NON-BRANCHING HYBRID TRANSIT NETWORK DESIGN
UNDER HETEROGENEOUS DEMAND

BY

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THESIS

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ABSTRACT

Daganzo (2010) proposed a hybrid structure combining grid scheme in the center and hub-spoke scheme in the periphery to design a competitive transit network on a square region. A nonlinear continuous optimization model is built upon this structure to minimize total transit system cost. One of this model’s main limitations is the uniform trip demand assumption, which constrains its applicability in real-world. Another pitfall is that lines branch in peripheral region, resulting in expanded facilities and longer waiting time for transit services. To bridge these gaps, a hybrid transit network layout with non-branching routes is proposed. To capture spatial heterogeneity of trip demand, method of deriving zone-to-zone trip demands from continuous demand density function is borrowed from Smith (2014). To obtain optimal network layout and vehicle dispatching frequencies, a mathematical model that minimizes transit system cost based on various zone-to-zone demands is formulated. By allowing different stop spacing in the central and peripheral regions, more flexibility is given to the structure in obtaining the solutions.

A series of scenarios under heterogeneous demand distribution in various transit mode choices and demand levels are designed to test impacts of these critical factors on optimal solutions. Results show that BRT is the most competitive mode while metro’s performance increases largely when demand is higher. In addition, as trip rate increases, system cost per passenger will be reduced. Compared with the branching hybrid model (Smith, 2014), the proposed model in this thesis shows better performance in cost saving. Several interesting future research topics are inspired by the outcomes of this thesis, including extensions to network structures, model extensions as well as multimodal hierarchical transit network design.
To my parents, for all their love and support
I would like to give special thanks to my advisor, Dr. Yanfeng Ouyang, for his instructions and support on my study and work during graduate school. His professionalism and insights in transportation engineering inspire my professional development and passion in this field.
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CHAPTER 1

INTRODUCTION

Public transit is widely considered as a promising sustainable transportation mode in urban areas, with several advantages including relieving traffic congestion, reducing and air pollution, and saving energy while still preserving social equity (Kepaptsoglou et al., 2009). Some outstanding benefits were presented by statistics. For example, it is estimated that transit reduced CO₂ emissions by 6.9 million metric tons in 2005 (Davis and Hale, 2007). And its contribution to traffic congestion mitigation was researched by Texas A&M Transportation Institute (Schrank et al., 2012). If all transit users traveled in private vehicles in 2011, an additional 865 million hours delay would have been generated in total in 498 U.S. urban areas, resulting in an additional $20.8 billion, equivalent to a 15% increase in congestion costs (Table 1.1).

Table 1.1. Increase in delay in 2011 if public transportation service eliminated (498 areas)

<table>
<thead>
<tr>
<th>Population Group and Number of Areas</th>
<th>Average Annual Passenger-Miles of Travel (Million)</th>
<th>Reduction Due to Public Transportation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hour of Delay Saved (Million)</td>
<td>Percent of Base Delay</td>
</tr>
<tr>
<td>Very Large (15)</td>
<td>43203</td>
<td>721</td>
</tr>
<tr>
<td>Large (32)</td>
<td>6407</td>
<td>80</td>
</tr>
<tr>
<td>Medium (33)</td>
<td>1598</td>
<td>12</td>
</tr>
<tr>
<td>Small (21)</td>
<td>455</td>
<td>3</td>
</tr>
<tr>
<td>Other (397)</td>
<td>4357</td>
<td>49</td>
</tr>
<tr>
<td>National Urban Total</td>
<td>56010</td>
<td>865</td>
</tr>
</tbody>
</table>

*Source: Texas A&M Transportation Institute

However, with rapid economic growth, demand in personalized mobility and private automobile ownerships increase and urban regions sprawl. These have resulted in a shift from transit to private transportation (Sinha, 2003; Pucher et al., 2007). The advantages of transit would diminish
as its market share decreases. In light of this, designing an efficient transit service network is vital to increase attractiveness and market share.

Transit planning is considered to be a very complex problem due to the complexity of the system. In general, it has several sub-problems as follows: (1) route network design; (2) frequencies setting; (3) timetable development; (4) vehicle scheduling; and (5) crew scheduling (Ceder and Wilson, 1986). Inputs and outputs of these planning activities are summarized in Table 1.2 to give a general idea of transit planning process.

Table 1.2. Inputs and Outputs in Transit Planning Process (Guihaire and Hao, 2008)

<table>
<thead>
<tr>
<th>Planning activities</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Route Network Design</strong></td>
<td>Demand data</td>
<td>Route changes</td>
</tr>
<tr>
<td></td>
<td>Supply data</td>
<td>New routes</td>
</tr>
<tr>
<td></td>
<td>Route performance indicators</td>
<td>Operating strategies</td>
</tr>
<tr>
<td><strong>Frequencies setting</strong></td>
<td>Subsidy available</td>
<td>Service frequencies</td>
</tr>
<tr>
<td></td>
<td>Buses available</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Service policies</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Current patronage</td>
<td></td>
</tr>
<tr>
<td><strong>Timetable development</strong></td>
<td>Demand by time of day</td>
<td>Trip arrival and departure times</td>
</tr>
<tr>
<td></td>
<td>Times for first and last trips</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Running times</td>
<td></td>
</tr>
<tr>
<td><strong>Vehicle scheduling</strong></td>
<td>Deadhead times</td>
<td>Bus operation schedules</td>
</tr>
<tr>
<td></td>
<td>Recovery times</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schedule constraints</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cost structure</td>
<td></td>
</tr>
<tr>
<td><strong>Crew scheduling</strong></td>
<td>Driver work rules</td>
<td>Driver schedules</td>
</tr>
<tr>
<td></td>
<td>Run cost structure</td>
<td></td>
</tr>
</tbody>
</table>

* Some of the activities may share the same input data

Optimal design of route network is the main focus of this thesis due to its significance. First, it is the first step in transit planning, which is the base for the following steps. Second, route network design is usually called strategic planning; its decisions may stay fixed for a long run since changes to infrastructures are relatively costly. Also, it influences the quality of transit service and both agency and user costs directly.

Competitive transit network design in Daganzo (2010) stands out as an innovative strategic approach to improve transit system’s effectiveness and efficiency. He proposes a hybrid structure to design route network topology and stop locations. His mathematical model offers optimal setting of
critical parameters for transit network design and operation, such as stop spacing, route network structure and dispatching frequencies. Nevertheless, there are still some pitfalls in this model including uniform trip demand assumption and branching lines in the peripheral region. To bridge these gaps, this thesis aims at exploring a new non-branching hybrid transit network structure and a model to find optimal values for critical parameters based on spatially heterogeneous trip demand. Scenario tests show that the proposed model performs very well in reducing system cost compared with branching hybrid structure.

The rest of this thesis is organized as follows. In Chapter 2, literature review on some intriguing research related to transit network design is presented. Works on hybrid transit network design and consideration of heterogeneous trip demand are highlighted. Chapter 3 describes the methodology of designing a non-branching hybrid transit network with a continuous optimization model under heterogeneous trip demand and testing its performances on improving effectiveness and efficiency of transit service. Results of scenario tests on this new model are presented and analyzed in Chapter 4. Chapter 5 concludes this thesis work and some potential future research opportunities are provided in Chapter 6. References and Appendix with proofs for model formulation are provided in the end.
CHAPTER 2
LITERATURE REVIEW

In this chapter, extensive review on transit network design problem is presented. Research works on some critical issues related to this thesis are described explicitly, including hybrid transit network design and heterogeneous trip demand.

2.1 Transit Network Design Problem

Plenty of literature review on transit planning are available from different perspectives. Chua (1984) summarized methods for network design and frequencies setting sub-problems on British urban bus services in the 1980s. Desaulniers and Hickman (2007) focused mainly on mathematical approaches but for every individual sub-problem. A global review was conducted by Guihaire and Hao (2008), who proposed a classification of 69 approaches dealing with sub-problems and descriptive analysis of each work’s models and the solution methods.

Magnanti et al. (1984) and Quak (2003) showed that even separated sub-problem is difficult to be solved due to its computational complexity. For example, optimization of transit route network alone has several difficulties, including non-linearity, multi-objectives, and combinatorial intractability (Baaj and Mahmassani, 1991). Similar conclusions were made by Gao et al. (2002) and Fan and Machemehl (2006). Started from single sub-problem and simple models, researchers have been integrating complexity and combination of sub-problems along the years.

Several factors have to be considered in the transit network design such as road network structure, service spatial coverage requirements and the objective of planning. And the route network evolves from simple structure to more complicated and innovative one. Back in 1920s, Pazs (1925) conducted the first research on designing single corridor. Later, Vuchic and Newell (1968) introduced more nodes and links to the extended transit network. Holroyd (1967) optimized transit route spacing on a square grid. In 1976, Byrne worked on a similar topic in designing optimal parallel lines. It is worthy mention that Byrne (1975) built a network design model in polar coordinates to find optimal line locations and headways in a radial structure. Optimal design for
polar network was also studied by many other researchers (Vaughan, 1986, Badia et al., 2014). Newell (1979) proposed a hub-and-spoke structure for transit network, in which the hub is a major corridor.

In recent years, more complicated and flexible transit network structure design models have been developed. Daganzo (2010) first proposed a hybrid structure concept with different schemes in center and peripheral area. While his work is based on an ideal square region, a number of extensions have been proposed to apply the hybrid concept in rectangular structure (Estrada et al.) and radial structure (Badia et al., 2014) as well as flexible transit systems (Nourbakhsh and Ouyang, 2012).

The literature mentioned above all used continuous analytical models based on simple network schemes and simplified assumptions, such as uniform demand density, uniform spacing or headway, and ideal configurations of service regions. Compared with discrete models, which are usually difficult to solve (Baaj and Mahmassami, 1990, 1995; Pattnaik et al., 1998), it is easier to study the behavior of transit network and design in a high level with continuous approaches. Analytical models usually consist of three main components: decision variables, objective functions and constraints. Continuous decision variables, such as spacing and headways, are used to control the optimal layout and operation strategies (Byrne, 1975, 1976; Daganzo, 2010). Typical objectives include minimizing user and operator costs separately or together (Ceder and Israeli, 1997; Ceder, 2001) and maximizing capacity or social welfare (Chang and Schonfeld, 1993). As for constraints, feasible ranges of variables, vehicle capacity, maximum number of routes or route length, financial constraints are usually considered (Chang and Schonfeld, 1993, Smith, 2014). This thesis will also adopt continuous approach in finding solutions for transit network design.

