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THE DENSITY OF CARBON DIOXIDE WITH A TABLE OF RECALCULATED VALUES

BY

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CIRCULAR NO. 13

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CONTENTS

THE DENSITY OF CARBON DIOXIDE WITH A TABLE OF RECALCULATED VALUES

I. Introduction

1. *Introduction.*—Recalulated values for the density of carbon dioxide for temperatures from 10 to 30 degrees and pressures from 720 to 770 mm. were published in the Journal of the American Chemical Society, Vol. 31, page 237 (1909). It was the purpose of this table to correct the errors which were inherent in the original Dietrich calculations first published in 1862 and which had become the common table of reference from that date. In the newly calculated table, the order or variation from the original Dietrich table was approximately four units in the third decimal place for the lower temperatures and pressures. This variation dropped to only about one unit in the higher ranges, although the initial value for carbon dioxide at zero degrees and 760 mm. pressure varied from the accepted value by approximately one unit in the second decimal. The calculated values, therefore, bearing the date of 1909, have, in the main, displaced those of the earlier Dietrich table. Following the publication of this table, however, a number of other factors have entered into the case. For example, in the Van Nostrand's Annual for 1922, the table has been extended, covering the same range of temperatures, but including barometric pressures from 700 to 770 mm. This extension of the table to cover lower barometric pressures is not altogether consistent in its calculated values with the original part of the table. This fact in itself might not be sufficient argument for recasting the table. However, it seems worth while at the present time to extend the table to cover a wider temperature range, namely, from 10 to 35 degrees, and also to incorporate in the formula for deriving the values given a factor which will take account of the deviation of carbon dioxide from the gas laws. Having decided upon the desirability of these changes, it was also deemed better to make the calculations for readings on a barometric brass scale than for readings on a glass scale. This latter change is of small moment, and indeed may become less significant as material both of brass and glass may be used in the future with a still lower coefficient of expansion than is at present obtainable. In any event, the new calculations made, as indicated, result in very slight deviations from the table as published in 1909, the difference being on the average only about one unit in the third

decimal place. However, it is evident that the extended table should be consistent throughout and should also be corrected with reference to the points thus noted.

2. *Formula Used for Calculation of 1909 Table.*—For a review of the derivation and theoretical discussion of the original formula as used in the 1909 table, reference should be made to the article already mentioned.* As a basis, however, for discussing the changes, it will be found desirable to repeat the original formula with a summary of the factors entering into the calculations.

Observed weight of 1 liter CO_2 at 0 deg. and 760 mm. (Guye, Jour. chim. phys., April, 1907), for Lat. 45 deg. = 1.9768.

Corrected value for CO_2, Lat. 41 deg., elevation 100 meters. 1.9768 × 0.9 996 007 = 1.9760.

Coefficient of expansion for CO_2, at constant volume (Chappuis, 1903) = 0.0 037 135.

Formula used:

$$V = \frac{v\,(P - w - b)}{760\ (1 + 0.0\,037\,135 \times t)}$$

in which P is the observed pressure, w the correction for tension of aqueous vapor, and b the correction for barometer, glass scale.

Hence, the formula for W, weight of 1 liter, observed volume, reduced to 0 deg. and 760 mm., would be

$$W = \frac{1.976\,(P - w - b)}{760\ (1 + 0.0\,037\,135 \times t)}$$

The original table was calculated on this basis.

II. Development of Formula for Calculation of New Table

3. *Correction for Deviation from Perfect Gas Law.*—First and most important in the derivation of the new formula is the inclusion of an expression to take care of the deviation from the gas laws. Various gases, including CO_2, deviate from the perfect gas law of Boyle and from Berthelot's equation of state due to pressure changes. Careful experiments on the behavior of gases over extended ranges of temperature and pressure have shown that the fundamental gas equation $pv = RT$, is not strictly applicable to any one gas, the deviations depending upon the nature of the gas and the conditions under which it is observed.

*Journal of American Chemical Society, vol. 31, No. 2, p. 237, Feb., 1909.

It has been shown that the gas laws are more nearly obeyed the lower the pressure, the higher the temperature, and the further the gas is removed from the critical state.

Starting with the assumption that for a perfect gas $\frac{pv}{RT} = 1$, Washburn* gives the following observed data for values of $\frac{pv}{RT}$ for carbon dioxide:

Temperature	Pressure	$\frac{pv}{RT}$
0°C.	0.1 atm.	0.9993
0°	1.0	0.9932

These values show that the experimental data bear out the theoretical statement for effect of pressure. At 1.0 atmosphere the value of $\frac{pv}{RT}$ calculated from Berthelot's equation is 0.9931, a difference of 0.01 per cent from the experimental value.

For a more nearly exact statement of the gas law a term should be included expressing the deviation from the gas law in terms of a constant, *B*, times the pressure, as

$$\frac{pv}{RT} = (1 + Bp)$$

Taking Washburn's experimental value of 0.9932 at 760 mm., it follows that

$$1 + B\,760 = 0.9932$$
$$B\,760 = -0.0068$$
$$B = -0.000\,008\,947$$

which is the constant of deviation.
From the general expression just given

$$pv = RT\,(1 + Bp)$$

At standard conditions of temperature and pressure

$$p_o v_o = RT_o\,(1 + Bp_o)$$

*Washburn, "Physical Chemistry," 2nd ed. 1921, p. 41, Table II.

Since R is the same in both equations,

$$\frac{p_o v_o}{(1 + Bp_o)T_o} = \frac{pv}{(1 + Bp)T}$$

whence

$$V_o = \frac{pvT_o\ (1 + Bp_o)}{p_oT\ (1 + Bp)}$$

Substituting the following values:

$p = (P - w - b)$ = observed pressure corrected for aqueous vapor and brass scale
p_o = 760 mm. Hg. absolute or standard pressure
T_o = 0 deg. C.
$T = t$ = observed temperature

the formula becomes

$$V_o = \frac{v\ (P - w - b)\ (1 + B\ 760)}{760\ t\ [1 + B\ (P - w - b)]}$$

To this must be added a correction factor for the coefficient of expansion for CO_2, which was taken in the original equation as 0.0037135. This factor will affect the observed temperature. The formula now becomes

$$V_o = \frac{v\ (P - w - b)\ (1 + B\ 760)}{760\ (1 + 0.0\,037\,135 \times t)\ \ [1 + B\ (P - w - b)]}$$

which will be observed to be the same as the original formula, with the addition of the fractional expression to correct for deviation from the gas laws.

