ROBOTS AS LANGUAGE USERS:  
A COMPUTATIONAL MODEL FOR PRAGMATIC WORD LEARNING

BY

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DISSERTATION

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The development of machines capable of natural linguistic interaction with humans has been an active and diverse area of research for decades. More recent frameworks, such as Cognitive Robotics, have been able to make progress on many long-standing problems in computational modeling of language acquisition – like that of symbol grounding – through the application of the principles of embodied cognition. Many of these systems have focused on modeling grounded word learning through statistical mappings between various sensor modalities, such as speech-to-vision or speech-to-motor control. However, the entire body of such systems has only been able to capture a tiny fraction of the developmental robustness or representational diversity observed in even the youngest of human word-learners. Children are capable of learning words in situations of extreme ambiguity, leveraging a variety of contextual knowledge to infer the targets of adults’ references. And unlike children, few cognitive robotics systems have any kind of understanding of the purpose of words outside of reference. The core premise of the following thesis is that this gap is, in part, due to computational models which ignore the communicative and intentional (i.e. pragmatic) aspects of language.

To address these issues, a computational framework for the learning of perceptually-grounded word meanings is presented. Our model is based on a representation of language as a useful behavior, embedded within an intentionally structured social interaction. Using techniques for inverse planning and control, the algorithms we have developed seek to understand the goal or purpose driving the behaviors of the interaction. We describe the application of these techniques to a set of human-robot interaction experiments, modeled after development studies demonstrating specific skills of children in the learning of word meanings under referential ambiguity. Through these experiments, we show how our framework allows the robotic agent to acquire knowledge about the physical and social task structure underlying the interaction, and leverage this in order to learn word meanings in many different cases of ambiguity. These include many novel situations where the robot
must make inferences due to the goal-directed actions of the speaker, or even knowledge of its own embodiment and potential role in the interaction. We will show finally how our robotic platform can be made to realize this role, actively taking part in its own learning experience, and begin to see language as something useful.
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The development of systems through which computers and other artificial agents are able to use language in the way humans do has been an active area of study for many decades. Early work focused on the recognition of human speech through the application of statistical methods and models, trained on large corpora of expertly annotated speech data. While automatic speech recognition (ASR) systems have seen incremental and steady progress over the years, most still remain inherently limited in their capabilities. Beyond lingering issues of robustness to noise, speaker variations, and poor accuracy for general word recognition tasks, ASR systems largely capture only the phonological and syntactic aspects of natural language. These systems for the most part have no understanding of the meaning of what is being said, or purpose of what is being said; i.e. the semantic and pragmatic aspects of language. Without these components, realistic linguistic use and ultimately linguistic interaction between humans and machines remains out of reach. Historically, extensions to basic ASR systems have attempted to integrate these aspects through a similar paradigm: rudimentary symbolic representations of meaning or dialog, constructed by human experts, are attached to the text strings translated from speech data. These approaches have proved as brittle and limited in capability as the symbol-manipulation systems upon which they were based.

One proposed explanation for the limitations of this paradigm comes from the embodied cognition hypothesis, which states that human cognition (and cognition in general) is a product of the functional and developmental processes of its physical embodiment. This embodiment includes not only the parts of the brain associated with high-level cognition of the type used by symbol-manipulation systems, but also those structures supporting sensory and motor capabilities. Furthermore, this embodiment is situated within a physical environment with which it is constantly interacting. Perception, action, and cognition are all part of a continuous, interconnected cycle that develops in real-time. Under this view, linguistic representations must be
embedded within this cycle, and are subject to the constraints, both physical and developmental, that are imposed by the agent’s embodiment. Items such as linguistic symbols are no longer preordained by the system designer and independent of the agent’s embodiment, but rather are grounded in perceptual representations specific to the agent’s sensorimotor abilities, and are formed continuously, as the agent interacts with and experiences its environment.

For artificial agents, this requirement of sensorimotor experience is often made achievable through the use of a robotic platform. The field of cognitive developmental robotics (CDR) is one such area in which principles and ideas of embodied cognitive development are applied to the construction of computational methods for artificial agents [1, 2, 3]. CDR generally values methods that are centered around biologically and developmentally feasible algorithms for learning and adaptation, rather than the traditional approach of expert-guided training of complex models using large corpora of data. With respect to language, approaches in this area have focused on sensorimotor integration and processes for acquiring the core components of the linguistic faculty: speech, syntax, and semantics. The rich set of sensorimotor information afforded by many robotic platforms has allowed for striking improvements at the level of semantics in particular.

On this specific topic, CDR has already proven its usefulness in grounded word learning. The challenges of grounded word learning include issues of both the structure of perceptual (sensorimotor) and conceptual (semantic) representations, and how these two components come to interact. Research over the past decade has produced robotic systems which are able to learn the meanings of words for objects [4, 5, 6], events [7], and actions [8, 9, 10] from real sensorimotor data, often in ways that capture aspects of the statistical processing capabilities seen in humans. Some of these systems are even capable of exploiting acquired linguistic knowledge to further their perceptual, cognitive, or interactive capabilities in ways that are far beyond the scope of traditional speech and language processing systems. However, when compared with the abilities of human language learners, these achievements appear to cover only limited portions of a human’s general word-learning competence, and do so in a piecemeal fashion. Most systems focus on learning words of a single type or category, and almost no progress has been made in representing how a word’s use is tied to its meaning. In addition, learning algorithms are driven primarily by statistical processing power, or a number of various domain-specific heuristics used to mitigate perceptual confusion. There has been little work in the direction of developing frameworks that
can more easily generalize across word categories or learning principles, and that will allow robots to interact linguistically with humans at a level that comes anywhere near even the most basic language users.

1.1 Current Issues with Early Language Acquisition Models

Given our basic intuitions about the immense complexity of the human language faculty, it is hardly surprising that even the most advanced computational models of language acquisition are unable to compete with the abilities of human learners. But what about the very youngest language learners? At their first 50 words, children have learned words for a wide variety of objects, events, and attributes (e.g. nouns, verbs, prepositions, adjectives, etc.), as well as a number of words that do not “stand” for anything at all (e.g. “hello”, “please”). They understand that language is used not only to reference and describe, but is also used to command and to question. They also understand that language is something that occurs within a social interaction that is surrounded by context and is richly structured. They are able to leverage this understanding in order to learn word meanings in situations where referents are non-ostensive or are highly ambiguous. Children exceed current systems in the domains of both the “what” and the “how” of early word learning.

Issues relating to both the kinds of things children can learn the words for, as well as the kinds of things children can use words to do (i.e. the “what”), we consider to be issues of the representation of meaning. Current systems have focused largely on meaning as “words for things”. These things have ranged from concrete objects [5] to actions [10], to spatial relationships [11], and attributes [12]. In each case, the representational structure of grounded meaning has focused on pairings between some sensory modality (vision, action) and speech. These purely referential representations of meaning are fundamentally dyadic, and consider only the speaker and the world s/he is describing. However, a significant part of a child’s early lexicon [13, 14] is composed of words like “hello”, “please”, “yes/no”, which have inherently social, or triadic meanings, involving the speaker, listener, and environment. Furthermore, the incorporation of a well-defined triadic interaction structure is crucial for models to be capable of representing and understanding the imperative and interrogative aspects of linguistic utterances. Such explicit representations of use and communicative function have been left largely
unconsidered by the vast majority of computational models to date.

The importance of understanding this speaker-listener interaction is even more apparent when considering the developmental disparities between children and current computational methods. The phrase “developmental disparities” is used here to refer to differences relating to the kind of situations in which the meaning of words can be successfully learned, and the processing mechanisms used to do so (i.e. the “how”). Current systems learn primarily in rigid interaction environments, usually with a tutor presenting a word to the learner who assumes the most visually salient object or event to be the intended referent. Such situations of unambiguous reference are not necessarily the norm for real-world child learners, especially those outside of Western, white, middle-class households [15] (even for Western, middle-class households, this kind of interaction accounts for only a fraction of the whole [16, 17]). The reality is that children are incredibly skilled at learning the meanings of words in situations where the intended referent is highly ambiguous, or is altogether not present. Understanding the exact nature of these skills is a long-standing problem in the field of language acquisition, and approaches to resolving referential ambiguity in artificial systems have typically involved the application of various preordained heuristics (e.g. mutual exclusivity [18]), and statistical processing to integrate information across experiences. Statistical techniques in particular have been favored in computational models, and have been used to moderate success in dealing with some aspects of referential ambiguity [4].

However, these computational methods have focused predominately on learning word meanings by measuring statistical coincidence, in ways that often assume batch processing capabilities and memory capacities far beyond the realm of biological or developmental plausibility. In addition, the most commonly applied learning heuristics have favored narrow domain-specific principles that do not reflect well our current understanding of the wide variety of information and skills children use to learn words under ambiguity [19]. Many of these theories of early language acquisition are based on evidence which suggests that children leverage a rich body of knowledge about the motivations and actions of the speaker [20, 21], contextual information about the scenario [22], and the social nature of the interaction between the speaker and listener [23, 24] in dealing with ambiguous referents. Integrating such a pragmatic competence might allow an artificial agent to resolve ambiguities in ways that are not only more developmentally plausible with respect to memory and processing capabilities, but are also capable of exploiting a wealth of contextual information that most current frameworks
In examining the nature of the disparities between real (human) and artificial (computational) language learners, a common theme emerges. In both categories of representational and developmental disparities, a primary factor seems to be the failure of current approaches to explicitly model linguistic interaction as an inherently social, communicative act. Under a framework where these ideas were included, a speech utterance would be treated as an action taken by the speaker to influence the listener — a premise which both parties would be assumed to understand and account for. Modeling this *pragmatic* aspect of language is the focus of the work outlined in this thesis.

1.2 Bridging the Gap

While it might seem perfectly obvious that language is an inherently social phenomenon, in many embodied systems little thought has been given to this aspect of the language learning process. Bridging the gap between what even the earliest child learners are capable of and what current CDR systems can do will require models that are *triadic* in nature — that is, they explicitly include both the speaker, listener, and their interaction context (environment). A visual representation of such a triadic interaction is shown in Figure 1.1. Instead of simply trying to augment the existing state-of-the-art models, the proposed approach will start by completely re-framing the language-learning problem in terms similar to the concept of a “language-game” as it was proposed by Wittgenstein [25].

In such a representation, language users are envisioned as players in a game, which could be competitive or cooperative in nature. The players make moves or sequences of moves in order to achieve some goal or reward. The goal can be thought of as the *intent* of an action or actions, both of which may physical, social, or communicative in nature. This intent might be to complete some physical task, perform a particular action, or elicit the attention of someone else to an object or an event — among many other potential purposes. For a listener who wishes to cooperate and help speakers to realize their goals, s/he must estimate the speaker’s *intent* based upon their utterance. Under this view, words derive their meaning from their communicative function. In a broad sense, this function is to get the listener to recognize the speaker’s intent with its exact meaning grounded in the specific state of this intent, an argument made most famously by Grice
The function of the speaker’s utterance is for the listener to recognize the intended object/event/action of reference, and potentially to help fulfill his/her request. This concept of intent is central to the aspect of language known as *pragmatics*, and serves as a foundational element to the pragmatic-based model of word learning proposed in this thesis.

For the purposes of the following work, *intent* shall be defined as a mental state of an agent reflecting a goal the agent hopes to realize through an action or sequence of actions, either linguistic or non-linguistic in nature. The power of such a formulation of linguistic interaction may be easy to understand in some aspects more than others. By explicitly acknowledging that words are used to “do” something, we can extend the meaning of words beyond standard lexical semantics, to something that encompasses its communicative function as well. This also provides the intuition that meaning is something that depends heavily on the task or social interaction in which it takes place. The less obvious consequence of this is that the concept of intent can provide us with a more principled way of integrating contextual information for the purpose of resolving ambiguity in the word-learning problem.

As previously discussed, children are able to make this inference based on their knowledge of the exogenous and endogenous motivations of the speaker, the speaker’s goal-directed actions, estimates of the speaker’s internal model of the world, and even estimates of the speaker’s model of the listener’s own mind [27]. This knowledge is the *common ground* [28] shared by the speaker and listener, and is a key to inference of intent. By viewing physical and communicative actions as simply two general ways to achieve some goal or
intent, knowledge acquired about one can be used to constrain the learning problem in the other. Learning the meaning of words is just one aspect of the overall process of construction and adaptation of this common ground during social interaction.

1.3 Purpose and Contribution of This Thesis

The fundamental goal of this thesis is to build a general pragmatic-based language learning framework that begins to bridge the gap between the abilities of current cognitive robotics models, and the actual abilities of the youngest language learners. We have described two primary factors contributing to this gap: representational disparities, what kinds of meanings can we learn; and developmental disparities, the ways in which we are able to learn. We have also discussed at a conceptual level a pragmatic framework, based around an intentional agent, which attempts to address some aspects of each of these issues. As it is unlikely that any computational model developed herein would be able to emulate a child’s word-learning abilities with complete accuracy, we will focus instead on capturing a limited set of developmental abilities demonstrated by early word learners that are still lacking in current computational frameworks. We will also explore some basic ways in which the pragmatic model can be used to stretch our representations of meaning to include pragmatic aspects, such as commands and requests, in addition to reference.

Generally speaking, the goal of the model presented here is to be capable of learning perceptually grounded meanings of words for basic objects and/or events, specifically in cases of referential ambiguity or non-ostentation, using various inferential abilities seen in humans. These include both the cross-situational statistics and lexical contrast techniques already seen in the computational literature, as well as the inference of intent from goal-directed actions, understanding of task structure, and knowledge about physical constraints. Furthermore, we seek to construct our computational framework in such a way that allows our agent to reason about its role in the interaction, use this ability to aid in actively resolving ambiguity, and through this, begin to understand the functional aspects of word meaning.

In addition to these specific experimental goals, we also impose a set of guiding restrictions on the computational models we use to keep them in line with basic principles of cognitive development. First, preference will be given to using techniques and algorithms that learn in an online
manner whenever possible, with as little supervision as possible with respect to model structure. Second, the models and algorithms used should be designed to be ultimately evaluated in real-world experimental human-robot interaction scenarios, in which noisy sensor data is the primary input and the only truly observable quantity.

To this end, we present a computational framework, based on statistical techniques of decision and control in addition to more traditional methods for speech and language processing, for the acquisition of perceptually grounded word meanings using pragmatic principles. By modeling language as a purposeful behavior that is embedded within a social interaction, we develop and apply techniques for inverse planning to understand the goals or intents that drive human behavior, which ultimately enables our agent to capture the kind of pragmatic inference abilities that are crucial to child word learners. Through its application to a set of human-robot interaction experiments, we intend to demonstrate the following contributions of this framework to current body of cognitive robotics systems for grounded word learning:

- A set of computational models and algorithms for basic grounded word learning that is inherently pragmatic and triadic.
- The ability to learn word meanings in situations of referential ambiguity from novel contextual information about the intentional structure of interactions.
- A representation of linguistic meaning that is capable of incorporating aspects of a word’s communicative function or use.
- The ability of our agent to apply understanding of functional aspects of language in order to actively guide process of word learning.

1.4 Thesis Organization

The rest of this thesis will proceed as follows. Chapter 2 contains a review of the relevant background material from the fields of cognitive robotics, developmental psychology, and machine learning, which includes an overview of topics from stochastic planning, as well as game theory. Chapter 3 details the core computational model and learning algorithms that comprise the pragmatic engine. The framework for integrating perceptual capabilities into the pragmatic model is presented in Chapter 4. The overall cognitive
architecture to be used in the human-robot interaction experiments, which includes the integrated pragmatic-perceptual framework, as well as various low-level signal processing and support algorithms, is outlined in Chapter 5. Chapter 6 describes the set of human-robot interaction experiments, details their setup, and presents and analyzes their results. In this chapter we also compare our work to other related research, and we discuss some of the limitations and issues with our model. Finally, the contributions of this thesis, and potential paths for future research are discussed in Chapter 7.
CHAPTER 2

REVIEW OF RELATED RESEARCH

2.1 Embodied Systems for Linguistic Interaction

The view that understanding the cognitive abilities of humans means also understanding the physical systems and processes that underly them was not lost on many of the early pioneers in artificial intelligence. In his 1950 paper [29], Alan Turing proposes that in order to actually create a machine capable of passing the Turing Test, a developmental approach might be preferable: “Instead of trying to produce a programme to simulate the adult mind, why not rather try to produce one which simulates the child’s?” He goes on to suggest that this learning could be achieved through use of something like an embodied agent. Around the same time, Norbert Wiener helped to shape the field of cybernetics around the study of how learning and feedback were supported and limited by the structure of their biological systems, i.e. their bodies [30, 31]. For Wiener and others who understood the importance of embodiment, cognition is not a set of fixed, isolated abilities, but rather a process with many different and highly interconnected aspects, which all continually adapt together with feedback from one another and the environment.

Under such a view, traditional ASR systems are limited in their linguistic abilities as far as they are limited in their general cognitive abilities. For many who consider cognition to be embodied, it would only be obvious that a system without perceptual representations of the world, such as vision or motor function, would also be incapable of effectively processing the semantic aspects of language. Systems without any kind of social or affective sense would likewise be unable to operate with humans at a pragmatic level. Even for systems endowed with fixed corpora of semantic and pragmatic knowledge by experts, the extremely limited scope of their understanding relegates them to narrowly defined application domains. Therefore, if we see that the physical disparity between humans and machines may be in some part responsible for their cognitive disparities, a reasonable approach might
be to first bring the embodiment of our artificial agents closer to that of humans.

2.1.1 Cognitive Developmental Robotics

Cognitive developmental robotics (CDR) is the result of applying such a philosophy to real-world systems. However, CDR does not simply entail expansion of the previously discussed expert knowledge bases to include information about additional sensory inputs. Rather, it takes into account the dynamic properties of adaptation and learning that are every bit as fundamental to the formation of the cognitive faculty as its physical form. CDR focuses on creating artificial cognitive capabilities that emerge through the gradual, continuous developmental processes of learning and adaptation, structured by the agent’s environmental and social interactions, as experienced through the sensorimotor system (for a more general overview of CDR and other embodied approaches, see [1, 2, 3]).

Even as a relatively new area of research, CDR systems have already been able to emulate a number of very basic and very important cognitive functions, which almost every human child masters with little effort, but were not considered under traditional AI paradigms. These include core abilities like joint attention [32], the acquisition of reaching and grasping skills [33], and the representation and learning of affordances [34, 35], to name only a small fraction. While these skills may seem extremely rudimentary in comparison to the highly developed and complex faculty of adult language, to those following a paradigm of embodied cognition, the latter is only made possible by the former. Without these core capabilities, the scope of natural language interaction with machines will continue to be limited to passive speech-to-text transcription devices, with no sense of what language means (semantics), or how it can be used (pragmatics).

Because of the new possibilities that its approaches offer, a primary subject of interest in the area of CDR has become language, and in particular, the topic of language acquisition receives a great deal of attention. The advantages of a more complete sensorimotor system and real-world physical embodiment have allowed researchers to begin exploring representations of semantic and pragmatic aspects of language, typically off limits to speech-only ASR systems. Machines now have the opportunity to be active participants and learners in the same real-world environments and social interaction scenarios as the children they seek to emulate.

One of the most significant capabilities of the CDR approach is that it has
allowed researchers to address the long-standing problem of symbol grounding. In the context of language, the problem of symbol grounding is fundamentally one of how linguistic symbols acquire their meaning [36]. The history of artificial intelligence has been dominated by approaches where the meaning of a linguistic symbol was itself grounded (by an expert) in another symbol upon which some fixed set of logical operations could be performed [37]. But the grounding of linguistic symbols in other kinds of symbols simply leads to problems of infinite regress, as most famously pointed out by John Searle in his *Chinese Room* thought experiment [38]. According to those taking an embodied view of cognition, meaning instead should be grounded ultimately in perceptual experiences, supported by a sensorimotor system, as is thought to be the case in humans.

2.1.2 Embodied Platforms for Natural Language

Initial experiments using embodied platforms to explore the issue of symbol grounding focused primarily on the association between sets of basic objects and the words describing them. The fundamental practical issues were the construction of perceptual (usually speech, vision, or action) representations from sensory data, and the learning of associations between these perceptual categories. Experimental scenarios consisted of an adult tutor presenting an object to the robot learner for visual inspection while simultaneously giving the word for the object as speech [39, 5, 40, 7]. Associations were acquired gradually, mostly by machine learning techniques based on statistical models. For these experiments, the robot was largely a passive and motionless agent, requiring an embodiment no more complex than a camera and microphone.

After the accomplishments of these initial systems, new frameworks and experiments were developed using representations of motor function to expand symbol-grounding abilities to include words describing actions and spatial relationships. One of the earliest experiments by Sugita and Tan [41] involved a mobile robot that was able to ground a small set of action and color words, and use them to compose simple two-word sentences with a recurrent neural network. In another series of experiments by Takano and Nakamura [42, 43], the authors developed a system through which a set of motion primitives were autonomously extracted from motion capture data and incrementally associated with linguistic symbols. Following the initial success of these and other similar experiments [8, 44, 45, 46], there has been a steadily increasing interest in using robots to study the special interaction
between language and action during cognitive development [47]. Indeed, recent results from neuroscience and psychology have demonstrated the close relationship between internal representations for language and action, as in the case of the discovery of so-called motor neurons [48, 49] and observation of Action Compatibility Effects (ACE) [50].

One particular example of experiments exploring the interaction between action and language representations are those dealing with grounding transfer. In these experiments, basic action-word groundings are exploited to transfer meaning to new words describing complex behaviors, without the need for direct representation or even demonstration of the behavior. In one experiment, Cangelosi [8] showed that an artificial agent could use previous knowledge of action-word pairings to learn multi-step actions from verbal instruction by transferring the existing groundings of component words to the new action. Work in our own lab [10] improved on the previous artificial neural network-based approach by using a generalized, dynamically expanding perceptual representation built on stochastic models, which was able learn both compositionally and hierarchically organized behaviors.

Despite such incremental improvements in representational complexity, most of the artificial agents produced have focused primarily on learning words for objects (and other vision-related concepts like shape, color, etc. [51, 12]) and basic actions. More recent work has succeeded in learning additional words relating to more abstract concepts such as affordances [35] and affected behaviors [44] of objects, as well as spatial relationships [11]. Words for which our notions of meaning are less easy to connect to specific perceptual symbols, such as “no”, are only beginning to be explored by researchers in this area [52]. Similarly, there has been little work in exploring the relationship between word meaning and use — a concept which will have to be an intrinsic feature for any future model hoping to represent functional utterances like negation.

Beyond the addition of richer and more complete sensorimotor information, other work has sought to bring more realism to the actual learning scenarios used in such experiments. Most, if not all, of the work mentioned so far was designed for and evaluated in contexts where the agent always knows what object the sample word is referring to. However, in real-world situations, young children are able to quickly and accurately learn words in a wide range of scenarios where referents are highly ambiguous. This has produced many different computational approaches which primarily have attempted to integrate heuristic principles for resolving ambiguity in very specific scenarios, or have used large training corpora in order to glean sta-
One of the most popular among these is the use of so-called “cross-situational statistics” [4]. In these approaches, observations across multiple episodes are collected, and the agent learns word-referent associations by computing the statistical regularities of word-referent co-occurrences. Accuracy is further improved through the integration of various heuristics thought to be employed by human learners, such as information about gaze direction and prosody [4] or the principle of mutual exclusion [53]. However, many of these methods rely heavily on batch learning techniques, with memory and processing requirements that may be beyond those exhibited by early learners. Additionally, social information served generally as a “spotlight” to improve accuracy of referent inference.

While this information is indeed an important tool employed by early word learners, in these examples the social dimension of language has in fact become disembodied. This is because the agent does not see itself or the speaker as an active, social agent, and does not understand the pragmatic, communicative aspect of the interaction. Some attempts at incorporating this interactive aspect have featured robot word learners who ask questions to resolve referential ambiguity [54]. Others have used models including representations of the speaker and listener’s beliefs as a way of integrating pragmatic information in an utterance understanding task [55]. Even so, the agents in these experiments still lack an explicit understanding of the goal-directed or intentional nature of linguistic utterances. Experiments by Frank [56] attempted to improve on previous associative methods [4] by using explicit models of the speaker’s referential intent to replicate observed phenomena like mutual-exclusion [18] and fast-mapping [57]. But producing such results appears to be more dependent on designer-imposed learning biases and memory/processing requirements, as the model and learning algorithms still do not give proper treatment to the dynamic, interactive aspects of real-world language learning. More recently studies [58, 53] have highlighted the importance of capturing both the dynamic, online processing aspects, as well as long-term statistical regularities, in models of word learning.

2.1.3 Language, Action, and Intent

In all of these computational models, the notion of intent has either been left out entirely, or implemented in a way that strips its intuitive role in understanding purposeful behavior. This is due in large part to the fact that
the intent behind language use is rarely just use itself, but rather something that is often embedded in a social interaction with larger goals. For humans, the task of understanding language appears to be deeply connected to the task of understanding action in general, an idea that the embodied approach of CDR is uniquely suited to explore. And indeed, many techniques and experiments have recently started to explore this connection in the context of tasks requiring joint human-robot action.

A framework developed by Taguchi [59] leverages a more complete model of intent in a word learning task. Their understanding of both agents’ beliefs and utterances as goal-directed communicative actions allows them to learn meanings for functional words like “what” and “which”. Unfortunately, their framework relies on explicit supervision in the form of corrective feedback from a human tutor, and does not appear to deal with intentional ambiguity. In other work by Lopes, Cederborg, and Oudeyer [60], an explicit model of goal-directed behavior is used to learn the meanings of such feedback signals in the context of a human-robot interaction. In both this and subsequent experiments [61], this communication model is learned simultaneously with the structure of the physical task that it is trying to describe. For these experiments, the focus is primarily on the acquisition of the word meaning models through continuous feedback with sometimes noisy or incorrect signals. The work that will be presented uses many of the same kinds of techniques and ideas, but focuses more on the learning of larger task structure models, and the use of these models to learn word meanings in more ambiguous and infrequent input.

Less frequently studied is the use of models that understand communicative actions as goal-directed behaviors, and the use of these models to study the acquisition of word meanings. As we will see in Section 2.2, such an understanding appears to be critical to the word learning abilities of children. The construction of computational models for teleological, or goal-directed understanding of language has recently begun to pick up interest [62, 63], but practical algorithms and implementations of these ideas for use in human-robot interaction experiments is something that remains to be seen. However, some general frameworks have been proposed, such as Pezzulo’s dynamic Bayesian network-based pragmatic engine [62], which will be used in this work as a basic starting point for the development of our own model.

Finally, it is also very much worth mentioning a number of cognitive robotics architectures and experiments for which the attribution and understanding of mental states (such as beliefs or intentions) in other agents is a
critical component, but do not focus on language acquisition in particular. The ability of an agent to make this attribution and reasoning is usually referred to as *Theory of Mind*, a term with a long tradition of use in the study of philosophy and psychology. Perhaps the most relevant among these is Scassellati’s work on development of a *Theory of Mind* module for use on the Cog humanoid robotic platform [64], which itself is based on ideas on the topic of Theory of Mind put forth by Leslie [65] and Baron-Cohen [66]. Central to his framework is the perception and understanding of gaze information, something that will also play an important role in some of our own human-robot interaction experiments presented in Chapter 6. Other cognitive architectures, such as Demiris’s HAMMER architecture [67] or Bicho, Louro, and Erlhagen’s work based on Dynamic Neural Field representations, have shown success in applying ideas about social and mental reasoning for the understanding of actions and behaviors in real-world interaction experiments.

