FAST OPTIMAL POWER FLOW ANALYSIS FOR LARGE-SCALE SMART GRID

BY

YI LIANG

THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical and Computer Engineering in the Graduate College of the University of Illinois at Urbana-Champaign, 2013

Urbana, Illinois

Adviser:

Associate Professor Deming Chen
ABSTRACT

Optimal power flow (OPF) plays an important role in power system operation. The emerging smart grid aims to create an automated energy delivery system that enables two-way flows of electricity and information. As a result, it will be desirable if OPF can be solved in real time in order to allow the implementation of time-sensitive applications, such as real-time pricing. We develop a novel algorithm to accelerate the computation of alternating current optimal power flow (ACOPF) through power system network reduction (NR). We formulate the OPF problem based on an equivalent reduced system and then compute its solution. The detailed optimal dispatch for the original power system is obtained afterwards using a distributed algorithm. Our results are compared with two widely used methods: full ACOPF and the linearized OPF with DC power flow and lossless network assumption, the so-called DCOPF. Experimental results show that for a large power system, our method achieves $7.01 \times$ speedup over ACOPF with only 1.72% error, and is 75.7% more accurate than the DCOPF solution. Our method is even 10% faster than DCOPF. Our experimental results demonstrate the unique strength of the proposed technique for fast, scalable, and accurate OPF computation. We also show that the proposed method is effective for smaller benchmarks.
To my parents, for their love and support
ACKNOWLEDGMENTS

I thank all my family and friends for their care and support. I would like to thank my adviser, Deming Chen, and my labmates for their enlightening discussions.
# TABLE OF CONTENTS

LIST OF TABLES ............................................. vi

LIST OF FIGURES ........................................... vii

LIST OF ABBREVIATIONS ................................. viii

CHAPTER 1  INTRODUCTION ................................. 1

CHAPTER 2  BACKGROUND OF POWER SYSTEM ANALYSIS AND OPTIMAL POWER FLOW ......................... 5
  2.1  Power Flow Analysis .................................. 5
  2.2  Optimal Power Flow .................................. 7
  2.3  Applications .......................................... 9

CHAPTER 3  FAST ACOPF ALGORITHM ...................... 11
  3.1  Approach Overview .................................... 11
  3.2  Generation of Similarity Descriptor .................. 12
  3.3  Congestion Forecast ................................... 14
  3.4  Similarity Identification and Grouping of Buses ... 17
  3.5  Network Reduction and Reduced System Generation 17
  3.6  Formulation and Calculation of the Reduced ACOPF Problem 19
  3.7  Congestion Check ...................................... 20
  3.8  Distributed ACOPF Computation for Each Subsystem 20

CHAPTER 4  EXPERIMENTAL RESULTS .................... 22
  4.1  Experimental Results for the IEEE 30-Bus Test System 22
  4.2  Experimental Results for Larger Benchmarks ......... 25

CHAPTER 5  CONCLUSION .................................... 28

REFERENCES ............................................... 29
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Test Benchmarks</td>
<td>22</td>
</tr>
<tr>
<td>4.2</td>
<td>Experimental Results of the IEEE 30-Bus Test System</td>
<td>24</td>
</tr>
<tr>
<td>4.3</td>
<td>Experimental Results of Accuracy Evaluation</td>
<td>25</td>
</tr>
<tr>
<td>4.4</td>
<td>Experimental Results of Computation Time</td>
<td>26</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

3.1 Overall Algorithm Flow ........................................... 13
4.1 Clustering Results for the IEEE 30-Bus Test System .... 23
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACOPF</td>
<td>Alternating Current Optimal Power Flow</td>
</tr>
<tr>
<td>DCOPF</td>
<td>OPF with DC assumption</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>ISO</td>
<td>Independent System Operator</td>
</tr>
<tr>
<td>NR</td>
<td>Network Reduction</td>
</tr>
<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
</tr>
<tr>
<td>PTDF</td>
<td>Power Transfer Distribution Factor</td>
</tr>
<tr>
<td>SCADA</td>
<td>Supervisory Control And Data Acquisition</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

The power system in United States is one of the largest and most complex cyber-physical systems in the world. In 2011, 40% of the total energy in the United States was consumed to generate electricity power [1]. To support its automation, power systems need to monitor, control, and secure the grid in real time for efficient and reliable operation. Nowadays, the emerging smart grid aims to enable two-way flows of information and electricity to create an automated and advanced energy system with different decision makers involved. Timely and accurate analysis and control of such a large system are vitally important for its operating reliability and efficiency. Inaccurate or slow analysis of the power system may result in uneconomic operation of the grid and potentially environmental pollution [2].

OPF has been widely used in power system planning and operation in the last 50 years, and seeks to optimize an objective function by adjusting a set of control variables subject to certain physical, operational, and policy constraints. However, even today, full ACOPF has not been widely adopted in real-time operations for large-scale power systems because of the high computational requirement. In the smart grid paradigm, the problem size grows tremendously with the integration of renewable energy, energy storage, and demand response. In addition, a more detailed model is needed to support various emerging applications, which further aggravates the computational
burden. With the advent of the wholesale electricity market, \textit{ACOPF} computation is now part of the core pricing mechanism for electricity pricing and trading. For example, an ultimate goal of the independent system operator (ISO) is to solve the security-constrained \textit{ACOPF} over large-scale power systems. Typically, this problem must be solved daily in 2 hours, hourly in 15 minutes, each 5 minutes in 1 minute by the ISO [3]. Currently, the problem is solved through various levels of approximations based on application and time sensitivity [4].

