trial may have to be repeated several times until the two pieces go together. A similar tendency to stick at the end of the hole also occurs when the plug is removed. The result is normally a loss of time whenever a plug is inserted in or removed from a close-fitting hole, or vice versa. This situation prevails in the following operations: holes checked with plug gages; shafts or rounds gaged with ring gages; bushings slipped in and out of jigs; boring bar pilots entered by hand in guide bushings; straight shank tools inserted in close-fitting drivers or spindle holes; an arbor support placed on a round milling machine arbor; workpieces placed on arbors, locating plugs, or in locating holes of fixtures.

Certain empirical means of alleviating or eliminating the sticking tendency described have been known and used for a long time. Some results have been obtained from grinding a short pilot or lead diameter on the end of the plug, or better, from an aligning groove near the end of the plug. This feature was incorporated in U.S. Patent No. 1,393,233 of October 11, 1921, to Albert A. Martell, illustrated by Fig. 1. Similar but more complete coverage is included in U.S. Patent 2,199,052 issued April 30, 1940, to F. W. M. Lee and illustrated by Fig. 2. Of particular interest is that the second patent extends the concept of the groove; it is placed in the end of the hole where it is not permitted on the plug.

The applications of these ideas heretofore have been largely by “rule of thumb.” In the literature, instructions for the design of the patented parts have commonly specified that the actual proportions be obtained by trial and error in each case.\(^{(1)}\)\(^*\)

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\(^{(1)}\) Parenthesized superscript numbers refer to the corresponding entries in References.
Some empirical designs have been offered for devices like those described.\(^2\) A few formulas without derivations have been given in recent years,\(^3\),\(^4\) but otherwise nothing has been found that constitutes a logical basis for design. This investigation has been undertaken to ascertain the conditions that cause plugs to stick and, from analysis of the conditions, to determine the limiting dimensions for non-sticking devices. The investigation has not been confined to the common non-sticking devices already in use and just described, but has encompassed some related devices and also has revealed the non-sticking possibilities of others heretofore not generally applied for such purposes.

![Diagram of plugs and rings with aligning grooves](image-url)
II. THEORY OF STICKING PLUGS

1. Basic Geometric Considerations

If a plug inserted in a hole is short enough or its diameter is sufficiently small with respect to its length, the plug will make contact with only one spot in the hole at a time in any position and consequently cannot stick. This limiting condition for the prevention of sticking is illustrated by the sketch of Fig. 3. The plug of diameter $d$ and length $l$ will not stick when inclined at any angle in the hole of diameter $D$ if

$$D \geq \sqrt{d^2 + l^2} \quad (1)$$

or

$$l \leq \sqrt{D^2 - d^2} \quad (2)$$

or

$$d \leq \sqrt{D^2 - l^2} \quad (3)$$

A specific situation related to the foregoing occurs when the angle to which the plug may be inclined with respect to the hole is limited. This is the case when a workpiece is placed on a plug and against a locating face or shoulder as depicted in Fig. 4. The angle $\theta$ in any position is limited by the contact between the workpiece and the shoulder of the plug.

To evaluate the conditions necessary to keep the workpiece from sticking as it goes over the plug, consider the schematic arrangement in Fig. 5. The line GO represents the opening of the hole of diameter $D$ in the workpiece. The line OA is along the face of the workpiece and represents the shortest distance from the hole to the outside edge of the piece. The outline FECB is a projection of the plug of diameter $d$ and length $l$. The line BA' is radial.

Fig. 3. The Pertinent Dimensions of a Short Plug Free in Any Position in a Hole

Fig. 4. A Diagram of a Workpiece Being Placed on a Plug and Against a Face
Fig. 5. The Conditions for a Workpiece Being Placed on a Plug and Against a Face

The angle OBA of Fig. 5 is a right angle, and $AB = OA \cos \theta = a \cos \theta$.

Then

$$AK = (a \cos \theta + d) \cos \theta$$
$$H = (a \cos \theta + d) \cos \theta + l \sin \theta$$
$$= a \cos^2 \theta + d \cos \theta + l \sin \theta$$

(4)

When $H = D + a$, the locus of point E intersects the line $GJ$. If $GJ$ is tangent to or above the curve so that $H \leq D + a$, the plug will slip into the hole without sticking.

The point E is highest above the x-axis when

$$\frac{dH}{d\theta} = 0 = 2a \cos \theta (- \sin \theta) - d \sin \theta + l \cos \theta$$

$$= \frac{l}{\sin \theta} - 2a - \frac{d}{\cos \theta}$$

and

$$l = 2a \sin \theta + d \frac{\sin \theta}{\cos \theta}$$
$$= \sin \theta \left(2a + \frac{d}{\cos \theta}\right)$$

(5)

Substitute Eq. 5 into Eq. 4 to get

$$H = D + a = a \cos^2 \theta + d \cos \theta + \sin^2 \theta \left(2a + \frac{d}{\cos \theta}\right)$$
$$= \frac{d}{\cos \theta} + 2a - a \cos^2 \theta$$

or

$$a \cos^2 \theta = d - (D - a) \cos \theta$$

(5a)

For angles of inclination less than 15 deg, which it is assumed will not be exceeded, $\theta < 0.262$ rad.

Also, $\sin 0.262$ rad. $= 0.259$, with the difference in the third decimal place.