It is hoped that this review has served to underscore the most significant issues involving current CDR approaches to early word learning, as they were presented in the introduction. To restate, our view is that they are of fundamentally two types: those relating to representations of meaning — *what* kinds of words can be learned and how they can be used; and those relating to acquisition of meaning — primarily the problem of *how* we can learn words in noisy, ambiguous real-world situations. The goal in Section 2.2 will be to briefly review the developmental literature relating to early language learning, and explore how ideas from social and pragmatic theories of early language acquisition might be used to structure and guide development of a new computational framework.

### 2.2 Pragmatic Models of Language Acquisition

One example of traditional reasoning behind how children come to learn the meanings of words was given by Augustine [68], later to be used by Ludwig Wittgenstein in framing his own foundational work on the philosophy of language [25]:

> When they called anything by name, and moved the body towards it while they spoke, I saw and gathered that the thing they wished to point out was called by the name they then uttered; and that they did mean this was made plain by the motion of the body, even by the natural language of all nations expressed by
the countenance, glance of the eye, movement of other members, and by the sound of the voice indicating the affections of the mind, as it seeks, possesses, rejects, or avoids. So it was that by frequently hearing words, in duly placed sentences, I gradually gathered what things they were the signs of and having formed my mouth to the utterance of these signs, I thereby expressed my will.

Wittgenstein notes that the conceptualization of words as merely standing for things does not begin to encompass all of the things we know as “language”. Such a view can not account for words that do not stand for specific things (e.g. “this/that”, “yes/no”), and ignores the effect that aspects like use and context have on a word’s meaning. We also know that perfect, ostensive teaching is not representative of the ambiguous situations in which children often learn words.

Issues of conceptual representation and referential ambiguity are certainly not ones faced by computational modelers alone. They are long-standing open problems in the fields of developmental psychology and linguistics, and their extensive study in these areas has produced numerous competing theories about their fundamental nature. Many of the approaches in CDR have been based on purely associative theories, or expanded theories of association guided by various “principles” and “biases” [69]. These principles are largely related to the learning of names for objects and other nouns, a trend that has been reflected in the computational models discussed in Section 2.1. Another competing class of ideas are the so-called “social-pragmatic” theories of language acquisition [27], which focus on how the social and pragmatic aspects of communicative interaction are understood and leveraged by learners. These treat language acquisition as just one particular aspect of a more general pragmatic competence, rather than an isolated cognitive faculty with its own special rules and principles. This aspect is obviously appealing to robotics researchers pursuing an embodied approach to language acquisition.

Social-pragmatic theories sometimes differ in various details, but are primarily focused around the core ability of intention reading. At the heart of this skill is the idea that a child understands human behavior, including communicative behavior, to be goal-directed (i.e. intentional). These goals might be to influence physical states of the environment or perhaps mental states of other social actors (and consequently their actions). In the case of linguistic communication, the purpose of an utterance is to get the in-
terlocutor to recognize one’s own intentional state [26]. When choosing an optimal action or utterance, the speaker must take into account the assumed knowledge, beliefs and motivations of the listener, who likewise takes into account similar information about the speaker in interpreting these utterances. In order to see how these ideas can be used to develop an improved model of early word learning, we begin exploring them in the context of the issues of representation and acquisition in early word learners.

2.2.1 Our First 50 Words

As stated, the current focus of language acquisition research in cognitive robotics has been primarily on learning names for objects, actions and events, with very little attention given to functional/social words. Many have justified this focus in the design of such systems by parroting arguments about the actual distributions of word categories observed in a child’s early lexicon. However, many studies have challenged these estimates [14], pointing to biases introduced when experimenters consider only utterances that are referential in nature. Numerous studies have shown that social and functional words such as “hello”, “yes/no”, etc., constitute a significant fraction of a child’s first 50 words [13, 14].

These types of words present an additional question that can not be addressed by nearly any current artificial system: how are meaning and use related? Even if any current systems were actual active users of language, it would have almost certainly been as a tool of reference. In fact, reference or declaration is only one of the ways that young children use language. They also use language to issue commands [70] and ask for guidance and information [71]. Therefore, one response to our question, possibly made most famously by Wittgenstein [25], is that meaning is use. Within a social-pragmatic framework, we have noted that utterances are made to get the listener to recognize the speaker’s intentional state. The intent itself could be to get the listener to share attention to an object (reference), to get the listener to take an action (command), or to get the listener to share the state of his/her own beliefs (question). As will be demonstrated in the experiments in Chapter 7, the ability to understand these kinds of uses can also affect the ability of a word learner to acquire meanings of words.
2.2.2 Early Word Learners’ Use of Social-Pragmatic Principles

With respect to referential ambiguity, “Constraints and Principles” approaches have focused on crafting a core set of heuristic principles that can explain a number of observed scenarios where children seem to effortlessly and accurately resolve this ambiguity. However, these heuristics are language-specific, often apply to only a subset of words, and are unable to adequately explain a number of situations where children resolve ambiguity even when the constraints do not apply. The social-pragmatic approach gives the explanation that a child’s ability to resolve referential ambiguity comes from his/her general ability to infer the speaker’s intent based on shared knowledge of each other’s beliefs, the state of the world, and contextual information about the interaction. The following examples show how social-pragmatic explanations of various observed phenomena compare to competing theories, and cases where pragmatic theories can offer explanations where others can not.

One such observation is the apparent use of lexical contrast principles to infer the proper referent [18]. In these scenarios, the child is presented with two objects — one with a known label and one without — and the speaker gives a previously unknown label. Results show that the child often maps the new word onto the object whose label is not known, a behavior that many explain as the result of an innate language-specific lexical contrast principle. This can also be explained as a result of a general pragmatic competence, whereby the child reasons if the adult intended the child to attend to the known object, s/he would have used the known label, based on their shared understanding of the child’s current linguistic knowledge.

The developmental literature is also replete with examples of ways in which children use their understanding of language users as social actors to learn the meaning of words — ways which are difficult to explain without appealing to pragmatic principles, and often times, embodiment. One clear example comes in a study where an adult used a novel verb before taking two separate actions [72]. For each action, the adult signified whether it had been a mistake (“Oops!”), or had gone as intended (“There!”), with children learning the verb in reference to the intended action. A similar result was achieved in the context of a finding game [21], in which an adult referenced an unseen object, hidden in a row of buckets, in advance of their search for the object. The adult then proceeded to pick objects out of the buckets, frowning at objects that were not the intended objects, stopping and smiling when the intended object was found. The children were found
to learn the correct referent, regardless of how many distractor objects were attended to first. In both of these examples, the use of intent by social-pragmatic theories offers a better explanation than the use of spatial or temporal proximity provided by associative accounts.

Learners have also been seen to use other information to infer intent in cases where it is not as explicitly provided as in the previous examples. Children were thought to be applying knowledge of an adult’s motivation or preferences in an experiment where the mother, who had previously interacted with three new toys in the presence of her child, was taken out of the room, and while a fourth, novel toy was presented to the child. Upon re-entering the room, the mother excitedly produced a label, which the child took to refer to the novel toy. The child made this inference on the basis of his knowledge that the mother would only act excitedly toward the object which was new to her [73]. Other kinds of information for inferring intent are more transient, such as knowledge of the speaker’s attentive state. A speaker can use his/her attention to highlight the intended object/event of reference [74], relying on foundational skills of joint attention.

Finally, children can also infer intent by understanding their active role in helping speakers achieve their general goals. In one example experiment, an adult first readied a toy for the child to play with, then presented the child with a novel object while shifting gaze between the child and the object. After saying “Widgit, Name”, the child interpreted the utterance as a request for him/her to use the new object to play with the toy. This was done in contrast to a scenario where the toy was not first conspicuously readied for play, and the child learned the word to simply refer to the novel object [23]. In more recent experiments, children were shown to be able to use information about the relative physical constraints of the themselves and the speaker to reason between an ambiguous object requested by the speaker [75, 24].

2.2.3 Constructing a New Language Engine

But how are these results important, and how can they guide us in the construction of a new, pragmatics-based computational framework for early language learning? We see that a wide variety of contextual information and shared knowledge about the social interaction are required to learn the meaning of words in cases of ambiguity. But whereas purely association-based accounts of word learning — and the majority of computational models — integrate limited, selective bits of this information in specific ways,
the pragmatic explanation suggests another organizational principle: intent. Understanding behavior as being produced to achieve a particular goal, children are able to leverage knowledge about the elements of the task structure that influence the specifics of that behavior (e.g. the goal itself, physical constraints, speaker/listener beliefs and preferences) in order to infer a speaker’s intent even when these behaviors are ambiguous.

These ideas suggest that any computational model that wishes to exploit these pragmatic principles in order to learn the meanings of words, must also be capable of representing and learning about the structure of the interaction in which it takes place. For children, these interactions, sometimes called “frames” [28], often include everyday, routine activities like diaper changing, feeding, and playing games. Frames are usually established well in advance of the word-learning they facilitate. Our pragmatic engine will be developed around a similar principle of first learning the structure of the interaction, and then using this knowledge to help resolve ambiguity during the process of word learning.

2.3 Mathematical Tools for Cognitive Modeling

At the computational level, our methods for implementing this basic pragmatic competence will be based on statistical models, specifically dynamic Bayesian models like the hidden Markov model and Markov decision process. Additionally, we will draw from traditional techniques for parameter estimation, as well as more advanced techniques like inverse reinforcement learning, and more broadly, from research in multi-agent systems and game theory. The following sections give an overview of these techniques, as they have been applied in modeling language acquisition and social interaction.

2.3.1 Statistical Machine Learning Fundamentals

Bayes’ Rule and Latent Variable Models

One of the most important techniques in our application of statistical models is inference of the value of one variable from the value of another. In the case where these variables are the values of observational data, this can be viewed as classification. At its heart is the fundamental Bayes’ rule, which gives the following relationship for two dependent random variables \( X \) and \( Y \):
\[ P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}. \]  

(2.1)

This allows one to estimate the distribution of the variable \( X \) in cases where \( X \) can not be directly observed, but \( Y \) can. Statistical models with this structure are often called \textit{latent variable models}, and are an important technique for many natural language applications where the values of a discrete latent variable might correspond to classes of speech features, or associate multi-modal sensory observations. Relationships and dependencies between multiple variables can be represented with a \textit{directed acyclic graph} called a Bayesian network. When these graphs describe the evolution of variables over time, they are known as dynamic Bayesian networks, and are of particular interest in the modeling of time-series data, like speech and action.

\textbf{Parameter Estimation and Learning}

In many applications of statistical models, one does not know in advance the exact value of the distribution, and would instead like to learn it from some set of training data. This is the problem of estimating the value of a variable \( \theta \) that parameterizes the distribution governing the observed data \( Y \). For one of the most popular techniques that we will focus on here, the estimate is made based on the parameter’s likelihood of generating the training set \( \hat{Y} = \{y_0, y_1, \ldots, y_T\} \). Assuming individual observations to be independent and identically distributed (and consequently, the joint distribution to be factorable), the \textit{maximum likelihood estimate} (MLE) of the parameter, \( \hat{\theta}_{\text{ML}} \), can be calculated by maximizing the log-likelihood function over the data:

\[ \hat{\theta}_{\text{ML}} = \arg\max_{\theta \in \Theta} \sum_{t=0}^{T} \log [P(y_t|\theta)]. \]  

(2.2)

For certain forms of the probability mass or distribution function \( p(y|\theta) \), the optimization can be computed quite easily. One example might be the simple Gaussian distribution, where the estimator is given by the sample mean and sample variance. In cases where the data is complicated in structure, or part of the data is missing/unobservable — as in the latent variable models discussed above — more sophisticated techniques are necessary to perform this optimization. One such class of methods used for hidden variable models are known as the \textit{expectation maximization algorithm} \cite{76}. The EM algorithm is an iterative procedure based on the alternation of an expectation (E) step and maximization (M) step. The E-step consists of taking
the expected value of the log-likelihood function over the hidden data set $X$, given the observed data set $\hat{Y}$ and the current estimate of the parameter $\theta^{(t)}$:

$$Q(\theta|\theta^{(t)}) = E_{X|Y,\theta} \left[ \log P(Y,X|\theta) \right].$$  \hspace{1cm} (2.3)

In the following M-step, the new parameter value $\theta^{(t+1)}$ is set to the value maximizing the current $Q(\theta|\theta^{(t)})$.

Another class of techniques we will utilize for parameter estimation are the stochastic gradient descent (SGD) methods. Techniques of this type are aimed at optimizing objective functions that can be expressed as sums of differentiable functions, such as the log-likelihood function (equation 2.2). Using a standard gradient descent method requires calculating the gradient of each term in the sum, with respect to each parameter in the parameter set. SGD limits this operation to a single data point or a small subset of data points at one time. This means, as opposed to EM techniques which require the complete observation set to calculate parameter estimates, SGD allows the model to be trained online, as data samples are gathered. The update rule for the ML estimate of $\theta$ using one-step SGD might look like:

$$\theta^{(t+1)} = \Pi_G \left( \theta^{(t)} + \epsilon_t \nabla \log \left[ P(y_t|\theta^{(t)}) \right] \right).$$  \hspace{1cm} (2.4)

In the case of constrained optimization, the operator $\Pi_G$ is used to represent the projection of the parameter estimate back onto the allowable constraint set after each gradient step.

Stochastic gradient algorithms have a number of drawbacks — one of the most significant is the need for proper setting and control of the learning rate in order to achieve acceptable performance. However, many of these issues can be mitigated using a wide variety of heuristics and modifications to the standard algorithm. But most importantly, the online and adaptive capabilities, combined with their simplicity, make SGD techniques particularly attractive for many of the learning tasks presented in this thesis.

Hidden Markov Models

A Hidden Markov Model (HMM), an extension of the basic Markov model, is a dynamic Bayesian network that is commonly used in modeling data with both spatial and temporal characteristics, such as speech and action. HMMs are composed of an underlying unobservable Markov process, $X_t$, and an observable process, $Y_t$. The unobservable process is parameterized
by initial state distribution \( \pi \) and transition matrix \( A \), where:

\[
[a]_{ij} = P(X_{t+1} = j | X_t = i), \quad (2.5)
\]

\[
\pi_i = P(X_1 = i). \quad (2.6)
\]

The distribution of the observable process at each time \( t \) is a stochastic function of the state of the Markov process at that time. This observation variable may be discrete, in which case it can be parameterized by the stochastic matrix \([b]_{jk} = P(Y_t = k | X_t = j)\), or continuous, in which case it is often drawn from a Gaussian (or mixtures of Gaussian) distribution, parameterized by \( \theta = \{\mu_j, \Sigma_j\}_{j=0}^N \), where \( \mu_j \in \mathbb{R}^d \) is the mean vector and \( \Sigma_j \in \mathbb{R}^{d \times d} \) is the covariance matrix.

Typically the three canonical problems associated with HMMs are the problems of classification, state estimation, and parameter estimation [77]. Classification refers to the problem of calculating the probability that a particular parameter set produced a given observation sequence. This calculation is often performed by means of the forward-backward algorithm [78].

This forward algorithm allows for recursive calculation of the joint probability of a particular value of the hidden state along with all observations up to time \( t \). Likewise, the joint probability of all observations from \( t + 1 \) on up to \( T \), given a specific value of the hidden state, can be calculated recursively by the backward algorithm. These recursions are given in the following:

\[
\alpha_{t+1}(j) = P(y_1, \ldots, y_{t+1}, X_{t+1} = j | \theta) \\
= \sum_{i=1}^{n} \alpha_t(i) a_{ij} f_j(y_{t+1} | \theta) \quad (2.7)
\]

\[
\beta_t(i) = P(y_{t+1}, \ldots, y_T | X_t = i, \theta) \\
= \sum_{j=1}^{n} a_{ij} f_j(y_{t+1} | \theta) \beta_{t+1}(j). \quad (2.8)
\]

Initial values for the forward probabilities are set to \( \alpha_1(j) = \pi_j f_j(y_1) \), and backward probabilities are set to \( \beta_T(i) = 1 \) for all \( i \). Calculating the corresponding most likely hidden state sequence of the model can be done using the Viterbi algorithm [79], a special case of the larger class of dynamic programming algorithms.

There are many ways of approaching the final task of parameter estima-
tion, but two of the most popular techniques are those of the Baum-Welch algorithm [80] (a special case of the EM algorithm) and stochastic gradient descent [81, 82]. Both methods have their own advantages and disadvantages. The Baum-Welch algorithm carries the primary advantage that each iteration of the algorithm is guaranteed to increase the value of the objective function. Its downside is that it requires the entirety of the training data to be present, a constraint which may not be suitable for online/incremental learning applications. The alternative is to use a stochastic gradient descent algorithm, such as the recursive maximum likelihood estimation (RMLE) algorithm [82]. Here, the parameter set $\theta$ is updated at each time step in the direction of the gradient of the incremental score:

$$\theta_{t+1} = \Pi_G (\theta_t + \epsilon_t \nabla \log [P(y_t|y_{t-1}, \ldots y_1, \theta_t)])$$  \hspace{1cm} (2.9)

where $\Pi_G$ is the projection operator on to the manifold of allowable parameter sets. The advantage here is that training is done online, as each data point is received. Unfortunately, assuring proper convergence using these techniques often requires careful tuning of the step size parameter, $\epsilon_t$.

Our choice to make use of the HMM in certain aspects of the work described in this thesis is based in no small part on its wide application to the domain of speech recognition, as well as the representation and learning of gestures and other motor primitives. In speech recognition, the HMM has long been a standard model for representing nearly every level of the language faculty, from fundamental phonological and morphological units [37, 83], to simplified syntactic structures such as context-free grammars [84]. This capability in representing time-series data has been just as readily applied to the domains of physical action, of which motor primitives — the short, reusable movements used to compose more complex gestures — are particularly interesting to us. Work on the topic of Programming-by-Demonstration (PbD), where a human tutor manually guides a robot’s actuators in order to teach it an action, has effectively used HMMs to automatically segment larger gestures into motor primitives [85], incrementally adding to its action repertoire when unknown primitives are discovered [86]. Further methods have been created for robot learners to generate novel examples of these learned motor primitives using their corresponding HMM parameter sets [87]. Previous work in our own lab has made use of this method and other HMM techniques in action-language integration experiments where a robot could produce complex actions from verbal instruction using previously learned motor primitive word groundings [10].
While many consumer-available speech recognition products are created using training data carefully selected by experts, HMMs trained in an unsupervised manner using unannotated linguistic corpora have been shown to capture latent orthographic [88] and phonetic [89] structure. These structures correspond strongly to linguistic concepts commonly constructed by humans (e.g. phonetic categories like nasals, fricatives, etc.), but are a result of an unguided process of self-organization within the model. This often-overlooked ability of the HMM to capture meaningful representational structure is one we see again in applications of action modeling. Figure 2.1 depicts how an HMM self-organizes to represent motor trajectories by fitting Gaussian-output distributions to piecewise-linear portions of the curve using its hidden structure to model their sequential relationships.

2.3.2 Decision, Control, and Planning

Even though these applications feature stochastic models of motor behaviors, their understanding of action differs little from their understanding of any other kind of sensory input. One of the key requirements of the pragmatics-based approach we propose is that we have a model for understanding action as an explicitly goal-directed or intentional behavior. These behaviors are the result of the sequential planning or decision making processes of the agent that takes into account the contextual information about
the current state of the world, other agents, and the uncertain dynamics
of its environment. The Markov decision process (MDP) is a well-studied
stochastic model that is capable of capturing many aspects of such problems.

Markov Decision Processes

We define an MDP using a tuple of four elements: a state space \( S = \{s_1, s_2, \ldots, s_N\} \), an action space \( \mathcal{A} = \{a_1, a_2, \ldots, a_M\} \), a state-action transition model \( T(s, s', a) = P(S_{t+1} = s'|S_t = s, A_t = a) \), and finally a reward function \( R(s, s', a) \) which gives the immediate reward received at state \( s' \) when transitioning from state \( s \) with action \( a \). This framework is an extension of the basic Markov model that allows us to model active agents, who produce behaviors to maximize some reward. Analogously, the hidden Markov model can be extended in the same way, yielding a new model called the partially observable Markov decision process (POMDP), with the added element of an observation model \( \Omega(o|s', a) = P(O_{t+1} = o|S_{t+1} = s', A_t = a) \).

For the purposes of this thesis, however, we will only consider fully observable MDPs.

Of particular interest in this application is the representation of the reward function. Through the reward, it is possible to encode the goals or intentions that drive the behaviors of a rationally acting agent. As mentioned previously, one such goal might be to reach a particular state, in which case an indicator function for that particular state, \( I(s^*) \), could be used to represent the reward function. Often, especially in scenarios where the state space is extremely large or heavily factored, the goal is some derived feature present in a number of states. In these cases, a common approach is to parameterize the reward function as a linear combination of some set of features:

\[
R(s, s', a) = \theta^T \psi(s, s', a),
\]

(2.10)

where \( \psi: \mathcal{S} \times \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^f \), and \( \theta \in \mathbb{R}^f \). Further on in this thesis, we will also discuss rewards and feature representations that depend on only the current state and action \( (\psi(s, a)) \), or simply the current state \( (\psi(s)) \). Such a representation of the reward function will prove to be especially useful when approaching the inverse reinforcement learning problem in situations where \(|\mathcal{S} \times \mathcal{S} \times \mathcal{A}| \gg f\).
Optimal Planning with MDPs

In nearly all applications of MDPs, the primary goal is to find a \textit{policy} function \( \pi : S \rightarrow A \) that maximizes some objective function. The policy function specifies the action \( a \) that the agent will choose when in state \( s \). This objective function is usually chosen to be an expected discounted sum of the reward function, often referred to as the \textit{return}, over some potentially infinite horizon:

\[
\hat{R}_t = \sum_{\tau=0}^{\infty} \gamma^\tau R(S_{t+\tau}, S_{t+\tau+1}, A_{t+\tau}),
\]

where \( \gamma \in [0, 1) \) is known as the \textit{discount} parameter. The most well-known technique for approaching this problem is the dynamic programming technique developed by Bellman [90]. This technique has many different variants, which center around the calculation of two quantities: the policy function \( \pi(s) \) and the \textit{value function} under that policy \( V^\pi(s) \):

\[
\pi(s) := \arg\max_a \{ \sum_{s'} T(s, s', a) [R(s, s', a) + \gamma V(s')] \},
\]

\[
V^\pi(s) := \sum_{s'} T(s, s', \pi(s)) \left[ R(s, s', \pi(s)) + \gamma V(s') \right],
\]

\[
= E^\pi \left[ \hat{R}| s_0 = s, \pi \right].
\]

As is shown here, the value function is the expected value of the future reward (return), given a particular policy. The \textit{optimal policy}, which we denote as \( \pi^*(s) \), is defined as the policy that maximizes the value function for all states. This optimal policy can be found through various applications of equations (2.12) and (2.13) above. In the technique of \textit{Value Iteration}, the policy update equation is substituted into the value function calculation to yield the combined equation

\[
V(s) := \max_a \{ \sum_{s'} T(s, s', a) [R(s, s', a) + \gamma V(s')] \},
\]

which is iteratively updated for all states until convergence. In the \textit{Policy Iteration} version of the algorithm, a policy update step is performed, after which value function updates are iteratively made until convergence. This procedure is then repeated until the policy update step results in no change for all states.

Rather than iteratively calculating equation (2.13), the value of \( V^\pi(s) \) for
a particular policy can be obtained through linear methods. Consider the case where the reward is a function of only the current state and action. We can then use a vector notation for the reward and value functions under a particular policy: $V^\pi, R^\pi \in \mathbb{R}^{|S|}$, where $R^\pi(s) = R(s, \pi(s))$. We also denote $T_\pi$ as the $|S| \times |S|$ stochastic matrix with entries given by $T(s, s', \pi(s))$. Using this notation, equation (2.13) can be expressed and evaluated as:

$$V^\pi = R^\pi + \gamma T_\pi V^\pi, \quad (2.16)$$
$$= (I - \gamma T_\pi)^{-1} R^\pi. \quad (2.17)$$

We will find this vector formulation and corresponding linear solution to be useful in the discussion of the inverse reinforcement learning problem presented later in this section.

Reinforcement Learning and Applications

But what about a scenario where the agent does not know the transition model or the reward function in advance? This is the problem of reinforcement learning (RL). One solution might be to use a simple Monte Carlo method to evaluate the equations above. First, let us define a helpful intermediate quantity $Q^\pi(s, a)$, the action-value function (more commonly referred to as the Q-function) as:

$$Q^\pi(s, a) = \mathbb{E} \left[ \sum_{t=0}^{\infty} R_t | s_0 = s, a_0 = a, \pi \right] \quad (2.18)$$
$$= \sum_{s'} T(s, s', a) \left[ R(s, s', a) + \gamma Q^\pi(s', \pi(s')) \right]. \quad (2.19)$$

We also denote $Q^*(s, a)$ to be the Q-function under the optimal policy $\pi^*(s)$.

Starting with a basic policy iteration algorithm, the $Q^\pi$ function can be estimated by generating a set of training episodes under policy $\pi$ with random initial state-action pairs $(s, a)$, and by simply averaging over the resulting returns sampled for each episode. This estimated $Q^\pi(s, a)$ can then be used easily to evaluate $V^\pi(s)$. However, one problem with this algorithm is that it is inefficient as it spends too much time evaluating each (sub-optimal) policy. One way to improve this might be to perform a policy update after every training episode.

A more pressing problem though is the fact that the return sampled from
a training episode is only used to update a single state-action pair. The technique of *temporal difference (TD) learning* developed by Sutton and Barto [91] addresses this problem by using the recursive Bellman equation to update the value function after each time step of an episode:

\[
V(s_t) = V(s_t) + \eta [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)],
\]

where \(r_{t+1}\) is the immediate reward experienced after leaving state \(s_t\), and the parameter \(\eta\) is the learning rate. As stated, the TD learning algorithm only estimates the value function for a fixed policy \(\pi\), and does not address the issue of how to update the policy function \(\pi\). As with the previous Monte Carlo methods, our approach is to apply the TD learning technique to estimate the action-value function \(Q^\pi(s, a)\). The result is called the *SARSA* algorithm, as it requires not only the current state-action pair and experienced reward, but also the next state-action pair, in order to realize the recursive bootstrapping procedure of the TD algorithm:

\[
Q(s_t, a_t) = Q(s_t, a_t) + \eta [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)].
\]

SARSA is referred to as an *on-policy* method, as the policy used in evaluating the action-value function is the same one used for choosing behaviors. However, it might also be desirable to learn about other policies for control, including the optimal policy. The most popular of the *off-policy* TD learning algorithms is the Q-learning algorithm [92], which updates its Q-function according to:

\[
Q(s_t, a_t) = Q(s_t, a_t) + \eta [r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)].
\]

In this particular form of the Q update, we see that we are in fact approximating the optimal Q-function \(Q^*\) directly. Unfortunately, both Q-learning and TD-learning suffer from many issues of finiteness, convergence, and instability. Much of the current research focuses on improving performance in these areas, and has produced extensions to the vanilla TD and Q learning algorithms, such as function approximation, as well as mixing of TD and Monte Carlo methods as in *TD(\lambda)* and *Q(\lambda)* [91].

For many applications in cognitive developmental robotics, temporal difference-style methods have been a popular choice due to their focus on gradual learning. Reinforcement learning approaches have been successfully applied
to learn basic motor primitives [93] and locomotion [94]. RL methods have also been used to develop more socially oriented skills. One example in this area was an experiment on developing joint attention [95], in which gaze-direction was learned by using TD-learning to build a map between head pose and target locations. Another example application was to the problem of imitation learning [96], which included the task of learning to imitate observed effect. This same paper also addresses the related but distinct problem of learning the tutor’s intended goal from his or her actions, even when these actions are incomplete or unsuccessful. Approaching this second task, while much more challenging, is nevertheless a critical component in developing our pragmatic model for early language acquisition.

