SAFE AND RELIABLE DYNAMIC COVERAGE CONTROL FOR MULTI-AGENT SYSTEMS

BY

GÖKHAN MEHMET ATINÇ

DISSERTATION

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Doctoral Committee:

Professor Geir E. Dullerud, Chair
Associate Professor Dušan M. Stipanović, Director of Research
Professor Petros G. Voulgaris, Co-Director of Research
Professor Naira Hovakimyan
Abstract

In this dissertation, we study the dynamic coverage problem for multi-agent systems, where the main objective of a group of mobile agents is to explore a given compact region. Qualitatively, the coverage goal can be described as gathering sensory information for each point in the compact domain up to a desired level. In order to achieve the coverage goal, we propose two different novel control schemes.

In supervised coverage control, we introduce a stationary supervisor that assists a group of coverage agents with the centralized coverage control input and the global trajectory tracking control input. The coverage control input ensures the coverage task is performed until the agents end up in local minima, and when they do, the global trajectory tracking control input ensures that the agents are deployed to uncovered regions. Our control scheme is designed such that the two control inputs are decoupled, meaning that only one of them is active at a given time. In addition to the coverage objective, we design control inputs for coverage agents for avoiding collisions and maintaining proximity to the supervisor.

In swarm-based coverage control, we consider groups of coverage agents that behave as swarms for completing the coverage task. Unlike the supervised coverage control, there is no stationary agent; all agents move as a group in order to explore a given domain. Moreover, contrary to the supervised coverage control scheme, there is no trajectory tracking; instead, agents are deployed to uncovered regions via swarming control where a leader agent selects a target position that is in an uncovered region while all other agents are commanded to swarm around the leader agent’s target position. In this scheme, coverage and swarming control inputs are also decoupled, meaning that only one of them is active.
at a given time. In addition to the coverage objective, we design control inputs for coverage agents for avoiding collisions with each other and static obstacles and maintaining proximity to each other.

For both control schemes, we introduce a smooth transition signal that enables the coverage agents to continuously transition between the coverage control and global trajectory tracking control in the case of supervised coverage control scheme and between the coverage control and swarming control in the case of swarm-based coverage control scheme. Through the decoupling of these control inputs, we attain simpler control problems that we analyze separately for different modes and provide stability results for the overall control schemes through Lyapunov-like analysis. Finally, to illustrate the effectiveness of our schemes, we provide numerical simulations for various scenarios.
To

#DirenGezi
#HerYerTaksimHerYerDireniş
#Unutursakkalbimizkurusun
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Chapter 1

Introduction

As technology advances, the control of autonomous systems develops into an increasingly popular research area. With the expansion in communication and computation capabilities, researchers, as well as engineers in industry, rely more and more on these systems for accomplishing tasks that are complex, or potentially dangerous for humans. One approach to accomplish these complex tasks is to design a single system, equipped with an advanced computation unit and many sophisticated, and naturally expensive, sensors and/or actuators. An alternative approach that is relatively new is to combine smaller inexpensive systems that have less computation capabilities, fewer sensors and actuators and communication capabilities, to create large-scale systems. Small systems may not be equipped to accomplish complex tasks individually; however these tasks may easily be accomplished by combining the individual subsystems into large-scale systems, referred as multi-agent systems, and designing control laws through careful analysis of the dynamics and capabilities of these large-scale systems.

The idea of employing smaller systems to create large-scale systems is intriguing; that being said, there are many challenges in control of these multi-agent systems. One of the most essential objectives in control design for multi-agent systems is the safety of the subsystems. Control laws must be designed to ensure that agents do not collide with each other or obstacles in the environment. Another critical objective in multi-agent systems is the reliability of the communication network. The control laws must guarantee that
the communication is reliable, so that the multi-agent system can efficiently accomplish a given task as a group. Additionally, the dynamics of the agents must be explicitly considered when designing control laws; some commands may not be feasible, or these commands may have to be modified in order to account for the unique dynamics of the individual subsystem. Finally, the collective behavior of the group must be taken into account in control design in order to maximize the efficiency and the capability of the multi-agent system.

Among various tasks that can be effectively performed by multi-agent systems, the research in this dissertation focuses on the coverage task. In essence, the coverage task refers to exploration of a given domain by a multi-agent system. After constructing the framework in which the coverage task is defined, the aforementioned objectives are considered in the context of the coverage problem, namely the safety of the group and the reliability of the communication, as well as other challenges that arise in the control design. Rigorous theoretical analysis of the control system is carried out for various novel approaches in coverage problem, and these approaches are validated through numerical simulations.

1.1 Outline

This dissertation is organized into 6 chapters. Following the literature review and the summary of contributions of this dissertation presented in Chapter 1, in Chapter 2, we discuss the building blocks that are essential in constructing the coverage schemes proposed in the following chapters. Namely, we describe the framework of the dynamic coverage objective, along with collision avoidance and proximity maintenance objectives. In Chapter 3, we present the first novel scheme proposed in this research, namely the supervised dynamic coverage control. Initially, we provide the stability analysis of the scheme for single integrator agents. Subsequently, we discuss how the scheme can be applied to wheeled mobile robots that can be described as single integrators via static feedback linearization. In the last section of this chapter, we propose an asynchronous variant of the supervised
coverage scheme in which agents can transition between modes independently from other agents. Collision avoidance, as well as proximity to the supervisor, are explicitly considered in this chapter. The second novel coverage control scheme of this research, namely the swarm-based dynamic coverage control, is presented in Chapter 4. First, we provide the stability analysis of the swarm-based scheme, including collision avoidance, obstacle avoidance and inter-agent proximity maintenance, for single integrator agents. Then, we consider the swarm-based coverage control of kinematic unicycle agents, and provide the stability analysis in this case. We conclude this chapter by expanding the swarm-based coverage scheme to multiple swarms, and in addition to collision/obstacle avoidance and proximity maintenance, we consider swarm avoidance. We illustrate the effectiveness of the two coverage control schemes with numerical simulations in Chapter 5. We provide concluding remarks on the proposed schemes and discuss possible future directions for future research in Chapter 6.

1.2 Background

Control of multi-agent systems has been a popular research area for the last couple of decades. In [1], [2], [3], [4] and [5], the authors discuss the consensus problem in multi-agent systems where agents make use of data from neighboring agents for computing the control inputs. Alternatively, gradient based methods have been utilized in [6] to solve the formation control problem in multi-agent systems, and in [7] to discuss flocking in multi-agent formations. Other works that propose gradient-based control design for multi-agent systems include [8], [9] and [10]. In [11], a decentralized overlapping control algorithm has been proposed for a group of unicycle agents. Trajectory tracking problem for a group of unicycle agents have been studied in [12] and [13]. In [14], [15] and [16], recent problems in control of multi-agent systems have been thoroughly reviewed.

Among different topics of research in control of multi-agent systems, in this dissertation, we limit our discussion to the coverage control problem. We refer to two main branches as the static coverage control, and the dynamic coverage control.
Static coverage control problem can be traced back to locational optimization problems ([17], [18]), where the main objective is the placement of sensors to cover a given region in an optimal fashion. In such problems, the solution is given by the Voronoi partitioning of a given domain. Inspired by the solutions to locational optimization problems, static coverage control has been proposed for a group of mobile sensing agents. The objective of static coverage control problems is to design control laws that will drive a group of sensing agents from any initial distribution to another distribution such that they can fully cover a given domain. This approach amounts to finding control laws that will deploy agents to the centroids of Voronoi cells in a Voronoi partitioning of the domain. There are several important works that deal with the static coverage problem; in [19], authors propose dynamic variants of the Lloyd algorithm ([20]) for designing gradient based control laws that iteratively calculate the control laws which drive single integrators to the centroids of the Voronoi cells. Similar approaches have been proposed in [21], [22], [23], [24], [25], [26], and [27] as well. In particular, Cortes et. al. consider the static coverage control for a group of agents that have prescribed constraints on the area of their sensing domain in [21]. In [22], Gao et. al. unify the averaging algorithms for multi-agent systems with static coverage control and cast the static coverage problem as an averaging problem over acyclic directed graphs. In [23], Schwager et. al. consider a decentralized static coverage problem where the agents adaptively estimate the underlying density function, in which valuable information about what to search for in the given domain is encoded, without utilizing any a priori information. Finally, in [24], authors propose a hybrid modeling of the static coverage problem for a group of agents obeying nonholonomic constraints.

In this dissertation, we focus on the dynamic coverage control problem. The objective in dynamic coverage control is to develop control strategies for a group of mobile sensing agents with limited range sensors such that each point in a given domain is explored, by some subset of the agents, sufficiently so that the coverage at each point in the domain reaches a prescribed level. Recently, there has been a number of works contributing to the research in dynamic coverage control. In [28] and [29], Hokayem et. al. discuss the dynamic coverage problem in convex polygonal regions from an algorithmic perspective.
In [30], Hussein et al. propose a coverage error function in order to formulate the dynamic coverage problem. The authors design gradient-type centralized control laws for minimizing the coverage error function. An essential drawback of the coverage error function is that it has local minima, thus the agents may become stationary although the whole domain is not sufficiently covered. In order to compensate for such situations, authors switch to global control laws that will deploy agents to relatively uncovered regions. However, the overall control law becomes discontinuous at switching, and there is no discussion on the stability of the switching behavior of the control law in this work. Other works that build on the control scheme introduced in [30] include [31], [32] and [33]. In [34] and [35], authors discuss some technical issues regarding the derivation of the coverage error function introduced in [30] in more detail, and propose control laws for the dynamic coverage problem with agents described by control affine nonlinear dynamics. In [36], Wang et al. propose the awareness coverage control for reformulating the dynamic coverage problem. A discrete information update is assumed to take place between agents whenever they get sufficiently close to each other. Song et al. combine the awareness coverage control with the estimation of the underlying density in [37]. In both works, the switching behavior of the control law designed in [30] is still present. Other works that deal with the coverage control problem for multi robot systems are [38], [39], [40], [41] and [42]. In [38], a static coverage control problem for agents with anisotropic sensors, where sensing regions are ellipses, are discussed. A similar problem is also considered in [39], where the sensing regions are modeled as circular arcs that lie along the heading directions of the robots. Experimental results on the proposed coverage control law for the agents with anisotropic sensors have been reported in [40]. In [43], a dynamic coverage control problem is discussed for multi-agent systems, where the accumulated information is considered to be decaying over time. In this setup, the agents have to move around the region of interest monitoring the region constantly in order to compensate for decayed information. Although this dissertation provides a more realistic framework for the coverage problem, the switching behavior of the control law is still an issue. Moreover, only coverage objective is considered in the work. In [44], Valicka et al. discuss coverage control, along with collision avoidance
and proximity maintenance, within the framework of multi-objective optimization, and in [45], experimental results are presented. The authors utilized multi-attribute utility copulas in selecting design parameters of the system. In these works, the control problem is regarded as a multi-objective optimization problem with multiple, possibly conflicting, objectives, such as coverage, collision avoidance, proximity, etc. In [46], Franco et al. propose a control scheme for single integrator agents inspired by the formulation of [30]. The difference is that, instead of switching, the control laws continuously weigh between the coverage and global deployment strategies and scale them accordingly. Moreover, the global deployment strategy relies on an hierarchical grid decomposition of the given domain. The efficiency of the scheme is depicted via simulations, but the stability of the scheme has not been discussed from a mathematical perspective. In [47], authors propose a vision-based coverage control scheme for kinematic unicycle agents, where the on-board camera is modeled as a wedge shaped anisotropic sensor. The continuous weighing of the coverage and global deployment strategies is also present in this work. The global deployment strategy is different than the one in [46]; in this work, the authors utilize image processing techniques to find islands of uncovered regions, which are referred to as blobs, on the coverage domain and assign the closest blob for each agent. The approach proposes a centralized strategy, but again, the stability of the scheme has not been discussed. Finally, in [48], authors propose a coverage scheme with variable gains, which saves energy, for handling the problem of persistent dynamic coverage problem, where the accumulated information decays, similar to the information decay considered in [43]. The analysis of the stability of the control system is not present in this work either. In [46], [47] and [48], only the coverage and the global deployment objectives are considered; collision/obstacle avoidance or proximity maintenances are not included in control design.

1.3 Summary of Contributions

In this dissertation, we propose two novel control schemes for the dynamic coverage problem in multi-agent systems.
The first scheme, the supervised dynamic coverage control scheme, combines coverage control and trajectory tracking control. Two components of the supervised coverage control are i) gradient-type coverage control law and ii) global control law. Coverage law is constructed to ensure that coverage agents accumulate sensory information until they end up at local minima of the coverage error function. The global control law is designed to deploy agents that end up in local minima to regions that are not explored as much. The main advantage of our approach is that we combine these two components to construct a continuous control law. Moreover, our proposed control law is such that the two components are decoupled, meaning that they are not active simultaneously. This decoupling provides us with simpler control problems that can be separately analyzed for different modes. To this purpose, we introduce a smooth transition signal that enables the coverage agents to continuously transition between the coverage control laws and the global control laws when necessary. This is where the supervisor comes into play. The tasks of the supervisor are i) keeping track of the coverage map, ii) providing the coverage agents with the centralized dynamic coverage control and when necessary, iii) providing global control laws in order to redeploy coverage agents.

Within the context of supervised coverage control scheme, via careful construction of Lyapunov-like functions, we explicitly consider various objectives; coverage, trajectory tracking, collision avoidance, and proximity to the supervisor. Via Lyapunov-like approach, we provide a mathematical analysis of the stability of the scheme that combines all of these objectives. To the best of our knowledge, the research on supervised coverage control presented in this dissertation is unique in providing a rigorous theoretical discussion on the overall dynamic coverage scheme, including avoidance and proximity objectives, as well as validation via numerical simulations. Our work on supervised coverage scheme not only provides a framework, that is suitable for real-time implementation, but also presents an elaborate mathematical study of the proposed scheme. Moreover, we propose a methodology to apply the supervised coverage scheme to wheeled mobile robots. Finally, we propose an asynchronous variant of the scheme, where the coverage agents transition between the coverage and trajectory tracking objectives independent
from other agents. We also provide the stability analysis of this scheme, and through numerical simulations, show that the coverage mission is completed in a shorter amount of time when the asynchronous scheme is employed.

The second scheme, the swarm-based coverage scheme, builds on the groundwork laid by the supervised coverage scheme, but it is significantly different than its predecessor. In this scheme, the coverage agents accomplish the coverage objective as a group without the need for a central supervisor. Instead of a supervisor, there is a leader agent, a nonstationary coverage agent just like any other, that selects an uncovered target point whenever the agents end up in local minima of the coverage error function. Unlike the supervised coverage scheme, there is no trajectory tracking; instead, the problem of deployment to uncovered regions, which we refer to as the swarming objective, amounts to finding control laws for stabilization to a point. In this sense, the swarm-based scheme is computationally less demanding; instead of selecting a point and designing reference trajectories for each coverage agent, the leader agent only provides the coverage agents with a target point, and all agents utilize control laws to swarm to a small neighborhood of the target point as a group. The communication of the group is maintained via inter-agent proximity functions; i.e., the presence of a central agent for maintaining the communication is not required. In this sense, we propose a distributed approach in the context of dynamic coverage control, which is one of the novelties of swarm-based scheme. Just as it is the case with the supervised coverage scheme, the swarm-based coverage control laws are continuous. The decoupling of coverage and swarming objectives is also present in this scheme, and it allows us to analyze the stability of the scheme rigorously, which is unique to our research to the best of our knowledge. Consequently, via careful construction of Lyapunov-like functions, we explicitly consider coverage, swarming, collision avoidance, obstacle avoidance and inter-agent proximity maintenance objectives, which is novel to our work. In addition, we analyze the stability of the swarm-based coverage scheme when applied to kinematic unicycle agents. Finally, we propose a novel multi-swarm variant of the swarm-based coverage scheme where there are multiple groups of agents that accomplish the dynamic coverage objective. In order to guarantee that each swarm as a
group avoids other swarms, we modify collision avoidance functions to construct swarm avoidance functions.
Chapter 2

Problem Definition

Safe dynamic coverage control consists of several elements that need to be addressed individually so that the overall picture can be completely described. To this purpose, this chapter is dedicated to the description of the building blocks of the dynamic coverage control scheme. First, the smooth transitioning signal is introduced and its significance is described. Then, the coverage objective is explained, as well as how coverage information is acquired and stored. Subsequently, avoidance and proximity objectives and the corresponding functions that provide the agents the means to accomplish these objectives are stated. Finally, a discussion on how the agents move to uncovered regions via global deployment control signals is provided. In each section, whenever necessary, different variants of the elements are described separately to account for both the supervised coverage scheme and the swarm-based coverage scheme.

2.1 Smooth Transition Signal

One of the novelties of our approach to dynamic coverage control is the fact that the control inputs include both the coverage and the global deployment control signals and whenever necessary, the agents can smoothly transition between the 2 signals to prioritize the corresponding objective. By doing the transitioning finitely many times, the agents can accomplish the dynamic coverage objective. To this purpose, we employ a smooth
function, $\gamma(t)$, to ensure that the agents can transition between objectives.

We can qualitatively discuss $\gamma(t)$ in the following way: $\gamma(t) \equiv 1$ as long as a Boolean condition is “false.” As soon as the condition becomes “true”, $\gamma(t)$ decreases from 1 to 0 in a predefined amount of time smoothly. $\gamma(t) \equiv 0$ as long as another Boolean condition is “false.” As soon as the second condition becomes “true”, $\gamma(t)$ increases smoothly from 0 to 1. Hence, an essential property of $\gamma(t)$ is that it is continuously differentiable.

In order to provide a concrete example of such a transitioning signal, we describe one period of a specific $\gamma(t)$ signal below. We denote the first transitioning time as $t_{s1}$ and the second transitioning time as $t_{s2}$. Also, let $\tau_{s1}$ denote the transition duration from $\gamma \equiv 1$ to $\gamma \equiv 0$ after first transitioning, and $\tau_{s2}$ denote the transition duration from $\gamma \equiv 0$ to $\gamma \equiv 1$ after second transitioning. Let initial time be $t'$:

$$
\gamma(t) = \begin{cases} 
1, & t' \leq t \leq t_{s1} \\
\frac{\cos (\omega_{s1}(t-t_{s1})) + 1}{2}, & t_{s1} \leq t \leq t_{s1} + \tau_{s1} \\
0, & t_{s1} + \tau_{s1} \leq t \leq t_{s2} \\
-\frac{\cos (\omega_{s2}(t-t_{s2})) + 1}{2}, & t_{s2} \leq t \leq t_{s2} + \tau_{s2}.
\end{cases}
$$

(2.1)

The $\gamma(t)$ signal described above is illustrated in Figure 2.1.

**Remark 1.** We want to explicitly distinguish between the regions where $\gamma(t)$ changes behavior; i.e., **Mode 1:** $\gamma(t) \equiv 1$, **Mode 2:** $0 \leq \gamma(t) \leq 1$, $\dot{\gamma}(t) \leq 0$, **Mode 3:** $\gamma(t) \equiv 0$ and **Mode 4:** $0 \leq \gamma(t) \leq 1$, $0 \leq \dot{\gamma}(t)$.

### 2.2 Sensing and Coverage

In the context of dynamic coverage control, we model the sensors that the agents utilize to cover a given domain in the following way:

$$
\tilde{S}_{i}(s) = \frac{M_{\text{cov}i}}{R_{\text{cov}i}^4} \max \left\{0, R_{\text{cov}i}^2 - s\right\}^2, \quad i = 1, \cdots, N,
$$

(2.2)
where $M_{\text{cov}_i}$ describes the maximum sensing level for each agent and $R_{\text{cov}_i}$ describes the sensing region. Qualitatively, $\tilde{S}_i(s)$ models a limited range sensor that attains its maximum value at $s = 0$ and the sensing level degrades radially up to a certain radius, given by $R_{\text{cov}_i}$. Vision based sensors, infrared cameras, and radars are among sensors that can be accurately modeled by this mathematical formula [31]. In Figure 2.2 we illustrate $\tilde{S}_i(||p_i - \tilde{p}||)$ for an arbitrary coverage agent.

To depict the time-varying radial behavior of the agent sensors, we modify the sensor model of each agent in the following way:

$$S_i(t, ||p_i - \tilde{p}||^2) \triangleq \gamma(t) \tilde{S}_i(||p_i - \tilde{p}||^2), \quad (2.3)$$

where $\tilde{p} \in D$ with $D \subset \mathbb{R}^2$ representing the compact set to be covered. In addition, we
have the following representation for the gradient of \( \tilde{S}_i \) with respect to its arguments:

\[
\tilde{S}_i' \triangleq \frac{\partial \tilde{S}_i}{\partial s} = -2 \frac{M_{\text{cov}_i}}{R_{\text{cov}_i}^4} \max \{0, R_{\text{cov}_i}^2 - s\}^2.
\] (2.4)

Using the sensor model (2.3), we formulate the sensory information accumulated by the coverage agents in the following way:

\[
Q(t, \mathbf{P}) = C^{**} - C^{**} e^{-k^*A},
\] (2.5)

where \( \mathbf{P} = [p_1^T \ldots p_N^T]^T \) is the overall position vector and \( k^* \) is a design variable. Also, \( A(t, \tilde{p}) = \int_0^t \sum_{i=1}^N S_i(t, \|p_i(\tau) - \tilde{p}\|^2) d\tau \).

Here, one may think of \( C^* \) as a proxy variable for quantifying how well a certain area is explored. If the coverage level at a particular point in a given domain reaches \( C^* \), we
consider that point to be sufficiently explored. Thus, we consider the coverage objective to be accomplished when the coverage level at every point in the domain reaches $C^*$. Moreover, $C^{**}$ is a design variable such that $C^* < C^{**}$. Note that $Q(t, P)$ given by (2.5) is an exponential function; thus, it has a horizontal asymptote at $C^{**}$. Hence, in order to ensure that the desired coverage level $C^*$ can be exactly attained at any point in a given domain, we design $Q(t, P)$ such that it’s horizontal asymptote is at a level that is slightly greater than $C^*$.

Using the sensor model and the accumulated information model given by (2.3) and (2.5) respectively, we can formulate the coverage objective for both the supervised and the swarm-based coverage control. Although the two formulations are similar, there is a subtle yet an essential difference between them, hence in this section, we state both formulations, as well as their differences.

In order to quantify the coverage objective for the supervised coverage control problem, we utilize the following area integral [49]:

$$e(t) = \int \int_{\mathcal{D}} h(C^* - Q(t, P)) \phi(\tilde{p}) \left[ S^* - \sum_{i=1}^{N} S_i + \sum_{i=1}^{N} \sigma(G_i) \right] d\tilde{p}_1 d\tilde{p}_2,$$  \hspace{1cm} (2.6)

where $\phi(\tilde{p}) \equiv 1$ is the density, $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function, $h(x) = (\max \{0, x\})^3$, $Q(t, P)$ is given by (2.5), $S_i$ is given by (2.3) and $G_i = \frac{||p_i - p_{irf}(t)||^2}{2}$. Also, $S^*$ is a positive number that satisfies $S^* > \sum_{i=1}^{N} M_{cov_i}$. Here, $p_{irf}(t)$ is a trajectory generated by the supervisor for each coverage agent.

Similarly, we formulate the coverage objective for the swarm-based coverage control problem using the following area integral:

$$e(t) = \int \int_{\mathcal{D}} h(C^* - Q(t, P)) \phi(\tilde{p}) \left[ S^* - \sum_{i=1}^{N} S_i + (1 - \gamma) \sum_{i=1}^{N} \sigma(D_i) \right] d\tilde{p}_1 d\tilde{p}_2,$$  \hspace{1cm} (2.7)

where $\phi(\tilde{p}) \equiv 1$ is the density, $S^* > \sum_{i=1}^{N} M_{cov_i}$, $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function, $h(x) = (\max \{0, x\})^3$, $Q(t, P)$ is given by (2.5), $S_i$ is given by (2.3), $\gamma$ is the transitioning signal given by (2.1), $D_i = \frac{(||p_i - p_{des}||^2 - d^2_{des})^2}{2}$, where $p_{des}$ is the desired point around which
the agents will end up when moving as a swarm, and $d_i$ is the desired distance between each agent and the desired point.

Although the coverage objective formulations for the two control problems are very similar, there is a crucial difference; in the supervised coverage scheme, we utilize 

$$\sum_{i=1}^{N} \sigma\left(\frac{\|p_i - p_{ref}(t)\|^2}{2}\right)$$

...
trajectories designated by the supervisor or moving as a swarm, thus moving away from local minima. Using this strategy finitely many times, we ensure that the global minimum of \( e(t) \) is reached in finite time in both formulations. Finally, the structures of the coverage error functions are reminiscent of the derivative of the error function utilized in [35] in that we use the positive term \( S^* \) that is an upper bound for \( \sum_{i=1}^{N} S_i \), but the reason for using \( S^* \) in this work is not as apparent; through simulations, we have seen that we get the best performance with the coverage control laws that are derived from the definitions of \( e(t) \) consisting the \( S^* - \sum_{i=1}^{N} S_i \) term, hence we opted for the formulations given by (2.6) and (2.7).

### 2.3 Collision Avoidance

One of the most fundamental ideas in the area of collision avoidance is the concept of avoidance control that was formulated in [50], and further developed in [51, 52] and [53]. Using the ideas proposed in the works by Leitmann et. al., analysis for collision avoidance can easily be integrated into Lyapunov-like stability analysis.

Collision avoidance functions may be described as being analogous to artificial potential fields. Artificial potential fields have first been introduced for the obstacle avoidance in manipulators by Khatib et. al in [54], and has widely been used in robotic applications. There are many works which propose similar approaches for collision avoidance in multi-agent systems; [8], [9], [10], [55], etc. For a survey on collision avoidance, we refer the readers to [56].

In this work, we distinguish between two collision avoidance objectives; inter-agent collision avoidance and static obstacle avoidance.
2.3.1 Inter-Agent Collision Avoidance

In order to guarantee collision avoidance between agents, we adopt the following approach from [57]: for each pair of agents we define the following avoidance functions:

\[
V_{ij}^{col}(p_i, p_j) = \left( \min \left\{ 0, \frac{\|p_i - p_j\|^2 - R_{col_i}^2}{\|p_i - p_j\|^2 - r_{col_i}^2} \right\} \right)^2,
\]

with \(i, j \in \{1, \ldots, N\}, i \neq j\) and \(R_{col_i} > r_{col_i} > 0\) for all \(i = 1, \ldots, N\), where \(N\) is the number of agents and \(p_i = [x_i, y_i]^T\) represents the position of the \(i^{th}\) coverage agent. \(R_{col_i}\) denotes the detection region in which agents can detect other agents, and \(r_{col_i}\) denotes the avoidance region, which is the smallest safe distance between the agents. Using these distances, we can define the avoidance and detection sets for each pair of agents in the following way:

\[
\Omega_{ij} := \{ \mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, \|p_i - p_j\| \leq r_{col_i} \},
\]

\[
D_{ij} := \{ \mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, \|p_i - p_j\| \leq R_{col_i} \},
\]

where \(\mathbf{P} = [p_1^T \cdots p_N^T]^T\) is the overall position vector. The overall avoidance and detection regions can then be defined as the union of pairwise avoidance and detection regions respectively:

\[
\Omega = \bigcup_{i,j \in \{1, \ldots, N\}, j \neq i} \Omega_{ij},
\]

\[
D = \bigcup_{i,j \in \{1, \ldots, N\}, j \neq i} D_{ij}.
\]

Let’s recall the definition for the avoidance of a set \(S\) [50]:

**Definition 1.** Consider the overall dynamical system

\[
\dot{\mathbf{P}} = f(\mathbf{P}, U(\mathbf{P})),
\]
where $\mathbf{P}$ is the concatenated overall position vector and $\mathbf{U} = \left[ u_1^T \cdots u_N^T \right]^T$ is the concatenated control input vector. System (2.11) avoids $\mathbf{S}$ if and only if for each solution $\mathbf{P}(t, \mathbf{P}_0)$ with $0 \leq t$, where $\mathbf{P}_0$ is the overall position vector at initial time $t = t_0$, $\mathbf{P}_0 \notin \mathbf{S}$ implies that $\mathbf{P}(t, \mathbf{P}_0) \notin \mathbf{S}$ for all $0 \leq t$.

Note that, by the virtue of Definition 1 and the structure of the avoidance function (2.8), for $\mathbf{P}_0 \notin \Omega$, agents will enter $\Omega$ if and only if $V_{\text{col}}^{ij} \to \infty$. Thus, if $V_{\text{col}}^{ij}$ can be shown to attain finite values for all $i, j \in 1, \cdots, N$, the agents are guaranteed to avoid the set $\Omega$, implying that collisions are guaranteed not to occur.

Defining $d_{ij} \triangleq \|p_i - p_j\|$, the partial derivatives of the collision avoidance functions with respect to the position of an agent are given by:

$$
\frac{\partial V_{\text{col}}^{ij}}{\partial p_i} = \begin{cases} 
0, & \text{if } R_{\text{col},i} \leq d_{ij} \\
\frac{4(R_{\text{col},i}^2 - r_{\text{col},i}^2)(d_{ij}^2 - R_{\text{col},i}^2)}{(d_{ij}^2 - r_{\text{col},i}^2)^3}(p_i - p_j), & \text{if } r_{\text{col},i} < d_{ij} < R_{\text{col},i} \\
\text{not defined,} & \text{if } d_{ij} = r_{\text{col},i} \\
0, & \text{if } d_{ij} < r_{\text{col},i}.
\end{cases}
$$

(2.12)

Note that in this work, we assume that there are no sensor uncertainties for the implementation of the collision avoidance functions. Also, note that since $V_{\text{col}}^{ij}$ are symmetric with respect to their arguments, the following relation holds:

$$
\frac{\partial V_{\text{col}}^{ij}}{\partial p_i} = -\frac{\partial V_{\text{col}}^{ij}}{\partial p_j}.
$$

(2.13)

2.3.2 Obstacle Avoidance

Obstacle avoidance may be considered as a special case of collision avoidance for agents, where instead of both parties implementing an avoidance control scheme, only one of the parties implements the scheme to avoid colliding with a static obstacle. Naturally, we employ an obstacle avoidance function that is very similar to the collision avoidance function given by (2.8). We implement the obstacle avoidance using the same avoidance
function in the following way:

\[
V^{\text{obs}}_{i_k}(p_i, p^o_k) = \left( \min \left\{ 0, \frac{\|p_i - p^o_k\|^2 - R^{2}_{\text{obs}}_i}{\|p_i - p^o_k\|^2 - r^{2}_{\text{obs}}_i} \right\} \right)^2,
\]

(2.14)

where \(p^o_k\) is the position of the center of the \(k^{th}\) obstacle with \(k \in 1, \ldots, N_o\) where \(N_o\) is the number of static obstacles, \(i \in 1, \ldots, N\), and \(R^{2}_{\text{obs}}_i > r^{2}_{\text{obs}}_i > 0\) for all \(i = 1, \cdots, N\). In this scheme, \(R_{\text{obs}}_i\) represents the obstacle detection region whereas \(r_{\text{obs}}_i\) represents the smallest safe distance between the agent and the center of the obstacle.

**Remark 2.** Note that due to the structure of the obstacle avoidance function, there is a restriction as to how large the static obstacle can be so that it can be safely avoided; as long as the radii of the smallest circle bounding the obstacles are less than or equal to \(r_{\text{obs}}_i\), agents can safely avoid collisions with the obstacles.

