CLASSICAL LOGIC AND ITS RIVALS

BY

BLAIR FRANKLIN GOODLIN JR.

DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Philosophy
with a minor in College Teaching
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2013

Urbana, Illinois

Doctoral Committee:

Professor Timothy G. McCarthy, Chair
Associate Professor Emeritus Hugh S. Chandler
Associate Professor Emeritus Steven J. Wagner
Associate Professor Emeritus Robert G. Wengert
ABSTRACT

Classical logic and a given nonclassical logic are, by definition, incompatible in some sense. In some cases, this incompatibility is innocuous. In other cases, the nonclassical logic is incompatible with classical logic on a fundamental level, such that the two logics can be seen as rival theories of logical entailment and only one of them can succeed. I will explore the structure of these cases of logical rivalry by considering three examples: Dummett’s antirealism, Putnam’s response to results of quantum mechanics, and Tye’s response to vagueness. I will show that, despite the differences between these cases’ motivations and methods, they nevertheless all conform to a particular framework in challenging classical logic. Moreover, these diverse cases all characterize classical logic as the result of an unwarranted generalization from a limited and apparently privileged realm of entailment.
To Victoria and Lenora

Everything they have taught me is more profound
than anything contained herein
ACKNOWLEDGMENTS

I would like to thank my advisor, Tim McCarthy, for all of his guidance and effort as this project finally comes to fruition. For many years, he has answered my questions and read my drafts with great patience. I could not have been successful without his help.

I would also like to thank the other members of my Doctoral Committee. Hugh Chandler has been a part of this project from the very beginning, joining Tim and me in conversations during our independent study that produced my original dissertation proposal. His comments are often as helpful as they are disarming. Steve Wagner taught the seminar that first exposed me to the idea of conflict between logics, setting me on the course that has led me to this point. Besides his expertise, I must also thank him for his encouragement over the years. Bob Wengert has done an incredible amount of work in the short time he has been a part of my committee. I am very grateful for the quality of his feedback and the speed with which he has supplied it.

The strength I have had to complete this degree is not my own. My parents, Debra Bazzini and H. Richard Powell, have been sources of constant encouragement throughout graduate school and all my life before it. My mother deserves special mention for having fostered my love of learning that endures to this day. She taught me how to read and how to take notes, laying the founda-
tion for the present work. My wife, Bernadette So-Goodlin, has been by my side since college. Her unwavering support has sustained me through my times of frustration. I could not have asked for a better companion and helpmate. My daughters, Victoria and Lenora, have been an inspiration to me and have offered their own brand of motivation. I can only hope to be half the man they think I am, and perhaps finishing graduate school is a step in that direction.
# TABLE OF CONTENTS

**CHAPTER 1: DEFINING RIVALRY**

1.1 Overview of the Project ........................................ 1
1.2 Defining Rivalry ........................................... 5
1.3 Other Challenges to Classical Logic ......................... 17
1.4 Rivalry as a Fundamental Challenge ....................... 24

**CHAPTER 2: AN OBJECTION TO RIVALRY** ................... 26

2.1 Introduction .............................................. 26
2.2 The Strict Inferentialist Objection ......................... 27
2.3 Examples of Strict Inferentialism ......................... 30
2.4 Responding to the Objection ............................ 41
2.5 Expanding the Response ................................. 56
2.6 Sketching a Case of Rivalry ............................ 58

**CHAPTER 3: DUMMETT'S CHALLENGE TO CLASSICAL LOGIC** ...... 60

3.1 Introduction .............................................. 60
3.2 Dummett’s Theory of Meaning .......................... 60
3.3 Correlating Abilities and Sentences .................... 71
3.4 The Limits of Abilities ................................... 74
3.5 The Challenge to Classical Logic ....................... 80
3.6 Dummett’s Challenge as Rivalry ....................... 85

**CHAPTER 4: AMENDING DUMMETT’S CHALLENGE** ............ 89

4.1 Introduction .............................................. 89
4.2 Amending Dummett’s First Stage ....................... 92
4.3 The Role of Spatiotemporal Focus ..................... 100
4.4 Another Challenge and Diagnosis .................... 108
4.5 Conclusion .............................................. 115

**CHAPTER 5: THE GENERAL STRUCTURE OF RIVALRY** ........ 117

5.1 Introduction .............................................. 117
5.2 The Framework of Rivalry ................................ 117
5.3 Dummett ............................................... 125
CHAPTER 1
DEFINING RIVALRY

1.1 OVERVIEW OF THE PROJECT

Given two logics, there is an excellent chance that they will disagree on the validity of at least one inference. Some cases of “disagreement” will be entirely uninteresting; e.g., $\frac{\varphi}{\Box \varphi}$ will be valid in S5 and not in classical logic, but this inference is invalid in classical logic because $\Box$ is not a logical constant in that system. In these uninteresting cases, we can explain away the disagreement somehow, perhaps by noting that that the logics differ in scope, as in this example, or by noting that the two logics are the result of two different projects, as would be the case if we compared S5 to a deontic logic. Each way of differentiating scopes and projects describes a new standard against which to judge a logic. So if we define a logic in general as a theory of entailment, then we might describe S5 more specifically as a theory of entailment involving necessity, and we would judge it according to how well it captures this limited case of entailment. We would not fault it for failing to capture entailment involving, say, obligation. These uninteresting cases are not considered cases of conflict just because we hold the logics to two different standards, where these standards are partially determined by the logics’ projects and scopes. In this way, we explain away any substantive disagreement between the logics in such cases. The
The core concept of my project will be that of *logical rivalry*, which arises when disagreement between two logics cannot be explained away, and so one of the logics must fail as a theory of logical entailment.

Cases of rivalry involve philosophical positions that claim to show that classical logic is flawed and endorse a rival logic in place of classical logic. I suggest that, given such a position, the force of its challenge to classical logic is indicated by the severity of the incompatibility between classical logic and the position's endorsed rival logic. This method may not be a rigorous way of quantifying the strength of a challenge to classical logic, but I do think that it allows us to determine which challenges present the greatest threat to classical logic's being a successful theory of logical entailment. In the course of this project, I will consider three such challenges to classical logic: Dummett's antirealism, Putnam's response to results of quantum mechanics, and Tye's response to vagueness. Each advocates our abandoning classical logic in favor of a rival logic—intuitionistic logic in the first case, three-valued logic in the second, and quantum logic in the third. Yet each challenge attacks classical logic's status as a theory of logical entailment on a different basis: Dummett claims that it fails to respect truth, Tye claims that it fails to respect the existence of vague sets, and quantum logic claims that it fails to respect physical reality. I will conclude that, despite these differences, all employ the same core objection to classical logic: that it is the result of an unwarranted generalization from some core set of inferences to the set of all inferences. Because I am analyzing the structure of these challenges, rather than their success or failure, I will be using relatively old sources for each of these. As we will see, the structure of these challenges and their characterization of classical logic remains the same throughout their
history, despite changes to supporting elements of the argument in responses to objections and other developments. Dummett’s later work in particular has become especially complex as he has responded to criticism, but his basic challenge remains the same as when it was stated more simply in earlier work.

In this chapter, I will present a formal definition of rivalry between classical first-order logic and an arbitrary nonclassical first-order logic. In the process of doing so, I will specify a formal language in which to express the logics and a translation function between the formal language and a natural language. I will define rivalry as an incompatibility between the classical and nonclassical logics’ entailment relations. I will also examine three other arguments in the literature that describe similar types of incompatibility—logical pluralism, alternate translation functions, and the problem of logical constants—and suggest that rivalry is a more fundamental kind of incompatibility than any of them.

That said, rivalry faces a sweeping objection. In ordinary language, when two apparently identical tokens of a word are used in significantly different ways, we conclude that the two “tokens” are actually instances of two different meanings of a given word. When we determine this, we write off any arising conflict as mere verbal disagreement. One may object that the same thing happens with rivalry. If an inference is valid in classical logic but not in the rival logic under investigation, then the logical constants appearing in that inference are used differently in the two logics. Since any logic is a formal system, albeit an interpreted one, any such difference in usage is significant, and so rivalry also reduces to a verbal disagreement.

In my second chapter, I will investigate and respond to this objection, including three specific instances of this objection due to Carnap, Quine, and
Davidson. Each of these instances claim that the meaning of a logical constant is exhausted by its use. To show that this claim fails, I will use Gödel's negative translation of the intuitionistic connectives to define connectives that would each be used in the same way as a corresponding classical constant, yet have a meaning different from that classical constant. I will apply this result to each of the three specific objections, and thus clear the way for my project to proceed.

As mentioned above, my project will examine the challenges to classical logic by Dummett, Putnam, and Tye. Dummett’s challenge is most deserving of special attention and will be the centerpiece of my dissertation. His position is notoriously dense and occasionally obscure; but, out of all three challenges, his position arguably offers the most robust philosophical underpinning for the resulting rival logic. His philosophy of language, which forms the foundation for his rival logic, involves our generalizing from a core set of inferences to a larger set of inferences. However, his explanation of this mechanism is unclear, which is problematic for me, because I want to make clear his claim that classical logic arises from a misguided application of this mechanism.

My third and fourth chapters will be devoted to Dummett’s position. In my third chapter, I will attempt to elucidate his position, while highlighting its relevance to my project and offering several criticisms and small corrections. In my fourth chapter, I will offer an amended version of his position, which will address the problem of unclear generalization described above. This revision will be my attempt to maintain as much of his original challenge as possible while making changes that Dummett would accept. In a sense, this will not be a new version of his position, but rather an adaptation of his position to my project.

In my fifth and final chapter, I will describe how the amended version of
Dummett’s challenge characterizes classical logic as the result of unwarranted generalization from a core set of inferences. I will then turn my attention to vagueness and quantum logic. In each case, I will describe the challenge, specify the way in which it employs generalization of inferences, and give an account of how one could argue that classical logic may arise from misapplication of that generalization. I will then investigate the implications of these challenges’ sharing a core characterization of classical logic, and suggest future directions for research.

1.2 DEFINING RIVALRY

Rivalry will be a formal incompatibility between two logics’ entailment relations. A description of this relationship, which will require a rigorous definition of a logic, will include an explicit mentioning of its entailment relation that will represent an attempt to capture entailment as it appears in natural language. I will first define a single formal language to be employed by all the logics we discuss. After using this formal language as the basis for an uninterpreted system, I will discuss a translation function between the natural and formal languages that will allow us to interpret the system as a full-fledged logic. Finally, I define the relationship of rivalry between two such logics.

1.2.1 The Formal Language

Let \( F \) be a set of formulas built from a set of symbols \( F \). We will identify the formal language described above with the set \( F \). Call \( F \) the *vocabulary* of the language \( F \). \( F \) will contain all and only the strings of symbols of \( F \) that are well-formed—the wffs—as defined by rules for the use of the members of \( F \). Shortly,
we will divide the symbols of $\mathcal{F}$ into categories according to the rules for their use. One reason we do this is because natural language operates in this way: In simple cases, nouns are words that name objects; verbs are words that operate on nouns to form sentences; adjectives operate on nouns to produce noun phrases, which verbs can operate on; adverbs operate on verbs to produce verb phrases, which can themselves operate on nouns or noun phrases to produce sentences. Since the whole point of formal logic is to see how the structure of a sentence contributes to its logical properties, we want the sentences of $\mathcal{F}$ to have expressive power similar to sentences of English. It would be wholly uninformative, for example, if $\mathcal{F}$ contained only sentence letters for every sentence of English.

We can categorize all the symbols in $\mathcal{F}$ by describing the kind of symbol(s) they operate on and what kind of symbol each one produces. To do this, we first define a set $\text{Syn}$ that will serve as a set of indices for subsets of $\mathcal{F}$. We will define $\text{Syn}$ inductively:

(i) $\mathbb{N} \in \text{Syn}$
(ii) $\langle \tau, \langle \sigma_1, \ldots, \sigma_n \rangle \rangle \in \text{Syn}$ iff $\tau \in \text{Syn}$ and $\sigma_i \in \text{Syn}$ for all $\sigma_i$

$\text{Syn}$ will have no other members. For the sake of convenience, we often abbreviate $\langle \sigma_1, \ldots, \sigma_n \rangle$ as $\vec{\sigma}$. We call the members of $\text{Syn}$ syntactic indices. The indices defined in (i) are the basic syntactic indices; all other members of $\text{Syn}$ are derived syntactic indices.

Every member of $\mathcal{F}$ is assigned exactly one syntactic index. Basic syntactic indices will be assigned to sentence letters and constant symbols, at minimum. We may assign basic syntactic indices to other symbols if it suits our purposes. Derived syntactic indices will be assigned to symbols based on the rules for
those symbols’ use. Suppose $f$ is a member of $\mathbf{F}$ and is assigned a derived syntactic index, say, $\langle \tau, \langle \sigma_0, \sigma_1 \rangle \rangle$. The number $\tau$ is the syntactic index of the string of symbols produced by $f$. The pair $\langle \sigma_0, \sigma_1 \rangle$ indicates the syntactic indices of the symbols upon which $f$ operates in the following way: $f$ can operate on any two symbols that each produce symbols of syntactic index $\sigma_0$ and $\sigma_1$, respectively. For example, if sentences and constants are assigned the indices 0 and 1, respectively, then two-place predicates will be assigned the index $\langle 0, \langle 1, 1 \rangle \rangle$, indicating that they can produce a sentence by operating on two constant symbols.\(^1\)

I will adopt the existing convention of writing the syntactic index $\langle \tau, \bar{\sigma} \rangle$ as $\tau/\bar{\sigma}$. Also, when discussing some arbitrary syntactic index $\tau/\bar{\sigma}$, this should be understood to range over the basic syntactic indices also, in which case $\bar{\sigma}$ would be empty. These two notational conventions reflect Ajdukiewicz’s original way of “cancelling out” syntactic indices, as one does with fractions, to determine whether a given expression was well-formed (see Ajdukiewicz 1961, §5). Let $\mathbf{F}_{\tau/\bar{\sigma}}$ be the set of all the symbols of $\mathbf{F}$ with syntactic index $\tau/\bar{\sigma}$. We call the set $\mathbf{F}_{\tau/\bar{\sigma}}$ a syntactic category of $\mathbf{F}$. Since each member of $\mathbf{F}$ has exactly one syntactic index, the syntactic categories form a partition of $\mathbf{F}$.

We are constructing $\mathbf{F}$ in such a way that its wffs will have this categorical structure, just as grammatical sentences of English do. Some may object that the structure of English words’ operations is more complicated than allowed for in the definition of Syn. For example, Lambek suggests that the location of operands matters for certain words in English (see Lambek 1958). So while

\[^1\text{Note that we do not require that two-place predicates produce a sentence only by operating on two constant symbols. Two-place predicates’ syntactic index of } \langle 0, \langle 1, 1 \rangle \rangle \text{ indicates that they can operate on any symbol with a basic syntactic index of 1 or a derived syntactic index of the form } \langle 1, \langle \sigma_1, \ldots, \sigma_n \rangle \rangle; \text{ i.e., any symbol that is a constant or names an object through some operation.}\]
poor operates on a noun to produce another noun, poor John is well-formed while John poor is not. If we must describe the location of an English expression’s operands as well as their syntactic indices, then we must add that information to our definition of syntactic indices for $\mathcal{F}$. Another argument for English’s grammar outrunning this structure is due to Lewis, who has argued that we need to draw a distinction between names and common nouns in English, so we cannot speak of one basic syntactic category of “terms” in $\mathcal{F}$ (see Lewis 1972).

There is no need to address these concerns here. If one were convinced by Lambek, we could have the odd-numbered positions of the sequence $\bar{\sigma}$ correspond to left-hand operands and the even-numbered positions of the sequence $\bar{\sigma}$ correspond to right-hand operands. If one were convinced by Lewis, we could easily make use of another basic index of Syn. I am not asserting that the way I have outlined is the correct way to partition $\mathcal{F}$—this would require far more knowledge of linguistics than I possess—but rather to illustrate a plausible way of doing so.

1.2.2 Logical Frameworks

We can now turn our attention to constructing the formal systems around $\mathcal{F}$ that we will later interpret to form a logic.

Let $\vdash$ be a two-place relation between subsets of the formal language $\mathcal{F}$, and $D$ be a nonempty subset of the vocabulary $\mathcal{F}$. We will call a structure of the form $\langle \mathcal{F}, \vdash, D \rangle$ a logical framework, given that $\vdash$ conforms to certain restrictions that will be described below.

Given a logical framework, the set $D$ will be the set of logical constant sym-
bols for that structure and will be called its dialect. For example, classical first-order logic’s dialect is \{ ∨, ∧, →, ¬, ∀, ∃\}; modal logic’s dialect is \{ ∨, ∧, →, ¬, □, ◊, ∀, ∃\}.²

Eventually, the relation \( \vdash \) will be interpreted as a given logic’s attempt to capture our ordinary notion of logical entailment. Knowing this, \( \vdash \) will be called an entailment relation. Note that \( \vdash \) is a relation that holds between subsets of \( F \), rather than holding between a subset and a formula. The entailment relation is characterized this way so we will have the option to more easily compare logical entailment with proof-theoretic and model-theoretic entailment. In the former case, logical entailment can hold between two sets of sentences as defined by the terms of a sequent calculus. In the latter case, we can require that the conclusion of any inference be a singleton.

Not just any relation qualifies as an entailment relation. We require that whenever any valid inference’s nonlogical symbols are uniformly replaced by other appropriate nonlogical symbols, the resulting inference is also valid. Because this is something we require of all logics, we establish this requirement at the level of logical frameworks: Suppose \( \varphi \in F \) and let \( D \) be a set of logical constants. If \( \lambda : F \rightarrow F \), \( \lambda \) will be a \textit{D-morphism} just in case \( \lambda(\varphi) \) is the result of substituting all occurrences of a given symbol in \( \varphi \) with another symbol of the same syntactic category, while any occurrence of a member of \( D \) in \( \varphi \) is left

²One last note on terminology: the word \textit{language} is used in various ways in the literature. In creating a theory of arithmetic with the Robinson’s axioms, for example, the tuple \( \langle s, +, <, 0 \rangle \) may be referred to as the “language of arithmetic.” Its purpose is to specify which nonlogical symbols will be used with the usual symbols of first-order logic to generate the wffs of arithmetic. One can see from this that this use of \textit{language} assumes which symbols are considered logical. We can also speak of a “first-order language,” where we are just specifying which symbols are the logical constants. In either case, \textit{language} is being used to specify the logical terms or to specify the nonlogical terms. My use of \textit{formal language} is an attempt to avoid this distinction and allow each logic to determine which symbols of the formal language will be treated logically. These symbols are that logic’s unique way of interpreting the formal language, and are thus that logic’s dialect.

9
unchanged.\(^3\) Let \(\Lambda\) be the set of all \(D\)-morphisms. For any set \(A = \{\alpha_1, \ldots, \alpha_n\}\), let \(\lambda(A) = \{\lambda(\alpha_1), \ldots, \lambda(\alpha_n)\}\). The relation \(\models\) will be an entailment relation, and thus \(L = \langle F, \models, D \rangle\) will be a logical framework, just in case the following two restrictions hold:

\[
\Gamma \models \Phi \iff \forall \lambda \in \Lambda (\lambda(\Gamma) \models \lambda(\Phi))
\]

\[
\Gamma \models \Phi \iff \forall \lambda \in \Lambda (\lambda(\Gamma) \models \lambda(\Phi))
\]

1.2.3 The Translation Function

Since logical entailment is a relation that holds between sets of statements of English, and a logic will be ordering statements of a formal language, we must define a translation function between the two languages. Let \(E\) be a set of grammatical statements of disambiguated English.\(^4\) Since a theory of logical entailment will simultaneously be a theory of how truth behaves with respect to the

\(^3\)Here is a more precise definition of \(D\)-morphisms: Let \(\varphi\) be a formula of \(F\). The formula \(\varphi\) is a string of symbols of the vocabulary \(\overline{F}\), built up according to the syntactic categories of those symbols. Let \(\hat{\varphi}\) be the sequence of these symbols in the order in which they appear in \(\varphi\). Let \(\lambda(\varphi)\) be the length of \(\hat{\varphi}\) and \(\pi_i\) be the projection function that will map \(\hat{\varphi}\) to the \(i\)th member of \(\hat{\varphi}\). Then \(\lambda\) is a \(D\)-morphism just in case all the following hold:

1. \(\lambda : F \rightarrow F\)
2. \(\text{len}(\hat{\varphi}) = \text{len}(\lambda(\hat{\varphi}))\)
3. \(\forall i \leq j \leq \text{len}(\hat{\varphi}) \left( \pi_i(\lambda(\hat{\varphi})) = \begin{cases} \pi_i(\hat{\varphi}) & \text{if } \pi_i(\hat{\varphi}) \in D \\ \alpha \in \overline{F}_\tau / \overline{F}_\sigma & \text{otherwise, where } \pi_i(\hat{\varphi}) \in \overline{F}_\tau / \overline{F}_\sigma \end{cases} \right)\)
4. \(\forall j < \text{len}(\hat{\varphi}) \left( \pi_j(\hat{\varphi}) = \pi_j(\lambda(\hat{\varphi})) \rightarrow \pi_j(\lambda(\hat{\varphi})) = \pi_j(\lambda(\hat{\varphi})) \right)\)

The third condition describes how \(\lambda\) will substitute only the nonlogical constants of a formula, where the set of nonlogical constants is relative to the logic \(L\). The fourth condition ensures that \(\lambda\) will do this uniformly.

\(^4\)The function described below will translate from disambiguated English to the formal language. From this description alone, it should be clear that ordinary English will require quite a bit of cleaning up before passing muster as “disambiguated English.”
logical constants, the sentences of $E$ must be those expressions of English that can possess a truth value.$^5$ Interjections and non-rhetorical questions, for example, will not appear in $E$.$^6$

For my purposes, I do not want to use translation functions from English sentences to their schemata. Rather, we will require that a translation function be one-to-one. I also want a translation function to have a certain respect, so to speak, for the links between the sentences of $E$: which predicates are represented by which symbols of $F$, whether or not a particular name has already been assigned a constant symbol, etc. For example, the stipulation that a translation function be one-to-one is enough to ensure that the sentences

- If Alice loves Alice then Bill loves Alice
- If Alice loves Alice then Carl loves Alice

will have distinct images. But we also want the translations to reflect the fact that the antecedents of the sentences are the same. So we require that the translation function use the same string of symbols to represent the antecedents in these sentences’ images. A translation function that does this will be called a linkage-preserving translation function.

Ideally, our translation function would be transparent: if $e \in E$, $\varphi \in F$, and $\rho$ is a transparent linkage-preserving translation function where $\rho: e \rightarrow \varphi$, then $\varphi$ should have as many symbols as $e$ has words, and the syntactic categories of the symbols of $\varphi$ should be the same as the syntactic categories of the words

---

$^5$To avoid a charge of question-begging against certain challenges to classical logic, the can used here should not be interpreted as implying that all sentences of $E$ actually have a truth value.

$^6$These are just examples of expressions that will not be in $E$, not an exhaustive list. As we will see, the decision of what sentence “can possess a truth value” dictates which sentences belong in $E$, and thus what theory of logical entailment we ought to adopt.
of $e$ to the greatest extent possible. The translation function we choose will not fulfill these conditions in all cases, however. For example, $E$ will contain sentences involving second-order quantification; since we are studying rivals to classical logic—a first-order theory—$\overline{F}$ will contain only first-order quantifiers. So there are no symbols of $\overline{F}$ with a syntactic index correlating to the way second-order quantifiers are used in English. In such cases, $\rho$ will map second-order-quantified English sentences to zero-place predicates of $F$.

It will also be convenient to require that linkage-preserving translation functions be onto. Doing so requires our paying attention to what formulas we want to have in $F$, since a linkage-preserving translation function’s being onto means that its inverse is defined for all formulas in $F$. But this will be unintuitive because there are formulas in $F$ that cannot be expressed as sentences of English as we ordinarily speak it. We have defined $\text{Syn}$ and $F$ in such a way that there is an infinite number of syntactic indices; a slight modification in the above definition would allow for an infinite number of syntactic categories.\(^7\) Moreover, we have not placed any limit on the length of the wffs of $F$, although it seems reasonable to dictate that they be of finite length. So there may be an infinite number of wffs of $F$ and $F$ may contain arbitrarily long wffs of finite length. We call $F$ maximal if it has such a structure. We call $E$ maximal just in case there is a one-to-one correspondence between it and $F$, where $F$ is maximal.

If $F$ is maximal, and if $\rho$ is one-to-one and onto, then $\rho$ would commit us to the existence of some very strange expressions of English. $E$ would contain sentences that we never could express because they would be interminably long.

\(^7\)Above, syntactic categories were defined with respect to the syntactic indices of the symbols of $\overline{F}$. So there were only as many syntactic categories as there were syntactic indices actually assigned to symbols of $\overline{F}$. Instead, we could dictate that there is a syntactic category for every syntactic index.
practically speaking. Certain sentences of \( E \) would contain practically unusable words whose rules for their grammatical usage would reflect the incredible complexity of the syntactic indices of their correlates in \( F \).

This situation seems to cry out for our rejecting \( F \)'s being maximal. But suppose we were to reject \( F \)'s being maximal. In order to justify the restriction, we would need to describe \( E \) as some well-defined set of useful sentences of English and then stipulate that \( F \) be just strong enough to capture those sentences. For example, we could define \( E \) as containing only those expressions of English that are actually truth-bearing. Such a definition would be ambiguous, of course. We would need to specify that we were referring to all sentences that had been spoken prior to a particular time by a particular population in a particular language defined by experts to be English. Suppose that, once we made this specification, we would have a well-defined set of sentences. Any satisfactory theory of logical entailment would need to describe how logical entailment behaved in these sentences. We will call \( E \) minimal if it contains only our actual truth-bearing expressions of English, given a fixed definition of what constitutes our “actual” truth-bearing expressions of English. Call \( F \) minimal just in case there is a one-to-one correspondence between it and \( E \), where \( E \) is minimal.

It now seems that we have two conflicting instincts. On the one hand, we do not want \( F \) to be maximal because doing so would place many more sentences in \( E \) than we need to analyze our ordinary concept of logical entailment. On the other hand, we do not want \( E \) to be minimal. If it were, our theory of logical entailment would apply only to those sentences; it could not speak to the validity of any inferences spoken after the fixed point (unless, of course, some-
one was just repeating previously-spoken sentences in an inference). We do not want to discuss the validity of inferences including only actual sentences, but the validity of inferences involving possible sentences also.

This conflict suggests that we make some decision on the membership of $E$ such that $E$ will be neither minimal nor maximal. I want to resist this suggestion. Any expansion of $E$ that is meant to include the “possible” truth-bearing expressions of English, short of making $E$ maximal, would involve labelling certain grammatical expressions of English as “impossible.” Any such distinction seems arbitrary. Although our notion of possibility here is ambiguous, the real problem is that it is not obvious how we ought to justify our claim that some expression is possible but not actual. There is a fact of the matter as to whether or not a given expression is actual: whether or not it had been expressed in accordance with the parameters given by our specification of “actuality.” There does not seem to be a similar fact to justify any given possible/impossible distinction over another.