Inverse Reinforcement Learning

The problem of inferring the reward function that an agent is attempting to maximize from demonstrations of its behavior is known as the problem of inverse reinforcement learning (IRL). The IRL problem is fundamentally ill-posed, as there are infinitely more reward functions for which a set of observed state-action trajectories is optimal [97]. Given this, most approaches to the IRL problem attempt to constrain or narrow the solution space by favoring particular reward functions. These techniques fall under two general categories. The first consists of gradient-based methods that minimize some dissimilarity function between the empirically observed policy and the optimal policy under a particular reward. The second category includes those techniques that cast IRL as a Bayesian inference problem, and use Monte Carlo methods to estimate a posterior distribution over the reward function. Each class of algorithms has a number of advantages and disadvantages, which will be discussed briefly here.

Gradient techniques are some of the earliest and most varied approaches to the IRL problem. In their review of IRL algorithms, Neu and Szepesvri make the case that these methods all fundamentally attempt to find a reward function such that the dissimilarity between the trajectories produced under the reward’s optimal policy and the observed trajectories is minimized [97]. Using the representation of the reward function from equation (2.10), the problem is formulated thusly:

\[ \theta^* = \arg\min_{\theta} J(\theta; O_D), \]  

(2.23)

where \( O_D = \{\xi_1, \ldots, \xi_N\} \) is the complete set of example demonstrations, and
\( \xi_i = \{s_t, a_t\}_{t=0}^{T_i} \) denotes the state-action sequences that are the individual observations. The exact form of the dissimilarity function \( J(\theta; O_D) \) being minimized is what distinguishes the various algorithms of this type, which are numerous [98, 99, 100, 101, 102]. These methods each provide advantages and disadvantages with respect to computational simplicity, robustness to noise, sensitivity to scaling, and generalization, among many other aspects. The particular approach adopted in this thesis follows from the maximum-likelihood gradient IRL method used by Lopes et al. [102]. In this case, the reward parameter is estimated through a similarity function: the log-likelihood. We use \((s_t, a_t)\) to denote a single state-action pair within the set of observed trajectories \( O_D \). Assuming a stationary policy of the agent over the observations, the likelihood can be factored into probabilities of individual state-action pairs:

\[
\theta^* = \arg\max_{\theta} \log P(O_D|\theta), \\
= \arg\max_{\theta} \log \prod_t P(a_t|s_t, \theta), \\
= \arg\max_{\theta} \sum_t \log P(a_t|s_t, \theta). 
\tag{2.24}
\]

Under the standard MDP formulation, the action taken at a particular state is fixed if an agent is acting in a way that maximizes expected reward. In order to find a gradient procedure for maximizing this function, however, we must switch to a differentiable, stochastic policy function. Under the assumption — ubiquitous to nearly all IRL techniques — that the probability with which an agent takes an action in a particular state is proportional to the optimal expected return of that decision, the stochastic policy \( \pi_\theta(a|s) \) is modeled as a Boltzmann or Softmax distribution:

\[
\pi_\theta(a|s) \triangleq P(A_t = a|S_t = s, \theta), \\
= \frac{e^{\alpha Q^*(s, a; \theta)}}{\sum_{a'} e^{\alpha Q^*(s, a'; \theta)}}, 
\tag{2.25}
\]

where \( Q^*(s, a; \theta) \) denotes the optimal Q-function for reward parameter \( \theta \). For the sake of simplicity, we have adopted the slight abuse of notation of [97], using \( \pi_\theta(a|s) \) to refer to the optimal stochastic policy under \( \theta \), in place of \( \pi_\theta^*(a|s) \). The parameter \( \alpha \) is used to control how strongly the agent favors actions with greater utility.
This softmax formulation of the policy allows the gradient of equation (2.24) to be taken with respect to the parameter vector $\theta$:

$$\nabla_\theta \left[ \sum_t \log P(a_t|s_t, \theta) \right] = \sum_t \frac{1}{\pi_\theta(a_t|s_t)} \nabla_\theta \pi_\theta(a_t|s_t). \tag{2.27}$$

Alternatively, the log-likelihood and its gradient can be reformulated by replacing the summation over all points in the dataset with a summation over all possible state-action pairs in the given MDP:

$$\nabla_\theta \log P(O_D|\theta) = \nabla_\theta \left[ \sum_{S \times A} \hat{\mu}_E(s) \hat{\pi}_E(a|s) \log \pi_\theta(a|s) \right]$$

$$= \sum_{S \times A} \hat{\mu}_E(s) \hat{\pi}_E(a|s) \frac{1}{\pi_\theta(a|s)} \nabla_\theta \pi_\theta(a|s), \tag{2.28}$$

where $\hat{\mu}_E(s)$ denotes the normalized empirical state occupancy count, and $\hat{\pi}_E(a|s)$ denotes the normalized empirical action probabilities for a given state, over the demonstration set. Assuming that $O_D$ consists of $N$ total observed state-action pairs $(s_t, a_t)$, we define the calculation of these empirical quantities thusly:

$$\mu_E(s) = \sum_t \mathbb{I}(s_t = s), \tag{2.29a}$$

$$\hat{\mu}_E(s) = \frac{\mu_E(s)}{N}, \tag{2.29b}$$

$$\pi_E(a|s) = \sum_t \mathbb{I}(s_t = s \land a_t = a), \tag{2.29c}$$

$$\hat{\pi}_E(a|s) = \frac{\pi_E(a|s)}{\mu_E(s)}. \tag{2.29d}$$

Taking the derivative of $\pi_\theta(a|s)$ with respect to $\theta$ in turn involves calculation of the derivative of $Q^*(s, a; \theta)$, which is non-trivial, due to the dependence of $Q^*$ on $\pi_\theta(a|s)$ itself. Neu and Szepesvri show that for reward functions of the form given in equation (2.10), this gradient exists and is given by:

$$\nabla_\theta \pi_\theta(a|s) = \alpha \pi_\theta(a|s) \left[ \Psi_\theta(s, a) - \sum_{a'} \pi_\theta(a'|s) \Psi_\theta(s, a') \right]. \tag{2.30}$$
The term $\Psi_\theta(s, a)$ is known as the *conditional feature expectation*, which is the expected sum of (discounted) features under policy $\pi_\theta$, starting from initial state-action $(s, a)$:

$$
\Psi_\theta(s, a) = E_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t \psi(s, a) | s_0 = s, a_0 = a\right].
$$

These feature expectations can be calculated in a similar fashion to the iterative estimation of the value function given in equation (2.13).

In this and other gradient IRL approaches, the gradient calculation is used to successively update the reward parameterization, according to the general form given in equation (2.4). Most gradient-IRL algorithms require that the “forward problem” of finding the optimal policy be solved at each step in order to calculate the gradient, resulting in significantly higher computation cost in comparison to solving the forward problem alone. Within the family of methods, there are also trade-offs in performance and speed. Both the Policy Matching (PM) approach [97] and the ML approach used here have objective functions that are non-convex, which can lead to suboptimal solutions under gradient approaches. While other methods like MaxEnt [101] are convex, PM and MLIRL have the advantage of simplicity and intuition in favoring rewards that reproduce the observed policy. But ultimately, PM/MLIRL and MaxEnt approaches have both been shown to stand above most others in terms of performance, with relatively little difference between the two.

The second class of IRL algorithms are those based in Bayesian inference. One of the earliest uses of this approach comes from Ramachandran and Amir’s Bayesian IRL formulation [103]. Here IRL is structured as a Bayesian estimation problem, where the goal is to estimate the posterior distribution over reward functions $R$ (represented as an $N$-dimensional vector over the state space), given a sequence of state-action pairs $O_D = \{(s_1, a_1), (s_2, a_2), \ldots, (s_T, a_T)\}$ generated by an optimally behaving demonstrator $D$. The posterior distribution is given by Bayes’ rule:

$$
P(R|O_D) = \frac{P(O_D|R)P(R)}{P(O_D)}. \quad (2.32)
$$

This algorithm retains many of the assumptions on the stationarity of the agent’s policy and form of the stochastic policy that were seen in the gradient-based IRL approaches. In his paper, Ramachandran shows that the reward function minimizing the squared-error loss is equal to the mean of this
posterior distribution. Clearly for the general case where each $R(s)$ is drawn from some continuously valued distribution, analytical calculation of the mean is intractable, requiring the use of Monte Carlo estimation methods. Because — as in gradient approaches — each sample of the reward requires re-solving the optimal planning problem of the MDP, the number of samples necessary for convergence in the BIRL algorithm makes it computationally prohibitive to use for MDPs with large state spaces. At the same time, the fact that it estimates a distribution over reward functions, rather than a single point in the parameter space, makes BIRL more robust to observed behaviors that are suboptimal.

The general framework of probabilistic inference of goals provided by BIRL, however, will still prove to be useful in our application. While the learning of a reward parameter may be more computationally tractable using gradient IRL, the notion of using a prior distribution over goals gives us a simple, intuitive way to estimate which of a set of previously learned tasks or goals an agent is attempting to optimize in a novel observation. In this way we are able to exploit the usefulness of Bayesian formulation in cases where the demonstration was inaccurate or incomplete relative to its actual goal, something that has been seen in a number of applications to intention inference [102, 104, 105]. Such scenarios are important, as they are very similar to many of the experimental scenarios discussed in Section 2.2 that we wish to emulate [20, 21].

2.3.3 Multi-Agent Systems and Game Theory

One final area of interest is the body of research in multi-agent systems, which fuses the previously discussed statistical models and algorithms with concepts from the area of game theory. Game theory is the study of behavior and decision making in environments featuring multiple competing or cooperating agents. For our purposes, we will discuss specific applications of game theoretic methods and ideas in the areas of linguistics, where it has seen extensive use in modeling and understanding pragmatics. These ideas are particularly relevant to our proposed application, keeping in line with the idea of communicative interactions as language games.

Extensive Form Games

We begin by limiting our focus to so-called extensive form games, where players make moves sequentially, i.e. one after another [106]. Such games
are usually represented by tree structures, like the one shown in Figure 2.2a, with branches representing player moves and leaf nodes representing ultimate payoffs for each of the players. Payoffs are functions of the particular sequence of moves made by each player and may be different, as is the case in competitive games, or similar, as in cooperative games (we consider only cooperative games). The question now, as is most often the case in game theory, is how do each of the players choose their moves, and why?

If we assume that each player knows the other’s possible payoffs as well as his/her own, behaves rationally (maximizes payoff), and that Player 2 can see Player 1’s move, reasoning about their strategies is fairly straightforward. For Player 2, having seen Player 1’s move and knowing her payoff, he simply selects the action that maximizes his own payoff. Player 1’s decision is not as simple, as she must first take into account what Player 2’s responses will be to each of her moves — a fundamental reasoning process in game theory. Knowing that Player 2 will move to maximize his own reward, Player 1 can predict her own ultimate payoff for both players’ moves, and choose an action to maximize this. Even if Player 2 employs a stochastic strategy instead of a deterministic one, Player 1 can still choose an action based on expected payoff. This entire process of reasoning (i.e. first determining Player 2’s moves in each case, then Player 1’s) is known as backward induction.

In this simple case, we can also reason by transforming our tree representation into a normal-form game. These representations are characterized

Table 2.1: Normal-form payoff matrix.

<table>
<thead>
<tr>
<th></th>
<th>$A'A'$</th>
<th>$A'B'$</th>
<th>$B'A'$</th>
<th>$B'B'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0,0</td>
<td>0,0</td>
<td>2,1</td>
<td>2,1</td>
</tr>
<tr>
<td>$B$</td>
<td>1,3</td>
<td>3,1</td>
<td>1,3</td>
<td>3,1</td>
</tr>
</tbody>
</table>

Nash equilibria are in italics, and the unique subgame perfect equilibrium is printed in bold.
by a *payoff matrix*, in which rows and columns correspond to the possible strategies of Player 1 and Player 2 respectively. The extensive form game of Figure 2.2a has been converted to the example payoff matrix given in Table 2.1, with the elements of Player 2’s policy pairs corresponding to responses to each of the possible preceding moves of Player 1. In the previous paragraph, we reasoned that Player 2 would always pick the best response to Player 1’s action, which is the strategy profile \((B', A')\) for Figure 2.2a, and that Player 1 would choose the best action given her beliefs about Player 2’s response policy — \(A\) in this case. This combined strategy profile is known as a *subgame perfect equilibrium*, as the behavior of the players in each subgame (subtree) is optimal. Subgame perfect equilibria (SPE) are a subset of another kind of equilibria known as *Nash Equilibria*. Nash equilibria (NE) are values of the combined strategy profile (i.e. elements of the payoff matrix) for which no player can make a unilateral change in strategy that yields a better payoff for that player. We see in Table 2.1 that there are two such NE, and that not all NEs are SPEs.

Games of Imperfect and Incomplete Information

What happens if Player 2 is unable to see Player 1’s move? Such scenarios are called games of *imperfect information*. In our tree representation, this is represented by a circle joining a set of a player’s nodes that the player can not distinguish between (shown in Figure 2.2b), called an *information set*. The process of backward induction used above can not see through such information sets, meaning we can not find a SPE. While backward induction can not be applied, we can still reason about strategies using the assumption of mutually understood rationality. In the game shown in Figure 2.2b, Player 1 would understand that whatever Player 2’s policy might be, picking \(A\) will always yield a greater payoff. Under this, Player 2, even though he can not see which node of the information set he is at, would assign a much greater probability that Player 1 had played \(A\), and choose the policy that maximized his expected reward. The resulting strategy profile is a *perfect Bayesian equilibrium*, which is defined over a strategy profile and a belief system. The PBE is satisfied when strategies are sequentially rational (i.e. they maximize expected payoff at every information set), and beliefs are consistent (i.e. probabilities of nodes in an information set given a strategy profile are computed using Bayes’ rule).

Consider now a third scenario where Player 2 does not know Player 1’s strategy or payoffs, referred to as a case of *incomplete information*. In some
cases it is possible for this game to be transformed into a game of imperfect information involving three players, where Player 1 is able to observe Player 0’s move, but Player 2 can not observe the move. Here, Player 0 is usually taken to represent a choice made by nature that selects Player 1’s type, which in turn fully defines the payoff function. One game of this form is the class of signaling games [107]. Signaling games consist of a sender and receiver, who we denote with $\sigma$ and $\rho$ respectively, with the sender being of some type $i$ chosen by nature. Only the sender is able to observe her type, which upon learning she chooses to take some action $a$. In a cooperative signaling game, $a$ is a message visible to the receiver, who then chooses a response $d$.

As with the game of imperfect information, we wish to apply the concept of perfect Bayesian equilibrium to our problem, this time in more mathematical detail. First, we define the beliefs of both the sender and the receiver over each other’s action strategies. The sender holds a belief about the receiver’s strategy $\rho(a,d)$, that specifies the probability of the receiver taking a response $d$ to message $a$. Being rational, the sender of type $i$ chooses a signaling strategy that maximizes her expected payoff $U_\sigma(i,a,d)$ given her belief $\rho$:

$$\sigma(i) \in \text{argmax}_a \sum_d \rho(a,d)U_\sigma(i,a,d).$$  

Likewise, the receiver holds a belief $\mu$ over the sender’s type given the message $a$ that has been received. We similarly consider a rational receiver that chooses the strategy maximizing his expected payoff $U_\rho(i,a,d)$ given the belief $\mu$:

$$\rho(a) \in \text{argmax}_d \sum_i \mu(i|a)U_\rho(i,a,d).$$  

Previously, we noted that two of the conditions of a PBE are that the players’ strategies maximize the expected payoff, given beliefs over the other player’s behavior policies. Equations (2.33) and (2.34) satisfy these conditions, known as conditions of sequential rationality, for the signaling game. The third condition is that the beliefs be consistent, meaning that beliefs over states in an information set should behave according to Bayes’s rule. This means that the belief $\xi(i|a)$ is a posterior probability calculated from the receiver’s belief over the sender’s strategy $\sigma(i,a)$:

$$\xi(i|a) = \frac{\sigma(i,a)P(i)}{\sum_j \sigma(i',a)P(i')}.$$  

38
Signaling games are of particular interest to us, as they have been used by Parikh [108] and others [109, 110] to model the way the listener can take into account the social and pragmatic nature of the communicative act in order to interpret ambiguous utterances. It is based on the previously discussed idea that an utterance is an action taken by the speaker to get the listener to recognize some intentional state of the speaker. Under Parikh’s model, shown in Figure 2.3a, this intentional state is an action defining the speaker’s type, i, and is hidden to the listener. Furthermore, interpretation of the utterance a, is an action taken by the listener, d, that attempts to correctly pick the speaker’s (hidden) type. In line with the cooperative nature of the communicative task, we define the payoff function to have zero or negative values for both players whenever \( i \neq d \), and some positive number for both when \( i = d \). The exact value of these cases, which we will call the defect and join cases respectively, may be determined in part by some cost associated with the message a (e.g. length, complexity, etc.).

As before, we begin approaching this problem by assigning prior belief probabilities, \( p \) and \( p' \), of the listener to the unknown variable, in this case the speaker’s intended meaning \( i \). Let \( \alpha \) and \( \alpha' \) be alternative messages for \( i \) and \( i' \) that are unambiguous, but yield lower potential payoffs. These can be thought to represent longer (i.e. costlier), less ambiguous sentences. From this, the players can construct a table of all possible strategies of the speaker, and can calculate their expected payoff under each interpretation of a by the listener by using the shared knowledge of \( p \). Figure 2.3b shows an example of such a table where \( p = 0.9 \). In this particular table we see that there is a clear optimal equilibrium strategy which allows the listener to select the correct interpretation in the case of an ambiguous utterance.

There are, however, some significant problems using this kind of approach for our application. In the most basic method used by Parikh, the agents do
not form or leverage explicit beliefs about the strategy of the other in their decision making. For this simple technique to be feasible, it requires that each possible referent have an unambiguous alternative description. A more advanced technique might be to use the idea of the perfect Bayesian equilibrium to instead have the speaker and listener hold beliefs about the other’s play. This approach would still entail both agents simultaneously choosing belief/strategy profiles to coordinate their behavior. For a learning agent, this is not an applicable solution, as the adult speaker likely already has a belief in mind about how the listener should respond. One final problem pointed out in [111] is that there is usually little said about how \( p \) is selected. For our particular application, we find this flexibility to be an advantage, and will show in our proposed model how this prior probability might serve as a connection point for integrating knowledge of motivation, beliefs, or context.

There is much work in game theory that considers agents who can learn. Nearly all algorithms used fall into one of two categories — fictitious play and reinforcement learning — both of which model how agents learn over successive repetitions of a particular game. In fictitious play, an agent keeps a history of how frequently opponents make a given move, and uses the derived belief on their stochastic policy to choose a rational strategy. In reinforcement learning, a history of experienced payoffs is kept and used to guide the player’s subsequent strategy toward those which have had best historical utility. Computational models employing reinforcement learning alone [112], or in combination with fictitious play [113], have been applied to the case of general signaling games. More interesting yet are learning algorithms that have been applied to signaling game models of pragmatic language use [114]. For each of these models, however, the learner is ultimately given access to the speaker’s hidden type or an explicit reinforcement signal, as well as the received message. Reality is not so forgiving to children, who often must learn in situations where true intended referents are never revealed, and the adult does not provide feedback to their guesses [115]. While we still find the basic framework of the signaling game to be useful, we will need to develop our own set of techniques in order to address the challenges of our specific task.
CHAPTER 3

A PRAGMATIC MODEL FOR EARLY WORD LEARNING

In the introduction of this thesis, a broad set of goals was outlined in order to guide the construction of a pragmatic model of early word learning. Inspired by observations of child word learners in the areas of developmental psychology and cognitive science, these goals included both desired capabilities to be replicated by the model, as well as overarching principles regarding the kinds of techniques to be used in the model. This chapter begins by outlining our approach at the highest level, and detailing the primary experiments against which the model will be evaluated. We proceed by presenting the core pragmatic model, and incrementally extended it in order to meet more complex goals. At each layer, we provide the mathematical formulation of the model, derive the associated algorithms for learning and inference, and discuss how the model applies to the relevant motivating experiments.

3.1 Overview and Motivation

The core problems with current computational models for grounded word learning, as we have presented them thus far, are of two types: representational and developmental. At a high level, we have discussed how a model based on principles of social-pragmatic theories of language acquisition might be used to address these issues. While such theories are complex and multi-faceted, we pull from them two key concepts, upon which our model will be built: triadic interactions and intentional behavior. Demonstrating that a model could faithfully capture all or any aspect of complex notions like “triadic”, “intent”, or even “pragmatics” is a task beyond the scope of this work. Instead, we proceed by presenting an overview of a framework built on a narrow, but well-defined, interpretation of these concepts, and provide a small set of developmental experiments that will serve as templates against which we will evaluate the learning capabilities of our model.
3.1.1 General Approach

The general approach for the model presented here is based on the idea of language-games or interaction “frames” (or “formats”, as used by Bruner [28]), which we depicted in Figure 1.1. As mentioned, the two critical components of the interaction frame are its triadic and intentional nature. “Triadic” refers to what the model includes — namely, the speaker (adult/tutor), the listener (child/robot), and the parts of the environment (world) that are contextually relevant. Environment comprises not only the physical world — its state, interaction dynamics, etc. — but also the mental world, which includes each agent’s beliefs, knowledge, preferences, motivations, intents, etc. A triadic interaction further dictates what the meaning of language is: a social tool to influence the mental states of others with respect to the shared environment.

The “intent” defines how the game or task is structured. It is the goal of the agents — what they are trying to achieve. This goal may be social, such as getting someone to pay attention to an object/event, or it may be physical, like putting an object into a basket. Or perhaps it may be some combination of the two. In every case, we assume intent to be a mental variable: that is, something not directly observable to other agents. While an agent can only observe the actions of others and the current physical states, we know that these actions are driven by the underlying intent, and shaped by the state of the world, the agent’s behavioral preferences, and shared beliefs about other agents. If we work under the critical assumption that these actions are chosen to let the agent best achieve its goals, we can develop a structured way of reasoning about the connection between action, intent, and context.

To further clarify the application of these concepts, consider the following example of a simple interaction format, shown in Figure 3.1. Here the world consists of an adult and child sitting at a table, upon which there are a number of objects, as well as a bucket. Within this environment, the adult performs a very regular task with the following structure:

1. The adult selects one of the objects, and fixes his/her gaze upon it.
2. The adult verbally produces the specific label for the object (e.g. “Dax!”).
3. The adult picks up the object, and places it into the bucket.

In this case, the overall “script” for the interaction is quite rigid, consisting of a fixed sequence of behaviors. What is variable, however, is the particular
Figure 3.1: Example interaction format. The speaker selects a target object, produces the label for the intended object, and then satisfies the task goals by moving the intended object into the bucket.
object that is selected, which will in turn affect the specifics of some of the actions (i.e. particular reach position, word produced, etc.). By learning about the general task structure, the child can learn about the connection between a word label, and a specific intent — namely to place a specific object in the bucket. This will be the general approach taken here: the robot first learns about the intentional structure of a task, and then uses this structure to learn the meanings of words.

3.1.2 Motivating Examples

In the previous example, there are multiple, redundant sources of information the learner could use to determine intent: gaze, reaching for an object, and moving it to the bucket. However, in many real-world interactions, a child may have access to only one of these sources, which itself may be ambiguous. As we have discussed, however, children often appear to be capable of learning meanings of words by inferring the underlying intent of interaction through a wide variety of other types of information. For the purposes of practicality, we will focus on three simple means by which they appear to do this, as demonstrated in the developmental literature.

Lexical Contrast

The first among these is often called “lexical contrast” [116], or under more strict interpretations, “mutual exclusivity” [18]. In scenarios that attempt to demonstrate and test this ability, the interaction format consists of a child, an adult, and a number of toys/objects that are present in the environment, some of which the child already knows the names of them. The adult ambiguously attends to some set of objects, only one of which has an unknown label, and then gives the label for the unknown object. Under a pragmatic account, the child reasons that: “If the adult had meant object $X$ s/he would have used word $A$, so they must be using label $B$ to refer to object $Y$”. In this most basic pragmatic reasoning, the listener is able to constrain ambiguity by reasoning about how a speaker would act if maximizing expected utility.

Goal-Directed Behaviors

It is also possible for a listener to infer intent from a speaker’s physical goal-directed actions. In one study, an adult established an interaction format
with a child consisting of a finding-game where the adult gave the label for
the object s/he was intending to find, then pulled objects out of a bucket,
inspected them, and handed them over to the child [21]. Children were
shown to be able to learn the correct word-referent pairing, even in cases
where the adult inspected and rejected multiple objects before finding the
intended one. In another case, children were able to infer the correct intent
when the adult was unable to retrieve the objects altogether (i.e. the object
could not be removed from the bucket), given that they knew which objects
were in each bucket [20]. In both experiments, the child leveraged knowledge
about the interaction format in order to resolve ambiguity, even when goal-
directed actions were not completed, or were unsuccessful.

Action Constraints
The final set of experiments we will look at are those that appear to demon-
strate a child’s ability to reason about constraints on a speaker’s physical
actions, as well as his/her own (potential) role in the scenario, in order to
handle ambiguity [75, 24]. In these, an adult attempts to physically complete
some task — known to both the adult and the child — such as grabbing
a particular object and placing it in a specific location. When the adult
is able to grab certain objects and not others, the child interprets a verbal
request from the adult as a reference to an object, and is able to physically
aid the adult in completing his/her goal. The proposed explanation for the
child’s reasoning is that the adult would not have verbally referred to an
object they were capable of reaching themselves, so the unreachable object
must be the intent. While these particular experiments did not address word
learning directly, the pragmatic principles demonstrated are quite relevant
to this thesis.

3.1.3 Modeling Approach
The model and techniques presented in this section form the foundational
elements that will allow us to begin to move beyond many of the current lim-
itations of grounded language acquisition systems. They have been inspired
in large part by the growing body of research focusing on the links between
the comprehension and production of action and language [50, 117]. Work in
cognitive developmental robotics in particular has begun to develop general
computational frameworks for language processing and acquisition that fo-
cus on the kinds of intentional, pragmatic reasoning principles that can only
be achieved through embodied, situated, interaction [118, 62]. However, little has been done in terms of creating detailed models and implementations for these ideas, or figuring out how they can be applied to problems such as language acquisition.

As we have just outlined, our approach in addressing this open issue will be to focus on interactions involving both physical and communicative behaviors with a common underlying intentional structure, which are of critical importance in a child’s linguistic development [27, 28]. At its foundation, our model will be built around techniques for inverse planning, where actions are understood in terms of the goals or rewards they maximize. It is these goals into which we embed our representation for the meaning of words, and they are therefore a critical component to the methods we use. Just as critical are the components that influence the ways in which actions are taken: beliefs about other agents, the world, and the interactions between these three components.

We begin by using a basic signaling-game model to build a representation of word meaning based on communicative utility, and derive a simple method for online learning of this word-meaning mapping from training examples featuring some amount of ambiguity. Following this, we extend the model to the realm of physical action by embedding it within a Markov decision process framework. We then provide a goal-based representation of the kinds of interaction formats we have just described, and show how we can apply techniques from inverse reinforcement learning [103, 102] to first acquire knowledge about the general format of the interaction, and later to learn words from ambiguous, yet goal-directed behaviors within the interaction. In Section 3.4, we develop a truly triadic pragmatic word-learning engine by unifying these two models through the agent’s understanding of its own embodiment and role in the interaction. Finally, we show how our agent can apply the same representations, and in service with its intentional inference capabilities, take an active role in the interactions and its own learning.