2.2 Hybrid Transit Network Design

In this section, the hybrid network design approach (Daganzo, 2010) is introduced as well as extensions based on this model conducted by others, including Estrada et al. (2011) in rectangular cities, Badia et al. (2014) in circular cities and Smith (2014) under heterogeneous demand in square cities.
2.2.1 Hybrid Transit Network Design in Square Region

In Daganzo’s work, the transit route network structure is a combination of grid scheme (Vaughan, 1986) in the center of a square region and hub and spoke scheme (Newell, 1979) in the peripheral region. Stops are located uniformly over the whole region with identical spacing. The service area is divided into two parts with different characteristics. For central region, every stop is served by two lines in perpendicular (double coverage); while for peripheral region, each stop is served only by one line (single coverage), see Figure 2.1. Lines branch in the peripheral area in order to guarantee uniform spacing far away from the center.

![Figure 2.1. Hybrid Route Network Layout in Square Zone (Daganzo, 2010)](image)

The study region is a square city with side length of D (km). There is a parameter denoting the ratio of double coverage, which is \(\alpha = \frac{d}{D}\), where \(d\) (km) is the side length of central square. When \(\alpha = 1\) (\(d = D\)), the structure is a grid network with all double coverage stops and when \(\alpha = \frac{s}{D}\) (\(d = s\)), the structure is a hub and spoke network with all single coverage stops. So this model covers the situations of traditional route network structures.

A non-linear optimization model was built with the objective of minimizing total system cost per unit time per trip. The total system cost is summation of agency cost and user cost. Agency cost includes capital cost for transit infrastructures, vehicle purchase and maintenance, and service operation cost. While the user cost is the total time lost for trips from origins to destinations; time cost for a particular trip consists of (i) access time by walking from the origin to the nearest stop and
from last stop to the destination; (ii) waiting time for next serving vehicle, including waiting at origin and intermediate transfer stops; (iii) riding time in the vehicle, and (iv) transfer penalty. To minimize the combined system cost, a trade-off between agency cost and user cost is made to save money for operators while to improve transit service to attract more users and reach a win-win situation. There are three decision variables in this mathematical model, which are spacing, headway (or frequency) and ratio of central area (or double coverage). In addition, several critical constraints are considered to cater to the reality. One of them is that vehicle occupancy should be smaller or equal to capacity. Finally, this model is applied to Barcelona’s transit network design case, in which three transit modes are tested: bus, BRT and metro. As a result, several intriguing conclusions are claimed. First, the larger the unit infrastructure cost is, the smaller $\alpha$ should be. Moreover, user cost overweighs agency cost. In addition, spatial concentration of stops has been studied and it is found that beyond a critical level, spatial coverage is counterproductive (Daganzo, 2010).

Although the Daganzo’s work based on hybrid concept is intriguing and has significant influences on transit network design, his model has several limitations due to simplified assumptions. One critical assumption is that origins and destinations are uniformly and independently distributed. However, in real world, origins and destinations tend to be clustered by different land use in different regions. Moreover, the model limits to the network of square shape with concentric central square, which is also not the case in reality. Lastly, stop spacing stays constant over the whole service region. In this context, several extension research have been conducted and will be discussed in the following session.

2.2.2 Hybrid Transit Network Design in Rectangular Region

Estrada et al. (2011) extended this hybrid concept and introduced more flexibilities in several parameters. First, the service region is allowed to be elongated into a rectangle with different side lengths $D_x, D_y$ in $(x, y)$ axes directions. Along with this modification, spacing and ratio of double coverage is also relaxed to be considered separately in $x$ and $y$ directions. Furthermore, the line spacing $s_x, s_y$, are allowed to be different from stop spacing $s$ and assumed as integer multiple of $s$ by introducing integer parameters $p_x, p_y$; i.e., $s_x = p_x s$ and $s_y = p_y s$. The transit network layout in this model is presented in Figure 2.2 and an example of lattice of lines and stops is shown in Figure 2.3, in which $p_x = 4, p_y = 2$. To reduce the complexity of model, $p$ values are set as constant when solving the model but it is tested on different values in simulation to explore its impact on optimal solutions.
This research expands applicability of hybrid structure concept in Daganzo (2010) to more general cities with elongated shape such as Buenos Aires, Oslo, Helsinki, Miami and Washington, DC (Estrada et al., 2011).

![Figure 2.2. Hybrid Route Network Layout in Rectangular Zone (Estrada et al., 2011)](image1)

With this analytical model with uniform demand assumption, Estrada et al. (2011) designed seven scenarios to verify model and test its performance. Three cases with sets of different \((p_x, p_y)\) values are solved and compared to test the impacts of introduction of simple stop transfer stops. However, the optimal cost does not vary much under different grid structures. It is worthy mention that one of the scenarios is based on real OD data in Barcelona, which is not uniform. However,
heterogeneous demand is not considered in the design approach. In addition, sensitivity analysis is conducted to explore how optimal solutions change when input parameters change. The results show that the critical factor influencing solutions is constraint on number of corridors while dimensions of service area slightly affect solutions. Comparison between this model and Daganzo’s original model is absent.

2.2.3 Hybrid Transit Network Design in Radial Region

In 2014, Badia et al. extended hybrid structure concept on transit network design in cities characterized by radial street pattern, such as Moscow, Paris, Madrid, Amsterdam, Milan and Berlin. The two types of schemes in their new structure are radial and circular lines in the center and hub and spoke scheme in the periphery. The objective function is also composed of agency and user costs and six decision variables are used to define the model. Five of them are spatial variables determining structure topology: the angle between radial lines in central area $\theta$ (rad), the angle between stops of circular lines $\theta_c$ (rad), stop spacing in radial lines $s_r$ (km), circular line spacing $s$ (km), and the central area ratio $\alpha=r_c/R$, where $r_c$ (km) is the radius of central area and $R$ (km) of the whole design area. The other decision variable is headway.

![Figure 2.4. Hybrid Route Network Layout in Radial Zone (Badia et al., 2014)](image.png)
Similar to Estrada’s work, although simple stops are introduced between adjacent radial lines and adjacent circular lines respectively, the angle between stops and the angle between lines are not formulated as continuous decision variables simultaneously in the model. Instead, line spacing is set to be multiple of stop spacing, or angle between radial lines is set to be multiple of angle between stops. By testing different parameter values, it is found that introduction of simple stops can reduce total cost. To be noted, the impacts of not only trip demand density but also its distribution (temporally and spatially) on optimal solutions are tested and discussed. For spatial heterogeneous demand distribution, a new model is built up to integrate centripetal distribution demand density. It is shown that more users cause less system cost per user.

Additionally, comparison between radial and grid hybrid design models is conducted in this work. By solving radial model and rectangular model presented in Estrada et al. (2011) under similar conditions, similar solutions and total costs are found in both structures while the ratio of center in radial case is less than that in grid structure in every scenarios.

2.3 Spatially Heterogeneous Trip Demand

In reality, trip demand distribution is usually heterogeneous, both in time and space, stochastic and elastic. However, for simplicity in computation, the majority of analytical models for transit network design have been assuming uniform deterministic inelastic demand (Byrne and Vuchic, 1976; Daganzo, 2010). This section focuses on other demand characteristics and how they were integrated in the models as well as their impacts on solutions.

There has been quite plenty of literature on spatial heterogeneous demand along the years. For radial and ring structure, the spatial demand heterogeneity is easier to be captured by a continuous function if the demand only varies in centripetal direction. Back in 1970s, Bryan (1975) had already studied optimization model of line locations and headways under heterogeneous demand in radial structure. In his work, a continuous trip demand to CBD area (center of the city) varying with origin locations throughout the radial network was defined and used in the model. Bryan only conducted the case study under radially varying demand density. Black (1979) also built a model on circular urban area with a negative exponential density function declining uniformly from the center to periphery. Six hypothetical cities with various values for the parameters in the density
function were studied and compared. Badia et al. (2014) also formulated an optimization model for hybrid network structure with continuous demand function distributed centripetally.

On the other hand, Vaughan (1986) used a discrete zone-to-zone travel demand derived from a continuous demand density function of commuters’ trip OD distribution. This is a common method to capture spatial heterogeneity when continuous function is difficult to obtain. Also, using zonal demand can simplify calculations of metrics in models, such as expected travel distance, access time, etc. While trip rates vary from zone to zone, demand in each zone remain uniformly distributed. Chien and Schonfeld (1997) and Chien and Spasovic (2001) proposed a series of work on transit network design based on heterogeneous zonal travel demand.

One intriguing work is conducted by Smith (2014), who combined the two methods mentioned above to capture demand heterogeneity while still keep simplicity in optimization model formulation. He formulated an optimization model on hybrid grid structure to minimize total transit network system per passenger with computed zonal travel demand. While inheriting general structure and metrics in Daganzo’s work, this model captures and integrates spatial demand heterogeneity into hybrid structure by dividing trips into four categories: (i) trips from central region to central region; (ii) trips from central region to peripheral region; (iii) trips from peripheral region to central region, and (iv) trips from peripheral region to peripheral region. Trip rates for each type are computed from a continuous demand density function for rectangular area proposed by Ouyang et al. (2014). This model overcomes the limitation of uniform demand assumption in Daganzo (2010) and optimal system cost is reduced compared with Daganzo’s model (Smith, 2014).
CHAPTER 3

METHODOLOGY

In this chapter, a hybrid transit network structure without branching lines in the periphery is proposed in section 3.1 and a nonlinear model is formulated to minimize transit system cost under spatially heterogeneous demand in section 3.2. Optimal solution can be found for route network structure, stop spacing, and dispatching frequency as well as fleet size. Finally, a series of scenarios are designed to test the proposed model’s performance and to compare it with existing hybrid model discussed in chapter 2.

3.1 Non-branching Hybrid Transit Network Layout Design

Hybrid networks proposed by Daganzo (2010), Estrada et al. (2011), Badia et al. (2014) and Smith (2014) all set bus route to branch in the peripheral region to insure the same stop spacing with central region. An obvious drawback of this kind of structure is that if the center is small, routes will branch into many ones in the peripheral areas. In this case, the users close to the edge will end up with a very large headway, which may be unacceptable and decline people to use the service. Moreover, branching lines will add up to large infrastructure length and in turn large agency cost. Motivated to overcome these pitfalls, the idea of designing a non-branching network comes up. Instead of insuring spatial coverage, model in this thesis provides a non-branching transit network structure, aiming to increase temporal coverage and reduce agency cost. Both central and peripheral routes share the same headway. Compared with previous branching network, this network may result in more walking distance while it can reduce waiting time and infrastructure length in the peripheral areas. Thus, it is interesting to research on the numerical approach and performance of this new hybrid transit network design model.