4. *Correction for Expansion.*—The second modification to be introduced in the new formula is an expression for the coefficient of expansion of CO_2 more nearly correct in all cases than the figure 0.0 037 135 used in the older formula. Chappuis* in his study of expansion coefficients gave for carbon dioxide, at constant volume, and temperatures from 0 to 40 deg., a coefficient, for 518 mm. pressure, of 0.0 036 972, and for 998 mm. pressure, of 0.0 037 299. In the original paper, by interpolation at a pressure of 758 mm., Parr obtained the ratio of expansion per degree of temperature as 0.0 037 135. This coefficient was the one

*Chappuis, Trav. et Mem. du Bur. Intern. des Poids et Mes., XIII, 190, 1903.

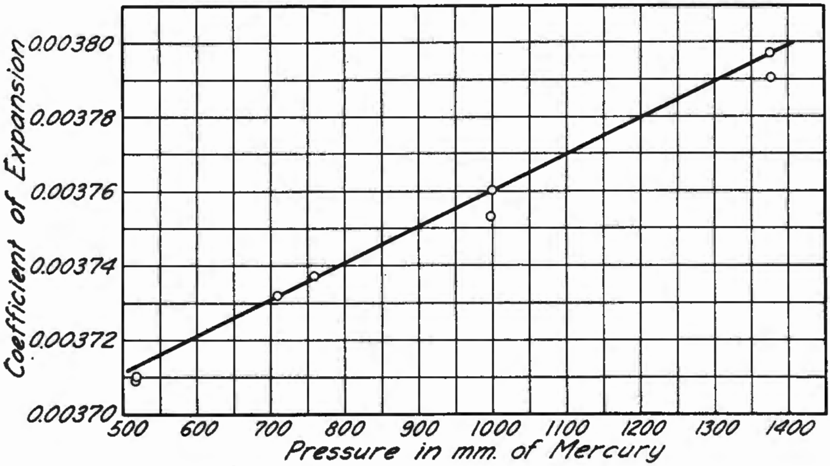

FIG. 1. COEFFICIENT OF EXPANSION OF CO_2

used in the earlier formula, and is correct for only the one pressure. It will be more accurate to express the coefficient in terms of a coefficient times the observed pressure plus a constant, all of which may be expressed by the symbol a. That portion of the denominator of the new formula which contains the expression for the coefficient of expansion, namely,

$$760\ (1 + 0.0\,037\,135 \times t)$$

may be written as

$$760\ (1 + a\,t)$$

where $a = (Kp + C)$, p again being equal to the observed pressure, $= (P - w - b)$ corrected, now giving as the expression

$$760\ [1 + \{K\,(P - w - b) + C\}\ t]$$

It now becomes nesessary to determine the values of K and C. Plotting pressures against coefficients gives a straight line through most of these points (see Fig. 1). For the purpose of this discussion those values marked* in Table 1 will be used.

Using the four values for coefficients marked,* we have six possible combinations for a simultaneous equation from which the value of K may be determined, as, for example,

$$0.0\,037\,536 = K\,998 + C$$
$$0.0\,037\,100 = K\,518 + C$$

Subtracting

$$0.0\,000\,436 = K\,480$$
$$K = 9083 \times 10^{-11}$$

Solving the six equations in this manner, we obtain six values for K, as follows:

Using pressures of 760 − 518, $K = 11\,157 \times 10^{-11}$
Using pressures of 1377 − 998, $K = 9762 \times 10^{-11}$
Using pressures of 1377 − 518, $K = 9383 \times 10^{-11}$
Using pressures of 998 − 518, $K = 9083 \times 10^{-11}$
Using pressures of 1377 − 760, $K = 8687 \times 10^{-11}$
Using pressures of 998 − 760, $K = 6974 \times 10^{-11}$

From these an average value of 91325×10^{-12} was taken for K.

Substituting this value of K in the four equations corresponding to the four values for coefficients marked,* and solving for C we get the following values:

0.0 037 052 694
0.0 036 675 930
0.0 036 624 577
0.0 036 648 455

which give a mean value for $C = 0.0\,036\,750\,414$.

The new formula as developed will now stand, showing sequence of derivation,

$$\frac{pv}{RT} = 1 + Bp \tag{1}$$

$$V_o = \frac{pv\ T_o\ (1 + Bp_o)}{P_o\ T\ (1 + Bp)} \tag{2}$$

$$V_o = \frac{v\ (P - w - b)\ (1 + B\ 760)}{760\,t\,[1 + B\ (P - w - b)]} \tag{3}$$

$$V_o = \frac{v\ (P - w - b)\ (1 + B\ 760)}{760\ (1 + a\ t)\ [1 + B\ (P - w - b)]} \tag{4}$$

$$V_o = \frac{v\ (P - w - b)\ (1 + B\ 760)}{760\,[1 + \{K\ (P - w - b) + C\}\ t]\ [1 + B\ (P - w - b)]} \tag{5}$$

TABLE 1

COEFFICIENT OF EXPANSION OF CO_2 AT CONSTANT PRESSURE

In the latest literature the following values are found for the coefficient of expansion of CO_2 at constant pressure, giving the change in volume per unit volume per degree Centigrade.

