ULTRACOLD FERMIONIC ATOMS IN DISORDERED POTENTIALS

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DISSERTATION
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Abstract

This thesis describes the construction of an experimental apparatus to study disorder physics using ultracold fermionic atomic gases. We use this apparatus to realize 3D Anderson localization for the first time for quantum matter waves. We provide the first measurement of how the mobility edge—a hallmark of 3-dimensionality—and of localization lengths depend on the disorder strength. In a second experimental study, we add an optical lattice to the disorder potential to realize the disordered Fermi-Hubbard model, which is a minimal model for strongly correlated electronic solids. The interplay of interactions and disorder is investigated. We find an Anderson-like disorder-driven metal-to-insulator transition as well as disorder-induced breaking of the Mott gap in the strongly interacting regime.
To my Parents and to Sveta
Acknowledgements

I first want to thank Brian, who taught me the important notion that experimental physics lives in the intersection space between equations and reality, and that both will gladly turn around and bite off your finger if you are not careful. In this dangerous situation, it is the necessity that one becomes a better scientist, and it is also the hope that one becomes a better human being.

I would like to thank my family and friends, and I would like to point out that, for many of us, graduate school is a rather self-centered experience in which we are only left to worry about ourselves while we do what we love among people whom we appreciate. Yet those who inspire and sustain us are left out of the author lists and honorable mentions. In summary, graduate school is a wonderful place and a privilege, and I am grateful that the fundamental laws of the universe have so conspired that I’ve had the chance to be here.

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# Table of Contents

**List of Figures** ............................................. viii

**Chapter 1 Introduction** .................................. 1
   1.1 Motivation ........................................... 1
   1.2 Experiments ......................................... 3
       1.2.1 Anderson localization ....................... 3
       1.2.2 Interacting fermions in a disordered lattice .. 4
   1.3 Limitations of the experimental method .............. 5
   1.4 Future directions .................................. 5
   1.5 Outline ............................................. 6

**Chapter 2 Atomic Physics Overview** ..................... 8
   2.1 Introduction ........................................ 8
   2.2 $^{40}$K electronic structure ....................... 9
   2.3 Magnetic interactions ................................ 10
       2.3.1 Hyperfine structure ........................ 12
       2.3.2 Zeeman effect ............................... 14
   2.4 Optical potentials .................................. 15
   2.5 Collisions .......................................... 16
       2.5.1 Interaction potential ....................... 17
       2.5.2 Lippman-Schwinger equation ............... 18
       2.5.3 $s$-wave scattering ......................... 21
       2.5.4 Anti-symmetrization and vanishing of interactions .. 21

**Chapter 3 $^{40}$K Apparatus** ............................ 23
   3.1 Overview of experimental sequence .................. 23
   3.2 Vacuum System ...................................... 23
       3.2.1 Preparation .................................. 27
   3.3 Lasers .............................................. 27
       3.3.1 ECDLs ....................................... 28
       3.3.2 Tapered Amplifiers ......................... 28
   3.4 MOT .................................................. 29
   3.5 Transfer ............................................ 30
   3.6 Forced RF cooling .................................. 30
       3.6.1 Losses and heating ......................... 31
   3.7 QUIC trap .......................................... 34
   3.8 Optical evaporation ................................ 35
5.7.2 Method verification ................................................. 98
    Fraction of atoms remaining vs. pulse duration ................. 99
    Double occupancy vs. characteristic density .................... 101
5.7.3 Findings .......................................................... 101

Appendix A  Multipole expansion calculation for QUIC trap fields . . . 104
    A.1 Multipole expansion ........................................... 104
    A.2 Trap fields ..................................................... 106
    A.3 QUIC trap ..................................................... 107

Appendix B  Drawings .................................................... 110

References ............................................................... 123
List of Figures

2.1 Aufbau diagram of the electron structure of $^{40}K$. .................. 10
2.2 Energies of the $^{40}K$ hyperfine levels. ................................. 11
2.3 Zeeman sub-levels of the ground state. ................................. 12

3.1 Vacuum system and magnetic traps. ................................. 24
3.2 Vacuum system baking ................................................. 26
3.3 Schematic of diode laser system. .................................. 28
3.4 Cooling trajectories .................................................. 32
3.5 QUIC trap geometry .................................................. 33
3.6 Printed parts for magnetic coil winding. ............................. 34
3.7 Magnetic trap assembly. .............................................. 36
3.8 Optical trap schematic. ............................................... 37
3.9 Optical speckle field. .................................................. 37
3.10 Flexure mount. ......................................................... 39
3.11 Disorder beam alignment and focusing. ............................. 40
3.12 In-situ measurement of speckle beam size. .......................... 41
3.13 Lattice laser system. ................................................ 42
3.14 Forward lattice beam rough alignment. ............................. 43
3.15 Lattice fine alignment. .............................................. 44
3.16 Lattice diffraction. ................................................. 44
3.17 Lattice calibration sequence. ..................................... 45
3.18 Lattice excitation spectra. ........................................ 46
3.19 Lattice calibration ................................................... 47
3.20 Imaging optics. ....................................................... 48
3.21 Current stabilization diagram. .................................... 49
3.22 Light intensity stabilization diagram. .............................. 50

4.1 Time-reversed closed trajectories in random scattering ............ 54
4.2 Anderson localized gas. .............................................. 56
4.3 Localization dynamics. .............................................. 59
4.4 Number loss from localized component. ............................ 62
4.5 Full range of AL data. .............................................. 62
4.6 Mobility edge ......................................................... 63
4.7 Effect of disorder on kinetic energy distribution. ................... 65
4.8 Schematic of combined confinement and speckle disorder potential. 65
4.9 localization length .................................................. 67
4.10 Simple model for effect of momentum on localization length. .... 67
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Band structure.</td>
<td>72</td>
</tr>
<tr>
<td>5.2 Phases in the clean lattice.</td>
<td>74</td>
</tr>
<tr>
<td>5.3 Interaction and tunneling energies.</td>
<td>77</td>
</tr>
<tr>
<td>5.4 Phase diagram of FHM in a combined lattice and parabolic confining potential.</td>
<td>79</td>
</tr>
<tr>
<td>5.5 Confinement-induced phases in the MI regime.</td>
<td>81</td>
</tr>
<tr>
<td>5.6 Probability density $\rho$ of the DFHM parameters.</td>
<td>85</td>
</tr>
<tr>
<td>5.7 DFHM phase diagram.</td>
<td>86</td>
</tr>
<tr>
<td>5.8 Disordered lattice experiment.</td>
<td>88</td>
</tr>
<tr>
<td>5.9 Transport measurement sequence.</td>
<td>91</td>
</tr>
<tr>
<td>5.10 Band-mapped images of gas before and after applied impulse</td>
<td>91</td>
</tr>
<tr>
<td>5.11 COM velocity of the gas after applied impulse.</td>
<td>93</td>
</tr>
<tr>
<td>5.12 Band-mapped lattice gas without momentum impulse.</td>
<td>94</td>
</tr>
<tr>
<td>5.13 Universal transport behavior.</td>
<td>95</td>
</tr>
<tr>
<td>5.14 Double occupancy measurement procedure.</td>
<td>98</td>
</tr>
<tr>
<td>5.15 Loss of atoms induced by near-resonant light.</td>
<td>99</td>
</tr>
<tr>
<td>5.16 Dependence of double occupancy $D$ on characteristic density $\tilde{\rho}$ in the clean lattice.</td>
<td>100</td>
</tr>
<tr>
<td>5.17 Disorder-induced double occupancy.</td>
<td>101</td>
</tr>
<tr>
<td>5.18 Occupancy calculated in the atomic limit.</td>
<td>102</td>
</tr>
<tr>
<td>A.1 QUIC trap schematic.</td>
<td>105</td>
</tr>
<tr>
<td>B.1 Vacuum system (dimensioned).</td>
<td>111</td>
</tr>
<tr>
<td>B.2 Collection cell assembly.</td>
<td>112</td>
</tr>
<tr>
<td>B.3 Transfer tube.</td>
<td>113</td>
</tr>
<tr>
<td>B.4 Custom cross.</td>
<td>114</td>
</tr>
<tr>
<td>B.5 Science cell.</td>
<td>115</td>
</tr>
<tr>
<td>B.6 Quad coil.</td>
<td>116</td>
</tr>
<tr>
<td>B.7 Quad coil holder.</td>
<td>117</td>
</tr>
<tr>
<td>B.8 Ioffe coil.</td>
<td>118</td>
</tr>
<tr>
<td>B.9 Ioffe coil holder bracket.</td>
<td>119</td>
</tr>
<tr>
<td>B.10 Flexure mount (dimensioned).</td>
<td>120</td>
</tr>
<tr>
<td>B.11 Breadboard.</td>
<td>121</td>
</tr>
<tr>
<td>B.12 Tapered amplifier body.</td>
<td>122</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Motivation

This thesis describes the construction of a new fermionic lattice experiment along with two complete measurements. It is only the third such experiment to produce results and the first to incorporate disorder. On our path to realizing an optical lattice, we performed the first direct observation of Anderson localization in 3D: a single particle phenomenon first predicted in 1958 and known to affect the transport properties of solids. By combining an optical speckle field and an optical lattice, we realize the disordered Fermi-Hubbard model—an effective model for strongly interacting electrons in disordered crystalline solids—which is notoriously difficult to analyze theoretically. Our measurements on the transport properties in the disordered lattice amount to a quantum simulation of the model. We find a metal-to-insulator transition driven by Anderson localization and a disorder-induced breaking of the Mott gap. Furthermore, we measure the extent to which interactions suppress the metal-to-insulator transition, thus addressing a 30-year-old question regarding the interplay between disorder and interactions.

Disorder phenomena are most intensely researched in the context of crystalline solids where the presence of lattice imperfections can dramatically affect electronic transport [1, 2]. Disorder was initially recognized for its role of slowing wave diffusion: a phenomenon known as weak localization [3]. It was later shown theoretically that non-interacting particle-waves scattering strongly from a random potential will halt their motion [4]. A complete absence of propagation, termed Anderson localization (AL) after its discoverer, can result from coherent scattering and destructive self-interference, leading to an insulating state. Since this discovery, a sustained interest in AL has resulted in a vast body of literature. Arguments from scaling theory have found a distinction between bulk and low-dimensional systems. Mobility edges and localization lengths have been calculated for lattice models, while more general cases have been treated analytically with perturbative expansion techniques.

Regardless of continued effort, direct observation of AL has until recently [5] remained elusive due to the complexity of condensed matter physics where distinguishing AL from any other mechanism yielding similar results is challenging. Furthermore, the universal presence of Coulomb interactions between electrons casts doubt on the relevance of AL as a single
particle effect in electronic solids. These complications, along with the fundamental role that disorder plays in condensed matter physics, have inspired our experimental interest.

We use an ultracold spin-polarized gas of fermionic atoms to realize a strictly non-interacting system, which we subject to an optical disorder potential. We observe that under certain conditions atoms fail to follow a classical diffusing path and do not propagate. This result amounts to a first direct observation of Anderson-localized matter in three dimensions [6].

The relevance of AL to the properties of electronic solids is potentially limited because of the interacting nature of electrons. While it has been established that weak localization is present in solids [2], the extent to which AL determines the transport properties remains unknown. The interplay between disorder and interactions is a subject of intense debate with theoretical predictions in apparent contradiction over basic outcomes [5]. At interaction strengths comparable to the kinetic energy, strong correlations between particles cause the failure of mean field and Hartree-Fock theories [7]. Disorder aside, theory in this regime is notoriously intractable because of the difficulty of reducing the many-body state to fewer degrees of freedom. Simultaneously, the problem is of great practical importance as it applies to many materials with superb transport properties and a potential for technological applications. For example, high-\(T_c\) superconductors display curious—and unexplained—properties both above and below the critical temperature [8]. Other strongly correlated materials such as the transition metal oxides display giant thermoelectric effects [9, 10] and giant magnetoresistance [11]. Our interest in disordered physics is particularly relevant to such materials because they are very often disordered [12–15] due to lattice defects, impurities, and dopants.

The open questions surrounding the interplay between disorder and interactions motivate the second measurement discussed in this thesis. The experimental study of a disordered interacting lattice (described in Chapter 5) is done in the spirit of quantum simulation: a successful strategy for testing condensed matter theories [16–18]. By exploring a mapping in which atoms play the role of electrons and optical potentials play the role of the (disordered) ionic lattice, we conduct a quantum simulation of the disordered Fermi Hubbard model (DFHM): a minimal effective model for strongly interacting electrons in disordered materials [7]. The power of this method is in the excellent control over the potential landscape (e.g., lattice tunneling, interaction, and disorder energies [17]). Measurements can give spatial [19–21], momentum, and quasimomentum distributions [22], site occupancy [23], and other properties that are not directly accessible for electrons in solids. These new observables may help unambiguously distinguish the effects of interaction from disorder: a task which has proven difficult for transport measurements in solids [24].

In the clean lattice, ultracold \(^{40}K\) atoms realize the Fermi-Hubbard model (FHM). The FHM successfully predicts the interaction-driven Mott insulating state, and cold atom experiments have realized this phase [19, 23]. Proximity to the Mott insulating phase is the
origin of many intriguing phenomena in condensed matter physics such as spin and charge ordering, strange metallic behavior [25], and—most notably—high-$T_c$ superconductivity [8]. Furthermore, it has been suggested that the FHM on its own is sufficient to explain anti-ferromagnetism [26] and even high-$T_c$ superconductivity [27]. The disordered Fermi-Hubbard model (DFHM) has been predicted to lead to a variety of disorder-induced non-Fermi-liquid, anti-ferromagnetic, and superconducting phases [28]. Although low enough entropy per particle has not been achieved with ultracold atoms to attack directly anti-ferromagnetism and high-$T_c$ superconductivity, there are still many open questions regarding physics above $T_c$. We design an experiment to approach some of these. The first part of the experiment addresses the controversial issue of whether interactions destroy Anderson localization. Additionally, occupancy in the Mott insulating regime is explored, probing the ability of disorder to break the Mott gap and lead to a “dirty metal” state with finite compressibility.

1.2 Experiments

1.2.1 Anderson localization

The first result described in this work is an experimental study of 3D Anderson localization [6]. The experiment is performed using a non-interacting ultracold gas of $^{40}K$ atoms prepared in the same spin state. Lack of interactions is guaranteed by the Pauli exclusion principle which forbids $s$-wave scattering of identical fermions [29], while $p$- and higher partial waves are strongly suppressed at nano-Kelvin temperatures [29]. These collisional properties ensure that the results can be understood in terms of single-particle physics. A disordered potential is created by a stationary speckle light field which has no local minima [30]. The dark regions in speckle—the regions of lowest potential—have a special property: they are never points of zero intensity, but rather continuous paths which change direction on a distance scale comparable to the speckle correlation length [31]. These dark paths lead away to the edge of the (finite) field or, on rare occasions, form closed loops with barriers for escape too low to be of importance. As a consequence, particles constrained to the dark regions will always be classically allowed to propagate arbitrarily far in the speckle.

In the AL experiment, the ultracold gas is released in the speckle field while supported against gravity by a magnetic field gradient. After an initial expansion lasting 20 – 40 ms, a fraction of the atoms becomes localized and exhibits no further motion. The spatial distribution of this localized component remains frozen for as long as the atoms can be observed (i.e., longer than 1 sec). Furthermore, the shape of the distribution is roughly exponential, which is a characteristic sign of AL. Observing the atoms halt their motion amounts to the first observation of the dynamic formation of the localized state. The fraction of atoms in the localized component is determined by a disorder strength dependent
energy cut-off called a mobility edge \cite{2}. We infer the mobility edge from the data using minimal assumptions. The mobility edge is a signature of three-dimensionality and has not been directly measured previously. It is also an important concept for understanding how AL affects transport: in gapped systems, the mobility edge may determine metallic or insulating properties. The inability of speckle to classically trap particles and the non-interacting nature of the atomic ensemble allow our observations to be interpreted strictly as single-particle wave interference physics. The dynamics of localization, along with the measured mobility edge and localization lengths, provide a new benchmark for theories of AL. Our data compare poorly with predictions from weak scattering theory and the self-consistent Born approximation, indicating a likely—and expected—failure of these methods in the strong scattering limit.

1.2.2 Interacting fermions in a disordered lattice

We realize the DFHM by trapping $^{40}K$ atoms prepared in two spin states in a cubic disordered optical lattice potential. The lattice is formed by standing waves of laser light \cite{32}. A disorder potential created using optical speckle randomizes the lattice on-site, tunneling, and interaction energies \cite{33}. The statistical properties of the random distribution are established through ex-situ microscopy of the speckle field. We examine how the interacting metal and Mott insulator change when disorder is present. One probe we employ is a transport measurement, in which the atoms are given a momentum impulse, similar to Ref. \cite{34}. The resulting (line-of-sight integrated) quasimomentum distribution is observed through a procedure known as band-mapping \cite{22}. We use the center-of-mass velocity, proportional to the first moment of the distribution, as a measure of the metallic properties of the system. This experiment finds an Anderson-like metal-to-insulator transition occurring at critical disorder height $\Delta_c$. Increasing interaction stabilizes the conducting phase, pushing $\Delta_c$ to higher values. This is a startling result: interactions and disorder are known to separately lead to metal-to-insulator transitions \cite{25}. Strong interactions can turn a metal into a Mott insulator (MI), and disorder is able to cause AL and also transform a metal into an insulator.

One of the central questions confronting materials science and condensed matter physics is whether the combination of disorder and localization is more effective at localization than either ingredient acting on its own \cite{5,24,35}. Our disordered lattice measurement amounts to the first systematic experimental study of this question.

When interactions are strong enough to support a Mott insulator, results from the impulse measurement become difficult to understand due to the presence of multiple phases and localized states present in the trap \cite{36–38}. Instead, a double occupancy measurement is used to quantify the fluctuations and compressibility of the disordered system. The Mott insulating state necessitates single occupancy, and the existence of disorder-induced doubly occupied sites can be interpreted as breaking of the Mott gap, resulting in a compressible
and possibly conducting state. Such behavior would be consistent with certain theoretical work [39–43], and can be understood from a simple energetic argument: above a certain disorder energy the variability in on-site energies begins to dominate the interaction energy penalty, inducing double occupancies. The double occupancies are measured by selectively ejecting the atoms on doubly occupied sites from the cloud. The data show good agreement with double occupancy calculated in the atomic limit.

1.3 Limitations of the experimental method

In discussing the experimental results, it is necessary to point out the limitations of the method. Unlike bulk crystals which are uniform on the order of $10^{23}$ lattice sites, cold atom systems are limited to $10^5$ particles. This small number limits the measurement resolution, as it is challenging to distinguish fewer than a few hundred atoms in our experimental setup. In contrast, the motion of a tiny fraction of the charge carriers in a solid can amount to a large signal-to-noise ratio, enabling measurements to be performed closer to equilibrium and in a perturbative fashion.

The ultracold atom system is decoupled from the environment and is not in equilibrium with a thermal bath. This isolation allows atoms can be cooled to such low absolute temperatures, but it is also the reason entropy produced in generating motion (necessary in transport measurements) remains in the system. The inevitable temperature increase poses difficulties for performing steady-state constant temperature or thermal gradient measurements. Furthermore, reliable temperature probes are absent for many interesting systems [38] (e.g., interacting atoms in an optical lattice), necessitating careful estimates for the entropy and the entropy increase during the measurement process.

Another complication is the presence of a confining potential which causes variable filling across the gas. Phases requiring fixed filling (e.g., Mott and band insulators) can therefore be realized for only part of the system (typically for about half the atoms for our measurements). Further complications stem from the finite energy range that the lattice imposes: particles with energy higher than the bandwidth can be energetically disallowed from the center of the trap [44]. Spatial dependence of energy and entropy [45] is another consequence of confinement. Much research has been dedicated to understanding these effects [46]. With the help of a universal scaling of the potential geometry, proposed phase diagrams elucidate what phases can be expected in the system [36, 38, 47].

1.4 Future directions

While the weak-scattering picture and self-consistent Born approximation are useful for understanding the qualitative features of AL, a treatment accurate in the strongly localized regime is needed to compare to the measured mobility edge and to unravel thermal averaging
of the localization length. Weak scattering predictions need to be taken with caution in light of their known failure in the strongly localized regime and their disagreement with the data. A theory that closely describes our measurement would have to address some details specific to the experiment, such as the effect of the finite extent of the disorder beam and the thermodynamics of the trapped gas after disorder is turned on (e.g., density of states and equilibrium properties) [48].

Experimentally, an effort is under way in our team to understand how the disorder length scale affects the physics of AL [49]. The speckle size is varied by changing the disorder beam waist before entering the focusing lens. Furthermore, the study probes whether the size of the localized cloud is commensurate with the size of single localize wavefunctions or, alternatively, particles are localized over small distances while centered at various locations in the speckle. Another point of interest for atoms in speckle disorder is their capability to thermalize and reach a steady state after a disturbance. This can be quantified by measuring cross-dimensional rethermalization rates for trapped interacting atoms [50]. A study on this issue will aim to establish the steady state as a relevant starting point for analyzing disordered systems. The influence of inter-particle interactions on the localization of fermions can be studied by introducing a second spin species and using a Feshbach resonance [51]; of particular interest is the impact of disorder on the BEC – BCS crossover (see Refs. [52, 53] and references therein).

Several new projects, both in preliminary stages, will focus on lattice physics. One intention is to probe the excitation spectrum of the disordered phases with the help of Raman spectroscopy [54]. Raman spectroscopy can give more detailed information than a bulk transport measurement due to its ability to target narrow quasimomentum ranges and to reveal the band structure. The second new project on the $^{40}$K experiment will seek to realize long-distance interactions in the lattice. Experimentally, the interaction can be realized using a highly excited Rydberg state of the atom.

### 1.5 Outline

The remainder of this thesis is organized as follows:

- Chapter 2 begins with the electronic structure of $^{40}$K and describes how the interaction of the valence electron with electromagnetic waves and static magnetic fields allows us to trap and cool atoms and to construct potential landscapes. A discussion of the low-energy collisional properties serves to justify the non-interaction assumption for the Anderson localization measurement.

- Chapter 3 details the design and construction of the experimental apparatus. The alignment and calibration procedures for the optical potentials are presented.
• Chapter 4 describes the Anderson localization measurement. The relevant physics is introduced, followed by a description of the measurement method and the data. The results are discussed in light of previous experimental and theoretical work and are compared to classical diffusion behavior.

• Chapter 5 begins with a discussion of lattice physics and the phases of matter that might be expected in non-interacting, interacting, uniform, and trapped systems. Experimental results on the disordered Fermi-Hubbard model are presented, including a transport and a site occupancy measurement. The data is compared to theoretical work, as well as to simple models and calculations.
Chapter 2

Atomic Physics Overview

2.1 Introduction

$^{40}K$ is an unstable isotope of potassium with a half-life of approximately $10^9$ years. Its decay (to $^{40}Ca$ or $^{40}Ar$) is the predominant source of background radioactivity on Earth. We chose to work with $^{40}K$ because of its fermionic nature, which is important to us for two reasons. First, identical fermions do not interact at low temperatures, thereby enabling us to create non-interacting ensembles to study Anderson localization. Second, we can use $^{40}K$ to emulate the many-body physics of electrons which are also fermions. Experiments exploiting the similarity between condensed matter and ultracold atom systems are called quantum simulations and have been performed to study a variety of condensed matter problems. Among those are the BEC – BCS crossover [55] and the Fermi-Hubbard [19, 23, 56], Bose-Hubbard [57], and disordered Bose-Hubbard [33] models. $^{40}K$ is a natural choice for such experiments because of the extensive knowledge and experience working with the atom in the cold atom community. Its collisional properties are well-studied [58], a Feshbach resonance has been mapped out [59], a degenerate Fermi gas has been achieved [60], and 3D optical lattice experiments have been realized [61]. An added benefit of working with potassium is the availability of cheap laser diodes that can be tuned to the resonant 767 nm wavelength. Over recent years, the cost of laser sources and optics for near-infra-red light has sunk substantially due to developments in the optical data transfer and storage industries.

The properties of $^{40}K$ as a compound fermion are determined by the even number of nucleons and odd number of electrons. The distinction from bosonic atoms (such as the $^{39}K$ and $^{41}K$ isotopes) becomes important at very low temperatures when the gas becomes quantum degenerate. The Pauli exclusion principle disallows simultaneous occupancy of a given quantum state by multiple identical fermions. Therefore, an ensemble of such particles will fill a “Fermi sea” in which low energy levels tend to receive unit occupancy, while all thermal dynamics happens near the highest filled energy levels. The thermal occupancy of energy states is given by the Fermi-Dirac distribution

$$f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_BT} + 1},$$

(2.1)

where $\epsilon$ is the energy eigenvalue, $T$ is the temperature, and $\mu$ is the chemical potential. This
distribution also describes electrons in solids and plays an important role in determining their transport properties. For example, an insulating phase may be produced in a weakly interacting system when all states within an energetically allowed band are occupied, leaving no possibilities for center-of-mass motion for the whole population. The fermionic nature of electrons also leads to great difficulties in theoretical description and understanding due to the great many quantum states populated. The antisymmetrization requirement for the fermionic wavefunction poses great challenges for the computational analysis of many-body systems. Quantum simulation experiments attack these difficulties by exploring the similarities between cold atoms and electrons (see Ref. [18] for a review of experimental studies).

