PARTICLE IMAGE VELOCIMETRY FOR NATURAL CONVECTION IN A CUBE AT HIGH RAYLEIGH NUMBERS

BY

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DISSERTATION

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The study of natural convection in enclosures has attracted the attention of numerous researchers over the past decades due to its applications in many practical engineering problems. Some examples include air conditioning and ventilation systems, electronic cooling devices, heat exchangers and thermal storage.

In the present work an experimental investigation of natural convection developed in an air filled cube of side $H = 0.35 \text{ m}$ is conducted using Particle Image Velocimetry (PIV) to measure the velocity field within the cavity and visualize the dominant flow structures. The low turbulence Rayleigh number regime $5.08 \times 10^7 \leq Ra \leq 3.40 \times 10^8$ is examined for the first time and experimental results including streamlines, the two velocity components in the vertical and horizontal direction, Reynolds stresses, swirling strength and vorticity are reported. In addition, Proper Orthogonal Decomposition (POD) was employed to analyze the flow and identify its major components. The estimated error from PIV measurements is within 1-2%, thus the high accuracy of the results can form experimental benchmark data and can be used to validate future CFD codes.

Flow visualization enabled the study of the evolution of the boundary layers along the isothermal and adiabatic walls. Furthermore, two secondary recirculations in the upper left and lower right corner of the cavity where measurements were conducted were also observed and reported. The two regions are anti-symmetric with the one in the upper left corner being initially
Finally, heat transfer measurements on the hot wall were conducted and Nusslet number for the corresponding Rayleigh number was estimated. A correlation between the two numbers was obtained and compared against other correlations in the literature. A deviation from the classical power law theory was observed and it is attributed to radiation and non-Boussinesq effects. A concurrent computer simulations effort was also conducted and the results are presented for comparison with experimental data.
To my adviser...for shaping the future.
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## LIST OF ABBREVIATIONS

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<tr>
<td>Cu</td>
<td>Copper</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition System</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusslet Number</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl Number</td>
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<tr>
<td>Ra</td>
<td>Rayleigh Number</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
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<tr>
<td>TKE</td>
<td>Turbulent Kinetic Energy</td>
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1 INTRODUCTION

This section includes the motivation that initiated this work. A literature review on the topic of natural convection in enclosures is also presented. This begins from the early work of Batchelor up to nowadays and the application of optical techniques such as Particle Image Velocimetry (PIV) in cavities to measure velocity fields and visualize flow structures at the same time.

1.1 Motivation

Hydrogen has the potential to be an attractive energy carrier, particularly for the transportation sector. It is clean, efficient and derived from diverse resources such as biomass, hydro, wind, solar, and geothermal [1]. Hydrogen storage is widely recognized as a critical aspect for the successful commercialization and acceptance of hydrogen powered automobiles. A tremendous technical challenge to overcome is to be able to store sufficient hydrogen on-board a wide range of vehicles, while meeting various customer requirements such as driving range, cost, safety and performance.

A canonical problem in the aforementioned application is buoyancy driven flow within a fluid-filled enclosure with opposite vertical walls kept at different temperatures. This flow is encountered in several applications in addition to hydrogen storage, such as air-conditioning in buildings, electronics cooling, furnaces, heat exchangers, solar collectors, and even geophysical and astrophysical phenomena [2].
1.2 Literature Review

The two-dimensional version of the problem was formulated by Batchelor [3], who set out to determine the effect of the fluid Prandtl number, Pr, the flow Rayleigh number, Ra, and the aspect ratio of the cavity on the heat transfer between the walls. An asymptotic analysis provided the dominant mode of heat transfer at various flow limits and confirmed that the heat transfer is dominated by conduction at low Ra. Batchelor also predicted that the fluid motion within the cavity is confined to the boundary layer on the vertical walls, and that heat transfer would be dominated by convection at large Ra.

Since the work of Batchelor, the problem of natural convection in enclosed cavities has received significant attention in the 1960s – 70s. Elder [4, 5] provided a plethora of experimental data for this type of flow, for both the laminar and turbulent flow regimes, for aspect ratios 1-60, Pr = 10^3 and Rayleigh numbers up to Ra ≈ 10^{10}. Gill [6] provided an analytical approach to the two-dimensional problem but focusing on the large Ra regime and on the temperature distribution within the convection core formed in the cavity, which he found to not be isothermal as previously suggested. A lot of the early experimental work on natural convection in enclosures is summarized by Ostrach [7], where a summary of average Nu correlations for rectangular enclosures of various aspect ratios is reported.

Several researchers have tackled both the two- and three-dimensional version of the problem using computational techniques. Davis [8] provided details of the computational method used to obtain an accurate solution of the equations describing two-dimensional natural convection in a square cavity with differentially heated side walls. Second-order, central difference approximations were used. Solutions for $10^3 \leq Ra \leq 10^6$ are believed to be accurate.
to better than 1% at the highest Rayleigh number and down to 0.1% at the lowest value. Markatos & Pericleous [9] studied the buoyancy-driven laminar and turbulent flow and heat transfer in a square cavity for $10^3 \leq \text{Ra} \leq 10^{16}$. The turbulence model used for $\text{Ra} \geq 10^6$ is a ($k - \epsilon$) two-equation model for turbulence, that includes gravity $\sim$ density gradient interactions. Fusegi et al. [10] studied the three-dimensional steady flow natural convection in a differentially heated cubical enclosure for $10^3 \leq \text{Ra} \leq 10^6$. They concluded that the third velocity component $w$ is an order of magnitude smaller than the dominant velocities $u$ and $v$ and the size of areas with non-zero $w$ velocity decreases as Rayleigh number increases. The overall Nusslet number indicated discrepancies from the 2D correlations. Tric et al. [11] studied the Rayleigh regime up to $\text{Ra} = 10^7$. The solutions were obtained with a pseudo-spectral Chebyshev algorithm with a spatial resolution supplied by polynomial expansions. The solutions are believed to be accurate better than $(0.03, 0.05)$% in relative global error for the corresponding Ra range ($10^3, 10^7$). All these studies provided benchmark numerical solutions and useful insight into flow structures and quantities.

In the 90s a significant amount of theoretical and computational work heavily focused on the transition between laminar and turbulent regime. Natural convection in enclosures undergoes a transition from a laminar to a turbulent state when the Rayleigh number exceeds a critical value. Often, the beginning of the transition to turbulence in these cavity flows is characterized by the occurrence of separate instabilities in the flow, each of which is identified by the appearance of a discrete frequency [12]. This is the well-known Hopf bifurcation, when the solution to the Navier-Stokes equations branches from a steady to unsteady solution with a single frequency of oscillation. The Hopf bifurcation can be seen as the initial phase of the transition to turbulence [13].
Henkes & Hoogendoorn [13] have numerically solved the unsteady, two-dimensional Navier-Stokes equations, to calculate the stability of the steady laminar-convection flow of air in a square cavity that is heated differentially over the vertical walls. They concluded that the flow looses stability at critical Rayleigh number value $\text{Ra}_{cr} = 2 \times 10^8$. They also showed that the evolution to steady state below the aforementioned critical value indicated internal gravity waves in the core with a frequency $f H / \sqrt{g \beta \Delta T H} = 0.107$. The solution for values greater than the critical Rayleigh number demonstrates two incommensurate frequencies $f H / \sqrt{g \beta \Delta T H} = 0.038$ and 0.555. They coupled the lower frequency with the core. They also observed an instability in the corner of the enclosure where the hot vertical boundary layer is redirected to a horizontal layer and interpreted it as a hydraulic jump. They finally related the higher frequency with the boundary layer along the vertical walls. Therefore, they concluded that this is a Tollmien-Schlichting instability, mostly found in the boundary layer along a semi-infinite hot plate in an isothermal environment. Another study [14] interpreted the sudden expansion of the flow as analogous to an internal hydraulic jump.

Ravi & Henkes [15] examined the corner phenomenon closely and revealed the oversimplification of the problem made in the previous studies. They concluded that the corner flow structure has no connection with a hydraulic jump phenomenon. They qualitatively depicted that the structure in the corner of the enclosure is caused solely by thermal effects. They suggested that the stable stratified core causes the temperature to undershoot in the vertical boundary layer. Consequently, colder fluid than the stratified core reaches the top of the enclosure and the resulting buoyancy force causes the plume. While Rayleigh number augments the plume grows stronger and a flow separation at the ceiling can be observed. The plume re-entra...
the vertical boundary layer causing re-circulation.

Henkes & Le Quéré [16] and Janssen & Henkes [17] concluded that the instability occurring in a jet-like fluid layer exiting from those corners of the cavity where the vertical boundary layers are turned horizontal is mainly shear-driven. They noticed the occurrence of vorticity concentrations which are very similar to Kelvin-Helmholtz vortexes in a plane jet, suggesting that the instability is a Kelvin-Helmholtz type instability. The aforementioned studies also concluded that the instability observed in the vertical boundary layers is indeed related to the instability occurring in the natural convection boundary along the vertical plate. However, they concluded that the instability in the cavity is strongly shear-driven which is different from the instability along the isolated vertical plate which is mainly buoyancy-driven.

In recent computational studies [18, 19] convection, conduction, and radiation, were coupled and results indicated that there is no instability present in the region where the hot vertical boundary layer is redirected horizontally. It was illustrated that radiation modifies the air-flow structures particularly at the top hot corner and the bottom cold corner. By accelerating the flow along the horizontal wall it increases turbulence intensity, especially in the areas where the horizontal layers impinge on the side walls. Therefore, accurate experimental results will boost the accuracy and validity of computational models and enhance our understanding of natural convection in enclosures.

One of the open challenges in studying this type of flows is the large Ra regime. Complex turbulent flow and large physical dimensions make accurate numerical simulations difficult. The same issue applies to experimental work, where Ra is limited by the physical size or achievable temperature differences and thermal boundary conditions that are practical for laboratory applications. There have been recent reports of more precisely controlled
experiments in order to provide higher quality data and also gain insight into the turbulent fluctuation quantities of these flows although Rayleigh range is limited. Tian & Karayiannis [20, 21] experimentally studied the low level turbulence natural convection in an air filled vertical square cavity at $Ra = 1.58 \times 10^9$ using Laser Doppler Anemometry (LDA). The results were obtained using high precision and can be useful as benchmark data for comparison with CFD codes. It was concluded that temperature and velocity components fluctuated independently in the boundary layer along the solid wall. A low base turbulent frequency was reported in the range 0.1-0.2 Hz. This frequency increased along the isothermal walls whereas in the core no mean flow or fluctuations were reported.

In the last decade with the development and commercialization of Particle Image Velocimetry (PIV) there is an increasing number of reports that applied the aforementioned optical technique in enclosures to obtained velocity vector information and as a visualization tool. Corvaro & Paroncini [22] analyzed the influence of the position of a small heating source placed on the bottom wall of a square cavity filled with air. A 2D PIV was employed to measure the velocity fields and dynamic structures generated by the heat transfer at the same Rayleigh numbers. In [23] an experimental analysis of the natural convection in a square enclosure is presented. The effect of the position of a hot strip on the dynamic structures generated by natural convection was analyzed at steady state and under laminar conditions using 2D PIV. In [24] an analysis of natural convection in an air filled enclosure with opposite heated and cooled walls is performed. Four different tilts of the cavity are investigated and a complete definition of the velocity field in the cavity is obtained through PIV. In a recent study [25], natural convection heat transfer from a heat generating horizontal cylinder enclosed in a square
cavity, where temperature difference exists across its vertical walls has been experimentally investigated and 2D PIV measurements of the flow structures inside the compartment were conducted.

In the present study, natural convection in an air-filled cube is considered in the range of $5.08 \times 10^7 \leq \text{Ra} \leq 3.40 \times 10^8$ and two-dimensional PIV is used to analyze the resulting flow-field. To the author’s best knowledge, experimental PIV data for this region for a cubical cavity have not been reported before.
2 EXPERIMENTAL SETUP

In this section the experimental setup built to study the flow within a fluid-filled enclosure with vertical walls kept at different temperatures is described. The setup is decomposed in its two major compartments and detailed information is provided for each of the components.

2.1 The Enclosure

In the present work a cubic cavity with two differentially heated vertical side walls is being considered. The cavity is filled with air at atmospheric pressure. The main components of the experiment are shown in Fig. 2.1 and schematically presented in Fig. 2.2. These are the test cavity and the 2D-PIV system.
Figure 2.1: Experimental Setup

Figure 2.2: Schematic of the Experimental Setup
The side of the cavity has a length of $H = 0.35 \text{ m}$. Two opposite vertical walls of the cavity are constructed of 0.003 m thick copper plate, while the remaining four sides are constructed of 0.0159 m thick Plexiglass, in order to reduce heat losses to the environment. The top plexiglass side of the enclosure has five 0.0095 m female NPT threads in a cross formation as illustrated in Fig. 2.3. Compression fittings were inserted in the threads to allow access to temperature profile probes for temperature measurements inside the cavity. Valves were connected to the two 0.0127 m female NPT threads on the top left and lower right corners. The valves were used for injecting the particles in the cavity for PIV experiments. On the front side three 0.0127 m female NPT threads and one of the same size on the back side were milled as depicted in Fig. 2.4 and Fig. 2.5 respectively. There was no work required to be done on the bottom plexiglass side of the enclosure (see Fig. 2.6).
Figure 2.3: Top View Drawing

Figure 2.4: Front View Drawing
Figure 2.5: Back View Drawing

Figure 2.6: Bottom View Drawing
The cold copper plate was milled with channels, and copper tubes 0.019 m in diameter were soldered in the channels as shown in Fig. 2.7. Through the tubes a mixture of 80% ethylene glycol in water by volume was circulated. The coolant was maintained at a constant temperature, $T_c$ using a NES-LAB RTE-140 isothermal bath [26]. The other copper plate (see Fig. 2.8) was heated by four 360 W ultra-thin heating sheets which were attached to the outside. A feedback thermocouple was used as input to an OMEGA CNi16D43 PID controller which regulated the heating sheets input so that the plate was kept at a uniform temperature, $T_h$. A 0.127 m thick insulating sheet was bonded to the remaining walls to reduce thermal losses to the environment and maintain the adiabatic wall heat transfer conditions. Insulation was removed from the side facing the CCD camera during the capturing of PIV images.

During the assembly process, the four plexiglass surfaces were glued and sealed together with silicon based sealant while bolts held the copper plates in place. A rubber gasket was placed between the copper plates and the plexiglass to prevent any air leaks and heat transfer between the copper and the plexiglass. A rendering of the enclosure is shown in Fig. 2.9.
Figure 2.7: Test Cavity Cold Wall View
Figure 2.8: Test Cavity Hot Wall View
2.2 Active Walls Temperature Measurements

The temperature of the two isothermal copper walls was measured by cementing five K-type thermocouples in a cross-formation on each wall as illustrated in Fig. 2.10. A high thermal conductivity compound was used to connect the thermocouples to the wall. The thermocouples had previously been calibrated in the NESLAB bath to yield an accuracy of ± 0.2 K. Throughout the duration of each experimental run, the temperature signals were digitized using the NI CompactDAQ USB chassis and the NI 9213 thermocouple module and recorded via the NI LabVIEW software.

Figure 2.10: Schematic of the Sensor Placement on Hot Wall
2.3 PIV set-up

Particle Image Velocimetry was employed to capture and measure the instantaneous velocity field in the flow. This was achieved by injecting olive oil particles $1 \mu m$ in diameter and illuminating them in a single plane for a short time interval (5-10 msec). A double cavity Nd:YAG laser with 190 mJ/pulse and wavelength 532 nm was used to excite the seeding in the enclosure. The dual Nd:YAG laser is the preferred light source for most PIV experiments. This integrated laser houses two separated laser heads whose pulses are combined to produce a coaxial pulse train using a built-in beam combination optics. The short pulse duration is fast enough to freeze even the motion of particles in flows that are supersonic without image streaking. The high energy pulse is able to illuminate small particles for air flows. For, two lasers are utilized any time between pulses is possible, with the full energy in each pulse.

A spherical lens was placed right after the aperture of the laser to control the thickness of the light sheet. In order to control the height of the light sheet two cylindrical lenses were utilize, as shown in Fig. 2.11. The laser sheet produced was not exceeding 0.001 m in thickness and it entered the cavity from the bottom side, through a small slit on the insulation after being reflected off a 45$^\circ$ angle mirror as shown in Fig. 2.2.
The TSI PowerView 11 MP was used to capture the images. The camera has 4008 × 2672 pixels in the digital image, 4.2 frame rate, i.e. frames/s, 12-bit dynamic output available with each pixel allowing for accurate imaging of both small and large particles in the flow. Finally, the camera was fitted with a 28 mm lens.
This section includes a description of the experimental procedure followed. This includes the velocity field measurements using PIV in the enclosure, the temperature measurements in the core and finally the heat flux measurements on the hot wall.