Temperature, deg. C.	Pressure, mm. Hg.	Coefficient	Observer	Source
0–40	518	0.003709	Chappuis	Van Nostrand, 1903
0–40	518	*0.0037100		
0–40	710	0.0037320	Chappuis	Chem. Yr. Bk., 1924
0–40	760	*0.0037370		
0–20	998	0.0037602	Chappuis	Van Nostrand, 1903
0–40	998	*0.0037536		
0–20	1377	0.0037972	Chappuis	Van Nostrand, 1903
0–40	1377	*0.0037906		

where $B = -8947 \times 10^{-9}$
$K = 91\ 325 \times 10^{-12}$
$C = 0.0\ 036\ 750\ 414$
P = observed pressure
t = observed temperature

5. *Correction for Density.*—In 1909 Parr used the value 1.9760 grams for the weight of 1 liter of pure dry carbon dioxide at 0 deg. and 760 mm.,* corrected for latitude of 41 deg., and an elevation of 100 meters.† The latest data on CO_2 in regard to density are based on practically the same information.

The following data are taken from the Chemists Year Book, 1924:

Density of CO_2. Data in grams per liter (1000.027 cc.) at 0 deg. C. under pressure of 760 mm. of mercury at 0 deg. C. and latitude 45 deg. at sea level.‡

Mol. Wt. (O = 16) = 44.00
Relative density = 22.133
Observed density = 1.9768 gm. per liter.

Phys. Chemische Tabellen, Vol. I, Landolt-Bornstein, p. 269 (1923), give

Observed density of CO_2
1.9769¶
1.9763§
1.9768** Latitude 41 deg.

taking the value $1.9768 \times 0.9\ 996\ 007 = 1.9760$.

*Journal of American Chemical Society, 31, No. 2, 237, Feb. 1909.

†"The latitude of 41 deg. is less than ½ deg. from New Haven, New York, Pittsburgh, Urbana Ill., Lincoln, Neb., Salt Lake City, etc." Wells, "Chemical Calculations," p. 40.

‡Guye. 1905.

¶Rayleigh, Proc. Roy. Soc., 62, 204, 1898.

§Leduc, Ann. Chim. Phys., 15, 1, 1898.

**Guye, Mem. de Geneve, 35, 569, 1908.

TABLE 2

EFFECT OF TYPE OF SCALE ON BAROMETER CORRECTION FACTOR *b* AND VALUE OF WEIGHT OF CO_2 *W* CALCULATED FROM ORIGINAL FORMULA

Pressure mm. Hg	Temperature deg. C.	*b*		*W*		
		By Glass Scale	By Brass Scale	By Glass Scale	By Brass Scale	Difference
720..........	10	1.24	1.16	1.7788	1.7789	0.0001
	20	2.48	2.32	1.6945	1.6948	0.0003
	30	3.73	3.49	1.6018	1.6024	0.0006
740..........	10	1.27	1.19	1.8289	1.8291	0.0002
	20	2.55	2.39	1.7427	1.7431	0.0004
	30	3.83	3.59	1.6480	1.6486	0.0006
770..........	10	1.32	1.24	1.9040	1.9042	0.0002
	20	2.64	2.48	1.8151	1.8155	0.0004
	30	3.97	3.73	1.7182	1.7188	0.0006

Since weight W = volume $\times$ density, the weight W of 1 liter observed volume reduced to 0 deg. and 760 mm. will be given by the formula

$$W = \frac{1.976\,(P - w - b)\,(1 + b\,760)}{760\,[1 + \{K\,(P - w - b) + C\}\,t\,]\,[1 + B\,(P - w - b)\,]}$$

6. *Correction for Barometer.*—The next modification of the original formula is in regard to the factor b, the correction for barometer. In the original formula correction was made for a glass scale. In the present formula correction is made for a brass scale. In order to show the effect of using the brass scale correction instead of that for the glass scale, and the significance of the different results calculated, a few values of the weight of CO_2 have been calculated for different portions of the table. In order to show the difference, the original formula may be used.

These results, Table 2, show a difference of from 0.0001 to 0.0006 in the value of W caused by use of glass or brass scale, respectively.

Having thus shown that the use of the correction coefficients for brass scale makes sufficient difference in final values to warrant a change from coefficients for glass scale to coefficients for brass scale in the original formula, the next step was to obtain the most accurate table of data for coefficients for barometer with brass scale.

Readings of the barometer require to be corrected for

(a) Capillary depression of the Hg. in the tube, if the tube is less than about 15 mm. bore

(b) Temperature of the Hg. column and scale

(c) Variation of "gravity" from the standard, owing to the difference in latitude from 45 deg., or to the height above sea level.

Corrections (b) and (c) are necessary, since the standard gravity, to which densities of gases are always reduced for comparison, signifies that at 45 deg. latitude, at sea level, and with the Hg. column at 0 deg. C.

Barometric readings can be reduced to 0 deg. C. as follows:* The corrected height $B_o = B_t \left[\frac{1 - (K - a)\,t}{1 + K\,t} \right]$ where

B_t = reading observed at temp. t deg. C.
K = coefficient of cubical expansion of mercury
a = coefficient of linear expansion of the scale

The factor $B_t \times \frac{(K - a)\,t}{1 + Kt}$ is to be substracted from B_t

$K = 1818 \times 10^{-7}$
$a = 184 \times 10^{-7}$ for brass scale
$a = 85 \times 10^{-7}$ for glass scale

The values given in Table 3 were not derived from this equation, but may be closely checked by its use if desired.

7. *Correction for Tension of Aqueous Vapor.*—The value of W, correction factor for aqueous vapor tension, is given in millimeters of mercury at 0 deg. C. The most recent literature does not quite agree upon data for tension of aqueous vapor. The tables of data found in current handbooks do, however, follow one of two authorities. These are given in Table 4. Since the values vary only in the second decimal in most cases, they have been averaged, and the mean taken as the standard for this work.

8. *Accuracy of Determined Value of Coefficient of Deviation.*—To check the determined values of the coefficient of deviation, $B = -8947 \times 10^{-9}$, a graph, Fig. 2, was made using values of pv and v given by Washburn.*

*International Meteorological Tables, 1890. Used in the Chemists' Year Book of 1924.
*Washburn, "Physical Chemistry," 2nd ed., 1921, p. 41, Table II.