3.2 Basic Pragmatic Model

For our first steps in the development of a basic pragmatic model for early word learning, we focus on the core learning faculties of cross-situational learning and lexical contrast, with the objective of later extending the model to capture more complex behaviors. To do this we start from the game-theoretic framework, and consider the very simple scenario where an adult
tutor produces a word describing an object s/he is attending to. Let us also introduce basic referential ambiguity by allowing this attention to be distributed over a confusion set of objects that contains the referent.

3.2.1 Mathematical Formulation

We begin by representing this scenario as an extensive-form game of incomplete information: the signaling game. However, as discussed in Section 2.3.3, traditional game-theoretic approaches of finding equilibria do not apply here. Instead, our approach is to first assume that the speaker $\sigma$ and the listener $\rho$ are rational agents, who seek to maximize their expected utility. Based only on her observation of the message, $a \in A$, the listener makes an interpretation, $d \in D$, of the speaker’s intended referent, $i \in I$. The speaker makes the decision about which $a$ to send based on his belief about how the listener will interpret the message. If the speaker is a rational agent, as we assume, the relationship between their policy and beliefs can be expressed as:

$$
\sigma(i, a) = P(A = a|I = i) \propto E[U_\sigma(i, d)|a].
$$

$$
E[U_\sigma(i, d)|a] = \sum_{d \in D} P(D = d|A = a)U_\sigma(i, d).
$$

But what is the agent’s utility $U_\sigma(i, d)$? This is a question that is wrapped up in the nature of intent itself, as it depends on the ability of the listener to make an inference, $d$, that satisfies the speaker’s intent, $i$. For the scenario discussed here, the intent is for the listener to attend to a particular object, which we will assume they are able to do only if the correct inference is made. Therefore, we let $U_\sigma(i, d) = U_\rho(i, d)$ be equal to 0 whenever $i \neq d$, and some positive value $R$ whenever $i = d$. From this we can write the expectation of the speaker’s utility as

$$
E[U_\sigma(i, d)|a] = \sum_{d \in D} P(D = d|A = a)U_\sigma(i, d),
$$

$$
= P(D = i|A = a) \cdot R.
$$

As noted in Section 2.3.3, and as we can see from equation (3.2), a Bayesian perfect equilibrium requires that the speaker hold some belief about how the listener will respond. For now, we will define such a belief as $\hat{\rho}(a, d) =$
\[ P(D = d | A = a). \]

The listener is also assumed to be rational, governed by a policy favoring moves that yield greater expected payoffs, similar to equation (3.1). However, the listener differs from the speaker in that s/he makes a decision after having observed the speaker’s action \( a \). The listener’s expected utility is given by:

\[
E[U_\rho(i, d)|a, z] = \sum_{i \in \mathcal{I}} P(I = i | A = a, Z = z) U_\rho(i, d). \tag{3.4}
\]

In equation (3.4), we introduce the variable \( Z \) to represent other information the listener may use to shape their prior belief about the speaker’s intent. In Sections 3.3 and 3.4, we will show how this can be done based on knowledge of the task structure and/or physical actions of the speaker. For the scenarios explored here, we consider simpler social cues, like gaze, as ways of constraining the set of possible intended referents. Here, \( Z \) might let us narrow down \( I \) to some subset of the entire set of objects present in the scene, \( \mathcal{I} = \mathcal{D} = \{1, 2, \ldots N_o\} \), which need not match the number of words used in the scenario in size (i.e. \( |A| \neq N_o \)).

We use the third condition for a perfect Bayesian equilibrium (PBE) — belief consistency — to calculate the posterior probability \( P(I | A, Z) \), the probability of a speaker’s intent given information about his/her possible focus of attention, \( Z \), and their use of message \( A \). First we apply Bayes’ theorem:

\[
P(I | A, Z) = \frac{P(A | I, Z) P(I | Z)}{\sum_{i'} P(A | I = i', Z) P(I = i' | Z)}, \tag{3.5}
\]

to yield an expression in terms of \( P(A | I, Z) \) and \( P(I | Z) \). The prior \( P(I | Z) \) is analogous to those used in previously discussed game-theoretic models of pragmatics. The conditional probability \( P(A | I, Z) \) is the probability that the speaker will use message \( A \) given intent \( I \). We make the simplifying assumption that the message and the speaker’s attentional state are independent given intent, \( P(A | I, Z) = P(A | I) \), which makes this an estimate of the stochastic policy of the speaker.

Calculation of this posterior distribution requires a belief about the speaker’s strategy \( P(A | I) \). We have already discussed in our brief review of game theory how common signaling game approaches of equilibrium selection do not apply well to a child learner. Alternatively, we have pointed out that the game-theoretic learning approach of choice — reinforcement learning — does not match the actual conditions of child learners who often learn without
any kind of explicit feedback or unequivocal knowledge of correct referent. So how can we begin to learn about the speaker’s linguistic strategy using only knowledge of the message, and some additional information on the intent prior $P(I|Z)$?

For our approach we assume that the listener, to the extent that s/he can, still plays in line with PBE. The first way we do this is by having the listener choose interpretations $d$ for each interaction episode that maximize expected payoff given a posterior belief calculated by equation (3.5). The second way is by allowing the listener to understand that the speaker is a rational agent given his/her beliefs about the listener’s response policy. In Section 3.2.1, we used equation (2.33) to note that the speaker’s policy (and the receiver’s belief about it), had only the requirement that it maximize the speaker’s expected utility. This idea can be approximated in a probabilistic setting like the one here, by using a softmax function to define a stochastic policy of the speaker that depends on his expected utility:

$$P(A = a|I = i) = \hat{\sigma}(i, a) = \frac{e^{\alpha E[U_\sigma(i,a)]}}{\sum_{a'} e^{\alpha E[U_\sigma(i,a')]}},$$

(3.6)

While the listener now knows how to make an interpretation for some given observations, $a$ and $z$, they are still faced with the problem of learning a model for the speaker’s strategy $\hat{\sigma}(i, a)$.

3.2.2 Learning Algorithm

Under equations (3.6) and (3.2), the problem for the listener is not so much the direct learning of the speaker’s policy as it is the learning about the speaker’s beliefs on the listener’s own policy $\hat{\rho}(a, d)$. We would like the listener to gradually bring his/her own beliefs about their response policy $\rho(a, d)$ in line with the speaker’s “prescribed” policy. A number of different approaches exist for solving this task. As this is essentially a hidden variable model, one way might be to apply EM-type algorithms to learn a parameterized approximation of the policy, similar to many of the popular association-based models discussed earlier [4]. However, such batch learning algorithms ignore the incremental and dynamic nature of language acquisition in real language learners that is essential in faithfully capturing phenomena like lexical contrast.

Instead, our initial attempts at approaching this problem will be based on a stochastic gradient descent approach. Here, we use a maximum-likelihood objective function, which offers us a way to estimate the stochastic policy.
model used by the speaker based only on the observed data that was generated using the model. Recall that stochastic gradient descent techniques for MLE optimize the log-likelihood function by updating the parameter set each time an observation is taken, as shown in equation (2.4).

To apply this method to our problem, we first define the model as a function of parameter vector $\phi \in \mathbb{R}^{|A| \times |I|}$, so that each $\phi_{j,k}$ is equal to the probability that the listener chooses interpretation $D = k$ after hearing message $A = j$:

$$
\phi_{j,k} \triangleq P(D = k|A = j), \quad (3.7)
$$

$$
\sum_k \phi_{j,k} = 1. \quad (3.8)
$$

Next, we derive the likelihood function of a single observation, at time $t$. Because the observations are dependent on a hidden variable $I$ — the speaker’s intent — the likelihood function takes the form:

$$
P(a_t|z_t, \phi) = \sum_i P(i, a_t|z_t, \phi) = \sum_i \hat{\sigma}(i, a_t; \phi)P(i|z_t), \quad (3.9)
$$

where the function $\hat{\sigma}(i, a, \phi)$ is the parameterized version of the listener’s estimate of the speaker’s stochastic policy originally given in equation (3.6). After each observation sample $(a_t, z_t)$, each parameter $\phi_{j,k}$ in the vector is updated by moving along its gradient of the log-likelihood function:

$$
\frac{\partial}{\partial \phi_{j,k}} \log \left[ P(a_t|z_t, \phi) \right] = \frac{\partial}{\partial \phi_{j,k}} \log \left[ \sum_i \hat{\sigma}(i, a_t; \phi)P(i|z_t) \right] = \frac{1}{\sum_i \hat{\sigma}(i, a_t; \phi)P(i|z_t)} \left( \sum_i P(i|z_t) \cdot \frac{\partial}{\partial \phi_{j,k}} \hat{\sigma}(i, a_t; \phi) \right). \quad (3.10)
$$

Finding the gradient involves finding the partial derivative of the softmax function, which fortunately produces a very simple closed form expression. First, let us define the indicator function $\mathbb{1}(x, y)$ as having value 1 when its two arguments are equal, and 0 otherwise. Derivatives of the speaker’s
stochastic policy with respect to parameter $\phi_{j,k}$ are given by:

$$\frac{\partial}{\partial \phi_{j,k}} \hat{\sigma}(i, a_t; \phi) = \alpha \sigma I(k, i) \cdot \hat{\sigma}(i, a_t; \phi) \left[I(j, a_t) - \hat{\sigma}(i, j; \phi)\right].$$  \hspace{1cm} (3.11)

Recall that in this parameterization the individual parameters correspond to stochastic response policies of the listener, requiring the constraint that the sum of all $\phi_{j,k}$s for a single $k$ must be equal to 1. After each update step, the parameter vector is projected back onto the allowable probability manifold specified by this constraint. This projection operation is denoted by $\Pi_G$, and is performed using the algorithm described in [40].

### 3.2.3 Discussion

The purpose of the material presented in this section was twofold. The first objective was to create a model capable of representing the associations between words and their meanings, and to derive an algorithm for incrementally learning these associations from potentially ambiguous observations. The second objective was for the model to be capable of demonstrating the emergence of so-called “lexical contrast” (or more strictly, mutual exclusivity) biases observed in child word learners[18] from basic pragmatic principles.

Our intuition about how this emerges comes from the model’s understanding of a word in terms of the goal it is trying to achieve, as exemplified in equations (3.5) and (3.6). Consider the case where our learner is presented with a novel word label, $a'$, and an ambiguous set of possible referent objects, many or most of which are already known to have other labels. In making its inference of the posterior probability, $P(I|A, Z, \phi)$, our agent reasons about the probability that the speaker would have used the new label, given each hypothesis about the intent. For objects with known words, the novel referent would likely not be as effective in getting the listener to recognize the intended referent. Therefore, $P(a'|I, \phi)$ (equation 3.6) would be low for such objects, while it might be close to chance for objects without known labels, ultimately resulting in a posterior probability that favored this later set of objects. It is in this way that the lexical contrast reasoning is captured by the basic model’s general pragmatic understanding of word use. In Section 3.3, we will see how these same kinds of principles can be applied to learning and reasoning about physical goal-directed behaviors.
3.3 Extended Pragmatic Model

So far, we have only considered the problem of word learning within very basic interaction formats, which involved the tutor turning their attention to a particular object and giving a label for that object. The learner was completely dependent on information contained in the gaze cue, as well as basic principles of lexical contrast emerging from the pragmatic model, to mitigate referential ambiguity. Now we begin to extend the model to be able to infer speaker’s intent from their physical, goal-directed behaviors [21, 23] in cases where attentional/gaze information is not available or is unreliable. The key in these cases is an understanding of the physical task structure, which must be first learned, and then applied to aid in resolution of ambiguity for word learning.

3.3.1 Mathematical Formulation

Figure 3.2 shows the proposed structure of our expanded model, which takes the form of a dynamic Bayesian (decision) network, and shares many features with other statistical models for behavior understanding [118, 119, 62]. At its core is a (multi-agent) Markov decision process, consisting of the true state of the world at each time $t$, denoted by $S_t$, actions taken by the agents $A_t$, and the hidden intentional states $I_t$ that determine which actions are chosen:

- $S_t$ is composed of the physical states of both of the agents, as well as any contextually relevant entities in the surrounding environment.
• $A_t$ are the actions available to each agent. These may be physical (e.g. moving one’s arm to push an object) or communicative (e.g. an utterance like “Dax!”) in nature.

• $I_t$ represents what the agent is trying to accomplish through its actions. It may be to realize a particular physical state, mental state of an agent, or both (e.g. to have a listener attend to the pushing of a particular object).

Two other important elements of the model are the “Belief” states of the agents, $B_t$, and the observations $O_t$. $B_t$ encodes the beliefs or knowledge of an agent about the aspects of the interaction that are not its specific intent, such as the current state of the world (which we assume to be perfectly known here), the knowledge about the goal structure of the current interaction/task, or expected linguistic conventions (i.e. $\hat{\rho}$ and the listener’s accompanying estimate of $\phi$ in Section 3.2). Because the mental states (intents, beliefs) and actions we have described are not necessarily shared between the agents, we use the superscripts $\sigma$ and $\rho$ to denote those variables as they pertain to the speaker and listener, respectively.

For completeness, we also include $O_t$ for cases where the true state of the world and actions can only be inferred from noisy sensor observations. This aspect will become more critical in later sections, as we focus on the issues of perception that come with real-world, embodied implementations. For the sake of the current discussion, we will assume to work with directly observable states and actions.

Another piece that is critical for defining this model, as with the MDP upon which it is built, are the physical transition dynamics. These are represented as a distribution $P(S_{t+1}|S_t, A_t)$ over the state of the world in the next time step, conditioned on the current state and actions of the agents. As with our discussion on MDPs in Chapter 2, we use the notation $T(s, s', a)$ to refer to the probability of a specific state-action-state transition. This model can capture the effects that various actions will produce in the environment, as well as the uncertainty of these effects. Practically, it can also be used to encode physical constraints and limitations on agents.

Generating Actions, Inferring Intent

We now come to the problem of how this extended model can be applied to provide our learner with the desired capability to resolve referential am-
bigness by understanding the speaker’s physical, goal-directed behaviors. We begin by defining the state and action spaces that will be necessary to represent the critical features of the interaction scenarios discussed at the beginning of this chapter. Our representation of the state space will clearly be dependent on the particular scenario for each of these, but in general it encompasses the states of the speaker/tutor, listener/learner, and any objects present in their shared environment. We will assume that our agent knows the state can be described as a factored representation, consisting of the set of states for these individual elements. Therefore, we can express the state \( S_t \) as a vector variable over these states, drawn over a domain \( S \), that is the Cartesian product of the state spaces for the individual elements and entities:

\[
S_t = (S_t^\sigma, S_t^\rho, S_t^{o,1}, \ldots S_t^{o,N_o}),
\]

(3.12)

\[
S = S^\sigma \times S^\rho \times S^{o,1} \times \ldots S^{o,N_o},
\]

(3.13)

where \( S^{o,n} \) refers to the state space of the \( n \)-th object out of \( N_o \) in the environment. For the experimental scenarios discussed in this thesis, we will primarily use the spatial location of the objects, or in the case of the agents, the end-effector, as the information encoded within the states. Specific details of how these states are defined and determined from the robot’s perceptual capabilities will be discussed in Chapter 5.

As we mentioned previously, there are two types of actions that we wish to represent within this framework: physical and communicative, which we treat as two disjoint sets. Physical actions, like the state space, will involve both of the agents and the objects within the environment. Unlike the state space, however, there are two separate action spaces for the speaker and listener. Rather than representing an agent’s actions as the movement of their end-effectors and creating a complex model of how these physical interact with various objects, we consider the action space to be the movements of the objects themselves. Assuming that the agent can only effect movement on one object at a time, we define physical actions to be an element of the union of the action sets for individual objects:
The set of actions for each of the agents is some subset of the complete actions over these objects, and they need not be equal to one another. In our implementation (detailed in Chapter 5) we will generally treat individual action spaces as movements in the four cardinal directions, plus a “no-movement” action. As done in the basic pragmatic model, we assume the set of communicative actions to be single word symbols, which we denote as \( A_m = \{1, 2, \ldots, M\} \). In the service of brevity in the following discussion, specific communicative actions will be identified as \( m \in A_m \), while unspecified actions, \( a \), are assumed to refer to physical actions.

We now return to the problem of word learning. In the scenarios motivating this stage of model development, the objective was to use observations of an agent’s goal-directed action to aid in resolving ambiguity during word learning, under the premise that both the communicative and physical action of the adult were driven by some underlying intent. For the sake of simplicity, let all unlabeled intents and actions in the following section refer to those of the speaker (\( A_t = A^\sigma_t \), \( I_t = I^\sigma_t \)) unless otherwise specified. Suppose that in a single episode of the new word-learning scenario, our agent observes the speaker taking some communicative action \( m \in A_m \), as well as an accompanying physical action, in the form of a state-action sequence, \( \{s_t, a_t\}_{t=0}^T \). Let us also assume that for all \( t \) over the course of this training episode, the speaker’s intent \( I_t \) does not change. We now rewrite the likelihood objective function of equation (3.9) to include the additional information about the speaker’s goal-directed actions:

\[
P(m|\{s_t, a_t\}_{t=0}^T, \phi) = \sum_i P(i, m|\{s_t, a_t\}_{t=0}^T, \phi) \\
= \sum_i P(m|i, \phi)P(i|\{s_t, a_t\}_{t=0}^T). 
\]

As before, our intuition is that we can exploit contextual information in order to reduce ambiguity in the inference of intent. Under our framework of (sequential) rational action, we can calculate the posterior over intentions using the same kind of softmax calculation of action probabilities (as in equa-
tion (2.26)) that was used in Chapter 2’s discussion of inverse reinforcement learning:

\[ P(i|s_t, a_t; t=0) \propto P(s_t, a_t; t=0|i), \]  
\[ P(s_t, a_t; t=0|i) = \prod_{t=0}^{T} P(s_t, a_t|i), \]  
\[ P(a|s, i) = \frac{e^{\alpha Q_i^*(s, a)}}{\sum_a e^{\alpha Q_i^*(s, a)}}, \]  

where \( Q_i^*(s, a) \) is the optimal Q-function under intent \( i \), as given in equation (2.19). Equations (3.17) and (3.18) give the probability that a specific reward function would have produced the speaker’s observed behavior, which we intuitively understand to be dependent on our speaker’s intent. This leaves us with the significant challenge of how to define intent, and more specifically, how it is to be used to parameterize \( R_i \) (and consequently \( Q_i^*(s, a) \)).

In our basic model, the purpose of utterances was for the listener to recognize and share attention to an intentional state that was simply one of the objects present in the environment. For these extended scenarios, however, the intent consists not only of the objects themselves, but also the target state of each of the objects that the speaker wishes to affect through physical action. In such cases, simple object labels alone are not sufficient to generate a reward function \( R \in \mathbb{R}^{|S|} \) needed to represent such an intent. At the same time, allowing the space of intents to span the combined state space defined in equation (3.13) would make little sense as a representation of word meaning, and it certainly would not be a computationally feasible inference problem.

This is where the key idea of the structured interaction format or task comes into play. For many of the situations in which children learn their first words, there is often some very regular task being performed, which varies in only some small aspect, such as the object(s) upon which it is being performed, as in the motivating examples for this section [20, 21]. This allows the learner to greatly constrain the space of intents (or reward functions) over which the inference takes place. Furthermore, knowledge acquired about regularities of the task being performed can be used by the listener to learn word meanings for the variable aspects of the format, even when performance of the task is incomplete or unsuccessful.

Even under the relatively narrow state and action spaces we have proposed
here, creating models capable of the autonomous construction and learning
of representations for complex interactions and their mapping into linguistic
structures would be an immensely difficult task. For the purposes of our
experiment, we assume knowledge of the variable elements of the
task to be some set of objects, which implicitly defines, as well as limits,
our representation of word meanings.

Under this key assumption, we now define the space of intents as \( I \in \{1, 2, \ldots N\} \), just as before. But even if we assume to know what reduced
representation of the task space is relevant for the given word, we still need to
determine its relationship to knowledge about the task being performed, how
this task knowledge is represented, and finally how these are used to generate
the reward representation needed for effective application of equation (3.18).

For this purpose, we present the following function for generating rewards
based on the specific intent, and knowledge about the physical task of the
interaction:

\[
R_i(s; \theta) = \theta^T \psi(g_i(s)). \tag{3.19}
\]

The reward function for each intent is generated according to a linear “feature-
based” formula, similar to the one given in equation (2.10). It consists of
the following components:

- \( g_i : S \rightarrow \hat{S} \): This function selects a subset \( \hat{S}_i \subseteq S \) of the state-space
  specific to intent \( i \).
- \( \psi : \hat{S} \rightarrow \mathbb{R}^f \): This function generates a set of features for each state in
  the reduced, intent-specific state space.
- \( \theta \in \mathbb{R}^f \): The column vector of reward weights for each feature. This
  represents the invariant aspects of the interaction task.

An example of \( g_i \) might be to simply select the state variable(s) correspond-
ing to the particular object \( i \): \( g_i(S_t) = S_t^{o,i} \). Whatever the implementation,
it is important that the domain of state variables be identical for all \( \hat{S}_i \). This
allows the feature function \( \psi \) and task parameter \( \theta \) to represent the aspects
of the goal/reward function that are invariant across intended objects.

We use this representation of the reward function to define the intent-
dependent optimal Q-function \( Q_i^*(s, a; \theta) \) given task parameter \( \theta \):
\[ Q^*_i(s, a; \theta) = R_i(s; \theta) + \sum_{s'} P(s'|s, a) V^*_i(s'; \theta) \]
\[ = \theta^T \psi_i(s) + \sum_{s'} P(s'|s, a) V^*_i(s'; \theta), \]  

(3.20)

where \( V^*_i(s; \theta) \), the optimal value function for reward \( R_i(s; \theta) \), is simply equal to \( \max_a Q^*_i(s, a; \theta) \). It should be noted that we only consider goals or rewards that can be represented as functions of the current state alone.

Finally, we rewrite the expression given in equation (3.18) in terms of this new parameter, \( \theta \), representing the knowledge about the physical task that is driving the observed state-action sequence:

\[ P\{s_t, a_t | t=0 \mid i, \theta\} = \prod_{t=0}^{T} P(a_t | s_t, i, \theta) \]  

(3.21)

\[ P(a | s, i, \theta) = \frac{e^{\alpha Q^*_i(s, a, \theta)}}{\sum_{a'} e^{\alpha Q^*_i(s, a', \theta)}}, \]  

(3.22)

3.3.2 Learning Algorithm

For the extended model, we have added a second learning objective. In order to exploit knowledge about the “fixed” aspect of the physical task in conjunction with the word learning techniques presented in Section 3.2, our agent must first learn what these physical task goals are. This is done by estimating the parameter \( \theta \in \mathbb{R}^f \) that encodes the feature weights used to generate the intent-dependent reward function. To estimate \( \theta \) from the training data, we use the maximum likelihood gradient IRL approach [102] to solve the following optimization problem:

\[ \theta^* = \arg \max_{\theta} \log \left[ P\{s_t, a_t | t=0 \mid i, \theta\} \right] \]
\[ = \arg \max_{\theta} \sum_{t=0}^{T} \log P(a_t | s_t, i, \theta). \]  

(3.23)

The objective function used here is nearly identical to equation (3.21) seen above. During the training period, we assume that the observations represent rational, successful, and unambiguous behaviors. As a result, the true intended target object is considered to be given.
For each such training sample — \(\{s_t, a_t\}_{t=0}^T, i\) — that is observed by the learning agent, a stochastic gradient update of the parameter \(\theta\) is performed. For the \(n\)-th such sample, the update equation is:

\[
\theta^{(n+1)} = \theta^{(n)} + \eta \cdot \Delta^{(n)},
\]

\[
= \theta^{(n+1)} + \eta \cdot \nabla_{\theta} \log \left[ P(\{s_t, a_t\}_{t=0}^T | i, \theta^{(n)}) \right].
\] (3.24)

Directly applying the gradient calculation of equation (2.28) is complicated somewhat by the fact that the parameter \(\theta\) and corresponding feature extraction function \(\psi\) do not operate over the same state space as the given state-action sequence \(\{s_t, a_t\}_{t=0}^T\), but rather a reduced state space \(\hat{S}\) produced by the function \(g_i : \mathcal{S} \rightarrow \hat{S}\). In order to reconcile this difference, we need to make an adjustment to the way that observation data is used to compile the statistics of the empirical distributions \(\hat{\mu}_E(s)\) and \(\hat{\pi}_E(a|s)\), which under the algorithm, must be defined over \(\hat{S}\). To do this, we modify our calculations of equations (2.29a) and (2.29c):

\[
\mu_E(s) = \sum_t \mathbb{I}(g_i(s_t) = s),
\] (3.25)

\[
\pi_E(a|s) = \sum_t \mathbb{I}(g_i(s_t) = s \land h_i(a_t) = a).
\] (3.26)

We define the function \(h_i : \mathcal{A} \rightarrow \hat{\mathcal{A}}_i\) similarly to \(g_i\), in that it selects a subset of the action space \(\hat{\mathcal{A}}_i \subseteq \mathcal{A}\) relevant to intent \(i\). We also note once more that the state and state-action counts are now vectors/matrices defined over the reduced state/action spaces \((\mu_E(s) \in \mathbb{Z}^{\mathcal{S}}_\hat{\mathcal{S}}, \pi_E(a|s) \in \mathbb{Z}^{\mathcal{A}_i \times \mathcal{S}}_\hat{\mathcal{S}})\), and not the complete spaces, \(\mathcal{S}\) and \(\mathcal{A}\), of which the observations \(s_t\) and \(a_t\) are elements.

Using these modified equations for updating the empirical state and action distributions, we return to equation (2.28) to calculate the gradient of the log-likelihood with respect to \(\theta\):

\[
\nabla_{\theta} \log P(\{s_t, a_t\}_{t=0}^T | i, \theta) = \nabla_{\theta} \left[ \sum_{\hat{S} \times \hat{\mathcal{A}}} \hat{\mu}_E(s) \hat{\pi}_E(a|s) \log \pi_\theta(a|s) \right]
\]

\[
= \sum_{\hat{S} \times \hat{\mathcal{A}}} \hat{\mu}_E(s) \hat{\pi}_E(a|s) \frac{1}{\pi_\theta(a|s)} \nabla_{\theta} \pi_\theta(a|s),
\] (3.27)

where \(\pi_\theta(a|s)\) denotes the optimal stochastic policy of an agent given task
parameter \( \theta \), as defined by equation (2.26). We again draw attention to the fact that the states and actions in the summation are elements of the reduced, intent-specific state and action spaces. We then recall the formulas for the gradient of the individual log probabilities given in equation (2.30):

\[
\nabla_\theta \log P(a|s, \theta) = \frac{1}{\pi_\theta(a|s)} \nabla_\theta \pi_\theta(a|s) = \frac{1}{\pi_\theta(a|s)} \cdot \pi_\theta(a|s) \cdot \alpha \cdot \left[ \nabla_\theta Q^*(s, a; \theta) - \sum_{a'} \nabla_\theta Q^*(s, a'; \theta) \right] = \alpha \cdot \left[ \Psi_\theta(s, a) - \sum_{a'} \pi_\theta(a'|s) \Psi_\theta(s, a') \right]. \tag{3.28}
\]

Following the technique presented by Lopes [102], we make the assumption that the policy remains unchanged for small variations in the reward function. Based on the vector formulation for finding the optimal value function \( V^*(s) \) in equation (2.17), this assumption yields a simplified expression for \( \Psi_\theta(s, a) \):

\[
\Psi_\theta(s, a) = \psi(s, a) + \gamma T_\pi(I - \gamma T_\pi)^{-1} \left[ \sum_{a'} \pi_\theta(a'|s) \psi(s, a') \right], \tag{3.29}
\]

where \( T_\pi \) is a square stochastic matrix representing the state transition probabilities under policy \( \pi_\theta \). Its entries are given by:

\[
[T_\pi]_{jk} = \sum_a P(s, a|\pi_\theta)P(s' = k|s = j, a). \tag{3.30}
\]

The gradient technique used here lends itself quite naturally to online learning implementations. As new state-action observation sequences — and corresponding intents — are received, they can be used to calculate some number of gradient steps. When many such training sequences are introduced incrementally, it may be desirable to introduce a heuristic for factoring past observation sequences into the calculation of the statistics \( \mu_E(s) \) and \( \pi_E(a|s) \), in order to aid in generalization and to prevent significant restructuring of the model as older samples are forgotten. We do this by further modifying equation (3.26) to perform the update as a weighted sum of new and old statistics:
\begin{align}
\mu_E^{(n)}(s) &= \beta \mu_E^{(n-1)}(s) + (1 - \beta) \sum_{t=0}^{T_n} \mathbb{I}(g_i(s_t) = s), \\
\pi_E^{(n)}(a|s) &= \beta \pi_E^{(n-1)}(a|s) + (1 - \beta) \sum_{t=0}^{T_n} \mathbb{I}(g_i(s_t) = s \land h_i(a_t) = a).
\end{align}

(3.31)

(3.32)

Here, \( n \) refers to the \( n \)-th training sequence (and its corresponding intent). The weighting parameter \( \beta \in (0, 1) \) controls the relative importance of new versus historical statistics used in the gradient calculation.