### 3.1 Velocity Field Measurements

The temperature on the bath and the PID controller was properly adjusted in order to achieve the desired Rayleigh number. Table 3.1 summarizes the temperatures measured on the active walls.

<table>
<thead>
<tr>
<th>Tb (K)</th>
<th>Tc (K)</th>
<th>Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td>303.3</td>
<td>293.0</td>
<td>5.08 × 10^7</td>
</tr>
<tr>
<td>308.4</td>
<td>293.0</td>
<td>7.34 × 10^7</td>
</tr>
<tr>
<td>313.1</td>
<td>293.0</td>
<td>9.42 × 10^7</td>
</tr>
<tr>
<td>329.3</td>
<td>293.0</td>
<td>1.50 × 10^8</td>
</tr>
<tr>
<td>340.2</td>
<td>293.0</td>
<td>1.81 × 10^8</td>
</tr>
<tr>
<td>348.5</td>
<td>293.0</td>
<td>2.00 × 10^8</td>
</tr>
<tr>
<td>361.4</td>
<td>293.0</td>
<td>2.25 × 10^8</td>
</tr>
<tr>
<td>395.3</td>
<td>278.0</td>
<td>3.40 × 10^8</td>
</tr>
</tbody>
</table>

Once the bath and the heaters were turned on the system was allowed
sufficient time to reach steady state operation. Steady state was defined when there is 0.1 K change in temperature on the active copper walls over an 1 hour interval. The time required to reach steady state depends on the temperature gradient desired to achieve. Estimated time is explained in Sec. 4.2. Steady state conditions were monitored using the DAQ system.

After ensuring steady state conditions, the olive oil particles were injected inside the cavity through a port located on the top. A Laskin nozzle [27] connected to compressed air supply was used to generate the $1 \mu$m in diameter particles. The particles were allowed sufficed time to disperse and get well mixed with air before capturing any PIV images. This process was in the order of minutes as indicated by the timescales in Table 4.7.

The PIV technique employed is based on the measurement of the differential displacement of fluid particles estimated from two consecutive scans of the slice which is illuminated by separated laser pulses. The time interval ($\Delta t$) between the two different laser pulses was set according to the following considerations. The time had to decrease while augmenting the Rayleigh number as the velocity flow field accelerated with increasing Raleigh number. A preliminary test was conducted in order to determine the time interval that exhibits the best average signal-to-noise ratio. Table 3.2 summarizes the time intervals employed in this work and the corresponding Rayleigh number.
### Table 3.2: $\Delta t$ between the two laser pulses

<table>
<thead>
<tr>
<th>$\Delta t$ (s)</th>
<th>$Ra$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>$5.08 \times 10^7$</td>
</tr>
<tr>
<td>0.01</td>
<td>$7.34 \times 10^7$</td>
</tr>
<tr>
<td>0.01</td>
<td>$9.42 \times 10^7$</td>
</tr>
<tr>
<td>0.005</td>
<td>$1.50 \times 10^8$</td>
</tr>
<tr>
<td>0.005</td>
<td>$1.81 \times 10^8$</td>
</tr>
<tr>
<td>0.005</td>
<td>$2.00 \times 10^8$</td>
</tr>
<tr>
<td>0.005</td>
<td>$2.25 \times 10^8$</td>
</tr>
<tr>
<td>0.005</td>
<td>$3.40 \times 10^8$</td>
</tr>
</tbody>
</table>

The images were captured by a CCD camera and stored on a PC for offline processing. Two thousand instantaneous fields were captured for each Rayleigh number. The acquisition frequency, which corresponds to the time between the two frames, was set at 3.75 Hz. These parameters along with the time between the two pulses were set in Insight 4G which also controls the synchronization between the laser emission and the CCD camera capture.
3.2 Temperature Measurements in the Core of the Enclosure

Temperature measurements in the core were conducted using the OMEGA temperature profile probes. They consisted of 5 smaller diameter K-type thermocouples placed inside a single outer sheath made of stainless steel. The accuracy of the K-type thermocouples was ± 0.2 K. The probes were 0.46 m long and 0.0016 m in diameter. The space between points was 0.05 m. This provided the possibility to profile the temperature at various points along the vertical axis.

![Figure 3.1: Cavity Top Wall View](image)

Two probes were placed in the downstream position of the flow, in an at-
tempt to minimize interference with the flow, inside the cavity entering from the top as illustrated in Fig. 3.1. Two different configurations were examined having probes at positions 1 & 2 and 1 & 3. Temperature was recorded using the DAQ system described in Sec. 2.2.
3.3 Heat Flux Measurements

Heat flux measurements were performed using the OMEGA thin-film heat flux sensors, model HFS-3. Each sensor functions as a self-generating thermopile transducer. They utilize a multi-junction thermopile construction and the carrier is a polyimide film which is bonded using a PFA lamination process. All sensors had an integrated K-type thermocouple for discrete temperature measurement essential to describe the heat flux.

Five sensors with nominal sensitivity $3.0 \mu V/W/m^2$ were placed in a cross formation on the hot plate as shown in Fig. 2.10.
This section summarizes various preliminary calculations and considerations. A thermal resistance analysis indicates that convection in the enclosure is orders of magnitude greater than conduction through the plate indicating that the thickness of the plate does not play any significant role. Furthermore, an estimate of the time required by the system to reach steady state is provided and the method of estimating the Nusslet number is also explained. Finally, considerations regarding seeding selection and its response time for PIV experiments are addressed.

4.1 Thermal Resistance

A thermal resistance analysis was conducted in order to investigate the dominant heat transfer mechanism. Fig. 4.1, illustrates the resistance network. Heat is being conducted through the plate and then natural convection develops inside the cavity.
Using the following correlation by Churchill and Chu [28] we can estimate the Nusslet number for the corresponding Rayleigh numbers examined in this study:

\[
\bar{Nu} = 0.68 + \frac{0.670Ra^{1/4}}{1 + (0.492/Pr^{9/16})^{4/9}} \quad Ra \leq 10^9
\] (4.1)

Consequentially, the convective heat transfer coefficient \( h_c \) can be computed from \( \bar{Nu} = \frac{hcH}{k} \) followed by the convective \( R_{conv} = \frac{1}{h_cA} \) and conductive \( R_{cond} = \frac{t}{k_{Cu}A} \) resistance. \( A \) is the area and \( w \) is the thickness of the plate.

Table 4.1 summarizes the estimated resistances for the Rayleigh numbers examined. It is apparent that convection is orders of magnitude greater than conduction.
Table 4.1: Convective and conductive resistances

<table>
<thead>
<tr>
<th>Ra</th>
<th>$R_{\text{conv}}$ (K/W)</th>
<th>$R_{\text{cond}}$ (K/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>2.305</td>
<td>0.000054</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>2.092</td>
<td>0.000054</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>1.953</td>
<td>0.000054</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>1.703</td>
<td>0.000054</td>
</tr>
<tr>
<td>$1.81 \times 10^8$</td>
<td>1.603</td>
<td>0.000054</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>1.548</td>
<td>0.000054</td>
</tr>
<tr>
<td>$2.25 \times 10^8$</td>
<td>1.478</td>
<td>0.000054</td>
</tr>
<tr>
<td>$3.40 \times 10^8$</td>
<td>1.304</td>
<td>0.000054</td>
</tr>
</tbody>
</table>
4.2 Estimated Time To Reach Steady State

The heat input provided from the heaters to the copper plate can be quantified by:

\[ P = mc_p(T_f - T_i)/t \]  \hspace{1cm} (4.2)

where \( m \), \( c_p \), \( T_i \) and \( T_f \) are the mass, specific heat, initial temperature and final (desired) temperature on the copper plate.

Assuming an initial temperature on the plate \( T_i = 288 \text{K} \) the essential time to achieve the desired temperature on the plate is shown in Table 4.2.

Table 4.2: Estimated time to reach desired Rayleigh number

<table>
<thead>
<tr>
<th>Ra</th>
<th>t (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.08 \times 10^7</td>
<td>46</td>
</tr>
<tr>
<td>7.34 \times 10^7</td>
<td>61</td>
</tr>
<tr>
<td>9.41 \times 10^7</td>
<td>77</td>
</tr>
<tr>
<td>1.50 \times 10^8</td>
<td>126</td>
</tr>
<tr>
<td>1.81 \times 10^8</td>
<td>157</td>
</tr>
<tr>
<td>2.00 \times 10^8</td>
<td>182</td>
</tr>
<tr>
<td>2.25 \times 10^8</td>
<td>220</td>
</tr>
<tr>
<td>3.40 \times 10^8</td>
<td>315</td>
</tr>
</tbody>
</table>

In addition, an estimate of time required for the flow in the cavity to reach steady state was achieved through the calculation of the diffusion time for each Rayleigh number.
Diffusion time is given by:

\[ t_{diff} = \frac{H^2}{\alpha} \]  

(4.3)

where \( H \) is the characteristic dimension of the enclosure and \( \alpha \) is the thermal diffusivity of air. Eq. 4.3 yields the following times tabulated in Table 4.3.

<table>
<thead>
<tr>
<th>Ra</th>
<th>( t_{diff} ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5.08 \times 10^7 )</td>
<td>107</td>
</tr>
<tr>
<td>( 7.34 \times 10^7 )</td>
<td>105</td>
</tr>
<tr>
<td>( 9.41 \times 10^7 )</td>
<td>103</td>
</tr>
<tr>
<td>( 1.50 \times 10^8 )</td>
<td>99</td>
</tr>
<tr>
<td>( 1.81 \times 10^8 )</td>
<td>95</td>
</tr>
<tr>
<td>( 2.00 \times 10^8 )</td>
<td>93</td>
</tr>
<tr>
<td>( 2.25 \times 10^8 )</td>
<td>90</td>
</tr>
<tr>
<td>( 3.40 \times 10^8 )</td>
<td>85</td>
</tr>
</tbody>
</table>

The two timescales were combined and doubled to ensure that steady state conditions were achieved before any heat transfer or PIV measurements were conducted. Table 4.4 indicates the total time anticipated.
Table 4.4: Estimated time to reach steady state

<table>
<thead>
<tr>
<th>Ra</th>
<th>$t_{total}$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>5.0</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>5.5</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>6.0</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>7.5</td>
</tr>
<tr>
<td>$1.81 \times 10^8$</td>
<td>8.5</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>9.1</td>
</tr>
<tr>
<td>$2.25 \times 10^8$</td>
<td>10.3</td>
</tr>
<tr>
<td>$3.40 \times 10^8$</td>
<td>13.3</td>
</tr>
</tbody>
</table>
4.3 Heat Flux

It is well known that the heat flux at the boundary of a solid in contact with an adjacent fluid, is given by Fourier’s law applied at the solid-fluid interface [29]:

\[ q''_s = -k_f \frac{\partial T}{\partial y} \bigg|_{y=0} \]  

(4.4)

where \( q''_s \) is the surface heat flux (W/m\(^2\)), \( k_f \) is the fluid thermal conductivity (W/m-K) and \( \left( \frac{\partial T}{\partial y} \right|_{y=0} \) is the normal component of the temperature gradient (K/m). The temperature gradient is evaluated on the fluid side.

The heat transfer coefficient, \( h \), is defined as:

\[ h \equiv \frac{q''_s}{\Delta T} \]  

(4.5)

where \( \Delta T \) is the temperature difference across the cavity \( \Delta T = T_h - T_c \).

The heat transfer coefficient is the primary quantity characterizing the convection heat transfer process. It can vary with position on the surface and with time in any given flow situation. Furthermore, it strongly depends on thermal boundary conditions, the geometry of the object and velocity.

A spatially average heat transfer coefficient, \( \bar{h} \) can be defined as:

\[ \bar{h} = \frac{q}{A_s(T_h - T_c)} \]  

(4.6)

where \( q \) is the total heat transfer and \( A_s \) is the total heat transfer surface area.

Using Eq. 4.6 the average Nusslet number, \( \bar{Nu} \) is defined as:

\[ \bar{Nu} = \frac{\bar{h}H}{k_f} \]  

(4.7)
where $H$ is the characteristic dimension of the cavity. The averaged $\bar{Nu}$ is a temporally and spatially averaged quantity.

Using Eq. 4.4 and Eq. 4.5 it can be shown that the local Nusslet number is a dimensionless temperature gradient at the wall:

$$Nu = \frac{hH}{k_f} = -\frac{H}{\Delta T} \frac{\partial T}{\partial y} \bigg|_{y=0} \quad (4.8)$$

Thus $\bar{Nu}$ can be interpreted as an average, dimensionless gradient at the surface-fluid interface.
4.4 PIV Considerations

4.4.1 Processing Scheme

Data collected for different Rayleigh numbers were processed in INSIGHT 4G [30] and Matlab [31]. Images were analyzed using the Fast Fourier Transform (FFT) correlation algorithm. A rectangular grid with 75% oversampling was selected. A Gaussian peak engine was also selected to detect the peak in the correlation map. This locates the correlation peak with sub-pixel accuracy by fitting a Gaussian curve to the highest pixel and its four nearest neighbors.

Furthermore, a Rohaly-Hart analysis [30] was performed to enhance the velocity estimation of the standard PIV processing. When a vector fails the signal-to-noise ratio (SNR) criterion, the algorithm adds the correlation map acquired from neighbor spots so that it obtains a good correlation peak in the summed collaboration map.

Moreover, a local median validation step was executed. In this method the velocity components of the reference vector are the median value of all the vectors in the neighborhood. The defined velocity tolerance, in pixels, determined the allowed difference between the current vector and its reference vector. In this case it was set at 2 pixels whereas the neighborhood area was fixed at $3 \times 3$ pixels.

In addition, vector conditioning was performed after the vector validation to fill any holes in the vector field due to low SNR or failure in the validation step. The Recursive filling method was used; in this method the holes are sorted by the number of valid neighbors found initially. It first fills the holes with the most valid neighbors. It then fills the holes with the second most valid neighbors and the process is continued until all the holes are processed.
Smoothing was also performed to the vectors which replaces every vector by its Gaussian-weighted mean of the neighbor vectors. The filter size was set at $3 \times 3$ and the standard deviation of the distribution was fixed at $\sigma = 0.8$.

Finally, a mask was applied to all images in order to exclude from the analysis areas outside the cavity. This step was necessary as these fields would produce null vectors. More information regarding vector processing, validation and smoothing can be found in [30].

4.4.2 Seeding Particles selection

Selecting the seed particles is the simplest but also one of most significant aspects of PIV. It is essential that the seed particles follow the flow field and ideally be neutrally buoyant for the given flow conditions. At the same time the particle must scatter the light sufficiently to allow detection by the CCD camera. A complete discussion and analysis on the general selection criteria of seed particles can be found in [32]. This subsection presents the considerations pertinent to this study.

In most applications, the seed particles are heavier than the fluid medium and will ultimately settle under gravity. When a particle falls in a surrounding fluid, the downward force of gravity balances the upward force induced by viscous drag. A constant velocity is achieved when these two forces are balanced, and this terminal velocity can be estimated using the following equation:

$$V_T = \frac{(\rho_p - \rho_g)d_p^2g}{18\mu}$$

(4.9)

where $\rho_p$ is the density of the particle, $\rho_g$ is the density of the surrounding air, $d_p$ is the diameter of the particle, $g$ is the gravitational acceleration and
\( \mu \) is the viscosity of the gas.

Considering the particles of 1 \( \mu \)m in diameter and density \( \rho_p \) of 900 Kg/m\(^3\), used in this study and the properties of the surrounding fluid, i.e. air, as listed in Table 4.5, the terminal velocity is estimated using Eq. 4.9 and listed in Table 4.5.