TABLE 3

VALUES OF BAROMETER CORRECTION FACTOR *b*, FOR BRASS SCALE TO REDUCE READINGS OF A MERCURIAL BAROMETER WITH A BRASS SCALE TO 0 DEG. C.

Values of *b* are given in millimeters of mercury. All data from 10 deg. to 30 deg. are taken from the latest handbooks. Temperatures between 30 deg. and 35 deg. are not given in handbooks, and were therefore calculated by averaging the increase per degree at each pressure, and adding this mean factor continuously from 30 deg. to 35 deg. C. Values for odd pressures are by interpolation.

Temperature deg. C.	Barometer Correction Factor *b*										
	720 mm. Hg.	725 mm. Hg.	730 mm. Hg.	735 mm. Hg.	740 mm. Hg.	745 mm. Hg.	750 mm. Hg.	755 mm. Hg.	760 mm. Hg.	765 mm. Hg.	770 mm. Hg.
10	1.16	1.17	1.18	1.185	1.19	1.20	1.21	1.215	1.22	1.23	1.24
11	1.28	1.28	1.29	1.30	1.31	1.33	1.35	1.36	1.37	1.375	1.38
12	1.39	1.40	1.41	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.49
13	1.51	1.52	1.53	1.54	1.55	1.56	1.57	1.58	1.59	1.60	1.61
14	1.63	1.64	1.65	1.66	1.67	1.68	1.69	1.705	1.72	1.73	1.74
15	1.74	1.75	1.77	1.78	1.79	1.80	1.81	1.825	1.84	1.85	1.86
16	1.86	1.87	1.88	1.895	1.91	1.92	1.94	1.95	1.96	1.975	1.99
17	1.97	1.98	2.00	2.015	2.03	2.045	2.06	2.07	2.08	2.095	2.11
18	2.09	2.10	2.12	2.135	2.15	2.165	2.18	2.195	2.21	2.225	2.24
19	2.21	2.23	2.24	2.255	2.27	2.285	2.30	2.315	2.33	2.345	2.36
20	2.32	2.34	2.36	2.375	2.39	2.405	2.42	2.435	2.45	2.465	2.48
21	2.44	2.45	2.47	2.49	2.51	2.525	2.54	2.56	2.58	2.595	2.61
22	2.56	2.57	2.59	2.61	2.63	2.645	2.66	2.68	2.70	2.715	2.73
23	2.67	2.69	2.71	2.73	2.75	2.765	2.78	2.80	2.82	2.84	2.86
24	2.79	2.81	2.83	2.85	2.87	2.89	2.91	2.925	2.94	2.96	2.98
25	2.91	2.93	2.95	2.97	2.99	3.01	3.03	3.05	3.07	3.09	3.11
26	3.02	3.04	3.06	3.085	3.11	3.13	3.15	3.17	3.19	3.21	3.23
27	3.14	3.16	3.18	3.205	3.23	3.25	3.27	3.29	3.31	3.335	3.36
28	3.26	3.28	3.30	3.325	3.35	3.37	3.39	3.415	3.44	3.46	3.48
29	3.37	3.39	3.42	3.445	3.47	3.49	3.51	3.535	3.56	3.585	3.61
30	3.49	3.51	3.54	3.565	3.59	3.61	3.63	3.655	3.68	3.705	3.73
31	3.60	3.63	3.65	3.684	3.709	3.73	3.751	3.777	3.802	3.828	3.854
32	3.72	3.74	3.77	3.803	3.829	3.85	3.872	3.899	3.925	3.952	3.978
33	3.83	3.86	3.89	3.922	3.948	3.971	3.993	4.021	4.047	4.076	4.102
34	3.95	3.98	4.01	4.041	4.068	4.092	4.114	4.143	4.170	4.199	4.227
35	4.07	4.10	4.13	4.160	4.188	4.212	4.235	4.265	4.293	4.323	4.351
	Temperature factors used from 30 to 35 deg. C.										
	0.1163	0.1172	0.118	0.119	0.1196	0.1205	0.121	0.122	0.1226	0.1237	0.1243

TABLE 4

VALUES OF AQUEOUS VAPOR TENSION CORRECTION FACTOR W

Temperature deg. C.	Aqueous Vapor Tension Correction Factor W		
	I	II	Mean Value (used in calculations)
10	9.1398	9.179	9.1594
11	9.7671	9.810	9.7885
12	10.4320	10.479	10.455
13	11.137	11.187	11.162
14	11.884	11.936	11.910
15	12.674	12.728	12.701
16	13.510	13.565	13.5375
17	14.395	14.450	14.4225
18	15.330	15.383	15.3565
19	16.319	16.367	16.343
20	17.363	17.406	17.3845
21	18.466	18.503	18.4845
22	19.630	19.661	19.6455
23	20.858	20.883	20.8705
24	22.152	22.178	22.165
25	23.517	23.546	23.5315
26	24.956	24.987	24.9715
27	26.471	26.505	26.448
28	28.065	28.103	28.084
29	29.744	29.785	29.7645
30	31.510	31.555	31.5325
31	33.366	33.416	33.391
32	35.318	35.372	35.345
33	37.369	37.427	37.398
34	39.523	39.586	39.5545
35	41.784	41.853	41.8185

Values in column I from International Bureau of Weights and Measures.
Values in column II calculated by Broch and Weibe from the results of Regnault and Juhlin.

Values of $\frac{pv}{RT}$ were plotted as ordinates and those of p as abscissas.

From this graph, values for several different pressures between 680 mm. and 770 mm. were obtained, and a second graph was made on an enlarged scale to show accurately the points between 720 mm. and 770 mm., the values bearing upon the table being calculated.