In the absence of a way to adjudicate between the extensions of the minimal version of $E$, I believe we ought to use a maximal version of $E$. Doing so allows us to avoid arbitrary decisions of what concept of possibility is relevant to our purposes. By making $E$ maximal, we are considering every possible extension of the minimal version of $E$. Also, making $E$ maximal allows us to make $F$ maximal, which simplifies the project because we need not worry about restricting the length of the wffs of $F$, or the complexity of the syntactic indices corresponding

---

8 Note that the phrase possible truth-bearing expressions of English does not mean the same as possibly truth-bearing expressions of English. The “possible truth-bearing expressions of English” are the truth-bearing expressions of English that are contingently expressible. If we identified $E$ with this set, we would exclude those sentences that are truth-bearing but were impossible to express. The “possibly truth-bearing expressions of English” are the truth-bearing expressions of English that are contingently truth-bearing. If we identified $E$ with this set, we would exclude those sentences to which it is impossible to assign a truth value.
to syntactic categories, etc. In what follows, little will actually depend on the
decision to have \( E \) be maximal; when things do, it will be noted.\(^9\)

Now, finally, we can define a logic. Given some logical framework \( L = \langle F, \models, D \rangle \)
and some linkage-preserving translation function \( \rho \), a *logic* \( \mathcal{L} \) will be identified
with the structure \( \langle \rho, L \rangle \).

Note that \( \models \) is still not identified with either model-theoretic consequence
or proof-theoretic consequence, even in the presence of a translation function
interpreting the logical framework. Both of these are formal notions explaining
why it would be the case that \( \Gamma \models \varphi \). That the logical entailment relation is
distinct from the proof-theoretic consequence relation should be obvious. Distingu-"ghing logical entailment from model-theoretic consequence requires justifi-
cation because the entailment relation has been shown to be equivalent to the
relation of model-theoretic validity under certain circumstances. Specifically, it
has been shown that

\[
\Gamma \models \varphi \iff \Gamma \models \varphi,
\]
given that the language of \( \Gamma \) and \( \varphi \) is rich enough to express elementary num-
ber theory (Quine 1986, 53–55).\(^{10}\) Despite this, I will not identify logical en-
tailment with model-theoretic validity because we have not established that \( E \)
is rich enough to express elementary number theory. \( E \)'s being maximal was a
simplifying assumption, not an established fact.

\(^9\)It will be interesting, later on, to discuss whether Dummett would claim that our problem
was that \( E \) should not have been made maximal; i.e., that we should not be investigating logical
entailment between sentences that cannot realistically be expressed (in the context of canonical
application). Or whether Dummett's claim is that \( E \) erroneously counts certain expressions as
truth-bearing. Or both.

\(^{10}\)Quine's notion of substitutional validity discussed in this reference is not exactly the same as
my definition of the entailment relation. However, I think we can define a translation between
the two by positing a further language whose wffs are ordered according to Quine's substitu-
tional validity, and a many-to-one translation function mapping wffs of \( F \) onto their schemes in
the new language.
1.2.4 Rivalry and Restricted Entailment

My project involves rivalry between classical and nonclassical logics, not between arbitrary logics. Going forward, I will refer to rivalry and rival logics without including this qualification.

Let classical logic be represented by $\mathcal{L}_c$ and its framework by $L_c$, where $\mathcal{L}_c = \langle \rho, L_c \rangle$ and $L_c = \langle \mathbf{F}, \models_c, D_c \rangle$. To define rivalry, we will use $D_c$-morphisms—a specific instance of the general idea of $D$-morphisms defined above—to define a restriction on the entailment relation of some nonclassical logic $\mathcal{L}_n$. Let $\Lambda_c$ be the set of all $D_c$-morphisms. We will say that $\Gamma \models_n \Phi$ iff both the following obtain:

(i) $\Gamma \models_n \Phi$

(ii) $\forall \lambda \in \Lambda_c \left( \lambda(\Gamma) \models_n \lambda(\Phi) \right)$

Suppose $\mathcal{L}_x$ is a logic, where $\mathcal{L}_x = \langle \rho, L_x \rangle$ and $L_x = \langle \mathbf{F}, \models_x, D_x \rangle$. Note that $\mathcal{L}_c$ and $\mathcal{L}_x$ share the same formal language and translation function, and that they both have exactly one entailment relation. If any of these structural elements differed between the logics, it would indicate that the two logics employ different methods for formalizing pretheoretic logical entailment. If the two logics disagree on the validity of a given inference, the possibility exists that this disagreement is the result of these differing methods of formalization. In such a case, we would say that the logics stand in methodological disagreement with one another.

Let a deviant logic be a nonclassical logic that disagrees methodologically with classical logic. We will call a nonclassical logic an alternative logic just in case it is not a deviant logic. A logic $\mathcal{L}_n$ will be an ally of classical logic just in
case it is an alternative logic and $\vdash_{n|c} = \vdash_{c}$. A logic $\mathcal{L}_n$ will be a rival of classical logic just in case it is an alternative logic and $\vdash_{n|c} \neq \vdash_{c}$.

1.3 OTHER CHALLENGES TO CLASSICAL LOGIC

The definition of rivalry uses classical logic as a reference point, like the origin of a coordinate system. My reason for doing this is both historical and practical: Classical logic has enjoyed a privileged status, even if only by convention, such that logics are compared to it. Because classical logic plays this role in the definition of rival logics, any rival logic is a rival to classical logic. So far, rivalry has been described as a purely formal relation between classical logic and a rival logic according to which the two logics’ entailment relations are incompatible with one another. But any logic is meant to be a theory of logical entailment, and any theory can capture its intended phenomena well or poorly. A rival logic thus represents a challenge to classical logic’s value as a theory of logical entailment by offering examples of how it captures logical entailment better than classical logic. In this section, I will discuss several other challenges to classical logic’s value as a theory of logical entailment: alternate translation functions, logical pluralism, and the problem of logical constants. I will show that all are unlike rivalry, in that the first two are cases of methodological disagreement and that the third is not a formal challenge to classical logic. Doing so will more clearly locate rivalry among other debates in the literature and lay the groundwork for my next section, in which I will contrast rivalry with those debates and argue that rivalry represents a more fundamental challenge to classical logic.

The first of these challenges is Susan Haack’s disagreement with classical
logic’s formalization of English sentences. Haack claims that the process of translating from the natural language to the formal language involves our choosing to ignore certain subtleties of our ordinary language. Because of this, we may end up with several different pieces of informal discourse represented by a single piece of the formal language. For example, “material implication, strict implication, relevant implication, and other formal conditions might all have some claim to represent some aspect of ‘if’, or … 2-valued and 3-valued and non-extensional disjunctions might all be possible projections of (some) uses of ‘or’ ” (Haack 1978, 230). It seems that a proper logic, according to Haack, would formalize these different uses of \textit{if} and \textit{or} with different logical constants, rather than translating all of them as “$\to$” and “$\lor$.” Let $\mathcal{L}_h$ be a theory of entailment that is nonclassical for the reasons Haack proposes. To make this example more concrete, suppose $\mathcal{L}_h$ differs from classical logic only because each interprets our English word \textit{if} differently, and does so in the way Haack describes. The question before us is how $\mathcal{L}_h$ differs from $\mathcal{L}_c$, and what kind of challenge this may present to classical logic as a theory of logical entailment.

The following are all statements of $E$:

\begin{enumerate}[(1.1a)]
    \item If the Earth is less massive than the Moon, then the Moon is more massive than the Earth.
    \item If sugar does not dissolve in water, then the sea is not sweet.
    \item If $2 = 5$, then Bigfoot exists.
\end{enumerate}

The classical logician would use $\rho$ to translate each of these, perhaps as:

$$L(e, m) \to M(m, e)$$
We add the information that the antecedents of all of these conditionals are false, and thus all of these would be valid under classical logic:

\[
\models c \neg L(e, m) \land (L(e, m) \rightarrow M(m, e)) \tag{1.3a}
\]

\[
\models c \neg W(a) \land (W(a) \rightarrow A(l)) \tag{1.3b}
\]

\[
\models c \neg = (2, 5) \land (= (2, 5) \rightarrow R(u)). \tag{1.3c}
\]

I assume that Haack would agree with the classical logician that (1.1a) is a genuine instance of a material conditional, and so (1.3a) is logically true. But Haack claims that apparently similar statements are not logically true; e.g., (1.3b) and (1.3c). The classical logician's mistake, Haack may claim, is that he assumes the included conditionals—(1.1b) and (1.1c)—are material conditionals when they are actually a strict conditional and a relevant conditional, respectively. What marks (1.1b) as a strict conditional, she would claim, is that it appeals to a putative natural law in its antecedent, and so the entire conditional can be understood as a natural law. What marks (1.1c) as a relevant conditional, she would claim, is the unknown truth value of the consequent, and so we demand that a conditional with such a consequent have an antecedent relevant to it.

Haack would claim that (1.1b) and (1.1c) ought to be translated using new connectives to represent strict and relevant implication. Let $\rightarrow$ and $\rightarrow^*$ be interpreted as the strict conditional and relevant conditional, respectively; $\mathcal{L}_h$ will have these two conditionals in its dialect in addition to $\rightarrow$. The above condi-
tionals then would be rendered in $L_h$ as:

\[
\models_h \neg M(e, m) \land (M(e, m) \rightarrow L(m, e)) \quad (1.4a)
\]

\[
\not\models_h W(a) \land (W(a) \rightarrow A(l)) \quad (1.4b)
\]

\[
\not\models_h \neg = (2, 5) \land (= (2, 5) \rightarrow R(u)). \quad (1.4c)
\]

We know that $L_c$ and $L_h$ disagree on whether or not (1.1b) and (1.1c) are logically true. But, as shown here, $L_c$ and $L_h$ also disagree on how (1.1b) and (1.1c) ought to be represented in the formal language. This latter disagreement makes the former lose its teeth.

$L_h$’s formalization of (1.1b) and (1.1c) is the result of $L_h$’s employing a disambiguation of English where material, strict, and relevant uses of if are disambiguated into three different connectives. Call this disambiguation $E'$. But then $L_h$ is a theory of entailment for sentences of $E'$, not $E$. $L_h$ must be using a translation function other than $\rho$ because the translation functions for $L_h$ and $L_c$ are defined on different domains, and because $E \nsubseteq E'$ and $E' \nsubseteq E$. Because the difference in $L_h$’s and $L_c$’s disambiguations manifests itself in a difference of translation functions, we can record the methodological disagreement between $L_h$ and $L_c$ using the above definition for a logic; i.e., $L_h = \langle \rho, L_h \rangle$ and $L_c = \langle \rho', L_c \rangle$.

Another challenge is Beall and Restall’s logical pluralism. It claims that our pretheoretic notion of validity is inherently ambiguous, but can be captured by what they call the Generalized Tarski Thesis: “An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion” (Beall and Restall 2006, 29). Each possible way of disambiguating our pretheoretic notion
of validity corresponds to a possible way of interpreting the “cases” described
in the Generalized Tarski Thesis. For example, a given inference may be valid if
and only if, in every model in which the premises are true, so is the conclusion.

Let $\mathcal{L}_p$ be a logic that is pluralistic in the sense given above. According to
Beall and Restall, logical entailment comprises several entailment relations, one
for each interpretation of the cases described in the Generalized Tarski Thesis.
Moreover, none of these interpretations would be more fundamental or primit-
ive than the others, so we cannot define one in terms of another. Here, as
with $\mathcal{L}_h$, we can record $\mathcal{L}_p$ and $\mathcal{L}_c$’s methodological disagreement by speci-
fying that $\mathcal{L}_p$’s framework include multiple entailment relations and dialects:
$L_p = \langle F, \vdash_1, \ldots, \vdash_n, D_1, \ldots, D_m \rangle$.\(^{11}\)

So far, we have considered two challenges that classical logic faces, shown
how they are cases of methodological disagreement, and how that method-
ological disagreement is manifested in a difference between the formalizations
these challenges offer compared to $\mathcal{L}_c$ and $L_c$. A given rival logic, call it $\mathcal{L}_r$,
will not differ from $\mathcal{L}_c$ in either of the ways outlined above; and so we know
that rivalry should not be confused with either of these challenges. I will now
consider a more subtle challenge to classical logic stemming from the problem
of defining logical constants. A logic exemplifying this challenge, call it $\mathcal{L}_m$,
will differ from $\mathcal{L}_c$ in its dialect only. Therefore, $\mathcal{L}_m$ will not have a structure
that marks it as being different in kind from a rival logic—unlike $\mathcal{L}_h$ and $\mathcal{L}_p$—
because $D$-morphisms allow rival logics to have dialects that differ from $D_c$.

---

\(^{11}\) $L_p$’s being accepted as a genuine logical framework requires a small and innocuous stretch-
ing of the definition of a logical framework to allow multiples entailment relations and dialects.
Note also that the number of dialects will vary based on the specific pluralistic logic. It may
be that one dialect serves as the set of logical constants for all the entailment relations, or that
each entailment relation has its own dialect. All that we know for certain is that a framework
must have more than one entailment relation in order to be interpreted by a pluralistic logic.
What differentiates $\mathcal{L}_m$ from $\mathcal{L}_r$ is its advocate’s attitude toward the constants that appear in $D_m$ but not in $D_c$. To illustrate this, I will contrast classical logic’s relation to a deontic logic with classical logic’s relation to a logic that differs from it only in the addition of a new quantifier, “there are infinitely many.”

Let $\mathcal{L}_d$ be a deontic logic. The deontic logician is trying to generate a theory of our correct inferences about obligations. To do this, she forms a dialect with greater expressive power than $D_c$ by adding two operators to $D_c$: $O$, interpreted as “it is obligatory that,” and $P$, interpreted as “it is permitted that.” The difference between $D_c$ and $D_d$ can be explained by a difference in the goals of classical logic and deontic logic. Classical logic is topic-neutral. It concerns reasoning in general, rather than reasoning about a particular subject. Deontic logic fails to present a challenge to classical logic because its variance from classical logic is explained by the deontic logician’s interest in capturing a certain subset of our inferences more specifically (with a more expressive language) than classical logic. Informally speaking, deontic logic represents an entirely different project from classical logic, while simultaneously assuming that classical logic holds more generally. This relationship explains why $\mathcal{L}_c$ and $\mathcal{L}_d$ are allies according to the above definition.

Now, let the quantifier $Q_0$ be interpreted as “there exist infinitely many.” Let $\mathcal{L}_m$ be a logic using this quantifier, where the dialect $D_m$ is just $D_c$ with $Q_0$ added to it. $\mathcal{L}_m$ and $\mathcal{L}_c$ will disagree on the validity of some inferences. For example, the axioms of Robinson arithmetic ($Q$) form a theory of the natural numbers with classical logic. The standard model of $Q$ is countably infinite. So, according to the Löwenheim-Skolem Theorem, there exist uncountable non-standard models of $Q$ also. Note that we can define a relation $<$, interpreted as
“less than,” in \( \mathbb{Q} \):

\[
x < y \iff \exists z (x + z = y \land z \neq 0).
\]

Only in \( \mathcal{L}_m \), however, can we eliminate these nonstandard models because we can express the idea that a given number has only finitely many predecessors:

\[
\forall y \neg \mathbb{Q}_0 x (x < y).
\]

So the inference \( \frac{Q}{\forall y \neg \mathbb{Q}_0 x (x < y)} \) is valid in \( \mathcal{L}_m \) but not in \( \mathcal{L}_c \).

Unlike \( \mathcal{L}_d \), \( \mathcal{L}_m \) is meant to be topic-neutral in the same way that \( \mathcal{L}_c \) is topic-neutral. When we fix a specific level of neutrality, we can develop a theory as to what it is that makes a member of \( \bar{F} \) a logical constant for that level of neutrality. That theory will then guide us in choosing the members of \( \bar{F} \) that should be included in a dialect of a logic for that level of neutrality. Any adequate theory of logical constanthood must count the members of \( D_c \) as logical constants, so we can interpret any theory of logical constanthood as answering the question of whether or not the dialect of a topic-neutral logic ought to be equivalent to \( D_c \) or just include \( D_c \) as a proper subset. This answer often revolves around features that we want to preserve in a logic. For example, Lindström proved that any extension of classical logic will either fail to be compact, fail to be complete, or the Löwenheim-Skolem theorem will fail to apply to it. If we consider these to be necessary attributes of a logic, then we cannot but have the members of \( D_c \) be the logical constants of a topic-neutral logic. On the other hand, we may decide that there are concepts that are so fundamental to our reasoning that they must be considered logical constants, even if this means accepting the consequences enumerated by Lindström. For example, because \( \mathcal{L}_m \) rejects the nonstandard models of \( \mathbb{Q} \), the Löwenheim-Skolem Theorem fails to apply to it; moreover, \( \mathcal{L}_m \) is not compact. So we lose these
supposedly-desirable traits of a logic. Despite this, one may still think that having \( Q \) without nonstandard models is worth the loss of these traits.

The problem of logical constants is one of determining the correct set of logical constants for a topic-neutral logic. Once we settle on such a set, any logic whose dialect differs from it cannot be a proper theory of topic-neutral logical entailment. Unfortunately, my definition above does not account for a logic’s topic-neutrality, as this is not a formal notion. As a result, both \( \mathcal{L}_d \) and \( \mathcal{L}_m \) count as allies of classical logic, despite the fact that \( \mathcal{L}_m \) represents a challenge to classical logic, albeit a challenge other than rivalry.

1.4 RIVALRY AS A FUNDAMENTAL CHALLENGE

Above, I describe three challenges to logic other than rivalry: Haack’s view on the translation of the natural language into the formal language, logical pluralism, and addition of extra constants to \( D_c \) with the stipulation that these are the only constants of a topic-neutral logic. I claim that rivalry is a more fundamental challenge to classical logic than any of these.

First, note that these challenges question our choice of the translation function \( \rho \), the number of entailment relations a logic ought to have, and whether or not \( D_c \) was an appropriate choice for the dialect of a topic-neutral logic, respectively. All of these are open questions about how we ought to construct a logic, but none of them are questions that we must answer in order to build \( \mathcal{L}_c \) and thus study rivals to classical logic. I am not claiming that \( \rho \) is the only way we could translate from the natural language to the formal language; I only claim that we must have some translation function from \( E \) to \( F \), and \( \rho \) is the one \( \mathcal{L}_c \) uses. By identifying \( \langle F, \models_c, D_c \rangle \) as the framework of classical logic—a frame-
work with only one entailment relation—I am formalizing the fact that classical logic is equipped with only one entailment relation; I am not endorsing logical monism in the process. By leaving $Q_0$ out of $D_c$, I am not claiming that $Q_0$ is not a logical constant; I am only noting that $Q_0$ is not considered a constant of classical logic, even if it ought to be a part of the dialect of a more accurate topic-neutral logic.

None of these points show that rivalry is more fundamental than the other challenges, but it does show that the analysis of rivalry is not dependent on these challenges being settled. So, at least, rivalry is on the same footing as these challenges. What makes rivalry more fundamental than these other challenges is that rivalry focuses only on the differences between the entailment relations of $L_c$ and a rival logic. The entailment relation is the core of the theory; the point of a logic is to have this formal relation correspond to our informal notion of logical entailment. By focusing only on the entailment relations, we are free to discuss the question “what is the correct theory of logical entailment?” without focusing on matters of the best way to formalize that informal notion and the structure surrounding it. The concept of rivalry assumes only that there is some correct theory of logical entailment. When we find a case of rivalry, we have found two candidates for a correct theory of entailment, and we cannot attribute that difference to a difference in dialect or translation function. We are thus assured that we are analyzing logical entailment itself, and not letting the mechanisms we use for this study to get in the way.

That said, some have objected that this assumption is flawed, that there is no sense in which a logic can be “correct.” We will turn our attention toward this objection in the next chapter.
2.1 INTRODUCTION

This chapter will discuss an objection to the very possibility of rivalry: that any putative case of rivalry reduces to a verbal disagreement because the meanings of the rival and classical constants differ. For example, when the intuitionist claims \( \models_i \varphi \lor \neg \varphi \) for some \( \varphi \in \mathbf{F} \), and the classical logician claims \( \models_c \varphi \lor \neg \varphi \), the objector would claim that the connective \( \lor \) has different meanings in these two statements. This objection conflicts with the most straightforward account of the meaning of a logical constant: the member of \( \mathbf{E} \) to which the constant corresponds under the translation function. Under this straightforward account, the intuitionist’s \( \lor \) and the classical logician’s \( \lor \) both correspond to “or.” Recall also that the translation function is the same for both logics by hypothesis, so the objector cannot claim that each logic has a unique mapping from \( \lor \) to some member of \( \mathbf{E} \). To argue that the intuitionistic and classical logicians actually disagree on the meaning of \( \lor \), and that rivalry fails as a result, the objector will need a more robust theory of meaning for the logical constants.

The objection I will describe is based on a theory of meaning for the logical constants according to which the meanings of the logical constants is deter-
mined by—but not necessarily identified with—their use. Since the rival and classical logician's constants necessarily differ in their use, the objector claims, they necessarily differ in meaning and so any apparent rivalry should be dismissed. In §2.2, I will describe the theory of meaning and accompanying objection in terms of introduction and elimination rules for the logical constants. In §2.3, I will describe how Carnap, Quine, and Davidson have argued for versions of this type of theory. I will offer a response to the objection in §2.4, using Gödel's negative translations to undermine the idea that the meanings of the logical constants can be identified with their use. If I am successful, I will have shown that the rival logician's constants do not necessarily have meanings different from the classical constants, and thus the objection fails to be an a priori objection to the possibility of rivalry. In §2.5, I will formulate and respond to a weaker but more intuitive version of the objection found in §2.2. Finally, I will argue in §2.6 that the rival logician must have a response to this weaker objection. He must explain why his constants can be used differently and yet his challenge does not reduce to a verbal disagreement.

2.2 THE STRICT INFERENTIALIST OBJECTION

Inferentialism is the theory that the meanings of logical constants consist in some aspect of their use. For example, an inferentialist may identify a constant's meaning with the inferences involving it that we count as valid and the sentences involving it that we count as logically true. But use is not a precise term. We do not want to discuss agents' actual use of a constant. For one thing, agents make mistakes. Also, we want to include hypothetical uses of the constant. So inferentialist theories are often couched in terms of dispositions to
use a constant in a particular way. A logical constant’s *inferential role* is the set of inferences involving the constant that agents are disposed to make.¹

Introduction and elimination rules for a given constant can represent these dispositions, and thus its inferential role. Introduction rules for the constant capture those dispositions to infer sentences that include the constant from sentences that do not. Elimination rules for a given constant capture those dispositions to infer sentences that do not include that constant from sentences that do.

Above, I showed that the meanings of logical constants are important to the analysis of rivalry: a challenge of rivalry will fail if the classical and rival logicians’ constants have different meanings. But there is not yet consensus on how we ought to capture the meanings of the logical constants. One option is to identify their meanings with their inferential roles. This view has its origins in Gentzen: “The introductions represent, as it were, the ‘definitions’ of the symbols concerned, and the eliminations are not more, in the final analysis, than the consequences of these definitions” (1935, 80). But viewing this as a theory of meaning for the logical constants would be to overread him. Popper, however, does take this position, claiming that a logical constant’s “inferential definition” captures its meaning, and the body of these definitions serve as a demarcation between logical and nonlogical constants (1946–1947). Since Prior’s “tonk” reaction to Popper’s position and others like it, inferentialism has been focused on how to formulate the rules for logical constants such that there is a boundary between the ordinary logical constants and constants like “tonk.” For exam-

¹Of course, agents may have dispositions to make incorrect inferences. The burden is on the inferentialist to explain how the inferential role of a logical constant does not include these and other undesirable inferences. However, I will grant the inferentialist that these inferences are excluded, and that the inferential role of a constant contains only inferences that are the product of appropriate dispositions.
ple, Hacking argues that the logical constants must respect certain basic facts about deducibility in their introduction and elimination rules (1979). Harman argues that certain inferences are immediately obvious, and the rules for logical constants should be based on these immediate implications and inconsistencies (1986). Prawitz argues that the rules should embody assertion conditions (1979).

For our purposes, we can set aside the questions of how or why a constant’s inferential role determines its meaning. We are interested in knowing whether two constants’ meanings differ. So if a constant’s meaning is connected to its inferential role, we can ask whether a difference in two constants’ inferential roles indicates a difference in their meanings. Let strict inferentialism be a theory according to which any difference in two constants’ inferential roles implies a difference in their meanings.²

According to strict inferentialism, any challenge of rivalry will be dismissed on the grounds that the classical logician’s constants and the rival logician’s constants have different meanings. For suppose $\Gamma \vdash_c \varphi$ but $\Gamma \not\vdash_r[c] \varphi$, and let $C$ be the set of logical constants present in the formulas in $\Gamma; \varphi$. Then we should be able to trace the disagreement over the validity of this inference to at least one $c \in C$, where the rival and classical logician disagree on the introduction and/or elimination rules for $c$.³ So we know that the rival logician’s inferential role for $c$ differs from the classical logician’s. This inference had been putative

²Not all inferentialist theories are strict. For example, Morton argues that classical and intuitionistic disjunction should not be counted as having different meanings because they share a core of inferences that constitute their meaning (1973).

³This way of describing the disagreement between the rival logician and classical logician may seem overly abstract. But we should avoid saying that every member of $C$ is necessarily being used differently by the rival and classical logician. For example, $\vdash_c T \land (\varphi \lor \neg \varphi)$ but $\not\vdash_r[c] T \land (\varphi \lor \neg \varphi)$, where $\varphi \in F$ and $T$ is some tautology upon which $L_c$ and $L_i$ agree. In this case, $C = \{ \land, \lor, \neg \}$. But $L_c$ and $L_i$ agree in all cases on the use of $\land$. 

29
evidence for rivalry between the logics. But under a strict inferentialist theory, this inference is evidence for the rival logician's discussing constants' having meanings different from the classical logician's constants' meanings.

2.3 EXAMPLES OF STRICT INFERENTIALISM

Several strict inferentialist theories have appeared in the literature. In this section, I will consider three prominent ones, due to Carnap, Quine, and Davidson. I will describe each of their theories, their conceptions of inferential roles, and how rivalry is precluded under each specific version of strict inferentialism.

2.3.1 Carnap

According to Carnap, the project of constructing a logic is part of a more general project of constructing a language. He describes two methods for doing this.

The first method of language-construction is to establish the formation rules for sentences of the language, define a consequence relation for that language, and only then interpret the meanings of the symbols of the language with a semantics.\footnote{The two methods I discuss are described in Carnap 1939. Carnap's order of presentation for these methods the reverse of mine. In his text, the method that I present here first is referred to as "the second method;" the second method I present, "the first method." I have reversed their order to make this discussion clearer.}

Under this method, the rival logician is in no position to offer an argument that his constants mean the same thing as the classical logician's constants. For example, suppose the rival logician rejected \textsc{lem}. The meaning of his constant $\lor$, say, and the meaning of the corresponding classical constant would be determined by the syntactical rules for their use. Since the rival consequence rela-
tion and the classical consequence relation differ with respect to $\lor$, the meanings of the two constants must also differ. As a result, rivalry is precluded by this method.

Carnap’s second method of constructing a logic begins with a claim that the logical constants have some meaning prior to the formulation of a logical calculus (Carnap 1939, 27). In this case, the “semantical rules”—for example, saying that $p \land q$ is true just in case $p$ is true and $q$ is true—constitute the meanings of the logical constants. One can then claim that a given logical calculus is either correct or incorrect based on whether or not it respects these semantical rules. These semantical rules can determine the introduction and elimination rules for logical constants in the following sense: Given a logical calculus, the semantical rules for the connectives will determine which inferences are valid and which are not. Using this information, we can read off the introduction and elimination rules for the connectives. So while Carnap claims that this method locates the constants’ meanings in their associated semantical rules, we can express those meanings using introduction and elimination rules, thus making it easier to compare a constant’s inferential role with a person’s use of that constant. And so Carnap’s position offers a clear example of how we could identify the meaning of a logical constant with its inferential role.