Table 4.5: Terminal velocity of the particles

<table>
<thead>
<tr>
<th>Ra</th>
<th>( \mu \times 10^{-6} ) (Kg/ms)</th>
<th>( \rho ) (Kg/m(^3))</th>
<th>( V \times 10^{-6} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.08 ( \times 10^7 )</td>
<td>18.4</td>
<td>1.18</td>
<td>26.6</td>
</tr>
<tr>
<td>7.34 ( \times 10^7 )</td>
<td>18.6</td>
<td>1.17</td>
<td>26.4</td>
</tr>
<tr>
<td>9.41 ( \times 10^7 )</td>
<td>18.7</td>
<td>1.17</td>
<td>26.2</td>
</tr>
<tr>
<td>1.50 ( \times 10^8 )</td>
<td>19.1</td>
<td>1.13</td>
<td>25.7</td>
</tr>
<tr>
<td>1.81 ( \times 10^8 )</td>
<td>19.3</td>
<td>1.12</td>
<td>25.4</td>
</tr>
<tr>
<td>2.00 ( \times 10^8 )</td>
<td>19.5</td>
<td>1.10</td>
<td>25.1</td>
</tr>
<tr>
<td>2.25 ( \times 10^8 )</td>
<td>19.8</td>
<td>1.08</td>
<td>24.7</td>
</tr>
<tr>
<td>3.40 ( \times 10^8 )</td>
<td>20.3</td>
<td>1.05</td>
<td>24.2</td>
</tr>
</tbody>
</table>

Another measure of a particle’s ability to follow the flow is the particle response time \( \tau \), which is defined by the following equation:

\[
\tau = \frac{d_p^2 \rho_p}{18 \nu \rho_g} 
\]

(4.10)

For the aforementioned particle properties and the properties of air as listed in Table 4.6, Eq. 4.10 yields the following response times.
Table 4.6: Response time of the particles

<table>
<thead>
<tr>
<th>Ra</th>
<th>$\nu \times 10^{-6}$ (m$^2$/s)</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$\tau \times 10^{-6}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>15.6</td>
<td>1.18</td>
<td>2.7</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>15.8</td>
<td>1.17</td>
<td>2.7</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>16.0</td>
<td>1.17</td>
<td>2.7</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>16.8</td>
<td>1.13</td>
<td>2.6</td>
</tr>
<tr>
<td>$1.81 \times 10^8$</td>
<td>17.3</td>
<td>1.12</td>
<td>2.6</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>17.7</td>
<td>1.10</td>
<td>2.6</td>
</tr>
<tr>
<td>$2.25 \times 10^8$</td>
<td>18.4</td>
<td>1.08</td>
<td>2.5</td>
</tr>
<tr>
<td>$3.40 \times 10^8$</td>
<td>19.3</td>
<td>1.05</td>
<td>2.5</td>
</tr>
</tbody>
</table>

One characteristic time scale of the flow under consideration is determined by the time scale of the core recirculation estimated by [33]:

$$t = \frac{H^2}{\nu} Ra^{-0.25}$$ (4.11)

where $H$ is the characteristic length-scale of the cavity and $\nu$ is the kinematic viscosity of air. For the different cases examined in this work Eq. 4.11 yields the following results summarized in Table 4.7.
Table 4.7: Timescale of the core

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>$\nu \times 10^{-6} \text{ (m}^2/\text{s})$</th>
<th>$t$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>15.6</td>
<td>107</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>15.8</td>
<td>96</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>16.0</td>
<td>89</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>16.8</td>
<td>76</td>
</tr>
<tr>
<td>$1.81 \times 10^8$</td>
<td>17.3</td>
<td>70</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>17.7</td>
<td>67</td>
</tr>
<tr>
<td>$2.25 \times 10^8$</td>
<td>18.4</td>
<td>63</td>
</tr>
<tr>
<td>$3.40 \times 10^8$</td>
<td>19.3</td>
<td>54</td>
</tr>
</tbody>
</table>

It is apparent that the time scale of the flow under examination is in the order of minutes. This establishes the particles as excellent candidates for following the flow.
In this section the results obtained from PIV analysis are presented. These include the streamlines of the flow in the enclosure, the velocity magnitude and the two velocity components in the x and y direction, the Reynolds stresses, the swirling strength and flow vorticity. Moreover the dimensionality of the flow is examined and temperature profiles of the core are also presented. Finally, a correlation between Nu and Rayleigh number is concluded from the heat transfer measurements.

5.1 PIV Measurements

5.1.1 Streamlines

In Figs. 5.1-5.8 the streamlines of the various Rayleigh numbers examined are depicted. As Rayleigh number increases different regions can be distinguished in the enclosure. The flow demonstrates different scalings in the two vertical boundary layers along the isothermal walls, the horizontal boundary layers along the adiabatic walls and the core region. A latter analysis (see Sec. 5.3) indicated a stratified core with a linear profile.
Figure 5.1: Streamlines for $Ra = 5.08 \times 10^7$

Figure 5.2: Streamlines for $Ra = 7.34 \times 10^7$
Figure 5.3: Streamlines for $Ra = 9.42 \times 10^7$

Figure 5.4: Streamlines for $Ra = 1.50 \times 10^8$
Figure 5.5: Streamlines for $Ra = 1.81 \times 10^8$

Figure 5.6: Streamlines for $Ra = 2.00 \times 10^8$
Figure 5.7: Streamlines for $Ra = 2.25 \times 10^8$

Figure 5.8: Streamlines for $Ra = 3.40 \times 10^8$
5.1.2 Velocity in the Horizontal Direction

Figures 5.9 - 5.16 show the ensemble averaged $\langle U \rangle$ velocity contours of the flow in the x-direction as a function of the Rayleigh number. It is apparent the strong dependence of the magnitude and the flow structures on the Rayleigh number.

We can distinguish the two boundaries along the top and the bottom adiabatic walls of the cavity. The boundary layer on the top wall has more of a jet-like form, whereas the boundary layer on the bottom is more of a plume shaped. Based on the geometry established (see Fig. 2.2) the flow on the bottom is moving in the positive direction whereas the flow on the top is considered to be negative.

The evolution of the weak re-circulation just below the top boundary layer is also depicted. As Rayleigh number increases it shrinks, accelerates and starts to expand again. This re-circulation suppresses the development of the top boundary which is smaller in size compared to the boundary layer on the bottom. This re-circulation is a topological phenomenon resulting from the horizontal top boundary layer after being redirected in the vertical direction by the cold isothermal wall. Consequently, a portion of the flow is reflected in the opposite direction than the direction of the initial boundary layer.

The anti-symmetric re-circulation develops in lower right corner of the enclosure, atop of the horizontal boundary layer for Rayleigh numbers $Ra \geq 1.50 \times 10^8$. It accelerates and becomes faster as Rayleigh number augments. This re-circulation is slower than the one developed in the upper left corner for the Rayleigh regime examined. As a result of this re-circulation the development of the horizontal bottom boundary layer is suppressed and the
two boundary layers on the top and the bottom of the enclosure tend to become equivalent in size.

The top horizontal boundary layer is faster compared to the one on the bottom due to energy addition while the flow ascends over the hot wall. Thus this boundary layer carries significant amount of momentum which results in a more violent impact when forced to change direction by the cold isothermal wall. Subsequently the re-circulation appears first in the upper left corner. With the increase of Rayleigh number, the bottom horizontal boundary layer builds enough momentum so that when redirected by the hot wall one part of the flow reflects in the opposite direction than the initial direction of the bottom boundary layer.
Figure 5.9: $\langle U \rangle$ Velocity for $Ra = 5.08 \times 10^7$ 

Figure 5.10: $\langle U \rangle$ Velocity for $Ra = 7.34 \times 10^7$
Figure 5.11: $\langle U \rangle$ Velocity for $Ra = 9.42 \times 10^7$

Figure 5.12: $\langle U \rangle$ Velocity for $Ra = 1.50 \times 10^8$
Figure 5.13: $\langle U \rangle$ Velocity for $Ra = 1.81 \times 10^8$

Figure 5.14: $\langle U \rangle$ Velocity for $Ra = 2.00 \times 10^8$
Figure 5.15: $\langle U \rangle$ Velocity for $Ra = 2.25 \times 10^8$

Figure 5.16: $\langle U \rangle$ Velocity for $Ra = 3.40 \times 10^8$
5.1.3 Velocity in the Vertical Direction

In Figs. 5.17 - 5.24 the ensemble averaged $\langle V \rangle$ profiles are depicted and the evolution of the two boundary layers along the vertical walls can be examined. The flow descending on the cold wall is considered negative and the ascending flow on the hot wall is positive in compliance with the system geometry.

We notice that the boundary layer on the hot wall is faster because of the heat addition and thus thinner for all the cases examined. For the low end of the Rayleigh number regime we examined, a slow moving region upstream of the boundary layer along the hot wall can be observed. This region extends in the x-direction and declines with the increase of Rayleigh number. This is a result, of sufficient amount of energy being added to the enclosure for high Rayleigh numbers to establish a strong, global re-circulation making the distinction of several regions within the enclosure clear.

For Rayleigh numbers $Ra \geq 1.5 \times 10^8$ one can observe the development of the y-component of the weak circulation in the upper left region discussed in Sec. 5.1.2. This re-circulation grows stronger with the increase of Rayleigh number and drains momentum out of the cold vertical boundary causing it to decelerate and diminish in size.
Figure 5.17: $\langle V \rangle$ Velocity for $Ra = 5.08 \times 10^7$

Figure 5.18: $\langle V \rangle$ Velocity for $Ra = 7.34 \times 10^7$
Figure 5.19: $\langle V \rangle$ Velocity for $Ra = 9.42 \times 10^7$

Figure 5.20: $\langle V \rangle$ Velocity for $Ra = 1.50 \times 10^8$
Figure 5.21: $\langle V \rangle$ Velocity for $Ra = 1.81 \times 10^8$

Figure 5.22: $\langle V \rangle$ Velocity for $Ra = 2.00 \times 10^8$
Figure 5.23: $\langle V \rangle$ Velocity for $Ra = 2.25 \times 10^8$

Figure 5.24: $\langle V \rangle$ Velocity for $Ra = 3.40 \times 10^8$
5.1.4 Mean Velocity Curves in the Horizontal & Vertical Direction

In Fig. 5.25 the ensemble averaged $\langle U \rangle$ velocity curves in the horizontal direction are illustrated. Three different regions can be distinguished in the plot. These are the flow at the top and the bottom of the enclosure in the top and bottom boundary layers and the slowly moving core. As discussed in Sec. 5.1.2, the flow is faster at the top of the enclosure compared to the flow in the bottom boundary layer. This is due to the energy addition to the flow as it ascends on the vertical isothermal hot wall. A lethargic region that corresponds to the core in the enclosure can be noticed stretching approximately between $0.075 \text{ m} \leq y \leq 0.20 \text{ m}$.

The ensemble averaged $\langle V \rangle$ curves in the horizontal direction are shown in Fig. 5.26. We can notice the boundary layers developed on the hot and cold isothermal walls and a passive, slowly moving region that extends from $x = 0.05 \text{ m}$ to $x = 0.20 \text{ m}$. The boundary layers along the isothermal walls start from the bottom of the hot wall and the top of the cold wall and decrease in size as the velocity magnitude increases. The formation process of these boundary layers is different from the boundary layers formed on a plate. For free convection over an isothermal plate the fluid in the boundary is taken from the ambient whereas for forced convection over a plate the speed of the boundary is reduced by the drag from the plate and fluid is pushed out at the same time. The boundary layer forms at the beginning of the plate. In the case of natural convection in enclosures, the vertical boundary derives from the horizontal boundary layer with some contribution from the ambient fluid as is being transfer to the horizontal boundary layer by the reversal motion discussed in Sec. 5.1.2
Figure 5.25: $\langle U \rangle$ Velocity Curves in the Horizontal Direction

Figure 5.26: $\langle V \rangle$ Velocity Curves in the Vertical Direction
5.1.5 Velocity Magnitude

Combining the results from Sec. 5.1.2 and Sec. 5.1.3 the velocity magnitude of the flow in the cavity was estimated and plotted in Figs. 5.27 - 5.34. The plots provide a complete picture of the velocity field in the enclosure and reveal an apparent dependence of the flow patterns on Rayleigh number.

Once again several regions can be distinguished in the cavity: Two horizontal boundary layers on the top and bottom adiabatic walls, two vertical boundary layers on the active isothermal walls, the stratified core and two re-circulations in the upper left and lower right corner regions.
Figure 5.27: Velocity Magnitude for Ra = 5.08 × 10^7

Figure 5.28: Velocity Magnitude for Ra = 7.34 × 10^7
Figure 5.29: Velocity Magnitude for \( Ra = 9.42 \times 10^7 \)

Figure 5.30: Velocity Magnitude for \( Ra = 1.50 \times 10^8 \)
Figure 5.31: Velocity Magnitude for $Ra = 1.81 \times 10^8$
Figure 5.33: Velocity Magnitude for $Ra = 2.25 \times 10^8$

Figure 5.34: Velocity Magnitude for $Ra = 3.40 \times 10^8$
5.1.6 Reynolds Stresses

It is known that the mean momentum or Reynolds equations are given by [34]:

$$\rho \frac{D \langle U_j \rangle}{Dt} = \left[ \mu \left( \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle U_j \rangle}{\partial x_i} \right) - \langle p \rangle \delta_{ij} - \rho \langle u_i u_j \rangle \right]$$

(5.1)

where $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \langle \mathbf{U} \rangle \cdot \nabla$ is the mean substantial derivative.

Eq. 5.1 is the momentum conservation equation in a generalized form. The terms in the square brackets represent three different stresses: the viscous stress, the isotropic stress $-\langle p \rangle \delta_{ij}$ resulting from the mean pressure field and stress arising from the fluctuating velocity field, $-\rho \langle u_i u_j \rangle$. The density term is dropped for convenience and conventionally we refer to $\langle u_i u_j \rangle$ as the Reynolds stress. Reynolds stress stems from momentum transfer by the fluctuating velocity field in the same sense as the viscous stress stems for momentum at the molecular level.

The Reynolds stresses are the components of a symmetric, second-order tensor. The diagonal components such as $\langle u^2 \rangle$ and $\langle v^2 \rangle$ are normal stresses while the components that lay off the diagonal, i.e. $\langle uv \rangle$ are shear stresses. Normal stresses in the x and y direction are displayed in Figs. 5.35-5.42 and Figs. 5.43-5.50 respectively, while shear stress is shown in Figs. 5.59-5.66.

$\langle u^2 \rangle$ Normal Stress

Figures 5.35 - 5.42 show the development of $\langle u^2 \rangle$ normal stress in the enclosure. The values obtained are within 30% of turbulence intensity. It is apparent that the magnitude depends on the Rayleigh number. One can observe that for the low end of the Rayleigh region examined the amount of $\langle u^2 \rangle$ produced is insignificant.

For Rayleigh numbers $Ra \geq 1.50 \times 10^8$ a region of increasing $\langle u^2 \rangle$ stress
can be observed on the top and bottom adiabatic walls. There is a distinct
difference between the top and the bottom wall with the magnitude on the top
wall being stronger. This is due to greater amounts of momentum transferred
by the fluctuating velocity field of the top horizontal boundary layer which
built up on kinetic energy after ascending over the hot wall.
Figure 5.35: $\langle u^2 \rangle$ Reynolds stress for $Ra = 5.08 \times 10^7$

Figure 5.36: $\langle u^2 \rangle$ Reynolds stress for $Ra = 7.34 \times 10^7$
Figure 5.37: $\langle u^2 \rangle$ Reynolds stress for $Ra = 9.42 \times 10^7$

Figure 5.38: $\langle u^2 \rangle$ Reynolds stress for $Ra = 1.50 \times 10^8$
Figure 5.39: $\langle u^2 \rangle$ Reynolds stress for $Ra = 1.81 \times 10^8$

Figure 5.40: $\langle u^2 \rangle$ Reynolds stress for $Ra = 2.00 \times 10^8$
Figure 5.41: $\langle u^2 \rangle$ Reynolds stress for $Ra = 2.25 \times 10^8$

Figure 5.42: $\langle u^2 \rangle$ Reynolds stress for $Ra = 3.40 \times 10^8$
Figures 5.35 - 5.42 show the development of $\langle v^2 \rangle$ normal stress in the enclosure. The values obtained are within 30% of turbulence intensity. For Rayleigh numbers $Ra \geq 1.5 \times 10^8$ significant amount of stress develops along the two isothermal active walls. An interesting feature is that stress is stronger on the cold wall for $Ra < 2.25 \times 10^8$. This is due to momentum deposition on the cold wall by the top horizontal boundary layer when redirected vertically by the cold active wall.

At Rayleigh number $Ra = 2.25 \times 10^8$ an inversion of trends can be noticed and the $\langle v^2 \rangle$ normal stress on the hot wall is stronger in magnitude. This is due to the small re-circulation developed upstream of the cold vertical boundary layer that drains momentum out of the boundary layers causing it to diminish in size.
Figure 5.43: $\langle v^2 \rangle$ Reynolds stress for $Ra = 5.08 \times 10^7$

Figure 5.44: $\langle v^2 \rangle$ Reynolds stress for $Ra = 7.34 \times 10^7$
Figure 5.45: $\langle v^2 \rangle$ Reynolds stress for $Ra = 9.42 \times 10^7$

Figure 5.46: $\langle v^2 \rangle$ Reynolds stress for $Ra = 1.50 \times 10^8$
Figure 5.47: $\langle v^2 \rangle$ Reynolds stress for $Ra = 1.81 \times 10^8$

Figure 5.48: $\langle v^2 \rangle$ Reynolds stress for $Ra = 2.00 \times 10^8$
Figure 5.49: \( \langle v^2 \rangle \) Reynolds stress for \( Ra = 2.25 \times 10^8 \)

Figure 5.50: \( \langle v^2 \rangle \) Reynolds stress for \( Ra = 3.40 \times 10^8 \)
Turbulent Kinetic Energy (TKE)

The turbulent kinetic energy is defined to be half of the Reynolds stress tensor:

\[
k \equiv \frac{1}{2} \langle \mathbf{u} \cdot \mathbf{u} \rangle = \frac{1}{2} \langle u_i u_i \rangle
\]  

(5.2)

It is the mean kinetic energy per unit mass in the fluctuating velocity field. Combining the two previous results the turbulent kinetic energy was estimated and depicted in Figs. 5.51 - 5.58.