**Algorithm 1** Maximum Likelihood Gradient IRL Task Training

1: RandomInit \( \theta : \theta_k \sim \mathcal{U}nif(0, 1) \)
2: \( Q^*(s, a; \theta) \leftarrow \text{PolicyIteration}(\theta, T, \psi) \)
3: \( \pi_\theta(s, a) \leftarrow e^{\alpha Q^*(s, a; \theta)}/Z \)
4: while \( \{(s_t, a_t)_{t=0}^{T_n}, i\} \) do
5: for \( t = 0 \) to \( T \) do
6: \( \mu_E(g_i(s_t)) \leftarrow \mu_E(g_i(s_t)) + (1 - \beta) \)
7: \( \pi_E(h_i(a_t)|g_i(s_t)) \leftarrow \pi_E(h_i(a_t)|g_i(s_t)) + (1 - \beta) \)
8: end for
9: \( \hat{\mu}_E \leftarrow \mu_E / \sum_s \mu_E(s) \)
10: \( \forall s : \hat{\pi}_E(a|s) \leftarrow \pi_E(a|s) / \sum_{a'} \pi_E(a'|s) \)
11: Calculate \( \Psi_\theta(s, a) \forall s, a \)
12: \( \Delta \leftarrow 0 \)
13: for \( (s, a) \in \hat{\mathcal{S}} \times \hat{\mathcal{A}} \) do
14: \( \Delta \leftarrow \Delta + \alpha \hat{\pi}_E(a|s) \cdot [\Psi_\theta(s, a) - \sum_{a'} \pi_\theta(a'|s) \Psi_\theta(s, a')] \)
15: end for
16: \( \theta \leftarrow \theta + \eta \Delta \)
17: \( Q^*(s, a; \theta) \leftarrow \text{PolicyIteration}(\theta, T, \psi) \)
18: \( \pi_\theta(s, a) \leftarrow e^{\alpha Q^*(s, a; \theta)}/Z \)
19: \( \mu_E \leftarrow \beta \cdot \mu_E; \ \pi_E \leftarrow \beta \cdot \pi_E \)
20: end while

The complete learning algorithm for the task parameter \( \theta \) is outlined in Algorithm 1. After training and convergence of the task parameter, we can use equations (3.15) and (3.18), along with the stochastic gradient learning rule of equation (3.10) to approach the problem of word learning using the agent’s physical goal-directed behaviors in order to resolve referential ambiguity. The algorithm for this is presented in Algorithm 2.

3.3.3 Discussion

The model we have presented here, constructed using a general Markov decision process methodology, attempts to exploit the integration of physical
Algorithm 2 Word-Meaning Learning from Goal-Directed Behaviors

Require: MDP \((T, \psi)\), Task Parameter \(\theta\)

1: \(Q^*(s, a; \theta) \leftarrow \text{PolicyIteration}(\theta, T, \psi)\)
2: \(\pi_\theta(s, a) \leftarrow e^{\alpha Q^*(s, a; \theta)}/Z\)
3: while \(\langle \{s_t, a_t\}_{t=0}^T, m \rangle \) do
4: \(Z_p \leftarrow 0\)
5: for \(i \in I\) do
6: \(P_i \leftarrow 1\)
7: for \(t = 0\) to \(T\) do
8: \(P_i \leftarrow P_i \cdot \pi_\theta(h_i(a_t)|g_i(s_t))\)
9: end for
10: \(Z_p \leftarrow Z_p + P_i\)
11: end for
12: \(\phi \leftarrow \phi + \eta_\phi \nabla_\phi \log \left[ \sum_i P(m|i, \phi) \cdot P_i/Z_p \right]\)
13: end while

and communicative behaviors within a common intentional structure of an interaction. This intent is encoded within the reward function of the MDP, posing the intent inference problem as one of inverse planning. We generate these rewards on the basis of two components: the general task being performed in the interaction (e.g. putting toys in a bucket), and the specific object being targeted in that task. Using this structure allows us to constrain the search space of rewards, making the inference problem not only computationally tractable, but meaningful, and of use to the word-learning problem as well.

Applying inverse reinforcement techniques, our learner incrementally estimates the parameter representing the general task goal. The estimated parameter \(\theta\), when combined with an intended target \(i\), are used to generate a specific value for the reward function \(R_i(s)\). We can then use these to produce an estimate about the intended target of an observed state-action sequence, \(P(I|\{s_t, a_t\}_{t=0}^T, \theta)\), and use this to aid in the disambiguation of the intent of a corresponding verbal description, much in the way \(P(I|Z)\) was used in the basic model.

Because behaviors are represented in terms of goals, state-action probabilities are evaluated on their potential for achieving the goal relative to other actions, which brings a number of advantages. Unlike trajectory-based action representations, variations in starting positions, or incomplete/unsuccessful demonstrations can still be used to properly infer the intended goal. In addition, the forward planning problem used to generate the action utilities [90] is capable of incorporating other contextual knowledge about possible constraints or restrictions on an agent’s abilities, as well as the
abilities of other agents, which we will explore in Section 3.4.

3.4 Triadic Pragmatic Model

In this section, we show how the extended model can be developed even further to reason pragmatically about a speaker’s intent based on knowledge of their physical action constraints, as well as the listener’s own role in the interaction [75, 24]. This is achieved not by adding new elements or layers of complexity to the model, but rather by a more general treatment of language, action, and the function of communication. In addition to its application to the proposed experiments, we also show how it can be used to give the agent a more interactive role in its own learning process.

3.4.1 Mathematical Formulation

In our presentation of both the basic and extended pragmatic models, we have operated under the key assumption that the utility of an utterance depends solely on correct interpretation by the listener. This implicitly fixed a word’s pragmatic function as entirely referential, and as a result, physical action and communicative action were effectively decoupled, though complementary. Intents had a “dual” nature: producing or achieving some particular event or state (i.e. the physical aspect), and sharing attention with the listener to this reference event/state/object (i.e. the communicative aspect). These intents were fundamentally the same, but they were inferred from mostly independent models and observations.

However, one thing that was the same for both physical and communicative intention inferences, was their basis in the principle of utility maximization. Each contained at its core the following calculation used to evaluate the likelihood of seeing a particular action given an intentional state:

\[
P(A | I, \text{Model}) = \frac{e^{\alpha \cdot E[\text{Utility} (A) | I, \text{Model}]}}{\sum_{A'} e^{\alpha \cdot E[\text{Utility} (A') | I, \text{Model}]}}.
\]  

(3.33)

In Section 3.3, we employed an expected utility function ubiquitous in sequential planning problems, the Q-function. A function of both the current state and action, and dependent on a given (optimal) action policy or strategy, we know that \( Q^*(s,a) \) is equal to the immediate reward for the given state-action, plus the expected value of the (discounted) future rewards re-
sulting from the state-action and given (optimal) policy. Assuming rewards to be only dependent on the state, as we have done thus far, we can express the intent-dependent, task-parameterized $Q_i^*(s, a; \theta)$ in a way similar to equation (3.20):

$$Q_i^*(s, a; \theta) = R_i(s; \theta) + \gamma \sum_{s'} P(s'|s, a)V_i^*(s'; \theta), \quad (3.34)$$

$$= \theta^T \psi(g_i(s)) + \gamma \sum_{s'} P(s'|s, a)V_i^*(s'; \theta), \quad (3.35)$$

where the optimal value function $V_i^*(s; \theta)$ is simply equal to $Q_i^*(s, \pi^*(s); \theta)$.

For communication, we likewise based the probability in part on an expectation of the utility/value over the state resulting from the action, shown in equation (3.2). The only difference between this expression and equation (3.35) is that the utility of an action is taken as an expectation over influenced mental states (i.e. inferred intentions) rather than physical ones. Assuming that communication does not affect the physical state of the world $S$, we might express this utility as a Q-function over communicative actions $a_m \in A_m$:

$$Q_i^*(s, a_m; \theta, \phi) = R_i(s; \theta) + \sum_d P(D = d|s, a_m, \phi)V_i^*(s, d; \theta). \quad (3.36)$$

Here, we have simply and naively augmented the value function in equation (3.35) to include the listener’s interpretive state in addition to the physical state $s$: $V_i^*(s, d; \theta)$. We recall from equation (3.3) that in our basic pragmatic model, this function would have been defined as some positive constant ($R = 1$ without loss of generality) if and only if $i = d$, and 0 otherwise.

Consider now the motivating experiments for this current section. In these scenarios, the function of the speaker’s utterance goes beyond reference to an object, and in fact constitutes a request for the object. The intent behind the utterance can not be satisfied simply by proper comprehension alone, but also requires some behavioral aid that perhaps only the listener can provide. At the same time, because communication is being used in service of some larger task goal, its expected utility is balanced against physical means of completing the goal when planning actions. This planning task of the speaker, and corresponding inference task of the listener, require each to be able to leverage knowledge about the physical and communicative abilities of both.
Before showing how our model can be used to do this, we define some useful notations:

- Let \( \pi^\sigma \) and \( \pi^\rho \) denote policies for the speaker and listener respectively. 
  \( \pi^\sigma : S \rightarrow A^\sigma \) and \( \pi^\rho : S \rightarrow A^\rho \), where the action spaces are those available to the speaker and listener respectively.

- Let \( \pi^\sigma_*(s; \theta) \) denote the speaker’s optimal policy under reward \( R_i(s; \theta) \) for intent \( i \) and task parameter \( \theta \) as defined in equation (3.19). Let \( \pi^\rho_*(s; \theta) \) denote the same for the listener.

- Let \( V^\sigma_*(s; \theta) \) refer to the value function at state \( s \) under intent \( i \) and task parameters \( \theta \), under a speaker following the optimal policy \( \pi^\sigma_*(s; \theta) \) above.

- Let \( V^\rho_*(s; \theta) \) be the same for the listener.

Each of the quantities defined here is already supported by the existing framework of extended pragmatic model presented in Figure 3.2. And following from our discussion in the paragraphs above, the deep and intuitive link between the planning (or understanding) of physical and communicative actions allows us to achieve our goals through the generalization of existing mathematical formulas rather than through the addition of more rules.

Returning to the case of a speaker’s request for the listener’s physical help in completing a desired task with a particular object, we assume that our cooperative listener will act optimally based on its interpretation of the speaker’s intent, \( d \), and knowledge of the task \( \theta \). Within our MDP framework, we could also say that the listener acts according to the optimal policy (over its own action space \( A^\rho \)) for reward \( R_d(s; \theta) \): \( \pi^\rho_*(s; \theta) \). While this reward drives the listener’s actions, it is not necessarily the same reward \( R_i(s; \theta) \) used to calculate the augmented value function \( V^\sigma_*(s, d; \theta) \).

Rather than evaluate the value function for every possible combination of \( i \) and \( d \), we assume that \( V^\sigma_*(s, d; \theta) = 0 \) for all \( i \neq d \). Note the difference between this assumption, and the similar but much stricter definition on the utility made in the basic model. This assumption in effect disregards any possibility that the listener might “accidentally” achieve goal \( i \) during its attempted pursuit of goal \( d \). As a result, the expected value in equation (3.36) becomes equal to the probability that the correct interpretation is made, times the value the listener can provide in completing that specific task.
In order to evaluate the intent of a speaker given their goal directed actions, we use the softmax calculation of the action probabilities from expected utilities, given in equation (3.33), once more:

\[
P(a|s,i,\theta,\phi) = \frac{e^{\alpha Q^*_i(s,a;\theta,\phi)}}{\sum_{a'} e^{\alpha Q^*_i(s,a';\theta,\phi)}}.
\] (3.37)

We define a unified utility function for the speaker \(Q^*_i(s,a;\theta,\phi)\) thusly:

\[
Q^*_i(s,a;\theta,\phi) = \begin{cases} 
R_i(s;\theta) + \gamma \sum_{s'} P(s'|s,a) V^*_\sigma(s';\theta) & \text{if } a \in A_p \\
R_i(s;\theta) + P(D = i|s,a,\phi) V^*_\rho(s;\theta) & \text{if } a \in A_m 
\end{cases},
\] (3.38)

where \(A_p\) and \(A_m\) refer to the sets of physical and communicative (i.e. words) actions available to the speaker. This expression is essentially a unification of equations (3.35) and (3.36), with a more explicit definition for \(V^*_i(s,d;\theta)\). It is important to note that the summation over actions in equation (3.37) above must include both the physical and communicative action sets.

This last point is what provides the critical intuition underlying the application of this expanded algorithm to the motivating experiments [75, 24]. Under our formulation, intended objects that are unreachable to the speaker will have physical action utilities that are extremely small. By comparison, if the same object is reachable to the listener, the expected utility of communicating one’s intent will be much greater, even if the speaker is not completely certain about the likelihood of proper interpretation by the listener. This means that given an observed communicative act, the posterior over intents will be higher for objects that are reachable only by the listener. Furthermore, the lexical contrast-type reasoning inherent to the basic pragmatic model remains a feature of this model, and can be used to further disambiguate in cases where multiple such objects are present.

### 3.4.2 Learning Algorithm

The paradigm for learning of word meanings will proceed in largely the same manner as the extended pragmatic model presented in the previous section: the agent will first learn the parameterized representation of the task \(\theta\), and then will use this during the second phase to learn word meanings in
ambiguous situations. This means that Algorithm 1 will remain completely unchanged in this application. While the basic structure of Algorithm 2 remains the same, there are a few small, but important, changes and additions that must be made. The most significant of these is the added calculation of the listener’s own optimal value function for each object under the learned task parameter. The change to the expected utility for words also requires an update of the original gradient calculation of equation (3.11) for learning the word-meaning map:

\[
\frac{\partial}{\partial \phi_{jk}} P(a_m | s, i, \phi) = \alpha \mathbb{I}(k, i) \cdot V^*_i(s; \theta) \cdot P(a_m | s, i, \phi) [\mathbb{I}(j, a_m) - P(j | i, \phi)].
\]

(3.39)

One caveat to notice here is the gradient’s sensitivity to scaling of the value function, and indirectly, the reward function. It may be beneficial therefore to scale the value function by \( \| \theta \|_{\infty} / (1 - \gamma) \) in the case of infinite-horizon MDPs, or \( \| \theta \|_{\infty} \) if absorbing states are used.

3.4.3 Becoming an Interactive Learner

One of the central themes of this thesis has been the need for computational models that can understand and exploit the usefulness of language. So far, our representations of meaning have focused primarily on straightforward lexical semantics. But in this section, we also explore how more pragmatic aspects of meaning, like reference and request, can be captured in subtle ways by our model. Such functionality is only made possible by our explicitly triadic representation of language and the interaction as a whole. We have already discussed how our agent might reason about itself as a tool used by the speaker in pursuit of his/her goals, but we have not given it the power to actually follow through and pursue this “hypothetical” purpose. To this end, we now briefly discuss how we can put our agent’s embodiment to work, not only to help achieve the goals of others, but also to become an active participant in the development and learning of its own linguistic representations.

In the fully triadic pragmatic model, the agent is able to reason about its own role and potential in helping the speaker achieve some goal, which combined with knowledge of the task structure and action constraints of both agents, allows it to infer the intent of an utterance in cases of ambiguity. Embedded in this reasoning is the implication that the listener will ultimately take physical action to aid in completing the goal. Fortunately,
a blueprint for how to perform this action is already built into the model. Algorithm 3 describes how simple this is to carry out.

Algorithm 3 Interactive Word-Meaning Learning Algorithm

**Require:** MDP \((T, \psi)\), Task Parameter \(\theta\), Language Parameter \(\phi\)

1. \(Q^* (s, a; \theta) \leftarrow \text{PolicyIteration}(\theta, T^\sigma, \psi)\)
2. \(Q^* (s, a; \theta) \leftarrow \text{PolicyIteration}(\theta, T^\rho, \psi)\)
3. \(\pi_\delta (s, a) \leftarrow e^{\alpha Q^*(s, a; \theta)} / Z\)

**while** \((s, a_m)\) **do**

5. Calculate \(P(i|s, a_m, \phi, \theta) \forall i\)
6. Calculate \(H(I|s, a_m, \phi, \theta)\)
7. **if** \(H(I|s_t, a_m, \phi, \theta) > \tau_H\) **then**
8. Choose \(i \sim P(i|s, a_m, \phi, \theta)\)
9. \(s' \leftarrow s_t\)
10. **while** \(s' \neq \text{argmax}_s V^\rho_i (s')\) **do**
11. Take greedy action \(\hat{a} = \text{argmax}_a \pi_\rho_i (a|s'; \theta)\)
12. Set \(s' \leftarrow \text{resulting state}\)
13. **end while**
14. **end if**
15. Observe reaction \(\{s_r, a_r\}_{\tau=0}^T\) or \(a_m'\)
16. \(\phi \leftarrow \phi + \eta_\phi \nabla_\phi \log \left[ \sum_i P(a_m'|i, \phi, \theta) \cdot P(i|\{s_r, a_r\}_{\tau=0}^T, \phi, \theta) \right]\)
17. **end while**

Besides the satisfaction that comes with watching our agent helping its tutor achieve his/her goals, how else can this be useful? Consider the special case, within the basic experimental scenario we have been using, where ambiguity can not be resolved beyond two or more candidate objects (i.e. the conditional entropy \(H(I|\cdot) \geq 1\) bit). Whereas normally this entropy could not be reduced, and as a result would hinder learning of word meanings, the ability of the agent to act gives it the potential to reduce ambiguity further. In this case, the agent could first randomly choose an intent \(i'\) according to \(P(I|A_m, S, \theta, \phi)\), then act according to policy \(\pi^\rho_i (s|\theta)\) and Algorithm 3 to attempt to satisfy the presumed intent. Assuming that the speaker will continue to act until his/her intent is satisfied, s/he is likely to produce one of the following kinds of behaviors, based on the correct choice of intended object, and if the agent successfully completes the task:

1. **Correct intent, successful action:** The speaker needs to produce no further action and may rest.

2. **Correct intent, unsuccessful/incomplete action:** If the object is now in a state reachable by the speaker, s/he may act to complete the task.
3. Incorrect intent: The speaker must verbally restate his/her request for the object, as it is still unreachable.

By acting upon the system to move it from \( s \) into some new state \( s_x \), and observing the behavior the speaker reacts with \( a_x \) (which might also be a state-action sequence), the listener has additional information to use in calculating the gradient. The information gain, or reduction in entropy following observation of the tutor’s reaction \( s_x, a_x \) to the attempted completion of goal \( \hat{i} \), is given by:

\[
IG(I; \hat{i} = H(I) - H(I|\hat{i})) = H(p_{\hat{i}}, 1 - p_{\hat{i}}),
\]

where \( p_{\hat{i}} = P(\hat{i}|a_m, s, \theta, \phi) \). It is straightforward to show that the information gain is maximized by selecting the maximum value for the posterior of \( i \) [120]. In the best-case scenario (outcomes 1 or 2 above), entropy can be eliminated almost entirely. In the worst-case scenario, with \( n \) completely ambiguous intents, the entropy may only be reduced by \( \log_2(n + 1/n) \) bits.

3.4.4 Discussion

In this section we have presented a so-called “triadic” model for pragmatic word learning. This model is in effect a unification of many of the principles and techniques of our initial “basic” and “extended” pragmatic models. As in the basic model, the inference of linguistic intent is driven by a mirrored reasoning about the ability of a particular word to affect the desired mental state (interpretation) in the listener. And as in the extended model, our agent understands that there is some physical task to which the intended referent is connected. We consider these two kinds of behaviors — physical and communicative — as two possible means to the same end. However, in the case of communicative action, the speaker sees the listener as a potential path to its goal, and s/he must balance the utility of requesting the user’s help against his/her own physical ability to achieve the goal.

It is this same reasoning that the listener must use inversely, in order to recognize the intent underlying an action. As with the extended scenario, our agent acquires knowledge about the physical task embedded in the interaction, encoded in \( \theta \), and uses this to generate representations of utility, \( Q_\sigma^*(s, a; \theta) \) and \( V_\sigma^*(s; \theta) \), which could then be used to infer a word’s intent.
from the speaker’s complementary goal-directed actions. A new line of reasoning emerges if our agent also generates these functions in terms of its own action potential, $Q^\rho_i(s, a; \theta)$ and $V^\rho_i(s; \theta)$. Words as goal-directed actions are most attractive for a speaker when the value function for the listener, $V^\rho_i(s; \theta)$, is much greater than his/her own, $V^\sigma_i(s; \theta)$. Therefore, given only the fact that the speaker chose to communicate *anything*, the listener may be able to limit the likely referent to these kinds of intents, without any need for observing some physical action. This kind of reasoning is based upon our agent’s understanding of its own embodiment, which influences the specific potential that it holds to the speaker, and the way in which the meanings of words are learned. It is this same embodiment that we use to allow our agent to actively drive its own word learning. This aspect of our model, while explored in only a very narrow scope, is one that we consider to hold some of the greatest potential for the future.
In the previous chapter, we detailed the model structure and set of learning algorithms used in creating our “pragmatic engine” for autonomous learning of word meanings. While the components of the model are rooted in real-world interactions between embodied agents, the focus has been almost entirely on cognitive processes operating on internal, symbolic representations. However, a core premise of this work has been that these symbols ultimately be grounded in the agent’s representations of perception and action, which are themselves connected to the embodiment of the agent, in our case a humanoid robot.

One of the fundamental challenges of implementing our methods in a real-world learning environment is the bridging of the noisy, continuous world of sensory experience with the internal, symbolic world of conceptual representations. How does the robot know that a particular word has been said, or a particular action has been taken? How does the robot even know what that word sounds like, how does it use that knowledge to recognize the observed speech segment, and how is the knowledge acquired in the first place? Before our agent can begin to learn the meanings of words, these questions must first be addressed. Earlier work of ours sought to do this through the development of a basic modeling framework, which we applied in a context of first learning basic action-word groundings, and then applying this grounded linguistic knowledge to bootstrap the learning of more complex actions and their linguistic descriptions [10].

4.1 Perceptual Simulators

Our approach was inspired by ideas of perceptual simulators and symbols proposed by Barsalou [121], for which we developed a two-level perceptual-conceptual model structure. At the top layer linguistic concepts link perceptual symbols, which index the various perceptual categories present in a given modality. Perceptual simulators are generative models that represent
Figure 4.1: Diagram depicting relationship between conceptual (blue), perceptual (red/green) models, and sensory observations in the proposed experiment. Also shown is the feedback of estimated concept state sequences as input to complex action model (purple).

the sensory experiences of the categories, and provide the link between the internal symbolic world and the world of noisy observations. Single-modality observation streams are treated as chains of perceptual events generated by a single perceptual simulator out of an entire lexicon of such models corresponding to that modality. These categories might correspond to static elements like objects, or to ones that include temporal structure, like words or basic actions. One of the overall goals of the framework was constructing such lexicons for words and actions from scratch.

The basis of this framework was the hidden Markov model (HMM). HMMs were used as the “perceptual simulators” due to their extensive application in both modeling speech [37] as well as action [86]. Our overall algorithm was based on similar online sequence clustering methods used for learning action representations [122, 42]. A graphical representation of the model structure used is shown in Figure 4.1. First, an observation sequence $A(t)$ is segmented into subsequences $A_i(t)$ which are assumed to be generated by a single element of the lexicon. The lexicon itself consists of a set of HMMs, $\mathcal{K} = \{\varphi_1, \ldots, \varphi_k, \ldots, \varphi_K\}$, where $\varphi_k$ represents the parameter set for HMM $k$. Individual samples in the subsequence $A_i(t)$ may be either $d$-dimensional real-valued vectors, or symbols drawn from a dictionary of size
d. Categorizing each $A_i(t)$ received as an element of the lexicon can be done by the following maximization:

$$a_i = \arg \max_{k \in K} P(A_i(t)|\varphi_k). \quad (4.1)$$

The problem for an agent that must learn incrementally from scratch is that the $\varphi_k$s are not known in advance, nor is their number. To construct a lexicon from scratch, we proposed the following basic lexicon learning algorithm based on a competitive-learning principle. Each time a new $A_i(t)$ is received, we attempt to classify it as generated by an existing model $\varphi_k$. If the winning model is judged to be a good enough fit, its parameters are adjusted to fit the new data, while all other models remain the same. If no model is judged to be a satisfactory fit, a new one is created, and its parameters are initialized using the sequence as its training data (cf. equation 2.2). While the expectation-maximization technique [76] is better suited to the initialization task, we use stochastic gradient techniques [82] for incremental updates.

One significant issue is determining the methods for judging how well a model fits the data, also known as the problem of novelty detection. For our application, we use a novelty detection heuristic similar to one used in [123]. During the classification step, we calculate both the log-likelihood $\hat{L}_k = \log [P(A_i(t)|\varphi_k)]$ and a length-normalized version of the log-likelihood $L$. Then a mapping $\Lambda_k = F(L_k, \varphi_k)$ is applied, the purpose of which is to account for the possible variations in goodness-of-fit that a given perceptual category can achieve. This mapping is given by

$$F(L_k, \varphi_k) = \int_{-\infty}^{L_k} N(x, \mu(\varphi_k), \sigma^2(\varphi_k))dx, \quad (4.2)$$

where the parameters of the Gaussian PDF $N(x, \mu, \sigma^2)$ are estimated for each $k$ from past values of $L_k$. A threshold $\tau_0$ is applied to this mapping to decide whether the best fitting model is a good enough fit. This entire procedure is more formally outlined in Algorithm 4.