Using the aforementioned definitions, we can define the avoidance and detection sets for each agent and each obstacle:

\[
\Omega^o_{i_k} := \{\mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, \|p_i - p^o_k\| \leq r_{\text{obs}}_i\},
\]

\[
D^o_{i_k} := \{\mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, \|p_i - p^o_k\| \leq R_{\text{obs}}_i\},
\]

(2.15)

where \(\mathbf{P} = [p^T_1 \cdots p^T_N]^T\) is the overall position vector. The overall avoidance and detection regions can then be defined as the union of pairwise avoidance and detection regions respectively:

\[
\Omega^o = \bigcup_{i \in \{1, \cdots, N\}, k \in \{1, \cdots, N_o\}} \Omega^o_{i_k},
\]

\[
D^o = \bigcup_{i \in \{1, \cdots, N\}, k \in \{1, \cdots, N_o\}} D^o_{i_k}.
\]

(2.16)

By the virtue of Definition 1 and the structure of the obstacle avoidance function (2.14), for \(\mathbf{P}_0 \notin \Omega^o\), agents will enter \(\Omega^o\) if and only if \(V^{\text{obs}}_{i_k} \to \infty\). Thus, if \(V^{\text{obs}}_{i_k}\) can be shown to attain finite values for all \(i \in 1, \cdots, N, k \in 1, \cdots, N_o\), the agents are guaranteed to avoid the set \(\Omega^o\), implying that agents are guaranteed to avoid obstacles.
Defining \( d_{ik}^o \triangleq \| p_i - p_k^o \| \), the partial derivative of the obstacle avoidance functions with respect to the position of an agent are given by:

\[
\frac{\partial V_{obs}^{ik}}{\partial p_i} = \begin{cases} 
0, & \text{if } R_{obs_i} \leq d_{ik}^o \\
\frac{4}{(d_{ik}^o - r_{obs_i})^3} \left( R_{obs_i}^2 - r_{obs_i}^2 \right) \left( d_{ik}^o - R_{obs_i} \right) (p_i - p_k^o), & \text{if } r_{obs_i} < d_{ik}^o < R_{obs_i} \\
\text{not defined,} & \text{if } d_{ik}^o = r_{obs_i} \\
0, & \text{if } d_{ik}^o < r_{obs_i}.
\end{cases}
\] (2.17)

**Remark 3.** It is important to note that collision avoidance functions described in this section may be generalized to operate with different shaped avoidance regions, such as elliptical ones. That being said, in this work, we opted for circular avoidance regions.

The main advantage of employing the collision avoidance functions given by (2.8) and (2.14) is that they operate locally, meaning that the agents do not need to know the global position of the agents and/or the obstacles to be able to avoid them. As long as there are no agents and/or obstacles within the detection region of an agent, these functions, as well as their gradients, attain zero values. Whenever an object is within the detection region of the agent, then these functions become “active” and the agent implements inter-agent collision and obstacle avoidance schemes via the gradients of the aforementioned avoidance functions.

### 2.4 Proximity Maintenance

One of the most essential objectives in a multi-agent setting is the proximity objective. Typically, agents communicate with each other wirelessly, hence it is critical for the agents to maintain a safe distance with their neighbors so that the communication quality is not degraded or lost completely. Works that deal with the proximity objective include, but are not limited to, [58], [59], [60] and [61].

In this work, we formulate the proximity constraint in two different ways: the first function, which is called the supervisor proximity function, is utilized in the supervised
dynamic coverage control scheme; it guarantees that the agents stay close to the stationary supervisor. The second function, which is called the inter-agent proximity function, guarantees that two agents stay close to each other.

### 2.4.1 Agent-Supervisor Proximity Maintenance

During the supervised coverage mission, it is assumed that the communication between the coverage agents is maintained via wireless communication through the supervisor; all agents exchange information with the supervisor and the supervisor sends necessary information about all agents to every agent. In order to maintain the communication, the coverage agents must stay sufficiently close to the supervisor. We formulate this distance constraint with a proximity objective for each coverage agent in the following way [45]:

\[
V_{sv}^i(p_i, p_{sv}) = \left( \max \left\{ 0, \frac{r_{sv}^2 - \|p_i - p_{sv}\|^2}{\|p_i - p_{sv}\|^2} \right\} \right)^2,
\]

(2.18)

where \(p_i\) is the position of \(i^{th}\) coverage agent, \(p_{sv}\) is the position of the supervisor and \(R_{svi} > r_{svi} > 0\) for all \(i = 1, \cdots, N\). In (2.18), \(r_{svi}\) denotes the degradation region in which the communication quality starts degrading, and \(R_{svi}\) denotes the loss region, which is the largest safe distance between the supervisor and a coverage agent. Using these distances, we can define the degradation and loss sets for each agent and the supervisor in the following way:

\[
\Delta_{sv}^i := \{ \mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, r_{svi} \leq \|p_i - p_{sv}\| \},
\]

\[
\mathcal{L}_{sv}^i := \{ \mathbf{P} : \mathbf{P} \in \mathbb{R}^{2N}, R_{svi} \leq \|p_i - p_{sv}\| \},
\]

(2.19)
where $\mathbf{P}$ is the overall position vector. The overall degradation and loss regions for the communication with the supervisor can then be defined in the following way:

$$
\Delta^{sv} = \left( \bigcap_{i=1, \ldots, N} \Delta^{sv}_i \right)^c = \bigcup_{i \in \mathbf{N}} \Delta^{sv}_i,
$$

$$
\mathcal{L}^{sv} = \left( \bigcap_{i=1, \ldots, N} \mathcal{L}^{sv}_i \right)^c = \bigcup_{i \in \mathbf{N}} \mathcal{L}^{sv}_i.
$$

(2.20)

Once again, by the virtue of Definition 1 and the structure of the proximity function (2.18), for $\mathbf{P}_0 \notin \mathcal{L}^{sv}$, agents will enter $\mathcal{L}^{sv}$ if and only if $V^{sv}_i \to \infty$. Thus, if $V^{sv}_i$ can be shown to attain finite values for all $i = 1, \ldots, N$, the agents are guaranteed to avoid the set $\mathcal{L}^{sv}$, implying that the proximity to the supervisor, thus the communication, will always be maintained by all agents.

Defining $d_i \triangleq \|p_i - p_{sv}\|$, the partial derivative of the supervisor proximity functions with respect to the position of an agent is given by:

$$
\frac{\partial V^{sv}_i}{\partial p_i} = \begin{cases} 
0, & \text{if } d_i \leq r_{sv_i} \\
\frac{4(R_{sv_i}^2 - r_{sv_i}^2)(r_{sv_i}^2 - d_i^2)}{(d_i^2 - R_{sv_i}^2)^3}(p_i - p_{sv}), & \text{if } r_{sv_i} < d_i < R_{sv_i} \\
\text{not defined}, & \text{if } d_i = R_{sv_i} \\
0, & \text{if } R_{sv_i} < d_i.
\end{cases}
$$

(2.21)

**Remark 4.** The quality of the communication network is actually regarded as a binary variable; there is full communication if the distance between an agent $i$ and the supervisor is less than $R_{sv_i}$, and there is no communication if it’s greater than $R_{sv_i}$. Thus, the communication does not actually degrade at $r_{sv_i}$; $r_{sv_i}$ merely represents the critical distance when the proximity gradient should start acting. When the distance is smaller than $r_{sv_i}$, there shouldn’t be any control laws for maintaining proximity since there is no immediate risk of communication loss.
2.4.2 Inter-agent Proximity Maintenance

During the swarm-based coverage mission, every agent is assumed to communicate with every other agent wirelessly. In order to maintain the quality of the communication, a distance constraint is imposed between the agents. This constraint is formulated with the inter-agent proximity function, which may be thought of as the dual of inter-agent collision avoidance functions [45]:

\[
V_{ij}^{prox}(p_i, p_j) = \left( \max \left\{ 0, \frac{r_{loss_i}^2 - \|p_i - p_j\|^2}{\|p_i - p_j\|^2 - R_{loss_i}^2} \right\} \right)^2,
\]  

(2.22)

with \( i, j \in \{1, \ldots, N\}, i \neq j \) and \( R_{loss_i} > r_{loss_i} > 0 \), for all \( i = 1, \cdots, N \). \( r_{loss_i} \) denotes the degradation region in which the communication quality starts degrading, and \( R_{loss_i} \) denotes the loss region, which is the largest safe distance between agents. Using these distances, we can define the degradation and loss sets for each pair of agents in the following way:

\[
\Delta_{ij} := \{ P : P \in \mathbb{R}^{2N}, r_{loss_i} \leq \| p_i - p_j \| \},
\]

\[
\mathcal{L}_{ij} := \{ P : P \in \mathbb{R}^{2N}, R_{loss_i} \leq \| p_i - p_j \| \},
\]  

(2.23)

where \( P \) is the overall position vector. The overall degradation and loss regions can then be defined in the following way:

\[
\Delta = \left( \bigcap_{i,j \in \{1, \ldots, N\}, j \neq i} \Delta_{ij}^c \right)^c = \bigcup_{i,j \in \mathbb{N}, j \neq i} \Delta_{ij},
\]

\[
\mathcal{L} = \left( \bigcap_{i,j \in \{1, \ldots, N\}, j \neq i} \mathcal{L}_{ij}^c \right)^c = \bigcup_{i,j \in \mathbb{N}, j \neq i} \mathcal{L}_{ij}.
\]  

(2.24)

Defining \( d_{ij} \triangleq \| p_i - p_j \| \), the partial derivatives of the proximity functions with respect
to the position of an agent are given by:

\[
\frac{\partial V_{\text{prox}}}{\partial p_i} = \begin{cases} 
0, & \text{if } d_{ij} \leq r_{\text{loss}_i} \\
\frac{4(R_{\text{loss}_i}^2 - r_{\text{loss}_i}^2)(r_{\text{loss}_i}^2 - d_{ij}^2)}{(d_{ij}^2 - R_{\text{loss}_i}^2)^3}(p_i - p_j), & \text{if } r_{\text{loss}_i} < d_{ij} < R_{\text{loss}_i} \\
\text{not defined}, & \text{if } d_{ij} = R_{\text{loss}_i} \\
0, & \text{if } R_{\text{loss}_i} < d_{ij}.
\end{cases}
\]  

(2.25)

Even though proximity functions are closely related to the inter-agent collision avoidance functions, there is one main difference; the proximity functions may not be implemented by the same local sensors that are utilized in implementing collision avoidance functions. For the supervised coverage control problem, it is assumed that as long as the distance constraint is satisfied, there is full communication between every coverage agent and the supervisor. Similarly, for the swarm-based coverage control problem, it is safe to assume that since the agents stay close to each other via inter-agent proximity functions, the underlying communication graph stays connected all the time, thus each agent indirectly knows the position of every other agent.

### 2.5 Target Point Selection

During the dynamic coverage mission, there will be situations where the coverage agents end up in local minima of the coverage error function. In such situations and in the case of the supervised coverage control, the supervisor is responsible for selecting target points for each coverage agent, and generating the trajectories that will direct them to the selected points. On the other hand, in the case of the swarm-based coverage control, the leader is responsible for selecting \( p_{\text{des}} \).

In supervised coverage control, whenever a trajectory is to be generated, the supervisor selects a target point in the domain \( D \) for each coverage agent. This point is selected
according to the following weighted formula:

\[ w_i = \mu_1 \| p_i - \tilde{p} \| + \mu_2 \frac{1}{(C^* - C|\tilde{p}|)^2}, \]  

(2.26)

where \( C|s \) represents the coverage level at \( s \), and \( 0 < \mu_1 \) and \( 0 < \mu_2 \) denote the weights.

The target point for each agent is selected as

\[ p_{des} = \arg \min_{\tilde{p}} (w_i). \]

(2.27)

Assume that the target point for an agent is selected at time \( t'' \). A continuously differentiable trajectory is then generated for that coverage agent with the following properties:

\[
\begin{align*}
p_{\text{ref}}(t'') &= p_i(t''), \\
p_{\text{ref}}'(t'') &= \dot{p}_i(t''), \\
p_{\text{ref}}'(t) &= 0, \quad t \geq t'' + \Delta t_{i_{\text{ref}}}, \\
\| p_{\text{ref}}(t'' + \Delta t_{i_{\text{ref}}}) - p_{\text{des}} \| &\leq \bar{\varepsilon}_i,
\end{align*}
\]

(2.28)

where \( \bar{\varepsilon}_i \) and \( \Delta t_{i_{\text{ref}}} \) are design parameters with \( 0 \leq \bar{\varepsilon}_i \) and \( 0 < \Delta t_{i_{\text{ref}}} \) and the trajectories are continuously differentiable. The supervisor designs trajectories according to the conditions dictated by (2.28) in order to make sure that the continuity of the control signals is maintained.

In swarm-based coverage control, the leader agent selects a target point in the domain \( \mathcal{D} \) according to the following weighted formula:

\[ w = \mu_1 \| p_1 - \tilde{p} \| + \mu_2 \frac{1}{(C^* - C|\tilde{p}|)^2}, \]

(2.29)

where \( C|s \) represents the coverage level at \( s \), and \( 0 < \mu_1 \) and \( 0 < \mu_2 \) denote the weights.

The target point is selected as

\[ p_{des} = \arg \min_{\tilde{p}} (w). \]

(2.30)
**Remark 5.** Without loss of generality, we denote the leader agent in a group of coverage agents with the subscript 1.

In application of both control schemes, the domain $\mathcal{D}$ is discretized into a set of equally sized square cells. Thus, the selection of the desired point amounts to finding the cell with the smallest cost. To this purpose, we initially use sort function in MATLAB to sort the cells according their cost in ascending order and then select the cell with the minimum cost. Note that we do not even need to find the exact point with the minimum cost for our scheme to work; finding a point that is not fully covered is sufficient. Thus, the uniqueness of $p_{des}$ or $p_{des}$ is not an issue.

It is important to point out that although the selection of target points is essentially the same for both the supervised and the swarm-based coverage schemes, in the first one the supervisor selects a target point and designs a sufficiently smooth trajectory for each coverage agent, whereas in the swarm-based scheme, only the leader agent selects a single target point and all other agents are commanded to swarm around that point, without going through the procedure of designing trajectories.

### 2.6 Transitioning Conditions

In this section, we explicitly state the Boolean conditions according to which the signal $\gamma(t)$ transitions from one mode to the other.

The first condition must provide transitioning from coverage mode to swarming mode. The transition should not take place as long as the coverage control is sufficiently large; in other words, as long as agents continue covering the area, they should keep on doing that without switching. The parameter that represents whether there is sufficient coverage going on or not is the decrease in coverage error. Hence, our first condition is the following:

$$C_1: \| \bar{e}(t + \Delta t_e) - \bar{e}(t) \| \leq \varepsilon, \quad (2.31)$$

where $\bar{e}(t) = \frac{\iint h(C^* - Q)d\bar{p}_1d\bar{p}_2}{C^* Area(D)}$ is the normalized coverage error. If the decrease in $\bar{e}(t)$ over a specified amount of time $\Delta t_e$ is less than a prescribed threshold $\varepsilon$, it is concluded that
agents are not doing much coverage; hence, sensors are turned off and agents transition to trajectory tracking mode in supervised coverage scheme and swarming mode in swarm-based scheme. This check is done every $\Delta t_e$ seconds. Condition $C1$ is the same for both the supervised and the swarm-based coverage schemes.

The second condition must provide transitioning from trajectory tracking mode to coverage mode in the case of supervised coverage scheme and from swarming mode to coverage mode in the case of swarm-based coverage scheme. The transition should take place whenever the coverage agents are sufficiently close to their designated positions. This condition is represented for the supervised coverage scheme in the following way:

\[
C'2': \quad \|p_i - \bar{p}_i\| \leq \bar{\varepsilon}_i, \quad \forall i = 1, \cdots, N,
\]  

(2.32)

where $\bar{p}_i$ are generated according to the formula given by (2.29) and $0 \leq \bar{\varepsilon}_i$ are design variables, given in (2.28).

A similar condition is represented for the swarm-based coverage scheme by the following:

\[
C''2': \quad \|p_i - p_{des}\| \leq \bar{\varepsilon}_i, \quad \forall i = 1, \cdots, N,
\]  

(2.33)

where $\bar{\varepsilon}_i := d_{des} + \varepsilon_i$, with nonnegative design variables $\varepsilon_i$. 

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Chapter 3

Supervised Dynamic Coverage Control

In this chapter, we focus on the supervised dynamic coverage control scheme. Supervised coverage control scheme is designed with the application in mind; in particular, the inclusion of a supervisor for assisting coverage agents, the design of transition signal $\gamma(t)$ along with selection of the transitioning conditions and the design of reference trajectories are all crucial in constructing continuous control laws that combine coverage and trajectory tracking objectives in a decoupled fashion.

The coverage control laws derived in this chapter are similar to the control schemes proposed by [30–32] in that they propose a gradient-type control law derived from a coverage error function that is analogous to $e(t)$ given by (2.6). That being said, there is a major difference between them; continuity is an essential property of the control laws derived in this chapter whereas the control laws of the aforementioned works depict switching behavior, thus they are discontinuous. Moreover, a rigorous stability analysis of the switching behavior is not present in these works. In order to ensure the continuity of the control signals proposed in this dissertation, we integrate the trajectory tracking objective into the control law, select transitioning conditions to smoothly transition between different operation modes, and design the $\gamma(t)$ signal that enables this transitioning.

Compared to the symmetry breaking control laws proposed in other works (e.g. [30–
as a solution for deploying agents that are trapped in local minima, our approach is
more systematic; through the inclusion of a supervisor that keeps track of coverage agents’
positions and the coverage map, we implement a control scheme where the supervisor
designs reference trajectories for deployment of agents while ensuring the continuity of
control signals. Moreover, via the incorporation of the terms $\sum_{i=1}^{N} \sigma(G_i)$, we construct
the coverage error function $e(t)$ such that the trajectory tracking objective is explicitly
included in the control scheme and also explicitly considered in the stability analysis.

To convey the whole story on the supervised coverage control scheme, we begin this
chapter with a discussion of the case where we only the coverage and trajectory tracking
portion are considered. In the next section, we include collision avoidance and proximity
maintenance in the stability analysis. Subsequently, we discuss how we can apply the
supervised coverage control scheme to wheeled mobile robots that can be modeled as
single integrators via feedback linearization. Finally, we conclude this chapter by proposing
an asynchronous variant of the supervised coverage scheme where coverage agents can
simultaneously be in different modes, and providing the stability analysis.

### 3.1 Supervised Coverage Control for Single Integrators

In this section, we discuss the stability of the supervised dynamic coverage control scheme
without the collision avoidance and proximity maintenance objectives. We consider a
multi-agent system with a single stationary supervisor (i.e., $\dot{p}_{sv} \equiv 0$) and $N$ coverage
agents that are single integrators; i.e., their equations of motion are described by

$$\dot{p}_i = u_i, \quad i = 1, \ldots, N. \tag{3.1}$$

We denote the overall position vector of the coverage agents by $P = \left[p_1^T \cdots p_N^T\right]^T$, and
the overall position vector at initial time $t = t_0$ by $P_0$.

**Remark 6.** We assume that the supervisor is positioned at a prescribed location and
stays stationary.

### 3.1.1 Main Results

Prior to the discussion on the technical details of the stability analysis, a brief explanation of how the coverage function is utilized in this section that will clarify in advance a crucial point regarding the stability analysis is in order.

The main approach that is taken in this section in analyzing the stability of the coverage problem is to regard \( e(t) \), given by (2.6), as a Lyapunov function and consider its derivative. However, we do not directly consider the time derivative of \( e(t) \); instead, we utilize a function, which we denote as \( \dot{e}(t) \), to analyze the stability of the control problem. In general, the time derivative of \( e(t) \), \( \dot{e}(t) \), is not equal to \( \dot{\hat{e}}(t) \). As discussed in [34] in more detail, \( \dot{e}(t) \) is indeed equal to \( \dot{\hat{e}}(t) \) plus some additional terms. It was proved in [34] that, if the sensing regions of coverage agents are circles and these regions do not overlap, the additional terms are exactly equal to 0. In the case of overlapping sensing regions, it has been discussed that the influence of the extra terms are minor and can be neglected [35]. Nevertheless, in order to acknowledge the fact that the function we are utilizing in this section is indeed different than the time derivative of the coverage function \( e(t) \), we use the "\( ^{\hat{\cdot}} \)" notation. Now, we state the main result for the supervised dynamic coverage control scheme.

**Theorem 1.** Consider a group of \( N \) single integrators and a supervisor on a compact domain \( D \subset \mathbb{R}^2 \) with the control inputs

\[
u_i = \gamma(t)u_{\text{cov}_i} + (1 - \gamma(t))u_{\text{glo}_i}, \tag{3.2}
\]

where \( \gamma(t) \) is the smooth transition signal given by (2.1), transitioning according to the
conditions C1 and C2′ given by (2.31) and (2.32), and \( u_{\text{cov}} \) and \( u_{\text{glo}} \) are given by

\[
\begin{align*}
    u_{\text{cov}} &= -K_{\text{cov}} I_{\text{cov}}^c, \quad I_{\text{cov}}^c = -2 \int_{D} h(C^* - Q) \gamma(t) S'_i(p_i - \tilde{p}) d\tilde{p}_1 d\tilde{p}_2, \\
    u_{\text{glo}} &= -K_{\text{glo}} (p_i - p_{i_{\text{ref}}}(t)) + \bar{p}_{i_{\text{ref}}}(t),
\end{align*}
\]

where \( h(\cdot) \), \( Q \) and \( S'_i \) are defined in Section 2.2, \( p_{i_{\text{ref}}}(t) \) is the reference provided by the supervisor and \( \bar{p}_{i_{\text{ref}}}(t) \) is a feedforward term. Then, there exist matrices \( 0 \prec K_{\text{cov}} \triangleq \begin{bmatrix} k_{\text{cov}_x} & 0 \\ 0 & k_{\text{cov}_y} \end{bmatrix} \) and \( 0 \prec K_{\text{glo}} \triangleq \begin{bmatrix} k_{\text{glo}_x} & 0 \\ 0 & k_{\text{glo}_y} \end{bmatrix} \) such that the following results hold:

(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in \( D \) reaches \( C^* \) at a finite time \( T_{\text{final}} \).

(ii) In the trajectory tracking mode, each agent follows their reference trajectories and travels to a sufficiently close neighborhood of \( p_{i_{\text{des}}} \) such that Condition C2′ is satisfied for all agents.

Proof. In order to discuss the stability of the control system, we compute the following function:

\[
\begin{align*}
    \dot{e}(t) &= -\int_{D} h(C^* - Q)(k^* C^* e^{-k^* A})(\sum_{i=1}^{N} S_i)(S^* - \sum_{i=1}^{N} S_i) + \sum_{i=1}^{N} \sigma(G_i)(d\tilde{p}_1 d\tilde{p}_2 \\
    &\quad + \sum_{i=1}^{N} \left( \int_{D} h(C^* - Q)[\gamma(t) S'_i(p_i - \tilde{p})] d\tilde{p}_1 d\tilde{p}_2 \\
    &\quad + \int_{D} h(C^* - Q)d\tilde{p}_1 d\tilde{p}_2(\sigma(G_i)(1 - \sigma(G_i))(p_i - p_{i_{\text{ref}}}(t)) + \bar{p}_{i_{\text{ref}}}(t)) \right). (3.4)
\end{align*}
\]
We will simplify some terms in (3.4) using the following notations:

\[ E_1 = - \int_D h(C^* - Q)(k^*C^{**}e^{-k^*A})(\sum_{i=1}^N S_i)(S^* - \sum_{i=1}^N S_i + \sum_{i=1}^N \sigma(G_i))d\tilde{p}_1d\tilde{p}_2 \]

\[ E_2 = -\dot{\gamma}(t) \sum_{i=1}^N \int_D h(C^* - Q)\tilde{S}_i d\tilde{p}_1d\tilde{p}_2, \]

\[ \kappa_i^{glo} = \left( \int_D h(C^* - Q)d\tilde{p}_1d\tilde{p}_2 \right) (\sigma(G_i)(1 - \sigma(G_i))), \]

\[ T_i^{glo} = \kappa_i^{glo}(p_i - p_{i,ref}(t))^T. \] (3.5)

Notice that \( E_1 \leq 0 \) and \( 0 < \kappa_i^{glo} \) except when the coverage reaches \( C^* \) over the whole domain. In order to simplify the notation, from this point onwards, we stop showing the explicit time dependence of the reference trajectories and their derivatives.

Using the notations of (3.3) and (3.5), we get

\[ \dot{\hat{e}}(t) = E_1 + E_2 + \sum_{i=1}^N T_i^{cov^T} \hat{p}_i + \sum_{i=1}^N T_i^{glo^T}(\hat{p}_i - \hat{p}_{i,ref}). \] (3.6)

Additionally, using (3.2), we can rewrite \( \dot{\hat{e}} \) as

\[ \dot{\hat{e}}(t) = E_1 + E_2 + \sum_{i=1}^N T_i^{cov^T} (\gamma(t)u_{cov_i} + (1 - \gamma(t))u_{glo_i}) \]
\[ + \sum_{i=1}^N T_i^{glo^T} (\gamma(t)u_{cov_i} + (1 - \gamma(t))u_{glo_i} - \dot{p}_{i,ref}). \] (3.7)

From (3.2), it can be seen that by ensuring that \( \gamma(t) \) transitions from 0 to 1 or vice versa whenever necessary, the control signal can transition smoothly between the coverage control input and the global trajectory tracking control input. This structure for the control law provides decoupling of the control signals for different objectives. Consequently, we can analyze the system in 4 different modes, that is, Mode \( i \) with \( i = 1, \cdots, 4 \):
Mode 1 ($\gamma(t) \equiv 1$): In this mode, the supervisor does not provide any reference trajectories for the coverage agents as the agents are in the coverage mode. Thus, we have $p_{i\text{ref}} = p_i$ and $\dot{p}_{i\text{ref}} = u_i$, $\ddot{p}_{i\text{ref}} \equiv 0$ and $\dot{\gamma}(t) = 0$. Hence, $u_i = u_{\text{cov}} = -K_{\text{cov}} T_{i\text{cov}}$. Also, note that $T_{i\text{glo}}^g \equiv 0$ and $E_2 \equiv 0$ in this mode. As a result, we get

$$\dot{\hat{e}}(t) = E_1 - \sum_{i=1}^{N} T_{i\text{cov}}^T K_{\text{cov}} T_{i\text{cov}} \leq - \sum_{i=1}^{N} T_{i\text{cov}}^T K_{\text{cov}} T_{i\text{cov}}, \quad (3.8)$$

since $E_1 \leq 0$. This implies that in this region the coverage mission is carried on as long as the rate of coverage is sufficiently high.

Mode 2 ($0 < \gamma(t) < 1$, $\dot{\gamma}(t) \leq 0$): In this mode, condition $C1$ is satisfied, so the agents are transitioning from coverage mode to trajectory tracking mode, but the supervisor has not yet provided the agents with reference trajectories. Hence, we have $p_{i\text{ref}} = p_i$, $\dot{p}_{i\text{ref}} = u_i$, $\ddot{p}_{i\text{ref}} \equiv 0$ and $\dot{\gamma}(t) \leq 0$. The control inputs are $u_i = \gamma(t) u_{\text{cov}} + (1 - \gamma(t)) u_{\text{glo}}$, and we have $T_{i\text{glo}}^g \equiv 0$. Thus, $\dot{\hat{e}}(t)$ becomes

$$\dot{\hat{e}}(t) = E_1 + E_2 - \gamma(t) \sum_{i=1}^{N} T_{i\text{cov}}^T K_{\text{cov}} T_{i\text{cov}} + (1 - \gamma(t)) \sum_{i=1}^{N} T_{i\text{cov}}^T (-K_{\text{glo}} (p_i - p_{i\text{ref}}) + \ddot{p}_{i\text{ref}})
- \gamma(t) \sum_{i=1}^{N} T_{i\text{glo}}^T K_{\text{cov}} T_{i\text{cov}} + (1 - \gamma(t)) \sum_{i=1}^{N} T_{i\text{glo}}^T (-K_{\text{glo}} (p_i - p_{i\text{ref}}) + \ddot{p}_{i\text{ref}})
- \sum_{i=1}^{N} T_{i\text{glo}}^T \dddot{p}_{i\text{ref}}
= E_1 + E_2 - \gamma(t) \sum_{i=1}^{N} T_{i\text{cov}}^T K_{\text{cov}} T_{i\text{cov}}. \quad (3.9)$$

For this mode, as we have verified through simulations as well, we can select $k^*, \omega_{s_2}$ (i.e., we can select $\omega_{s_2}$ to design the transition duration of $\gamma(t)$, $\tau_{s_1}$) such that the following inequality holds:

$$E_1 + E_2 \leq 0. \quad (3.10)$$
Then, we have
\[
\dot{e}(t) \leq -\gamma(t) \sum_{i=1}^{N} T_i^{\text{cov}} T_i^{\text{cov}}. \tag{3.11}
\]

In effect, the sensors of the coverage agents are turned off within a prescribed amount of time. This implies that starting from the instant the agents exit Mode 2 until the instant they enter Mode 4, agents do not acquire new sensory information.

**Mode 3 (\(\gamma(t) \equiv 0\))** : At the beginning of this mode, the supervisor selects target points for each agent according to the procedure described in Section 2.5 and provides each agent with the corresponding reference trajectory. In this mode, we have \(\dot{\gamma}(t) \equiv 0, T_i^{\text{cov}} \equiv 0\) and \(E_2 \equiv 0\). Hence, \(u_i = -K_{\text{glo}} T_i^{\text{glo}} + \bar{p}_{\text{ref}}(t)\) where the feedforward term is designed to be \(\bar{p}_{\text{ref}}(t) \equiv \dot{p}_{\text{ref}}\). Then, we have

\[
\dot{e}(t) = E_1 + \sum_{i=1}^{N} T_i^{\text{cov}} (-K_{\text{glo}} (p_i - p_{\text{ref}}) + \dot{p}_{\text{ref}}) - \sum_{i=1}^{N} T_i^{\text{glo}} (K_{\text{glo}} (p_i - p_{\text{ref}}) - \dot{p}_{\text{ref}} + \dot{p}_{\text{ref}})
\]

\[
\leq - \sum_{i=1}^{N} K_i^{\text{glo}} (p_i - p_{\text{ref}})^T K_{\text{glo}} (p_i - p_{\text{ref}}),
\tag{3.12}
\]

since \(E_1 \leq 0\). Note that in Mode 3, there is no additional coverage since the sensors of the agents are turned off; instead, the agents follow designated trajectories that will take them to the vicinity of the assigned target uncovered points. In this mode, we can see that by using the functions \(G_i\) that have been previously defined, we can show the agents’ convergence to the assigned trajectories. Also, by the virtue of the assumptions on the trajectories given by \([2.28]\), for sufficiently high \(k_{\text{glo}}\), we guarantee that the agents end up within \(\bar{\varepsilon}_i\) neighborhoods of the target points \(p_{i_{\text{des}}}\) within \(\Delta t_{i_{\text{ref}}}\) seconds after the instant Mode 3 starts, and when they do, \(\dot{p}_{i_{\text{ref}}} = 0\) for each coverage agent.

**Mode 4 (0 < \(\gamma(t) < 1\), 0 ≤ \(\dot{\gamma}(t)\))** : In this mode, condition \(C2^\prime\) is satisfied, so the agents are transitioning from trajectory tracking mode to coverage mode. We have \(p_{\text{ref}} = p_i, \dot{p}_{\text{ref}} = u_i\) with \(u_i = \gamma(t) u_{\text{conv}} + (1 - \gamma(t)) u_{\text{glo}}, \bar{p}_{\text{ref}} = 0\) and \(0 \leq \dot{\gamma}(t)\). Also, note that
\[ I_i^{glo} \equiv 0. \] Then, we have

\[
\dot{e}(t) = E_1 + E_2 - \gamma(t) \sum_{i=1}^{N} T_i^{cov} K_{cov} T_i^{cov} + (1 - \gamma(t)) \sum_{i=1}^{N} T_i^{cov} (-K_{glo}(p_i - p_{ref}) + \bar{p}_{ref})
\]

\[
- \gamma(t) \sum_{i=1}^{N} T_i^{glo} K_{cov} T_i^{cov} + (1 - \gamma(t)) \sum_{i=1}^{N} T_i^{glo} (-K_{glo}(p_i - p_{ref}) + \bar{p}_{ref})
\]

\[
- \sum_{i=1}^{N} T_i^{glo} \dot{p}_{ref}
\]

\[
\leq -\gamma(t) \sum_{i=1}^{N} T_i^{cov} K_{cov} T_i^{cov}.
\] (3.13)

since \( E_1 \leq 0 \) and \( E_2 \leq 0 \) due to the fact that \( 0 \leq \dot{\gamma}(t) \). Again, the negative semi-definiteness of \( \dot{e}(t) \) is maintained in this mode. Moreover, the sensors are turned on again; this implies that agents begin accumulating new sensory information as soon as they enter Mode 4.

