EXAM PREPARATION LEARNING

BY

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DISSERTATION

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Abstract

This thesis investigates student learning through practice exams. A series of experiments were conducted using a web-based platform that provided students with an organized structure to study past exam problems. We establish the learning obtained from doing these practice exams (Chapter 1) and then manipulate the feedback mechanisms (Chapter 2 and 4) and duration of the treatment (Chapter 3). The results show that all students benefit from practice exams and worked out solution feedback. However, investing more resources in this learning tool might not result in better learning gains. A comparison between experiments suggests that, beyond the quality of the practice exams and solution feedback, motivation and learning goals may be crucial to enhancing student learning during exam preparation.
To my family.
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Chapter 0

Introduction

For the past forty years, Physics Education Research (PER) has been devoted to understanding students’ difficulties [1, 2, 3] and developing activities and instruction to improve students’ understanding in physics. Interactive classrooms that promote active learning have been introduced and widely implemented, and their effects carefully documented [4, 5, 6]. PER research extends beyond the classroom with internet and multimedia technology. Innovations including Just-In-Time Teaching, pre-lectures and inverted classrooms have been developed to provide students with more effective learning opportunities outside of the classroom, which in turn improves their in-class experience. Despite a wide range of Physics Education Research on student learning inside and outside of the classroom, relatively little work has focused on student learning during exam preparation. In the present work, a series of experiments were conducted using a web-based platform that provided students with an organized structure to study past exam problems including multiple feedback mechanisms. We establish the learning obtained from doing these practice exams (Chapter 1) and then manipulate the feedback mechanisms (Chapter 2 and 4) and duration of the treatment (Chapter 3). The results show that all students benefit from practice exams. However, investing more resources in the learning tool might not result in better learning gains. A comparison between experiments suggests that other factors (beyond the learning tool) may be crucial to enhancing student learning during exam preparation. To provide context for our investigations of student learning during exam preparation, it may be useful to provide an overview of PER related to students learning in the other aspects of the course including work before lecture, during lecture in recitation, homework and lab.

The “flipped” classroom, currently gaining a lot of attention, is based on PER research on activities designed for students to engage in before coming to the lecture. These activities can improve learning during the lecture by preparing students with necessary information (pre-lectures) and by setting their attention to the right features (prepare for future learning). Pre-lectures [7, 8] are used to introduce the key material to students before coming to class through multimedia [9]. By equipping students with necessary basic information, they can quickly learn from the material in the lecture and, because of that, the lecture time can be spent on more interesting activities in a more meaningful ways. Aside from providing students with the necessary information, problem situations or contrasting examples can be introduced to direct students attention to the important features of the material. Schwartz and Bransford [10] used contrasting cases to create a “time for telling.” In being directed to the right features, students can gain more knowledge from later
learning resources [11]. Activities before lectures can also be used to inform instructors of the level of the students and their difficulties on the topics (just-in-time teaching). Novak et al [12] had students answer pre-flight questions that were due before class. From students answers, instructors can adjust the lectures to suit the students level and to focus on students difficulties. These activities before lectures help prepare both students and instructors to be ready for the coming lectures.

Many in-class activities are designed to improve student learning by enhancing different learning processes or tackling different problems. Research has shown that students learn better in class if they are actively engaging with their peers and the lecture material. One problem with large lecture halls is that communication is one way and students become only passive listeners. To solve this problem, Peer Instruction [4, 13], based on active and cooperative learning [14], uses challenging conceptual questions to start conversations among students. Similarly, Interactive Lecture Demonstrations emphasize observing real physics demonstrations, predicting the outcomes and discussing the demonstrations with their peers [6, 15]. Classroom polling can also enhance active learning by increasing the interaction between students and instructors during the lecture [5, 16]. Through these changes, the traditional lecture can be transformed into an interactive learning environment [17, 18, 19, 20].

Significant research has also shown the benefits of a completely different classroom format. Laws [21] created a laboratory-based classroom without formal lecture [22]. Her studies show that students can learn all of the introductory physics material through organized hands-on experiments. Case-based learning [23, 24] is another kind of class driven by the learners. It focuses on real-world problems. Another new kind of classroom is inverted classroom where traditional events inside the classroom happen outside the classroom (at home) and vice versa. With technology, lectures can be delivered to students online and different activities such as group discussions can be done in the normal class time [25, 26, 27, 28]. These different classroom approaches open up more possibilities for improving students learning.

Many post-lecture activities have also shown to improve student understanding. The most common activity outside of classroom is homework [29]. Homework has been shown to have positive effects on learning and a stronger effect happens when the teachers feedback is provided [30]. This could be a problem for large classrooms. However with current technology, online homework with immediate feedback is possible and it can enhance students learning as much as traditional homework [31, 32, 33]. Aside from improving the learning, Ramdass and Zimmerman [34] believe that an important role of homework is to help developing self-regulation skills. Another activity is cooperative group learning [35, 36] which uses cooperative learning [37] to promote problem solving skills. To tackle misconceptions in physics [38, 39], tutorials were created by the University of Washington Physics Education Research group [40] based on Posner and Strikes conceptual change theory [41, 42]. Lastly, experiment and laboratory is important for learning especially in physics. Active Learning Laboratories created by Thornton and Sokoloff [43] are based on discovery learning where students need to predict, observe and explain the experiment. These instructions,
activities and pedagogies have been studied and incorporated in physics courses widely. There are also learning activities students chosen to do beyond the class requirement.

Working on practice exams is a popular activity during the few days before an exam. Students commonly use old exams from past semesters as a study guide and to practice for the upcoming exam. The belief in learning from practice exams is widespread. In a recent student survey, practice exams and video solutions ranked the highest over lecture, discussion, homework, and lab on the role in their learning (Figure 0.1). However, there is little research on learning done during exam preparation. This thesis will focus on understanding the effectiveness of practice exams and various feedback systems in learning physics concepts.

![End of Term Survey: Question 9 (N = 286)](image)

Figure 0.1: Student rating (Physics 102 Spring 2014) about the important role practice exams play in their learning (A=Essential, B=Very Important C=Important D=Not Very Important E=Useless). Lecture (A: 30%, B: 24%, C: 35%, D: 9%, E: 1%), Discussion (A: 24%, B: 29%, C: 25%, D: 15%, E: 7%), Homework (A: 12%, B: 26%, C: 34%, D: 24%, E: 4%), and Lab (A: 3%, B: 6%, C: 25%, D: 42%, E: 24%)

Despite student ratings on the importance of practice exams, there is relatively little research from the PER community on the most effective use of practice exams. Fortunately, we can gain some insight by looking at existing research from other fields. A general review on the role of practice
exams is provided by Kulik, Kulik and Bangert [44]. By evaluating the findings from several studies on practice exams, they found that practice exams can improve the scores on aptitude and achievement tests. However, the impact of practice exams depends on three factors: similarity to the actual exam, number of practice problems and the ability level of the population.

Learning during exam preparation can be very different from traditional lectures. At this stage, students are not completely novices. They have learned some relevant knowledge and skills from class activities. Learning during this stage is not about learning new material. It is rather the time to understand the already learned material deeper or to make stronger connections between those ideas or to be more fluent with the class material. Another skill learned from practice exams includes problem categorization, which is an indicator for expert-like behavior [45, 46]. Practicing can also improve speed and accuracy by acquiring schema automation [47, 48]. Normal class activities, such as lectures or labs, might not be able to serve these purposes. Instead, practice exams, the tool that students commonly use, might be the answer.

The most obvious benefit of taking a practice exam is in helping students realize what they need to study. Indeed, Black and Wiliam [49] found that formative assessment was the key element to improving student learning. Bol and Hacker [50] showed that practice exams do help strong students better assess their understanding. However results from Rebello [51] suggested that weaker students may not have the skills to take advantage of the formative assessment provided by practice exams.