Although a highly nonlinear full \textit{ACOPF} would provide the most accurate control settings in power system operations, due to the high computational demands of \textit{ACOPF}, \textit{DCOPF} is widely used. However, since \textit{DCOPF} uses a linear approximation of the power flow equations and the lossless \textit{DC} power flow assumption (the so-called \textit{DC} power flow assumption), it is not accurate, and the assumption of neglecting reactive power and power losses largely limits its application to real-world problems [5]. Currently, people use various approximation techniques and engineering judgments to explore reasonable solutions to the \textit{ACOPF} problem [4]. However, today’s inaccurate approximation may unnecessarily cost billions of dollars annually because of the use of inaccurate \textit{OPF} solutions [2]. It may also result in environmental pollution from unnecessary emissions and wasted energy [2]. As a result, accelerating \textit{ACOPF} computation while maintaining high accuracy is very important.

A wide variety of optimization techniques have been examined to solve the non-convex \textit{ACOPF} problems, such as quadratic programming [6], linear programming [7] and the interior point method [8, 9]. Alternative ap-
proaches include genetic algorithms (GA) [10], evolutionary programming [11], steepest descent-based methods [12] and particle swarm optimization [13]. However, these methods are not computationally efficient, and cannot be used in large-scale power system for real-time operation. A distributed algorithm for ACOPF problem was proposed in [14], where the OPF problem for the original systems was decomposed into per-area instances. This approach assumes the decoupling between different regions, which is not true for a densely interconnected power system. It can also result in very large border regions, which slow down the convergence and may even cause the problem of non-convergence. In addition, the convergence is not guaranteed unless the objective function of the OPF problem is convex with respect to the border region variables, which is not always true in reality.

There are NR techniques to reduce the computational burden by finding an equivalent system. Some traditional reduction methods, such as the Ward equivalent technique [15], are usually performed by computing the admittance and eliminating unnecessary elements that are not in the study area. The reduced model may lose sparsity and may not yield the same power flow pattern as the original one. In addition, this technique is only used for power flow analysis. Alternatively, sensitivity matrix-based methods, such as the power transfer distribution factor (PTDF) -based method, are used for NR [16, 17]. The method proposed in [16] preserves the same power flow pattern as that in the original system at the operation set point where the reduction is performed. This method has the operation set point dependence and yields significant error when the system operates at a different set point. In [17], another NR method was proposed to derive an equivalent system that does not depend on the set point. However, both [16] and [17] are proposed
for power system long-term planning studies. The speed of generating the equivalent system is not fast enough, which is not suitable for the real-time power system operating purpose.

In this thesis, a new method based on NR is proposed to solve for ACOPF for the large-scale smart grid. My contributions are:

- We propose a novel method to partition the power network that can efficiently reduce the error brought by NR and a fast analytical approximation method to identify the parameters of the equivalent system without using DC power flow assumption.

- Instead of only considering the reduced equivalent system, we propose a distributed method to efficiently recover the detailed solution for the original system. Congestion and the transmission capacity of lines are considered in the algorithm to ensure the feasibility of the ACOPF solution.

- We provide an effective methodology for scalable computation of the ACOPF problems with high accuracy and speed.

The rest of this thesis is organized as follows. In Chapter 2, we give the necessary background and the ACOPF formulation. Chapter 3 describes the framework and the algorithm of the NR-based ACOPF solution method. We present the numerical results in Chapter 4 and the conclusions in Chapter 5.
CHAPTER 2

BACKGROUND OF POWER SYSTEM ANALYSIS AND OPTIMAL POWER FLOW

Over the past 50 years, the steady-state OPF problem was well formulated and many variations of ACOPF formulations were studied. In this chapter, we begin with the background of power system analysis. We will introduce power flow analysis and ACOPF formulation.

2.1 Power Flow Analysis

The power flow equations constitute the steady-state model of the power system and are widely used to compute the system states once the injections and the withdrawals at each network node are specified.

We consider a power system with $N + 1$ buses and $L$ lines. We denote by $\mathcal{N} \triangleq \{0, 1, 2, \cdots, N\}$ the set of buses, with the bus $0$ being the slack bus, and by $\mathcal{L} \triangleq \{\ell_1, \ell_2, \cdots \ell_L\}$ the set of transmission lines that connect the buses in the set $\mathcal{N}$. We associate with each line $\ell \in \mathcal{L}$ the ordered pair $\ell = (i, j)$. The series admittance of line $\ell$ is denoted by $g_{\ell} - jb_{\ell}$. Each bus $i$ is characterized by the voltage phasor:

$$E_i = V_i e^{j\theta_i}, \quad (2.1)$$
where $V_i$ is the nodal voltage magnitude and $\theta_i$ is the nodal voltage phase angle. The net injected complex power at each bus $i$ is

