DIRECT NUMERICAL INVESTIGATION AND REDUCED-ORDER MODELING OF 3-D HONEYCOMB ACOUSTIC LINERS

BY

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DISertation

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Abstract

Single degree-of-freedom conventional acoustic liners are widely installed in jet engines to reduce internal engine noise. They work by converting acoustic energy into vorticity-bound fluctuations. Despite being widely used, effective design-stage models of acoustic liners placed in high sound amplitude conditions, possibly with a turbulent grazing flow, are not available due to the near-liner flow complexity and diagnostic challenges. The work presented in this thesis uses direct numerical simulations (DNS) of a compressible, viscous fluid to understand the inherent fluid mechanics and guide reduced-order-model development.

While there are numerous orifices and cavities in a general conventional acoustic liner sample, only the one orifice and one cavity is investigated in this work. Resolved simulations of the sound-induced flow through a circular orifice with a 0.99 mm diameter are examined. The detailed investigations are split into two steps: the first step neglects any grazing flow. The no-flow simulation data identify the role the orifice wall boundary layers play in determining the orifice discharge coefficient which is an important indicator of liner non-linearity. It is observed that when the liner behavior is not well described by linear models, the orifice boundary layers contain secondary vorticity generated from its separation from the corner on the high-pressure side of the orifice. Quantitative comparisons of the simulation-predicted impedance match available data for incident sound amplitude of 130 dB at frequencies from 1.5 kHz to 3.0 kHz. At amplitudes of 140 – 160 dB the simulation impedance are in agreement with analytical predictions when using simulation-measured quantities, including the discharge coefficient and root-mean-square velocity through the orifice, although no experimental data for this liner exist at these conditions. The simulation data are also used to develop two time-domain models for the acoustic impedance wherein the velocity profile through the orifice is modeled as the product of the fluid velocity and a presumed radial shape, $\xi V(r)$. The models perform well, predicting the in-orifice velocity and pressure, and the impedance, except at the most non-linear cases where it is seen that the assumed shape $V(r)$ can affect the back-plate pressure predictions. These results suggest that future time-domain models that take the velocity profile into account, by modeling the boundary layer thickness and assuming a velocity profile shape, may be successful in predicting the non-linear response of the liner.
The second step introduces a grazing flow where the detailed interaction of an incident acoustic field and a Mach 0.5 laminar and turbulent grazing flow with a cavity-backed circular orifice is studied. All results are for tonal excitation at 130 dB from 2.2 – 3.0 kHz, or at 3 kHz with 130 – 160 dB acoustic amplitude. The results suggest that the liner experiences a drag increase over the baseline geometry with acoustic excitation and that facesheet shear stress measurements, while dominant at low acoustic amplitudes, contribute less at higher acoustic amplitudes. The DNS data further show that the orifice discharge coefficient can be semi-empirically modeled effectively using an acoustic-hydrodynamic scaling. The results indicate that experimental in situ impedance measurements can be contaminated by microphone-orifice interaction. Finally, the time-domain model without grazing flow was extended to include grazing flow by properly modeling the discharge coefficient and the turbulent boundary layer effect. Reasonable agreement of the liner impedance prediction was found with the DNS data. Discrepancies of the prediction suggest the future improvement of the model development.
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Chapter 1

Introduction

1.1 Background

Reducing the noise radiated by aircraft gas turbine engines remains an active and challenging goal. Several noise sources are associated with commercial turbo-fan engines (Hubbarb, 1995), each with its own method of noise reduction. For the so-called core-associated noise, which includes tonal and broadband noise from the fan, compressor and turbine, as well as broadband combustion noise, acoustic liners are an efficient noise reduction strategy and are installed on all modern engines. However, international regulations periodically reduce the maximum allowable noise levels from commercial aircraft while fuel consumption minimization drives these engines towards larger bypass ratios with fixed overall fan diameters and hotter combustor exit temperatures which, together, reduce the area over which liners can be installed and increases the combustion noise contribution, respectively. There is thus increasing pressure to develop improved liner designs. Further, novel airplane concepts have proposed liners be installed on external aircraft surfaces to further reduce the noise of podded or buried engines (Posey et al., 2006).

Over the past decades, significant progress in the design and understanding of acoustic liners has been made, mostly through theoretical analysis and experiments. To design better acoustic liners, engineers have examined different liner geometry configurations, testing environments, and construction materials. Through numerous experiments, crucial parameters in liner designs have been recognized and applied to liner construction. Many useful empirical/semi-empirical models have also been proposed. In pursuit of an improved understanding of the liner’s working fluid mechanisms, experiments continue to uncover their fundamental behavior and their interaction with external flows. Nevertheless, as the complexity of the problem increases, especially when a turbulent grazing flow effect is taken into consideration, traditional intrusive experiments face unprecedented difficulties (e.g., due to the small scale of the liner orifice) to accurately visualize and quantify the flow near the liner. Consequently, numerical simulations are well placed to advance the understanding and modeling of acoustic liners for improved future designs.
1.2 Literature Review

Research on acoustic liners has spanned several decades and can be usefully categorized as below.

(I) Theoretical Approach Pure analytical modeling of the impedance, \( Z = R + iX \), which associates dissipative and inertial fluid behavior with the resistive \((R)\) and reactive \((X)\) components, respectively, has early origins for low sound amplitudes at single frequencies normally incident upon the orifice (Rayleigh, 1894; Sivian, 1935) by considering the ‘slug’ of fluid confined within the orifice to be acted upon by a periodic, externally imposed pressure force due to the incident sound on one side and the cavity-affected pressure on the other. Along the walls a simple viscous model provides damping while end corrections account for added mass and other effects. The theoretical design procedures represent ideal approaches for the analysis of duct acoustic propagation and radiation. However, these methods require knowledge of the source modal characteristics, which are difficult to estimate without careful experiments. When there is little information about the source characteristics, either because the particular turbomachinery is still in the early design stage or component test data are not available, theoretical approaches are often only rough approximations.

(II) Experimental Approach Most of the early research on acoustic liners was performed experimentally. Early investigations used monochromatic, low-amplitude sound waves, typically less than 120 dB, and linearized equations to develop a validated theory of the acoustic impedance. The experimental study of Ingard & Labate (1950) showed that the sound-induced movement of the fluid near the orifice exhibited four qualitatively different motions depending on the sound amplitude, frequency, orifice diameter, and orifice thickness. Due to the small size of the orifice, typically around one mm in diameter, experimental in situ measurement of the flow in and around it is difficult so most liner investigations focus on the impedance. The impedance \( Z \) can be thought of as the ‘effective’, averaged boundary condition the incident sound experiences rather than as a detailed quantity associated with a point measurement. The experimental measurement of \( Z \) can be obtained using several methods. In the absence of a grazing flow a normal incidence tube can use a two microphone technique, such as discussed by Zorumski & Tester (1976). When the sound is at grazing incidence, or when there is a grazing flow, alternative methods such as the two-microphone Dean’s method (Dean, 1974) or through solving an associated inverse problem (Jones et al., 2004; Watson et al., 2005; Howerton & Parrott, 2009; Watson & Jones, 2009). Engine scale tests often use an insertion loss method (Motsinger & Kraft, 1995). Experimental data of near-orifice velocities of a slit orifice in a grazing flow measured with laser-doppler velocimetry are available in Charwat & Walker (1983).
(III) Empirical/Semi-empirical Approach  The empirical approach is established on extensive experimental testing and is a reliable way to provide data for particular designs. However, extensive testing can be time consuming and expensive and the tests are not always representative of the real installation environment. Extrapolation of empirical data to novel designs is challenging. In the semi-empirical approach, the theoretical and empirical procedures are combined to provide a rational prediction based on experience when source modal characteristics are unknown, at the expense of rigor. The success of semi-empirical methods rests on the modeling assumptions made and on the available experimental data. Most liners currently under consideration were designed by a semi-empirical process. At higher amplitudes, or at frequencies near cavity resonance, the theoretical models fail to accurately predict $Z$ because of fluid nonlinearity. Several approximate prediction methods at these non-linear conditions, often using empirical data for guidance, have been proposed. The more widely used models invoke some form of a discharge coefficient to account for the changing momentum flux through the orifice due to boundary layer growth along the walls (Melling, 1973; Howe, 1979; Zorumski & Tester, 1976; Cummings & Eversman, 1983; Hersh et al., 2003). Alternative nonlinear models use discrete vortex dynamics (Jing & Sun, 2002) or perturbative theories (Zinn, 1970; Tang & Sirignano, 1973). When a grazing flow is present, as is typically the case for aircraft engine liners, the complexity of the fluid mechanics increases and less is known about the flow details and about how to effectively model it. Models proposed by Rice (1976) and by Zorumski & Tester (1976) leverage experimental data to estimate the parametric dependence of the impedance on the grazing flow Mach number, a summary of which is given in Motsinger & Kraft (1995).

(IV) Numerical Approach  High-fidelity simulation techniques, such as direct numerical simulation (DNS) and large-eddy simulation (LES), are becoming prospective candidates for acoustic liner eduction. A unique advantage of DNS and LES is the availability of non-intrusive space-time data. With the increase of available computational power, computational aeroacoustics (CAA) is emerging as a tool capable of predicting liner performance. The compressible Navier-Stokes equations are solved directly and the results are post-processed similar to the experimental data. The input parameters (e.g., liner geometries) can be easily changed, and the simulations may include realistic conditions. However, one disadvantage of numerical liner eduction method is the long wall-clock time required for each computation. Increasingly powerful computers reduce the time but careful selection must still be exercised to maximize available resources. Further, highly accurate simulation techniques with complex geometry capabilities are still an active area of research. (Reichert et al., 2012)

Two typical simulation challenges were summarized by Tam (2004): (1) The magnitude of the acoustic
incidence perturbations are usually orders of magnitude (i.e., 130 dB incident sound $p' \approx 0.001p_{\infty}$) smaller than the fluctuations from a turbulent flow (i.e., $p' \sim 0.01p_{\infty}$) and therefore a scheme with low numerical noise, dispersion and dissipation is required; (2) An acoustic liner has multiple scales (i.e., the diameter of the orifice $d$, the momentum thickness $\theta$, the wavelength of the incidence $\lambda$), some of which may be orders of magnitude larger than others (i.e., the ratio of 3.0 kHz planewaves and mm-scale orifices $\lambda/d \approx 100$). Therefore, the choice of high spatial resolution and a sufficiently large computational domain can be challenging.

Purely numerical approaches to modeling acoustic liners are useful tools for understanding the detailed fluid mechanics as shown in the early work by Tam & Kurbatskii (2000) and Tam et al. (2001) on the behavior of slit liners under normally incident sound over a range of sound amplitudes and frequencies. Corresponding experiments were conducted at the same conditions and the acoustic absorption coefficient was compared between the two studies, with good agreement. The two-dimensional numerical simulations identified two regions separated by whether discrete vortex shedding was observed and, following Ingard & Labate (1950), the vortex bound energy was quantified and related to the loss of acoustic energy. Zhang & Bodony (2011c) revisited the slit liner simulations of Tam and co-workers and showed that a numerical model of an acoustic liner, when coupled with Dean’s impedance education method (Dean, 1974), can accurately predict the impedance of a slit liner. Tam et al. (2008a) extended their analysis to three-dimensional rectangular orifice liners exposed to single frequency and broadband acoustic excitation. Their simulations included only a small number of orifices to keep the computational cost manageable and were able to predict the impedance over a range of incident sound conditions which matched well with corresponding experimental data. A cavity-backed circular orifice attached to a larger tube has been studied numerically by Roche et al. (2009) where the exchange of resistance from wall-dominated viscous effects to separation-induced vorticity production was examined along with the influence of several numerical modeling assumptions on the predicted dissipation.

Numerical studies of low speed grazing flows by Tam et al. (2008b) for a slit liner demonstrated the influence the boundary layer has on the ejected vorticity. At grazing flow Mach numbers up to 0.85 past a slit liner Zhang & Bodony (2011c) validated their results with Jing et al. (2001) and further showed that the Rice (1976) model demonstrated a Mach number scaling for the impedance that was consistent with the numerical data and developed simple, analytical functions with parametric Mach number dependence for several flow quantities associated with the orifice. Roche et al. (2010) performed the 3-D simulations for both normal and grazing-incidences at 140 dB and found the local admittance is intrinsic to the Helmholtz resonator. They also found higher resistance and lower reactance for the
resonator with an addition of the grazing flow. More recently, Tam et al. (2013) performed a simulation of a series of adjacent slit liners backed by resonators under a Mach 0.3 grazing flow. They observed the liner can produce high-frequency self-noise and can potentially increase aerodynamics drag.

Among most of the previous computational studies on liners, a very fundamental element was missing: previous simulations were always performed and validated in a quiescent condition or under a laminar boundary layer while the real conditions for these acoustic liners are turbulent boundary layers. Experimental work of the liner studies under turbulent boundary layer seems relatively easier to perform but a detailed flow visualization also requires high resolution techniques at sub-millimeter scales. Previous empirical models either without grazing flow or with laminar grazing flows are still in use but the extension to turbulent boundary layers can be difficult as the link between the basic fluid mechanics and the models remains unclear. High resolution experiments (i.e., using laser Doppler velocimetry or PIV) of the detailed flow inside or near the acoustic liner are always highly appreciated but current measuring techniques are a bottleneck.

Under these circumstances, numerical investigations provide a useful tool with which to study the dynamics of the sound-turbulence-orifice interaction with a backing-cavity. This work aims to achieve a fundamental understanding of the interactions of a turbulent boundary layer, orifice and sound from a first-principles perspective. The geometry of the model is realistic while a temporally-evolving turbulent boundary layer is included, rather than a spatially-evolving one. Extensive care is taken to resolve the smallest scales of the turbulent boundary layer and the flow structures inside or near the orifice as well. Cases are performed under a laminar boundary layer with the same Mach number for the purposes of comparison. Detailed discussions on the impedance prediction and sound-turbulence and orifice interaction are given.

1.3 Motivation and Objectives

1.3.1 Current research gaps

- There are surprisingly little data on the detailed flow in and around a circular orifice backed by a hexagonal cavity, such as the types commonly used in aircraft engines.

- While much is known about how to model the impedance of ‘conventional liners’ at low sound amplitudes and at single frequencies, extending these models to the time domain for broadband acoustic excitation and/or high amplitude incident sound has not been successful.
- The effect of a turbulent grazing flow has been semi-empirically modeled but the detailed working mechanisms of liners under these conditions remains unclear.

- There are very few works on time-domain models for liner impedance prediction, especially under the effect of grazing flow.

1.3.2 Structure of the thesis

With the goal of closing the current research gaps, the present work discusses numerical investigations of the sound-induced flow through a circular orifice which is backed by a hexagonal cavity, a geometry commonly used by industry. Care is taken to resolve the flow around the orifice and to demonstrate the simulation’s validity through comparison with experimentally-measured impedances for the same liner geometry at the same acoustic excitation conditions. With the aid of the DNS data, reduced-order time domain models are proposed. Investigations and modeling of the flow properties start off from the no-flow cases and then move to the next stage with grazing (turbulent) flow conditions. Following a description of the numerical methods in Chapter 2, a series of detailed simulation results including flow quantities and impedance predictions are discussed in Chapter 3. Based on the flow phenomena and measured simulation data, a time-domain model of the flow through the orifice is constructed in Chapter 4. In Chapter 5, a Mach 0.5 compressible turbulent boundary layer is added towards the simulation along with investigations of the flow properties and impedance predictions. Chapter 6 extends the time-domain model of impedance prediction under grazing flow conditions. Conclusions and future research plans are given in Chapter 7.
Chapter 2

Numerical Methods

2.1 Governing Equations

The compressible Navier-Stokes equations in a non-orthogonal coordinate system \( \xi = \Xi(x) \) is solved, with \( \xi = (\xi, \eta, \zeta)^T \), written in conservative form

\[
\frac{\partial}{\partial t} \left( \frac{\rho}{J} \right) + \frac{\partial}{\partial \xi_j} (\rho U) = 0 \quad (2.1)
\]

\[
\frac{\partial}{\partial t} \left( \frac{\rho u_i}{J} \right) + \frac{\partial}{\partial \xi_j} \left( \rho u_i U_j + \hat{\rho} \hat{\xi}_j \hat{\xi}_{i,k} - \tau_{kj} \hat{\xi}_{j,k} \right) = 0 \quad (2.2)
\]

\[
\frac{\partial}{\partial t} \left( \frac{\rho E}{J} \right) + \frac{\partial}{\partial \xi_j} \left( \left( \rho E + p \right) U_j - \hat{\xi}_{j,i} \{ u_k \tau_{ik} - q_i \} \right) = 0 \quad (2.3)
\]

where \( J = | \partial x / \partial \xi | \) is the Jacobian of the grid transformation, \( \hat{\xi}_{j,i} = J^{-1} \partial \xi_j / \partial x_i \) are the Jacobian-weighted metrics, and \( U_i = u_j \hat{\xi}_{i,j} \) are the contravariant velocities. A Newtonian fluid with Fourier’s law of heat conduction are assumed such that

\[
\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (2.4)
\]

\[
q_i = -k \frac{\partial T}{\partial x_i} \quad (2.5)
\]

with \( \lambda = \mu_B - 2\mu / 3 \) and \( \mu_B = 0.6\mu \) for air. The temperature dependence of \( \mu \) and \( k \) are given by power laws

\[
\frac{\mu}{\mu_0} = \frac{k}{k_0} = \left( \frac{T}{T_0} \right)^n \quad (2.6)
\]

with \( n = 0.666 \). When the equations are non-dimensionalized with respect to the ambient density \( \rho_\infty \), the ambient speed of sound \( a_\infty \), the orifice diameter \( d \), and the ambient viscosity \( \mu_\infty \) the equation of state for a calorically perfect ideal gas is

\[
p = \gamma - 1 \gamma \rho T \quad (2.7)
\]
with \( \gamma = \frac{C_p}{C_v} \), the ratio of specific heats, fixed at 1.4. This non-dimensionalization yields the Reynolds number \( \text{Re} = \rho_\infty a_\infty d/\mu_\infty \) and Prandtl number \( \text{Pr} = \mu C_p/k \). The Prandtl number is assumed to be 0.72 while the Reynolds number is given later.

2.2 Numerical Schemes

The equations are discretized using a node-centered summation by part (SBP) finite difference approach of the form

\[
\frac{\partial}{\partial \xi} \approx P^{-1}Q
\]

(2.8)

where \( \{P, Q\} \) are explicit SBP matrices (Strand, 1994) that are fourth order accurate in the interior and third order accurate globally. The divergence of the viscous fluxes is computed by repeated first derivatives. Simultaneous-approximation-term (SAT) boundary conditions are used on all computational non-periodic boundaries to enforce isothermal no-slip (Svärd & Nordström, 2008) and far-field (Svärd et al., 2007) conditions. These boundary conditions are numerically stable and have been shown to be accurate in aeroacoustic problems by Bodony (2010). A sponge region located adjacent to the far-field boundary is also used to help impose the incident sound waves and to further reduce unwanted reflections (Bodony, 2006) by using a specified target state, \( Q_{\text{target}} \). The SAT formulation follows the idea that a penalty term is added to the right-hand-side of the governing equations. The penalized equation with SAT and sponge terms is

\[
\frac{\partial Q}{\partial t} = R(Q) + \sigma_v P^{-1} E_1 (Q - g_v) + \frac{\sigma_i}{\text{Re}} P^{-1} E_1 I(Q - g_i) - A_s \kappa (Q - Q_{\text{target}})
\]

(2.9)

where \( \sigma_i \) and \( \sigma_v \) are the penalty parameters for the inviscid and viscous boundary conditions, respectively, and \( E_1 = (1, 0, \ldots, 0)^T \). \( R(Q) \) represents the divergence of the fluxes in the governing equations, \( A^+ \) is a Roe matrix and \( I \) is the identity matrix. Also, \( A_s \) is the amplitude of the sponge, \( n_s \), the spatial strength and \( \kappa \) is the scaled coordinate which ranges from 0 at the interior of the sponge to 1 at the exterior. Parameters \( \sigma_i \) and \( \sigma_v \) should satisfy the conditions \( \sigma_i \leq -2 \) and \( \sigma_v \leq -\frac{1}{\frac{\mu_0}{\rho_0} \frac{5\mu}{\rho}} \) respectively for stability.