Consider

$$\cos \theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} \ldots$$

In this case, since $\theta$ is of the order of $10^{-3}$, terms of $\theta^4$ and higher may be neglected. Also, as a close approximation

$$\cos^3 \theta = 1 - \frac{3\theta^2}{2}.$$  

Equation 5a may be approximated by

$$a \left(1 - \frac{3\theta^2}{2}\right) d = (D - a) \left(1 - \frac{\theta^2}{2}\right) \text{ and } \theta = \sqrt{\frac{2(D - d)}{2a + D}}$$

Substitute for $\cos \theta$ and $\sin \theta$ in terms of $\theta$ in Eq. 5 to get

$$l = \sqrt{\frac{2(D - d)}{2a + D}} \left(2a + \frac{d}{1 - \frac{D - d}{2a + D}}\right)$$
$$= \sqrt{\frac{2(D - d)}{2a + D}} \left(4a^2 + 4ad + dD\right)$$

Since $D - d$ is small, let $dD = \theta^2$, and

$$l = (2a + d) \sqrt{\frac{2(D - d)}{2a + D}}$$

and as a further approximation

$$l = \sqrt{2} \frac{2a + d}{D - d} \left(D - d\right) = \sqrt{2} \frac{2a + d}{C}$$

where $(D - d) = C$.

This is a practical approximation to the length, $l$, of a non-sticking plug extending from a face-

plate.

The converse of the foregoing case is one where-in the workpiece has a projection of definite length $l$ and diameter $d$ that must be inserted in a locating hole of diameter $D$. This situation is suggested by the diagram of Fig. 6. The workpiece has a flange along whose face is a radial line $BA$. The diameter $d$ projecting from the flange face of the workpiece on the shoulder of the plug, is longer than $OA$, and makes contact with the workpiece at $A$. To simplify the problem, the workpiece is considered stationary and the plug enters it, bearing against point $O$. As the plug enters the hole, the angle of inclination $\theta$ varies as does the position of the line $BA'$, but $BA'$ is held in contact with the workpiece at point $A$ which moves along $BA'$.

The point $E$ is highest above the x-axis when

$$\frac{dH}{d\theta} = 0 = 2a \cos \theta (- \sin \theta) - d \sin \theta + l \cos \theta$$

$$= \frac{l}{\sin \theta} - 2a - \frac{d}{\cos \theta}$$

and

$$l = 2a \sin \theta + d \frac{\sin \theta}{\cos \theta}$$

$$= \sin \theta \left(2a + \frac{d}{\cos \theta}\right)$$

(5)
is outlined by BCEF. The face of the workpiece is ultimately located against the face A’AQPR T of a fixture, and the projecting diameter on the workpiece is located in hole OKJG of diameter D. The projecting diameter of the workpiece may not be changed from specifications, but the hole of diameter D can be altered to eliminate sticking. A typical alteration is the countersink to depth m as shown. A counterbore or spherical surface might be put in instead of a countersink. The important question is: what minimum depth, m, is necessary to keep the plug from binding in the hole?

As the plug enters the hole in Fig. 6, an element BC is assumed to maintain contact with point O, and the point A on the edge of the workpiece flange stays on line A’Q. The point E has a locus as indicated by the dashed curve, for which

\[ y = (a + d) \cos \theta + l \sin \theta - \frac{a + m \sin \theta}{\cos \theta} \]  
(7)

\[ \frac{dy}{d\theta} = l \cos \theta - (a + d) \sin \theta - \frac{m + a \sin \theta}{\cos^2 \theta} = 0 \]

If \( \cos \theta \) is assumed to be 1, since angle \( \theta \) is small when the plug enters the hole,

\[ \sin \theta = \frac{l - m}{2a + d} \]  
(8)

and  

\[ y_{max} = d + l \left( \frac{l - m}{2a + d} \right) - m \left( \frac{l - m}{2a + d} \right) \]

But \( y_{max} \leq D \) if the point E is not to bind against the element GJ of the hole, and as an approximation

\[ m \geq l - \sqrt{(D - d) (2a + d)} \]  
(9)

2. The Basic Causes of Sticking

Friction is the cause of a plug sticking in a hole when the axes are not in alignment. The condition existing is illustrated in Fig. 7. A plug has entered a distance \( m \) into a hole and is being pushed by a force \( F_A \) having a component \( F_H \) parallel to the axis of the hole and \( F_V \) at right angles. The force \( F_A \) intersects the axis of the hole a distance \( p \) from the
face of the receiver. The plug is inclined at an angle \( \alpha \) and makes contact in the hole at points A and B.

For equilibrium, the horizontal forces are:

\[
F_H = F_{r_2} + F_{r_1} \cos \alpha + F_1 \sin \alpha
\]  
(10)
The vertical forces are

\[
F_z + F_V + F_{r_1} \sin \alpha = F_1 \cos \alpha
\]  
(11)
Moments around point A are

\[
m_{F_2} + \frac{D}{2} F_H = p F_V + D F_{r_2}
\]  
(12)
and

\[
F_2 = \frac{p}{m} F_V - \frac{D}{m} \left( \frac{1}{2} F_H - F_{r_2} \right)
\]  
(12a)

When a plug is a close fit in a hole, the angle \( \alpha \) is small and \( F_H \) as expressed in Eq. 10 is substantially equal to \( F_{r_2} \) plus \( F_{r_1} \). When \( m \) is small, \( F_{r_1} \) approaches \( F_{r_2} \) in size and \( \frac{D}{2} F_{r_1} - F_{r_2} \) is small. For larger values of \( m \), \( F_H \) becomes smaller because \( F_{r_1} \) and \( F_{r_2} \) cannot exceed \( m F_1 \) and \( m F_2 \), respectively. Consequently, the size of \( F_2 \) as given by Eq. 12a depends mostly upon \( p/m F_V \). For a given moment of \( p F_V \) applied to the plug, the force \( F_2 \) drops off from a large value as \( m \) increases from a small value, \( F_1 \) decreases correspondingly.