To check the value of B take any pressure, say 700 mm., and calculate $\frac{pv}{RT}$ as follows:

$$\frac{pv}{RT} = 1 + (-0.000\,008\,947 \times 700)$$
$$= 1 - (0.000\,008\,947 \times 700)$$
$$= 0.993\,737$$

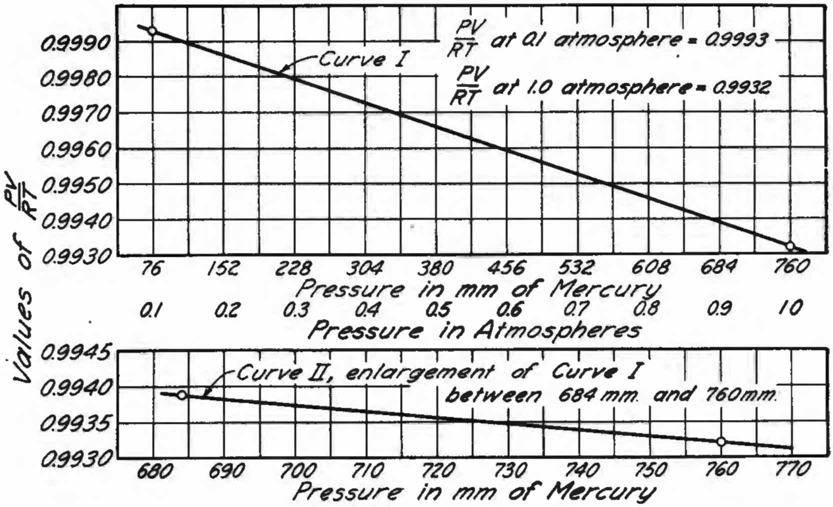

FIG. 2. CURVES FOR VALUES OF $\frac{pv}{RT}$

The value of $\frac{pv}{RT}$ read from the chart is 0.993736.

In like manner calculations at pressures of 740 mm. and 770 mm., respectively, gave comparisons between calculated and graph values of $\frac{pv}{RT}$ as follows:

	Pressure = 740 mm.	Pressure = 770 mm.
Calculated value of $\frac{pv}{RT}$ =	0.9 933 793	0.99 312
Graph value of $\frac{pv}{RT}$ =	0.99 338	0.99 312

These results prove that the value of B as used in this work is correct within nesessary limits, to values determined experimentally.

9. *Comparison of Values in Old and New Tables.*—A comparison between the old and new tables of values between 10 and 30 deg. C. for the weight of carbon dioxide shows that use of the new formula results in an appreciable lowering of all values in the original table. The difference in values shows a lowering of from 0.0004 milligrams per cubic

centimeter to 0.0016 milligrams, the maximum change. No comparison can be made for temperatures greater than 30 deg. C. since that temperature was the limit in the original table. To indicate the change made in values a brief table of differences follows, showing the amount by which the new table lowers the former values at the temperature and pressure indicated:

Temp.	Differences in mm. per c. c.					
deg. C.	720 mm.	730 mm.	740 mm.	750 mm.	760 mm.	770 mm.
10	0.0010	0.0009	0.0007	0.0006	0.0005	0.0004
15	0.0012	0.0010	0.0009	0.0007	0.0007	0.0006
20	0.0013	0.0012	0.0011	0.0009	0.0009	0.0007
25	0.0014	0.0013	0.0013	0.0012	0.0011	0.0010
30	0.0016	0.0015	0.0014	0.0013	0.0012	0.0011

TABLE 5

WEIGHT OF CARBON DIOXIDE

Calculated from 1.976 = weight of 1 liter of CO_2 at 0 deg. C., 760 mm., and 41 deg. latitude. Corrected for aqueous vapor, barometer brass scale, and deviation from gas law.

Temperature deg. C.	Weight of CO_2—Milligrams per cubic centimeter												
	720 mm. Hg.	722 mm. Hg.	724 mm. Hg.	726 mm. Hg.	728 mm. Hg.	730 mm. Hg.	732 mm. Hg.	734 mm. Hg.	736 mm. Hg.	738 mm. Hg.	740 mm. Hg.	742 mm. Hg.	744 mm. Hg.
10	1.7778	1.7828	1.7878	1.7929	1.7979	1.8030	1.8080	1.8130	1.8180	1.8231	1.8281	1.8331	1.8382
11	1.7695	1.7745	1.7795	1.7846	1.7896	1.7946	1.7996	1.8046	1.8096	1.8146	1.8197	1.8247	1.8297
12	1.7612	1.7662	1.7712	1.7762	1.7812	1.7862	1.7912	1.7962	1.8012	1.8062	1.8112	1.8162	1.8212
13	1.7529	1.7579	1.7628	1.7678	1.7728	1.7778	1.7827	1.7877	1.7927	1.7976	1.8026	1.8076	1.8126
14	1.7445	1.7495	1.7544	1.7594	1.7643	1.7693	1.7742	1.7792	1.7841	1.7891	1.7941	1.7990	1.8040
15	1.7361	1.7411	1.7460	1.7509	1.7558	1.7608	1.7657	1.7707	1.7756	1.7805	1.7855	1.7904	1.7954
16	1.7276	1.7326	1.7375	1.7424	1.7473	1.7522	1.7571	1.7621	1.7669	1.7719	1.7768	1.7817	1.7867
17	1.7191	1.7240	1.7289	1.7338	1.7387	1.7436	1.7485	1.7534	1.7583	1.7632	1.7681	1.7730	1.7780
18	1.7103	1.7154	1.7202	1.7251	1.7300	1.7349	1.7398	1.7446	1.7495	1.7544	1.7593	1.7642	1.7691
19	1.7017	1.7067	1.7115	1.7164	1.7213	1.7261	1.7310	1.7358	1.7407	1.7455	1.7504	1.7553	1.7602
20	1.6931	1.6980	1.7028	1.7077	1.7125	1.7173	1.7222	1.7270	1.7318	1.7367	1.7415	1.7464	1.7513
21	1.6843	1.6891	1.6939	1.6988	1.7036	1.7084	1.7132	1.7180	1.7229	1.7277	1.7325	1.7374	1.7422
22	1.6754	1.6802	1.6850	1.6898	1.6946	1.6994	1.7042	1.7090	1.7139	1.7187	1.7235	1.7283	1.7331
23	1.6663	1.6711	1.6759	1.6807	1.6855	1.6903	1.6952	1.6998	1.7046	1.7094	1.7142	1.7190	1.7238
24	1.6572	1.6620	1.6668	1.6715	1.6763	1.6811	1.6859	1.6906	1.6954	1.7001	1.7049	1.7097	1.7145
25	1.6481	1.6529	1.6576	1.6623	1.6671	1.6719	1.6766	1.6813	1.6861	1.6908	1.6956	1.7003	1.7051
26	1.6386	1.6433	1.6480	1.6527	1.6575	1.6623	1.6670	1.6717	1.6764	1.6811	1.6858	1.6906	1.6954
27	1.6290	1.6337	1.6384	1.6431	1.6479	1.6527	1.6573	1.6620	1.6667	1.6714	1.6761	1.6808	1.6856
28	1.6194	1.6241	1.6288	1.6335	1.6383	1.6430	1.6476	1.6523	1.6570	1.6617	1.6664	1.6711	1.6759
29	1.6098	1.6145	1.6192	1.6239	1.6286	1.6333	1.6379	1.6426	1.6473	1.6520	1.6567	1.6614	1.6661
30	1.6003	1.6049	1.6096	1.6143	1.6189	1.6236	1.6283	1.6329	1.6376	1.6423	1.6470	1.6516	1.6563
31	1.5902	1.5949	1.5995	1.6042	1.6088	1.6135	1.6182	1.6228	1.6274	1.6321	1.6367	1.6414	1.6461
32	1.5802	1.5845	1.5894	1.5940	1.5987	1.6033	1.6080	1.6126	1.6172	1.6219	1.6265	1.6312	1.6358
33	1.5697	1.5741	1.5789	1.5835	1.5882	1.5928	1.5974	1.6020	1.6066	1.6112	1.6158	1.6205	1.6251
34	1.5592	1.5637	1.5684	1.5730	1.5776	1.5823	1.5868	1.5914	1.5960	1.6006	1.6052	1.6098	1.6144
35	1.5487	1.5533	1.5579	1.5625	1.5670	1.5717	1.5762	1.5808	1.5854	1.5899	1.5945	1.5991	1.6037