First, the layout and geometric parameters that define the non-branching transit network are shown in Figure 3.1. The transit service region is a square with sides of D (km). Same with previous hybrid grid network structures introduced in Chapter 2, the service region is divided into two areas (central and peripheral) with grid scheme in the central area and hub and spoke scheme in the
peripheral region. The central square is concentric with service region. As the layout shows, two perpendicular transit lines serve one stop in the central square with sides of \( d \leq D \) while only one line serve one stop in the periphery. The ratio of the center area to the whole area is \( \alpha^2 \), where \( \alpha = \frac{d}{D} \). The distinction of this non-branching transit network is that routes do not branch in the peripheral region. There are two groups of lines: N-S lines and E-W lines. As Figure 3.2 shows, take N-S direction for illustration, bus lines go through from the top edge to the bottom edge of the whole region. To be noted, while most of lines use exclusive transit infrastructure corridors, two lines in N-S direction overlap with two lines in E-S direction at the four corners of peripheral region, sharing the same infrastructure.

Similar to models in Daganzo (2010) and Smith (2014), the stop spacing within central square is identical, denoted by \( s_c \). However, stop spacing in peripheral area may vary from the center edge to the whole region boundaries. A parameter \( \beta_j \), where \( j = 0, 1, 2, \ldots \), is introduced to define cordons which represent a square with side length of \( \beta_j \cdot D \); \( \beta_0 \) cordon is exactly the boundaries of the central square. The spacing between \( \beta_j \) and \( \beta_{j+1} \) cordons is denoted by \( s_p \) (km), which can be mathematically represented as \( s_p = \frac{(\beta_{j+1} - \beta_j) \cdot D}{2} \). Stops in peripheral region are located at intersections of these \( \beta_j \) cordons and routes, in which \( \beta_j \) is obtained by both \( s_p \) and \( \beta_0 \) cordon. In summary, \( \alpha, s_c, s_p \) are the three decision variables that determine network topology. Another decision variable is headway or bus line operation frequency. It is assumed that headway is identical throughout the whole study area. Optimal values for these four variables will be obtained by solving the model formulated in the next section.
Figure 3.1. General layout of non-branching hybrid transit network structure

Figure 3.2. Transit lines

(a) N-S Direction
(b) E-W Direction
3.2 Optimization Model Formulation

In this section, total system cost per unit time is considered to be minimized. System cost in transit network consists of costs from two parties. One is agency investment and cost, which includes transit infrastructure investment, vehicle ownership cost and vehicle operation and maintenance costs. The other stakeholders are transit users, who pay mostly time and energy for mobility. Agency and user metrics will be introduced in the following part. Some of the metrics depend on trip demand, so demand is discussed first here.

3.2.1 Trip Demand

For this new hybrid structure, heterogeneous demand is considered in modelling. The transit trip demand from origin O \((x_1, y_1)\) to destination D \((x_2, y_2)\) is described by a continuous demand density function as equation (3.1), which was proposed by Ouyang et al. (2014).

\[
\delta(x_1, y_1, x_2, y_2) = \prod_{i=1}^{2} \left( a_1 + a_2 \sum_{j=1}^{2} \exp \left[ - (a_3 j x_i - b_{ij})^2 - (a_4 j y_i - b_{ij})^2 \right] \right).
\] (3.1)

Using the same idea of Smith (2014) to capture spatial heterogeneity of the demand, trips are grouped into four categories based on regions in which origin and destination fall into. The respective demands of these four types of trips are denoted as: (i) \(\lambda_{c-c}\) : trip demand from region to central region; (ii) \(\lambda_{c-p}\) : trip demand from central region to peripheral region; (iii) \(\lambda_{p-c}\) : trip demand from peripheral region to central region; and (iv) \(\lambda_{p-p}\) : trip demand from peripheral region to peripheral region. Since the concept for separating the center and periphery is the same with the one in Smith (2014), the calculation of total number of passengers for each trip type can be borrowed from it as follows:

\[
\lambda_{c-c} = \int_{y_2=LB}^{UB} \int_{x_2=LB}^{UB} \int_{y_1=LB}^{UB} \int_{x_1=LB}^{UB} \delta(x_1, y_1, x_2, y_2) dx_1 dy_1 dx_2 dy_2; \quad (3.2)
\]

\[
\lambda_{c-p} = \int_{y_2=0}^{D} \int_{x_2=0}^{D} \int_{y_1=LB}^{UB} \int_{x_1=LB}^{UB} \delta(x_1, y_1, x_2, y_2) dx_1 dy_1 dx_2 dy_2 - \lambda_{c-c}; \quad (3.3)
\]

\[
\lambda_{p-c} = \int_{y_2=LB}^{UB} \int_{x_2=LB}^{UB} \int_{y_1=0}^{D} \int_{x_1=0}^{D} \delta(x_1, y_1, x_2, y_2) dx_1 dy_1 dx_2 dy_2 - \lambda_{c-c}; \quad (3.4)
\]

\[
\lambda_{p-p} = \int_{y_2=0}^{D} \int_{x_2=0}^{D} \int_{y_1=0}^{D} \int_{x_1=0}^{D} \delta(x_1, y_1, x_2, y_2) dx_1 dy_1 dx_2 dy_2 - \lambda_{c-c} - \lambda_{c-p} - \lambda_{p-c}, \quad (3.5)
\]

where,
\[ UB = \frac{(1 + \alpha)D}{2}, \]  
\[ LB = \frac{(1 - \alpha)D}{2}. \]  
\[ UB = \frac{(1 + \alpha)D}{2}, \]  
\[ LB = \frac{(1 - \alpha)D}{2}. \]  

3.2.2 Agency Metrics

The agency metrics are the same with existing hybrid models, but most of them need recalculation due to the change of network layout. As mentioned before, agency cost is related to three parameters: (i) transit infrastructure length \( L \) (km) (3.8), with regard to its construction and maintenance; (ii) total vehicle-distance traveled by all vehicles per hour \( V \) (veh km/h) (3.9), which captures the operation and maintenance costs; and (iii) fleet size \( M \) (veh) (3.10), which is used to calculate vehicle ownership investment. These variables are derived in Appendix A. Final formulations of these metrics are:

\[ L = \frac{3\alpha D^2 - \alpha^2 D^2}{s_c} - 4D(1 - \alpha), \]  
\[ V = \frac{D^2(6\alpha - 2\alpha^2)}{s_c H}, \]  
\[ M = \frac{V}{v_c} \star PHF, \]  
\[ v_c = \frac{E}{T}, \]  

where \( v_c \) (km/h) is the commercial speed of transit service; PHF is peak hour factor, which describes the gratitude of rush hour; \( E \) is the total expected travel distance by all passengers; \( T \) is the total in-vehicle time by all passengers. Details of these parameters will be given in next section.

3.2.3 User Metrics

User cost occurs as time and energy consumed from the beginning to the end of a trip. The components of a trip consist of: walking access time \( A \) (h); waiting time for buses \( W \) (h); riding or in-vehicle time \( T \) (h) and transferring efforts. All metrics are obtained by assuming that all passengers take the shortest path between origin and destination despite involving more transfers. Another assumption is that users choose the closest stop from origin as a boarding stop and the closest stop to the destination as an alighting stop. With regard to transfer costs, a fixed penalty parameter \( \delta_t \) for transfers is introduced as an equivalent walking distance. Some important user
metrics are calculated using the following equations (see Appendix A for proofs). To be noted, all these metrics consider total trips in the system.

\[
A = \lambda_{c-p} + (\lambda_{c-p} + \lambda_{p-c}) \left( \frac{s_p}{4v_w} + \frac{s_c(3 + \frac{1}{a})}{8v_w} \right) + \lambda_{p-p} \left( \frac{s_p}{2v_w} + \frac{s_c(1 + \frac{1}{a})}{4v_w} \right),
\]

(3.12)

where \(v_w\) denotes passenger walking speed.

\[
W = \lambda_{c-c}H \left( 1 - \frac{s_c}{aD} \right) + (\lambda_{c-p} + \lambda_{p-c})H \left( 1 - \frac{s_c}{2aD} \right) + \lambda_{p-p}H \left( \frac{5}{4} - \frac{s_c}{2aD} \right);
\]

(3.13)

\[
E = \frac{2}{3}aD\lambda_{c-c} + \left( \frac{3D}{8} + \frac{11}{24}aD \right)(\lambda_{c-p} + \lambda_{p-c}) + \left( \frac{3D}{4} + \frac{1}{6}aD \right)\lambda_{p-p};
\]

(3.14)

\[
T = \frac{E}{v} + \lambda_{c-c} \left[ \frac{2\tau_aD}{3s_c} + \tau' \left( 2 - \frac{2s_c}{aD} \right) \right] + (\lambda_{c-p} + \lambda_{p-c}) \left[ \frac{5\tau_aD}{6s_c} + \frac{\tau(1-a)D}{4s_p} + \tau' \left( 2 - \frac{s_c}{aD} \right) \right] + \lambda_{p-p} \left[ \frac{2\tau_a}{3s_p} + \frac{3D}{4} + \frac{1}{6}aD \right] + \tau' \left( \frac{5}{2} - \frac{s_c}{aD} \right),
\]

(3.15)

where

\(v\) denotes vehicle cruising speed,

\(\tau\) denotes the time lost per stop due to the door operation, deceleration and acceleration

\(\tau'\) denotes delay per passenger due to boarding. It is assumed that passenger alighting time is less than boarding time, so only boarding delay is considered here.

\[
e_T = \lambda_{c-c} \left( 1 - \frac{2s_c}{aD} \right) + (\lambda_{c-p} + \lambda_{p-c}) \left( 1 - \frac{s_c}{aD} \right) + \lambda_{p-p} \left( \frac{3}{2} - \frac{s_c}{aD} \right),
\]

(3.16)

where \(e_T\) denotes the total transfers occurring in the system per hour.

3.2.4 Optimization Model

Different from proposed models in Daganzo (2010) and Smith (2014), this model considers minimizing the total system cost per unit time for simplicity in metrics calculation. The total system cost, denoted by \(Z\), consists of two parts: agency cost \(Z_A\) and user cost \(Z_U\). Since agency cost is in monetary unit while user cost is in time unit, \(Z_A\) is transformed into time cost by introducing a parameter, which is passengers’ value of time \(u\) ($/hour). Parameters \(s_L, s_V,\) and \(s_M\) respectively denote the unit costs of three parts related to agency cost, defined as \(L, V,\) and \(M\) in last section. To obtain \(Z_U\), four parts should be added up: \(A, W, T,\) and transfer penalty in time unit \(\delta_{et}/v_w\).