2.3 Numerical Filtering

The overset grids utilize high-order Lagrange polynomial interpolation for passing flow data between them (Sherer & Scott, 2005). Determination of the overlap points and interpolation stencils uses the pre-processing tool BELLERO developed at Wright-Patterson AFB by Sherer et al. (2006). While accurate, the use of
Figure 2.1: Transfer function for the implicit filter. Legend: ‘—’ $\alpha_f = 0.49$, ‘−−’ $\alpha_f = 0.47$, ‘−·−’ $\alpha_f = 0.45$.

Lagrange polynomials can generate numerical instabilities which are not dissipated by the centered finite difference scheme and repeated first derivatives used to compute the viscous terms. In order to remove the undesired numerical instabilities, a tenth order implicit filter is selected Visbal & Gaitonde (2002). The filtered values $\hat{f}$ are obtained by solving the tridiagonal system

$$
2\alpha_f \bar{f}_{i-1} + 2\bar{f}_i + 2\alpha_f \bar{f}_{i+1} = \frac{(1-2\alpha_f)}{512}(f_{i-5} + f_{i+5}) + \frac{5(-1 + 2\alpha_f)}{256}(f_{i-4} + f_{i+4}) \\
+ \frac{45(1 - 2\alpha_f)}{512}(f_{i-3} + f_{i+3}) + \frac{15(-1 + 2\alpha_f)}{64}(f_{i-2} + f_{i+2}) \\
+ \frac{(105 + 302\alpha_f)}{256}(f_{i-1} + f_{i+1}) + \frac{2(193 + 126\alpha_f)}{256}f_i
$$

(2.10)

Note in Eq. (2.10) $\alpha_f$ is a free coefficient between $[-0.5, 0.5]$ for numerical stability. Figure 2.1 plots the transfer function for the implicit filter against the modified wavenumber. As the filter coefficient $\alpha_f$ increases, the cut-off wavenumber is getting close to $\pi$. The simulations performed in this work applies a tenth order filtering with the choice of $\alpha_f = 0.49$. 
2.4 Time Integration

The time integration uses the standard non-linear fourth order of Runge-Kutta (RK4) scheme running at a constant time step given by

\[
k_1 = h_t f(x_n, y_n) \\
k_2 = h_t f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\
k_3 = h_t f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\
k_4 = h_t f(x_n + h, y_n + k_3) \\
y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) + O(h_t^5),
\]

where \( h_t \) is the time step. The CFL number is kept below 0.5 for the stability of these simulations.

2.5 Simulation Environment

The core simulation uses an in-house code developed by Professor Daniel J. Bodony in Rocstar Center at University of Illinois at Urbana-Champaign. The code shows a good parallel scaling up to 20,000 cores using Message Passing Interface (MPI) for simulations in single grid. When applied to multiple over-lapping grids, up to 4096 cores are used to achieve an efficient performance.
Chapter 3

Numerical Investigation of a 3-D Honeycomb Liner with Circular Apertures

3.1 Introduction

In this chapter, investigation of a conventional 3-D honeycomb liner is conducted via the numerical methods mentioned in Chapter 2. Through the simulation of the compressible Navier-Stokes equations, details of acoustically-excited flow through a circle orifice are presented under different acoustic enforcing conditions. One of the most significant objectives is to understand the flow features near/inside the orifice of the liner via visualization. These features play a critical role in the reduced-order time-domain modeling, which is further discussed in Chapter 4. Comparisons of the numerically predicted impedance values with the experimentally measured and empirically/semi-empirically predicted ones are revealed as a validation of the simulation fidelity. Simulations also provide the impedance predictions beyond the capabilities of the experimental measurement.

3.2 Computational Model

3.2.1 Geometric model

The conventional liner geometry investigated was also studied by Jones et al. (2004) at the NASA Langley Research Center and is sketched in figure 3.1. A rigid facesheet of thickness $\tau$ is perforated with cylindrical holes of diameter $d$ and fixed to a honeycomb core comprised of regular, hexagonal cells. The distance between parallel sides of the cells is $\ell$ and the cells have depth $H$. A rigid backplate closes the cells from below and the cell walls are rigid so that neighboring cells cannot communicate. The specific liner properties used for this study correspond to the 6.4% porosity liner tested in Jones et al. (2004), with parameters given in table 3.1. Due to manufacturing the orifices are not regularly arranged relative to the cells and, on occasion, an orifice may lie directly above a cell wall so that it is split between cells. The cells do not permit any inter-cell communication so that the liner is locally reacting. We further focus on cases without grazing flow to establish the methodology; including a grazing flow, which is always present in aeroengine
applications, will be considered in Chapter 5.

Several simplifying geometrical assumptions were made for the computations. A 6.4% porosity liner with 0.99 mm diameter holes corresponds to, roughly, 6–7 orifices per cell on average, with an inter-orifice spacing several times larger than the orifice diameter. We assume that the orifices within a cell do not communicate and act independently, similar to that found by Tam et al. (2010) and suggested by Melling (1973), so that a single orifice is taken to sit in the center of a hexagonal cell, as shown in figure 3.2(a). The top surface of the facesheet is, as in the experiments, the end wall of a normal incidence tube. For the simulations this region is modeled by a rectangular cylinder with a base of size $L_1 \times L_2$ and a height of $H = 100$ mm. The cylinder base lengths are $L_1 = \sqrt{3} \ell / 2$ and $L_2 = \ell$ by the properties of a regular hexagon. With this choice the periodicity of the liner is altered from its original form to that shown in figure 3.2(b), with an overall

### Table 3.1: Numerical values for liner geometry.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orifice diameter</td>
<td>$d$</td>
<td>0.99</td>
</tr>
<tr>
<td>Facesheet thickness</td>
<td>$\tau$</td>
<td>0.64</td>
</tr>
<tr>
<td>Honeycomb ‘diameter’</td>
<td>$\ell$</td>
<td>5.49</td>
</tr>
<tr>
<td>Cell depth</td>
<td>$D$</td>
<td>38.10</td>
</tr>
</tbody>
</table>
Figure 3.2: Schematic of a unit cell (a) and its assumed periodicity (b).

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>Grid Size</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cartesian</td>
<td>$141 \times 141 \times 181$</td>
<td>Above the facesheet</td>
</tr>
<tr>
<td>2</td>
<td>Orthogonal</td>
<td>$121 \times 61 \times 247$</td>
<td>Orifice and 7$d$ above and below it</td>
</tr>
<tr>
<td>3</td>
<td>Non-orthogonal</td>
<td>$121 \times 121 \times 201$</td>
<td>Cavity</td>
</tr>
<tr>
<td>4</td>
<td>Cartesian</td>
<td>$41 \times 41 \times 223$</td>
<td>Cut away orifice centerline</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>8,739,272 points</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Grid information used in the numerical simulation.

porosity of 0.99%. Reynolds number based on $\rho a_\infty d/\mu_\infty = 23,110.25$.

3.2.2 Grid design

To discretize the geometry of figure 3.2(a) an overset grid approach is used wherein multiple, overlapping grids co-exist and information is passed between them via interpolation. Four grids were used: the first three account for the geometry, namely the rectangular region above the facesheet, the hexagonal cell, and the circular orifice, while an additional rectangular grid was used to remove the polar axis of the cylindrical grid. Each grid was logically rectangular. The minimum grid spacing in the orifice is $\Delta r_{\min} = 0.0081d$ at the orifice walls. In the $z$-direction, a uniform grid spacing $\Delta z_{\min} = 0.02d$ is used 1$d$ offset above and below the orifice. At 7$d$ away from the orifice, the resolution decreases smoothly but still keeps $\Delta z_{\min} \leq 0.1d$. The ratio of the grid spacing of the coarse mesh and the fine mesh at their interface is around 2. (See Table 3.2)
### Table 3.3: Incident sound parameters investigated.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>×</td>
</tr>
<tr>
<td>2.0</td>
<td>×</td>
</tr>
<tr>
<td>2.5</td>
<td>×</td>
</tr>
<tr>
<td>3.0</td>
<td>×</td>
</tr>
</tbody>
</table>

3.2.3 Incident acoustic waves

Acoustic waves are incident normally upon the orifice and are generated within the simulation through the far-field boundary condition and are reinforced by the sponge zone, as mentioned earlier. In addition to damping outward traveling acoustic waves, the sponge zone helps impose planar acoustic waves of the form

\[ q'(x, t) = A \left[ 1, \frac{k}{\rho_\infty a_\infty^3 |k|}, 1/a_\infty^2 \right]^T \exp\{i(k \cdot x - \omega t)\}, \]

with assumed real part, where \( q' = [\rho', u', p']^T \), and \( A = 10^{\text{SPL}/20 - 9.701} \) for air at standard temperature and pressure. The wavenumber \( k \) and frequency \( \omega \) are related by \( |k| = \omega/a_\infty \) and the direction of \( k \) is normal to the facesheet. These waves are imposed only at top of the domain and propagate to the orifice by the fully non-linear equations of motion.

3.3 Flow Properties near the Orifice

The simulations start with the fluid initially at rest. The incident plane wave of specified amplitude and frequency impinges upon the facesheet, excites the fluid near the orifice, and is mostly reflected back. After 10 cycles the flow reaches a statistically stationary state, as determined by several pressure and velocity probes placed within the computational domain, and data are then collected for post-processing. (See, for example, figures 3.5 and 3.6 below.) In all, seven cases were run with the parameters given in table 3.3; the frequency sweep at fixed 130 dB amplitude corresponds to conditions at which experimental data are available (Jones et al., 2004) while amplitude sweep at and above 140 dB does not. Grid convergence of the data is discussed in Appendix B.

3.3.1 Instantaneous data

Instantaneous frames of vorticity isosurfaces are shown in figure 3.3 for four simulations at a fixed frequency of 3 kHz and incident sound amplitudes from 130 dB to 160 dB. The corresponding figures at 130 dB fixed
amplitude but variable frequency are shown in figure 3.4.

At constant frequency, one effect of increasing the incident sound amplitude is to increase the distance vorticity penetrates into the surrounding fluid, both within and external to the cavity. At 130 dB the vorticity is confined to the orifice and does not separate, which is indicative of linear behavior as discussed by Sivian (1935). At 140 dB the vorticity is able to separate from the orifice walls and forms discrete vortex rings that appear stable and travel away from the orifice. At this amplitude, and above, a linear behavior cannot be expected. At 150 dB the discrete vortex loops continue to be formed but are increased in amplitude and begin to experience self-sustaining disturbances which distort the loops but do not cause them to undergo transition or to ‘burst.’ At 160 dB the vorticity within the loops intensifies and undergoes a stronger instability which causes transition, leading to apparently turbulent vortex loops. (The degree to which the loops are turbulent is shown below in figure 3.15.) At all amplitudes very little vorticity exists except that which is created at the orifice.

At a lower amplitude of 130 dB, increasing the frequency from 1.5 kHz to 3.0 kHz causes the orifice-generated vorticity to remain increasingly confined to the orifice. At the lowest frequency, which is near the liner’s Helmholtz resonance frequency of 1.1 kHz, discrete vortex loops are shed from the orifice and travel away from it. At 2 kHz their strength is substantially reduced until, at 2.5 kHz and above, all vorticity is confined to the orifice.

Time traces of pressure fluctuations $p' = p - p_\infty$ are shown in figures 3.5 and 3.6, at constant frequency and amplitude, respectively, for probes located on the facesheet surface away from the orifice, in the center of the orifice, and on the backplate of the cavity. From the figures roughly 10 cycles are required before a statistically steady state is reached. At fixed frequency (figure 3.5) the amplitude of all pressure traces increases with increased incident sound amplitude, reaching roughly 1% and 4% of $p_\infty$ at 150 dB and 160 dB, respectively. At 130 dB the traces are sinusoidal, indicative of linear behavior. Any deviation from a purely monochromatic response is difficult to discern at 140 dB, is barely evident at 150 dB, and clearly evident at 160 dB. In all cases the surface facesheet pressure fluctuations are larger than the orifice center fluctuations which are, in turn, larger than the backplate pressure fluctuations. Their ratio is evidently a function of incident sound amplitude, as is their relative phase. At fixed amplitude the pressure traces in figure 3.6 show significant non-linearity for the 1.5 kHz case, which is close to the liner resonant frequency, but are predominantly single-frequency for all other incident sound frequencies. The relative amplitudes of the three traces continues to be a function of frequency but their relative phase is less variable. The vertical velocities shown in figures 3.7 and 3.8, taken along the centerline of the orifice mid-way from the facesheet top and bottom, exhibit trends that are qualitatively similar to the pressure traces. Observe that the peak
Figure 3.3: Vorticity fields due to different intensity sound at fixed 3 kHz frequency. Figures (a)–(d) are isosurfaces of $|\omega d/a_\infty|$ at values 0.075, 0.15, 0.25, 1.0, respectively, colored by $u_r/a_\infty$ with ranges shown. Figures (e)–(h) are two-dimensional contours of $|\omega d/a_\infty|$ with ranges shown.
Figure 3.4: Vorticity fields due to different frequency sound at fixed 130 dB amplitude. Figures (a)–(d) are isosurfaces of $|\omega d/a_\infty|$ at constant value of 0.075, colored by $u_r/a_\infty$ with ranges shown. Figures (e)–(h) are two-dimensional contours of $|\omega d/a_\infty|$ with ranges shown.
velocities approach 30% of the ambient sound speed at 160 dB incident sound amplitude with significant harmonic content. By 150 dB the maximum velocity is 15% of $a_\infty$ and reduces to 1% by 130 dB, 3 kHz, and rises to 4% of $a_\infty$ at 130 dB, 1.5 kHz as the Helmholtz resonance frequency is approached.

### 3.3.2 Phase averaged data

The flow variability between acoustic periods is low so phase averaged data are now presented, denoted by angled brackets $\langle \cdot \rangle$. The velocity field within the orifice, taken at the midplane of the facesheet, at phases $\phi = 0$, $\pi/2$, $\pi$, and $3\pi/2$ are shown in figures 3.9 and 3.10. Among all profiles there are broad similarities, as can be seen, but several differences in detail exist between the profiles. At constant frequency (figure 3.9) the effect of increasing the sound amplitude is to increase the thickness of the transition layer separating the boundary-influenced portion of the profile from the core flow. The near-wall fluid lags the centerline velocity in a manner qualitatively similar to Stokes’ second problem (White, 1991). Further, near the walls there is a region of reversed flow at the peak velocity phases, $\phi = 0$ and $\pi$, roughly 0.05$d$ units thick for amplitudes greater than an equal to 140 dB whose radial size is not a strong function of sound amplitude. At fixed 130
Figure 3.6: Pressure fluctuations due to different frequency sound at fixed 130 dB amplitude. Legend same as figure 3.5.
Figure 3.7: Vertical velocity fluctuations at the center of the circular aperture due to different amplitude sound at fixed 3 kHz frequency.
Figure 3.8: Vertical velocity fluctuations at the center of the circular aperture due to different frequency sound at fixed 130 dB amplitude.
dB amplitude the phase profiles of figure 3.10 are qualitatively similar, but do not exhibit any reversed flow at the peak velocity phases. The transition layer thickness decreases with increasing frequency, as does the peak velocity.

We now look more closely at the orifice boundary layer through its displacement thickness $\delta^*$, defined in the usual manner by

$$δ^*(ϕ) = \int_0^{d/2} \left( 1 - \frac{(ρw)}{(ρw)_{CL}} \right) \, dr \tag{3.2}$$

where the phase-averaged momentum has been used. The results are shown in figure 3.11, for $|δ^*|$ since the flow changes direction, where several trends are visible. At fixed amplitude (figure 3.11(a)) the boundary
Figure 3.10: Phase averaged velocity profiles at the center of the circular aperture due to different frequency sound at fixed 130 dB amplitude.
layer thickness is a function of frequency and phase where at $\phi = 0, \pi$ the change in the flow direction causes the definition of the boundary layer to no longer be useful since $\langle \rho w \rangle_{CL} \rightarrow 0$. As the flow accelerates away from these two phases the boundary layer thins and then thickens until the next flow reversal causes $\delta^*$ to be ill defined. Whether the flow is into or out of the cavity makes little difference on $\delta^*$. It is generally true that the higher the frequency the thinner the boundary layer but the data show there is some phase angle dependence, with the boundary layer growing more quickly the lower the frequency but subsequently changing its growth rate depending on the frequency. Observe that $\delta^* \approx 0$ for some phase angles, indicating reversed flow. At fixed frequency (figure 3.11(b)) it is seen that the boundary layer grows more quickly at higher amplitudes and looses its monotonicity in the 160 dB case. The intra-cycle variation is minimal.

Because of the flow reversal it is challenging to assign a single value of the boundary layer thickness for a given amplitude-frequency pair. However, if we examine the flow through the orifice more closely it is possible to define phases in which a substantial flow exists through the orifice, as is done in figure 3.12. In the figure the displacement thickness for the 160 dB, 3 kHz case is shown as a function of time and plotted along with the orifice center vertical velocity $w$, as indicated. If we let $w_{\text{peak}}$ be the peak velocity achieved by the in-orifice flow and $w_{\text{peak}} - w_{\text{rms}}$ to be the root-mean-square velocity fluctuation level then we can define the periods of time when a jet exists as follows. The data shows that there is a lag between the boundary layer growth and the centerline velocity change so that we quantify the jet as being present when $w$ first exceeds $w_{\text{peak}}$ and ending when $w$ falls below $w_{\text{rms}}$. The times when the jet is present are shown in figure.
Figure 3.12: Correlation of orifice jet with its boundary layer thickness.

Table 3.4: Flow times associated with ‘upward’, ‘downward’, and ‘transition’ motion of orifice jet. All times are percentages of the acoustic period.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_U$</th>
<th>$T_D$</th>
<th>$T_{T,U\rightarrow D}$</th>
<th>$T_{T,D\rightarrow U}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 dB, 1.5 kHz</td>
<td>32.5</td>
<td>32.4</td>
<td>17.5</td>
<td>17.6</td>
</tr>
<tr>
<td>130 dB, 2.0 kHz</td>
<td>32.5</td>
<td>32.8</td>
<td>17.4</td>
<td>17.3</td>
</tr>
<tr>
<td>130 dB, 2.5 kHz</td>
<td>32.1</td>
<td>32.2</td>
<td>17.8</td>
<td>17.9</td>
</tr>
<tr>
<td>130 dB, 3.0 kHz</td>
<td>32.3</td>
<td>32.1</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>130 dB, 3.0 kHz</td>
<td>32.3</td>
<td>32.1</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>140 dB, 3.0 kHz</td>
<td>32.2</td>
<td>33.3</td>
<td>17.5</td>
<td>17.0</td>
</tr>
<tr>
<td>150 dB, 3.0 kHz</td>
<td>30.4</td>
<td>30.3</td>
<td>19.8</td>
<td>19.5</td>
</tr>
<tr>
<td>160 dB, 3.0 kHz</td>
<td>34.8</td>
<td>34.1</td>
<td>16.0</td>
<td>15.1</td>
</tr>
</tbody>
</table>

3.12 and identified by the ‘U’ and ‘D’ labels for ‘up’ and ‘down’, respectively. The time intervals when a proper jet is not in the orifice is shown by the dashed line segment of the $w$ trace and labeled by a ‘T’. The distribution of times the orifice spends in the states is given in table 3.4 where there is little difference seen between flow into and out-of the orifice, except at the highest amplitude, and between the ‘up-to-down’ and ‘down-to-up’ transitions.