2.2 $^{40}K$ electronic structure

In its ground state, $K$ has three filled electron shells and a single valence electron in the $4S$ orbital (Fig. 2.1) which determines many of the relevant properties of the atom for the measurements we describe. A closed-cycle optical transition to the $4P$ excited state can be addressed with commercial near-infra-red diode lasers. The interaction of the atom with far-off-resonant laser light can be understood almost entirely through a two-level model for the electron. Fig. 2.2 shows the relevant energy levels and their hyperfine splittings. The $4S \rightarrow 4P$ transition has a 6 MHz linewidth [62] and gives the main contribution to light shifts due to far-off-resonant laser wavelengths used in the experiment (532 nm, 782 nm, and 1064 nm). Spin-orbit coupling sees a splitting of the $4P$ state into $4P_{1/2}$ and $4P_{3/2}$, with the oscillator strength of the $4S_{1/2} \rightarrow 4P_{1/2}$ transition equal to half that of the $4S_{1/2} \rightarrow 4P_{3/2}$ transition [32]. Collisional properties are also largely determined by the quantum state of the valence electron: the $s$-wave symmetry of the ground state wavefunction determines the van der Waals nature of the scattering potential at large interatomic distances.

The hyperfine interaction of the nuclear and electron spins causes a splitting of the ground $4S_{1/2}$ state. The large angular momentum of the nucleus ($I = 4$, where $I$ is the nuclear angular momentum quantum number) results in a range of Zeeman sub-levels (Fig. 2.3). Of those, the $|F, m_F\rangle = |9/2, 9/2\rangle$ and $|F', m_{F'}\rangle = |11/2, 11/2\rangle$ are central to laser cooling and imaging because they assemble a closed cycling transition when addressed with $\sigma^+$ polarized light. In order to be cooled to degeneracy, atoms are usually prepared in a mixture of the $|9/2, 9/2\rangle$ and $|9/2, 7/2\rangle$ states. At low temperatures, interactions between identical species are suppressed, and only the inter-species collision is allowed. In this collision, conservation of angular momentum prevents spin exchange. Therefore the mixture remains stable over time, and no other Zeeman states become populated. Furthermore, both $|9/2, 9/2\rangle$ and $|9/2, 7/2\rangle$ are low-field seeking states and are therefore magnetically trappable. This is of particular importance, as the first stage of evaporative cooling is performed in a magnetic trap. This stage uses a radio-frequency magnetic field (RF) to selectively eject from the trap the atoms.
with high energy. The RF transfers atoms from the $|\frac{9}{2}, \frac{9}{2}\rangle$ and $|\frac{9}{2}, \frac{7}{2}\rangle$ states to $|\frac{7}{2}, \frac{7}{2}\rangle$, which is a high-field seeking untrapped state.

### 2.3 Magnetic interactions

The magnetic dipole moments and angular momenta of the nucleus and the valence electron predominantly determine how the atom interacts with magnetic fields. The magnetic Hamiltonian $\mathcal{H}_M$ is

$$\mathcal{H}_M = -\mu_e \cdot \mathbf{B}_e - \mu_I \cdot \mathbf{B}_e - \mu_{atom} \cdot \mathbf{B}$$

$$= \mathcal{H}_{so} + \mathcal{H}_{hf} + \mathcal{H}_Z. \quad (2.2)$$

The first term approximately describes the electron spin-orbit coupling, in which the magnetic dipole moment of the electron $\mu_e$ experiences an energy shift due to the magnetic field $\mathbf{B}_e$ created by its own motion. Although spin-orbit coupling is in essence a relativistic effect requiring a covariant theory, treatment in terms of $\mathcal{H}_{so}$ is sufficient for qualitative understanding and is a standard development in quantum mechanics texts (see for example Ref. [64]). $\mathcal{H}_{so}$ is responsible for the splitting between the $4P_{1/2}$ and the $4P_{3/2}$ states. $\mathcal{H}_{hf}$ describes the hyperfine interaction between the nuclear magnetic moment $\mu_I$ and $\mathbf{B}_e$. The last term in Eq. (2.2) gives the Zeeman effect of an external field $\mathbf{B}$ on the magnetic dipole moment of the atom $\mu_{atom}$. The $\mathcal{H}_{hf}$ and $\mathcal{H}_Z$ components of the Hamiltonian are discussed below, and the $g$-factors relating the angular momenta with the corresponding dipole moments are derived.
Figure 2.2  Energies of the $^{40}K$ hyperfine levels (adopted from Ref. [63]).
2.3.1 Hyperfine structure

In the parameter regime explored in our experiment, \( \mathcal{H}_{hf} \) dominants \( \mathcal{H}_Z \) in the magnetic field Hamiltonian (Eq. 2.2). At 1.3 \( h \)-GHz the hyperfine splitting of the ground 4S state is much larger than the Zeeman shifts, which scale as 1.4 \( h \)-MHz/G, where \( h \) is Planck’s constant (Fig. 2.3). Therefore, for the typical fields of a few Gauss in the experiment, the Zeeman coupling can be considered a perturbation to hyperfine structure. The goal of this subsection is to rewrite \( \mathcal{H}_{hf} \) in terms of angular momentum operators and obtain expressions for the energy levels displayed in Fig. 2.2.

The hyperfine Hamiltonian is given by

\[
H_{hf} = -\mu_I \cdot \mathbf{B}_e. \tag{2.3}
\]

In the following, let us deal with the simplest case of a s-electron. Consider the electron as a distribution of magnetization \( \mathbf{M} \):

\[
\mathbf{M} = -g_s \mu_B \mathbf{S} |\psi(r)|^2, \tag{2.4}
\]

where \( g_s \) is the \( g \)-factor of the electron, \( \mu_B \) is the Bohr magneton, \( \mathbf{S} \) is the electronic spin operator, and \( \psi(r) \) is the spatial part of the electron wavefunction. \( g_s \) has been the topic of intense research in the nuclear field and is known to extraordinary precision. For our purposes, \( g_s = 2 \) to a very good approximation. An estimate for \( \mathbf{B}_e \) can be obtained from...
a classical electromagnetism result for a uniformly charged sphere:

\[ \mathbf{B}_e = \frac{2}{3} \mu_0 \mathbf{M} = \frac{2}{3} \mu_0 g_s \mu_B \mathbf{S} |\psi(0)|^2. \] (2.5)

Here \( \mu_0 = 4\pi 10^{-7} \text{ N/A}^2 \) is the permeability of free space. The nuclear magnetic moment is

\[ \mu_I = g_I \mu_N \hbar, \] (2.6)

where \( g_I \) is the nuclear \( g \)-factor, and \( \mathbf{I} \) is the nuclear angular momentum operator. \( g_I \) has not been calculated to adequate precision, but accurate measurements exist. For \( ^{40}K \),

\[ g_I = -0.3241 \pm 0.0001 \] [65]. With the above substitutions, the hyperfine Hamiltonian can be rewritten as

\[ H_{hf} = \frac{2}{3\hbar} g_I \mu_N \mu_0 g_s \mu_B |\psi(0)|^2 \mathbf{I} \cdot \mathbf{S} = A \mathbf{I} \cdot \mathbf{S}, \] (2.7)

where the constant factors have been lumped into \( A \). More generally, for \( l \neq 0 \), \( \mathbf{S} \) is replaced by the total electronic angular momentum operator \( \mathbf{J} \):

\[ H_{hf} = A \mathbf{I} \cdot \mathbf{J}. \] (2.8)

We define the total angular momentum \( \mathbf{F} = \mathbf{I} + \mathbf{J} \) which commutes with the Hamiltonian \( H_{hf} \). The energy splitting can be evaluated in the eigenbasis \( |IJFM_F\rangle \):

\[ E_{hf} = A \langle \mathbf{I} \cdot \mathbf{J} \rangle = \frac{A}{2} \left[ F(F+1) - I(I+1) - J(J+1) \right], \] (2.9)

where \( F, I, \) and \( J \) are the angular momentum quantum numbers related to the angular momentum operators by

\[ \mathbf{F}^2 |IJFM_F\rangle = \hbar^2 F(F+1) |IJFM_F\rangle \]
\[ \mathbf{I}^2 |IJFM_F\rangle = \hbar^2 I(I+1) |IJFM_F\rangle \]
\[ \mathbf{J}^2 |IJFM_F\rangle = \hbar^2 J(J+1) |IJFM_F\rangle. \] (2.10)

Formula (2.9) reveals the simple structure behind the relative magnitudes of the energy shifts in terms of simple fractions. However, it is not very useful for evaluating the absolute values of the energies because it is difficult to evaluate \( A \) to reasonable precision.
2.3.2 Zeeman effect

Now we can consider the effect of an external magnetic field as a perturbation\(^1\) to the energy levels in Eq. (2.9). The \(F\) states are shown to split according to the component of total angular momentum \((m_F)\) along \(B\). The \(B\)-dependence of the energy shifts is also calculated. Knowledge of the energy shifts is important experimentally because it allows us to manipulate the atoms. Atom populations can be transferred between neighboring \(m_F\) states with the help of RF magnetic fields. Alternatively, as practised in our experiment, \(m_F\) levels from one hyperfine manifold can be coupled via GHz frequencies to the \(m_F\) levels of another. This technique is used in forced RF evaporation, as well as in controlling the spin composition of an atomic ensemble. Furthermore, the Zeeman shift enables magnetically trapping the atoms, and understanding the energetics is crucial to the design and performance of the magnetic trap.

To first order, the energy shift is the expectation value of \(H_Z = -\mu_{\text{atom}} \cdot B\) in the basis \(|IJFM\rangle\). The magnetic moment of the atom \(\mu_{\text{atom}}\) can be written as

\[
\mu_{\text{atom}} = -\frac{1}{\hbar} (g_J \mu_B J - g_I \mu_N I) \approx -\frac{1}{\hbar} g_J \mu_B J,
\]

where

\[
\begin{align*}
\mu_B &= \frac{e\hbar}{2m_e} = 9.27401 \times 10^{-24} \text{ J/T} \\
\mu_N &= \frac{e\hbar}{2m_p} \quad (2.12)
\end{align*}
\]

are the Bohr and nuclear magnetons. Because of the small electron-to-proton mass ratio \((m_e/m_p = 1/1836)\), \(\mu_{\text{atom}}\) is well approximated by the electronic magnetic moment alone. We can write the Zeeman Hamiltonian as

\[
H_Z = \frac{1}{\hbar} g_J \mu_B J \cdot B. \quad (2.13)
\]

The electronic \(g\)-factor \(g_J\) can be derived considering the interaction of the magnetic field with the electron:

\[
H_Z = -\mu \cdot B = -\frac{\mu \cdot J}{J(J+1)} J \cdot B = \frac{1}{\hbar} \langle L \cdot J \rangle + g_s \langle S \cdot J \rangle J(J+1) \mu_B B J_z, \quad (2.14)
\]

where we have used \(\mu = -\mu_B L - g_s \mu_B S\). Comparing to Eq. (2.13) and setting \(g_s = 2\), we find

\[
g_J = \frac{\langle L \cdot J \rangle + g_s \langle S \cdot J \rangle}{J(J+1)} = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}. \quad (2.15)
\]

\(^1\)An exact diagonalization of the magnetic dipole Hamiltonian is also possible. The result is the Breit-Rabi formula \([66]\) which is correct for any magnetic field strength.
Note, however, that \( \mathbf{J} \) does not commute with the hyperfine Hamiltonian and therefore \( m_J \) is not a good quantum number. We can rewrite \( \mathcal{H}_Z \) in terms of the projection of \( \mathbf{J} \) along \( \mathbf{F} \):

\[
\mathcal{H}_Z = \frac{1}{\hbar} g_J \mu_B \frac{\langle \mathbf{J} \cdot \mathbf{F} \rangle}{F(F+1)} \mathbf{F} \cdot \mathbf{B} = \frac{1}{\hbar} g_F \mu_B \mathbf{F} \cdot \mathbf{B} = \frac{1}{\hbar} g_F \mu_B BF_z, \tag{2.16}
\]

where

\[
g_F = \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} g_J. \tag{2.17}
\]

For the \( 4S_{1/2} F_{7/2} \) state, we calculate \( g_J = 2 \) and \( g_F = -\frac{2}{9} \). For the \( 4S_{1/2} F_{9/2} \) state, \( g_J = 2 \) and \( g_F = \frac{2}{9} \). Since \( L = 0 \) for these states, the angular momentum is given by the sum of the nuclear spin and the electron spin. In the extended state (i.e., when the \( z \)-components of nuclear and electronic angular momentum line up) \( m_F = 9/2 \), the magnetic moment becomes \( 1 \mu_B \approx g \mu_B \), which can be expected since the magnetic moment of the atom is almost solely due to the spin of the electron.

### 2.4 Optical potentials

Light far detuned from the \( 4S \rightarrow 4P \) transition resonance is used to create conservative potential landscapes for the atoms. These potentials are very nearly additive in nature, and are proportional to light intensity, allowing for straightforward control and adjustment. Light with wavelength shorter than the transition resonance (blue detuning) repels the atoms and longer wavelength (red detuning) attracts them. Using polarization and focusing optics and phase masks, light can be manipulated to design various potential geometries for specific experimental applications. For example, an optical trap for the atoms is created at the overlap region of two focused Gaussian 1064 nm laser beams crossed at 30°. The standing wave created by a retro-reflected red-detuned (782.2 nm) laser beam produces a lattice potential in which the atoms are attracted to the intensity nodes. A speckle field of blue-detuned (532 nm) light is used to create the random potential necessary for studying disorder physics.

The dependence of the dipole force on intensity and detuning can be understood through a classical model of the electron as a driven damped harmonic oscillator. The oscillating electric field of the light wave induces an electric dipole moment in the atom, resulting in an energy offset. The oscillation of the induced dipole is damped by spontaneous scattering.
The potential $U_{dip}$ and scattering rate $\Gamma_{sc}$ are given by [67]

\[
U_{dip} = -\frac{3\pi c^2}{2\omega_0^2} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I \approx \frac{3\pi c^2 \Gamma}{2\omega_0^2} \frac{I}{\Delta}
\]

\[
\Gamma_{sc} = \frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\omega}{\omega_0} \right)^3 \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right)^2 I \approx \frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\Gamma}{\Delta} \right)^2 I
\]

Here $\omega$ is the angular frequency of the electromagnetic field, $\omega_0$ is the resonant frequency of the transition, $\Delta = \omega_0 - \omega$ is the detuning, $\Gamma = 6.02$ MHz is the natural linewidth, $I$ is the light intensity, and $c$ is the speed of light. The potential can be understood as resulting from the real part of the electron polarizability, while scattering comes from the imaginary part. $U_{dip}$ can be attractive or repulsive when the field is respectively in phase (red detuning) or out of phase (blue detuning) with the induced dipole moment. The $\frac{1}{\omega_0 - \omega}$ and $\frac{1}{\omega_0 + \omega}$ terms are known as “rotating” and “counter-rotating”. The counter-rotating term is often omitted from the sum when it is small in comparison to the rotating term, resulting in the rotating wave approximation [32].

The classical oscillator model gives the correct result for $U_{dip}$ for a quantum mechanical two-level atom. The generalization to a multilevel atom requires a quantum mechanical treatment in order to understand the coupling strengths for the transitions to multiple excited states. In this general case, complexities may arise such as a dependence of $U_{dip}$ on the ground state and on light polarization. However, in the case of far-off-resonant linearly polarized light—as used for optical trapping, lattice, and speckle disorder—the expression in Eq. (2.18) remains relevant [67].

Eq. (2.18) suggests a recipe to produce a good conservative potential. For a given magnitude of the potential, using larger detuning and higher optical power would result in less spontaneous scattering and consequently less heating. Exploring this advantageous trend is usually limited by the cost and technical difficulties of dealing with laser powers in excess of a few tens of Watts.

### 2.5 Collisions

The control over and understanding of pairwise interactions between atoms is of great importance for both measurements described in this thesis: lack of interactions in the spin-polarized gas guarantees the single-particle nature of localization in the AL measurement, while in the lattice measurement the very effect of interactions on localization is studied.

In the range of temperatures we explore, atoms interact through a quantum mechanical collision. In the following we examine the interaction beginning with the time-dependent Schrödinger equation. A standard approach is to describe the colliding atoms by the wave-
function $\Psi$ of a particle with reduced mass $m$ interacting with a stationary potential $V_{\text{int}}$. The induced electric moments of the atoms contribute the most to $V_{\text{int}}$ and dominate magnetic interactions. The interaction at large interatomic distances is determined by the induced dipole–induced dipole attraction and has the well-known van der Waals $r^{-6}$ functional form. The electrostatic nature of the potential allows us to initially consider only the spatial part of the wavefunction $\psi(\mathbf{r})$ and temporarily neglect the spin degrees of freedom. The problem reduces to a time-independent one, where the scattering wavefunctions are solutions to the Lippman-Schwinger equation \cite{68}.

In the limit of low energy, only the $s$-waves are shown to contribute to scattering. The relevance of this limit for ultracold atoms is established through a semi-classical argument showing that higher partial wave scattering requires kinetic energies far too high for the temperatures of interest. The $s$-wave nature of scattering results in a great simplification: the low energy collision can be described by a single parameter, the scattering length $a_s$ \cite{29}, which can be measured experimentally. This allows us to replace the interaction potential, however complex, by an effective contact potential characterized by the same $a_s$. In addition to providing physical insight, the effective potential simplifies interaction energy calculations such as used in the derivation of the Hubbard model (see Section 5.2.3).

Finally, the role of spin and the significance of Fermi-Dirac statistics is examined. The anti-symmetrization requirement for identical fermions allows $s$-wave scattering to occur only for particles with different spin, while interactions vanish for a spin-polarized gas. This feature of fermionic atoms is of practical importance for scientific measurements as the cold atom system can be prepared in an interacting or non-interacting ensemble by simply adjusting the spin composition.

### 2.5.1 Interaction potential

At short internuclear distances the pairwise interaction potential is strongly repulsive owing to the repulsion between the filled electron shells and the Pauli exclusion principle. At medium and large distances atoms attract due to their mutual polarizability. The potential depth depends on the alignment of the spins of the valence electrons. In the singlet state it is of the order of $10^3 k_B \cdot \text{K}$, which is comparable to the melting temperatures of solids. In the triplet state the potential depth is only a few hundred $k_B \cdot \text{K}$.

At large distances the total potential depth is irrelevant because the atoms interact predominantly through the electric dipole potential \cite{29}:

$$V_{\text{int}} = \frac{1}{4\pi \epsilon_0 r^3} [\mathbf{d}_1 \cdot \mathbf{d}_2 - 3(\mathbf{d}_1 \cdot \hat{\mathbf{r}})(\mathbf{d}_2 \cdot \hat{\mathbf{r}})]. \quad (2.19)$$

Here, $\mathbf{d}_1 = -e \mathbf{r}_e$ is the electric dipole moment operator for the first atom ($\mathbf{r}_e$ is the coordinate of the electron), and $\hat{\mathbf{r}} = \mathbf{r}/r$ is the unit vector with $\mathbf{r}$ being the vector separation between the nuclei. The atoms do not have a permanent dipole moment, and the ground
states are to a very good approximation eigenstates of parity. Consequently, diagonal matrix elements of the electric dipole operator vanish, and there will be no contribution to the interaction energy to first order in perturbation theory. The leading contribution is of second order and can be understood as an induced dipole – induced dipole interaction energy. At large distances, $V_{\text{int}}$ can be written as the van der Waals potential $V_{\text{int}} = C_6/r^6$ with $C_6 = 3897$ a.u. for $^{40}\text{K}$ \cite{69}.

The $r^{-6}$ dependence can be expected from a classical model of linear polarizability \cite{70}. The induced dipole moment of the first atom $d_1$ is approximately proportional to the electric field $E_2$ due to the dipole moment of the second atom. The electric field of a dipole falls off as $r^{-3}$, therefore $d_1 \propto r^{-3}$. The interaction energy can be written as

$$V_{\text{int}} = -d_2 \cdot E_1 \propto r^{-6}$$

in agreement with the functional form of the van der Waals potential.

### 2.5.2 Lippman-Schwinger equation

In the center-of-mass reference frame, the quantum mechanical collision between two atoms can be described as a single atom scattering off a potential $V$ and satisfying the time-dependent Schrödinger equation

$$\left[ i \hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2mr^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{\hbar^2 l(l + 1)}{2mr^2} - \mathcal{H}_0 \right] |\psi; t\rangle = V_{\text{int}} |\psi; t\rangle$$

$$\left( i \hbar \frac{\partial}{\partial t} - \mathcal{H}_0 \right) |\psi; t\rangle = \left( V_{\text{int}} + \frac{\hbar^2 l(l + 1)}{2mr^2} \right) \frac{V}{\mathcal{V}} |\psi; t\rangle. \quad (2.21)$$

The $\mathcal{H}_0$ term represents the free particle kinetic energy. It is customary to group the interaction potential $V_{\text{int}}$ together with the centrifugal term, as done in the last line.

The potential can be understood to turn on in adiabatically slow fashion such as $V \to \lim_{\eta \to 0} e^{\eta t} V$. This allows the assumption that $|\psi; t\rangle$ evolved from the free particle wavefunction $|\phi; t\rangle$ in the distant past. $|\phi; t\rangle$ satisfies the homogeneous Schrödinger equation

$$\left( i \hbar \frac{\partial}{\partial t} - \mathcal{H}_0 \right) |\phi; t\rangle = 0. \quad (2.22)$$

Both wavefunctions are energy eigenvectors. Since we are considering elastic scattering,
they have the same energy eigenvalue $E$:

$$
|\psi; t\rangle = |\psi\rangle e^{-iEt/\hbar}
$$

$$
|\phi; t\rangle = |\phi\rangle e^{-iEt/\hbar}.
$$

(2.23)

The complete solution to the time-dependent problem can now be written as

$$
|\psi; t\rangle = |\phi; t\rangle - \frac{i}{\hbar} \int_{-\infty}^{0} \theta(t - t') e^{-i\mathcal{H}_0(t-t')/\hbar} \lim_{\eta \to 0} \frac{V e^{i\eta t'} |\psi; t'\rangle dt'}{(E - \mathcal{H}_0 + i\eta)^{-1}}.
$$

(2.24)

where $\theta(t - t')$ is the Heavyside step function, and the kets have been evaluated at $t = 0$. Expression (2.24) is called the Lippmann-Schwinger equation and is a convenient starting point for discussions of scattering because time dependence is absent. The quantity $(E - \mathcal{H}_0 + i\eta)^{-1}$ represents the free propagation of atoms and is the Green’s function for the kinetic energy operator $\mathcal{H}_0$.

We project both sides of Eq. (2.24) onto the momentum basis $|k\rangle$ [71]:

$$
\langle k | \psi \rangle = \langle k | \phi \rangle + \frac{i}{\hbar} \langle k | (E - \mathcal{H}_0) ^{-1} V | \psi \rangle
$$

$$
\psi(k) = \phi(k) + \frac{i}{\hbar} \frac{1}{E - \mathcal{H}_0} \langle k | V | \psi \rangle.
$$

(2.25)

Here $k$ is the wavenumber vector, $m$ is the reduced mass, and $\phi(k') = (2\pi)^3 \delta(k' - k)$.

Since $|\psi\rangle$ appears on both sides of Eq. (2.25), we can substitute $\psi$ in the last term of the equation for the right side of Eq. (2.24):

$$
\psi(k) = \phi(k) + \frac{i}{\hbar} \frac{1}{E - \mathcal{H}_0} \langle k | V | \psi \rangle + \frac{1}{E - \mathcal{H}_0} \frac{1}{E - \mathcal{H}_0} \langle k | V | \psi \rangle.
$$

(2.26)

$\psi$ still appears in Eq. (2.26), and we can keep making the same substitution. We arrive at an infinite sum of terms containing $V \frac{1}{E - \mathcal{H}_0} V$, $V \frac{1}{E - \mathcal{H}_0} \frac{1}{E - \mathcal{H}_0} V$, ..., known as the Born series. The $n$-th term in the series has the physical interpretation of describing a free particle propagating and scattering off the potential $n - 1$ times.

Taking only the first two terms of the expansion amounts to making the first Born
approximation.

\[
\psi(k) = \phi(k) + \frac{i}{\hbar} \frac{1}{E - \frac{\hbar^2 k^2}{2m}} \langle k | V | \phi \rangle = \phi(k) + \frac{i}{\hbar} \frac{1}{E - \frac{\hbar^2 k^2}{2m}} \sum_{k'} \langle k | V | k' \rangle \langle k' | \phi \rangle = \phi(k) + \frac{i}{\hbar} \frac{1}{E - \frac{\hbar^2 k^2}{2m}} V(k, k)
\]

(2.27)

In the low energy limit, we can take \( V(k, k) \rightarrow V(0, 0) \) and \( E \rightarrow 0 \) \[29\].

\[
\psi(k) = \phi(k) - \frac{2m}{\hbar^2} \frac{1}{k^2} V(0, 0)
\]

(2.28)

Using the Fourier transform \( \int dk_3 e^{i k_3 \cdot r} \frac{1}{k^2} = \frac{1}{4\pi r} \), we obtain the scattering part of the wavefunction

\[
\psi_{sc}(r) = - \frac{mV(0, 0)}{4\pi \hbar^2 r} \equiv \frac{a_s}{r}.
\]

(2.29)

In the last line we have defined the scattering length \( a_s \)—the single parameter controlling the low-energy two-particle collision. The potential matrix element is \( V(0, 0) = \int dr_3 V(r) \).

The scattering length within the first Born approximation is

\[
a_s = \frac{m}{4\pi \hbar^2} \int dr_3 V(r).
\]

(2.30)

For a practical purpose, Eq. (2.30) is not suited for calculating \( a_s \) within reasonable precision since the scattering length is a very sensitive function of \( V(r) \). Instead, \( a_s \) is determined experimentally. For the \( \left| \frac{9}{2}, \frac{9}{2} \right> \) and \( \left| \frac{9}{2}, \frac{7}{2} \right> \) states used in our experiment, the triplet scattering length is approximately 174 \( a_0 \) \[59\] (\( a_0 = 52.917721092(17) \) pm is the Bohr radius).