It can be observed that TKE starts to develop first on the vertical boundary layers on the active walls and is stronger in magnitude on the cold wall mainly because of the contribution of \( \langle v^2 \rangle \) normal stress. It is apparent that for the last two cases examined this does not hold true and TKE is stronger on the hot wall. This is a result of the re-circulation developed in the upper left region of the enclosure which drains momentum out of the boundary layer causing it to diminish.

Comparing the top and bottom walls is obvious that the magnitude of turbulent kinetic energy is stronger on the top adiabatic wall since the contribution of \( \langle u^2 \rangle \) stress in that region is stronger as discussed in Sec. 5.1.6.
Figure 5.51: TKE for $Ra = 5.08 \times 10^7$

Figure 5.52: TKE for $Ra = 7.34 \times 10^7$
Figure 5.53: TKE for Ra = $9.42 \times 10^7$

Figure 5.54: TKE for Ra = $1.50 \times 10^8$
Figure 5.55: TKE for $Ra = 1.81 \times 10^8$

Figure 5.56: TKE for $Ra = 2.00 \times 10^8$
Figure 5.57: TKE for $Ra = 2.25 \times 10^8$

Figure 5.58: TKE for $Ra = 3.40 \times 10^8$
\langle uv \rangle Shear Stress

In Figs. 5.59 - 5.66 the development of shear stress in the enclosure is illustrated. There is a distinct difference between the low end of the Rayleigh regime examined, i.e. $Ra \leq 9.42 \times 10^7$ and the rest of the cases tested.

For Rayleigh numbers $Ra \leq 9.42 \times 10^7$ we notice that initially shear starts to develop in the lower left region of the enclosure. Negative shear is developed close to the bottom adiabatic wall and positive shear in a part of the re-circulation atop of the bottom horizontal boundary layer.

For Rayleigh numbers $Ra \geq 1.5 \times 10^8$ is clear that negative shear develops along the walls of the enclosure due to injections and sweeps and positive shear in the re-circulation that slips over the horizontal boundary layers due to inward and outward interactions. The magnitude of the both negative and positive shear becomes stronger with the increase of Rayleigh number.
Figure 5.59: $\langle uv \rangle$ Reynolds stress for $Ra = 5.08 \times 10^7$

Figure 5.60: $\langle uv \rangle$ Reynolds stress for $Ra = 7.34 \times 10^7$
Figure 5.61: $\langle uv \rangle$ Reynolds stress for $Ra = 9.42 \times 10^7$

Figure 5.62: $\langle uv \rangle$ Reynolds stress for $Ra = 1.50 \times 10^8$
Figure 5.63: $\langle uv \rangle$ Reynolds stress for $Ra = 1.81 \times 10^8$

Figure 5.64: $\langle uv \rangle$ Reynolds stress for $Ra = 2.00 \times 10^8$
Figure 5.65: $\langle uv \rangle$ Reynolds stress for $Ra = 2.25 \times 10^8$

Figure 5.66: $\langle uv \rangle$ Reynolds stress for $Ra = 3.40 \times 10^8$
5.1.7 Vorticity

Vorticity $\omega(x, t)$ is defined as the curl of velocity:

$$\omega = \nabla \times U \tag{5.3}$$

and it equals twice the angular velocity or rate of rotation of the fluid at $(x, t)$. Vorticity was estimated from the PIV velocity data and is presented in Figs. 5.67 - 5.74.

Results indicate a counter-clockwise vortex activity along the two isothermal walls when the flow ascends on the hot wall and descends on the cold wall. Comparing the two we notice that the activity is stronger on the hot wall due to higher velocities and shear with the slow moving core. For Rayleigh numbers $Ra \geq 7.34 \times 10^7$ counter-clockwise vortexes start to develop on the top adiabatic wall in the region where the boundary layer on the hot wall is redirected by the adiabatic wall. The phenomenon becomes stronger as Rayleigh number elevates and we notice that for $Ra \geq 2.25 \times 10^8$ this area merges into the region along the hot isothermal wall.

Another interesting feature is the clock-wise vortexes mainly along the top adiabatic and cold walls although some traces of clock-wise vortexes can also be seen on the bottom wall and in the downstream region on the hot wall. These vortexes are a result of shear between the flow and the walls of the enclosure.
Figure 5.67: $\Omega_z$ for $Ra = 5.08 \times 10^7$

Figure 5.68: $\Omega_z$ for $Ra = 7.34 \times 10^7$
Figure 5.69: $\Omega_z$ for $Ra = 9.42 \times 10^7$

Figure 5.70: $\Omega_z$ for $Ra = 1.50 \times 10^8$
Figure 5.71: $\Omega_z$ for $Ra = 1.81 \times 10^8$

Figure 5.72: $\Omega_z$ for $Ra = 2.00 \times 10^8$
Figure 5.73: $\Omega_z$ for $Ra = 2.25 \times 10^8$

Figure 5.74: $\Omega_z$ for $Ra = 3.40 \times 10^8$
5.1.8 Vortex Identification

Accurate assessment of the population trends of prograde and retrograde vortices requires the identification and extraction of such structures from the background turbulence. There are a number of ways this can be achieved. One method of identifying vortexes advecting at similar speeds is the Galilean decomposition via removal of a fixed advection velocity from an instantaneous velocity. A major drawback of this technique is that one must remove a broad range of advection velocities in order to reveal all the structures embedded in the flow since the advection velocity of a vortex in a wall-bounded flow can depend on its relative position to the wall [35]. Alternatively, the analysis of the local velocity-gradient tensor as suggest by [36, 37] can be employed. Swirling strength can also be used for vortex identification. It is Galilean invariant and does not identify irrotational regions of intense shear. However although it is adept at identifying vortices embedded in turbulent velocity fields it does not yield the sense of rotation. In this work the swirling-strength parameter of the form below as proposed by [35] was adopted:

\[ \Lambda_{ci} \equiv \lambda_{ci}(x,y) \frac{\omega_z(x,y)}{|\omega_z(x,y)|} \]  

(5.4)

where \( \omega_z \) is the instantaneous fluctuating vorticity. In three dimensions, the local velocity gradient tensor will have one real eigenvalue (\( \lambda_r \)) and a pair of complex conjugate eigenvalues (\( \lambda_{cr} \pm i\lambda_{ci} \)) when the discriminant of its characteristic equation is positive [38]. When this is true, the particle trajectories about the eigenvector corresponding to \( \lambda_r \) exhibit a swirling, spatial motion [37]. The period required for a particle to swirl once about the \( \lambda_r \) axis is represented by \( \lambda_{ci}^{-1} \).

If the flow is pure shear flow, the particle orbits are infinitely-long ellipses
and the orbit period is also infinite, corresponding to $\lambda_{ci} = 0$. Thus, $\lambda_{ci} > 0$ corresponds to shorter, more circular ellipses, i.e. eddies [38]. Vortices in the counter-clockwise direction are considered to be positive and in the clockwise direction negative. Zhou et al. [36] showed that the strength of any local swirling motion is quantified by $\lambda_{ci}$. They defined it as the swirling strength of the vortex.

In Figs. 5.75 - 5.82 the swirling-strength of the flow examined in this work is depicted. We notice that for the low end of the Rayleigh number regime examined, i.e. $Ra \leq 9.42 \times 10^7$, there is prograde vortex activity in two regions in the enclosure in the upper right and lower left corner where the vertical boundary layers are redirected by the adiabatic walls. This is a topological phenomenon and is also due the fact that the velocity gradients are greater in that area. The flow on the hot wall builds up on kinetic energy and accelerates whereas on the cold wall the flow experiences gravitational acceleration. This effect depends on Rayleigh number and it becomes stronger in magnitude as it augments.

For Rayleigh numbers $Ra \geq 1.81 \times 10^8$ prograde vortexes appear for the first time in the region where the re-circulation after the top horizontal boundary layer is present. For this Rayleigh number regime, the top boundary layer becomes strong enough to initiate this vortex activity after being reflected by the cold wall and create the re-circulation discussed in Sec. 5.1.2.
Figure 5.75: $\Lambda_{ci}$ for $Ra = 5.08 \times 10^7$

Figure 5.76: $\Lambda_{ci}$ for $Ra = 7.34 \times 10^7$
Figure 5.77: $\Lambda_{ci}$ for $Ra = 9.42 \times 10^7$

Figure 5.78: $\Lambda_{ci}$ for $Ra = 1.50 \times 10^8$
Figure 5.79: $\Lambda_{ci}$ for $Ra = 1.81 \times 10^8$

Figure 5.80: $\Lambda_{ci}$ for $Ra = 2.00 \times 10^8$
Figure 5.81: \( \Lambda_{ci} \) for \( \text{Ra} = 2.25 \times 10^8 \)

Figure 5.82: \( \Lambda_{ci} \) for \( \text{Ra} = 3.40 \times 10^8 \)
5.1.9 Symmetry in the Flow

The symmetry of the flow about the center of the cavity where measurements were taken was investigated in order to conclude to deviation from two dimensions since planar two-dimensional PIV was used in this work.

Using the mass continuity equation, in the case of an incompressible flow, and solving for the gradient of the third component ($w$) of the velocity field we have:

$$\nabla \vec{V} = 0 \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial w}{\partial z} = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \quad (5.5)$$

The gradients for the velocity components in the x and y direction, i.e. $u$ and $v$ respectively were estimated using the data obtained from PIV measurements. Thus $\frac{\partial w}{\partial z}$ can easily be determined from Eq. 5.5.

Figs. 5.83 - 5.90 depict the gradient of $w$ in the center of the cavity. We observe that values for the derivative range between $-0.0005 \ & 0.0005$. The small values indicate that the third velocity component does not rapidly change in the plane of symmetry within the enclosure and not much information is lost from conducting 2D PIV.
Figure 5.83: Gradient of \( w \) velocity component for Ra = 5.08 \times 10^7

Figure 5.84: Gradient of \( w \) velocity component for Ra = 7.34 \times 10^7
Figure 5.85: Gradient of \((w)\) velocity component for \(Ra = 9.42 \times 10^7\)

Figure 5.86: Gradient of \((w)\) velocity component for \(Ra = 1.50 \times 10^8\)
Figure 5.87: Gradient of \((w)\) velocity component for \(Ra = 1.81 \times 10^8\)

Figure 5.88: Gradient of \((w)\) velocity component for \(Ra = 2.00 \times 10^8\)
Figure 5.89: Gradient of \((w)\) velocity component for \(Ra = 2.25 \times 10^8\)

Figure 5.90: Gradient of \((w)\) velocity component for \(Ra = 3.40 \times 10^8\)
5.2 Examination of the Corner Effect

As already discussed in Sec. 1.2 a lot of work in the literature was conducted on the study of the instability observed in the corner of the enclosure where the hot vertical boundary layer is redirected to a horizontal layer.

Initially the instability was interpreted as a hydraulic jump by Henkes & Hoogendoorn [13]. Ravi & Henkes [15] revealed the oversimplification of the problem in the previous study and they depicted that the structure in the corner of the enclosure is caused by thermal effects. Henkes & Le Quéré [16] and Janssen & Henkes [17] concluded that the instability is mainly shear driven suggesting that is a Kelvin-Helmholtz type instability.

As illustrated in Fig. 5.91 for the upper half of the enclosure and in Fig. 5.92 for the lower half no hydrodynamic instabilities are observed. This is for \( Ra = 3.40 \times 10^8 \) where the phenomena are the most intense. Our findings confirm a recent computational study by Ibrahim et al. [18] that the corner instabilities disappear when wall and gas radiation are considered as radiation redistributes the energy to other walls.

Radiation modifies the flow structures both at the top hot and the bottom cold corner. It accelerates the flow along the horizontal wall and it increases turbulence intensity. Especially in the areas where the horizontal layers impinge on the side walls.
Figure 5.91: Flow vector field in the upper half of the enclosure

(a) Cold wall

(b) Hot wall
Figure 5.92: Flow vector field in the lower half of the enclosure
5.3 Temperature Measurements in the Core

The temperature profiles at various regions inside the enclosure, measured as described in Sec. 3.2, are tabulated for all Rayleigh numbers examined. The averaged value over 30 000 data points is depicted along with the standard deviation ($\sigma$). A polynomial fit was performed to the data points and the resulting polynomial degree and $R^2$ of the fit are also shown. The data points and the fit are also graphically illustrated. In the plots the following dimensionless parameters are employed:

$$Y = \frac{y}{H}, \quad \theta = \frac{T - T_c}{T_h - T_c}$$

where $H$ is the characteristic dimension of the enclosure, $y$ is the height of the cavity and $T$ is the temperature measured.

It can be observed that the data lines collapse on top of each other for Rayleigh numbers $Ra \geq 1.5 \times 10^8$ indicating that the flow developed in the enclosure is symmetric about the center of the cavity. The discrepancies present between the linear profiles for $Ra \leq 1.5 \times 10^8$ indicate that the flow within the enclosure is still going through a transition which ends between $9.42 \times 10^7 \leq Ra \leq 1.5 \times 10^8$. 

5.3.1 Temperature Profile for $\text{Ra} = 5.08 \times 10^7$

Table 5.1: Probe 1

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<th>T (K)</th>
<th>$\sigma$ (K)</th>
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Table 5.4: Temperature profile data fit

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Figure 5.93: Temperature profile for Ra = 5.08 \times 10^7
5.3.2 Temperature Profile for Ra = 7.34 \times 10^7

Table 5.5: Probe 1

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Table 5.8: Temperature profile data fit

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<th>R²</th>
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Figure 5.94: Temperature Profile for $Ra = 7.34 \times 10^7$. 
5.3.3 Temperature Profile for $Ra = 9.42 \times 10^7$

Table 5.9: Probe 1

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<td>0.02</td>
</tr>
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<td>250</td>
<td>302.5</td>
<td>0.03</td>
</tr>
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Table 5.10: Probe 2

<table>
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<th>y (mm)</th>
<th>T (K)</th>
<th>$\sigma$ (K)</th>
</tr>
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<tr>
<td>50</td>
<td>299.3</td>
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</tr>
<tr>
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<td>0.03</td>
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<td>0.01</td>
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<td>200</td>
<td>301.6</td>
<td>0.03</td>
</tr>
<tr>
<td>250</td>
<td>302.3</td>
<td>0.02</td>
</tr>
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</table>

Table 5.11: Probe 3

<table>
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<th>$\sigma$ (K)</th>
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<td>0.06</td>
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<tr>
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<td>0.05</td>
</tr>
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</table>
Table 5.12: Temperature profile data fit

<table>
<thead>
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<th>Slope</th>
<th>Intercept</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>3.6035</td>
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<td>0.9995</td>
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<td>2</td>
<td>1</td>
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<td>-1.0164</td>
<td>0.9989</td>
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</table>

Figure 5.95: Temperature profile $Ra = 9.42 \times 10^7$
5.3.4 Temperature Profile for $Ra = 1.50 \times 10^8$

Table 5.13: Probe 1

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
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</tr>
<tr>
<td>100</td>
<td>305.6</td>
<td>0.03</td>
</tr>
<tr>
<td>150</td>
<td>307.0</td>
<td>0.02</td>
</tr>
<tr>
<td>200</td>
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<td>0.01</td>
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<td>309.7</td>
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</table>

Table 5.14: Probe 2

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<th>$y$ (mm)</th>
<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.05</td>
</tr>
<tr>
<td>100</td>
<td>305.5</td>
<td>0.02</td>
</tr>
<tr>
<td>150</td>
<td>306.8</td>
<td>0.04</td>
</tr>
<tr>
<td>200</td>
<td>308.2</td>
<td>0.04</td>
</tr>
<tr>
<td>250</td>
<td>309.6</td>
<td>0.04</td>
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</tbody>
</table>