3.2 Supervised Coverage Control with Collision Avoidance and Proximity Maintenance

In this section, we discuss the stability of the complete control scheme; the supervised dynamic coverage control scheme together with collision avoidance and proximity maintenance objectives.

3.2.1 Main Results

We have already discussed in Section (3.1) that when sensing regions overlap, equation (3.4) would not be exactly equal to the time derivative of \( e(t) \). In this section, we set \( R_{cov_i} = \frac{r_{col}}{2} \). This implies that, if collision avoidance can be guaranteed, the agents’ sensing regions will never overlap, hence the extra terms derived in [34] will automatically be 0, which in turn implies that the time derivative of \( e(t) \) given by (3.17) is exact. Thus, we drop the “\(^*\)” notation in this section. Subsequently, we can state the main result for
the supervised dynamic coverage control scheme with guaranteed collision avoidance and proximty maintenance.

**Theorem 2.** Consider a group of $N$ single integrators and a supervisor on a compact domain $\mathcal{D} \subset \mathbb{R}^2$ with the control inputs

$$u_i = \gamma(t)u_{\text{cov}} + (1 - \gamma(t))u_{\text{glo}} + u_{\text{col}} + u_{\text{sv}}, \quad (3.14)$$

where $\gamma(t)$ is the smooth transition signal given by (2.1), transitioning according to the conditions $C1$ and $C2'$ given by (2.31) and (2.32), and $u_{\text{cov}}, u_{\text{glo}}, u_{\text{col}}$ and $u_{\text{sv}}$ are given by

$$u_{\text{cov}} = -K_{\text{cov}}T_{\text{cov}}^i, \quad T_{\text{cov}}^i = -2\int\limits_{\mathcal{D}} h(C^* - Q)\tilde{S}'(p_i - \tilde{p})d\tilde{p}_1d\tilde{p}_2,$$

$$u_{\text{glo}} = -K_{\text{glo}}(p_i - p_{\text{ref}}) + \tilde{p}_{\text{ref}},$$

$$u_{\text{col}} = -K_{\text{col}}T_{\text{col}}^i, \quad T_{\text{col}}^i = \sum_{j \neq i}^{N} \frac{\partial V_{\text{col}}^{ij}}{\partial p_i},$$

$$u_{\text{sv}} = -K_{\text{sv}}T_{\text{sv}}^i, \quad T_{\text{sv}}^i = \frac{\partial V_{\text{sv}}^i}{\partial p_i}, \quad (3.15)$$

where $h(\cdot), Q$ and $\tilde{S}'$ are defined in Section 2.2, $p_{\text{ref}}$ is the reference trajectory provided by the supervisor, $\tilde{p}_{\text{ref}}(t)$ is a feedforward term, and $V_{\text{col}}^{ij}$ and $V_{\text{sv}}^i$ are given by (2.8) and (2.18) respectively. Assume that $P_0 \notin \Omega$ and $P_0 \notin L_{\text{sv}}$, where $\Omega$ and $L_{\text{sv}}$ are overall avoidance region and the communication loss region for the agents and the supervisor, given by (2.10) and (2.20). Then, there exist matrices $0 < K_{\text{cov}} \triangleq \begin{bmatrix} k_{\text{cov}ix} & 0 \\ 0 & k_{\text{cov}iy} \end{bmatrix}$, $0 < K_{\text{glo}} \triangleq \begin{bmatrix} k_{\text{glo}ix} & 0 \\ 0 & k_{\text{glo}iy} \end{bmatrix}$, $0 < K_{\text{col}} \triangleq \begin{bmatrix} k_{\text{col}ix} & 0 \\ 0 & k_{\text{col}iy} \end{bmatrix}$ and $0 < K_{\text{sv}} \triangleq \begin{bmatrix} k_{\text{sv}ix} & 0 \\ 0 & k_{\text{sv}iy} \end{bmatrix}$ such that the following results hold:

(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in $\mathcal{D}$ reaches $C^*$ at a finite time $T_{\text{final}}$.

(ii) In the trajectory tracking mode, each agent follows their reference trajectories and
travels to a sufficiently close neighborhood of $p_{\text{ideal}}$ such that Condition C2’ is satisfied for all agents.

(iii) Agents avoid collisions with other agents; i.e., $P_i \notin \Omega$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$.

(iv) Agents maintain proximity to the supervisor; i.e., $P_i \notin \mathcal{L}^{\text{sv}}$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$.

Proof. Consider the following Lyapunov-like function:

$$V(t) = e + V^{\text{col}} + V^{\text{sv}}$$

$$= e(t) + \frac{w_{\text{col}}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V_{ij}^{\text{col}} + w_{\text{sv}} \sum_{i=1}^{N} V_{i}^{\text{sv}},$$

where $0 < w_{\text{col}}, w_{\text{sv}}$ are design parameters, $e$ is given by (2.6), $V_{ij}^{\text{col}}$ is given by (2.8) and $V_{i}^{\text{sv}}$ is given by (2.18). The time derivative of $e(t)$ is given by

$$\dot{e}(t) = -\int_{D} h(C^* - Q)(k^* C^{**} e^{-k^*A})(\sum_{i=1}^{N} S_i)(S^* - \sum_{i=1}^{N} S_i + \sum_{i=1}^{N} \sigma(G_i))d\tilde{p}_1 d\tilde{p}_2$$

$$+ \sum_{i=1}^{N} \left( \int_{D} h(C^* - Q)[\dot{\gamma}(t)\tilde{S}_i + 2\gamma(t)\tilde{S}_i'(p_i - \hat{p})^T \hat{p}_i] d\tilde{p}_1 d\tilde{p}_2ight.$$

$$+ \int_{D} h(C^* - Q)d\tilde{p}_1 d\tilde{p}_2 (\sigma(G_i)(1 - \sigma(G_i))(p_i - p_{\text{ref}})(\hat{p}_i - \hat{p}_{\text{ref}}))^T \right).$$

Using equations (3.5) and (3.15), and taking the time derivative of $V$, we get

$$\dot{V} = \dot{e}(t) + \dot{V}^{\text{col}} + \dot{V}^{\text{sv}}$$

$$= E_1 + E_2 + \sum_{i=1}^{N} T_{i}^{\text{con}}^T \dot{p}_i + \sum_{i=1}^{N} T_{i}^{\text{glob}}^T (\hat{p}_i - \hat{p}_{\text{ref}}) + w_{\text{col}} \sum_{i=1}^{N} \left( \sum_{j \neq i}^{N} \frac{\partial V_{ij}^{\text{col}}}{\partial p_i} \right) \dot{p}_i$$

$$+ w_{\text{sv}} \sum_{i=1}^{N} \frac{\partial V_{i}^{\text{sv}}}{\partial p_i} \dot{p}_i.$$  

\(3.18\)
As we did in Section 3.1, we will analyze the system in 4 different modes; that is Mode $i$ with $i = 1, \ldots, 4$:

**Mode 1 ($\gamma(t) \equiv 1$):** In this mode, the supervisor has not provided the agents with reference trajectories since they are in coverage mode. Thus, we have $p_{i, \text{ref}} = p_i$ and $\dot{p}_{i, \text{ref}} = u_i$, $\bar{p}_{i, \text{ref}} \equiv 0$ and $\dot{\gamma}(t) = 0$. Hence, $u_i = u_{\text{cov}_i} + u_{\text{col}_i} + u_{\text{sv}_i}$. Also, note that $T_i^{\text{glo}} \equiv 0$ in this mode. The derivative of $V$ becomes

$$
\dot{V} = E_1 + E_2 + \sum_{i=1}^{N} \left( T_i^{\text{cov}} + u_{\text{col}} T_i^{\text{col}} + w_{\text{sv}} T_i^{\text{sv}} \right) \left( -K_{\text{cov}} T_i^{\text{cov}} - K_{\text{col}} T_i^{\text{col}} - K_{\text{sv}} T_i^{\text{sv}} \right)
$$

$$
\leq - \sum_{i=1}^{N} \begin{bmatrix} T_i^{\text{cov}} & T_i^{\text{col}} & T_i^{\text{sv}} \end{bmatrix} \mathcal{P}_i \begin{bmatrix} T_i^{\text{cov}} \\ T_i^{\text{col}} \\ T_i^{\text{sv}} \end{bmatrix},
$$

where

$$
0 \preceq \mathcal{P}_i = \begin{bmatrix} K_{\text{cov}_i} & K_{\text{col}_i} & K_{\text{sv}_i} \\ w_{\text{col}} K_{\text{cov}_i} & w_{\text{col}} K_{\text{col}_i} & w_{\text{col}} K_{\text{sv}_i} \\ w_{\text{sv}} K_{\text{cov}_i} & w_{\text{sv}} K_{\text{col}_i} & w_{\text{sv}} K_{\text{sv}_i} \end{bmatrix}
$$

and $E_1 + E_2 \leq 0$. This implies that in this mode, if the coverage agents’ initial positions are such that they are outside each others’ avoidance regions and the communication loss region, they will never collide and they will maintain proximity to the supervisor since $0 \preceq \mathcal{P}_i$, implying $V$ attains finite values, which in turn means $V^{\text{col}}$ and $V^{\text{sv}}$ will attain finite values. Moreover, it can be seen as that as long as there are no collision avoidance and/or proximity gradients, the agents continue the coverage mission since in this case, $\dot{V}(t)$ satisfies

$$
\dot{V} \leq - \sum_{i=1}^{N} T_i^{\text{cov}} K_{\text{cov}_i} T_i^{\text{cov}}.
$$

**Mode 2 ($0 < \gamma(t) < 1$, $\dot{\gamma}(t) \leq 0$):** In this mode, we have $p_{i, \text{ref}} = p_i$, $\dot{p}_{i, \text{ref}} = u_i$, $\bar{p}_{i, \text{ref}} \equiv 0$ and $\dot{\gamma}(t) \leq 0$. Hence, $u_i = \gamma(t) u_{\text{cov}_i} + (1 - \gamma(t)) u_{\text{glo}_i} + u_{\text{col}_i} + u_{\text{sv}_i} = \gamma(t) u_{\text{cov}_i} + u_{\text{col}_i} + u_{\text{sv}_i}$ since $u_{\text{glo}_i} \equiv 0$. Note that $T_i^{\text{glo}} \equiv 0$ in this mode. Then, we can compute the time derivative of $V$ as
\[ \dot{V} = E_1 + E_2 + \sum_{i=1}^{N} \left( T_i^{cov} + w_{col} T_i^{col} + w_{sv} T_i^{sv} \right) \left( -\gamma(t) K_{cov,i} T_i^{cov} - K_{col,i} T_i^{col} - K_{sv,i} T_i^{sv} \right) \]

\[ = E_1 + E_2 - \sum_{i=1}^{N} \begin{bmatrix} T_i^{cov} & T_i^{col} & T_i^{sv} \end{bmatrix} \mathcal{P}_{i_2}(t) \begin{bmatrix} T_i^{cov} \\ T_i^{col} \\ T_i^{sv} \end{bmatrix} \]

(3.22)

where

\[ 0 \preceq \mathcal{P}_{i_2}(t) = \begin{bmatrix} \gamma(t) K_{cov,i} & K_{col,i} & K_{sv,i} \\ \gamma(t) w_{col} K_{cov,i} & w_{col} K_{col,i} & w_{col} K_{sv,i} \\ \gamma(t) w_{sv} K_{cov,i} & w_{sv} K_{col,i} & w_{sv} K_{sv,i} \end{bmatrix} . \]

(3.23)

Note that \( \mathcal{P}_{i_2}(t) \) is a time-varying matrix, hence we cannot immediately deduce the positive semi-definiteness. To show the positive semi-definiteness of \( \mathcal{P}_{i_2}(t) \), we can decompose it in the following way:

\[ \mathcal{P}_{i_2}(t) = W_{sv}^i J_{sv}^i K_{sv}^i(t), \]

(3.24)

with

\[ W_{sv}^i = \begin{bmatrix} I_{2 \times 2} & 0 & 0 \\ 0 & w_{col} I_{2 \times 2} & 0 \\ 0 & 0 & w_{sv} I_{2 \times 2} \end{bmatrix}, \quad J_{sv}^i = \begin{bmatrix} I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\ I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\ I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \end{bmatrix}, \quad K_{sv}^i(t) = \begin{bmatrix} \gamma(t) K_{cov,i} & 0 & 0 \\ 0 & K_{col,i} & 0 \\ 0 & 0 & K_{sv,i} \end{bmatrix} \]

(3.25)

where \( I_{2 \times 2} := \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \) and \( I_{2 \times 2} \) is the 2 by 2 identity matrix. Notice that \( W_{sv}^i \) and \( K_{sv}^i(t) \) are diagonal matrices. Moreover, it can be seen that for positive values of \( w_{col} \) and \( w_{sv} \), \( W_{sv}^i \) is positive definite. For \( 0 < \gamma(t) < 1 \), for positive gain matrices \( K_{cov,i}, K_{col,i} \) and \( K_{sv,i} \), \( K_{sv}^i(t) \) is positive definite, and at the beginning or end of Mode 2, i.e., for \( \gamma(t) \equiv 1 \) or \( \gamma(t) \equiv 0 \), \( K_{sv}^i(t) \) is positive semi-definite. Finally, by definition, \( J_{sv}^i \) has nonnegative eigenvalues. Hence, in Mode 2, the positive semi-definiteness of \( \mathcal{P}_{i_2}(t) \) is maintained. Moreover, in order render \( \dot{V} \) negative semi-definite in Mode 2, as we have verified through
simulations as well, we can select $k^*$ and $\omega_{s_2}$ such that

$$E_1 + E_2 \leq 0,$$

for all $0 \leq \gamma(t) \leq 1$. Then, $\dot{V}$ satisfies

$$\dot{V} \leq - \sum_{i=1}^N \left[ T_{i}^{\text{cov}T} T_{i}^{\text{col}T} T_{i}^{\text{sv}T} \right] P_{i_2}(t) \left[ \begin{array}{c} T_{i}^{\text{cov}} \\
T_{i}^{\text{col}} \\
T_{i}^{\text{sv}} \end{array} \right]. \quad (3.26)$$

Thus, in Mode 2, the positive semi-definiteness of $P_{i_2}(t)$ is preserved, meaning that agents continue gathering sensory information as long as there are no collision avoidance and/or proximity gradients, no collisions occur and proximity to supervisor is maintained by each coverage agent. Note that at the instance $\gamma(t)$ becomes 0, so does $I_{i}^{\text{cov}}$, as well as $E_1$ and $E_2$. This implies that the sensors of the coverage agents are turned off.

Mode 3 ($\gamma(t) \equiv 0$): In this mode, $\dot{\gamma}(t) \equiv 0$, $I_{i}^{\text{cov}} \equiv 0$ and $E_2 \equiv 0$. Moreover, a trajectory is generated for each coverage agent by the supervisor. Hence, $u_i = u_{glo_i} + u_{col_i} + u_{sv_i}$. The time derivative of $V$ becomes

$$\dot{V} = E_1 + E_2 - \sum_{i=1}^N \left[ \sqrt{\kappa_i^{\text{glo}} (p_i - \hat{p}_{i_{ref}})^T} T_{i}^{\text{col}T} T_{i}^{\text{sv}T} \right] P_{i_3} \left[ \begin{array}{c} \sqrt{\kappa_i^{\text{glo}} (p_i - \hat{p}_{i_{ref}})} \\
T_{i}^{\text{col}} \\
T_{i}^{\text{sv}} \end{array} \right]$$

$$+ \sum_{i=1}^N \left( w_{col} T_{i}^{\text{col}T} + w_{sv} T_{i}^{\text{sv}T} \right) \hat{p}_{i_{ref}} + \sum_{i=1}^N T_{i}^{\text{glo}T} (\hat{p}_{i_{ref}} - \hat{\dot{p}}_{i_{ref}}), \quad (3.27)$$

where

$$0 \preceq P_{i_3} = \left[ \begin{array}{ccc} K_{glo_i} & K_{col_i} & K_{sv_i} \\
w_{col} K_{glo_i} & w_{col} K_{col_i} & w_{col} K_{sv_i} \\
w_{sv} K_{glo_i} & w_{sv} K_{col_i} & w_{sv} K_{sv_i} \end{array} \right], \quad (3.28)$$
and $E_1 + E_2 \leq 0$. At this point, the supervisor has freedom in designing the trajectories that satisfy the conditions given by (2.28). The feedforward terms are selected as $\bar{p}_{i_{\text{ref}}} \equiv \dot{p}_{i_{\text{ref}}}$, and the trajectories are designed such that the following inequality holds:

$$\sum_{i=1}^{N} \left( w_{\text{col}}^{T} T_{i}^{\text{col}} + w_{\text{sv}} T_{i}^{\text{sv}} \right) \dot{p}_{i_{\text{ref}}} \leq 0. \quad (3.29)$$

If this is the case, $\dot{V}(t)$ satisfies

$$\dot{V}(t) \leq - \sum_{i=1}^{N} \left[ \sqrt{\kappa_{i_{\text{glo}}}^{\text{glo}}} (p_{i} - p_{i_{\text{ref}}})^{T} T_{i_{\text{col}}}^{\text{col}} T_{i_{\text{sv}}}^{\text{sv}} \right] P_{i_{\text{a}}} \begin{bmatrix} \sqrt{\kappa_{i_{\text{glo}}}^{\text{glo}}} (p_{i} - p_{i_{\text{ref}}}) \\ T_{i_{\text{col}}}^{\text{col}} \\ T_{i_{\text{sv}}}^{\text{sv}} \end{bmatrix}. \quad (3.30)$$

Then, in Mode 3, no collisions occur, proximity to the supervisor is maintained by each agent, and when the agents are not in the detection region and/or about to leave the communication region (i.e., distance greater than $r_{sv}$), they follow the designed trajectories until they reach an $\varepsilon_i$ neighborhood of $p_{i_{\text{des}}}$. 

**Remark 7.** Since the supervisor knows the states of the coverage agents, it can always regenerate trajectories if necessary, in order to ensure that (3.29) holds.

**Mode 4 ($0 < \gamma(t) < 1$, $0 \leq \dot{\gamma}(t)$):** In this mode, we have $p_{i_{\text{ref}}} = p_{i}$, $\dot{p}_{i_{\text{ref}}} = u_{i}$, $\bar{p}_{i_{\text{ref}}} \equiv 0$ and $\dot{\gamma}(t) \leq 0$. Hence, $u_{i} = \gamma(t) u_{\text{cov}} + (1 - \gamma(t)) u_{\text{glo}} + u_{\text{col}} + u_{\text{sv}} = \gamma(t) u_{\text{cov}} + u_{\text{col}} + u_{\text{sv}}$, since $u_{\text{glo}} \equiv 0$. Note that $T_{i_{\text{glo}}}^{\text{glo}} \equiv 0$ in this mode. Then, we can compute the time derivative
of \( V \) as

\[
\dot{V} = E_1 + E_2 + \mathcal{T}^{cov}_i \left( \gamma(t) K_{cov} I_{cov}^i + K_{col} I_{col}^i + K_{sv} I_{sv}^i \right) \\
- w_{col} \sum_{i=1}^{N} \mathcal{T}^{col}_i \left( \gamma(t) K_{cov} I_{cov}^i + K_{col} I_{col}^i + K_{sv} I_{sv}^i \right) \\
- w_{sv} \sum_{i=1}^{N} \mathcal{T}^{sv}_i \left( \gamma(t) K_{cov} I_{cov}^i + K_{col} I_{col}^i + K_{sv} I_{sv}^i \right)
\]

\[
= E_1 + E_2 - \sum_{i=1}^{N} \begin{bmatrix} I_{cov}^i & I_{col}^i & I_{sv}^i \end{bmatrix} P_i(t) \begin{bmatrix} I_{cov}^i \\ I_{col}^i \\ I_{sv}^i \end{bmatrix},
\]

(3.31)

where

\[
0 \preceq P_i(t) = \begin{bmatrix} \gamma(t) K_{cov} & K_{col} & K_{sv} \\ \gamma(t) w_{col} K_{cov} & w_{col} K_{col} & w_{col} K_{sv} \\ \gamma(t) w_{sv} K_{cov} & w_{sv} K_{col} & w_{sv} K_{sv} \end{bmatrix},
\]

(3.32)

and \( E_1 + E_2 \leq 0 \). Just as it was the case in Mode 2, \( P_i(t) \) is a time-varying matrix, hence we cannot immediately deduce the positive semi-definiteness. However, since \( P_i(t) = P_i(t) \), and we have already shown that \( P_i(t) \) is positive semi-definite for \( 0 \leq \gamma(t) \leq 1 \), we conclude that \( P_i(t) \) is also positive semi-definite. Hence, \( \dot{V} \) satisfies

\[
\dot{V} \leq - \sum_{i=1}^{N} \begin{bmatrix} I_{cov}^i & I_{col}^i & I_{sv}^i \end{bmatrix} P_i(t) \begin{bmatrix} I_{cov}^i \\ I_{col}^i \\ I_{sv}^i \end{bmatrix}.
\]

(3.33)

Thus, in Mode 4, the agents continue gathering sensory information as long as they don’t end up at local minima of \( V \), no collisions occur and proximity to the supervisor is maintained by each coverage agent. Moreover, the sensors are turned on again; this implies that agents begin accumulating new sensory information as soon as they enter Mode 4.
Remark 8. We included Section 3.1 to have a complete discussion on the supervised dynamic coverage control scheme and the technical details of the coverage error function. For the remainder of this dissertation, we will always consider the coverage objective together with the collision avoidance objective, thus the overlapping of sensing regions will not be a problem for the control schemes.

3.3 Supervised Coverage Control for Wheeled Mobile Robots

In this section, we apply the supervised coverage control scheme to multi-agent systems where the agents are wheeled mobile robots modeled as kinematic unicycles. For this purpose, we utilize a feedback linearization method for kinematic unicycle agents to describe the agents as single integrators. Then, based on the linearized system, we modify the definitions of the coverage, collision avoidance and proximity functions.

3.3.1 Kinematic Unicycle Equations of Motion

We model wheeled mobile robots as kinematic unicycle agents, described by the following equations:

\[
\begin{bmatrix}
\dot{p}_i \\
\dot{\theta}_i
\end{bmatrix} =
\begin{bmatrix}
\cos (\theta_i) & 0 \\
\sin (\theta_i) & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
u_i \\
\nu_i
\end{bmatrix},
\]

(3.34)

where \(p_i = \begin{bmatrix} x_i & y_i \end{bmatrix}^T \) for \(i = 1, \ldots, N\), \(x_i, y_i \in \mathbb{R}\) are the Cartesian coordinates of the agents, \(\theta_i \in [0, 2\pi)\) are the orientations of the agents with respect the world frame and \(u_i, \nu_i\) are linear and angular control inputs, respectively.
### 3.3.2 Feedback Linearization

In this section, we follow the feedback linearization technique proposed by De Luca et al. in [62]. We consider a point $B_i$ on the robot that has the following coordinates:

$$
x_{B_i} = x_i + l_i \cos(\theta_i), \quad y_{B_i} = y_i + l_i \sin(\theta_i), \quad i = 1, \cdots, N, \quad (3.35)
$$

where $x_i$ and $y_i$ represent the coordinates of the centroid of the $i^{th}$ unicycle robot, $\theta_i$ is the orientation of the $i^{th}$ robot with respect to the world frame and $l_i$ is the distance of point $B_i$ to the centroid along the axis of the robot that is perpendicular to the wheel axis. Note that $l_i \neq 0$ and they are design variables, meaning that we are free to choose where points $B_i$ will be located with respect to the centroids of the unicycles. In this work, we choose the same distance for all robots; i.e., $l_i = l$ for all $i = 1, \cdots, N$.

After the coordinate transformation, we have the following kinematic equations for $B_i$:

$$
\dot{x}_{B_i} = \dot{x}_i - l_i \sin(\theta_i) \dot{\theta}_i = u_i \cos(\theta_i) - l \sin(\theta_i) \nu_i
$$

$$
\dot{y}_{B_i} = \dot{y}_i + l \cos(\theta_i) \dot{\theta}_i = u_i \sin(\theta_i) + l \cos(\theta_i) \nu_i, \quad (3.36)
$$

which can be written in the following way:

$$
\begin{bmatrix}
\dot{x}_{B_i} \\
\dot{y}_{B_i}
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta_i) & -l_i \sin(\theta_i) \\
\sin(\theta_i) & l_i \cos(\theta_i)
\end{bmatrix}
\begin{bmatrix}
u_i \\
u_i
\end{bmatrix}
:= \Gamma_i
\begin{bmatrix}
u_i \\
u_i
\end{bmatrix}. \quad (3.37)
$$

It can be seen that $\det(\Gamma_i) = l_i \neq 0$ for all $i = 1, \cdots, N$, hence $\Gamma_i$’s are always invertible. Using the invertibility of $\Gamma_i$, we define the control signals in the following way:

$$
\begin{bmatrix}
u_i \\
u_i
\end{bmatrix} = \Gamma_i^{-1}
\begin{bmatrix}
u_{x_i} \\
u_{y_i}
\end{bmatrix} =
\begin{bmatrix}
u_{x_i} \cos(\theta_i) + \nu_{y_i} \sin(\theta_i) \\
\frac{1}{l_i} (\nu_{y_i} \cos(\theta_i) - \nu_{x_i} \sin(\theta_i))
\end{bmatrix}. \quad (3.38)
$$
Thus, we end up with the following equations of motion for $B_i$ for $i = 1, \ldots, N$:

\[
\begin{align*}
\dot{x}_{B_i} &= v_{x_i} \\
\dot{y}_{B_i} &= v_{y_i} \\
\dot{\theta}_i &= \frac{1}{l_i} (v_{y_i} \cos(\theta_i) - v_{x_i} \sin(\theta_i)) .
\end{align*}
\] (3.39)

By using a coordinate transformation, we end up with a single integrator model for the unicycle robot where the evolution of the orientation is prescribed via the transformation. Such a model is especially suitable for the coverage control problem discussed in this work since our objective is not to control the orientation but rather control the motion of the robots on the plane; in that sense, orientation is a degree of freedom that we’re not imposing motion on, but instead utilizing to control the motion of the agents on $\mathbb{R}^2$.

### 3.3.3 Modification of Functions

Now that we have attained a single integrator model of the unicycle agent through feedback linearization, we can apply the coverage, collision avoidance and proximity maintenance control laws discussed in Section 3.2. However, note that the agents themselves are still unicycle agents; points $B_i$ act as single integrators. Thus, we need to make sure that the control inputs are designed according to $B_i$ rather than the centroids of the unicycle agents. To this purpose, we modify the definitions of the functions that have been utilized for single integrators accordingly; whenever we use $p_i$, instead we use $p_{B_i}$. For instance, the sensor model is modified as

\[
S_i(t, \|p_{B_i} - \tilde{p}\|^2) \triangleq \gamma(t) \tilde{S}_i(\|p_{B_i} - \tilde{p}\|^2),
\] (3.40)

where $p_{B_i} = [x_{B_i} \ y_{B_i}]^T$. The accumulated information $Q(t, p)$ given by (2.5) becomes

\[
Q(t, P_B) = C^* - C^* e^{-k^* A},
\] (3.41)
with \( \mathbf{P_B} = \begin{bmatrix} p_{B_1}^T & \cdots & p_{B_N}^T \end{bmatrix}^T \), and

\[
A(t, \tilde{p}) = \int_0^t \sum_{i=1}^N S_i(t, \|p_{B_i}(\tau) - \tilde{p}\|^2) d\tau.
\]  

(3.42)

Similarly, the coverage error, avoidance and proximity functions are modified in the following way:

\[
e(t) = \iint_D h(C^* - Q(t, \mathbf{P_B})) \phi(\tilde{p}) \left[ S^* - \sum_{i=1}^N S_i + \sum_{i=1}^N \sigma(G_i) \right] d\tilde{p_1} d\tilde{p_2}.
\]  

(3.43)

with \( G_i = \frac{\|p_{B_i} - p_{i\text{ref}}\|^2}{2} \),

\[
V_{ij}^{\text{col}}(p_{B_i}, p_{B_j}) = \left( \min \left\{ 0, \frac{\|p_{B_i} - p_{B_j}\|^2 - R_{\text{col}}^2}{\|p_{B_i} - p_{B_j}\|^2 - r_{\text{col}}^2} \right\} \right)^2,
\]  

(3.44)

and

\[
V_i^{\text{sv}}(p_{B_i}, p_{\text{sv}}) = \left( \max \left\{ 0, \frac{r_{\text{sv}}^2 - \|p_{B_i} - p_{\text{sv}}\|^2}{\|p_{B_i} - p_{\text{sv}}\|^2 - R_{\text{sv}}^2} \right\} \right)^2.
\]  

(3.45)

**Remark 9.** Utilization of the modified functions imply that the sensors are not placed at the centroids of the agents, but at the offset points \( B_i \).

Naturally, the weighted formula given by (2.26) becomes

\[
w_i = \mu_1 \|p_{B_i} - \bar{p}\| + \mu_2 \frac{1}{(C^* - \bar{C} _{\tilde{p}})^2},
\]  

(3.46)

and the conditions on the reference trajectories, given by (2.28), are modified as

\[
p_{i\text{ref}}(\tau'') = p_{B_i}(\tau''),
\]

\[
\dot{p}_{i\text{ref}}(\tau'') = \dot{p}_{B_i}(\tau''),
\]

\[
\ddot{p}_{i\text{ref}}(t) \equiv 0, \quad t \geq \tau'' + \Delta t_{i\text{ref}},
\]

\[
\|p_{i\text{ref}}(\tau'' + \Delta t_{i\text{ref}}) - \bar{p}_i\| \leq \bar{\varepsilon}_i.
\]  

(3.47)
Finally, condition \textbf{C2} of Section \ref{sec:2.6} given by (2.32), becomes

\[ C'_{uni} : \|p_{Bi} - \tilde{p}_i\| \leq \tilde{\varepsilon}_i. \]  

(3.48)

### 3.3.4 Discussion on Orientation

The aforementioned linearization technique provides a convenient representation of a unicycle agent in the context of coverage control. However, the time evolution of the orientation of the unicycle is not explicitly handled. In this section, we discuss how \( \dot{\theta}_i \) behaves for each unicycle agent in the presence of control inputs given by (3.14). For notational simplicity, we will drop the subscripts from the variables, and discuss the behavior of the signals for one agent.