For any given moment of \( p F_V \) applied to the plug, the significant magnitude of the component \( F_H \) of the force applied to the plug is that which causes the plug to slide farther into the hole. When movement of the plug into the hole is imminent in any position, \( F_{r_1} = \mu F_1 \) and \( F_{r_2} = \mu F_2 \), where \( \mu \) is the coefficient of friction. Equations 10 and 11 may be written as

\[
F_1 (\mu \cos \alpha + \sin \alpha) = F_H - \mu F_2
\]
and

\[
F_1 (\cos \alpha - \mu \sin \alpha) = F_V + F_2
\]
If the first is divided by the second,

\[
\frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha - \mu \sin \alpha} = \frac{F_H - \mu F_2}{F_V + F_2}
\]
For convenience, let

\[
\frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha - \mu \sin \alpha} = \phi
\]
Let \( D - d = C \). Also, in Fig. 7, \( \overline{AG} = \frac{d}{\cos \alpha} \).

Let \( D - \overline{AG} = D - \frac{d}{\cos \alpha} = C_1 \). For small angles \( C_1 \ll C \). Then

\[
\tan \alpha = \frac{C_1}{m} \quad \sin \alpha = \frac{C_1}{\sqrt{m^2 + C_1^2}} \quad \cos \alpha = \frac{m}{\sqrt{m^2 + C_1^2}}
\]
Let

\[
\frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha - \mu \sin \alpha} = \frac{\mu m + C_1}{m - \mu C_1} = \phi
\]  
(13)
and

\[
\phi F_V + \phi F_2 = F_H - \mu F_2
\]  
(14)
Equation 12 becomes

\[
m F_2 - \mu D F_2 = F_2 (m - \mu D) = p F_V - \frac{D}{2} F_H
\]
and

\[
F_2 = \frac{p F_V - \frac{D}{2} F_H}{m - \mu D}\n\]  
(15)
For convenience, designate \( m - \mu D = \psi \). Substitute the value for \( F_2 \) in Eq. 15 and in Eq. 14 and obtain:

\[
\phi F_V + \frac{\phi p F_V}{\psi} = \frac{\phi D}{2 \psi} F_H - \frac{\mu p}{\psi} F_V + \frac{\mu D}{2 \psi} F_H
\]
from which

\[
F_H = 2 F_V \left[ \frac{\psi + \rho (\phi + \mu)}{2 \psi + \phi (\phi + \mu)} \right]
\]  
(16)

\[
= 2 F_V \left[ \frac{\rho}{\psi} - \frac{\phi (2 \psi D - \phi)}{2 \psi + \phi (\phi + \mu)} \right]
\]  
(16a)
when \( m = \mu D, \psi = 0, \) and \( F_V = \tan \delta = \frac{D/2}{\rho} \). Thus, when the plug has entered the hole a distance \( m = \mu D \), if the force \( F_A \) is directed toward point \( A \) or toward any point to the left of \( A \), the plug will be pushed on farther. If \( m \) is greater than \( \mu D \), \( \psi \) is positive; and if \( m \) is smaller, \( \psi \) is negative. Also, as long as \( \frac{2 \rho}{D} > \phi \), \( \tan \delta < \frac{D/2}{\rho} \) when \( m > \mu D \); and \( \tan \delta < \frac{D/2}{\rho} \) when \( m < \mu D \). This means that under these conditions, \( F_A \) may lie more and more to the right of point \( A \) as \( m \) grows longer than \( \mu D \) and still enable the plug to proceed farther into the hole. On the other hand, the smaller the distance the plug is in the hole, the smaller the angle \( \delta \) must be and the larger \( F_H \) must be in proportion to \( F_V \) to make the plug proceed.

The factor \( \phi \) rises to infinity when \( m = \mu C_1 \), a very small amount; but it falls rapidly and asymptotically to a value of \( \mu \) as \( m \) increases.

Thus, as a plug first enters a hole, the force required to push it along must be many times that tending to tilt it. However, no matter how high the tilting force, the plug can be pushed along if sufficient force is exerted in the direction of the axis of the hole. This is borne out in practice because a stuck plug can be forced in or out by a press.
When a person inserts a close-fitting plug by hand in a hole, he may inadvertently push the plug too much to one side and thus apply a torque equivalent to \( pF_v \) in Fig. 7 while he is attempting to push the plug forward. The force necessary for further progress exceeds that available and the plug is stuck. The operator may then release the plug, and the condition depicted in Fig. 8 exists.

When a plug is subject to the forces \( F_H \) and \( F_V \) of Fig. 7, the metal in the plug and ring is strained by stresses induced by forces \( F_1 - F_{f1} \) and \( F_2 - F_{f2} \). When the exciting forces \( F_H \) and \( F_V \) are released, the material tends to return to the unstrained state, which requires a slight displacement of the plug. The opposed tendencies for movement at A and B set up forces as indicated by Fig. 8.

The force \( F_{f1} \) cannot exceed \( \mu F_1 \), and \( F_{f2} \) cannot exceed \( \mu F_2 \), but both friction forces may be smaller. If the resultant of \( F_{f1} \) and \( F_1 \) can pass through point B, so that \( \alpha \leq \tau \), the angle of friction, the forces at A and B hold the plug in equilibrium. The plug is said to be stuck. To loosen it a force must be applied in the opposite direction from \( F_V \). Often this force pushes the plug over in the opposite direction, and it becomes stuck again.