Other benefits from practice exams stem from the testing effect [52, 53, 54]. Studies have shown many benefits from the testing effect which includes improvement in retention [55], organization of knowledge [56, 57], transfer of knowledge [58, 59] and self-monitoring [60].

Beyond formative assessment and the testing effect, one may expect that taking practice exams provides an opportunity for gaining a deeper understanding of physics. Schwartz and Martin [11] showed that a problem set organized for introducing concepts in statistics can improve the learning during subsequent learning session. Even though the treatment group (work on organized problem set) and traditional (tell-and-practice) group performed about the same right after the treatment, the conventional group performed better than the traditional group after the subsequent learning resources. A similar analogy to the subsequent learning resource in a practice exam context might be the solution feedback. Epstein et al [61] found that practice without feedback does not improve performance. This suggests the importance of solution feedback.

A logical learning resource associated with practice exam problems are “good” solutions. Solution feedback can provide the necessary information required to solve the problem [62]. Again, there is not a lot of literature on this in PER. However, we may draw on education research showing students can learn effectively from Worked Examples [47]. Based on this idea, students may be able to learn from “good” worked out solutions to practice exam problems. Cognitive Load Theory [63] tells us that good solutions should reduce extraneous cognitive load in the learning process. These solutions should contain the necessary information required to solve the target problems in an organized way and avoid the split-attention effect and the redundancy effect. Zhongzhou and Gladding, in their work on grounded cognition, suggest that good visual representation should
activate the perceptual symbols necessary for constructing the represented concepts [64].

Solutions can be an important learning resource, however their effectiveness depends on both the quality of the solution, and the students skill at learning from them. One challenge is to determine the characteristics of “good” solutions. In part, good solutions avoid attributes that research has shown to have a negative impact on learning such as solutions that split learners attention, contain too much or too little information [65], or are too different from the target problems [50]. Furthermore, the expert-reversal effect [66] should also be considered since advanced learners can potentially view the same solutions differently from novices. Kulik and Kulik [67] and Hattie and Timperley [68] also suggest that different types of feedback can be optimized differently depending on feedback goals. These suggest that “good” solutions must be adjusted to suit different feedback goals and students with different aptitude levels.

To summarize, despite our physics students’ perception that practice exams are important to their learning, relatively little research has been done on student learning during exam preparation. The literature that does exist suggests that students ability to use the exam for formative assessment is limited, especially for students that need it most. Furthermore, the exam questions and answers and even the full solutions may be of limited value in helping students gain a better understanding of the material. The main goal of this thesis is to understand how much students learn from practice exams, and what types of learning resources can improve their learning. We begin by designing a series of experiments to measure the effect of solving practice problems on a students ability to answer similar problems. Our first two experiments both measured a positive performance gain from solving practice exams. The gains were observed in both an immediate post-test and on an exam taken a few days later. The first experiment, paired problem experiment, also suggested that the quality of the feedback can impact the gain.

The second experiment, tutor experiment, was conducted to investigate the impact of providing feedback beyond simply telling students the correct answer. Three treatment conditions (worked out solutions, worked out solutions with customized homework, and both feedback mechanisms plus one-on-one tutoring) were compared. All three groups performed better than the control. However, the results indicated that tutoring was not superior to the other options. Instead, participants with customized homework targeting individual weak topics showed the most improved performance on the related midterm problems.

The third experiment, distributed practice exam experiment, focused on the timing and duration of the treatment. The practice exams and worked out solutions in this experiment were provided earlier (since the second week) with higher frequency (weekly across the semester) than in previous experiments resulting in more than two hundred practice problems. Despite the positive results from the previous two experiments and the modifications made on the exam preparation tool, a learning gain from practice exams was not found.

One possible cause for the null result was the absence of the re-testing problems. The topics on the practice exams in experiment three depended on the content taught in class each week, thus the practice exams were different each week. Unlike in experiment one and two where participants had
opportunity to solve similar problems after receiving the solutions, participants in this experiment did not have that opportunity.

The final experiment was designed to test the effect from re-testing right after watching the solution videos. No differences between groups were found. However, despite the fact that the tool consisted of the same components (practice problems from past midterms and worked-out solution videos as the feedback), the learning gain in this experiment was much larger than all previous experiments.

A careful comparison between the setting of these four experiments and their learning gains suggests another factor that could affect the learning gain. In experiment one and two, the exam preparation tools were provided right before the exam for participants to use later, on their own time. The tool used in experiment three allowed participants to use the tool even more freely over the semester. On the contrast, participants in the last experiment were asked to use the tool during the experiment session with a set time constraint. The situations in these four experiments might have differently motivated participants to learn from provided learning material.

The results from these experiments suggested that instructors should use practice exams with worked-out solution videos provided as immediate feedback. These studies also showed that students can make use of the provided practice exams and solutions, but the learning gain also depends on the students themselves. The last experiment (chapter 4) showed that practice exams and worked-out solution videos (similar to chapter 3) can improve students performance on the exams. However, the important factors causing the large learning gain here might not be from greater use of practice exams and the solution videos, but rather, the motivation of the students in learning from the exam preparation tool.
Chapter 1

Practice Tests

Exams can be more than just summative assessment at the end of the course. They have the potential to be used as a learning tool. One possibility is to use them as practice exams during exam preparation period. As supported by research on testing effect, these practice exams can be an additional learning opportunity to student. However, before developing an exam preparation tool with a large selection of practice exams and sophisticate feedback system, we first need to establish the learning gain from solving similar exam problems as well as the additional learning gain from full solution feedback.

1.1 Testing Effects

Among the many strategies used during physics-exam preparation, practicing on the past midterm exams might be the most commonly used by students (see figure 1). At the University of Illinois, physics midterm exams from the past ten semesters are available for students. These exams can be used as a study guide for the topics covered in the exams. Students can also use these exams as a formative self-assessment to determine which area or skills they need to improve. Aside from these mediated effects, many studies on the testing effect have shown direct benefits from testing including improvement on retention and on knowledge organization.

Kulik, Kulik and Bangert [44] have shown that doing practice tests can increase students’ performance on the actual test and that the size of the gains depends on the number of practice tests given and the ability level of the population. This finding is supported by the recent research on testing effects [53, 54, 52]. This testing can happen in the form of both voluntary practice (flashcards or old exam problems, practice exams) and mandatory tests (weekly quizzes or midterms). The effects of testing include both direct and mediated effects [60]. Direct effects include improving retention, producing better organization of knowledge, improving transfer of knowledge, and metacognitive monitoring [52].

The first direct effect from testing is on retention. Studies have shown that testing without feedback can improve retention. Words and pictures [55], Swahili-English translation pairs [69],

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and reading passages [60], have been used as learning materials. These studies found that subjects can retain learning material longer if they were presented with a free recall test and that subjects presented with more tests can retain learning material longer than subjects with only one test. Not only the retention, testing can also improve the organization of knowledge. Masson and McDaniel [56] found that immediate recall test can improve the organization of knowledge on the delayed recall test. Similar results were shown by Zaromb and Roediger [57]. Both experiments were done with word lists and free recall tests. Their subjects, under the testing condition, were able to recall a larger number of clusters with more words in them.

Since testing leads to better retention with more organized knowledge, it is not a surprise that testing could also lead to better knowledge transfer. Enhanced transfer of knowledge in the testing condition was found in word-word paired association [58] and in bird classification [59] experiments. The recall on words that were presented but were not retrieved (were not tested) is improved in the delayed recall test. The classification knowledge of birds was transferred to new exemplars.