Eq. (2.30) is useful for defining the effective potential \( V_{eff} \). Since scattering is controlled by \( a_s \) only, any potential satisfying Eq. (2.30) will give an identical scattering cross-section and interaction energy. It is therefore useful to define a simple contact potential

\[
V_{eff} = \frac{4\pi \hbar^2 a_s}{m} \delta^3(r - r'),
\]

(2.31)

where \( \delta^3(r - r') \) is the 3D Dirac delta function. From Eq. (2.31) it becomes evident that positive \( a_s \) corresponds to attractive \( V_{eff} \), and negative \( a_s \) corresponds to repulsive interaction. The usefulness of the effective potential becomes evident in constructing the Hubbard Hamiltonian described in Section 5.2.3 where interaction energies due to wavefunction overlap are evaluated.
2.5.3 $s$-wave scattering

By making the first Born approximation and taking the $k \to 0$ limit, we have arrived at a spherically symmetric scattering wavefunction, implying that $s$-wave scattering dominates low energy collisions. The reason why higher order collisions vanish can be seen through a semi-classical argument. A finite angular momentum state ($l > 0$) will possess additional kinetic energy according to Eq. (2.21). The long-range interaction, combined with the centrifugal term $V = -\frac{C_6}{r^6} + \frac{\hbar^2(l+1)^2}{2m_r^2}$, has a maximum at $r_c = \left[\frac{m_6C_6}{\hbar^2(l+1)}\right]^{1/4}$. Higher partial wave collisions ($l > 0$) can only take place if the particles have enough relative momentum to overcome the centrifugal barrier, which can be estimated as $V(r_c)$:

$$V(r_c) = 2\left[\frac{\hbar^2(l+1)}{2m}\right]^{3/2}\sqrt{C_6}. \quad (2.32)$$

For $l = 1$ ($p$-wave collisions), the barrier is approximately $150 \, k_B \cdot \mu K$, indicating the irrelevance of such collisions for the typical experimental temperatures of a few hundred nK. In this regime, the vanishing of the $p$-wave collision cross-section for the $|\frac{9}{2}, \frac{9}{2}\rangle + |\frac{9}{2}, \frac{9}{2}\rangle$ state of $^{40}K$ has been calculated [58] and measured [50].

2.5.4 Anti-symmetrization and vanishing of interactions

In the above discussion we have neglected the effects of quantum statistics. Due to the fermionic nature of $^{40}K$, a wavefunction describing two atoms must be antisymmetric under particle exchange. This has profound consequences for the interactions between particles in the same spin state. If we were to write the total scattering wavefunction $\Psi$ for two $^{40}K$ atoms prepared in the $|\frac{9}{2}, \frac{9}{2}\rangle$ spin state we would arrive at

$$\Psi = \psi_{sc}(r)|\frac{9}{2}, \frac{9}{2}\rangle_1|\frac{9}{2}, \frac{9}{2}\rangle_2. \quad (2.33)$$

For the spin part of the wavefunction, particle exchange is equivalent to exchanging the quantum numbers inside the kets, leaving $\Psi$ unchanged. Since $\Psi$ is symmetric, the spin statistics theorem requires that the spatial wavefunction be antisymmetric under particle exchange. $\psi(r)$ is written in the center-of-mass reference frame and therefore exchanging the two atoms amounts to the operation $r \to -r$, which in spherical coordinates is equivalent to $r \to r$, $\theta \to \pi - \theta$, and $\phi \to \pi - \phi$. Antisymmetrizing the $s$-wave scattering function in Eq. (2.29) results in cancellation:

$$\Psi = \frac{1}{\sqrt{2}}\left(-\frac{a_s}{r} + \frac{a_s}{r}\right)|\frac{9}{2}, \frac{9}{2}\rangle_1|\frac{9}{2}, \frac{9}{2}\rangle_2 = 0. \quad (2.34)$$

The scattering wavefunction vanishes, signifying that identical fermions in the same spin species will not interact through an $s$-wave collision. This has several implications for the
experiments described in this work. Since interactions are essential for thermalization and cooling, atoms must be prepared as a mixture of (at least two) spin species to achieve low temperatures. Alternatively, spin-polarized ensembles provide an excellent medium to study non-interacting physics such as AL. The path to realizing such physics is to cool to the target temperature in a spin mixture, and subsequently expel the atoms of the unwanted spin species.
Chapter 3

$^{40}K$ Apparatus

The experimental apparatus described in this chapter is in many aspects an iteration of previous designs. It shares many hardware features (i.e., vacuum system layout, transfer method, lattice and disorder geometry, imaging, and computer control), as well as sequence procedures, with the $^{87}Rb$ apparatus in our lab (detailed in Refs. [72, 73]), which in turn is an iteration of the original $^{40}K$ experiment in JILA (detailed in Ref. [62]). The rest of this chapter contains descriptions of the electric, optical and mechanical systems which compose the experiment, with an emphasis on the new designs and practices.

3.1 Overview of experimental sequence

The starting point for measurements of scientific interest is an ultracold gas of atoms. Such measurements usually require a certain spin composition, atom number and temperature—all quantities which are explicitly measurable. The atoms are prepared to meet these benchmarks in a series of steps, referred to as the experimental sequence.

The experimental sequence begins with laser cooling about a billion $^{40}K$ atoms to approximately 1 mK. Cooling happens in a magneto-optical trap (MOT) [32] in the collection cell (see Fig. 3.1 for layout). The atoms are then transferred into a quadrupole magnetic trap and moved to the science cell by mechanically translating the magnetic coils along an automated track. Once in the science cell, the atoms are transferred into a quadrupole-Ioffe configuration (QUIC) magnetic trap [74]. The long, background-collision-limited lifetime in this trap allows for radio-frequency (RF) cooling of the atoms before their transfer to an optical trap and subsequent evaporative cooling. The sequence lasts approximately 200 sec and results in a degenerate Fermi gas prepared for loading into some combination of speckle disorder and optical lattice potentials. Properties of the atomic system are measured destructively through an absorption image of the gas, ending the experimental sequence.

3.2 Vacuum System

The vacuum system features two all-glass cells of custom design (Fig. 3.1), prepared by Technical Glass of Aurora, CO. Dimensioned drawings of the cells are included in Section B in the Appendix (Figs. B.2 and B.5). Laser cooling takes place in the collection cell, where
Figure 3.1  Top view of vacuum system and magnetic traps. The collection cell houses the MOT. Magnetic field for the MOT is provided by the quadrupole trap which is mounted on a movable cart. The cart is shown half way through its path to the science cell. In the QUIC trap, the atoms undergo evaporative cooling, before being transferred to the optical dipole trap. Vacuum is maintained by two ion pumps. The relatively wider nipple connecting the science cell to the 40 L/s pump ensures a high conductance path and high pumping efficiency. In contrast, the path from the collection cell to the 20 L/s pump is constrained by a narrow nipple because a relatively higher pressure is necessary for optimal operation of the MOT.
The $^{40}K$ vapor pressure is sustained by the first of four enriched dispensers. The supplier (Alvatec) has quoted 10 g of $K$ per dispenser, enriched to 10% $^{40}K$ fraction, which has proven sufficient for constant operation since 2008. It is activated through resistive heating and is left on at all times. A constant heating current of 5.23 A passes through the body of the dispenser, resulting in a 0.275 V voltage drop across the leads. While operation has been trouble-free, initial installation into the collection cell and initiating operation posed some problems. The first problem concerns the protective construction of the dispenser. To protect the contents from oxidation, the dispenser opening slit is factory-sealed with indium. Craftsmen at Technical Glass found it difficult to spot weld the dispenser to the electrical leads without melting the seal. They resolved the problem by immersing the far end of the lead into liquid nitrogen during spot welding. Another problem appeared when we first activated a dispenser, attempting to observe fluorescence from a laser beam tuned to resonance with $^{40}K$. While we observed elevated pressure in the vacuum system, fluorescence never followed. When we increased the heating current to 8 A, the dispenser deposited a film of metal—not $K$—onto the collection cell wall. Other groups have also reported receiving the wrong dispenser from Alvatec, as well as unsuccessful attempts to obtain fluorescence [75].

To maintain sufficient vapor pressure for MOT operation, the collection cell is heated to 36°C using resistive tape glued around the perimeter of its windows. The cell also houses four yet unused $Rb$ dispensers manufactured by SAES. The collection and science cells are connected through a long thin “transfer” tube (see Fig. B.3 for dimensioned drawings). Its small radius limits the mass flow between the two cells, enabling a steady pressure differential. A relatively high pressure is maintained in the collection cell, necessary for the MOT, while a low pressure in the science cell guarantees a long lifetime (660 ± 100 sec). The lifetime is measured by observing how the number of atoms in the QUIC trap changes with time. A cold gas is used for the measurement, ensuring that thermal escape is not the dominant loss mechanism.

Vacuum is maintained with the help of a constantly operational low-impedance connection to a 40 L/sec ion pump (Varian). A second ion pump (Varian, 20 L/sec) was used during initial pump-down, but now is constantly kept off. The pressure in the vacuum system is below the sensitivity of the vacuum pump gauges ($10^{-10}$ Torr).

Atoms are transferred to the science cell using a magnetic trap mounted on a 800 mm Parker linear automated stage. Although it causes mechanical vibrations, the mechanical transfer has many advantages such as simplicity of implementation and reliability. In addition, the coils can be moved away after the transfer, clearing optical access to the science cell.
Figure 3.2  Vacuum system baking. The vacuum system is prepared for a low-temperature bake (upper image). The vacuum hose attached to the all-metal valve is visible, connecting the vacuum system to a turbo pump. The baking oven (half-way built) is made up of heat-resistant bricks, which are laid both below and around the vacuum system. Lower image shows the vacuum system after the bake.
3.2.1 Preparation

Before assembly, all components of the vacuum system are washed with Alconox detergent in an ultrasonic cleaner, then rinsed consecutively with distilled water, HPLC grade acetone, and methanol. The stainless-steel components are then air-baked at 450°C for 6 hours. According to conventional wisdom, the air bake causes hydrogen absorbed in the stainless steel to diffuse out of the material. Also, the metal surface is covered by an oxidized layer, which is thought to prevent further outgassing.

The vacuum system is initially assembled without the collection cell for a medium-temperature vacuum bake. As we found out in our unsuccessful first attempt, baking the dispensers at 300°C (quoted as safe by Alvatec) results in breaking the indium seals of the dispensers, emptying their contents. An oven is built around the vacuum system (Fig. 3.2) for the purposes of the bake. The heating elements are situated at the top of the oven, and all glass parts (especially glass-to-metal seals) are covered in aluminum foil to prevent direct radiative heating and possible thermal stress fracture.

Initial vacuum pump-down is achieved by means of a pumping station attached to the vacuum system by an all-metal valve. The pumping station consists of a turbo pump backed by a dry-scroll pump. After 20 hours of pumping, the pressure decreases to $1 \times 10^{-6}$ Torr. The pump is left operational while the oven is maintained at 300°C for 10 days. By the end of the bake the pressure falls to $1 \times 10^{-8}$ Torr (measured by an ion gauge).

Once the high temperature bake is completed, the vacuum system is vented and the collection cell is attached. The baking procedure is repeated, this time for 5 days and at 200°C. During this final bake the all-metal valve is closed after initial pump-down, the pump is permanently unattached from the system, and the ion pumps are switched on. After a gradual cool-down (12 h) to room temperature, we run a titanium sublimation pump for 6 hours on 30 minute cycles, running 45 A for 3 minutes per cycle. By the end of this procedure the pressure in the vacuum system has fallen below the sensitivity of the ion pump gauges.

3.3 Lasers

For frequencies within a few linewidths of the transition, the light-atom coupling is strongly dependent on the detuning, and the lasers need to be frequency stabilized. In such applications (e.g., MOT and imaging) we use external cavity diode lasers which can be actively stabilized and locked onto an atomic transition. Other tasks require far-detuned light and typically require laser power in the Watt range. For that we use a variety of commercially available units providing light at 1064 nm, 532 nm, and the 700-800 nm range.
3.3.1 ECDLs

Two external cavity diode lasers for trapping and repump light are built in-house using anti-reflection coated laser diodes (Eagleyard Ridge Waveguide EYP-RWE-0780-02000-1300-SOT12-0000). The external cavity is formed between the back face of the diode and a 1500 line-per-inch holographic grating (Edmund Optics SM2) in the Littrow configuration. Each laser is frequency-stabilized and offset-locked through an independent spectroscopy set-up: saturated-absorption [76] for trapping and polarization [77] for the repump frequency.

In spite of protection circuitry designed to prevent electrostatic discharge, a number of laser diodes failed in the $^{40}K$ and $^{87}Rb$ experiments sharing the lab. The problem was traced back to the seasonal drop in humidity typical for the Illinois winter, during which the relative humidity frequently dropped below 10%. The situation was only aggravated by the closed temperature control system for the lab. Lowered humidity caused frequent electrostatic discharge from people handling the equipment. During one year, 3 diodes were replaced in the $^{40}K$ apparatus before an active humidification system was installed in 2010. Since then, no diodes have failed in the $^{40}K$ experiment.

3.3.2 Tapered Amplifiers

Tapered amplifiers (TAs) are a low-cost solution for the sub-Watt power and quick frequency adjustment requirements for operating the MOT. The in-house design for the laser unit (see Fig. B.12 for a dimensioned drawing of the mount) accepts a C-mount package from Eagleyard Photonics. The nominally 765 nm TA chips (EYP-TPA-0765-01500-3006-CMT03-0000) were available with center wavelength of 767 nm per request from the vendor. The chip mounts directly onto the copper mount, which accommodates standard Thorlabs collimation optics for input and output coupling. Thermal and electrical contact between the chip and the mount is enhanced by a thin layer of indium. To create this layer, pieces of indium wire pressed against the copper with a home-made Teflon tool. The TA chip is completely enclosed between the copper mount and the coupling optics for better mechanical
insulation.

Although rated at 2.5 A, the TAs are operated at maximum 2.2 A to increase longevity. At this current and 8 mW of input light, they supply approximately 0.8 W, of which at most 0.33 W couple into a single-mode polarization-maintaining fiber. The coupling efficiency is limited by the highly non-Gaussian mode of the beam. Additionally, the rejected light heats the fiber and causes the coupling efficiency to drift for about 30 min before thermalization is reached. No TA chips have failed as of present, with the two original chips operational since 2007.

3.4 MOT

The quadrupole field for the MOT is created by the two coils in anti-Helmholz configuration. The coils are water-cooled and have 40 turns each, running 10 A of current to create an estimated 10 G/cm gradient. The field zero overlaps with the three mutually orthogonal laser beam paths. Each path is simultaneously traversed by overlapping trap and repump beams. The beams are joined on an edge mirror before being split and telescoped up in size to match the collection cell windows. Trap and repump frequencies are detuned −20 MHz from the $F = |9/2\rangle \rightarrow F'$ and $F = |7/2\rangle \rightarrow F'$ D2 transitions, respectively.

We roughly double the number of trapped atoms in the MOT with the use of a dark spot in the repump beams. When aligned properly, the dark spot creates a region of space roughly the size of the laser-cooled gas where no repump light is present. As a result, many cold atoms find themselves in the dark ($F = |7/2\rangle$) state and stop scattering light from the trapping laser, suppressing light-induced collision losses. The dark spot MOT is the standard mode of operation of the experiment. One disadvantage is the long fill time: it takes approximately 45 sec for the MOT to fill to 70% of its maximum value.

Another method to increase the atom number is to shine a blue laser onto the collection cell. Blue light quickly raises the vapor pressure by causing atoms to de-adsorb from the inside surfaces of the vacuum system. Because the pressure falls to normal within milliseconds of turning off the light, the lifetime of the atoms in later stages of the sequence remains unaffected. In our apparatus, a 10 mW 405 nm diode loosely aimed at the cell resulted in doubling of the atom number while keeping fill times unchanged. However, this enhancement diminishes within 20 – 50 experimental runs. The 10 seconds of blue light per run resulted in significant depletion the collection cell walls of stored potassium, which is generally believed to be important for a stable operation of the experiment. Use of the blue diode was discontinued in the interest of long-term number reproducibility.
3.5 Transfer

In the transfer procedure, the atoms are loaded into the quadrupole trap, moved to the science cell, and loaded into the QUIC trap. Loading the quadrupole trap begins with the optical molasses [32] stage, which aims to lower the temperature of the magneto-optically trapped atoms. For this stage, the quadrupole magnetic field is switched off, and a set of 3 mutually orthogonal “shim” coils is used to cancel out any net forces on the gas and to maintain an optimal position for loading into the quadrupole trap. Such net forces may arise from local imbalance in the intensities of the first and second passes of the retro-reflected laser beams or from stray magnetic fields. Initial optimization for the shim coils can be done by eye: since there is no confinement in the optical molasses, atoms left in it visibly expand over a few tenths of a second. A net force on the gas would cause it to accelerate during expansion. Good initial settings can be achieved by adjusting the shim coil fields until no center-of-mass velocity is visible.

The last step before ramping the quadrupole fields is optical pumping. In this step, all magnetic fields are switched off with only one shim coil remaining on. This coil provides a quantization axis for the atoms while they are addressed with $\sigma^+$-polarized near-resonant light propagating in the direction of magnetic field. This light drives the cycling transition toward the stretched state $|F, m_F\rangle = |9/2, 9/2\rangle$, which possesses the largest magnetic moment, and therefore experiences the largest magnetic trapping forces. The atoms are finally transferred to the quadrupole trap by shutting off all light and ramping the quadrupole field gradient to 200 G/cm. The cart then moves the quadrupole trap towards the science cell where the lifetime of the atoms is much longer. The transfer is concluded by turning on the QUIC trap simultaneously with switching off the quadrupole trap.

The efficiency of transfer depends on a multitude of parameters (e.g., laser detunings, quadrupole and shim coil currents, timing, and laser intensities). However, all of these parameters can be straightforwardly optimized in view of a single measure: total atom number transferred to the science cell. We assess the overall transfer efficiency at approximately 25%. This estimate is acquired by a round-trip recapture procedure in which we transfer the atoms to the science cell, then bring them back to the collection cell, and measure the fluorescence as a fraction of initial MOT level. One advantage of this estimate is that it does not rely on an accurate calculation of atom number in the MOT. It does, however, assume that the transfer efficiency is the same in both directions—a condition which we have not been able to confirm.

3.6 Forced RF cooling

Forced radio-frequency (RF) cooling is achieved by selectively transferring (“cutting”) atoms with high energies into a magnetically untrapped state while the remaining atoms
rerethermalize to lower temperature. The static magnetic field $B$ and the driving RF magnetic field are related by 1284.8 MHz $- (1 + \frac{7}{9}) \times 1.4$ MHz/G × $B$ for the $| \frac{9}{2}, \frac{9}{2} \rangle \rightarrow | \frac{7}{2}, \frac{7}{2} \rangle$ transition, and 1284.8 MHz $- (\frac{7}{9} + \frac{7}{9}) \times 1.4$ MHz/G × $B$ for the $| \frac{9}{2}, \frac{7}{2} \rangle \rightarrow | \frac{7}{2}, \frac{7}{2} \rangle$ transition. Because of the spatial dependence of the trapping $B$-field, a given frequency ("RF knife") corresponds to a shell of constant potential energy. Only atoms with energy above the knife will be cut from the trap. Cooling progresses as the frequency is swept to higher values and the RF knife approaches the center of the trap. The RF signal is generated using a DDS (part number ADF 9854) to reference a phase-locked loop (PLL, part number ADF 4360–5). After amplification, the signal is broadcast via an antenna positioned approximately 1.5 cm below the science cell. The antenna consists of a standard 0.5” diameter copper UHV gasket which has been slit through and stub-tuned to 1.2 GHz.

In approaching very low temperatures, this cooling method leads to the complication that the ratio between populations in different spin species changes. To circumvent this problem, the last cooling stage is performed in an optical dipole trap.

### 3.6.1 Losses and heating

In achieving a high phase space density, cooling competes with various loss and heating processes. The three major mechanisms are background gas collisions, three-body recombination, and Majorana losses [32]. The first two processes are easily kept under control: background collisions are greatly suppressed due to the excellent vacuum maintained in the science cell, while three-body recombination loss is suppressed by Fermi statistics [78]. Furthermore, three-body loss rates depend on the atom density cubed and therefore become important only for very low temperatures and strong confinement.

Majorana losses can strongly affect the cooling efficiency at every temperature. They occur when atom spins are unable to adiabatically follow the direction of the confining magnetic field and undergo a spin-flip transition to a magnetically untrapped state. For a particle with a (magnetically trapped) spin up and a (magnetically untrapped) spin down state, the condition for adiabaticity is $\frac{\hbar \omega_t}{\mu_0 B} \ll 1$ [79], where $\omega_t$ is the trap confinement frequency and $\mu_0$ is the Bohr magneton. The condition amounts to the requirement that the Larmor precession frequency is much larger than $\omega_t$. Majorana spin flips become especially likely when there is a zero at the trap center, such as at the center of a quadrupole trap. While an atom may not strictly pass through the zero of the magnetic field, passing close to the zero may cause it to experience a locally parabolic potential with very high $\omega_t$. By passing arbitrarily close to the field zero, the effective $\omega_t$ can become arbitrarily large, and the adiabatic condition will eventually fail. This mechanism adversely affects cooling efficiency both for high temperature—due to the high thermal velocity of the atoms—and for low temperature—due to the elevated atom density close to the field zero.

This shortcoming of quadrupole traps is well-known [80]. In practice, however, Majorana
losses have turned out to be sufficiently mild that quadrupole traps have been used for initial stages of cooling $^{87}\text{Rb}$ [72] or for sympathetic cooling of $^{40}\text{K}$. Schemes for “plugging” the field zero with a blue-detuned laser beam have also proven successful for achieving low temperature [81]. Based on this knowledge, we attempted RF cooling in the quadrupole trap. Unfortunately, the necessary cooling efficiency was never achieved. At that time our experiment was the first effort to directly (not sympathetically) cool $^{40}\text{K}$ in a magnetic trap. From ours and other groups’ experience [75] it has since become clear that Majorana losses greatly undermine cooling in the quadrupole trap. The evaporation trajectory which describes the cooling efficiency (Fig. 3.4) shows a factor of 10 reduction in temperature for a factor of 100 reduction in atom number. We attempted to plug the trap with a 300 mW beam at 760 nm, focused to 120 $\mu$m waist (parameters similar to those used in the $^{87}\text{Rb}$ experiment described in Ref. [72]) with little success.

The large Majorana loss of $^{40}\text{K}$ atoms can be traced back to the large nuclear angular momentum ($I = 4$) and the multitude of Zeeman sublevels resulting from it. Calculating the losses for an atom with many trapped states is not straightforward [80]. However, a relatively large loss can be expected due to the stricter adiabaticity condition, which can be generalized as $\frac{\hbar \omega}{P g B} \ll 1$, where $g$ is the hyperfine $g$-factor [80].
Figure 3.5 QUIC trap geometry. The black arrow points in the “axial” direction, which is also the “front” imaging direction. To second order in the multipole expansion around the minimum, the magnetic field magnitude $|B|$ (and therefore the confining potential) possesses axial symmetry. The axial confinement is the weakest confinement, and is provided by the curvature of the magnetic field from the Ioffe coil.
Figure 3.6  Printed parts for magnetic coil winding. After it is bent in place, the quad coil is potted in epoxy. To force the wire to conform to the desired shape while curing, the coil is compressed on both sides by 3D-printed plastic parts (shown in white). Supported by a thick plastic plate on one side, each radial set of turns is squeezed in place by a separate ring on the other. The four concentric rings are shown on the right. The area of each ring in physical contact with the wire is colored in copper color.

3.7 QUIC trap

The Majorana loss is greatly alleviated by implementing a QUIC trap [74]. This trap is a kind of Ioffe-Pritchard trap [82]: a family of trap geometries possessing a non-zero magnetic field offset at the trap minimum. The QUIC trap consists of a pair of coils in the quadrupole configuration (“quad” coils), in addition to a third (“Ioffe”) coil lying in a perpendicular plane (Fig. 3.5). The three coils are connected in series such that the current in the Ioffe coil runs in the opposite direction to the current in either quad coil at their closest point. The resulting fields are very similar to those obtained in a “baseball” trap [82]. The magnetic field offset at the trap minimum is controlled by an approximately uniform bias field. To avoid the mechanical complication of adding extra hardware, the cart quadrupole coils are used to provide the bias.

The mechanical design is crucial to the development and evolution of the experimental apparatus. Built relatively early in the process, it permanently constrains optical access and occupies valuable space close to the collection cell. The QUIC configuration was chosen with the intention of initially using only the quadrupole coils to create a tight trap with a small footprint. The Ioffe coil was designed and made, but left aside as a fall-back option. Adding the Ioffe coil increased the cooling efficiency (Fig. 3.4) and enabled low enough temperatures for efficient transfer into a crossed dipole optical trap (OT).
The mechanical design of the QUIC trap (Fig. 3.7) was motivated by the effort to preserve optical access. To this end, all magnetic coils were assigned conic shapes, and the support structure was hidden behind the seams of the science cell optical surfaces. Realising this compact design is a challenge in its own, especially forcing the 1/16” square copper tubing to comply to the complex conic shape. A detailed CAD drawing of the coil allowed the design of complementary components to constrain the winding path (Fig. 3.6). These components were 3D-printed and used to hold the tubing in place during winding. They remained in place until the epoxy cured, turning the coil into a single solid piece. For the support structure, 7075 aluminum (black in Fig. 3.7) and titanium (yellow) were chosen for their strength, rigidity, and non-ferromagnetic properties.