$$S_{net_i} = P_{net_i} - jQ_{net_i},$$

(2.2)

where the net power injection at each node $i$ is $P_{net_i} = P_{gi} - P_{Li}$ and $Q_{net_i} = Q_{gi} - Q_{Li}$, where $P_{gi}(Q_{gi})$ is the real (reactive) power generated and $P_{Li}(Q_{Li})$ is the real (reactive) power consumed by the load at bus $i$. Equivalently, for each bus $i$, there are four real variables, $P_{net_i}$, $Q_{net_i}$, $V_i$, and $\theta_i$. The power flow equations express the relationship that these variables must satisfy when the power system operates in the steady state. We denote by $Y$ the $(N+1) \times (N+1)$ nodal admittance matrix, with $Y_{ij}$ as the element in row $i+1$ and column $j+1$. We adopt the convention that $Y = G - jB$, where $G$ is the conductance matrix and $B$ is the susceptance matrix. Then we have

$$I = YE,$$

(2.3)

where $I = [I_0, I_1, \cdots, I_N]^T$ is the vector of nodal current injection phasors; and $E = [E_0, E_1, \cdots, E_N]^T$ is the vector of nodal voltage phasors measured with respect to the ground node.

In power systems, we have three types of buses: (1) slack bus 0 with $V_0$ and $\theta_0$ specified; (2) $P,V$-bus with $P_{net_i}$ and $V_i$ specified; and (3) $P,Q$-bus with $P_{net_i}$ and $Q_{net_i}$ specified. At each bus two of the four variables are known and the other two are unknown. At each bus $i$, the net complex power is given by

$$S_{net_i} = P_{net_i} - jQ_{net_i} = E_i^* I_i = E_i^* \sum_{k=0}^{N} Y_{ik} E_k.$$

(2.4)
Therefore the power balance equations at each bus can be formulated as follows by separating the real and imaginary part,

\[ P_{\text{net}_i} = \sum_{k=0}^{N} V_i V_k [G_{ik} \cos \theta_{ik} - B_{ik} \sin \theta_{ik}], \]  
(2.5)

\[ Q_{\text{net}_i} = \sum_{k=0}^{N} V_i V_k [G_{ik} \sin \theta_{ik} + B_{ik} \cos \theta_{ik}], \]  
(2.6)

where \( i \in \mathcal{N} \), and \( \theta_{ik} = \theta_i - \theta_k \) is the voltage angle difference between bus \( i \) and \( k \). The complex power flow in the transmission line \( \ell = (i,j) \) can be formulated as

\[ S_{ij} = E_i^* I_{ij}, \]  
(2.7)

The goal of power flow analysis is to solve the above nonlinear equations and obtain the voltage phasors and power flow in branches that represent the state of the system.

2.2 Optimal Power Flow

OPF is used to optimize the steady-state performance of a power system in terms of an objective function under certain equality and inequality constraints. With specified reference bus angle, line admittance, shunt capacitances, and \( P_{\text{net}_i} \) and \( Q_{\text{net}_i} \) at \( P,Q\)-bus, the ACOPF problem can be formulated as follows:
\[
\begin{align*}
\min_u & \quad f(x, u) \\
\text{s.t.} & \quad g(x, u) = 0, \quad h(x, u) \leq 0
\end{align*}
\]
where \(u\) is the vector of independent (or control) variables and \(x\) is the vector of dependent (or state) variables. Here,

\[
u = [P_m, V_m, t_{e}], \quad \text{for } \forall P, V-\text{bus } m,
\]
\[
x = [V_r, \theta_r, \theta_m], \quad \text{for } \forall P, V-\text{bus } m \text{ and } \forall P, Q-\text{bus } r,
\]
where \(t_e\) is the vector of transformer tap settings. The equality constraints \(g(x, u) = 0\) consist of nonlinear power balance equations in (2.5) and (2.6). The inequality constraints \(h(x, u) \leq 0\) typically include

\[
V_i^{\text{min}} \leq V_i \leq V_i^{\text{max}},
\]
\[
P_{g_i}^{\text{min}} \leq P_{g_i} \leq P_{g_i}^{\text{max}},
\]
\[
Q_{g_i}^{\text{min}} \leq Q_{g_i} \leq Q_{g_i}^{\text{max}},
\]
\[
S_{\ell_k} \leq S_{\ell_k}^{\text{max}},
\]
\[
t_{\ell_k}^{\text{min}} \leq t_{\ell_k} \leq t_{\ell_k}^{\text{max}},
\]
for \(\forall i \in \mathcal{N}\) and \(\forall \ell_k \in \mathcal{L}\). Here, \(P_{g_i}\) and \(Q_{g_i}\) are the active power generation and reactive power generation of the generator at bus \(i\). \(S_{\ell_k}\) and \(t_{\ell_k}\) are the power flow and the transformer tap setting on \(\ell_k\).
2.3 Applications

OPF is an efficient tool in power system operations and it has many applications. Below are two popular applications.