The last direct effect from testing is on self-monitoring. It is an important skill for learning and it has also been shown to be improved by testing [70]. In their experiment, subjects under the no-testing condition were over confident in predicting their future performance. These direct effects from testing required no additional learning resources. Thus, we should be able to observe the learning gain from providing practice exams alone.

Beyond these direct effects, testing can also induce other beneficial behaviors or consequences. Frequent testing encourages students to study regularly throughout the course [71] and to learn more during the next study session [72, 73]. It also helps prevent interference from prior material when learning new material [74].

Lastly, the most important effect from testing is to provide necessary formative assessment to the students [49]. Similar to improvement on self-monitoring, formative assessment can be used to identify gaps in knowledge. However, in order for the formative assessment to be fully activated, informative feedback is required. Testing without corrective feedback can lead to no improvement [61] and the remembering of false answers [75]. These mediated effects depend on the supporting activities or learning materials. Especially, for the formative assessment to enhance the learning, proper feedback, such as the solutions, is required.

Both the direct and mediated testing effects described here support the findings of improvement from practice testing reported in Kulik, Kulik and Bangert [44]. This suggests that having practice exams available should be beneficial to students. Furthermore, suitable learning resources could provide additional support and make better use of the formative assessment.

However, most experiments on testing effects were done with fact recall tests or simple associations [76]. Unlike these learning tasks, physics problem solving could require multiple concepts and strategies. Even though testing proved to be beneficial in many learning contexts, the testing effects in physics problem solving should be tested.
1.2 Experiment 1: Paired Problem Experiment

In 2009, we developed an activity to help students study for a midterm exam in the introductory calculus-based electricity and magnetism course at the University of Illinois. The activity was available online and provided students with practice exam problems, grouped by topic. The practice exam problems in the tool were organized in pairs of similar problems such that the pairs required the same pieces of knowledge in order to solve them, but their text and figures were slightly different. There were 44 problems (22 pairs) covering most topics that appeared in the first hour exam (see Appendix A). The similar-paired problems were presented to students in consecutive order. After every answer submission, immediate feedback was given. There were two types of immediate feedback: the correct answers and worked out solutions. In this experiment, the immediate learning gain from the two types of feedback on practice problems was measured, as well as the effect on the following midterm.

1.2.1 Population and Design

To measure the impact of the exam preparation activity, the tool was made available in fall 2009 to approximately one thousand students enrolled in the introductory calculus based electricity and magnetism course at the University of Illinois. The exam preparation tool for the first midterm exam covered four topics: Gauss’ law, Coulomb’s law, electric potential and capacitors. It was made available online a week before the midterm. An announcement e-mail was sent to student with a link to the activity, however, participation was voluntary and no course credit was given for completing the activity.

Students were allowed to choose among the four main topics on the midterm. They were then presented with a problem to solve, and multiple choice options for the answer. After students tried solving a problem and submitted their answer, they were immediately told if their answer was correct or not and had a chance to look at the feedback (which was either the correct choice or a worked out solution). Then they would be presented with a similar problem. The objective of this setup is to see how much students learn from previous problems. For each paired problem, there are four possible arrangements of problem order and types of feedback. To be fair to all students, the problem order and feedback for the 22 problem pairs are arranged such that every student will have access to the full solutions of the first problem a quarter of the time and to the second problem another quarter of the time (this proportion applies to correct answers as well) as shown in table 1.1.

With this design, we can compare the effect of solving similar problems and receiving different versions of feedback. For example, to consider the effect of solving problem B on the performance of problem A, we can use the average score of problem A from groups 1 and 2 as an initial base score. The average score of problem A from group 3 is the performance after solving problem B with only correct answer feedback. The average score of problem A from group 4 is the performance after solving problem B with the full solution feedback. With these measurements, we can compare the
Table 1.1: Order in which problems are seen by four different groups, as well as their access to answers and solutions after answering the question. Four groups of students are required in order to capture students performance on every problem for all three situations (first of the pair, after its pair with answer, and after its pair with the solution option). Problem A, B, C and D in the table are used to emphasis that all four groups should receive the same amount of answer or solution immediate effect of all 44 problems.

To measure the effectiveness of the exam-prep tool on a real exam, another 13 similar problems were included in the course midterm exam. These 13 problems were also selected from old exam problems that matched paired problems used in the Exam-Prep Tool.

1.2.2 Data Analysis

For each student, we collected his or her answers, the times at which students opened and answered problems and whether they choose to look at the solutions when they were available. Our records showed that 70% of all students (769 out of 1100 students) tried at least one problem and about 50% (494 out of 1100 students) tried at least half. There were 160 students who finished all 44 problems before the midterm.

**Learning from similar problems and different levels of feedback**

By considering the performance from all 4 groups across the 44 problems, we can see the effect of solving similar problems and of having different levels of feedback. On average, participants scored 58.8±0.2% on problems that appeared first in the pairs indicated as base line. If they had access to only the correct answer of the first problem, the average score on the second problem was 63.5±0.03%. If they had access to the full solution of the first problem, the average score on the second problem was 66.0±0.3%. Hence, working on a similar problem can improve performance by 4.7±0.4% with just the correct answer and by 7.2±0.4% with access to the full solution. The uncertainty included in this comparison is only the variation due to variation in student ability.

One interesting feature in figure 1.1 is the large variation of student improvement on different sets of paired problems. A qualitative analysis of the most improved and least improved problem pairs provides us with some insight. From initial observations, most of the questions that showed large gains were problem pairs that have nearly identical solutions. For these pairs, a straightforward translation of the solution from the first problem could be used to get the answer to the second problem. After examining all 22 problem pairs closely, we categorized 5 pairs as having virtually identical solutions. The average gain of these pairs is 15±3%. The average gain on the remaining
Figure 1.1: Average scores on the first problems of the pairs (x-axis) plotted against the average scores on the second problems of the pairs (y-axis) after being exposed to either the final answers or full solution feedback of the first problems.

17 pairs is 4±1%. Using a $\chi^2$ analysis, we found that the gain from identical solution problems and the gain from the rest are significantly different with $\chi^2$ (d.o.f. = 1) = 14.9 ($p < 0.001$).

A careful examination of the 17 problem pairs that did not show a large improvement suggests two explanations. In some cases the lack of improvement can be attributed to limitations in the question format (e.g., multiple-choice questions that did not have effective distracters, so guessing played an important role). However, some responses suggest that students obtained only a very superficial understanding from the paired problems. Problems in each pair look nearly identical at the surface and were categorized as requiring identical knowledge, but the final formula of the first problem might not straightforwardly translate for the second problem.

Based on data from the 22 problem pairs, we draw the following conclusions. Doing old exam problems can improve a student's performance on subsequent similar problems and having access to the solutions for the exam problems can increase that level of improvement. However, in general...
this improved performance is restricted to problems with a high degree of similarity in problem solving processes, suggesting a relatively superficial level of learning.

Formative Assessment

![Figure 1.2: Students scores in the exam preparation tool (x-axis) compared to their scores on the midterm exam (y-axis)](image)

Another important value of studying old exam problems is their ability to provide students with formative assessment [49]. The top performing students often report this as a very effective technique. However, low-achieving students may not be as skilled at interpreting their performance on a practice exam. A web-based exam preparation tool may be able to help low-achieving students by making the formative assessment more explicit. In particular, the exam preparation tool could use the students performance on practice problems to assess their overall preparedness for the exam, as well as recommend specific topics that the students would benefit from studying more. To assess the suitability of old exam problems for providing general and specific formative assessment, we next look at the correlation between student performance on the practice exam questions and on the actual exam. Since the ability to predict student performance is strongly dependent on the number of questions they do, we restricted this analysis to the 160 students who completed all 44 exam-prep exercises. Figure 1.2 shows a scatter plot of the 160 students that completed all 44 exam-prep
exercises. On the horizontal axis is their performance on the exam preparation tool exercises and on the vertical axis is their performance on the exam. There is a statistically significant correlation between the two scores ($r = 0.56$). This correlation is low compared to the values of $r = 0.9$ one would expect if giving two identical exams. However, this is not surprising, considering that many students are using this tool at the beginning of their preparation (so they can study more and gain improvement outside of the tool), and we have already seen that studying these practice problems can improve their score. Indeed, it is reassuring to see that although the difficulty of the exam preparation exercises and the exam questions was approximately equal, nearly all of the students performed significantly higher on the exam than they did on the exam preparation tool with very few students performing significantly below that line on the exam. Thus, the tool can be seen to provide a minimum assessment (the dotted line) of the students preparedness for the exam.