Table 5.15: Probe 3

<table>
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<th>$y$ (mm)</th>
<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
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<td>0.06</td>
</tr>
<tr>
<td>100</td>
<td>305.4</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>306.7</td>
<td>0.06</td>
</tr>
<tr>
<td>200</td>
<td>308.2</td>
<td>0.05</td>
</tr>
<tr>
<td>250</td>
<td>309.7</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Table 5.16: Temperature profile data fit

<table>
<thead>
<tr>
<th>Probe</th>
<th>Polynomial Degree</th>
<th>Slope</th>
<th>Intercept</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3.3986</td>
<td>-0.8872</td>
<td>0.9995</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.4531</td>
<td>-0.8954</td>
<td>0.9997</td>
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<td>1</td>
<td>3.2708</td>
<td>-0.8203</td>
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Figure 5.96: Temperature profile for $Ra = 1.50 \times 10^8$
5.3.5 Temperature Profile for $Ra = 1.81 \times 10^8$

<table>
<thead>
<tr>
<th>Table 5.17: Probe 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (mm)</td>
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<tr>
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</tr>
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<tr>
<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.18: Probe 2</th>
</tr>
</thead>
<tbody>
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<td>$y$ (mm)</td>
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</tr>
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<tr>
<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.19: Probe 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (mm)</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
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<tr>
<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
</tbody>
</table>
Table 5.20: Temperature profile data fit

<table>
<thead>
<tr>
<th>Probe</th>
<th>Polynomial Degree</th>
<th>Slope</th>
<th>Intercept</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3.2863</td>
<td>-0.8331</td>
<td>0.9997</td>
</tr>
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<td>2</td>
<td>1</td>
<td>3.3504</td>
<td>-0.8475</td>
<td>0.9997</td>
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<tr>
<td>3</td>
<td>1</td>
<td>3.1484</td>
<td>-0.7683</td>
<td>0.9994</td>
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</table>

Figure 5.97: Temperature profile $Ra = 1.81 \times 10^8$
5.3.6 Temperature Profile for $Ra = 2.00 \times 10^8$

Table 5.21: Probe 1

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>T (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>309.2</td>
<td>0.05</td>
</tr>
<tr>
<td>100</td>
<td>311.4</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>313.7</td>
<td>0.03</td>
</tr>
<tr>
<td>200</td>
<td>315.9</td>
<td>0.05</td>
</tr>
<tr>
<td>250</td>
<td>318.1</td>
<td>0.04</td>
</tr>
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</table>

Table 5.22: Probe 2

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<tr>
<th>$y$ (mm)</th>
<th>T (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>309.0</td>
<td>0.05</td>
</tr>
<tr>
<td>100</td>
<td>311.2</td>
<td>0.02</td>
</tr>
<tr>
<td>150</td>
<td>313.2</td>
<td>0.05</td>
</tr>
<tr>
<td>200</td>
<td>315.5</td>
<td>0.05</td>
</tr>
<tr>
<td>250</td>
<td>317.8</td>
<td>0.05</td>
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</tbody>
</table>

Table 5.23: Probe 3

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>T (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>308.9</td>
<td>0.03</td>
</tr>
<tr>
<td>100</td>
<td>311.4</td>
<td>0.04</td>
</tr>
<tr>
<td>150</td>
<td>313.5</td>
<td>0.04</td>
</tr>
<tr>
<td>200</td>
<td>315.9</td>
<td>0.03</td>
</tr>
<tr>
<td>250</td>
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<td>0.04</td>
</tr>
</tbody>
</table>
Table 5.24: Temperature profile data fit

<table>
<thead>
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<th>Probe</th>
<th>Polynomial Degree</th>
<th>Slope</th>
<th>Intercept</th>
<th>R²</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>1</td>
<td>3.1278</td>
<td>-0.7384</td>
<td>0.9996</td>
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</table>

Figure 5.98: Temperature profile for Ra = 2.00 × 10⁸
5.3.7 Temperature Profile for Ra = 2.25 \times 10^8

Table 5.25: Probe 1

<table>
<thead>
<tr>
<th>y (mm)</th>
<th>T (K)</th>
<th>σ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>312.2</td>
<td>0.02</td>
</tr>
<tr>
<td>100</td>
<td>315.0</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>317.7</td>
<td>0.03</td>
</tr>
<tr>
<td>200</td>
<td>320.6</td>
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Table 5.26: Probe 2

<table>
<thead>
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<th>y (mm)</th>
<th>T (K)</th>
<th>σ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>312.0</td>
<td>0.05</td>
</tr>
<tr>
<td>100</td>
<td>314.6</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>317.2</td>
<td>0.005</td>
</tr>
<tr>
<td>200</td>
<td>320.1</td>
<td>0.05</td>
</tr>
<tr>
<td>250</td>
<td>322.9</td>
<td>0.02</td>
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</tbody>
</table>

Table 5.27: Probe 3

<table>
<thead>
<tr>
<th>y (mm)</th>
<th>T (K)</th>
<th>σ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>311.9</td>
<td>0.03</td>
</tr>
<tr>
<td>100</td>
<td>314.8</td>
<td>0.04</td>
</tr>
<tr>
<td>150</td>
<td>317.5</td>
<td>0.04</td>
</tr>
<tr>
<td>200</td>
<td>320.5</td>
<td>0.04</td>
</tr>
<tr>
<td>250</td>
<td>323.5</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table 5.28: Temperature profile data fit

<table>
<thead>
<tr>
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<th>Polynomial Degree</th>
<th>Slope</th>
<th>Intercept</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
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<td>3.2276</td>
<td>-0.7364</td>
<td>0.9999</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.3097</td>
<td>-0.7447</td>
<td>0.9995</td>
</tr>
<tr>
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<td>1</td>
<td>3.1376</td>
<td>-0.6979</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

Figure 5.99: Temperature profile for Ra = 2.25 \times 10^8
5.3.8 Temperature Profile for $Ra = 3.40 \times 10^8$

Table 5.29: Probe 1

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>310.5</td>
<td>0.05</td>
</tr>
<tr>
<td>100</td>
<td>315.3</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>320.6</td>
<td>0.05</td>
</tr>
<tr>
<td>200</td>
<td>326.4</td>
<td>0.06</td>
</tr>
<tr>
<td>250</td>
<td>332.2</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.30: Probe 2

<table>
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<tr>
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<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>310.0</td>
<td>0.04</td>
</tr>
<tr>
<td>100</td>
<td>314.7</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>319.7</td>
<td>0.03</td>
</tr>
<tr>
<td>200</td>
<td>325.5</td>
<td>0.04</td>
</tr>
<tr>
<td>250</td>
<td>331.4</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.31: Probe 3

<table>
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<tr>
<th>$y$ (mm)</th>
<th>$T$ (K)</th>
<th>$\sigma$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>310.1</td>
<td>0.03</td>
</tr>
<tr>
<td>100</td>
<td>315.2</td>
<td>0.04</td>
</tr>
<tr>
<td>150</td>
<td>320.2</td>
<td>0.03</td>
</tr>
<tr>
<td>200</td>
<td>326.2</td>
<td>0.05</td>
</tr>
<tr>
<td>250</td>
<td>332.1</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table 5.32: Temperature profile data fit

<table>
<thead>
<tr>
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<th>Polynomial Degree</th>
<th>Slope</th>
<th>Intercept</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2.8703</td>
<td>-0.6445</td>
<td>0.9986</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2.9080</td>
<td>-0.6405</td>
<td>0.9972</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2.8366</td>
<td>-0.6262</td>
<td>0.9982</td>
</tr>
</tbody>
</table>

Figure 5.100: Temperature profile for Ra = 3.40 × 10⁸
5.4 Heat Flux Measurements

In Fig. 5.101 the estimated Nusslet number as a function of the corresponding Rayleigh number is illustrated.

![Graph showing Nu as a function of Ra](image)

**Figure 5.101:** Nu as a function of Ra over the range examined in the present work

A power fit of the form \( \text{Nu} = a \text{Ra}^b \) was applied to the data values. Results were compared against other correlations available in literature as illustrated in Table 5.33.
Table 5.33: Present power fit as compared to literature

<table>
<thead>
<tr>
<th>Reference</th>
<th>a</th>
<th>b</th>
<th>Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>0.1983</td>
<td>0.2772</td>
<td>$5.08 \times 10^7 - 3.40 \times 10^8$</td>
</tr>
<tr>
<td>[9]</td>
<td>0.082</td>
<td>0.299</td>
<td>$10^6 - 10^{12}$</td>
</tr>
<tr>
<td>[39]</td>
<td>0.364</td>
<td>0.25</td>
<td>–</td>
</tr>
<tr>
<td>[40]</td>
<td>0.163</td>
<td>0.282</td>
<td>$10^3 - 10^6$</td>
</tr>
</tbody>
</table>

Also, Caton [41] proposed the following:

$$Nu = 0.18 \left( \frac{Pr}{0.2 + PrRa} \right)^{0.29},$$ \hspace{1cm} (5.7)

valid for $1 \lesssim H/L \lesssim 2$, $10^{-3} \lesssim Pr \lesssim 10^5$ and $10^3 \lesssim RaPr/(0.2 + Pr) \lesssim 10^5$.

Markatos & Perikleous [9] conducted a 2D computational analysis without invoking the Boussinesq approximation and assuming that $\rho$ is proportional to $1/T$, the densities were obtained from the temperature fields. Bejan [39] and Caton [41] also performed a 2D dimensional analysis invoking the Boussinesq approximation. Fusegi et al. [40] carried out a 3D, Boussinesq, numerical analysis. In the present work, the properties in estimating the Ra number were calculated at the bulk temperature, $T_b = (T_h + T_c)/2$ considering air as a real gas.

Discrepancies between our scaling relation and Fusegi et al. [40] are attributed to radiation that was not included in the aforementioned study. As illustrated in Appendix A radiation comprises 13-15% of the overall heat input to the enclosure. That percentage was added to the points estimated by the Fusegi et al. [40] correlation and “corrected” values are plotted in green in Fig. 5.101. It is apparent that for values in the low end of the Rayleigh number regime we examined there is a very good agreement between our ex-
experimental data and the values predicted by [40] when radiation is taken into account. Agreement is within 1%. As Rayleigh number increases we notice a deviation between experimental data and “corrected” values up to 3%. This is attributed to non-Boussinesq effects, i.e. Boussinesq approximation holds only for $Ra \leq 1.33 \times 10^8$ since $\Delta T \geq 28.6^\circ C$ [42] in our case.
6 UNCERTAINTY IN MEASUREMENTS

This section encompasses an error propagation analysis in order to determine the uncertainty in measurements and calculations. These include the uncertainty in defining the Rayleigh number examined in each case and the temperature in the core of the enclosure, the error in the heat flux measurements on hot plate and finally the uncertainty in PIV data.

6.1 Uncertainty in Rayleigh Number

The thermocouples placed on the two copper plates were calibrated to an accuracy of $\pm 0.2\, \text{K}$. In total, 22,000 data points were recorded at each location and the recording frequency was set at 3 Hz. Table 6.1 summarizes the average Rayleigh number examined each time and the uncertainty in the average value.
Table 6.1: Uncertainty in Rayleigh number

<table>
<thead>
<tr>
<th>Ra</th>
<th>±δ Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.08 × 10^7</td>
<td>0.14 × 10^7</td>
</tr>
<tr>
<td>7.34 × 10^7</td>
<td>0.14 × 10^7</td>
</tr>
<tr>
<td>9.42 × 10^7</td>
<td>0.13 × 10^7</td>
</tr>
<tr>
<td>1.50 × 10^8</td>
<td>0.12 × 10^8</td>
</tr>
<tr>
<td>1.81 × 10^8</td>
<td>0.12 × 10^8</td>
</tr>
<tr>
<td>2.00 × 10^8</td>
<td>0.11 × 10^8</td>
</tr>
<tr>
<td>2.25 × 10^8</td>
<td>0.10 × 10^8</td>
</tr>
<tr>
<td>3.40 × 10^8</td>
<td>0.11 × 10^8</td>
</tr>
</tbody>
</table>
6.2 Uncertainty in Temperature Measurements in the Core of the Enclosure

All thermocouples in the sheath of each temperature profile probe were accurate to ±0.2 K. The temperature was recorded at five different locations along the vertical axis in the enclosure (see Sec. 3.2). The total number of recorded data points at each location was 22,000 and the recording frequency was fixed at 3 Hz.

Furthermore, the 95% confidence interval for the slope and the intercept of the linear fit of the temperature values in the core of the enclosure was estimated. The lower and upper bound of the interval for each Rayleigh number examined are listed in Tables 6.2-6.9.

Table 6.2: 95% Confidence interval for the linear fit coefficients for $Ra = 5.08 \times 10^7$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.097 [3.651, 4.543]</td>
<td>-1.319 [-1.508, -1.131]</td>
</tr>
<tr>
<td>3.617 [3.286, 3.947]</td>
<td>-1.006 [-1.136, -0.876]</td>
</tr>
</tbody>
</table>

Table 6.3: 95% Confidence interval for the linear fit coefficients for $Ra = 7.34 \times 10^7$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.766 [3.487, 4.045]</td>
<td>-1.085 [-1.196, -0.974]</td>
</tr>
<tr>
<td>3.838 [3.565, 4.111]</td>
<td>-1.075 [-1.181, -0.969]</td>
</tr>
<tr>
<td>3.340 [3.112, 3.568]</td>
<td>-0.875 [-0.963, -0.787]</td>
</tr>
</tbody>
</table>
Table 6.4: 95% Confidence Interval for the Linear Fit Coefficients for \( Ra = 9.42 \times 10^7 \)

<table>
<thead>
<tr>
<th>Slope Intercept</th>
<th>Intercept</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.604 [3.458, 3.749]</td>
<td>-0.999 [-1.055, -0.942]</td>
<td></td>
</tr>
<tr>
<td>3.681 [3.409, 3.954]</td>
<td>-0.996 [-1.101, -0.892]</td>
<td></td>
</tr>
<tr>
<td>3.539 [3.323, 3.754]</td>
<td>-1.016 [-1.103, -0.929]</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5: 95% Confidence interval for the linear fit coefficients for \( Ra = 1.50 \times 10^7 \)

<table>
<thead>
<tr>
<th>Slope Intercept</th>
<th>Intercept</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.399 [3.260, 3.537]</td>
<td>-0.887 [-0.940, -0.834]</td>
<td></td>
</tr>
<tr>
<td>3.453 [3.409, 3.568]</td>
<td>-0.895 [-0.939, -0.851]</td>
<td></td>
</tr>
<tr>
<td>3.271 [3.109, 3.433]</td>
<td>-0.820 [-0.881, -0.759]</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: 95% Confidence interval for the linear fit coefficients for \( Ra = 1.80 \times 10^8 \)

<table>
<thead>
<tr>
<th>Slope Intercept</th>
<th>Intercept</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.286 [3.175, 3.398]</td>
<td>-0.833 [-0.875, -0.790]</td>
<td></td>
</tr>
<tr>
<td>3.350 [3.253, 3.448]</td>
<td>-0.847 [-0.884, -0.811]</td>
<td></td>
</tr>
<tr>
<td>3.148 [3.006, 3.291]</td>
<td>-0.768 [-0.822, -0.715]</td>
<td></td>
</tr>
</tbody>
</table>
Table 6.7: 95% Confidence interval for the linear fit coefficients for $Ra = 2.00 \times 10^8$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.260 [3.210, 3.309]</td>
<td>-0.791 [-0.809, -0.773]</td>
</tr>
<tr>
<td>3.321 [3.225, 3.418]</td>
<td>-0.793 [-0.828, -0.758]</td>
</tr>
<tr>
<td>3.128 [3.013, 3.243]</td>
<td>-0.738 [-0.781, -0.696]</td>
</tr>
</tbody>
</table>

Table 6.8: 95% Confidence interval for the linear fit coefficients for $Ra = 2.25 \times 10^8$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.166 [3.114, 3.217]</td>
<td>-0.736 [-0.755, -0.718]</td>
</tr>
<tr>
<td>3.246 [3.113, 3.380]</td>
<td>-0.745 [-0.792, -0.697]</td>
</tr>
<tr>
<td>3.077 [2.961, 3.193]</td>
<td>-0.698 [-0.740, -0.656]</td>
</tr>
</tbody>
</table>

Table 6.9: 95% Confidence interval for the linear fit coefficients for $Ra = 3.40 \times 10^8$

<table>
<thead>
<tr>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.870 [2.676, 3.065]</td>
<td>-0.645 [-0.716, -0.573]</td>
</tr>
<tr>
<td>2.908 [2.627, 3.189]</td>
<td>-0.641 [-0.743, -0.538]</td>
</tr>
<tr>
<td>2.837 [2.614, 3.059]</td>
<td>-0.626 [-0.708, -0.5442]</td>
</tr>
</tbody>
</table>
6.3 Uncertainty in Heat Flux Measurements

Heat flux added to the enclosure was recorded using the heat flux sensors as discussed in Sec. 3.3. In total, 22,000 data points were recorded at each heat flux sensor location. The recording frequency was set at 3 Hz.