Initially, we show that each control signal, \( u_{cov}, u_{glo}, u_{col} \) and \( u_{sv} \), is bounded. To this purpose, let us first consider \( u_{cov} \). By definition,

\[ u_{cov} = -2K_{cov} \iint_{\mathcal{D}} h(C^* - Q)\gamma(t)\tilde{S}'(p_B - \tilde{p})^T d\tilde{p}_1 d\tilde{p}_2. \]  

(3.49)

Note that \( h(C^* - Q) \) is bounded, \( \gamma(t) \) is bounded, \( \tilde{S}' \) is bounded, and due to the fact that the term is a compact area integral, there exists a bound \( \beta_{cov} > 0 \) such that

\[ \|u_{cov}\| \leq \beta_{cov}. \]  

(3.50)

Similarly, by definition, we have,

\[ u_{glo} = -K_{glo}(p_B - p_{ref}) + \tilde{p}_{ref}, \]  

(3.51)

where the feedforward term \( \tilde{p}_{ref}(t) \) is equal to \( \dot{p}_{ref}(t) \) in Mode 3, and 0 otherwise. Since the reference trajectory is generated by the supervisor, the derivative of the trajectory is designed to be bounded. Thus, there exists a \( \beta_{glo} > 0 \) such that

\[ \|u_{glo}\| \leq \beta_{glo}. \]  

(3.52)
Due to the continuity and monotonicity of the collision avoidance functions, there exists a constant \( r \in (r_{col}, R_{col})^* \) (63 64), such that

\[
\inf_{t \geq 0} \|p_{Bi} - p_{Bj}\| \geq r, \quad \forall i \in \{1, \cdots, N\}, i \neq j.
\] (3.53)

Thus, based on the definitions for the partial derivatives of \( V_{ij}^{col} \), given by (2.12), the gradient terms are always bounded since there are no collisions. Hence, for an agent \( i \), there exists a \( \beta_{coli} > 0 \) such that

\[
\|u_{coli}\| = \left\| K_{col} \sum_{i \neq j} \frac{\partial V_{ij}^{col}}{\partial p_{Bi}} \right\| \leq \beta_{coli}.
\] (3.54)

To simplify the notation, we say that there exists a \( \beta_{col} > 0 \) such that

\[
\|u_{col}\| \leq \beta_{col}.
\] (3.55)

Using a similar argument, there exists a constant \( R \in [r_{sv}, R_{sv})^* \) such that

\[
\sup_{t \geq 0} \|p_{Bi} - p_{sv}\| \leq R, \quad \forall i \in \{1, \cdots, N\}.
\] (3.56)

Thus, using the definition of the gradient of \( V_{i}^{sv} \) given by (2.21), for each agent, there exists a \( \beta_{prox_i} > 0 \) such that

\[
\|u_{sc_i}\| = \left\| K_{sv} \sum_{i=1}^{N} \frac{\partial V_{i}^{sv}}{\partial p_i} \right\| \leq \beta_{sv_i}.
\] (3.57)

Again, to simplify the notation, we say that there exists a \( \beta_{sv} > 0 \) such that

\[
\|u_{sv}\| \leq \beta_{sv}.
\] (3.58)

We have thus shown that each control signal is bounded. By construction, we also have

\*Although we assume that \( r_{col_i} = r_{col} \) and \( R_{col_i} = R_{col} \) for all \( i = 1, \cdots, N \) for simplicity, the discussion can easily be generalized to the case where each agent has different detection and avoidance radii.
\[ \|\gamma(t)\| \leq 1. \] This implies the following inequality:

\[ \|v_x\| \leq \beta_{max}, \quad \|v_y\| \leq \beta_{max}, \quad (3.59) \]

where \( \beta_{cov} + \beta_{glo} + \beta_{col} + \beta_{sv} := \beta_{max} \). The same inequality holds for every agent; i.e.,

\[ \|v_{x_i}\| \leq \beta_{max}, \quad \|v_{y_i}\| \leq \beta_{max}, \quad i = 1, \cdots, N. \quad (3.60) \]

Using the transformed equations of motion for the unicycle agents, given by (3.39), and the bounds on the control signals, given by (3.60), we can derive the following bound on the derivative of the orientation of each agent:

\[ \|\dot{\theta}_i(t)\| = \left\| \frac{v_{y_i} \cos (\theta_i) - v_{x_i} \sin (\theta_i)}{l} \right\| \leq \frac{1}{l} \left( \|v_{y_i} \cos (\theta_i)\| + \|v_{x_i} \sin (\theta_i)\| \right) \leq \frac{2\beta_{max}}{l}. \quad (3.61) \]

Note that the above bound is a very conservative bound; in implementation of the supervised coverage control scheme, the bound on the control signal is much smaller.

### 3.4 Asynchronous Supervised Coverage Control

In the supervised coverage control scheme of Section 3.1, the coverage agents are always in the same mode; whenever Condition \( C1 \) or \( C2' \) are satisfied, all agents transition to the corresponding modes. There may occur situations where an agent may not have reached a local minima of \( e(t) \) whereas the other agents have, so since that one agent is still actively covering the domain, all other agents have to wait for it to reach local minima in order to transition to trajectory tracking mode. In these situations, coverage agents may have to stay idle without doing any coverage, before transitioning to other modes. Naturally, this would cause the agents to complete the coverage objective in longer time.

To overcome this issue, in this section, we propose an asynchoronous variant of the supervised dynamic coverage scheme. The main difference between the supervised coverage scheme and the asynchronous variant is that the coverage agents may transition between
modes independently of other agents. By doing this, the duration in which agents stay idle is decreased, thus enabling the coverage agents to complete the coverage mission faster.

One thing that is different in asynchronous supervised coverage scheme is the condition that enables a coverage agent to transition from coverage mode to trajectory tracking mode. We modify this condition in the following way:

\[ C_{1_{as}}: \quad \| \bar{e}_i(t + \Delta \bar{e}_i) - \bar{e}_i(t) \| \leq \varepsilon, \tag{3.62} \]

where \( \bar{e}_i(t) = \frac{\iint_D h(C^*-Q_i)d\bar{p}d\bar{p}_2}{C^*+\text{Area}(D)} \) is the normalized individual coverage error, with

\[ Q_i(t, p) = C^{**} - C^{**}e^{-k^*A_i}, \tag{3.63} \]

and

\[ A_i(t, \bar{p}) = \int_0^t S_i(t, \|p_i(\tau) - \bar{p}\|^2) d\tau. \tag{3.64} \]

If the decrease in \( \bar{e}_i(t) \) over a specified amount of time \( \Delta \bar{e}_i \) is less than the prescribed threshold \( \varepsilon \), it is concluded that the agent is not doing much coverage; hence, sensors are turned off and the agent transitions to trajectory tracking mode. This check is done every \( \Delta \bar{e}_i \) seconds. Additionally, each agent has its own \( \gamma_i(t) \) signal instead of a common transitioning signal. Now, we state the main result on the stability of the asynchronous supervised coverage scheme.

### 3.4.1 Main Results

**Theorem 3.** Consider a group of \( N \) single integrators and a supervisor on a compact domain \( D \subset \mathbb{R}^2 \) with the control inputs

\[ u_i = \gamma_i(t)u_{cov_i} + (1 - \gamma_i(t))u_{glo_i} + u_{col_i} + u_{sv_i}, \tag{3.65} \]

where \( \gamma_i(t) \) is the smooth transition signal given by \( \text{[2.1]} \), transitioning according to the conditions \( C_{1_{as}} \) and \( C_{2'} \) given by \( \text{[3.62]} \) and \( \text{(2.32)} \), and \( u_{cov_i}, u_{glo_i}, u_{col_i} \) and \( u_{sv_i} \) are
where \( h(\cdot), Q \) and \( \tilde{S}'_i \) are defined in Section 2.2, \( p_{\text{ref}} \) is the reference trajectory provided by the supervisor, \( \tilde{p}_{\text{ref}}(t) \) is a feedforward term, and \( V_{ij}^{\text{col}} \) and \( V_{i}^{\text{sv}} \) are given by (2.8) and (2.18) respectively. Assume that \( P_0 \notin \Omega \) and \( P_0 \notin \mathcal{L}^s \), where \( \Omega \) and \( \mathcal{L}^s \) are overall avoidance region and the communication loss region for the agents and the supervisor, given by (2.10) and (2.20). Then, there exist matrices \( 0 \prec K_{\text{cov}_i} \equiv \begin{bmatrix} k_{\text{cov}_ix} & 0 \\ 0 & k_{\text{cov}_iy} \end{bmatrix} \), 

\( 0 \prec K_{\text{glo}_i} \equiv \begin{bmatrix} k_{\text{glo}_ix} & 0 \\ 0 & k_{\text{glo}_iy} \end{bmatrix} \), 

\( 0 \prec K_{\text{col}_i} \equiv \begin{bmatrix} k_{\text{col}_ix} & 0 \\ 0 & k_{\text{col}_iy} \end{bmatrix} \) and \( 0 \prec K_{\text{sv}_i} \equiv \begin{bmatrix} k_{\text{sv}_ix} & 0 \\ 0 & k_{\text{sv}_iy} \end{bmatrix} \) such that the following results hold:

(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in \( \mathcal{D} \) reaches \( C^* \) at a finite time \( T_{\text{final}} \).

(ii) In the trajectory tracking mode, each agent follows their reference trajectories and travels to a sufficiently close neighborhood of \( p_{\text{des}} \) such that Condition C2' is satisfied.

(iii) Agents avoid collisions with other agents; i.e., \( P \notin \Omega \) for all \( t \in [t_0, T_{\text{final}}] \), \( i = 1, \cdots, N \).

(iv) Agents maintain proximity to the supervisor; i.e., \( P \notin \mathcal{L}^s \) for all \( t \in [t_0, T_{\text{final}}] \), \( i = 1, \cdots, N \).

Proof. In order to analyze the stability of the asynchronous supervised control system,
we will consider the most general case; i.e., we will assume that there is at least one agent in each mode. To this purpose, consider the following Lyapunov-like function:

\[ V(t) = e + V_{col} + V_{sv} \]
\[ = e(t) + \frac{w_{col}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V_{ij} + w_{sv} \sum_{i=1}^{N} V_{sv}^i, \]  

(3.67)

where \( 0 < w_{col}, w_{sv} \) are design parameters, \( e \) is given by (2.6), \( V_{ij} \) is given by (2.8) and \( V_{sv}^i \) is given by (2.18). Let us define the following notations:

\[
E_1 = -\int\int_{D} h(C^* - Q)(k^* C^{**} e^{-k^* A})(\sum_{i=1}^{N} S_i)(S^* - \sum_{i=1}^{N} S_i) + \sum_{i=1}^{N} \sigma(G_i)d\tilde{p}_1d\tilde{p}_2
\]

\[
E_{im_2} = -\hat{\gamma}_i(t)\int\int_{D} h(C^* - Q)\tilde{S}_i d\tilde{p}_1d\tilde{p}_2, \quad i = 1, \ldots, N_m, \quad m = 1, \ldots, 4,
\]

\[
\kappa_{glo}^i = \left(\int\int_{D} h(C^* - Q)d\tilde{p}_1d\tilde{p}_2\right)(\sigma(G_i)(1 - \sigma(G_i))),
\]

\[
T_{glo}^i = \kappa_{glo}^i(\hat{p}_i - \hat{p}_{i_{ref}}(t))^T,
\]  

(3.68)

where \( N_m \) is the number of agents that are in Mode \( m \). Notice that \( E_1 \leq 0 \).

Using the notations of (3.66) and (3.68), we can write the derivative of \( V(t) \) in the following way:

\[
\dot{V}(t) = \dot{e}(t) + \dot{V}_{col} + \dot{V}_{sv}
\]
\[
= E_1 + \sum_{m=1}^{4} \sum_{i=1}^{N_m} E_{im_2} + \sum_{m=1}^{4} \sum_{i=3}^{N_m} T_{col}^i \dot{\hat{p}_i} + \sum_{i=1}^{N_3} T_{glo}^i (\hat{p}_i - \hat{p}_{i_{ref}})
\]
\[
+ w_{col} \sum_{m=1}^{4} \left(\sum_{j \neq i}^{N} \frac{\partial V_{ij}^T}{\partial \hat{p}_i}\right) \dot{\hat{p}_i} + w_{sv} \sum_{m=1}^{4} \sum_{i=1}^{N_m} \frac{\partial V_{sv}^T}{\partial \hat{p}_i} \dot{\hat{p}_i}. \]  

(3.69)

For agents that are in Mode 1, the supervisor has not provided the agents with reference trajectories since they are in coverage mode. Thus, we have \( p_{i_{ref}} = p_i \) and \( \dot{p}_{i_{ref}} = u_i \).
\( \tilde{p}_{i\text{ref}} \equiv 0 \) and \( \dot{\gamma}(t) = 0 \). Hence, \( u_i = u_{\text{con}i} + u_{\text{col}i} + u_{\text{sv}i} \). Also, note that \( T_i^{\text{glo}} \equiv 0 \) and \( E_{i12} \equiv 0 \) for the agents in Mode 1.

For agents that are in Mode 2, we have \( p_{i\text{ref}} = p_i, \dot{p}_{i\text{ref}} = u_i, \tilde{p}_{i\text{ref}} \equiv 0 \) and \( \dot{\gamma}_i(t) \leq 0 \). Hence, \( u_i = \gamma_i(t)u_{\text{con}i} + (1 - \gamma_i(t))u_{\text{glo}i} + u_{\text{col}i} + u_{\text{sv}i} = \gamma_i(t)u_{\text{con}i} + u_{\text{col}i} + u_{\text{sv}i} \) since \( u_{\text{glo}i} \equiv 0 \). Note that \( T_i^{\text{glo}} \equiv 0 \) in this mode. Moreover, as we have verified through simulations as well, we can select \( k^* \) and \( \omega_{s2} \) such that

\[
E_1 + \sum_{i=1}^{N_2} E_{i22} \leq 0, \quad (3.70)
\]

for all \( 0 \leq \gamma_i(t) \leq 1 \).

For agents that are in Mode 3, \( \dot{\gamma}_i(t) \equiv 0, T_i^{\text{con}} \equiv 0 \) and \( E_{i32} \equiv 0 \). Moreover, a trajectory is generated for each coverage agent by the supervisor. Hence, \( u_i = u_{\text{glo}i} + u_{\text{col}i} + u_{\text{sv}i} \). The supervisor has freedom in designing the trajectories that satisfy the conditions given by (2.28). The feedforward terms are selected as \( \tilde{p}_{i\text{ref}} \equiv \dot{p}_{i\text{ref}}, \) and the trajectories are designed such that the following inequality holds for all agents in Mode 3:

\[
\sum_{i=1}^{N_3} \left( w_{\text{col}}T_i^{\text{col}T} + w_{\text{sv}}T_i^{\text{sv}T} \right) \hat{p}_{i\text{ref}} \leq 0. \quad (3.71)
\]

Finally, for agents that are in Mode 4, we have \( p_{i\text{ref}} = p_i, \dot{p}_{i\text{ref}} = u_i, \tilde{p}_{i\text{ref}} \equiv 0 \) and \( \dot{\gamma}_i(t) \leq 0 \). Hence, \( u_i = \gamma_i(t)u_{\text{con}i} + (1 - \gamma_i(t))u_{\text{glo}i} + u_{\text{col}i} + u_{\text{sv}i} = \gamma_i(t)u_{\text{con}i} + u_{\text{col}i} + u_{\text{sv}i} \) since \( u_{\text{glo}i} \equiv 0 \). Note that \( T_i^{\text{glo}} \equiv 0 \) in this mode. Moreover, since \( \dot{\gamma}_i(t) \leq 0 \), we have

\[
E_1 + \sum_{i=1}^{N_4} E_{i42} \leq 0. \quad (3.72)
\]

\[\text{†}\]In this work, we assume \( \omega_{s2} \) is the same for all \( \gamma_i(t) \) functions but the results can easily be generalized to transition functions with different frequency values.
As a result, we have the following inequality for the derivative of $V(t)$:

$$
\dot{V}(t) \leq - \sum_{i=1}^{N_1} \left[ \begin{array}{ccc} I_{cov}^T & I_{col}^T & I_{sv}^T \end{array} \right] P_{i1} \left[ \begin{array}{c} I_{cov}^T \\
I_{col}^T \\
I_{sv}^T \end{array} \right] - \sum_{i=1}^{N_2} \left[ \begin{array}{ccc} I_{cov}^T & I_{col}^T & I_{sv}^T \end{array} \right] P_{i2}(t) \left[ \begin{array}{c} I_{cov}^T \\
I_{col}^T \\
I_{sv}^T \end{array} \right]$$

$$- \sum_{i=1}^{N_3} \left[ \sqrt{\kappa_i^{gl}} (p_i - p_{i,ref}) \right]^T I_{i}^T P_{i3} \left[ \begin{array}{c} \sqrt{\kappa_i^{gl}} (p_i - p_{i,ref}) \end{array} \right]$$

$$- \sum_{i=1}^{N_4} \left[ I_{cov}^T I_{col}^T I_{sv}^T \right] P_{i4}(t) \left[ \begin{array}{c} I_{cov}^T \\
I_{col}^T \\
I_{sv}^T \end{array} \right],
$$

where $P_{i1}$, $P_{i2}$, $P_{i3}$ and $P_{i4}$ are all positive semi-definite matrices given by (3.20), (3.23) and (3.28), and (3.32) respectively.

Thus, we conclude that the agents in Mode 1 continue covering the compact area $D$ as long as the condition $C1_{as}$ is not satisfied, agents in Mode 2 and 4 continue covering until they transition to their corresponding modes while preserving the negative semi-definiteness of $\dot{e}$, and the agents in Mode 3 follow their corresponding trajectories generated by the supervisor agent until they reach $\bar{\varepsilon}_i$ neighborhoods of $p_{i,des}$, while maintaining the the negative semi-definiteness of $\dot{e}$. Note that $e(t)$ never increases as long as (3.70) is satisfied, and the coverage error decreases due to all agents but the ones in Mode 3; agents in Mode 3 do not contribute to the accumulated coverage information. The supervisor can always regenerate trajectories to ensure that (3.71) is satisfied. Finally, we conclude that inter-agent collisions are avoided and proximity to the supervisor is maintained by each coverage agent.

### 3.5 Summary

In this section, we presented a control scheme for dynamic coverage control problem in multi-agent systems. We formulated a framework where a supervisor assists the coverage
agents with designated trajectories whenever the coverage agents are trapped in local minima. In the design of the control scheme, we utilized smooth transitioning signals to differentiate between different operation modes, and to decouple control laws so that operation in each mode is not affected by control signals corresponding to other modes. Moreover, by including collision avoidance and proximity functions, we proposed a supervised dynamic coverage scheme with guaranteed collision avoidance and proximity maintenance. After discussing the stability of our approach via Lyapunov-like analysis, we provided a discussion on the application of the supervised coverage control scheme to wheeled mobile robots modeled as kinematic unicycles. Finally, we proposed an asynchronous variant of the supervised coverage control scheme where coverage agents can simultaneously be in different modes, and proved the stability of the asynchronous variant. We illustrate the effectiveness of the supervised coverage control scheme and compare it with the asynchronous variant via numerical simulations in Chapter 5.
Chapter 4

Swarm Based Dynamic Coverage Control

In this chapter, we focus on the swarm-based coverage control scheme. The method of the swarm-based coverage control is similar to that of the supervised coverage scheme; the agents cycle between the coverage and the global deployment modes until they accomplish the coverage objective. Moreover, the control laws derived in this chapter are also continuous; there is no discontinuous switching behavior in the implementation of the control scheme. That being said, there are differences between the swarm-based coverage scheme and the supervised coverage scheme.

In contrast to the supervised coverage scheme, there is no stationary supervisor in the swarm-based coverage scheme. Instead, there is a leader agent equipped with the same set of sensors utilized in implementing collision avoidance, obstacle avoidance and proximity maintenance just like any other coverage agent. The communication network is not between the supervisor and each coverage agent; instead, the communication is maintained via inter-agent proximity functions between each agent and every other agent. In that sense, the swarm-based scheme is distributed; rather than having a central agent that communicates with every other agent, the connectivity of a swarm is maintained via inter-agent proximity functions.

\[ \text{In this dissertation, an all-to-all communication is assumed; our scheme can be extended to consider} \]
Another difference between the supervised scheme and the swarm-based scheme is that in the supervised scheme, the supervisor provides each coverage agent with a target point and a reference trajectory that will take each agent to the target point whereas in the swarm-based scheme, only the leader agent selects a target point. There is no trajectory generation in the swarm-based scheme; the problem is that of stabilization to a point instead of trajectory tracking.

The advantage of the supervised scheme is that, when assigning each agent with a target point, if the trajectory tracking control is successful, each agent will contribute to the decrease in the coverage error, hence this will result in faster completion of the coverage mission. That being said, this advantage comes at the cost of (i) having a stationary supervisor and (ii) the computational burden of selecting target points and designing trajectories, which may be significant if the number of agents is large.

On the other hand, in the swarm-based scheme, since only one point is selected for the swarm, and the problem is stabilization to a point instead of following a trajectory, the computational burden would be insignificant even in the presence of a large number of coverage agents. Moreover, there are no stationary agents, so the limitations in the application of the supervised scheme due to the presence of a supervisor do not exist for the swarm-based scheme. One drawback of the swarm-based scheme compared to the supervised scheme is that, since only one target point is selected and all other agents swarm around that point, only the leader agent is guaranteed to contribute to the decrease in coverage error; the other agents do not necessarily end up in uncovered regions. This may cause the coverage mission to be completed in a longer time. That being said, this issue can easily be handled by either implementing a smarter point selection algorithm in swarming mode or having more agents and/or multiple swarms. Increasing the number of agents and/or swarms would not result in a significant increase in computational burden due to the distributed nature of the swarm-based scheme.

We begin this chapter by discussing the swarm-based dynamic coverage control scheme for single integrator agents and present the stability analysis of the scheme. Subsequently, more general communication topologies, which is a subject of future research.
we discuss how the scheme is applied to kinematic unicycle agents, and present the stability analysis for this case as well. In addition, we discuss how we control the orientation in kinematic unicycle agents in the context of swarm-based coverage control. Finally, we discuss a multi-swarm variant of the swarm-based coverage scheme where there are multiple swarms instead of a single swarm. In all cases, in addition to coverage and swarming, we explicitly consider collision avoidance, static obstacle avoidance and inter-agent proximity maintenance, and in the case of multi-swarm coverage scheme, we consider swarm avoidance.

4.1 Swarm-Based Coverage Control for Single Integrators

In this section, we discuss the stability of the swarm-based dynamic coverage control scheme with collision avoidance, obstacle avoidance and proximity maintenance. We consider a multi-agent system of $N$ coverage agents that are single integrators; i.e., their equations of motion are described by

$$\dot{p}_i = u_i, \quad i = 1, \cdots, N. \quad (4.1)$$

We denote the overall position vector of the coverage agents by $P = \begin{bmatrix} p_1^T & \cdots & p_N^T \end{bmatrix}^T$, and the overall position vector at initial time $t = t_0$ by $P_0$. Moreover, without loss of generality, we denote the leader agent with the subscript 1. Finally, we assume that the static obstacles on the coverage domain obey the restrictions described in Remark 2.

4.1.1 Main Results

In this section, just like we have done for the supervised coverage scheme, we set $R_{cov_i} = \frac{r_{coli}}{2}$. This implies that, if collision avoidance can be guaranteed, the agents’ sensing regions will never overlap, hence the extra terms derived in [34] will automatically be 0, just as it was the case in the supervised scheme in Section 3.2. Now, we state the main result of
Theorem 4. Consider a group of $N$ single integrators on a compact domain $\mathcal{D} \subset \mathbb{R}^2$ with $N_o$ static obstacles, with the control inputs

$$u_i = \gamma(t)u_{cov_i} + (1 - \gamma(t))u_{swa_i} + u_{col_i} + u_{obs_i} + u_{prox_i}, \quad (4.2)$$

where $\gamma(t)$ is a smooth transition signal transitioning according to the conditions $C1$ and $C2''$ given by $(2.31)$ and $(2.33)$, and $u_{cov_i}$, $u_{swa_i}$, $u_{col_i}$, $u_{obs_i}$ and $u_{prox_i}$ are given by

$$u_{cov_i} = -K_{cov_i}T_{i}^{cov}, \quad T_{i}^{cov} = -2 \iint_{\mathcal{D}} h(C^* - Q)\gamma(t)\tilde{S}'_i(p_i - \tilde{p})d\tilde{p}_1d\tilde{p}_2,$$

$$u_{swa_i} = -K_{swa_i}\arctan((\|p_i - p_{des}\|^2 - d_{i,des}^2)(p_i - p_{des}),$$

$$u_{col_i} = -K_{col_i}T_{i}^{col}, \quad T_{i}^{col} = \sum_{j \neq i}^{N} \frac{\partial V_{i, j}^{col}}{\partial p_i},$$

$$u_{obs_i} = -K_{obs_i}T_{i}^{obs}, \quad T_{i}^{obs} = \sum_{k=1}^{N_o} \frac{\partial V_{i, k}^{obs}}{\partial p_i},$$

$$u_{prox_i} = -K_{prox_i}T_{i}^{prox}, \quad T_{i}^{prox} = \sum_{j \neq i}^{N} \frac{\partial V_{i, j}^{prox}}{\partial p_i}, \quad (4.3)$$

where $h(\cdot)$, $Q$ and $\tilde{S}'_i$ are defined in Section 2.2. $V_{i, j}^{col}$, $V_{i, j}^{obs}$ and $V_{i, j}^{prox}$ are given by $(2.8)$, $(2.14)$ and $(2.22)$ respectively, $p_{des}$ is the target point selected by the leader agent in swarming mode and $d_{i,des}$ is the ideal distance of each agent to $p_{des}$. Assume that $P_0 \notin \Omega$, $P_0 \notin \Omega^o$ and $P_0 \notin \mathcal{L}$, where $\Omega$, $\Omega^o$ and $\mathcal{L}$ are overall avoidance region, overall obstacle avoidance region and the communication loss region for the agents, given by $(2.10)$, $(2.16)$ and $(2.24)$ respectively. Then, there exist matrices $0 \prec K_{cov_i} \triangleq \begin{bmatrix} k_{cov_i x} & 0 \\ 0 & k_{cov_i y} \end{bmatrix}$, $0 \prec K_{swa_i} \triangleq \begin{bmatrix} k_{swa_i x} & 0 \\ 0 & k_{swa_i y} \end{bmatrix}$, $0 \prec K_{col_i} \triangleq \begin{bmatrix} k_{col_i x} & 0 \\ 0 & k_{col_i y} \end{bmatrix}$, $0 \prec K_{obs_i} \triangleq \begin{bmatrix} k_{obs_i x} & 0 \\ 0 & k_{obs_i y} \end{bmatrix}$ and $0 \prec K_{prox_i} \triangleq \begin{bmatrix} k_{prox_i x} & 0 \\ 0 & k_{prox_i y} \end{bmatrix}$ such that the following results hold:
(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in $D$ reaches $C^*$ at a finite time $T_{\text{final}}$.

(ii) In swarming mode, each agent travels to a sufficiently close neighborhood of $p_{\text{des}}$ such that Condition C2" is satisfied for all agents.

(iii) Agents avoid collisions with other agents; i.e., $P \notin \Omega$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$.

(iv) Agents avoid all obstacles; i.e., $P \notin \Omega^o$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$.

(v) All agents stay connected; i.e., $P \notin \mathcal{L}$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$.

Proof. Consider the following Lyapunov-like function:

$$V = e + V^{\text{col}} + V^{\text{obs}} + V^{\text{prox}}$$  \hspace{1cm} (4.4)

where $e$ is the coverage area integral given by [2.7], $V^{\text{col}}$ is given by

$$V^{\text{col}} = \frac{w_{\text{col}}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V^{\text{col}}_{ij},$$  \hspace{1cm} (4.5)

with $0 < w_{\text{col}}$, $V^{\text{obs}}$ is given by

$$V^{\text{obs}} = w_{\text{obs}} \sum_{i=1}^{N} \sum_{k=1}^{N_o} V^{\text{obs}}_{ik},$$  \hspace{1cm} (4.6)

with $0 < w_{\text{obs}}$, and $V^{\text{prox}}$ is given by

$$V^{\text{prox}} = \frac{w_{\text{prox}}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V^{\text{prox}}_{ij},$$  \hspace{1cm} (4.7)

with $0 < w_{\text{prox}}$. In order to simplify some terms in taking the derivative of $V(t)$, we define
the following notations:

\[
\mathcal{I}_{swa}^i = (\|p_i - p_{des}\|^2 - d_{i,des}^2)(p_i - p_{des}),
\]

\[
E_1 = -\int_{D} \dot{h}(C^* - Q(t, P))(kC^{**}e^{-kA})(\sum_{i=1}^{N} S_i) \left[ S^* - \sum_{i=1}^{N} S_i + (1 - \gamma) \sum_{i=1}^{N} \sigma(D_i) \right] dp_1 dp_2,
\]

\[
E_2 = -\dot{\gamma}(t) \sum_{i=1}^{N} \int_{D} h(C^* - Q)\dot{S}_i dp_1 dp_2,
\]

\[
E_3 = -\gamma(t) \sum_{i=1}^{N} \sigma(D_i),
\]

\[
\kappa_{swa}^i = 2 \int_{D} h(C^* - Q(t, P))(1 - \gamma)\sigma(D_i)(1 - \sigma(D_i)).
\]  

(4.8)

Using the notations of (4.3) and (4.8), we can write the derivative of \(V(t)\) in the following way:

\[
\dot{V} = E_1 + E_2 + E_3 + \sum_{i=1}^{N} (\mathcal{I}_{cov}^i + \kappa_{swa}^i \mathcal{I}_{swa}^T + \mathcal{I}_{col}^i + \mathcal{I}_{obs}^i + \mathcal{I}_{prox}^i) \dot{p}_i.  \tag{4.9}
\]

Note that \(E_1 \leq 0\). Now, we analyze the system in 4 modes; i.e., \textbf{Mode }i \text{ with } i = 1, \ldots, 4:

\textbf{Mode 1 (}\gamma(t) \equiv 1\text{)}: In this mode, since \(\gamma(t) \equiv 1\), we have \(\kappa_{swa}^i \equiv 0\) and \(u_i = u_{cov} + u_{col} + u_{obs} + u_{prox}\). Moreover, we have \(E_2 \equiv E_3 \equiv 0\). Finally, note that \(E_1 \leq 0\) by definition. Using the definitions of (4.3), we can write the derivative of \(V\) in the following way:

\[
\dot{V} = E_1 - \sum_{i=1}^{N} (\mathcal{I}_{cov}^i + w_{col} \mathcal{I}_{col}^T + w_{obs} \mathcal{I}_{obs}^T + w_{prox} \mathcal{I}_{prox}^T) u_i
\]

\[
\leq -\sum_{i=1}^{N} \begin{bmatrix} \mathcal{I}_{cov}^i & \mathcal{I}_{col}^T & \mathcal{I}_{obs}^T & \mathcal{I}_{prox}^T \end{bmatrix} \begin{bmatrix} \mathcal{I}_{cov}^i \\ \mathcal{I}_{col} \\ \mathcal{I}_{obs} \\ \mathcal{I}_{prox} \end{bmatrix} M_{i1} \begin{bmatrix} \mathcal{I}_{cov} \\ \mathcal{I}_{col} \\ \mathcal{I}_{obs} \\ \mathcal{I}_{prox} \end{bmatrix},  \tag{4.10}
\]
with

\[ 0 \preceq \mathbf{M}_{i} = \begin{bmatrix}
K_{\text{cov}i} & K_{\text{col}i} & K_{\text{obs}i} & K_{\text{prox}i} \\
\text{w}_{\text{col}}K_{\text{cov}i} & \text{w}_{\text{col}}K_{\text{col}i} & \text{w}_{\text{col}}K_{\text{obs}i} & \text{w}_{\text{col}}K_{\text{prox}i} \\
\text{w}_{\text{obs}}K_{\text{cov}i} & \text{w}_{\text{obs}}K_{\text{col}i} & \text{w}_{\text{obs}}K_{\text{obs}i} & \text{w}_{\text{obs}}K_{\text{prox}i} \\
\text{w}_{\text{prox}}K_{\text{cov}i} & \text{w}_{\text{prox}}K_{\text{col}i} & \text{w}_{\text{prox}}K_{\text{obs}i} & \text{w}_{\text{prox}}K_{\text{prox}i}
\end{bmatrix}. \quad (4.11)