After a plug has entered a certain distance into a hole, the tendency to stick decreases and then ceases. When the angle \( \lambda \) is greater than the angle of friction \( \tau \), the resultants of \( F_1 - F_{f1} \) and \( F_2 - F_{f2} \) form a couple that releases the plug. Therefore, if a plug is so designed that it can enter a hole a distance \( l \) such that \( \lambda = \tau \) before it engages both sides of the hole, it will not stick. From Fig. 8, the distance the plug must be in the hole for sticking just to cease is

\[
l = d \tan \lambda = d \tan \tau = \mu d \quad (17)
\]

Also

\[
\tan \alpha = \frac{D - \frac{d}{\cos \alpha}}{m} = \frac{C_1}{m} \quad (18)
\]

and

\[
l = \frac{dC_1}{m} + \sqrt{m^2 + C_1^2} = \mu d \quad (19)
\]

when \( C_1 \) is of the order of \( 10^{-4} \) and \( m \) of \( 10^{-1} \) in.,

\[
m \approx \mu d = l \quad (19a)
\]

The solution of Eq. 18 for \( \alpha \) is lengthy; but when \( \alpha \) is very small, as it normally is for close-fitting members, a practical approximation is that

\[
\tan \alpha = \frac{D - \frac{d}{\mu d}}{C} = \frac{\mu d}{C} \quad (20)
\]

Equations 17, 19a, and 20 are the important ones from a design standpoint, because they provide a basis for determining limiting conditions.

A particular case of importance is that of the minimum hole diameter in which a specified plug will not stick in any position. A plug is most likely to stick as it begins to enter a hole, as shown in Fig. 9, where it is in contact at points A and B at the end of the hole. In that position, the limiting condition for sticking is \( \alpha = \beta = \tau \), the angle of friction. Thus \( d/D = \cos \tau \), and the minimum diameter to avoid sticking is:

\[
D = \frac{d}{\cos \tau} = d \sec \tau = d \sqrt{1 + \mu^2} \quad (21)
\]
This is related to the coefficient of friction \( \mu \) as follows:

<table>
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<th>( \mu )</th>
<th>( \sec \tau )</th>
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<td>0.1</td>
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</table>

Another specific case of importance in design is that wherein a plug is in contact with a partial hole and makes contact not at points 180 deg apart but at some other radial angle as depicted in Fig. 10. In this case, the plug will not stick sidewise in the segment of the hole when

\[
\phi < 180 - 2\tau_1 \quad \text{or} \quad \psi = 180 - \phi \geq 2\tau_1 \quad (22)
\]

The angle \( \tau_1 \) is a projection of the angle of friction.

3. Experimental Support of the Theory of Sticking

Experiments were conducted with a variety of ring and plug gages to verify the theory of sticking. First, the coefficient of friction was determined between each mating plug and ring. A ring was held in a chuck on the spindle of a dividing head with the axis of the ring at right angles to the axis of the head as shown in Fig. 11. A snug-fitting bar was inserted in the ring gage and levelled with a precision spirit level by adjusting the dividing head. The ring and mating plug were thoroughly cleaned with carbon tetrachloride, the plug inserted in the ring, and the dividing head spindle turned until the plug slipped. Ten trials were made for each plug and ring combination. The coefficient of friction obtained in that way is the tangent of the angle at which the plug slipped.

To determine how far a plug must be inserted in a ring before the tendency to stick ceases, the arrangement shown in Fig. 12 was used. This consists of a heavy vee block in which a plug is fastened. A micrometer stop is clamped alongside the plug. The ring and plug were cleaned with carbon tetrachloride, and the ring was slipped over the
plug to the stop and then cocked to try to make it stick. If the ring stuck, it was removed. The stop was adjusted to allow the ring to cover a small amount more of the plug. The ring was then placed over the plug again and cocked. This procedure was repeated, each time increasing the length of plug allowed to enter the ring, until sticking was observed to cease. Then the distance from the outside face of the ring to the plug was measured with a depth micrometer and subtracted from the “miked” length of the ring. Ten trials were made for each pair and the values averaged. This value of \( m \), together with measured values of \( d \) and \( C \) for each combination, was substituted in Eq. 19 to calculate the value of \( \mu \).

The sizes of rings and plugs tested and the results obtained are shown in Table 1. The coefficients of friction by direct tests and sticking tests differ only in the third decimal place in each case. The small discrepancies can quite reasonably be attributed to experimental errors, and the results of the experiment substantiate the theory that a plug ceases to stick in a hole when the distance it has entered is \( l = \mu d \approx m \) as stated by Eqs. 17 and 19a.

<table>
<thead>
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<th>Ring Diam in.</th>
<th>Plug Diam in.</th>
<th>Clearance in.</th>
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<th>( \mu )</th>
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<td>Direct Tests</td>
<td>0.139</td>
<td>0.144</td>
</tr>
</tbody>
</table>
III. DESIGN OF NON-STICKING PLUGS

Figure 13 shows the outline of a plug with a pilot diameter $A$ of length $l_1$, a groove of diameter $B$ and length $l_2$, and a major diameter $d$. This is a typical non-sticking design. For the design to be successful, the dimensions assigned must fulfill several conditions.

The first condition is that the length $l_1$ of the pilot must be short enough so that the diameter $A$ will not stick in the intended hole of diameter $D$ in any position. The requirements for this condition are given by Eq. 2 which is rewritten for this case as

$$l_1 = \sqrt{D^2 - A^2} \quad (23)$$

If the clearance between $D$ and $A$ is designated by $C_2$, or $D = A + C_2$,

$$l_1 = \sqrt{C_2^2 + 2AC_2}$$

But $C_2^2$ is small compared to $A$ and $D$ and may be neglected. For practical purposes

$$l_1 = \sqrt{2AC_2} = 1.4\sqrt{AC_2} \quad (24)$$

The second condition is that the plug entering the hole shall not bind at the edge of diameters $d$ and $A$ as depicted in Fig. 14. This condition is met if

$$\frac{l}{EB} \geq \tan \tau = \mu \quad (25)$$

where

$$EB = \frac{d + A}{2}$$

and

$$l \geq \frac{\mu}{2} (d + A) \quad (26)$$

As $A$ is made larger and approaches $d$, the distance $l$ must be increased to prevent sticking. The smaller $A$ is, the shorter $l$ may be to meet this second condition.