By conditionally averaging when the jet is present an average displacement thickness can be determined, the values of which are given in table 3.5 for the ‘up’, ‘down’, and overall stages. By this metric the boundary layer thins as the incident sound frequency increases at 130 dB fixed amplitude while it grows rapidly between 130 dB and 140 dB at 3 kHz fixed frequency, followed by a slower growth with increasing amplitude to 160 dB.

The orifice wall boundary layer separates the jet into an inviscid core with a viscous outer annulus. An engineering measure of the boundary layer influence is the discharge coefficient, $C_D$, which can be usefully
approximated by

\[ C_D \approx \frac{\pi(d/2 - \delta^*)}{\pi d^2/4} = \left(1 - \frac{\delta^*}{d/2}\right)^2. \]  
(3.3)

While useful for steady or nearly steady flows, the discharge coefficient definition, along with that of the boundary layer, fails when the flow is reversing direction as a well defined jet does not exist. From the averaged displacement thickness data one can construct a similarly averaged discharge coefficient the values of which are shown in table 3.5 for the present simulations. As the liner operates close to resonance or at high acoustic amplitudes the discharge coefficient reduces to roughly 0.70 while at less vigorous acoustic loadings it reaches 0.86.

The origin of the reversed flow in the phase averaged profiles can be more easily seen in the contour and streamline plots shown in figures 3.13 and 3.14 for the constant frequency and constant amplitude cases, respectively, for phase \( \phi = 0 \). At 130 dB, 3 kHz (figure 3.13(a)) the flow enters and exists the orifice cleanly, with a small recirculation zone near the bottom quarter of the orifice. As the amplitude increases the size of the recirculation zone increases until it reaches the facesheet by 140 dB, and then thickens at 150 dB. By 160 dB, in figure 3.13(d), several circular flow patterns appear in the boundary layer, indicating the repeated appearance of flow separation and generation of secondary vorticity along the orifice walls. When the amplitude is held constant but the frequency changed from 1.5 kHz to 3.0 kHz, as shown in figure 3.14, the near wall reversed flow diminishes considerably as the boundary layer thins and the flow weakly separates from the orifice lip. The phase-averaged data for \( \phi = 3\pi/2 \) are very similar to those at \( \phi = \pi/2 \) and are thus not shown.

At 160 dB the sound-induced jet appears highly unsteady and evidently turbulent. The degree to which this is true is shown in figure 3.15 for the azimuthal spectrum of \( u'_\theta \) computed at location \( r/d = 0.2, z/d = 2 \), as a function of the phase angle \( \phi \). Referring to figure 3.9(d), we observe that the most energetic portion of the jet lags the peak velocity found at the orifice. At \( \phi = 0 \) the flow is changing directions to moving out of the cavity, reaching its peak velocity at \( \phi = \pi/2 \), and starts to again change directions at \( \phi = \pi \). The
Figure 3.13: Phase-averaged velocity and streamlines at $\phi = \pi/2$ due to different amplitude sound at fixed 3 kHz frequency.
Figure 3.14: Phase-averaged velocity and streamlines at $\phi = \pi/2$ due to different frequency sound at fixed 130 dB amplitude.
spectra, however, show that the peak energy is attained closer to $\phi = \pi$, having risen continuously from $\phi = 0$, as the jet propagates into the region above the orifice. During the time the jet forms and decays the energy spectra show a faster increase in the higher mode numbers than for the $n = 1$ mode. Beyond the jet’s peak levels of turbulence, when the flow has already changed direction, the spectrum decays approximately uniformly with $n$, with a slightly higher rate-of-decay for the higher modes.

### 3.3.3 Scattered pressure field

When vorticity is created at the orifice it has an opportunity to generate sound through interaction with the orifice or with neighboring vorticity. The theory of sound generated by vorticity is well described (Howe, 2003) by the acoustic wave equation driven by the divergence of the density-weighted Lamb vector $\rho \omega \times u$, but suffers the same conceptional complications associated with Lighthill’s acoustic analogy (Lighthill, 1952). Experimentally measuring liner self-noise is challenging though a few liners are known to ‘whistle’ when subjected to grazing flow (Petrie & Huntley, 1980). From the simulation databases it is relatively straightforward to compute the self noise through determining the scattered pressure field, $p'_s$, via the simple
decomposition

\[ p'_s = p' - p'_i - p'_r \]  \hspace{1cm} (3.4)

where \( p'_r \) is the reflected acoustic field in the absence of the orifice and \( p'_i \) is the incident pressure field. To determine the scattered field numerically, separate calculations were performed without the orifice using only the domain above the facesheet (grid 1 in table 3.2) but at the same conditions. The pressure from these auxiliary calculations, say \( p_{aux} \), were saved and used to compute the scattered pressure via \( p'_s = p - p_{aux} \). An example scattered field for the 160 dB, 3 kHz data is shown in figure 3.16 where the hydrodynamic influence is clearly visible near the orifice.

Near the orifice the scattered pressure contains significant hydrodynamic components and extracting the radiating component is not possible. Instead, the scattered pressure is collected on a planar surface located \( z = 20 \) diameters above the orifice and, using a Kirchhoff surface, is projected to larger values of \( z \). The
domain size above the facesheet, of dimensions \( L_1 \times L_2 \), is sufficiently narrow that only the planar mode is propagating with all others evanescent. Computing the OASPL of the scattered field results in the data shown in figures 3.17 and 3.18. At constant frequency the scattered pressure field stays 31 dB below the incident sound at 130 dB and 25 dB below at 160 dB. At constant amplitude (figure 3.18) the scattered pressure is seen to be a strong function of frequency with its loudest contribution coming closer to the natural resonance mode of the liner. For all conditions the scattered pressure is more than 20 dB below the incident field. (It was confirmed that the results are independent of the Kirchhoff surface location and the value of \( z \) at which the OASPLs are computed.)

When making in situ pressure measurements on the liner facesheet surface the microphone location must be placed outside the hydrodynamic influence of the orifice, otherwise the pressure measurements do not yield reliable impedance data since Dean’s (1974) method requires the pressures to be dominated by acoustic fluctuations. From the simulation data the root-mean-square of the scattered pressure field gives a reasonably good estimate of the hydrodynamic influence of the orifice, with data shown in figure 3.19 for a ray which passes through the orifice center on the surface of the facesheet. In all seven cases the scattered pressure field peaks just inside the orifice and decays rapidly outward. Beyond \( r/d = 2 \) the orifice influence is lost and the point can be considered to be in the acoustic field. Note that the scattered pressure root-mean-square does not decay to zero away from the orifice, indicating the presence of (very weak) acoustic
waves which travel outward, tangential to the facesheet from the orifice.

3.4 Comparison against Experiment

There exists limited experimental data against which to validate the current simulations, except for measurements of the liner’s acoustic impedance, \( Z \), which is defined as the ratio of the complex-valued Fourier coefficients of the acoustic pressure and velocity,

\[
Z = \frac{\hat{p}^a}{\hat{v}^a}
\]

(3.5)

where the frequency is assumed to be the same as that of the incident sound field. The acoustic impedance is an apparent property—not a point measurement—and is best considered as an effective boundary condition the incident acoustic field experiences when it impinges upon the surface. As discussed in Chapter 1 several methods exist for deducing the impedance and that due to Dean (1974) is chosen here because of its relative simplicity and its acceptance in normal incidence scenarios. The method assumes one dimensional, single frequency propagation inside the cavity and ignores the influence of the hydrodynamic field present near the orifice.
Figure 3.19: Root-mean-square scattered pressure on the facesheet surface.
For Dean’s method pressure signals at two locations are needed: one on the facesheet, which is not influenced by the hydrodynamic pressure field, and the other on the cavity backplate. Calling these points A and B, respectively, the characteristic acoustic impedance is given by

\[
\frac{Z}{\rho_\infty a_\infty} = -i \frac{\hat{p}_A}{|\hat{p}_B|} \frac{e^{i\phi}}{\sin kH}
\]

(3.6)

where \(\hat{p}\) is the Fourier coefficient of \(p - p_\infty\), \(\phi\) is the phase angle between \(\hat{p}_A\) and \(\hat{p}_B\), and \(k = |k|\) is the modulus of the wavenumber vector.

The facesheet surface pressures were measured 5 radii away from the orifice center, outside the hydrodynamic influence, while the backplate pressures were measured directly below the orifice center. It was verified that the backplate pressure was uniform across the cavity. The resulting porosity normalized impedance values are shown in figure 3.20 for the 130 dB conditions for which experimental data from Jones et al. (2004) are also available. The impedance has been split into its real and imaginary parts according to \(Z/(\rho_\infty a_\infty) = \theta + i\chi\) and multiplied by the porosity \(\sigma\). There are theoretical arguments for \(\sigma Z\) being porosity independent. Melling (1973), for example, estimates that the characteristic resistance should be

\[
\frac{R}{\rho_\infty a_\infty} = 1 - \frac{1.2}{C_D} \left( \frac{1 - \sigma^2}{\sigma} \right) \frac{\rho_\infty}{2} w_{\text{rms}}
\]

(3.7)

where \(R\) is the resistance due to wall friction, \(w_{\text{rms}}\) is the in-orifice velocity root-mean-square, and \(C_D = 0.76\) as discussed by Motsinger & Kraft (1995). For 130 dB and greater the wall friction is negligible, as shown by Zhang & Bodony (2011b), leaving a term proportional to \(\sigma^{-1}\) for \(\sigma \ll 1\). The reactance, on the other hand, is porosity dependent and an approximate expression for it due to several investigators (Ingard, 1953; Melling, 1973) and summarized by Motsinger & Kraft (1995) is

\[
\frac{X}{\rho_\infty a_\infty} = -\coth kH + \frac{k(\tau + \epsilon d)}{\sigma}
\]

(3.8)

where \(\tau\) is, again, the facesheet thickness. The parameter \(\epsilon\) is a dimensionless end correction and is approximately equal to 0.85\((1 - 0.7\sqrt{\sigma})\) and \(k = \omega/a_\infty\) is the wavenumber.

At 130 dB there is quantitative consistency between the simulation and measurements for frequencies up to 2.5 kHz, as shown in figure 3.20, but a 50% difference is found in the resistance for the 3 kHz case. Differences in the reactance predictions are typically smaller than those for the resistance predictions, and are similar in magnitude to those found by Tam et al. (2008a) for a rectangular-orifice liner. Although the underlying reason is not known, it has been found experimentally for this liner that increased uncertainty for
measuring $Z$ exists at 3 kHz (Jones et al., 2004) but not enough to account for the differences observed. The analytical model by Melling (1973), Eq. (3.7), using the simulation data, over-predicts the simulation data by up to 50% near resonance while the semi-empirical model for the reactance, Eq. (3.8), does a remarkably good job for the data shown in figure 3.20(b).

Impedance predictions for higher amplitudes at 3 kHz are shown in figure 3.21, at conditions for which no experimental data for this liner are available. It is commonly assumed (Motsinger & Kraft, 1995) that the resistance is proportional to the acoustic velocity at high acoustic amplitudes, as given by Eq. (3.7). Experimental data also suggest that the reactance should diminish with increasing sound pressure level due to a reduction in the orifice end correction with increasing acoustic velocity. An empirical curve fit suggests that the reactance should reduce until 157 dB, approximately, after which it remains constant (Ingard, 1953; Motsinger & Kraft, 1995). The present data are qualitatively consistent with these trends. Further, the analytical expressions in Eq. (3.7) and (3.8) are in general agreement with the simulation-predicted impedances, when using the simulation-measured velocity root-mean-square. It should be noted that the reactance expression, Eq. (3.8), is independent of amplitude while the simulation data are not.
Figure 3.21: Normalized impedance predictions under different SPL at a fixed 3 kHz frequency. Resistance curve from Eq. (3.7) while reactance curve is from Eq. (3.8).
Chapter 4

Time-domain model of the 3-D Honeycomb Liner with Circular Apertures

4.1 Introduction

With the flow in and around the orifice documented and compared to existing data and analytical predictions in Chapter 3, one objective this chapter is to utilize the simulation databases to inform possible ways in which a time-domain model of the liner can be constructed. By following the work of Cummings & Eversman (1983) and Hersh et al. (2003) we modify, and simplify, their models based on the simulation data. Of particular interest from the simulations is the behavior of the orifice wall boundary layers which play a key role in controlling the effective non-linearity of the orifice and, hence, the impedance.

4.2 Governing Equation

As discussed in Hersh et al. (2003) the starting point for the time-domain model is the momentum equation normal to the orifice face applied to the control volume $V$ shown in figure 4.1. Assuming downward (in the figure) the displacement of the control volume is $\xi(t)$ and the density within $V$ is constant, the momentum equation can be approximated by

$$\rho \tau S \frac{d^2\xi}{dt^2} + \rho \frac{1 - C_D}{C_D} S \left| \frac{d\xi}{dt} \right| \frac{d\xi}{dt} + P_c S = P_0 Se^{i\omega t}$$

(4.1)

where $C_D$ is the (instantaneous) discharge coefficient of the orifice jet, $S = \pi d^2/4$ is the cross-sectional area, and $P_c$ and $P_0$ are the average surface pressures the cavity and facesheet sides, respectively. Note that the neither the wall friction nor the radiation resistance have been included since, at amplitudes greater than 130 dB, they are unimportant. (The wall friction data were presented in Zhang & Bodony (2011a).)
4.2.1 Cavity pressure evaluation

The incident sound is assumed to be monochromatic with frequency $\omega$ for purposes of comparison with the current data. The in-cavity pressure, however, may not be because of non-linearity in Eq. (4.1) and is instead found by treating the cavity pressure fluctuations as a linear, inviscid medium subjected to the boundary condition

$$\frac{\partial p}{\partial z} = -\rho \frac{d^2 \xi}{dt^2} V(r) H(d/2 - r)$$  \hspace{1cm} (4.2)

where $H(x)$ is the Heaviside function and $V(r)$ is a shape function to be discussed below. After further assuming the cavity is cylindrical with diameter equal to hydraulic diameter of the hexagonal cavity and that no azimuthal variation exists, the pressure in the cavity may be found to be

$$p'(r, z, t) = \sum_{m=0}^{\infty} D_m(t) J_0 \left( \frac{j_1 m}{L} r \right) \cosh[k_z(H + z)]$$  \hspace{1cm} (4.3)

where

$$D_m(t) = \frac{\int_{r=0}^{a} \beta(r, t) J_0 \left( \frac{j_1 m}{L} r \right) r \, dr}{k_z \sinh(k_z H) \int_{r=0}^{L} J_0^2 \left( \frac{j_1 m}{L} r \right) r \, dr}.$$  \hspace{1cm} (4.4)

The term

$$\beta(r, t) = -\rho \ddot{\xi} V(r) H(d/2 - r)$$  \hspace{1cm} (4.5)
is the inertial force applied to the control surface facing the cavity but weighted by $V$ to account for the influence of the boundary thickness. Evaluating $(\pi d^2/4)^{-1} \int_{r=0}^{d/2} p'(r, z, t) \, dr$ at $z = H$ yields $P_c(t)$. When the cavity height is sufficiently small, $H \omega \ll 2\pi c_{\infty}$, this expression becomes equivalent to the more traditional assumption that $P_c = -\rho \xi S$, that is, the pressure in the cavity rises when the fluid within the orifice displaces into the cavity a distance $\xi$.

4.2.2 Discharge coefficient

When $C_D$ is close to one the non-linear term proportional to $|\dot{\xi}|$ is negligible and a linear equation of motion of $\xi(t)$ is found, and has been well described by Sivian (1935). When the liner acts non-linearly, which is typically the case in application, the complexity of the flow within the orifice has resisted simple modeling and several semi-empirical approaches are available (Hersh et al., 2003; Cummings & Eversman, 1983). The modeling approaches differ in how they account for the dependency of $C_D$ on the flow conditions. Before addressing how to model $C_D$ itself we consider first whether Eq. (4.1) is a valid equation on which to base a liner model by computing the $C_D(t)$ data directly from the simulation data using Eq. (3.3) and comparing $\dot{\xi}$ with the fluid velocity taken at the orifice center. Figure 4.2 shows the comparison for the three most non-linear cases, as judged by the values of $C_D$ from table 3.5, along with the predictions when using a linear assumption with $C_D \equiv 1$. It is clearly seen that a linear assumption is not appropriate and significantly over-predicts the fluid velocity. Generally good predictions are available when accurate discharge coefficients are used as, most notably, the peak velocity predicted reduces, relative to a linear prediction. Characteristics of the non-sinusoidal velocity fluctuations seen in the simulation data are reflected in the model predictions as seen in figure 4.2(a) and (c). At 150 dB, in figure 4.2(b), the comparison is less favorable as the multiple frequencies seen in the simulation are not present in the $\dot{\xi}$ prediction; however, the dominant frequency component is well captured.

In general one does not have $C_D(t)$ data available and must instead model it. Typically a constant value of $C_D$ around 0.76 is assumed or taken from experimental data of steady flow through the orifice, as summarized in Motsinger & Kraft (1995). Frequency and amplitude-dependent values have been deduced from experimental data by Hersh et al. (2003). Since our expression for the in-cavity pressure, Eq. (4.3), depends on the shape of the velocity profile through $V(r)$, we propose to model not $C_D(t)$ itself but the velocity profile directly and in the form of $\dot{\xi}V(r)$ such that $C_D(t)$ can be deduced. It is unlikely that any universal function $V(r)$ exists which can account for all dynamics seen in the near-orifice flow, especially for those cases where secondary vorticity is generated within the orifice, as was seen in figure 3.3b.

The phased averaged velocity fields shown in figures 3.7 and 3.8, taken at the point of maximum velocity,
Figure 4.2: Comparison of predicted $\dot{\xi}$ with fluid velocity at the orifice center. Legend: $\cdots$, simulation data; $\cdots$, Eq. (4.1) with $C_D(t)$; $\cdot \cdot \cdot$, Eq. (4.1) with $C_D \equiv 1$. 

(a) 130 dB, 1.5 kHz

(b) 150 dB, 3.0 kHz

(c) 160 dB, 3.0 kHz
suggest that the velocity profile is a relatively smooth function of radius which, depending on the flow condition, peaks either along the orifice centerline or closer to the orifice wall. For the present simulations the boundary layer thickness is also several times smaller than the facesheet thickness.