This implies that in this mode, if the coverage agents’ initial positions are such that they are outside each others’ avoidance regions and loss regions, they will never collide and the communication network will never be lost; since \( 0 \preceq \mathbf{M}_{i1} \), \( V \) will always attain finite values, which in turn means \( V^{\text{col}} \) and \( V^{\text{prox}} \) will attain finite values. Similarly, the agents will not collide with static obstacles since \( V^{\text{obs}} \) will attain finite values. Furthermore, as long as the avoidance and/or proximity gradients are zero, the agents continue the coverage mission until they reach a local minima of \( V \) since in this case, \( \dot{V}(t) \) satisfies

\[ \dot{V} \leq -\sum_{i=1}^{N} \mathcal{I}_{i}^{\text{cov}T} K_{\text{cov}i} \mathcal{I}_{i}^{\text{cov}}. \quad (4.12) \]

**Mode 2** (\( 0 \leq \gamma(t) \leq 1, \ \dot{\gamma}(t) \leq 0 \)): In this mode, \( \dot{\gamma}(t) \leq 0 \). Consequently, we get \( u_i = \gamma(t)u_{\text{cov}i} + (1 - \gamma(t))u_{\text{swa}} + u_{\text{col}i} + u_{\text{obs}i} + u_{\text{prox}i} \). Then, the time derivative of \( V \) is given by

\[
\dot{V} = -\sum_{i=1}^{N} \left[ \mathcal{I}_{i}^{\text{cov}T} \mathcal{I}_{i}^{\text{swa}} (p_i - p_{\text{des}})^T \mathcal{I}_{i}^{\text{col}T} \mathcal{I}_{i}^{\text{obs}T} \mathcal{I}_{i}^{\text{prox}T} \right] \mathbf{M}_{i2}(t) \left[ \begin{array}{c}
\mathcal{I}_{i}^{\text{cov}} \\
\mathcal{I}_{i}^{\text{swa}} (p_i - p_{\text{des}}) \\
\mathcal{I}_{i}^{\text{col}} \\
\mathcal{I}_{i}^{\text{obs}} \\
\mathcal{I}_{i}^{\text{prox}}
\end{array} \right] + E_1 + E_2 + E_3, \quad (4.13)
\]

\[ \text{The desired target position } p_{\text{des}} \] and final distances of agents to the target position, \( d_{i\text{des}} \), may be generated at any time during Mode 1; i.e., as long as \( \gamma(t) \equiv 1 \).
with

$$
\mathcal{M}_i^2(t) = \begin{bmatrix}
\gamma(t)K_{\text{cov}_i} & (1 - \gamma(t))K_{\text{swa}_i} & K_{\text{col}_i} & K_{\text{obs}_i} & K_{\text{prox}_i} \\
\gamma(t)K_{\text{cov}_i} & (1 - \gamma(t))K_{\text{swa}_i} & K_{\text{col}_i} & K_{\text{obs}_i} & K_{\text{prox}_i} \\
\gamma(t)w_{\text{col}_i}K_{\text{cov}_i} & (1 - \gamma(t))w_{\text{col}_i}K_{\text{swa}_i} & w_{\text{col}_i}K_{\text{col}_i} & w_{\text{col}_i}K_{\text{obs}_i} & w_{\text{col}_i}K_{\text{prox}_i} \\
\gamma(t)w_{\text{obs}_i}K_{\text{cov}_i} & (1 - \gamma(t))w_{\text{obs}_i}K_{\text{swa}_i} & w_{\text{obs}_i}K_{\text{col}_i} & w_{\text{obs}_i}K_{\text{obs}_i} & w_{\text{obs}_i}K_{\text{prox}_i} \\
\gamma(t)w_{\text{prox}_i}K_{\text{cov}_i} & (1 - \gamma(t))w_{\text{prox}_i}K_{\text{swa}_i} & w_{\text{prox}_i}K_{\text{col}_i} & w_{\text{prox}_i}K_{\text{obs}_i} & w_{\text{prox}_i}K_{\text{prox}_i}
\end{bmatrix},
$$

$$
c_i^{\text{swa}} := \sqrt{\kappa_i^{\text{swa}} \arctan \left( \left\| p_i - p_{\text{des}} \right\|^2 - d_{\text{des}}^2 \right) / \left( \left\| p_i - p_{\text{des}} \right\|^2 - d_{\text{des}}^2 \right)},
$$

(4.14)

Note that $\mathcal{M}_i^2(t)$ is a time-varying matrix. To show that $\mathcal{M}_i^2(t)$ is a positive semi-definite matrix, we can decompose it in the following way:

$$
\mathcal{M}_i^2(t) = W_i^{\text{swa}} J_i^{\text{swa}} K_i^{\text{swa}}(t),
$$

(4.15)

with

$$
W_i^{\text{swa}} = \begin{bmatrix}
I_{2 \times 2} & 0 & 0 & 0 & 0 \\
0 & I_{2 \times 2} & 0 & 0 & 0 \\
0 & 0 & w_{\text{col}_i}I_{2 \times 2} & 0 & 0 \\
0 & 0 & 0 & w_{\text{obs}_i}I_{2 \times 2} & 0 \\
0 & 0 & 0 & 0 & w_{\text{prox}_i}I_{2 \times 2}
\end{bmatrix},
$$

$$
J_i^{\text{swa}} = \begin{bmatrix}
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2}
\end{bmatrix},
$$

$$
K_i^{\text{swa}}(t) = \begin{bmatrix}
\gamma K_{\text{cov}_i} & 0 & 0 & 0 & 0 \\
0 & (1 - \gamma)K_{\text{swa}_i} & 0 & 0 & 0 \\
0 & 0 & K_{\text{col}_i} & 0 & 0 \\
0 & 0 & 0 & K_{\text{obs}_i} & 0 \\
0 & 0 & 0 & 0 & K_{\text{prox}_i}
\end{bmatrix},
$$

(4.16)
where $\mathbf{1}_{2 \times 2} := \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{I}_{2 \times 2}$ is the 2 by 2 identity matrix. Notice that $\mathbf{W}_{\text{swa}}^i$ and $\mathbf{K}_{\text{swa}}^i(t)$ are diagonal matrices. Moreover, it can be seen that for positive values of $w_{\text{col}}$, $w_{\text{obs}}$ and $w_{\text{prox}}$, $\mathbf{W}_{\text{swa}}^i$ is positive definite. For $0 < \gamma(t) < 1$, for positive gain matrices $K_{\text{cov}}^i, K_{\text{swa}}^i, K_{\text{col}}^i, K_{\text{obs}}^i$ and $K_{\text{prox}}^i$, $\mathbf{K}_{\text{swa}}^i(t)$ is positive definite, and at the beginning or end of Mode 2, i.e., for $\gamma(t) \equiv 1$ or $\gamma(t) \equiv 0$, $\mathbf{K}_{\text{swa}}^i(t)$ is positive semi-definite. Finally, by definition, $\mathbf{J}_{\text{swa}}^i$ has nonnegative eigenvalues. Hence, in Mode 2, the positive semi-definiteness of $\mathcal{M}_{i_2}^2(t)$ is maintained. Moreover, in order to render $\dot{V}$ negative semi-definite in Mode 2, as we have verified through simulations as well, we can select $k^*$ and $\omega_{s_2}$ such that $E_1 + E_2 + E_3 \leq 0$ for all $0 \leq \gamma(t) \leq 1$. Then, $\dot{V}$ satisfies

$$\dot{V} \leq - \sum_{i=1}^{N} \begin{bmatrix} \mathcal{T}_{\text{cov}}^i \\ \mathcal{T}_{\text{col}}^i \\ \mathcal{T}_{\text{obs}}^i \\ \mathcal{T}_{\text{prox}}^i \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{swa}}^i(p_i - p_{\text{des}}) \\ \mathbf{c}_{\text{swa}}^i(p_i - p_{\text{des}}) \end{bmatrix} \mathcal{M}_{i_2}^2(t) \begin{bmatrix} \mathcal{T}_{\text{cov}}^i \\ \mathcal{T}_{\text{col}}^i \\ \mathcal{T}_{\text{obs}}^i \\ \mathcal{T}_{\text{prox}}^i \end{bmatrix}.$$ (4.17)

Thus, in Mode 2, collisions among agents are avoided, obstacles are avoided and the connectivity of the communication network is maintained by each coverage agent. Note that at the instance $\gamma(t)$ becomes 0, $\mathcal{T}_{\text{cov}}^i$, as well as $E_1$ and $E_2$, become 0, too. This implies that the sensors of the coverage agents are turned off.

Notice that during the transition period, there are two possibly conflicting objectives, the coverage objective and the swarming objective, that are simultaneously active since both the coverage and swarming control signals act on the agents. That being said, due to the fact that $0 \preceq \mathcal{M}_{i_2}^2(t)$, $V$ is guaranteed not to increase and this guarantee is sufficient; we do not expect the multi-agent system to accomplish either the coverage objective or the swarming objective in the transition mode. We only want to guarantee that $\dot{V}$ does not attain positive values, and that is exactly what is guaranteed in Mode 2.
Mode 3 ($\gamma(t) \equiv 0$): In this mode, $\dot{\gamma}(t) \equiv 0$, $I_i^{\text{cov}} \equiv 0$, $E_2 \equiv 0$ and $E_3 \equiv 0$. Moreover, a desired target position has already been generated for the multi-agent group as well as desired distances $d_{i\text{des}}$ for each agent. The control signal for each agent is $u_i = u_{\text{swa}_i} + u_{\text{col}_i} + u_{\text{obs}_i} + u_{\text{prox}_i}$. Therefore, $\dot{V}$ becomes

$$\dot{V} = E_1 - \sum_{i=1}^{N} \left[ c_i^{\text{swa}} (p_i - p_{\text{des}})^T \begin{bmatrix} T_{i}^{\text{col}} & T_{i}^{\text{obs}} & T_{i}^{\text{prox}} \end{bmatrix} \right] M_i \begin{bmatrix} c_i^{\text{swa}} (p_i - p_{\text{des}}) \\ T_{i}^{\text{col}} \\ T_{i}^{\text{obs}} \\ T_{i}^{\text{prox}} \end{bmatrix}, \quad (4.18)$$

where

$$0 \preceq M_i = \begin{bmatrix} K_{\text{swa}_i} & K_{\text{col}_i} & K_{\text{obs}_i} & K_{\text{prox}_i} \\ w_{\text{col}} K_{\text{swa}_i} & w_{\text{col}} K_{\text{col}_i} & w_{\text{col}} K_{\text{obs}_i} & w_{\text{col}} K_{\text{prox}_i} \\ w_{\text{obs}} K_{\text{swa}_i} & w_{\text{obs}} K_{\text{col}_i} & w_{\text{obs}} K_{\text{obs}_i} & w_{\text{obs}} K_{\text{prox}_i} \\ w_{\text{prox}} K_{\text{swa}_i} & w_{\text{prox}} K_{\text{col}_i} & w_{\text{prox}} K_{\text{obs}_i} & w_{\text{prox}} K_{\text{prox}_i} \end{bmatrix},$$

$$c_i^{\text{swa}} := \sqrt{\kappa_i^{\text{swa}} \arctan (\|p_i - p_{\text{des}}\|^2 - d_{i\text{des}}^2)(\|p_i - p_{\text{des}}\|^2 - d_{i\text{des}}^2)}, \quad (4.19)$$

and $E_1 \leq 0$. Thus, $\dot{V}$ satisfies

$$\dot{V} \leq - \sum_{i=1}^{N} \left[ c_i^{\text{swa}} (p_i - p_{\text{des}})^T \begin{bmatrix} T_{i}^{\text{col}} & T_{i}^{\text{obs}} & T_{i}^{\text{prox}} \end{bmatrix} \right] M_i \begin{bmatrix} c_i^{\text{swa}} (p_i - p_{\text{des}}) \\ T_{i}^{\text{col}} \\ T_{i}^{\text{obs}} \\ T_{i}^{\text{prox}} \end{bmatrix}. \quad (4.20)$$

The assumption is that at initial time $t_0$, $P_0 \notin \mathcal{L}$. This implies that there is a ball of radius $R_{\text{loss}}$ such that $P_0 \in B_{R_{\text{loss}}}(\overline{P}_0)$, where $\overline{P}_0$ is the average of the initial positions of the agents. We have shown in Modes 1 and 2 that proximity is maintained, which implies that $P \in B_{R_{\text{loss}}}(\overline{P}) \forall i = 1, \cdots, N$ for $t \in [t_{1i}^k, t_{3i}^k]$, where $t_{1i}^k$ is the instant when Mode $i$ starts for the $k^{th}$ time. In Mode 3, by the virtue of inequality (4.20), $V(t)$ is guaranteed to have a negative semi-definite derivative, which implies that it will not attain infinite
values. As a result, we can guarantee that $V_{prox}(t)$ will attain finite values, which in turn implies that network communication will not be lost; i.e., we have $P \in B_{R_{loss}}(P)$ for all $t \in [t^k_3, t^k_4]$.

Using the assumption that at initial time $t_0$, $P_0 \notin \Omega$ and $P_0 \notin \Omega^o$, we have shown that collisions do not occur in Modes 1 and 2. By the virtue of the inequality (4.20) and using the aforementioned approach, by guaranteeing that $V(t)$ attains finite values, we prove that no collisions occur and agents avoid obstacles in Mode 3.

Using inequality (4.20), we conclude that as long as there are no collision avoidance and/or proximity gradients, we have

$$
\dot{V} \leq - \sum_{i=1}^{N} (p_i - p_{des})^T c^{s_{swa}}_i K^{s_{swa}}_i (p_i - p_{des}).
$$

(4.21)

Note that $p_i = p_{des}$ for all $i = 1, \cdots, N$ is a local minima for $V(t)$. However, at most one agent may end up at the desired position when the multi-agent group moves as a swarm, which is designated to be the leader agent. Let us assume that the leader agent ended up at $p_{des}$. Then, since collisions are guaranteed not to occur, the distances of other agents to the leader will be greater than $r_{col}$. This implies that no other agent can be inside the ball of radius $r_{col}$ around $p_{des}$. The ideal behavior of the agents other than the leader agent is that they end up on a circle of radius $d$ around $p_{des}$; however, based on the values of $d_{des}$, a probable scenario is that they end up in an annulus around $p_{des}$, where the lower and upper bounds are given by $r_{col}$ and $R_{loss}$. Finally, even in the pathological scenario where an agent other than the leader ends up in the vicinity of $p_{des}$, we can guarantee that all agents stay inside the ball with radius $R_{loss}$ around $p_{des}$ and there will not be any collisions; thus, the multi-agent group still moves as a swarm and ends up in the neighborhood of a relatively uncovered point in the coverage domain $\mathcal{D}$.

**Mode 4** ($0 \leq \gamma(t) \leq 1, \ 0 \leq \dot{\gamma}(t)$): In this mode, we have $0 \leq \dot{\gamma}(t)$. Thus, we have $E_2 + E_3 \leq 0$, and $E_1 \leq 0$ by definition. The control input for the agents become $u_i =$
\[ 
\gamma(t) u_{\text{cov}} + (1 - \gamma(t)) u_{\text{swa}} + u_{\text{col}} + u_{\text{obs}} + u_{\text{prox}}, \]

Then, the time derivative of \( V \) satisfies

\[ 
\dot{V} \leq - \sum_{i=1}^{N} \begin{bmatrix} T_i^{\text{cov}} & c_i^{\text{swa}}(p_i - p_{\text{des}})^T & T_i^{\text{col}} & T_i^{\text{obs}} & T_i^{\text{prox}} \end{bmatrix} \mathcal{M}_{i4}(t) \begin{bmatrix} T_i^{\text{cov}} \\ c_i^{\text{swa}}(p_i - p_{\text{des}}) \\ T_i^{\text{col}} \\ T_i^{\text{obs}} \\ T_i^{\text{prox}} \end{bmatrix}, \tag{4.22} 
\]

where

\[ \mathcal{M}_{i4}(t) \equiv \mathcal{M}_{i2}(t), \]

\[ c_i^{\text{swa}} := \sqrt{\kappa_i^{\text{swa}}} \arctan \left( \frac{\|p_i - p_{\text{des}}\|^2 - d_{\text{des}}^2}{\|p_i - p_{\text{des}}\|^2 - d_{\text{des}}^2} \right). \tag{4.23} \]

Since \( \mathcal{M}_{i4}(t) \equiv \mathcal{M}_{i2}(t) \) and we have already shown that \( \mathcal{M}_{i2}(t) \) is positive semi-definite, so is \( \mathcal{M}_{i4}(t) \). Thus, in Mode 4, no collisions occur, obstacles are avoided and network communication maintained by each coverage agent. Note that at the instance \( \gamma(t) \) becomes 1, the sensors of the coverage agents are turned back on.

Just as it was the case in Mode 2, during this transition period, there are two possibly conflicting objectives, the coverage objective and the swarming objective, that are active since both the coverage and swarming control signals act on the agents. That being said, due to the fact that \( 0 \preceq \mathcal{M}_{i4}(t) \), \( V \) is guaranteed not to increase and this guarantee is sufficient; we do not expect the multi-agent system to accomplish either the coverage objective or the swarming objective in the transition mode. We only want to guarantee that \( \dot{V} \) stays non-positive, and that is exactly what is guaranteed in Mode 4.

**Remark 10.** Without loss of generality, we set \( d_{\text{des}} = 0 \) and \( d_{\text{des}} = d_{\text{des}} \), which implies that the leader agent will converge to a small neighborhood of \( p_{\text{des}} \), whereas the other agents will converge the circle of radius \( d_{\text{des}} \) around \( p_{\text{des}} \).

**Remark 11.** Since we assume that at initial time \( t_0 \) we have \( P_0 \notin \mathcal{L} \), this implies that there is an open ball of radius \( R_{\text{loss}} := \min_i R_{\text{loss},i} \) such that \( P_0 \in B_{R_{\text{loss}}}(P_0) \). Thus, another way to look at the proximity objective would be to guarantee that \( P \in B_{R_{\text{loss}}}(P) \).
for all $t \in [t_0, T_{\text{final}}]$. In this sense, the multi-agent system can be described to behave as a swarm.

**Remark 12.** Through inequality (4.20), it can be seen that the selection of $d_{\text{ides}}$'s is crucial; we should select them such that $\max_i r_{\text{col}, i} := r_{\text{col}} \leq d_i \leq R_{\text{loss}}$ for every agent in the swarm. Hence, it is a reasonable assumption that lower and upper bounds of $r_{\text{col}, i}, R_{\text{col}, i}, r_{\text{loss}, i}$, and $R_{\text{loss}, i}$ are known apriori by the agents.

**Remark 13.** It is possible that the agents end up at local minima of $V(t)$ before approaching the vicinity of their corresponding target points in swarming mode. That being said, this happens very rarely in implementation. Moreover, even if it happens, due to the structure of our scheme, the agents would transition to Mode 1, and continue with the coverage objective. If there is not enough coverage, the agents would transition back to Mode 3, and by going through this cycle, the agents would be able to find an uncovered point that they can reach.

Hence, we have shown that by transitioning between coverage and swarming objectives finitely many times, the agents cover the given domain in finite time, while avoiding collisions with each other and static obstacles and maintaining proximity to each other.

### 4.2 Swarm-Based Coverage Control for Kinematic Unicycles

In this section, we discuss the stability of the swarm-based dynamic coverage control scheme when applied to agents that are described as kinematic unicycles. Additionally, we consider inter-agent collision avoidance, obstacle avoidance and inter-agent proximity maintenance. Finally, we comment on the control of the angular velocity of the unicycle agents.
4.2.1 Main Results

We consider a multi-agent system of \(N\) coverage agents that are kinematic unicycles; i.e., their equations of motion are described by

\[
\dot{q}_i = \begin{bmatrix} \dot{p}_i \\ \dot{\theta}_i \end{bmatrix} = g_i(q_i) u_i = \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix}, \quad i = 1, \ldots, N, \quad (4.24)
\]

where \(u_{i1}\) is the linear velocity and \(u_{i2}\) is the angular velocity of the unicycle agent. We denote the overall position vector of the coverage agents by \(P = [p_1^T \ldots p_N^T]^T\), and the overall position vector at initial time \(t = t_0\) by \(P_0\). Moreover, without loss of generality, we denote the leader agent with the subscript 1. Finally, we assume that the static obstacles on the coverage domain obey the restrictions described in Remark 2.

Note that in coverage control, collision/obstacle avoidance and proximity maintenance, the control problem is that of controlling the position of an agent on the \(x - y\) plane. Similarly, in swarming mode, the problem is to deploy an agent from a specific position on the domain to another position. In this sense, the orientation control of a unicycle coverage agent is not considered in the control design for these objectives, and thus it must be considered separately. To this purpose, we initially consider the position control part of the problem, and then focus on the orientation part in the next section. Next, we state the main result of this section.

**Theorem 5.** Consider a group of \(N\) kinematic unicycle agents on a compact domain \(D \subset \mathbb{R}^2\) with the control inputs

\[
u_{i1} = \gamma(t) g_i^T u_{\text{cov}}, \quad (1 - \gamma(t)) g_i^T u_{\text{swa}}^i + g_i^T u_{\text{col}}^i + g_i^T u_{\text{obs}}^i + g_i^T u_{\text{prox}}, \quad (4.25)
\]

where \(\gamma(t)\) is a smooth transition signal as described in Section 2.1, transitioning according to the conditions \(C1\) and \(C2'\) given by (2.31) and (2.33), \(g_i^\rho = \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}\), and \(u_{\text{cov}}, u_{\text{swa}}\),
\( u_{\text{col}}, u_{\text{obs}}, u_{\text{prox}}, I_x \) and \( I_y \) are given by

\[
\begin{align*}
  u_{\text{cov}} &= -K_{\text{cov}} I_{\text{cov}}, \\
  I_{\text{cov}} &= -2 \int_{\mathcal{D}} h(C^* - Q) \gamma(t) \tilde{S}_i'(p_i - \bar{p}) d\bar{p}_1 d\bar{p}_2, \\
  u_{\text{swa}} &= -K_{\text{swa}} \arctan(\|p_i - p_{\text{des}}\|^2 - d_{i,\text{des}}^2)(p_i - p_{\text{des}}), \\
  u_{\text{col}} &= -K_{\text{col}} I_{\text{col}}, \\
  I_{\text{col}} &= \sum_{j \neq i} N \frac{\partial V_{\text{col}}^{ij}}{\partial p_i}, \\
  u_{\text{obs}} &= -K_{\text{obs}} I_{\text{obs}}, \\
  I_{\text{obs}} &= \sum_{k=1}^{N_o} \frac{\partial V_{\text{obs}}^{ik}}{\partial p_i}, \\
  u_{\text{prox}} &= -K_{\text{prox}} I_{\text{prox}}, \\
  I_{\text{prox}} &= \sum_{j \neq i} N \frac{\partial V_{\text{prox}}^{ij}}{\partial p_i},
\end{align*}
\]

(4.26)

\( h(\cdot), Q \) and \( \tilde{S}_i' \) are defined in Section 2.2, \( V_{\text{col}}^{ij}, V_{\text{obs}}^{ij} \) and \( V_{\text{prox}}^{ij} \) are given by (2.8), (2.14) and (2.22) respectively, \( p_{\text{des}} \) is the target point selected by the leader agent in swarming mode and \( d_{i,\text{des}} \) is the ideal distance of each agent to \( p_{\text{des}} \). Assume that \( P_0 \notin \Omega, P_0 \notin \Omega^o \) and \( P_0 \notin \mathcal{L} \). Then, there exist matrices \( 0 \prec K_{\text{cov}} \triangleq \begin{bmatrix} k_{\text{cov}_{ix}} & 0 \\ 0 & k_{\text{cov}_{iy}} \end{bmatrix}, 0 \prec K_{\text{swa}} \triangleq \begin{bmatrix} k_{\text{swa}_{ix}} & 0 \\ 0 & k_{\text{swa}_{iy}} \end{bmatrix}, 0 \prec K_{\text{col}} \triangleq \begin{bmatrix} k_{\text{col}_{ix}} \ 0 & k_{\text{col}_{iy}} \end{bmatrix}, 0 \prec K_{\text{obs}} \triangleq \begin{bmatrix} k_{\text{obs}_{ix}} \ 0 & k_{\text{obs}_{iy}} \end{bmatrix}, 0 \prec K_{\text{prox}} \triangleq \begin{bmatrix} k_{\text{prox}_{ix}} \ 0 & k_{\text{prox}_{iy}} \end{bmatrix} \) such that the following results hold:

(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in \( \mathcal{D} \) reaches \( C^* \) at a finite time \( T_{\text{final}} \).

(ii) In the swarming mode, each agent travels to a sufficiently close neighborhood of \( p_{\text{des}} \) such that Condition C2" is satisfied for all agents.

(iii) Agents avoid collisions with other agents; i.e., \( P \notin \Omega \) for all \( t \in [t_0, T_{\text{final}}], i = 1, \cdots, N. \)

(iv) Agents avoid all obstacles; i.e., \( P \notin \Omega^o \) for all \( t \in [t_0, T_{\text{final}}], i = 1, \cdots, N. \)

(v) All agents stay connected; i.e., \( P \notin \mathcal{L} \) for all \( t \in [t_0, T_{\text{final}}], i = 1, \cdots, N. \)
Proof. Consider the following Lyapunov-like function:

\[ V = e + V^{\text{col}} + V^{\text{obs}} + V^{\text{prox}} \]  (4.27)

where \( e \) is the coverage area integral given by (2.7), \( V^{\text{col}} \) is given by

\[ V^{\text{col}} = \frac{w_{\text{col}}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V_{ij}^{\text{col}}, \]  (4.28)

with \( 0 < w_{\text{col}} \), \( V^{\text{obs}} \) is given by

\[ V^{\text{obs}} = w_{\text{obs}} \sum_{i=1}^{N} \sum_{k=1}^{N_{o}} V_{ik}^{\text{obs}}, \]  (4.29)

with \( 0 < w_{\text{obs}} \), and \( V^{\text{prox}} \) is given by

\[ V^{\text{prox}} = \frac{w_{\text{prox}}}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V_{ij}^{\text{prox}}, \]  (4.30)

with \( 0 < w_{\text{prox}} \). Taking the time derivative of \( V(t) \) and using the notations of (4.8) and (4.26), we get

\[ \dot{V} = E_{1} + E_{2} + E_{3} + \sum_{i=1}^{N} (I_{i}^{\text{cov}} + \kappa_{i}^{\text{swa}} I_{i}^{\text{swa}} + w_{\text{col}} T_{i}^{\text{col}} + w_{\text{obs}} T_{i}^{\text{obs}} + w_{\text{prox}} T_{i}^{\text{prox}}) g_{i}^{T} \dot{p}_{i}. \]  (4.31)

We analyze the system in 4 modes; i.e., **Mode** \( i \) with \( i = 1, \cdots, 4 \):

**Mode 1** (\( \gamma(t) \equiv 1 \)): In this mode, since \( \gamma(t) \equiv 1 \), we have \( \kappa_{i}^{\text{swa}} \equiv 0 \) and \( u_{i1} = g_{i}^{T} u_{\text{cov}i} + g_{i}^{T} u_{\text{col}i} + g_{i}^{T} u_{\text{obs}i} + g_{i}^{T} u_{\text{prox}i} \). Moreover, we have \( E_{2} \equiv E_{3} \equiv 0 \). Finally, note that \( E_{1} \leq 0 \)
by definition. Using the definitions of (4.3), we can write the following inequality for $\dot{V}(t)$:

$$
\dot{V} = E_1 - \sum_{i=1}^{N} (T_{i}^{\text{cov}} + w_{\text{col}} T_{i}^{\text{col}} + w_{\text{obs}} T_{i}^{\text{obs}} + w_{\text{prox}} T_{i}^{\text{prox}}) g_i \upsilon_i,
$$

$$
\leq - \sum_{i=1}^{N} \begin{bmatrix} T_{i}^{\text{cov}} & T_{i}^{\text{col}} & T_{i}^{\text{obs}} & T_{i}^{\text{prox}} \end{bmatrix} G_{i} \begin{bmatrix} T_{i}^{\text{cov}} \\ T_{i}^{\text{col}} \\ T_{i}^{\text{obs}} \\ T_{i}^{\text{prox}} \end{bmatrix},
$$

(4.32)

with

$$
0 \preceq G_{i} = \begin{bmatrix}
g_i p K_{\text{cov}}, g_i p^T & g_i p K_{\text{col}}, g_i p^T & g_i p K_{\text{obs}}, g_i p^T & g_i p K_{\text{prox}}, g_i p^T \\
w_{\text{col}} g_i p K_{\text{cov}}, g_i p^T & w_{\text{col}} g_i p K_{\text{col}}, g_i p^T & w_{\text{col}} g_i p K_{\text{obs}}, g_i p^T & w_{\text{col}} g_i p K_{\text{prox}}, g_i p^T \\
w_{\text{obs}} g_i p K_{\text{cov}}, g_i p^T & w_{\text{obs}} g_i p K_{\text{col}}, g_i p^T & w_{\text{obs}} g_i p K_{\text{obs}}, g_i p^T & w_{\text{obs}} g_i p K_{\text{prox}}, g_i p^T \\
w_{\text{prox}} g_i p K_{\text{cov}}, g_i p^T & w_{\text{prox}} g_i p K_{\text{col}}, g_i p^T & w_{\text{prox}} g_i p K_{\text{obs}}, g_i p^T & w_{\text{prox}} g_i p K_{\text{prox}}, g_i p^T
\end{bmatrix}.
$$

This implies that in this mode, if the coverage agents’ initial positions are such that they are outside each others’ avoidance regions and communication loss regions, they will never collide and the communication network will never be lost since $0 \preceq G_{i}$, hence $V$ will always attain finite values, which in turn means $V^{\text{col}}$ and $V^{\text{prox}}$ will attain finite values. Similarly, agents will avoid obstacles since $V^{\text{obs}}$ will also attain finite values. Furthermore, as long as there are the avoidance and/or proximity gradients are zero, the agents continue the coverage mission until they reach local minima of $V$ since in this case

$$
\dot{V} \leq - \sum_{i=1}^{N} T_{i}^{\text{cov}} g_i p K_{\text{cov}}, g_i p^T T_{i}^{\text{cov}}.
$$

**Mode 2** ($0 \leq \gamma(t) \leq 1, \dot{\gamma}(t) \leq 0$): In this mode, we have $\dot{\gamma}(t) \leq 0$. Hence, $u_i = \gamma(t) g_i p^T u_{\text{cov}}, + (1 - \gamma(t)) g_i^p T u_{\text{swa}}, + g_i^p T u_{\text{col}}, + g_i^p T u_{\text{obs}}, + g_i^p T u_{\text{prox}}$. Then, the time derivative
of $V$ is given by

$$
\dot{V} = -\sum_{i=1}^{N} \left[ \mathcal{T}_i^{cov} c_{i}^{swa}(p_i - p_{des})^T + \mathcal{T}_i^{col} \mathcal{T}_i^{obs} \mathcal{T}_i^{prox} \right] \mathcal{G}_i(t) + E_1 + E_2 + E_3,
$$

with

$$
\mathcal{G}_i(t) = \begin{bmatrix}
\gamma g_i^p K_{conv,1}^T g_i^{pT} (1-\gamma) g_i^p K_{swa,1}^T g_i^{pT} g_i^p K_{col,1}^T g_i^{pT} g_i^p K_{obs,1}^T g_i^{pT} g_i^p K_{prox,1}^T g_i^{pT} \\
\gamma g_i^p K_{conv,2}^T g_i^{pT} (1-\gamma) g_i^p K_{swa,2}^T g_i^{pT} g_i^p K_{col,2}^T g_i^{pT} g_i^p K_{obs,2}^T g_i^{pT} g_i^p K_{prox,2}^T g_i^{pT} \\
\gamma w_{col} g_i^p K_{conv,1}^T K_{swa,1}^T g_i^{pT} w_{col} g_i^p K_{col,1}^T g_i^{pT} w_{col} g_i^p K_{obs,1}^T g_i^{pT} w_{col} g_i^p K_{prox,1}^T g_i^{pT} \\
\gamma w_{obs} g_i^p K_{conv,1}^T K_{swa,1}^T g_i^{pT} w_{obs} g_i^p K_{col,1}^T g_i^{pT} w_{obs} g_i^p K_{obs,1}^T g_i^{pT} w_{obs} g_i^p K_{prox,1}^T g_i^{pT} \\
\gamma w_{prox} g_i^p K_{conv,1}^T K_{swa,1}^T g_i^{pT} w_{prox} g_i^p K_{col,1}^T g_i^{pT} w_{prox} g_i^p K_{obs,1}^T g_i^{pT} w_{prox} g_i^p K_{prox,1}^T g_i^{pT}
\end{bmatrix},
$$

$$
c_{i}^{swa} := \sqrt{\kappa_i^{swa} \arctan(\|p_i - p_{des}\|^2 - d_{i,des}^2)(\|p_i - p_{des}\|^2 - d_{i,des}^2)},
$$