4.2 Learning of Speech and Action Representations

The lexicon learner is a critical component to the successful implementation and usage of our pragmatic word leaning model in a real-world human-robot interaction scenario. In this section, we demonstrate the basis of its
Algorithm 4 Lexicon Creation Algorithm

1: \[ K \leftarrow 0 \]
2: \[ \textbf{while } A_i(t) \textbf{ do} \]
3: \[ \textbf{for } k = 1 \textbf{ to } K \textbf{ do} \]
4: \[ L_k = (1/T_i) \log [P(A_i(t)|\varphi_k)] \]
5: \[ \Lambda_k = F(L_k, \varphi_k) \]
6: \[ \textbf{end for} \]
7: \[ \textbf{if } \{ k \in K : \Lambda_k > \tau_0 \} = \{ \emptyset \} \textbf{ then} \]
8: \[ \text{Increment } K \text{ by 1; Create new model } \varphi_K \]
9: \[ \varphi_K = \text{train}(A_i(t), \varphi_K) \]
10: \[ \mu(\varphi_K) = L_K; \sigma(\varphi_K) = \sigma_0 \]
11: \[ \textbf{else} \]
12: \[ \hat{k} = \text{arg max}_{k \in K : \Lambda_k > \theta_0} L_k \]
13: \[ \varphi_{\hat{k}} = \text{update}(A_i(t), \varphi_{\hat{k}}) \]
14: \[ \text{Update } \mu(\varphi_{\hat{k}}) \text{ and } \sigma(\varphi_{\hat{k}}) \text{ using } L_k \]
15: \[ \textbf{end if} \]
16: \[ \textbf{end while} \]

usefulness through its application to a simple experiment involving the autonomous learning of perceptually-grounded action-words. In this scenario, the lexicon learner was used to incrementally construct representations of single words and simple gestures using speech and motor data from an iCub humanoid robot [124]. At the same time, internal linguistic symbols were grounded in these representations by learning associations between simultaneously presented words and actions. Finally, we show how this rudimentary semantic representation can be used to bootstrap the learning of more complex behaviors by exploiting the embodied nature of the robot’s representation of action.

For this experiment, a set of 13 words was learned from a stream of semi-continuous speech by first transforming the audio signal into a sequence of Mel-frequency cepstral coefficients (MFCCs) of length 13. The signal energy for each window was then low-pass filtered and thresholded in order to segment the speech stream into subsequences corresponding to single-word utterances. Each segment was further processed by a phonetic classifier, based on a 10th-order HMM trained without supervision on a separate two-minute long speech sample using the EM algorithm (cf. Poritz [89]). These segments were transformed to discrete symbol strings by using the Viterbi [79] algorithm to make an estimate of the most likely hidden state sequence. It was these strings that were used as input to the lexicon learner algorithm. The word lexicon elements were discrete observation, 5 state HMMs, with left-to-right transition models [37]. The resulting confusion matrix after
each of the 13 words was presented 10 times (in random order) is shown in Figure 4.2a. The overall confusion rate for the experiment was a reasonable 6.15%.

Six basic actions were also used to evaluate the lexicon learner’s performance. These actions were demonstrated through direct manual manipulation of the arms of an iCub humanoid robot, 40 times each in random order with brief pauses taken between, and included: moving the hand to the right, left, up and down, as well as raising the hand above the shoulder and lowering it again. Streaming joint angle values from the robot’s arm were converted to Cartesian position and velocity estimates for the end-effector (hand), and used as inputs to the lexicon learner. The $L_2$ norm of the velocity was low-pass filtered and used as the energy signal in segmentation. Action segments were then processed so that all positions were relative to the initial position of the segment. Elements of the basic action lexicon were four-state HMMs with Gaussian output distributions, identical to the action representation shown in Figure 2.1. The resulting confusion matrix for the data set is given in Figure 4.2b. While there were no between-class confusions, the lexicon learner did learn two more action categories than the ground-truth. These categories corresponded to stylistic variations of the basic actions demonstrated.

The result of equation (4.1) is the transformation of a set of noisy sensory sequences, $A_i(t)$, into a set of perceptual symbols, $a_i \in \{1, 2, \ldots, K\}$. The process of language grounding was performed using a simple latent variable model like that of [6]. Here, speech and action representations were fused via an internal conceptual state: each concept generates (multi-modal) perceptual symbols according to some probability mass function (PMF). The model parameters are the collection of these PMFs, expressed as a set of matrices, $O$, with each matrix in the set representing a different modality:

$$[O_a]_{m,k} = P(a = k | c = m).$$

The underlying conceptual state for $i$th observation, $c_i$, is an element of the set $C = \{1, \ldots, M\}$. Grounding occurs through the learning of the parameters of this model, $\{O_a, O_b\}$, estimated via a stochastic gradient descent technique on a maximum likelihood objective function, as given in [6]. Online learning steps are performed given observation pairs $a_i, b_i$, determined to be temporally synchronous (i.e. segments overlapping in time). This kind of model, which we will refer to as a basic “generative model”, is quite commonly applied in various grounded language acquisition experiments [9, 4],
Figure 4.2: Confusion matrices for word and action lexicon learning tasks. Ordinate labels are given only denote the “ground-truth” action/word categories intended by the tutor, and were not provided to the robot during training.
and is therefore one that we will adopt as a baseline for comparison when evaluating the performance of our pragmatic model in the chapters to come. In this experiment, we used this model and techniques to learn associations between the sets of words and actions above. The results of the parameter estimation after 100 training samples are shown in Figures 4.3a and 4.3b.

After having learned a basic mapping between actions and words using traditional associative methods, our final goal was for the agent to learn a set of “complex” actions from verbal instruction. Using the same lexicon learning algorithm as before, but with estimated concept sequences as the input, our robot learned a set of complex actions composed of sequences of basic actions by leveraging learned action-word groundings. After hearing instructive sentences like “Wave [is] raise, left, right, left, right, lower”, our agent was able to produce an action wave even though he had never seen the complete complex action produced. Using methods developed by Calinon [87], we were able to use the basic action lexicon’s HMMs in reverse
to generate example sequences. This same generative ability was used for the complex action to chain together sequences of basic actions to produce the previously unseen composite action “wave”. A comparison between an example produced by the human tutor and one constructed by the robot is shown in Figure 4.4.

The fundamental purpose of this small experiment was twofold. The first goal was simply to confirm that the algorithm could be practically implemented and perform its intended function — the autonomous learning of basic perceptual representations from noisy sensorimotor experience. Its second purpose was to provide a more concrete demonstration of how the model can be practically applied to speech and motor function. This will be relevant primarily for speech in application to the pragmatic model. We will use the same incremental lexicon learning algorithm to build representations of word symbols (which we denoted as discrete speech actions \( a_m \) in the pragmatic model), from segmented streams of speech.

These techniques will, however, be less applicable to creating representations of action. The primary reason for this is the goal-based representation of action in our pragmatic model. We will see in the coming chapter that while this representation is embedded in a relatively low-level perception of space, and its parameterization \( \theta \) is learned in an online manner, its connection to sensory and perceptual data is ultimately fixed. This is due in large part to the relative lack of computationally tractable methods for solving the inverse planning or reinforcement learning problems in continuous state.
and action spaces. However, some recent techniques for learning among a set of candidate reward functions in continuous spaces, using Gaussian mixture models [125], hold the possibility for future extension to our model.
In this chapter, we will more concretely detail how the models and algorithms developed in Chapters 3 and 4 are applied to the general set of human-robot interaction scenarios presented in Chapter 6. After first discussing the particular embodiment of our agent — the iCub humanoid robot — we will outline how the various elements of the pragmatic model are implemented. For those parts of the model for which we have made simplifying assumptions about the structure of perceptual representations, we will detail the processing capabilities that support their function.

5.1 The iCub Humanoid Robot

In our real-world implementations, we will be using the iCub as our embodied platform [124]. The iCub is a humanoid robot with the capability to move its head, neck, torso, arms, legs, and hands through 53 motors, 18 of which are in the hands alone. In addition to standard position/velocity control, strain gauges in the hips and shoulders allow for force/torque/impedance control. Beyond its proprioceptive capabilities, the iCub is also outfitted with two color cameras that can stream $640 \times 480$ images at a rate of 30 frames per second. Two microphones are embedded in a head casing featuring artificial asymmetrical pinnae, allowing for better sound localization. Interfacing with the robot is facilitated by the YARP middleware package [126]. Owing to its open-source philosophy, both of these packages have come to see widespread use as standard hardware/software platforms for cognitive robotics research. As a result, there exists a large number of implementations of common algorithms shared among the community, many of which we use in support of this proposed work (e.g. Cartesian and gaze controllers [127, 128]). Because of its extensive sensorimotor periphery and a software system that supports efficient real-time communication with the robot, the iCub is well suited for our proposed application.
5.2 Application of the Pragmatic Model

In the discussion of the pragmatic engine, the elements of the model shown in Figure 3.2 were treated in largely abstract terms, with only a general description of their relationship to the human-robot interaction scenario. These elements were the states of the environment, intentional states, and physical and linguistic (words) actions. Also included were a number of models and functions that determine the dynamics of the states and actions, such as the state-action transition model, the task and word meaning parameters, and the mapping from states to features. In this section, we detail more clearly how these are all defined and implemented in the human-robot interaction experiments. This includes the algorithms and procedures for how they are processed and estimated using the noisy perceptual data streamed from the robot.

5.2.1 Physical Elements

World State

Chapter 3 defined the world or environment state \( S \) as the Cartesian product of the individual state spaces for the speaker, listener, and all objects: \( S = S_\sigma \times S_\rho \times S_{o1} \times \ldots S_{oN} \). For all of the experiments considered in this thesis, an agent or object’s state will correspond to its spatial location, or the location of its end-effector in the case of agents. We will further define this location to refer only to the \( X \) and \( Y \) positions of the object with respect to the robot’s root reference frame. This simplification relies on our assumption that the objects are all situated on a common \( Z \) coordinate plane (i.e. the table). Each possible value for an object’s state corresponds to the location of that object at a particular \( (x, y) \) location. For our implementation, these locations are spaced evenly in the \( x \) and \( y \) directions across the workspace, which we assume to be bounded between \( (x_{min}, x_{max}) \) and \( (y_{min}, y_{max}) \). This forms a \( M_x \times M_y \) grid, and results in object state spaces that are each \( M_x \cdot M_y \) large.

An object’s state at any given time must be determined from some continuous-valued location measurement \( (X(t), Y(t)) \). One approach might be to simply find the state corresponding to the grid center location \( (c_x, c_y) \) that is closest to the measurement. Such winner-take-all state decisions might have negative effects on the calculation of the empirical observation statistics \( \hat{\mu}_E \) and \( \hat{\pi}_E \) that are needed for learning of the task parameter in equation (3.28),
especially when measurements \((X(t), Y(t))\) are nearly equally close to many state centers. A better method might be to smooth these statistical calculations by calculating a probability mass function (PMF) over the states:

\[
P(S_t = s^{(k)} | X(t), Y(t)) \propto P(X(t) | S_t = s^{(k)}) P(Y(t) | S_t = s^{(k)})
\]

\[
= \mathcal{N}(X(t); c_{x_k}, \sigma_x^2) \cdot \mathcal{N}(Y(t); c_{y_k}, \sigma_y^2).
\]  

(5.1)

The specific values chosen for \(\sigma_x\) and \(\sigma_y\) will influence the amount of smoothing that occurs across states.

Actions

The actions discussed for our pragmatic model were of two types: physical and linguistic. The latter types were the set of symbols corresponding to spoken words, and their representation and perception is covered by the methods and models described in Chapter 4. For now we focus on physical actions, which we previously assumed to be the union of the spaces of actions that an agent could take on individual objects:

\[A^p_\sigma = (A_{o1} \cup \ldots \cup A_{oN}).\]

Note that these action spaces are dependent on the particular agent, and may not be the same for each agent. Furthermore, the possible actions for each object may also not be the same.

For our human-robot interaction experiments, we will consider the set of physical actions to be some discrete set of possible spatial movements of the objects. We will use the set of actions corresponding to movements in the four ordinal directions (up, down, left, right), as well as the case where there is no (or nearly no) movement. As with the state space above, observations of actions must be determined on the basis of continuous-valued perceptual data. In this case, the perceptual data are the discrete-time difference measurements \((\dot{X}(t), \dot{Y}(t))\), where \(\dot{X}(t) = X(t) - X(t - 1)\). As before, we would like to calculate a PMF over this set of discrete actions, rather than trying to produce a single point estimate of the observed action. Unlike before, however, our action classes are determined by movement in some particular direction (i.e. downward), not necessarily a particular range of values for \(\dot{X}\) or \(\dot{Y}\). Therefore, we first find the normalized movement vector:
\[ D(t) \triangleq (\dot{X}(t), \dot{Y}(t)), \]
\[ \hat{D}(t) = \frac{D(t)}{\|D(t)\|_2}. \] (5.2)

Ideally, the probability of \( \hat{D} \) being one of the ordinal movements should be proportional to how close it is to the unit vector for that movement \( \delta_a \):

\[ \hat{P}(a|\hat{D}(t)) \propto e^{\sigma_d \|\hat{D}(t) - \delta_a\|_2^2}. \] (5.3)

However, we must also account for the no-movement case, which corresponds to small magnitudes of \( D \). To do this, we heuristically let the probability of a “no-move” action be proportional to the magnitude of the original \( D(t) \):

\[ \hat{P}(a_0|D(t)) \propto e^{\sigma_d \|D(t)\|_2^2}. \] (5.4)

As with the state probabilities, the specific value chosen for the parameter \( \sigma_d \) will control the smoothness of the PMF across the actions.

State and Action Dynamics

One of the critical components of our pragmatic engine is the model of state and action dynamics, or the model of how actions influence the evolution of the world state from one time step to the next. This “transition” model is the same one that is central to the Markov decision process (MDP) upon which much of our framework is based. We define the following notation for the transition model, \( T \):

\[ [T]_{ijk} = P(S_{t+1} = k|S_t = i, A_t = j), \] (5.5)

where \( \sum_k T_{ijk} = 1 \). For the most part, we consider the state and action models to be factored on the basis of the objects they represent. This means that actions on a particular object will affect only the state of that object, and no others, resulting in much simpler computation for calculations involving \( T \). Relaxing this assumption would surely be beneficial to the
model’s ability to represent complex physical interactions, but it would also cause a significant increase in computational complexity.

Recalling the grid-based state space for individual objects, and our representation of actions as single movements along this grid, we describe the following implementation for the state-action transition model. Let \( s^{(k)} \) be the currently occupied state for an object, and let \( s^{(l)} \) be the state directly above it in the grid. We then define:

\[
P(S_{t+1} = s | S_t = s^{(k)}, A_t = a_{up}) = \begin{cases} 
\zeta & \text{if } s = s^{(l)} \\
\zeta/8 & \text{if } s \neq s^{(l)} \text{ and } d(s, s^{(l)}) \leq \sqrt{2},
\end{cases} \tag{5.6}
\]

where \( d(s, s') \) is the Euclidean distance in grid units between state \( s \) and \( s' \). This means that there is some probability that a move along the grid will result in reaching the “ideal” location, plus all adjoining locations on the grid. The formulation in equation (5.6) is the same for moves in all four directions, as well as the “no move” case. A typical value used for the \( \zeta \) above in our implementations is 0.9.

There are also situations in many of our human-robot interaction experiments where an object may not be moved freely in some direction, for reasons such as the presence of a physical barrier or other constraints on its motion. In these cases, transition probabilities into states corresponding to locations of obstructions are set to 0, and the values of the PMF are re-normalized. If the object is in a location where an agent is unable to reach it or otherwise act upon it, transition probabilities are set to 1 for self-transitions, and 0 for all else, for each action.

5.2.2 Intentional and Belief Elements

In our framework, we have used the word “intent” to refer to the goal an agent is trying to reach through its behavior, a concept which is embedded in the MDP and its formulation of the reward function. However, we have noted that one of the core premises of this thesis is that even though reward functions may span the entire state space (or even more complex spaces), the number of actual potential rewards we search over is far smaller. The interactions which we target are structured in a way that rewards can be generated from more compact sets of parameters and variables, which have complex connections to linguistic representations. In order to greatly simplify the problem of learning these connections between words and intents,
we have assumed a particular structure for the interaction format, its general task, and the variable aspects of the intent. Under this assumption, intent states are the possible objects for which some general task is being performed. This was captured in our formulation for the reward function given in equation (3.19):

\[ R_i(s; \theta) = \theta^T \psi(g_i(s)). \]  

(5.7)

**Intents**

The space of possible values for the intentional state are the indices of possible objects upon which the desired task is to be performed: \( \mathcal{I} \in \{1, 2, \ldots, N\} \). The set of potential intended objects does not necessarily have to be equal to the set of all objects present in the current environment, but we do assume it to be a subset of this larger set for our implementation. All of the objects have some corresponding spatially defined state, as detailed in equation (5.1). An object is perceived visually, requiring our robot to be capable of segmenting the objects within an image and estimating their location through some means (e.g. stereopsis). We will detail the processing system for performing these functions in Section 5.3.

One of the key functions of the intentional state is to serve as an argument of the reward generating function above. Specifically, we have defined a function \( g_i(s) \) that effectively selects the subset of the complete state space over all objects that are specific to a particular intended object: \( g_i : \mathcal{S} \rightarrow \hat{\mathcal{S}}_i \). In some of our experiments, \( \hat{\mathcal{S}}_i \) may be strictly equal to the individual state space for the object, \( \mathcal{S}_i \). For others, it may also include the states of other “auxiliary” objects that are important to the overall task, but are not part of the set of possible intents. The same is also true for the function \( h_i : \mathcal{A} \rightarrow \hat{\mathcal{A}}_i \), which performs the same kind of selection over the space of actions.

**State Features and Task Parameters**

Our interaction formats also include an invariant, “task” component, which is the goal to be satisfied for an intended object, regardless of the specific object that has been selected. After an intent-specific reduced state space \( \hat{\mathcal{S}}_i \) has been selected, the function \( \psi(\hat{s}) \) extracts the value of the “features” for each state: \( \psi : \hat{\mathcal{S}} \rightarrow \mathbb{R}^f \). The values of the task parameter vector
\( \theta \in \mathbb{R}^J \) are the weights applied to each feature before summing to find the reward value at that state. For some scenarios, we use a straightforward implementation of the feature function, where \( f = |\hat{S}| \), and \( \psi(s) = 1 \) only for the \( s \)-th feature, and 0 for all else. In this case, the task parameter becomes effectively a standard vector for the reward values at each state.

In other cases, specifically those for which the task involves both an intentional and auxiliary object, we use a more complex feature space. This includes the above features for each of the intended and auxiliary objects, as well as features that are functions of both objects. These might be features such as the distance between the objects, or their relative positions. In either of these cases, we will see that the unsupervised inverse reinforcement learning (IRL) technique will learn what features in this more complex representation are relevant to the observed task.

5.3 Visual Processing

Proper perception of many of the elements of this model rely on complex capabilities for the processing of visual information. These include the ability to not only segment and track multiple objects within a stream of images, but also the ability to determine its spatial location and movements. There are also other kinds of social information useful to our experiments, such as gaze, that must be determined through visual processing. Many of these processing capabilities are built upon one another (as depicted in Figure 5.1), and therefore require computationally efficient algorithms in order to be capable of real-time processing for successful interaction. In this section, we discuss the techniques we use for the object segmentation, tracking, and localization problems, as well as our method for generating joint attentional saliency information from gaze features based on an artificial neural network.

5.3.1 Object Segmentation and Tracking

In order to vastly simplify the complexity of the object segmentation and tracking task, we first make a handful of assumptions based on our training scenarios:

1. The interaction environment consists of a largely white background and workspace (table), on which the objects are placed.

2. The objects are generally colorful in nature, in order to ease the problem of separating foreground pixels from the background.
Figure 5.1: Block diagram picturing the flow of data among the various modules of the visual processing system.

(a) Example of typical visual environment.
(b) Example segmentation map generated by the algorithm.

Figure 5.2: Sample images of the robot’s view of the environment during the human-robot interaction experiments.

3. The objects are generally free of “internal” details, or high-contrast areas that lie within the visual outline of the object.

4. An object does not move in such a way that there is little/no overlap between its bounding box from one frame to the next.

An example image of the robot’s view of a visual environment satisfying these assumptions is pictured in Figure 5.2a.

A diagram of the segmentation and tracking processing chain is shown in Figure 5.3. At its core, the algorithm thresholds the saturation of the image to isolate colorful regions, then further separates these regions based on edges detected with the common Canny algorithm [129]. After mor-
Figure 5.3: Block diagram picturing the modules and processes of the object segmentation and tracking algorithm.

Phological operations (dilation/erosion) aimed at filtering out speckle noise, we are left with a number of blobs, mostly corresponding to various objects. Parallel to this operation is the back-projection of color histograms for previously segmented objects onto the image [130], producing an array of images equal to the number of such objects, which are then normalized for each pixel across objects. The resulting images approximate the probability of a particular pixel belonging to an object.

Following this, the CamShift algorithm [131] is used to adjust previous estimates of an object’s bounding rectangle. Under the 4th assumption listed above, this allows us to track an object as it moves through the visual scene, and further refine the probability map by zeroing out pixels lying outside the bounding box. After this step, taking the argmax of the array across objects yields a single image with guesses of the object index at each pixel. These estimates are used to paint the initial segmented blobs from the thresholding operations above, where applicable. Finally, the image of these labeled regions is passed on to a watershed segmentation step [132] that produces a final labeled image (example shown in Figure 5.2b). Color histograms and bounding boxes are calculated for any new objects that have appeared, and these are added to the current list of actively tracked objects.

For most of these fundamental techniques, existing implementations contained in the OpenCV [133] software library were used. The simplicity and efficiency of these algorithms allows for the real-time processing of 320 × 240 color images streaming at 15 frames per second.
5.3.2 Estimation of Location and Motion

Given a consistent labeling of segmented objects from image to image, it is now possible for us to reliably keep track of an object’s spatial location and movement over the course of the interaction. To do this, we make use of the iCub’s binocular vision capabilities, and the OpenCV implementation of the Semi-Global Block Matching (SGBM) algorithm to compute a dense disparity/depth map for the visual environment [134]. This depth map can be transformed through the iCub’s forward kinematics to produce a Cartesian XYZ for each pixel relative to the root reference frame of the robot [135].

Combined with the segmentation map, a list of 3-D spatial locations for pixels corresponding to each object can be generated. But in order to do this, there are a number of questions and challenges to be addressed. The first is picking a method for producing a single point estimate for an object’s position from the point cloud of single-pixel estimates. The most intuitive method might be to simply calculate the mean or median value of the list of points, or some other method for approximating the geometric center of the point cloud.

The second major challenge is producing a location estimate that is reliable, given the potentially significant noise and inaccuracy in the single-point measurements of the cloud. While calculating a statistic such as mean or median can handle some amount of uncertainty, they generally require both the points and their noise to be normally distributed around the object center, which is the case much of the time. But often times, situations in which the object is occluded, is moving, or has a lack of defining visual features, will result in 3-D point estimates that contain significant clusters at other spatial locations, in addition to a number of other outliers. This can produce location estimates that vary significantly from frame-to-frame as an object moves or comes close to other objects.

To mitigate this problem, we return to some of the same simplifying assumptions we used in the development of our segmentation and tracking algorithm, specifically those dealing with the location of our objects on a table, and their expected movement from frame to frame. But this time we consider their spatial, rather than spectral, implications. Since we assume to know the presence of objects on a flat table, we are able to discard points for the objects that are estimated to be below the general plane of the table ($Z < -0.3$). Next, we assume that the movement of the object in space will not be greater than roughly 3 m/s in any direction. For our
robotic implementation, images are streamed from the cameras at 15 frames per second, which means we do not expect the object’s position to differ by more than 0.2 m from one frame to the next. Therefore, we reject as outliers any estimated points for an object whose Euclidean distance to the previous estimated location of the object is greater than 0.2 m. After this rejection step, we simply estimate the new spatial location of the object using the mean or median of the point cloud as before.

5.3.3 Gaze Estimation and Joint Attention

Another supporting skill that is critical to a real-world robotic implementation is that of joint attention. The joint attentional faculty allows agents to socially construct shared attentional frames, which is the entirety of objects or events that the interacting agents commonly know to be possible targets of attentional focus within the scenario. This faculty includes both the ability to recognize another’s attentional focus, and the complementary ability to use one’s own attentional focus to direct the attention of another [136, 74]. As we have discussed in previous chapters, recognizing possible attentional states of a speaker is often a key piece of information in narrowing the range of possibilities for his/her referential intent. We therefore seek to implement some small processing mechanism for using joint attentional cues to serve as a “social spotlight” in service of the word-learning problem [4]. The goal of this capability is only for the purpose of partially reducing the ambiguity of the speaker’s intent in the initial tests of our pragmatic model — where no other information about intent is available — and is not used to reduce it completely.

The active and passive aspects of joint attention — the ability to direct another’s attention and the ability to follow this direction, respectively — are two sides of the same coin. Recognition of another’s attentional focus is in large part the recognition of their directive action. Humans are able to direct attention both through linguistic behaviors (e.g. “Look at the ball!”) as well as non-linguistic behaviors (e.g. pointing). The latter category of behaviors is perhaps the most important for the pre-/early-linguistic children that are our focus. Primary among these is use of gaze-direction for direction and inference of attentional focus.

Determining the object or event of focus from gaze requires an agent first to visually estimate the head and eye pose of the other agent, and then map this gaze-direction onto some area of the environment. This map can be constructed on the basis of predetermined geometric knowledge, or it
can be learned through sensory experience. Our approach, like those used in most CDR experiments, will be learning-oriented. Previous work in the cognitive robotics field has yielded frameworks for learning gaze-direction using a variety of learning techniques, such as reinforcement learning [95] and neural networks [32], and learning styles — agents that behave passively [137] and actively [138] in their interactions.

We have previously developed such a system based around neural networks trained via Hebbian learning [139]. Its target scenario features a human and robot seated at a table with various toys placed on top of it. In each training episode, the tutor first fixed his gaze on a specific object, and then manually interacted with the object (by shaking it), in order to make it salient to the robot. The robot’s task was threefold: estimate the head pose, detect and estimate the location of an interesting/salient event, and use these to learn a map from head pose to spatial locus of attention.

The first task, visually estimating head pose, is arguably the most well-studied part of this entire problem. Approaches vary mostly on their methods of facial feature extraction and ways of transforming these features into descriptions of head pose (see [140] for a more complete review of these techniques). Computationally expensive approaches based on structural models or template matching are not suited to our agent who must react in real-time, and is already under significant processing burden from other tasks. Because of these considerations, we use a technique similar to the one used in [141], based on a prior geometric model of the face. But instead of head pose representation based on head tilt/pan and eye vergence angles, we construct a representation that can be quickly calculated from a single frame. First, we run the image through a color filter designed to select skin tones. Following a thresholding, we look for the largest “blob” of skin-colored pix-
els, which we assume to be the head. We then look for the empty blobs within the head contour that are most likely to be the eyes (cf. Figure 5.4a, some head angles may result in only one eye contour being visible). Finally we estimate the position of the bridge of the nose between the eyes, relative to a bounding box placed around the head. We use this estimate to derive a representation for head pose that captures head azimuth/elevation information:

\[ \hat{p} = \frac{(A_1 + A_2)}{(A_3 + A_4)} - 1 \]  
\[ \hat{q} = d - d_0, \]

where the pseudo-azimuth \( \hat{p} \) and pseudo-elevation \( \hat{q} \) are calculated using the derived metrics depicted in Figure 5.4b, with \( A \) values giving areas (in pixels), and \( d \) values giving distances (in pixels). The quantity \( d_0 \) is the baseline distance from the bottom of the face bounding box to the eye midpoint, corresponding to a tutor head pose that is fixated on the robot’s eyes.