Imagine the rival logician’s task, if he is trying to argue that his constants have the same meaning as the classical logician’s, even when he denies LEM. He is claiming that there is some $\phi \in \mathbf{F}$ such that $\not\models_{r,c} \phi \lor \neg \phi$, and thus the classical logician is wrong to claim that $\models_{c} \phi \lor \neg \phi$ for any $\phi \in \mathbf{F}$ because classical logic fails to respect the semantical rules for disjunction. The rival logician could argue that the symbol $\lor$ has the same meaning in his logic as well as classical
logic because the semantical rule for $\lor$ is the same in both cases: $p \lor q$ is true just in case $p$ is true or $q$ is true.

But rivalry is precluded under this second method of constructing a logic, also. Semantical rules are not specific enough to ensure that two constants' meanings are the same: "Since the assignment of the meaning[s] is expressed in words, and [are], in consequence, inexact, no conclusion arrived at in this way can very well be otherwise than inexact and ambiguous" (Carnap 1937, xv). In the above example, the semantical rule for $\lor$ is expressed in the metalanguage, and so the word true is ambiguous. Specifically, it is unclear whether it is meant to be understood as verification or not. One disambiguation would be captured best by an intuitionistic calculus; the other, classical. Of course, these are only two of the many different possible disambiguations of true. The rival logician made the mistake of thinking that the semantical rule for $\lor$ expressed above was precise enough that he and the classical logician agreed on it. If the rival logician could express his semantical rule for $\lor$ unambiguously with introduction and elimination rules, and if the classical logician did the same, then it would become obvious that the rival logician's $\lor$ and the classical logician's $\lor$ had different semantical rules. Ironically, the method of constructing a logic allowed the debate between the intuitionist's and classicalist's conceptions of truth to be briefly hidden by their common use of the word true in the formulation of the semantical rules. Because a connective's meaning consists in its semantical rules under this picture of constructing a logic, the two connectives would have different meanings. We would then have evidence that the rival logician's constants had meanings different from the classical logician's.

Carnap would conclude that rivalry is precluded under both methods of
constructing a logic. The first method leads to conventionalism about logic, so the rival logician loses the ability to say that classical logic is wrong in any sense. The second method entails that the rival logician's constants and the classical logician's constants have different meanings.

2.3.2 Quine

According to Quine, we can see that a logical constant's meaning is exhausted by its inferential role by reflecting on what it means to understand that constant, and he uses a hypothetical case of radical translation to show what this understanding consists in. Suppose that we are trying to translate a native's language by radical translation, and the native's language contained a word \textit{nat} that we suspected corresponded to the negation operator \textit{not} in our language. According to Quine, we would formulate a test for our hypothesis by first noting which truth function our word \textit{not} is meant to capture: For some sentence \(e\) of our language, we say that the sentence “not \(e\)” is true (false) just in case the sentence “\(e\)” is false (true). The test for our hypothesis must be a test to determine whether the native's word \textit{nat} also captures this truth function. Since we only have access to the native's behavior, and not his belief that a given sentence is true or false, we formulate our test in terms of assent or dissent. So we determine if the native's \textit{nat} corresponds to \textit{not} by outlining our own patterns of assent and dissent for sentences involving \textit{not}: We assent (dissent) to “not \(e\)” just in case we would dissent (assent) to “\(e\).” We then observe the native's patterns of assent and dissent for sentences involving \textit{nat}, and determine if they match with ours for \textit{not}. We could question the native as part of our testing procedure also, by taking some sentence to which the native assents (dissents),
changing it to include *nat*, and rephrasing it as a question to see if the native dissents (assents) to this new sentence. In all of these cases, the stimulation would be kept constant. Given that the native assents (dissents) to a sentence *n* of the his language, his dissent (assent) to “nat *n*” will be a piece of evidence confirming that we can translate his *nat* as our *not*.

These patterns of assent and dissent for negation can be given in terms of introduction and elimination rules. When we described Carnap's semantical rules in terms of introduction and elimination rules, those rules were said to describe the behavior of truth with respect to the logical constants. In this case, the introduction and elimination rules describe the behavior of assent with respect to the logical constants.

We can also extend this theory to other logical connectives. For example, suppose we were determining whether a native's words *ond* and *ar* correspond to conjunction and disjunction, respectively. A native's assent to a sentence containing *ond* will confirm *ond*'s corresponding to conjunction just in case the native also assents to all of the immediate subsentences of that sentence, and his dissent is confirming just in case he dissents from some immediate subsentence. Describing the assent and dissent patterns that would confirm *ar*’s corresponding to disjunction is more complicated. Quine states that the native's dissent to a sentence containing *ar* will confirm its correspondence to disjunction just in case the native also dissents to each subsentence (Quine 1960, 58). While Quine does not explicitly describe how the native's assent to *ar*-sentences confirms *ar*’s correspondence to disjunction, I think we can fill in the argument. Quine does not want to say that a native's assent to a sentence containing *ar* will confirm *ar*’s corresponding to disjunction just in case the native also assents to
at least one of the immediate subsentences of that sentence—this would de-
scribe intuitionistic disjunction, and Quine claims that we reason classically. If
the native assents to some sentence, and assents to an ar-sentence with it as an
immediate subsentence, that will confirm ar’s corresponding to or. And this is
exactly what we would expect, given the usual introduction rule for disjunction.
The elimination rule for classical disjunction is:

\[
\begin{array}{c}
A \lor B \\
C \\
C
\end{array}
\]

So Quine would adopt the position that a native’s assent to an ar-sentence would
confirm ar’s corresponding to disjunction only if all of the following obtain:

(i) there exists a sentence of the native’s language \( n \) such that, if the
native were to assent to one of the immediate subsentences of the
ar-sentence, he would assent to \( n \) also
(ii) if the native were to assent to the other immediate subsentence of
the ar-sentence, he would assent to \( n \) also
(iii) the native actually assents to \( n \)

Under this method of translating the native’s connectives, we cannot trans-
late any of the native’s sentences as conflicting with our logical truths. Suppose
we had a hypothesis that \( \text{ond} \) should be translated as \( \text{and} \) and \( \text{nat} \) by \( \text{not} \). As
described above, we would begin the process of testing this hypothesis by spec-
ifying our own patterns of assent and dissent for these constants. Note that
these patterns would reflect the fact that we never assent to a sentence of the
form “\( e \) and not \( e \).” So if the native were to assent to a sentence of the form
“\( n \) ond nat \( n \),” this would be evidence that his assent pattern for \( \text{ond} \) did not
match with ours for \( \text{and} \), or his assent pattern for \( \text{nat} \) did not match with ours

---

5Quine assumes that we reason classically. For the time being, we will grant him this.
for not, or both. And so some part of our hypothesis is false; we cannot understand the native as using one of our constants (in translation) with a different inferential role.

According to Quine, we are obligated to identify the meanings of the logical constants this way by the principle of charity, which dictates that we not translate the native as rejecting our logical laws. Suppose that, according to a putative translation manual, the native assented to a sentence “n ond nat n” that we translate as “e and not e.” The stimulation surrounding his assent is irrelevant, since our logical truths are stimulus-analytic and our logical falsehoods stimulus-contradictory, according to Quine. Suppose further that there is nothing to suggest that this is some non-truth-functional use of and and not, and that the native persists in his assent despite the linguist’s offers for him to recant. We then have good reason to reject this manual because Quine interprets a commitment to the principle of charity as a commitment to maximize the number of true beliefs, and minimize the number of false beliefs, that we attribute to the native. If we were to adopt this manual, we would be attributing a false belief to the native: that “e and not e” is true. Attributing this one false belief to the native is acceptable to charity, but in accepting this translation manual, we would also be accepting that the native’s use of and was not the same as ours. Thus, we would lose justification for saying that the native believes that all sentences of the form “a and not a,” except “e and not e,” were false. Indeed, the same would go for any logical falsehood of ours that included and. We would also lose justification for saying that the native believes logical truths including and were true. What’s worse, this may only be half of the story because the above sentence contains not also. The consequences of translating
the native's *nat* as *not* are similar to the above case with *ond* and *and*.

Any putative case of rivalry will be dismissed under Quine's account, since the patterns of assent and dissent for the rival and classical constants differ. He claims that the hypothesized patterns of the native's assent and dissent provided all we needed to understand the native's words—specifically, whether they corresponded to our logical connectives *not*, *or*, and *and*, or to some other truth function that may or may not be reflected in some logical constant of our language. And Quine believes that there is nothing else to which the rival logician can appeal in his defense: “There is no residual essence of [disjunction] in addition to the sounds and notations and the laws in conformity with which a man uses those sounds and notations” (Quine 1986, 81). Under this view, our *or* and the rival logician's *or* are different constants. The fact that they happen to be visually and aurally identical is irrelevant. “Here, evidently, is the [rival] logician's predicament: when he tries to deny the doctrine he only changes the subject” (Quine 1986, 81).

2.3.3 Davidson

Quine argues that we have no way of translating the rival logician's supposed denials of our logical laws into our own language such that the rival constants have the same meanings as the classical constants. The question of whether we can translate the rival logician as contradicting our logic is distinct from, but related to, the question of whether we can *understand* the rival logician as contradicting our logic. Davidson argues that we cannot understand a rival logician as doing this.

We could attempt to understand a foreign language by using a translation
manual from that language to our own. But we would succeed only if we also knew that the language into which the manual translates the foreign language is actually our own and that we can interpret our own language (Davidson 1973, 129). It is unclear how we ought to support these two further assumptions. Luckily, they can be avoided by not mentioning our own language. So if we are after a way of understanding the rival logician that avoids the need to justify these requirements, we should find a way of interpreting the rival logician's speech that does not require us to mention our language. According to Davidson, we can do this with a theory of truth for the rival logician's language that is expressed in our language, provided that the theory satisfies several restrictions. First, such a theory must be recursive, so as to enable an interpreter to interpret any of the potentially infinite number of sentences someone may express and do so on the basis of a finite number of rules. So the theory will have base axioms for determining the interpretations of the simple sentences, and recursive axioms that determine the interpretation of a complex sentence based on the interpretations of simple sentences it contains. Second, the theory should have as consequences all of the T-sentences that describe truth conditions for the sentences of the language to be interpreted.

Such a theory can serve as a theory of meaning, Davidson argues, because the totality of the T-sentences, their relations to one another, and the interdependence of the truth conditions give a sentence its meaning. How well a theory of this type describes the meanings of the interpreted language's sentences is a function of how many of these T-sentences are true. Success is measured this way because of how Davidson interprets the principle of charity: “We

---

6 Even under these circumstances, Davidson argues, we would not have a theory of interpretation because one of the things we would demand from such a theory is that it describe how the interpretation of a sentence depended on the interpretations of its parts.
want a theory that . . . maximizes agreement, in the sense of making [the natives] right, as far as we can tell, as often as possible” because “the more sentences we conspire to accept or reject…, the better we understand the rest, whether or not we agree about them” (Davidson 1973, 136–7).

So how do we maximize agreement with the natives? “We look for the best way to fit our logic, to the extent required to get a theory satisfying Convention T, onto the new language” (Davidson 1973, 136). Most of the T-sentences entailed by the theory will be generated by the recursive axioms, rather than given by the base axioms. A theory whose recursive axioms did not capture our logical constants would fail to maximize agreement between us and the native.

Davidson’s theory of meaning for the logical constants can be expressed in a way similar to Carnap’s theory. For Davidson, the logical constants are those that are defined in the recursive clauses of the theory of truth, and these are T-schemata, which are similar in form to Carnap’s semantical rules. However, we will change the account of what is described by the introduction and elimination rules. For Carnap, the rules described how truth behaved with respect to the logical constants. But Davidson’s position makes essential use of radical interpretation, where we only have access to the native’s utterances. We can take the native’s utterances to be a manifestation of the native’s belief that the sentence is true, according to Davidson, so it may be more appropriate to say that the rules describe how belief—specifically, the belief that the sentences involved are true—behaves with respect to the logical constants for Davidson’s theory.

So where does this leave the rival logician? Suppose the rival logician were to utter “e and not e,” where e is some sentence of his language. By asserting
this sentence, we understand him to have the belief that it is true. We would need to make sure that he did not simply misspeak, of course. For our language, which we are assuming is classical, we have the following two clauses in its theory of truth:

\[ T(\varphi \text{ and } \psi) \iff T(\varphi) \text{ and } T(\psi) \]

\[ T(\text{not-}\varphi) \iff \text{not-}T(\varphi) \]

So if the rival logician is attempting to speak our language, which would be required if he is trying to tell us an interesting fact about conjunction, then his words \textit{and} and \textit{not} must correspond to \textit{and} and \textit{not} in our language. But these words in our language are used as in the above two T-sentences. So if the rival logician's words are the same as ours, then we would be disagreeing with him. According to the principle of charity, this should make us less likely to accept the identify translation manual from the rival logician's language into our own. Specifically, this may make us less sure that the above T-sentences translate the rival logician's words \textit{and} and \textit{not}. We may then, in turn, be less sure that we agree with the rival logician on the truth of other sentences that involve \textit{and} and \textit{not}.

Nevertheless, Davidson's theory does not fully preclude rivalry. Davidson seeks to maximize agreement between the classical and rival logician. It may be that agreement is maximized even when some sentences are disagreed upon. The recursive clauses of the theories of truth for their languages would differ in such a case. This difference would be a case of rivalry if the rival logician can argue that he and the classical logician are seeking to describe the same concept of truth. The rival logician will be successful if he can argue that his
A disagreement between logics fails to be a case of rivalry if it reduces to a verbal dispute. Carnap, Quine, and Davidson each provide a way of justifying a strict inferentialist objection to rivalry, according to which any disagreement between logics reduces to a verbal dispute. One way to cast doubt on this identification is by appealing to Gödel's result that classical logic can be embedded in intuitionistic logic.\footnote{One may respond to strict inferentialism in other ways. For example, one may advocate a moderate inferentialism, as mentioned above, according to which only a core of inferences was relevant to forming a constant's inferential role. One could then argue that the inferences upon which the classical and rival logician disagreed were not relevant to determining the constant's inferential role, so the classical and rival constants did not have different meanings. I will not be pursuing this line of argument here, however, because it is unclear to me how one ought to draw a boundary between the relevant and irrelevant inferences.} We can use Gödel's negative translation to define quasiclassical logical constants with the same inferential roles as classical constants, but with different meanings. These quasiclassical constants do not represent a direct counterexample to the strict inferentialist objection; it claims that sameness of inferential role is a necessary condition for sameness of meaning, while Gödel's negative translation will be used to show that sameness of inferential role is not a sufficient condition for sameness of meaning. But if there is some other factor besides inferential role relevant for determining the sufficient conditions for the sameness of meaning, it may be that the logical constants can differ in inferential role without differing in meaning.
2.4.1 Adapting Gödel’s Negative Translation

First, we will review Gödel’s result. Let $\vdash_c$ and $\vdash_i$ denote provability in classical and intuitionistic logic, respectively. Gödel defines a translation “$\prime$” from statements of classical logic to statements of intuitionistic logic:\(^8\)

\[
\begin{align*}
\varphi' &= \varphi \quad \text{(where $\varphi$ is atomic)} & (\neg \varphi)' &= \neg (\varphi') \\
(\varphi \lor \psi)' &= \neg (\neg \varphi' \land \neg \psi') & (\varphi \land \psi)' &= \varphi' \land \psi' \\
(\varphi \rightarrow \psi)' &= \neg (\varphi' \land \neg \psi') & (\forall x \varphi)' &= \forall x \varphi' \\
(\exists x \varphi)' &= \neg \forall x \neg \varphi'
\end{align*}
\]

If we let $\Gamma = \{\gamma, \ldots\}$ and $\Gamma' = \{\gamma', \ldots\}$, then

$$\Gamma \vdash_c \varphi \iff \Gamma' \vdash_c \varphi'$$

because of DeMorgan’s rule and the logical equivalence $\varphi \rightarrow \psi \equiv \neg \varphi \lor \psi$. In the restricted case where $\Delta$ and $\psi$ contain only negation and conjunction as logical constants,

$$\Delta \vdash_c \psi \iff \Delta \vdash_i \psi.$$

But the “$\prime$” translation maps inferences that are classically valid onto inferences that are classically valid and contain only negation and conjunction as logical constants.

\(^8\)Gödel’s actual translation in his article used different symbols to denote classical negation ($\sim$) and intuitionistic negation ($\neg$), and classical conjunction (.$\,$) and intuitionistic conjunction ($\land$) (1933, 287). Since I am interested in discussing these issues on a background of a common vocabulary, I will be using the same symbol for both negations and conjunctions. Also, note that the translations for the quantifiers are from Troelstra’s discussion of Gödel’s article (1986, 283).
constants. So we can now see that

\[ \Gamma \vdash_c \varphi \iff \Gamma' \vdash_i \varphi'. \]

Gödel goes on to show that, based on this result, “the system of intuitionistic arithmetic and number theory is only apparently narrower than the classical one, and in truth contains it, albeit with a somewhat deviant interpretation” (Gödel 1933, 295).

We will apply this result to our project by defining new logical constants, which we will call *quasiclassical*, in terms of logical constants upon which classical and intuitionistic logic agree:

\[ \varphi \lor_q \psi \overset{\text{def}}{=} \neg (\neg \varphi \land \neg \psi) \]

\[ \varphi \rightarrow_q \psi \overset{\text{def}}{=} \neg (\varphi \land \neg \psi) \]

\[ \exists_q x \varphi \overset{\text{def}}{=} \neg \forall x \neg \varphi. \]

We will also define a translation function “*” that is logically the same as Gödel’s “r” but which employs the quasiclassical constants:

\[ \varphi^* = \varphi \quad \text{(where } \varphi \text{ is atomic)} \]

\[ (\neg \varphi)^* = \neg (\varphi^*) \]

\[ (\varphi \lor \psi)^* = \varphi^* \lor_q \psi^* \]

\[ (\varphi \land \psi)^* = \varphi^* \land \psi^* \]

\[ (\varphi \rightarrow \psi)^* = \varphi^* \rightarrow_q \psi^* \]

\[ (\forall x \varphi)^* = \forall x \varphi^* \]

\[ (\exists x \varphi)^* = \exists_q x \varphi^* \]

Note that “*” replaces all classical constants with their quasiclassical counterparts, if defined, and leaves the other classical constants the same.
According to classical logic, the classical constants have the same inferential roles as their quasiclassical counterparts under the “∗” translation. For it should be obvious, based on Gödel’s result, that

\[ \Gamma \vdash_c \varphi \iff \Gamma^* \vdash_c \varphi^* . \]

Moreover, since the truth tables for \( \lor_c \) and \( \lor_q \) are the same,

\[ \Gamma \models_c \varphi \iff \Gamma^* \models_c \varphi^* . \]

Proof-theoretic entailment is sufficient for logical entailment, as is model-theoretic entailment. We can thus be assured that

\[ \Gamma \models_c \varphi \iff \Gamma^* \models_c \varphi^* . \]

This result is expected, given that we know we can express classical logic using a subset of \( D_c \) if we define some logical constants in terms of others, such as defining \( \varphi \rightarrow \psi \) as \( \neg \varphi \lor \psi \). In such a case, agreement is assured because of those definitions. The “∗” function, along with an inverse of it, play the same role in this case: ensuring that a classical logician using the quasiclassical connectives will not disagree with a classical logician using the ordinary set of connectives.

These examples highlight a crucial aspect of our discussion of rivalry: we cannot determine the nature of two logics’ disagreement without being aware of the functions we use to interpret the similarity (or dissimilarity) of the two logics’ constants. We will now use the “∗” translation to define an interpretation.
function describing how the intuitionistic logician could interpret the classical logician. We can conclude that

\[ \Gamma \models_c \varphi \iff \Gamma^* \models_i \varphi^* , \]

based on reasoning similar to the way we concluded that \( \Gamma \models_c \varphi \iff \Gamma^* \models_c \varphi^* \). But we can also conclude this by viewing “*” as an interpretation function: If an intuitionistic logician were to understand the classical logician in the way described by the “*” translation, then the intuitionistic logician would agree with any claim made by the classical logician regarding logical entailment. In other words, \( \Gamma \models_c \varphi \implies \Gamma^* \models_i \varphi^* \). If the classical logician understands the intuitionistic logician’s words via the identity interpretation, the classical logician would agree with any statement made by the intuitionistic logician. In other words, \( \Gamma^* \models_i \varphi^* \implies \Gamma \models_c \varphi \). Note that the logicians’ agreement is not a contingent result of their happening to not come across a disagreed-upon inference. It is literally the case that there is no possible statement one logician could make with which the other would disagree.

The upshot of the “*” translation is that the classical and intuitionistic logicians would each regard his own constants as having the same inferential roles as the corresponding constants of the other. But, as I will show in the next section, their meanings differ.

One may object that describing the classical and quasiclassical connectives as having the same inferential role relies on an illicit extension of the concept of inferential role. Specifically, equating the inferential roles of the classical and quasiclassical connectives raises doubts about which connective(s) we are ac-
ually using in a given formula. When we express, say, a disjunction, we cannot
tell whether we are using $\vee_c$ or $\vee_q$ simply by looking at how that disjunction
participates in our inferences. Even though Gödel's negative translation, and
thus the definition of $\vee_q$, was motivated by intuitionistic concerns, we can de-
fine arbitrarily many new logical constants in the same way $\vee_q$ was defined. So
our situation is worse than we originally thought: we were unsure if our dis-
junctions were instances of $\vee_c$ or $\vee_q$, but now we see that our disjunctions
could be instances of many more connectives. Something must be wrong with
equating the inferential roles of the classical and quasiclassical connectives, the
objector will argue, if it entails that we have so little grip on what connective we
are using in a given formula. My use of the negative translation to define the
quasiclassical constants was founded on a policy of allowing logical constants
to be defined non-primitively. Declaring non-primitive constants to be logi-
cal constants requires a very different way of understanding what an inferential
role is. Classical and quasiclassical constants have the same inferential role in
an extended sense, but they do not have the same inferential role in the usual
sense.

We still have a response to this objection. The objector and I are discussing
two different concepts of inferential role. But I also think that what the objector
calls an “extension” of the concept of an inferential role is actually our ordinary
concept of an inferential role, and the objector is artificially restricting it. An
inferential role is just a description of a sentence's behavior in our inferences. It
makes sense, then, that sentences logically equivalent to one another may have
the same inferential role. But it cannot hold generally that logically equivalent
sentences have the same inferential role. For example, given a sentence $\varphi$, we
can construct a sentence logically equivalent to it by forming a finite but very long conjunction with \( \varphi \) as one conjunct and tautologies for the others. If this sentence is long enough, then it may not actually participate in our inferences in the same way that \( \varphi \) does because we lack the computational abilities necessary to employ it in actual reasoning.\(^9\) This example shows us that it cannot be the case that logically equivalent sentences have the same inferential role in all cases. But the objector goes too far in claiming that no two sentences can have the same inferential role in virtue of their logical equivalence. If we know that two sentences are equivalent, then we would be justified in substituting one for the other in any given inference. That said, a person's substitution may be less justified in cases where the logical equivalence is less obvious. But in defining \( \vee_q \), I have chosen a definition whose logical equivalence to classical disjunction is just about as obvious as we can make it.

2.4.2 Applying the Response

The previous section argued that \( \lor_c \) and \( \lor_q \) have the same inferential role, given a relatively neutral conception of what constitutes an inferential role. In this section, I will show that this result also holds for Carnap's, Quine's, and Davidson's more specific conceptions of what constitutes an inferential role.

I think that my response applies straightforwardly to Carnap's version of the "change of meaning" objection. According to Carnap, a logical constant's inferential role is determined by its semantical rules. Since \( \varphi \lor_c \psi \) and \( \varphi \lor_q \psi \) are

\(^9\)Here I am imagining \( \varphi \) to be very short, perhaps even atomic, while the logically equivalent sentence is far longer than \( \varphi \)—too long to write down, perhaps even too long for a modern computer to parse. Even if I grant that the sentence can be parsed by some computer more powerful than our current technology, the point remains: these sentences' inferential roles cannot be identical because because the actual inferences in which they can participate will differ for us.
logically equivalent, they have the same inferential role.

For Quine and Davidson, applying the above argument takes more work. We must show how a classically-reasoning linguist using classical constants and performing radical translation, or radical interpretation, would understand an intuitionistically-reasoning native using quasiclassical constants to be using classical constants. Specifically, we will show that $\lor_c$ and $\lor_q$ have the same inferential role for Quine's and Davidson's own conceptions of how inferential roles are determined, and that this is a result of Quine's and Davidson's each being committed to some version of the principle of charity.

We will consider Quine first. Suppose that a classically-reasoning linguist is performing radical translation on a intuitionistically-reasoning native. For Quine, a constant’s inferential role was given in terms of assent and dissent; the linguist sought to translate a native connective by matching the native’s patterns of assent and dissent for that connective to one of his own. Quine’s version of the change of meaning objection stated that the linguist cannot translate the native’s disjunction as his own classical disjunction because their inferential roles differ. Gödel’s negative translation argument shows us that there is a way for the intuitionistic native to agree with every statement of the classical linguist, and so the patterns of assent and dissent for his intuitionistic disjunction would match those of the linguist’s classical disjunction. Gödel’s actual argument showed how this would happen in situations where the native understood the linguist’s sentence $\phi \lor \psi$ to correspond to the sentence $\neg (\neg \phi \land \neg \psi)$ in his own language. The only problem with this picture is that the linguist’s and the native’s sentences are of different complexities. To address this, I de-

---

10Gödel’s translation also describes how the intuitionistic native would understand sentences involving other connectives. But I will focus on disjunction only, to make this section consonant with the last one.
fined $\lor_q$ as a constant of the same complexity as $\lor_c$. But Gödel's argument still applies when we use the quasiclassical constants: If, in general, the native understood the linguist's sentences of the form $\varphi \lor \psi$ to correspond to sentences of the form $\varphi \lor_q \psi$ in his own language, then the native would agree with any of the linguist's disjunctions. Moreover, Quine's commitment to the principle of charity requires him to adopt a translation manual based on Gödel's negative translation, since Quine interprets the principle of charity to require the linguist to avoid translating the native as rejecting his logical laws. Since the negative translation manual does not translate the intuitionistic native as rejecting the linguist's classical laws, it should be preferred to the identity manual according to which the native violates one of the linguist's classical laws. So $\lor_c$ and $\lor_q$ will have the same inferential role, under Quine's interpretation of inferential roles, and the manual under which this holds is preferable to one in which it does not hold, due to Quine's commitment to his version of the principle of charity.

Now we consider the Davidsonian case, and a situation in which we reason classically and the natives reason intuitionistically. According to Davidson's version of the principle of charity, we must maximize agreement between us and the natives. I argued above that the best way to do this is to carefully choose the recursive axioms of the theory of truth for the native's language. Clearly there will be sentences upon which we disagree. Based on this disagreement, Davidson would argue, we ought to translate the natives as using connectives different from ours, lest we lose the ability to justify our agreement on other sentences.

If Davidson seeks to maximize agreement in this situation, he ought to ad-
vocate the use of the Gödel negative translations in our theory of truth for the native's language. For example, if we suspected that the native's word or corresponded to disjunction, but we found that he disagreed with certain sentences of the form “e or not e,” then we could add the following recursive clauses to our theory of truth for his language in order to maximize agreement between us:

\[
\begin{align*}
T(\text{not-}\varphi) & \iff \text{not-}T(\varphi) \\
T(\varphi \text{ and } \psi) & \iff T(\varphi) \text{ and } T(\psi) \\
T(\varphi \text{ or } \psi) & \iff \text{not-}(\text{not-}T(\varphi) \text{ and } \text{not-}T(\psi)).
\end{align*}
\]

Note that we can change these to clauses to involve \(\lor_c\) and \(\lor_q\). Under that version of the negative translation, we would perceive the native's connective \(\lor_q\) being used just as our connective \(\lor_c\) is used. Therefore, given Davidson's concept of inferential role, \(\lor_q\) and \(\lor_c\) must have the same inferential role under this manual. Moreover, Davidson's commitment to the principle of charity would commit him to using a negative translation in formulating a translation manual between our language and the native's because it maximizes agreement.