A power fit of the form $\text{Nu} = a \text{Ra}^b$ was applied to the data values and the coefficients $a$ & $b$, $R^2$ and fit standard error (RMSE) are reported in Table 6.10.

Table 6.10: Nusselt number power fit

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>$R^2$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0321, 0.2201)</td>
<td>(0.2606, 0.3401)</td>
<td>0.9710</td>
<td>0.8551</td>
</tr>
</tbody>
</table>

(95% confidence bounds) (95% confidence bounds)
6.4 Accuracy of PIV Measurements

Each component of a PIV system involves specific aspects that affect the accuracy of the final result. In the remainder of this section a number of these components are reviewed and the way they affect the velocity measurements is discussed.

6.4.1 Tracer Dynamics

Firstly, the error due to the addition of tracer particles was taken into account. Although it is generally assumed that the particles are sufficiently small to follow the flow the fidelity to represent the local fluid motion is possible to a certain degree.

Nominally, we refer to tacking with error of a few percent as high fidelity [32]. When the fluid is accelerating, any difference between the particle and the fluid velocity will create a drag force. This analysis allowed for considerable drag. In PIV, errors due to various other parameters are of the order of 1%, thus a similar accuracy is essential in the particle tracking.

The slip velocity error is related to the acceleration of the fluid through the simple particle dynamics equation [32]:

\[
\vec{v}_p \approx \frac{\vec{u} - \vec{v}_p}{\tau_p} + \vec{g}
\]

(6.1)

where \( \vec{u} - \vec{v}_p \) is the mean setting (rising) velocity relative to the fluid introduced by gravitational acceleration, given by:

\[
\vec{v}_p - \vec{u} = \frac{\bar{\rho} - 1}{\bar{\rho}} \frac{\tau_0}{\phi} \vec{g}
\]

(6.2)

where \( \bar{\rho} = \frac{\rho_p}{\rho_f} \) is the density ratio of the particle to the fluid and \( \phi \) is a
modifying factor for the low-Reynolds drag on a motionless solid sphere in an unsteady flow.

The term $\tau_p$, in Eq. 6.1 is a time constant in a general form in which the effect of the density ratio $\tilde{\rho}$ and the effect of high finite Reynolds number in decreasing the time constant was taken into account. It can be estimated by:

$$\tau_p = \frac{\tilde{\rho} - 1}{\tilde{\rho}} \tau_o$$

(6.3)

where $\tau_0$ is the respond time defined in section 4.4.2.

As a first approximation the modified factor was taken to be equal to be $\phi = 1$. Thus the slip velocity was estimated using Eq. 6.2. This value was then used to evaluate the finite particle slip Reynolds number and factor $\phi$ using the following equations:

$$Re_p = \frac{|\vec{v}_p - \vec{u}|d_p}{\nu_f}$$

(6.4)

$$\phi = 1 + \frac{3}{16}Re_p$$

(6.5)

The results were used to establish a second approximation as shown in Table 6.11. The second approximation results for the $\phi$ factor were used to estimate $\tau_p$ in Eq. 6.3. Results are displayed in Table 6.12 and were compared to the cutoff time constant give by:

$$\tau_{cutoff} = \frac{d^2p}{8\nu_f \epsilon_{cutoff}^2}$$

(6.6)

where $\epsilon_{cutoff} = 0.14$ is the cutoff Stokes number and the value was adopted from Fig. 2 in [43].
Table 6.11: Second approximation results

| Ra     | $|\vec{v}_p - \vec{u}| \times 10^{-5}$ (m/s) | $Re_p \times 10^{-6}$ | $\phi$ |
|--------|------------------------------------------|-----------------------|--------|
| $5.08 \times 10^7$ | 2.656                                | 1.706                | 1.00   |
| $7.34 \times 10^7$ | 2.639                                | 1.670                | 1.00   |
| $9.42 \times 10^7$ | 2.622                                | 1.635                | 1.00   |
| $1.50 \times 10^8$ | 2.570                                | 1.530                | 1.00   |
| $1.80 \times 10^8$ | 2.535                                | 1.463                | 1.00   |
| $2.00 \times 10^8$ | 2.511                                | 1.417                | 1.00   |
| $2.25 \times 10^8$ | 2.472                                | 1.347                | 1.00   |
| $3.40 \times 10^8$ | 2.419                                | 1.459                | 1.00   |

Table 6.12: Slip error time constant

<table>
<thead>
<tr>
<th>Ra</th>
<th>$\tau_p (\mu s)$</th>
<th>$\tau_{cutoff} (\mu s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>2.71</td>
<td>0.41</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>2.69</td>
<td>0.40</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>2.67</td>
<td>0.40</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>2.62</td>
<td>0.38</td>
</tr>
<tr>
<td>$1.80 \times 10^8$</td>
<td>2.58</td>
<td>0.37</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>2.56</td>
<td>0.36</td>
</tr>
<tr>
<td>$2.25 \times 10^8$</td>
<td>2.52</td>
<td>0.35</td>
</tr>
<tr>
<td>$3.40 \times 10^8$</td>
<td>2.47</td>
<td>0.33</td>
</tr>
</tbody>
</table>

A comparison between the two time constants indicates that we are well above the cutoff frequency at which the mean square amplitude of the particle velocity reduces to 50% from the mean square amplitude of the fluid oscillation thus significant slip error was not introduced to the measured
6.4.2 Image Mapping

A significant uncertainty is introduced in the measurements by imaging itself. Most of the times the image magnification $M_0$ is considered to be a constant by which the measured displacement $\Delta X$ in the image domain is converted to a displacement $\Delta x$ in the object domain. In PIV measurements, the magnification varies over the depth of the light sheet which leads to an uncertainty in $M_0$. The relative variation is proportional to $\Delta z_0/z_0$, where $\Delta z_0$ is the thickness of the laser sheet and $z_0$ is the distance between the laser sheet and the imaging lens.

In the present work the thickness of the laser sheet was measured to be $\Delta z_0 = 1$ mm and $z_0 = 1187.5$ mm which leads to a variation of $8.4 \times 10^{-4}$ or $0.084\%$ in the magnification for a given point in the image plane. This is well below the typical value of $0.3\%$ recorded by [32].

Moreover, in the case of planar PIV measurements wherein the out-of-plane component $\Delta z$ is not negligible contaminates the in-plane measurement. This perspective error is minimized when the laser sheet coincides with the plane of symmetry. In the present work the laser sheet was placed at the center of symmetry of the enclosure which resulted in minimal out of plane motion as discussed in Sec. 5.1.9.

6.4.3 Interrogation

Uncertainties in interrogation can generally be classified as either random errors or bias errors [44]. A brief description of these two errors is given below:
Bias errors are not random in nature and can degrade both the accuracy of instantaneous PIV measurements and any statistics estimated using biased results. The most significant bias error associated with a PIV measurement is peak or pixel locking that describes the biasing of particle displacements towards integer pixel values. Peak locking is attributable to both the choice of sub-pixel estimator and under-resolved optical images [45].

A Gaussian sub-pixel estimator is superior to both centroid and quadratic fits in terms of mitigating peak-locking effects [46]. In 2003, Roesgen proved that a sinc interpolation kernel completely suppresses spurious spectral side-lobes in the correlation, yielding minimal peak-locking influences for adequately resolved particle images [47].

Peak locking can be significant bias error in the case when particles are under-resolved, i.e. the ratio of particle image diameter to pixel size \((d_r/d_{pix})\) is less than 2, and independent of the choice of sub-pixel estimator. In such cases the correlation peak is not adequately resolved thus the sub-pixel estimator cannot accurately determine the sub-pixel displacement of the particles. As a result the values "lock" toward integer pixel values.

In this work the particle image is between 1 and 2 pixels in diameter. This indicates that the pixel-biasing effects are within acceptable limits in regard to the quality of the measured flow patterns and the statistics of the flow [32]. For diameters smaller than 1 pixel the basing effects become sever as the correlation-peak amplitude is strongly affected. On the contrary, for particle diameters larger than 2 pixels random error generally dominate and their amplitude is proportional to the particle image diameter. Thus the optimal diameter would be around 2 pixels.
Random Errors

Random errors in PIV are associated with electronic noise in cameras, shot noise (which is independent from pixel to pixel and in time), and random errors associated with properly identifying the sub-pixel displacement [45]. These errors can influence the accuracy of instantaneous PIV measurements but due to their random nature they can be averaged either in space or time over a large ensemble.

The random error for the optimal particle diameter for the estimated sub-pixel displacement was estimated to be 0.1 pixel units for $32 \times 32$ pixel interrogation areas, which is the case in this study, in simulation studies [48, 49] and a theoretical analysis [50]. Westerweel [50] also showed that the minimum theoretical error based on the statistical properties of the image is about 0.04 pixels and that the value obtained from simulation studies depended strongly on the illumination light sheet profile and the particle image intensity distribution.

6.4.4 Vector Placement

Since the molecules mark a finite volume the displacement vector should be placed in the center of the particle image pair. In the case when the interrogation window contains more than one particle image pairs the displacement is measurement from the center of the interrogation domain. For a finite number of image pairs though an uncertainty is introduced between the actual location where the displacement vector is placed and the center of the interrogation window is.

For $N_I$ particle images within a square uniform interrogation domain of dimension $D_I$ the rms difference between the center of the interrogation do-
main and the location of the particle image is $\frac{D_I}{\sqrt{12N_I}}$ [32]. In this study $N_I = 2000$ and $D_I = 32$ leading to 0.2 rms difference between the center of the interrogation window and the actual location of the particle. This has no consequences when the displacement is uniform.

6.4.5 Flow Kinematics

PIV measurements also have the ability to obtain the spatial derivatives of the instantaneous velocity flow field. The largest gradients are present at the smallest length scales in turbulent flows, characterized by the Kolomogorov length scale. Kolmogorov scale $\lambda_K$ is defined as $\lambda_K \equiv (\nu^3/\epsilon)^{1/4}$ where $\epsilon$ is the turbulence dissipation rate and $\nu$ is the kinematic viscosity. The dynamic spatial range (DSR) of digital PIV is not capable of resolving both the integral length scale and the Kolmogorov length scale in a single recording. Thus, the vorticity fields are in general under-resolved. For vortical patterns that are adequately resolved, the error in the amplitude of the measured vorticity can be reduced to about 10% but significant errors occur for small-scale vortical structures [32]. Generally, the vortical patterns estimated from local circulation provide a qualitative description of the vortical flow pattern.

6.4.6 Sampling Error

A sufficiently high number of PIV recordings is essential when evaluating statistical properties of a flow. It was shown in [32] that the error converges proportional to $N_F^{-1/2}$ where $N_F$ is the number of image pairs. This is provided that the subsequent PIV recordings are statistically independent.

Given that the turbulence level is larger than the random fluctuations in the PIV measurements, that is, the relative sampling error in the mean
velocity is given by [32]:

\[
\frac{\text{var}[\bar{u}]^{0.5}}{U} \geq \frac{\sigma_u/U}{N_f^{0.5}}
\]  

(6.7)

where the numerator is the turbulence intensity. Eq. 6.7 is multiplied by 2 for a 95\% reliability interval for the Student t-statistics. In this work turbulence intensity levels up to 30\% were recorded thus an error in the measured mean flow of about 1\% is introduced.

6.4.7 Uncertainty in Ensemble Average Statistics

Standard deviation of the sample mean was estimated as a criterion for uncertainty. Table 6.13 summarizes the uncertainty (95\% confidence levels) in recorded mean values for each Rayleigh number examined.

Table 6.13: Standard deviation as a criterion for uncertainties

| Ra            | SD\(_{(\bar{u})}\) (%) | SD\(_{(\bar{v})}\) (%) | SD\(_{|U|}\) (%) |
|---------------|-----------------------|------------------------|-----------------|
| \(5.08 \times 10^7\) | 0.14                  | 0.05                   | 0.14            |
| \(7.34 \times 10^7\) | 0.15                  | 0.05                   | 0.15            |
| \(9.42 \times 10^7\) | 0.16                  | 0.05                   | 0.16            |
| \(1.50 \times 10^8\) | 0.11                  | 0.02                   | 0.10            |
| \(1.80 \times 10^8\) | 0.11                  | 0.03                   | 0.11            |
| \(2.00 \times 10^8\) | 0.07                  | 0.01                   | 0.07            |
| \(2.25 \times 10^8\) | 0.12                  | 0.03                   | 0.12            |
| \(3.40 \times 10^8\) | 0.15                  | 0.03                   | 0.14            |
7 PROPER ORTHOGONAL DECOMPOSITION ANALYSIS

Proper Orthogonal Decomposition (POD) has been increasingly employed for the analysis of complex flow fields obtained either experimentally or through numerical simulations. It can be applied to reduce large data sets obtained by data-intensive numerical simulations or experiments employing whole-field techniques by only retaining the most salient features of the flow field [51]. Furthermore, POD can also be used as a power-based filtering method to reconstruct the flow field with only the most dominant flow structures [52]. This section addresses the way POD method was employed to provide quantitative comparison of flow patterns for different Rayleigh numbers.

7.1 Background

In the field of fluid mechanics, POD is a powerful tool that decomposes a flow field $U(x, t)$ into its POD modes $\phi_j(x)$. This new base can be used to reconstruct the flow field separating uncorrelated inhomogeneous flow structures from uncorrelated random fluctuations which are both superimposed in the mean flow field $\bar{u}(x)$:

$$U(x, t) = \bar{u}(x) + u(x, t)$$ (7.1)

$u(x, t)$ are the deviations from the mean which form an eigenvalue problem. The eigenvectors of the problem are the $\phi_j(x)$ POD modes and $\lambda_j$ are the corresponding eigenvalues. This eigenvalue value problem is called the
Fredholm integral equation as described by in [53]:

\[ \int_x R(x, x') \phi_j(x') dx' = \lambda_j \phi_j(x) \]  

(7.2)

whose kernel is the two-point (spatial) correlation function:

\[ R(x, x') = \langle u(x, t) u(x', t) \rangle. \]  

(7.3)

The systematic fluctuations occur repeatedly thus in order to capture the periodicity of the coherent structures in the flow a high number of snapshots is required.

Analytical Description

For a flow with \( F \) field data points, the spatial velocity distribution for time step \( t^i \) is given by:

\[
\mathbf{u}^i = \begin{bmatrix}
    u \left( x^{(1)}, t^i \right) \\
    u \left( x^{(2)}, t^i \right) \\
    \cdots \\
    u \left( x^{(F)}, t^i \right)
\end{bmatrix}
\]  

(7.4)

Then the velocity matrix:

\[
\mathbf{U} = \begin{bmatrix}
    \mathbf{u}^1 & \mathbf{u}^2 & \cdots & \mathbf{u}^K
\end{bmatrix}
\]  

(7.5)

is a collection of all the velocity vectors \( \mathbf{u}^i \) \((i = 1 : K)\) over \( K \) time steps.

Moreover, using the velocity matrix the covariance matrix can be built:

\[
\mathbf{R} = \mathbf{U} \mathbf{U}^T.
\]  

(7.6)
Furthermore, the corresponding eigenvalue problem can be formulated:

$$ \mathbf{R} \psi = \lambda \psi. \quad (7.7) $$

This leads to the following diagonal matrix:

$$ \lambda = \begin{bmatrix} \lambda_1 & 0 \\ & \lambda_2 \\ & & \ddots \\ & & & \lambda_K \end{bmatrix} \quad (7.8) $$

of the eigenvalues $\lambda_j$ and to the matrix:

$$ \psi = [\phi_1 \phi_2 \ldots \phi_K]^T \quad (7.9) $$

which is the eigenvectors $\phi_j \ (j = 1:K)$ that represent the POD modes.

Each mode contributes to the overall signal power by:

$$ P_j = \frac{\lambda_j}{||\lambda||} \quad (7.10) $$

The powers $P_j$ also decrease with increasing index $j$ since the eigenvalues of $\lambda$ are organized in a decreasing order.

The weighting coefficients are estimated using matrix $\psi$:

$$ a^i = \psi u^i. \quad (7.11) $$

By superimposing the product between the weighting coefficients $\alpha_j^i$ and the POD modes $\phi_j$ the transient part of the flow field can be reconstructed:

$$ u^i = \sum_{j=1}^{K} \alpha_j^i \phi_j = \psi^T a^i. \quad (7.12) $$
When the reconstruction of the flow stops after a certain number of POD modes $K_P$ then:

$$u_R(x) = \sum_{j=1}^{K_P} \alpha_j \phi_j(x). \quad (7.13)$$

In this case $u_R(x)$ is not an exact reconstruction of the flow field. However, the main part of the energy is often contained within the first modes and the residual can often be neglected [54].