So the objective function is:

\[
\min_{s_c, s_p, H, a} \{ Z = Z_A + Z_U \},
\]

(3.17)
where

\[
Z_A = \frac{(L_L + V_V + M_M)}{u} \quad (3.18)
\]

\[
Z_U = A + W + T + \frac{\delta e_t}{v_w} \quad (3.19)
\]

This objective function is subject to several constraints:

\[
s_c \geq s_{\text{min}} \quad (3.20)
\]

\[
s_p \geq s_{\text{min}} \quad (3.21)
\]

\[
H \geq H_{\text{min}} \quad (3.22)
\]

\[
\frac{s_c}{D} \leq \alpha \leq 1 \quad (3.23)
\]

\[
\text{Occ} \leq C \quad (3.24)
\]

where \(C(\text{pax})\) is vehicle capacity, \(\text{Occ} \text{ (pax)}\) is the maximum number of passengers in vehicle, namely occupancy in peak hour, and

\[
\text{Occ} = \frac{PHF s_c H}{a D} \max \left\{ \frac{\max \{\lambda_{c-p}, \lambda_{p-c}\} + \lambda_{p-p}}{2}, \left( \frac{\lambda_{p-p}}{2} \right)^2 \right. \left. + \frac{\lambda_{c-c}}{4} + \left( \frac{\lambda_{p-c} + \lambda_{c-p} + \lambda_{p-p}}{2} - \frac{\lambda_{p-p}}{8} \right) \right\} \quad (3.25)
\]

Constraints (3.20) and (3.21) ensure stop spacing to be larger than a minimum value determined by city streets spacing and regular practices. Constraint (3.22) enforces that headway should be larger than a certain lower bound, which may vary with different modes and for different issues. For example, for metro, the headway must guarantee a safe distance between two sequential vehicles. Constraint (3.23) captures the feasible range of \(\alpha\). The last constraint (3.24) indicates that maximum occupancy should not exceed the vehicle capacity, which is straightforward.

### 3.3 Scenario Design and Solver

To test this new model’s applicability and performance, a series of scenarios are designed in this section. The test region is an ideal square city with \(D=10\) km. Several aspects are considered, including trip demand volumes and spatial distribution pattern, which will be explicitly illustrated in section 3.3.1. In addition, for the convenience to compare results of proposed model in this thesis with previous hybrid models, it is essential to remain consistency in input data, including trip demand, constant parameters in agency and user metrics and modal characteristics. Details about input data and solving method are presented here.
3.3.1 Trip Demand Volume and Distribution

Similar to Smith (2014), this study adopted continuous demand density function developed in Ouyang et al. (2014), in which parameter values in demand functions of 4 heterogeneous spatial demand distribution patterns were given: (i) mono-centric distribution, with both origins and destinations clustered in the center of the city, for example, the majority of passengers live and work in the center; (ii) twin city, where the O-D demand is clustered in two adjacent regions near the city center; (iii) asymmetrically sprawled distribution, with both origins and destinations clustered near peripheries; (iv) commuter distribution, with origins clustered in one area (center or periphery) and destinations clustered in the other area, one example is that most people live in peripheral region and work in the center. Since in this hybrid model, central region with double coverage is concentric with service region, only design for the mono-centric demand distribution pattern is optimal. So all scenario tests are conducted under mono-centric demand. The parameter values in demand function of mono-centric pattern based on total rate of 10,000 passengers per hour is shown in Table 3.1. Figure 3.3 shows the marginal distributions of trip demand for the mono-centric pattern.

For total demand volume, a low demand level with 20,000 trips per hour and a high demand level with 80,000 trips per hour are used as demand inputs in designed scenarios, same as Smith (2014) does. To obtain real demand density function for low and high demand levels, equations (3.1) should be multiplied by 2 and 8, respectively (Smith, 2014).

Table 3.1. Demand distribution parameter values for mono-centric pattern (Ouyang et al., 2014)

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
</tr>
<tr>
<td>Mono-centric</td>
<td>0.0016</td>
</tr>
</tbody>
</table>
Figure 3.3. Marginal distributions of trip demand for the mono-centric distribution pattern

(a) In low demand level (20,000 pax/h)

(b) In high demand level (80,000 pax/h)
3.3.2 Parameter Values

Assignments of all parameters used in the model are mostly borrowed from Daganzo (2010) and Smith (2014) for convenience in comparison analysis. A summary of parameter values for different modes is shown in Table 3.2. In addition, several parameters’ values are assumed to be identical despite of modal variation. 20 ($/hour) is believed to be a reasonable value for time value \( u \) (Daganzo, 2010). Passengers are assumed to walk at a speed of \( v_w \), which equals 2 km per hour. Minimum spacing for \( s_c \) and \( s_p \) are set as 0.15 km while minimum headway is 1 min. Peak hour factor is set to be 2.5 for general cases.

Table 3.2. Modal Characteristics (Daganzo, 2010; Smith, 2014)

<table>
<thead>
<tr>
<th>Modes</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C(pax)</td>
</tr>
<tr>
<td>Bus</td>
<td>120</td>
</tr>
<tr>
<td>BRT</td>
<td>150</td>
</tr>
<tr>
<td>Metro</td>
<td>1000</td>
</tr>
</tbody>
</table>

3.3.3 Solver

The software used to program and solve this nonlinear optimization model is MATLAB R2013a, where the solver “fminsearchcon” (D’Errico, 2012) is plugged in to obtain optimal solutions. This solver is designed based on “fminsearch”, a built-in optimization toolbox in MATLAB, which can solve nonlinear unconstrained optimization problems by using Nelder-Mead simplex algorithm (Smith, 2014). What “fminsearchcon” do is basically implemented simple bound constraints on top of “fminsearch”.

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CHAPTER 4
RESULTS AND ANALYSES

In this chapter, optimal solutions obtained by non-branching model proposed by this thesis and branching model in Smith (2014) in three transit modes and two demand levels under mono-centric pattern are presented. Comparison analysis between different scenarios is conducted to verify its applicability and test its performance. Results shows that the proposed model performs better than branching model on reducing transit costs. Non-branching structure and the introduction of another spacing variable are two main contributors to cost saving.

4.1 Comparison between Three Transit Modes

In mono-centric scenario with demand distributed as Figure 3.3 shows, the optimal solutions for proposed model are presented in Table 4.1. Optimal solutions for different transit modes under this scenario are compared. When the metro service is in low demand level, the total system cost is much larger than other modes, with disadvantages in both user cost $Z_U^*$ and agency cost $Z_A^*$. The reasons can be found as follows: The square of optimal network structure parameter $(\alpha^*)^2$ indicates the fraction of service area that receives double coverage, which is 30% for bus, 26% for BRT and 20% for metro in low demand level. While this fraction increases to 35% for bus, 33% for BRT and 26% for metro in high demand cases. As changes in $(\alpha^*)^2$ indicate, the size of central area decreases from bus to metro. In addition, optimal stop spacing in central area $s_c^*$ as well as optimal spacing between cordons in peripheral area $s_p^*$ for metro are much larger than other modes. All decision variables $\alpha^*$, $s_c^*$, and $s_p^*$ show that a smaller and sparser structure for metro is preferred due to the high unit cost of infrastructure. So the optimization model balances the high agency cost by shrinking the central area and expanding spacing between routes. The agency metrics verify this claim. The optimal infrastructure length $L^*$ is far less than other modes (238.93 km, 194.56 km, 77.32 km for bus, BRT, metro in low demand level). The optimal total vehicle travel distance $V^*$
shows the same trend since it is positively correlated to $L^*$. However, metro has its own advantage in travelling speed. The result in optimal commercial speed $v_{c}^*$ verifies this characteristic (18.19 km/h, 26.18 km/h, 35.73 km/h for bus, BRT, metro in low demand level). As a result, metro saves the optimal in-vehicle time $T^*$, almost half of that in bus mode. Nevertheless, this advantage does not help metro stand out in saving user cost since $s_{e}^*$ and $s_{p}^*$ expands. Users have to walk longer to the stops. Thus, the optimal access time $A^*$ increases significantly. Also, the transfer penalty $\delta_t$ is set to be larger in metro, thus $(\frac{\delta_{t} T_v}{v_{e}})^*$ is the largest among all modes. These results do not imply that metro is not an efficient transit mode in all conditions. In fact, the performance of metro improves from low to high demand level and the difference of costs between other modes decreases as shown in Table 4.1. Moreover, the occupancy rate $\text{Occ}^*/C$ is quite low (38% in low demand level and 55% in high demand level) while other modes have very high occupancy rates, almost to 100%. That is to say, metro is not fully used in this two demand levels and may overweigh other modes in higher demand level. $H^*$ for metro is larger than others, with 4.5 min in low demand level and 2.8 min in high demand level. This reflects the larger stopping penalty in metro mode. In conclusion, the optimal solutions for metro illustrate how this model makes a trade-off between agency cost and user cost to obtain a system optimum.

For the other two transit modes: bus and BRT, optimal solutions have small variations while BRT have a slight advantage on user cost saving, which makes it the most competitive mode in both low demand level and high demand level. This finding is similar to the conclusions in Daganzo (2010) and Smith (2014). As shown in Table 4.1, $s_{e}^*$ is around 500 m and $s_{p}^*$ is around 250 m for both modes. These results are within reasonable range of stop spacing. The optimal headway is similar, approximately 4 min in low demand level and 1.4 min in high demand level. The advantage of BRT lies in saving $T^*$ (5 min less than bus) even though some other costs are slightly larger. There are two main reasons for this result. One is obviously the high cruising speed, and the other is that stop spacing is slightly larger for BRT, hence, stopping delay is less. Note that BRT’s advantages magnify when demand is higher. The total cost is 3 min per passenger less than bus in low demand while the difference becomes 5 min per passenger in higher demand level. See Figure 4.1 and Figure 4.2 for detailed costs comparison between modes.
Some general findings for all three modes are analyzed. One critical finding is that $s_p^*$ is smaller than $s_c^*$ in every scenario, that is to say, on a certain route, stops are denser in the peripheral part than stops in the central part. This is reasonable because in non-branching hybrid structure, route spacing increases from center edges to the peripheries. Thus, stops should be denser to guarantee enough service spatial coverage and balance passenger access time. Another finding is that user cost dominates the system cost, similar to results in Daganzo (2010) and Smith (2014), with the ratio $Z_u^*/Z_*$ around 80%. The ratio will change if different time value is used.