2.3.1 Minimization of Generation Cost

In the case of minimizing the generation cost, the objective function \( f \) is usually considered as the total active power generation cost:

\[
f = \sum_{i \in \mathcal{N}_G} f^i(P_{gi}),
\]  

where \( \mathcal{N}_G = \{i \mid \text{bus } i \text{ is connected to a generator}\} \), and \( f^i(P_{gi}) \) is the active power generation cost at bus \( i \). \( f^i(P_{gi}) \) is usually modeled by a quadratic function,

\[
f^i(P_{gi}) = a_i P_{gi}^2 + b_i P_{gi} + c_i,
\]

where \( a_i, b_i, c_i \) are the cost coefficients. If this problem can be solved accurately in real time, optimal control operations will be updated timely to achieve the lowest generation cost, and potentially a large amount of money can be saved.

2.3.2 Minimization of Line Loss

In this case, the objective function \( f \) is considered as the total loss on transmission lines [18]:

\[
f = \sum_{(i,j) \in \mathcal{L}} G_{ij}(V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)).
\]
Solving this ACOPF problem in real time will enable timely adjustment of control settings to reduce the line loss, which can improve the economic efficiency of power system operation. In 2011, around 7% of the electricity generated was lost in the transmission lines in the U.S., which is worth about $3.23 billion. As a result, it is important to solve this ACOPF problem quickly and accurately.
CHAPTER 3

FAST ACOPF ALGORITHM

In this chapter, we present the NR-based algorithm for solving the ACOPF for the large-scale smart grid. We will take the objective function of minimizing the active power generation cost, which is shown in (2.12), as an example to illustrate this method.

3.1 Approach Overview

The goal of this approach is to accelerate the computation of the ACOPF solution by reducing the number of variables in the ACOPF; thus reducing the size of the ACOPF problem. With NR, the size of $\mathbf{x}$, $\mathbf{u}$ and the admittance matrix $\mathbf{Y}$, are reduced down to the size of $\mathbf{x}^{eq}$, $\mathbf{u}^{eq}$, and $\mathbf{Y}^{eq}$ in the newly formulated ACOPF problem for the reduced equivalent system. We denote by $\mathcal{N}^{eq} \triangleq \{0, 1, 2, ..., N^{eq}\}$ the set of buses in the reduced system and by $\mathcal{L}^{eq} \triangleq \{\ell_1, \ell_2, \ell_3, ..., \ell_{L^{eq}}\}$ the set of transmission lines that connect the buses in set $\mathcal{N}^{eq}$. Similarly, power balance equations and line flow equations are formulated as

$$S^{eq}_{i} = P_{i} - jQ_{i} = E_{i}^{eq}^{*} I_{i}^{eq}$$

$$= E_{i}^{eq}^{*} \sum_{j=0}^{N^{eq}} Y_{ij}^{eq} E_{j}^{eq}, \forall i \in \mathcal{N}^{eq},$$

(3.1a)

$$S_{ij}^{eq} = E_{i}^{eq}^{*} I_{ij}^{eq}, \forall (i, j) = \ell \in \mathcal{L}^{eq}.$$  

(3.1b)
In order to keep the equivalence between the reduced $ACOPF$ problem and the original $ACOPF$ problem, the power injection pattern and the power flow pattern should be maintained. The goal of $NR$ is to find an aggregation function that maps the variables in the original system to the variables in the reduced system, and that minimizes the mismatch between the original system and the reduced equivalent system. However, it is impossible to use existing methods to analytically or numerically find the exact aggregation function in real time for the large-scale smart grid. In this approach, the aggregation function is analytically approximated by linearizing the $AC$ power balance equations. Therefore, we propose the $NR$-based $ACOPF$ computation algorithm.

The overall algorithm flow of the proposed method is shown in Figure 3.1. We first generate the similarity descriptors and the congestion indicators. By clustering, we group buses into subsystems. $NR$ is performed to generate an equivalent reduced system. Then, the $ACOPF$ problem is formulated and calculated for the equivalent reduced system. After checking the feasibility of the solution, detailed control settings are recovered by solving $ACOPF$ for each subsystem.

### 3.2 Generation of Similarity Descriptor

In order to identify the similarity between buses and to group similar buses into one subsystem, a novel bus similarity descriptor containing voltage, load/generator model, and surrounding network topology information is introduced here. The traditional descriptor in power system applications only considers the bus itself and ignores the interactions with its adjacent
buses in its local network. Such isolation of buses cannot fully reflect bus features. We assign each bus $i \in \mathcal{N}$ the similarity descriptor

$$D_i = \tau_i \cdot (V_i, \theta_i, M_{G_i}, M_{L_i}, \Gamma_i),$$  \hspace{1cm} (3.2)$$

where

$$\tau_i = (\tau_{V_i}, \tau_{\theta_i}, \tau_{M_{G_i}}, \tau_{M_{L_i}}, \tau_{\Gamma_i})$$  \hspace{1cm} (3.3)$$
is the weight vector; $M_{G_i}$ and $M_{L_i}$ represent the generator model and load model at bus $i$; $\Gamma_i$ is defined by

$$
\Gamma_i = \left( \sum_{j, (i,j) \in \mathcal{L}} \frac{V_i}{V_j \cos \theta_{ij}}, \sum_{j, (i,j) \in \mathcal{L}} \frac{V_i}{V_j \sin \theta_{ij}} \right),
$$

which is the local topology descriptor. In this approach, we use the polynomial “ZIP” load model [19] to describe the load model. The generators are modeled as synchronous generators and inductor generators[20, 21]. All the parameters that describe the generator’s and load’s features are included in the descriptor. The similarity descriptor provides a measure of how “close” two buses are. It allows us to identify buses that can be merged together in the reduced network by using clustering algorithms. The generation of similarity descriptors is done offline and will be updated when required.