![Graph showing the comparison of student performance on exam questions based on how the student performed on the paired practice exam questions.](image)

**Figure 1.3**: Thirteen exam questions were matched with 13 of the paired problems on the Exam-Prep Tool. Students who got both practice problems correct score 15% higher than students who got both problems wrong.

In addition to assessing a students general preparedness for the exam, it would be helpful if the tool could identify particular strengths and weaknesses. Such information could be helpful in identifying particular areas that the student should focus on studying. Figure 1.3 shows a comparison of student performance on exam questions based on how the student performed on the paired practice exam questions. Regardless of the improvement after the use of the tool, students who answered both problems right on the tool did about 15% better on the paired midterm problems than students who answered both problems wrong.
One explanation for the above difference is simply that better students do better on both the practice problems and the actual exam. To determine if the exam prep can help differentiate weaknesses in an individual student, we created Figure 1.4. The diamonds represent how students perform relative to the rest of the class on topics for which they got both paired problems correct. The squares show how they performed relative to the rest of the class on topics for which they got both paired problems wrong. The first thing we see is that students using the tool two or more days before the exam scored 6% higher than the average student no matter how they did on the Exam-Prep Tool. Therefore, there is little predictive power from the tool. This is attributed to the fact that these students have time to improve their areas of weaknesses. However, for students using the tool on the day of the exam, there is a drastic difference in exam performance. They scored about 8% higher than the rest of the class on topics they did well on and 13% worse than the rest of the class on topics they did poorly on. These results suggest that the exam preparation tool can provide some formative assessment to students and guide them to focus on the topics they need to study more.

![Figure 1.4: Students relative performance on 13 exam problems for topics the exam preparation tool predicted they would perform well on (diamonds), compared with topics the tool predicted they would not perform well on (squares).](image)

However, the performance gap between students who answered both problems right and who answered both problems wrong also depends on the base performance of the students. To minimize any systematic effects (stronger students always perform better), the performance on the 13 paired problems in the exam were grouped by student performance in the prerequisite mechanics course,
which on average has a high correlation with performance in the electricity and magnetism course. As shown in figure 1.5, “A” students performed almost equally, no matter how they did on the tool. This is consistent with the idea that strong students are able to learn well from practice exams. However, “B” to “D” students show a significant spread based on the performance on the tool. This suggests that low-performance students may require more guidance beyond the practice exams.

![Figure 1.5: Students performance on matched midterm problems (y-axis) that they were predicted to do well on (diamonds) or poorly on (squares) compared with their grade in the prerequisite mechanics course (x-axis)](image)

1.3 Experiment 1: Conclusions

The results of the experiment confirmed that practice exams can an effective learning tool and that providing worked out solutions significantly increased student learning. However, the immediate gain from worked out solutions were limited to problems with identical solution steps. This encouraged us to find and implement “better” feedback systems to increase the learning gain on the problems with non-identical solution steps (Chapter 2).

Another observation from this experiment is that students who practiced earlier showed high performance on the exam in the topics they performed poorly in the practice exam. If this is not purely a self-selection effect, then encouraging students to use the practice exam earlier should help students improve in their weak topics. Also, many studies show that there is a benefit of distributed
practice exam [77, 78, 76]. This suggests that practice exam should be introduced earlier or even distributed across the semester (Chapter 3).

In the following two experiments, we implemented more informative feedback systems (one-on-one tutor and customized practices) and distributed practice exams throughout the whole semester.
Chapter 2

Feedback System

Practice exams can be used as formative assessment to identify knowledge gaps, but to fill those gaps, informative feedback plays an important role. The result from previous experiments showed that worked out solutions are a better kind of feedback than providing only the correct final answer. In this chapter we investigate if student learning can be increased with a different type of feedback.

One manipulation that could be made is to provide more detail in the worked out solutions. However, providing more relevant information in the solutions might not always be better. Due to limitations in short term memory, as suggested by cognitive load theory [80], and time constraints during exam preparation, it might not be effective to provide students with complete solutions explaining all relevant topics in full details.

What is a good way to present the solutions? What is a good amount of information to present in a solution? Cognitive load theory can provide a good guideline for how to create effective feedback solutions. For example, solutions should not split readers attention (which can cause unnecessary cognitive load). In the following section, cognitive load theory will be discussed along with three possible feedback systems in a practice exam scenario: worked example feedback, customized practice problem feedback and one-on-one tutoring.

2.1 Cognitive Load Theory

Cognitive load theory [48, 81, 82] starts with the idea that our working memory is very limited. It can store about seven elements, operate on two to four elements at a time and can only retain these elements for about twenty to thirty seconds [83, 84, 85]. However, these limits apply to only novel information. Working memory does not show these limitations when dealing with information retrieved from cognitive schemata and automation stored in long-term memory. These schemata and automation processes require fewer resources from working memory.

The difference between experts and novices is the knowledge stored in these schemata. Chase and Simon [86] found that the difference between chess experts and novices are these schemata in the long-term memory, not the capacity of the working memory. Chess experts can remember chess
Acquiring the schemata of complex cognitive tasks is difficult if the cognitive load exceeds the learners cognitive capacity [80, 81]. Cognitive load theory identifies three types of cognitive load: intrinsic load, germane load and extraneous load. Intrinsic load is the load caused by the complexity of the schemas needed to be learned. It is dependent on the number of elements needed to be processed simultaneously by the learner [82] and it cannot be altered. Germane load is the load required for the processing, construction and automation of schemas. This is where the learning begins. Many strategies were shown to increase the germane load in worked examples context [87] such as increasing variability [88] and prompting self-explanation [89]. Lastly, extraneous load is the load caused by poorly designed instructions that interfere with schema acquisition [65], for example, instructions that required learners to mentally integrate mutually referring text and diagrams [90]. It should be avoided or minimized in any instructions. The total cognitive load consists of these three types of cognitive load, but not all of them need to be minimized in order to create a good instruction.

To create effective instructions, cognitive load theory suggests that the instructions should not overwhelm the learners with too much cognitive load [80]. They should induce high germane cognitive load and low extraneous cognitive load [91, 65]. Furthermore, instructions should encourage schema construction as well as schema automation [47, 48].

Another learning process in cognitive load theory’s view is to be fluent in processing schema, called schema automation. Schema automation can be acquired through practice [63]. In problem solving, Sweller and Cooper [47, 92] found that automation can only be acquired after the schema itself has been acquired and both of them facilitate the transfer. After schema automation has been acquired, schema processes require little to no cognitive resources.

The dynamic between expertise and the level of the instruction also plays an important role in learning. The effectiveness of any instruction technique depends on the level of the learners. The expert reversal effect was introduced by Kalyuga et al [93]. They found that effective instructions for inexperienced novices can lose their effectiveness and even have negative consequences when used on more experience learners [94]. This problem can be solved by implementing adaptive learning tool to provide each student with the right level of instructions.