To show the positive semi-definiteness of $\mathcal{G}_i(t)$, we can decompose it in the following way:

$$
\mathcal{G}_i(t) = W_{uni}^i J_{uni}^i K_{uni}^i(t),
$$

with

$$
W_{uni}^i = \begin{bmatrix}
I_{2 \times 2} & 0 & 0 & 0 & 0 \\
0 & I_{2 \times 2} & 0 & 0 & 0 \\
0 & 0 & w_{col} I_{2 \times 2} & 0 & 0 \\
0 & 0 & 0 & w_{obs} I_{2 \times 2} & 0 \\
0 & 0 & 0 & 0 & w_{prox} I_{2 \times 2}
\end{bmatrix},
$$

$$
J_{uni}^i = \begin{bmatrix}
I_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & I_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & I_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & I_{2 \times 2} & 1_{2 \times 2} \\
1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & 1_{2 \times 2} & I_{2 \times 2}
\end{bmatrix},
$$

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\[
K_{\text{uni}}^i(t) = \begin{bmatrix}
\gamma g_i^p K_{\text{cov}} g_i^p & 0 & 0 & 0 & 0 \\
0 & (1 - \gamma) g_i^p K_{\text{swa}} g_i^p & 0 & 0 & 0 \\
0 & 0 & g_i^p K_{\text{col}} g_i^p & 0 & 0 \\
0 & 0 & 0 & g_i^p K_{\text{obs}} g_i^p & 0 \\
0 & 0 & 0 & 0 & g_i^p K_{\text{prox}} g_i^p \\
\end{bmatrix}, \quad (4.36)
\]

where \( I_{2 \times 2} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \) and \( I_{2 \times 2} \) is the 2 by 2 identity matrix. Notice that \( W_{\text{uni}} \) is a diagonal matrix with positive eigenvalues. When \( 0 \leq \gamma(t) \leq 1 \), for positive gain matrices \( K_{\text{cov}}, K_{\text{swa}}, K_{\text{col}}, K_{\text{obs}}, \) and \( K_{\text{prox}}, K_{\text{uni}}^i(t) \) is positive semi-definite. Finally, by definition, \( J_{\text{uni}}^i \) has nonnegative eigenvalues. Hence, in Mode 2, the positive semi-definiteness of \( G_{i2}(t) \) is maintained. Moreover, in order render \( \dot{V} \) non-positive in Mode 2, we can select \( k^* \) and \( \omega_{s2} \) such that \( E_1 + E_2 + E_3 \leq 0 \) for all \( 0 \leq \gamma(t) \leq 1 \). Hence, \( \dot{V} \) satisfies

\[
\dot{V} \leq -\sum_{i=1}^{N} \begin{bmatrix} T_i^{\text{cov}} & c_i^{\text{swa}}(p_i - p_{\text{des}})^T \\ T_i^{\text{col}} & T_i^{\text{obs}} & T_i^{\text{prox}} \end{bmatrix} G_{i2}(t) \begin{bmatrix} T_i^{\text{cov}} \\ T_i^{\text{col}} \\ T_i^{\text{obs}} \\ T_i^{\text{prox}} \end{bmatrix}. \quad (4.37)
\]

Thus, in Mode 2, no collisions occur, obstacles are avoided and network communication is maintained by each coverage agent. Note that at the instance \( \gamma(t) \) becomes 0, \( T_i^{\text{cov}} \), as well as \( E_1 \) and \( E_2 \), become 0. This implies that the sensors of the coverage agents are turned off. \( V \) is guaranteed not to increase in this mode; we do not expect the multi-agent system to accomplish either the coverage objective or the swarming objective in the transition mode. We only want to guarantee that \( \dot{V} \) does not attain positive values.

**Mode 3 (\( \gamma(t) \equiv 0 \)):** In this mode, \( \dot{\gamma}(t) \equiv 0, T_i^{\text{cov}} \equiv 0, E_2 \equiv 0 \) and \( E_3 \equiv 0 \). Moreover, a desired target position has already been generated for the multi-agent group as well as desired distances \( d_{i\text{des}} \) for each agent. The control signal for each agent is \( u_{i1} = g_i^{p^T} u_{\text{swa}} + \)
\[ \dot{V} = E_1 - \sum_{i=1}^{N} \left[ \epsilon_i^{sua} (p_i - p_{des})^T \mathcal{I}_i^{col^T} \mathcal{I}_i^{obs^T} \mathcal{I}_i^{prox^T} \right] G_i \]

where

\[ G_i = \begin{bmatrix} g_i^{p} K_{swa_i} g_i^{pT} & g_i^{p} K_{col_i} g_i^{pT} & g_i^{p} K_{obs_i} g_i^{pT} & g_i^{p} K_{prox_i} g_i^{pT} \\
 w_{col_i} g_i^{p} K_{swa_i} g_i^{pT} & w_{col_i} g_i^{p} K_{col_i} g_i^{pT} & w_{col_i} g_i^{p} K_{obs_i} g_i^{pT} & w_{col_i} g_i^{p} K_{prox_i} g_i^{pT} \\
 w_{obs_i} g_i^{p} K_{swa_i} g_i^{pT} & w_{obs_i} g_i^{p} K_{col_i} g_i^{pT} & w_{obs_i} g_i^{p} K_{obs_i} g_i^{pT} & w_{obs_i} g_i^{p} K_{prox_i} g_i^{pT} \\
 w_{prox_i} g_i^{p} K_{swa_i} g_i^{pT} & w_{prox_i} g_i^{p} K_{col_i} g_i^{pT} & w_{prox_i} g_i^{p} K_{obs_i} g_i^{pT} & w_{prox_i} g_i^{p} K_{prox_i} g_i^{pT} \end{bmatrix}, \]

\[ \epsilon_i^{sua} := \sqrt{\kappa_i^{sua}} \arctan \left( \frac{\|p_i - p_{des}\|^2 - d_{i,des}^2}{\|p_i - p_{des}\|^2 - d_{i,des}^2} \right), \]

and \( E_1 \leq 0 \). Thus, \( \dot{V} \) satisfies

\[ \dot{V} \leq - \sum_{i=1}^{N} \left[ \epsilon_i^{sua} (p_i - p_{des})^T \mathcal{I}_i^{col^T} \mathcal{I}_i^{obs^T} \mathcal{I}_i^{prox^T} \right] G_i \]

The assumption is that the agents’ initial positions are such that they are not inside each others’ loss regions. This implies that there is a ball of radius \( R_{loss} \) such that \( \mathbf{P}_0 \in B_{R_{loss}}(\mathbf{P}_0) \). We have shown that in Modes 1 and 2, proximity is maintained, which implies that \( \mathbf{P} \in B_{R_{loss}}(\mathbf{P}) \) for \( t \in [t^k_1, t^k_3] \), where \( t^k_1 \) is the instant when Mode \( i \) starts for the \( k^{th} \) time. In Mode 3, by the virtue of inequality (4.40), \( V(t) \) is guaranteed to have a nonpositive derivative, which implies that it will attain finite values. As a result, we can guarantee that \( V_{prox}(t) \) will attain finite values, which in turn implies that network communication will not be lost; i.e., we have \( \mathbf{P} \in B_{R_{loss}}(\mathbf{P}) \) for all \( t \in [t^k_3, t^k_4] \).

Using the assumption that the agents’ initial positions are not inside each others’
avoidance regions and obstacle avoidance regions, we have shown that collisions do not occur and obstacles are avoided in Modes 1 and 2. By the virtue of the inequality \(4.40\) and using the aforementioned approach, by guaranteeing that \(V(t)\) attains finite values, we prove that no collisions occur and obstacles are avoided in Mode 3 as well. Moreover, as long as there are no avoidance and/or proximity gradients, we have
\[
\dot{V} \leq - \sum_{i=1}^{N} c_{swa}^i (p_i - p_{des})^T g_i^p K_{swa} g_i^p (p_i - p_{des}).
\]

**Mode 4** (\(0 \leq \gamma(t) \leq 1, \ 0 \leq \dot{\gamma}(t)\)): In this mode, we have \(0 \leq \dot{\gamma}(t)\). Thus, we have \(E_2 + E_3 \leq 0\), and \(E_1 \leq 0\) by definition. The control input for the agents become
\[
\dot{u}_i = \gamma(t) g_i^p u_{cov_i} + (1 - \gamma(t)) g_i^p u_{swa_i} + g_i^p u_{col_i} + g_i^p u_{obs_i} + g_i^p u_{prox_i}.
\]
Then, the time derivative of \(V\) satisfies
\[
\dot{V} = - \sum_{i=1}^{N} \begin{bmatrix} \mathcal{I}^T_{cov} & \mathcal{I}^T_{swa} & \mathcal{I}^T_{col} & \mathcal{I}^T_{obs} & \mathcal{I}^T_{prox} \end{bmatrix} \begin{bmatrix} \mathcal{G}_{12}(t) \end{bmatrix}, \tag{4.41}
\]
where
\[
\mathcal{G}_{12}(t) \equiv \mathcal{G}_{14}(t),
\]
\[
c_{swa}^i \equiv \sqrt{\kappa_{swa}^i \arctan(\|p_i - p_{des}\|^2 - d_{i_{des}}^2)(\|p_i - p_{des}\|^2 - d_{i_{des}}^2)}.
\]

Just as it was the case in Mode 2, \(\mathcal{G}_{14}(t)\) is a time-varying matrix, hence we cannot immediately deduce the positive semi-definiteness. However, since \(\mathcal{G}_{14}(t) \equiv \mathcal{G}_{12}(t)\) and we have already shown that \(\mathcal{G}_{12}(t)\) is positive semi-definite, so is \(\mathcal{G}_{14}(t)\). Thus, in Mode 4, no collisions occur, obstacles are avoided and network communication is maintained by each coverage agent. Note that at the instance \(\gamma(t)\) becomes 1, the sensors of the coverage agents are turned on.

Just as it was the case in Mode 2, during this transition period, there are two possibly conflicting objectives, the coverage objective and the swarming objective, that are active.
since both the coverage and swarming control signals act on the agents. That being said, due to the fact that $0 \preceq G_{14}(t)$, $\dot{V}$ stays non-positive, thus $V$ is guaranteed not to increase.

### 4.2.2 Discussion on Orientation

Notice that the control problem in Mode 3, i.e., swarming Mode, is essentially the problem of point stabilization for kinematic unicycle agents with one difference; in the context of coverage control scheme, the stabilization problem is going from a point on the domain to another point without any constraints on the initial and final orientation, whereas in general, the point stabilization problem for unicycle agents considers the orientation as well. The problem of point stabilization for kinematic unicycles is historically known to be a challenging problem, and many different approaches have been proposed; in [65], a discontinuous time-invariant feedback is considered for the rendezvous problem of nonholonomic agents, whereas in [66], a time-varying continuous feedback control is proposed. Another approach, adopted by [67], is to use a dynamic extension and linearize the agent dynamics when the velocities of the agents are non-zero. The static feedback linearization of the unicycles proposed by [62] and utilized in the application of the supervised coverage scheme to wheeled mobile robots in Section 3.3.2 is yet another approach to control of unicycle agents. A detailed review of different approaches in control of nonholonomic agents is outside the scope of this dissertation, hence we conclude our discussion by stating that the problem of stabilization to a point in the context of swarm-based coverage scheme is a relatively easier problem, and the angular velocity control that will be discussed in this section solves this problem.

As we have previously stated, the coverage, avoidance, swarming and proximity objectives do not explicitly take the orientation of the agents into account, hence we have freedom in designing the orientation control. We utilize the approach of [35] and [12].

The Lyapunov-like function $V(t)$ given by (4.27) is a function of the positions of the agents only; it does not depend on the orientation. In order to design the angular velocity
control, let’s rewrite $\dot{V}(t)$ in the following way:

$$\dot{V}(t) = \sum_{i=1}^{N} (B_{ix} \cos(\theta_i) + B_{iy} \sin(\theta_i)u_{i1}),$$

where

$$\begin{bmatrix} B_{ix} \\ B_{iy} \end{bmatrix} = I_i^{cov} + \kappa_i^{swa} I_i^{swa} + w_{col} I_i^{col} + w_{obs} I_i^{obs} + w_{prox} I_i^{prox}. \tag{4.44}$$

The rate of decrease of $V(t)$ can be maximized if the orientation of each robot can be aligned with the gradient of $V(t)$. To this purpose, $u_{i1}$ has been designed in [35] in the following way:

$$u_{i1} = -k_i (B_{ix} \cos(\theta_i) + B_{iy} \sin(\theta_i)),$$

where $k_i$ is a design parameter. Using the linear velocity in (4.45), $\dot{V}$ can be written as

$$\dot{V}(t) = -k_i (B_{ix} \cos(\theta_i) + B_{iy} \sin(\theta_i))^2,$$

$$= k_i \left( B_{ix}^2 + B_{iy}^2 \right) \sin^2 \left( \theta_i + \arctan \left( \frac{B_{ix}}{B_{iy}} \right) \right). \tag{4.46}$$

Thus, the decrease in $\dot{V}(t)$ would be maximized if the orientation of each agent is equal to

$$\tilde{\theta}_{i,des} = \frac{\pi}{2} - \arctan \left( \frac{B_{ix}}{B_{iy}} \right). \tag{4.47}$$

In the context of swarm-based scheme, we slightly modify the desired orientation $\theta_{i,des}$. Instead of $B_{ix}$ and $B_{iy}$, we utilize the following terms:

$$\begin{bmatrix} I_{ix} \\ I_{iy} \end{bmatrix} = I_i^{cov} + (1 - \gamma(t)) \arctan (\|p_i - p_{des}\|^2 - d_{i,des}^2) (p_i - p_{des}) + I_i^{col} + I_i^{obs} + I_i^{prox}. \tag{4.48}$$
Thus, our modified desired orientation becomes

\[
\dot{\theta}_{\text{des}} = \frac{\pi}{2} - \arctan \left( \frac{I_i x}{I_i y} \right).
\]  

(4.49)

**Remark 14.** Note that when we consider the each gradient term in (4.48) separately, they are in the same direction as the corresponding gradient term in (4.44) except that each term is scaled differently. Due to these differences in scaling, \( I_{ix} \) and \( I_{iy} \) do not always align with \( \mathcal{B}_{ix} \) and \( \mathcal{B}_{iy} \). What this implies is, mathematically speaking, the decrease in \( \dot{V} \) is not always maximized as it is the case when \( \mathcal{B}_{ix} \) and \( \mathcal{B}_{iy} \) are utilized. Nevertheless, the coverage scheme still performs well.

Compared to the desired orientation proposed in [35], we have an additional term in the desired orientation. We modify the desired orientation in the following way:

\[
\theta_{\text{des}}(t) = \dot{\theta}_{\text{des}} + (1 - \gamma(t)) \theta_t \cos (\omega t) + \frac{\pi}{2} - \arctan \left( \frac{I_{ix}}{I_{iy}} \right) + (1 - \gamma(t)) \theta_t \cos (\omega t),
\]  

(4.50)

where \( \theta_t \) and \( \omega_t \) are design parameters and \( \gamma(t) \) is the transitioning signal. As we have previously discussed, the control problem in Mode 3 is that of stabilization to a point for unicycle agents, but unlike the more general stabilization problem of unicycles, we do not impose a motion on the orientation. Hence, we can use this extra degree of freedom to ensure that the agents indeed end up at the desired positions in Mode 3. To this purpose, we add the sinusoidal term \((1 - \gamma(t)) \theta_t \cos (\omega t)\), which is non-zero only in Mode 3, to rotate an agent in place whenever they get stuck. In effect, we force the agents to sweep a range of directions in order to get out of the deadlock whenever the gradient of \( V(t) \) is perpendicular to the orientation of the agent.

Finally, we select the control for the orientation in the following way:

\[
u_{i2} = -k_{\theta_i} (\theta_i - \theta_{\text{des}}(t)) + \dot{\theta}_{\text{des}}(t),
\]  

(4.51)

where \( \theta_{\text{des}}(t) \) is given by (4.50).
4.3 Multi-Swarm Coverage Control

In this section, we will consider the coverage control problem for groups of swarms of single integrator agents. Instead of one multi-agent group behaving as a swarm, we consider the case where there are several groups where each group behaves as a swarm independently of other groups. The assumption is that there is communication between the leaders of each group, hence the overall combined coverage information about the domain is shared among the groups. The communication among the members of the same group is maintained via proximity functions.

An additional challenge for multi-swarm coverage control problem is the problem of swarm avoidance. In addition to collision avoidance among members of the same swarm, we want to design control laws to ensure that each swarm as a group avoids other groups. Thus, the control laws should prevent the ball encircling one swarm from overlapping the ball encircling another swarm. To implement this, we utilize the avoidance functions given by (4.8). For the multi-swarm coverage scheme, we assume that the leader agents are equipped with extra sensors that have larger ranges than the sensors utilized for accomplishing collision avoidance and obstacle avoidance objectives, thus they can accomplish the swarm avoidance objective.

Before stating the main result, we need to modify some of the notation to account for multiple swarms.

4.3.1 Modification of Functions

Without loss of generality, we assume that there are $S$ swarms with single integrator agents. We denote the size of the swarm by $N_s$, with $s \in \{1, \cdots, S\}$. Since we are considering multiple swarms, we have to modify our notations to be able to distinguish between members of different swarms. To start, we denote the dynamics of the agents in the following way:

$$
\dot{\begin{bmatrix} p_i^s \\ \dot{p}_i^s \end{bmatrix}} = u_i^s, \quad i = 1, \cdots, N_s, \quad s \in \{1, \cdots, S\},
$$

(4.52)
where $N_s$ is the number of agents in $s^{th}$ swarm. Following the same logic, we update our notation for the sensing functions:

$$\tilde{S}_s^i(z) = \frac{M_{\text{cov}}^i}{R_{\text{cov}}^2} \max \{0, R_{\text{cov}}^2 - z\}^2, \quad (4.53)$$

where $i = 1, \cdots, N_s$, $s \in \{1, \cdots, S\}$, and $M_{\text{cov}}^i$ and $R_{\text{cov}}^i$ describe the maximum sensing level and the sensing region for the $i^{th}$ agent in $s^{th}$ swarm, respectively. The modified time-varying sensor function becomes

$$S_s^i(t, \|p_s^i - \bar{p}\|^2) \Delta \gamma(t) \tilde{S}_s^i(\|p_s^i - \bar{p}\|^2), \quad (4.54)$$

where $\bar{p} \in \mathcal{D}$ with $\mathcal{D} \subset \mathbb{R}^2$ representing the compact set to be covered. Using the sensor function definition of (4.54), the accumulated information is represented as

$$Q(t, P) = C^{**} - C^{**} e^{-k^* A}, \quad (4.55)$$

where $A(t, \bar{p}) = \int_0^t \sum_{s=1}^S \sum_{i=1}^{N_s} S_s^i(t, \|p_s^i(\tau) - \bar{p}\|^2) d\tau$ and $P$ is the overall position vector.

**Remark 15.** Note that $P$ and $A(t, \bar{p})$ have exactly the same definitions as before; the only difference is since we have multiple swarms, we change the notation to be able to differentiate between members of different swarms.

In order to formulate the coverage objective for the multi-swarm problem, we utilize the following area integral:

$$e(t) = \iint_{\mathcal{D}} h(C^* - Q(t, P)) \phi(\bar{p}) \left[ S^* - \sum_{s=1}^S \sum_{i=1}^{N_s} S_s^i + (1 - \gamma) \sum_{s=1}^S \sum_{i=1}^{N_s} \sigma(D_s^i) \right] d\bar{p}_1 d\bar{p}_2, \quad (4.56)$$

where $\phi(\bar{p}) \equiv 1$ is the density, which is to assumed to be equal everywhere, $S^* > \sum_{s=1}^S \sum_{i=1}^{N_s} M_{\text{cov}}^i$, $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function, $h(x) = (\max \{0, x\})^3$, $\gamma$ is the transitioning signal introduced in Section 2.1, $p_{s_{\text{des}}}^i$ are the desired points around which the agents will end up when moving as swarms, $d_{i_{\text{des}}}^s$ are the desired distances between
each agent in the swarm and the desired points, and $D_i^s = \frac{\left(\|p_i^s - p_{des_i}^s\|^2 - d_{des_i}^2\right)^2}{2}$. The notations for collision avoidance, obstacle avoidance and proximity functions are modified in the following way:

\begin{align}
V_{s_i,j}^{col}\left(p_i^s, p_j^s\right) &= \left(\min\left\{0, \frac{\|p_i^s - p_j^s\|^2 - R_{col_i}^2}{\|p_i^s - p_j^s\|^2 - r_{col_i}^2}\right\}\right)^2, \quad (4.57) \\
V_{s_i,w}^{col}\left(p_i^s, p_j^w\right) &= \left(\min\left\{0, \frac{\|p_i^s - p_j^w\|^2 - R_{col_i}^2}{\|p_i^s - p_j^w\|^2 - r_{col_i}^2}\right\}\right)^2, \quad (4.58) \\
V_{s_i,j}^{prox}\left(p_i^s, p_j^s\right) &= \left(\max\left\{0, \frac{r_{loss_i}^2 - \|p_i^s - p_j^s\|^2}{\|p_i^s - p_j^s\|^2 - R_{loss_i}^2}\right\}\right)^2, \quad (4.59) \\
V_{s_i,w}^{prox}\left(p_i^s, p_j^w\right) &= \left(\max\left\{0, \frac{r_{loss_i}^2 - \|p_i^s - p_j^w\|^2}{\|p_i^s - p_j^w\|^2 - R_{loss_i}^2}\right\}\right)^2, \quad (4.60) \\
V_{s_i,k}^{obs}\left(p_i^s, p_k^o\right) &= \left(\min\left\{0, \frac{\|p_i^s - p_k^o\|^2 - R_{obs_i}^2}{\|p_i^s - p_k^o\|^2 - r_{obs_i}^2}\right\}\right)^2, \quad (4.61)
\end{align}

where $i = 1, \ldots, N_s$, $j = 1, \ldots, N_w$, $s, w \in \{1, \ldots, S\}$, $s \neq w$, $R_{col_i}^s > r_{col_i}^s > 0$ are collision detection and avoidance radii, $R_{obs_i}^s > r_{obs_i}^s > 0$ are obstacle detection and avoidance radii and $R_{loss_i}^s > r_{loss_i}^s > 0$ are communication loss and degradation radii for each agent.

Note that since each swarm is maintaining the communication network among the group members only, $V_{s_i,w}^{prox}\left(p_i^s, p_j^w\right) \equiv 0$ for $s \neq w$, $i = 1, \ldots, N_s$ and $j = 1, \ldots, N_w$.

The notations for avoidance/detection and communication degradation/loss regions are modified in the following way:

\begin{align}
\Omega_{i,j} &:= \{P : P \in \mathbb{R}^{2N}, \|p_i^s - p_j^w\| \leq r_{col_i}^s\}, \\
D_{i,j} &:= \{P : P \in \mathbb{R}^{2N}, \|p_i^s - p_j^w\| \leq R_{col_i}^s\}, \quad (4.62)
\end{align}

\begin{align}
\Omega &= \bigcup_{i=1,\ldots,N_s, j=1,\ldots,N_w} \Omega_{i,j}, \\
D &= \bigcup_{i=1,\ldots,N_s, j=1,\ldots,N_w} D_{i,j}, \quad (4.63)
\end{align}
Ω_{\nu k}^o := \{ P : P \in \mathbb{R}^{2N}, \| p_i^\nu - p_k^\nu \| \leq r_{obs_i}^\nu \},
D_{\nu k}^o := \{ P : P \in \mathbb{R}^{2N}, \| p_i^\nu - p_k^\nu \| \leq R_{obs_i}^\nu \},
(4.64)

Ω^o = \bigcup_{i=1,\cdots,N_s,k=1,\cdots,N_o}^{s \in \{1,\cdots,S\}} \Omega_{\nu k}^o,
D^o = \bigcup_{i=1,\cdots,N_s,k=1,\cdots,N_o}^{s \in \{1,\cdots,S\}} D_{\nu k}^o, \quad (4.65)

Δ_{i^s j^w} := \{ P : P \in \mathbb{R}^{2N}, r_{loss_i}^s \leq \| p_i^s - p_j^w \| \},
L_{i^s j^w} := \{ P : P \in \mathbb{R}^{2N}, R_{loss_i}^s \leq \| p_i^s - p_j^w \| \},
(4.66)

Δ = \bigcup_{i=1,\cdots,N_s,j=1,\cdots,N_w}^{s, w \in \{1,\cdots,S\}} Δ_{i^s j^w},
L = \bigcup_{i=1,\cdots,N_s,j=1,\cdots,N_w}^{s, w \in \{1,\cdots,S\}} L_{i^s j^w}. \quad (4.67)

Remark 16. Without loss of generality, we designate the agent with the smallest loss radius as the agent with the subscript 1, i.e., the leader agent. Thus, \( \min_{s} R_{loss_i}^s = R_{loss_1}^s \) for \( s \in \{1,\cdots,S\} \).

4.3.2 Swarm Avoidance Functions

To begin, let us denote the swarm avoidance and detection radii of the leader agent of the \( s^{th} \) swarm by \( r_{gro}^s \) and \( R_{gro}^s \), respectively. We set the swarm avoidance radius to be the following:

\[ r_{gro}^s := 2R_{loss_1}^s + \varepsilon_{gro}, \quad i = 1, \cdots, N_s, \]

where \( s \in \{1,\cdots,S\} \) and \( 0 < \varepsilon_{gro} \) is a small positive number. The swarm avoidance functions are implemented by the leader agents of the swarms. We define the swarm avoidance functions in the following way:

\[ V_{s_i s_j}^{gro}(p_i^s, p_j^s) = \left( \min \left\{ 0, \frac{\| p_i^s - p_j^s \|}{\| p_i^s - p_j^s \|^2} - \frac{R_{gro}^s}{2} \right\} \right)^2, \]

(4.69)
where \( s \in \{1, \ldots, S\} \), \( j = 1, \ldots, N_s \), and

\[
V_{s,wj}^{gro}(p^s_i, p^w_j) = \left( \min \left\{ 0, \frac{\|p^s_i - p^w_j\|^2 - R_{gro}^s}{\|p^s_i - p^w_j\|^2 - r_{gro}^s} \right\} \right)^2,
\]

with \( s, w \in \{1, \ldots, S\} \), \( s \neq w \), \( i = 1, \ldots, N_s \) and \( j = 1, \ldots, N_w \). Note that, by the virtue of equation (4.68), \( \|p^s_i - p^w_j\| \) would always be less than \( r_{gro}^s \) in the presence of inter-agent proximity functions, thus \( V_{s,wj}^{gro}(p^s_i, p^w_j) \equiv 0 \) for all \( j = 1, \ldots, N_s \). Moreover, since the swarm avoidance functions are implemented by the leader agents in the swarms only, we have \( V_{s,wj}^{gro}(p^s_i, p^w_j) \equiv 0 \) for \( i \neq 1, j = 2, \ldots, N_s \) and \( V_{s,wj}^{gro}(p^s_i, p^w_j) \equiv 0 \) for \( i \neq 1, j = 1, \ldots, N_w \).

Hence, the swarm detection and avoidance regions are defined in the following way:

\[
\Omega_{1sjw}^{gro} := \{ P : P \in \mathbb{R}^{2N}, \|p^s_1 - p^w_j\| \leq r_{gro}^s \},
\]

\[
D_{1sjw}^{gro} := \{ P : P \in \mathbb{R}^{2N}, \|p^s_1 - p^w_j\| \leq R_{gro}^s \},
\]

\[
\Omega^{gro} = \bigcup_{j \in 1, \ldots, N_w} \bigcup_{s,w \in \{1, \ldots, S\}} \Omega_{1sjw}^{gro}, \quad D^{gro} = \bigcup_{j \in 1, \ldots, N_w} \bigcup_{s,w \in \{1, \ldots, S\}} D_{1sjw}^{gro}.
\]

Note that, by the virtue of Definition 1 and the structure of the swarm avoidance function (4.70), for \( P_0 \notin \Omega^{gro} \), agents will enter \( \Omega^{gro} \) if and only if \( V_{s,wj}^{gro}(p^s_i, p^w_j) \to \infty \). Thus, if \( V_{s,wj}^{gro}(p^s_i, p^w_j) \) can be shown to attain finite values for all \( j \in 1, \ldots, N_w \), \( s, w \in \{1, \ldots, S\} \), the agents are guaranteed to avoid the set \( \Omega^{gro} \). The distance of an agent to the leader agent will always be less than \( R_{loss}^s \) because of the inter-agent proximity functions. In addition, we set \( r_{gro}^s \) to be greater than \( 2R_{loss}^s \). Thus, the avoidance of \( \Omega^{gro} \) by the agents would imply that swarms avoid other swarms.

Defining \( d_{ij}^{sw} \triangleq \|p^s_i - p^w_j\| \), the partial derivatives of the swarm avoidance functions
with respect to the position of the leader agent of the $s^{th}$ swarm are given by:

$$
\frac{\partial V_{gro}}{\partial p_s^i} = \begin{cases}
0, & \text{if } R_{gro}^s \leq d_{ij}^{sw} \\
\frac{4(R_{gro}^s - r_{gro}^s)(d_{ij}^{sw^2} - R_{gro}^s)}{(d_{ij}^{sw^2} - r_{gro}^s)^3}(p_s^i - p_j^w), & \text{if } r_{gro}^s < d_{ij}^{sw} < R_{gro}^s \\
\text{not defined,} & \text{if } d_{ij}^{sw} = r_{gro}^s \\
0, & \text{if } d_{ij}^{sw} < r_{gro}^s.
\end{cases}
$$

(4.73)

It is also important to note that the swarm avoidance functions are non-cooperative; only the leaders of the swarms implement the swarm avoidance scheme. What this means is, the leader agent of a swarm avoids a member of another swarm, but the member of the swarm does not try to avoid the leader. In this sense, the gradients of the swarm avoidance functions with respect to the states of the agents other than the leader agents are automatically zero.

### 4.3.3 Main Results

Now we can state main result for the multi-swarm coverage problem for single integrators.