It is apparent that the dominant structures are represented by the initial most powerful modes. Such a truncated reconstruction of the original data results in a power-based filtered signal, still containing inhomogeneous flow behavior [51]:

$$U_R(x, t) = \bar{u}(x) + u_R(x, t). \quad (7.14)$$

A more complete discussion on POD can be found in [55]. Proper Orthogonal Decomposition can also be referred as the Karhunen-Loeve decomposition (KLD), the principle component analysis (PCA) and the singular value decomposition (SVD) [56].
7.2 Energy Distribution

The energy distribution over the POD modes was estimated for each Rayleigh number examined. Fig. 7.1 and Fig. 7.2 illustrate the total energy and cumulative energy as a function of POD modes respectively.

From Fig. 7.1, it can be concluded that TKE distributes over a higher number of modes as Rayleigh number increases. It is easy to observe that, for Rayleigh numbers at the low end of the range examined most of the energy is distributed over the first $\sim 16$ modes. There is an apparent reversal of trend after that and modes in Rayleigh numbers at the high end of our range have more energy compare to those in the low Rayleigh number regime.

In the same line Fig. 7.2 depicts the cumulative energy up to a specific number of modes for each Rayleigh number. One can observe that TKE spreads over a larger number of modes as Rayleigh number increases. For instance, it is apparent that for 50% of the cumulative energy it requires $\sim 10$ modes for $Ra = 7.34 \times 10^7$, $\sim 13 - 15$ for the Rayleigh numbers $9.42 \times 10^7 - 2.00 \times 10^8$, $\sim 18$ for $Ra = 2.25 \times 10^8$ and finally $\sim 25$ for $Ra = 3.40 \times 10^8$. 
Figure 7.1: Total Energy as a Function of Number of Modes

Figure 7.2: Cumulative Energy as a Function of Number of Modes
7.3 Spacial Eigenmodes

7.3.1 Horizontal (x) Direction

Figs. 7.3-7.10 show the first eigenmode in the x-direction and the associated energy with the mode for each Rayleigh number. It is apparent that for Rayleigh numbers at the high end of the regime we examined, i.e. $Ra \geq 1.80 \times 10^8$, there is no significant contribution to the total energy from the horizontal direction. The flow structures for Rayleigh numbers $Ra = 7.34 \times 10^7$ and $Ra = 9.42 \times 10^7$ present great similarities, although the energy associated with the mode for the $Ra = 7.34 \times 10^7$ is significantly higher. Also the two modes have contrary result in the flow since one is positive ($Ra = 7.34 \times 10^7$) and the other one is negative ($Ra = 9.42 \times 10^7$).

The second eigenmode is illustrated in Figs. 7.11 - 7.18. There is a clear resemblance between the structures for Rayleigh numbers $Ra = 7.34 \times 10^7$ and $Ra = 9.42 \times 10^7$. Once again the two modes are different in sign indicating that they affect the global flow in the enclosure in an opposite manner. There is a change in trend in this case since now the mode for $Ra = 9.42 \times 10^7$ is positive and the mode for $Ra = 7.34 \times 10^7$ is negative. In addition, the two modes are fairly close in energy content in this case. There are also visible similarities in the structures between $Ra = 1.80 \times 10^8$ and $Ra = 2.00 \times 10^8$. Although magnitude wise the mode for $Ra = 2.00 \times 10^8$ appears to be stronger, this has no physical interpretation, unless is combined with the POD coefficient (see Sec. 7.1) to reconstruct the flow field. The two modes have different signs indicating the adverse impact they have in the flow.

In Figs. 7.19- 7.26 the third eigenmode is shown. An interesting feature in this case is the structures observed for Rayleigh numbers $Ra = 1.50 \times$
$10^8$, Ra = $1.81 \times 10^8$, and Ra = $2.00 \times 10^8$. There is a flow structure associated with the boundary layer on the top adiabatic wall and another one with conflicting result related to the reflected recirculation by the cold isothermal wall after the top adiabatic wall is redirected vertically. The energy associated with the mode is within 1% difference for all the three Rayleigh numbers indicating that they are of the same nature. The flow structures related with the aforementioned flow regions, i.e. the boundary layer on the top adiabatic wall and the reflected re-circulation, for the above Rayleigh numbers, demonstrate high periodicity as they are clearly present in modes 6 and 7 as depicted in Figs. 7.43-7.58.

In the case of higher modes we begin to loose coherency as illustrated in Figs. 7.59-7.82 for modes 8, 9 and 10.
Figure 7.3: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 5.08 \times 10^7

Figure 7.4: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 7.34 \times 10^7
Figure 7.5: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.6: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$.
Figure 7.7: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $1.81 \times 10^8$

Figure 7.8: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $2.00 \times 10^8$
Figure 7.9: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$.

Figure 7.10: First Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$.
Figure 7.11: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 5.08 \times 10^7 \)

Figure 7.12: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 7.34 \times 10^7 \)
Figure 7.13: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$

Figure 7.14: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$
Figure 7.15: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 1.81 \times 10^8

Figure 7.16: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 2.00 \times 10^8
Figure 7.17: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$.

Figure 7.18: Second Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$. 

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Figure 7.19: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $5.08 \times 10^7$.

Figure 7.20: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $7.34 \times 10^7$. 

Figure 7.21: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.22: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.23: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.24: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.25: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.26: Third Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.27: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 5.08 \times 10^7 \)

Figure 7.28: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 7.34 \times 10^7 \)
Figure 7.29: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.30: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 
Figure 7.31: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$.

Figure 7.32: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$. 

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Figure 7.33: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.34: Forth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.35: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 5.08 \times 10^7

Figure 7.36: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 7.34 \times 10^7
Figure 7.37: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.38: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.39: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$.

Figure 7.40: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$. 

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Figure 7.41: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.42: Fifth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.43: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( \text{Ra} = 5.08 \times 10^7 \)

Figure 7.44: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( \text{Ra} = 7.34 \times 10^7 \)
Figure 7.45: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.46: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 
Figure 7.47: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 1.81 × 10^8

Figure 7.48: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 2.00 × 10^8
Figure 7.49: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.50: Sixth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.51: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 5.08 \times 10^7 \)

Figure 7.52: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for \( Ra = 7.34 \times 10^7 \)
Figure 7.53: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 9.42 × 10^7

Figure 7.54: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 1.50 × 10^8
Figure 7.55: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.56: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.57: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.58: Seventh Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.59: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$.

Figure 7.60: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$.
Figure 7.61: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$

Figure 7.62: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$
Figure 7.63: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.64: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.65: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $\text{Ra} = 2.25 \times 10^8$

Figure 7.66: Eighth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $\text{Ra} = 3.40 \times 10^8$
Figure 7.67: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$.

Figure 7.68: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$. 

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Figure 7.69: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.70: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.71: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $1.81 \times 10^8$

Figure 7.72: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $2.00 \times 10^8$
Figure 7.73: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $2.25 \times 10^8$

Figure 7.74: Ninth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = $3.40 \times 10^8$
Figure 7.75: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$ 

Figure 7.76: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.77: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$

Figure 7.78: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$
Figure 7.79: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 1.81 × 10^8

Figure 7.80: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for Ra = 2.00 × 10^8
Figure 7.81: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.82: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
7.3.2 Vertical (y) Direction

In Figs. 7.83 - 7.90 the first mode in the vertical direction is shown. One can observe that there is contribution in this direction for all Rayleigh numbers in contrast to the horizontal direction where there is no contribution from Rayleigh numbers $Ra = 1.80 \times 10^8$, $Ra = 2.25 \times 10^8$ and $Ra = 3.40 \times 10^8$. Another feature that can be noted is that for low end Rayleigh numbers, i.e. $Ra \leq 9.43 \times 10^8$, flow structures appear on the cold wall whereas for the rest of the cases flow structures are present on both the cold and the hot walls. Moreover, for $Ra \leq 9.43 \times 10^8$ the flow structure on the cold wall shows great similarities, just like in the horizontal direction cases. For $Ra = 7.34 \times 10^7$ the flow structure has the opposite impact than the structures for $Ra = 5.08 \times 10^7$ and $Ra = 9.43 \times 10^7$.

In mode 2 (Figs. 7.91 - 7.98) we observe the same trend as in mode 1, with coherent structures present on the cold wall for Rayleigh numbers $Ra \leq 9.43 \times 10^7$ and for the rest of the cases structures are present on both the active walls. In mode 3 (Figs. 7.99 - 7.106) flow structures appear on the hot wall for $Ra \leq 9.43 \times 10^7$ for the first time. In addition, the similarities between the flow structures on the cold wall for Rayleigh numbers $Ra = 1.50 \times 10^8$, $Ra = 1.80 \times 10^8$ and $Ra = 2.00 \times 10^8$ are apparent. We can distinguish two structures with contradictory behavior. Additionally, the energy for this mode is less than 1% different for the three aforementioned Rayleigh numbers thus the structures should be of the same nature.

For mode 4 (Figs. 7.107 - 7.114), we observe that for Rayleigh numbers $Ra = 7.34 \times 10^7$, $Ra = 9.43 \times 10^7$ and $Ra = 1.50 \times 10^8$ we have the same structures appearing on the cold wall. In Figs. 7.115 - 7.122, we notice that the flow structure on the hot wall for $Ra = 1.50 \times 10^8$ is being periodic as
it is also present on the hot wall for $Ra = 1.81 \times 10^8$ and $Ra = 2.00 \times 10^8$.

There is high periodicity also among the structures in the flow on the cold wall for $Ra = 1.50 \times 10^8$, $Ra = 1.81 \times 10^8$ and $Ra = 2.00 \times 10^8$ for modes 6, 7 and 8 as shown in Figs. 7.123 - 7.146.

In the case of higher modes we observe that we begin to lose coherency as illustrated in Figs. 7.147-7.162 for modes 9 and 10 and no further conclusions can be extracted.
Figure 7.83: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for \( \text{Ra} = 5.08 \times 10^7 \)

Figure 7.84: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for \( \text{Ra} = 7.34 \times 10^7 \)
Figure 7.85: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.86: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.87: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$.

Figure 7.88: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$. 

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Figure 7.89: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 2.25 × 10^8

Figure 7.90: First Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 3.40 × 10^8
Figure 7.91: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.92: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.93: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.94: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.95: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.96: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.97: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$.

Figure 7.98: Second Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$. 

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Figure 7.99: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 5.08 × 10^7

Figure 7.100: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 7.34 × 10^7
Figure 7.101: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 9.42 × 10^7

Figure 7.102: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 1.50 × 10^8
Figure 7.103: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$.

Figure 7.104: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$. 

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Figure 7.105: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.106: Third Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.107: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.108: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.109: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 9.42 × 10^7

Figure 7.110: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 1.50 × 10^8
Figure 7.111: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.112: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.113: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.114: Forth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.115: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$.

Figure 7.116: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$. 

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Figure 7.117: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.118: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 
Figure 7.119: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for \( Ra = 1.81 \times 10^8 \)

Figure 7.120: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for \( Ra = 2.00 \times 10^8 \)
Figure 7.121: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$}

Figure 7.122: Fifth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.123: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.124: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.125: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$

Figure 7.126: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$
Figure 7.127: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.128: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.129: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.130: Sixth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.131: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$.

Figure 7.132: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$. 
Figure 7.133: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.134: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 
Figure 7.135: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$.

Figure 7.136: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$. 
Figure 7.137: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $\text{Ra} = 2.25 \times 10^8$

Figure 7.138: Seventh Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $\text{Ra} = 3.40 \times 10^8$
Figure 7.139: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.140: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.141: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 9.42 \times 10^7

Figure 7.142: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 1.50 \times 10^8
Figure 7.143: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $\text{Ra} = 1.81 \times 10^8$

Figure 7.144: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $\text{Ra} = 2.00 \times 10^8$
Figure 7.145: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.146: Eighth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.147: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.148: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.149: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$

Figure 7.150: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$
Figure 7.151: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 1.81 \times 10^8$

Figure 7.152: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.00 \times 10^8$
Figure 7.153: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.154: Ninth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
Figure 7.155: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 5.08 \times 10^7$

Figure 7.156: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 7.34 \times 10^7$
Figure 7.157: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 9.42 \times 10^7$.

Figure 7.158: Tenth Spatial Eigenmode in the x-direction as a Function of Rayleigh Number for $Ra = 1.50 \times 10^8$. 

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Figure 7.159: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 1.81 × 10^8

Figure 7.160: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for Ra = 2.00 × 10^8
Figure 7.161: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 2.25 \times 10^8$

Figure 7.162: Tenth Spatial Eigenmode in the y-direction as a Function of Rayleigh Number for $Ra = 3.40 \times 10^8$
This section includes a description of the OpenFOAM (Open Field Operation and Manipulation) CFD toolbox and the model we built to simulate and study natural convection in an enclosure. Results are presented for the eight Rayleigh numbers in the range $5.08 \times 10^7 \leq \text{Ra} \leq 3.40 \times 10^8$ and compared to experimental data.

8.1 OpenFOAM

The OpenFOAM (Open Field Operation and Manipulation) CFD toolbox is an open source CFD software package which incorporates a large number of features to solve anything from complex fluid flows involving chemical reactions, turbulence and heat transfer, to solid dynamics and electromagnetics. OpenFOAM has a large user base across most areas of engineering and science, from both academic and commercial organizations. Tools for meshing and for pre- and post-processing are also included and almost everything run in parallel as standard, enabling users to take full advantage of computer hardware at their disposal. More information on OpenFOAM can be found in OpenFOAM documentation [57].
8.2 Reynolds averaged Navier-Stokes Equations

The equations governing viscous incompressible flow, whether turbulent or laminar, are [58]:

\[
\partial_t \bar{u}_i + \bar{u}_j \partial_j \bar{u}_i = -\frac{1}{\rho} \partial_i \bar{P} + \nu \nabla^2 \bar{u}_i. \tag{8.1}
\]

\[
\partial_t \bar{u}_i = 0. \tag{8.2}
\]

Eq. 8.1 expresses the momentum conservation and Eq. 8.2 expresses the incompressibility of fluid volumes, which in this case is equivalent to mass conservation. In this section the shorthand \( \partial_i \) is used for the partial derivative \( \frac{\partial}{\partial x_i} \) and \((u_1, u_2, u_3) = (u, v, w)\).

The total velocity can be decomposed into a sum of its mean and a fluctuation, \( \tilde{u}(x, t) = U(x, t) + u(x, t) \), where \( U \equiv \bar{u} \). By substituting this in Eq. 8.1 and Eq. 8.2 we obtain:

\[
\partial_t (U_i + u_i) + (U_j + u_j) \partial_j (U_i + u_i) = -\frac{1}{\rho} \partial_i (P + p) + \nu \nabla^2 (U_i + u_i). \tag{8.3}
\]

\[
\partial_t (U_i + u_i) = 0. \tag{8.4}
\]

The average of these equations is obtained by drawing a bar over each term, noting the rules \( \bar{U} = U \) and \( \bar{u} = 0 \):

\[
\partial_t U_i + U_j \partial_j U_i = -\frac{1}{\rho} \partial_i P + \nu \nabla^2 U_i - \partial_j u_j \tilde{u}_i \tag{8.5}
\]
\[ \partial_i U_i = 0. \quad (8.6) \]

Eqs. 8.5 and Eq. 8.6 are the Reynolds averaged Navier-Stokes equations (RANS). These are the same as Eqs. 8.1 and Eq. 8.2 for the total instantaneous velocity, except from the last term of the momentum equation (Eq. 8.5). For more information on RANS please see [58].