2.4.3 Differences in Meanings

The above discussion shows that \(\lor_c\) and \(\lor_q\) have the same inferential role in each of these specific cases of strict inferentialism. In this section, I will show that the procedures by which a competent speaker would substantiate his belief in a disjunction are different from the procedures by which one would sub-
stantiate his belief in a quasidisjunction with the same disjuncts. I then will have shown that the senses of $\lor_c$ and $\lor_q$ differ, and that we therefore cannot identify a connective's meaning with its inferential role. This method is clearly inspired by Frege, but I do not need his full theory. I only need to assume that a difference in substantiating procedures indicates a difference in sense, and a difference in sense indicates a difference in meaning. Neither of these theses is contentious.

Suppose that Karl is a competent speaker of $\mathbf{E}$, and that $\mathbf{F}$ is a formalization of $\mathbf{E}$ in which our discussion can proceed. Let $\alpha, \beta \in \mathbf{F}$, and suppose that Karl believes that $\alpha$ and $\beta$ are both true. There may come a time when Karl may be called upon to substantiate his belief that $\alpha$ is true, that $\beta$ is true, or both; e.g., another competent speaker may believe that one or both are false, and may ask Karl to provide evidence for his belief. Let $a$ be some procedure that Karl may employ to substantiate his belief that $\alpha$ is true. For example, to substantiate his belief that the sentence *J. S. Bach died in 1750* is true, Karl may look up Bach's death date in a respectable music reference book. Note that the purpose of these procedures is neither to describe the source of Karl's beliefs, nor to describe a condition for Karl's understanding of a sentence. We will call $a$ an *evidential procedure*, say that $a$ *evinces* $\alpha$, and write this as $a \triangleright \alpha$. It may be that an evidential procedure involves performing a procedure that constitutes an evidential procedure itself. We use subset notation to indicate this. For example, if a speaker could substantiate $\beta$ by performing two evidential procedures $b$ and $c$, we may say that $a \triangleright \beta$ and $b, c \subseteq a$.

The evincing relation should bear a resemblance to the familiar notion of proof that is often used to describe intuitionistic connectives. But here I am
Trying to capture a less formal notion. Saying that $a \Rightarrow \alpha$ ought to be ambiguous between any of the following:

- $a$ shows that $\alpha$ is true
- $a$ justifies a speaker’s belief in $\alpha$
- $a$ makes a speaker’s utterance of $\alpha$ correct.

Evidential procedures themselves are not rigorous, either. We will not yet require that evidential procedures be algorithms or even be performable by a speaker; e.g., an evidential procedure may take an infinite amount of time to complete. Moreover, we allow that a given evidential procedure may be an evidential procedure for sentences with different senses.

The evincing relation must carry some normative force, despite its lack of rigor. We assume that Karl is a competent speaker of the language; as such, other speakers would agree that $a$ provides evidence for Karl’s belief—or any other competent speaker’s belief—that $\alpha$ is true. We can thus make claims like “$a \Rightarrow \alpha$” without needing to relativize the evincing relation to a particular speaker.

We will impose one restriction on evidential procedures, corresponding to the usual assumption that a contradiction cannot be true: there is no evidential procedure $a$ such that $a \Rightarrow \bot$. That is, there are no procedures that an agent would use to substantiate a contradiction. We will call this restriction evidential noncontradiction.

We now have enough of an apparatus to define a notion of “recursive evincibility” inspired by recursive realizability (see Kleene 1945). Conjunction is the
only connective that allows for a straightforward definition:

\[ e \triangleright \alpha \land \beta \overset{\text{def}}{=} \langle a, b \rangle \subseteq e \text{ and } a \triangleright \alpha \text{ and } b \triangleright \beta \]

That is, an evidential procedure evinces a conjunction just in case the evidential procedure contains two sub-procedures evincing the two conjuncts. We place the sub-procedures in an ordered pair to know which sub-procedure evinces which conjunct.

The other definitions will be more complicated. Under the Kleene/BHK interpretation, some constants are seen as operations on proofs: the conditional, for example, transforms a proof of the antecedent into a proof of the consequent. To describe the recursive envincability clauses for the other connectives, we will make a similar move by appealing to the fact that evidential procedures are composed of evidential procedures, not pieces of evidence.

The traditional proof-theoretic definition of negation is:

\[ e : \neg \alpha \overset{\text{def}}{=} e : \alpha \rightarrow \bot, \]

meaning that \( e \) is a proof of \( \neg \alpha \) if it transforms any putative proof of \( \alpha \) into a proof of a contradiction. This interpretation is suitable because we recognize, or stipulate, that there can be no proof of a contradiction. Informally, we might say that a person equipped with a proof of \( \neg \alpha \) is equipped to reject any putative proof of \( \alpha \). When presented with a putative proof of \( \alpha \), he can transform it into a proof of \( \bot \), showing that the original “proof” was not actually a proof of
α at all. An evidential procedure for α ought to do the same:

\[ a \vdash \neg \alpha \overset{\text{def}}{=} \text{for any evidential procedure } b, \ a \vdash (b \not\vdash \alpha) \]

This definition is suitable only because envyincability is not a purely formal relation.\(^\text{11}\) The definition for \(a \vdash \neg \alpha\) is the same in its effect as the formal definition of \(e : \neg \alpha\). The latter has more formal detail, but \(e\) proves \(\neg \alpha\) by being a recipe for rendering any putative proof of \(\alpha\) unacceptable. The former is the same, but is silent as to the method of rendering any \(b\) unacceptable.

Finally, we define universal quantification. This definition is not needed for our discussion of classical and quasiclassical disjunction, but I will include it for the sake of completeness. Under the traditional proof-theoretic interpretation, a proof of a universal quantification \(\forall x \varphi(x)\) is a function that maps the name of a term \(t\) onto a proof of \(\varphi(t)\). This definition tells us that there exists a proof of every instantiation of the universal quantification. So we define:

\[ a \vdash \forall x \alpha(x) \overset{\text{def}}{=} \text{for every object } t, \text{ and some evidential procedure } e \ a \vdash (e \vdash \alpha(t)). \]

This definition tells us that, by possessing the evidential procedure \(a\) that evinces \(\forall x \alpha(x)\), we are equipped to substantiate any instantiation of that universal quantification.

\(^{11}\)The problem of interpreting negation is one of the main reasons why this discussion is inspired by recursive realizability, rather than Kripke frames. Under that interpretation, where \(\models\) here denotes the forcing relation,

\[ k \models \neg \alpha \overset{\text{def}}{=} \neg \exists \ell \geq k \ (\ell \models \alpha). \]

That is, \(\neg \alpha\) is true at stage \(k\) if there is no later stage at which \(\alpha\) is true. But this does not explain why the agent at stage \(k\) would think that \(\alpha\) is not true at some later stage. By couching the discussion in terms of evidential procedures, we can describe what it is about a competent speaker—namely, a procedure he may employ that rejects all putative substantiations of \(\alpha\)—that explains his belief that \(\alpha\) is not true.
tification. We should also require that $a$ evinces these instances and nothing else. This definition is inspired by the understanding of a universal quantification as a long conjunction.

We are now equipped to show that the senses of classical and quasiclassical disjunction differ because $\varphi \lor_c \psi$ and $\varphi \lor_q \psi$ must be evinced by different evidential procedures.

The recursive clause for classical disjunction is straightforward:

$$e \triangleright a \lor_c b \overset{\text{def}}{=} a \subseteq e \text{ and } a \triangleright a, \text{ or } b \subseteq e \text{ and } b \triangleright \beta, \text{ or both.}$$

The recursive clause for quasiclassical disjunction is not straightforward. Remember that

$$a \lor_q b \overset{\text{def}}{=} \neg (\neg a \land \neg b)$$

so we need to determine which evidential procedure evinces $\neg (\neg a \land \neg b)$. Using the above clause for negation, we know

$$e \triangleright \neg (\neg a \land \neg b) \overset{\text{def}}{=} \text{ for any evidential procedure } f \text{, } e \triangleright (f \not\triangleright \neg a \land \neg \beta).$$

In other words, possessing $e$ requires that any putative evidential procedure $f$ of $\neg a \land \neg \beta$ be rejected. To determine just what kind of evidential procedure must be rejected, we employ the clause for conjunction:

$$e \triangleright \neg (\neg a \land \neg \beta) \overset{\text{def}}{=}$$

for any evidential procedures $g$ and $h$, $e \triangleright (g \not\triangleright \neg a)$ and $e \triangleright (h \not\triangleright \neg \beta)$.

We could apply the clause for negation once more so as to further analyze $g$ and
But I think at this point it would be clearer to describe $e$ in these terms: $e$ is the evidential procedure according to which we reject all evidential procedures that have us rejecting all substantiations of $\alpha$ and all substantiations of $\beta$.

We can finally show that the senses of $\lor_c$ and $\lor_q$ differ. For a disjunction $\alpha \lor_c \beta$ and a quasiclassical disjunction with the same disjuncts $\alpha \lor_q \beta$, the set of their possible evidential procedures must differ. In the case of classical disjunction, either $a$ or $b$ will evince $\alpha \lor_c \beta$, assuming $a \supset \alpha$ and $b \supset \beta$. But neither $a$ nor $b$ will evince the quasiclassical disjunction $\alpha \lor_q \beta$. Moreover, $a$’s and $b$’s not evincing $\alpha \lor_q \beta$ is not the result of any simplifying assumption that evidential procedures are canonical. So there is at least one procedure a competent speaker could use to substantiate the classical disjunction that would not substantiate a quasiclassical disjunction with the same disjuncts. Since the ways a speaker would substantiate these sentences must differ, even as the disjuncts are kept same, the senses of the connectives must differ. Thus, the meanings of $\lor_c$ and $\lor_q$ must differ.

2.5 EXPANDING THE RESPONSE

According to strict inferentialism, any difference in two constants’ inferential roles implies a difference in their meanings. In the last section, I showed that two constants’ inferential roles can be identical and yet their meanings can differ. I concluded that a constant’s inferential role does not exhaust its meaning. By rejecting an identification of the constants’ meanings with their inferential roles, we reject strict inferentialism. In this section, I will turn my attention to a weaker version of inferentialism, which claims that significant difference in inferential role indicates difference in meaning. This position seems to be one
we actually employ when speaking English. We realize another speaker's word means something different than ours—contrary to what we may have assumed—when he uses the word differently. Despite the initial intuitiveness of this objection, making this a viable objection to rivalry requires much more work on the part of the weak inferentialist.

Without strict inferentialism's identification of a constant's meaning with its inferential role, the weak inferentialist must claim that difference in meaning is indicated by significant difference in inferential role. The problem for the weak inferentialist is figuring out just what constitutes “significant” difference. Compounding this problem is the fact that we have no reason to think that all parts of an inferential role contribute to a constant's meaning. Consider again the example above: when hearing another speaker use some word whose meaning I know, the speaker can use that word in ways different from the ways I would use it and yet these uses would not necessarily count against my belief that our words had the same meaning. In making this claim, I am not appealing to accidents on the speaker's part, where I consider his “use” of the word in question to be an error. I claim that this holds even if the speaker's use indicates a full-fledged disposition to use the word in question in these different ways. These differences in use can come from various sources: The word may have a certain connotation for him that it does not have for me; the word may be associated with a certain level of formality of speech that it lacks for me; or it may just be that he prefers one word over some synonym for personal reasons. The upshot of all this is that words may have some kind of core meaning for me that is preserved in a core subset of its inferential role, such that someone's apparent use of a word actually involves a change in meaning if his use of it indicates a lack
of a disposition to make an inference contained in this subset. The weak infer-
entialist’s challenge to is specify which inferences form the core inferential role 
that determines the meaning of a given logical constant. Without this speci-
fication, which seems to be a very difficult thing to do, the weak inferentialist 
objection is merely skeptical.

2.6 SKETCHING A CASE OF RIVALRY

I began this chapter with the undebatable claim that rivalry is possible only if 
the relevant classical and rival constants have the same meaning. Strict infer-
entialist theories argued that a logical constant’s meaning should be identified 
with its inferential role; and since the rival and classical constants have differ-
ent roles by hypothesis, then rivalry is not possible. I then defended rivalry 
by showing how a logical constant’s meaning is not exhausted by its inferen-
tial role, thus derailing the strict inferentialist’s objection. Weak inferentialism 
claimed that a logical constant’s meaning consisted in some yet-unspecified set 
of significant inferences. I pointed out that the specification of significant in-
ferences must be done, and that the task is not as straightforward as it may 
appear.

Despite all this, so far I have only shown that strict and weak inferentialism 
do not produce an a priori refutation of rivalry. So while both inferentialists 
cannot simply show that the rival logician’s constants’ meanings are different 
from the classical constants’ meanings, just by virtue of his using them differ-
ently, we cannot conclude that rivalry is possible. It is up to the rival logician to 
show that the meanings of his constants do not differ from those of the classical 
constants, thus refuting the change of meaning objection.
We thus have two requirements that a challenge or rivalry must fulfill. The first is a formal requirement, that the rival logic must be a rival to classical logic as defined in the previous chapter. The second is a philosophical requirement, described above, that the rival logician can show that his logical constants' meanings do not differ from the corresponding classical logical constants' meanings. In the next chapter, we will examine Dummett's challenge to classical logic and show how he fulfills these requirements.
3.1 INTRODUCTION

In this chapter, I will show that Dummett's challenge to classical logic is a case of rivalry. The previous two chapters described two requirements that Dummett's account must fulfill: a formal requirement that the logic Dummett advocates is a rival logic as defined in my first chapter, and a philosophical requirement that Dummett has a response to the objection to rivalry described in my second chapter. To show that Dummett fulfills these, I will offer a critical overview of Dummett's account. For clarity, I will divide his account into three stages. The first is Dummett's description of his theory of meaning. Dummett will use this theory as the foundation for his rejection of bivalence, which constitutes the second stage of his account. The third stage of Dummett's account is his diagnosis of why we might have thought that classical logic was correct in the first place. After describing Dummett's account, I will describe how it constitutes a case of rivalry.

3.2 DUMMETT'S THEORY OF MEANING

A theory of meaning ought to be built around truth, according to Dummett, because a sentence's meaning should be identified with its truth conditions.
To correctly analyze the connection between truth and meaning, he claims, we must analyze what it means to know the meaning of a sentence. This shift from talking about meaning to talking about knowledge of meaning is based on two ideas Dummett imports from Frege: that the truth conditions of a sentence are its sense, and that the sense of a sentence is what we know when we know the meaning of the sentence. To build a theory of meaning around truth, Dummett will need an explanation of what it means to say that a person knows the truth conditions of a given sentence.

If knowing the meaning of a sentence is knowing its truth conditions, what kind of knowledge is this? One option is that our understanding of a sentence consists in explicit knowledge: to know the meaning of a sentence is to know its truth conditions, and knowledge of its truth conditions is “manifested by [an] ability to state that condition” (1976, 45). This option has intuitive appeal, since speakers of a language are usually able to express explicitly the truth conditions for the sentences of their language; for example, by stating the T-schema for a sentence. Also, when a person is unfamiliar with the meaning of a sentence, we can explain its meaning by explicitly stating the sentence’s truth conditions. Despite the appeal, it cannot hold generally that knowledge of sentences’ truth conditions is explicit. The only way one could express the meaning of a sentence would be by expressing its truth conditions, but those truth conditions are expressed with a sentence whose meaning would need to be explained by a further expression involving truth conditions, and so on. What this picture lacks is some way of explaining the meaning of a sentence.

Knowledge of a sentence’s truth conditions must be implicit knowledge, according to Dummett—the kind of knowledge that we can attribute to the speaker
on the basis of some evidence other than an ability to state explicitly the truth conditions for that sentence. Consider an example not involving meaning: a touch-typist may not be able to specify which letters belong on every key when she is presented with a typewriter with blank keys. But we would still say that she knows the locations of all the letters on the keys, as evidenced by her ability to type on that blank keyboard. She has implicit, not explicit, knowledge of the letters’ locations. We should bear in mind that “implicit knowledge” does not refer to the psychological mechanisms of the speaker. In fact, Dummett insists that if “a robot be devised to behave in just the ways that are essential to a language-speaker, an implicit knowledge of the correct theory of meaning for the language could be attributed to the … robot with as much right as to a human speaker” (1976, 37). However, I suspect that Dummett would not claim that any “speaker” that could pass a Turing Test would be considered to have the relevant kind of implicit knowledge. An automaton programmed to use sentences just as we do would not know the meanings of these sentences. I suspect this is a form of the problem of other minds, for only a minded speaker can know the meanings of the sentences it uses. To avoid superimposing the problem of other minds onto the question of whether a speaker knows the meaning of a sentence, the “ways that are essential to a language-speaker” should include the requirement that the language-speaker has learned the language in the same way we do. If the speaker has learned the language in the same way that we have, we can be sure that the speaker is not simply an automaton.¹

¹I suspect that learning a language also requires a mind. But suppose a zombie were to behave in just the ways that are essential to a language-learner. I want to avoid begging any questions, so I must admit that, under my account, we may be forced to accept that the zombie was a speaker despite its lack of consciousness. But we want a description of a theory of meaning, not a theory of mindedness; and if we have been fooled into thinking that the zombie possessed a mind, so be it.
One consequence of claiming that knowledge of sentences’ meanings is implicit, and thus requires manifestation, is that we have no justification for believing a sentence has some part of its meaning that cannot be put into use: “if two individuals agree completely about the use to be made of [a] statement, then they agree about its meaning” (1983, 98), so “there must be an observable difference between the behaviour or capacities of someone who is said to have that knowledge and someone who is said to lack it” (1983, 99). Dummett supports this by pointing to language-learning: when we learn a language, what we learn is how to use the sentences of that language; and “our proficiency in making the correct use of the statements and expressions of the language is all that others have from which to judge whether or not we have acquired a grasp of their meanings” (1983, 99). A putative speaker's use of a sentence will be the evidence for his knowing the meaning of that sentence, and we will need an account of what counts as evidence for that knowledge. To accomplish this, the theory of meaning must have two parts, according to Dummett. One is the theory of reference. In a theory of meaning where the meaning of a sentence consists in its truth conditions, the theory of reference will be “an inductive specification of the truth-conditions of sentences of the language” (1976, 40). The other is the theory of sense, which will explain what counts as evidence for the implicit knowledge a speaker possesses when he understands a sentence. The theory of sense will “lay down in what a speaker’s knowledge of any part of the theory of reference is to be taken to consist, by correlating specific practical abilities of the speaker to certain propositions of the theory” (1976, 40). We will now clarify the structure of this “inductive specification” by contrasting it with two other views.
It is often assumed that a sentence's truth conditions are determined by the truth conditions of its components. By claiming that truth conditions determine a sentence's meaning, it stands to reason that Dummett would also accept that a sentence's meaning ought to be determined by the meanings of its components. Let such a view be called componentialism. A further question is whether a sentence's components' meanings alone determine the meaning of that sentence, or if there is some other factor that contributes to the determining of the sentence's meaning. Let radical componentialism be the view that there is no such other factor; i.e., only the sentence's components' meanings determine its meaning. This is the first type of theory of meaning with which I will contrast Dummett's theory.

Radical componentialism cannot be correct, Dummett would argue, because such a theory cannot explain what it means to know the truth conditions of certain sentences. We can draw a distinction between those sentences whose truth value is determined by the truth values of other sentences—logically complex sentences, for example—and those whose truth conditions do not involve the truth conditions of other sentences. Dummett sharpens this distinction. He defines the set of reducible sentences, which includes the sentences of the former kind.2 Those sentences that are not reducible, which includes most of the latter kind of sentences, are labelled barely true. The radical componentialist

---

2Dummett's definition of reduction is:

The thesis that statements of a class $M$ are reducible, in this sense, to statements of another class $R$ takes the general form of saying that, for any statement $A$ in $M$, there is some family $\bar{A}$ of sets of statements of $R$ such that, for $A$ to be true, it is necessary and sufficient that all the statements in some set belonging to $\bar{A}$ be true; a translation is guaranteed only if $\bar{A}$ itself, and all the sets it contains, are finite. In such a case we may say that any statement of $M$, if true, must be true in virtue of the truth of certain, possibly infinitely many, statements in $R$ (1976, 56–7).
has an account of how a speaker knows the meaning of a reducible sentence: A person knows the truth conditions of a reducible sentence when he has implicit knowledge of how its truth depends on the truth of the other sentences to which it reduces. But it cannot be the case that all of our sentences are reducible, for the same reason that it cannot be the case that knowledge of all our sentences is explicit: the reduction must end at some point, and it is only at this point that we will have an explanatory account of meaning. So reducible sentences may reduce to other reducible sentences, but eventually there must be a reducible sentence that reduces to a barely true sentence. However, the radical componentalist has no way of describing the meaning of a barely true sentence and thus no way of accounting for what it means to know the meaning of a reducible sentence that reduces to that barely true sentence.

We can now turn our attention to the other view with which we will contrast Dummett’s position. A view that is componentalist, but admits that a sentence’s meaning is determined by the meanings of its components and the meanings of outside sentences is a holistic theory.\(^3\) We can define a spectrum of theories by varying the outside sentences that play this role. One end of this spectrum is radical componentalism, the limit case in which no outside sentences contribute to determining the meaning of a sentence. The other end of the spectrum would be radical holism. According to this view, all sentences of a language contribute to determining the meaning of any given sentence of that language.\(^4\)

\(^3\)Using the term *holistic* here may seem unusual, since Dummett often contrasts his view with holism and yet Dummett’s own view will be considered holistic, as will be explained below. But Dummett himself has admitted that his view is holistic in a moderate sense (see 1987, 271–274). See Hansen (2001) for a longer discussion of this.

\(^4\)Dummett is clearly alluding to Quine when he describes radical holism. According to Dummett, Quine’s brand of radical holism is a radically holistic theory with the additional thesis that “no truth-value assignment to any sentence is ever mandatory in the light of experience: a re-
Dummett rejects radical holism on the grounds that it fails to account for how we can know a sentence's determinate individual content—that attribute a sentence possesses, described by the theory of meaning, which serves to determine a sentence's correct use. Call this the sentence's *core content*. If we cannot know that a sentence's core content is determinate, we lose the ability to say that a use of a sentence is correct. If we cannot know that a sentence's core content is individual, we lose the ability to say that a use of a particular sentence is correct when that use does not involve certain other sentences.

Dummett admits that radical holism can give some account of a sentence's meaning, but not its core content:

its meaning simply consists in the place which it occupies in the complicated network which constitutes the totality of our linguistic practices. The only thing to which a definite content may be attributed is the totality of all that we are, at a given time, prepared to assert; and there can be no simple model of the content which that totality of assertions embodies; nothing short of a complete knowledge of the language can reveal it (1983, 100).

So we cannot explain a particular sentence's significance to linguistic practice because we can only interpret linguistic practice monolithically.

What's worse, radical holism requires knowledge of an entire language in order to know the meaning of a certain sentence, yet it cannot explain how
we come to have the complete knowledge of the language required to possess
knowledge of a sentence's core content. Under a radically holistic theory, it is
impossible for a speaker to learn the meaning of any sentence whose meaning
depends upon her knowing the meaning of other sentences, since she would
have needed to learn the meanings of those outside sentences first. Once again,
we see how language-learning is key to formulating a proper theory of meaning
for our language.

The radical holist could try to respond to this objection by claiming that the
process of learning a sentence's meaning involves first learning some "initial
meaning" which is then expanded as the student learns more of the language.
The idea is that the sentence's initial meaning is a kernel of meaning that per-
sists even as the student learns more uses for the sentence. The student then
possesses the full meaning when he has learned the entire language.

This response fails, however. Suppose the student did know the initial mean-
ing of some sentence. Since we assume that componentialism holds, the stu-
dent would also need to know the meanings of the sentence's components. Fi-
nally, since this is a holistic view and not the limit case of radical compone-
tialism, the student will need to know the meanings of some other sentences. I
will grant the radical holist that the student knows the meaning of these further
sentences, even though it is unclear how he came to have them, or if the stu-
dent possesses these meanings fully or initially or whatever, or how the amount
of knowledge of these sentences he possesses affects the amount of the mean-
ing of the sentence in question he possesses. As the student learned more of the
language, the content of the sentence would change: it may participate in in-
ferences involving newly-learned sentences, for example, and these would need
to be licensed by the sentence’s content. We expect this, since we are assuming that the sentence’s initial meaning is properly “smaller,” so to speak, than its full meaning.

A picture of meaning according to which a speaker grasps a sentence’s meaning partially, but grows to grasp it fully, seems like a picture of meaning according to each stage of the speaker’s language learning is a conservative extension of the previous stage. But the radical holist wants to claim that we cannot determine the entirety of the sentence’s use simply by possessing its initial content. So it seems that a radical holist must claim that some intermediate fragment is not a conservative extension of the fragment learned immediately before it. Otherwise the full language is a conservative extension of the initial fragment described above and thus all uses of the sentence in the full language are determined by its initial content.

So the radical holist must hold that, while the sentence’s initial content may license some use of that sentence in the context of the initial fragment, that same use may be illegitimate at some later stage. In other words, we cannot know that an sentence is correct in any intermediate fragment. For example, given some sentence, that sentence can only be judged to be “really correct” once the entire language is known, even if it is apparently correct at some intermediate state. The only explanation for why a particular use of the sentence could be correct at an earlier stage and incorrect at a later stage is that the sentence’s meaning changed. If this kind of language-learning allows a sentence’s content to change this way, the later content does not seem to be an extension of the first; it is simply a different content. So the radical holist was incorrect when he said that the “initial content” somehow reflected the sentence’s mean-
Dummett’s actual position is holistic, but neither radically componentialist nor radically holistic. Dummett calls his moderate holism molecularism, according to which “individual sentences carry a content which belongs to them in accordance with the way they are compounded out of their own constituents, independently of other sentences of the language not involving those constituents” (1983, 104). So a molecular theory of meaning built on truth would include a theory of reference to explain how the truth conditions of a sentence are determined by the truth conditions of its components and the sentences of the language that have those components as components. At the same time, it will describe what implicit knowledge a speaker possesses when he understands a sentence, based on the meaning of the sentence and its constituents.

So far, I have described the location of Dummett’s molecular theory of reference on a spectrum of componentalist theories of reference. Dummett locates his position on another spectrum by distinguishing his theory of sense from atomistic theories of sense, which would describe what constitutes evidence of our knowing the meaning of a sentence by describing what constitutes evidence of our knowing the meanings of the words in that sentence. In contrast, if our theory of reference is molecular, the theory of sense will describe what constitutes evidence of our knowing the meaning of a sentence by describing what counts as evidence of our knowing the meanings of the constituent sentences of that sentence.

Dummett admits that atomistic theories of sense are tenable, but faults them for requiring that the smallest linguistically significant act is the utterance of a word. Since we commonly view the utterances of sentences as the smallest lin-
guistically significant act, according to Dummett, and this is just what molecular theories of sense require, we ought to prefer them over atomistic theories of sense (1976, 38).