The third condition is that the plug diameter $d$ be guided into the hole diameter $D$ at an angle less than the angle $\alpha$ defined previously in Eq. 20. From Eq. 20 and Fig. 13, the limiting condition here is expressed by

$$\tan \alpha = \frac{D - d}{\mu d} = \frac{d - A}{2l} \quad (27)$$

or

$$l = \frac{\mu d (d - A)}{2 (D - d)} \quad (28)$$

Fig. 13. The Outline of a Plug with a Pilot Diameter and Aligning Groove Illustrating Two Conditions of Design to Prevent Sticking

Fig. 14. An Illustration of the Design Condition to Prevent Binding on the Edges of the Pilot and Body Diameters of a Plug
From Eqs. 26 and 28,
\[ l = \frac{\mu d^2}{2 (D - d)} = \frac{\mu}{2} (d + A) \]  
(29)
which reduces to
\[ A = \frac{d (2d - D)}{D} = \frac{2d^2}{D} - d = D - 3C + \frac{2C^2}{D} \]  
(30)
The term \( 2C^2/D \) is negligible, and for practical purposes,
\[ A_{\text{min}} = D - 3C = d - 2C \]  
(30a)
This expression specified a minimum value for \( A \), which may be made as large as \( d \) and still satisfy the second and third conditions. However, as \( A \) is made smaller, \( C_2 \) increases, and \( l_1 \) as expressed by Eq. 24 can be made larger. Thus, an \( A \) diameter smaller than \( d \) is advantageous because it allows the pilot on the end of the plug to have more wear surface.

If Eq. 30 is substituted in Eq. 29,
\[ l = \frac{\mu d^2}{D} \]  
(31)
and if \( A = d \)
\[ l = \mu d \]  
(32)
which is very slightly greater than Eq. 31. Also, from Fig. 13,
\[ l_2 = l - l_1 \]  
(33)
Another dimension that needs to be established for the plug of Fig. 13 is the diameter \( B \). An approximation from Figure 15 is
\[ \left( \frac{A + B}{2} \right)^2 + l^2 = D^2 \]
or
\[ B = 2\sqrt{D^2 - l^2} - A \]  
(34)
If \( A = d \), \( l = \mu d \). Since \( D - d \) is small, let \( D = d \), and
\[ B = d (2\sqrt{1 - \mu^2} - 1) \]  
(35)
For a value of \( \mu = 0.2 \), \( B = 0.96d \). A convenient design value is
\[ B = 0.95d \]  
(36)
Example 1. An aligning groove and land are to be designed for a plug having a diameter of 1.9995 in. to be inserted into a hole of 2.000 in. diam without sticking. The coefficient of friction is assumed to be 0.2.
First, the dimensions will be calculated to the full limits. From Eq. 30:
\[ A = \frac{1.9995 (2 \times 1.9995 - 2)}{2} = 1.9985 \text{ in.} \]
From Eq. 31:
\[ l = \frac{0.2 (1.9995)^2}{2} = 0.3998 \text{ in.} \]
From Eq. 23:
\[ l_1 = \sqrt{4 - (1.9985)^2} = 0.0775 \text{ in.} \]
From Eq. 33:
\[ l_2 = 0.3998 - 0.0775 = 0.322 \text{ in.} \]
From Eq. 34:
\[ B = 2\sqrt{4 - (0.3998)^2} - 1.9985 = 1.9215 \text{ in.} \]
The dimensions by the approximated formulas are:
\[ A = d = 1.9995 \text{ in.} \]
From Eq. 32:
\[ l = 0.2 (1.9995) = 0.3999 \text{ in.} \]
From Eq. 24:
\[ l_1 = \sqrt{2 \times 1.9995 \times 0.0005} = 0.0447 \text{ in.} \]
From Eq. 33:
\[ l_2 = 0.3999 - 0.0447 = 0.3552 \text{ in.} \]
From Eq. 36:
\[ B = 0.95 \times 1.9995 = 1.8995 \text{ in.} \]
\[ \cong 1.90 \text{ in.} \]
From the foregoing calculations it is seen that when \( A = d \), a pilot length of only about 0.045 in. is allowable, compared with 0.078 in. when the minimum size of \( A \) is used. The total length \( l \) of the groove and pilot is practically the same in both cases, and there is little difference specified for the diameter of the groove.
Diameter \( A \) should be given a tolerance to keep it within the range of \( d \) and \( A_{\text{min}} \). The dimension \( l_1 \) should have a negative tolerance when dimension \( A \) has a positive tolerance, and vice versa. Also, dimension \( l_1 \) should be given a positive tolerance.
When the length \( l \) is designed on the basis of a larger coefficient of friction than actually encountered, the plug enters the hole beyond the point at
which sticking ceases before full engagement occurs. However, the tendency to stick decreases inward from the end of the hole and is relatively small as the point of cessation is approached. If a value for the coefficient of friction somewhat smaller than the actual is used in proportioning the length $l$, some tendency to stick may remain but be so slight as to be scarcely noticeable. A low design value for $\mu$ gives a small length $l$ which is advantageous in keeping the projection short on the end of the plug. Figure 16 shows an experimental plug of 1 in. nominal diameter. The pilot is ground to the same diameter as the body of the plug and is about 0.015 in. long. The length $l$ is 0.110 in., corresponding to a coefficient of friction of 0.11 in the formula $l = \mu d$. The plug readily enters a hole with 0.0002 to 0.0003 in. clearance.

![Fig. 16. An Experimental Plug with an Aligning Groove and Pilot Diameter at One End Conforming to Design Theory](image)

Nomograms are given in Figs. 17 and 18 for rapidly calculating the length of pilot and groove $l = \mu d$ and length of pilot $l_1 = \sqrt{2A/C}$.

Where the plug cannot be altered, a modification of the hole as shown in Fig. 19 is suggested.