The second task is for the robot to detect interesting visual events, created by the tutor’s active interaction with an object. In this context, an object’s “interest” is determined by its colorfulness (relative to a white background) and any motion. For color saliency, we use one of the more standard transformations proposed by Itti, Koch, and Niebur [142], which produces new values of the red, green and blue channels from the original RGB pixel values \( r, g, \) and \( b \):

\[ R = r - (b + g)/2, \]
\[ G = g - (b + r)/2, \]
\[ B = b - (r + g)/2. \]

A motion salience map is constructed based on the squared-difference between the current and previous grayscale images, \( G_t \) and \( G_{t-1} \). This image is spatially low-pass filtered using a Gaussian kernel to reduce salience caused by edge jitter, and then saturated in order to mitigate the exaggerated salience values produced by high-contrast areas and quick motions. This step is necessary to balance the weight of focus placed on the kind of low-speed motions we expect to see from a tutor’s realistic interaction with an object. The last step is to apply a thresholded version of the combined
color salience map $T(C_t)$ as a mask to the post-processed motion salience image $H_t$ and run it through a leaky integrator to get the final combined salience map $S_t$:

$$S_t = T(C_t) \cdot [H_t + (1 - \alpha)S_{t-1}], \quad (5.10)$$

where $\alpha$ is the leakage constant, which has values on the order of $1 \times 10^{-3}$ for our experiments. This leaky integration selects for objects exhibiting sustained activity by building up salience values for moving objects over time. When a point or “blob” of points exceeds a set salience threshold $\tau$, its $(x,y)$ location (or the location of the blob’s centroid) is paired with the current estimate of the head pose $(\hat{p}, \hat{q})$ and used as a training sample for the map.

The map itself is called upon in two different situations. The first is the training situation described above, where both input and output values for the map are provided. The second is the prediction situation, where the agent only has access to a head pose $(\hat{p}_0, \hat{q}_0)$, and must use the map to estimate the location the tutor is attending to. For our application, we would like these situations to be in free variation within a larger online learning scenario. To do this, we need a model and a learning rule capable of incremental updating and on-the-fly prediction. Specifically, we use an artificial neural network composed of an input, output, hidden input, and hidden output layers. The hidden layers are connected by a set of weights $w_{ij}$, with $i$ indexing the outer hidden neurons, and $j$ indexing the inner. Activations of the inner and outer hidden layers are generated by the input and output neurons using radial basis functions (RBFs). These RBFs are normalized Gaussian functions centers $c_i$ and $c_j$, and widths $\Sigma_i$ and $\Sigma_j$. Even with their relatively simple structure, RBFNNs are capable of approximating any arbitrary mapping, such as our target mapping, $(\hat{p}, \hat{q}) \rightarrow (x, y)$.

We assign the centers $c_j$ and $c_i$ for hidden inner and outer neurons so that they are evenly distributed across the input and output spaces. Internal weights $w_{ij}$ are updated each time a training sample is received using a generalized Hebbian learning-like rule [143], with learning rate $\eta$ and leakage coefficient $\epsilon$ (a typical heuristic used to mitigate the effect of outliers):

$$w_{ij}^{(t+1)} = \epsilon w_{ij}^{(t)} + \eta(F_{ij}(\hat{p}, \hat{q}, x_t, y_t) - F_i(x_t, y_t)W_j^TF(x_t, y_t)). \quad (5.11)$$

The term $F_{ij}(\hat{p}, \hat{q}, x_t, y_t)$ above is the complete activation of each neuron at time $t$, which can be factored into the inner and outer hidden layer activa-
tions \( F_j(\hat{p}, \hat{q}, t)F_i(x_t, y_t) \). As stated, the individual \( F \)'s are calculated from Gaussian RBFs — for example, \( F_i(x_t, y_t) \) is given by \( \mathcal{N}(x, y; c_i; \Sigma_i) \), where \( c_i = [c_{x_i}, c_{y_i}] \) and \( \Sigma_i \) is diagonal. It is worth noting that only one hidden layer is theoretically needed to learn our arbitrary mapping. Because early sampling of input and output spaces is quite sparse, however, the smoothing of activations on both input and output aids in generalization, even for low numbers of training samples.

In non-training episodes, the gaze location \((x, y)\) can be predicted using the estimated head pose and the current value of the learned weights. To do this, we first multiply the trained weight matrix \( W \) by the inner activation vector \( F_j(\hat{p}_t, \hat{q}_t) \) and re-normalize to generate the predicted outer activation, which we denote \( \hat{F}_i(x_t, y_t) \). The following equation shows how to calculate the predicted output activation \( \hat{x} \), with a similar result holding for \( \hat{y} \) as well:

\[
\hat{x} = \sum_{i=1}^{N} c_i \hat{F}_i(x_t, y_t).
\]  

(5.12)

While exact values for \( \hat{x} \) and \( \hat{y} \) are mathematically pleasing, they may not be as practically useful for our application, where we would like to estimate the possible object(s) of attention. One naive approach might be to derive a distribution over objects based on their distance from \((\hat{x}, \hat{y})\), but calculating this distance correctly would require further processing to correct for spatial distortion caused by orthogonal projection. Fortunately, such information is already built into our network, which learns the shape of this arbitrary mapping, distortion included. Instead of calculating attention as a specific point, we reorganize the predicted outer activations \( \hat{F}_i(x_t, y_t) \) into a grid based on their corresponding \([c_{x_i}, c_{y_i}]\) values. We then up-sample this to our original full-image resolution and low-pass filter to produce what itself can be thought of as a saliency map, which we denote with \( J_t \). Using the same thresholded color salience map as before (cf. equation 5.10) to mask \( J_t \), we can calculate a salience score for each object blob, consisting of set of pixels \( B_k \):

\[
z_k = \sum_{b \in B_k} J_t(b) \cdot T(C_t(b)).
\]  

(5.13)

The joint saliency scores for each object \( z_k \), can be used to construct a probability distribution for the attentional state through use of a softmax or other typical non-linear activation function. Figures 5.5a and 5.5b show an example calculation of \( J_t \) and its application to selecting the object of maximum attentional salience. This probability distribution can be used
Figure 5.5: Joint attentional saliency map constructed from activations of learned RBFNN and the object of most likely attention based on the joint attentional saliency map.

as a prior on the speaker’s intent, and used to narrow the set of possible referential targets in the basic object-word learning scenario to come.
In this chapter, we present the results of the application of the models and methods developed in the previous chapters to a set of human-robot interaction experiments. These are based on a select group of experiments — found in the developmental literature concerning early language acquisition — that were used as the motivating examples during the development of our computational framework. While our scenarios are not exact reproductions, they are intended to evaluate the same kinds of core pragmatic abilities thought to be used by children to resolve referential ambiguity in these experiments. These include not only the principles of cross-situational learning and lexical contrast present in current embodied word-learning frameworks, but also those that are more specific to our model, such as learning from goal-directed behaviors, or leveraging information about action constraints and triadic interaction structure.

6.1 Experiment I: Pragmatic Learning of Basic Object-Words

The experiment we perform is of a very simple nature, primarily attempting to evaluate the fundamental soundness of the learning algorithms derived for the basic pragmatic model. The goal of our framework in this case is to learn a set of words and how they are used to refer to various objects in the visual space (i.e. their meaning). In our experiments, we seek to demonstrate the ability of our model to learn from observations featuring referential ambiguity through the use of basic statistical processing capabilities and emergent lexical contrast principles, in line with the results of previous work [4, 53]. Finally, for these initial experiments and those that follow, training data is presented continuously, requiring the use of real-time perception and online learning techniques developed in the previous chapters.
6.1.1 Setup and Scenario

Our very first experimental setup was composed of a human tutor and robot learner situated at either side of a table, on which a number of simple objects had been placed. The general physical setup of the interaction is pictured in Figure 6.1. For the initial experiment, the objects were a green wallet, a gray stuffed rat toy, an orange ball, a red funnel, and a green plastic donut. Two more objects, a purple dart gun and a blue plastic donut, were introduced at a later phase in the experiment. The complete list of objects used for all experiments, their descriptions, and word label(s), along with an example image for each are given in Table 6.1. The interaction itself was based upon repetition by the tutor of the following script, similar to the format depicted graphically in Figure 3.1:

1. The tutor selects one of the objects on the table as his/her intended referent for the training episode.

2. The tutor then directs his/her gaze toward this intended target.

3. Upon completion of the gaze fixation, the tutor verbally produces the label of the target object.

4. After a brief pause (2-3 seconds), the tutor returns his/her gaze to the robot.

While it was not provided as direct knowledge to the robot, the experiment was divided into two phases, each following this script. During the
Table 6.1: List of objects used for human-robot interactions.

<table>
<thead>
<tr>
<th>Interaction Objects</th>
<th>Description</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey rat</td>
<td>“rat”</td>
<td></td>
</tr>
<tr>
<td>Red funnel</td>
<td>“funnel”</td>
<td></td>
</tr>
<tr>
<td>Green wallet</td>
<td>“wallet”</td>
<td></td>
</tr>
<tr>
<td>Green donut</td>
<td>“donut”</td>
<td></td>
</tr>
<tr>
<td>Orange ball</td>
<td>“ball”</td>
<td></td>
</tr>
<tr>
<td>Blue donut</td>
<td>“ring”</td>
<td></td>
</tr>
<tr>
<td>Purple gun</td>
<td>“gun”</td>
<td></td>
</tr>
<tr>
<td>Blue seal</td>
<td>“seal”</td>
<td></td>
</tr>
<tr>
<td>Orange block</td>
<td>“block”</td>
<td></td>
</tr>
<tr>
<td>Pink shape</td>
<td>“shape”</td>
<td></td>
</tr>
<tr>
<td>Green cap</td>
<td>“cap”</td>
<td></td>
</tr>
</tbody>
</table>
first phase, the script was performed using only the first five objects in Table 6.1, while the other two (the blue donut and gun) were not yet present. After sufficient training for the basic learning of these associations, the second “phase” began, and the other two objects were introduced. The experiment then proceeded in the same manner as before, with the exception that the two new objects were now possible intentional targets within the script. The purpose of this phase is to demonstrate the effective emergence of “lexical contrast” principles within the pragmatic model. This is based on our intuition that previous learning about word-meanings for the initial object set will help to significantly improve intention inference and learning speed for newly presented object-word pairs.

6.1.2 Implementation Details

The basic pragmatic model and learning algorithm outlined in Section 3.2, along with the algorithms for construction of speech representations (Chapter 4), and visual perception (Chapter 5) were implemented on the iCub humanoid robot. The object segmentation and tracking algorithm produced integer labels for each of the objects in the visual scene, which were used as the set of possible intended referents for the tutor’s utterances. The representation of words themselves were constructed using the sample implementation as was done for the action-word learning experiments presented in Chapter 4. This included the transformation of the speech signal, first into a 13-element MFCC feature vector, then into a sequence of phonetic symbol probabilities, using the 10-state “phone” HMM, which were finally used as inputs for the lexicon learning algorithm.

The other critical detail in the implementation of this experiment is the use of gaze information, which we discussed in Section 3.2 to help shape the prior over intents $P(I|Z)$. The salience values $z_k$ from equation (5.13) for each object are used to construct the prior probability for an intended object given the gaze information:

$$P(I = k|Z) = \frac{z_k + \varepsilon}{\sum_k z_k + \varepsilon}. \quad (6.1)$$

The “smoothing factor”, $\varepsilon > 0$, serves two purposes. The first is to ensure that the prior $P(I|Z)$ has full support of the entire set of $I$. This makes proper inference of the intent using the complete posterior $P(I|A,Z,\phi)$ possible in cases where the gaze information is inaccurate, but the listener has
Figure 6.2: Basic generative model for production of words from meaning.

high confidence in the meaning of an utterance. The second purpose is an experimental one: it allows us to artificially control the ambiguity of our training samples, in order to ensure the need for handling cross-situational statistical information in the learning of word-meaning associations. In our experiment, we set the value of $\varepsilon$ to $0.5/N_{o}^{(t)}$, where $N_{o}^{(t)}$ is the number of objects in the workspace at time $t$.

During the experiment, sensory data was continuously streamed to the robot, and the learning of object-words was performed in an incremental, online manner. Training samples for the word-meaning map consisted of a speech lexicon classification symbol, and a gaze-based probability distribution over the possible objects present in the visual scene. These training samples were generated whenever an utterance was perceived, and used to perform a single-step gradient update on the estimated map parameter $\phi$, according to equations (3.10) and (3.11).

For the purpose of evaluating the performance of our model in this basic object-word learning experiment, we define the following “generative” model of word-meaning learnings as a baseline for comparison. In this model, shown in Figure 6.2, the mapping parameter directly encodes the probability of a word being generated given a particular intentional state (i.e. meaning):

$$\lambda_{jk} \equiv P(A = j | I = k).$$

(6.2)

This general type of structure is common in statistical models of the word-meaning learning problem [40, 4, 56] — including the “concept” model presented for action-word learning in Chapter 4 — and so serves as good point of comparison. However, it should be noted that in these examples, successful estimation of the parameters relies in some part on the use of algorithms that operate over the entire corpus of training data. This stands in opposition to one of the fundamental premises underlying this work and the work of others [53], which is that learning is a continuous, adaptive process. Acknowledging that a direct comparison to these techniques is therefore not
possible, we propose the following simple stochastic gradient-based rule for estimating the model parameters in an online learning scenario:

\[
\lambda_{jk}^{(t+1)} = \lambda_{jk}^{(t)} + \eta \cdot \frac{\partial}{\partial \lambda_{jk}} \log P(a_t|z_t, \lambda^{(t)}),
\]

(6.3)

\[
\frac{\partial}{\partial \lambda_{jk}} \log P(a_t|z_t, \lambda) = \frac{1}{P(a_t|z_t, \lambda)} \sum_i P(I = i|z_t) \cdot \frac{\partial}{\partial \lambda_{jk}} P(a_t|I = i, \lambda)
\]

\[
= \frac{P(I = k|z_t)}{P(a_t|z_t, \lambda)} \cdot \mathbb{I}_{\{a_t = j\}}.
\]

(6.4)

### 6.1.3 Results and Discussion

The results of this human-robot interaction experiment are presented in Figures 6.3 and 6.4. In Figure 6.3a, the word confusion matrix for the speech lexicon learning algorithm that was discussed in Chapter 4 is pictured. The algorithm effectively and autonomously constructed a speech representation that was able to learn the seven spoken words of the experiment without any classification errors. Figure 6.3b shows the final estimate of the learned word-meaning map — parameter \( \phi \) of equation (3.9) — at the end of the experiment. In it, we see the learning of the proper associations between the word lexicon items and objects achieved by our techniques.

However, Figures 6.4a and 6.4b are most essential for understanding and evaluating the success of the model in achieving its goals. Figure 6.4a graphs the probability of inference for the actual intentional state of the speaker over the course of the experiment, given three different types of models and information. Shown in black is the probability of correct inference based solely on gaze information. Due to inaccuracy or ambiguity inherent to the gaze estimation algorithm, as well as our own artificial smoothing, it is rare for this probability to be greater than 0.5, and in many cases (around 30%) it is not even the most likely among intents. Yet we can see that the posterior probability of the actual intent given gaze and word information quickly rises to unity as the model learns the word-meaning mapping.

Besides intention recognition accuracy, we would also like to evaluate the performance of our model and algorithms in terms of how close their estimate for the word-learning map, \( \hat{\phi} \), is to the ground truth, \( \phi^* \). Because the
Figure 6.3: Results of the basic object-word learning experiment.
(a) Probability of correct intent inference.

\[ D_{JS}(P(i|a, \hat{\phi}) || P(i|a, \phi^*)) \]

(b) Jensen-Shannon divergence between actual and estimated mappings.

Figure 6.4: Performance over the course of the object-word learning experiment.
parameters of the map represent a probability distribution over intents — conditioned on a linguistic symbol — it may be useful for us to apply a metric that measures the divergence between the estimated and ground-truth distributions. For this purpose, we employ a divergence metric based on the Jensen-Shannon divergence. The Jensen-Shannon divergence is a symmetric divergence for probability distributions \( P \) and \( Q \) that is itself based on the Kullback-Leibler divergence:

\[
D_{JS}(P||Q) = \frac{1}{2} \cdot [D_{KL}(P||(P + Q)/2) + D_{KL}(Q||(P + Q)/2)] 
\]

\[
D_{KL}(P||Q) = \sum_i P_i \log \frac{P_i}{Q_i}.
\]

We then define our derived distance metric as the following:

\[
D(P(I|A, \hat{\phi})||P(I|A, \phi^*)) = \frac{1}{N_a} \sum_j D_{JS}(P(I|A = j, \hat{\phi})||P(I|A = j, \phi^*)). 
\]

The divergence measure between the estimated and ground-truth maps is pictured in Figure 6.4b. We can see that the model is able, for the most part, to successfully learn the word-meaning map, in spite of significant ambiguity, simply on the basis of statistical processing.

This steady convergence of our word-meaning map estimate is disrupted in our experiment, however, when novel potential referents and words are introduced. In these same two figures, we see that our baseline generative model is able to do little better than gaze-only information when inferring intent for a novel object-word pair, and as a result, is slow to learn mapping between these items. By comparison, the basic pragmatic model is able to immediately resolve this ambiguity, by discounting possible targets for which it has learned other words to be more effective communicators. This allows the model to very quickly re-converge toward the correct word-meaning parameter value. This capability is similar to that of the model presented in [53], but in the case of our model, demonstration of the lexical contrast principle is an emergent result of a more general ability for pragmatic understanding of behavior.
6.2 Experiment II: Learning from Intentional Behavior

We now move on to the set of experiments for testing the aspects of our model that constitute some of its most significant contributions. These are the experiments in which our learner must learn and exploit knowledge about the dual physical and communicative structure of the interaction in order to resolve the tutor’s ambiguous linguistic references. In particular, we apply the extended pragmatic model and algorithms presented in Section 3.3 to a set of scenarios where the tutor is repeatedly performing some physical task with a particular object, which is accompanied by a verbal description. After learning a goal-centered representation of the task, our learning agent is able to learn the meaning of the verbal descriptions, even in cases where the physical component of the speaker’s behavior is incomplete, unsuccessful, or otherwise ambiguous, in terms of its intended target.

6.2.1 Experiment IIa: Searching Game

The first of these experiments is built on the “searching game” of Tomasello, Strosberg, and Akhtar [20]. In the original experiment, the adult tutor verbally announced her intention to find a particular, novel object in a row of buckets all containing such novel objects. The adult then began her search, retrieving objects one by one, rejecting and replacing objects, until the intended target was found. The episode concluded with the adult handing the target object over to the child. The purpose of this experiment was to demonstrate the inadequacy of basic attentional or saliency cues (e.g. the adult’s temporally proximal interaction with an object) in resolving ambiguity, and the necessity for the child to understand the entirety of the adult’s behaviors in the context of the particular task (i.e. retrieval and hand-over).

For our purposes, we designed an experiment in the spirit of [20], but with some major simplifications. As in the original, the procedure consisted of a training phase where the task was introduced to the learner, and a subsequent phase where language was used in conjunction with the searching process detailed above. In the task training phase, the training episodes were produced according to the following script:

1. **Selection**: The tutor selects one of the objects on the table as his/her intended referent for the training episode.

2. **Retrieval**: The tutor reaches for the intended target object, grasps it, and moves it to a location in front of himself/herself.
3. **Hand-over:** After a brief pause of 1-2 seconds (i.e. the “inspection”), the tutor then moves the object to another position in front of the robot.

4. **Return:** Following another brief pause (2-3 seconds), the object is then returned to its initial location, and the training episode is concluded.

A visualization of this interaction format is provided in Figure 6.5. It should be noted that there is no linguistic element to the task training phase, and that training samples consist only of successful demonstration (i.e. no searching) of the complete task. Additionally, the training samples are performed using different target objects (which are provided to the learning algorithm) and a number of random starting locations.

After enough task training samples for satisfactory learning have been provided, the experiment then moves on to the word-learning phase. For each episode in this phase, the tutor first verbally announces his/her intent (e.g. “Donut!”), and then attempts to perform the task according to the script above, with the difference being that many objects may be retrieved and subsequently rejected before the intended object is ultimately found and handed over. When rejecting an object, the tutor places it to the side (not necessarily at its original location), before moving on to the next. A total of six objects were used as the possible targets in our experiment: a pink triangle, a red funnel, a green spray-can cap, an orange block, an orange plastic donut, and blue seal (animal) toy. For each episode, one to two confuser objects were included in the search before ultimate completion of
6.2.2 Experiment IIb: Placement Game

A second experiment we performed attempts to use goal-directed behavior as a way of inferring referential intent in the absence of more ostensive cues, under a slightly different set of circumstances. In work by Akhtar, Carpenter, and Tomasello [21], this pragmatic capability was demonstrated by children in another kind of finding game. After establishing their intention to retrieve some particular object from a set of locked containers, as well as the location of each of the specific objects within the containers, the adult once again announced her target, and then attempted to complete the task. This time, upon reaching the container of the intended object, it was found to be locked, and the adult was unable to retrieve the object. Nevertheless, the children were shown to still be capable of learning the correct referents for the set of given words. There are many important aspects to these results, but one of the most significant is that it was not necessary for the child to see the adult successfully complete the task with an object to recognize it as the intent.

As in the previous scenario, we seek to emulate this underlying principle rather than the exact experimental conditions. To this end, we set up the following “Placement Game”, where the tutor must place some intended object directly next to some other auxiliary object. The intended objects are located in compartments that limit their potential motion to be along one direction (either into or out of the container). Likewise, the auxiliary object is constrained so that it can not be placed into these compartments, which

![Figure 6.6: Interaction format for Experiment IIb. After selection and verbal description of the target object, the speaker moves the auxiliary object to the setup location (blue). Following the setup, task is completed by sliding the target object out of its compartment — the sides of which are shown in black — and into place next to the auxiliary object (green).](image)

the task with the intended object. Sample pictures and word labels for each of these objects are again given in Table 6.1.
means that both objects must be moved in order to successfully perform the task. The task script, which is depicted in Figure 6.6, proceeds as follows:

1. **Selection:** The tutor selects one of the objects on the table as his/her intended referent for the training episode.

2. **Setup:** The tutor moves the auxiliary object to a location that is aligned with the reachable path of the intended object.

3. **Placement:** The tutor then moves the intended object along its allowable path to the target position relative to the auxiliary object — in this case, directly next to it.

4. **Return:** Following another brief pause (2-3 seconds), the objects are returned to their initial locations, and the training episode is concluded.

This experimental scenario is similar to the template [21] in the sense that the tutor must first complete a setup action (i.e. reaching and unlocking the container), in order to complete another subsequent action (retrieving the object inside the container). To avoid the challenge of representing and perceiving a complicated task such as unlocking a container, we have restructured the problem to retain the important idea of performing “unblocking” actions in order to complete a task given physical constraints. One final difference is the continuous visibility of objects in our experiment, versus the template’s objects which are not visible while in the containers. In the interaction episodes during the word learning phase, however, the saliency of objects are not modulated through gaze or explicit motion of the target object. The visual persistence of the object is needed only for the perception of its location, something that [21] assumes the child retains knowledge of even when the object is hidden.

The task training phase consisted of complete, successful demonstrations of the script above with the same objects that were used in Experiment IIa (Table 6.1). Following sufficient training samples for the convergence of the task reward parameter, the word learning phase then began. In these task episodes, the tutor once again announces his/her intent (“Donut!”) and then attempts to complete the task with the selected object. Here, only the *Selection* and *Setup* steps in the script are successfully completed, corresponding to the steps in the template scenario of the tutor reaching for and attempting to unlock the container. The target object is not acted upon by the tutor, and the intent must be inferred from the setup action alone.
Table 6.2: Perceptual processing and learning parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x_{min}, x_{max}) )</td>
<td>((-0.9, -0.4))</td>
</tr>
<tr>
<td>( (y_{min}, y_{max}) )</td>
<td>((-0.4, 0.4))</td>
</tr>
<tr>
<td>( M_x )</td>
<td>10</td>
</tr>
<tr>
<td>( M_y )</td>
<td>16</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>( (x_{max} - x_{min}) / (2 \cdot M_x) )</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>( (y_{max} - y_{min}) / (2 \cdot M_y) )</td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>( 1e \cdot -4 )</td>
</tr>
<tr>
<td>( \eta_\theta )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \eta_\phi )</td>
<td>0.2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2</td>
</tr>
</tbody>
</table>

During any single task episode, the target object is one of three randomly chosen objects from the complete set that are located in the compartments, meaning that the baseline (i.e. without action information) ambiguity can only be resolved to a set of three objects, or around \( \log_2 3 \sim 1.6 \) bits of entropy.

6.2.3 Implementation Details

Many of the implementation details are common to both of the interaction games, and others still are common to Experiment I as well. This includes the specifics of the speech/word lexicon learning algorithm implementation, which we leave unchanged from the previous experiments. As was also done in Experiment I, the possible target objects are segmented from the visual stream of the robot, given labels, and tracked from one frame to the next. In the experiments dealing with goal-directed action, we also track the spatial location of an object using the stereo vision capabilities of the robot and the methods presented in Section 5.3. The values (or probability distributions) for the state and action spaces are then computed according to equations (5.1) and (5.4). Table 6.2 lists the specific parameter values used in our implementation of these equations, as well as some of the other parameters used in the word learning stage, such as the learning rate \( \eta \) and softmax parameter \( \alpha \).

The state-action sequences that correspond to task training and word learning episodes are segmented on the basis of physical activity by the tutor and the environment, a task that is made trivial due to extended pauses taken by the tutor between training episodes. In the script for the word learning phase of the experiment, the announcement of the intended object
always precedes the demonstration of goal-directed action. Therefore, when an
utterance is heard and corresponding word symbol, \( m_t \in A_m \), is gener-
ated, this is matched with the next state-action sequence to be observed,
and used as a training sample, \( (m_t, \{s_t, a_t\}_{t=0}^T) \).

Between the two experiments explored in this section, there are a number
of relatively minor implementation differences, mostly in the specifics of the
task features and parameterization, as well as physical constraints on action
that are imposed through the state-action transition model.

Experiment IIa Details

For the Searching Game, there are no special considerations for the tran-
sition model \( T \) beyond what is detailed in Section 5.2. Likewise, we use a
straightforward set of features for each state, where the vector produced by
the feature-generating function \( \psi : \hat{S} \rightarrow \mathbb{R}^f \) is of length \( f = |\hat{S}| \), and has
a value of 0 for all but the \( s \)-th element, which has value 1. As mentioned
previously, this means that each element of \( \theta \) effectively encodes the exact
value of the reward function for each state in the (reduced) state-space. This
reduced state space is transformed from the complete state-space by intent-
dependent mapping \( g_i(s) \), which simply selects the individual state-space
corresponding to object \( i \).

Experiment IIb Details

Experiment IIb requires a slightly different and more complex implementa-
tion for many of these components. We begin with \( g_i(s) \), which reduces \( S \)
to a space composed of the intended object’s state, as well as the state of
the auxiliary object: \( \hat{S} = \{S_i \times S_{aux}\} \). The function \( h_i(a) \) transforms to a
similarly composed set of actions. The state-action transition model \( T \) is
defined such that the intended objects may not be moved laterally (in the Y
direction of the root frame) due to the constraints of the compartments, and
the auxiliary object may not be moved past the \( X \) locations \( (X > -0.6m) \)
at which the compartments begin. Finally, we define the feature vector \( \psi(s) \)
as the concatenation of three smaller feature vectors. The first two are the
same state-selector type of features used above, for both the intended and
auxiliary objects’ state spaces. The third is also a 0/1-valued vector corre-
sponding to the relative distance between the two objects with respect to
their locations on the spatial grid.
6.2.4 Results and Discussion

Both of the experiments presented in this section consist of two general phases: a task learning phase aimed at learning the intent-independent aspects of the physical task, and a word-learning phase, where a verbal description is provided alongside an incomplete, unsuccessful, or otherwise ambiguous attempt to perform the task using the specific object of reference. Here, we present the results of the task training, as well as demonstrations of the successful inference of intent from goal-directed behaviors, for each of the two experimental scenarios. We then use the results of the word-learning episodes from both to learn a single word-meaning mapping.