4.2 Comparison between Low and High Demand Levels

Impacts of total trip demand volume on optimal solutions of some critical variables and parameters are discussed in this section. As Table 4.2 shows, $x^*$ increases with larger demand volume in all modes. On the contrary, $s_c^*$ decreases with a higher percentage than $x^*$ when demand is higher. These results indicate that with high transit trip demand, a larger central grid region should
be designed in the hybrid structure with denser stops. For change in $H^*$, the result is straightforward that higher demand requires a smaller headway to reduce user cost such as waiting time.

In terms of cost part, both agency cost and user cost per passenger decrease as demand increases. Although $T^*$ and transferring time cost per passenger $(\frac{\delta \epsilon T}{v_w})^*$ increases slightly, it does not offset the saving in $A^*$ and $W^*$. Considering the reduction in system cost per passenger, it is observed that the reduction rate increases from bus mode to BRT and metro mode. This result reflects that when demand is large, advantages of larger capacity transit mode such as BRT and metro stand out.

For other metrics, some changes reflect how higher demand affects total cost. First, $L^*$, $V^*$, and $M^*$ all increase by large rates (up to 296%). However, the agency cost per passenger is reduced since demand changes at a larger rate (300% from 20,000 pax/h to 80,000 pax/h). As stops are denser, passenger will suffer a longer stopping delay. Moreover, more passengers at each stop will increase boarding time. As a result, average riding time is larger with higher demand. $A^*$ and $W^*$ are both reduced due to less walking distance and smaller headway. Other slight changes are not discussed here, see Table 4.2 for details.

4.3 Comparison between Branching Structure Model and Non-Branching Structure Model

This section discusses how the proposed non-branching model performs compared with branching model in Smith (2014). The comparison is conducted under mono-centric demand distribution pattern. Note that heterogeneous demand is captured by the same method with the model in Smith (2014), thus the differences of results mostly come from the variations in network structure and model formulation.

Based on characteristics of these two structures, non-branching structure has several advantages. As discussed in Chapter 2, in branching structure, users in the peripheral area need to wait multiple headways. This drawback is eliminated in non-branching structure, which can save waiting time when headway is the same. Another advantage is the reduction in infrastructure length $L_0$, which is intuitive. The sacrifice in non-branching model is route spatial coverage. It may result in less stop spatial coverage and increase access time. However, it may not perform worse in user cost
due to separation of stop spacing decision variables. While Smith’s model only allows uniform spacing over the whole service region, this model separates the stop spacing decision variables in central and peripheral regions. This flexibility may help find better optimal solutions for stop spacing in the center and offset the impact of increasing route spacing in peripheral area by adopting a smaller $s_p$.

Inferences made above can be verified by result comparison. Table 4.3 shows optimal solutions of Smith’s model under mono-centric pattern. Results of this model are shown in Table 4.1. Percentage changes in all variables and metrics from branching model to non-branching model are calculated and presented in Table 4.4. As cost results shown in Table 4.4 indicate, both agency cost and user cost are significantly reduced and total system cost is reduced by around 20% in low demand level and 18% in high demand level. So it is proved that non-branching structure model can have a better performance in cost saving. For details, $\alpha^*$ is slightly larger while $s_c^*$ is larger (from 468 m to 596 m for BRT mode in low demand) and $s_p^*$ is smaller (from 468 m to 257 m for BRT mode in low demand). Reduction in $L^*$ is observed to be quite large. This benefit is from not only the advantages of non-branching structure mentioned above, but also the larger stop spacing in the center, which results in less number of routes. As a consequence, $V^*$ and $M^*$ are both reduced since $V^*$ is positively correlated to $L^*$ while $M^*$ is positively correlated to $V^*$. The reductions in these three agency metrics decrease agency cost significantly (18%, 30%, 36% 16%, 19%, 28%).

There is also a large reduction in user cost (around 17%). All user cost components ($A^*$, $W^*$, $T^*$ and $(\frac{\delta v}{v v_0})^*$) decrease. For access time, $A^*$ is reduced due to decreased spacing in peripheral area. Even though $s_c^*$ is larger, total access time decrease. Considering waiting time, a reduction in $W^*$ is not surprising. While $H^*$ stays similar (-6%, 11%, 18% in low demand level and -10%, -8%, -3% in high demand level), the elimination of multiple headway saves waiting time. To be noted, different formulation is used to calculate number of transfers. In this model, trips with 0 transfer are considered to tilt to reality while Smith (2014) assumes all trips have at least 1 transfer. This may decrease number of transfers, the transfer penalty and total waiting time. However, the influence is partial. Number of transfer $e_T^*$ would also be reduced even if same formulation is adopted since the number of P-P trips decrease due to a larger $\alpha^*$. So this difference in formulation will not change the trend but only degree. In terms of in-vehicle time, saving in $T^*$ is found in non-branching model due
to the reduction in travel distance in the peripheral area. Transfer cost \( \left( \frac{\delta_c e^T}{v_w} \right)^* \) decreases with reduction in \( e^T \). In conclusion, this non-branching structure model can save more on all cost components.
4.4 Tables of Results

Table 4.1. Optimal solutions under mono-centric demand pattern for three modes in low and high demand level

<table>
<thead>
<tr>
<th>Demand Pattern</th>
<th>Mono-Centric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Level</td>
<td>Low</td>
</tr>
<tr>
<td>Mode</td>
<td>Bus</td>
</tr>
<tr>
<td>Decision Variables</td>
<td></td>
</tr>
<tr>
<td>$a^*$</td>
<td>0.5439</td>
</tr>
<tr>
<td>$s_c^*$ (km)</td>
<td>0.5194</td>
</tr>
<tr>
<td>$s_p^*$ (km)</td>
<td>0.2562</td>
</tr>
<tr>
<td>$H^*$ (min)</td>
<td>3.9238</td>
</tr>
<tr>
<td>Trip Demand Metrics</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{c-c}^*$ (pax/h)</td>
<td>10487</td>
</tr>
<tr>
<td>$\lambda_{c-p}^*$ (pax/h)</td>
<td>3995</td>
</tr>
<tr>
<td>$\lambda_{p-c}^*$ (pax/h)</td>
<td>3995</td>
</tr>
<tr>
<td>$\lambda_{p-p}^*$ (pax/h)</td>
<td>1522</td>
</tr>
<tr>
<td>Performance Metrics</td>
<td></td>
</tr>
<tr>
<td>$v_c^*$ (km/h)</td>
<td>18.19</td>
</tr>
<tr>
<td>$e_T^*$ (transfer/pax)</td>
<td>0.89</td>
</tr>
<tr>
<td>$O_{cc}^*$ (pax)</td>
<td>120</td>
</tr>
<tr>
<td>$O_{cc}^*/C$</td>
<td>100%</td>
</tr>
<tr>
<td>Agency Metrics</td>
<td></td>
</tr>
<tr>
<td>$L^*$ (km)</td>
<td>238.93</td>
</tr>
<tr>
<td>$V^*$ (veh km/h)</td>
<td>7865.09</td>
</tr>
<tr>
<td>$M^*$ (veh)</td>
<td>432</td>
</tr>
<tr>
<td>User Metrics</td>
<td></td>
</tr>
<tr>
<td>$A^*$ (min/pax)</td>
<td>9.75</td>
</tr>
<tr>
<td>$W^*$ (min/pax)</td>
<td>3.71</td>
</tr>
<tr>
<td>$T^*$ (min/pax)</td>
<td>16.61</td>
</tr>
<tr>
<td>$(\delta_{eT} I_{v_w})^*$ (min/pax)</td>
<td>0.80</td>
</tr>
<tr>
<td>Costs</td>
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<tr>
<td>$Z_A^*$ (min/pax)</td>
<td>5.28</td>
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<tr>
<td>$Z_U^*$ (min/pax)</td>
<td>30.87</td>
</tr>
<tr>
<td>$Z^*$ (min/pax)</td>
<td>36.15</td>
</tr>
<tr>
<td>$Z_U^<em>/Z^</em>$</td>
<td>85%</td>
</tr>
</tbody>
</table>

* In terms of data consistency to compare with branching model’s data, all user metrics and costs solution values are divided by total number of passengers. Same for all other tables in this chapter.
Table 4.2. Percentage changes in metrics between low and high demand level

<table>
<thead>
<tr>
<th>Demand Pattern</th>
<th>Mono-centric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Bus</td>
</tr>
<tr>
<td>α*</td>
<td>9%</td>
</tr>
<tr>
<td>$s_c^*$ (km)</td>
<td>-15%</td>
</tr>
<tr>
<td>$s_p^*$ (km)</td>
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<tr>
<td>$v_c^*$ (km/h)</td>
<td>-5%</td>
</tr>
<tr>
<td>$e_{T^*}$ (transfer/pax)</td>
<td>2%</td>
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<tr>
<td>$O_{cc^*}$ (pax)</td>
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<tr>
<td>$L^*$ (km)</td>
<td>29%</td>
</tr>
<tr>
<td>$V^*$ (veh km/h)</td>
<td>277%</td>
</tr>
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<td>$M^*$ (veh)</td>
<td>296%</td>
</tr>
<tr>
<td>$A^*$ (min/pax)</td>
<td>-18%</td>
</tr>
<tr>
<td>$W^*$ (min/pax)</td>
<td>-66%</td>
</tr>
<tr>
<td>$T^*$ (min/pax)</td>
<td>7%</td>
</tr>
<tr>
<td>$(\delta_{te}e_{T^<em>})^</em>$ (min/pax)</td>
<td>2%</td>
</tr>
<tr>
<td>$Z_{A^*}$ (min/pax)</td>
<td>-7%</td>
</tr>
<tr>
<td>$Z_{U^*}$ (min/pax)</td>
<td>-10%</td>
</tr>
<tr>
<td>$Z^*$ (min/pax)</td>
<td>-10%</td>
</tr>
</tbody>
</table>
Table 4.3. Optimal solutions by model in Smith (2014) under mono-centric demand pattern