8.2.1 \( \kappa - \omega \) SST (Shear Stress Transport) turbulence model

The \( \kappa - \omega \) shear stress transport (SST) turbulence model [59, 60, 61] comprises of two equations [59, 62]:

One for the specific turbulence kinetic energy \( \kappa \) \((m^2 \text{s}^{-2})\) (Eq. 8.7):

\[
\frac{\partial}{\partial t} (\rho \kappa) + \frac{\partial}{\partial x_i} (U_i \rho \kappa) = \frac{\partial}{\partial x_j} \left( \mu_\kappa \frac{\partial}{\partial x_j} \kappa \right) + \tilde{P}_\kappa - \beta^* \rho \omega \kappa \quad (8.7)
\]

and one for \( \omega \), the specific turbulence dissipation rate \((\text{s}^{-1})\):

\[
\frac{\partial}{\partial t} (\rho \omega) + \frac{\partial}{\partial x_i} (U_i \rho \omega) = \frac{\partial}{\partial x_j} \left( \mu_\omega \frac{\partial}{\partial x_j} \omega \right) + P_\omega - \beta \rho \omega^2 + \\
2 \rho (1 - F_1) \frac{1}{\omega} \frac{1}{\sigma_{\omega^2}} \frac{\partial}{\partial x_j} \kappa \frac{\partial}{\partial x_j} \omega. \quad (8.8)
\]

The effective viscosities \((\text{kg m}^{-1} \text{s}^{-1})\) are given by [60]:

\[
\mu_\kappa = \mu + \mu_t \frac{1}{\sigma_\kappa}, \quad (8.9)
\]

\[
\mu_\omega = \mu + \mu_t \frac{1}{\sigma_\omega}, \quad (8.10)
\]

where \( \mu_t \) is the modified turbulent viscosity \((\text{kg m}^{-1} \text{s}^{-1})\), and \( \sigma_\kappa \) & \( \sigma_\omega \) are
the diffusion constants of the model. The Reynolds stresses\( \tau_{ij} \) (kg m\(^{-1}\) s\(^{-1}\)) are computed as usual in two-equation models using the Boussinesq expression [60, 62]:

\[
\tau_{ij} = -\rho U_i U_j = 2\mu_i S_{ij} - \frac{2}{3}\rho \kappa \delta_{ij}. \tag{8.11}
\]

In Eq. 8.11, \( S_{ij} \) is the mean rate of deformation component (s\(^{-1}\)) and \( \delta_{ij} \) is the Kronecker delta. The rate of production of \( \omega \), \( P_\omega \) (kg m\(^{-3}\) s\(^{-2}\)), is given by [62]:

\[
P_\omega = \gamma \left[ 2\rho S_{ij} \cdot S_{ij} - \frac{2}{3}\rho \omega \left( \frac{\partial}{\partial x_j} U_i \right) \delta_{ij} \right]; \tag{8.12}
\]

where \( \gamma \) is a model constant.

The blending function \( F_1 \) is defined by [62]:

\[
F_1 = \tanh \left( \min \left[ \max \left( \frac{L}{y}; \frac{500\nu_t}{y^2 \omega} \right); \frac{4\rho \sigma_{\omega,2\kappa}}{X y^2} \right] \right)^4, \tag{8.13}
\]

whereas \( F_1 \) varies from unity at the wall to zero outside wall boundary layers. \( L = \frac{\kappa^{1/2}}{(0.09\omega)} \) is the turbulence length scale (m), dived by the shortest distance to the near wall (m). The positive portion of \( X \) is limited as [61]:

\[
X = \max \left( \frac{1}{\omega} \frac{\rho - 1}{\sigma_{\omega,2} \partial_{x_i} \omega; 10^{-10}} \right). \tag{8.14}
\]

Each coefficient of the Menter’s model (\( \phi \equiv \sigma_\kappa, \sigma_\omega, \beta, \gamma \)) is evaluated from [59]:

\[
\phi = \phi_1 F_1 + \phi_2 (1 - F_1), \tag{8.15}
\]

where \( \phi_1 \) is related to the adjusted \( \kappa - \omega \) model and \( \phi_2 \) is connected with the standard \( \kappa - \epsilon \) whose constrains values are given as follows [62]:
\[ \sigma_{\kappa,1} = \frac{1}{0.85034}, \quad \sigma_{\kappa,2} = 1.0, \]  
\[ \sigma_{\omega,1} = 2.0, \quad \sigma_{\omega,2} = \frac{1}{0.85616}, \] (8.16)  
\[ \beta_1 = \frac{3}{40}, \quad \beta_2 = 0.0828, \quad \beta^* = \frac{9}{100}, \quad \kappa = 0.41, \] (8.18)  
\[ \gamma_1 = \left( \frac{\beta_1}{\beta^*} \right) - \left( \frac{\sigma_{\omega,1} \kappa^2}{(\beta^*)^{1/2}} \right) = 0.5532, \] (8.19)  
\[ \gamma_2 = \left( \frac{\beta_2}{\beta^*} \right) - \left( \frac{\sigma_{\omega,2} \kappa^2}{(\beta^*)^{1/2}} \right) = 0.44. \] (8.20)  

By limiting the eddy viscosity we can improve the performance in flows with adverse pressure gradients and wake regions [62]:

\[ \mu_t \frac{1}{\rho} = \frac{\alpha_1 \kappa}{\max (\alpha_1 \omega, SF_2)} = \nu_t, \] (8.21)

given that \( S = (2S_{ij}S_{ij})^{1/2} \) is the invariant measure of the strain rate, \( \alpha_1 = 0.31 \) is a constant, \( \nu_t \) is the kinematic eddy viscosity (m\(^2\) s\(^{-1}\)) and \( F_2 \) is the bending function given by [62]:

\[ F_2 = \tanh \left( \left[ \max \left( \frac{2L}{y}; \frac{500\nu_t}{y^2 \omega} \right) \right]^2 \right). \] (8.22)

For other regions of the boundary layer, the eddy viscosity can be evaluated using:

\[ \mu_t = \rho \left( \frac{\kappa}{\omega} \right). \] (8.23)
To prevent the build-up for turbulence in stagnation regions, $\tilde{P}_\kappa$, the effective rate of production is limited as follows [61]:

$$\tilde{P}_\kappa = \min (P_\kappa; 10\beta^* \rho_\kappa \omega), \quad (8.24)$$

where $P_\kappa$, the rate of production ($\text{Kg m}^{-1} \text{s}^{-3}$), is expressed by [63]:

$$P_\kappa = 2\mu_t S_{ij} \cdot S_{ij} - \frac{2}{3} \rho_\kappa \left( \frac{\partial}{\partial x_j} U_i \right) \delta_{ij}. \quad (8.25)$$
8.3 Building the Model in OpenFOAM

8.3.1 The Physics of the Flow

The only source of the flow generation is the temperature difference between the walls of the enclosure and the air inside the cavity. The temperature difference between the layers of air causes a density variation. Although the flow in this situation is incompressible, in order to consider this density variation a compressible solver was employed. One can use the Boussinesq approximation to solve the flow as an incompressible flow with density variation. The flow can be considered steady-state when the boundary conditions remain constant. The properties of air inside the cavity are saved in constant/thermophysicalProperties file and air is considered to be a perfect gas.

8.3.2 Governing Equations

To solve the Navier Stokes equations in a steady state buoyancy driven flow condition we choose the solver code buoyantSimpleFoam. Source code can be found in this address: $FOAM APP/solvers/heatTransfer/buoyantSimpleFoam.

This solver is a steady-state solver for buoyant, turbulent flow. The buoyantSimpleFoam.C consists of a runTime loop which recalls the equations UEqn.H (momentum), hEqn.H (energy) and pEqn.H (dynamic pressure) in every iteration and uses the pressure-velocity SIMPLE corrector.

The density field is in the createFields.H and can be read from basic-Thermo thermophysical properties. One can set the velocity field from startTime directory and compressible phi is estimated from compressible-CreatePhi.H. A turbulence model can be created by defining a new class of
rho, U, phi, thermo() from compressible:RASModel. Using the gravitational acceleration \( g \) one can create the \( g.h \) field. Gravitational acceleration can be defined in enviromentalProperties and the mesh specifications. Finally, the dynamic pressure \( pd \) is set from startTime and the total pressure is defined as: \( p = pd + rho \times gh + pRef \) in which \( pRef \) is a reference pressure set in the initial conditions.

The general equations used in the solve are listed below:

**Momentum Equations**

\( UEqn.H \) is the solver for the momentum equations. The code solves the following differential equation:

\[
\nabla (\Phi U) - (\nabla \Phi) U - \nabla \mu_{eff} \nabla U - \nabla (\mu_{eff} (\nabla U) T) = -\nabla pd - (\nabla \rho) gh.
\] (8.26)

The \( \text{divDevRhoReff()} \) is the source term for the momentum equation and is defined in the \textit{compressible}/\textit{RASModel.H}.

**Energy Equations**

The solver for this equation is \( hEqn.H \) and it solves the differential equation below:

\[
\nabla (\Phi h) - (\nabla \Phi) h - \nabla \alpha \nabla h = \nabla \left( \frac{\Phi}{\rho P} \right) - p \nabla \left( \frac{\Phi}{\rho} \right).
\] (8.27)

**Dynamic Pressure Equations**

The equation for Dynamic Pressure in steady-state compressible flow is solved in the file \textit{pEqn.H}. In the code the flux and velocity are calculated on the cell boundaries and the equation for the flux and pressure is solved as below:
Once the residuals are calculated, the pressure is estimated as \( p = pd + \rho g h + p_{Ref} \).

Finally, the corrected velocity is calculated as:

\[
U = rUA \ast (\nabla pd + \nabla \rho \ast gh)
\]  

(8.29)

where \( rUA = 1.0/UEqn().A() \)

**Continuity Equation**

For steady state compressible flow the continuity equation is:

\[
\nabla (\rho U) = 0
\]  

(8.30)

The solver calculates the continuity error and that is the way it checks the mass continuity. The `initContinuityErrs.H` declares and initializes the cumulative continuity error. Using the condition `maxresidual < convergenceCritirion` in the file `convergenceCheck.H`.

**Equation of State**

The basic thermodynamic properties are calculated using the code `$FOAM SRC/thermophysicalModels/basic/basicThermos/basicThermo.H`, based on the perfect gas assumption \( p = \rho RT \). The code defines a class of properties which are calculated as `volScalarField` based on the model defined in `thermophysicalProperties` file.
8.4 Results

Figures 8.1-8.8 illustrate the velocity field obtained computationally. Several regions in the enclosure are distinct: The two vertical boundary layers on the isothermal walls, the stratified core and the two horizontal boundary layers on the adiabatic walls. The two recirculations in the upper left and lower right corners of the enclosure, observed experimentally, are not present in the computational results.

On the contrary, two flow structures are apparent in the upper right corner of the cavity, where the hot boundary layer is redirected by the adiabatic wall. These flow structures become faster while Rayleigh number increases. Furthermore, for Rayleigh numbers $Ra \geq 1.5 \times 10^8$ one can observe the development of a recirculation in the vicinity of the cold boundary layer that drains momentum out of the boundary layer causing it to decelerate and diminish in size. The same trend was observed experimentally, as discussed in Sec. 5.1.3.

The velocity magnitude obtained from computer simulations is in an excellent agreement with the velocity magnitude measured using PIV. It can be observed that the boundary layer on the hot wall is significantly faster compared to the one on the cold wall. This observation is in agreement with the experimental results.
Figure 8.1: Velocity Magnitude $|U|$ for $Ra = 5.08 \times 10^7$

Figure 8.2: Velocity Magnitude $|U|$ for $Ra = 7.34 \times 10^7$
Figure 8.3: Velocity Magnitude $|U|$ for $Ra = 9.42 \times 10^7$

Figure 8.4: Velocity Magnitude $|U|$ for $Ra = 1.50 \times 10^8$
Figure 8.5: Velocity Magnitude $|U|$ for $Ra = 1.81 \times 10^8$

Figure 8.6: Velocity Magnitude $|U|$ for $Ra = 2.00 \times 10^8$
Figure 8.7: Velocity Magnitude $|U|$ for $Ra = 2.25 \times 10^8$

Figure 8.8: Velocity Magnitude $|U|$ for $Ra = 3.40 \times 10^8$
CONCLUSIONS

An experimental investigation of natural convection in an air filled cube was conducted in the range $5.08 \times 10^7 \leq \text{Ra} \leq 3.40 \times 10^8$. In order to capture the flow structures and measure the velocity of the flow inside the cavity, 2D-PIV was used and data was recorded in the middle plane of the enclosure. The results show the evolution of the boundary layers along the isothermal and adiabatic walls. An interesting feature observed is the development of the two secondary re-circulations in the upper left and lower right corners of the cavity where measurements were conducted. The two regions are anti-symmetric with the one in the upper left corner being initially faster. Other experimental results reported include Reynolds stresses, vorticity and swirling strength. The estimated error from PIV measurements is within $1-2\%$, thus the high accuracy of the results can form experimental benchmark data and can be used to validate CFD codes.

Proper Orthogonal Decomposition was employed to analyze the flow and identify its major components that carry significant amount of energy. Furthermore, heat transfer measurements on the hot wall were conducted and the corresponding Nu number at each Rayleigh number was estimated. A correlation between the two numbers was obtained and compared against other computationally derived correlations in the literature. Deviations were attributed to radiation and non-Boussinesq effects and our findings can be used to validate future algorithms that take the two aforementioned parameters into account. A concurrent computer simulations effort was also conducted.
and the results were compared with experimental data. The velocity magnitude estimated computationally is in a good agreement with the magnitude measured using PIV. Finally, major flow structures observed experimentally are also present in the computational results.
A ESTIMATING CONTRIBUTION FROM RADIATION

In Fig. A.1 we call surface 1 the hot copper plate, the cold copper plate is surface 2 and finally the four plexiglass sides of the cube is surface 3.

Figure A.1: Cube: Surface 1 is the Hot Side, Surface 2 is the Cold Side and Surface 3 is the Four Plexiglass Sides

A.1 Estimating View Factors

The view factors for the heat exchange between the cold and hot plate, i.e. $F_{12}$ and $F_{21}$ are the same because the area of the plates is the same. The view factor for two finite parallel plates is: $F_{12} = F_{21} = 0.1998 \approx 0.2$.

Using the summation rule: $F_{11} + F_{12} + F_{13} = 1$ and the fact that surface 1 is a flat surface so that $F_{11} = 0$, view factor $F_{13} = 0.8$.

By symmetry of the cube $F_{23} = F_{13}$. We can find $F_{31}$ which is also equal to $F_{32}$ by symmetry using the reciprocal rule: $A_1 F_{13} = A_3 F_{31} \Rightarrow F_{31} = 0.05 = F_{32}$.

By applying the summation rule once again we obtain: $F_{33} = 0.9$.
A.2 Blackbody Emissive Power, Radiosity of a Surface & Radiant Heat Transfer from Each Surface

The Blackbody emissive power for $i$ surface can be estimated using the following equation:

$$E_{bi} = \sigma T_i^4$$  \hspace{1cm} (A.1)

where $\sigma = 5.6704 \times 10^{-8}$ Wm$^{-2}$K$^{-4}$ is the Stefan-Boltzmann constant.

The radiosity of surface $i$ is given by:

$$J_i = \epsilon_i E_{bi} + (1 - \epsilon_i) \sum_{k=1}^{n} J_k F_{ik}$$  \hspace{1cm} (A.2)

where $i = 1, 2, ..., n$ is the number of surfaces and $\epsilon$ is the emissivity of a surface. The values used in this calculation are $\epsilon_1 = \epsilon_2 = 0.07$ and $\epsilon_3 = 0.86$.

The radiant heat transfer from each surface is obtained using the following equation:

$$\dot{Q}_i = \frac{\epsilon_i A_i}{1 - \epsilon_i} (E_{bi} - J_i)$$  \hspace{1cm} (A.3)

Table A.1 summarizes the estimated radiant heat transfer from each surface and what percentage of that corresponds to the overall heat input as measured by the heat flux sensors.
Table A.1: Radiant heat transfer from each surface

<table>
<thead>
<tr>
<th>Ra</th>
<th>$\dot{Q}_1$ (W)</th>
<th>$\dot{Q}_2$ (W)</th>
<th>$\dot{Q}_3$ (W)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.08 \times 10^7$</td>
<td>0.27</td>
<td>-0.25</td>
<td>-0.001</td>
<td>12.5</td>
</tr>
<tr>
<td>$7.34 \times 10^7$</td>
<td>0.42</td>
<td>-0.39</td>
<td>-0.002</td>
<td>12.5</td>
</tr>
<tr>
<td>$9.42 \times 10^7$</td>
<td>0.57</td>
<td>-0.52</td>
<td>-0.003</td>
<td>12.7</td>
</tr>
<tr>
<td>$1.50 \times 10^8$</td>
<td>1.16</td>
<td>-0.98</td>
<td>-0.011</td>
<td>13.1</td>
</tr>
<tr>
<td>$1.81 \times 10^8$</td>
<td>1.63</td>
<td>-1.32</td>
<td>-0.020</td>
<td>13.4</td>
</tr>
<tr>
<td>$2.00 \times 10^8$</td>
<td>2.02</td>
<td>-1.57</td>
<td>-0.028</td>
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</tr>
<tr>
<td>$2.25 \times 10^8$</td>
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<td>-2.01</td>
<td>-0.044</td>
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</tr>
<tr>
<td>$3.40 \times 10^8$</td>
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<td>-3.40</td>
<td>-0.139</td>
<td>15.1</td>
</tr>
</tbody>
</table>
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