4.2.3 Models for the velocity profiles

Two simple models for the velocity profile shape $V(r)$ were chosen. Model A uses the analytical solution for oscillating flow within a cylindrical pipe of radius $a = d/2$ under a constant frequency pressure gradient, available in White (1991), for example, for which the shape of the velocity fluctuations are given by

$$V(r) = \sqrt{1 - \frac{2}{\sqrt{r^*}} e^{-B} \cos B + \frac{e^{-2B}}{r^*}}$$

(4.6)

where $r^* = r/a$, $B = (1 - r^*)\sqrt{\omega^*/2}$, and $\omega^* = \omega a^2/\nu$. Model B, on the other hand, utilizes a plausible velocity profile with the boundary thickness $\delta^*$ as a free parameter and is given by

$$V(r) = \frac{1}{\tanh(a/\delta^*)} \tanh \left( \frac{a - r}{\delta^*} \right)$$

(4.7)

where $\delta^*$ was discussed above. The motivation for these two models stems from the desire to make the time domain liner modeling as transparent as possible so that different physical processes are not lumped together into one or more semi-analytical terms. Towards this we view it more effective to separate out the velocity profile shape, its boundary layer thickness, and the orifice velocity. Model A is a fully analytical approach with no tunable constants which gives the profile and boundary layer thickness but is valid under single frequency conditions and assumes no variation in the flow along the orifice walls, i.e., it is fully developed, which is clearly not true. Model B uses a fixed velocity profile which resembles the data but moves the modeling to the orifice boundary layer thickness $\delta^*(t)$. Instead of pursuing a possible boundary layer model here, although several models are available (White, 1991; Dwyer, 1968; Phillips, 1996), we consider the ‘best case’ in which the exact data are available to evaluate the efficacy of such an approach.

4.3 Results and Analysis

4.3.1 Velocity profiles

Comparisons of the two models for $V(r)$ against the phase averaged velocity profile, taken at the phase of maximum velocity out of the cavity, are shown in figures 4.3 and 4.4. At constant amplitude model A under
predicts the boundary layer thickness for frequencies close to resonance, but improves at frequencies far away from resonance, and has a maximum velocity off the centerline and closer to the orifice walls. At 3 kHz in figure 4.3(d) model A is a very good predictor of the velocity profile. By construction model B has the proper boundary layer thickness, since $\delta^*$ is taken from the simulation data, but has its maximum velocity at the orifice centerline, which is nowhere true. For higher amplitudes these conclusions hold qualitatively for the profiles shown in figure 4.4 except for the highest amplitude scenario in figure 4.4(d) where neither profile is a faithful representation of the data.

Whether models A or B are useful depends on the degree to which they can predict the time-dependent quantities measurable within the liner and the impedance. Figures 4.5 through 4.9 show comparisons of the two models to the simulation data for the centerline orifice velocity and impedance for all seven cases considered. The velocity predictions at 130 dB show reasonable agreement, with amplitude matching within 5%, except for the 1.5 kHz case where the amplitude differences grow to 10%. Model A over-predicts while model B under-predicts by the same amount. At 3 kHz (figure 4.6) both models again perform well up to 140 dB, beyond which model B is inferior as it under-predicts the peak velocities by 15%.

### 4.3.2 Cavity pressure

The corresponding backplate pressure predictions at 130 dB (figure 6.5) are more strikingly different than their velocity counterparts. At 130 dB, 1.5 kHz, for example, both models over-predict the back-plate pressure and are out-of-phase with it; recall that model B under-predicted the velocity at this condition. By 2 kHz the differences in pressure have diminished significantly and continue to improve as the frequency is increased. The 3 kHz cases in figure 6.6 show reasonable agreement out to 140 dB. At 150 dB the pressure amplitude is correctly predicted but the signal traces are of a different waveform than seen in the simulation data. This observation continues to 160 dB where neither model shows the same, multi-peaked pressure trace found in the simulations. Observe that in all cases model A predicts a higher backplate pressure than does model B.

### 4.3.3 Impedance prediction

Model predictions of the liner impedance are shown in figure 4.9 for all seven cases using Dean’s (1974) method in the form given in Eq. (3.6). Models A and B perform well overall (i.e., reproduces the simulation data) but show inaccuracies at the most nonlinear conditions. Model A, where $V(r)$ is based on the root-mean-square solution from periodic pipe flow, severely under-predicts the resistance at 130 dB, 1.5 kHz while over-predicting the reactance at 160 dB, 3 kHz. Model B, on the other hand, where $V(r)$ is given by
Figure 4.3: Root mean square of the vertical velocity profiles at fixed 130 dB amplitude. Legend: — present data; — model A (Eq. (4.6)); — model B (Eq. (4.7)).
Figure 4.4: Root mean square of the vertical velocity profiles at fixed 3000 Hz frequency. Legend: — present data; — model A (Eq. (4.6)); — model B (Eq. (4.7)).
a hyperbolic tangent with simulation-determined displacement thickness, performs well near resonance but not at 160 dB, 3 kHz.

Both models exhibit the correct trends with frequency and amplitude and differ in their predictive accuracy only when the liner is near its most non-linear states. Near resonance where there is little secondary vorticity created near the orifice walls (figure 3.14(a)) and the velocity profile is nearly flat at phases $\phi = \pi/2$ and $3\pi/2$ the hyperbolic tangent model performs well when the displacement thickness is measured from the simulation. In contrast, model A’s profile has a too-thin boundary layer and misses the resistance considerably. At 160 dB, when the near-orifice secondary vorticity is strong and the near-wall profiles complex, having the correct displacement thickness is insufficient; one must have also the correct profile shape. At this condition neither model A nor B have both properties correct and so their predictive accuracy suffers. These results illustrate the influence the assumed profile $V(r)$ can have on the quality of the results at non-linear conditions.
Figure 4.5: Vertical velocity fluctuations at the center of the aperture. (SPL = 130 dB) Legend: — present data; — model A (Eq. (4.6)); — model B (Eq. (4.7)).
Figure 4.6: Vertical velocity fluctuation at the center of the aperture. ($f = 3000$ Hz)

Legend: — present data; □ — model A (Eq. (4.6)); ○ — model B (Eq. (4.7)).
Figure 4.7: Pressure fluctuations at the backplate of the cavity at 130 dB. Legend: — present data; — model A (Eq. (4.6)); — model B (Eq. (4.7)).
Figure 4.8: Pressure fluctuations at the backplate of the cavity at 3000 Hz. Legend: — present data; —model A (Eq. (4.6)); —model B (Eq. (4.7)).
Figure 4.9: Impedance Predictions. Legend: ×: Experimental Data ($\sigma = 6.4\%$); +: Experimental Data ($\sigma = 15.0\%$); ■: present data; □: Model A; ○ Model B; +: 6.4\% porosity data and ×, 15.0\% porosity data from Jones et al. (2004).
Chapter 5

Numerical Investigation of a 3-D Honeycomb Liner with Grazing Flow

5.1 Introduction

In this chapter the objective is to investigate a 3-D honeycomb liner under a grazing flow using the direct numerical approach to understand how the liner behaves when beneath a boundary layer, and to develop a reduced-order model that is sensitive to this change. The liner geometry to be investigated remains exactly the same as was studied by Jones et al. (2010) at the NASA Langley Research Center, and in Chapter 3 as well. While the experimental work focused on the global measurement of the important parameters of the acoustic liners, specifically the acoustic impedance, the simulations emphasize more on the local behavior of the liner. Special care is taken to document the interaction between the laminar/turbulent boundary layer, orifice and sound.

5.2 Experimental Methods

5.2.1 Experimental setup

Figure 5.1 shows a drawing of the NASA Langley facility used to study the impedance of acoustic liners in the presence of a grazing flow. The long rectangular cross-sectional duct that connects with a plenum chamber to an out-of-frame vacuum tank is known as the Grazing Flow Impedance Tube (GFIT) and is able to generate a maximum mean flow Mach number of 0.6. Eighteen tonal acoustic drivers are mounted next to the gas chamber upstream of the test window and are capable of producing single frequency acoustic fields ranging in frequency from 0.4 to 3.0 kHz, and at amplitudes up to 150 dB. The acoustic liners are exposed to the grazing flow and multiple flush-mounted microphones are installed on the wall opposite the liner sample to measure the acoustic pressure signals. Note that the design of the duct cuts off the higher-order duct acoustic modes in both horizontal and vertical dimension above 3 kHz. A more detailed description can be found in Jones et al. (2010).
5.2.2 Liner impedance eduction method

To understand how the impedance is measured experimentally, this section describes the procedure used at NASA Langley. The experimental liner impedance eduction method uses an indirect method by numerically solving the Pridmore-Brown second-order ordinary differential equation,

\[
\frac{d^2 P}{dy^2} + \frac{2k_x}{k - M k_x} \frac{dM}{dy} \frac{dP}{dy} + [k^2 - 2kk_xM - (1 - M^2)k_x^2]P = 0 \tag{5.1}
\]

with proper boundary conditions, where \( k = \omega/c \) and \( p = P(y)e^{-ik_x x} \). The axial propagation constant, \( k_x \), can be computed from the measured sound pressure level and phase decay rates, using the equation,

\[
k_x = -\frac{d\phi}{dx} + \frac{dSPL}{dx} \frac{i}{20 \log_{10}(e)} \tag{5.2}
\]

The impedance of the acoustic liner is computed as,

\[
\zeta = -ikP \left(1 - \frac{k_x}{k} M \right)^2 \left(\frac{dP}{dy}\right)^{-1} \tag{5.3}
\]

Detailed descriptions of the procedure for the liner impedance eduction can be found in Jones et al. (2010).

5.3 Computational Model

Attention is now turned to the computational model used to study the grazing flow-orifice interaction and, in particular, the three steps required to create it. The first step aims to determine several important parameters of the base flow in the GFIT. It has been experimentally verified, that, to a very good approximation, the liner influence on the mean flow within a duct is minor and can, to first order, be neglected. (Jones et al., 2010). The second step is to generate a fully developed turbulent boundary layer with bulk properties
determined from the first step. The final step invokes computation of the boundary layer-orifice interaction. As with the earlier study in Chapters 3 and 4 without a grazing flow, as much detail as possible is retained in the simulations, relative to the experiments; however, some simplifications are made. These were:

- The experimentally-tested liner contains thousands of cells. Since the cells are not connected, we assume these cells do not interact with each other and retain only 1 cell in the simulation. (Same approximation as in the no-flow cases.)

- The experimental liner has a porosity 8.7% which means 8-9 orifices per cell on average, with an inter-orifice spacing at least 2 times larger than the orifice diameter. We assume that orifices within a cell do not interact strongly so that a single orifice is retained in the simulations and is taken to sit in the center of a hexagonal cell. This results in a corresponding porosity 0.99%. Final comparisons will be normalized by the porosity. (Same approximation as in the no-flow cases.)

- We replace the grazing incidence of the acoustic field found in the experiment with a normal incidence because of the tiny size of the orifice relative to the acoustic wavelength. Tam & Kurbatskii (2000) suggested that a 2-D slit liner with small orifices relative to the wavelength is insensitive to the angle of the incidence.

- The Reynolds number based on the momentum thickness $\delta_\theta$ in the experiments is roughly 23,100, as estimated using the RANS simulation discussed in Section 5.3.1 below. Because the orifice diameter (0.99 mm) is of the same order of magnitude as the large scale, energetic structures found in the boundary layer, the orifice is not expected to interact strongly with the smaller structures. We thus expect a weak sensitivity to the boundary layer Reynolds number and simulate a reduced value of 2,300.

### 5.3.1 Base flow simulation

The construction of the computational model for the flow simulation in GFIT in figure 5.2(a) is based on the geometric data supplied by NASA Langley (Jones et al., 2010). It should be noted that only one quarter of the full duct is configured and symmetric boundary conditions are applied at the center to save computational resources. The computational grid uses a non-uniform structured mesh in which more grid points are clustered near the duct wall to ensure high resolution there. The compressible Reynolds Averaged Navier-Stokes (RANS) equations are solved with a realizable $k-\varepsilon$ turbulence model for the closure of the problem. The boundary conditions used are shown in figure 5.2(a) where the experimentally unknown plenum stagnation pressure is varied until the predicted cross-duct Mach number profile matched that
measured by a Pitot-static probe downstream of the liner section. The value of Mach 0.5 is chosen because of the available liner impedance data in Jones et al. (2010). Once the flow is steady, the critical features (i.e., momentum thickness of the boundary layer) of the grazing flow in the duct are estimated. A quantitative comparison between the simulation and experimental data is shown in figure 5.2(b). Although the centerline Mach number at the exit plane is around 5% underestimated by the simulation, the velocity profiles have similar shapes, and there are several possible uncertainty sources that preclude making a better comparison, including (a) laminar-to-turbulent transition region within the duct and (b) corner flow effects not captured by the $k-\varepsilon$ model. Detailed comparisons of the data are provided in table 5.1, which suggest that the simulation results have a reasonable agreement with the test conditions in the GFIT. One objective of this simulation is to obtain details of the flow features at the test section, where the only known experimental parameter is the centerline Mach number. These data will guide our turbulent boundary layer simulation described next.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>exp. data</th>
<th>sim. results</th>
<th>error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline Mach number at the liner test section</td>
<td>0.50</td>
<td>0.52</td>
<td>4.0</td>
</tr>
<tr>
<td>Centerline Mach number at the exit plane</td>
<td>0.61</td>
<td>0.57</td>
<td>6.7</td>
</tr>
<tr>
<td>Visual boundary layer thickness</td>
<td>21.2 mm</td>
<td>20.0 mm</td>
<td>5.7</td>
</tr>
<tr>
<td>Displacement thickness</td>
<td>2.6 mm</td>
<td>2.74 mm</td>
<td>5.4</td>
</tr>
<tr>
<td>Momentum thickness</td>
<td>2.0 mm</td>
<td>2.03 mm</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 5.1: Comparisons of the turbulent boundary layer features between the simulation results and experimental data.
<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>Grid Size</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cartesian</td>
<td>$501 \times 401 \times 116$</td>
<td>Above the facesheet</td>
</tr>
<tr>
<td>2</td>
<td>Orthogonal</td>
<td>$121 \times 91 \times 336$</td>
<td>Orifice and $10d$ above and below it</td>
</tr>
<tr>
<td>3</td>
<td>Non-orthogonal</td>
<td>$121 \times 121 \times 206$</td>
<td>Cavity</td>
</tr>
<tr>
<td>4</td>
<td>Cartesian</td>
<td>$51 \times 51 \times 336$</td>
<td>Cut away orifice centerline</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>$31,312,838$ points</td>
</tr>
</tbody>
</table>

Table 5.2: Grid information used in the numerical simulation.

5.3.2 Turbulent grazing boundary layer

The ultimate simulation of the sound-induced flow aims to include the grazing flow as the background. Therefore, a Mach 0.5 compressible turbulent boundary layer is generated according to the parameters used in the experiments with a reduced Re$_\theta$. The detailed procedure and analysis of the simulation results are shown in Appendix A. For the purpose of comparison, a compressible laminar boundary layer with the same freestream Mach number and momentum thickness is also created.

5.3.3 Grid design

The grazing flow simulations contain either a laminar or a fully developed turbulent boundary layer grazing past a single-orifice cavity sitting below the facesheet as is shown in Figure 5.3. To discretize the geometry an overset grid approach similar to that described in Chapter 3 is used wherein multiple, overlapping grids co-exist and information is passed between them via interpolation. Four grids are used: the first three account for the geometry, namely the rectangular region above the facesheet, the hexagonal cell, and the circular orifice, while an additional rectangular grid is used to remove the polar axis of the cylindrical grid. Each grid is logically rectangular. The minimum grid spacing in the orifice is $\Delta r_a = 0.0075d$ at the orifice walls. In the wall normal direction $y$-direction, a uniform grid spacing $\Delta y_{\min} = 0.01d$ is used $1d$ offset above and below the orifice. At $10d$ away from the orifice, the resolution decreases smoothly but still keeps $\Delta z_{\max} \leq 0.1d$. Also note that the size of the upper rectangular box is set to be sufficiently large for fully uncorrelated boundary layer statistics, as noted in Appendix A.

5.3.4 Simulation setup

The grazing flow simulations start with either a laminar or fully developed turbulent layer. The incident acoustic plane wave of specified amplitude and frequency impinges normally upon the facesheet, excites the fluid near the orifice, and is mostly reflected back. After several initial cycles, the flow reaches a statistically steady state. In all, 9 different cases are classified into four different sets and are shown in table 5.3. Set (A) contains one simulation with a turbulent boundary layer and the cavity-backed orifice, but no impinging
sound is enforced and is for reference only. All of the other cases include the planar incident sound: Set (B) is for validation and contains the cases at fixed 130 dB level but the frequency varies from 2.2 kHz to 3.0 kHz with an increment of 0.4 kHz. Set (C) contains the cases at fixed frequency 3.0 kHz but the amplitude varies from 130 dB to 160 dB in increments of 10 dB. This set is used to study the effects caused by the changing of the sound amplitude on the interaction between the boundary layer, sound, and the orifice. Set (D) contains the cases with fixed frequency 3.0 kHz with either laminar or turbulent boundary layers at either 130 dB or 160 dB SPL level. The ultimate goal is to investigate the behavior of the sound-induced flow induced by a laminar and a turbulent boundary layer. Note that some simulations appear in more than one set.

5.4 Vorticity Visualization

For visualization, vorticity contours taken through the orifice are shown for all the cases from set (B) to set (D). (See Figures 5.4 to 5.6) We first discuss how the boundary layer affects the sound induced-flow inside or near the orifice. Unlike the previous simulations without grazing flow, the presence of the boundary layer immediately breaks the symmetry of the sound-induced flow about the facesheet in the flow direction. The
<table>
<thead>
<tr>
<th>No.</th>
<th>Grazing Flow Type</th>
<th>Amplitude</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Turbulent boundary layer</td>
<td>No sound</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>2.2 kHz</td>
</tr>
<tr>
<td>B2</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>2.6 kHz</td>
</tr>
<tr>
<td>B3</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>C1</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>C2</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>C3</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>C4</td>
<td>Turbulent boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>D1</td>
<td>Laminar boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>D2</td>
<td>Laminar boundary layer</td>
<td>130 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>D3</td>
<td>Turbulent boundary layer</td>
<td>160 dB</td>
<td>3.0 kHz</td>
</tr>
<tr>
<td>D4</td>
<td>Turbulent boundary layer</td>
<td>160 dB</td>
<td>3.0 kHz</td>
</tr>
</tbody>
</table>

Table 5.3: List of the simulations completed in this chapter.

Present investigations reveal that, at sound amplitude 140 dB or above, the vorticity is able to separate from the orifice walls and travel away from the orifice, like the no-flow cases. With grazing flow, most of the ejected vorticity is immediately washed downstream during the outflow cycle while during the inflow cycle there exists a bias flow towards the downstream side of the cavity. At 150 dB, the internally-ejected vorticity is unsteady and appears to undergo an instability where as, for the no-flow case, the ejected vorticity is relatively stable. At 160 dB the vorticity apparently undergoes a stronger instability which causes transition and leading to a strong turbulent jet for the inflow cycle; the outflow jet can hardly penetrate the high speed grazing flow and therefore the vorticity generated quickly merges into the boundary layer.

From another perspective the sound-induced flow can also affect the grazing flow near the orifice, especially near the downstream region, when the sound amplitude is high enough. Such behavior was not observed in Set (B) where, at 130 dB, the sound is not able to generate a flow with sufficiently high momentum to leave the orifice walls. At 140 dB or above, however, the vorticity generated sheds and eventually results in the flow separation. For the outflow cycle, the ejected vortices are immediately washed downstream. Further observation reveals these vortices are usually attached to the facesheet surface and their influence range increases as the amplitude of the sound increases.