### 3.3 Congestion Forecast

In a power system, congestion occurs whenever the provision of transmission services required by the preferred generation/demand schedule exceeds the physical capability of the grid. In this thesis, we only consider the restrictions imposed by the physical transmission capacity of the line. Congestion may increase the total generation cost because it may prevent cheap electricity generation from being dispatched. If we neglect congestion in the original system, it is possible that the calculated $ACOPF$ solution after $NR$ is not feasible. Therefore, it is very important to properly group buses in order to preserve the congestion profile. We propose a new method to ensure that the line flow constraints are not violated in the original system when applying the control settings derived from the $ACOPF$ solution of the reduced network.
Congestion forecast is a heuristic method that predicts where congestion is going to occur. It takes power system field measurement data and the load forecast result $S_{L_i}^f$ as inputs. In addition, it takes the uncommitted transfer capability ($UTC$) of the successfully calculated ACOPF cases as feedback. $UTC$ in line $\ell = (i, j)$ is defined by

$$u_{(i,j)} \triangleq S_{(i,j)}^{\text{max}} - |S_{(i,j)}|, \quad \text{where } \ell = (i, j) \in \mathcal{L}. \quad (3.5)$$

In order to predict congestion, we assign a congestion indicator for bus $i \in \mathcal{N}$ based on the following heuristics:

1. If the power generation capacity at bus $i$ plus the total $UTC$ of the transmission lines connected to bus $i$ are larger than its demand, then bus $i$ can either accommodate itself or import power from other generators. The reverse is also true.

2. Motivated by OPF, power systems will force cheap generators to generate as much power as they can and export it to reduce the overall cost until some factors, such as loss or congestion on the lines, limit the benefit of increasing generation output. We use the derivative of the cost function with respect to the power generation $\lambda_{P_{g_i}} = \frac{\partial f}{\partial P_{g_i}}$ at current operational state to evaluate it.

3. Based on different loading and generating conditions, the system will update the control settings to the new optimal control settings by solving the ACOPF problem. Depending on the system condition, $UTC$ changes correspondingly. We denote $u_\ell$ as the original $UTC$ and $\tilde{u}_\ell$ as $UTC$ after applying new optimal control settings. By comparing these two $UTC$s, we find the lines that became congested and accordingly
predict which lines are going to get congested.

Based on the above heuristics, we define

\[ \phi^c_i = 1 - e^{\gamma_{\phi_i} \left( \sum_{(i,j) \in L} u_{(i,j)} + S_{\text{max}}^i - S_{L_i}^i \right)} , \]  
\[ (3.6) \]

\[ \alpha^c_i = (\frac{\lambda^f_{g_i}}{\max_i \lambda^f_{g_i}})^{\gamma_{\phi_i}} , \]  
\[ (3.7) \]

\[ \beta^c_i = \min_{(i,j) \in L} \beta_{ij} , \]  
\[ (3.8a) \]

\[ \beta_{ij} = \begin{cases} 
1, & u_{(i,j)} < \bar{u}_{(i,j)} \\
\left( \frac{\bar{u}_{(i,j)}}{u_{(i,j)}} \right)^{\gamma_{\beta_i}}, & \text{otherwise} 
\end{cases} \]  
\[ (3.8b) \]

where \( \phi^c_i \) indicates the impact of supply and demand balance on congestion; \( \alpha^c_i \) reflects the impact of power generation cost on congestion. \( \beta^c_i \) indicates the possibility of lines connected to bus \( i \) getting congested after applying the optimal control settings. \( \gamma_i = (\gamma_{\phi_i}, \gamma_{\alpha_i}, \gamma_{\beta_i}) \) is the weight vector. Note that \( \gamma_{\phi_i}, \gamma_{\alpha_i} > 0 \) and \( 0 < \gamma_{\beta_i} < 1 \). We define the congestion indicator \( C_i \):

\[ C_i = \phi^c_i \ast \alpha^c_i \ast \beta^c_i . \]  
\[ (3.9) \]

It is obvious that \( \phi^c_i, \alpha^c_i, \beta^c_i \in [0, 1] \), thus \( C_i \in [0, 1] \). The congestion indicator \( C_i \) for bus \( i \) is assigned to be 1 when bus \( i \) is connected to lines that are susceptible to congestion. We tend to isolate bus \( i \) if \( C_i \) is close to 0 and group \( i \) into a subsystem if \( C_i \) is close to 1.
3.4 Similarity Identification and Grouping of Buses