![Fig. 19. A Cross-Section of a Hole with an Aligning Groove and Pilot Diameter](image)

![Fig. 17. A Nomogram for Calculating the Length of Groove and Pilot of a Plug](image)
Fig. 18. A Nomogram for Calculating the Pilot Length of a Plug

Example:
A = 1.750
C = 0.0003
1 = 0.032

Formula:
I = \sqrt{2AC}

Formula:
I = -MAC
Example:
\( \mu = 0.200 \)
\( d = 2.000 \)
\( D_2 = 2.040 \)

Formula:
\[ D_2 = d \sec \tau = d \sqrt{1 + \mu^2} \]

Fig. 20. A Nomogram for Calculating the Pilot or Clearance Diameter of a Hole
Example:

\( \mu = 0.05 \) \quad \text{Result in point } "A"
\( d = 1.00 \) \quad \text{on construction line } "p"
\( D-d = 0.0005 \) \quad \text{Result in point } "B"
\( D-D = 0.002 \) \quad \text{on construction line } "Q"
\( l = 0.600 \)

Coefficient of friction \( \mu " \)

\( \frac{\mu d (D-d)}{2(D-D)} \)

Clearance \( C = D - d \)

Fig. 21. A Nomogram for Calculating the Length of an Aligning Groove in a Hole
4. Design of Non-Sticking Locating Holes

Sometimes when a plug must be inserted in a hole, it is not permissible to put an aligning groove and pilot on the plug. Such an occasion arises where the plug is a workpiece on which a groove is not permitted, but the piece must be inserted in a close-fitting fixture hole, collet, or ring gage. For such circumstances, a pilot diameter at the end of the hole with an adjacent groove as indicated in Fig. 19 has been suggested.

If a plug of diameter \( d \) is to be inserted in diameter \( D_1 \), then according to the conditions expressed in Eq. 21, the minimum diameter to avoid sticking is

\[
D_1 = d \sec \tau = d \sqrt{1 + \mu^2} \quad (37)
\]

As the plug enters further into the hole and reaches diameter \( D \), it must be restrained by diameter \( D_1 \), so that it cannot become inclined with the hole more than the angle \( \alpha \) defined in Eq. 20 if sticking is to be avoided. The relationship is expressed as:

\[
tan \alpha = \frac{D - d}{\mu d} = \frac{D_1 - D}{2l} = \frac{d \sec \tau - D}{2l} \quad (38)
\]

and

\[
l = \frac{\mu d (D_1 - D)}{2 (D - d)} \quad (39)
\]

Example 2. Let \( \mu = 0.1 \), \( d = 0.9995 \), and \( D = 1.000 \). Then

\[
D = 0.9995 \times 1.005 = 1.0045
\]

\[
l = \frac{0.1 \times 0.9995 (1.0045 - 1.000)}{2 \times 0.0005} = 0.450 \text{ in.}
\]

In Figs. 20 and 21 are presented nomograms for the solutions of Eqs. 37 and 39.

As has been indicated, the tendency to stick decreases after a plug enters a hole, and is relatively small for an appreciable distance before the plug reaches the point where sticking ceases entirely. Several trial grooved rings were made of hardened steel with the dimensions shown in Fig. 22 to ascertain the effects of reducing the dimensions below the limits that would be specified by Eqs. 38 and 39. The ring of Fig. 22a has a pilot diameter of 0.7503 in. arrived at by assuming a coefficient of friction of 0.20 as a basis for the solution of Eq. 38. The length of 0.360 in. corresponds to the use of a value of \( \mu = 0.02 \) in Eq. 39. As expected, the plug showed no tendency to stick in the pilot diameter. As the plug entered the 0.7503 in. diameter, some tendency to stick was noted but it was small and not objectionable for practical purposes.

The pilot diameters of 1.0008 in. and 1.0013 in. for rings of Fig. 22b and 22c correspond to values for a coefficient of friction of 0.04 and 0.05, respectively, as bases for solving Eq. 38. For the diameters shown, the groove lengths of 0.360 and 0.235 are enough to hold the plugs at very small angles so they should not stick even with an excessive coefficient of friction as they enter the holes. However, for both examples b and c, it was found practically as difficult to insert the plug as it would be in a straight hole without any aligning provisions. In both the latter cases, the sticking was most pronounced in the pilot diameters.

The results obtained with the examples of Fig. 22 indicate that the pilot diameter is more critical than the length of the groove. In other words, the pilot diameter would be calculated on the
basis of an adequate coefficient of friction, but for the length of the groove a relatively small coefficient of friction may be assumed. If the pilot diameter is sufficiently large, the main purpose of the groove is to provide clearance for machining the hole.

In any event, since the pilot diameter must be large in proportion to the diameter of the hole, the aligning groove must necessarily take up a substantial part of the length of the hole. In some cases that may impair the hole as a locating device. To overcome that deficiency two other designs have been developed.

The design of Fig. 23 for a hole of diameter $D$ has a narrow band of that diameter at the entrance of the hole. This band lies only within the angle $\phi$. The rest of the circle at the opening is relieved to a minimum distance $H_1$ diametrically opposite the band. The conditions to prevent sticking as the plug of diameter $d$ just enters the opening are stated by Eqs. 21 and 22. The distance $H_1 = d \sec r$, and the angle $\phi = (180 - 2\tau_1)$ deg. An angle $\phi$ equal to about 150 deg has been found practical for hardened steel rings and plugs.