To facilitate the optimization of the design parameters (e.g., coil positions, sizes, and number of turns), an Excel file was developed, containing a multipole expansion calculation for the confinement frequencies and the field offset at the trap minimum. The calculation is detailed in Appendix A. Serving as input parameters for the calculation, the coil dimensions in the Excel file are directly referenced by the CAD (Autodesk Inventor) model in a parametric fashion: all drawings of coils and support structure are redrawn automatically upon updating the Excel file.

3.8 Optical evaporation

For the final cooling stage, it is advantageous to transfer the atoms to an optical trap. Since the trap depth is the same for both spin species, $|\frac{9}{2}, \frac{9}{2}\rangle$ and $|\frac{9}{2}, \frac{7}{2}\rangle$ atoms experience no relative sag, and evaporative cooling does not change their number ratio. Furthermore, the optical trap allows easy spin manipulation, as each species can be addressed with a single microwave frequency in the presence of a uniform magnetic field.

To load the OT, the magnetic trap is turned off linearly, while the OT is simultaneously ramped to maximum power. Evaporative cooling takes place as the trap depth is lowered and high energy atoms are allowed to escape, while the remaining rethermalize to a lower temperature [83, 84]. The cooling trajectory shown in Fig. 3.4 results in a gas of $8 \times 10^4$ atoms with $T/T_F = 0.15$.

A 20 W IPG Photonics fiber laser (YLM-20-1064-LP) is used for the dipole trap. The light has 1064 nm wavelength and 50 cm coherence length (measured with a Michelson interferometer). To form the OT, two passes of the dipole beam intersect at 30° inside the science cell (Fig. 3.8), 40 μm below the magnetic trap minimum. At the intersection, both beam passes are focused to a 120 μm radius. The polarizations are linear and mutually orthogonal to avoid interference at the intersection. This is a necessary precaution: other groups have reported anomalous interference-related heating of the atoms, although the self-intersecting beam path might be longer than the coherence length.
Figure 3.7  Magnetic trap assembly. Technical drawings are presented in Appendix B.
Figure 3.8  Optical trap schematic.

Figure 3.9  Optical speckle field intensity $I$ (in the focal plane) normalized by the averaged intensity $\langle I \rangle$.

3.9 Disorder

The disordered landscape is created by the light shift from a stationary speckle field overlapped with the optical trap (Fig. 3.5). The speckle (Fig. 3.9) forms at the focus of a 532 nm
beam passing through a plano-convex lens with 13 mm back focal length and 15 mm diameter (Lightpath Industries Gradium GPX-15-15). A 0.25 mm thick holographic diffuser (from Luminit, LLC) mounted to the flat side of the focusing lens scatters the light through a 0.5 degree range of angles. At the focal plane, the beam profile has an overall Gaussian envelope of approximately 170 µm. The statistical properties of the speckle are described by the autocorrelation function $\Gamma(\delta r)$ [30]

$$
\Gamma(\delta r) = \frac{\langle I(r)I(r + \delta r) \rangle}{\langle I(r) \rangle^2},
$$

where $I(r)$ is the spatially varying laser field intensity. $\Gamma(\delta r)$ can be described to an excellent approximation by a Gaussian:

$$
\Gamma(r) = \frac{1}{2} \left[ 1 + e^{(\sigma^2_x + \sigma^2_y)/2\sigma^2_{xy} e^{\sigma^2_z/2\sigma^2_z}} \right],
$$

where $z$ is the direction of light propagation. For the measurements described in this work, $\sigma_{xy} = 270 - 300$ nm and $\sigma_z = 1600 - 1700$ nm. The speckle size is determined by the wavelength $\lambda_\Delta$, the waist of the input beam, and the Rayleigh range (400 µm) [30]. Although elongated along $z$, the speckle pattern is nearly isotropic in its structure. For in-depth discussion of how to produce a computer model of the speckle possessing the correct quantitative measures, see Ref. [73]. An important feature of speckle is the behavior of the dark regions. Those regions are non-intersecting curves of zero intensity (i.e., vortices) permeating the speckle field. Their behavior has been extensively studied [31], and is important for the interpretation of the measurements described in Chapter 4.

To allow adjustment in the $x - y$ plane, the disorder lens tube is held by a flexure mount (Fig. 3.10), adjustable by up to 2 mm in each direction. A mechanical drawing of the flexure mount is shown in the Appendix in Fig. B.10. 100 TPI threading on the lens tube enables vertical adjustment with 1.5 cm range. Moving the flexure mount after initial rough alignment is avoided because the set-screw positioning is hysteretic. Instead, final tuning is accomplished by steering the beam using the last (micrometer-actuated) mirror in the beam-path.

Initial alignment of the disorder beam is challenging due to the large detuning and consequently small potential that it produces. Two approaches have been used, both requiring an extensive search in the $x - y$ plane. In the first approach, a cold gas is held for a number of seconds in a shallow OT, while the disorder beam undergoes a sinusoidal modulation at twice the trap frequency with maximum amplitude. The modulation results in heating and number loss, which becomes more severe as the disorder beam is moved closer to the OT. An equally successful approach is to pulse the disorder beam for 2 ms after releasing the atoms from the OT and observe the gas position after long-time (20 ms) expansion.

The fine alignment signal is produced by pulsing the disorder beam for 100 µs and
Figure 3.10  Flexure mount. Clockwise from upper right: flexure mount assembly attached to the QUIC trap supporting structure (top view); side section view of trap, flexure mount and lens outlined in red; assembly with set screw grooves visible on flexure mount arm; picture of assembly next to an inch ruler.
measuring the RMS radius of the gas in expansion time (Fig. 3.11). When the beam is well overlapped with the OT, the atoms scatter from the disorder and the momentum distribution in the \(x-y\) plane broadens. The beam position corresponding to maximum RMS radius \(\sigma_x\) therefore gives the best spatial overlap with the atoms. Once alignment in the \(x-y\) plane is completed, the maximum \(\sigma_x\) is recorded the lens is adjusted in the vertical (\(z\)) direction. A new maximum \(\sigma_x\) is found and the procedure is repeated until the focusing plane is aligned to the OT.

In order to calibrate the average disorder energy, an in-situ measurement of the speckle beam size is necessary. For this purpose, we leave the disorder beam stationary and move the OT by adjusting the location where the second pass of the dipole beam intersects the first pass. The position of the OT is determined by an in-trap image of the atoms. In a subsequent experimental run, we release the atoms from the OT and allow them to expand in the disorder for 20 ms before taking an absorption image. The interpretation of the image is somewhat involved because it requires some knowledge of the localization physics we investigate in Chapter 4. In particular, we use the fact that disorder localizes a fraction of the gas which remains stationary. The remaining (mobile) atoms expand in a ballistic fashion, and their center of mass (COM) is accelerated by the repulsive action of the speckle beam envelope. The COM of the mobile atoms as a function of trap position provides the signal for measuring the disorder beam size, as displayed in Fig. 3.12. The fit to the data has the functional form of a derivative of a Gaussian with a waist equal to the waist of the speckle beam envelope.
Figure 3.12 Typical measurement of speckle beam size. The displacement of the centroid of the expanding gas is plotted against the initial (trapped) position. The measured beam waist is 184 ± 10 µm.

3.10 Lattice

3.10.1 Hardware

Light for the lattice comes from a Ti:Sapphire laser (Tekhnoscan TIS-SF-07) tuned to 782.2 nm (Fig. 3.13). When pumped with 12 W, it outputs a maximum of 1.2 W, although more than 0.85 W has not been necessary. As the lattice shares the optical table with the rest of the experiment, it is subject to mechanical vibration induced by the motion of the cart. Nevertheless, it has shown excellent long-term stability. The wavelength has not been observed to change in months of operation. To maintain the power output, the input coupler (two external knobs) has needed adjustment a couple of times a week, and the output couplers (four internal knobs) have only been optimized twice in 2.5 years of operation. Since installation, lasing action has never disappeared and it has not become necessary to perform the full alignment procedure. The Verdi V18 which pumps the Ti:Sapphire laser has not been so problem-free. A reoccurring error message raised doubt about the state of the diode bars, and the whole unit was replaced as a part of warranty coverage.

Light for each lattice beam is streamed off from the Ti:Sapphire laser beam by three consecutive AOMs which offset the frequency by +80 MHz, −80 MHz, and +75 MHz. Different frequencies prevent possible interference between the lattice beams as they intersect at the atoms. A further precaution against interference is orienting the relative polarizations of the lattice beams perpendicularly to each other. Unfortunately there are only two
Figure 3.13  Lattice laser system.
such distinct orientations, which takes away the freedom to optimize beam polarizations for maximum transmission through the glass surfaces of the science cell. Polarization is adjusted by a half-wave plate and cleaned up by a polarizing beamsplitter cube after the lattice beam exits the optical fiber (Fig. 3.13 inset). Further downstream, only unprotected gold mirrors are used in order to preserve polarization. The final optical component is a retro-reflecting mirror, mounted on a Thorlabs VM1 mount with graduated screws.

3.10.2 Alignment

The first step of the lattice beam alignment is finding the first pass of the beam relative to the trap. This is accomplished by tuning the Ti:Sapphire laser wavelength to the atomic resonance to induce fluorescence, effectively imaging the beam (Fig. 3.14). With the help of a wave-meter, the wavelength is set within 0.004 nm of the transition. To increase the chance of the beam hitting the atoms, the largest possible gas is used: atoms are imaged straight out of the QUIC trap without any cooling, with the lattice beam on during 20 ms of expansion. Fluorescence detection is done simultaneously with two orthogonal imaging directions, completely determining the beam position and allowing easy overlap with the known position of the OT.

Finer adjustment is achieved by tuning the beam to the lattice wavelength $\lambda_l = 782.2$ nm, turning it into a dipole trap. When the beam is on, a cold gas (200 – 500 nK) released
Figure 3.15  Lattice fine alignment. Left: forward alignment signal is given by the distance the atoms travel after receiving a momentum kick from the beam. The beam position is adjusted by turning the graduated screw of a mirror mount. The fit to the signal is a derivative of a Gaussian, centered at the best alignment position. Right: retro alignment is accomplished by pulsing the beam and observing the atoms diffract (see Fig. 3.16). Alignment signal is given both by the fraction of atoms diffracted (symmetric signal) and, similarly to forward alignment, by the position of the undiffracted atoms (anti-symmetric signal).

Figure 3.16  Lattice diffraction by each of the three beams.

from the OT becomes trapped (Fig. 3.14), providing an accurate estimate of the position of the lattice beam. For finest alignment, the lattice beam is used to impart momentum to the atoms in the OT. The beam position is tuned until the impulse results in no deflection of the gas in expansion time (Fig. 3.15).

To form a standing wave, the beam is retro-reflected using an unprotected gold mirror. An excellent starting point for retro alignment is achieved by coupling the beam back into the fiber output using only the last mirror. To obtain a signal for fine alignment, the lattice beam is pulsed on for 100 – 200 µs immediately after the gas is released from the OT, causing diffraction (Fig. 3.16). The sudden application of the lattice potential couples states in the trap to quasimomentum states in a number of bands (similarly to Ref. [57]). A beam intensity corresponding to 30 – 40 $E_R$ lattice depth is typically used. The diffraction peaks are separated by $nhk$ in momentum space, and the peak centroids will appear a distance
The lattice is calibrated by studying the excitation spectrum of the non-interacting spin-polarized gas, similarly to Ref. [85]. Since the excitation spectrum is specific to the lattice depth, the latter can be inferred from a comparison between measured and calculated excitation spectra. Furthermore, it is sufficient to only consider the highest frequency $\nu_{\text{max}}$ in the ground-to-second excited band feature. $\nu_{\text{max}}$ corresponds to the energy separating the $q = 0$ state (band center) of the ground band from the $q = \pm q_B$ states (band edges) in the second excited band (Fig. 5.1).

The experimental procedure for measuring the excitation spectrum is summarized in Fig. 3.17. A spin-polarized gas of atoms is loaded into a single lattice beam and subjected to 250 ms of single frequency modulation. When the modulation frequency matches the interband transition frequency, the atoms are heated, resulting in measurable number loss and temperature increase (Fig. 3.18). The spectrum feature corresponding to $\nu_{\text{max}}$ (blue lines

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hk t/M \quad \text{apart, where} \quad t \quad \text{is the expansion time. For them to appear spatially separated as in Fig. 3.16, both} \quad T \quad \text{and} \quad T_F \quad \text{should not exceed} \quad 1 \ E_R/k_B.
\]

The fraction of scattered atoms is used as a signal for fine retro optimization. The number of atoms populating the scattering peaks is a function of both light intensity and pulse time, and for fermions it is difficult to calculate. However, the exact dependence is unimportant, because the fraction of scattered atoms is necessarily a symmetric function of the relative position of the atoms and the retro position, with the center of symmetry given by the best alignment. It is easy to verify this method by comparing the scattered fraction data to central cloud position in expansion time, which should remain unchanged at optimal alignment.

3.10.3 Calibration

The lattice is calibrated by studying the excitation spectrum of the non-interacting spin-polarized gas, similarly to Ref. [85]. Since the excitation spectrum is specific to the lattice depth, the latter can be inferred from a comparison between measured and calculated excitation spectra. Furthermore, it is sufficient to only consider the highest frequency $\nu_{\text{max}}$ in the ground-to-second excited band feature. $\nu_{\text{max}}$ corresponds to the energy separating the $q = 0$ state (band center) of the ground band from the $q = \pm q_B$ states (band edges) in the second excited band (Fig. 5.1).

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Figure 3.18  Lattice excitation spectra for $s = 6.3\ E_R$ (a), $s = 36.5\ E_R$ (b), and $s = 28.1\ E_R$ (c and d). Driving frequency resonant with the inter-band transition causes number loss and heating. Consequently, the excitation spectrum becomes evident in both atom number and gas size measurements.  

(a), (b), and (c): The number loss feature corresponding to $\nu_{max}$ is shown as a blue line. The finite slope of the line determines the uncertainty in the measured lattice depth.  

d: The RMS radius of the gas in the $x$ (black squares) and $y$ (red circles) directions is correlated with the number signal in c.
in Figs. 3.18a, b, and c) has a finite frequency width. The beginning and ending frequency of the feature correspond to the low and high estimates for the lattice depth, providing an estimate for the statistical error in $s$. The modulation spectrum at a given beam intensity is a single data point in the calibration, and the final calibration is determined by fitting a straight line with zero intercept through at least 3 data points (Fig. 3.19). Alternatively, when left as a free parameter, the intercept from the fit is consistent with zero.

### 3.11 Imaging

Absorption imaging is the main measurement tool in both data-taking and optimization procedures. A resonant $\sigma^+$-polarized beam is sent through the atoms, addressing the cycling transition. The atoms resonantly scatter light out of the beam and cast a shadow, which is imaged onto a Princeton Instruments Pixis 1024 camera. The CCD array is shared for two imaging directions (Fig. 3.20). In front imaging, each 13 $\mu$m pixel corresponds to 3.12 $\mu$m at the atoms, or 4.17 magnification. Side imaging, only used in alignment procedures, has 2.2 magnification.

Absorption imaging returns the optical depth (OD) of the cloud, which is proportional to the density integrated along the imaging direction. Three consecutive images are necessary to obtain the OD: $S(x, y)$ contains the imaging beam with the shadow of the cloud, $L(x, y)$ contains the imaging beam without the atoms, and $D(x, y)$ is taken without imaging beam.
to measure the background lighting. The OD is given by

$$OD = \ln \left( \frac{L - D}{S - D} \right). \quad (3.3)$$

Interference fringes in the imaging beam lead to a major reduction of the signal-to-noise ratio. Caused by the large number of nearly parallel surfaces in the beam-path, the fringes are written onto the OD image by two different mechanisms. First, mechanical vibrations can slightly change the fringe pattern for $L$ and $D$, resulting in imperfect subtraction. Additionally, fringes can create regions of intensity high enough to saturate the atoms, leading to a perceived decrease in OD.

A number of measures can be taken to decrease the effect of fringes. A quarter-wave plate ($\lambda/4$) is placed in the front imaging path to reduce interference due to light reflected downstream. To reduce mechanical vibration, all shutters are placed on sorbathane pads except the imaging shutter, which is suspended from a fixture above the optical table. The imaging light intensity must be chosen carefully to avoid saturation by the bright spots in the beam, while the dark spots bring enough light to dominate the dark count of the CCD. As a rule of thumb, the range of appropriate intensities spans around an order of magnitude. OD fringes can also be taken out by a numerical algorithm, as described in Ref. [86]. The algorithm uses background images ($L$) to build an orthogonal basis set describing the fringes. An inner product of the basis set with the OD image determines the basis set expansion describing the fringes, enabling their direct subtraction from the image.
Figure 3.21 Current stabilization diagram. For the cart-mounted quadrupole coils, current is provided by a 21 V 240 A supply (Agilent 6682A), and two high-power FETs (APT10M07JVFR) are used in parallel configuration. For the QUIC trap, the power supply is capable of 875 A at 5 V (Agilent 6680), and 8 FETs are used in parallel. All FETs are mounted on a cold plate with water cooling.

3.12 Control

The experiment is controlled through an FPGA and two 8-channel National Instruments boards (NI 6733). The FPGA control software is written in-house [72] and uses typeset scripts which it compiles into a series of commands sent to the FPGA and the analog boards. The ±10 V outputs of the analog boards are used as set-point voltages for the servos controlling magnetic coil currents and light intensities. Analog signals which do not require the precision of the analog boards (shim coil set-points) are provided by a 28-channel level-setting DAC controlled by the FPGA. The FPGA is also responsible for all digital signalling, as well as passing commands to the direct digital synthesizers (DDSs) used in frequency generation.

3.12.1 Current stabilization

Electric currents running through magnetic coils are actively stabilized and controlled through standard PI loop servos (Fig. 3.21). The current paths are designed such that the last component before the power supply “−” is a field-effect transistor (FET). The servos operate by controlling the FET gate voltage to match the current readout to a given reference. Large currents (e.g., cart coils and magnetic trap) are read out by Danfysik Ultrastab 867 probes, while smaller currents (e.g., shim, magnetic impulse, anti-gravity, and imaging coils) are read out by F.W. Bell Hall probes. Time evolution of the magnetic
current is controlled by adjusting the reference voltage using a DAC board. Current profiles with sinusoid, power law, and exponential profiles can be easily generated. In all cases, the supply voltage is adjusted such that about 1 V is dropped across the FETs. This ensures enough dynamic range for servo operation, while keeping dissipated power relatively low. To maintain this voltage drop for high current applications, the voltage is adjusted in real time through a floating voltage input port in the Agilent supplies.

### 3.12.2 Light intensity stabilization

Similarly to current, light intensities are stabilized using a PI loop (Fig. 3.22). The system is actuated through a variable gain amplifier (VGA, part number ADL 5330) which amplifies an RF signal (generated by a DDS, part number AD 9959) according to a reference voltage provided by the DAC. The signal is further amplified by 30 dB by a high power amplifier and sent into an acousto-optic modulator which controls the light intensity. The servo compares a computer-controlled reference to a photodiode signal from the stabilized beam, allowing the implementation of various functional shapes for the light intensity. When fiber coupling is involved, as is the case for the lattice beams, input light for the photodiode is picked off after the fiber output so that the servo can correct for short term drifts in fiber coupling efficiency. Such drifts are often caused by the light heating the front facet of the fiber input or by a drift of the polarization. The latter problem is easily resolved by using a polarization-maintaining fiber and ensuring that the laser polarization is well aligned with the axis of the fiber.
Chapter 4

Anderson localization

4.1 Introduction

Anderson localization (AL) is an interference phenomenon in which waves fail to propagate in a disordered medium [1, 2, 4, 5, 87]. Historically, AL has been associated with condensed matter physics and is especially important to technological applications because of the ubiquity of impurities and defects in materials [1]. However, AL is a more general phenomenon that applies to both classical and quantum waves. While it has been observed in many systems, experimental confirmation of some of the basic phenomenology of AL is still lacking. In this chapter, we detail an ultracold atom measurement which attempts to close some of the long-standing gaps in the observation of 3D AL. These results have been published in Ref. [6] and have been followed by similar work published in Ref. [88].

AL has been observed in a variety of physical systems. Manifestations of interference-induced localization have been demonstrated for light [89–91], ultrasound [92], water gravity waves [93], and in 1D [94, 95], and in momentum space [96] for ultracold atom gases. We observe 3D AL of an ultracold spin-polarized atomic Fermi gas in a disordered potential created using optical speckle. While in 1D and 2D particles can be localized no matter how weak the disorder, AL is not an inevitable consequence in 3D [97]. The requirement that a sufficient disorder strength must be attained to achieve AL is a distinctive feature of 3D compared with lower dimensions. A threshold in the energy spectrum (i.e., a mobility edge) separates lower-energy localized states from higher-energy extended states. As our experiments are performed with a thermal ensemble of atoms, the mobility edge manifests itself as an energy cut-off separating the localized from the mobile atoms in the ensemble. The localized and mobile populations are clearly distinguishable from images of the atom gas, enabling us to infer the mobility edge based on minimal assumptions. We use this advantage to map out the disorder energy dependence of mobility edge for the first time.

The fact that the atoms are spin-polarized—and thus strictly non-interacting—means that the dynamics of motion is only guided by AL, allowing us to observe the localized state during its formation, providing unprecedented insight into the dynamics of localization. This process, which has not been previously studied or observed, is directly contrasted to classical diffusion.
This study of AL provides an excellent starting point for investigations more directly linked to the original condensed matter problem. Some relevant work is described in Chapter 5, studying how interactions affect AL for lattice fermions. In the future, the unique control possible over ultra-cold disordered gases may enable measurements to shed new light on other aspects of localization that are not well understood or masked by inter-particle interactions or dissipation (see Ref. [5] for a review).

We observe three-dimensional Anderson localization of non-interacting ultracold matter by allowing a spin-polarized atomic Fermi gas to expand into a disordered potential (Fig. 4.2). Localization is characterized by the emergence of a two-component density distribution consisting of a mobile component and a localized component that does not diffuse. The behavior of the localized component is qualitatively consistent with several features of 3D Anderson localization. It is shown to be incompatible with simple trapping and classical diffusion, implying a strong deviation from classical physics, and implicating interference as the cause of localization. As predicted for 3D Anderson localization, we demonstrate that a mobility edge exists that increases with the disorder strength. For the localized gas, we measure a characteristic length $\xi$ which can be interpreted as a thermal average of localization lengths of single particle states lying below the mobility edge. $\xi$ is observed to decrease with disorder strength and increase with particle energy, as expected from weak scattering predictions [98].

Unlike in previous work on 1D AL using ultracold atoms, we work in 3D and simultaneously employ non-interacting particles and a disordered potential with finite-range correlations. In Ref. [94], optical speckle was used to create disorder, but collisions between particles affected the short-time dynamics [99]. In our experiment, inter-particle interactions are eliminated by operating at temperatures far below the 150 $\mu$K threshold for $p$-wave collisions between spin-polarized $^{40}K$ atoms [50]. In Ref. [95], a Feshbach resonance was used to remove inter-particle interactions, but the potential consisted of a quasi-periodic lattice with perfect, long-range correlations that can affect the localization transition [100, 101].

The disordered potential in our work is characterized by an approximately Gaussian autocorrelation function with $\zeta_z = 270$ nm and $\zeta_x = 1600$ nm root mean square (RMS) radii transverse to and along the direction of the speckle field propagation. Since the size of the localized gas is much larger than the autocorrelation radii in each direction, the atoms probe the full statistical properties of the speckle potential. Thus their motion cannot be expected to depend on some local detail of the random potential, but rather on the general properties of speckle.

### 4.2 Anderson localization physics

AL is a non-interacting wave-interference phenomenon. It was originally described as the inability of a particle to scatter out of an initial state due to a disordered potential cou-
pling it to other states [4]. This analytical derivation is exact and is a general theorem of scattering applicable to both real and momentum space. An observation of AL in momentum space is described in Ref. [96]. Over years of research, various theoretical approaches have contributed to the understanding of wave propagation through disorder. In the weak scattering approximation, quantum corrections to classical diffusion have been calculated using diagrammatic techniques [102]. While adequate in describing weak localization, this approach fails in the limit of strong scattering and predictions for localization lengths and the mobility edge for AL are to be taken with caution. Unsurprisingly, our observations contradict some weak scattering predictions.

Non-perturbative approaches have also been developed to study localization. Weak localization and coherent backscattering lend themselves to analysis in analogy with random walks [103], which reveal the critical dimension in the problem. Rigorous arguments on dimensionality effects are also obtained from scaling theory [97]. Furthermore, numerical work exists, which does not rely on approximations. However, such work typically employs the Anderson model—a disordered lattice model—which is not directly applicable to the speckle disorder case we investigate.

4.2.1 Coherent scattering

Disorder affects wave propagation due to self-interference. This mechanism becomes apparent in the path integral picture, in which a particle will take every available trajectory during its coherent evolution. In this approach, the amplitudes of less likely trajectories will cancel out due to destructive interference. Conversely, constructive interference will amplify certain possibilities. In random scattering, closed paths will receive such amplification: for every closed trajectory, there exists another one which follows the same path with opposite direction (Fig. 4.1). As these are simply time-reversed paths, they share the same phase and add constructively. The coherent sum of the corresponding amplitudes $A$ results in doubling of the likelihood $I$ for such trajectories over the incoherent sum:

\[
I_{\text{coh}} = |A + A|^2 = 4 |A|^2,
\]

\[
I_{\text{incoh}} = |A|^2 + |A|^2 = 2 |A|^2.
\]

Such closed trajectories contribute to the possibility that a particle remains stationary. In a disordered crystal, this mechanism, referred to as weak localization, slows down electron propagation and reduces the electric conductivity [102]. Particles become Anderson localized and transport halts completely in the limit of strong scattering, the conditions for which are discussed in the following subsection.
Figure 4.1 Two time-reversed trajectories (blue and red) exist for every closed path that the particle (black circle) can take. The disorder is represented by random scattering centers (green circles). The amplitudes of the trajectories add constructively, resulting in suppressed motion.