In the similarity identification process, each point is represented by $\Psi_i$, which is defined by

$$\Psi_i = \begin{cases} 0, & C_i < \delta \\ D_i, & C_i \geq \delta \end{cases}$$  \hspace{1cm} (3.10)$$

where $\delta$ is a threshold for congestion indicators. System operators select $\delta$ to meet their accuracy and performance requirements. A hot start K-means algorithm is used to cluster the buses [22]. The most recent historical clustering result is used as the starting point to improve the convergence speed of the K-means algorithm. Due to the special physical features of the slack bus and transformers, we isolate the slack bus and make sure that lines with transformers are not grouped into subsystems unless its tap ratio is close to 1. After the clustering process, the system is then divided into $S$ subsystems. The set $\mathcal{N}$ is divided into $S$ subsets, where $\mathcal{N}_k \in \mathcal{N}$ and $\mathcal{N}_k \cap \mathcal{N}_m = \emptyset$ for $\forall k, m \leq S$. Subsystem $k$ contains all the buses in $\mathcal{N}_k$. Let $c_k \in \mathcal{N}_k$ denote the centroid bus in subsystem $k$. Value at bus $c_k$ is the average value of the cluster.

3.5 Network Reduction and Reduced System Generation

The NR process follows the following strategy:

(a) Buses inside one subsystem are aggregated into one bus;

(b) Lines between two subsystems are aggregated into one line; and

(c) Lines inside one subsystem are ignored.
The power network parameters are approximated to maintain the same power injection pattern and power flow pattern as the original system. We propose a fast method to approximate the aggregation function.

3.5.1 Power Demand and Generation in the Equivalent System

Power demand $S_{Lk}^{eq}$ and power generation $S_{gk}^{eq}$ at bus $k$ in the equivalent system is calculated as follows:

$$S_{Lk}^{eq} = \sum_{i \in N_k} S_{Li} \quad \text{and} \quad S_{gk}^{eq} = \sum_{i \in N_k} S_{gi},$$

(3.11)

where $S_{Li}$ and $S_{gi}$ are the power demand and power generation at bus $i$ in the original system.

3.5.2 Bus Voltage in the Equivalent System

Since subsystem $k$ is aggregated into bus $k$ in the equivalent system, $E_k^{eq}$ is approximated by the voltage of the centroid bus $c_k$ in subsystem $k$.

$$E_k^{eq} = E_{c_k}.$$  (3.12)

3.5.3 Equivalent Line Admittance Approximation

Traditionally, many approaches identify the parameters of the equivalent system by calculating the sensitivity matrix. However, that kind of approach is computationally expensive, especially for large-scale power systems, as the calculation of the sensitivity matrix may take from minutes to hours to complete. In this approach, we approximate the parameter by the linearized power balance equations. We merge the power balance equations (2.5) and (2.6) of the buses in one subsys-
tem and generate power balance equations for the single equivalent bus. Similarly, we merge the line flow equations from (2.7) where line flows in the equivalent system are

$$S_{mn} = \sum_{r \in N_m, v \in N_n} S_{rv}. \quad (3.13)$$

In order to maintain the same power injection pattern and the same power flow pattern, by using Taylor’s expansion, the line admittance matrix is approximated by

$$Y_{ij}^{eq} = \sum_{s \in N_i, t \in N_j} \left( \frac{V_s V_t \cos \theta_{st}}{V_c_i V_c_j \cos \theta_{c_i c_j}} G_{st} + j \frac{V_s V_t \sin \theta_{st}}{V_c_i V_c_j \sin \theta_{c_i c_j}} B_{st} \right). \quad (3.14)$$

### 3.6 Formulation and Calculation of the Reduced ACOPF Problem

In order to perform ACOPF computation, a new objective function and a set of new constraints after NR are generated based on (3.11)-(3.14). Several generators are aggregated into a single bus in the equivalent system. The cost function of an equivalent generator is greedily changed to a piecewise function:

$$f^k(P_{eq}^g) = \min_{i \in N_k} \sum_i f_i^i(P_{gi})$$

s.t. $$P_{eq}^g = \sum_{i \in N_k} P_{gi}, \quad \forall k \in N^{eq}. \quad (3.15)$$

The equality constraints, which are the power balance equations, are changed to

$$S_i^{eq} = P_i^{eq} - jQ_i^{eq} = E_i^{eq} \sum_{k=0}^{N^{eq}} E_{ik}^{eq} Y_{ik}^{eq}. \quad (3.16)$$

Inequality constraints are relaxed based on (3.11)-(3.14). For constraints on state variables and power generation limits, the minima of the lower bounds are used as the new lower bounds, and the maxima of the upper bounds are used as the new upper bounds. Line limits in the equivalent system are relaxed to
the sum of the corresponding line limits in the original system. We find the optimal solution for the reduced equivalent system by performing \textit{ACOPF} analysis. Further computation is needed to find the optimal solution to the original system.

### 3.7 Congestion Check

Based on the \textit{ACOPF} solution for the reduced system, the interchanged power between different subsystems is obtained. The feasibility of the \textit{ACOPF} solution for the reduced system is then efficiently checked in parallel by performing power flow analysis for each subsystem while considering the original constraints. In order to consider power interchange, we add an additional equality constraint that models the power interchange activities between different subsystems to the constrained power flow analysis problem for each subsystem. If there is no solution for the constrained power flow analysis in a subsystem, it indicates that there exists a congested line in that subsystem.

As shown in Figure 3.1, if there are congested lines detected, we isolate the related buses, remove the congested lines out of the subsystem and go back to the \textit{NR} step.