Dummett does not specify what constitutes linguistic significance, but we can understand it in a way that builds a stronger case against atomism. He says that besides “unimportant exceptions,” we think of utterances of sentences as the smallest linguistically significant act (1976, 38). On a weak reading of this, utterances of words are linguistically significant but utterances of sentences are more linguistically significant somehow. The only way I can think to justify this reading is to claim that sentences and words both possess meanings, and that the possession of meaning is required for linguistic significance. But this explanation should be unsatisfying. It makes our attribution of linguistic significance look arbitrary because there is no explanation for why an utterance of any meaning-bearing piece of language should be linguistically significant. Moreover, meaning-bearing is not enough to ensure linguistic significance. After all, the roots of a word are etymologically significant just because they possess meaning, but uttering a root of a word is often linguistically insignificant.

A better reason for thinking sentences to be the smallest linguistically significant act is because words cannot be truth bearers. If words cannot be truth bearers, then they cannot function in a theory of reference based on truth, like the one Dummett advocates. The theory of reference must be more primitive than the theory of sense, since we require that the theory of reference have an inductive structure that the theory of sense coordinates with a description of what counts as a manifestation of the implicit knowledge of sentences’ meanings. We can thus be confident that a molecular theory of meaning, rather than
an atomistic theory of meaning, is the most accurate way to structure the theory of meaning for our language.

Dividing the theory of meaning into the theory of reference and the theory of sense is necessary because of our desire to have a theory of meaning that is both molecular and respectful of a speaker’s knowledge of a sentence’s truth conditions being implicit. As a result, we were forced to accept that evidence of a speaker’s implicit knowledge of a particular sentence’s truth conditions is something specific (though not necessarily unique) to that sentence. In the next section, we will focus on determining just what kind of abilities will count as evidence for a speaker’s possessing knowledge of a sentence’s meaning.

3.3 CORRELATING ABILITIES AND SENTENCES

According to Dummett, a sentence is true only if we can recognize its truth conditions’ obtaining. This thesis can be traced to Frege’s thesis that a sentence’s sense determines its reference. Dummett says the following about the relationship between senses and references of words, but the same holds for senses and references in general, including those of sentences:

I understand [a] term by knowing what is needed, for any given object, to establish that the object is the referent of the term, but, of course, I cannot be expected to be able to determine that question at a glance. At least from Frege’s standpoint, there cannot even be a requirement of effective decidability: as long as I can recognize something as settling the question, it is unnecessary that I should be able in all cases to employ some procedure which will lead to a settlement of it (1981, 488).

Given Frege’s identification of a sentence’s reference as its truth value, Dummett is here suggesting that understanding a sentence consists in knowing how
to determine whether its truth conditions obtain. I suggest that this “knowing how” consists in possessing an ability to apply a procedure at the end of which we recognize whether the sentence's truth conditions obtain. Let such a procedure be called a verification procedure for the sentence. The theory of sense will correlate to each sentence the ability to apply a verification procedure for that sentence. Granted, the above quote seems to imply that understanding a sentence does not require our being able to apply a verification procedure, but only our being able to recognize a verification procedure when presented with a description of it. But Dummett's point is that we can understand a sentence even if we cannot actually apply a verification procedure for it because the limits of our understanding are determined by the actual abilities we possess to apply verification procedures as well as certain potential abilities we possess by virtue of some extending of our actual abilities. In this way, even a speaker's recognition of a verification procedure is underwritten by an ability to perform that procedure.\(^5\)

The shape of a given sentence's verification procedure is partially determined by that sentence's being either barely true or reducible. In either case, knowledge of the sentence's meaning is implicit. In the case of barely true sentences, “our model for such knowledge … is the capacity to use the sentence to give a report of observation” (Dummett 1976, 57). If a person knows the meaning of a sentence, he therefore knows its truth conditions. If the truth conditions obtain at some moment, and he can recognize those truth conditions' obtaining, then an outward sign of his knowing the truth conditions will be a use of the

---

5Some may balk at this, since it seems as if we understand sentences that we could never actually verify. As I will explain below, Dummett claims that some of these sentences are ones we do not actually understand. For other such sentences, we may possess the potential to perform the procedure even if, at a given moment in time, we may not be able to actually perform it.
sentence as an assertion that the truth conditions obtain, which Dummett calls a “report of observation.” The verification procedure for a barely true sentence will then be a procedure by which the speaker places himself in a position to make such a report, followed by the report itself.

Dummett claims that grasping a reducible sentence’s truth conditions “will consist in an implicit grasp of the way in which its truth depends upon the truth of statements in [the set of sentences to which it reduces]” (1976, 57). But a reducible sentence need not reduce to a barely true sentence—it may reduce to another reducible sentence. For any given reducible sentence, there is a chain of reductions which will eventually lead to a set of barely true sentences, so the verification procedure for a reducible sentence will be constructed from the verification procedures of the barely true sentences to which it reduces. This verification procedure must also make it clear that the speaker is aware that those barely true sentences are the sentences which lie at the end of this chain of reductions.

There appears to be an obvious flaw in Dummett’s position: a person can apparently be using a sentence to report a direct observation but the truth conditions may not actually obtain. One may be tempted to conclude that Dummett is assuming that direct observations succeed. To be sure, understanding a sentence requires that we be able to successfully make a direct observation. So the skeptic can object that Dummett insists that a person have the ability to use a sentence to report a direct observation, but we have no way of being sure that any putative report is successful. I will gloss over this here; I think this is actually a problem with implementing Dummett’s theory of meaning, rather than a problem with the theory itself. More precisely, this is a problem that affects
our ability to ascribe to a putative speaker an ability to use a given sentence to report a direct observation. Even if the skeptic is correct, and we cannot always accurately ascribe this knowledge to the speaker, that does not affect the structure of the theory of meaning. The Fregean picture of sense requiring recognition, so that a person cannot understand a sentence without being able to recognize its truth conditions’ obtaining, is correct. (So says Dummett.) If we are unable to ascribe understanding of a sentence to a speaker, that does not impugn the theory of meaning; it only impugns our performance in ascribing abilities to a person.

3.4 THE LIMITS OF ABILITIES

Based on the above account, truth will be bivalent just in case a competent speaker of our language is able to perform a verification procedure for every sentence of it. According to Dummett, this is what most speakers assume: We have a “propensity to assume a realistic interpretation of all sentences of our language, that is, to suppose that the notion of truth applicable to statements made by means of them is such that every statement of this kind is determinately either true or false, independently of our knowledge or means of knowing” (1976, 62). This propensity would be innocuous if our ability to verify sentences extended so far as to allow us to verify any sentence, but this does not seem to be the case. Above, we described Dummett’s argument that a person knows the meaning of a barely true sentence only if he is able to use the sentence to report a direct observation. Now, there are sentences that, by hypothesis, we could never use in this way. These undecidable sentences can come from “the use of quantification over an infinite or unsurveyable domain (e.g.
over all future times); the use of the subjunctive conditional, or of expressions explainable only by means of it; the possibility of referring to regions of space-time in principle inaccessible to us” (1976, 46).

The advocate of bivalence—call her a bivalentist—needs some way of arguing that we can perform undecidable sentences’ verification procedures. According to Dummett, since we cannot use an undecidable sentence ourselves to report a direct observation, the bivalentist instead imagines the observations being made by “some being with a different spatio-temporal perspective, or whose observational and intellectual powers transcend our own, such powers being modeled on those which we possess, but extended by analogy” (1976, 60). For example, we often consider sentences about the past to be understood just as well as sentences about the present. We may be able to verify a sentence in the present tense, but understanding sentences about the past requires us to use our fallible memory. We understand past-tense sentences, Dummett’s bivalentist would claim, because we understand the sentence’s verification procedure as being performed by a being whose memory is not as fallible as ours, whose powers of observation are as clear in the case of past-tense sentences as ours are in the case of present-tense sentences. Given an undecidable sentence, the bivalentist claims that we understand it because the superhuman abilities needed to perform its verification procedure have analogues among our actual

---

6Dummett’s opponent is usually characterized as being a realist. Surely a realist is an advocate of bivalence, but I think this statement is true of more than just realists. As I understand it, a realist is one who rejects the above picture of truth being relative to our ability to recognize it. He takes issue not with Dummett’s rejection of bivalence, which occurs below, but rather with Dummett’s recognition-immanent theory of meaning. In other words, his main disagreement with Dummett occurs earlier in Dummett’s argument. At this point in Dummett’s argument, the recognition-immanence of truth has already been established (supposedly). So the advocate of bivalence who remains must be one that accepts the recognition-immanence of truth and who also believes in bivalence. Thus, she must claim that we can perform a verification procedure for any sentence.
abilities. A grasp of these actual abilities, as well as an understanding of the analogy between them and the superhuman abilities, provide us with the conception of how the undecidable sentence's verification procedure might be applied. According to the bivalentist, this conception is enough to underwrite our understanding of that undecidable sentence.

The problem is that we can only account for our “understanding” of the undecidable sentences “by imputing to us an apprehension of the way in which those sentences might be used by beings very unlike ourselves” (1976, 62). Since we could never find ourselves in a position to use this sentence to report a direct observation, this account fails to show us how we could have “come to be able to assign to our sentences a meaning which is dependent upon a use to which we are unable to put them” (1976, 62). In other words, this account fails to show us that we understand those sentences. Bivalence requires that we can use those sentences in principle and so puts us in a situation where we are understanding certain sentences based on a illusory analogical relationship with other sentences; “that is, that we systematically misunderstand our own language” (1976, 62).

I think that Dummett’s diagnosis can be strengthened and be made more clear by offering a more complete account of the analogical extending by which he claims we extend the abilities we actually possess. An example will be helpful. Consider the following sentences:

\( s_1 \) I can walk from my office to the water fountain in 150 steps.
\( s_2 \) I can walk from my office to New York in 1,500,000 steps.
\( s_3 \) I can walk on the surface of a sphere for an infinite number of steps.
Under Dummett’s theory, if I understand these sentences, then for each sentence I must possess some ability that allows me to verify it. I certainly understand $s_1$: I have the ability to walk 150 steps, and this is the ability that allows me to see the water fountain at the end of my walk.\footnote{One may argue that I must also have the ability of recognizing the water fountain or maybe the ability to count my steps. Assume that the ability to walk 150 steps is the only ability whose exercising allows one to recognize $s_1$’s truth conditions obtaining.} I can show that I have this ability by walking out my office door and taking 150 steps toward the water fountain. To understand $s_2$, I must have the ability to walk 1,500,000 steps. However, I am not physically fit. Unlike my ability to walk 150 steps, I do not actually have the ability to walk 1,500,000 steps. So how could I justify my belief that I understand $s_2$? Reasoning according to Dummett’s diagnosis goes like this: I understand $s_1$ for reasons indicated above; I can state what ability would allow me to understand $s_2$ regardless of whether or not I possess that ability; I then recognize a similarity between these two abilities. I can walk 150 steps; the only thing that prevents me from walking more—viz., 1,500,000—is my lack of physical fitness. So I could have the ability to walk 1,500,000 steps if I were to get into better shape. Similar reasoning may lead me to think that I also understand $s_3$: I can walk 150 steps; the only thing that prevents me from walking more—viz., an infinite number—is my finite lifespan. So I could have the ability to walk an infinite number of steps if I were to live forever.\footnote{Technically, what prevents me from possessing the ability that will allow me to recognize $s_3$’s truth conditions’ obtaining is my inability to walk an infinite number of steps in a finite amount of time, since Dummett claims that verification procedures must be decidable. However, I think that the force of the analogy is best felt when we compare these abilities without reference to this restriction. The rest of my argument can proceed either way.}

Dummett describes this process as analogical extension, which I think is a misleading way of characterizing what’s going on. Certainly there is analogical reasoning going on here; the ability to walk 150 steps is like the ability to walk
1,500,000 steps and the ability to walk an infinite number of steps, insofar as they are all an exercise of the more general ability of walking. Dummett argues that we try to widen the number of sentences we understand by claiming that we understand sentences which can be verified through abilities we do not possess actually but are nevertheless analogous to abilities we do possess actually. This analogy-based stretching of our understanding can only extend so far, he claims, and it breaks down when we try to explain how we might understand undecidable sentences. In the case of undecidable sentences, the abilities necessary to verify them are simply disanalogous to any ability we may possess actually. In the above example, walking an infinite number of steps is disanalogous to walking 150 steps. Now, I agree with Dummett that these abilities have no analogues among the abilities we possess actually. But I think that Dummett places too little emphasis on why these are disanalogous. Abilities’ being analogous or disanalogous should be based on more than the presence of a general ability of which they are apparently instances. For example, the ability to walk 150 steps is not analogous to the ability to walk an infinite number of steps just because they are both instances of walking. After all, the “ability” to walk forwards and backwards at the same time is written as an instance of the more general ability of walking. But no human could possess this ability. In this case, I have set up what appears to be an analogy. But there is no relevant analogy between the ability to walk 150 steps and this other ability; the only similarity we see is a similarity in the words we use to describe each of them (they both begin with the ability to walk). But this is how we describe the abilities, not a property of the abilities themselves, so we cannot use it as a basis of an extension.
Analogical extension is better understood as being underwritten by the ways in which we come to possess abilities. One comes to possess the ability to walk 150 steps by following a certain process. Since this ability is gained early in one’s walking career, this process will be relatively rudimentary: learning to keep one’s balance, to move one’s feet in a particular way, to manage one’s energy, etc. This process would also include physical changes in the learner, such as an increase in stamina. The process by which an individual comes to possess the ability to walk 151 steps is similar to the process just described. This second process would be nearly identical to the first, but would require the learner to gain slightly more mastery in all the areas described above. Similarly, the process by which an individual came to possess the ability to walk 1,500,000 steps would require even greater mastery in these areas. Nevertheless, we can conceive of this advanced process as being an extension of the rudimentary process first described. The ability to walk an infinite number of steps is not analogous to these abilities. Rather than claiming that there is no process by which we could gain his supernatural ability—which seems true, but also question-begging—let us assume that an individual possesses this ability. This individual could not have come to possess this supernatural ability via a process that is an extension of the rudimentary process described above.\footnote{If the force of this claim seems weak, recall that this ability, insofar as it is a semantic ability, is best described as “the ability to walk a infinite number of steps in a finite amount of time,” as described in my previous note.} An individual can come to possess a supernatural ability only via some supernatural process dis-analogous to the processes by which we gain the abilities involved in legitimate examples of analogical extension.
3.5 THE CHALLENGE TO CLASSICAL LOGIC

According to classical logic, for any sentence $\varphi$, either $\varphi$ or $\neg \varphi$ is true. Above, we showed that a sentence is true only if we can recognize this, so if classical logic is correct then for any sentence $\varphi$, we can recognize either $\varphi$’s or $\neg \varphi$’s truth conditions’ obtaining. Since we are unable to recognize undecidable sentences’ truth conditions’ obtaining, they cannot be true and so it appears that classical logic is wrong.

This is Dummett’s challenge to classical logic in a nutshell. But there are several steps we must follow before we can describe his challenge fully. The first is to understand how a theory of meaning carries normative weight at all, much less enough to ensure that we ought to believe its account of truth when it conflicts with classical logic’s. We will do this by characterizing the relationship between linguistic practice and a theory of meaning.

In order for language to be a tool for communication, the units of linguistic practice—viz., sentences—must possess meanings. Moreover, communication can be successful only if the speaker’s sentences mean what he believes them to mean. How a sentence’s meaning relates to the ideas that the speaker intends to communicate is far beyond the scope of this project. But I do think we can say that a competent speaker will know the meanings of the sentences he uses and will use a sentence to communicate an idea just because the sentence has that meaning. In other words: given a sentence, we use it in a particular way just because we believe it has a particular meaning. If that belief is incorrect, and so a sentence has no meaning at all or a meaning other than we think it does, then communication will not be successful.\footnote{Communication can be unsuccessful, albeit in a different sense, if the sentence’s truth value is not well-defined, but this is beyond the scope of this project.}
lief that a sentence has a particular meaning requires some kind of justification; otherwise we have no reason to think that we are actually communicating as we intend or even at all. It falls to the theory of meaning to provide this justification. A theory of meaning will describe how meaning behaves in our language; e.g., how the meaning of a complex sentence is determined by its parts. If a theory of meaning is to provide justification for our belief that a sentence has a particular meaning, it must also explain how a sentence possesses the meaning it does. An explanatory theory of meaning will then serve to underwrite the assumption that our language is a tool for communication.

In the process of describing sentences’ meanings and how they have those meanings, a theory of meaning will certify certain utterances as being correct or incorrect. Note that the terms correct and incorrect here are different from true and false. If a speaker is convinced that snow is black and utters “Snow is black,” then he has uttered a false sentence. The speaker makes a mistake about how the world is. A speaker's utterance is incorrect, as I am using the term, when he makes a mistake about the meaning of the sentence he utters. For example, one way that a theory of meaning determines which utterances are correct or incorrect is by determining which sentences of our language are meaningful.\textsuperscript{11} Since language is a tool of communication by hypothesis, and meaningless sentences cannot be used for communication, the use of a meaningless sentence in a language would be an incorrect use of that expression, relative to a particular theory of meaning.\textsuperscript{12}

\textsuperscript{11}I am not sure if sentences are necessarily meaningful. For example, if we accept that It is noontime on the sun is meaningless, we could say that it is a sentence without meaning or that it does not even count as a sentence.

\textsuperscript{12}I do not see how it could be used correctly at all; maybe it is better, rather than say that it is an incorrect use, I should say that it is not a correct use in the sense that it fails to fulfill our intent to communicate.
An expression can also be incorrect relative to a theory of meaning when a speaker uses some sentence under the assumption that it has a particular meaning when it actually has a different meaning. If an English speaker travels to Germany and wants to express his belief “Snow is white” but instead utters the sentence Schnee ist rot, he has said something incorrect. Now, clearly Schnee ist rot is false because it means “Snow is red” and snow is not red. But his utterance is not incorrect because he asserted something false. Rather, we trace the problem to his mistakenly believing that Schnee ist rot meant “Snow is white.” So his utterance is incorrect relative to that theory of meaning just because, according to that theory of meaning, Schnee ist rot does not mean “Snow is white.”

We now know what it means to say that a given assertion is incorrect relative to a theory of meaning. But it should be clear now that we could posit different theories of meaning for a language. And so currently we are unable to say that a given assertion is correct, simpliciter, because that assertion may be correct relative to one theory of meaning and incorrect according to another. What we need is some way of adjudicating between theories of meaning, with the hope of finding which theory of meaning is the right one for our language. We could then speak of a given utterance as being correct without needing to relativize this correctness to a particular theory of meaning. Indeed, I think we must have a way of adjudicating between conflicting theories of meaning because we demand that we have a theory of meaning to underwrite our assumption that our language is communicative. In fact, this underwriting role shows us how to make this adjudication: A theory of meaning must be explanatory in

---

[13] Now we can see how correctness is distinct from, but somehow more fundamental than, truth. Uttering a false sentence is blameworthy only if that sentence correctly reflects the speaker’s intention.
order to fulfill this role, so we choose the theory of meaning for our language that best explains how our sentences have the meanings they do.

I think that any disagreement over correctness can be couched in terms of conflict between theories of meaning, even when it may not be obvious that a speaker is hypothesizing a theory of meaning to underwrite his assertion. For example, the “Schnee ist rot” case above could be understood as an example of conflicting theories of meaning. The speaker clearly possessed some informal theory of meaning—since this is required for language to be communicative, and clearly his intent was to communicate—according to which *Schnee ist rot* means “Snow is white.” More than likely, the speaker does not have a well-worked-out theory of meaning according to which his utterance of *Schnee ist rot* is correct. But we assume that his assertion was an attempt to say something true, and this requires that he was under the impression that, at the very least, *Schnee ist rot* meant “Snow is white.”

An important objection to this picture is the skeptical challenge that we cannot formulate the correct theory of meaning for our language. If we could not formulate an explanatory theory of meaning, it must be because our language is actually meaningless or because our language is meaningful but we lack the ability to formulate an explanatory theory of meaning. I will simply dismiss the first version of the challenge. Claiming that our language is meaningless implies that our linguistic practices are not instances of communication, which seems to fly in the face of empirical data. Dummett responds to the second version by making a simplifying assumption that our language can be systematized in a way that allows us to formulate a theory of meaning (Dummett 1976, 65). I would go further and argue that our language’s being systematizable is required
for its being a tool of communication. It makes no sense to say that there is a correct theory of meaning for our language, and it can thus be a tool of communication, but it is impossible for us to have access to that theory of meaning. If we cannot grasp the theory of meaning for our language, even in principle, then we have no hope of knowing which linguistic practices are correct and which are incorrect. If this happens, then our linguistic practices are communicative but not in a way that we can grasp. So our language may be a “tool for communication” in some sense, but not in a sense that we can understand. In other words, we cannot use our own language to communicate as we intend. A fundamentally inaccessible theory of meaning therefore cannot fulfill our demand that we have a theory of meaning to underwrite our assumption that our language is communicative.

We now know what it means to say that a sentence is correct or incorrect according to a theory of meaning, and how to choose the proper theory of meaning for our language. We already knew what it means to say that a sentence is wrong according to a logic: it contradicts the laws of that logic. We can now explain how these accounts allow us to say that a given logic is correct or incorrect, and how Dummett’s account represents a challenge to classical logic.

Logics codify correct linguistic practice. A person’s use of a contradictory sentence, say, the ball is red and the ball is not red is not incorrect “because it violates classical logic,” as if classical logic somehow determines what linguistic practices are correct and which are incorrect. Correct linguistic practice is prior to any codification of it, and a logic is a regimentation of correct practice and perhaps a heuristic for it. So any authority classical logic apparently has is derived from whatever makes correct linguistic practices correct. I explained
above how a theory of meaning ultimately determines which linguistic practices are correct, and how we can determine which theory of meaning is correct for our language. And so this is where we ultimately find the source of normativity for our linguistic practices: the correct theory of meaning will determine the correct linguistic practices, whose regimentation will be the correct logic.

If we assume that classical logic is an accurate regimentation of a particular set of linguistic practices, we can ask if those practices are correct. To ask this question, we must determine whether those practices are the ones certified by the theory of meaning for our language. But the linguistic practices regimented by classical logic could only be approved by a theory of meaning built around a recognition-transcendent notion of truth. No theory of meaning for our language of this kind can be the correct theory of meaning for our language for reasons given above, according to Dummett, so the linguistic practices regimented by classical logic cannot be correct. Classical logic is an accurate regimentation of incorrect linguistic practices. Since the whole point of a logic is to capture correct linguistic practices, classical logic fails.

3.6 DUMMETT’S CHALLENGE AS RIVALRY

My first two chapters described two requirements for rivalry. The first was a formal requirement: the rival logic, understood as a formal system, must stand in a particular relationship to classical logic. The second was a philosophical requirement: the rival logician must have a way of responding to the objection that his particular challenge to classical logic amounted to a purely verbal disagreement. By fulfilling both of these requirements, a rival logic will present a genuine challenge to classical logic.
Dummett’s logic fulfills the formal requirement for rivalry as described in chapter 1. That requirement stated that a logic was a rival to classical logic just in case it disagreed with classical logic over the validity of inferences, even while classical logic and the rival logic agreed that the logical constants in those inferences were actually logical constants according to certain minimal conditions for logical constanthood. The motivation for this requirement was to ensure that the rival logic’s challenge to classical logic could not be dismissed as the result of a difference in dialect or translation function. Dummett’s project involves a widening of intuitionistic semantic principles, but his logic is formally identical to intuitionistic logic. In chapter 1, I showed that intuitionistic logic was a rival to classical logic, so Dummett’s logic fulfills the formal requirement for rivalry also.

Dummett has a response to the philosophical requirement for rivalry. The motivation for this requirement was an objection prevalent in the literature, and described at the beginning of chapter 2, which charged that rivalry was impossible because the rival logician’s logical constants’ use differed from the classical logician’s logical constants’ use. This difference in use implies a difference in meaning, according to the objection, and so the apparent rivalry reduces to a verbal disagreement. The philosophical requirement for rivalry is a requirement to respond to this specific objection. In doing so, the rival logician will have advanced a position according to which his challenge to classical logic more than verbal disagreement.

A difficult way of responding to this objection would be to show that a difference in two logical constants’ uses does not imply a difference in their meanings. I took a step in this direction in the second part of chapter 2, where I
argued that we cannot identify a logical constant's use with its meaning. But I think that a fully-formed version of this response would require an account of what the meanings of the logical constants actually consist in—a daunting project, and one I will not pursue here.

Dummett's position allows for a different response to the objection. He can grant that a difference in two constants' uses implies a difference in their meanings, but even the classical logician must admit that we would not take the incorrect uses of these constants into account when comparing them. One way to draw the distinction between correct and incorrect uses is by appealing to the theory of meaning for our language. Dummett's theory of meaning rules as incorrect the instances of \( \text{LEM} \) about which Dummett and the classical logician disagree. There would be no difference in the correct uses of classical disjunction and Dummett's disjunction. The objector thus loses the evidence he would use to show that Dummett's disagreement with classical logic is a case of verbal disagreement.

It is unclear how the objector may attempt to rebut this response without arguing that Dummett's larger argument fails. Besides this strategy, one may argue that Dummett's standard of incorrectness described above is irrelevant here. Dummett's argument concerns the meaning of \( \text{or} \), one may argue, but the meanings under discussion here concern the meanings of the logical constants \( \lor_c \) and \( \lor_i \) (classical disjunction and intuitionistic disjunction, respectively).

This rebuttal fails, however. The logical constants in the formal language are meant to correspond to the logical constants in the natural language—\( \text{or} \), in this case. Claiming that a flawed disjunction in the natural language can be a correct disjunction in the formal language is to sever the connection between
the two languages and to regard the formal language as a purely formal system.

One is welcome to do this as a project of pure mathematics, but it is no longer
logic as we understand it.

Although Dummett has the means of fulfilling the formal and philosophical
requirements for rivalry, this is distinct from concluding that his challenge suc-
ceeds and so has proven that classical logic is wrong. There is no question that
the logic he advocates fulfills the formal requirement for rivalry. But the many
attacks on his larger argument in the literature do not allow us to say defini-
tively that his position is correct. Since his larger argument forms the basis for
his response to the philosophical requirement for rivalry, we thus cannot con-
clude that his response to the second requirement is successful. But this does
not impact my project. I am not trying to show that there are rival logics that
successfully show that classical logic is wrong; I am trying to show that rivalry is
possible and explore the structure of these challenges. Even if we cannot say for
certain that Dummett’s larger argument succeeds, it certainly has merit, and so
his arguments do provide good reason to think that rivalry is possible, and that
his theory represents a genuine challenge to classical logic. That said, I do think
that the structure of Dummett’s account can be made clearer. In the next chap-
ter, I will offer a modest addition to Dummett’s account and a supplemental
challenge to classical logic based on this addition that will make the structure
of Dummett’s account more apparent.
CHAPTER 4

AMENDING DUMMETT’S CHALLENGE

4.1 INTRODUCTION

In the last chapter, I distinguished between three stages of Dummett’s account. The first is his development of a theory of meaning that will serve as the background for the challenge proper. In this stage, Dummett concludes that understanding a sentence consists in the possession of a semantic ability or abilities corresponding to the sentence that allow us to verify and falsify the sentence. Dummett’s opponent in this stage is the realist, who argues for a recognition-transcendent notion of truth.¹ The second stage is Dummett’s argument that bivalence does not hold because we do not possess enough semantic abilities to verify and falsify every sentence of our language. Dummett’s opponent in this stage is the bivalentist, who claims we do possess, at least in principle, the semantic abilities necessary to understand our entire language. Note that we assume that the bivalentist agrees with Dummett on the success of the first stage, and her objections are different from the realist’s. The third stage is Dummett’s diagnosis of why one may have erroneously thought that bivalence held.