Experiment IIa Results and Discussion

The results of the task training phase for the Searching Game are presented in Figure 6.7. Three training samples (Figure 6.7a) were presented, and three of the gradient steps of equation (3.28) were taken for each demonstration. Figure 6.7b shows the estimated task parameters as the vector of feature weights, while Figure 6.7c displays it according to the reward value at each location of the state-space grid. From these, we can see that the reward function is nearly zero at all states, except for a small few, which correspond to the two locations where the tutor moves the objects during the retrieval and hand-over steps of the script. These invariances of the task are captured by the IRL algorithm, even under the variations in starting point and path of movement across the training observations.

The training episodes during the word-learning phase consisted of a series of movements of various objects toward the “retrieval” position, followed by either a rejection (not-intended), or a hand-over (intended) of the object, the latter of which concludes the training episode. In order to properly use the observed behavior to resolve ambiguity for word learning, the goal-centered (intentional) model is essential. Consider the data from an sample episode shown in Figure 6.8. In this example, cues such as motion salience of an object will be ambiguous (as three objects are moving over the course of the episode), while others, such as temporal synchrony, will be inaccurate (Objects 4 and 6 are moved prior to Object 2, which is the intended object). Direct models of trajectory, such as the HMM-based representation used in [87] and our own work in Chapter 4, will also be inadequate, given the variations in position and path seen in Figure 6.8a.

Figure 6.8b shows how the goal-based representation of behavior is able to
(a) Example demonstrations used in the training of the task.

(b) Learned feature weight vector for the reward function.

(c) Reward values for the object location-based state space.

Figure 6.7: Training examples and learned task parameterization for the “Searching Game”.

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correctly infer the intention of the tutor, by considering the entirety of the observed episode. This plot graphs the evolution of the learner’s estimate of the intended target as the episode proceeds in time. The likelihoods of Objects 4 and 6 rise and then fall as they are moved to the retrieval location in front of the tutor and then rejected. Finally, as Object 2 is retrieved and then moved to the hand-over location, its estimated probability as the intended object goes toward unity. This same kind of behavior is observed over the course of the other seven episodes in the word-learning phase of Experiment IIa. In each, the correct intent was inferred to within 1%.

Experiment IIb Results and Discussion

Sample trajectories as well as the final results of the task training phase for the Placement Game are displayed in Figures 6.9 and 6.10. Because the absolute and relative positions/movements of the two objects are an integral part of this scenario, the training sample trajectories are plotted with respect to each other. We can see that, in line with our intuition about the task, there are few regularities in potential behavioral goals with respect to the absolute frames of reference, while the opposite seems to be true for the relative positions of the objects. Here, trajectories appear to converge to the same general location.

These observations seem to be captured by the IRL algorithm, which produced the estimate of the task reward pictured in Figure 6.10 from six training samples, with three gradient steps taken for each. While not as strongly differentiated as the learned parameter for the Searching Game, the
Figure 6.9: Observed training trajectories of the relevant objects in the “Placement Game”, viewed in the three potential frames of interest.

Figure 6.10: Learned feature weights for the “Placement Game”. Blue, red, and black weights correspond to features relating to the auxiliary object position, target object position, and relative position of the objects, respectively. The spatial organization of this last set of feature weights is also shown.
features with the strongest weights are clearly those corresponding to the relative positions of the intended and auxiliary object, which is displayed in a spatial representation in Figure 6.10b. It is worth noting that the relative importance of these features to the task representation is an emergent and unsupervised determination of the IRL algorithm.

The interaction episodes for the word-learning phase, as with the previous experiment, were designed to test our algorithm’s ability to make inferences about intent from various types of ambiguous behavior. In this case, the ambiguity stems from the fact that only the steps of selecting the target object and moving the auxiliary object to a “setup” position reachable by that object, are observed. Figure 6.11 gives an example of such an episode, where the tutor moves the auxiliary object into a setup position for Object 3. Motion salience or temporal synchrony are again not capable of disambiguating the intended object, as no action is performed on any of the target objects. By comparing the trajectory of this movement (which moves from top to bottom along the $Y$ axis), with the estimated probability over intent throughout the course of the episode (Figure 6.11b), we also see that proximity to, or motion toward, a particular object is not necessarily a unambiguous indicator of intent. In this case, an understanding of the task, and how it drives rational action, is necessary for the inference of intent. The effectiveness of this method held across the set of 18 sample demonstrations, the average posterior probability of the correct intent, $P(I|\{s_t, a_t\}_{t=0}^{T}, \theta)$, was slightly greater than 98%.

Figure 6.11: Sample object trajectories over a single episode during the word-learning phase of Placement Game scenario. The corresponding estimates of intent at each time-step, given the entire state-action sequence up to that time.
Experiment IIc Word-learning Results

We now present the results of integrating the intention inferences from goal-directed actions in these two interaction games into the word-learning problem, as detailed in Algorithm 2. The eight training episodes from Experiment IIa, as well as the 17 samples from Experiment IIb, were used together to train a single object-word map. The result of the speech lexicon learning algorithm for the six words describing the six objects is presented in Figure 6.12a. The estimated word-meaning map parameter, $\phi$, between the elements of the speech lexicon and the set of visually segmented objects is shown in Figure 6.12b. Figure 6.12c plots the divergence between the estimated map and the ground-truth mapping over the course of the entire word-learning phase for our extended pragmatic model using information from goal-directed action for intent inference. This is compared to a salience-only model (like the basic pragmatic model used in Experiment I), which can not resolve ambiguity beyond the confuser set of three objects present in any single episode, as mentioned in our description of the scenario.

Analyzing these results, we see the desired performance from the lexicon learning algorithm, as well as our pragmatic word-learning algorithms. It is in these experiments that we begin to observe one of the primary contributions of this thesis. By learning about the physical task or interaction in which the word-learning problem is embedded, we are able to reason about a speaker’s intent from their goal-directed actions in situations where other possible information sources for determining referents, such as the visual or motion salience of an object, fall short. These include scenarios where the speaker’s actions are ambiguous (Experiment IIa), or non-ostensive with regards to the object of intent (Experiment IIa). These methods and interactions have also been somewhat unique, in that neither ostensive reference nor explicit feedback is necessary for successful word learning, a skill that mirrors the robust, hypothesized social-pragmatic capabilities observed in child word learners [27].

6.3 Experiment III: Learning from Social Interaction

In the final set of experiments, we attempt to test the ability of our pragmatic model to invoke a triadic understanding of interactive behavior — involving the speaker, listener, and physical environment — in the service of learning object-word meanings under referential ambiguity. At the same time, we hope for these experiments to demonstrate how the model can cap-
Figure 6.12: Results of combined experiment for word-learning from understanding of goal-directed behavior.
ture pragmatic meaning that extends beyond simple reference to an object, and into a request for a joint action between speaker and listener upon that object, framed by the context of the learned interaction. Finally, we show how the representation of the learner’s own ability to take action, embedded in the triadic model, allows the listener to become an active participant in the task, and to improve the process of word-learning.

The scenario for this set of experiments is based on the interactions presented in [75, 24], where a speaker is attempting to complete some task in the presence of a listener. The key feature to these scenarios are the locations of various objects, potentially to be used in the task, in one of two distinct physical spaces. These are the “Speaker’s” and the “Listener’s” areas, which designate the object locations that the speaker and listener, respectively, are capable of reaching. During the interaction, when the listener hears a verbal request for an object, s/he will favor intentional inferences for objects in areas reachable to himself/herself, and not to the speaker. Under a pragmatic view, objects that are reachable by the speaker are discounted by reasoning that the speaker would have had better chances of completing the task simply by acting his/herself, rather than enlisting the aid of the listener. This requires the listener to exploit knowledge of the task structure, the action constraints of both agents, as well as linguistic conventions. As we mentioned previously, the experiments in [75, 24] do not explicitly deal with the problem of word learning, but we use their fundamental focus on pragmatic resolution of referential ambiguity for the purposes of our own word-learning experiments.

6.3.1 Scenario and Setup

The setup of our interaction scenario bears similarities to both our previous interactions, as well as these new template experiments. As before, the environment consists of a human tutor and robot learner, seated at a table upon which many objects have been placed. Conceptually, the table has two regions, corresponding to the locations that are reachable/unreachable to the tutor, the boundaries of which we assume to be known to our learning agent. We consider all areas of the work space to be reachable to this learner. The objects in this experiment are reused from the previous two experiments: a plastic donut, funnel, wallet, seal toy, ceramic mug, and a plastic block. The task to be completed, however, was simpler in comparison, and involved only the movement of a selected object to a particular location in front of the tutor. The interaction for task training was performed according to this
Figure 6.13: Interaction format for Experiment III. During task training, the speaker selects and moves various objects to a target location (green). In the word-learning phase, the speaker may verbally request that the listener help move an intended object (black), that is located in an area unreachable to the speaker (red).

Script:

1. **Selection:** The tutor selects one of the objects on the table as his/her intended referent for the training episode.

2. **Retrieval (task):** The tutor reaches for the intended target object, grasps it, and moves it to a location in front of himself/herself.

3. **Retrieval (word):** The tutor produces a verbal request for the listener to help in moving an item to a particular location.

4. **Return:** Following another brief pause (2-3 seconds), the object is then returned to its initial location, and the training episode is concluded.

A graphical representation of this interaction scenario is again given in Figure 6.13. During training, task demonstrations were only performed physically, on objects within the reachable area of the tutor. In the subsequent word learning phase, the selection step was followed instead by a verbal request for objects outside the tutor’s reach, using the proper label for the selected object. For each training episode, four of the six total objects were present on the table — two in the area reachable by the tutor, and two outside of it. The intended object is selected from this latter category.

Because there may be multiple objects within the unreachable area, it is sometimes not possible for the listener to completely resolve ambiguity based on word usage alone, especially early on in the interaction, when little or nothing is yet known about word meanings. In such situations, the learner can select one of the potential targets, and take physical action to help the
speaker achieve the task with the intended target. Then using the reaction of the tutor, the learner could update its estimate of intent, according to the methods detailed in Algorithm 3. In our experiment, the speaker either reacts by renewing his/her request for the object (implying that the guess of the robot was not correct), or by acting to complete the task with the intended object now in the reachable area.

6.3.2 Implementation Details

The implementation of the computational and perceptual processes for this experiment was mostly the same as that used previously for Experiment IIa, but with a few important changes and additions. As noted in Section 3.4 and Algorithm 3, the learning agent must retain optimal Q-functions under the task parameter $\theta$ for both itself and the speaker. In the current scenario, we embed the action constraints on the speaker in the stochastic state-action transition model $T^\sigma$ by setting the probabilities of all self-transitions to 1 for states corresponding to objects located in the half of the table nearest to the robot ($X > -0.6$). Because of the need for a larger model of the workspace in order to accommodate the different regions of allowable movements for the tutor, the spatial processing parameters for the limits on the $X$ dimension were changed to $x_{\text{min}} = -1.1$ m and $x_{\text{max}} = -0.45$ m.

The extension of this experiment allows the robot to join in the task in order to get feedback from the speaker if it determines its uncertainty about the intended referent to be significant enough. We use a value of 1 bit of entropy over $P(I|S, A_m, \theta, \phi)$ as the threshold for choosing whether to make such an action. Ideally, if the robot decided to take action, it would generate such actions according to lines 10-13 in Algorithm 3. For our specific experiment, where there are no constraints on the robot’s movements in the task space, the optimal movement trajectory is simply a straight line between the target object’s current location and the goal location. Because of this, the control of the action sequence performed by the robot was implemented using the actionPrimitives and Cartesian Controller [127] modules available in the iCub’s open-source code library. These allow us to command the robot to move its arm to the particular location of the object, grasp the object, move it to the goal location, and finally release the object. In order to maintain proper function and persistence of the visually tracked objects while the robot is completing this motion — which often causes objects to go out of view due to rotation of the torso — the segmentation and tracking algorithm is paused for the duration of the action.
As in the previous experiment, training samples for the physical task are taken to be the segments of activity that are preceded and followed by extended pauses by the tutor. The generation of training data for the word-learning phase is more similar to that of Experiment I. Upon hearing the tutor’s utterance at time $t$, the current snapshot of the state $s_t$ is paired with the perceived speech lexicon symbol, $m_t \in A_m$. In the case where the robot then produces an action itself, the next observed segment of either speech or physical action activity is taken as the tutor’s feedback for use in Algorithm 3.

6.3.3 Results and Discussion

The reward training procedure in this experiment was very straightforward, given the simplicity of task. In total, six different observations of the task were presented to the gradient-IRL learning algorithm. Plots of both the raw feature weights, as well as spatially-organized reward function are shown in Figure 6.14.

The results of the word learning phase of Experiment III are presented in Figures 6.15 and 6.16. Over the course of the 24 total training episodes, six different spoken words were used to refer to six distinct objects. The HMM-based representation for this word lexicon was learned incrementally, and the number of classifications of the various words as each lexical element is seen in Figure 6.15a. As in previous experiments, because of the relatively simple word lexicon being used, no confusions between the different words were observed. Figure 6.15b shows the final estimate of the mapping between the
Figure 6.15: Results of object-word learning in the triadic interaction scenario.

(word lexicon elements and the set of objects in the experiment, and confirms
the learning of correct associations between the objects and spoken words.  

Figure 6.16a and 6.16b respectively show the probability of correct intent inference, and the divergence between the learned word-meaning map and the ground truth over the course of the experiment. In each of these, the performance of the fully triadic pragmatic model is compared to the performance of the “basic” pragmatic model that only has saliency information as a means of reducing the ambiguity of intent. For this experiment, no gaze information is provided, so the salience information can only produce a prior over intents that is uniformly distributed across the four objects present in any given episode. In Figure 6.16a we see that even at the outset, the triadic
Figure 6.16: Performance of the object-word learning algorithm over the course of the triadic interaction scenario.
model can immediately resolve the range of possible intents from this set of four objects to the set of two unreachable objects. As learning progresses, and knowledge about word meanings develops, the accuracy of intent inferences rises well beyond the level of 50% for the triadic model, while the basic model barely manages to perform above chance. The difference in the speed and quality of this learning is also reflected in the estimated/ground truth divergence graph in Figure 6.16b.

Finally, we compare these results to those of our extended triadic interaction experiment, in which the robot may choose to take an active role in the completion of the task and resolution of ambiguity. Out of the 24 training episodes, the robot chose to make such an action six times, largely
in the earliest stages of the experiment, and was correct in its initial guess three of these times. The result of the subsequent feedback of the tutor on intention-inference and word-learning performance is reflected in Figure 6.17. In these, we see primarily an increase in the speed with which it is able to learn the correct word-meaning map, and begin making more accurate inferences about the underlying intent of the speaker’s utterances.

Taken together, these results show how our pragmatic model is able to capture the abilities of human listeners, both adult [75] and child [24], to make inferences about a speaker’s intent by leveraging knowledge about the relative constraints/capabilities of both the speaker and listener with respect to a particular task. When applied to the task of word learning under referential ambiguity, the pragmatics-based language engine is able to improve performance beyond what is currently achievable based on simple salience cues or cross-situational statistics. This result, which constitutes one of the major contributions of this work, is made possible by an agent that understands, to some degree, its own embodiment. It is this embodiment that we use to drive word-learning performance even further, by giving our robot the power to take an active role in what then becomes a truly interactive scenario. Finally, in doing so, we provide a realization for the idea that this kind of model has the potential to represent linguistic function that extends beyond simple reference.

6.4 General Discussion

Summary and Relationship to Previous Work

In this chapter, we have presented three sets of human-robot interaction experiments with the purpose of testing the ability of our pragmatics-based computational model to learn perceptually grounded object-words in situations where the intended referent is ambiguous. The robot learner in these scenarios was presented with very little “spotlighting” information (such as gaze or visual salience of a particular object), and instead had to infer the speaker’s intent via other principles, such as lexical contrast or observation of goal-directed behaviors. For some of these scenarios, the gradual acquisition of linguistic knowledge earlier on in the experiment allowed for improved inferential abilities later on. In others, the learning of contextual knowledge about the physical task structuring an interaction, into which the use of language was embedded, was key.
The first experiment confirmed the ability of our model and learning algorithms to handle the basic statistical processing capabilities necessary for learning object-word associations. Additionally, it demonstrated our model’s capturing of the principle of lexical contrast, one of the fundamental word-learning abilities present in many previous models [4, 144, 56]. Of particular relevance are models that focus on the role of incremental learning [53], or pragmatic, utility-driven reasoning [145]. Unlike these models, however, our focus has been on the application of this model in an artificial cognitive system, where the purpose is to ground linguistic symbols in the multi-modal, situated perceptual representations of our embodied agent.

This situated context aspect becomes even more important in our second set of experiments, where the robot’s ability to understand linguistic intent is bound not to a specific perceptual cue (such as gaze), but rather a larger meaningful task that drives the speaker’s behavior, which itself must be learned for a particular interaction context. By learning about the physical task, we can use observations of a speaker’s purposeful action to resolve referential ambiguity in the word-learning problem in ways not addressed in previous models. Newer models based in embodied and situated cognitive agents have begun to use an understanding of action to address other kinds of linguistic ambiguity, such as determining what perceptual information is relevant in a form-meaning pair [46]. Here, the physical and linguistic actions of a task are unified at the level of perception in a way that is much stronger than our own, while our model captures the connection between their explicitly goal-directed nature.

It is in the third set of experiments that this deep connection of our model becomes more fully realized and exploited. Like the previous two experiments, the robot uses previously acquired knowledge about language and the task structure. But now, by using an agent’s understanding of the potential of its own embodiment in the interaction, we are able to use our pragmatic model to learn word meanings in previously unexplored situations. Unlike in previous scenarios, there is no way to appeal to gaze or movement-based salience cues for resolving intent. In this case, the proper inference of intent is based on a teleological reasoning that involves both language and action [63], where the robot itself is a means by which the human speaker is able to achieve his/her goal. And it is through this understanding that a new kind of functional representation for the meaning of words emerges. Here a word is not only a tool for influencing the attentional state of a listener (i.e. reference), but also a way to influence the listener’s participation in the interaction (i.e. request or command).
In our final experiment we begin to explore how we might truly realize the potential of such a communicative function for the purpose of enhancing the abilities of our learning agent. For episodes within Experiment III in which the referent is still ambiguous across a number of objects, we have our agent make a guess as to the speaker’s intended object, and complete the task using this object. Essentially, the robot takes control of the system at the initial state \( s_t \), for which the observed utterance \( a_m \) is unable to distinguish between the possible underlying rewards. It then drives it to a different state \( s' \), with the hope that the new state-action observation \( (s', a') \), based on the speaker’s response, will be more useful in disambiguating their intention. In this situation our pragmatic model functions very similarly to other active-learning frameworks for estimating an agent’s reward function [102, 60, 146], some of which also focus on social learning [147]. In relation to these frameworks, our model is one of the very few that has explored the use of these ideas in application to the problem of language acquisition. The capacity for active learning is a product of the explicitly purposeful representation of language already embedded within our pragmatic model. We consider this aspect of our model to be a small, but significant step on the way to constructing artificial cognitive agents for whom language is something useful.

Limitations

While our implementation of this model in these specific experiments has demonstrated many new kinds of word-learning capabilities, it also has a number of limitations, constraints, and areas in need of further development. Many of these are necessary restrictions on scope and complexity that are required for feasible study of our model’s core functionality. Others are limitations of the currently available computational techniques for stochastic decision and planning in continuous state and action spaces. Two of the most pressing are the rigidity of our perceptual representations (along with their processing techniques), and the relatively limited role of embodiment as it is implemented in our current framework.

One of the first major issues is with the way we represent and perceive the sensory experience of the robot. In order to develop tractable and practical applications of our pragmatic framework to the target human-robot word-learning scenarios, we made some relatively strong assumptions about the structure of the interaction as it related to the representation of both tasks and perception. These assumptions included a state space based on visually
segmented objects, and their spatial locations. While the IRL algorithm enables our agent to extract the features of the task that are relevant to the goal (as shown in Experiment IIb, Figure 6.10a) in an unsupervised manner, it was fundamentally constrained in the types of goals it could represent.

Relaxing these constraints constitutes a major computational challenge for our model, as well as most other models of decision, control, and especially inverse planning. As we scale the size and complexity of our model’s representations of state, action, and feature spaces, IRL techniques become intractable due to the necessity of solving the optimal “forward” planning problem for each possible intent. This is a significant impediment in our ability to scale the current model beyond the relatively small, simple experimental scenarios we have presented here. However, this is an important and actively-researched problem, and there have been many promising developments in scaling IRL techniques to very large representations [148].

In addition to the task component, we were also limited in our representation of meaning by explicit use of objects as the intentional component connected to word symbols. This is related to another critical problem with our current implementation, which is the lack of sophistication in the visual representation of objects. The color histograms used in the segmentation and tracking algorithms were the extent of these representations, and the sole means by which the meanings of words were grounded in the robot’s perceptual experiences. As a result, these object labels referred to only a specific visual object within a scene, rather than a broader, perceptually organized category. More practically, this representation placed a great burden on the performance of the segmentation and tracking task throughout the experiment, making these algorithms a significant weak point in the overall robustness of our computational system. Improving the quality of the visual processing and representation is one of the most critical problems needing to be addressed.

Another significant issue is that of the robot’s representation, understanding, and use of its own embodiment within the model. One of the core contributions of this work was to show how such an embodiment and its internal representation could be exploited to unlock new word learning capabilities. However, in our current framework, we have given our robot very explicit knowledge about its embodiment and its potential function. In keeping with the principles of cognitive development that we have tried our best to adhere to in the construction of our model, it would be desirable to allow our robot, at the very least, to learn some of the basic parameters of some representation through autonomous exploration, as done in a number of other robotic
systems (see [149] for a review).

Some of the other limitations that we see in our framework are indicative of much broader challenges that are topics for long-term research. These include issues such as autonomous formation of representations for continuous state, action, and reward spaces, dealing with multi-word utterances, and co-development of perceptual and conceptual representations, to name but a few. We will discuss the potential for future exploration of these topics, as they relate to the pragmatic model we have set forth here, in the concluding chapter.
In this thesis, we have shown how techniques for stochastic planning and control can be applied to a cognitive robotics framework to create a model for perceptually grounded word learning that captures many of the social and pragmatic aspects of the same word learning ability in children. This work was driven by what we believed to be two of the most significant disparities in the capabilities of robotic systems, relative to their child counterparts. The first was their ability to learn word meanings in cases where the referent of the word was ambiguous with respect to simple salience or gaze information. The second was their ability to represent the function or usefulness of utterances beyond basic reference.

To address these issues, we have constructed a computational framework that explicitly models the triadic and intentional nature of social interactions, in which words are understood as purposeful actions. The purpose of these actions is to modulate the listener’s understanding of the speaker’s goals, such as sharing attention to some object within the environment. We use a signaling game as the foundation of our model, in which word meanings are represented in terms of how likely they are to produce a certain interpretation by the listener. Through this representation, we apply techniques for inverse planning to recast the word learning problem from one of association to one of intentional inference, which we use to derive a basic algorithm for online word learning.

We further extend this goal-centered representation of language by embedding it within a larger, Markov decision process-based framework for rational action. This allows us to capture the ways in which children are able to learn language by understanding their role in the context of a situated, intentional social interaction. The role may now be for the listener to recognize some physical task the speaker is performing, or perhaps even to get the listener to take part in the physical task. We apply techniques of inverse reinforcement learning to allow our agent to learn goal-centered representations of these tasks from observations of the speaker. This creates
a common ground of knowledge between the speaker and listener that can then be used to infer the intent of a speaker from physical and/or communicative actions, which is the critical skill that lies at the heart of a child’s pragmatic word-learning abilities.

Because the capabilities of our model are only meaningful and useful in the context of situated, social interactions, they were evaluated within a set of human-robot interaction experiments, implemented on an iCub humanoid robot. Nearly all elements of our model are, in some way, embedded in representations of perception and action, some of which we provide a fixed structure and processing capabilities for. Others, such as representations of words, are learned through a process of autonomous, incremental construction of models for sensory experience. In terms of language, we focused on the learning of words that are grounded in representations of the speech signal, and corresponding meanings that are grounded in visually segmented objects.

Through these experiments, we demonstrated the ability of our pragmatic model to successfully learn these perceptually grounded object-word meanings in scenarios featuring referential ambiguity. These included well-studied situations requiring skills for basic statistical and contrastive reasoning, for which the performance of our model matched well with the capabilities seen in many of the current approaches to the problem. But the significant contribution of our pragmatic framework lies in its ability resolve referential ambiguity in new kinds of situations, where these more common principles alone are not enough. These included interactions whose intentional structure also involved some physical task that is being performed. We showed how our model could be applied to infer a speaker’s intent based on their goal-directed physical behaviors by first learning about the intentional regularities of the task the speaker was performing.

In the final set of experiments, we demonstrated how, by integrating an understanding of its own embodiment, our robot could reason about its own potential role in the task in order to resolve ambiguity. We also saw how a new kind of pragmatic meaning emerges from our model, in which an utterance was used not simply for the purpose of reference, but request. One of the final contributions of this work was to show how the robot could use its embodiment to physically realize its requested participation in the interaction, and in doing so actively enhance its own word learning capabilities.
7.1 Future Work

As we noted in our general discussion of the model’s application to the set of human-robot interaction experiments, there are a number of interesting and challenging ways in which the proposed framework can be developed further. These include addressing specific technical challenges that are likely to arise in the extension of our model to new scenarios, as well as more fundamental questions about what needs to be done to move our linguistic representation beyond its single word, object-focused implementation.

One of the most exciting and challenging topics for future research is the development of more advanced representations for the state, action, and reward components of the model. A critical contribution of this thesis was in the application of inverse planning techniques to provide our agent with new and interesting ways to solve the problem of referential ambiguity in word learning. Currently, most of these techniques scale poorly as the size of their representation increases, and few have approached the even more challenging task of working in continuous state and action spaces. In order for our learner to be able to handle the representational complexity necessary for modeling more realistic interactions and linguistic usage, we will need to develop better means for extracting the relevant aspects of the interaction structure, and more efficient approximations of the optimal planning problem. Two potential opportunities include integration with recently developed incremental learning techniques for task representations [46, 150], and scalable methods for IRL that rely on real-time dynamic programming approximations [148].

Another significant open issue, which we mentioned early on in this thesis, is the lack of representations for meaning that extend beyond reference to a particular perceptual category. Words with meanings that are largely social or functional (e.g. “Hello!”, “yes/no”, etc.) have been particularly neglected. We have attempted, in a small way, to begin addressing this issue by representing words in terms of their communicative function, which we used to capture some pragmatic aspects of meaning, such as request. While the fundamentally triadic nature of our language model lends itself to meanings that are rooted in the mental states of other agents, significant work needs to be done in order to make these ideas practically implementable, especially as we move toward multi-word utterances. Given the focus of our own approach on embodied cognitive systems, one possible candidate framework to explore is that of Embodied Construction Grammar [151].
7.2 Final Remarks

At the outset of this thesis, we set for ourselves the goal of developing a pragmatics-based model of language acquisition, in order to address some important issues we perceived to be facing current cognitive systems approaches. Certainly, these are only one small part of the complete body of open problems in this area. Likewise, we do not believe our model to be a complete or accurate computational representation of the complex cognitive processes that underly the language acquisition capabilities of children, or even those specific processes that fall under the heading of “pragmatics”. What we do believe, however, is that the basic pragmatic framework presented here provides a valuable starting point for the integration of ideas about the intentional and social nature of communication into future cognitive robotics models of language acquisition.
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