<table>
<thead>
<tr>
<th>Demand Pattern</th>
<th></th>
<th>Mono-Centric</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Demand Level</td>
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<td>High</td>
</tr>
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<td>Mode</td>
<td>Bus</td>
<td>BRT</td>
</tr>
<tr>
<td><strong>Decision Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>0.512</td>
<td>0.489</td>
<td>0.407</td>
</tr>
<tr>
<td>$s^*$ (km)</td>
<td>0.414</td>
<td>0.468</td>
<td>0.919</td>
</tr>
<tr>
<td>$H^*$ (min)</td>
<td>4.16</td>
<td>3.44</td>
<td>3.75</td>
</tr>
<tr>
<td><strong>Trip Demand Metrics</strong></td>
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<td>$\lambda_{c-c}^*$ (pax/h)</td>
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<td>8950</td>
<td>6351</td>
</tr>
<tr>
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<td>4429</td>
<td>4919</td>
</tr>
<tr>
<td>$\lambda_{p-c}^*$ (pax/h)</td>
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<td>4429</td>
<td>4919</td>
</tr>
<tr>
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<td>3809</td>
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<tr>
<td><strong>Performance Metrics</strong></td>
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<td></td>
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<tr>
<td>$v_c^*$ (km/h)</td>
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<td>21.8</td>
<td>33</td>
</tr>
<tr>
<td>$e_T^*$ (transfer/pax)</td>
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<td>1.05</td>
<td>1.1</td>
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<tr>
<td>$\text{Occ}^*$ (pax)</td>
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<td>112</td>
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<td>265</td>
<td>127</td>
</tr>
<tr>
<td>$V^*$ (veh km/h)</td>
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<td>9159</td>
<td>3667</td>
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<tr>
<td>$M^*$ (veh)</td>
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<td>420</td>
<td>111</td>
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<tr>
<td><strong>User Metrics</strong></td>
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</tr>
<tr>
<td>$A^*$ (min/pax)</td>
<td>12.4</td>
<td>14</td>
<td>27.6</td>
</tr>
<tr>
<td>$W^*$ (min/pax)</td>
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<td>4.2</td>
<td>5.3</td>
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<tr>
<td>$T^*$ (min/pax)</td>
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<td>13.7</td>
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<td>$\left(\frac{\delta t e_T}{v_w}\right)^*$ (min/pax)</td>
<td>0.94</td>
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<td><strong>Costs</strong></td>
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<td>22.4</td>
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<td>$Z_U^*$ (min/pax)</td>
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<td>41.7</td>
<td>68.1</td>
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Table 4.4. Percentage change in optimal solutions from the model in Smith (2014) to non-branching model under mono-centric demand pattern

<table>
<thead>
<tr>
<th>Demand Level</th>
<th>Low</th>
<th>High</th>
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<tr>
<td>Mode</td>
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</tr>
<tr>
<td><strong>α</strong></td>
<td>6%</td>
<td>5%</td>
</tr>
<tr>
<td><strong>s_c</strong> (km)</td>
<td>25%</td>
<td>27%</td>
</tr>
<tr>
<td><strong>s_p</strong> (km)</td>
<td>-38%</td>
<td>-45%</td>
</tr>
<tr>
<td><strong>H</strong> (min)</td>
<td>-6%</td>
<td>11%</td>
</tr>
<tr>
<td><strong>v_c</strong> (km/h)</td>
<td>15%</td>
<td>20%</td>
</tr>
<tr>
<td><strong>e_T</strong> (transfer/pax)</td>
<td>-15%</td>
<td>-17%</td>
</tr>
<tr>
<td><strong>Occ</strong> (pax)</td>
<td>6%</td>
<td>29%</td>
</tr>
<tr>
<td><strong>L</strong> (km)</td>
<td>-22%</td>
<td>-27%</td>
</tr>
<tr>
<td><strong>V</strong> (vch km/h)</td>
<td>-11%</td>
<td>-26%</td>
</tr>
<tr>
<td><strong>M</strong> (vch)</td>
<td>-23%</td>
<td>-39%</td>
</tr>
<tr>
<td><strong>A</strong> (min/pax)</td>
<td>-21%</td>
<td>-18%</td>
</tr>
<tr>
<td><strong>W</strong> (min/pax)</td>
<td>-24%</td>
<td>-15%</td>
</tr>
<tr>
<td><strong>T</strong> (min/pax)</td>
<td>-12%</td>
<td>-16%</td>
</tr>
<tr>
<td>(<strong>δ_e_T</strong> / v_w) (min/pax)</td>
<td>-15%</td>
<td>-17%</td>
</tr>
<tr>
<td><strong>Z_A</strong> (min/pax)</td>
<td>-18%</td>
<td>-30%</td>
</tr>
<tr>
<td><strong>Z_U</strong> (min/pax)</td>
<td>-17%</td>
<td>-17%</td>
</tr>
<tr>
<td><strong>Z</strong> (min/pax)</td>
<td>-17%</td>
<td>-20%</td>
</tr>
</tbody>
</table>
CHAPTER 5
CONCLUSIONS

In this thesis, the objective mentioned in introduction chapter is accomplished: developing a model to design optimal non-branching hybrid transit network under spatially heterogeneous demand with objective of minimizing transit system cost.

This thesis contributes to hybrid grid network design initially proposed by Daganzo (2010) in three aspects. First, non-branching structure will help reduce total system cost compared with branching structure in three ways. Without branching lines, multiple headway is eliminated for peripheral users, and in turn waiting cost will be reduced. Moreover, infrastructure length, vehicle travel distance as well as fleet size will be reduced since routes do not branch to cover every stop in the peripheral region. In turn, agency cost is reduced. In addition, user cost may be saved with less in-vehicle travel distance. Second, this study considered trip demand’s spatial heterogeneity. By overcoming simplification of uniform distribution assumption in Daganzo’s model, this model extends the applicability of hybrid structure design on various demand distributions. Last but not least, the model allows different values for spacing in central area and peripheral area. This relaxation on uniform spacing assumption helps to find better optimal solutions.

This thesis follows four main steps to obtain results. First, a review on related research works is conducted and presented in Chapter 2. Topics includes: (i) general transit network design problem; (ii) hybrid transit network design and extension research; (iii) transit network design approaches under spatially heterogeneous demand. These literatures provide with inspirations and methods in the following model formulation, trip demand calculation as well as future work. Second, a non-branching hybrid network layout is designed in a square region with three parameters which determine route network topology and stop locations. They include the ratio parameter of central area, $\alpha$; the stop spacing inside central area, $s_c$; and spacing between cordons to design stop locations in peripheral area, $s_p$. Along with headway, these four parameters are chosen to be decision variables in the optimization model for network design. The third step is to formulate objective and constraints for optimization model to find solutions of these four decision variables. The objective is to minimize system cost, which is a summation of all cost components of agency cost and user cost.
The last step is to conduct scenario tests on this model in order to verify its applicability and test the performance. These scenarios are in: (a) three different modes; (b) two total trip demand levels; and (c) mono-centric demand distribution pattern.

Results of scenario tests are obtained by using a plugged in solver in MATLAB and are analyzed and compared in Chapter 4. Generally speaking, this non-branching model shows a good performance. Optimal solutions are reasonable and can verify how this model acts to deal with different scenarios. Moreover, the optimal design by this model can reduce transit system cost about 20% compared with Smith’s model. This difference shows the advantages of non-branching scheme over branching scheme as well as separate spacing variables over single one. Some critical findings in results analysis are:

- In terms of mode choice, BRT is the most competitive mode in this non-branching transit network while metro has a potential to be a better choice for a high demand case.
- A smaller and sparser route network is preferred by transit modes with larger capacity, higher speed but higher infrastructure cost, such as metro. To offset the larger stopping penalty, headway is set to be larger.
- Spacing between cordons in peripheral area is set to be much smaller than stop and route spacing in the center to offset the reduction in route spatial coverage.
- Both agency cost and user cost per passenger are reduced when demand volume increases.
- Both agency cost and user cost are reduced compared with branching model.
CHAPTER 6
FUTURE WORK

Many future research opportunities can be found from three aspects: extensions to network structure, model formulation adjustments and multimodal hierarchical design.

6.1 Network Structure Extensions

As chapter 2 introduces, Estrada et al. (2011) did an extension work to hybrid structure in Daganzo (2010) to cater for rectangular region. Their work is also based on branching scheme in the peripheral area. While this thesis work applies a non-branching scheme in a square region and obtains a good performance in reducing total cost, it will also be interesting to extend it to rectangular region. Several modifications are needed. First, since the service region is elongated, route and stop spacing as well as the ratio of central area need separate variables in N-S and E-W directions. Additionally, the demand formulation needs modification since metrics in different directions should be calculated separately. To be explicit, demand heterogeneity in this thesis is captured by calculating only four types of trips (C-C, C-P, P-C and P-P) within and between two zones (central and peripheral) based on the fact that the four quadrants in peripheral region are identical. However, in rectangular region, quadrants in North-South direction have a different shape with quadrants in East-West direction. So it is essential to separate peripheral region into 4 zones (quadrants in N, S and quadrants in E, W) and calculate 32 types of trip demand within and between these zones (Figure 6.1).
Another interesting topic in structure extensions is to add simple stops between transfer stops. In the works of hybrid transit network design in rectangular region (Estrada et al., 2011) and radial region (Badia et al., 2014), introduction of simple stops are proved to be helpful in reducing total system cost. Inspired by this idea, this non-branching transit network also can integrate simple stops to see if system cost will be reduced.

6.2 Model Extensions

The objective in this model is minimizing total system cost, which is summation of agency cost and user cost. Nevertheless, in some cases, there may be a prioritization based on different considerations. So a weighted summation may be preferable. In addition, the values for some parameters in this model can be changed, such as value of time, unit construction cost, capacity, etc. It is very interesting to explore how changes in these parameters affect model’s performance and final transit system cost.

6.3 Multimodal Network Design

Different transit modes are tested using this model and the results show their own advantages and disadvantages. While bus is more flexible in construction and operation, more rapid mode may benefit users more especially in high demand condition. However, due to the high infrastructure cost and long construction cycle, rapid transit such as BRT and metro may not spread
as dense as a bus network. An idea of building a hierarchical transit network integrating multiple modes based on this non-branching hybrid structure is inspired. In chapter 4, the design outcomes of different modes show that the more rapid transit prefers a smaller and sparser central grid region. With this principle, the new hierarchy structure will be a stack-up of transit networks with multiple levels (Figure 6.2). In higher level, transit mode (such as metro) tends to be faster but with larger route and stop spacing. While in lower level, high accessibility is guaranteed. An example of hierarchical design of metro and bus with non-branching structure is shown as Figure 6.3.

Figure 6.2. Multimodal hierarchical networks

Figure 6.3. Hierarchical network design of metro and bus with non-branching structure
REFERENCES


Gao, Z., Sun, H., Shan, L. A continuous equilibrium network design model and algorithm for transit systems. Transportation Research Part B 38, 235–250.