I claimed that Dummett describes bivalence as arising from unwarranted

¹Referring to Dummett’s opponent here as “the realist” serves to give that opponent a name, not to assert that any variety of realism would disagree with Dummett at this point or in this way.
idealization: it diagnoses our faith in bivalence as coming from a misrepresent-
tation of the extent to which we can extend our abilities to apply verification
processes for a sentence. Warranted idealization can occur, according to Dum-
mett, when we draw analogies between the abilities we possess actually and
those we possess only in principle. I will refer to these as our actual abilities
and our potential abilities, respectively. Eventually, the analogies we draw be-
come strained, and we mistakenly attribute to ourselves abilities that we do not
possess actually, and which have no analogues among the abilities we possess
actually. These are our unobtainable abilities.

In this chapter, I will propose an amendment to Dummett’s theory of mean-
ing. According to Dummett, we understand a sentence just in case we pos-
sess two abilities: the ability to verify the sentence and the ability to falsify
the sentence. I will show that these abilities each can be decomposed into
two abilities: the ability to identify the objects named in that sentence, and
the ability to recognize whether the relevant predications of those objects oc-
cur. I will also show these further semantic abilities require, as a manifestation
condition, that speakers direct their attention toward an appropriate region of
spacetime. Based on this amended first stage, I will offer amended second and
third stages that follow the same methodology as Dummett’s original second
and third stages, but which take advantage of my additions.

To be clear: I am not suggesting that Dummett’s challenge to classical logic
is flawed, or that my amended version of his challenge replace his original one.
That said, my amended version offers two advantages. First, it will offer a novel
mechanism with which to characterize the idealization of our semantic abil-
ities. Analogical extension is difficult to comprehend, and I suggest that my
mechanism is more clear. Using this other extension mechanism may result in different lines being drawn between the actual and potential abilities, and the potential and unobtainable abilities. But Dummett's point—that where the lines fall is unimportant to his challenge as long as the set of unobtainable abilities is nonempty—will survive and still serve as the basis for an attack on bivalence. The second advantage of my amended version is that it will allow us to compare abilities' degrees of idealization, making it possible to take a chain of abilities and identifying the point at which to draw the line between the potential and unobtainable abilities, in theory. Dummett does say that we do not need to know precisely where this line lies. In order to mount a challenge against classical logic, according to Dummett, all we need to show is that the set of unobtainable abilities is nonempty.\footnote{Strictly speaking, we also need an assurance that the members of the unobtainable abilities correspond to sentences of our language such that not possessing those abilities implies that we do not understand these sentences.} I do agree with him on this point, but I also think that we have a sense that a given semantic ability involves more or less idealization than another. The force of Dummett's challenge is amplified when we have a way of capturing this intuition.

Even after describing these advantages, I must clarify my motivations for amending Dummett's challenge and explain its place in my larger project. As shown in the last chapter, Dummett's challenge is an example of rivalry. My amended version of the challenge makes additions to his theory of meaning, but I am not motivated by a desire to improve Dummett's challenge. My additions to Dummett's theory will provide more structure upon which Dummett can base his challenge while remaining faithful to spirit of the original challenge. In that sense, my amended version of Dummett's challenge is not a new challenge, but a more detailed presentation of Dummett's original challenge.
This additional detail will be used in the next chapter to describe Dummett’s characterization of classical logic, and compare it to characterizations of classical logic from other cases of rivalry.

4.2 AMENDING DUMMETT’S FIRST STAGE

In his original first stage, Dummett describes a speaker’s understanding a sentence $\varphi$ in terms of his ability to apply a verification procedure for $\varphi$. The following example that will show how spatiotemporal location and a speaker’s ability to identify the subject of a predicate are crucial to understanding his ability to apply a verification procedure for a sentence. Because of their importance, they must be included in our discussion to allow our accurately ascribing understanding to speakers.

Suppose Karl is standing in front of, and looking at, the Empire State Building. He would then seem to be in a good position to verify or falsify the sentence *The Empire State Building is grey*, hereafter $\text{Grey(EB)}$. We may tentatively describe the ability he possesses to verify $\text{Grey(EB)}$ as the ability to recognize whether the Empire State Building is grey. We will denote it as $\left[\text{Grey(EB)}\right]$. For the sake of argument, let’s assume that Karl possesses $\left[\text{Grey(EB)}\right]$ actually, so there is no question of whether we are idealizing Karl’s semantic abilities. Since Karl can actually verify $\text{Grey(EB)}$, Karl therefore understands $\text{Grey(EB)}$, but only in a partial sense, according to Dummett, until Karl also possesses the ability to falsify $\text{Grey(EB)}$.\(^3\)

---

\(^3\)In general, Karl’s understanding a sentence does not require his being able to actually carry out that sentence’s verification procedure. Here we consider a simplified case, where we do not yet need to address the complications of whether Karl’s not being able to actually execute a sentence’s verification procedure affects his understanding of that sentence.
Suppose now that Karl flies to Paris and stands in front of the Eiffel Tower. He would then seem to be in a good position to verify or falsify the sentence *The Eiffel Tower is painted a bronze color*, hereafter *Bronze(ET)*. We may tentatively describe the ability he possesses to verify *Bronze(ET)* as the ability to recognize whether the Eiffel Tower is bronze colored. We will denote it as \([\text{Bronze(ET)}]\).

Again, assume that he possesses this ability actually. Since Karl can actually verify *Bronze(ET)*, Karl therefore partially understands *Bronze(ET)*, according to Dummett.

Based on Dummett's position, the following questions are roughly equivalent:

- On what grounds can we say that Karl can verify and falsify *Grey(EB)*?
- How can Karl show that he understands this sentence?
- In what way can one conclude that Karl possesses \([\text{Grey(EB)}]\)?

In all cases, our answers will reference Karl's exercising of \([\text{Grey(EB)}]\) and describe his use of *Grey(EB)* to report a direct observation. Yet our actual answers to these questions will differ depending on whether Karl is in New York or Paris—even when we assume he understands *Grey(EB)* in both cities. In New York, his understanding of *Grey(EB)* was underwritten only by \([\text{Grey(EB)}]\). In Paris, he still possesses \([\text{Grey(EB)}]\) but his distance from the Empire State Building prevents us from saying that he simply exercises \([\text{Grey(EB)}]\) to underwrite his understanding of *Grey(EB)*. If his understanding of *Grey(EB)* requires his exercising \([\text{Grey(EB)}]\), then his understanding of *Grey(EB)* while in Paris must be underwritten by \([\text{Grey(EB)}]\) and an additional ability: the ability to travel from

---

4This “reporting” is important. It’s an action that we, as arbiters of Karl’s understanding, understand to indicate an awareness on Karl’s part of the connection between the sentence *Grey(EB)* and the verification procedure involving \([\text{Grey(EB)}]\).
Paris to New York somehow. So Grey(EB) is understood more immediately, in a sense, in New York than in Paris. We will say that it is proximately verifiable and proximately falsifiable in New York City, but neither proximately verifiable nor proximately falsifiable in Paris.\(^5\)

The upshot of this is that we are unable to discuss the ability that underwrites our understanding of a sentence without also mentioning the circumstances in which that ability is exercised. Even if a speaker possesses the semantic ability \([\varphi]\), the speaker can perform a verification of \(\varphi\) only if the speaker exercises \([\varphi]\) in circumstances that allow this ability to succeed. Karl may possess \([\text{Grey} (\text{EB})]\), but he must exercise \([\text{Grey} (\text{EB})]\) while looking at the Empire State Building in order to convince us that he understands *The Empire State Building is grey*.\(^6\) It follows that a speaker may exercise \([\varphi]\) under certain circumstances and fail to perform a verification of \(\varphi\). If Karl exercises \([\text{Grey} (\text{EB})]\) while looking at the Eiffel Tower, he will not be able to perform a verification of Grey(EB). But we may not count this failure as evidence that Karl did not understand Grey(EB). His failure is not due to his not possessing \([\text{Grey} (\text{EB})]\) or because Grey(EB) is without truth value, but only because he exercised \([\text{Grey} (\text{EB})]\) under inappropriate circumstances.

\(^5\)There is a difference between a sentence's being proximately verifiable and its being true, and its being proximately falsifiable and its being false—even if we understand truth and falsity in justificationist terms. Saying that Grey(EB) is proximately verifiable in New York means: assuming Grey(EB) can be verified at all, it can be verified most immediately in New York. This definition must not be understood as meaning that proximate verifiability is conditioned on a sentence's actually being true. Rather, we determine where one would verify the sentence if it were true. A false sentence like *The Empire State Building is red* is proximately verifiable in New York, because the most appropriate place to attempt its verification is in New York.

\(^6\)Dummett claims that using a sentence to report a direct observation is the model manifestation of implicit knowledge of that sentence's meaning, so we focus here on direct observations. One may still object that one need not observe, say, the Empire State Building to knows its color: one need only observe some kind of authoritative photograph or other source. In ordinary cases, this kind of observation ought to be sufficient. However, the question of what constitutes an acceptable manifestation is a complicated one to answer, and a separate one at that. So we will consider only the model case.
This example reveals a manifestation condition for a semantic ability: that a speaker be observing an appropriate region of spacetime. For example, Karl can exercise $\text{Grey(EB)}$ in the way relevant to manifesting understanding of $\text{Grey(EB)}$ only if he is directing his attention toward the Empire State Building. In addition, he must be able to identify the Empire State Building; we will turn our attention to this further condition shortly.

In essence, the above story illustrates how truth is regulated by a principle more specific than Dummett’s Principle C (“if a statement is true, there must be something in virtue of which it is true”). The realist may appeal to Principle C in order to support a correspondence theory of truth, where the “something in virtue of which” a sentence is true is some feature of the world. The above story, in the hands of the realist, would suggest that that feature of the world can be located at some restricted region of spacetime. Dummett claims that the realist has his conceptual priority backwards: the configuration of the world does not tell us which sentences are true; rather, the true sentences tell us about the configuration of the world. So the above story, in the hands of Dummett, would suggest that each sentence’s contribution to a full description of the world is restricted to a particular region of spacetime. The realist would claim that the greyness present in the region of spacetime occupied by the Empire State Building underwrites the truth of $\text{Grey(EB)}$. Dummett would claim that the truth of $\text{Grey(EB)}$ assures us of the presence of greyness in the region of spacetime occupied by the Empire State Building.\(^7\)

Having invoked Dummett’s understanding of Principle C, I will introduce two convenient pieces of shorthand: we will say that a **verifier** or **falsifier** of a

\(^7\)Dummett and the realist would disagree as to the nature of spacetime. The realist would claim that spacetime exists independently of us, while Dummett would claim that spacetime is the spatiotemporal grid that makes communication possible (see Dummett 2004, 51 and 55).
sentence occupies a particular region of spacetime, similar to the way that a realist may say that a truthmaker for a sentence occupies a certain region of spacetime. To say that a verifier or falsifier for a sentence is located at a spatiotemporal location is to say that a speaker's belief in that sentence's truth or falsity is justified—and thus his understanding of that sentence partially underwritten—because he is directing his attention toward that location and exercising his ability to perform a verification (or falsification) of this sentence. Speaking of verifiers and falsifiers appeals to the intuition that a verification procedure succeeds only if a speaker is focusing his attention on an appropriate region of spacetime, and this connection obtains based on something particular to that region of spacetime. That said, verifiers are not objects in any sense, even as we may casually refer to them as being “located” in a particular region of spacetime.

So far, we have shown that semantic abilities have manifestation conditions specifying what region(s) of spacetime a speaker must be observing. Before moving on to the amended version of Dummett’s first stage, we must talk about another type of ability a speaker must possess.

Consider Dummett’s original first stage. There, understanding a sentence consisted in the possession of two abilities. Using the example of Grey(EB), these abilities would be:

(i) the ability to verify Grey(EB)
(ii) the ability to falsify Grey(EB)

This understanding is partial, not full, because Dummett requires that the speaker must possess the ability to verify and the ability to falsify a sentence. So a speaker who understands some sentence will also need to have access to a falsifier for that sentence, along with an ability to apply the relevant procedure for the sentence.
Or, to use the language from the above paragraph,

(i) the ability to recognize the presence of greyness in the region of spacetime occupied by the Empire State Building

(ii) the ability to recognize the absence of greyness in the region of spacetime occupied by the Empire State Building

Expressed this way, we can see that these abilities no longer assure understanding of Grey\((EB)\). The ability to recognize the presence of greyness in the region of spacetime occupied by the Empire State Building is the same practical ability as the ability to recognize the presence of greyness in the region of spacetime occupied by, say, Notre Dame de Paris. Both of these are names of one ability—the ability to recognize the presence of greyness.\(^9\) There’s nothing wrong with one ability being involved in the meaning of multiple sentences. The ability to recognize greyness is a piece of what a speaker must possess in order to understand both *The Empire State Building is grey* and *Notre Dame de Paris is grey*, the latter hereafter Grey\((ND)\). But a speaker may understand one of these sentences, and not understand the other, if he were unable identify the Empire State Building or Notre Dame. This contrast is more striking if a speaker had no exposure to French; such a speaker could not understand Grey\((ND)\) because the subject phrase of the sentence would be gibberish to him.\(^{10}\) The problem, it seems, is a speaker’s possessing the abilities of recognizing the presence and absence of greyness ensures that he can recognize the predication appearing in Grey\((EB)\), but he cannot understand Grey\((EB)\) unless he can recognize the subject of the

---

\(^9\)The same goes for the ability to recognize the absence of greyness, but for the sake of simplicity we will set that aside.

\(^{10}\)If the example involving Notre Dame is not sufficiently evocative, consider a sentence describing an object as grey, where the name of the object is given in an obscure, untransliterated language.
sentence also. So we must amend Dummett’s account to include subject identification. A speaker must be able to recognize the Empire State Building, but not in any context whatsoever. Rather, we need a requirement that establishes a connection between subject identification and verification procedures. We can thus limit the requirements for subject identification by claiming that a putative speaker must be able to identify the subject of the given sentence in the context of a verification of that sentence, and he must be able to identify the subject of the given sentence in the context of a falsification of that sentence. One would expect that a speaker be able to identify the subject of a given sentence in far more contexts than those described in these requirements. Indeed, Dummett’s larger philosophy of language includes an account of what a speaker must know in order to know the meaning of a name. But here we are not concerned with giving an account of whether a speaker understands, say, the name \textit{Notre Dame de Paris}. Rather, we are concerned with ensuring that a speaker’s abilities to apply a verification procedure for a sentence are sufficient to underwrite his understanding of that sentence.

Note that we cannot simply assume that a speaker can identify the subject of a predicate, even if it is described explicitly in the semantic ability in question; e.g., “the ability to observe the presence of greyness in the region of spacetime occupied by the Empire State Building.” That is, even if a speaker were to possess this ability, we have no guarantee that the speaker would be able to identify the Empire State Building. To suggest otherwise is to fail to see the ambiguity of this ability. “The region of spacetime occupied by the Empire State Building” defines a particular region of spacetime, but this definition is a name for that particular region of spacetime. A speaker may apparently manifest this
ability while being unfamiliar with the Empire State Building, believing himself to be manifesting, say, “the ability to observe the presence of greyness in the region of spacetime occupied by the building at the southwest corner of the intersection of 34th Street and 5th Avenue.” This speaker’s manifesting of this ability is indistinguishable from his manifesting “the ability to observe the presence of greyness in the region of spacetime occupied by the Empire State Building.” Yet this speaker would not understand Grey(EB) because he would not know the meaning of the phrase the Empire State Building.

We are now ready to amend Dummett’s first stage. To make his account more explicit, and to include subject identification, I propose the expanded Dummettian thesis that understanding a sentence $\varphi$ consists in possessing the following abilities:

(i) the ability to identify those objects mentioned in a verification of $\varphi$, written $Id^+[\varphi]$

(ii) the ability to identify those objects mentioned in a falsification of $\varphi$, written $Id^-[\varphi]$

(iii) the ability to recognize the relevant predications of the above-mentioned objects appearing in a verification of $\varphi$, written $Rec^+[\varphi]$

(iv) the ability to recognize the relevant predications of the above-mentioned objects appearing in a falsification of $\varphi$, written $Rec^-[\varphi]$

Let us refer to the first two as examples of \textit{identificational abilities} and the second two as examples of \textit{recognitional abilities}.

Earlier in this chapter, I described a manifestation condition for semantic abilities: that a speaker be observing an appropriate region of spacetime. Call the region of spacetime being observed by a speaker his \textit{spatiotemporal focus}. 

99
While spatiotemporal focus is not explicitly mentioned in the above list, its influence is still felt in ways we will explore in the next section.

4.3 THE ROLE OF SPATIOTEMPORAL FOCUS

In this section, we will formalize spatiotemporal focus so as to understand how it comports with the logical constants. We have two reasons to analyze the connection between spatiotemporal focus and semantic abilities. First, we learned from the above story that a putative speaker must have an appropriate spatiotemporal focus in order for his action to count as evidence of his understanding a particular sentence. Second, a formalization of spatiotemporal focus will allow us to resolve an ambiguity we may encounter when attempting to identify which semantic ability a putative speaker is manifesting in a given situation.

This ambiguity arises because semantic abilities are just practical abilities that we use for semantic ends. For example, \( \text{Rec}^+ \left[ \text{Grey(EB)} \right] \) is just the ability to recognize a predication of greyness, which is a non-semantic, practical ability. By describing \( \text{Rec}^+ \left[ \text{Grey(EB)} \right] \) as a semantic ability, we are not describing it as an entirely new ability. Rather, we are noting the connection between the ability to recognize greyness and Grey(EB). We partition off certain uses of our ability to recognize greyness—those in which our exercising of that ability occurs in the context of a verification procedure for Grey(EB)—and refer to those uses as being uses of the semantic ability \( \text{Rec}^+ \left[ \text{Grey(EB)} \right] \). All of this follows from Dummett’s theory of meaning. Up to this point, we have not described a way to distinguish between manifestations of two different semantic abilities when they share the same underlying practical ability. For example, \( \text{Rec}^+ \left[ \text{Grey(EB)} \right] \) and \( \text{Rec}^+ \left[ \text{Grey(ND)} \right] \) are different semantic abilities, but since they share an un-
derlying practical ability, a manifestation of either will be a manifestation of the ability to recognize greyness.\footnote{We cannot appeal to a speaker’s intent when we attempt to determine whether a given manifestation is a manifestation of $Rec^+[\text{Grey(EB)}]$ or $Rec^+[\text{Grey(ND)}]$. Dummett’s theory of meaning is built around the idea that a putative speaker understands a language just in case a community that speaks that language can judge that he understands the language. So understanding can only be manifested in actions observable to others.} The way we differentiate between a putative speaker’s manifesting $Rec^+[\text{Grey(EB)}]$ rather than $Rec^+[\text{Grey(ND)}]$ is by noting whether he is directing his attention toward the Empire State Building or Notre Dame when he declares that greyness is present.

To have a rigorous account of spatiotemporal focus, we must first define what regions of spacetime are relevant. Let a \textit{spatiotemporal coordinate} be an ordered quadruple $(s_1, s_2, s_3, t)$ that we interpret as a point in spacetime, with each of the $s_n$’s representing that point’s location along a spatial dimension and $t$ representing that point’s location along a temporal dimension.\footnote{I am claiming that, for a given semantic ability, a speaker’s having an appropriate spatiotemporal focus is manifestation condition for that ability. We could include more information in the manifestation conditions, such as location in a possible world. In that case, our coordinates would be ordered quintuples that included spatiotemporal coordinates and the index of the world in which the area was located. To keep my discussion simple, for the time being I will use spatiotemporal coordinates only.} Let $\mathcal{P}$ be the set of all sets of spatiotemporal coordinates that are both connected and open, and call the members of $\mathcal{P}$ \textit{spatiotemporal patches}. In a sense, spatiotemporal patches are a more general case of the “spacetime worms” one may use to capture an ordinary object’s enduring through an interval of time. Indeed, any spacetime worm of any object is a member of $\mathcal{P}$. The main difference between patches and worms is that patches may not be occupied. We are not looking to capture an object during an interval of time, only a region of space during an interval of time.

A proper account of spatiotemporal focus requires two mappings: one from a patch to the set of sentences whose verifiers are located there, and another
from a patch to the set of sentences whose falsifiers are located there. These
mappings must be relativized to a possible distribution of verifiers and falsifiers
across the patches. Let a procedural specification \( s \) be an ordered pair \( \langle s^+, s^- \rangle \),
where \( s^+, s^- \subseteq F \), such that \( s^+ \) and \( s^- \) are those sentences that can be simult-
aneously verified and falsified, respectively. Given Dummett’s rejection of di-
aletheism, \( s^+ \) and \( s^- \) will be disjoint. We will require that these specifications
be complete: \( s^+ \cup s^- = F \), so while there could be as many as \( 2^{|F|} \) procedural
specifications, in practice there will be fewer because of extralogical connec-
tions between sentences.\(^{13}\) For example, verifying \( \text{Bronze(ET)} \) implies that we
cannot verify \( \text{The Eiffel Tower is painted pink} \). Let \( S \) be the set of all procedural
specifications. We will index the procedural specifications, and carry that index
over into the members of each ordered pair; so \( S = \{ s_1, s_2, \ldots \} \) and \( s_i = \langle s_i^+, s_i^- \rangle \).

For every \( s_i \in S \), let \( \pi_i \) be an ordered pair of mappings \( \langle \pi_i^+, \pi_i^- \rangle \). Let \( \pi_i^+ : P \rightarrow \wp(F) \) such that, for \( p \in P \), if \( \varphi \in \pi_i^+(p) \), then a speaker can successfully manifest \( \text{Rec}^+[\varphi] \) if he is directing his attention toward \( p \). Let \( \pi_i^- : P \rightarrow \wp(F) \) such that, for \( p \in P \), if \( \varphi \in \pi_i^-(p) \), then a speaker can successfully manifest \( \text{Rec}^-[\varphi] \) if he is
directing his attention toward \( p \). For example,

\[ \text{Grey(EB)} \notin \pi_i^+ (\text{The front of the Eiffel Tower at noon on July 12, 2010}) \]

but

\[ \text{Bronze(ET)} \in \pi_i^+ (\text{The front of the Eiffel Tower at noon on July 12, 2010}). \]

\(^{13}\)The set of procedural specifications does not constitute an exhaustive catalog of possible
verificationist truth specifications. If so, then the requirement that procedural specifications
be complete would amount to an endorsement of bivalence. Rather, the procedural specifica-
tions should be seen as a catalog of the extralogical connections between the sentences of our
language. Dummett would claim that any verificationist truth specification would be a proper
subset of a procedural specification
We can also describe $\pi_i^+$ and $\pi_i^-$ by saying that a sentence $\varphi$ will be proximately verifiable at $p$ under procedural specification $s_i$ just in case $\varphi \in \pi_i^+ (p)$ and proximately falsifiable at $p$ under procedural specification $s_i$ just in case $\varphi \in \pi_i^- (p)$. For any spatiotemporal patch $p$, we call $\langle \pi_i^+(p), \pi_i^-(p) \rangle$ its semantic map under specification $s_i$, and refer to $\pi_i^+(p)$ and $\pi_i^-(p)$ as its positive semantic map and negative semantic map under procedural specification $s_i$, respectively.

Note that we map patches onto sentences, rather than the other way around, because we do not assume that a given sentence has a “canonical verifier,” as if any given sentence $\varphi$ is proximately verifiable at only one spatiotemporal patch. Sentences may have multiple verifiers and falsifiers; i.e., for a given sentence $\varphi$, there may be patches $p$ and $p'$ such that a speaker's verification of $\varphi$ could involve his directing his attention toward $p$ or his verification of $\varphi$ could involve his directing his attention toward $p'$. For example, suppose $p, p' \in P$, and let $p'$ be the patch such that $\langle s_1, s_2, s_3, t \rangle \in p$ just in case $\langle s_1, s_2, s_3, t + \varepsilon \rangle \in p'$, where $\varepsilon$ is some arbitrarily small amount of time. Informally, $p'$ is spatially the same as $p$, but slightly shifted temporally. If $\text{Grey}(\text{EB}) \in \pi_i^+(p)$, then it ought to be the case that $\text{Grey}(\text{EB}) \in \pi_i^-(p')$ also. If we can confirm the color of the Empire State Building at one instant, we ought to be able to do the same one instant later.

Verifiers are ubiquitous: for any $p, p' \subseteq P$, if $p \neq p'$ then $\pi_i^+(p) \cup \pi_i^-(p)$ contains at least one sentence that is not a member of $\pi_i^+(p') \cup \pi_i^-(p')$. In other words, there is no $P \subset P$ such that $\cup_{p \in P} (\pi_i^+(p) \cup \pi_i^-(p)) = F$. Ubiquity formalizes the fact that each spatiotemporal patch contains at least one unique verifier or one unique falsifier to contribute to our project of determining how much of $F$ we understand. For example, for any patch $p$, verifiers and falsifiers for The patch $p$ is nonempty and The patch $p$ is empty would both be located at $p$, if
under different procedural specifications.

We will define $\pi^+_i$ and $\pi^-_i$ recursively. Since Dummett advocates a componentalist semantics, and because we assume that a falsification for a sentence is also a verification for its negation, our base cases will use the atomic sentences and their negations. We will then continue our definition of $\pi^+_i$ and $\pi^-_i$ by explaining how proximate verifiability and proximate falsifiability comport with the logical constants. To respect Dummett’s verificationism, we will express these definitions in terms of confirmation, represented as conditional probability, utilizing the probabilistic semantics introduced by Popper and developed by Field (1959; 1977). In order to adapt this probabilistic semantics to the current context, we need to modify the way in which evidence is handled.

Consider the case of a sentence’s being certain. According to Field’s semantics, a sentence $A$ is certain just in case $\forall B \ P(A \mid B) = 1$, where $B$ ranges over the set of all sentences.\(^\text{14}\) This definition is too general for our purposes, because we are relativizing a sentence’s conditional probability to some spatiotemporal focus. The spirit of Field’s definition is that a sentence is certain when the conditional probability of that sentence is 1 regardless of what evidence is considered. To relativize certainty to a spatiotemporal focus, we will specify evidence by referencing semantic maps. Given a patch $p$, a sentence $\varphi$ is certain under $p$ just in case, for every $\gamma \in \pi^+_i(p)$, $P(\varphi \mid \gamma) = 1$.\(^\text{15}\) That is, $\varphi$ is certain under $p$ just in case $\varphi$’s conditional probability remains 1 regardless of the evidence we gather from $p$. One should not interpret this definition as suggesting that every $\gamma$ makes $\varphi$ certain, as if $\varphi$ has a low absolute probability but any $\gamma$ is good

\(^{14}\)If $B = \neg A$, then $P(A \mid B) \neq 1$, but I do not believe this should count against $A$’s being certain. Therefore, I assume $\neg A$ is excluded from the set of sentences over which $B$ ranges. I cannot see where Field makes this exclusion, so any errors from this assumption are mine.

\(^{15}\)Unlike Field’s definition of certainty, here we do not need to exclude $\neg \varphi$ from the possible values of $\gamma$ because we cannot have $\varphi, \neg \varphi \in \pi^+_i(p)$, according to Dummett.
enough to make $\varphi$ certain. Rather, $\varphi$'s certainty consists in its conditional probability remaining 1 regardless of the other evidence we consider. So every $\gamma$ is a piece of evidence that we consider in a search for disconfirming evidence for $\varphi$. Finding none, with $\varphi$'s probability remaining 1 throughout the process, it is certain at $p$.