4.2.2 Ioffe-Regel criterion

Given the properties of the speckle potential and the temperature range we explore, AL is achievable for many particles in the gas. While in 1D and 2D particles are Anderson localized for any finite disorder in an extended system [97], AL in 3D is conditional on the Ioffe-Regel criterion, which requires that the quantum wavelength exceeds the Boltzmann mean free path $l_B$ [98]. $l_B$ is defined as the average distance required for a particle to completely erase the memory of the initial direction of propagation. While there is no easy way to estimate $l_B$ in the strong scattering limit, it is reasonable to assume that a lower limit is set by the speckle autocorrelation length: $l_B \approx \left(\frac{\zeta_x^2}{\zeta_z}\right)^{1/3}$ (see Section 3.9 for speckle characterization). Because we use a thermal ensemble of particles, the gas contains a range of particle energies and wavevectors. Using $2\pi/\Lambda_{dB}$ (where $\Lambda_{dB} = h/\sqrt{2\pi M k_B T}$ is the thermal deBroglie wavelength of a particle with mass $M$) as a characteristic wavevector, the Ioffe-Regel criterion corresponds to $T \approx 300$ nK. Because a spread of particle wavelengths is present in the gas and since the Ioffe-Regel criterion is not a precise constraint, localization is possible even for temperatures somewhat above this limit.

4.2.3 Mobility edge and dimensionality

Dimensionality ($D$) affects the phenomenology of AL. In 1 and 2D all particle states are localized for infinitesimal disorder in an extended system [1]. This is not the case in 3D where states with kinetic energy above a cut-off value (the mobility edge) are free to propagate.
Although it has not been measured, the mobility edge is known to exist based on arguments from scaling theory [97]. Insight into why 3D is unique can be gained by considering the related problem of random walks, which can be used to model a particle scattering from the random potential [103]. It is known that for $D > 2$, a random walk has a finite likelihood of never returning to the origin [104, 105]. This makes $D = 2$ the critical dimension, as is the case for AL [97]. In this simplified model of AL, the dimensionality of the walk is a crucial factor in the likelihood of a particle halting its motion.

The mobility edge is an important concept in the understanding of disorder physics. Its presence suggests that disorder can drive a metal-to-insulator transition in gapped systems: for particles occupying the ground band of a lattice, an insulating state will be produced when the mobility edge lies in the band-gap. Alternatively, at $T = 0$ an insulating state will arise when the mobility edge exceeds the Fermi energy irrespective of the existence of a gap. These considerations have attracted interest in the mobility edge dependence on disorder strength, and theoretical predictions exist for the cases of binary and bounded disorder within the Anderson model [106–109] and for speckle disorder in the weak scattering limit [99]. The critical behavior of states at the mobility edge has been studied as well [110, 111]. Unfortunately none of the calculations apply directly to the measurement we describe: the Anderson model is a lattice model, and the weak scattering assumption breaks down for strong localization.

### 4.3 Measurement overview

For measurements of AL, the atoms are cooled in a dipole trap to 170–1500 nK; the gas is spin polarized after cooling to eliminate interactions between atoms. Quantum statistics do not play a significant role in the measurements discussed here since the lowest temperature we sample corresponds to roughly one-half of the Fermi temperature. As shown schematically in Fig. 4.2a, a disordered potential generated using a single, focused 532 nm optical speckle beam (as in Ref. [33]) is slowly turned on over 200 ms while the atoms are trapped in the OT. The atoms experience a repulsive potential proportional to the speckle intensity. The speckle beam propagates in the vertical $z$ direction, while we refer to the transverse directions (i.e., in the focal plane of the lens) as $x$ and $y$. The disorder strength is characterized by the average potential height $\Delta$ which is the potential energy in the middle of the speckle beam, averaged over a volume larger than the correlation volume $\zeta_{xy}^2 \zeta_z$.

By adjusting the 532 nm laser intensity, $\Delta$ can be continuously varied in the range 0 – 1000 $k_B\cdot nK$ ($k_B$ is Boltzmann’s constant) within a 10% systematic uncertainty. After the speckle field is turned on, the optical trap is suddenly switched off, and the gas is allowed to expand in the disordered potential while supported against gravity by a magnetic field gradient.
Figure 4.2 A Schematic of the experimental set-up, showing an ultracold gas expanding into an optical speckle field (green) and separating into localized (blue) and mobile (red) components. The disorder beam propagates in the vertical ($z$) direction, along which the speckle autocorrelation is elongated. The envelope of the speckle field—a Gaussian profile with a 170 µm waist along $x$ and $y$ and characterized by a 400 µm Rayleigh length along $z$—is several times larger than the dimensions of the localized component. B–D Slices along $x$ (B) and $z$ (C) through the center of a typical spatial distribution (D) of a localized gas. The image in D shows a 480 µK gas that has expanded for 20 ms through the disordered potential with $\Delta = 240 \, k_B \cdot nK$. The filled curves show independent fits to the mobile (red) and localized (blue) components.
4.3.1 Speckle properties

In a 3D speckle field created using a single laser beam passing through a holographic diffuser, there are no local intensity minima that can trap and classically localize particles. Instead, there are 1D regions of strictly zero intensity (i.e., optical vortices [31]) which permeate the speckle. Optical vortices change their direction on the correlation length scale, providing unobstructed 3D paths throughout the speckle field. While some vortices can form rings that can trap particles in a finite volume and therefore lead to a percolation threshold, earlier calculations [112] and our simulations show that the threshold is so low in energy such that less than 0.02% of the particles are trapped for all of the data presented here. This property allows us to interpret absence of propagation as strictly arising from wave interference. In contrast, using speckle to create disorder in 1D [113, 114] and 2D [115–119] necessarily leads to a percolation threshold. In 1D, the threshold for unbounded disorder is, strictly speaking, infinite, although in practice speckle beams have a finite size, and the speckle height distribution has a hard cut-off corresponding to the total beam power brought to a focus. In 2D, the system is not as pathological and the percolation threshold is approximately $\Delta/2$ [115]. When such a significant threshold is present, special care must be taken to distinguish AL effects from classical trapping.

Speckle can also be created using two intersecting laser beams polarized along the same direction (perpendicular to the plane spanned by the beams) [88]. With this technique, the autocorrelation properties of the field can be tuned over a wider range. However, the scheme poses strict requirement on the mechanical rigidity of the optics (which need interferometric stability) and on the quality of light polarization. The second constraint is particularly important, as light intensity with in-plane polarization will create an additional speckle field not interfering with the field with out-of-plane polarization. The resulting intensity landscape is the scalar sum of the two different speckle fields and the disorder potential will possess a percolation threshold which depends on the relative intensities.

4.4 Expansion measurement

The effects of AL on transport are studied by releasing the atomic gas in the disordered potential and allowing it to freely evolve. Absorption images at different hold times reveal the evolution of the density distribution. As observed for the typical image shown in Fig. 4.2d, a two-component profile emerges for intermediate disorder energies. The mobile component has a profile similar to that of a thermal gas, expanding at a faster rate than thermal velocity. In stark contrast, the stationary localized component only undergoes expansion for the first 20 – 25 ms before motion halts completely and the atoms remain locked in a static spatial profile for hundreds of milliseconds. Data is analyzed through numerical fits to the optical depth (OD) image. The fit parameters reveal relevant observables such as
atom number in each component and location and size of the spatial distribution.

4.4.1 Heuristic fits

Images of the gas after expansion in the speckle potential are analyzed by first fitting only the mobile component to a Gaussian profile. This is accomplished by excluding the region containing the localized component using a rectangular mask. The size of the mask is approximately twice the size of the localized component along \( x \) and runs across the entire image in \( z \). Changes in the width of the mask on the order of 50\% do not significantly affect the fitting parameters and the measured localized fraction. The mobile component is removed from the image by subtracting the Gaussian fit, and the residual localized component is fit to a heuristic model. The model function \( OD(x, z) \) describing the optical depth is

\[
OD(x, z) = \Omega e^{-\frac{(x-x_c)^2}{2\sigma_x^2}-\frac{|z-z_c|}{\xi_z}}. \tag{4.2}
\]

In the \( z \)-direction, \( OD(x, z) \) is exponential, reflecting the expected shape for a localized wavefunction. The functional form along the \( x \)-direction describes the in-trap distribution of the gas which remains unchanged after release from the trap. The centers of the fits \( x_c \) and \( z_c \) to the mobile and localized components are treated as independent free parameters. By integrating the fitted column density in each component we extract the fraction of localized atoms \( f \).

Strongly localized single particles are known to generally possess exponential density profiles [1] with localization lengths \( \xi \) that depend on the particle energy, \( \Delta \), and the microscopic disorder parameters [98]. However, a theoretical distribution applicable to our experiment (accounting for a thermal average over particle energies and localization lengths) is unresolved. Nevertheless, due to the good agreement between the observed profile and the exponential fit, it is reasonable to assume that \( \xi_z \) is a thermally averaged localization length. This implies that the localized profile represents a sum of single-particle states centered at the trap position and commensurate in size with the observed final distribution. This interpretation is justified experimentally by our team in work beyond the scope of this thesis [49].

4.4.2 Observing localization

The dynamics of propagation is revealed by the evolution of the localization length \( \xi_z \). As shown in Fig. 4.3, the size of the localized component becomes fixed after it rapidly expands along \( z \) for the first 25 ms after release from the optical trap. The transverse size is approximately constant at the in-trap size. While data is shown out to 140 ms in Fig. 4.3C, we have checked that the localized component does not expand over 1000 ms. The apparent lack of diffusion cannot be explained classically due to the practical absence of a percolation threshold in the disorder potential.
Figure 4.3  Dynamics of the localized component of a 390 nK gas released into the speckle potential with $\Delta = 600 \ k_B \cdot \text{nK}$. (A,B) Slices along $x$ (A) and $z$ (B) through an OD image taken before release from the trap (black) and after the gas has expanded for 40 ms (red) and 140 ms (blue) duration (i.e., hold time). The scale bar indicates 200 $\mu$m. The optical depth for the in-trap image was reduced by a factor of 15 by detuning the imaging laser 12 MHz from the atomic transition. The decrease in optical depth between 40 and 140 ms is due to atoms slowly leaving the localized component. (C) The thermally averaged localization length $\xi_z$ (red circles) and RMS size $\sigma_x$ (black squares) of the localized component for variable hold time in the speckle potential. Each point is determined from an average of 6 experimental shots; the error bars (not visible for every point) in all figures represent the standard error. To compare these data to the results of a numerical simulation of classical motion (solid lines), the simulated RMS radius along $z$ is converted to a localization length by assuming an exponential profile.
The expansion of the localized component is inconsistent with classical dynamics. We numerically simulated classical trajectories in the 3D speckle potential used for the data in Fig. 4.3C for a range of particle energies, averaging over randomly sampled initial positions; the computed dynamics are diffusive along all directions. To simulate the expansion of the gas, we average the diffusion constants over a thermal ensemble of energies consistent with the initial momentum distribution. The simulated RMS radius along $z$ is converted to a localization length for comparison to the data by assuming an exponential profile. The motion in $x$ and $y$ reaches the asymptotic regime within 10 $\mu$m in $z$. Therefore, for the localization lengths we measure, localization solely in $z$ cannot explain the absence of diffusion in the transverse directions given the simulated diffusion rates.

It would be instructive to calculate quantum corrections to the diffusion rate in the spirit of weak localization [98, 120, 121]. However, such corrections still result in diffusive behavior, which is qualitatively different from our observations.

As evident in Fig. 4.3C, the simulated size after expansion is inconsistent with the observed dynamics of the localized component. We interpret the localized component as being comprised of particles with low enough energy to be Anderson localized; atoms with higher energy constitute the mobile component. In 3D, localization arises via self-interfering trajectories formed by multiple scattering events (primarily at shallow angles) from the disordered potential. Whether or not this complex interference results in localization depends on the particle energy compared with the mobility edge $E_c$. In contrast to the effective mobility edge observed in 1D [94] that results from correlations in the disordered potential [99, 122, 123], $E_c$ strongly depends on $\Delta$.

For energies below the mobility edge, particle wavefunctions are localized and decay exponentially with a localization length $\xi$. Wavefunctions with an overall exponential envelope are characteristic of strong localization [2] and have been observed for cold atoms in 1D [94, 95] and in computer simulations [124]. As evident in Fig. 4.3D, the spatial distribution we observe is consistent with a thermal average of such wavefunctions with localization lengths distributed such that the overall profile is approximately exponential along the $z$-direction. The single-particle $\xi$ depends on the energy $E$, $\Delta$, and the microscopic nature of the disorder, and is known to diverge when $E$ is close to $E_c$ [98, 110]. Along $x$, the density profile—which remains unchanged from the Gaussian in-trap profile—is consistent with transverse localization lengths much smaller than the initial size of the gas.

### 4.4.3 Number decay from localized component

A decay of atom number from the localized component becomes evident in Fig. 4.3A,B, occurring in a way that leaves the shape of the spatial distribution unchanged. The number of atoms $F$ remaining in the localized population as a fraction of the total initial population is plotted in Fig. 4.4. An exponential fit of the form $F = F_0 + F_i e^{-t/\bar{t}}$ reveals a time constant
\( \bar{t} = 39 \pm 4 \) ms and a non-zero offset \( F_0 = 0.19 \pm 0.02 \). This decay is not yet understood. It may in part arise from the atoms sampling lower intensity regions of the speckle field where the mobility edge decreases.

## 4.5 Mobility edge

The bimodal nature of the density distribution is direct evidence that a mobility edge exists. In further support of this picture—that a kinetic energy cut-off \( E_c \) separates localized from mobile states—we are able to collapse the full range of expansion data (Fig. 4.5) onto a single curve describing the mobility edge.

The mobility edge is calculated from the fraction of localized atoms. The number of atoms in the localized and mobile components is determined from a fit (4.2) to the spatial distribution of the atoms after a 20 ms hold in the disorder: at this time the localized profile has been fully formed, while the quickly expanding mobile gas still has sufficient OD to yield a good numerical fit. The localized fraction \( f \) is plotted in Fig. 4.6A. For \( T = 240 \) nK and \( \Delta > 550 \) \( k_B \)-nK, the error bars extend to \( f = 1 \), signifying that no mobile atoms are distinguishable within the sensitivity of the experiment. For all temperatures, \( f \) monotonically increases with \( \Delta \), consistent with a mobility edge shifting to higher energies in the kinetic energy distribution. Conversely, for fixed \( \Delta \), \( f \) decreases with \( T \) as more particles are thermally excited above \( E_c \). The mobility edge is determined from each point in Fig. 4.6A by calculating the momentum cut-off \( \sqrt{2ME_c} \) required to achieve the measured fraction of localized particles for a 3D spherically symmetric Gaussian momentum distribution:

\[
f = \frac{\pi}{2 (Mk_BT)^3} \int_0^{\sqrt{2ME_c}} dp \, p^2 e^{-p^2/(2Mk_BT)}.
\]

This expression contains an approximation: in assuming the equivalence between an energy and a momentum cut-off, we neglect the finite width of the spectral function [120, 125, 126].

\( E_c \) calculated according to Eq. (4.3) is shown in Fig. 4.6B. For a given \( \Delta \), there are four values of \( E_c \): one for each temperature. Each point on the graph represents the average \( E_c \), and the error bars span the range of values. A power-law fit through the data in Fig. 4.6B yields \( E_c \propto \Delta^{0.59 \pm 0.02} \). This result compares poorly to the prediction that \( E_c \propto \Delta^2 \) based on the self-consistent Born approximation [120] and weak scattering theory [98]. The disagreement is likely due to the failure of these theories in the regime of strong localization. To our knowledge, no predictions exist for \( E_c \) in the strong scattering regime.

### 4.5.1 Kinetic energy distribution

In calculating \( E_c \) we have assumed that the speckle field does not significantly affect the kinetic energy distribution. This assumption is verified experimentally by measuring the
Figure 4.4  Number loss from localized component. The fraction $F$ is the ratio between the atoms in the localized component and the total initial atom number.

Figure 4.5  Images for the complete data set (temperature ranging 200 – 1500 nK and disorder ranging 0 – 1000 $k_B \cdot nK$). Each image is the average of a minimum of 5 experimental shots. Fringes that arise from technical noise are removed using standard image processing techniques; this procedure is not applied to images used for determining the properties of the localized component.
Figure 4.6  (A) The fraction of atoms $f$ in the localized component is measured after 20 ms of expansion in the disordered potential for varying $\Delta$ and $T = 240 \pm 20$ nK (blue circles), $480 \pm 20$ nK (green squares), $1130 \pm 60$ nK (orange triangles), and $1470 \pm 230$ nK (red diamonds). Each point is determined from fits to 5 images, and the error bars give the standard error. (B) From the data in (A), the mobility edge $E_c$ is determined at each $\Delta$. Each point is a weighted average, accounting for the uncertainty in $T$ and localized fraction. The error bars represent the range of $E_c$. The dashed line is a power law fit.
(line-of-sight integrated) momentum distribution $n(p)$ of the gas after suddenly releasing it from the combined trap and disorder potential. The kinetic energy of a particle in the trap is given by $\frac{p^2}{2M}$, where $p$ is the momentum of the particle after the release. We find that whether disorder is present or not, $n(p)$ remains Gaussian to an excellent approximation as can be expected for a Boltzmann thermal distribution. Furthermore, the size of the Gaussian fit is unaffected (Fig. 4.7), implying that disorder does not change the kinetic energy distribution. This might be somewhat surprising in light of the increase in potential energy that the atoms must experience due to the atomic wavefunctions probing the bright regions in the speckle field. However, a simple argument based on the virial theorem supports our findings.

4.5.2 Virial theorem

The virial theorem holds for a collection of particles with bounded position and momentum vectors: a condition satisfied by the atoms in the trap. For each particle, the virial theorem states that the kinetic energy $E_K$ satisfies [127]

$$\langle E_K \rangle = -\frac{1}{2} \langle \nabla V(r) \cdot r \rangle,$$

(4.4)

where $V(r)$ is the potential and the angular brackets indicate time averaging. Separating the potential into a trap ($V_t = \frac{1}{2} M \omega^2 r^2$) and a speckle ($V_s$) component results in

$$\langle E_K \rangle = -\frac{1}{2\tau} \left[ \int_0^\tau d\tau \nabla \left( \frac{1}{2} M \omega^2 r^2 \right) \cdot r + \int_0^\tau d\tau \nabla V_s \cdot r \right].$$

(4.5)

Although it awaits rigorous proof, the second term in the brackets can be expected to vanish for fine enough disorder, based on the isotropic properties of speckle (see Fig. 4.8). Consequently, the virial theorem reduces to the well-known form for a central $r^2$ potential:

$$\langle E_K \rangle = \langle V_t \rangle.$$  

(4.6)

Assuming that the system becomes ergodic after a long enough time $\tau$, the time average for a single particle can be understood as an expectation value for the ensemble average.

Eq. (4.6) tells us that if we compare a particle in the clean parabolic trap to a particle having the same classical turning points in the disorder trap, their time-average kinetic energies will be equal. Therefore, if the disorder does not change the classical turning points of particles in the ensemble, it will also not affect the kinetic energy distribution, irrespective of the disorder potential height.
Figure 4.7  Effect of disorder on kinetic energy distribution. Gas size is given by the RMS radius of a Gaussian fit to $n(r)$ in expansion time. Measurement is performed with a spin-polarized gas of approximately $10^5$ atoms.

Figure 4.8  Schematic of combined confinement and speckle disorder potential. The contributions to the virial from the disorder (underlined term in Eq. (4.5)) at points A and B will mutually cancel out.
4.6 Localization lengths

The dependence of the measured localization length on the disorder strength and particle energy (controlled by adjusting the temperature) is shown in Fig. 4.9. While data are only shown for a limited range of $\Delta$ and $T$, the observed qualitative behavior—that $\xi_z$ monotonically increases with $T$ and decreases with $\Delta$—is characteristic of the entire range of parameters explored here. These trends are consistent with a weak-scattering picture, in which particle trajectories are deflected by independent scattering events during propagation through the disordered potential. As the energy increases, the particle must propagate further for the momentum to be deflected enough to form a self-interfering localized path (Fig. 4.10). Since the average energy is proportional to $T$, $\xi$ can be expected to increase with temperature. For increasing disorder strength, scattering occurs more frequently as the particle propagates. The localization length shrinks since the momentum can be significantly altered over a shorter length scale. The general monotonic trends are consistent with a weak-scattering picture, in which the localization length is controlled by $l_B \propto \sqrt{E/\Delta^2}$ [98] at low energy.

A disparity between localization lengths for the $x$ and $z$ directions can be foreseen from the weak-scattering theory results for $\xi$ [98]. For a given wavevector $k$,

$$\xi = \frac{3l_B/2}{\frac{3}{\pi} - k^2l_B^2}. \quad (4.7)$$

For strong disorder and correspondingly small values of $l_B$, Eq. (4.7) reduces to $\xi = \frac{\pi}{2}l_B$.

In the Boltzmann approximation for the transport of matter waves [98], $l_B$ can be written as

$$\frac{1}{kl_B} \propto \Theta(k\zeta - 1) \left[ \frac{1}{(k\zeta)^5} + \frac{1}{(k\zeta)^2} \right]. \quad (4.8)$$

The Heaviside step function $\Theta(k\zeta - 1)$ captures the Ioffe-Regel criterion for the localization length $\xi$ to possess a finite value. When this condition is satisfied (as in AL), Eq. (4.8) becomes

$$kl_B \propto \frac{(k\zeta)^5}{1 + (k\zeta)^3}. \quad (4.9)$$

From the asymptotics of Eqs. (4.7) and (4.9), we obtain $\xi \propto \zeta^5$ for small values of $\zeta$ and $\xi \propto \zeta^2$ for large $\zeta$. Thus, in the weak scattering limit, the ratio between the localization lengths $\xi_z/\xi_x$ due to anisotropic autocorrelation function can be expected to be at least $(\zeta_z/\zeta_x)^2 \approx 36$.

4.7 Conclusion and outlook

Our measurement is the most direct observation of 3D AL to date, and the first to provide estimates for the mobility edge and localization lengths. It also the first measurement to
Figure 4.9 Localization length for fixed disorder height $\Delta = 480 \ k_B \cdot nK$ (black circles) as a function of temperature $T$, and as at fixed $T = 480 \ nK$ (green squares) as a function of disorder disorder $\Delta$. The green and black points are an average of 5 and 10 experimental realizations, respectively. The error bars in $\xi$ represent standard error. The error bars in $T$ are from the uncertainty in the thermal expansion velocity.

Figure 4.10 Simple model for effect of momentum on localization length. On the average, a particle (black circle) with larger momentum (red arrows) changes its direction of motion less than a particle with smaller momentum (blue arrows) for the same number of elastic scattering events. Vector changes in momentum due to scattering are shown in green.
allow a glimpse into the dynamic formation of the localized state. We find a disagreement with predictions for the mobility edge from weak scattering theory, calling into question the applicability of these theories to the AL regime and the universal nature of their conclusions. Meanwhile, some questions about our experiment remain unanswered, such as the origin of the uniform loss of atomic density from the localized component and the role of the finite extent of the disorder potential. Other questions, such as the dependence of localization lengths on speckle size, are addressed in a follow up measurement described in Ref. [49]. The ultimate goal of our disorder work is to approach physics relevant to the condensed matter field, such the effect of disorder on transport and its interplay with interactions in lattice systems. A first study in this research direction is presented in the following chapter.
Chapter 5

Disordered lattice

5.1 Introduction

We study the effect of interactions and disorder on the transport properties of fermions in a lattice. The problem is of importance to modern materials science because it treats materials with strongly interacting electrons. Such materials often display superb transport properties (e.g., high-\( T_c \) superconductivity \[8\], giant thermoelectricity \[9, 10\], and magnetoresistance \[11\]) with great promise for technological applications. It is the calling of condensed matter physics and materials science to understand how these properties come about so they can be optimized and better materials can be designed. However, disordered strongly interacting systems are very difficult to understand. Their properties are defined by the interplay of strong correlations and Anderson localization, each of which is a difficult problem surrounded by open questions \[2, 8, 25\]. The effect of interaction on localization has been a topic of intense research since the discovery of AL, and contradicting predictions exist. The main question we address in this chapter is: do interactions enhance or inhibit localization and to what extent? We attack this question through an experimental study of the disordered Fermi-Hubbard model (DFHM). A synopsis of our results can be found in Ref. \[128\].

The DFHM is a minimal effective model describing strongly correlated electrons with disorder due to dopants, impurities, and defects \[7\]. The Hamiltonian \( \mathcal{H}_{\text{DFHM}} \) is:

\[
\mathcal{H}_{\text{DFHM}} = - \sum_{(ij), \sigma} t_{ij} c^\dagger_i c_j + \sum_i U_i n_i,↓ n_i,↑ + \sum_{i, \sigma} \epsilon_i n_i,\sigma,
\]

where \( t_{ij} \) is the tunneling energy between nearest-neighbor sites \( i \) and \( j \), \( U_i \) is the energy of interaction between atoms of different spin states residing on the same site, \( \epsilon_i \) is the occupation energy, and \( c_i \) and \( n_i \) are the fermionic destruction and number operators. Summation is performed over all sites \( i \), spin states \( \sigma = \{↑, ↓\} \), and nearest neighbors \( (ij) \). In the single-band model discussed here, the Pauli exclusion principle forbids multiple atoms of the same spin state to simultaneously occupy the same lattice site.