**APPENDIX A**

**PROOFS**

**Result 1.** The total length of the two-way infrastructure system is given by (3.8):

\[
L = \frac{3\alpha D^2 - \alpha^2 D^2}{s_c} - 4D(1 - \alpha).
\]

**Proof.** Figure A.1 shows half of the route network layout. The network is assumed to be large enough so that number of routes is large enough for approximated calculation. Only infrastructures in N-S direction is shown because those in E-W direction can be obtained exactly by rotating the picture 90°. The infrastructure system has two parts: center and periphery. Since the configurations of these two parts are different, so we may consider them separately. In N-S direction, there are \( n \) routes,

\[
n = \frac{d}{s_c} = \frac{aD}{s_c}.
\]

![Figure A.1. Route layout in N-S direction](image)

For central region, denote the infrastructure length as \( L_c \),

\[
L_c = 2 \cdot d \cdot n = \frac{2a^2 D^2}{s_c}.
\]
For peripheral region, in x direction, due to symmetry, we first consider the infrastructure length in half of north quadrant, denoted by region k. In region k, infrastructure lengths change gradually from 0 to \( \frac{D(1-\alpha)}{2} \) with uniform rate, thus, the expected total length in x direction in region k is,

\[
L_k = \frac{1}{2} \left( 0 + \frac{D(1-\alpha)}{2} \right) \cdot \frac{n}{2} = \frac{D^2(\alpha-a^2)}{8s_c}.
\]

Thus, the expected total length in x direction is \( 4L_k \). In y direction, every route has a length of \( D(1-\alpha) \). So the total infrastructure length in N-S direction within peripheral region is,

\[
L_{N-S} = \frac{D^2(\alpha-a^2)}{2s_c} + n \cdot D(1-\alpha) = \frac{3D^2(\alpha-a^2)}{2s_c}.
\]

To get the total length in the whole peripheral region, we need to double \( L_{N-S} \) and extract the length of overlapping parts due to rotation. So final result of total length of the two-way infrastructure system is,

\[
L = 2L_{N-S} - 4D(1-\alpha) + L_c = \frac{3D^2(\alpha-a^2)}{s_c} - 4D(1-\alpha) + \frac{2a^2D^2}{s_c} = \frac{3\alpha D^2-a^2D^2}{s_c} - 4D(1-\alpha).
\]

**Result 2.** The total vehicle-distance traveled per hour is given by (3.9):

\[
V = \frac{D^2(6\alpha-2a^2)}{s_c H}.
\]

**Proof.** The distance traveled is the length of routes to be covered in an hour, which is twice the total length of infrastructure system multiplied by number of dispatch per hour. In this structure, the headway for center and periphery are the same. So number of dispatches per hour is exactly \( 1/H \) for all routes. To be noted, there are 4 routes going through the edges of center square and their routes overlap in peripheral region. Thus \( 4D(1-\alpha) \) is added to total infrastructure length.

\[
V = \frac{2(L+4D(1-\alpha))}{H} = \frac{D^2(6\alpha-2a^2)}{s_c H}.
\]

**Result 3.** The total number of transfers is given by (3.16):

\[
e_T = \lambda_{c-c} \cdot \left( 1 - \frac{2s_c}{aD} \right) + \left( \lambda_{c-p} + \lambda_{p-c} \right) \cdot \left( 1 - \frac{s_c}{aD} \right) + \lambda_{p-p} \cdot \left( \frac{3}{2} - \frac{s_c}{aD} \right).
\]

**Proof.** Unlike Daganzo (2010) and Smith (2014), it is assumed that if passengers’ origin and destination fall in the influence area of the same bus route, their trips should be considered as zero transfers. So number of transfers should be computed depending on number of users with zero transfers, number of users with 1 transfers and number of users with 2 transfers, denoted by \( P_0, P_1, P_2 \) respectively. To capture the influence area of bus routes easily, the parts of routes in peripheral
region are transformed into straight lines. The simplified layout and influence areas are shown in Figure A.2. It is quite straightforward that the size of each route influence area is identical.

![Figure A.2. An example of influence area for one route](image)

Number of transfers need to be considered separately for different types of trips.

(a) C-C trip

For this kind of trips, zero transfers happens when origin and destination fall in the same influence strip. Suppose origin is located randomly in central region, then if destination fall in the areas of the two orthogonal strips shown in Figure A.3, then users need no transfers. Otherwise, 1 transfer has to be made. There is no 2 transfers situation in C-C trip. So the probability of 0 transfers is \( \frac{2}{n} \), the probability of 1 transfers is \( 1 - \frac{2}{n} \).

![Figure A.3. Destination range with no transfers for C-C trip](image)
(b) C-P trip

For this kind of trips, assume origin is located randomly in the central region, then if
destination falls into peripheral influence areas (as shadows shown in Figure A.4) of the two
orthogonal routes, then no transfer is needed. So probability of 0 transfer is \( \frac{4}{4n} \). As a result,
the probability of 1 transfer is \( 1 - \frac{1}{n} \), since there is no 2 transfers situation.

![Figure A.4. Destination range with no transfers for C-P trip](image)

(c) P-C trip

For this kind of trips, assume origin is located randomly in the peripheral region, then if
destination falls into the same influence strip in the center, no transfer will occur (Figure
A.5). So probability of 0 transfer is \( \frac{1}{n} \). Same as C-P trip, there is no 2 transfers situation. So
the probability of 1 transfer is \( 1 - \frac{1}{n} \).

![Figure A.5. Destination range with no transfers for P-C trip](image)
(d) P-P trip

For this kind of trips, assume origin is located randomly in the peripheral region, then if destination falls into the intersection area of same route influence area and peripheral region, as the shadow indicates, then users need no transfers (Figure A.6). So the probability of 0 transfer is the ratio of shadow and total peripheral area, which is $\frac{1}{2n}$. 1 transfer occurs when the destination falls in the adjacent cordon so the probability of 1 transfer is $\frac{1}{2}$. The probability of 2 transfers can be obtained by $1 - \frac{1}{2} - \frac{1}{2n}$.

![Figure A.6. Destination range with no transfers for P-P trip](image)

Thus, $P_0, P_1, P_2$ can be obtained as follows:

\[
P_0 = \lambda_{c-c} \ast \frac{2}{n} + (\lambda_{c-p} + \lambda_{p-c}) \ast \frac{1}{n} + \lambda_{p-p} \ast \frac{1}{2n},
\]

\[
P_1 = \lambda_{c-c} \ast \left(1 - \frac{2}{n}\right) + (\lambda_{c-p} + \lambda_{p-c}) \ast \left(1 - \frac{1}{n}\right) + \lambda_{p-p} \ast \frac{1}{2},
\]

\[
P_2 = \lambda_{p-p} \ast \left(\frac{1}{2} - \frac{1}{2n}\right).
\]

Therefore, the total number of transfers is:

\[
e_T = 1 \ast P_1 + 2 \ast P_2 = \lambda_{c-c} \ast \left(1 - \frac{2}{n}\right) + (\lambda_{c-p} + \lambda_{p-c}) \ast \left(1 - \frac{1}{n}\right) + \lambda_{p-p} \ast \frac{1}{2} + 2 \ast \lambda_{p-p} \ast \left(\frac{1}{2} - \frac{1}{2n}\right) = \lambda_{c-c} \ast \left(1 - \frac{2sc}{aD}\right) + (\lambda_{c-p} + \lambda_{p-c}) \ast \left(1 - \frac{sc}{aD}\right) + \lambda_{p-p} \ast \left(\frac{3}{2} - \frac{sc}{aD}\right).
\]

**Result 4.** The combined total walking time at the origin and destination is given by (3.12):
Proof. The expected walking distance from/to one stop in central region, denoted by $d_c$, is obviously $s_c/4$. The walking distance from/to a stop in peripheral region varies with routes, so expected distance is calculated. With parameters in Result 1, the average walking distance in a route between 2 stops in peripheral region can be calculated as

$$d_p = \frac{s_p}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \left( s_c + \frac{s_c}{a} \right) = \frac{s_p}{2} + \frac{s_c(1 + \frac{1}{a})}{8}. $$

Thus, for C-C trips, expected walking time per trip is $2d_c/v_w$; for C-P or P-C trips, expected walking time per trip is $d_c + d_p/v_w$; and for P-P trips, expected walking time per trip is $2d_p/v_w$.

Therefore, the total walking time is calculated:

$$A = \lambda_{c-c} \cdot \frac{s_c}{2v_w} + (\lambda_{c-p} + \lambda_{p-c}) \cdot \left( \frac{s_p}{4v_w} + \frac{s_c(3 + \frac{1}{a})}{8v_w} \right) + \lambda_{p-p} \cdot \left( \frac{s_p}{2v_w} + \frac{s_c(1 + \frac{1}{a})}{4v_w} \right).$$

**Result 5.** The total waiting time of all users including waiting at the origin and all transfer stops is given by (3.13):

$$W = \lambda_{c-c} H \left( 1 - \frac{s_c}{2aD} \right) + (\lambda_{c-p} + \lambda_{p-c}) H \left( 1 - \frac{s_c}{2aD} \right) + \lambda_{p-p} H \left( \frac{5}{4} - \frac{s_c}{2aD} \right).$$

Proof. The waiting time at stops depends on the headway and number of transfers. Similar to Daganzo (2010), it is assumed that the headways are low so that people arrive independent of the schedule. So expected waiting time at one stop is half the headway. In terms of headway, the network structure in this paper results in simpler calculation since there is no bunching bus route and all headways are identical. Thus, the waiting time at origin or one transfer stop is $H/2$. Waiting time for a certain trip $W'$ can be easily obtained as follows:

$$W' = \begin{cases} \frac{H}{2}, & 0 \text{ transfer trips} \\ H, & 1 \text{ transfer trips} \\ \frac{3H}{2}, & 2 \text{ transfers trips} \end{cases}$$

Thus, total waiting time for all users is calculated:
\[ W = P_0 H + P_1 H + P_2 H = \frac{H}{2} \left[ \lambda_{c-c} \left( 1 - \frac{2}{n} \right) + \left( \lambda_{c-p} + \lambda_{p-c} \right) \left( 1 - \frac{1}{2n} \right) \right] + \frac{3H}{2} \left[ \lambda_{p-p} \left( 1 - \frac{1}{2n} \right) \right]. \]

\[ H \left[ \lambda_{c-c} \left( 1 - \frac{1}{n} \right) + \left( \lambda_{c-p} + \lambda_{p-c} \right) \left( 1 - \frac{1}{2n} \right) \right] + \frac{3H}{2} \left[ \lambda_{p-p} \left( 1 - \frac{1}{2n} \right) \right] = \lambda_{c-c} H \left( 1 - \frac{1}{n} \right) + \lambda_{c-p} H \left( 1 - \frac{1}{2n} \right) + \lambda_{p-p} H \left( \frac{3}{2} - \frac{1}{2n} \right) = \lambda_{c-c} H \left( 1 - \frac{s_e}{2aD} \right) + \lambda_{p-p} H \left( \frac{3}{4} - \frac{s_e}{2aD} \right). \]