Another simpler strategy to match students with the right learning material can also be implemented through practice exams. Formative assessment in the form of practice exams can be used to direct student’s attention to the right learning tool (solutions) and to skip what they already mastered. By making the solutions accessible after the practice exams, the expert reversal effect can be avoided.

Learning from practice exams and solution feedback is supported by the cognitive load theory. Both types of learning, schema acquisition and schema automation, can be naturally acquired through practice exams and solution feedback. Furthermore, the practice-then-solution setting can help avoid the expert reversal effect.
Cognitive load theory not only provides some overview guideline on how to create effective instructions, research on cognitive load theory also suggests many effective learning tools, including the one we implemented in this experiment, worked examples.

2.2 Worked Examples

Cognitive load theory provides several possible advantages of using worked examples to learn problem solving skill [47, 92]. First, worked examples provide an opportunity to improve student learning by increasing germane cognitive load and decreasing extraneous load [87]. Second, worked examples can avoid unnecessary search patterns for solutions, and guide novice learners to the most efficient and appropriate way to solve problems [95]. Note that in the view of cognitive load theory, borrowing knowledge is as good as constructing knowledge [96]. Lastly, good worked examples can also direct students to the important features of the problem and the solution steps (increase the germane cognitive load) without unnecessarily diverting the students attention (decrease the extraneous cognitive load).

However, there is more than one recipe for effective worked examples. Variations of worked examples with different focuses have been studied. Process-oriented worked examples are designed to emphasize the principles and strategies underlining the problem solving process [97]. High-variability worked examples have also shown advantages over low-variability worked examples and conventional practice problems [88]. Worked examples with errors are designed to highlight the comparison between examples [98]. Despite various designs of worked examples, they are all intended to help students acquire problem-solving schemas.

Even though many studies on worked examples were done when the material was first introduced to novices, those results can also be applied to the learning (schema acquisition) from worked examples used as feedback in the practice exams. In fact, Reisslein, Atkinson, Seeling and Reisslein [99] found that problem-then-example sequences are beneficial for high-prior-knowledge students. Furthermore, following schema acquisition, schema automation should, then, be encouraged [47, 92] and acquiring schema automation can be done through practice [63]. Practice exams and worked-example feedback can serve the purpose of learning (filling the knowledge gap equates to schema acquisition and sharpening the skill equates to schema automation) during exam preparation (high prior knowledge).

2.3 Customized Practice Problems

A possible improvement on practice exams and worked out solutions is to make it adaptive for each learner. If the learner has already mastered a topic, additional practice on that topic might not significantly improve their performance. On the other hand, providing additional practice problems on the topics for which they had difficulty might be more beneficial.

Kulik, Kulik and Bangert [44] showed that the amount of practice is positively correlated
with performance. However, due to the expert-reversal effect [93], practicing on already-mastered problems can be counter-productive. Focusing practice on topics that need schema acquisition and/or schema automation can be more effective than practicing on all problems. The idea of customized practice is similar to Karpicke and Roedigers experiment [100]. They showed that having students restudy only problems that they got wrong is as effective as restudying all problems. Unlike adaptive testing which tries to effectively measure skill level [101, 102], customized practice is intended to effectively help develop skill acquisition and skill automation. In fact, through continued use of customized practice problems, students can master a topic (Schroeder in press).

In addition to the learning gain from practice problems and worked out solutions, providing customized practice problems based on a student’s prior performance should allow them to develop needed skills and result in a higher learning gain. For the following experiment, customized practice problems were provided to students who incorrectly answered similar problems on the initial practice exam. These students can be assumed to either need to learn the problem solving schema or need to acquire schema automation. Providing more worked examples and additional practice problems can help students acquire these.

2.4 Tutors

Having a private tutor can be considered the gold standard among learning strategies. In problem solving, scaffolding can benefit from having a tutor. Tutors can influence and maintain the learners interest on the task initially and during the learning process. Tutors can adjust the complexity of the task to suit the learner. Tutors can also direct the learners attention to the critical features and demonstrate solutions to the task [103].

Furthermore, tutoring can reduce extraneous load by preventing students from spending too much time on irrelevant information. Tutoring can also induce germane load by challenging students to cognitively work on activities that will contribute to learning such as self-explanation or reflecting on their input [104].

Finally, tutors can be adaptive. They can adjust their role to suit students needs. Tutors can interact and explain things to students at a suitable level [105]. They can answer students specific questions and detect students misconceptions or misunderstandings. They can immediately assess students understanding at every steps of the tutoring process.

Tutors can be considered an enhancement on the practice exams with worked examples system. In this experiment, the tutoring treatment was tested. If tutoring can increase the learning gain of the exam preparation tool, then, the exam preparation tool can be improved by implementing some of the tutoring strategies such as scaffolding and adjusting the solutions to suite the students levels.
2.5 Experiment 2: Tutor Experiment

Many studies support the advantages of worked examples, customized practice problems, and tutoring. This experiment was designed to compare these feedback treatments intended to improve the learning gain found in the first practice exam experiment. The primary question investigated in this study is: how much do additional treatments improve learning?

2.5.1 Setting

The experiment procedure is summarized in figure 2.1. We compared three different sets of exam preparation activities based on practice exams: 1. Practice exams with solution feedback (Practice Only), 2. Practice exams with solution feedback and customized homework (Practice+Homework) and 3. Practice exams with solution feedback, customized homework and tutor sessions with an experienced PER member (Practice+Homework+Tutor). The experiment was a clinical study meeting three times in the week before the second midterm exam. Three sets of practice exam problems were used, one for each session, with all exams being equal in difficulty and coverage. All problems were multiple-choice, taken from past exams with minor changes. The solution feedback contained the basic strategy needed to solve the problems and a list of steps and equations to get to the final answer. The customized-homework consisted of similar problems with the correct answer already marked. The participants could work on the problems and check the answers by themselves after the experiment session. Participants with the tutor condition had a one-hour session with a tutor after every practice exam to go over the practice problems. The answers and confidence level on all of the practice problems were collected. The score on the following midterm exams of all participants was analyzed.
Figure 2.1: Schematic of the experiment setting. The sequence of the activities of each group flowed from top to bottom. Participants in each group received one type of treatment throughout the experiment.
2.5.2 Population

The experiment was done in the calculus-based Introduction to Classical Mechanics course at the University of Illinois at Urbana Champaign. This course is required for both physics and engineering majors, typically taken in their first year at the University. In fall 2012, students who scored less than the average score in the first midterm were invited by e-mail to participate in the experiment. The invitation was sent out two weeks before the second midterm. The 76 students who replied (black bars in figure 2.2) were randomly assigned into four groups: 1. Practice exam with solution feedback, 2. Practice exam with solution feedback and related homework, 3. Practice exam with solution feedback, related homework and tutoring sessions and 4. No treatment as the control group. The number of participants for each group is shown in figure 1.2. Three participants who did not show up at the first session and two participants who dropped the class (one from the Practice-Only group and one from the Tutor group) were discarded from the analysis. Seven participants who did not complete all of the tasks were also excluded. The average scores on the first midterm of the 44 participants who completed all three treatment sessions (yellow bars in figure 2.2) and the 20 participants in the control group are shown in figure 2.3).
The students in the first three groups, received an e-mail telling them that they had been selected to participate in the experiment and that they will receive compensation if they participate in all three sessions. We chose to tell students about the compensation after they were selected to filter out students who might participate in the experiment only for the money. The students in the control group received an email explaining that the experiment was full and that they were not able to participate. Randomly selecting registered-students as the control minimized the self-selection effect. Note that the exams for the past ten semesters and the exam keys were available to all students in the course. So, many of the students in the control group will have solved the same questions that students worked on in the treatment groups.