In itself, \( \mathcal{H}_{\text{DFHM}} \) contains physics of strong interactions and disorder, and reduces to simpler effective models in various limits. For \( U_i = 0 \), Eq. (5.1) reduces to the Anderson
model of single particles in a disordered lattice. When all energies are not site-dependent, Eq. (5.1) becomes simply the Fermi-Hubbard model (FHM) of strong interactions. In turn, the first term of the FHM gives band theory, and the second and third terms combined give the atomic limit.

A lot of research effort has been dedicated to the theoretical analysis of the DFHM. However, progress has been limited by the complexity of the problem, which combines the physics of AL—a hard problem in its own right—with the challenges of the FHM [5]. A major obstacle in studying the low-temperature phase space is the fermion minus-sign problem which requires that any many-body wavefunction be anti-symmetrized [129]. One example of the depth of these simple-to-write models is the debate of whether or not they describe high-$T_c$ superconductivity [27, 28]: a discussion entirely relevant to the disordered problem as well [25].

In addition to theoretical analysis, the DFHM also poses problems for experimental research. It is generally difficult to establish the relevance of effective models to a given condensed matter system because of the missing degrees of freedom. In solids there are phonons, screening, and complicated lattice geometries with numerous—sometimes degenerate—bands. Additionally, disorder in the solid is difficult to control and characterize. In our cold atom approach we offer many simplifications: we work in a single band of a simple cubic lattice, disorder is completely known and easy to control, and we measure quantities (e.g., site occupancy, quasimomentum distribution, and excitation spectra) inaccessible in condensed matter systems. Our confidence in this approach rests on extensive experience within our research group of working with disordered cold atom systems, as exemplified in Refs. [22, 33, 34, 38, 130, 131].

In the following, we will introduce the relevant lattice physics in order of increasing complexity. Insight from the related single-particle, uniform, and clean systems is found to reach surprisingly far in explaining the full interacting disordered trapped system. The atom density in the lattice is maintained by a confinement potential induced both by the Gaussian envelope of the lattice beams and the optical trap. The discrete translational symmetry of the lattice is broken, resulting in spatially variable filling. Despite this complication, it is possible to gain some intuitive understanding by first considering single particle physics in a uniform lattice. Some of the phenomenology can be expected to persist in the presence of interactions and confinement. Reassuringly, this method agrees quantitatively with dynamic mean-field theory (DMFT) results. On the other hand, the atom density is easily calculable in the opposite limit of strong interactions (i.e., the atomic limit).

### 5.2 Clean lattice physics

The properties of atoms in a clean uniform lattice are determined by their interaction with the periodic potential and with each other. Stationary single-particle states in the periodic
potential obey Bloch’s theorem, which leads to a dispersion relation different from the free-particle one. It is characterized by discontinuities at wavevectors corresponding to multiples of the inverse of the lattice spacing [132]. The discontinuities split the dispersion into bands separated by band-gaps.

This single-particle picture remains valid when many non-interacting particles populate the lattice. They fill the band according to Fermi-Dirac statistics. Metals and band insulators can be explained in this picture. Interactions can be added in a perturbative fashion (e.g., using the relaxation time approximation [132] or Fermi-liquid theory [133]) to explain finite conductivity in metals, thermoelectric effects, etc.

Perturbative methods break down at large interaction strengths, and models must be devised to treat kinetic and interaction potential energy on the same footing. The simplest such model is the Hubbard model. It successfully predicts Mott insulating and antiferromagnetic phases and is also proposed as a minimum model for high-$T_c$ superconductivity.

Compared with electronic solids, cold atom systems have the extra complication of confinement. Combined with sufficient atom number, confinement can lead to phase inhomogeneities in the atom gas, such as a Fermi-liquid coexisting with a Mott or a band insulator. Furthermore, localized states emerge at large distance from the trap center. Theoretical tools have been developed to address these complications (see Ref. [38] for a review) and facilitate the analysis of experimental data.

5.2.1 Band theory

A single particle in a periodic potential obeys Bloch’s theorem and has a wavefunction of the form [132]

$$\psi_q = u_q(x) e^{iqx},$$  \hspace{1cm} (5.2)

where $u(x)$ has the periodicity of the potential, and $q$ is a reciprocal lattice vector. Although the theorem is valid for any dimensionality, it is straightforward to initially consider the 1D case and then generalize to the separable 3D simple cubic problem. The quantity $hq$, called quasimomentum, is similar to momentum in free space: it denotes an eigenstate of the Hamiltonian and is associated with translational symmetry. Furthermore, total quasimomentum is conserved in elastic collisions. However, $hq$ is not equivalent to momentum because translational symmetry in the lattice is not continuous but discrete. The reciprocal lattice vector is defined modulo $2\pi/d$ because only values within that range correspond to distinct quasimomenta. In the range $q \in [-q_B; q_B]$, where $q_B = \frac{\pi}{d}$, the number of quasimomentum states per spin species is equal to the number of lattice sites [132, 134]1.

---

1The finite number of quasimomentum states is imposed by the finite size of the lattice and the finite phase space density $n(r, q)$. For a 3D lattice in the semi-classical approximation, $n(r, q) = \frac{1}{4\pi^3}$. Although $q$-states are finite in number, $q$ is a continuous variable because the boundary conditions for the atom in an optical lattice are free. In contrast, $q$ is made discrete in standard textbook approaches by imposing a
Figure 5.1 Band structure.

The dispersion relation $E(q)$ and the wavefunctions $\psi_q(x)$ are obtained directly from Schrödinger’s equation

$$\mathcal{H}\psi(x) = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x) = E\psi(x). \quad (5.3)$$

Here $V(x) = s\cos(2q_Bx)$ is the optical lattice potential with Bravais wavevector $2q_B$. Throughout this work we refer to optical potential heights in units of the lattice recoil energy $E_R = \hbar^2\pi^2/2md^2 = 390 \, k_B \cdot nK$, where $m$ is the atomic mass of $^{40}K$ and $d = \lambda l/2$ is the lattice spacing.

Using the Fourier expansion $\psi(x) = \sum_q C_q e^{iwx}$, (5.3) becomes

$$\frac{\hbar^2}{2M} \sum_q C_q^2 q^2 e^{-iwx} + s\cos(2q_Bx) \sum_q C_q e^{iwx} = E \sum_q C_q e^{iwx}$$

$$\frac{\hbar^2}{2M} C_q q^2 + \frac{s}{2} (C_{q-2q_B} + C_{q+2q_B}) = E C_q. \quad (5.4)$$

The last line presents a system of equations relating the wavefunction $\psi_q(x)$ to the eigenenergy $E(q)$. An approximate solution can be found numerically by only considering a few equidistant $q$-values and solving for $E$ and the coefficients $C_q$.

The resulting dispersion is displayed in Fig. 5.1. For a shallow lattice the dispersion resembles the free-particle case $E(q) = \hbar^2q^2/2M$, except for wavevectors matching the periodicity of the lattice $q = n\pi/d$, where $n$ is an integer. States with such wavevectors scatter strongly off the lattice potential which causes the dispersion to depart from the periodic boundary condition [132, 134].
free particle case. The dispersion relation is discontinuous and consists of bands separated by energy offsets at the edges, i.e., band-gaps. At intermediate and large lattice depths ($s \approx 4E_R$ and higher), the ground band is well-approximated by

$$E(q) = 2t \left[ 1 - \cos \left( \frac{\pi q}{q_B} \right) \right]. \quad (5.5)$$

The term bandwidth refers to difference between minimum and maximum energy in the ground band. In 1D, 2D, and 3D the bandwidth for the simple cubic lattice is hence $4t$, $8t$, and $12t$, respectively.

The band-gap can be estimated using Eq. (5.4). The states at the band edge have the same periodicity as the lattice (see Eq. (5.2)), and therefore possess Fourier content predominantly in the $C_{\pm qB}$ component. Neglecting the remaining Fourier coefficients reduces Eq. (5.4) to

$$\left( \frac{\hbar^2 q^2}{2M} - E \right) C_{qB} + UC_{-qB} = 0$$

$$\left( \frac{\hbar^2 q^2}{2M} - E \right) C_{-qB} + UC_{qB} = 0. \quad (5.6)$$

The allowed solutions are $E_{\pm} = \frac{\hbar^2 q^2}{2M} \pm s$, and therefore the band-gap is approximately twice the lattice depth.

The presence of the band-gap enables the existence of an insulating state in a system of non-interacting fermions. For a system of $N$ lattice sites, there are $Nq$-states in the ground band. Each one of those states can be occupied by at most one fermion of a certain spin species. Therefore if there are $N$ fermions with $|\uparrow\rangle$ and $N$ with $|\downarrow\rangle$ and if the temperature is much lower than the band-gap, the ground band will be filled (Fig. 5.2). The system will not exhibit center-of-mass (COM) motion in response to an external force, provided that the perturbation is not strong enough to drive particles into higher bands. Net transport is absent because for every particle with a given quasimomentum there is another particle with equal and opposite quasimomentum. $q$-states that can support COM motion only exist in the excited bands, which are separated by the band-gap. Population in the excited bands is exponentially suppressed by the temperature, leading to insulating properties.

### 5.2.2 Interactions as a perturbation

Interactions can be included into this picture in a perturbative fashion. An example is the Hartree-Fock approximation, which relies on the assumption that the interacting ground state is the same as the one of the non-interacting system, and the corrected energy is the expectation value in this state [135]. This approach provides an excellent approximation when interactions are small. Although electron – electron interactions are strong, per-
Figure 5.2 Conducting and insulating phases in the clean lattice can be characterized by particle occupancy in both real (right) and reciprocal (left) space. Blue and red represent two different spin species. Due to Pauli’s exclusion principle, two particles of the same species cannot occupy the same site unless one of them is in a higher excited band. Insulating properties in the non-interacting case arise when all states are occupied. At half filling, interactions enforce single occupancies with each particle contributing uniformly to all \( q \)-states.
Perturbative approaches are successful because screening and Coulomb blockade result in a mean free path four orders of magnitude larger than the lattice spacing for metals at room temperature [134]. It also applies to atoms in optical lattices where interactions can be adjusted.

Perturbative approximations can explain finite electrical resistivity, thermal conductivity, and thermoelectricity [132]. Excitations in the system can be understood through Fermi-liquid theory [134, 136] which states that interactions between particles result in an inertial reaction in the surrounding medium. This effect can be captured by treating the system as non-interacting particles with a different (renormalized) effective mass.

Such constructs fail for strong interactions as the assumption of periodicity of the potential—central to the validity of Bloch’s theorem—is undermined by scattering. Mean field and Hartree-Fock theories fail when fluctuations are high, interaction ranges are small, and at low spatial dimensions [137].

### 5.2.3 Hubbard model

The failure of mean field corrections to the non-interacting picture motivates the treatment of interaction energy on equal footing with kinetic energy in the Hamiltonian. The simplest approach is the Fermi Hubbard model which can be derived from the effective Hamiltonian for fermions in the ground band of a lattice interacting via the contact potential described in Eq. (2.31) [138]

$$\mathcal{H} = \sum_{\sigma} \int d^3x \hat{\psi}^\dagger_\sigma(x) \left[ -\frac{\hbar^2}{2M} \nabla^2 + V(x) \right] \hat{\psi}_\sigma(x) + \frac{4\pi\hbar^2a_s}{M} \int d^3x \hat{\psi}^\dagger_\uparrow(x) \hat{\psi}^\dagger_\downarrow(x) \hat{\psi}_\downarrow(x) \hat{\psi}_\uparrow(x).$$

(5.7)

Here $\hat{\psi}$ is the second-quantized fermionic operator, $\sigma = \{\uparrow, \downarrow\}$ is the spin index, and $a_s$ is the scattering length. For single atoms, the energy eigenstates are Bloch waves. A superposition of the Bloch states can be used to create the basis of Wannier functions $\omega_i(x) = \sum_k e^{ik \cdot x_i} \psi_k(x)$, where $i$ denotes the lattice site and $x_i$ is the site location. The Wannier functions are maximally localized states centered at individual lattice sites. The field operator can be expanded in the Wannier basis using the creation operator $\hat{c}$ as $\hat{\psi}(x) = \sum_i \hat{c} \omega(x - x_i)$, yielding the Hubbard Hamiltonian [139]

$$\mathcal{H}_{\text{FHM}} = -\sum_{\sigma,(ij)} t \left( \hat{c}_{\sigma,i}^\dagger \hat{c}_{\sigma,j} + \text{h.c.} \right) + U \sum_i \hat{c}_{\uparrow,i}^\dagger \hat{c}_{\uparrow,i} \hat{c}_{\downarrow,i} \hat{c}_{\downarrow,i}^\dagger$$

$$= -t \sum_{\sigma,(ij)} \hat{c}_{\sigma,i}^\dagger \hat{c}_{\sigma,j} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}. \quad (5.8)$$
The angular brackets denote a summation over nearest neighbors. The interaction \( U \) and tunneling \( t \) energies are given by

\[
t = \int dx \omega_i^*(x - x_i) \left[ -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + V(x) \right] \omega_j(x - x_j)
\]

\[
U = \frac{4\pi \hbar^2 a_s}{M} \int dx |\omega_i(x)|^4.
\]

The overlap integral in Eq. (5.9) involves only the nearest-neighbor sites \( i \) and \( j \), and is referred to as the tight-binding approximation for calculating band structure. On its own, the tunneling term in the Hubbard Hamiltonian is equivalent to band theory and represents the kinetic energy of the system: scattering off the lattice potential is completely included. Potential energy in the Hubbard model is only due to the contact interaction term \( U \).

Closed-form approximations can be obtained for \( U \) and \( t \) in the limit of a deep lattice. In this limit, the potential at each site can be approximated by a parabola with a Gaussian ground state, yielding

\[
U \approx \frac{4\pi \hbar^2 a_s}{M} \left( \frac{\pi}{2d^2} \sqrt{s E_R} \right)^{3/2}.
\]

\[
t \approx \frac{4E_R}{\sqrt{\pi}} \left( \frac{s}{E_R} \right)^{3/4} e^{-2\sqrt{s/E_R}}.
\]

Values of \( t \) and \( U \) and the ratio of \( U \) to the bandwidth are displayed in Fig. 5.3 as a function of lattice depth \( s \). In the transport and double occupancy measurements, we choose the interaction strength by adjusting \( s \) such that we obtain the desired value of \( U/12t \).

The FHM is a minimum model for strongly interacting electronic materials [25], and it is the simplest model which cannot be reduced to a single-particle theory [7]. Although simple to derive and write down, the FHM is notoriously difficult to solve, mainly because of the “sign problem” [129], which requires that a many-body eigenfunction of the Hamiltonian be antisymmetric with respect to exchanging any two particles. This limits exact diagonalization to a few particles, while larger systems have to be approached using approximations such as dynamical mean field theory [129], high-temperature expansions [141], and statistical methods (e.g., quantum Monte Carlo at half filling [142]).

Unlike Hartree-Fock and Fermi-liquid theories, which can handle interactions only in a perturbative manner [137], the FHM correctly predicts the Mott insulating (MI) state [25]: a hallmark of strong correlations. The MI has been observed in cold atom systems with fermions [19, 23]. The state occurs when the system is occupied by two spin species at half filling, and the interaction strength exceeds the bandwidth, i.e., \( U > 12t \). The energy penalty \( U \) for two atoms of different species sharing the same site causes double occupancy to be suppressed by the Boltzmann factor \( e^{-U/k_BT} \). At low enough temperature, the atoms
Figure 5.3  Above: interaction $U$ and tunneling $t$ energies. Below: interaction energy in units of bandwidth.
will be localized to single sites, which translates into a uniform distribution in reciprocal space. One can so imagine, as an extension of the band theory picture, that the ground band is uniformly filled by both species. Adding an extra particle to the system will necessarily lead to a double occupancy. This additional particle will then obey a similar dispersion relation as the ground band, however, this new “Mott band” will be offset by energy $U$. A gap opens between the bands as soon as $U$ exceeds the bandwidth $12t$, leading to insulating behavior.

The Hubbard model has been shown to include anti-ferromagnetism [26] and has been conjectured as a minimum model for high-$T_c$ superconductivity [27]. For both of these phenomena, however, the characteristic energy scales require lower temperature than what is currently accessible with cold atoms.

### 5.2.4 Confinement

The discussion so far pertains to uniform systems such as crystalline solids. A parabolic confinement potential is typically present in cold atom experiments, breaking the translational symmetry and causing variable filling across the lattice sites. However, insight from the discussion of uniform systems is still relevant, and the analogy between optical lattices and solids persists. The state of the cold atom system can be understood as a composition of coexisting phases, each of which resembles the phases in the uniform lattice [36, 38, 47]. Which phases are present in the trapped system is determined by a combination of the Hubbard tunneling and interaction energies $t$ and $U$, the number of atoms $N$, the harmonic confinement frequency $\omega$, and the lattice spacing $d$. This complication is addressed through the introduction of a universal phase diagram for the trapped Fermi Hubbard model (Fig. 5.4). On the diagram, the dimensionless interaction strength $U/t$ defines the abscissa, and the characteristic density $\tilde{\rho} = N \left( \frac{M\omega^2d^2}{12t} \right)^{3/2}$, also dimensionless, defines the ordinate. $\tilde{\rho}$ has a simple geometric interpretation as $\frac{4\pi}{3}N_0$ [85], where $N_0$ is the number of lattice sites within a potential height of half the bandwidth:

$$\frac{1}{2} M \omega^2 \zeta^2 = 6t$$

$$N_0 = \frac{4\pi}{3} \left( \frac{\zeta}{d} \right)^3 = \frac{4\pi}{3} \left( \frac{12t}{M \omega^2 d^2} \right)^{3/2}.$$  \hspace{1cm} (5.12)

The phase diagram is universal because the dimensionless variables $U/t$ and $\tilde{\rho}$ relate all four relevant energy scales: $U$, $t$, $\epsilon_F$, and the confinement energy $\frac{1}{2} M \omega^2 r^2$.

The non-magnetic phase diagram of the trapped strongly interacting lattice system (Fig. 5.4) is calculated in Ref. [36] using DMFT. Within the parameters shown, it consists of four distinct regions: band insulating “B”, Fermi-liquid “L”, MI core “Mc”, and MI shell “Ms”. With the exception of “L”, all regions are characterized by a coexistence of multiple phases:
Figure 5.4  Phase diagram of FHM in a combined lattice and parabolic confining potential (reproduced from Ref. [38]). “L” (blue region) denotes a Fermi-liquid conducting phase, “B” (red) marks the presence a band insulator, “Mc” (grey) and “Ms” (white) mark the presence of a MI core and shell, respectively. The special points on the diagram are discussed in the text. Inset: cartoon representation of the Mc region describing a MI core with half filling, surrounded by a Fermi-liquid shell.
a Fermi-liquid\(^2\) is always present at the edge of the gas where filling is low, and in the “Ms” region it is also present at the trap center. At any point within the “L” region, a system will have variable filling and, although it is characterized by the same phase throughout, density-related properties can be expected to vary in space. The complications caused by this non-uniformity must be considered in the analysis of experimental data.

Some special points and features of the diagram can also be reproduced—both qualitatively and quantitatively—using simple arguments from energetics. These arguments are based on the interplay between the relevant energy scales. Subsection 5.2.3 addresses the minimum interaction strength necessary to produce a MI: setting the \( U/t \) coordinate of the green point in Fig. 5.4 to 12. Also based on energetics, a functional form can be obtained for the curve (yellow) separating the “Mc” and “Ms” regions. When more and more particles are added to a system of given bandwidth and confinement, a double occupancy will eventually occur in the trap center. The first such double occupancy will signify the formation of a Fermi-liquid core.

The ordinate of the green point is then \( \tilde{\rho}_{\text{green}} \left( \frac{U}{t} = 12 \right) = 5.9 \), in approximate agreement with the graph.

Similar arguments can be made about the minimum \( \tilde{\rho} \) to produce a band insulator at the trap center in the absence of interactions (magenta point in Fig. 5.4). The condition is that all sites within the radius \( \bar{\zeta} \) such that \( \frac{1}{2} M \omega^2 \bar{\zeta}^2 = \frac{12}{t} \) are doubly occupied. Similarly to Eq. (5.13),

\[
N = \frac{8 \pi \bar{\zeta}^3}{3 d^3} = \frac{8 \pi}{6} \left( \frac{U}{t M \omega^2} \right)^{3/2},
\]

also in good agreement with the DMFT calculation.

### Localized states

The confinement potential introduces localized states which have no analog in the uniform system [22, 44]. These states are energetically constrained to regions of space (i.e., shells)

\(^2\)The authors of Ref. [36] generally refer to the interacting metallic phase as a Fermi-liquid, although, strictly speaking, the term only applies when \( U/t \) is small. In the strongly-correlated regime, the properties of this (possibly conducting) phase are not well-known.
some distance away from the trap center. Their potential energy exceeds the maximum allowed kinetic energy $12t$ and are therefore classically disallowed from the trap center. Such states are necessarily populated when a MI is present, since a minimum occupancy requirement for the MI is that the chemical potential $\mu$ lies above the bandwidth. Fig. 5.5 shows a schematic of a possible distribution of phases in the trap, along with a population of localized particles. Studies have shown that localized states affect transport $[143, 144]$, and therefore understanding their dynamics is crucial to understanding the transport properties of the MI in a trapped atomic system.

### 5.3 Disordered lattice physics

The most important disorder-related phenomenon that affects transport in lattices is AL $[1, 24]$. It is studied in Chapter 4 strictly as a single particle phenomenon in a random potential. AL had been extensively studied in the disordered lattice through the Anderson model $[2]$: a tight banding model with randomized nearest-neighbor hopping and on-site energies. There are few unsolved questions in the Anderson model as its analysis is simpler than the speckle case. Instead, research effort concentrates on the interacting disordered problem which applies to the transport physics of doped cuprates and transition metal oxides $[13, 25]$. Studies have determined that AL remains relevant, but there is disagreement to what extent interactions affect this single-particle mechanism $[5]$. The problem combines the difficulties of analyzing strong correlations and AL. Much effort has concentrated on
the effective disordered Fermi Hubbard model (DFHM) in an attempt to extract universal features of the interplay between interactions and disorder. In particular, computational studies have addressed the question whether interactions stabilize or suppress the disordered conducting phase: an issue that our measurements aim to resolve experimentally.

5.3.1 Anderson model

A state with insulating properties can emerge in the non-interacting system due to Anderson localization. When sufficient disorder is present to satisfy the Ioffe-Regel criterion, Anderson localized states will emerge in the system. Localized and extended states will be separated by a threshold energy called a mobility edge. For strong enough disorder, the mobility edge can lie above the Fermi energy and all states in the system will be localized, resulting in insulating behavior.

The lattice introduces new aspects to the disorder problem. Firstly, the periodicity of the potential dramatically alters the dispersion relation of particles, which now inhabit bands separated by band-gaps [132]. Consequently, when the mobility edge rises above the conduction band, a metal-to-insulator transition occurs. Additionally, the cubic symmetry of the lattice disallows tunneling in the diagonal direction. This can be expected to affect AL by constraining the possible interference paths to chains linked by nearest-neighbor tunneling. Another fundamental difference from speckle disorder comes from the possibility of classical trapping: a particle can find itself constrained to a region completely surrounded by energetically disallowed sites. This brings out a connection to the percolation problem [145]. Despite these complications, non-interacting particles on a disordered lattice are relatively well-understood, as they realize the Anderson model (AM) with a Hamiltonian

\[ \mathcal{H}_{AM} = \sum_{\langle ij \rangle, \sigma} t_{ij} c_{i, \sigma}^\dagger c_{j, \sigma} + \sum_i \epsilon_i n_i, \]  

(5.15)

where site-to-site variability in \( t_{ij} \) is usually neglected. The AM has been extensively studied and many features such as localization length [98, 124], mobility edge [106–109], density of states [146], and critical exponents [108, 110, 111] have been calculated. Different methods of analysis disagree over the behavior of the mobility edge away from the band center. While some studies claim an initial increase with energy [107, 147, 148], others find quadratic decrease [149], or a weak decrease along with a re-entry behavior in which extended states near the band edge exist for an intermediate disorder strength, surrounded by localized states at higher and lower disorder [106, 109, 150–152]. Ref. [2] provides a review of pre-1990 theoretical and experimental work.
5.3.2 Disordered interacting physics

On their own, interactions and disorder have a similar effect of suppressing transport in the lattice: strong interactions can cause Mott insulating behavior, while a non-interacting particle may become Anderson localized. The result of combining disorder and interactions leads to physics beyond the Anderson model [24, 35], and has been an heavily researched topic since the discovery of AL. This issue is generally relevant to solids since Coulomb interactions between electrons are inevitable, as are lattice imperfections. Recently, the problem has attracted the most attention in connection with doped Mott materials in which both disorder and interactions are particularly strong.

As summarized in Ref. [5], different and seemingly conflicting theoretical predictions exist. Among those are suggestions that interactions destroy AL in 1D [153]. In this analysis, interactions are included through a mean-field non-linearity in the Schrödinger equation. Under similar conditions, the authors of [154] find quasi-periodic yet spatially localized modes. Other studies show that an isolated pair of interacting particles [155–157] propagates further than what would be expected for a single particle, indicating a possible mechanism for the destruction of AL. Hartree-Fock theory in 2D [158] shows that weak interactions increase conductance in the strongly localized regime while decreasing conductance for weak disorder. Additionally, strong interactions are always found to decrease the conductance in this approximation. In work specific to fermions [159, 160], disorder has been predicted to cause AL-like transition of the weakly interacting many-electron state. The transition happens at a finite temperature and results in exact vanishing of transport. In related work [161], a Fermi-liquid theory of weakly interacting electrons in disordered metals is developed.