### 3.8 Distributed \textit{ACOPF} Computation for Each Subsystem

The \textit{ACOPF} solution for the reduced system gives the sum of the control variables inside each subsystem. To decide the optimal dispatch inside the subsystem, it is still an \textit{ACOPF} problem but with a smaller size and interchange power specified. Thus, \textit{ACOPF} is computed to find the optimal settings for each subsystem. Finally, we obtain the detailed solution to the original \textit{ACOPF} problem. With the nature of such a coarse-grained framework, we are able to distribute the com-
putation of $ACOPF$ for $S$ subsystems to $S$ processors to improve the speed.

We use the primal-dual interior point method to solve the $ACOPF$ problem. It is worth mentioning that this framework works with different solvers and is in parallel with the performance of the optimization problem solvers.
To test the proposed fast $ACOPF$ computation algorithm for large-scale smart grids, we use two standard IEEE test power systems and two modified large test systems published in Matpower [23]. They are summarized in Table 4.1. In our tests, we use the total active power generation cost as the objective function of the $ACOPF$ problem. We run all the tests on a laptop, which has an Intel Core2 Duo Processor of 2.26 GHz and 2 GB memory.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 30-bus</td>
<td>30</td>
<td>41</td>
<td>6</td>
</tr>
<tr>
<td>IEEE 300-bus</td>
<td>300</td>
<td>411</td>
<td>69</td>
</tr>
<tr>
<td>Case 3120sp</td>
<td>3,120</td>
<td>3,693</td>
<td>505</td>
</tr>
<tr>
<td>Case 21k</td>
<td>21,084</td>
<td>25,001</td>
<td>2,692</td>
</tr>
</tbody>
</table>

4.1 Experimental Results for the IEEE 30-Bus Test System

The IEEE 30-bus standard load-flow test system is used as a benchmark here. Figure 4.1 shows the network of the IEEE 30-bus system and it is partitioned into 6 subsystems. Two modified IEEE 30-bus systems with 5%
more load demand and 10% more load demand and a modified IEEE 30-bus system with congestion are used to demonstrate the robustness of the proposed method including congestion forecast and congestion check. Real power costs for these 30-bus test systems were adapted from [16].

Figure 4.1: Clustering Results for the IEEE 30-Bus Test System

In this experiment, the congestion indicator threshold is set as 10% of the max value of all the congestion indicators $C_i$s. For the non-congested test system, no additional bus is isolated. As shown in Figure 4.1, the dash lines are the boundaries of the subsystems. To illustrate the capability of the congestion forecast module, we set the transmission capacity in line 2-5 to be 32 MW while the active power flow in this line was 63.01 MW in the
standard case. Then bus 5 is isolated and the solid line shows the isolation.

Table 4.2: Experimental Results of the IEEE 30-Bus Test System

<table>
<thead>
<tr>
<th></th>
<th>Standard 30-bus</th>
<th>With 5% DI</th>
<th>With 10% DI</th>
<th>With Congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial ($/h)</td>
<td>875.28</td>
<td>940.40</td>
<td>1008.05</td>
<td>875.28</td>
</tr>
<tr>
<td>ACOPF ($/h)</td>
<td>802.20</td>
<td>854.41</td>
<td>907.59</td>
<td>947.44</td>
</tr>
<tr>
<td>DCOPF ($/h)</td>
<td>806.97</td>
<td>859.70</td>
<td>913.44</td>
<td>967.67</td>
</tr>
<tr>
<td>PM ($/h)</td>
<td>802.35</td>
<td>854.54</td>
<td>907.69</td>
<td>947.63</td>
</tr>
<tr>
<td>DC error* ($/h)</td>
<td>4.77</td>
<td>5.26</td>
<td>5.75</td>
<td>20.23</td>
</tr>
<tr>
<td>DC error*</td>
<td>0.595%</td>
<td>0.612%</td>
<td>0.634%</td>
<td>2.135%</td>
</tr>
<tr>
<td>PM error* ($/h)</td>
<td>0.15</td>
<td>0.13</td>
<td>0.10</td>
<td>0.19</td>
</tr>
<tr>
<td>PM error*</td>
<td>0.018%</td>
<td>0.015%</td>
<td>0.011%</td>
<td>0.020%</td>
</tr>
<tr>
<td>Improvement**</td>
<td>96.86%</td>
<td>97.53%</td>
<td>98.26%</td>
<td>99.06%</td>
</tr>
</tbody>
</table>

*PM: Proposed Method; DC error: DCOPF error; DI: Demand Increase.
*PM/DCOPF Error Compared to ACOPF
**PM Accuracy Improvement Compared to DCOPF

Table 4.2 shows the experimental results for the IEEE 30-bus test system. Initially, we set the power generation to be \{260.9, 40.0, 0.0, 0.0, 0.0, 0.0\} (MW), which is a feasible setting for the test system, and the generation cost is 875.28 $/hr. By using the proposed method, the optimal setting is \{178.91, 48.50, 21.18, 21.14, 11.93, 11.40\} (MW), with the generation cost of 802.35 $/hr. Note that bus 1 is the slack bus and we don’t control the active power output. The total generation cost is reduced by 8.31%. The proposed method has 0.016% error on average compared to the most accurate full ACOPF. The proposed method reduces the error by 97.93% on average compared to DCOPF. The error of DCOPF is about 43 times larger than
the proposed method in congestion-free test systems. In the congestion case, \textit{DCOPF} has a larger error which is about 2.1\% and the error of \textit{DCOPF} is about 106 times larger than the proposed method. The proposed method handles congested systems much better than \textit{DCOPF} does.