**Result 6.** Total expected in-vehicle travel distance is given by (3.14):

\[ E \equiv \frac{2}{3} \alpha D \lambda_{c-c} + \left( \frac{3D}{8} + \frac{11}{24} \alpha D \right) \left( \lambda_{c-p} + \lambda_{p-c} \right) + \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p}. \]

**Proof.** Assume two points located randomly within a square \( d^2 \), then the expected distance between them in L1 metric is \( \frac{2d}{3} \). So for C-C trips, the expected in-vehicle travel distance per trip is

\[ E_{c-c} = \frac{2}{3} \alpha D. \]

For C-P or P-C trips, there are two parts of expected vehicle travel distance: (a1) travel distance from a random stop within central square to a random stop on its edge; (b1) travel distance from a random stop on central square edge to a stop on the same route in the peripheral region. For part a1, the expected distance includes \( \frac{1}{3} \alpha D \) and \( \frac{1}{2} \alpha D \) in X, Y directions (\( \frac{1}{3} \alpha D \) in X and \( \frac{1}{2} \alpha D \) in Y; or \( \frac{1}{3} \alpha D \) in Y and \( \frac{1}{2} \alpha D \) in X), so expected total distance for part a1 is \( \frac{5}{6} \alpha D \). For part b1, the expected travel distance is the same as half of average route length in a peripheral quadrant. Since the route length in a peripheral quadrant is distributed uniformly from \( \frac{D(1-a)}{2} \) to \( D(1-a) \), so expected distance for part b1 is \( \frac{3D(1-a)}{8} \).

\[ E_{c-p} = E_{c-c} = \frac{5}{6} \alpha D + \frac{3D(1-a)}{8} = \frac{3D}{8} + \frac{11}{24} \alpha D. \]

For P-P trips, there are three parts of expected vehicle travel distance: (a2) travel distance from a random stop in peripheral region to a stop on the same route on central square edge; (b2) travel distance from the latter stop in part a2 to a random stop on central square edge; (c2) travel distance from the latter stop in part b2 to a stop on the same route in the peripheral region. The values of part a2 and c2 are both the same with part b1, that is \( \frac{3D(1-a)}{8} \); for part b2, the expected distance is

\[ d_{b2} = \begin{cases} 
\frac{1}{3} \alpha D, & \text{if two stops on the same edge} \\
\alpha D, & \text{if two stops on adjacent edges} \\
\frac{4}{3} \alpha D, & \text{if two stops on opposite edges}
\end{cases} \]
The probabilities for these three situations are \( \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \) respectively. Thus,
\[
d_{b2} = \frac{1}{4} \alpha D + \frac{1}{2} \alpha D + \frac{1}{4} \frac{4}{3} \alpha D = \frac{11}{12} \alpha D.
\]
Thus,
\[
E_{p-p} = \frac{11}{12} \alpha D + \frac{3D(1-a)}{4} = \frac{3D}{4} + \frac{1}{6} \alpha D.
\]

With above results, total expected in-vehicle travel distance is
\[
E = \lambda_{c-c} \cdot E_{c-c} + (\lambda_{c-p} + \lambda_{p-c}) \cdot E_{c-p} + \lambda_{p-p} \cdot E_{p-p} = \frac{2}{3} \alpha D \lambda_{c-c} + \left( \frac{3D}{8} + \frac{11}{24} \alpha D \right) (\lambda_{c-p} + \lambda_{p-c}) + \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p}.
\]

**Result 7.** The expected total in-vehicle time is given by (3.15):
\[
T = \frac{E}{v} + \lambda_{c-c} \left[ \frac{2xAD}{3sc} + \tau' \left( 2 - \frac{2sc}{ad} \right) \right] + (\lambda_{c-p} + \lambda_{p-c}) \left[ \frac{5xAD}{6sc} + \frac{\tau(1-a)D}{4sp} + \tau' \left( 2 - \frac{sc}{ad} \right) \right] + \\
\lambda_{p-p} \left[ \frac{2x}{3sp} \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) + \tau' \left( \frac{5}{2} - \frac{sc}{ad} \right) \right].
\]

**Proof.** There are three parts of in-vehicle time: (i) bus travelling time \( T_1 \), which is the travelling time between stops; (ii) bus stopping time \( T_2 \), which considers the decelerating time when bus approaches stop and accelerating time when bus leaves stop; (iii) stop delay \( T_3 \), which dues to passenger boarding delay. Since it is assumed that bus travels at constant speed on the way, \( T_1 = \frac{E}{v} \).

Total stopping time equals delay per stop times number of stops while number of stops equals expected travelling distance \( \bar{E} \) times number of stops per unit distance \( \bar{n} \). Since the spacing can be different in central and peripheral regions, \( \bar{n} \) and \( \bar{E} \) are calculated separately for different regions. In central region, it is obvious that \( \bar{n} \) equals \( \frac{1}{sc} \), while in peripheral region, \( \bar{n} \) equals the ratio of total number of stops and total infrastructure length in peripheral region. Using the results derived above,
\[
\bar{n} = \begin{cases} 
\frac{1}{sc}, & \text{in central region} \\
\frac{D(1-a)2Dsp}{C2sc} \approx \frac{2}{3sc}, & \text{in peripheral region}
\end{cases}
\]
\[
\bar{E} = \begin{cases} 
\frac{2}{3} \alpha D \lambda_{c-c} + \frac{5}{6} \alpha D (\lambda_{c-p} + \lambda_{p-c}), & \text{in central region} \\
\frac{3D(1-a)}{8} \left( \lambda_{c-p} + \lambda_{p-c} \right) + \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p}, & \text{in peripheral region}
\end{cases}
\]
Thus,
\[ T_2 = \frac{\tau}{s_c} \left[ \frac{2}{3} \alpha D \lambda_{c-c} + \frac{5}{6} \alpha D \left( \lambda_{c-p} + \lambda_{p-c} \right) \right] + \frac{2\tau}{3s_p} \left[ \frac{3D(1-\alpha)}{8} \left( \lambda_{c-p} + \lambda_{p-c} \right) + \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p} \right]. \]

To calculate \( T_3 \), total number of passengers boarding per hour is calculated first:

\[ \text{#boarding per hour} = 1*P_0 + 2*P_1 + 3*P_2; \]

\[ \text{#boarding per hour} = P_0 * 1 + P_1 * 2 + P_2 * 3 = \lambda_{c-c} * \frac{2}{n} + \left( \lambda_{c-p} + \lambda_{p-c} \right) * \frac{1}{n} + \lambda_{p-p} * \frac{1}{2n} + 2 \left( \lambda_{c-c} * \left( 1 - \frac{2}{n} \right) + \left( \lambda_{c-p} + \lambda_{p-c} \right) * \left( 1 - \frac{1}{n} \right) + \lambda_{p-p} * \frac{1}{2} \right) + 3 \lambda_{p-p} * \left( \frac{1}{2} - \frac{1}{2n} \right) = \]

\[ \lambda_{c-c} \left( 2 - \frac{2s_c}{aD} \right) + \left( \lambda_{c-p} + \lambda_{p-c} \right) \left( 2 - \frac{s_c}{aD} \right) + \lambda_{p-p} \left( \frac{5}{2} - \frac{s_c}{aD} \right); \]

\[ T_3 = \tau' \text{#boarding per hour} = \tau' \lambda_{c-c} \left( 2 - \frac{2s_c}{aD} \right) + \tau' \left( \lambda_{c-p} + \lambda_{p-c} \right) \left( 2 - \frac{s_c}{aD} \right) + \tau' \lambda_{p-p} \left( \frac{5}{2} - \frac{s_c}{aD} \right); \]

\[ T = T_1 + T_2 + T_3 = \frac{E}{v} + \frac{\tau}{s_c} \left[ \frac{2}{3} \alpha D \lambda_{c-c} + \frac{5}{6} \alpha D \left( \lambda_{c-p} + \lambda_{p-c} \right) \right] + \frac{2\tau}{3s_p} \left[ \frac{3D(1-\alpha)}{8} \left( \lambda_{c-p} + \lambda_{p-c} \right) + \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p} \right] + \]

\[ \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) \lambda_{p-p} \left( \frac{5}{2} - \frac{s_c}{aD} \right) + \tau' \left( \lambda_{c-c} - \frac{2s_c}{aD} \right) + \tau' \left( \lambda_{c-p} + \lambda_{p-c} \right) \left( 2 - \frac{s_c}{aD} \right) + \tau' \lambda_{p-p} \left( \frac{5}{2} - \frac{s_c}{aD} \right) + \]

\[ \lambda_{p-p} \left[ \frac{2\tau}{3s_p} \left( \frac{3D}{4} + \frac{1}{6} \alpha D \right) + \tau' \left( \frac{5}{2} - \frac{s_c}{aD} \right) \right]. \]

**Corollary 1.** The expected commercial speed obeys (3.11):

\[ v_c = \frac{E}{\alpha T}. \]

**Proof.** Commercial speed is the radio of total travel distance and total operation time. The operation time is the same with passenger in vehicle time, so the result is claimed.

**Corollary 2.** The fleet size is calculated as the number of vehicles in operation during the rush hour, given by (3.10):

\[ M = \frac{V}{v_c} \times PHF. \]

**Proof.** It is essential to guarantee fleet size can cover vehicles need in the peak hour. \( \frac{V}{v_c} \) is the expected number of vehicles in a certain hour. So a peak hour factor is considered in the equation.

**Result 8.** The expected vehicle occupancy on the critical load point during the rush hour is approximately given by (3.25):
\[
\text{Occ} = \frac{PHFS_cH}{aD} \times \max \left\{ \frac{\max\{\lambda_{c-p}, \lambda_{p-c}\}}{2}, \frac{\lambda_{p-p}}{32}, \frac{\lambda_{c-c}}{4}, + \left(\frac{\lambda_{p-c} + \lambda_{c-p} + \lambda_{p-p} - \lambda_{p-p}}{2}\right) \right\}
\]

**Proof.** Since critical loads are on the same links in Smith’s (2014) model, see proof in his thesis appendix.