TABLE 5 (Continued)

WEIGHT OF CARBON DIOXIDE

Temperature deg. C.	Weight of CO_2—Milligrams per cubic centimeter												
	746 mm. Hg.	748 mm. Hg.	750 mm. Hg.	752 mm. Hg.	754 mm. Hg.	756 mm. Hg.	758 mm. Hg.	760 mm. Hg.	762 mm. Hg.	764 mm. Hg.	766 mm. Hg.	768 mm. Hg.	770 mm. Hg.
10	1.8432	1.8482	1.8533	1.8583	1.8634	1.8684	1.8734	1.8785	1.8835	1.8885	1.8935	1.8986	1.9036
11	1.8347	1.8397	1.8447	1.8497	1.8548	1.8598	1.8648	1.8688	1.8748	1.8798	1.8848	1.8899	1.8949
12	1.8262	1.8312	1.8362	1.8412	1.8462	1.8512	1.8562	1.8611	1.8661	1.8711	1.8761	1.8811	1.8861
13	1.8175	1.8225	1.8275	1.8325	1.8375	1.8424	1.8474	1.8524	1.8574	1.8624	1.8674	1.8724	1.8774
14	1.8089	1.8139	1.8188	1.8238	1.8288	1.8337	1.8387	1.8436	1.8486	1.8536	1.8586	1.8636	1.8686
15	1.8003	1.8052	1.8102	1.8151	1.8200	1.8250	1.8299	1.8349	1.8399	1.8449	1.8498	1.8548	1.8598
16	1.7916	1.7965	1.8015	1.8064	1.8113	1.8162	1.8212	1.8260	1.8310	1.8359	1.8408	1.8458	1.8507
17	1.7829	1.7878	1.7927	1.7977	1.8026	1.8075	1.8124	1.8171	1.8220	1.8269	1.8318	1.8367	1.8416
18	1.7739	1.7789	1.7838	1.7887	1.7936	1.7985	1.8034	1.8081	1.8130	1.8179	1.8228	1.8276	1.8325
19	1.7650	1.7699	1.7748	1.7797	1.7845	1.7894	1.7943	1.7991	1.8039	1.8088	1.8137	1.8185	1.8234
20	1.7561	1.7610	1.7658	1.7707	1.7755	1.7804	1.7852	1.7901	1.7949	1.7997	1.8046	1.8094	1.8143
21	1.7470	1.7519	1.7567	1.7615	1.7664	1.7712	1.7760	1.7808	1.7856	1.7905	1.7953	1.8001	1.8050
22	1.7379	1.7427	1.7476	1.7524	1.7571	1.7620	1.7668	1.7716	1.7764	1.7812	1.7860	1.7909	1.7957
23	1.7286	1.7334	1.7382	1.7430	1.7478	1.7526	1.7574	1.7621	1.7669	1.7717	1.7766	1.7813	1.7861
24	1.7192	1.7240	1.7288	1.7336	1.7384	1.7432	1.7479	1.7527	1.7575	1,7623	1.7671	1.7718	1.7766
25	1.7099	1.7147	1.7195	1.7242	1.7290	1.7338	1.7385	1.7433	1.7480	1.7528	1.7575	1.7623	1.7671
26	1.7001	1.7049	1.7096	1.7144	1.7191	1.7239	1.7286	1.7338	1.7381	1.7428	1.7476	1.7523	1.7570
27	1.6903	1.6951	1.6998	1.7045	1.7093	1.7140	1.7187	1.7234	1.7282	1.7329	1.7376	1.7423	1.7471
28	1.6806	1.6853	1.6900	1.6947	1.6994	1.7041	1.7088	1.7135	1.7182	1.7229	1.7276	1.7323	1.7371
29	1.6708	1.6755	1.6802	1.6849	1.6895	1.6942	1.6989	1.7036	1.7083	1.7130	1.7177	1.7224	1.7271
30	1.6610	1.6657	1.6704	1.6750	1.6797	1.6844	1.6890	1.6937	1.6984	1.7031	1.7077	1.7124	1.7171
31	1.6507	1.6554	1.6600	1.6647	1.6694	1.6741	1.6787	1.6833	1.6880	1.6926	1.6973	1.7019	1.7066
32	1.6404	1.6451	1.6497	1.6544	1.6590	1.6637	1.6683	1.6729	1.6776	1.6822	1.6868	1.6915	1.6961
33	1.6297	1.6344	1.6390	1.6436	1.6483	1.6529	1.6575	1.6621	1.6667	1.6714	1.6760	1.6806	1.6852
34	1.6190	1.6237	1.6283	1.6329	1.6375	1.6421	1.6467	1.6513	1.6559	1.6605	1.6651	1.6697	1.6744
35	1.6083	1.6130	1.6176	1.6222	1.6268	1.6314	1.6359	1.6405	1.6451	1.6497	1.6543	1.6589	1.6635