Next, we move to compare the effect of the turbulence within the boundary layer by performing simulations with laminar boundary layers with the same freestream velocity and momentum thickness. Comparisons in Figure 5.6 shows that the laminar boundary layer has a very limited interaction with the flow inside the orifice while the turbulent boundary layer involves more interactions. At the 160 dB sound level, the in-cavity jet is much less uniform for a turbulent grazing flow than it is for a laminar one. At lower amplitudes the influence of the boundary layer quality is stronger.
5.5 Velocity Profiles near the Orifice

The velocity field inside and near the orifice is, from the point of view of the acoustic impedance, the most important consequence of the flow-orifice-sound interaction. In this section we document several different measures of the near-orifice velocity for use later in developing a reduced-order impedance model. We first examine the velocity profiles at the center cut of the orifice.

5.5.1 Mid-facesheet velocity distributions

The impact of the grazing flow on the in-orifice flow can be quickly appreciated by considering the profiles of the orifice-normal velocity taken in the orifice, along the facesheet mid-plane, as shown in Figure 5.7. Conditions without grazing flow (corresponding to the work in Chapter 3) and with laminar and turbulent boundary layers are shown for 3 kHz, 130 dB incident sound. The no-flow cases illustrate the azimuthal symmetry the velocity profile has when a grazing flow is absent and, from the work described in Chapters 3 and 4, the profile can be well approximated by hyperbolic tangent curves with a fitted boundary layer thickness or with periodic pipe flow root-mean-square profiles. When a grazing flow is present in the $+x$ direction, the azimuthal symmetry is clearly broken and the inflow and outflow cycles are not similar, as
Figure 5.5: Vorticity contours near the orifice of the liner under turbulent boundary layer (Set C). (3.0 kHz) Slices taken at $x = 0.0, z = 0.0$.

shown in Figure 5.7(b-c, e-f). Note also that the peak velocity has increased by a factor of 2 and that is concentrated on the downstream side of the orifice during the inflow cycle. During the outflow cycle the flow is more azimuthally symmetric but not as much as for the no-flow cases. The laminar-versus-turbulent boundary layer appears also to affect the magnitude of the in-orifice velocity, with the turbulent case being higher. Further, Figure 5.7(a)(d) reveal that, except the flow direction, the flow is also symmetric about the center of the orifice. With grazing flow, the inflow and outflow velocity profiles are completely different. The inflow region is mostly concentrated at the downstream half of the orifice while the outflow shows a different phenomenon. Figure 5.7(b)(c)(e)(f) also indicate that the turbulent boundary layer case may cause higher local maximum velocity fluctuations.

5.5.2 Orifice velocity time histories

To further quantitatively analyze the biased velocity profiles, three probes (see Figure 5.8, namely A, B and C), are placed at different locations in the mid-plane of the orifice to record the time history of the velocity fluctuations after the initial transition period. Figure 5.9 reveals the time history of velocity fluctuations at 130 dB case with different grazing flow conditions. Since the velocity profile has an axisymmetric feature at
a quiescent condition, the time history of the velocity fluctuations at A and C collapse perfectly with each other except for a very slight phase lag between probe B and probes A and C. With a laminar boundary layer present, the velocity fluctuations at location A, B and C still remain sinusoidal but with different magnitudes and phases. At probe A, the velocity fluctuation is always positive, indicating an outflow-only motion. Although probe B sits in the center of the orifice, the velocity fluctuation observed has a positive constant mean value that suggests a biased outflow motion on top of which sits the sound-induced fluctuations. Probe C sits 0.36d downstream from the origin and reveals a strong motion that is roughly symmetric about $v/a_\infty = 0$. For the turbulent boundary layer-induced flow in Figure 5.9(c), the same qualitative picture emerges as in the laminar case but with large deviations. Again probes A and B show a net outflow and biased outflow, respectively, with roughly the same velocity magnitude peaks, albeit with an increased level of high-frequency (relative to the forcing frequency) fluctuations arising from the boundary layer. Probe C now shows a net inflow character with peak velocities that are roughly 2 times those for the laminar boundary layer case.

Figure 5.10 and Figure 5.11 show the effect of changing the sound amplitude or the frequency on the velocity history recorded at probes A, B and C. The velocity fluctuation magnitude increases as the amplitude
of the incident sound goes up. The maximum magnitude of the fluctuation reaches roughly 10%, 18% and 30% of the ambient speed of sound for 140 dB, 150 dB and 160 dB, respectively. The additional fluctuations caused by the turbulent boundary layer play a less significant role as the acoustic field becomes stronger, and eventually overwhelms the hydrodynamic field at 160 dB. Also note that at higher SPLs, the discrepancies between the fluctuations at different locations lessens, indicating a more azimuthally-symmetric in-orifice flow. At fixed 130 dB amplitude, the frequency of the incident sound does not appear to influence the in-orifice flow by a large amount.

5.5.3 Phase averaged velocity profiles

At the axis planes ($\varphi = 0^\circ, 180^\circ$; see Figure 5.8) of the orifice the phase averaged velocity can be easily seen in the contours and streamlines shown in figures 5.12 and 5.13. Plots at phase angles of $\phi = \pi/4$, $\phi = \pi/2$, $\phi = 3\pi/4$, and $\phi = \pi$ cover the inflow cycle while $\phi = 5\pi/4$, $\phi = 3\pi/2$, $\phi = 7\pi/4$, and $\phi = 2\pi$ indicates the corresponding phases for the outflow cycle. We first investigate all the cases at different SPLs but with turbulent boundary layer, as listed in set (C). At 130 dB, the streamlines indicate that a big vortex is formed in the middle of the orifice due to the high speed grazing flow. This main vortex exists nearly
for the whole inflow cycle, $\phi : 0 \rightarrow \pi$. The inflow cycle also reveals that fluid enters the cavity only via the downstream side of the orifice. The outflow cycle is different: at the beginning, fluid leaves the cavity only via the upstream side of the orifice due to the in-orifice vortex. After the peak outflow phase ($\phi = 3\pi/2$) the main vortex disappears and the fluid is able to leave the cavity via the whole orifice. The streamlines also indicate that the fluid leaving the orifice is immediately carried downstream. When the outflow cycle ends, the main vortex again appears in the middle of the orifice. At high SPLs, due to an increase of the amplitude of the incident sound, more momentum is added for the fluid to penetrate into the cavity and the vortex is overwhelmed, leaving its influence reduced. At 140 dB, the size of the vortex shrinks and is found to be attached to the left wall of the orifice. The size of the vortex continues to reduce at 150 dB and eventually disappears at 160 dB. Between 150 dB and 160 dB, the inflow cycle’s upstream boundary layer thickens while the downstream boundary layer thins due to the grazing flow. During the outflow cycle, however, only a slight difference between the up- and downstream orifice boundary layers is found. At 160 dB the streamlines at phases $\phi = 3\pi/2$ and $\phi = 7\pi/4$ indicate a strong vertical momentum of the fluid penetrating into the boundary layer.

Comparisons between the laminar boundary layer and the turbulent boundary layer cases are shown in Figure 5.13. At 130dB, the size of the center vortex found in the laminar boundary layer case is reduced relative to the turbulent case, which indicates more fluid is allowed into the cavity. Such difference is mainly caused by the different mean velocity profiles between a laminar and a turbulent boundary layer. With the same freestream velocity and momentum thickness the laminar boundary layer velocity profile typically has
Figure 5.9: Velocity fluctuations with different grazing flow conditions. (130 dB 3.0 kHz) Legend: ‘−−’ Probe A; ‘−’ Probe B; ‘− · −’ Probe C.

(a) No Flow  (b) Laminar BL  (c) Turbulent BL

a lower gradient at the wall. Therefore, the shear layer at the lid of the orifice is weaker. For the same reason, the fluid ejecting out of the cavity can therefore penetrate farther away from the liner facesheet. At 160 dB the cases for laminar and turbulent boundary layer bear almost no difference except the inflow cycle shows a slightly thicker upstream boundary layer at \( \phi = \pi/2 \) and \( \phi = 3\pi/4 \) under turbulent grazing flow.

### 5.6 Discharge coefficient

#### 5.6.1 Instantaneous discharge coefficient

The rate of the fluid that reaches into the cavity, which is closely related to the discharge coefficient, will eventually determine the acoustical behavior of the liner. Previous work (Zhang & Bodony (2012), Chapters 3 and 4) showed that a useful way to approximate the (average) discharge coefficient by taking the mean boundary layer thickness formed inside the orifice in quiescent condition. With the presence of the grazing flow, difficulties immediately arise as the flow pattern is no longer axisymmetric. Therefore, we must use a different way to predict the discharge coefficient when a grazing flow is present. Figure 5.14 demonstrates the basic idea of the discharge coefficient prediction in which the ratio between the core inflow/outflow area and the area of the orifice from the DNS results is used, resulting in the inflow and outflow formulas of,

\[
C_{D_i} = \frac{4A_i}{\pi d^2} \quad C_{D_o} = \frac{4A_o}{\pi d^2},
\]

The task to predict the discharge coefficient now requires an evaluation of the core flow area. We define the region where the magnitude of the velocity fluctuations exceed the spatially averaged inflow/outflow region velocity as the core inflow/outflow region. Because of the flow reversal it is challenging to assign a single
value for the core flow area. Further, the different features we observed in the flow streamlines for the inflow and outflow cycle also require us to differentiate the discharge coefficient between the inflow and outflow cycle. Therefore, we split the whole acoustic cycle into four different periods as we did without grazing flow. (Zhang & Bodony, 2012) In Figure 5.15, the labels ‘D’ and ‘U’ represent downward and upward motion respectively while label ‘T’ represents transition period. Separation of these periods is via the fluctuation of the mass flow rate. The data are presented in the next section.
Figure 5.11: Velocity fluctuations at different frequencies. (130 dB) Legend: ‘ ‘ Probe A; ‘ ‘ Probe B; ‘ ‘ Probe C.

Figure 5.12: Contour plot of \( v/a_\infty \) velocity with streamlines under turbulent boundary layer at different SPLs.
Figure 5.13: Comparison of the contour plot of $v/a_\infty$ velocity with streamlines under laminar and turbulent boundary layers.

(a) 130 dB Laminar BL
(b) 130 dB Turbulent BL
(c) 160 dB Laminar BL
(d) 160 dB Turbulent BL

Figure 5.14: Schematic of the definition for core flow region.

(a) Inflow cycle
(b) Outflow cycle

Figure 5.14: Schematic of the definition for core flow region.
Figure 5.15: Analysis of instantaneous discharge coefficient with mass flow rate. (160 dB)
5.6.2 Averaged discharge coefficient

By conditionally averaging the instantaneous discharge coefficient at the inflow and outflow periods, we obtain the (averaged) inflow and outflow discharge coefficient shown in Figure 5.16. Note that the inflow discharge coefficient is always lower than the outflow discharge coefficient at the same SPL. Both discharge coefficients show a linear increase with the increase of the amplitude of the sound except at 160 dB outflow period, where the discharge coefficient reaches nearly 0.76. We can further average the inflow and outflow discharge coefficients to obtain a globally averaged discharge coefficient $\overline{C_D}$ for the whole acoustic cycle. This parameter is especially useful for reduced-order modeling and will be discussed later. Comparisons of the global discharge coefficient with the no grazing flow, laminar boundary layer and turbulent boundary layer cases are shown in Figure 5.17. At fixed SPL, the grazing flow condition acts to lower the discharge coefficient value by ‘blocking’ the fluid from going into and out of the orifice. The influence of the grazing flow diminishes with increasing SPL so that the difference between the no-flow, laminar, and turbulent flow values of $C_D$ also diminishes. Also note that the discharge coefficient is slightly higher for the cases with laminar boundary than the ones with the turbulent boundary layer. Another interesting observation is that the trend of the global discharge coefficient behaves oppositely whether there is a grazing flow: with grazing flow the discharge coefficient increases as the SPLs increases while a decreasing trend is observed for the cases at quiescent condition.

To explore the reason for the different trends, we first review the analysis of the flow inside the orifice without grazing flow from Chapters 3 and 4. In the low SPL cases the fluid smoothly goes into the cavity with little energy loss at the boundary walls while the high SPL cases the local flow separation creates a self-
induced vorticity-generation mechanism. No other mechanism can significantly interfere the sound-induced flow motion except the solid walls. With grazing flow, a completely different mechanism is found. Recall that the grazing flow induces a strong vortex inside the orifice, which plays a role of resisting fluid going into or coming out of the cavity. In order to overcome the resistance and penetrate deeper into the cavity, more energy is required from the incident sound. Therefore, the capability of the fluid entering or leaving the orifice is mostly determined by the momentum carried by the normal incident sound. This explains why the discharge coefficient under grazing flow is proportional to the amplitude of the sound. Under these circumstances, the effects caused by the flow separation at higher SPLs are negligible compared with the effect brought by the grazing flow.

5.6.3 Flow angle

With the presence of the grazing flow, the angles at $\pi/2$ and $3\pi/2$ shown in Figure 5.18 can be used to estimate how much fluid penetrates or leaves the cavity. For the inflow cycle, we mark the location of the streamline that hits downstream the corner of the orifice, along the orifice mid-plane and compute the inflow angle; for the outflow cycle, we trace the streamline that starts from the upstream corner and record the distance from the facesheet when it reaches right above the downstream corner. Figure 5.19 reveals the relation between the amplitude of the incident sound versus the inflow and outflow angles. The inflow angle shows a linear with SPL (in dB) increase while the outflow angle reveals a stronger, monotonically-increasing relation.
The correlation between the flow angle and the corresponding discharge coefficient is given in Figure 5.20. The inflow and outflow angle represents two different trends but both can be fitted by a hyperbolic tangent function. As the angle goes to $\pi/2$, it should be expected that the grazing flow effect is negligible and the discharge coefficient should converge to a constant below 1. Here, the constant is set to be 0.76 as suggested by Motsinger & Kraft (1995).

### 5.7 External Drag Analysis

Novel airplane concepts have proposed acoustic liners be installed on external aircraft surfaces to further reduce air frame noise. With high speed grazing flow, the additional drag induced by these liners could be substantial and should be taken into consideration. We therefore numerically predict the total drag with and without an orifice via the original definition of the drag,

\[
\mathbf{F}_{\text{w/o orifice}} = \int_{S_{\text{surface}} \cup S_{\text{orifice}}} \mathbf{f} dS
\]

\[
\mathbf{F}_{\text{w orifice}} \approx \int_{S_{\text{surface}}} \mathbf{f} dS + \int_{S_{\text{neck}}} p \mathbf{n} dS + \int_{S_{\text{cavity}}} p \mathbf{n} dS
\]

respectively, where $D_1$ denotes the shear stress contribution, $D_2$ denotes the pressure contribution on the deck of the orifice and $D_3$ denotes the pressure contribution of the pressure component analysis from the cavity. An integration schematic is given in Figure 5.21.
5.7.1 Wall shear stress

We now investigate more closely towards the total drag by considering the shear stress contribution first. A detailed numerical integration suggests both the shear stress contribution from the neck of the orifice and the cavity are negligible compared to the shear stress from the fracesheet of the liner. Therefore, the only part remaining in the wall shear stress contribution towards the drag is from the facesheet surface of the liner. Further, in order to maintain the 8.7% porosity rate from the experiment, $S_{\text{surface}}$ is chosen to be $11.5S_{\text{orifice}}$, where $S_{\text{orifice}}$ is the area of the orifice. Meanwhile, the analysis of the orifice-free wall shear stress is evaluated by selecting a surface with the same area $S_{\text{surface}}$ but far away from the orifice. Without the orifice, wall shear stress distribution is supposed to be uniform everywhere on the facesheet surface if sufficiently averaged in time. Because of the presence of the orifice it is natural to investigate the wake created by the interaction between the orifice and the boundary layer. Figure 5.22 shows the wall shear stress distribution at different SPLs with the presence of the orifice. Since the mean flow is parallel to the surface, the wake region created by the orifice is very limited without sound interaction. Only those regions adjacent to the rim of the orifice in the downstream direction reveal significant wall shear stress increase (See Figure 5.22(a)). At 130 dB the vorticity is confined to the orifice and does not separate, which indicates no further interaction with the downstream region. Hence, little significant change in wall shear stress distribution is found. At 140 dB, the area of relatively higher wall shear stress region increases but only restricted in the downstream location. At 150 dB and 160 dB, this higher wall shear stress region shows a '<' shape and eventually spreads upstream and compasses the whole orifice. (See Figures 5.22(d) and 5.22(e)) Comparisons between the wall shear stress from the laminar boundary layer and turbulent boundary layer are shown in Figure 5.23. Observe that a much lower wall shear stress is seen for the corresponding laminar
cases due to a completely distinct nature of the velocity profiles between laminar and turbulent boundary layers near the wall. However, a similar trend of the wall shear stress increase with the interaction from the incident sound is found despite of the type of grazing flow.

5.7.2 Surface drag

With the pointwise wall shear stress distribution evaluated with and without the presence of the orifice, we may compute the surface drag via evaluating the integral in Eq. (5.5) and (5.6) respectively. Corresponding comparisons at different cases are shown in Figure 5.24. Without the orifice, the total surface drag caused by the wall shear stress experiences almost no change as the amplitude of the sound increases, as is expected. Although the existence of the orifice always positively affects the shear stress distribution, a slight decline of the surface drag has been ‘surprisingly’ found when there is no sound. Further analysis into the integral
unveils the answer that the removal of an orifice area exceeds the effect of the induced higher surface shear stress. At 130 dB the surface drag with the orifice recovers and is found to be almost equal to the corresponding case without the orifice. At 140 dB and above, however, the surface drag with the orifice increases as an increase of the incident sound amplitude with an approximately 15% increase at 160 dB.

### 5.7.3 Total drag

After proper scaling between the pressure induced drag and viscous wall shear stress induced drag, further analysis towards the total drag with the orifice can be done term by term listed in Eq. (5.6). Figure 5.25 reveals the total drag with the presence of the orifice, with a dramatic increase trend as an increase of the amplitude of the sound. Observe that the total drag always increases even without any sound, meaning the pressure induced the drag exceeds the missing part of the viscous surface drag due to an orifice removal. Also note that at 160 dB the total drag induced by the orifice exceeds twice amount of the surface drag without an orifice. Figure 5.26 shows the total drag increase under different types of boundary layers. Although the total drag for the laminar boundary layer cases is typically smaller, the increase rate due to the orifice is comparable with the turbulent boundary layer cases. Figure 5.27 shows the total drag induced by different sound frequencies at 130 dB amplitude, with very little change is observed. These findings further confirm that the amplitude of the sound is a key factor in deciding the total pressure, rather than the frequency, at least away from Helmholtz resonance. Figure 5.28 reveals the pressure distribution of orifice wall where high pressure distribution is found at the downstream edge of the orifice. Further analysis towards the sources of the total drag is given in Figure 5.29. For no sound or 130 dB cases, surface drag takes up more than 90% of the total drag. At higher amplitude cases, however, the surface drag plays a less important role and eventually drops below 60% at 160 dB case.