As the plug proceeds into the hole, it may make contact with the top of diameter $D$ at a distance $l_1$ from the end. To prevent the plug sticking between the band and the top of diameter $D$, $l_1 \geq \mu d$ to satisfy Eq. 19a. Then as the plug enters farther, the lower part of the hole is relieved an additional length. $l_2 = l_1$ to prevent sticking if the plug is raised. Again, the unrelieved part of the circle is $(180 - 2\tau_1)$ deg or about 150 deg for hard steel.
In Fig. 27 a cross-section is shown of a plug beginning to enter a threaded ring and inclined at the maximum angle permitted by $D_R$. The plug does not stick, according to Eq. 21, if $H_1 \geq d$ see $\tau$. Thus, the minimum condition is that

$$D_R = 2H_1 - D = 2d\sqrt{1 + \mu^2} - D \quad (42)$$

Since the difference between $D$ and $d$ is small, a practical minimum is

$$D_R = D (2\sqrt{1 + \mu^2} - 1) \quad (43)$$

Example 3. Let $\mu = 0.2$, $d = 0.9995$, $D = 1.0000$.

Then

$$l_1 = l_2 = 0.2 \times 0.9995 = 0.1999 \text{ in.}$$

$$l = 0.40 \text{ in. approx.}$$

If $\tau$ is assumed to be 10 deg, $\phi = 160$ deg.

The nomogram of Fig. 17 can be used for calculations for this design. Figure 24 shows the dimensions to which the ring photographed in Fig. 25 was machined. This ring takes a 1.0001 in. diam plug without any sticking.

Another design of non-sticking ring sketched in Fig. 26 has a helical groove of lead $L$, minimum root diameter $D_R$, and width $L - t$ for 1½ turns from the opening.

According to Eq. 22, a plug of diameter $d$ entering the ring of Fig. 26 will not stick in the helical diameter $D$ through an angle of $(180 - 2\tau_1)$ deg from the entrance because the plug can be extricated easily sideways. After that, the helix must have advanced at least a distance $\mu d$ to prevent sticking. Thus, the lead of the helix must be

$$L = \left(\frac{360}{180 - 2\tau_1}\right) \mu d \quad (40)$$

In 1½ turns, the helical cut extends from the end of the hole a total distance of

$$L_w = \frac{3}{2} L = \frac{3\mu d}{2} \left(\frac{360}{180 - 2\tau_1}\right) \quad (41)$$

In Fig. 28 a plug is entering the hole in a position 180 deg from that in Fig. 27 and is inclined at an angle $\alpha$, the maximum amount allowed by diameter $D_R$. Then

$$\tan \alpha = \frac{AB}{BC} = \frac{D_R - D}{L} = \frac{2d\sqrt{1 + \mu^2} - 2D}{L} \quad (44)$$
If $D_R$ is designed to Eq. 43,
\[
\tan \alpha = \frac{2D}{L} \left( \sqrt{1 + \mu^2} - 1 \right) \tag{45}
\]
Also from triangles of Fig. 28,
\[
\frac{L}{2} - t = EF = FH \sin \alpha = (d - GH) \sin \alpha
= \frac{d - D \cos \alpha}{\sin \alpha} \tag{46}
\]
and the width of the thread to prevent sticking on the back is
\[
t = \frac{L}{2} - \frac{d - D \cos \alpha}{\sin \alpha} \tag{47}
\]
Example 4. Let $\mu = 0.2$, $d = 0.9995$, $D = 1.0000$, and $r_1 = 10$ deg. Then
\[
L = \frac{(360/160) \times 0.2 \times 0.9995}{\sin \alpha} = 0.45 \text{ in.}
\]
\[
L_w = \frac{3 \times 0.45}{2} = 0.68 \text{ in.}
\]
\[
D_R = 1.000 \left( 2 \sqrt{1 + 0.04} - 1 \right) = 1.040 \text{ in.}
\]
\[
\tan \alpha = \frac{2 \times 0.9995 \sqrt{1 + 0.04} - 2}{0.45}
= \frac{0.03898}{0.45} = 0.0866 \quad \text{(By Eq. 44)}
\sin \alpha = 0.0863 \quad \cos \alpha = 0.9963
\]
\[
t = 0.225 - \frac{0.9995 - 1 \times 0.9963}{0.0863}
= 0.225 - \frac{0.0023}{0.0863} = 0.19 \text{ in.}
\]
In accordance with the theory, a ring was made to dimensions shown in Fig. 29. Figure 30 is a picture of this ring. No sticking whatever occurs when a 1.0001 in. diam plug is inserted.

5. Design of Faceplate Plug Locators

A common means of workpiece location is on a plug and against a faceplate, as indicated in Fig. 4. For a specified clearance $C$ between the plug and hole, the plug may have a length $l = \sqrt{2 (2a + d) C}$ according to Eq. 6. Without the restraining shoulder adjacent to the plug, the plug length could only be $l_t = \sqrt{2dC}$, as indicated by Eq. 24.

Example 5. A workpiece has a 1.0005 in. diam hole to be located on a 1.0000 in. diam plug. A flange on the workpiece around the hole is 11 in. in diam.

What is the allowable length of plug if the flange locates on a faceplate adjacent to the plug?

The allowable plug length is:
\[
l = \sqrt{2 (11) \times 0.0005} = 0.105 \text{ in.}
\]

What is the allowable length of plug to avoid sticking if the effect of the flange face is not taken into account? In that case, the plug length is only
\[
l_t = \sqrt{2 \times 1 \times 0.0005} = 0.032 \text{ in.}
\]

A way of reducing the tendency to stick is to relieve locating plugs by cutting away three equidistant segments as indicated in Fig. 31. The points farthest apart on the plug and most likely to cause sticking in engagement with a workpiece are those corresponding to A and B in Fig. 31. In other words, if a piece with a hole slightly larger than the plug is placed over the plug and tilted, the piece may bind on the plug at two points such as A and B.