Table 6

Weight of Carbon per Cubic Centimeter of Carbon Dioxide

Calculated from 1.976 = Weight of 1 liter of CO_2 at 0 deg. C., 760 mm., and 41 deg. latitude. Corrected for aqueous vapor, barometer brass scale, and deviation from gas law.

Temperature deg. C.	Weight of Carbon—Milligrams per cubic centimeter of CO_2												
	720 mm. Hg.	722 mm. Hg.	724 mm. Hg.	726 mm. Hg.	728 mm. Hg.	730 mm. Hg.	732 mm. Hg.	734 mm. Hg.	736 mm. Hg.	738 mm. Hg.	740 mm. Hg.	742 mm. Hg.	744 mm. Hg.
10	0.4848	0.4862	0.4875	0.4889	0.4903	0.4917	0.4930	0.4944	0.4958	0.4972	0.4985	0.4999	0.5013
11	0.4826	0.4839	0.4853	0.4867	0.4880	0.4894	0.4908	0.4921	0.4935	0.4948	0.4962	0.4976	0.4990
12	0.4803	0.4817	0.4830	0.4844	0.4858	0.4871	0.4885	0.4889	0.4912	0.4926	0.4940	0.4953	0.4967
13	0.4780	0.4794	0.4807	0.4821	0.4835	0.4848	0.4862	0.4875	0.4889	0.4902	0.4916	0.4930	0.4943
14	0.4757	0.4771	0.4784	0.4798	0.4812	0.4825	0.4839	0.4852	0.4866	0.4879	0.4893	0.4906	0.4920
15	0.4734	0.4748	0.4761	0.4775	0.4788	0.4802	0.4815	0.4829	0.4842	0.4856	0.4869	0.4883	0.4896
16	0.4711	0.4725	0.4738	0.4752	0.4765	0.4779	0.4792	0.4806	0.4819	0.4832	0.4846	0.4859	0.4873
17	0.4688	0.4701	0.4715	0.4728	0.4742	0.4755	0.4769	0.4782	0.4795	0.4809	0.4822	0.4835	0.4849
18	0.4664	0.4678	0.4691	0.4704	0.4718	0.4731	0.4745	0.4758	0.4771	0.4785	0.4798	0.4811	0.4825
19	0.4641	0.4654	0.4667	0.4681	0.4694	0.4707	0.4721	0.4734	0.4747	0.4760	0.4774	0.4787	0.4800
20	0.4617	0.4631	0.4644	0.4657	0.4670	0.4683	0.4697	0.4710	0.4723	0.4736	0.4749	0.4763	0.4776
21	0.4593	0.4606	0.4619	0.4633	0.4646	0.4659	0.4672	0.4685	0.4699	0.4712	0.4725	0.4738	0.4751
22	0.4569	0.4582	0.4595	0.4608	0.4622	0.4635	0.4648	0.4661	0.4674	0.4687	0.4700	0.4713	0.4727
23	0.4544	0.4557	0.4570	0.4583	0.4597	0.4610	0.4623	0.4636	0.4649	0.4662	0.4675	0.4688	0.4701
24	0.4519	0.4532	0.4545	0.4558	0.4572	0.4585	0.4598	0.4611	0.4624	0.4637	0.4650	0.4663	0.4676
25	0.4494	0.4508	0.4520	0.4533	0.4547	0.4560	0.4572	0.4585	0.4598	0.4611	0.4624	0.4637	0.4650
26	0.4468	0.4481	0.4494	0.4507	0.4520	0.4533	0.4546	0.4559	0.4572	0.4585	0.4598	0.4611	0.4624
27	0.4442	0.4455	0.4468	0.4481	0.4494	0.4507	0.4520	0.4533	0.4545	0.4558	0.4571	0.4584	0.4597
28	0.4416	0.4429	0.4442	0.4455	0.4468	0.4481	0.4493	0.4506	0.4519	0.4532	0.4545	0.4557	0.4571
29	0.4390	0.4403	0.4416	0.4429	0.4442	0.4454	0.4467	0.4480	0.4493	0.4505	0.4518	0.4531	0.4544
30	0.4364	0.4377	0.4390	0.4403	0.4415	0.4428	0.4441	0.4453	0.4466	0.4479	0.4492	0.4504	0.4517
31	0.4337	0.4350	0.4362	0.4375	0.4388	0.4400	0.4413	0.4426	0.4438	0.4451	0.4464	0.4476	0.4489
32	0.4310	0.4322	0.4335	0.4347	0.4360	0.4373	0.4385	0.4398	0.4410	0.4423	0.4436	0.4449	0.4461
33	0.4281	0.4293	0.4306	0.4319	0.4331	0.4344	0.4356	0.4369	0.4382	0.4394	0.4407	0.4419	0.4432
34	0.4252	0.4265	0.4277	0.4290	0.4302	0.4315	0.4328	0.4340	0.4353	0.4365	0.4378	0.4390	0.4403
35	0.4224	0.4236	0.4249	0.4261	0.4274	0.4286	0.4299	0.4311	0.4324	0.4336	0.4349	0.4361	0.4374

This table has been calculated from the preceding table of Weights of Carbon Dioxide, using the gravimetric factor for Carbon in Carbon Dioxide.