### 5.8 Pressure Statistics

Analysis of the pressure statistics over the orifice is performed in this section. Time traces of the pressure of the 5 different probes above the orifice are shown in Figure 5.30. Temporal pressure autocorrelations are shown in Figure 5.31 against the non-dimensional time interval where $T_n$ denotes the natural time period of the liner. When there is no sound, the pressure autocorrelations at probes A, B, C and D reveal a high frequency fluctuation and do not differ much from each other. The flow motion is dominated by the turbulence rather than the orifice at these locations. The pressure autocorrelation at probe E is, however, obviously different from the trends recorded at A, B, C and D, which reveals a more orifice-dominated
motion. This means that the orifice only allows fluid entering the cavity via the downstream region of the orifice. When the incident sound is enforced, the presence of the orifice with the aid of the incident sound starts to alter the trend of the pressure auto-correlations in a sequence from the downstream to the upstream. Figure 5.31 shows that the high frequency fluctuations at start to decay as the SPL of the incident sound increases. Above 150 dB, the orifice is nearly dominated by the incident acoustic field. Further investigation of the pressure autocorrelation can be done via the study of the integral time scale $T$,

$$T = \int_{0}^{\infty} R(\tau) \, d\tau$$  \hspace{1cm} (5.7)

where $R(\tau)$ is the temporal pressure autocorrelation. At each point above the orifice, the integral time scale is computed and shown in Figure 5.32. It should be noted that short times scales dominate the orifice when there is no acoustic forcing. These short time scales corresponds to the nature of the turbulence. At the far end of the orifice in the streamwise direction, the integral time scales behave differently with an apparent increase. In this region, the flow is dominated by the cavity. The presence of the incident sound increases the integral time scales as the sound pressure increases. The role of the turbulence decreases and the SPL of the acoustic incident sound increases. At 160 dB, more than 80% of the entire orifice is influence by the acoustic field.

5.9 Direct impedance prediction

Impedance is usually regarded as one of the most important parameters for acoustic liners and therefore many engine designs largely rely on this parameter. The liner’s acoustic impedance, $Z$, which is defined as the ratio of the complex-valued Fourier coefficients of the acoustic pressure and velocity,

$$Z = \frac{\hat{p}_a}{\hat{v}_a}. \hspace{1cm} (5.8)$$

5.9.1 Dean’s method

In this section Dean’s method (Dean, 1974) is used to predict the impedance of the acoustic liner. Note that only acoustic pressure signals at the facesheet and backplate of the cavity should be collected, however, a need complicated by the presence of the turbulent grazing flow. The characteristic acoustic impedance is given by

$$\frac{Z}{\rho_\infty a_\infty} = -i \frac{|\hat{p}_F|}{|\hat{p}_B|} \frac{e^{i\phi}}{\sin kH}$$  \hspace{1cm} (5.9)
where the subscripts $F$ and $B$ denotes the facesheet and backplate respectively, $\hat{p}$ is the Fourier coefficient of $p - p_\infty$ at the frequency of interest, $\phi$ is the phase angle between $\hat{p}_F$ and $\hat{p}_B$, and $k = |k|$ is the modulus of the wavenumber vector. The porosity factor should also be considered since this simulation retains only one orifice, as per the analysis in Chapter 3. The normalized impedance can still be split into its real part $\theta$ and imaginary part $\chi$ and multiplied by the porosity $\sigma$.

**Pressure analysis**

Due to the presence of the turbulent boundary layer, the facesheet surface pressures are measured 20 radii away from the orifice center, outside the hydrodynamic influence caused by the interaction between the orifice and grazing flow. However, the hydrodynamic noise caused by the turbulent boundary layer always exists. Figure 5.33 reveals the time history of the pressure fluctuations and its pressure sound level (PSL). The spectrum ($\Delta f = 1.0$ Hz) maintains at a constant level ($\sim 90$ dB) till 20 kHz and sharply drops at higher frequencies. The corresponding Overall Sound Pressure Level (OASPL) is roughly 135 dB. With the presence of the turbulent boundary layer, it is challenging to predict the impedance without the influence from the turbulence-induced pressure fluctuations. Therefore, 420 probes are placed at the facesheet of the liner with a minimum distance $8d$ between each other. Similarly, 9 probes at the backplate of the cavity are placed with a minimum distance $2d$. (Details are illustrated in Figure 5.34.) In order to see the influence from the turbulence, we arbitrarily pick 2 probes each from the facesheet and backplate pressure and record the time histories at 130 dB (3.0 kHz). Figure 5.35 reveals two different features of the facesheet and backplate pressure. At the facesheet, the pressure time history at different locations can be significantly different due to the randomness of the turbulence. The backplate pressure is nearly uniformly distributed at the backplate as the signals collapse with each other. This means that the geometry of the cavity allows only planar wave modes propagation. Also note that both signals are no longer sinusoidal under the presence of the turbulent boundary layer. We further investigate the relation between the theoretical, ensemble-averaged and instantaneous pressure fluctuations at different SPLs. Corresponding results are shown in Figure 5.36.

The theoretical expression is given by

$$p' = 10^{(SPL/20-9.701)} e^{i \omega t} \quad (5.10)$$

where SPL is scale of decibel (dB). At 130 dB, the instantaneous pressure fluctuations is largely contaminated by the turbulent noise. At this SPL, the strength of the acoustic field is comparable to the hydrodynamic field caused by the turbulent boundary layer. At 140 dB or higher, the acoustic component starts to overwhelm the hydrodynamic noise and eventually, at 160 dB, little discrepancy between the theoretical expression and
the instantaneous pressure fluctuations is observed. However, it should be noted that the ensemble-averaged signals match remarkably well with the theoretical expression even at 130 dB. A comparison of the facesheet pressure beneath different types of boundary layers is shown in Figure 5.37. Note that significant difference is observed at 130 dB while little discrepancy exists at 160 dB. This again supports the dominant contribution to the total pressure from the acoustic field at high SPLs.

5.9.2 Accuracy and precision

Due to the presence of the turbulent boundary layer, the impedance measured at a single point on the surface may not represent the entire liner. Moreover, we are particularly interested in the regions that can cause prediction error. We will split the discussion into two parts: the averaged prediction and local pointwise prediction. Since the pressure distribution is uniform at the backplate, we sample the backplate pressure at the center point direct below the orifice. The facesheet pressure collected at each probe among the 420 mounted on the facesheet, along with the backplate pressure, can each yield an impedance value. The averaged prediction is the ensemble averaged impedance, which can minimize the influence caused by turbulent boundary layer.

Comparison with the experimental data

Corresponding experiments were carried at NASA Langley Research center for the impedance measurement. (Jones et al., 2010) An inverse method is derived to solve for the impedance value for the liners with the same geometry but different porosity. To normalized the values, the comparisons of the impedance is multiplied by the porosity. (\(\sigma = 0.99\%\) for DNS; \(\sigma = 8.7\%\) for the experiment) Figure 5.38 reveals the comparisons of the impedance prediction versus the experimental eduction data. While the simulation predicted impedance value shows a smooth varying trend at different frequencies, the experimental reveals a much stronger varying feature. Figure 5.39 reveals the impedance prediction at different SPLs, in which the resistance increases as the SPL increases while the reactance does not change much. The trend is consistent with the previous findings where no grazing flow condition was included in Chapter 3. At 130 dB, the numerical predicted impedance value is compared with experimental data using different eduction models. A more interesting point in Figure 5.39 can be found when comparing the impedance value under different grazing flow conditions. At 130 dB, the resistance value largely increases when there is grazing flow. The change of the grazing flow condition from laminar boundary layer towards the turbulent boundary layer also dramatically increases the resistance value. Since the acoustic power and the turbulent power is roughly at the same level at 130 dB, a possible reason is that the turbulent eddies on the facesheet are
responsible for preventing the flow impinging into the cavity, which results in an increase of the resistance value. However, such change is not very apparent at 160 dB in which the acoustic strength overwhelms the turbulent hydrodynamic field. Consequently, the effect of the turbulent eddies plays a trivial role in 160 dB cases.

5.9.3 Prediction accuracy

Scattered prediction

Figures 5.40 through 5.42 reveal the scattered impedance prediction. It should be noted that the uncertainty range at 130 dB is not small, primarily caused by the facesheet turbulent noise. As the SPL of the incident sound increases, the acoustic pressure dominates the total facesheet pressure which results in the impedance prediction converging into a single point. Therefore, it is expected that the impedance prediction has a higher precision at high SPLs. Figures 5.41(a) to 5.41(d) show the scattered impedance prediction under different grazing flow conditions. While a significant variance can be found for the 130 dB case, the prediction at 160 dB rarely differs between the laminar boundary layer and the turbulent boundary layer. It should be noted that when the acoustic field overwhelms the hydrodynamic field, the grazing flow condition almost has little influence towards the impedance prediction. Also note that at 130 dB the frequency of the incident sound has little impact on the impedance prediction.
Figure 5.22: Wall shear stress distribution with the presence of the orifice and turbulent boundary layer.

(a) No sound

(b) 130 dB

(c) 140 dB

(d) 150 dB

(e) 160 dB
Figure 5.23: Wall shear stress distribution with the presence of the orifice and laminar/turbulent boundary layer.
Figure 5.24: Comparisons of the surface drag with and without the orifice under a turbulent boundary layer under different SPLs.

Figure 5.25: Drag increase due to the present of the interaction between sound and orifice under turbulent boundary layer.
Figure 5.26: Drag increase due to the present of the interaction between sound and orifice under laminar/turbulent boundary layer.

Figure 5.27: Drag increase due to the present of the interaction between sound and orifice.
Figure 5.28: Pressure distribution at the orifice walls under turbulent boundary layer. (160 dB)

Figure 5.29: Drag source component percentage analysis. Legend: ‘−□−’ surfacewall shear stress; ‘−△−’ orifice neck pressure difference; ‘−○−’ cavity pressure difference.

Figure 5.30: Schematics of the pressure probes at the inlet of the orifice.
Figure 5.31: Pressure auto-correlation at different locations above the surface. (3.0 kHz)
Figure 5.32: Contour plot of the integral time scale of the pressure over the orifice. (3.0 kHz)
Figure 5.33: Time history and PSL of the facesheet pressure fluctuations under the turbulent boundary layer. (No sound)

Figure 5.34: Schematic of the microphone locations.
Figure 5.35: Time history of the facesheet and the backplate pressure at two different locations. (130 dB, 3.0 kHz)

(a) Facesheet Pressure  
(b) Backplate Pressure

Figure 5.36: A comparison of the facesheet pressure fluctuations between the DNS results and the analytical expression. Legend: ‘−’, DNS data; ‘−−’, analytical expression (Eqn. 5.10).

(a) 130 dB, 3 kHz (Turbulent BL)  
(b) 140 dB, 3 kHz (Turbulent BL)  
(c) 150 dB, 3 kHz (Turbulent BL)  
(d) 160 dB, 3 kHz (Turbulent BL)
Figure 5.37: A comparison of the facesheet pressure fluctuations between the DNS results and the analytical expression. Legend: ‘−’, Instantaneous DNS data; ‘− · −’, ensemble averaged DNS data; ‘−−’, analytical expression (Eqn. 5.10).
Figure 5.38: Normalized impedance prediction at 130 dB with turbulent boundary layer. Legend: Experiments by Jones et al. (2010): ‘□’, ‘△’, ‘▽’, ‘+’ experimental impedance eductions using different methods (Jones et al., 2010); ‘.’ present simulation data.

Figure 5.39: Normalized impedance prediction at different SPLs and grazing flow conditions. Legend: ‘♦’ No grazing flow; ‘■’ Laminar boundary layer; ‘.’ Turbulent boundary layer.
Confidence region

Quantitative analysis of the scattered prediction is given in Figures 5.43 to 5.45 in which the radius of the 95% confidence region is shown. At different SPL the radius of the confidence region reveals a exponential decay when the amplitude of the sound increases as is shown in Figure 5.43. The frequency hardly affects the confidence region as the radius of the confidence region stays at the same level. (See Figure 5.44). Also note that the dramatic change of the confidence region under different SPLs reveals the potential prediction error and is found to be consistent with the previous pressure analysis.
Figure 5.41: Comparisons of the scattered impedance predictions under different boundary layers.

**Convergence**

When there is no grazing flow, it is observed that 10 statistically steady acoustic cycles would be sufficient for the prediction to reach a 10% prediction error. The 10 cycles period for the acoustic pressure sampling is doubted with the presence of the turbulent boundary layer. We only investigate the 130 dB cases as the turbulent boundary layer affects them most. Detailed sampling procedure is shown in Figure 5.46. To avoid transitional process, the initial 5 cycles are not counted. Figure 5.47 reveals the trend of the prediction values with error bars. As the number of the acoustic cycles increases above 20, the prediction value does not vary much. Also note that the error bar decays at the number of cycles increases. Obviously, using 10 cycles in acoustic impedance prediction under turbulent boundary layer may not be sufficient.
5.9.4 Error analysis

While it is always better to locate the facesheet probe as far away as possible from the center of the orifice, the space available for the pressure probes in a real engine test could be very limited. Therefore, it would be inevitable to mount a facesheet pressure probe in the near field of the orifice with significant hydrodynamic pressure interactions. In the no grazing flow cases, we proposed the relatively unaffected distance to be at least two times of the diameter of the orifice (Zhang & Bodony, 2012). It is therefore useful to know the locations where the facesheet pressure measurement will not be affected by the presence of the grazing flow. To demonstrate this, the absolute impedance prediction error map is provided in Figures 5.48 and 5.49 where the reference impedance value is the ensemble averaged value measured 20 radii away from the orifice. It is noted that the impedance prediction can yield an 100% error if the facesheet probes are mounted within $1.5d$ from the center of the orifice. Near the orifice the relative error becomes very obvious in the upstream and
downstream locations while the error reduces in the spanwise locations. In this situation, a probe mounted in the spanwise direction would have a much smaller prediction error than the one in the streamwise direction if they have the same distance towards the center of the orifice. While the sound boundary layer and orifice interaction at 160 dB case is the strongest, we observe the error near the orifice is the smallest. Note that at low SPLs a small disturbance could easily introduce a significant error in the impedance prediction while the overwhelming acoustic field is not easily affected. Also note that the error of the reactance and resistance prediction also differs significantly in percentage.
Figure 5.46: Time history of facesheet and backplate pressure at two different locations. (130 dB, 3.0 kHz)
Figure 5.47: Prediction convergence using different number of acoustic cycles at 130 dB.
Figure 5.48: Impedance prediction error near the orifice of the liner at different SPLs. (3.0 kHz) Left column: Normalized resistance; Right column: Normalized reactance. (Values in percentage)
Figure 5.49: Impedance prediction error near the orifice of the liner at different frequencies. (SPL = 130 dB) Left column: Normalized resistance; Right column: Normalized reactance. (Values in percentage)
Chapter 6

Time Domain model of a 3-D Honeycomb Liner with Grazing Flow

The data presented in Chapter 5 identifies the basic features of the interaction between the grazing flow, the orifice, and the incident sound. In this chapter that information is used to develop a simplified model of the interaction for use in predicting the acoustic impedance. The model developed follows that presented in Chapter 4 but has an increased amount of empiricism because of the additional flow complexity.

6.1 Model Construction

6.1.1 Governing equation

The time-domain model of the liner without grazing flow was studied in Chapter 4 and, there, the approximate governing equation was written as

\[
\rho r \frac{d^2 \xi}{dt^2} + \rho \frac{1 - C_D}{C_D} \frac{d\xi}{dt} \frac{d\xi}{dt} + P_c S = P_0 S e^{i\omega t}
\]  

(6.1)

where \(\xi(t)\) is the displacement of the control volume in the cavity at the center point and \(C_D\) is the discharge coefficient of the orifice jet. With the turbulent boundary layer, it is hard to see the geometric center of the control volume, i.e., the center of the orifice. Instead, we define the spatially averaged displacement to investigate the flow features inside the orifice. Corresponding formula can be written as

\[
\eta(t) = \frac{1}{S \tau} \int_V \xi(x, t) dV,
\]

(6.2)

where \(S\) is the area of the orifice, \(\tau\) is the thickness of the facesheet and \(x\) denotes the point inside the orifice. Further, a random force is included to the governing equation and therefore we propose the extended fits as,

\[
\rho r S \frac{d^2 \eta}{dt^2} + \rho \frac{1 - C_D}{C_D} \frac{d\eta}{dt} \frac{d\eta}{dt} + P_c S = P_0 S e^{i\omega t} + \sum_j T(\omega_j) S e^{i(\omega_j t + \phi_j)}
\]

(6.3)
Figure 6.1: Wall pressure spectrum at the facesheet of the liner. (No sound) Legend: ‘−’ DNS data; ‘−−’ Goody’s empirical model (Goody, 2004); ‘· · ·’ Maestrello’s empirical model (Mastrello, 1969).

where $T_j$ represents the amplitude of the turbulent boundary layer fluctuation at frequency $\omega_j$ and $\phi_j$ is the random phase.

6.1.2 Modeling the turbulent boundary layer

Since the orifice is surrounded by a flat facesheet, one direct way to model the turbulence effect over the orifice is to assume the boundary-layer-indexed fluctuations over the orifice are the same as those that would be present over the solid wall. The study of the wall pressure spectra is relatively mature with plenty of useful existing models. A detailed summary and comparisons of the existing models of the wall pressure frequency spectra can be found in Hwang et al. (2009). Among all the existing models, we pick two: Goody’s empirical model (Goody, 2004) and Maestrello’s empirical model (Mastrello, 1969) and compare them with our existing DNS data. Figure 6.1 reveals the comparisons between the DNS data and the existing model predictions. It is observed that the DNS data agrees reasonably well with both Goody’s and Maestrello’s models but with the former one performing better. Therefore we will use Goody’s model for our reduced order model.

6.1.3 Modeling the discharge coefficient

Another missing piece of the puzzle is the discharge coefficient. The DNS studies in Chapter 5 reveal that the discharge coefficient differs significantly at the inflow and outflow cycles. Therefore, it is reasonable
to model them separately. Rogers & Hersh (1975) proposed that both the inflow and outflow discharge coefficient, namely $C_{Di}$ and $C_{Do}$, are strong functions of $v_i/U_\infty$ and $v_o/U_\infty$, where $v_i$ and $v_o$ are the core inflow and outflow velocity respectively. They proposed the 'lid model' with a power law relation for the inflow/outflow discharge coefficients. These formula are reasonable when $C_{Di}$ and $C_{Do}$ are small but fail when $v_i/U_\infty$ or $v_o/U_\infty$ exceeds 0.3 as the expressions yield a prediction over 1. Therefore, we propose the modification

$$C_{Di} = \beta_1 \tanh \left[ \alpha_1 \left( \frac{v_i}{U_\infty} \right)^{\gamma_1} \right] \tag{6.4}$$

$$C_{Do} = \beta_2 \tanh \left[ \alpha_2 \left( \frac{v_o}{U_\infty} \right)^{\gamma_2} \right] \tag{6.5}$$

where $v_i/U_\infty$ and $v_o/U_\infty$ goes as $(v_i/U_\infty)^{\gamma_1}$ or $(v_o/U_\infty)^{\gamma_2}$ when $v_i/U_\infty \ll 1$ or $v_o/U_\infty \ll 1$. When $v_i/U_\infty$ and $v_o/U_\infty$ is large, the prediction goes to a constant $\beta_1$ or $\beta_2$, meaning a maximum discharge coefficient is achieved. The parameters are obtained using least-square curve fitting. Using the incompressible Bernoulli equation, we can relate the inflow and outflow core velocity with the amplitude of the incident sound.

$$v_i = \sqrt{\frac{2 \times 10^{\text{SPL}/20-9.701}}{\rho_\infty}} \tag{6.6}$$

The mass conservation of the flow yields

$$v_i C_{Di} = v_o C_{Do} \tag{6.7}$$

$$v_o = \frac{C_{Do}}{C_{Di}} \sqrt{\frac{2 \times 10^{\text{SPL}/20-9.701}}{\rho}} \tag{6.8}$$

Therefore, $C_{Di}$ can be explicitly predicted by the amplitude of the incident sound and $C_{Do}$ is given in Eq. (6.5) implicitly. Figure 6.2 shows the comparison of the extended fit of the discharge coefficient to the measured velocity, for the inflow and outflow cycles separately. The new fits improve the predictive accuracy at higher sound pressure levels. It is also noteworthy that the DNS and experimental data of Rogers & Hersh (1975) are consistent at the cross-over point of $v_{i,o}/U_\infty \sim 3 \times 10^{-2}$. 