Certainty will play a role in the below definitions, as will a complementary concept: Given a patch $p$, a sentence $\varphi$ is groundless under $p$ just in case, for every $\gamma \in \pi_i^+(p)$, $P(\varphi \mid \gamma) = 0$. A sentence's being groundless under $p$ does not amount to a claim that $p$ cannot contribute any evidence to confirm its being true. Rather, a sentence is groundless under $p$ just in case any evidence from $p$ confirms, in the strongest possible sense, that the sentence is not true.

We can now define $\pi_i^+$ and $\pi_i^-$. Let $\pi_i^+: \mathcal{P} \rightarrow \wp(F)$ and $\pi_i^-: \mathcal{P} \rightarrow \wp(F)$. Dummett claims that the verifiability of atomic and negated atomic sentences is primitive. To capture this, let $F^a$ be the subset of $F$ containing only the atomic sentences of $F$ and their negations, and let $\alpha^+: \mathcal{P} \rightarrow \wp(F^a)$ and $\alpha^- : \mathcal{P} \rightarrow \wp(F^a)$. For each $p \in \mathcal{P}$, let $\alpha^+(p)$ be the set of atomic and negated atomic sentences that are proximately verifiable at $p$, and $\alpha^-(p)$ be the set of atomic and negated atomic sentences that are proximately falsifiable at $p$. We then use the following as our base cases:

$$\varphi \in \pi_i^+(p) \iff \varphi \in \alpha^+(p) \quad \text{(for atomic or negated atomic $\varphi$)} \quad \text{(Base}^+)$$

$$\varphi \in \pi_i^-(p) \iff \varphi \in \alpha^-(p) \quad \text{(for atomic or negated atomic $\varphi$)} \quad \text{(Base}^-)$$

Now, we detail clauses for the logical connectives. The positive clause for conjunction is adapted from a postulate appearing in Field and van Fraassen (1977; 1981). The positive clause for disjunction is adapted from a postulate
from van Fraassen (1981):

\[
\phi \land \psi \in \pi_i^+(p) \iff \text{for every } \gamma \in \pi_i^+(p), \ P(\phi \mid \gamma) \cdot P(\psi \mid \phi \land \gamma) = 1 \quad \text{(Conj\textsuperscript{+})}
\]

\[
\phi \lor \psi \in \pi_i^+(p) \iff \text{for every } \gamma \in \pi_i^+(p), \ P(\phi \mid \gamma) + P(\psi \mid \gamma) - P(\phi \land \psi \mid \gamma) = 1 \quad \text{(Disj\textsuperscript{+})}
\]

For the conditional, van Fraassen suggests that \( \phi \rightarrow \psi \in \pi_i^+(p) \) just in case

\[ P(\psi \mid \phi \land \gamma) = 1 \text{ for every } \gamma \in \pi_i^+(p). \]

By defining a relation of extension between patches, we can say something stronger. We will say that a patch \( p' \) extends \( p \), written \( p \leq p' \) or \( p' \geq p \), just in case both of the following conditions obtain: for every \( \phi \in \pi_i^+(p) \), then \( \phi \in \pi_i^+(p') \); and for every \( \psi \in \pi_i^-(p) \), then \( \psi \in \pi_i^-(p') \). To say that a conditional is proximately verifiable at \( p \) is to say that data available at \( p \) guarantee that \( \psi \) is proximately verifiable wherever \( \phi \) is proximately verifiable, but this guarantee extends only as far as the data are still accessible. To say that data accessible at one patch is accessible in another is to say that the latter extends the former, so we can create the positive clause for the conditional as:

\[
\phi \rightarrow \psi \in \pi_i^+(p) \iff \text{for every } p' \geq p \text{ and } \gamma \in \pi_i^+(p'), \ P(\psi \mid \phi \land \gamma) = 1 \quad \text{(Cond\textsuperscript{+})}
\]

We have already accounted for negated atomic sentences, but still require a clause for negated compound sentences. Field claims that

\[ P(\neg \phi \mid \psi) = 1 - P(\phi \mid \psi). \]

This equivalence implies that a compound sentence is groundless just in case its negation is certain, suggesting \( \neg \phi \in \pi_i^+(p) \) just in case

\[ P(\phi \mid \gamma) = 0 \text{ for any compound } \phi \text{ and any } \gamma \in \pi_i^+(p). \]

Instead, we will use the above clause for the conditional to adapt the traditional intuitionistic identification of \( \neg \phi \) with
\( \varphi \rightarrow \bot \) to the current definition, and thus say something stronger:

\[ \neg \varphi \in \pi^+_i(p) \iff \text{for every } p' \geq p \text{ and } \gamma \in \pi^+_i(p'), P(\varphi \mid \gamma) = 0 \quad \text{(for compound } \varphi) \]

\( \text{(Neg}^+ \text{)} \)

Given a universal quantification \( \forall x \varphi(x) \) with instances \( \{ \varphi_1, \ldots \} \), let \( \Phi \) be a sequence of all the instances of \( \forall x \varphi(x) \) and \( \Phi_i = \langle \varphi_1, \cdots, \varphi_i \rangle \) be an initial subsequence of \( \Phi \). We then say:

\[ \forall x \varphi(x) \in \pi^+_i(p) \iff \text{for every } \gamma \in \pi^+_i(p), \lim_{i \to |\Phi|} P(\forall x \varphi(x) \mid \varphi_1 \wedge \cdots \wedge \varphi_i \wedge \gamma) = 1 \]

\( \text{(Univ}^+ \text{)} \)

The clause for the existential quantifier is straightforward:

\[ \exists x \varphi(x) \in \pi^+_i(p) \iff \text{for every } \gamma \in \pi^+_i(p), P(\varphi(a) \mid \gamma) = 1 \text{ for some subject } a \]

\( \text{(Exist}^+ \text{)} \)

Dummett upholds noncontradiction, so we are free to identify a proof of a negated compound sentence as showing that the compound sentence is false. The above clause for negation is relativized to \( p \), so we can identify \( \neg \varphi \)'s being proximately verifiable at \( p \) with \( \varphi \)'s being proximately falsifiable at \( p \), assuming \( \varphi \) is compound:

\[ \varphi \in \pi^-_i(p) \iff \neg \varphi \in \pi^+_i(p) \quad \text{(for compound } \varphi) \]

\( \text{(Rec}^- \text{)} \)

Finally, we stipulate that no other sentences are in \( \pi^+_i(p) \) or \( \pi^-_i(p) \), and our definition is complete.
The above definition suggests the following account of logical entailment:

$$\Gamma \models \varphi \iff \text{for every } p \in \mathcal{P}, \text{if } \Gamma \subseteq \pi^+_i(p) \text{ then } \varphi \in \pi^+_i(p)$$

According to this relation, entailment extends only as far as our abilities to verify sentences of $\mathcal{F}$. So, while this relation may not be the same one described by Dummett, I claim that it is one he would endorse.

We now have a robust theory of how spatiotemporal focus comports with the logical constants. Above, I mentioned that it may be difficult to identify the ability a speaker is manifesting when two abilities share an underlying practical ability. To solve this problem, we appeal to the fact that the manifestation conditions for a given recognitional ability $Rec^+[\varphi]$ (or $Rec^-[\varphi]$) are determined, at least in part, by the spatiotemporal patches containing verifiers (or falsifiers) for $\varphi$. So we associate with each recognitional ability $Rec^+[\varphi]$ (or $Rec^-[\varphi]$) an equivalence class of patches $P$, where $p \in P$ just in case $\varphi \in \pi^+_i(p)$ (or $\varphi \in \pi^-_i(p)$). We can then identify a semantic ability with its underlying practical ability and equivalence class.

### 4.4 ANOTHER CHALLENGE AND DIAGNOSIS

Above, I offered an amended version of Dummett’s first stage and gave reasons for why it represented a more detailed version of Dummett’s account while keeping the spirit of his argument intact. In this section, I will describe a challenge and diagnosis that center on the relationship between spatiotemporal focus and semantic abilities, and that share the same goals as the challenge and diagnosis described by Dummett in the second and third stages of his original
account. The additional diagnosis and Dummett’s diagnosis will complement each other to more fully describe how we may have come to believe that truth is bivalent. Nothing in my amended first stage contradicts Dummett’s original first stage, so this additional challenge and diagnosis sit alongside Dummett’s.

One may object that a new diagnosis is unnecessary, because Dummett’s diagnosis is separate from his challenge to classical logic and so his argument can succeed regardless of the success of the diagnosis. According to this objection, Dummett’s diagnosis is meant only to pacify the classical logician by explaining how he could have fallen into his error.

I believe that Dummett’s diagnosis carries greater rhetorical weight than this objection claims. Dummett’s semantics entail that the classical logician systematically misunderstands his own language—an audacious conclusion, one that may imperil his challenge. If his semantics leads to an implausible account of how we use our language, that could be seen as evidence for his theory’s being flawed. A diagnosis softens the blow of this conclusion by explaining how it is entirely understandable that the classical logician would have mistakenly thought that truth was bivalent. And so a diagnosis defuses the objection that the audacity of Dummett’s conclusion counts as evidence against his challenge.

Besides its rhetorical importance, the diagnosis is also critical to my project because it serves as the basis for Dummett’s characterization of classical logic. In the next chapter, I will use the below diagnosis to show how Dummett characterizes classical logic as an unwarranted idealization from inferences underwritten entirely by actual and potential abilities to inferences underwritten partially by unobtainable abilities. I will then come to my final conclusion, that this general formula for characterizing classical logic—that it arises from an un-
warranted idealization from inferences of a privileged class—is shared by other examples of rivalry.

Bivalence obtains only if we understand all sentences of $F$, which requires that we possess $\text{Rec}^+[\varphi]$ and $\text{Rec}^+[\neg\varphi]$ for any $\varphi \in F$. Possessing these recognitional abilities must involve some idealization, since the ubiquity of verifiers entails that this can be possible only if we are able to direct our attention toward any $p \in P$. Because there are certain patches toward which we could never direct our attention, there must be certain sentences that we cannot understand. This argument is similar to the one Dummett offers for why we do not possess the semantic abilities necessary to understand sentences concerning cases of quantification over infinite domains, counterfactuals, and inaccessible regions of spacetime. In that argument, he did not explain exactly how we extended our semantic abilities through idealization. Instead, he gave examples of putative semantic abilities that we could not possess. I will follow the same pattern.

I am reluctant to say that sentences whose verifiers lie in inaccessible patches are counterexamples to bivalence. A patch’s inacessibility is not a feature of the patch itself, but of that patch’s relation to a speaker. It’s difficult to describe a patch’s being inaccessible in an absolute sense. The only way I can see to do this is by determining which patches are inaccessible for every speaker. This, however, requires full knowledge of the limits of idealization of semantic abilities for every speaker.

In contrast, the verifiers for barely true counterfactuals seem to be located at genuinely inaccessible patches because the verifiers seem to be located in counterfactual reality. Now, the inaccessibility of these patches could be seen as the result of our coordinate system, rather than the patches themselves. Our co-
ordinate system for describing patches is limited to the actual world, so we have no way to locate the patches containing the verifiers for counterfactuals. But suppose we were to include a modal index in our coordinate system. Dummett would claim that the patches in other possible worlds were inaccessible because having the ability to observe them would constitute having “direct insight into counterfactual reality,” which we clearly do not possess, even in principle. Even if we included a modal index, but set aside Dummett’s claim, I would still hesitate to grant that we do not possess these semantic abilities. These counterfactual patches would be inaccessible patches as described above: unobservable but not because of some feature of the patch itself but because of the patch’s relation to a speaker.

The verifiers for sentences involving quantification over infinite domains must be located in unobservable patches because those patches appear to be infinitely long temporally or infinitely large spatially. Unlike the patches containing verifiers for inaccessible regions of spacetime and counterfactuals, these patches are unobservable due to a feature they possess intrinsically. Here we find the limits to our semantic abilities: If $\varphi$ is a sentence involving quantification over an infinite domain, then any patch $p$ such that $\varphi \in \pi^+_i(p)$ is unobservable. A speaker could not manifest $\text{Rec}^+[\varphi]$ because he could not fulfill the manifestation condition of observing $p$, and so we are unable to ascribe understanding of $\varphi$ to him. In other words, we do not understand $\varphi$. As a result, bivalence fails.

Having described an amended version of Dummett’s first stage and an additional challenge to classical logic based on that amended first stage, I will now propose an additional diagnosis. This diagnosis will serve the same purpose in
my amended challenge as Dummett’s diagnosis serves in his original challenge.

If there is any justification to be had for extending our semantic abilities beyond those we possess actually, it must be on the basis of abilities we possess actually, and that this sentence has instances \( \varphi(c_1), \ldots, \varphi(c_n) \). Dummett claims that “we gain our understanding of quantification over finite, surveyable domains by learning the procedure of conducting a complete survey, establishing the truth-value of every instance of the quantified statement” (1976, 61), so for every \( p \in \mathcal{P} \),

\[
\forall x \varphi(x) \in \pi_i^+(p) \iff \varphi(c_1) \land \cdots \land \varphi(c_n) \in \pi_i^+(p).
\]

Moreover, Dummett describes the process of verification as one in which we check the instances of \( \forall x \varphi(x) \) individually, so if

\[
\varphi(c_1) \land \cdots \land \varphi(c_j) \in \pi_i^+(p)
\]

and

\[
\varphi(c_1) \land \cdots \land \varphi(c_k) \in \pi_i^+(p')
\]

where \( j \leq k \), then \( p \leq p' \). Suppose a putative speaker attempted to verify \( \forall x \varphi(x) \). According to Dummett, he must verify this sentence’s instances. As the speaker verifies the instances, he manifests a series of semantic abilities: \( \text{Rec}^+ [ \varphi(c_1) ] \), \( \text{Rec}^+ [ \varphi(c_2) ] \), and so on. Simultaneously, with each instance the speaker verifies, he continues to manifest another series of semantic abilities: \( \text{Rec}^+ [ \varphi(c_1) \land \varphi(c_2) ] \), \( \text{Rec}^+ [ \varphi(c_1) \land \cdots \land \varphi(c_m) ] \). At some point, we must idealize our abilities. Suppose \( \text{Rec}^+ [ \varphi(c_1) \land \cdots \land \varphi(c_m) ] \) is the limit of our actual
abilities. Knowing that we possess $Rec^+ [\varphi(c_1) \land \ldots \land \varphi(c_m)]$, the idealization seems to rest on the instance $\varphi(c_{m+1})$; i.e., we need only show that we possess $Rec^+ [\varphi(c_{m+1})]$ in order to show that we possess $Rec^+ [\varphi(c_1) \land \ldots \land \varphi(c_{m+1})]$.

This, I think, is the beginning of a new diagnosis, describing how it is that we come to believe that we can limitlessly extend our recognitional abilities relevant to verifying sentences involving quantification over infinite domains. Dummett claims that we believe that this idealization is not bounded by medical possibility. By couching Dummett’s diagnosis in terms of recognitional abilities, we gain a greater appreciation for why we would disregard medical possibility: the process we follow in verifying $\varphi(c_1) \land \ldots \land \varphi(c_m)$ involves observing some spatiotemporal patch, and it seems to be a modest extension to allow us to verify one more instance, such that we verify $\varphi(c_{m+1})$, thus verifying $\varphi(c_1) \land \ldots \land \varphi(c_{m+1})$, and manifesting the corresponding recognitional abilities. The belief in limitless extension is reminiscent of the sorites paradox: Knowing that we possess $Rec^+ [\varphi(c_1) \land \ldots \land \varphi(c_m)]$ actually, and believing that we thus possess $Rec^+ [\varphi(c_1) \land \ldots \land \varphi(c_{m+1})]$ in principle, then we believe there is no limit to what recognitional abilities we possess in principle. Moreover, the bivalentist’s case seems to grow stronger as the number of verified instances grows larger. For if we possess $Rec^+ [\varphi(c_1) \land \ldots \land \varphi(c_m)]$ actually, for a large value of $m$, then we have already shown that we have a powerful ability to observe spatiotemporal patches, and so the extension of those abilities to verify one more instance seems less significant than for smaller values of $m$.

At this point, we have only diagnosed how we might come to think that we understand bounded quantifications over arbitrarily large domains, not quantifications over infinite domains. The furthest we can idealize according to the
above method is to imagine applying a verification procedure that required a patch that was infinitely long temporally—this is a consequence of Dummett’s claim that we understand quantifications over finite domains by verifying each instance of the quantification. But we may think that this sequential verification is just one possible way we could have idealized our abilities so as to understand quantifications over arbitrarily large domains. Rather than verifying a quantification’s instances sequentially, we may think we could have verified all of them simultaneously. In idealizing simultaneous verification, we idealize our powers of observation along spatial dimensions, rather than the temporal dimension. This second process of idealization allows for the verification of quantifications over infinite domains, but the idealization of our actual abilities for verifying quantifications results in recognitional abilities that allow observations of patches that are infinitely large temporally, not infinitely large spatially, according to Dummett. So we mistakenly think we understand quantifications over infinite domains by assuming that the two processes of idealization are interchangeable.

All this may be the symptom of a deeper misunderstanding, that verifiers operate in a componentialist fashion—for conjunctions, at the very least. Under this view, to verify a conjunction one must observe only the patches containing the conjuncts’ verifiers. For example, suppose that \text{Grey(ND)} \in \pi_i^+(p) and \text{Bronze(ET)} \in \pi_i^+(p') under some procedural specification \( s_i \). One may assume that \text{Grey(ND)} \land \text{Bronze(ET)} \in \pi_i^+(p \cup p') , but \( p \cup p' \) is not a spatiotemporal patch because it is not a connected set. Rather, a verifier for \text{Grey(ND)} \land \text{Bronze(ET)} would lie in a spatiotemporal patch \( q \) such that \( p, p' \subseteq q \). To understand what \( q \) would look like, we need to remember that the purpose of
introducing spatiotemporal patches is to locate verifiers, which formalize the fact that we demand a putative speaker direct his attention toward a region of spacetime in order to manifest a recognitional ability and thus partially prove he understands a given sentence. The purpose of this whole apparatus is to make more rigorous our expectations about how verification procedures ought to proceed. So the shape of $q$ is determined by some verification procedure for $\text{Grey}(\text{EB}) \land \text{Bronze}(\text{ET})$, and will likely involve a subpatch describing travel from the New York to Paris.\footnote{I write \textit{``some verification procedure''} here because I expect that there are multiple verification procedures for $\text{Grey}(\text{EB}) \land \text{Bronze}(\text{ET})$. Even if were happened to be interested in privileging one verification procedure for a given sentence as canonical, there would likely be multiple spatiotemporal patches associated with it, and thus multiple semantic maps containing the sentence.} This misunderstanding is most likely the result of thinking that verifiers make sense outside the context of verificational procedures. The picture becomes one in which verifiers are located in spacetime and are thus available to participate in verificational procedures. Here, I believe, is the ultimate diagnosis for why the bivalentist believes what she does. Just as the realist posits truth conditions that obtain independently of our ability to observe whether or not they obtain, the bivalentist posits verifiers that exist independently of our ability to observe them.

4.5 CONCLUSION

In this chapter, I presented a more specific way of describing the semantic abilities that Dummett claims one must possess in order to understand a given sentence. One of the advantages of this more specific version of Dummett’s account is that it gives us a more rigorous way of understanding how our semantic abilities are idealized. In the original version of Dummett’s account,
idealization was described in terms of analogical extension of actual semantic abilities. This version lacked a way of comparing two potential semantic abilities and thus say that one required more idealization than the other one. In the amended version of Dummett’s account, idealization of actual recognitional abilities will be accompanied by an increase in the set of patches to which we have access: as we add recognitional abilities to our stock of available abilities, those abilities’ manifestation conditions will describe patches we add to our set of accessible patches.

So far, our discussion has centered on bivalence. In the next chapter, we will describe the consequences that the failure of bivalence has for classical logic. In doing so, we will describe Dummett’s characterization of classical logic and compare it to two other cases of rivalry.
CHAPTER 5

THE GENERAL STRUCTURE OF RIVALRY

5.1 INTRODUCTION

In previous chapters, I described two requirements for rivalry. In this chapter, I will describe a general framework for rivalry that will show how a challenge of rivalry fulfills these requirements. In describing this framework, I will appeal to the challenge of rivalry advanced by Dummett to illustrate various parts of the framework. I will then consider the challenges of Dummett, Putnam's quantum logic, and Tye's three-valued logic arising from his semantics for vagueness and show that each one is a case of rivalry that conforms to the framework of rivalry. The most important part of this process is an illustration of how each challenge includes an account of the meanings of the logical constants, and uses this account to animate the entire challenge.

5.2 THE FRAMEWORK OF RIVALRY

A challenge of logical rivalry begins with motivating principles that determine initial conditions on the project of forming a theory of logical entailment. For example, Dummett argues for a theory of meaning according to which truth is identified with warranted assertibility; since a logic should describe how truth comports with the logical constants, identifying truth with warranted assert-
ibility may affect which logical laws will be acceptable. For the rival logician’s challenge to have any force, these principles must be normative. The rival logician must support these principles so that anyone, even someone inclined to endorse classical logic, must accept them. These principles will then determine the rival logician’s responses to the formal and philosophical requirements of rivalry.

In order to define a logic as a rival to classical logic, we described a restriction of the putative rival logic’s entailment relation. Any entailment relation must treat entailment schematically, in the following sense: given a valid inference, the result of replacing the nonlogical symbols of the inference with other syntactically appropriate symbols will yield a valid inference. We formalized this by defining $D$-morphisms, each of which described a way to vary the nonlogical elements of a wff while keeping the members of $D$—the set of logical constants—fixed. Given a putative rival logic $\mathcal{L}_r$ with entailment relation $\models_r$, we defined a restricted entailment relation $\models_{r|c}$ using $D_c$-morphisms, a special case of $D$-morphisms in which the members of $D_c$—the classical logical constants—are held fixed while the other elements vary. The motivation for defining this restriction is to ensure that we are making a fair comparison between classical logic and the putative rival logic. We must see which inferences are counted as valid by the putative rival logic when only the members of $D_c$ are treated as logical constants. Formally, $\Gamma \models_{r|c} \varphi$ just in case both the following obtain, where $\Lambda_c$ is the set of all $D_c$-morphisms:

(i) $\Gamma \models_r \varphi$

(ii) $\forall \lambda \in \Lambda_c (\lambda(\Gamma) \models_r \lambda(\varphi))$
A logic will be a rival to classical logic just in case there is some inference \( \Gamma \vdash_c \varphi \) such that \( \Gamma \vdash_{r|c} \varphi \) (or vice versa).

Note that this definition hinges on comparing the logics’ treatments of the classical logical constants, since both the classical and rival entailment relations are defined in terms of \( D_c \)-morphisms. That is, this comparison will be appropriate only if there is a common core of logical constants used in both the definition of \( \vdash_c \) and the definition of \( \vdash_{r|c} \). The set of classical logical constants seems to serve this purpose, but in chapter 2 we discussed ways in which one may object that there can be no such common core of logical constants—once the entailment relations of the logics diverge, the objections claims, the rival and classical logical constants must differ in meaning. According to this objection, the \( D_c \)-morphisms can preserve a given symbol’s status as a logical constant across logics, but it cannot preserve such a symbol’s meaning across logics. So any account of rivalry must include some mechanism to explain why the meanings of the logical constants in \( D_c \) do not differ from the meanings of their counterparts in \( D_r \). In all the cases I consider below, this is accomplished by the challenge itself, which determines a particular realm or subject as paradigmatic for logical entailment. Within this realm, classical logic holds. For example, Dummett’s challenge is that truth is warranted assertibility, so the paradigmatic realm is the one in which our ability to verify sentences is certain; i.e, where the domain of discourse is finite. Indeed, whenever the domain of discourse is finite, classical logic holds, according to Dummett. How then does this paradigmatic realm provide evidence that the rival constants’ and classical constants’ meanings do not differ? Recall that the rival logician’s motivating principle is normative, implying that his rival logic is the correct theory of logi-
cal entailment for our language. This conclusion holds for any part of our language also, including the paradigmatic realm. At the same time, the challenge determines the borders of the paradigmatic realm. So the challenge simultaneously describes a particular realm as paradigmatic, and describes the rival logic and classical logic as both holding within that realm. To respond to the philosophical requirement of rivalry, then, the rival logician will point to the agreement between the rival logic and the classical logic in the paradigmatic realm. If those logics agree in that realm, then the differences in use that the inferentialist objector offers as evidence will not come from it, but rather from some other realm less central to our concept of logical entailment. For example, Dummett's logical constants and the classical logical constants disagree in their uses only when the domain of discourse is infinite. But this realm is not paradigmatic just because warranted assertibility begins to break down in this context.

So the challenge's motivating principle itself, as normative initial conditions for the project for forming a logic, produces responses to both the formal and philosophical requirements of rivalry. For the former, the challenge will show that the rival logic holds generally. For the latter, the challenge will provide an account of the meanings of the logical constants.¹ This account takes the form of a demarcation of a paradigmatic realm where both the rival logic and classical logic hold, and provides evidence that the rival logical constants and the classical logic constants disagree in their uses only when the domain of discourse is infinite. But this realm is not paradigmatic just because warranted assertibility begins to break down in this context.

¹Referring to this as an account of the meanings of the logical constants, rather than a theory of meaning for the logical constants is deliberate, although this terminology is mine and admittedly arbitrary. I mean to draw a contrast between a theory of meaning for the logical constants, which I think of as a complete description of the meanings of the logical constants, and an account of the meaning of the logical constants, which I think of as a less-complete collection of statements about the meanings of the logical constants. The rival logician would need to provide a theory of meaning for the logical constants if he had to show that the meanings of his constants were the same as the classical constants. Since the requirement is weaker, needing only to show that they do not differ, an account will suffice.
classical logical constants do not differ in their meanings.

Above, we discussed the challenge's response to the formal requirement before we discussed the challenge's response to the philosophical requirement. This ordering made sense for expository purposes: any satisfactory response by the rival logician to the formal requirement of rivalry will involve a demonstration that his logic is a rival logic. This demonstration requires reference to a set of logical constants shared by the rival and classical logicians. The philosophical requirement was then introduced as a demand for the rival logician to show that the meanings these constants possess in the context of the rival logic do not differ from the meanings these constants possess in the context of classical logic. But it should now be clear that the challenge's response to the philosophical requirement is conceptually prior to its response to the formal requirement—the rival logician must have in hand an account of the meanings of the logical constants before his logic can fulfill the definition of a rival logic.

I showed above how a challenge's response to the philosophical requirement is a demarcation of a paradigmatic realm. This demarcation is the essence of a challenge of rivalry, and shows us how to understand a case of logical rivalry as a challenge to classical logic: it claims that classical logic does not hold in the general case, but only in the limited case of the paradigmatic realm.

The final element of a challenge of rivalry is its diagnosis of how one could have come to believe that classical logic held in the general case. In my discussion of Dummett, I tried to make clear the importance of his diagnosis. It was more than an attempt to placate the classical logician, I claimed, and served as a response to a potential objection: that Dummett's challenge must be flawed if it had as a consequence that we were wrong about something so fundamental
as logic.\textsuperscript{2} I believe that the framework just described—especially the demarcation of the paradigmatic realm—accounts for diagnoses made by Dummett and other rival logicians.

A rival logician's diagnosis is an explanation of how we could have come to believe that classical logic held in the general case. Recall that a challenge's motivating principles is normative—even the classical logician must accept them. These principles are the basis for the demarcation of the appropriate paradigmatic realm, so the classical logician must also be aware that classical logic holds in the paradigmatic realm, and must be aware that the normative principles grant the paradigmatic realm a special status. If the classical logician believes that classical logic holds beyond this paradigmatic realm, it must be because she believes that the principles under which logical entailment operates in the paradigmatic realm can be generalized to the general case somehow. That is, the classical logician overestimates the power of the paradigmatic status granted by the rival logician's principles. For example, Dummett claims that truth is warranted assertibility and so the paradigmatic realm comprises cases where the domain of discourse is finite. This realm involves sentences we understand based on abilities we possess actually or potentially. The claim that classical logic holds outside of this realm, according to Dummett's diagnosis, results from our assuming that we possess in some sense analogues of these abilities that will allow us to understand sentences involving quantifications over infinite domains.