Table 6 (Continued)

Weight of Carbon per Cubic Centimeter of Carbon Dioxide

Temperature deg. C.	Weight of Carbon—Milligrams per cubic centimeter of CO_2												
	746 mm. Hg.	748 mm. Hg.	750 mm. Hg.	752 mm. Hg.	754 mm. Hg.	756 mm. Hg.	758 mm. Hg.	760 mm. Hg.	762 mm. Hg.	764 mm. Hg.	766 mm. Hg.	768 mm. Hg.	770 mm. Hg.
10	0.5027	0.5040	0.5054	0.5068	0.5082	0.5096	0.5109	0.5123	0.5137	0.5150	0.5164	0.5178	0.5192
11	0.5004	0.5017	0.5031	0.5045	0.5058	0.5072	0.5086	0.5100	0.5113	0.5127	0.5140	0.5154	0.5168
12	0.4980	0.4994	0.5008	0.5021	0.5035	0.5049	0.5062	0.5076	0.5089	0.5103	0.5117	0.5130	0.5144
13	0.4957	0.4970	0.4984	0.4998	0.5011	0.5025	0.5038	0.5052	0.5066	0.5079	0.5093	0.5106	0.5120
14	0.4933	0.4947	0.4960	0.4974	0.4988	0.5001	0.5015	0.5028	0.5042	0.5055	0.5069	0.5082	0.5096
15	0.4910	0.4923	0.4937	0.4950	0.4964	0.4977	0.4991	0.5004	0.5018	0.5031	0.5045	0.5058	0.5072
16	0.4886	0.4899	0.4913	0.4926	0.4940	0.4953	0.4967	0.4980	0.4994	0.5007	0.5020	0.5034	0.5047
17	0.4862	0.4876	0.4889	0.4903	0.4916	0.4929	0.4943	0.4956	0.4969	0.4982	0.4996	0.5009	0.5022
18	0.4838	0.4851	0.4865	0.4878	0.4892	0.4905	0.4918	0.4931	0.4944	0.4958	0.4971	0.4984	0.4998
19	0.4814	0.4827	0.4840	0.4854	0.4867	0.4880	0.4893	0.4907	0.4920	0.4933	0.4946	0.4959	0.4973
20	0.4789	0.4803	0.4816	0.4829	0.4842	0.4856	0.4869	0.4882	0.4895	0.4908	0.4922	0.4935	0.4948
21	0.4764	0.4778	0.4791	0.4804	0.4817	0.4830	0.4844	0.4857	0.4870	0.4883	0.4896	0.4909	0.4923
22	0.4740	0.4753	0.4766	0.4779	0.4792	0.4805	0.4818	0.4832	0.4845	0.4858	0.4871	0.4884	0.4897
23	0.4714	0.4727	0.4740	0.4754	0.4767	0.4780	0.4793	0.4806	0.4819	0.4832	0.4845	0.4858	0.4871
24	0.4689	0.4702	0.4715	0.4728	0.4741	0.4754	0.4767	0.4780	0.4793	0.4806	0.4819	0.4832	0.4845
25	0.4663	0.4676	0.4689	0.4702	0.4715	0.4728	0.4741	0.4754	0.4767	0.4780	0.4793	0.4806	0.4819
26	0.4637	0.4650	0.4662	0.4676	0.4688	0.4701	0.4714	0.4728	0.4740	0.4753	0.4766	0.4779	0.4792
27	0.4610	0.4623	0.4636	0.4649	0.4662	0.4674	0.4687	0.4700	0.4713	0.4726	0.4739	0.4752	0.4765
28	0.4583	0.4596	0.4609	0.4622	0.4635	0.4647	0.4660	0.4673	0.4686	0.4699	0.4712	0.4724	0.4737
29	0.4557	0.4569	0.4582	0.4595	0.4608	0.4620	0.4633	0.4646	0.4659	0.4672	0.4685	0.4697	0.4710
30	0.4530	0.4543	0.4556	0.4568	0.4581	0.4594	0.4606	0.4619	0.4632	0.4645	0.4657	0.4670	0.4683
31	0.4502	0.4515	0.4527	0.4540	0.4553	0.4566	0.4578	0.4591	0.4604	0.4616	0.4629	0.4641	0.4654
32	0.4474	0.4487	0.4499	0.4512	0.4524	0.4537	0.4550	0.4562	0.4575	0.4588	0.4600	0.4613	0.4626
33	0.4445	0.4457	0.4470	0.4482	0.4495	0.4508	0.4520	0.4533	0.4545	0.4558	0.4571	0.4583	0.4596
34	0.4415	0.4428	0.4441	0.4453	0.4466	0.4478	0.4491	0.4503	0.4516	0.4529	0.4541	0.4554	0.4566
35	0.4386	0.4399	0.4412	0.4424	0.4437	0.4449	0.4461	0.4474	0.4487	0.4499	0.4512	0.4524	0.4537

This table has been calculated from the preceding table of Weights of Carbon Dioxide, using the gravimetric factor for Carbon in Carbon Dioxide.

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The College of Engineering (Curricula: Architecture, Ceramics; Architectural, Ceramic, Civil, Electrical, Gas, General, Mechanical, Mining, Municipal and Sanitary, and Railway Engineering; Engineering Physics)

The College of Agriculture (Curricula: General Agriculture; Floriculture; Home Economics; Landscape Architecture; Smith-Hughes—in conjunction with the College of Education)

The College of Education (Curricula: Two year, prescribing junior standing for admission—General Education, Smith-Hughes Agriculture, Smith-Hughes Home Economics, Public School Music; Four year, admitting from the high school—Industrial Education, Athletic Coaching, Physical Education

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The School of Music (four-year curriculum)

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The College of Medicine (in Chicago)

The College of Dentistry (in Chicago)

The School of Pharmacy (in Chicago)

The Summer Session (eight weeks)

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