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6.2 Comparison with the DNS data

6.2.1 Spatially averaged velocity spectra

With the predicted $T_j$ in Eq. (6.3) using Goody’s model and the full cycle-averaged $C_D$ ($C_D = 0.5(C_{Di} + C_{Do})$), we can solve Eq. (6.3) numerically to get a spatially averaged velocity fluctuation and compare the model against the DNS data. Figure 6.3 shows the prediction of the spectra of the spatially averaged velocity fluctuations at 130 dB. The predictions show a decent match with the DNS data of the velocity fluctuations at the peak frequency (forcing frequency). The amplitudes at other frequencies are also reasonably consistent with the simulation data.
Figure 6.4: Spatially averaged velocity fluctuations at the center depth of the liner neck. Legend: ‘−’ present simulation data; ‘−−’ Reduce-order model prediction.
6.2.2 Backplate pressure spectra

From the detailed derivation in Appendix D, the total pressure in the cavity can be evaluated as the sum of the incident sound created by the impinging acoustic wave and the internally-reflected sound,

\[ p(r, z, t) = p_i(r, z, t) + p_r(r, z, t) \]

\[ = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \int_{r=0}^{\infty} \hat{\beta}(r, \omega_m) J_0 \left( \frac{\hat{j}_{1n} r}{L} \right) r \, dr \, J_0 \left( \frac{\hat{j}_{1n} r}{L} \right) \left( e^{i\mu_n z} + e^{-i\mu_n (z+2H)} \right) e^{i\omega_m t} d\omega_m. \]  

(6.9)

Note that the cut-off frequency for this cavity is approximately 20 kHz, which means only those acoustic waves with \( f \geq 20 \) kHz can have higher modes. However, the pressure spectra reveal that the fluctuations above 10 kHz are very weak. This is consistent with the uniform planar pressure distribution at the backplate of the cavity found in DNS studies. Hence, it is quite reasonable for us to assume only the fundamental mode (planar mode) propagates in the cavity. With this approximation we note that \( j_{10} = 0 \), \( \mu_n(\omega_m) = k_m = \frac{\omega_m}{c} \).

At \( z = -H \), we have the backplate pressure time-history being predicted to be,

\[ p(r, -H, t) = 2\rho_0 \sigma \int_{-\infty}^{+\infty} \hat{\zeta}(\omega_m) e^{-ik_m H} \frac{e^{i\omega_m t}}{ik_m} d\omega_m. \]  

(6.10)

Comparison of the pressure spectra at the backplate are shown in Figures 6.5 and 6.6. While the peak value agrees and the spectra below 10 kHz agrees pretty well, discrepancies can be found at higher frequencies. For most of the cases, the model over predicts the level of the spectra in this frequency range but the only exceptions are at 150 dB or 160 dB, where the strong jet may create secondary flow and causing higher pressure modes at the backplate. However, these factors were not included in this time domain model.

6.2.3 Impedance prediction

The reduced order model predictions of the liner impedance are shown in Figure 6.7 and are compared with the DNS predictions. It is observed that at 130 dB the predictions of the turbulent boundary layer cases exhibit a very good comparison with the DNS data, both in resistance and reactance prediction. At 3.0 kHz, the model does a better job of the resistance prediction than the reactance prediction for all the cases. Also note that an over-prediction of the reactance is found with increasing discrepancy with the DNS predictions as the SPL increases. While the discrepancies are unknown at this point, improvements of the liner model might be made via a more accurate modeling of the discharge coefficient.
Figure 6.5: Comparisons of the backplate pressure spectra. (130 dB) Legend: ‘−’ present simulation data; ‘--’ Reduced order model using Eq. (6.10).

(a) 130 dB, 2.2 kHz (b) 130 dB, 2.6 kHz (c) 130 dB, 3.0 kHz

Figure 6.6: Comparisons of the backplate pressure spectra. (3.0 kHz) Legend: ‘−’ present simulation data; ‘--’ Reduced order model using Eq. (6.10).

(a) 130 dB, 3.0 kHz (Turbulent BL) (b) 140 dB, 3.0 kHz (Turbulent BL)

(c) 150 dB, 3.0 kHz (Turbulent BL) (d) 160 dB, 3.0 kHz (Turbulent BL)
Figure 6.7: Impedance Predictions. Legend: solid symbols, DNS predictions; open symbols, 1-D Reduce-order Model predictions. ‘♦’ No grazing flow; ■’ Laminar boundary layer; •’ Turbulent boundary layer.
6.2.4 Summary

The model discussed in this chapter points a straightforward path to the impedance prediction of the honeycomb liner under a turbulent boundary layer. The kernel of the model lies in the solution of the second order ordinary differential equation. Unlike the reduced-order model in quiescent condition, two additional factors should be taken into consideration: (1) the presence of the turbulent boundary layer is able to add more uncertainties to the flow motions; (2) the grazing flow, regardless of its condition (laminar to turbulent) can strongly influence the discharge coefficient. Note that under a laminar grazing flow, the former factor can be neglected as there is no turbulent motion while under a turbulent grazing flow, the choice of the empirical models from the well-established turbulent boundary layer studies is a reasonable choice. On the other hand, the grazing flow, along with the acoustic field, can alter the discharge coefficient which eventually affects the non-linearity of the governing equation for the reduced-order model. Unfortunately, studies of the discharge coefficient of the orifice under grazing flow conditions are relatively immature. Moreover, the only existing empirical/semi-empirical models are not universally valid and typically fails at large $v_i/U_\infty$ values. Therefore, an extension of the current empirical model was proposed with a comparison between both the experimental and DNS data. The extension of the empirical formula keeps the original feature of the prediction at low $v_i/U_\infty$ but yields a much better prediction at high $v_i/U_\infty$ values. With the incorporation of the discharge coefficient and turbulence modeling, the model for the liner impedance is closed and is capable of yielding reasonable predictions. Nevertheless, this is never the end. Further investigations of the physics behind the flow-sound-orifice interaction is useful not only for better models of the discharge coefficient but also helps to move on to the study of multiple orifices under the grazing flow.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

Through simulation of the compressible Navier-Stokes equations, details of the acoustically-excited flow through a circular orifice are presented. The orifice, which has a diameter of 0.99 mm, a 0.64 mm thickness, and is backed by a hexagonal cavity of depth 38.1 mm, is a small portion of an acoustic liner commonly used on aeroengines for noise reduction and tested at the NASA Langley Research Center. The orifice is subjected to incident acoustic waves ranging from 130 dB to 160 dB at fixed 3 kHz frequency, and from 1.5 kHz to 3.0 kHz at fixed 130 dB amplitude. The Helmholtz resonance frequency of the liner is 1.1 kHz. These numerical investigations include no-flow, laminar grazing flow and turbulent grazing flow.

7.1.1 Acoustic liners without grazing flow

1. The simulations predict an acoustic impedance at 130 dB that matches experimental data, and is consistent with empirical correlations.

2. From the simulation databases it is observed that the in-orifice boundary layers are critical in affecting the non-linearity of the liner. Depending on the incident sound parameters secondary vorticity is found within the boundary layers which thickens them, leading to a reduced discharge coefficient and increased non-linearity of behavior.

3. When measured at the mid-orifice the simulation-predicted discharge coefficient varies with phase of the incident sound with an average value that ranges from 0.86 to 0.69. The flow into and out of the cavity is highly symmetrical except at the highest amplitude of 160 dB.

4. The scattered sound produced the orifice ranged between 20 and 30 dB below the incident sound amplitude.

5. Two simple, but related, time domain models for the orifice fluid velocity were constructed. They differed in their assumed shape of an in-orifice velocity profile, based on information from the simulations.
The model which used a hyperbolic tangent profile but had the correct boundary layer displacement thickness, as measured from the simulation, performed better over the parameter range than did one which used the velocity profile from oscillating pipe flow, indicating the importance the orifice boundary layers have on the acoustic impedance. These results suggest that future time domain models focus on accurately modeling the boundary layer thickness rather than on modeling end corrections or other quantities.

7.1.2 Acoustic liners with grazing flow

When a grazing flow is added, several additional conclusions are relevant:

1. Vorticity and velocity visualizations show that the grazing boundary layer can significantly alter the flow field inside the orifice when compared to no-flow cases. The state of the boundary layer is also important.

2. The discharge coefficient is greatly affected by the grazing flow with an increasing trend found with increase of the SPL. Moreover, the outflow discharge coefficient is greater than the inflow discharge coefficient.

3. The additional total drag caused by the interaction among the sound, boundary layer and orifice has three main parts. At low SPLs the wall shear stress dominates the total drag while pressure difference from the neck of the orifice and the cavity increases as the SPL level goes up. Further, external drag induced by the boundary layer might be significantly increased at high SPLs.

4. Impedance prediction shows a decent agreement with experimental data, depending on the impedance eduction method used.

5. The time domain model for the impedance prediction under turbulent boundary layer was extended from the no-flow model. The new model contained the turbulence effect along with a semi-empirical modeling to predict the discharge coefficient.

7.2 Future Work

Progressions of this work can be extended into the following aspects:

- **Broadband noise investigation**
  Previous studies mainly focused on the effect caused by the tonal sound at different SPLs and frequencies. The liners behave differently at different SPLs and frequencies. In a real situation, the jet engine
noise has a wide broadband distribution. It would be interesting to investigate the liner behavior under broadband noise at different OASPLs. The optimal design of the liner would also be a quite interesting topic for future studies.

- **Multiple orifice liner simulation**

  Another critical question that needs to be addressed is the hydrodynamic interactions between the neighboring orifices. In this work, the orifices are assumed to work independently. The eventual comparisons were made under linear hypothesis by multiplying the liner porosity. However, higher porosity liner usually means a shorter distance between different orifices. The interactions between the orifices will certainly increase. Further, the additional grazing flow may create a significant wake region in which other orifices can be strongly affected. Note that previous conclusions points out that single orifice liner simulations agrees well with both the experimental and empirical models while the results start to deviate when there is grazing flow. Although several possible reasons has been found, the direct evidence remains unclear.

- **Boundary layer effect investigations**

  Currently, the simulation is composed of a fixed freestream Mach number temporal turbulent boundary layer with a relatively low Reynolds number. A series of improvements can be made such parameter studies as the boundary layer thickness, the freestream Mach number and the Reynolds number. Studies of these effects would probably reveal a better idea of the influence of the boundary layer towards the liner.

- **Prediction of the discharge coefficient**

  The discharge coefficient plays a key role in reduced order liner modeling. However, an accurate prediction of the discharge coefficient, especially under grazing flow, could be a challenging but remarkable job. In this work, the discharge coefficient was modeled semi-empirically using DNS data. If possible, a deep investigation of the discharge coefficient via analytical methods would be a great breakthrough in liner reduced order models.
Appendix A

Numerical Simulation of a Mach 0.5 Temporal Turbulent Boundary Layer

A.1 Introduction

The actual work condition for these acoustic liners is under a high speed grazing turbulent boundary layer. To achieve this goal, the ultimate simulation will couple the sound induced flow via the orifice with a fully developed compressible turbulent boundary layer. Therefore, one of the prerequisites of our upcoming liner simulations is to configure a fully developed turbulent boundary layer with desired conditions. In this chapter a Mach 0.5 compressible temporal turbulent boundary layer is generated. Data from NASA Langley requires the desired momentum thickness to be 2.03 mm which corresponds to \( \text{Re}_\theta = 23100 \) for air at 15\(^\circ\)C. With current computational resources, this Reynolds number is too high to yield a well-resolved turbulent boundary layer via DNS. Thus, the Reynolds number (\( \text{Re}_\theta \)) is decreased to around one-tenth of its original value (\( \text{Re}_\theta \sim 2310 \)). The DNS data of the turbulent boundary layer will be used as an initial condition of the background flow in the 3-D honeycomb liner simulation.

A.2 Simulation Setup

To generate a temporal, fully developed turbulent boundary layer, a rectangular computational domain is constructed. Let \( x, y, \) and \( z \) denote the streamwise wall normal and spanwise directions, respectively. The dimension in streamwise and spanwise direction are \( L_x = 300d \) and \( L_z = 80d \) respectively, where \( d \) is the orifice diameter. Periodic boundary conditions are applied in both the \( x- \) and \( z- \) directions. In the wall normal direction, \( L_y \) is set to approximately 150d. Isothermal no-slip boundary condition is applied for the wall \( (y = 0) \). At the top of domain, a sponge zone with \( 4\delta_{99} \approx 80d \) is imposed to absorb the acoustic waves and flow structures. In future simulations, the sponge zone will also impose the normal planewave travelling towards the boundary layer. The grid construction uses structure mesh and the detailed grid information is provided in table A.1. A uniform mesh spacing is used in both \( x- \), \( z- \) and the first 11 points in the \( y- \) direction. A smooth grid stretching function is introduced to generate non-uniform grid spacing in the \( y \)
direction. The temporal turbulent boundary layer starts off from unstable boundary layer eigenfunctions. (See Malik (1990).)

A.3 Results and Analysis

A.3.1 Flow visualization

Snapshots of instantaneous vorticity are plotted in figure A.1 at different times. Initially, the boundary layer stays laminar and the isosurface of the vorticity stays as a flat sheet. (See figure A.1(a).) The initial Tollmien-Schlichting and oblique disturbances are distributed randomly in the computational domain. (See figure A.1(b).) As the instability disturbances grow, spanwise variations start to appear, resulting in Λ vortices. Meanwhile, due to the their unstable nature, these vortices will break down and form local turbulent spots in the form of hairpin-like eddies. (See figure A.1(c).) These turbulent spots appear randomly and will spread downstream and, as a result, some turbulent spots will inevitably coalesce. (See figure A.1(d).) The process continues until a fully developed turbulent boundary layer appears. (See Figs. A.1(f) to A.1(o).)

The time history of the momentum, displacement and visual thickness of the boundary layer thickness is shown in figure A.2. When the boundary layer stays laminar, the thickness remains relatively thin and grows as $\sqrt{\nu t}$. In the process of transition, the boundary layer thickens at a faster rate. When the boundary layer become into a fully developed turbulent boundary layer, the boundary layer has a shape factor of $H = \delta^*/\theta \approx 1.4$. For the future 3-D liner simulation coupled with the turbulent boundary layer, the desired ratio of the momentum thickness to the diameter of the aperture is $\theta/d = 2.06$ and is marked in the figure. Further quantitative analysis at this condition is given below.

A.3.2 Mean velocity profile

At $Re_\theta = 2310$, the mean velocity profiles are shown in figure A.3, and compared against the well-known one-seventh law. A more common way to express the mean turbulent velocity profile is shown in figure A.4 where the coordinates are defined as $y^+ = u_\tau y/\nu$ and $u^+ \equiv u/u_\tau$ where $u_\tau \equiv \sqrt{w/\rho}$. Although $M_\infty = 0.5$ is only weakly compressible, the Van Driest transformation is applied for better comparison with
Figure A.1: Vorticity magnitude at different time and momentum thickness. (Colored by the local Mach number)
Figure A.2: Time history of the momentum, displacement and visual thickness of a Mach 0.5 boundary layer. Legends: ‘−−’ Visual thickness $\delta_{99}$; ‘− · −’ Displacement thickness $\delta^*$; ‘−’ Momentum thickness $\theta$; ‘○’ marked at $U_\infty t/d = 1738$ where ratio of the momentum thickness to the hole diameter $\theta/d = 2.06$.

the incompressible turbulent boundary layer theory and $u^+_c$ is introduced as

$$u^+_c = \int_0^{U^+} \frac{\langle \rho \rangle}{\rho_w} \sqrt{\langle u^i \rangle} \, du^+$$  \hspace{1cm} (A.1)

In the remaining part of the work, $u^+_c$ will be replaced by $u^+$ unless otherwise noted. Figure A.4 reveals that the present DNS data matches the experimental and previous simulation data well. Inside the viscous sublayer, $y^+ \leq 5$, the velocity profile follows the trend $u^+ = y^+$. The data also coincides with the previous experimental and numerical results in the log-law region ($y^+ \in (25, 200)$). In the outer region, the boundary layer starts to deviate from the log-law which shows a turbulent boundary layer wake. The location of the wake mostly depends on the $Re_\theta$.

### A.3.3 Turbulent stresses

Measurements of the streamwise, wall-normal, and spanwise stresses are plotted in figure A.5(a) to figure A.5(c). The Reynolds stresses are always normalized by inner scales $u_\tau$ and Van Driest transformation is again applied,

$$\frac{\langle \rho \rangle \langle u^i_+ u^j_+ \rangle}{\rho_w u^+_\tau} = \frac{\langle \rho \rangle \langle u^i_\tau u^j_\tau \rangle}{\rho_w u^+_\tau} \quad i = 1, 2, 3.$$  \hspace{1cm} (A.2)
Figure A.3: Spatially ensembled average of the local Mach number. Legends: ‘—’ Visual thickness $\delta_{99}$; ‘· · ’ Displacement thickness $\delta^*$; ‘⋯’ Momentum thickness $\theta$. ($U_\infty t/d = 1308$)

Figure A.4: Semilog plot of the mean velocity profiles. Legends: ‘—’ Present DNS data ($Re_\theta = 2310$); ‘—’ high resolution LES data from Wang & Wang (2012) ($Re_\theta = 1770$); ‘· · ’ high resolution LES data from Wang & Wang (2012) ($Re_\theta = 3550$); ‘—’ Experiment of Degraaff & Eaton (2000) ($Re_\theta = 1430$); ‘—’ Experiment of Degraaff & Eaton (2000) $Re_\theta = 2900$; ‘—’ Experiment of DeGraaff and Eaton $Re_\theta = 5200$. 

\[ u^+ = y^+ \]
\[ u^+ = \frac{1}{\kappa} \log y^+ + C \]
\[ \kappa = 0.41, C = 5.2 \]
Although the previous experimental and simulations has different Reθ with the current simulation, profiles of \( \langle u'^2 \rangle \) reveals a very good match with both of them in the viscous sublayer region. Outside the viscous sublayer, the profiles have a strong dependence on Reθ. A peak for \( \langle u'^2 \rangle \) profiles is always found close to the wall and the maximum value given by the present DNS simulation is 8.06 and the location is found to be \( y^+ = 14.05 \). According to the empirical fit for the streamwise normal stress by Klewicki \textit{et al.} (1994), the magnitude of the peak value depends on Reθ and follows:

\[
\langle u'^2 \rangle_{\text{max}} = 8.5 \times 10^{-9} \text{Re}_\theta^2 + 4.8 \times 10^{-4} \text{Re}_\theta^2 + 6.86. \tag{A.3}
\]

Eq. (A.3) yields \( \langle u'^2 \rangle_{\text{max}} = 8.01 \) for the present DNS condition Reθ = 2310. Mochizuki \& Nieuwstadt (1996) found the peak value location following the expression

\[
y^+(\langle u'^2 \rangle_{\text{max}}) = 1.7 \times 10^{-4} \text{Re}_\theta + 14.4 \tag{A.4}
\]

which yields \( y^+(\langle u'^2 \rangle_{\text{max}}) = 14.79 \) for the present DNS condition. Therefore, both the maximum value and the location where the maximum value is achieved have a good match with the empirical data. Erm \& Joubert (1991) suggests that the wall normal stress also has a peak value which depends on Reθ and the peak location deviates away from the wall as increases Reθ. The behavior given by the present DNS data are also consistent with both the experimental and previous numerical results (See figure A.5(b)). Although there are no experimental data for the spanwise normal stress, comparison made with the previous numerical results also suggests a good consistency in figure A.5(c). The Reynolds shear stress is shown in figure A.5(d) and its behavior is similar to both of the wall normal and spanwise normal stress. Note the maximum value achieves approximately 0.94 and its location is at \( y^+ \approx 80 \). The Reynolds shear stress never exceeds unity and goes to zero either very close to the wall or far away in the outer region due to the no-slip boundary condition or no turbulence effect respectively.