4.2 Experimental Results for Larger Benchmarks

We also test this algorithm on larger benchmarks, including the IEEE 300-bus test system, case 3120sp, and case 21k from Matpower. In this approach, a 300-bus system is reduced to an 89-bus system with 112 lines; the 3120-bus system is reduced to a 449-bus system with 565 lines; and the 21k-bus system is reduced to a 4628-bus system with 5824 lines.

<table>
<thead>
<tr>
<th>Table 4.3: Experimental Results of Accuracy Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>\textit{ACOPF} ($/h$)</td>
</tr>
<tr>
<td>\textit{DCOPF} ($/h$)</td>
</tr>
<tr>
<td>\textit{PM} ($/h$)</td>
</tr>
<tr>
<td>\textit{DC} error ($/h$)</td>
</tr>
<tr>
<td>\textit{DC} error *</td>
</tr>
<tr>
<td>\textit{PM} error ($/h$)</td>
</tr>
<tr>
<td>\textit{PM} error *</td>
</tr>
<tr>
<td>Improvement **</td>
</tr>
</tbody>
</table>

\textit{PM}: Proposed Method; \textit{DC} error: \textit{DCOPF} error.
\*\textit{PM/DCOPF} Error Compared to \textit{ACOPF}
\**\textit{PM} Accuracy Improvement Compared to \textit{DCOPF}
Table 4.3 shows the accuracy of proposed method. The proposed method has 0.54% error on average (1.72% error for the 21k-bus system) compared to the most accurate full ACOPF. The proposed method reduces the error by 77.6% on average (75.7% for the 21k-bus system) compared to DCOPF. As the size of the power system increases, the error of obtaining the optimal generation cost also increases. For the 21k-bus system, 7.06% error was observed in DCOPF. The power system will unnecessarily lose $193,012 per hour, which is $1.69 billion per year. This method can provide an accurate solution to ACOPF problems that can reduce the error by 75.7% compared to DCOPF. Thus we can save $146,110 per hour, which is $1.28 billion per year.

Table 4.4: Experimental Results of Computation Time

<table>
<thead>
<tr>
<th></th>
<th>IEEE 30-bus</th>
<th>IEEE 300-bus</th>
<th>Case 3120sp</th>
<th>Case 21k</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACOPF (s)</td>
<td>0.6510</td>
<td>1.312</td>
<td>15.250</td>
<td>2552.8</td>
</tr>
<tr>
<td>DCOPF (s)</td>
<td>0.4720</td>
<td>0.5109</td>
<td>5.6130</td>
<td>400.7</td>
</tr>
<tr>
<td>PM (s)</td>
<td>0.4946</td>
<td>0.7966</td>
<td>7.2547</td>
<td>364.0</td>
</tr>
</tbody>
</table>

PM Speedup Compared to ACOPF

|                      | 1.32×       | 1.63×        | 2.12×       | 7.01×    |

PM Speedup Compared to DCOPF

|                      | 0.85×       | 0.64×        | 0.77×       | 1.10×    |

PM: Proposed Method.

Table 4.4 shows the computation time of the proposed method. Compared to full ACOPF, the proposed method achieves 1.32×-2.12× speedup for small benchmarks (30-bus, 300-bus, 3120-bus) and 7.01× speedup for the largest
benchmark (the 21,000-bus test system). The proposed method is slower than \textit{DCOPF} for small benchmarks, but is faster than \textit{DCOPF} for the largest benchmark.

The proposed method achieves better accuracy for all test systems compared to \textit{DCOPF}. For large systems, the proposed method has the advantage over \textit{DCOPF} in terms of both accuracy and speed.
CHAPTER 5

CONCLUSION

ACOPF is very important in power system operation. In some applications, it cannot be approximated by DCOPF because of the DC power flow assumption. In addition, the poor accuracy of DCOPF results in great loss of social welfare. Therefore, a faster ACOPF algorithm needs to be developed for large-scale smart grids.

In this thesis, we propose a fast ACOPF analysis framework through power system network reduction to speed up the computation of ACOPF problems. This distributed framework works with different ACOPF solvers, such as primal-dual interior point method. We demonstrate that this approach can achieve 1.32× to 7.01× speedup over full ACOPF while just introducing 0.54% error on average. With congestion forecast and check, as long as ACOPF can converge to the optimal solution, our proposed method can find an optimal solution, which demonstrates its robustness. Compared to the widely used DCOPF, we reduce the error by 77.6% on average. It can potentially save millions of dollars in smart grid operation. Also, experimental results show that the computation time of the proposed algorithm grows almost linearly. The proposed method can be used to solve ACOPF for large-scale power systems in many applications, such as operational reliability analysis and power market management.
REFERENCES