**Theorem 6.** Consider a group of $S$ swarms, each with $N_s$ single integrator coverage agents on a compact domain $D \subset \mathbb{R}^2$ with static obstacles obeying the restrictions described in Remark 2, with the control inputs

$$
u_i^s = \gamma(t)u_{cov_i}^s + (1 - \gamma(t))u_{swa_i}^s + u_{coli}^s + u_{obs_i}^s + u_{proxi}^s + u_{gro_i}^s,
$$

(4.74)

where $\gamma(t)$ is a smooth transition signal transitioning according to the conditions C1 and
Then, there exist matrices $0 < K_{\text{cov}_i} \triangleq \begin{bmatrix} k_{\text{cov}_{ix}} & 0 \\ 0 & k_{\text{cov}_{iy}} \end{bmatrix}$, $0 < K_{\text{swa}_i} \triangleq \begin{bmatrix} k_{\text{swa}_{ix}} & 0 \\ 0 & k_{\text{swa}_{iy}} \end{bmatrix}$, $0 < K_{\text{col}_i} \triangleq \begin{bmatrix} k_{\text{col}_{ix}} & 0 \\ 0 & k_{\text{col}_{iy}} \end{bmatrix}$, $0 < K_{\text{obs}_i} \triangleq \begin{bmatrix} k_{\text{obs}_{ix}} & 0 \\ 0 & k_{\text{obs}_{iy}} \end{bmatrix}$, and $0 < K_{\text{gro}_i} \triangleq \begin{bmatrix} k_{\text{gro}_{ix}} & 0 \\ 0 & k_{\text{gro}_{iy}} \end{bmatrix}$ such that the following results hold:

(i) Agents sufficiently cover the given domain; i.e., coverage level at every point in $D$ reaches $C^*$ at a finite time $T_{\text{final}}$.

(ii) In the swarming mode, each agent travels to a sufficiently close neighborhood of $p_{\text{des}}$ such that Condition C2" is satisfied for all agents.

(iii) All agents avoid collisions; i.e., $P \notin \Omega$ for all $t \in [t_0, T_{\text{final}}]$.

(iv) Agents avoid all obstacles; i.e., $P \notin \Omega^o$ for all $t \in [t_0, T_{\text{final}}]$, $i = 1, \cdots, N$. 

\[
\begin{align*}
\beta^s(i) &= \begin{cases} 
1, & \text{if } i = 1 \\
0, & \text{if } i \neq 1 
\end{cases} \\
\text{such that the following results hold:}
\end{align*}
\]
(v) All swarms avoid collisions; i.e., \( \mathbf{P} \notin \Omega^{gro} \) for all \( t \in [t_0, T_{final}] \).

(vi) All agents stay connected; i.e., \( \mathbf{P} \notin \mathcal{L} \) for all \( t \in [t_0, T_{final}] \), \( i = 1, \cdots, N \).

**Proof.** Consider the following Lyapunov-like function:

\[
V = e + V^{col} + V^{obs} + V^{prox} + V^{gro} \tag{4.76}
\]

where \( e \) is the coverage area integral given by (4.56), \( V^{col} \) is given by

\[
V^{col} = \frac{w_{col}}{2} \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left( \sum_{j \neq i} V^{col}_{s_i s_j} + \sum_{w \neq s} \sum_{j=1}^{N_w} V^{col}_{s_i w_j} \right), \tag{4.77}
\]

with \( 0 < w_{col} \), \( V^{obs} \) is given by

\[
V^{obs} = w_{obs} \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left( \sum_{k=1}^{N_o} V^{obs}_{s_i k} \right), \tag{4.78}
\]

with \( 0 < w_{obs} \), and \( V^{prox} \) is given by

\[
V^{prox} = \frac{w_{prox}}{2} \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left( \sum_{j \neq i} V^{prox}_{s_i s_j} + \sum_{w \neq s} \sum_{j=1}^{N_w} V^{prox}_{s_i w_j} \right), \tag{4.79}
\]

with \( 0 < w_{prox} \), and \( V^{gro} \) is given by

\[
V^{gro} = w^{gro} \sum_{s=1}^{S} \left( \sum_{j=2}^{N_s} V^{gro}_{s_1 s_j} + \sum_{w \neq s} \sum_{j=2}^{N_w} V^{gro}_{s_1 w_j} \right) + \frac{w^{gro}}{2} \sum_{s=1}^{S} \sum_{w \neq s} V^{gro}_{s_1 w_1}, \tag{4.80}
\]

with \( 0 < w^{gro} \). Note that \( \sum_{j=1}^{N_w} V^{prox}_{s_i w_j} \equiv 0 \) for \( w \neq s \) since the inter-agent proximity functions are active among the members of the same swarm only. Moreover, by the virtue of equation (4.68), \( \sum_{s=1}^{S} V^{gro}_{s_1 s_j} \equiv 0 \) for all \( j = 2, \cdots, N_s \). In order to simplify some terms
in taking the derivative of \( V(t) \), we define the following notations:

\[
\mathcal{I}_{swa}^s = (\|p_i^s - p_{des}\|^2 - d_{des}^2)(p_i^s - p_{des}),
\]

\[
E_1 = -\int_{\mathcal{D}} \dot{h}(C^* - Q(t, P))(kC^{**}e^{-kA}) \left( \sum_{s=1}^{S} \sum_{i=1}^{N_s} S_i^s \right) \left[ \sum_{s=1}^{S} \sum_{i=1}^{N_s} S_i^s \right] d\tilde{p}_1 d\tilde{p}_2,
\]

\[
E_2 = -\gamma(t) \sum_{s=1}^{S} \sum_{i=1}^{N_s} \int_{\mathcal{D}} h(C^* - Q) \tilde{S}_i^s d\tilde{p}_1 d\tilde{p}_2,
\]

\[
E_3 = -\gamma(t) \sum_{s=1}^{S} \sum_{i=1}^{N_s} \sigma(D_i^s),
\]

\[
\kappa_{swa}^s = 2 \int_{\mathcal{D}} h(C^* - Q(t, P))(1 - \gamma)\sigma(D_i^s)(1 - \sigma(D_i^s)).
\] (4.81)

Once again, note that \( \sum_{s=1}^{S} \sum_{i=1}^{N_s} \frac{\partial V^\text{prox}}{\partial p_i^s} \equiv 0 \) for \( w \neq s \), \( \sum_{j=1}^{N_w} \frac{\partial V^\text{prox}}{\partial p_i^s} \equiv 0 \) for \( i = 1, \cdots, N_s \), and \( \sum_{j=1}^{N_w} \frac{\partial V^\text{prox}}{\partial p_i^s} \equiv 0 \) for \( i \neq 1 \). Then, by slight abuse of the notations of (4.75) and (4.81), the derivative of \( \dot{V}(t) \) can be written in the following way:

\[
\dot{V}(t) = \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left( \mathcal{I}_{cov}^s + \kappa_{swa}^s \mathcal{I}_{swa}^s + \mathcal{I}_{col}^s + \mathcal{I}_{obs}^s + \mathcal{I}_{prox}^s + \mathcal{I}_{gro}^s \right) \dot{p}_i^s + E_1 + E_2 + E_3.
\] (4.82)

Note that \( E_1 \leq 0 \). Now, we analyze the system in 4 modes; i.e., Mode \( i \) with \( i = 1, \cdots, 4 \):

**Mode 1 (\( \gamma(t) \equiv 1 \)):** In this mode, since \( \gamma(t) \equiv 1 \), we have \( \kappa_{swa}^s \equiv 0 \) and \( u_i^s = u_{\text{cov}} + u_{\text{col}} + u_{\text{obs}} + u_{\text{prox}} + u_{\text{gro}} \). Moreover, we have \( E_2 \equiv E_3 \equiv 0 \). Finally, note that \( E_1 \leq 0 \) by definition. Using the definitions of (4.75) and (4.81), we have the following inequality for
the derivative of $V(t)$:

$$
\dot{V}(t) \leq -\sum_{s=1}^{S} \sum_{i=1}^{N_s} \begin{bmatrix}
T^T_{\text{cov}^s} & T^T_{\text{col}^s} & T^T_{\text{obs}^s} & T^T_{\text{prox}^s} & T^T_{\text{gro}^s}
\end{bmatrix}
N^s_{i1},
$$

with

$$
0 \preceq N^s_{i1} =
\begin{bmatrix}
K^s_{\text{cov}i} & K^s_{\text{col}i} & K^s_{\text{obs}i} & K^s_{\text{prox}i} & \beta(i)K^s_{\text{gro}i}
\end{bmatrix}
\begin{bmatrix}
w_{\text{col}}K^s_{\text{cov}i} & w_{\text{col}}K^s_{\text{col}i} & w_{\text{col}}K^s_{\text{obs}i} & w_{\text{col}}K^s_{\text{prox}i} & w_{\text{col}}\beta(i)K^s_{\text{gro}i}
w_{\text{obs}}K^s_{\text{cov}i} & w_{\text{obs}}K^s_{\text{col}i} & w_{\text{obs}}K^s_{\text{obs}i} & w_{\text{obs}}K^s_{\text{prox}i} & w_{\text{obs}}\beta(i)K^s_{\text{gro}i}
w_{\text{prox}}K^s_{\text{cov}i} & w_{\text{prox}}K^s_{\text{col}i} & w_{\text{prox}}K^s_{\text{obs}i} & w_{\text{prox}}K^s_{\text{prox}i} & w_{\text{prox}}\beta(i)K^s_{\text{gro}i}
w_{\text{gro}}\beta(i)K^s_{\text{cov}i} & w_{\text{gro}}\beta(i)K^s_{\text{col}i} & w_{\text{gro}}\beta(i)K^s_{\text{obs}i} & w_{\text{gro}}\beta(i)K^s_{\text{prox}i} & w_{\text{gro}}\beta(i)K^s_{\text{gro}i}
\end{bmatrix},
$$

where $\beta(i) = 1$ if $i = 1$ and $\beta(i) = 0$ if $i \neq 1$. This implies that in this mode, if the coverage agents’ initial positions are such that they are outside each others’ avoidance regions and loss regions, they will never collide and for each swarm, the swarm communication network will never be lost among the members of the swarm; since $0 \preceq N^s_{i1}$, $V$ will always attain finite values, which in turn means $V^\text{col}$ and $V^\text{prox}$ will attain finite values. Similarly, the agents will not collide with static obstacles since $V^\text{obs}$ will attain finite values. Moreover, swarms will avoid each other since $0 \preceq N^s_{i1}$, hence $V^\text{gro}$ will also attain finite values. Finally as long as the agents don’t end up in local minima of $V(t)$ and the avoidance and/or proximity gradients are zero, the agents continue the coverage mission, since in this case, $\dot{V}(t)$ satisfies

$$
\dot{V} \leq -\sum_{s=1}^{S} \sum_{i=1}^{N_s} \begin{bmatrix}
T^T_{\text{cov}^s} & T^T_{\text{col}^s} & T^T_{\text{obs}^s} & T^T_{\text{prox}^s} & T^T_{\text{gro}^s}
\end{bmatrix}
K^s_{\text{cov}i} T^\text{cov}^s, \quad (4.85)
$$

**Mode 2** ($0 \leq \gamma(t) \leq 1, \ \dot{\gamma}(t) \leq 0$): In this mode, $\dot{\gamma}(t) \leq 0$. As a result, we have $u^s_i = \gamma(t)u^s_{\text{cov}i} + (1 - \gamma(t))u^s_{\text{swa}i} + u^s_{\text{col}i} + u^s_{\text{obs}i} + u^s_{\text{prox}i} + u^s_{\text{gro}i}$. Then, the time derivative of $V$ is
given by

\[
\dot{V} = - \sum_{s=1}^{S} \sum_{i=1}^{N_s} \begin{bmatrix} I_{covs}^T & c_s^{swa} (p_i^s - p_{des}^s)^T & I_{cols}^T & I_{obs}^T & I_{prox}^T & I_{gros}^T \end{bmatrix} \mathcal{N}_{i_2}^{s} (t) \begin{bmatrix} I_{covs}^T \\ c_s^{swa} (p_i^s - p_{des}^s) \\ I_{cols}^T \\ I_{obs}^T \\ I_{prox}^T \\ I_{gros}^T \end{bmatrix} + E_1 + E_2 + E_3,
\]

with

\[
\mathcal{N}_{i_2}^{s} (t) = \begin{bmatrix} \gamma K_{cov}^s & (1-\gamma) K_{swa}^s & K_{col}^s & K_{obs}^s & K_{prox}^s & \beta(i) K_{gros}^s \\ K_{cov}^s & (1-\gamma) K_{swa}^s & K_{col}^s & K_{obs}^s & K_{prox}^s & \beta(i) K_{gros}^s \\ w_{col} K_{cov}^s & (1-\gamma) w_{col} K_{swa}^s & w_{col} K_{col}^s & w_{col} K_{obs}^s & w_{col} K_{prox}^s & w_{col} \beta(i) K_{gros}^s \\ w_{obs} K_{cov}^s & (1-\gamma) w_{obs} K_{swa}^s & w_{obs} K_{col}^s & w_{obs} K_{obs}^s & w_{obs} K_{prox}^s & w_{obs} \beta(i) K_{gros}^s \\ w_{prox} K_{cov}^s & (1-\gamma) w_{prox} K_{swa}^s & w_{prox} K_{col}^s & w_{prox} K_{obs}^s & w_{prox} K_{prox}^s & w_{prox} \beta(i) K_{gros}^s \\ \gamma w_{gro} \beta(i) K_{cov}^s & (1-\gamma) w_{gro} \beta(i) K_{swa}^s & w_{gro} \beta(i) K_{col}^s & w_{gro} \beta(i) K_{obs}^s & w_{gro} \beta(i) K_{prox}^s & w_{gro} \beta(i) K_{gros}^s \end{bmatrix},
\]

\[
c_s^{swa} := \sqrt{K_{swa}^s} \arctan \left( \frac{\| p_i^s - p_{des}^s \|}{\| d_i^{des} \|} - d_i^{des} \right),
\]

Note that \( \mathcal{N}_{i_2}^{s} (t) \) is a time-varying matrix. To show that \( \mathcal{N}_{i_2}^{s} (t) \) is positive semi-definite, we can decompose it in the following way:

\[
\mathcal{N}_{i_2}^{s} (t) = W_{multi}^s J_{multi}^s K_{multi}^s (t),
\]

90
with

\[
W_{\text{multi}}^{si} = \begin{bmatrix}
  I_{2 \times 2} & 0 & 0 & 0 & 0 \\
  0 & I_{2 \times 2} & 0 & 0 & 0 \\
  0 & 0 & w_{\text{col}} I_{2 \times 2} & 0 & 0 \\
  0 & 0 & 0 & w_{\text{obs}} I_{2 \times 2} & 0 \\
  0 & 0 & 0 & 0 & \beta(i) w_{\text{gro}} I_{2 \times 2}
\end{bmatrix}, \quad J_{\text{multi}}^{si} = \begin{bmatrix}
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} \\
  I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2} & I_{2 \times 2}
\end{bmatrix},
\]

\[
K_{\text{multi}}^{si}(t) = \begin{bmatrix}
  \gamma K_{\text{cov}} & 0 & 0 & 0 & 0 \\
  0 & (1-\gamma) K_{\text{swa}} & 0 & 0 & 0 \\
  0 & 0 & K_{\text{col}} & 0 & 0 \\
  0 & 0 & 0 & K_{\text{obs}} & 0 \\
  0 & 0 & 0 & 0 & K_{\text{prox}} \\
  0 & 0 & 0 & 0 & \beta(i) K_{\text{gro}}
\end{bmatrix}, \quad (4.89)
\]

where \( \beta(i) = 1 \) if \( i = 1 \) and \( \beta(i) = 0 \) if \( i \neq 1 \), \( I_{2 \times 2} := \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \) and \( I_{2 \times 2} \) is the 2 by 2 identity matrix. Notice that \( W_{\text{multi}}^{si} \) and \( K_{\text{multi}}^{si}(t) \) are diagonal matrices. Moreover, it can be seen that for positive values of \( w_{\text{col}}, w_{\text{obs}}, w_{\text{prox}} \) and \( w_{\text{gro}} \), \( W_{\text{multi}}^{si} \) is positive definite for \( i = 1 \) and positive semi-definite otherwise. Similarly, for \( 0 < \gamma(t) < 1 \) and positive gain matrices \( K_{\text{cov}}^{si}, K_{\text{swa}}^{si}, K_{\text{col}}^{si}, K_{\text{obs}}^{si}, K_{\text{prox}}^{si}, K_{\text{gro}}^{si} \), \( K_{\text{multi}}^{si}(t) \) is positive definite for \( i = 1 \) and positive semi-definite otherwise, and at the beginning or end of Mode 2, i.e., for \( \gamma(t) \equiv 1 \) or \( \gamma(t) \equiv 0 \), \( K_{\text{multi}}^{si}(t) \) is positive semi-definite. Finally, by definition, \( J_{\text{multi}}^{si} \) has nonnegative eigenvalues. Hence, in Mode 2, the positive semi-definiteness of \( N_{i_2}^{si}(t) \) is maintained. Moreover, in order render \( \dot{V} \) negative semi-definite in Mode 2, as we have verified through simulations as well, we can select \( k^* \) and \( \omega_{s_2} \) such that \( E_1 + E_2 + E_3 \leq 0 \).
for all \(0 \leq \gamma(t) \leq 1\). Then, \(\dot{V}(t)\) satisfies\

\[
\dot{V} = - \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left[ T_{i}^{\text{cov}^sT} e_{i}^{\text{swa}^s}(p_{i}^s - p_{\text{des}}^s)^T T_{i}^{\text{col}^sT} T_{i}^{\text{obs}^sT} T_{i}^{\text{prox}^sT} T_{i}^{\text{gro}^sT} \right] N_{i}^{s}(t)
\]

Thus, in Mode 2, collisions among agents are avoided, obstacles are avoided, the connectivity of each swarm communication network is maintained among the members of the swarm. Note that at the instance \(\gamma(t)\) becomes 0, \(T_{i}^{\text{cov}^s}\) as well as \(E_1\) and \(E_2\), become 0. This implies that the sensors of the coverage agents are turned off. Due to the fact that \(0 \preceq N_{i}^{s}(t), V\) is guaranteed not to increase and this guarantee is sufficient; we do not expect the multi-agent system to accomplish either the coverage objective or the swarming objective in the transition mode. We only want to guarantee that \(\dot{V}\) does not attain positive values, and that is exactly what is guaranteed in Mode 2.

**Mode 3 \((\gamma(t) \equiv 0)\) :** In this mode, \(\dot{\gamma}(t) \equiv 0, T_{i}^{\text{conv}^s} \equiv 0, E_2 \equiv 0, E_3 \equiv 0\) and \(E_1 \leq 0\). Moreover, a desired target position has already been generated for the multi-agent group as well as desired distances \(d_{\text{des}}^s\) for each agent. The control signal for each agent is \(u_{i}^s = u_{\text{swa}_i}^s + u_{\text{col}_i}^s + u_{\text{obs}_i}^s + u_{\text{prox}_i}^s\). Therefore, \(\dot{V}\) satisfies

\[
\dot{V} \leq - \sum_{s=1}^{S} \sum_{i=1}^{N_s} \left[ c_{i}^{\text{swa}^s}(p_{i}^s - p_{\text{des}}^s)^T T_{i}^{\text{col}^sT} T_{i}^{\text{obs}^sT} T_{i}^{\text{prox}^sT} T_{i}^{\text{gro}^sT} \right] N_{i}^{s}(t)
\]

\begin{equation}
(4.91)
\end{equation}
with

\[ 0 \leq N_{i3}^s = \begin{bmatrix}
  K^s_{\text{swa}_i} & K^s_{\text{col}_i} & K^s_{\text{obs}_i} & K^s_{\text{prox}_i} & \beta(i)K^s_{\text{gro}_i} \\
  w_{\text{col}}K^s_{\text{swa}_i} & w_{\text{col}}K^s_{\text{col}_i} & w_{\text{col}}K^s_{\text{obs}_i} & w_{\text{col}}K^s_{\text{prox}_i} & w_{\text{col}}\beta(i)K^s_{\text{gro}_i} \\
  w_{\text{obs}_i}K^s_{\text{swa}_i} & w_{\text{obs}_i}K^s_{\text{col}_i} & w_{\text{obs}_i}K^s_{\text{obs}_i} & w_{\text{obs}_i}K^s_{\text{prox}_i} & w_{\text{obs}_i}\beta(i)K^s_{\text{gro}_i} \\
  w_{\text{prox}_i}K^s_{\text{swa}_i} & w_{\text{prox}_i}K^s_{\text{col}_i} & w_{\text{prox}_i}K^s_{\text{obs}_i} & w_{\text{prox}_i}K^s_{\text{prox}_i} & w_{\text{prox}_i}\beta(i)K^s_{\text{gro}_i} \\
  w_{\text{gro}_i}(i)K^s_{\text{swa}_i} & w_{\text{gro}_i}(i)K^s_{\text{col}_i} & w_{\text{gro}_i}(i)K^s_{\text{obs}_i} & w_{\text{gro}_i}(i)K^s_{\text{prox}_i} & w_{\text{gro}_i}(i)K^s_{\text{gro}_i} 
\end{bmatrix}, \]

\[ c_i^{\text{swa}} := \sqrt{k_i^{\text{swa}} \arctan (\|p^s_i - p^s_{\text{des}}\|^2 - d^2_{\text{des}})}(\|p^s_i - p^s_{\text{des}}\|^2 - d^2_{\text{des}}). \quad (4.92) \]

Once again, since \( N_{i3}^s \) is positive semi-definite, we conclude that the proximity is maintained within each swarm, collisions among agents are avoided, obstacles are avoided by each agent and swarms avoid other swarms. Moreover, in the absence of avoidance and proximity gradients, \( \dot{V}(t) \) satisfies

\[ \dot{V} \leq -\sum_{s=1}^{S} \sum_{i=1}^{N_s} (p^s_i - p^s_{\text{des}})^T c_i^{\text{swa}} K^s_{\text{swa}_i} (p^s_i - p^s_{\text{des}}). \quad (4.93) \]

Thus, each swarm moves as a group to the vicinity of the corresponding target point in Mode 3.

**Mode 4 (0 \leq \gamma(t) \leq 1, \ 0 \leq \dot{\gamma}(t))**: In this mode, we have \( \dot{\gamma}(t) \leq 0 \). Thus, we have \( E_2 + E_3 \leq 0 \), and \( E_1 \leq 0 \) by definition. Hence, \( u^s_i = \gamma(t)u^s_{\text{cov}_i} + (1 - \gamma(t))u^s_{\text{swa}_i} + u^s_{\text{col}_i} + u^s_{\text{obs}_i} + u^s_{\text{prox}_i} + u^s_{\text{gro}_i} \). Then, the time derivative of \( V \) is given by

\[ \dot{V} = -\sum_{s=1}^{S} \sum_{i=1}^{N_s} \begin{bmatrix}
  c_{i}^{\text{swa}}(p_i^s - p_{\text{des}})^T & c_{i}^{\text{swa}}(p_i^s - p_{\text{des}})^T & I^T_{\text{cov}_i} & I^T_{\text{cov}_i} & I^T_{\text{obs}_i} & I^T_{\text{obs}_i} & I^T_{\text{prox}_i} & I^T_{\text{prox}_i} & I^T_{\text{gro}_i} & I^T_{\text{gro}_i}
\end{bmatrix} N_{i4}^s(t) \]

\[ (4.94) \]
with

\[
N_{i_4}^s(t) = N_{i_2}^s(t),
\]
\[
c_i^{swa} := \sqrt{\kappa_i^{swa}} \arctan \left( \frac{\|p_i^s - p_{des}^s\|^2 - d_{des}^2}{\|p_i^s - p_{des}^s\|^2 - d_{des}^2} \right).
\]  

(4.95)

Just as it was the case in Mode 2, \( N_{i_4}^s(t) \) is a time-varying matrix, hence we cannot immediately come to the conclusion that it is positive semi-definite. However, since \( N_{i_4}^s(t) \equiv N_{i_2}^s(t) \) and we have already shown that \( N_{i_2}^s(t) \) is positive semi-definite, so is \( N_{i_4}^s(t) \). Thus, in Mode 4, no collisions occur, obstacles are avoided, for each swarm, the swarm communication is maintained among the members of the swarm and swarms avoid other swarms. Note that at the instance \( \gamma(t) \) becomes 1, the sensors of the coverage agents are turned back on.

Just as it was the case in Mode 2, during this transition period, there are two possibly conflicting objectives, the coverage objective and the swarming objective, that are active since both the coverage and swarming control signals act on the agents. Since we have \( 0 \preceq N_{i_4}^s(t) \), \( V \) is guaranteed not to increase in the transitioning mode and this guarantee is sufficient.

Remark 17. In order to express the time derivative of \( \dot{V}(t) \) in a compact way, we have slightly abused the notations of (4.75) and (4.81). For instance, there are some terms in (4.83) that are identically zero; e.g., \( T_{i}^{gro} \equiv 0 \) for \( i \neq 1 \). Nevertheless, the expressions are correct, albeit redundant.

4.4 Summary

In this section, we presented an alternative control scheme for dynamic coverage control problem in multi-agent systems. We formulated a framework where the agents accomplish the coverage objective by moving as a swarm. In the context of swarm-based coverage scheme, instead of including a stationary supervisor that selects target points for the coverage agents, we designated one of the agents as the leader agent, and this agent selects...
a single target point for the swarm, while all other agents move to a vicinity of the target point as a group. In the design of the control scheme, we utilized smooth transitioning signals to differentiate between different operation modes, and to decouple control laws so that operation in each mode is not affected by control signals corresponding to other modes. We also considered collision avoidance and obstacle avoidance functions to ensure safe operation of the agents. Moreover, we utilized inter-agent proximity functions for maintaining the connectivity of the network among the members of the swarm. In this sense, we designed a distributed scheme for accomplishing the coverage objective.

After discussing the stability of our approach via Lyapunov-like analysis, we applied the swarm-based coverage control scheme to kinematic unicycle agents, and discussed the stability of the scheme in this case. Finally, we proposed multi-swarm variant of the swarm-based coverage control scheme where we employed multiple swarms for accomplishing the coverage objective. Additionally, we introduced swarm avoidance functions that are implemented by the leader agents in swarms, which guarantee that the swarms avoid other swarms. We also provided a discussion on the stability of the multi-swarm variant. We illustrate the effectiveness of the swarm-based coverage control scheme in Chapter 5.
Chapter 5

Simulations

In this chapter, we illustrate the effectiveness of the proposed schemes, the supervised coverage scheme and the swarm-based coverage scheme, via numerical simulations performed on MATLAB. First, we consider the supervised coverage scheme on a group of single integrators. Then, we compare the results with the asynchorous variant. After illustrating the supervised coverage scheme applied to wheeled mobile robots, we show the simulation results for the swarm-based coverage scheme applied to both single integrators and kinematic unicycles. Following the simulation results for the multi-swarm scenario, we conclude this chapter by providing a discussion on the implementation issues for the simulations.

5.1 Supervised Coverage Control

Initially, we present the simulation results for the supervised coverage control scheme.

5.1.1 Single Integrators

In this section, we present the simulation results for the supervised coverages scheme and its asynchoronous variant for single integrator agents. We consider a problem with 4 coverage agents and 1 supervisor. The compact domain $\mathcal{D}$ is given by a rectangular domain, $[-1.5, 33.5] \times [-1.5, 33.5]$ minus two smaller rectangular domains, $[0.95, 5.5] \times [0.95, 5.5]$.
These rectangular domains can be thought of as obstacles, however, in our simulations, they are not considered as physical obstacles; instead, we assume that these regions are not part of the coverage domain, hence the agents are not required to cover these regions. Coverage agents do not accumulate any sensory information in these regions, thus the information acquired in these regions do not contribute to the control signals. Due to the fact that they are not physical obstacles, agents may briefly enter these regions and/or pass over these regions (e.g. in Mode 3). We remove these regions from the rectangular domain in order to illustrate the effectiveness of our control scheme in nonconvex coverage domains, which are quite difficult to handle by other approaches.

Nonasynchronous Scheme

We initially consider the nonasynchronous supervised scheme. We assume that the control gains, and other parameters are the same for all coverage agents (e.g., \( k_{cov_i} = k_{cov_y} = k_{cov} \), \( \bar{\varepsilon}_i = \bar{\varepsilon} \forall i = 1, \ldots, N \), etc.). The time step of the simulation is fixed at 0.0015 seconds. We utilize the \textit{trapz} MATLAB function for calculating double integrals on the plane. The position of the supervisor are the initial positions of the agents are given by

\[
p_{sv}(t) \equiv \begin{bmatrix} 16 \\ 16 \end{bmatrix}, \quad p_1(0) = \begin{bmatrix} 12 \\ 12 \end{bmatrix}, \quad p_2(0) = \begin{bmatrix} 12 \\ 20 \end{bmatrix}, \quad p_3(0) = \begin{bmatrix} 20 \\ 20 \end{bmatrix}, \quad p_4(0) = \begin{bmatrix} 20 \\ 12 \end{bmatrix}, \quad (5.1)
\]

The parameters of the simulation and their values are given in Table 5.1. We depict the results of our simulations in Figures 5.1, 5.2, 5.3, 5.4 and 5.5.

In Figure 5.1 we depict the trajectories of the agents changing over time. During certain intervals, the agents’ trajectories seem to change approximately linearly on the \( x-y \) plane. These intervals indeed correspond to the durations in which agents operate in Mode 3, where they are assigned trajectories to follow by the supervisor. The coverage maps of the agents corresponding to the trajectories of Figure 5.1 are shown in Figure 5.2. It can be clearly observed that over time, the coverage level of \( D \) increases. One thing to notice is that only 0.065951% is not sufficiently covered at \( t = 200 \) seconds. However,
Table 5.1: Simulation parameters of Nonasynchronous Supervised Coverage Scheme

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^{**}$</td>
<td>42</td>
<td>$C^*$</td>
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<tr>
<td>$k^*$</td>
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</tr>
<tr>
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<td>$\tau_{s1}, \tau_{s2}$</td>
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</tr>
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<td>$\varepsilon$</td>
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<tr>
<td>$R_{sv}$</td>
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<td>$r_{sv}$</td>
<td>14.75</td>
</tr>
<tr>
<td>$k_{cov}$</td>
<td>$3.224 \times 10^{-4} - 2 \times 10^{-2}$ $^a$</td>
<td>$\delta t_{ref}$</td>
<td>3.5</td>
</tr>
<tr>
<td>$k_{col}$</td>
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<td>0.2</td>
</tr>
<tr>
<td>$k_{sv}$</td>
<td>0.003632</td>
<td>$\mu_2$</td>
<td>80</td>
</tr>
<tr>
<td>$k_{glo}$</td>
<td>$20 - 26^b$</td>
<td>$\Delta t_e$</td>
<td>3.75</td>
</tr>
</tbody>
</table>

$^a$ $k_{cov}$ varies based on a gain-scheduling algorithm.

$^b$ $k_{glo}$ varies based on a gain-scheduling algorithm.

Figure 5.1: Snapshots of Agent Trajectories over time (Nonasynchronous Supervised)
Figure 5.2: Snapshots of Coverage Map changing over time (Nonasynchronous Supervised), \( \bar{e}(t = 0.002) = 0.9981, \bar{e}(t = 50) = 0.1079, \bar{e}(t = 200) = 6.5951 \times 10^{-4}, \bar{e}(t = 393.9060) = 0 \)
as can be seen in the last plot in Figure 5.2, it takes another 193.9060 seconds to fully cover the region. We plot the normalized error $\bar{e}(t)$ in Figure 5.3. Moreover, note that the coverage level at some points in $D$ is greater than $C^* = 40$, as can be seen most clearly in the coverage map corresponding to $t = 393.9060$ seconds. The reason for this is that, due to the structure of $Q(t,p)$ and the fact that it has a horizontal asymptote at $C^{**}$, the coverage level is actually allowed to go over $C^*$. However, by the virtue of the structure of $h(\cdot)$, in calculating the coverage error, we consider a maximum coverage level of $C^* = 40$.