2.5.3 Procedure

The participants attended three sessions on three different days. At the beginning of each session, they answered some survey questions about their exam preparation. Then, they worked on the practice exam for one hour in an exam-like environment. For each problem, they had to choose an answer and their confidence in their answer. We used a set of three practice exams, one for each session (see Appendix B). The order of the practice exams was random for each student. After
one hour, the practice exams were graded and given back immediately along with the worked-out solutions. Then the participants that did not get tutoring left for the day.

Participants in the Practice+Homework condition received a maximum of three related homework problems in addition to the worked-out solutions. These related homework problems were selected based on the participants answers and confidence in the practice exam problems, with problems that the students got wrong but had high confidence in being given out first, followed by any other problems they got wrong. These homework problems were similar to the problems on the practice exams. The final answers, but not the worked-out solutions, were also provided with the homework-problem text. Participants took these homework problems home, where they were expected to solve the homework problems showing their work and hand them in during the next experiment session. Most students did complete the homework between sessions.

Each participant in the Practice+Homework+Tutor condition spent an hour with a tutor immediately after working on the practice exam. Over the course of the three sessions, each participant received 3 hours of individual tutoring. The participants could also work on the practice problems with the tutor or ask the tutor any physics questions. At the end of the session, the tutor gave the participant a maximum of three related homework problems based on the tutors judgment.

After three treatment sessions, all of the participants and the control took the second midterm. The problems on the midterm covered the same topics as the practice exams but they were not similar to any problems used in the experiment.

2.5.4 Results

The students scores on the three practice exams showed gradual improvement (figure 2.4). All three participant groups scored higher from session to session, and there was no significant difference between the groups. This is consistent with our earlier findings, since the problems on the three versions of the exam were designed to have similar solutions. If we consider the scores from the first practice exam and the third practice exam, all three participant groups showed significant gain (figure 2.5) \( t_{practice\ only}\ (d.o.f. = 17) = 3.4, p < 0.01 \) \( t_{practice+HW}\ (d.o.f. = 17) = 4.6, p < 0.001 \) \( t_{tutor}\ (d.o.f. = 7) = 3.9, p < 0.01 \).
Figure 2.4: Practice exam scores across treatment sessions
On the actual midterm exam, the participants performed significantly better than the control. On average, the participants showed a positive change from the first midterm exam to the second midterm exam. In contrast, students in the control group showed a drop (figure 2.6). On average, the participants scored 3.2±1.5% higher and the control scored 2.8±2.5% lower from the first to the second midterm. The drop in score of the control group was consistent with the average score drop of the entire class (-5.9±0.4%). It is important to note that none of the midterm problems had solution steps identical to any of the problems provided in the treatment sessions. By comparing the participants to the control group, we conclude that the treatments significantly helped students improve their exam performance ($t(d.o.f. = 62) = 2.85, p < 0.01$).
Figure 2.6: The first and second midterm exam scores (before and after the treatment) from participants who completed the three treatment sessions and the control.
Although there was a difference between the three treated groups and the control, participants with a personal tutor did not outperform the other two participant groups. Due to a 30% drop out rate of the tutor treatment group and the fact that the participants in the control group could not quit the experiment, this self-selection effect cannot be ignored. When we reanalyze the data to include all invited participants, the gains of the three participant groups were reduced to 4.4±2.1% compared to the control group ($t(d.o.f. = 72) = 2.15, p < 0.05$). Note that participants who completed three sessions scored about 6% higher ($t(d.o.f. = 62) = 2.85, p < 0.01$) compared to the control group (figure 2.6).

The data presented thus far shows that students in the treatment group outperform those in the control group. Fakcharoenphol [62] showed that practice problems can lead to improvement on the midterm exam problems that have identical solutions to the practice problems. The present findings extend this work by showing similar improvement can be made using practice problems that are similar but not necessarily identical to those on the midterm exam. This study also investigated the learning gain obtained from three different treatments, and no significant differences were found. In order to better discern the effect of the different treatments, the problems given on the midterm were divided into two groups, based on if the key concepts necessary for solving that problem were covered in the practice test or not. Two judges independently categorized the problems with roughly $\frac{1}{2}$ of the problems being placed in each category. The agreement rate between two judges
was initially 79% and the disagreements were discussed and resolved. Figure 2.8 shows that the performance gain of the treatment group relative to the control group was about twice as large on the related problems (15%) compared to the unrelated problems (8%). The following analysis will focus on the results of the 10 midterm exam problems categorized as related to the treatment activities, in order to better understand the impact of the different treatments.

Figure 2.8: The performance on the second midterm categorized by similarity to the practice exam

Figure 2.9 shows the impact that targeted homework activities had on student exam performance. The first two bars show that, for students that got a particular topic wrong, those students that also completed a targeted homework problem on that topic scored 10% higher than those who were not given a homework assignment targeting that topic. For each problem, we categorize participants into three groups: those who needed and received related homework, those who needed but did not receive the homework and those who did not need the homework. We assume that participants who answer incorrectly on the practice problem need the corresponding related homework and participants who answer correctly on the practice problem do not need that related homework. Participants who did not need the related homework scored the highest on average across all related midterm problems, while participants who needed and received the homework scored significantly higher than participants who needed the homework but did not receive it ($t(d.o.f. = 9) = 2.99, p < 0.05$).
The impact of the related-homework activities can also be observed in student performance on the practice exams (figure 2.10). Looking at the cases where students got a question wrong on the practice test (N=445), participants who received the related homework during the previous session performed about 10% better on the targeted problems on the next session than participants who did not get the homework ($t(d.o.f. = 443) = 2.05, p < 0.05$). We conclude that providing targeted homework activities can significantly improve student performance on that topic.
Despite the fact that participants in the tutor condition spent time one-on-one with a tutor and received the same set of related homework, they did not perform better than the other two treatment groups. Participants in the tutor group spent three extra hours, one hour at the end of every session, working one-on-one with tutors. The tutors in this experiment were members of the Physics Education Research group at University of Illinois at Urbana Champaign. In the session, the tutors answered participants physics questions and explained physics concepts. They helped participants work through the practice problems. They also assigned related homework problems, used in the Practice+Homework group, and checked them with the tutee in the subsequent experiment sessions. Students in this group received more support than the other treatments, so why did the additional support not result in a larger learning gain than the other groups?

One explanation is that the tutors did not do a good job. Although this may be true, it is a somewhat unsatisfactory conclusion, as the tutors were all highly trained in physics education, and were engaging in best practices based on the student responses. The tutors not only knew the solution to all of the practice problems, they also discussed with the students common misconceptions and mistakes before every session. Within the session, tutors encouraged students to
show their work and to explain their reasoning. Students were encouraged to ask if they did not understand any part of the material. The tutors also asked the students to do similar problems to gauge their understanding. At the end of the session, the tutors gave their students homework problems targeting each students weak topics. Anecdotally, the tutors commented that during the sessions many of the students were unable to concentrate due to lack of sleep, or concern about completing work for other courses. This observation is consistent with the relatively high attrition rate of 30% for the tutor-treatment group.

Survey questions administered before each session indicate that students from all three treatment groups spent about the same amount of time preparing for the exam, which included the time spent with tutors. The total time spent between the 1\textsuperscript{st} and 3\textsuperscript{rd} sessions is shown in figure 2.11. All three groups spent, on average, 10 hours total. Related homework and tutoring time did not significantly change the total time participants spent preparing for the exam. However, the participants in groups with related homework (practice exam, solution and homework) spent more time on materials provided from the experiment (bottom section of the time bars) than participants without homework ($t(d.o.f. = 34) = 2.8, p < 0.01$). Also, participants in the tutor group spent significantly less time beyond the experiment material than other two treatment groups (top section of the time bars) ($t(d.o.f. = 42) = 2.25, p < 0.05$). Although the three groups spent about the same total time preparing for the exam, they distributed the time on activities differently.
Figure 2.11: Self-reported time spent preparing for the exam between the first and third session

It is difficult to draw strong conclusions about the impact of 3 hours of tutoring based on the data collected in this experiment. However, the data does suggest two factors that should be considered in designing an effective tutoring treatment. First, the high attrition rate and the relatively poor condition of students attending the sessions suggest that student time constraints seem to be a very important factor in their learning. Second, the large reduction in time spent studying outside the course suggests that students being tutored may be overly reliant on the tutor to ensure they understand the material, instead of taking responsibility for themselves.