When I discussed Dummett, I broke up his account into three stages: the discussion of his theory of meaning, which constituted his motivating princi-
ples for the project of logic; his challenge to classical logic, which included responses to the formal and philosophical requirements of rivalry; and his diagnosis. It should be clear from the above discussion that any challenge of rivalry will have these elements. The advantage of the current framework is to unify all of these elements. Although it may not have been apparent in the discussion of Dummett occupying the previous two chapters, the stages of his argument are not discrete units. According to the above framework, Dummett’s challenge—as well as any other instance of rivalry—will consist in normative initial principles for the project of logic according to which we demarcate a paradigmatic realm. Note that the framework does not simply codify the elements of a challenge of rivalry as exemplified by Dummett, but shows how the demarcation of the paradigmatic realm animates the challenge’s response to the formal requirement of rivalry, its response to the philosophical requirement of rivalry, and its diagnosis.

We can now describe two different categories of motivating principles the rival logician may employ to demarcate the paradigmatic realm, and distinguish between two types of logical rivalry based on these two categories of principles. Recall that one function of the paradigmatic realm is to provide an account of meaning for the logical constants. As we consider more examples of rivalry below, it will become clear that a given instance of rivalry will involve an account of meaning for the logical constants that emphasizes either sense or reference. A challenge of rivalry that focuses on sense will be called internalist; one that focuses on reference, externalist. Dummett, for example, presents an internalist challenge. His project begins by asking what conception of truth can be employed in a theory of meaning. If we understand what it means to
know the truth conditions of a sentence, he claims, we will understand what it means to know the meaning of a sentence. In previous chapters, I have discussed in detail Dummett’s view on what it means to know the truth conditions of a sentence; in short, a speaker knows the truth conditions of a sentence just in case she possesses the ability to use that sentence correctly. A full account of the correct use of sentences is given by Dummett’s full theory of meaning—including the theory of force, which was not discussed above—and claims that the paradigmatic use of a sentence is as a report of a direct observation. Hence, the familiar distillation of Dummett’s theory: a sentence’s truth conditions are its conditions of warranted assertibility. Combined with his motivating principle, he will conclude that knowing the meaning of a sentence is knowing its conditions of warranted assertibility. When comparing the rival and classical constants, then, we judge them as “assertibility functions.”

Having described the general framework for challenges of rivalry, I will now show how several examples of rivalry fit into it. I have discussed Dummett at length in earlier chapters to elucidate the structure of his challenge but will still offer brief comments on Dummett in order to make clear the way in which his challenge fits the framework. I will then discuss Putnam’s challenge based on quantum mechanics and Tye’s challenge based on vagueness. I will not examine these cases with the same level of detail found in my treatment of Dummett, since the overall framework of rivalry has been described above. Instead, I will show how these challenges also conform to the framework.
5.3 DUMMETT

Dummett’s motivating principle is that truth should be identified with warranted assertibility, which I have described in previous chapters. The task now is to show how this principle can motivate a response to the formal and philosophical requirements for rivalry in the context of the framework described above.

On its own, the identification of truth with warranted assertibility is not enough to produce a rival logic because it does not involve the logical constants. Dummett assumes that we are comfortable with the idea that the truth value of a sentence is a function of the truth values of its components. But even if we accept that a sentence is true just in case it is warrantedly assertible, we cannot be guaranteed that the warranted assertibility of a sentence is a function of the warranted assertibility of its components. We can make some progress in this direction by noting that Dummett endorses componentialism. But this still does not get us where we need to be. Under classical logic, we might accept the following clause as a description of how truth comports with disjunction:

\[
\varphi \lor \psi \text{ is true } \iff \varphi \text{ is true or } \psi \text{ is true}
\]

The same goes for the other disquotational clauses for the logical constants. Identifying truth with warranted assertibility (abbreviated “w.a.” here) does not allow us to say

\[
\text{“}\varphi \lor \psi\text{” is w.a. } \iff \text{“}\varphi\text{” is w.a. or “}\psi\text{” is w.a.}
\]

and make similar substitutions in the other disquotational clauses, even af-
ter we make the claim that componentialism applies to warranted assertibility. Mere componentialism is not enough to guarantee that warranted assertibility comports with the logical constants in the same way truth is described to behave in the clauses. It may be, for example, that

\[
\varphi \lor \psi \quad \iff \quad \text{it is not the case that } \neg \varphi \text{ is w.a. and } \neg \psi \text{ is w.a.}
\]

which Dummett must avoid. This clause is inspired by Gödel’s negative translation discussed in an earlier chapter. If warranted assertibility comports with the logical constants according to it, classical logic will result. Dummett does not address this issue, but we can gesture toward a way to resolving it. After accepting that componentialism holds for warranted assertibility, we ought to specify that the ordinary disquotational clauses with \textit{w.a.} swapped for \textit{true} describe its behavior. This specification is appropriate for two reasons. First, it is the most natural and familiar way of specifying how truth—whether it be warranted assertibility or not—comports with the logical constants. Familiarity is rarely a means of support for a premise, but in this case it is appropriate because we want our theory of logical entailment to hew to our ordinary concepts as closely as possible. Second, the intuitionist and classical logician can agree on these clauses, albeit with different interpretations. Dummett means to show that both logics hold in the paradigmatic realm, so utilizing clauses on which both logicians agree seems fitting.

With this more specific brand of componentialism in place, Dummett’s challenge fits in the framework described above. Given Dummett’s argument about the limits of our semantic abilities, and the fact that his motivating principle dictates that a logic must describe how warranted assertibility comports with
the logical constants, we know classical logic will not hold in the general case. As mentioned above, the paradigmatic realm will comprise those cases where the domain of discourse is finite, since it is in these cases that there is no questioning our ability to verify the sentences in that realm. Classical logic and Dummett's rival logic will both hold in this paradigmatic realm, thanks in part to the decision to use the ordinary disquotational clauses for the logical constants. The logics' coinciding in the paradigmatic realm forms a foundation for a claim that the meanings of the logical constants do not differ, despite the failure of classical logic in the general case. Classical logic's holding in the paradigmatic realm also serves as the basis for Dummett's diagnosis: the classical logician makes an unwarranted extension from this limited case to the general case, attributing to himself semantic abilities he cannot possess. Dummett calls this a diagnosis, and yet it seems to be only part of full diagnosis. The paradigmatic realm's status is not enough to explain the extension. To extend the medical analogy, this extension is the pathogenesis of classical logic, and we should also describe its etiology. The origin of this extension is the classical logician's being so taken by her power to verify in the paradigmatic realm that she assumes her power to verify extends beyond that realm. This kind of idealization seems common, especially considering that all semantic abilities are practical abilities, according to Dummett. We are so accustomed to extending our abilities through various processes and means—exercise and technology, for example—that we can easily imagine a being possessing the abilities lying at the logical extreme of this extension.
5.4 PUTNAM’S QUANTUM LOGIC

In contrast to Dummett, Putnam presents an externalist challenge to classical logic. His externalist views are clearer in some of his other writings—that *water* on Earth and Twin-Earth are not synonymous despite the similarity of their senses, for example. In the current context, the externalist character of his challenge can be seen in the motivating principles he proposes for the project of logic: that a theory of logical entailment must conform to the world. Classical logic is flawed, then, because it fails to respect empirical results from the study of quantum mechanics.

The impact of quantum mechanics on logic is analogous to the impact of relativity on geometry, according to Putnam. Before the advent of relativity, Euclidean geometry had the character of necessary truth. More specifically, the parallel postulate enjoyed this status. If we agree that lines are defined as the shortest distance between two points—and so are *geodesics*—then the parallel postulate will hold just so long as space is not curved. Space’s having curvature at all—zero or otherwise—was unknown prior to relativity, which explains why the parallel postulate had the character of necessary truth. Relativity showed us not only that space can be curved, but also that space is curved by massive objects. Where space is curved, the distance between geodesics will vary, so the distance between two parallel lines can vary, which contradicts the parallel postulate and thus shows that it is not a necessary truth after all. We may

---

3 It must be noted that Putnam has since rejected the view I will describe. For the sake of simplicity, I will still refer to the view as his. I will be discussing the features of this particular challenge to classical logic, which is independent of its author.

4 To be precise, Putnam covers just this one version of the parallel postulate. Other versions of the parallel postulate will be unscathed by the results of relativity; e.g., that parallel lines never meet.
have thought otherwise, according to Putnam, because the average curvature of space is nearly zero in the context of our ordinary empirical observations. But this nearly-zero curvature of space is still a consequence of relativity, so we have a convergence of the Euclidean and non-Euclidean geometries in the context of our ordinary empirical observations. This case of “rivalry” in geometry has the same structure as a case of logical rivalry described by the above framework. Non-Euclidean geometry holds in the general case and Euclidean geometry fails in the general case. Non-Euclidean and Euclidean geometry both hold for our ordinary empirical observations, which is deserving the label of “paradigmatic realm” because Putnam’s principle for choosing a geometry is that it best matches the world, and this limited case is the one in which we have the firmest grasp on the external world.

Note that we have chosen in this example to have our ordinary concept of straight lines correspond to geodesics; i.e., the shortest paths between two points. When relativity proves that space has nonzero curvature, Putnam claims that we ought to ensure that the properties we attribute to lines fits these results. This principle, that geometry ought to conform to empirical results, leads to our rejection of the parallel postulate because geometry best fits the world if we do so. The same principle ought to guide our construction of a logic, according to Dummett, and empirical results from quantum mechanics require that we reject the law of distribution in order to have our logic best fit the world.

In quantum mechanics, physical systems are represented as Hilbert spaces where each normalized vector of this space represents a possible state of the system. For a Hilbert space $H$, let $S(H)$ be the set of all subspaces of $H$. We can arrange $S(H)$ into a lattice ordered by inclusion, with the zero-dimensional
origin as its minimum and \( H \) as its maximum. Call this lattice \( L(H) \). According to the above framework, a challenge of rivalry begins with motivating principles meant to guide the project of logic. Putnam’s motivating principle is that the correct theory of logical entailment ought to best fit the world. In the current context, that means our logic must conform to \( L(H) \), where we interpret the subspaces of \( H \) as propositions about states of the system. At this point, we have a lattice of subspaces interpreted as propositions and relations between them. What we need is a lattice of propositions ordered by logical entailment.\(^5\)

Putnam’s charge that logical entailment best fit \( L(H) \) is a charge that our lattice of propositions be isomorphic to \( L(H) \). It is in this sense that we will “read off” a logic from this lattice: we will interpret relations between the subspaces of \( L(H) \) as logical constants. We will now define a mapping \( m \) from the set of propositions to members of \( L(H) \) to accomplish these interpretations. First, note two special propositions: let \( \top \) denote a tautology and \( \bot \) denote a contradiction. Then \( m(\top) \) is the maximum of \( L(H) \) and \( m(\bot) \) its minimum, and for any atomic \( \varphi \), \( m(\varphi) \) will be a one-dimensional subspace in \( L(H) \). We must also define the span of two subspaces: given a space \( H \) and any number of its subspaces \( h_1, \ldots \), \( \text{span}_H(h_1, \ldots) \) will be the smallest subspace \( h' \) of \( H \) such that \( h_1, \ldots \subseteq h' \). We complete the definition by accounting for several logical con-

\(^5\)The logical entailment described here is restricted, in a sense, because it is defined only with respect to \( H \). Logical entailment can be defined more properly across Hilbert spaces. However, this weaker version of entailment more closely matches the argument Putnam presents, and the consequences of his challenge will be felt nonetheless.
stants:

\[ m(\varphi \lor \psi) = \text{span}_H(m(\varphi), m(\psi)) \]  \hspace{1cm} (5.1) \\
\[ m(\varphi \land \psi) = m(\varphi) \cap m(\psi) \]  \hspace{1cm} (5.2) \\
\[ m(\neg \varphi) = \text{the orthocomplement of } m(\varphi) \]  \hspace{1cm} (5.3)

Call the resulting lattice of propositions \( m(L(H)) \). Putnam claims that a theory of logical entailment that respects the results of quantum mechanics can be “read off” \( m(L(H)) \), interpreting the universal and existential quantifiers in terms of conjunction and disjunction, respectively.\(^6\) The resulting logic will be a rival to classical logic because it will lack distributivity. For example, according to quantum mechanics we cannot simultaneously assert that a given particle has a specific momentum and a specific position at an instant. Let \( \varphi \) be the proposition that a given particle has a particular momentum, and let \( \psi_1, \psi_2, \ldots \) be propositions attributing a position to the particle, where \( \psi_i \neq \psi_j \) if \( i \neq j \), and where \( \psi_1, \psi_2, \ldots \) together describe every possible position the particle can have. The proposition that the particle has a certain momentum described in \( \varphi \) and some unspecified position is

\[ \varphi \land (\psi_1 \lor \psi_2 \lor \cdots) \]

Because the \( \psi_i \)'s account for every possible position of the particle, \( m(\psi_1 \lor \psi_2 \lor \cdots) = H \). Note also that, for any \( h \in S(H) \), \( h \cap H = h \). Therefore, according

\(^6\)Putnam notes that this method is not original (see 1979, 179).
to the above definition of the mapping $m$,

$$m(\phi \land (\psi_1 \lor \psi_2 \lor \cdots)) = m(\phi)$$

Contrast this with the proposition

$$(\phi \land \psi_1) \lor (\phi \land \psi_2) \lor \cdots$$

asserting that some unspecified state obtains in which the particle has a momentum described by $\phi$ and a position specific to that state. We cannot know a particle's position and momentum at a given instant; i.e., $m(\phi \land \psi_i) = \bot$ for any particular $\psi_i$. Therefore,

$$m((\phi \land \psi_1) \lor (\phi \land \psi_2) \lor \cdots) = \bot$$

This shows that $\phi \land (\psi_1 \lor \psi_2 \lor \cdots)$ does not logically entail $(\phi \land \psi_1) \lor (\phi \land \psi_2) \lor \cdots$, contrary to classical logic.

Putnam's motivating principle is that the correct theory of logical entailment ought to best fit the world. As predicted by the framework for rivalry, and as foreshadowed by our discussion of geometry, the paradigmatic realm will be the one in which we have the firmest grasp on the external world; i.e., whenever the domain of discourse contains only macroscopic objects. We define the paradigmatic realm this way because it comprises most of our ordinary experience with the world. Quantum mechanics predicts that the phenomena described above that lead to the failure of distributivity only occur at the microscopic scale, so quantum logic and classical logic will both hold in the paradigmatic realm. In
the above description of the framework for rivalry, I showed how the demarcation of a paradigmatic realm can be used to respond to the philosophical requirement of rivalry, and how the demarcation motivates a challenge's diagnosis. I will now describe how Putnam's demarcation accomplishes both of these.

The philosophical requirement for rivalry, essentially, is a requirement to show that the change in the use of the classical logical constants does not imply a change in their meanings. Putnam does claim that “we simply do not possess a notion of ‘change of meaning’ refined enough” to address the question of whether the meanings of the logical constants change from classical logic to quantum logic, and that “even if we were to develop one, that would be of interest only to philosophy of linguistics and not the philosophy of logic [emphasis in the original]” (1979, 190). This claim could be seen as a rejection of the philosophical requirement itself. In the process of arriving at this conclusion, however, Putnam does offer a more straightforward response to the philosophical requirement. Putnam notes that certain “basic properties” of disjunction

---

7Recall that this is distinct from the strict inferentialist objection to rivalry I addressed in chapter 2. Putnam does offer his own defense of quantum logic against Carnap's version of the strict inferentialist objection, but here I am concerned with the part of Putnam’s position that constitutes a response to the philosophical requirement.
and conjunction hold in both classical logic and quantum logic:

\[
p \vdash p \lor q
\]

\[
q \vdash p \lor q
\]

if \( p \vdash r \) and \( q \vdash r \) then \( p \lor q \vdash r \)

\[
p, q \vdash p \land q
\]

\[
p \land q \vdash p
\]

\[
p \land q \vdash q
\]

while adopting quantum logic involves “merely changing our minds” about distribution: “Only if it can be made out that [the law of distribution] is ‘part of the meaning’ of ‘or’ and/or ‘and’ (which? and how does one decide?) can it be maintained that quantum mechanics involves a ‘change in the meaning’ of one or both of these connectives” (1979, 190). Even as he rejects the idea that we have a robust enough account of the meanings of logical constants, and therefore cannot judge if there has been a change in meaning, he also sketches an account of meaning for the logical constants according to which the meanings have not changed because only certain parts of a constant’s use contribute to its meaning; viz., the “basic properties” upon which classical and quantum logic agree, and which hold in the paradigmatic realm.

As with Dummett, a description of Putnam’s diagnosis is straightforward when we have in hand his demarcation of the paradigmatic realm: the classical logician must accept Putnam’s motivating principle about logic fitting the world, so a belief that classical logic holds in the general case must come from an extension to the general case of the classical logic holding in the limited case
of the paradigmatic realm. If the classical logician were to claim that this extension held—even in the face of Putnam's motivating principle—it must be because he mistakenly thought that the paradigmatic realm was privileged, in that it could serve as the base for this extension. The classical logician's error, Putnam may say, lies in his being enthralled by the view of the world with which he is most familiar, thinking that the logical laws holding there held in the general case.

5.5 TYE’S SEMANTICS OF VAGUENESS

According to Tye, a theory of vagueness ought to embrace vagueness by accepting that certain sentences are indefinite, and specify how our semantics ought to account for this. The motivating principle for his challenge is that our logical constants ought to allow for indefinite truth values. This kind of semantics and resulting logic will be appropriate just because “it concedes that the world is, in certain respects, intrinsically, robustly vague; and it avoids, at all levels, a commitment to sharp dividing lines” (1999, 293). Because Tye views vagueness as the result of a feature of the world, and demands that a theory of logical entailment conform to it, his challenge will be externalist in the same way Putnam's challenge was externalist.8

The centerpiece of Tye's account is his claim that the extensions of vague predicates are vague sets. According to Tye, a set is vague just in case it ful-

---

8 Tye will claim that the extensions of vague predicates are vague sets, and insists that these sets are examples of ontic vagueness. For that reason, we categorize challenge as externalist. However, it seems that other types of theories of vagueness could use this kind of semantic also, with a different explanation for why the sets are vague. That said, any such alternative theory of vagueness would need to be one that was willing to abandon classical logic in order to take full advantage of his semantics.
fills two criteria. First, the set must have *borderline members*, objects for which “there is no determinate, objective fact of the matter about whether they are in the set or outside of it” (1999, 283). Tye claims that this criterion is supported by our ordinary conception of the extensions of vague predicates. Second, a set is vague only if “there is no determinate fact of the matter about whether there are objects that are neither members, borderline members, nor non-members” (1999, 284). Suppose otherwise, Tye argues: If we assume that there are no such objects, then there are sharp borders between the members and borderline cases, and between the borderline cases and the non-members, which contradicts our ordinary notions about the extensions of vague predicates. If we assume that there are such objects, we have introduced “gratuitous metaphysical complications” because this type of object is not required to account for our pretheoretic ideas about vague predicates (1999, 283). Moreover, it sets a precedent according to which we might continue to ask a “potentially endless” series of questions about the existence of further categories of objects, he claims (1999, 283).

After defining the extensions of vague predicates as vague sets, Tye is then equipped to establish conditions for three truth values: Given a sentence $P(a)$ where $a'$ is the object named by $a$, $P^+$ is the extension of $P$, and $P^-$ is the antiextension of $P$, $P(a)$ is true (denoted $T$) iff $a' \in P^+$; $P(a)$ is false (denoted $F$) iff $a' \in P^-$; and $P(a)$ is indefinite (denoted $I$) iff there is no determinate fact of the matter about whether $a' \in P^+$ or $a' \in P^-$. Tye then defines his three-valued logical constants as follows:
Under Tye's three-valued logic, classical tautologies will not be logically true. For example, $\varphi \lor \neg \varphi$ has the value $I$ when $\varphi$ has the value $I$. In addition, classical contradictions will not be logically false. For example, $\varphi \land \neg \varphi$ has the value $I$ when $\varphi$ has the value $I$. Tye seeks to minimize this disagreement with classical logic, noting that classical tautologies are never false under any interpretation in his system, labeling them \textit{quasi-tautologies} in his system; and classical contradictions are never true under any interpretation in his system, labeling them \textit{quasi-contradictions} in his system.

Despite his efforts at minimization, I think Tye's disagreement with classical logic is clear. However, one more example will bring the disagreement into sharper focus. Consider the inference:

$$\varphi \lor \neg \varphi \quad \therefore \quad \psi$$

which is valid under classical logic. Tye claims that one sentence is logically entailed by another sentence “so long as it cannot be true that the first is anything other than true when the latter is true” (1999, 283). Under the truth assignment where $\varphi$ has the value $T$ or $F$ and $\psi$ has the value $I$, the premise will have
the value $T$ and the conclusion $I$. Thus, by Tye's own definition, this inference will not be a case of logical entailment because the premise can be true and the conclusion something other than true. This inference will serve as evidence that classical logic cannot hold in the general case.\footnote{To be fair, I think Tye made an error in defining logical entailment this way without also introducing a notion of "quasi- entailment," that one sentence quasi-entails another just in case the latter cannot be false if the former is true. Nevertheless, the disagreement with classical logic is apparent. I have chosen to discuss an inference, rather than relying only on the cases of (quasi-)tautologies and (quasi-)contradictions to make the discussion mesh with my other examples of rivalry.}

Tye's motivating principle is that we must respect the vagueness in the world by accounting for $I$ in a theory of logical entailment. As surprising as it may seem, Tye's principle grants paradigmatic status to the realm in which all extensions are precise sets. Recall that vague sets are defined with respect to precise sets. They must have borderline members—objects for which there is no fact of the matter about whether they are in the set or outside of it. This definition refers to two precise sets: objects that are definitely members and objects that are definitely non-members of the set. Also, in order for a set to be vague, there must be no fact of the matter about whether there are objects that are neither members, borderline members, nor non-members. Again, this condition makes reference to the sets of definite members and definite non-members. A similar situation arises in Tye's definition of the truth value $I$. According to his definition, a sentence is valued $I$ just in case there is no fact of the matter about whether the sentence is true or false. We must conclude that the definite truth values $T$ and $F$ are conceptually prior the indefinite truth value $I$.

Within this paradigmatic realm, Tye's rival logic and classical logic will both hold. Even more important to the formulation of a response to the philosophical requirement is the fact that Tye's three-valued logical constants and their
classical counterparts disagree in a weak sense outside of the paradigmatic realm. Where $l$ is admitted as a truth value, the classical constants will be undefined, rather than yielding outputs different from their three-valued counterparts. Tye can therefore respond to the philosophical requirement by claiming that, given two logical constants and a truth-assignment to an ordered set of component sentences upon which they both operate to form a compound sentence, those constants will differ in meaning just in case they output different truth values.

For example, consider the following three versions of the conditional:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$\phi \rightarrow_c \psi$</th>
<th>$\phi \rightarrow_v \psi$</th>
<th>$\phi \rightarrow_l \psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>I</td>
<td>T</td>
<td>T</td>
<td>I</td>
<td>T</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>F</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>I</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

where $\rightarrow_c$ is ordinary two-valued conditional, $\rightarrow_v$ is the three-valued conditional introduced by Tye, and $\rightarrow_l$ is the three-valued conditional introduced by Łukasiewicz (see Łukasiewicz 1920). It is clear that $\rightarrow_v$ and $\rightarrow_l$ disagree when $\phi$ and $\psi$ both take the value $l$: $\phi \rightarrow_v \psi$ takes the value $l$ and $\phi \rightarrow_l \psi$ takes the value $T$. One would then have solid grounds for claiming that $\rightarrow_v$ and $\rightarrow_l$ differed in meaning, since they produce different outputs given the same inputs. In contrast, $\phi \rightarrow_c \psi$ is undefined when $\phi$ and $\psi$ both take the value $l$. Here it is more difficult to argue that $\rightarrow_c$ and $\rightarrow_v$ differ in meaning because they value $\phi \rightarrow \psi$ differently. Instead of assigning $\phi \rightarrow \psi$ a value different from the one assigned by $\rightarrow_v$, as $\rightarrow_l$ does, $\rightarrow_c$ does not assign any value to $\phi \rightarrow \psi$. Using the
same kind of reasoning Tye employs when claiming that we ought to be satisfied with the status of classical tautologies as quasi-tautologies because they are never false—even if they are not always true—I think Tye would claim that there is no change in meaning between $\rightarrow_c$ and $\rightarrow_v$ because their (defined) outputs never differ for the same inputs.$^{10}$

As with the other examples of rivalry, Tye's diagnosis will center on his paradigmatic realm. Tye's motivating principle is normative, so even the classical logician must accept that vague sets can be extensions of vague predicates. In order for the classical logician to make the claim that classical logic holds in the general case, it must be the result of an extension from the paradigmatic realm. The mistake the classical logician makes, Tye would say, lies in being ensnared by the fact that $T$ and $F$ are the only legitimate truth values in the paradigmatic realm, and concluding that $T$ and $F$ are the only legitimate truth values for the whole of our language. This is likely the result of the fact that ordinary language can be precisified to some extent, even if we make a simple, arbitrary precisification by considering the definite members of a vague predicate's extension to be its “approximate extension” and all other objects its “approximate antiextension.”

5.6 CONCLUSIONS

The examples of rivalry due to Dummett, Putnam, and Tye show that the most important part of a challenge to classical logic is the account of meaning for

---

10 I use $\rightarrow_1$ in the above example as a contrast to the classical conditional. Tye is likely to reject it because it is not conservative with respect to truth values, unlike his connectives. Specifically, if $\varphi$ and $\psi$ were to be valued $I$ and then gain definite truth values $T$ and $F$, respectively, then $\varphi \rightarrow_1 \psi$ will go from $T$ to $F$. 

140
the logical connectives. By categorizing these as internalist or externalist, we were able to see how the impact of such an account is felt in other areas of the challenge. This result could be surprising; ordinarily, we might think of the philosophical requirement as something to which a rival logician must react after making his challenge proper. However, the above discussion shows that advancing an account of meaning for the logical constants can be a challenge to classical logic itself, if that account of meaning has implications that restrict classical logic to a subset of our inferences.

It is also striking to see how all the examples involve the same characterization of classical logic as an unwarranted generalization from a subset of our inferences. Indeed, the philosophical requirement imposes a structure on cases of rivalry according to which all rival logics will characterize classical logic in this way. In a sense, this characterization of classical logic is merely an artifact of the structure of a challenge of rivalry. Nevertheless, having this characterization spelled out seems to present a vulnerability for the very concept of rivalry that the classical logician may be able to exploit in a defense of classical logic. The three examples of rivalry considered here differ in their motivation, the form of the accompanying rival logic, and their response to the philosophical requirement. Despite this, they each attempted to mount a challenge of rivalry—the strongest type of challenge to classical logic, I claimed—which necessitated a framework according to which classical logic is characterized in a specific way. This result suggests that classical logic can be challenged on a fundamental level only if the challenge is made against a backdrop of that specific characterization. If the advocate of classical logic can show that this characterization is essentially flawed, despite the apparent normativity of the rival logi-
cians’ motivating principles, he will have shown that classical logic cannot be challenged on this fundamental level.