The total shear stress considers both the effect of viscous and Reynolds stress and plotted in figure A.6. The DNS data reveal that the ratio of the total shear stress to the wall shear stress is very close to unit when \( y^+ \leq 100 \). Also note that the viscous shear stress and the Reynolds shear stress contribute almost equally to the total shear stress at \( y^+ \approx 15 \). The viscous shear stress dominates very close to the wall but decays very rapidly when \( y^+ \) increases. The Reynolds shear stress, however, increases as \( y^+ \) increases and reaches its maximum value at \( y^+ \approx 80 \) and then decays at almost the same rate as the total shear stress. These trend matches the experiment very well except the small discrepancy of the total wall shear stress near the wall, where the experimental data slightly fluctuates between the unit.
Figure A.5: Profiles of Reynolds stresses normalized by the friction velocity. Legends: ‘−’ Present DNS data ($\text{Re}_\theta = 2310$); ‘−⋯’ high resolution LES data from Wang & Wang (2012) ($\text{Re}_\theta = 1770$); ‘−⋅−’ high resolution LES data from Wang & Wang (2012) ($\text{Re}_\theta = 3550$); ‘−△−’ Experiment of Degraaff & Eaton (2000) ($\text{Re}_\theta = 1430$); ‘−○−’ Experiment of Degraaff & Eaton (2000) ($\text{Re}_\theta = 2900$); ‘−♦−’ Experiment of Degraaff & Eaton (2000) ($\text{Re}_\theta = 5200$).
A.3.4 Auto-correlations

To show the correlations in terms of the streamwise and spanwise directions, the two-point correlation functions in \(x\)– and \(z\)– direction are computed using

\[
R_{\alpha\alpha}(\delta_x) = \sum_{k=1}^{N_x-1} \langle \alpha_k \alpha_{k+m} \rangle, \quad m = 0, 1, \cdots, k - 1
\]

(A.5)

\[
R_{\alpha\alpha}(\delta_z) = \sum_{k=1}^{N_z-1} \langle \alpha_k \alpha_{k+n} \rangle, \quad n = 0, 1, \cdots, k - 1
\]

(A.6)

where \(N_x\) and \(N_z\) are the numbers of the grid points in \(x\) and \(z\) direction and \(\alpha\) represents the fluctuations of one of the primitive variables \(\rho, u, v, w, p\). \(\Delta x\) and \(\Delta z\) represents the uniform grid spacing in \(x\) and \(z\) direction and we define \(\delta_x = m\Delta x\), \(\delta_z = n\Delta z\). The corrections are non-dimensionalized by \(R_{\alpha\alpha}(0)\) and plotted in figure A.7. The figure reveals that the correlations of all these variables decay rapidly towards zero when \(\delta_x\) or \(\delta_z\) exceeds \(2\delta_{99}\). Results confirm the computation domain is sufficiently large and the variables are decorrelated over a distance more than \(\delta_{99}\).

A.3.5 1-D power spectra

The 1-D power spectra of all the primitive variables \((\rho, u, v, w, p)\) in both streamwise and spanwise direction will be helpful for further investigation of the accuracy of the present simulation. Three different \(y^+\) locations \((y^+ = 10, y^+ = 25\) and \(y^+ = 53\)) are selected and the spectra are computed in the following way:

\[
E_{\alpha\alpha}(k_x\Delta x) = 1 + 2 \sum_{k_r=1}^{(N_x-1)/2} R_{\alpha\alpha}(k_r\Delta x) \cos \left( \frac{2\pi mk_r}{N_x-1} \right) \quad m = 0, \cdots, (N_x - 1)/2,
\]

(A.7)

\[
E_{\alpha\alpha}(k_z\Delta z) = 1 + 2 \sum_{k_r=1}^{(N_z-1)/2} R_{\alpha\alpha}(k_r\Delta z) \cos \left( \frac{2\pi nk_r}{N_z-1} \right) \quad n = 0, \cdots, (N_z - 1)/2.
\]

(A.8)

where \(k_x\Delta x = n/(N_x - 1)\) and \(k_x\Delta x = n/(N_x - 1)\). The spectra of all the variables in \(x\) direction shown in figure A.8 reveals a monotone decrease. The spectrum of \(E_{vv}\) in \(z\) direction, however, reveals that the peak achieves \(k_z\delta_{99} \in (20, 100)\).
Figure A.7: Correlations of density, $u, v, w$ velocity and pressure in $x$ and $z$ directions respectively. ($\theta/d = 2.06$) Legends: ‘$-$’ $y = 10y^+$, ‘$--$’ $y = 25y^+$, ‘$\cdot \cdot \cdot$’: $y = 53y^+$. (Re$\theta = 2310$)
Figure A.8: Spectra of density, $u$, $v$, $w$ velocity and pressure in $x$ and $z$ directions respectively. ($\theta/d = 2.06$)

Legends: '−−' $y = 10y^+$, '−−' $y = 25y^+$, '−−−' $y = 53y^+$. 
Appendix B

Grid Refinement

The 150 dB and 160 dB, 3.0 kHz, cases were investigated using two grids with different levels of refinement to determine the grid sensitivity of the results shown. The baseline grid contained approximately 8.7 million grid points, as documented in the main text, while the refined mesh included over 22.4 million, with most of the additional points located within a circular cylinder encompassing the orifice and extending 16 diameters above and below the facesheet. The refined grid simulations were run to statistical stationarity and the near-orifice flow quantities were compared to the baseline. The 150 dB liner comparison, which showed the most sensitivity to the grid, is shown in figure B.1 for the orifice center pressure and velocity traces. There is a 5% difference in the peak levels of $p'$ and $w$ and a reduction in the harmonic pressure component.

![Figure B.1: Grid refinement study on orifice center pressure (a) and velocity (b) for the 150 dB, 3 kHz liner. Legend: ‘−’, baseline mesh; ‘− o −’, refined mesh.](image-url)
Appendix C

Cavity Pressure Derivation (I)

In this section, the formula for evaluating the cavity pressure is derived. Here are the assumptions required:

• Density is constant inside and outside the cavity. \( \rho \equiv \rho_0 \).

• The flow field is axisymmetric. (No variation in \( \theta \) direction) \( \frac{\partial}{\partial \theta} = 0 \).

• The pressure fluctuation has harmonic time dependence, i.e., \( p(r, z, t) = \hat{p}(r, z)e^{-i\omega t} \).

Since the flow field is axisymmetric and pressure equation satisfies wave equation in the cylindrical coordinates,

\[
\frac{\partial^2 p}{\partial t^2} - c^2 \nabla_c^2 p = 0, \quad \text{where} \quad \nabla_c^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.
\]

Now let \( p = \hat{p}(r, z)e^{i\omega t} \), and substitute it into the wave equation, which yields the Helmholtz equation,

\[ \nabla_c^2 \hat{p} + k^2 \hat{p} = 0, \]

where \( k \) is the wavenumber satisfying the dispersion relation \( k = \frac{\omega}{c} \).

Apply the method of separation of variables again and let \( \hat{p}(r, z) = R(r)Z(z) \),

\[ Z''(z) - k^2 Z(z) = 0 \Rightarrow Z_n(z) = \cosh[k_n(H + z)] \]

\[ r^2 R''(r) + r R'(r) + (k^2_z + k^2) r^2 R(r) = 0 \Rightarrow R_n(r) = J_0(\tilde{k}_n r). \]

where \( \tilde{k}_n \) satisfies \( \tilde{k}_n^2 = k^2 + k^2_z \) and \( J_0 \) is the Bessel’s function of the first kind of order zero.

Apply the lateral no-penetration wall boundary condition,

\[ \frac{\partial p}{\partial r} = 0 \quad \text{at} \quad r = L, \]
it yields

\[ J'_0(\tilde{k}_n L) = 0 \quad \Rightarrow \quad J_1(\tilde{k}_n L) = 0. \]

where \( \tilde{k}_n = \frac{j_{1n}}{L} \) where \( j_{1n} \) is the \( n^{th} \) zero of the Bessel function of the first kind of order 1. Note that \( j_{10} = 0 \). Therefore,

\[ k^2_z = \tilde{k}^2_n - k^2 = \left( \frac{j_{1n}}{L} \right)^2 - k^2 \quad n = 0, 1, 2, 3, \ldots. \]

Therefore,

\[ \hat{p}(r, z) = \sum_{n=0}^{\infty} C_n(t) J_0(\frac{j_{1n}}{L} r) \cosh[k_z(H + z)] \]

\[ p(r, z, t) = \hat{p}(r, z)e^{i\omega t} = \sum_{n=0}^{\infty} C_n(t) J_0(\frac{j_{1n}}{L} r) \cosh[k_z(H + z)] \]

The coefficients \( C_n(t) \) are determined by applying the boundary condition on the moving surface

\[ \frac{\partial p}{\partial z} = -\rho_0 \frac{\partial v_z}{\partial t} \quad \text{at} \quad z = 0. \]

Now let \( v_z = \dot{\xi} V(r) H(d/2 - r) \) where \( V(r) \) is the \( z \)-velocity profile and \( H \) is the Heaviside function and substitute \( \hat{p} \) and \( v_z \) into the equation above, it yields

\[ \sum_{n=0}^{\infty} C_n(t) k_z J_0(\frac{j_{1n}}{L} r) \sinh(k_z H) = \beta(r, t)e^{-i\omega t}. \]

where \( \beta(r, t) = -\rho_0 \dot{\xi} V(r) H(d/2 - r) \).

Apply the orthogonality property of the Bessel functions, we get

\[ C_m(t) = \frac{\int_{r=0}^{a} \beta(r, t) J_0(\frac{j_{1m}}{L} r) r \, dr}{k_z \sinh(k_z H) \int_{r=0}^{L} J_0^2(\frac{j_{1m}}{L} r) r \, dr} e^{-i\omega t}. \]

\[ p(r, z, t) = \sum_{m=0}^{\infty} C_m(t) J_0(\frac{j_{1m}}{L} r) \cosh[k_z(H + z)]e^{i\omega t}. \]
For simplicity, the final expression can be written as

\[ p(r, z, t) = \sum_{m=0}^{\infty} D_m(t) J_0 \left( \frac{j_{1m}}{L} r \right) \cosh[k_z(H + z)], \]

where

\[ D_m(t) = \frac{\int_{r=0}^{a} \beta(r, t) J_0 \left( \frac{j_{1m}}{L} r \right) r \, dr}{k_z \sinh(k_z H) \int_{r=0}^{L} J_0^2 \left( \frac{j_{1m}}{L} r \right) r \, dr}. \]
Appendix D

Cavity Pressure Derivation (II)

In this section, the formula for evaluating the cavity pressure with broadband noise is derived. Here are the assumptions required:

- Density is constant inside and outside the cavity. $\rho \equiv \rho_0$.
- The flow field is axisymmetric. (No variation in $\theta$ direction) $\frac{\partial}{\partial \theta} = 0$.
- The pressure fluctuation has harmonic time dependence, i.e., $p(r, z, t) = \int_{-\infty}^{+\infty} \hat{p}_m(r, z) e^{i\omega_m t} d\omega_m$.
- Split the total pressure $p$ into the incident pressure $p_i$ and scattered pressure $p_s$, where $p = p_i + p_s$.

Since the flow field is axisymmetric, the incident pressure satisfies wave equation in the cylindrical coordinates,

$$\frac{\partial^2 p_i}{\partial t^2} - c^2 \nabla_c^2 p_i = 0,$$
where $\nabla_c^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

Now let $p_i(r, z, t) = \int_{-\infty}^{+\infty} \hat{p}_m(r, z) e^{i\omega_m t} d\omega_m$, and substitute it into the wave equation, which yields,

$$\int_{-\infty}^{+\infty} (\nabla_c^2 \hat{p}_m + k_m^2 \hat{p}_m) e^{i\omega_m t} d\omega_m = 0,$$

where $k_m$ is the wavenumber satisfying the dispersion relation $k_m = \frac{\omega_m}{c}$.

Apply the method of separation of variables again and let $\hat{p}_m(r, z) = R_m(r) Z_n(z)$,

$$Z_n''(z) + \mu_n^2(\omega_m) Z_n(z) = 0 \implies Z_n(z; \omega_m) = e^{i\mu_n(\omega_m)z}$$
$$r^2 R_m''(r) + r R_m'(r) + [-\mu_n^2(\omega_m) + k_m^2] r^2 R(r) = 0 \implies R_n(r; \omega_m) = J_0(k_n(\omega_m)r).$$
where \( \tilde{k}_n(\omega_m) \) satisfies \( \tilde{k}_n^2(\omega_m) = k_m^2 - \mu_n^2(\omega_m) \) and \( J_0 \) is the Bessel’s function of the first kind of order zero. Apply the lateral no-penetration wall boundary condition,

\[
\frac{\partial p_i}{\partial r} = 0 \quad \text{at} \quad r = L,
\]

it yields

\[
J_0(\tilde{k}_n(\omega_m)L) = 0 \quad \Rightarrow \quad J_1(\tilde{k}_n(\omega_m)L) = 0.
\]

where \( \tilde{k}_n(\omega_m) = j_{1n}/L \) where \( j_{1n} \) is the \( n^{th} \) zero of the Bessel function of the first kind of order 1. Note that \( j_{10} = 0 \). Therefore,

\[
\mu_n^2(\omega_m) = k_m^2 - \tilde{k}_n^2(\omega_m) = k_m^2 - \left( \frac{j_{1n}}{L} \right)^2 \quad n = 0, 1, 2, 3, \cdots
\]

Therefore,

\[
\hat{p}_m(r, z) = \sum_{n=0}^{\infty} \alpha_n(\omega_m) J_0\left( \frac{j_{1n}}{L} r \right) e^{i\mu_n(\omega_m)z}
\]

\[
p_s(r, z, t) = \int_{-\infty}^{+\infty} \hat{p}_m(r, z)e^{i\omega_m t}d\omega_m
\]

\[
= \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \alpha_n(\omega_m) J_0\left( \frac{j_{1n}}{L} r \right) e^{i\mu_n(\omega_m)z} e^{i\omega_m t}d\omega_m
\]

The coefficients \( \alpha_n(\omega_m) \) are determined by applying the boundary condition on the moving surface

\[
\frac{\partial p}{\partial z} = -\rho_0 \frac{\partial v_z}{\partial t} \quad \text{at} \quad z = 0.
\]

Note that we replace \( \eta \) with \( \xi \) where \( \eta \) is considered a spatial averaged displacement. Therefore, the velocity profile modeling is not required. \( (V(r) \equiv 1) \). Now let \( v_z = \eta(t)H(d/2 - r) \) where \( H \) is the Heaviside function.

Substitute \( \hat{p} \) and \( v_z \) into the equation above, it yields

\[
\int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \alpha_n(\omega_m) \mu_n(\omega_m) J_0\left( \frac{j_{1n}}{L} r \right) e^{i\omega_m t}d\omega_m = \beta(r, t).
\]
where $\beta(r, t) = -\rho_0 \ddot{\eta}(t) H(d/2 - r)$.

Now we take the Fourier transform of $\beta(r, t)$ and $\ddot{\eta}(t)$ with respect to $t$, we have

$$\beta(r, t) = \int_{-\infty}^{+\infty} \hat{\beta}(r, \omega_m) e^{i\omega_m t} d\omega_m$$

$$\ddot{\eta}(t) \equiv \hat{\ddot{\eta}}(\omega_m) = \int_{-\infty}^{+\infty} \hat{\ddot{\eta}}(\omega_m) e^{i\omega_m t} d\omega_m$$

By direct comparing, we have

$$\hat{\beta}(r, \omega_m) = -\rho_0 H(d/2 - r) \hat{\ddot{\eta}}(\omega_m)$$

$$\sum_{n=0}^{\infty} i \alpha_n(\omega_m) \mu_n(\omega_m) J_0 \left( \frac{j_1 n}{L} r \right) = \hat{\beta}(r, \omega_m)$$

Apply the orthogonality property of the Bessel functions, we get

$$\alpha_n(\omega_m) = \frac{\int_{r=0}^{a} \hat{\beta}(r, \omega_m) J_0 \left( \frac{j_1 n}{L} r \right) r \, dr}{i \mu_n(\omega_m) \int_{r=0}^{L} J_0^2 \left( \frac{j_1 n}{L} r \right) r \, dr}.$$ 

$$p_i(r, z, t) = \int_{-\infty}^{+\infty} \int_{r=0}^{a} \hat{\beta}(r, \omega_m) J_0 \left( \frac{j_1 n}{L} r \right) r \, dr \frac{J_0 \left( \frac{j_1 n}{L} r \right)}{J_0^2 \left( \frac{j_1 n}{L} r \right)} J_0 \left( \frac{j_1 n}{L} r \right) e^{i \mu_n(\omega_m) z} e^{i\omega_m t} d\omega_m.$$ 

Also note the boundary condition at the backplate of the cavity, we have

$$\frac{\partial p}{\partial z} = \frac{\partial p_i}{\partial z} + \frac{\partial p_s}{\partial z} = 0 \quad \text{at} \quad z = -H.$$ 

Therefore, the scattered pressure can be written as

$$p_s(r, z, t) = \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \int_{r=0}^{a} \hat{\beta}(r, \omega_m) J_0 \left( \frac{j_1 n}{L} r \right) r \, dr \frac{J_0 \left( \frac{j_1 n}{L} r \right)}{J_0^2 \left( \frac{j_1 n}{L} r \right)} J_0 \left( \frac{j_1 n}{L} r \right) e^{-i \mu_n(\omega_m)(z+2H)} e^{i\omega_m t} d\omega_m.$$
The total pressure in the cavity can be evaluated as

\[ p(r, z, t) = p_i(r, z, t) + p_s(r, z, t) \]

\[ = \int_{-\infty}^{+\infty} \int_{r=0}^{r=a} \hat{\beta}(r, \omega_m) J_0 \left( \frac{j_{1n} r}{L} \right) r \, dr \int_{r=0}^{\infty} \hat{\beta}(r, \omega_m) J_0 \left( \frac{j_{1n} r}{L} \right) r \, dr \left( e^{i\mu_n z} + e^{-i\mu_n (z+2H)} \right) e^{i\omega_m t} d\omega_m. \]

Also note the cut-off frequency for this cavity is approximately 20 kHz, which means only those acoustic waves with \( f \geq 20 \) kHz can have higher modes. This is consistent with the dominant planar pressure at the backplate of the cavity found in DNS. Hence, it is quite reasonable for us to assume only the fundamental mode (planar mode) propagates in the cavity. Note \( j_{10} = 0, \mu_n(\omega_m) = k_m = \frac{\omega_m}{c}. \)

At \( z = -H \), we have

\[ p(r, -H, t) = 2\rho_0 \sigma \int_{-\infty}^{+\infty} \frac{\hat{\chi}(\omega_m)}{i k_m} e^{-ik_m H} e^{i\omega_m t} d\omega_m. \]  

(D.1)


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