2.6 Experiment 2: Conclusion

The idea of providing more practice opportunities and learning tools on the most needed topics is supported by the results of this experiment. As we expected from our previous work, participants perform better during the next treatment session on similar problems with identical solution steps. However, in this experiment, participants demonstrated they can also transfer the knowledge to the midterm problems with different solutions steps. Customized homework exercises, specifically selected for each student, improved the student’s performance on related midterm problems.

One-on-one tutoring, on the other hand, does not guarantee an extra boost in this practice
exam scenario. Despite the extra time the participants spent with tutors whom we considered more experienced than average tutors and the extra time they used to work on the related homework, these participants performed only as well as the practice only group. The difference in “other time” suggests that tutoring or the idea of having a tutor can change the way these students prepared for the exam or at least the way they self-report their time. These results suggest that better forms of feedback (more engaging, more tailored to individual needs) don’t necessarily result in better learning gain.
Chapter 3

Distributed Practice Exams

In the paired problem experiment (chapter 1), students who used the online exam preparation tool for at least two days before the exam performed well on the exam in both strong-predicted and weak-predicted topics, whereas students who began studying less than two days before the exam performed poorly on the weak-predicted topics. These results may be explained by the self-selection effect, but they might indicate that students who begin studying exams earlier will perform better. In the first experiment, students who began studying earlier may have had sufficient time to improve their weak topics as identified by the practice exam.

One way to encourage students to begin practicing earlier is to distribute practice exams earlier and with greater frequency. Under this system, students can practice more problems and their misconceptions or weak topics can be identified earlier. After those misconceptions are fixed and the skills in those topics are strengthened, the later practice exams can be used to help retain the already learned material.

3.1 Distributed Practice

Cramming is a common strategy that students often use to prepare for an exam. Although it may be better than not studying at all, research has shown that distributed practice is far more effective [77, 78, 76]. Students false confidence in their understanding of the material might be due to the effectiveness of the cramming strategies (immediate-repeated practice) during the learning period. However, frequent (short delay intervals) practice diminishes retention of the learned material [53].

Distributing learning over time typically benefits long-term retention more than cramming, this finding is called distributed-practice effect [76]. The time interval between practice sessions and the lagging time until the final test can affect the outcome of the final test (Delaney et al., 2010). In an experiment on the testing effect in a distributed-schedule setting, Carpenter et al [106] showed that distributed testing is more effective than distributed re-studying. Furthermore, they found that delayed treatment (over 16 weeks) is better than immediate treatment (over 1 week) on a retention test thirty-six weeks later. In fact, Cepeda et al [77] found that the optimal time interval between practice actually depends on the lagging time between the final practice and the test. To promote long-term retention, distributed practice spaced with longer lags is encouraged.

For learning purposes, the benefit from distributed-practice is mainly in retention. Bahrick [107] conducted an experiment on English-Spanish word training with 0, 1 and 30-day practice-
recall intervals. Even though participants in the shorter interval conditions achieved higher correct recall during the training sessions, the participants in 30-day interval condition yielded the highest retention on the final test 30 days later.

<table>
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<th>Inter-session interval (days)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Following the 30-day interval</th>
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<td></td>
<td></td>
<td>33</td>
</tr>
<tr>
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<tr>
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<td>82</td>
<td>95</td>
</tr>
</tbody>
</table>

Figure 3.1: Results from Bahrick (1979)

Because of the distributed-practice effect, distributing practice exams throughout the semester might yield higher learning gains on exams, in contrast to typical ‘cramming’ strategies. The present work tests this hypothesis. We investigate whether students can be encouraged to use the practice exams earlier and whether distributing practice exams with worked solutions at regularly spaced intervals provides a larger gain than what ordinary students do before the midterm exam.

3.2 Experiment 3: Distributed Practice Exam Experiment

In fall 2013, a series of practice exams with worked-out video solutions were distributed over the course of the semester to participants. The practice exam problems were taken from old midterm exams in the calculus-based electromagnetism course. Each week, participants worked on practice problems already covered in class and then received feedback in the form of a solution video. There were three conditions: practice exams in an exam-like environment, online practice exams and the control that had no access to any experiment materials. The scores on three midterm exams and on the final exam were analyzed.
3.2.1 Population

All Students enrolled in Physics 212 (a calculus-based electromagnetic course) were invited to participate in this semester-long experiment. Fifty-two students registered but eight later dropped the class. We randomly assigned them into groups. Fourteen students were placed in the “practice exam in exam-like environment” group. These participants were told to come to the lab to take a practice exam for one hour every week. Sixteen students were placed in the “online practice exam” group. These students were told to work on the practice exams online during a two-day period each week. Participants in these two groups also had access to the online video solutions to the exam problems after the appropriate practice exam deadline for their group had passed. There were fourteen students in the control group. These students were told that the experiment was full and that they could not participate in the experiment.

After these groups were assigned, the participants were told that they would get paid if they participated in all of the practice exam sessions. They had to participate in at least 12 of 14 sessions to receive full compensation of $100. Additional absence would decrease the compensation. If they participate in less than 5 sessions, they would not receive any compensation.

3.2.2 Material

The practice problems used in the experiment were former midterm problems from the past ten semesters. All physics 212 students had access to these problems through the class website or the printed copy from the book store. Only some of those problems were selected to use in the practice exams. The recorded performance on those problems when they appeared in a midterm exam was used to select the problems. Midterm problems with an average correct score between 60% and 80% were selected. About 15 practice problems were assigned each week. These problems can be solved using the knowledge already covered in class a week before. About half of the practice exam problems were about recently covered topics. The other half of the practice exam problems were about previously covered topics. The practice problems were cumulative up to the previous week material. For example, the forth practice exam covered class material from the first week to the third week with roughly half of the questions focusing on the third week material. Therefore, the practice exam for each week could look very different from the previous week due to the new material covered in class. There were more than two hundred practice problems with video solutions distributed over fourteen weeks.

The video solutions were narrated slideshows. The solutions consisted of some explanation of the required concepts and a brief step-by-step walkthrough of the solution. Each solution was about 3 to 5 minutes long. Participants could access these solutions any time after the deadline.

Smartphysics is an online platform for distributing physics contents, homework, and quizzes. This is the same system used in the Physics 212 course for homework and prelectures. For this experiment, we used it to distribute weekly practice exams and solution videos. The participants usage logs and answers on the practice exams were collected.
3.2.3 Procedure

Participants in the exam-like condition were asked to come to the computer lab for an hour once a week. They received a paper copy of the practice exam to work on and were asked to enter their answers into Smartphysics. Their paper copies were photocopied at the end of the session. One hour later, their answers were graded and they could see their results along with the video solutions.

Participants in the online condition could access the weekly online practice exams and submit their answers any time two days before the deadline. After the deadline, their answers were graded and they could see their results along with the video solutions.

3.2.4 Results

Despite the compensation and the extra learning opportunity, a large portion of participants skipped many practice exam sessions. The participation rate was less than we anticipated, with average participation of 72% in the in-person group and 59% in the online group. The participation of the in-person group was significantly higher than the online group ($t(d.o.f. = 13) = 3.2, p < 0.01$).