The operation of the supervised coverage control scheme can clearly be observed in this figure. On certain intervals in time, coverage error decreases monotonically. On these intervals, the coverage agents operate in either Mode 1, Mode 2 or Mode 4. On other intervals, when the coverage error stays stationary, the coverage agents operate in Mode 3; they are each assigned target points by the supervisor and supposed to follow their corresponding trajectories. Due to the fact that the coverage agents’ sensors are turned off in Mode 3, no new sensory information is acquired, thus the coverage error stays constant in Mode 3. Note that the control laws for coverage agents are designed such that they accumulate sensory information mainly in Mode 1; however, they also accumulate new information while they turn their sensors off and on, i.e., in Mode 2 and Mode 4. In Figures 5.4 and 5.5, we depict the distances between coverage agents and the distances between the supervisor and the coverage agents, respectively. It can be seen that no collisions occur; inter-agent distances always stay above $r_{col}$. The distances to the supervisor can be seen to always stay below $R_{sv}$ in Figure 5.5, thus proximity to the supervisor is maintained by each coverage agent.

**Asynchronous Scheme**

In the asynchronous scheme, we consider the same coverage problem as the one in the nonasynchronous variant. All the initial conditions of the agents are the same, and all the parameters of the systems are the same with one exception; the value of $\epsilon$ is different for the coverage agents in the asynchronous scheme. The reason for this exception is the condition for transition from coverage mode to trajectory tracking mode is different for the
Figure 5.3: Normalized Error $\bar{e}(t)$, $\bar{e}(t = 393.9060) = 0$ (Nonasynchronous Supervised)

Figure 5.4: Inter-agent Distances (Nonasynchronous Supervised)
Figure 5.5: Distances of Agents to Supervisor (Nonasynchronous Supervised)

Table 5.2: Simulation parameters of Asynchronous Supervised Coverage Scheme

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<td>$C^{**}$</td>
<td>42</td>
<td>$C^*$</td>
<td>40</td>
</tr>
<tr>
<td>$k^*$</td>
<td>2.5</td>
<td>$M$</td>
<td>9</td>
</tr>
<tr>
<td>$r_{col}$</td>
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<td>$\tau_{s1}, \tau_{s2}$</td>
<td>0.1, 0.1</td>
</tr>
<tr>
<td>$R_{col}$</td>
<td>3.7</td>
<td>$\bar{\xi}$</td>
<td>0.003</td>
</tr>
<tr>
<td>$R_{cov}$</td>
<td>2.3</td>
<td>$r_{sv}$</td>
<td>14.75</td>
</tr>
<tr>
<td>$R_{sv}$</td>
<td>24.75</td>
<td>$k_{cov}$</td>
<td>$3.224 \times 10^{-4} - 2 \times 10^{-2}$ $\Delta t_{ref}$</td>
</tr>
<tr>
<td>$k_{col}$</td>
<td>1.1804</td>
<td>$\mu_1$</td>
<td>0.2</td>
</tr>
<tr>
<td>$k_{sv}$</td>
<td>0.003632</td>
<td>$\mu_2$</td>
<td>80</td>
</tr>
<tr>
<td>$k_{glo}$</td>
<td>$20 - 26^b$</td>
<td>$\Delta t_e$</td>
<td>3.75</td>
</tr>
</tbody>
</table>

$a$ $k_{cov}$ varies based on a gain-scheduling algorithm.

$b$ $k_{glo}$ varies based on a gain-scheduling algorithm.
asynchronous scheme; as described in (3.62), the agents transition to trajectory tracking mode based on their own normalized coverage error, given by \( \bar{e}_i(t) = \frac{\int \int h(C^*-Q_i)dp_1dp_2}{C^* Area(D)} \). Other parameters of the asynchronous scheme are given in Table 5.2. The results of the simulations for the asynchronous scheme are given in Figures 5.6, 5.7, 5.8, 5.9 and 5.10. In Figure 5.6, we depict the trajectories of the agents changing over time. We see the expected behavior in coverage agents; whenever they get stuck at local minima, they are deployed to uncovered regions via reference trajectories. The difference in the asynchronous scheme is that, an agent does not have to wait for all the agents to get stuck; if an agent itself does not have enough coverage over \( \Delta t_e \), then only that agent transitions to the trajectory tracking mode even if the other agents are still in other modes.

![Figure 5.6: Snapshots of Agent Trajectories over time (Asynchronous Supervised)](image)
The coverage maps of the agents corresponding to the trajectories of Figure 5.6 are shown in Figure 5.7. Just like it the nonasynchronous variant, the coverage level of $D$ increases over time. Notice that at $t = 200$ seconds, $\bar{e}(t) = 6.5951 \times 10^{-4}$, whereas this value is $2.8331 \times 10^{-5}$ for the asynchronous case, which is much smaller. The improvement in completion time of the coverage objective that the asynchronous scheme provides can be observed in Figure 5.8.

![Figure 5.7: Snapshots of Coverage Map changing over time (Asynchronous Supervised), $\bar{e}(t = 0.002)$ = 0.9981, $\bar{e}(t = 50)$ = 0.0496, $\bar{e}(t = 200)$ = 2.8331 $\times$ 10$^{-5}$, $\bar{e}(t = 345.0525)$ = 0](image)

The completion time for the nonasynchronous supervised scheme is 393.9060 seconds, whereas the agents complete the coverage objective with the asynchronous variant in 345.0525 seconds. Moreover, as can be clearly seen in Figure 5.8, $\bar{e}(t)$ decreases much faster in asynchronous variant compared to the nonasynchronous variant as expected.
Figure 5.8: Normalized Error $\bar{e}(t), \bar{e}(t = 345.0525) = 0$ (Asynchronous Supervised) vs. $\bar{e}(t = 393.9060) = 0$ (Nonsynchronous Supervised)

Finally, in Figures 5.9 and 5.10 no collisions between agents occur, and the agents always

Figure 5.9: Inter-agent Distances (Asynchronous Supervised)
maintain their distance to the supervisor, thus maintaining their communication.

![Graph showing distances to supervisor](image)

**Figure 5.10:** Distances of Agents to Supervisor (Asynchronous Supervised)

**Remark 18.** In the technical discussions, we set $R_{cov}$ to $\frac{r_{col}}{2}$ so that the sensing regions of the agents do not overlap. For the simulations of both the nonasynchronous and the asynchronous supervised coverage control of single integrators, we set $r_{col} = R_{cov}$. It can easily be seen from the simulations results that although the overlapping of the sensing regions is allowed, the operation of the control scheme is not hindered in either case.

### 5.1.2 Wheeled Mobile Robots

In this section, we illustrate the simulation results of the supervised coverage scheme applied to wheeled mobile robots. The covered domain is the same as the single integrator case. As discussed in Section 3.3 via feedback linearization, we describe wheeled mobile robots as single integrators. Unlike the supervised scheme for the single integrators, in this simulation, we set $R_{cov}$ to $\frac{r_{col}}{2}$ to illustrate that in this case, the scheme works just as well, and in accordance with the theoretical results, the overlapping of sensing regions are prevented. The offset distance $l_i$, which determines the position of the offset points
$B_i$ on the robots, is set to 0.21 for all agents. The time step of the simulation is fixed at 0.001 seconds. Other simulation parameters are given in Table 5.3. The results of the simulations are given in Figures 5.11, 5.12, 5.13, 5.14, 5.15 and 5.16. In Figure 5.11, the trajectories of the wheeled mobile robots over the coverage domain are depicted as they change over time. Note that the zig-zag behavior that is typical in unicycle agents is not present in the trajectories. The reason for this is, we actually plot the motion of the offset points $B_i$ on the robots, not the centroids. Through feedback linearization, we have shown that the equations of motion for these points are just like single integrators, hence, instead of zig-zag plots, we get the trajectories given in Figure 5.11. The coverage maps changing over time are given in Figure 5.12. Notice that the time it takes for the coverage task
Figure 5.12: Snapshots of Coverage Map changing over time (Supervised for Wheeled Mobile Robots), $\bar{e}(t = 0.002) = 0.9991$, $\bar{e}(t = 50) = 0.1172$, $\bar{e}(t = 200) = 0.0101 \times 10^{-5}$, $\bar{e}(t = 674.6560) = 0$
Table 5.3: Simulation parameters of Supervised Coverage Scheme for Wheeled Mobile Robots

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^{**}$</td>
<td>42</td>
<td>$C^*$</td>
<td>40</td>
</tr>
<tr>
<td>$k^*$</td>
<td>2.5</td>
<td>$M$</td>
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<tr>
<td>$r_{col}$</td>
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<td>$\tau_s$, $\tau_{s2}$</td>
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<td>$R_{col}$</td>
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<td>$\varepsilon$</td>
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<tr>
<td>$R_{cov}$</td>
<td>1.9</td>
<td>$\bar{\varepsilon}$</td>
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<tr>
<td>$R_{sv}$</td>
<td>24.75</td>
<td>$r_{sv}$</td>
<td>17.5</td>
</tr>
<tr>
<td>$k_{cov}$</td>
<td>$3.28 \times 10^{-4} - 2 \times 10^{-2}$</td>
<td>$\Delta t_{ref}$</td>
<td>4.5</td>
</tr>
<tr>
<td>$k_{col}$</td>
<td>1.2</td>
<td>$\mu_1$</td>
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<td>$k_{sv}$</td>
<td>0.003632</td>
<td>$\mu_2$</td>
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<tr>
<td>$k_{glo}$</td>
<td>$20 - 32$</td>
<td>$\Delta t_e$</td>
<td>4.75</td>
</tr>
</tbody>
</table>

$^a$ $k_{cov}$ varies based on a gain-scheduling algorithm.

$^b$ $k_{glo}$ varies based on a gain-scheduling algorithm.

to be completed is 674.6560 seconds, much longer compared to the previous cases. The main reason for this difference is the fact that we’ve set the sensing radius to a smaller value than the ones in previous cases. Thus, we’ve imposed the wheeled mobile robots to complete the coverage objective with sensors that have smaller range. By doing this, we’ve also prevented the sensing regions to overlap. We depict $\bar{e}(t)$ of the supervised coverage scheme for wheeled mobile robots in Figure 5.13. The transitioning behavior of the scheme can easily be observed in this figure. As we’ve already stated, the rate of decrease is slower compared to the previous simulations since we impose smaller sensing radii. It can easily be observed in Figures 5.14 and 5.15 that the safety of the agents and the reliability of the communication network are guaranteed; inter-agent collisions are always avoided, and all agents maintain their proximity to the supervisor. Finally, in Figure 5.16, we depict $v_{x_i}$ and $v_{y_i}$, the control signals we’ve attained via feedback linearization, given by (3.39). We’ve previously discussed that the bound on $\|\dot{\theta}_i\|$ for each agent, given by (3.61), is a conservative bound. The maximum value attained for $\|v_{x_i}\|$ or $\|v_{y_i}\|$ is 487.8065, i.e., $\beta_{max} = 487.8065$. In Figure 5.16, it can be seen that this maximum value is attained for a very short period of time around $t = 100$ seconds. Particularly, the maximum value is attained due to the fact that $u_{cov}$ becomes large at for agent #3. In addition, where collisions are imminent, $u_{col}$ dominates the magnitude of the control signal of the agent.
Figure 5.13: Normalized Error $\bar{e}(t)$, $\bar{e}(t = 674.6560) = 0$ (Supervised for Wheeled Mobile Robots)

Figure 5.14: Inter-agent Distances (Supervised for Wheeled Mobile Robots)
Figure 5.15: Distances of Agents to Supervisor (Supervised for Wheeled Mobile Robots)

Figure 5.16: $v_x$ & $v_y$ (Supervised for Wheeled Mobile Robots)
that is in danger of collision, hence the control signals may increase for brief durations while avoiding collisions. When there is no danger of collisions, the norms of the control signals are on the order of 50, i.e., much smaller than $\beta_{\text{max}}$. Thus, the conservativeness of the bound on $\dot{\theta}_i$ can easily be observed in Figure 5.16.

5.2 Swarm-Based Coverage Control

In this section, we present the simulation results for the swarm-based coverage control scheme. In contrast to the simulations of the supervised coverage scheme, in swarm-based scheme, we consider physical obstacles as well. Existence of physical obstacles increases the complexity of the coverage problem, and in general, algorithmic approaches do not perform well for domains with obstacles. The swarm-based coverage scheme, as we have proved in Chapter 4, works well even in the presence of obstacles. The following simulations will validate the technical results.

5.2.1 Single Integrators

We consider the swarm-based coverage control for a multi-agent group of 4 coverage agents. The compact domain $\mathcal{D}$ is given by the same rectangular domain as before, i.e., $[-1.5, 33.5] \times [-1.5, 33.5]$. The two circular obstacles with radii 3.04 are located at

$$p_1^o = \begin{bmatrix} 7 \\ 7 \end{bmatrix}^T, \quad p_2^o = \begin{bmatrix} 24.5 \\ 24.5 \end{bmatrix}^T.$$  \hspace{1cm} (5.2)

The initial conditions for the agents are the same as in the previous simulations. Note that in this scheme, we do not have a central supervisor agent. Due to the presence of several objectives, i.e., coverage, swarming, collision avoidance, obstacle avoidance and proximity maintenance, the simulation becomes complex, hence, we have used a time step that varies between 0.002 seconds and 0.00025 seconds. The simulation parameters of the swarm-based coverage scheme for single integrators are given in Table 5.4. The simulation results of the swarm-based scheme for single integrators are depicted in Figures 5.17, 5.18, 5.19.
Table 5.4: Simulation parameters of Swarm-based Coverage Scheme for Single Integrators

<table>
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<td>4.6</td>
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<tr>
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<td>$k_{prox}$</td>
<td>1.362</td>
<td>$k_{swa}$</td>
<td>2</td>
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</table>

$^a$ $\varepsilon = 1.5$ for the leader agent, and 0.5 for other agents.

$^b$ $k_{cov}$ varies based on a gain-scheduling algorithm.

$^c$ $d_{des} = 0$ for the leader agent, and 5 for other agents.

In Figure 5.17, we depict the trajectories of the agents over time. It can be observed that the agent had to move a lot more than the supervised scheme case in order to complete the coverage objective. This behavior is expected; the agents move as a group as they stay within close proximity of each other, so they cannot go to opposite corners of the domain. Thus, in some sense, they explore the domain sequentially. Moreover, we set $R_{cov}$ to $\frac{r_{col}}{2}$, and $r_{col}$ is small compared to previous simulations. Consequently, the time to finish the coverage objective is much longer compared to the other cases. The coverage maps over time for the swarm-based scheme and the normalized error $\bar{e}(t)$ can be seen in Figures 5.18 and 5.19. In Figure 5.18, the regions that are occupied by the obstacles are depicted as fully covered. This was done to make sure that the normalized coverage error was properly offset. Note that although the coverage error is already small at $t = 300$ seconds, i.e., $\bar{e}(t = 300) = 6.9870 \times 10^{-4}$, the coverage task is completed in 924.0185 seconds. The disadvantage of the swarm-based scheme is apparent through this behavior; since the leader agent moves to an uncovered region and all agents swarm around

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$^b$ By sequential, we do not mean that the region is partitioned and algorithmically explored; instead, we mean that since they cover the given domain as a group, they have to do this region by region.
Figure 5.17: Snapshots of Agent Trajectories over time (Swarm-based for Single Integrators)
Figure 5.18: Snapshots of Coverage Map changing over time (Swarm-based for Single Integrators), $\bar{e}(t = 1.9960) = 0.5951$, $\bar{e}(t = 300) = 6.9870 \times 10^{-4}$, $\bar{e}(t = 600) = 1.5188 \times 10^{-5}$, $\bar{e}(t = 924.0185) = 0$
Figure 5.19: Normalized Error $\bar{e}(t)$, $\bar{e}(t = 924.0185) = 0$ (Swarm-based for Single Integrators)

it, the leader agent is the only agent that is guaranteed to contribute to the decrease in the normalized error. Thus, it is possible that the other agents do not contribute much to the decrease in coverage error, especially when the majority of the domain is covered and small uncovered regions are left. In Figure 5.20, we can see that the agents avoid collisions. The swarming behavior of the multi-agent group can be observed in Figure 5.21. The inter-agent distances are always less than the loss radius $R_{\text{loss}} = 19$. Hence, the all-to-all communication is maintained by the swarm. Finally, in Figure 5.22, it can be seen that the agents avoid all the obstacles. The distance between an agent and any obstacle always stays above $r_{\text{obs}}$. As we’ve previously discussed, the presence of obstacles may cause problems with algorithmic coverage schemes, but the swarm-based coverage scheme handles this case fairly well.
Figure 5.20: Inter-agent Distances (Swarm-based for Single Integrators)

Figure 5.21: Inter-agent Distances (Swarm-based for Single Integrators)
5.2.2 Kinematic Unicycles

In this section, we depict the results of the numerical simulations for the swarm-based coverage scheme applied to kinematic unicycle agents. In contrast to the supervised scheme for wheeled mobile robots where we’ve described unicycles as single integrators, in this section, we explicitly consider the nonholonomic constraint of the unicycle agents. The coverage domain is the same as before, with 2 circular obstacles of radii 2.52 located at

\[
p_1^o = \begin{bmatrix} 7.25 \\ 7.25 \end{bmatrix}^T, \quad p_2^o = \begin{bmatrix} 24.4 \\ 24.4 \end{bmatrix}^T.
\]

The initial conditions for the agents are the same as the previous simulations. In this simulation, we have used a time step that varies between 0.00125 seconds and 0.0005 seconds. The simulation parameters of the swarm-based coverage scheme for single integrators are given in Table 5.5. The simulation results of the swarm-based scheme for single integrators are depicted in Figures 5.23, 5.24, 5.25, 5.26, 5.27 and 5.28. In Figure 5.23, we depict the trajectories of the agents over time. Notice the zig-zag behavior; this is typical in unicycle
Figure 5.23: Snapshots of Agent Trajectories over time (Swarm-based for Kinematic Unicycles)
Table 5.5: Simulation parameters of Swarm-based Coverage Scheme for Kinematic Unicycles

<table>
<thead>
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<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<td>$C^*$</td>
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</tr>
<tr>
<td>$k^*$</td>
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<td>$\tau_{s1}, \tau_{s2}$</td>
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<td>$R_{cov}$</td>
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<td>$\varepsilon$</td>
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<td>$r_{loss}$</td>
<td>13</td>
</tr>
<tr>
<td>$R_{obs}$</td>
<td>4.4</td>
<td>$r_{obs}$</td>
<td>2.8</td>
</tr>
<tr>
<td>$k_{cov}$</td>
<td>$3.28 \times 10^{-4} - 2 \times 10^{-2}$ (^a)</td>
<td>$k_\theta$</td>
<td>50</td>
</tr>
<tr>
<td>$k_{col}$</td>
<td>1.362</td>
<td>$\mu_1$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{obs}$</td>
<td>1.362</td>
<td>$\mu_2$</td>
<td>25000</td>
</tr>
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<td>$k_{prox}$</td>
<td>1.362</td>
<td>$\Delta t_e$</td>
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<td>$k_{swa}$</td>
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<td>$d_{des}$</td>
<td>0.46 (^b)</td>
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<tr>
<td>$c_\theta$</td>
<td>1.5</td>
<td>$\omega_\theta$</td>
<td>10</td>
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</table>

\(^a\) $k_{cov}$ varies based on a gain-scheduling algorithm.
\(^b\) $d_{des} = 0$ for the leader agent, and 4.6 for other agents.

agents. We did not observe this behavior in the case of supervised scheme for wheeled mobile robots since we had single integrator dynamics for the offset points, but in this simulation, we have nonholonomic kinematics for the agents. Thus, we can observe the zig-zag paths. In Figure 5.24, we present the coverage maps changing over time. Note that the mission is completed faster, in 610.5858 seconds, compared to the case with the single integrators. The main reason is in this simulation, we assumed a larger sensing radius. The regions occupied by the obstacles are set to $C^*$ coverage level on the coverage map so that the normalized error can be properly offset. Once again, due to the structure of $Q(t, p)$ and the fact that it has a horizontal asymptote at $C^{**}$, the coverage level is actually allowed to go over $C^*$. Since the sensing regions are large enough, whenever the coverage agents are close to the obstacles, from a mathematical point of view, they gather information in the regions occupied by the obstacles. However, this information does not contribute to the coverage control at all. In Figure 5.25, we depict the normalized coverage error for the swarm. Note that the decrease in the normalized error is initially slow. This behavior is a result of the nonholonomic dynamics of the agents; if the combined gradient of the objectives is perpendicular to the orientation of an agent, the agent comes
Figure 5.24: Snapshots of Coverage Map changing over time (Swarm-based for Kinematic Unicycles), $\epsilon(t = 0.00125) = 0.9939, \epsilon(t = 50) = 0.6521, \epsilon(t = 211) = 0.0725, \epsilon(t = 610.5858) = 0$
to a halt since there is no control input in the heading direction. This is the reason why the agents cannot initially do much coverage; only after they are deployed to uncovered regions can they move more freely. In Figure 5.26, we can see that the agents avoid collisions. The swarming behavior of the multi-agent group can once again be observed in Figure 5.27. The inter-agent distances are always less than the loss radius $R_{loss} = 19$. Hence, the all-to-all communication is maintained by the swarm. Finally, in Figure 5.28, it can be seen that the agents avoid all obstacles. The distance between an agent and any obstacle always stays above $r_{obs}$. As we’ve previously discussed, the presence of obstacles may cause problems with algorithmic coverage schemes, but the swarm-based coverage scheme handles the obstacles even in the case of kinematic unicycle agents.
Figure 5.26: Inter-agent Distances (Swarm-based for Kinematic Unicycles)

Figure 5.27: Inter-agent Distances (Swarm-based for Kinematic Unicycles)
5.2.3 Multi-Swarm

In the last section of this chapter, we present the simulations for the multi-swarm swarm-based coverage control scheme. The coverage domain is given by the rectangular region, \([-1.5, 46.8] \times [-1.5, 46.8]\). We consider a problem with 2 swarms, each with 3 coverage agents. The locations of 5 circular obstacles are given by

\[
p_1^o = \begin{bmatrix} 9 \\ 9 \end{bmatrix}, \quad p_2^o = \begin{bmatrix} 36.3 \\ 9 \end{bmatrix}, \quad p_3^o = \begin{bmatrix} 22.65 \\ 22.65 \end{bmatrix}, \quad p_4^o = \begin{bmatrix} 36.3 \\ 36.3 \end{bmatrix}, \quad p_5^o = \begin{bmatrix} 9 \\ 36.3 \end{bmatrix}. \tag{5.4}
\]

The initial conditions for the agents are given by

\[
p_1^1 = \begin{bmatrix} 18.65 \\ 6 \end{bmatrix}, \quad p_2^1 = \begin{bmatrix} 26.65 \\ 6 \end{bmatrix}, \quad p_3^1 = \begin{bmatrix} 22.65 \\ 12 \end{bmatrix}, \\
p_1^2 = \begin{bmatrix} 26.65 \\ 42 \end{bmatrix}, \quad p_2^2 = \begin{bmatrix} 22.65 \\ 36 \end{bmatrix}, \quad p_3^2 = \begin{bmatrix} 18.65 \\ 42 \end{bmatrix}. \tag{5.5}
\]

Figure 5.28: Distances of Agents to Obstacles (Swarm-based for Kinematic Unicycles)
In this simulation, we have used a time step that varies between 0.001 seconds and 0.000425 seconds. The other parameters of the multi-swarm simulations are given in Table 5.6. We report on the results in Figures 5.29, 5.30, 5.31, 5.32, 5.33, 5.34, 5.35, and 5.36.

The trajectories of the swarms over time can be seen in Figure 5.29. The sensing regions of the agents are small compared to previous simulations. Moreover, the coverage domain is larger. Thus, although there are 2 swarms with 6 agents in total, the coverage objective is completed in longer time. Moreover, the presence of swarm avoidance functions limit the maneuverability of the swarms. In swarming mode, the swarms cannot be deployed to any arbitrary points; the target points must be sufficiently far away from the other swarms so that they can reach their destinations. Thus, in swarming mode, we have prioritized between swarms in target selection; the first swarm always selects a point, but if the other swarm cannot find a point that is sufficiently far away from the other swarm, it stays in place. In this sense, the multi-swarm scheme would certainly benefit from a better
Figure 5.30: Snapshots of Coverage Map changing over time (Multi-swarm), $\bar{e}(t = 0.001) = 0.9990$, $\bar{e}(t = 19.9990) = 0.0740$, $\bar{e}(t = 674.8374) = 1.4822 \times 10^{-5}$, $\bar{e}(t = 1211.1923) = 0$
Table 5.6: Simulation parameters of Multi-Swarm Swarm-based Coverage Scheme

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<td>C**</td>
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<td>C*</td>
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<td>k*</td>
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<td>r_{col}</td>
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<td>τ_{s1}, τ_{s2}</td>
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<td>R_{col}</td>
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<td>ε</td>
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<td>R_{cov}</td>
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<td>r_{obs}</td>
<td>1.8</td>
</tr>
<tr>
<td>k_{cov}</td>
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<td>k_{gro}</td>
<td>0.362</td>
</tr>
<tr>
<td>k_{col}</td>
<td>1.362</td>
<td>μ_1</td>
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<td>k_{prox}</td>
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<td>Δt_e</td>
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<tr>
<td>R_{gro}</td>
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<td>r_{gro}</td>
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</table>

a $k_{cov}$ varies based on a gain-scheduling algorithm.
b $d_{des} = 0$ for the leader agent, and 3.3 for other agents.

target selection algorithm. In Figure 5.30, we depict the coverage maps corresponding to the trajectories of Figure 5.29 and the normalized error $\bar{e}(t)$ is given in Figure 5.31. The value of $\bar{e}(t)$ is 0.0740 at $t = 19.9990$ seconds, but it becomes 0 at $t = 1211.1923$ seconds. It can be seen from Figure 5.32 that inter-agent collisions are avoided. Similarly, it can be observed in Figure 5.33 that obstacles are avoided by the agents. In Figure 5.34, we limit the y axis of the plot for clarity. In Figure 5.35, it can be seen that proximity is maintained by each swarm separately. In the first plot, the inter-agent distances for swarm #1 are given, and it can be seen that the maximum inter-agent value is less than $R_{loss}$. The same is true for swarm #2, as can be seen in the second plot in Figure 5.35.

Finally, in Figure 5.36, we show the distances of the agents to the swarm leaders. It can be seen in the first plot that the distances between the agents in swarm #2 and the leader agent in swarm #1 are always greater than $r_{gro}$. More importantly, the distances are always greater than $R_{loss}$. The same holds for the distances of agents in swarm #1 and the leader in swarm #2. These plots clearly show that the swarms avoid each other.
Figure 5.31: Normalized Error $\bar{e}(t)$, $\bar{e}(t = 121.1923) = 0$ (Multi-swarm)

Figure 5.32: Inter-agent Distances, $d_{ij}^{sw} = \|p_i^s - p_j^w\|$ (Multi-swarm)
Figure 5.33: Distances of Agents to Obstacles (Multi-swarm)

Figure 5.34: Distances of Agents to Obstacles in detail (Multi-swarm)
Figure 5.35: Inter-agent Distances within swarms, $d_{ij}^{sw} = \|p_i^s - p_j^w\|$ (Multi-swarm)

Figure 5.36: Distances of Agents to Leaders, $d_{ij}^{sw} = \|p_i^s - p_j^w\|$ (Multi-swarm)
5.3 Implementation Issues

In the implementation of the schemes, there are two issues that need to be explained in order to give the overall picture. The first one is the gain scheduling issue that we have mentioned in the footnotes in previous sections. Due to the structure of our control laws, as the coverage error decreases, the control gains that are initially selected may not be high enough after a while. In such situations, the agents may easily select new control gains in advance for coverage or trajectory tracking/swarming control laws in a gain-scheduling type of scheme, while maintaining the continuity of the control signals. We have indeed employed such a strategy for the simulations of this section.

The second issue arises whenever the agents do not reach to a close vicinity of their target points within a given time. This may happen due to several reasons; the gains may not initially be high enough, the agents may end up in local minima in Mode 3 due to the avoidance and/or proximity maintenance objectives, or in supervised coverage scheme, the supervisor may select target points that are fairly close to each other for different agents. To get out these deadlock situations, in our simulations, we employ emergency routines. The emergency routine gets the agents out of deadlock situations, allowing them to transition to their corresponding modes, and continue the coverage mission.

5.4 Summary

In this chapter, we validated the effectiveness of both the supervised and the swarm-based coverage control schemes. We compared the nonasynchronous and the asynchronous supervised scheme for single integrators, and illustrated the improvement in completion time of the coverage objective for the asynchronous variant. We also presented the simulations for the supervised scheme applied to wheeled mobile robots. Moreover, we simulated the swarm-based coverage scheme for both single integrators and kinematic unicycle agents. We depicted the swarm behavior of the multi-agent group, and the effectiveness of our scheme even in the presence of physical obstacles. Finally, we reported on the simulations of the multi-swarm swarm-based coverage scheme.
Chapter 6

Conclusions

In this dissertation, we proposed two novel control schemes within the context of dynamic coverage control. Initially, we formulated a framework where a supervisor assists the coverage agents with designated trajectories whenever the coverage agents are trapped in local minima. In designing continuous control inputs, we utilized smooth transitioning signals to differentiate between different operation modes, and to decouple control laws so that operation in each mode is not affected by control signals corresponding to different modes. Moreover, by including collision avoidance and proximity functions, we proposed a supervised dynamic coverage scheme with guaranteed collision avoidance and proximity maintenance. We provided a discussion on the application of the centralized supervised coverage control scheme to wheeled mobile robots modeled as kinematic unicycles. Subsequently, we constructed an asynchronous supervised coverage scheme that shortens the task completion since the agents can transition to different modes without waiting for other agents. We provided the stability analysis for the proposed scheme via Lyapunov-like approach. We illustrated the effectiveness of our approaches for both single integrator agents and wheeled mobile robots that can be modeled as single integrators via feedback linearization.

The second scheme proposed in this research, the swarm-based coverage scheme, acts as a step towards distributed dynamic coverage control. In this scheme, we constructed a control methodology that allows the agents complete the coverage task as a group without
the need for a supervisor. The communication of the group is maintained via inter-agent proximity functions. Moreover, there is no need for trajectory design; the agents swarm to a target point as a group without having to follow trajectories. Additionally, by constructing Lyapunov-like functions that combine various objectives, namely coverage, swarming, collision avoidance, obstacle avoidance and inter-agent proximity maintenance, and decoupling the coverage and swarming objectives using smooth transitioning signals, we partitioned the overall control problem into simpler problems that we rigorously analyzed to prove the stability of the overall scheme. We also analyzed the stability of the swarm-based coverage scheme when applied to kinematic unicycle agents. Finally, we proposed an extension of the swarm-based coverage scheme to the multi-swarm case, in which each swarm, as a group, avoids other swarms. To accomplish this, we constructed swarm avoidance functions using collision avoidance functions. We validated our approaches for both single integrator agents and kinematic unicycle agents through numerical simulations.

6.1 Future Directions

Although the work presented in this dissertation is fairly complete, there are a number of directions in which the research can be extended. Through the construction of the supervised and swarm-based coverage schemes, we have built a general framework in control design; thus, these schemes can be applied to agents with nonlinear dynamics, which we haven’t considered in this work. Specifically, it would be a challenging, yet interesting research problem to apply the schemes of this dissertation to agents with velocity constraints, such as unmanned planes. Information decay is not considered in either of the schemes; thus, incorporation of information decay would render the schemes more realistic. Although the swarming-based scheme is a step towards distributed dynamic coverage, it is not completely distributed; we haven’t explicitly considered the topology of the communication graphs. A generalization to different topologies where the exchange of information is truly distributed would be a significant improvement of the swarm-based scheme. The global deployment algorithms utilized in this dissertation may be
improved through algorithmic approaches, so that the supervisor or the leader agent can make smarter decisions in target selection. Finally, real-time implementation on multi-agent systems, such as autonomous lawn-mowers or reconnaissance robots would be the ultimate method of validation of the proposed schemes.
Bibliography