If the arcs of contact of the plug were of such length that contact would be made at points like $A'$ and $B'$, the resultant forces $P'R_2$ would be opposed and deter the workpiece from being twisted on the plug to a freer position. On the other hand, if the cutaway portions are sufficiently large so that contact is made at A and B where the resultant forces
the hole may be displaced in any direction with respect to the center of the plug a distance \((D-d)/2\). If the same workpiece is placed over a three-sided relieved plug of nominal diameter \(d\), as in Fig. 32, a maximum displacement \(s\) in any of three directions is possible. From triangles in Fig. 32,

\[
\left(\frac{F}{2}\right)^2 = \left(\frac{d}{2}\right)^2 - m^2 = \left(\frac{D}{2}\right)^2 - (m + s)^2
\]

from which

\[
s = -2m \pm \sqrt{4m^2 + (D^2 - d^2)}
\]

Also

\[
m = \frac{d}{2} \sin (30 + \sigma)
\]

If the plug is designed with \(\sigma = 15\) deg,

\[
m = \frac{d}{2} \sin (30 + 15)
\]

and

\[
s = \frac{\sqrt{2D^2 - d^2} - d}{2\sqrt{2}}
\]

\(F_x\) are in line, a slight additional applied force is enough to break the equilibrium and release the piece. If the cutaway portions are still larger, the piece will not stick of its own accord.

The limiting position allowing twisting of the piece is given by the diagram of Fig. 31 as that in which

\[
2\alpha + 120 = 180 - 2\delta
\]

or

\[
\sigma = 30 - \delta = 30 - \tau
\]

where \(\tau\) is the angle of friction between the materials in plug and workpiece. If a plug is relieved except for arcs equal to \(2\alpha\), a workpiece can readily be twisted on it to a position of greatest freedom. For a coefficient of friction \(\mu = 0.27\), \(\tau = 15\) deg = \(8\). Then \(\sigma = 15\) deg and \(2\alpha = 30\) deg. Also \(\varphi = 60 - \sigma = 45\) deg, and \(2\varphi = 90\) deg, the angle of relief.

A three-sided relieved plug provides more loading freedom than a full plug but is a less accurate locator. If a workpiece with a hole of diameter \(D\) is placed over a full plug of diameter \(d\), the center
Consider that
\[ 2D^2 - d^2 = D^2 + (D^2 - d^2) = D^2 + (D + d)(D - d) \]
\[ = D^2 + 2d(D - d) + (D - d)^2 \]

Also
\[ d(D - d) = Dd - d^2 \]
\[ D(D - d) - d(D - d) = D^2 - 2Dd + d^2 = (D - d)^2 \]

Since \( D - d \) is small, of the order of \( 10^{-3} \) to \( 10^{-4} \), \((D - d)^2\) is negligible, of the order of \( 10^{-6} \), and for practical purposes, \( 2d(D - d) \approx 2D(D - d) \) and \( 2D^2 - d^2 \approx [D + (D - d)]^2 \). Substituting this in Eq. 51,

\[ s = \frac{D - d}{\sqrt{2}} \]  

(52)

The difference between the radial displacement with a full round plug, which is the same as the minimum error with a three-sided plug, and the maximum displacement with a three-sided plug is

\[ E = s - \frac{D - d}{2} = 0.207 (D - d) \]

Figure 33 depicts a workpiece with a hole of diameter \( D \) being placed over a three-sided plug of nominal diameter \( d \). The plug is designed with an angle \( \sigma = 15 \, \text{deg} \), so that if the workpiece sticks in any position, it can be twisted readily to the position of greatest freedom. The plug extends from the faceplate against which the workpiece is located and which limits the angle to which the workpiece can be tilted as it is placed over the plug. The allowable length \( l_1 \) of the plug diameter \( d \) to avoid sticking in the freest condition may be arrived at by the same method by which Eq. 6 was derived. The allowable length is

\[ l_1 = \sqrt{2} \left( 2a + v \right) (V - v) \]  

(53)

But \( v = m + d/2 \), and \( V = v + s + \frac{D - d}{2} \)

\[ = m + \frac{D}{2}. \]

\[ s = \frac{D - d}{\sqrt{2}} \quad \text{and} \quad m = \frac{d}{2 \sqrt{2}}. \]

So

\[ l_1 = \sqrt{2} \left( 2a + m + d/2 \right) \left( s + \frac{D - d}{2} \right) \]

\[ = \sqrt{2.414} \left( 2a + 0.8535d \right) (D - d) \]  

(54)

\[ = \sqrt{2.414} \left( 2a + 0.8535d \right) C \]  

(54a)

Example 6. Let the workpiece described in Example 5 be placed over a relieved plug with a 1.000 locating diameter. Then

\[ l_1 = \sqrt{2.414 \times 10.8535 \times 0.0005} \]

\[ = \sqrt{0.0131} = 0.115 \, \text{in.} \]

This is only slightly more than an allowable length \( l = 0.105 \) for a full plug of the same diameter. Also

\[ E = 0.207 \times 0.0005 = 0.00012 \, \text{in.} \]

The relieved plug permits about 24 percent more displacement than the full round plug.

The analysis indicates that a relieved plug can only be made slightly longer than a full round plug if sticking is to be avoided. A relieved plug also provides appreciably less locating accuracy and has appreciably less wearing surface.
IV. SUMMARY

A close-fitting plug tends to stick when it enters a hole. The distance that the plug must enter before sticking ceases has been found to be the product of the diameter of the plug times the coefficient of friction. Sticking can be prevented by making the plug so short that diagonally opposite points on its diameter do not contact opposite sides of the hole, or by providing a means to lead the plug into the hole beyond the point where sticking ceases before full contact is made.

On the basis of the evaluation of the causes of sticking, formulas have been derived and are given in this bulletin for the proper design of common types of plugs and rings used for gages, fixtures, jigs and other tools.

The authors express their appreciation to Scully-Jones and Company, Chicago, for making the models for this investigation.

V. REFERENCES

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