Disordered Fermi Hubbard model

More recent efforts rely heavily on the 3D DFHM to extract universal physics of the interplay between interactions and disorder. The DFHM is of special interest to the cold atom community because ultracold atoms in disordered lattices almost exactly realize the model.

Disorder can be included in the FHM (Eq. (5.8)) by introducing a statistical variability in the terms of the Hamiltonian. This sets the scene for new phenomenology as the interplay between Anderson localization and Fermi-Hubbard physics may give rise to interesting new phases in strongly interacting materials. An example is the strange metal phase with a resistivity which varies linearly with temperature over a very large range [162]. Other puzzling properties have been discovered in low-dimensional systems, the most famous being a disorder-induced metallic phase in 2D [163, 164].

While these phenomena are readily accessible in ultracold atom experiments, there are many questions that remain out of reach as they arise at temperatures an order of magnitude lower than the current state-of-the-art. At low enough temperatures ($T < t^2/U$), one can
expect the emergence of anti-ferromagnetic ordering, at least in the clean system. Mean field calculations on the DFHM produce a phase diagram that includes generalized MI, high-$T_c$ and low-$T_c$ superconductors, and disordered magnetic phases [28]. In other work [40, 165], disorder is found to stabilize paramagnetic and anti-ferromagnetic metallic phases at weak interactions, while strong disorder leads to AL and suppresses the anti-ferromagnetic long-range order. Unfortunately, this low-temperature regime is not accessible for current cold atom experiments.

Another curious aspect of high-$T_c$ materials manifests itself in experimental studies: there exist microscopic spatial inhomogeneities in the superconducting gap. Since disorder is so commonly present in such materials, a connection between the inhomogeneities and disorder is difficult to establish or exclude [15]. The FHM by itself is believed by some to be insufficient for explaining these inhomogeneities [166]. In view of this rich phenomenology and many open questions, our interest in disorder physics is well-justified.

Combining speckle disorder and an optical lattice opens the opportunity to study disordered materials physics by emulating the DFHM. In the mapping between solids and cold atoms, the optical lattice plays the role of the crystal lattice of ions, the speckle beam plays the role of impurities and defects, while $^{40}K$ atoms in two different Zeeman sub-levels of the ground state play the role of spin-up and spin-down electrons. The strength of the interactions between the atoms is controlled by varying the lattice depth, and the filling is set by a combination of interaction strength and confining potentials.

The site dependence of the energies in the DFHM (Eq. (5.1)) reflects the disordered nature of the model. The presence of statistical spread in both $\epsilon_i$ and $t_{ij}$ (termed on- and off-diagonal disorder, respectively) has been shown to be of physical importance in bosonic systems [33] as the absence of either leads to modified phenomenology. A similar distinction can also be expected for fermions according to Ref. [42] where off-diagonal disorder results in a disconnection of the Anderson and Mott insulator phases. Both kinds of disorder are simultaneously present in our experiment. The statistical distributions of $t_{ij}$, $U_i$, and $\epsilon_i$ are known and are presented in Fig. 5.6.

Predictions based on the DFHM can provide a basis of comparison for our experimental data. The non-magnetic ground state of the model has been examined in a number of papers [39–43, 167, 168]. A phase diagram has been proposed at the expense of some uncontrolled approximations, the consequences of which have not been established due to the absence of a small parameter in the calculation.

The regime described in these studies is experimentally accessible. In particular, Ref. [42] addresses ultracold $^{40}K$ in an optical lattice with speckle disorder, providing a very direct point of reference. The phase diagram (Fig. 5.7) consists of three distinct regions: an MI-like phase with properties dominated by interactions, an Anderson insulator-like phase dominated by disorder, and a metallic phase which separates the two insulators. The various phases are distinguished by their local density of states: AL leads to a point-like
Figure 5.6  Probability density $\rho$ of the DFHM parameters at $s = 14 E_R$ and $\Delta = 1 E_R$. (adopted from Ref. [42]). The standard deviation of the $\epsilon$-distribution is equal to $0.9\Delta$, where $\Delta$ is the mean disorder energy. The average of the ratio of tunneling energy along two different lattice directions (blue and green curves) does not deviate by more than 10% even for moderately strong disorder ($\Delta \approx U$). The disorder does not shift the most probable values of $t$ and $U$ from the values in the clean lattice.

spectrum, while MI is characterized by a gap at the Fermi level. These observables are not yet directly accessible in our experiment. Instead, we test transport and occupancy, which are not explicitly calculated in Ref. [42]. Another complication in comparing to these results comes from the lattice geometry: the calculations are done for an infinite uniform lattice, while the cold atom system is finite and trapped. Furthermore, the calculation is done on a Bethe lattice, which is a tree-like construct with no closed loops and consequently different coordination number from the simple cubic optical lattice. Keeping in mind these differences, we experimentally address a key feature of the phase diagram: the initially positive slope of the metal-to-Anderson insulator transition line, implying a metallic phase stabilized by interactions.

5.4 Atomic limit

The absence of a reliable temperature probe in the strongly interacting optical lattice is a major complication for quantum simulations [38]. Knowledge of $T$ is indispensable in establishing that the system is in the correct regime to probe interesting low-temperature physics such as MI or anti-ferromagnetism. We circumvent this problem by using a calculation to estimate $T$ in the MI region. Although it is strictly correct in the limit of vanishing tunneling $t$, the calculation has shown good agreement with site occupancy data in the MI regime [23]. In addition to estimating $T$, we use the calculation to obtain estimates for the temperature in the lattice and for the fraction of atoms on doubly occupied sites $D$.

In the atomic limit, atoms can be thought of as residing on separate well-like lattice sites in chemical and thermal equilibrium. The energy offset of the wells $\epsilon_i$ increases with the radial distance from the center of the trap due to the harmonic confinement. Disorder can
be incorporated into the atomic limit, and its effect on double occupancy can be studied. To achieve this, a random number generator is used to assign values to $\epsilon_i$ from the known statistical distribution [33].

All thermodynamic quantities can be calculated from the on-site grand canonical partition function $Z_i$.

$$Z_i = 1 + 2ze^{-\beta \epsilon_i} + z^2 e^{-2\beta \epsilon_i - \beta U}$$

Here $z$ is the fugacity and $\beta = 1/k_B T$. In the order of their appearance in Eq. (5.16), the three Boltzmann terms correspond zero, single, and double site occupancy. For the purpose of the atomic limit, all spin statistics effects are taken into account by the double occupancy term where only one antisymmetric state is possible. $Z_i$ can be evaluated numerically for the whole system with modest computational effort. From $Z_i$ we derive the likelihood for a vacancy $v_i$, single $s_i$ and double $d_i$ occupancy, as well as the entropy $S_i$.

$$\langle v_i \rangle = 1/Z_i$$
$$\langle s_i \rangle = 2ze^{-\beta \epsilon_i}/Z_i$$
$$\langle d_i \rangle = z^2 e^{-2\beta \epsilon_i - \beta U}/Z_i$$
$$S_i = \frac{\partial}{\partial T} kT \ln Z_i$$

Figure 5.7  DFHM phase diagram (adopted from Ref. [42]).

For given lattice and trapping parameters these are completely determined by the over-
all temperature $T$ and the fugacity $z$. The total atom number $N$ is calculated from the expectation values of site occupancies:

$$N = \sum_i (\langle s_i \rangle + 2 \langle d_i \rangle).$$  \hspace{1cm} (5.18)

The energy parameters $\epsilon_i$ and $U$ are determined by the lattice depth $s$, the confinement frequency $\omega$, and the disorder energy $\Delta$. Setting $T$ and $z$ uniquely defines the system, and choosing the correct values will result in a solution which is a good description of the experimentally observed system. However, these quantities are not directly measurable. Instead, the total number $N$ is measured, and the total entropy $S$ is estimated experimentally. In order to obtain the correct solution, $T$ and $z$ are varied in the calculation until the known $N$ and $S$ values are achieved. This procedure is called “entropy matching” and can be performed systematically using a fixed-point iteration algorithm.

A lower bound for $S$ is given by the entropy of the gas before the lattice load. It can be derived from the chemical potential $\mu$, which in turn is determined from $N$, $\omega$, and $T$ as measured using time-of-flight imaging [37]:

$$\frac{S}{k_B} = \frac{(k_BT)^2}{(\hbar \omega)^3} \mu Li_3 \left( -e^{\mu/k_BT} \right) - 4 \left( \frac{k_BT}{\hbar \omega} \right)^3 Li_4 \left( -e^{\mu/k_BT} \right) =$$

$$= -N \frac{\mu}{k_BT} + 4N \frac{Li_4 \left( -e^{\mu/k_BT} \right)}{Li_3 \left( -e^{\mu/k_BT} \right)}.$$  \hspace{1cm} (5.19)

Although the above formula is derived for non-interacting fermions, it provides an excellent approximation for interacting atoms in the trap.

An upper bound on $S$ can also be obtained experimentally. Although loading the lattice is designed to be as close to adiabatic as possible, heating always occurs due to spontaneous scattering from the lattice laser as well as from dissipation processes that accompany changes in the spatial density distribution. Loading the atoms in the lattice and subsequently unloading them back into the harmonic potential causes the gas to undergo these non-adiabatic processes twice, and therefore measuring the final entropy yields an upper bound on $S$.

### 5.5 Experiment overview

The experimental result described in the remainder of this chapter is a central result of this thesis. A shorter description has been posted to the arXiv [128]. We study the transport and occupancy properties of disordered strongly correlated systems by experimentally realizing the DFHM. Our results allow a close comparison to theoretical predictions from condensed matter physics, making them relevant to a large body of literature concerning some of the major open problems in the field.
Figure 5.8  Atoms are initially cooled in a QUIC trap (shown in copper color) and then transferred to a 1064 nm crossed optical trap (white) where they are evaporatively cooled to degeneracy. The trap overlaps with the lattice beams (red), and the speckle beam (green). The focusing lens for the speckle light is shown in blue-grey. The black arrow points in the direction of imaging (toward the CCD camera). Relative to the lattice coordinates, the speckle beam propagates in the \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}} \right] \) direction, and the imaging beam propagates in the \( \left[ \frac{1}{2}, \frac{1}{2}, -\frac{1}{\sqrt{2}} \right] \) direction. Inset: The disordered lattice landscape populated by \(^{40}K\) atoms of two different angular momentum species.
5.5.1 Realization

Experiments are performed using 40,000 to 80,000 ultra-cold $^{40}$K atoms cooled to degeneracy in a 1:1 mixture of the $|F, m_F\rangle = |\frac{9}{2}, \frac{9}{2}\rangle$ and $|\frac{9}{2}, \frac{7}{2}\rangle$ Zeeman sub-levels of the ground-state hyperfine manifold. The atoms are loaded into the ground band of a 3-dimensional optical lattice of cubic symmetry (Fig. 5.8) created by three orthogonal pairs of counter-propagating laser beams ($\lambda_l = 782.2$ nm) forming standing waves. The lasers are red detuned from the atomic transition ($\lambda_a = 766.7$ nm), and therefore the atoms are attracted to the anti-nodes of the standing wave. Similarly to Refs. [6, 33, 34], the disorder is created by a speckle light field: a single blue detuned ($\lambda_d = 532$ nm) laser beam passes through a diffuser and focuses onto the atoms through a large aperture lens. The stationary speckle field has an overall Gaussian envelope with radius 170 $\mu$m and speckle correlation lengths $\zeta_x = 300$ nm in the focal plane and $\zeta_z = 1700$ nm in the propagation direction. The light shift from the speckle field results in a disordered potential of average energy $\Delta$. Superimposed on the lattice, the disorder introduces a statistical spread in $\epsilon_i$, $t_{ij}$, and $U_i$, and the DFHM (Eq. (5.1)) is realized. Relative to the lattice coordinates, the speckle beam propagates in the $[\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}]$ direction, resulting in an approximately isotropic distribution of lattice energies [33]. The standard deviation in $\epsilon_i$ is approximately equal to $\Delta$. Furthermore, due to the relatively small speckle size, the energies are nearly uncorrelated [72].

Experimentally, we control the ratio of tunneling to interactions by tuning the lattice potential depth (see Fig. 5.3). When the lattice depth is increased, $U$ is enhanced, while $t$ is suppressed.

5.5.2 Methods

Two separate measurements were performed. The first one is a transport measurement testing the ability of the system to redistribute its mass in response to an external force, similar to Ref. [34]. The behavior of the system immediately after the applied impulse is studied in an attempt to avoid the consequences of dissipation and confinement effects that would take place if further evolution is allowed. Therefore the behavior of the atoms is similar to the linear response of a system in equilibrium. This measurement reveals that the introduction of disorder obstructs transport, leading to an Anderson insulating phase. The interaction dependence of the transition is explored by measuring the critical disorder energy $\Delta_c$ across a range of lattice depths $s$. When presented in terms of the unitless $\Delta_c/t$ and $U/t$, the data reveals the main result of this study: that critical disorder increases with interactions and therefore the metallic phase is stabilized by interactions.

In the strongly interacting regime, where transport is strongly suppressed due to the MI phase, site occupancy is probed. The fraction of atoms residing on doubly occupied sites is measured by ejecting such atoms from the trap using light-induced collisions. Interactions
normally suppress double occupancy [23] and decrease site-to-site number fluctuations. At half filling, strong interactions, and low temperature ($U > 12t$ and $U \gg k_B T$) the MI phase occurs in which number fluctuations vanish and every site is occupied by one atom. Conversely, double occupancy is required for a conventional conducting phase at half filling. We find good agreement between double occupancy data and atomic limit results, revealing the power of this incomplete but instructive model.

5.6 Transport measurement

To characterize the transport properties of the gas, we look at how the quasimomentum distribution $n(q)$ changes in response to an applied impulse, similar to Ref. [34]. In a conducting phase (e.g., metal or Fermi liquid), the atoms would acquire center-of-mass (COM) momentum, which manifests itself as an imbalance in $n(q)$. In contrast, atoms in an insulating state (e.g., band, Mott, and Anderson insulators) will not respond to the impulse. The absence of motion in the band insulator can be explained by particles filling all available states in the ground band. Unoccupied quasimomentum states that may support COM motion only exist in the excited bands, separated by a band-gap (see Section 5.2.1). Similarly, a Mott insulator occurs when all the quasimomentum states are occupied and the Fermi energy lies in the band-gap. However, the gap in this strongly interacting case is the Mott gap, and the maximum occupancy in the ground band is achieved at half filling. A fundamentally different mechanism determines the insulating properties in the AL phase: although unoccupied quasimomentum states may be present in the Brillouin zone, the motion of the particles is suppressed due to coherent scattering from the disorder potential.

The transport measurement is a suitable probe for a system entirely in the Fermi-liquid phase. Experimental conditions such as the degree of degeneracy $T/T_F$, atom number $N$, average confinement frequency $\omega$, and the resulting characteristic density $\tilde{\rho}$ are summarized in Table 5.1. The conditions are chosen such that the gas is in the Fermi liquid phase for all data. The gas is elongated along one direction with the aspect ratio of the root mean square (RMS) radii estimated to be approximately $1.8 : 1 : 1$ for all $s$.

In the Fermi-liquid regime, transport data is relatively easy to interpret: although site filling varies throughout the gas, the population in confinement-induced localized states is minimal and the whole system responds to external forces. In contrast, at higher lattice depths, band and Mott insulating phases exist only in regions of the trap, leading to complications in understanding transport. For example, it is unclear whether the arguments for absence of conduction from Subsections 5.2.1 and 5.2.3 hold up for a finite inhomogeneous system. Furthermore, these phases necessitate a finite occupation of localized states (see Subsection 5.2.4) far from the trap center which also possess unclear transport properties.

We impart momentum on the atoms by applying a magnetic field gradient for 2 ms.
Figure 5.9  Transport measurement sequence. The lattice (black) and the speckle (green) are loaded simultaneously over 200 ms via an exponential ramp. A magnetic field gradient (magenta) is pulsed on for 2 ms. Immediately thereafter the lattice and the speckle are turned off via a 300 $\mu$s linear ramp. Finally, the optical trap (cyan) is turned off 10 ms before imaging (magenta).

Figure 5.10  Band-mapped images of gas at $s = 4 E_R$ before and after applied impulse.
This duration is short compared to $1/\omega$, ensuring that the atoms do not slosh in the trap before they are released. The magnitude of the magnetic field has been chosen such that the atoms exhibit significant COM motion while always remaining within the first Brillouin zone (5.11a inset). The direction of the impulse is the same for both angular momentum species, with the $m_F = \frac{7}{2}$ atoms experiencing $7/9$ of the impulse magnitude seen by $m_F = \frac{9}{2}$ atoms. Due to this difference in accelerations, the distance between two atoms of either species accelerating from a stop to the maximum recorded COM velocity $v_{\text{COM}}$ is less than $0.7 \, \mu\text{m}$. This distance is a fraction of the typical $r \approx 5 \, \mu\text{m}$ RMS radius of the gas along the impulse direction, signifying that most atoms will stay in contact most of the time.

After the impulse we apply band-mapping: we turn off the lattice linearly over $300 \, \mu\text{s}$, which is fast compared to the inverse of the band-gap frequency but slow compared to the inverse of the bandwidth [22, 169]. After a free expansion lasting $10 \, \text{ms}$, an image of the band-mapped gas (Fig. 5.10) represents $n(q)$, integrated along the imaging direction. From the images we calculate the first moment and hence the COM, which we compare to the position of the gas before the impulse to determine $v_{\text{COM}}$ and the no-motion baseline.

Fig. 5.11 shows $v_{\text{COM}}$ data for lattice depths $s = 4, 5, 6, 7 \, E_R$ and disorder $\Delta$ in the range $0 - 1.5 \, E_R$. Two trends become evident from the data: $v_{\text{COM}}$ decreases monotonically both for increasing $\Delta$ and $s$. The first trend signifies a disorder-driven metal-to-insulator transition. The second trend is driven by two cooperating but distinct mechanisms: interactions and effective mass. In a deeper lattice, interactions are stronger, leading to decreased conductivity, while broadening the quasimomentum distribution $n(q)$. This effect is important to our investigation of how interactions affect the metal-to-insulator transition. The second mechanism is a consequence of the bands flattening with increasing $s$, leading to a higher effective mass for the atoms. As shown later in this section, normalizing $s$ by the bandwidth $12t$ serves to isolate this somewhat trivial single-particle physics from the more interesting interaction effects.

Band-mapped images of the gas before the momentum impulse inform us about the width of the distribution. Looking at the sample images for $s = 4 \, E_R$ and $s = 10 \, E_R$ (Fig. 5.12), we note that the deeper lattice results in a wide $n(q)$ occupying most of the Brillouin zone. The reason is the higher interaction strength ($U_{10E_R} = 0.5 \, E_R$, compared to $U_{4E_R} = 0.1 \, E_R$) which tends to spatially constrain the atoms to single sites, thus broadening

<table>
<thead>
<tr>
<th>$s , (E_R)$</th>
<th>$T/T_F$</th>
<th>$N$</th>
<th>$\omega , (\text{rad/sec})$</th>
<th>$\tilde{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$0.14 \pm 0.03$</td>
<td>$37,000 \pm 2,500$</td>
<td>$670 \pm 15$</td>
<td>$0.9 \pm 0.1$</td>
</tr>
<tr>
<td>5</td>
<td>$0.18 \pm 0.03$</td>
<td>$41,200 \pm 3,000$</td>
<td>$700 \pm 15$</td>
<td>$1.6 \pm 0.2$</td>
</tr>
<tr>
<td>6</td>
<td>$0.16 \pm 0.02$</td>
<td>$47,100 \pm 6,500$</td>
<td>$730 \pm 10$</td>
<td>$3.1 \pm 0.5$</td>
</tr>
<tr>
<td>7</td>
<td>$0.17 \pm 0.02$</td>
<td>$48,700 \pm 1,900$</td>
<td>$760 \pm 15$</td>
<td>$5.3 \pm 0.3$</td>
</tr>
</tbody>
</table>

Table 5.1 Experimental conditions for transport data. The uncertainties represent the standard deviation in the measured quantities.
Center-of-mass velocity $v_{\text{COM}}$ of the gas after applied impulse. $v_{\text{COM}}$ is observed to decrease with disorder $\Delta$ for all lattice depths $s$. Full data range is displayed for $s = 4 \ E_R$ (blue squares), $s = 5 \ E_R$ (red circles), $s = 6 \ E_R$ (green triangles), and $s = 7 \ E_R$ (orange diamonds). Grey band displays uncertainty in determining $v_{\text{COM}} = 0$ (standard deviation of the mean $v_{\text{COM}}$ when no impulse is applied). Critical disorder $\Delta_c$ is defined as the point where an exponential fit dotted line crosses the $v_{\text{COM}} = 0$ uncertainty band.

the distribution in reciprocal space. Within the $s = 4 \ E_R$ data we note that disorder introduces little change in the $n(q)$, with the $\Delta = 1.5 \ E_R$ image providing no clue about the localized state of the atoms. The absence of disorder-induced broadening of $n(q)$ reaffirms the Anderson nature of the metal-to-insulator transition and distinguishes it from the band and Mott insulator transitions.

To gain quantitative insight into the effect of interactions on the metal-to-insulator transition, we measure the characteristic disorder energy $\Delta_c$ of the transition. We define $\Delta_c$ as the disorder energy at which $v_{\text{COM}}$ falls to the level of resolution of the measurement ($v_{\text{res}} \approx 0.05 \text{ mm/sec}$). We estimate $\Delta_c$ by fitting the data to the heuristic model $A e^{-\Delta/\overline{\Delta}}$ (with $A$, and $\overline{\Delta}$ as free parameters), and setting $A e^{-\Delta_c/\overline{\Delta}} = v_{\text{res}}$. The model describes the data very well at low $\Delta$ (from $\Delta = 0$ to about $2\overline{\Delta}$), and it is this scale of $\Delta$ which determines the fitting parameters. Choosing $\Delta_c$ as a hard cut-off based on these parameters is not unphysical because the band-gap enforces a similar upper bound on populated quasimomentum states.

From $\Delta_c$ we obtain the universal behavior of the metal-to-insulator transition across all lattice depths (Fig. 5.13) by plotting the dimensionless quantities $\Delta/12t$ and $U/12t$ implied by the Hamiltonian [42, 102]. Displayed in this manner, the data are applicable to any disordered Hubbard system. The resulting $s$-independent plot suggests that interactions push the metal-to-insulator transition to higher disorder and effectively stabilize the metallic
Figure 5.12  Band-mapped lattice gas without momentum impulse. a Representative images at $s = 4 E_R$, $\Delta = 0 E_R$ (i) and $\Delta = 1.46 E_R$ (ii), and at $s = 7 E_R$, $\Delta = 0 E_R$ (iii) and $\Delta = 1.46 E_R$ (iv) illustrate that disorder does not strongly affect the quasimomentum distribution, which broadens at higher interaction strengths. The color bar shows the measured optical depth (OD). b Traces through the images in a along the dashed line. The quasimomentum $\tilde{q}$ projected along this axis in the imaging plane is measured in units of the maximum allowed quasimomentum in the Brillouin zone $q_{\text{max}}$. The solid blue and orange lines correspond to images (i) and (iii), and the blue and orange shaded regions to images (ii) and (iv).
Figure 5.13  Universal transport behavior. The critical disorder strength $\Delta_c/12t$ is shown for varying interaction strength $U/12t$, which is controlled by tuning the lattice potential depth $s$. The error bars show the uncertainty in the fit to the data in Fig. 5.11 used to determine $\Delta_c$. The percolation threshold is shown as a dashed line.
phase. The data exhibit good qualitative agreement with theory [42], but $\Delta_c$ values are approximately three times smaller than the predicted disorder energies for the AL transition. This discrepancy may be explained by the Bethe lattice geometry used in the calculation and the presence of the trap in the experiment. $\Delta_c$ can also be compared to numerous theoretical predictions based on the Anderson model for non-interacting particles in a cubic lattice. One result based on the method of kinetic equations [148] predicts that all states will be localized for a disorder strength no higher than $\Delta/12t \approx 0.8$, which is consistent with the disorder strength observed for the most weakly interacting point (at $s = 4 E_R$). Another group of studies, also pertaining to the Anderson model, finds that most states localize at relatively higher disorder $\Delta/12t \approx 2$ [106, 109, 147, 151, 152].

We can understand the localization behavior qualitatively by considering the effect of interactions on the $n(q)$. At $U = 0$ and less than half filling, both particle species occupy an unfilled band and the gas has metallic properties. A finite interaction mixes the single-particle states to create a set of many-particle eigenstates. Consequently, higher quasimomentum states receive population and $n(q)$ broadens. This mechanism is ultimately reflected in the creation of the MI when $U$ exceeds $12t$ and population is evenly distributed across the first Brillouin zone. Interaction-induced broadening of $n(q)$ is consistent with the trends visible in Fig. 5.12. On the other hand, we know that disorder inhibits transport only for particles with kinetic energies below the mobility edge [106]. By transferring population to higher quasimomenta, interactions thereby cause $\Delta_c$ to increase. Alternatively, disorder can be understood as Anderson localizing the many-particle eigenstates, in agreement with Refs. [159, 160]. We conclude that in order to understand the effect of disorder on the strongly interacting system, it is useful to start by asking what happens to the many-body eigenstate. The attempt to arrive at general understanding about the many-body system based on the single particle picture leads to the prediction that AL is destroyed by interactions: a conclusion which contradicts our observations.

To confirm the AL nature of the transition, it is necessary to compare $\Delta_c$ to the site percolation threshold $\Delta_p$. In the model of site percolation, particles hopping between nearest neighbors can propagate arbitrarily long distances as long the fraction of energetically allowed sites exceeds 0.31: a number specific to the 3D cubic lattice. To calculate the disorder energy $\Delta_p$ necessary to reach this fraction, we refer to the atomic limit, namely neglecting the tunneling term in the Hamiltonian, and follow the prescription outlined in Ref. [170]. The result (dashed line in Fig. 5.13) is a poor description of the data, overestimating $\Delta_c$ by more than an order of magnitude at high lattice depth. The failure of the percolation hypothesis underlines the Anderson-like nature of the transition. In contrast, a similar measurement on interacting bosons in a disordered optical lattice [34] finds a percolation transition from a superfluid to an insulating state.
5.7 Double occupancy measurement

We are interested to study the effect of disorder when interactions exceed the bandwidth and an MI phase is present. Probing this regime with the transport measurement described in Section 5.6 is complicated due to a population of confinement-induced localized states with unclear transport properties (see Subsection 5.2.4). An alternative method is devised to measure a defining property of the MI state—site occupancy.

For \( U > 12t \), a system with one particle per site undergoes a metal-to-Mott insulator transition due to a gap appearing in the energy density of states at the Fermi level. Each lattice site becomes occupied by one particle of either species, while double occupancy is exponentially suppressed by temperature. For this measurement, we access the MI regime by loading a gas of 80,000 atoms with \( T/T_F = 0.22 \pm 0.2 \) into a \( s = 10 \) \( E_R \) lattice, resulting in \( \tilde{\rho} = 19 \). Following the procedure of entropy matching described in Subsection 5.4, we use the atomic limit to estimate a temperature of 50 nK. The temperature is smaller than the interaction energy \( U_{10E_R} = 200 \ k_B \cdot nK \), implying that interactions dominate thermal fluctuations. Theory suggests that a region in the center of the trap is occupied by a MI, surrounded by a conducting phase with radially decreasing filling [36]. Such inhomogeneity is inherent to any trapped system supporting a MI, even at \( T = 0 \).

5.7.1 Experimental method

Insight into the nature of the disordered strongly correlated state at \( U > 12t \) can be gained by measuring double occupancy as a function of \( \Delta \). At half filling, the suppression of double occupancy is a necessary condition for the emergence of the MI state. We test the ability of disorder to disrupt this strongly correlated state and generate double occupancies. The fraction of atoms residing on doubly occupied sites \( D \) is measured by inducing a light-assisted collision between such atoms, thus ejecting them from the trap. We construct an atomic limit model for the system, providing an important benchmark for a verification of the measurement method, while also serving as a means to understand the dependence of \( D \) on \( \Delta \).

The experimental procedure designed to expel the atoms from doubly occupied sites is shown in Fig. 5.14. The procedure relies on a photo-assisted collision loss which is induced by a laser beam detuned 22 MHz blue of the atomic transition. The laser pulse is applied after the lattice depth is quickly (200 \( \mu s \)) increased from the value of interest (\( s = 4 \) – \( 10 \) \( E_R \)) to \( s = 32 \) \( E_R \). At this depth the lattice sites become tight traps with confinement frequencies of 150 kHz causing an enhanced overlap of the atomic wavefunctions on doubly occupied sites. The long nearest-neighbor tunneling time \( t/h \approx 360 \) ms guarantees that all dynamics is frozen out for the duration of the loss-inducing laser pulse (0 – 5 ms). The atoms on those sites undergo light-assisted collisions and are expelled from the trap [171] when subjected to the near-resonant pulse. This collision loss constitutes a double occupancy
Figure 5.14  Double occupancy measurement procedure. Lattice (black) and disorder (green) loading occur simultaneously, followed by a sharp (200 µs) increase of lattice depth to $32 E_R$. The atoms are exposed to a near-resonant pulse of duration $\tau$ (red) and subsequently unloaded from the combined lattice and disorder potential in a 30 ms exponential ramp. Imaging (magenta) occurs 10 ms after releasing the gas from the optical trap (cyan).

measurement. In addition to collision losses, near-resonant light also causes heating loss. However, the two processes are clearly distinguishable (Fig. 5.15) as their characteristic time scales differ by an order of magnitude [172].

The data on double occupancy fraction (Fig. 5.17) is obtained by taking alternating shots with and without a 0.5 ms light pulse, and comparing the atom number. Additional verification data for this method is presented in the following subsection.

5.7.2 Method verification

The double occupancy probe relies on light-induced collision losses as an experimental tool. We develop this tool first by resolving the time-dependence of the atom number decrease, similarly to Ref. [172]. We successfully identify a fast and a slow loss mechanism—due to loss of atoms on doubly occupied sites and due to off-resonant heating, respectively. Next, we measure $D$ for $s = 4 E_R$ and for $s = 10 E_R$ and variable $\tilde{\rho}$ to observe qualitative agreement with the inhomogeneous phase diagram in Ref. [36]. We benchmark our findings against results from the atomic limit calculation (see Subsection 5.4) and find good quantitative agreement as well.
Figure 5.15 Fraction of atoms $\alpha$ remaining after near-resonant pulse of duration $\tau$. The $s = 4 \, E_R$ data (red) display a fast initial loss due to light-assisted collision between atoms residing on doubly occupied sites. This short-timescale feature is absent in the $s = 10 \, E_R$ data (black). Red and black curves are double and single exponential fits, respectively.

**Fraction of atoms remaining vs. pulse duration**

Fig. 5.15 shows the fraction of remaining atoms as a function of pulse duration $\tau$ for $s = 4 \, E_R$ (red) and $s = 10 \, E_R$ (blue). The $s = 4 \, E_R$ data shows a sharp initial drop, followed by a slower number loss. In contrast, the $s = 10 \, E_R$ data only shows the long time scale process. As a function of pulse duration $\tau$, the number of atoms $N$ at $s = 4 \, E_R$ is well-described by a double exponential of the form

$$N(\tau) = N_s e^{-\tau/\bar{\tau}_s} + N_l e^{-\tau/\bar{\tau}_l}. \quad (5.20)$$

Here $N_s$ and $N_l$ are the number of atoms taking part in the short and long processes respectively described by the time constants $\bar{\tau}_s$ and $\bar{\tau}_l$. Fitting returns $\bar{\tau}_s = 0.24 \pm 0.07$ ms and $\bar{\tau}_l = 7.9 \pm 0.9$ ms for $s = 4 \, E_R$. Since there is no short timescale process for $s = 10 \, E_R$, a single exponential fit is used, returning $\bar{\tau}_l = 6.8 \pm 0.8$ ms.

The origin of the two processes is revealed by comparing the time constants to the inverse of the off-resonant scattering rate $(2.18)$. On the average, atoms scatter approximately one photon every 0.1 ms, and it takes on the order of 10 ms to scatter a number of photons with a total energy equal to the trap depth. Consequently, we can associate $\bar{\tau}_s$ with...
Figure 5.16 Dependence of double occupancy $D$ on characteristic density $\tilde{\rho}$ in the clean lattice. $D$ increases monotonically with $\tilde{\rho}$ in the $10\ E_R$ lattice (black), in good agreement with atomic limit calculation (dashed line). For $s = 4\ E_R$ (red point) weaker interactions lead to high double occupancy even at low $\tilde{\rho}$. The systematic uncertainty in determining the $D = 0$ line is shown as a grey band.

In terms of the function parameters, the double occupancy fraction $D$ is given by $N_s/(N_s + N_l)$. To reduce the uncertainty in $D$, while avoiding taking large data sets as in Fig. 5.15, we restrict the measurement to the $\tau = 0$ and $\tau = 0.5\ ms$ points. The double occupancy fraction can be expressed as

$$D = \frac{\alpha_p - \alpha}{\alpha_p},$$  

(5.21)

where $\alpha = N(0.5)/N(0)$ is the ratio of atom number before and after the pulse, and $\alpha_p = N_p(0.5)/N_p(0)$ is the same ratio when no double occupancy is present. As opposed to the double exponential fit, calculating $D$ in this fashion does not depend on the detailed knowledge of the theoretical model for number loss and all of its parameters. Instead, Eq. 5.21 only assumes that the short time scale $\tilde{\tau}_s < 0.5\ ms$ and results in a systematic underestimate of 10% of $D$ compared to the exact expression (5.20). $\alpha_p$ is determined experimentally by performing the double occupancy measurement on a spin-polarized gas, where the Pauli exclusion principle sets $D = 0$ and no short-time number loss in present. The systematic error in $D = 0$ is given by the standard error in measuring $\alpha_p$ (grey band).
Disorder-induced double occupancy. Disorder causes $D$ to increase moderately. This behavior is captured by the atomic limit curves. The dashed curve is calculated using the low entropy estimate $(S = (2.0 \pm 0.2) \, k_B \text{ per particle})$ and the dotted curve comes from the high entropy estimate $(S = (2.8 \pm 0.3) \, k_B \text{ per particle})$.

**Double occupancy vs. characteristic density**

Fig. 5.16 presents further support for the agreement between the atomic limit and the double occupancy measurement method. The experiment is performed with 80,000 atoms cooled to $T/T_F = 0.17$, loaded into $s = 10 \, E_R$ lattice, and subjected to harmonic confinement with $\omega/2\pi$ ranging from 100 to 220 Hz. With increasing $\tilde{\rho}$, double occupancy becomes energetically favorable, as lattice sites far from the center of the gas acquire potential energy due to the increased confinement. This simple energetics argument is presented in Subsection 5.2.4 and is found to reproduce the phase diagram of Ref. [36] (Fig. 5.4). Energetics is also captured by the atomic limit calculation (Section 5.4), which can be used to calculate $D$ for a given temperature estimate. Reassuringly, the atomic limit calculation shows good agreement with the double occupancy data.

**5.7.3 Findings**

Measuring the $\Delta$-dependence of $D$ amounts to taking a vertical slice through the phase diagram Fig. 5.7. Our main goal is to look for the predicted MI-to-disordered strongly-correlated metal transition [42]. Unlike the transport measurement, this experimental method does not distinguish conducting from insulating properties. Instead, we look for
increase in $D$—a signature of disorder breaking the Mott gap. Whether a broken gap is accompanied by conduction is not clear from existing DMFT predictions. In these studies, the phases are not determined by their transport properties but rather by the density of states.

The data in Fig. 5.17 shows a moderate increase in double occupancy fraction $D$ as $\Delta$ is increased from 0 to $1.5 \ E_R$. We interpret this behavior as a breaking of the Mott gap. Even at large $\Delta$, $D$ remains well below the values for $s = 4 \ E_R$ (red point). Because the specific density $\tilde{\rho}$ is kept equal for all data, higher double occupancy is only due to a lower interaction strength ($U_{4E_R} = 0.1 \ E_R$ compared to $U_{10E_R} = 0.5 \ E_R$) allowing the formation of a band insulator [36] at the trap center.

Disorder-induced double occupancy is not surprising in view of energetics: the increase happens over a scale of $\Delta$ comparable to $U_{10E_R} = 0.5 \ E_R$. The randomization of on-site energies can create instances where double occupancy is energetically favorable. This effect can be quantified using the atomic limit calculation described in Subsection 5.4. Disorder is built into the model by the known probability density of on-site energies $\rho$ (see Ref. [33] and Fig. 5.6) and the reasonable assumption that $\epsilon_i$ are spatially uncorrelated [72]. As seen in Fig. 5.17, the calculation agrees well with the measurement. The high and low

---

**Figure 5.18** Occupancy calculated in the atomic limit for variable disorder $\Delta$. Each pixel corresponds to a lattice site. Sites with average occupancy higher than 1 are colored black. Images correspond to the data points in Fig. 5.17.
limits of entropy are estimated experimentally, following the procedure in Subsection 5.4. 2D slices through the center of the calculated distribution are shown in Fig. 5.18. Sites with average occupancy higher than 1 (colored black) become more common as $\Delta$ increases. This tendency is accompanied by a growth of the RMS radius of the gas, which is due to more distant lattice sites becoming energetically accessible when sites closer to the center are elevated in energy by the disorder.

Although disorder energetics offer a plausible explanation, the increase in $D$ could also be due to more trivial heating effects, e.g., from the lattice beams. Measuring $T$ directly in the lattice is fraught with challenges [38] but a simple experimental procedure can be used to quantify the extent of non-adiabaticity. Starting with an atom gas of known entropy, we load and immediately unload the lattice (see Subsection 5.4). This results in an entropy increase from $(2.0\pm0.2) \, k_B$ to $(2.8\pm0.3) \, k_B$ per particle, irrespective of $\Delta$. We conclude that disorder does not cause any additional heating, although its effect on absolute temperature in the lattice—and therefore thermal excitations and double occupancy—remains unknown.

The discovery that disorder breaks the Mott gap is an important tile in the puzzle of disordered strongly correlated systems. It complements the observation that interactions stabilize the metallic phase, as confirmed by the transport measurement of Section 5.6. The picture that emerges is that disorder and interactions compete to produce an insulating state, and each is capable, within limits, to negate the effect of the other.
Appendix A

Multipole expansion calculation for QUIC trap fields

The confining properties of the magnetic trap can be calculated from a multipole expansion around the point of symmetry for the quadrupole coil pair. The result of the calculation is an approximation for the location of the trap minimum, the confining frequencies and the field offset, given as a function of coil positions, sizes and electric current. While easy to evaluate, the trap parameters thus obtained agree well with more accurate numerical calculations, making the multipole expansion a valuable tool for trap design.

A.1 Multipole expansion

Maxwell’s equations for a static magnetic field in free space admit the existence of a scalar potential $\Psi$ such that ([82])

$$B = \nabla \Psi$$
$$\nabla^2 \Psi = 0.$$  \hfill (A.1)

$\Psi$ can be expressed in spherical harmonics ($Y_{lm}$) as

$$\Psi = \sum_{l,m} a_{lm} r^l Y_{lm} \equiv \sum_{l,m} \Psi_{lm}. \hfill (A.2)$$

For geometries with axial symmetry (coils and straight wires) we only need the $m = 0$ terms

$$\Psi_{l0} = r^l P_{l0} \cos(\theta) \quad \text{... spherical coordinates}$$
$$= p_l(\rho, z) \quad \text{... cylindrical coordinates}, \hfill (A.3)$$

where $p_l$ are the Legendre polynomials (tabulated in Table I in [82]). From eq. (A.1), the magnetic field becomes

$$B = \nabla \sum_l a_{l0} p_l = \sum_l a_{l0} \partial_z p_l \hat{\zeta} + \sum_l a_{l0} \partial_\rho p_l \hat{\rho}. \hfill (A.4)$$
The axial and radial components of $B$ can be written separately as

$$B_z = \sum_{l} a_l 0 \partial_z p_l = \sum_{n=0}^{\infty} b_n B_{zn}(\rho, z)$$

$$B_{\rho} = \sum_{l} 0 a_{l} \partial_{\rho} p_l = \sum_{n=0}^{\infty} b_n B_{\rho n}(\rho, z).$$  \hspace{1cm} (A.5)

$B_{zn}$ and $B_{\rho n}$ are tabulated in table II of ref. [82].

For a circular wire of radius $R$, centered at $z = A$ (fig. A.1), the exact solution for the field along $z$ is ([173])

$$B_z(z, \rho = 0) = \frac{\mu_0 I R^2}{2[R^2 + (A - z)^2]^{3/2}} = \frac{\mu_0 I R^2}{2(R^2 + A^2)^{3/2}} \sum_{n=0}^{\infty} g_n(A, R) (R^2 + A^2)^{n} z^n$$

$$B_{\rho}(z, \rho = 0) = 0.$$  \hspace{1cm} (A.6)

Here $g_n$ are the homogeneous polynomials, tabulated in table III in [82]. Since eq. (A.5) is a polynomial expansion in $\rho$ and $z$, it has to be identical to the Taylor expansion in $z$ when
\( \rho = 0 \). Comparing eqs. (A.5) and (A.6) gives the following values for the prefactors \( b_i \):

\[
\begin{align*}
    b_0 &= \frac{\mu_0 I R^2}{2 (R^2 + A^2)^{3/2}} \\
    b_1 &= \frac{3 \mu_0 I R^2 A}{2 (R^2 + A^2)^{5/2}} \\
    b_2 &= \frac{3 \mu_0 I R^2 (4 A^2 - R^2)}{4 (R^2 + A^2)^{7/2}} \\
    b_3 &= \frac{5 \mu_0 I R^2 A (4 A^2 - 3 R^2)}{4 (R^2 + A^2)^{9/2}},
\end{align*}
\]

(A.7)

where \( \mu_0 = 4 \pi 10^{-7} \text{ N/A}^2 \) is the permeability of free space. The axial and radial fields become

\[
\begin{align*}
    B_z &= b_0 + b_1 z + b_2 \left( z^2 - \frac{\rho^2}{2} \right) + b_3 \left( z^3 - \frac{3}{2} z \rho^2 \right) + \cdots \\
    B_\rho &= -b_1 \frac{\rho}{2} - b_2 \rho z - b_3 \left( \frac{3}{2} \rho z^2 - \frac{3}{8} \rho^2 z \right) + \cdots
\end{align*}
\]

(A.8)

Note the \( b_3 \) term of \( B_\rho \) is given incorrectly by table II in ref. [82] (missing a \( z \)).

### A.2 Trap fields

In trap design we are interested in the field offset, the gradients and the curvatures of the magnetic field magnitude close to the trap minimum. We approximate those by calculating the contribution from the first three orders of the multipole expansion (we will disregard terms \( b_3 \) and higher). Thus we will be interested in \( \frac{\partial B}{\partial x_i} \) and \( \frac{\partial^2 B}{\partial x_i^2} \), where \( x_i \equiv \{x, y, z\} \).

\[
\begin{align*}
    B &= \sqrt{B_x^2 + B_y^2 + B_z^2} \\
    \frac{\partial B}{\partial x} &= \left( B_x^2 + B_y^2 + B_z^2 \right)^{-1/2} \left( B_x B_x' + B_y B_y' + B_z B_z' \right) \\
    \frac{\partial^2 B}{\partial x^2} &= \left( B_x^2 + B_y^2 + B_z^2 \right)^{-3/2} \left( B_x B_x'' + B_y B_y'' + B_z B_z'' \right) + \\
    &\quad + \left( B_x^2 + B_y^2 + B_z^2 \right)^{-1/2} \left( B_x B_x B_x'' + B_y B_y B_y'' + B_z B_z B_z'' \right)
\end{align*}
\]

(A.9)

For the QUIC geometry (fig. A.1), these will be evaluated at the trap minimum. As will be seen in section A.3, the position of the trap minimum is generally different from the point of reference of the multipole expansion (the center of the coordinate system). Instead, it will occur at position \( x = y = 0; \ z = z_0 \). Since the multipole expansion is correct only for a small volume close to the origin, the calculated trapping parameters should only by trusted when \( z_0 \) is small.

Another important trap parameter is the overall trap depth. To calculate the depth,
a detailed knowledge of the magnetic fields far from the trap minimum is necessary. This calls for a full-fledged calculation of \( B \) in the volume enclosed by the coils.

### A.3 QUIC trap

A QUIC trap consists of a set of quadrupole coils (along the x-axis) with current running in opposite directions; a single (Ioffe) coil (centered along z-axis); and a coil/set of bias coils providing a uniform field along the z-axis (fig. A.1). The contributions to the field from all coils is shown below.

**Ioffe coil:**

\[
B_z = b_{0_{iof}} + b_{1_{iof}} z + b_{2_{iof}} \left( z^2 - \frac{\rho^2}{2} \right) \\
B_x = -b_{1_{iof}} \frac{x}{2} - b_{2_{iof}} zx \\
B_y = -b_{1_{iof}} \frac{y}{2} - b_{2_{iof}} zy.
\]

\[ (A.10) \]

**Quadrupole Coils:**

\[
B_x = 2b_{1_{quad}} x \\
B_y = -b_{1_{quad}} y \\
B_z = -b_{1_{quad}} z.
\]

\[ (A.11) \]

Note \( b_{1_{quad}} \) is computed for each quadrupole coil separately.

**Bias field:**

\[
B_z = b_{0_{bias}} \\
B_x = 0 \\
B_y = 0
\]

\[ (A.12) \]

The total field is the vector sum of the fields from each coil. From symmetry it can be seen that the fields in the x- and y-directions vanish for \( x = y = 0 \). The minimum of the total field will occur in \( x = y = 0; z = z_0 \), where \( z_0 \) is the minimum of the field in the x-direction.

\[
B_z = b_{0_{iof}} + b_{1_{iof}} z + b_{2_{iof}} \left( z^2 - \frac{\rho^2}{2} \right) - b_{1_{quad}} z + b_{0_{bias}} \\
\]

\[
B_z(x = y = 0, z) = b_{0_{iof}} + b_{1_{iof}} z + b_{2_{iof}} z^2 - b_{1_{quad}} z + b_{0_{bias}} \\
\Rightarrow z_0 = \frac{b_{1_{quad}} - b_{1_{iof}}}{2b_{2_{iof}}}
\]

\[ (A.13) \]
The other magnetic field components are

\[
B_x = -b_{1 \text{iof}} \frac{x}{2} - b_{2 \text{iof}} \frac{z}{2} x + 2 b_{1 \text{quad}} x \\
B_y = -b_{1 \text{iof}} \frac{y}{2} - b_{2 \text{iof}} \frac{z}{2} y - b_{1 \text{quad}} y. \tag{A.14}
\]

Let \( B_0 = B(x = y = 0; z = z_0) \).

\[
B_0 = b_{0 \text{iof}} + b_{0 \text{bias}} + (b_{1 \text{iof}} - b_{1 \text{quad}}) \frac{b_{1 \text{quad}} - b_{1 \text{iof}}}{2b_{2 \text{iof}}} + b_{2 \text{iof}} \left( \frac{b_{1 \text{quad}} - b_{1 \text{iof}}}{2b_{2 \text{iof}}} \right)^2 = \\
= b_{0 \text{iof}} + b_{0 \text{bias}} - \frac{1}{4} \frac{(b_{1 \text{quad}} - b_{1 \text{iof}})^2}{b_{2 \text{iof}}} \tag{A.15}
\]

\( B_0 \) has to be greater than zero in order for it to be a minimum in the field magnitude. The directional derivatives become

\[
\begin{align*}
\partial_x B(x = y = 0; z = z_0) &= 0 \\
\partial_y B(x = y = 0; z = z_0) &= 0 \\
\partial_z B(x = y = 0; z = z_0) &= b_{1 \text{iof}} - b_{1 \text{quad}} + 2b_{2 \text{iof}} z_0 = 0. \tag{A.16}
\end{align*}
\]

All derivatives vanish, which means we are in a global minimum. Now examine the second derivatives:

\[
\begin{align*}
\partial_x^2 B(x = y = 0; z = z_0) &= \frac{1}{B_0} \left( -\frac{b_{1 \text{iof}}}{2} - b_{2 \text{iof}} z_0 + 2 b_{1 \text{quad}} \right)^2 - b_{2 \text{iof}} = \\
&= \frac{1}{B_0} \left( \frac{3}{2} b_{1 \text{quad}} \right)^2 - b_{2 \text{iof}} \tag{A.17}
\end{align*}
\]

\[
\begin{align*}
\partial_y^2 B(x = y = 0; z = z_0) &= \frac{1}{B_0} \left[ -z_0 b_{2 \text{iof}} - \frac{b_{1 \text{iof}}}{2} - b_{1 \text{quad}} \right]^2 - b_{2 \text{iof}} = \\
&= \frac{1}{B_0} \left( \frac{3}{2} b_{1 \text{quad}} \right)^2 - b_{2 \text{iof}} \tag{A.18}
\end{align*}
\]

\[
\begin{align*}
\partial_z^2 B(x = y = 0; z = z_0) &= \frac{1}{B_0^2} \left( b_{1 \text{iof}} + 2 b_{2 \text{iof}} z_0 - b_{1 \text{quad}} \right) + \\
&\quad \frac{1}{B_0} \left( b_{1 \text{iof}} + 2 b_{2 \text{iof}} z_0 - b_{1 \text{quad}} \right)^2 + 2 b_{2 \text{iof}} = \\
&= 2 b_{2 \text{iof}}. \tag{A.19}
\end{align*}
\]
From here we can calculate the oscillation frequencies for a $^{40}K$ atom in the ground state:

$$\omega_i = \sqrt{\frac{\mu_B \partial_i^2 B}{m}}, \quad (A.20)$$

where $i = \{x, y, z\}$, $\mu_B$ is the Bohr magneton, and $m$ is the atomic mass.

Note that $b_{0\text{bias}}$ appears in the offset $B_0$. This allows the control of the radial frequencies $\omega_x$ and $\omega_y$ by simply ramping the bias field. Meanwhile $\omega_z$ remains unaffected, as it is set only by the curvature of the Ioffe coil. The $z$-direction confinement is typically the weakest, and hence the QUIC trap is referred to as cigar-shaped. In our design, the $\omega_z$ has a maximum value of 27 Hz.

At high temperatures the atomic cloud is large and it mostly sees the quadrupole field envelope, characterized by $b_{1\text{quad}}$. Desired values for that is $b_{1\text{quad}} \approx 300G/cm$. 

109
Appendix B

Drawings

All dimensions are given in inches.
Figure B.1  Vacuum system (dimensioned).
Figure B.2  Collection cell assembly.
Figure B.3  Transfer tube.
Figure B.4  Custom cross.
Figure B.5  Science cell.
Figure B.6  Quad coil.
Note drawing identical to Quad Coil Holder_CK, except lacking channel feature here. Use same tolerances as with Quad Coil Holder_CK.

Figure B.7  Quad coil holder.
Figure B.8  Ioffe coil.
Figure B.9  Ioffe coil holder bracket.
Figure B.10  Flexure mount (dimensioned).
Figure B.11 Breadboard.
Figure B.12  Tapered amplifier body.
References