![Participation rate for both experimental groups](image)

Figure 3.2: Participation rate for both experimental groups

In our tutor experiment, students in all three conditions spent about the same total time preparing for the exam. Additional learning activities or materials imposed on the students did
not increase the total time that they spent on studying for the exam. They substituted their usual learning time with the time that they spent on the additional experimental activities.

In the present experiment, with the same experimental materials available to both in-person group and online group, Figure 3.2 shows that different participation conditions motivated participants differently. The fact that the participation rate of the online group is smaller than the in-person group suggests that time constraints or the intrinsic work required by the practice exams are not the only factors for the low participation rate. Although the in-person group had to put in more effort to participate than the online group, its participation rate was higher.

Figure 3.3: Average scores on the three midterms and on the final of participants who completed the course

The average scores of the three groups on the three midterm exams and on the final exam are shown in Figure 3.3. There was no significant difference between the groups. The high dropout rate and fluctuations from relatively small statistics, limit the sensitivity of the experiment. Even though some students personally praised the treatment for helping them understand the course material, their exam scores did not support their opinion of their understanding.

One explanation for the null result, is that the poor participation rate is diluting the impact of the learning material. To investigate this hypothesis we examined only students that participated the same week as the actual exam, in addition to all students in the control group (Figure 3.4).
Across all three groups, there is no significant difference in scores within each of the four exams. Following the results of the previous two experiments, it would have been expected that the use of the learning tool would improve the learning gain over the control. However, these selective participants did not show the learning gain.

Figure 3.4: Average scores on the three midterms and on the final of students who participated the same week as the actual exam

3.3 Experiment 3: Discussion

Despite the time that students spent on these practice exams and video solutions, the results of this experiment did not show a learning gain from the distributed practice exam. This is in contrast to literature showing positive effects for distributed practice exams [107]. Indeed, given the possibility of a self-selection bias in the treatment groups (participants were more motivated students) if anything one would expect to observe a larger gain. However, this is not reflected in Figure 3.4. Regardless of whether this sample of participants is biased or not, no learning gain was observed.

Comparing the experimental setups in the paired problem experiment (chapter 1) and the tutor experiment (chapter 2) with this experiment, several differences were identified. First, the worked out solutions in this experiment are narrated slideshows instead of static worked out solutions. Second, since the content tested in the practice exams is accumulative, the next practice exams can
be very different from previous week. Lastly, due to the experimental setup that allowed participants to freely access the practice exams and video solutions after the deadline, the treatment can be less effective. These three important changes might be the reason why no learning gain was found.

The first possible explanation for the lack of learning gain is that these practice problems and video solutions are not good enough. This is hard to believe since all of the practice problems were selected from the same set of old exams. Furthermore, the problem selection in this experiment was much larger than the last two experiments. Students also had more time and opportunities to practice. Aside from the practice problems, the newly-created video solutions should convey information better than the static solutions used in the previous two experiments. All in all, the assumption that these practice problems and video solutions are not good is unlikely.

A second explanation could be the students’ motivation to participate which was influenced by the schedule of the experiment. The first two experiments happened only once, right before the midterm. The upcoming midterm motivated the students to use the practice exams and worked out solutions provided in the experiment. On the contrary, participants in the third experiment always had access to the weekly practice problems and solution videos after the targeted week. Even with unlimited time to use the practice exams, only half of them used the practice exams in the last few weeks of the semester. There was no penalty for skipping any or all treatment sessions except receiving less or none compensation. A freely-accessible exam preparation tool might not motivate students to use the tool seriously enough, on a regular basis.

The third assumption might be that these practice exam problems, without similar follow up problems, induced false confidence. Watching the video solutions without retesting with new problems might impair or hinder the learning gain found in previous experiments. By testing them right after watching the solution videos, students might have had a more accurate self-assessment on their understanding of the topic and their learning from the solution videos. This possibility is explored further in chapter 4.

The most interesting question from this experiment is why no learning gain can be observed from participants who received two hundred practice exam questions and worked out video solutions distributed across the semester. Despite the positive results from previous two experiments (where the treatments were optional as in this experiment), distributed practice exams in this experiment were shown to be ineffective. Students motivation is important, but motivating students to use the tool might be beyond our control in this experiment setting. Since the exam preparation tool is not part of the course, students may not take the tool seriously. In chapter 4, our last assumption (retesting after watching the video solution) will be tested to see if it is the cause for the null result.
Chapter 4

Maximizing Student Learning in a Practice Exam Context

Formative assessment conducted through hundreds of practice exam problems and the learning tool in the form of multimedia worked out solutions [108, 9, 87] might not be as effective as hypothesized. In the distributed practice experiment, students participated in a series of practice exam sessions over the course of the semester. After taking these weekly practice exams, the multimedia solutions were immediately available to the students. Despite hundreds of practice problems and multimedia video solutions, no learning gain was observed. The results from this experiment suggest that more practice problems and “better” feedback systems do not guarantee improved learning.

The results of the paired problem experiment suggested that students should immediately gain something from solving similar practice problems and watching worked-out solution videos. However, no actual gain on later exams was seen in the distributed exam experiment. Two possible explanations for this finding include 1) students did learn but were not able to retain the knowledge the period between the practice exam sessions and the midterm. 2) students did not learn as much because the tool in distributed practice experiment was freely accessible over a long period of time, affecting their motivation to use the tool (lack of pressure or immediacy).

One difference between the experiments with a positive result and the one with a null result is the re-test problems. The practice problems in the distributed practice experiment differed from week to week, following the concepts taught in class that week. Note that we re-tested a few problems in practice exams the following week and found that most students did not even recognize that these problems were repeated problems. This suggests that whatever participants learned from the solution videos, was forgotten within a week.

It is known that practice tests improve retention [52, 60]. However, it is not clear if testing after every solution video is important in helping students retain the newly learned material. Without re-testing problems, students might watch the video solutions passively. Furthermore, students normally gain confidence from reading over solutions or watching the explanation of solutions [109, 60]. This activity might make students more fluent with the solutions while not improving their understanding of how to solve problems on their own. If the increase in confidence exceeds their actual ability to solve problems, students might put less effort in acquiring deeper understanding. These assumptions of passive learning and over confidence might explain why, in experiment three, semester-long practice exams with video solutions (without re-test problems) did not show any learning gain.

One question that arose out of the distributed practice exam experiment (chapter 3) was: to
what extent does re-testing problems contribute to learning in a practice exam context? In the following experiment, I will investigate this hypothesis through the use of two experiment conditions: practice exams and solutions either with follow-up similar problems (paired problems) or without follow-up problems. Student confidence after viewing each solution will be measured along with the performance on zero-transfer (identical solution steps) problems and near-transfer problems on the posttest. The result of this experiment can provide some insights on follow up problems and practice exam system in general.

4.1 Experiment 4: Retesting Experiment

To test the effect of paired problems on learning gain, an experiment on practice exams was conducted with students in a calculus-based introductory electromagnetic course in spring 2014. The same six practice problems were used in the pretest, learning tool and posttest. Both groups received the same video solutions twice. The only difference between the two groups was that one group received additional paired problems right after viewing each video solution during the learning phase. This immediate re-testing should provide an opportunity for students to assess themselves, avoid false confidence obtained and pay more attention to the parts that they need.

This experiment was co-conducted for testing another experimental condition, delayed re-testing. The idea for delaying re-testing problems was to prevent students from solving the re-test problem using the solution in their short-term. The result on this experimental condition is included within this paper, but it was not our main interest.

4.1.1 Population

All participants were recruited from in a calculus-based introductory electromagnetic course at the University of Illinois in spring 2014. Only students whose scores were below the mean on the first midterm exam were invited. The experiment was advertised two weeks before the second midterm as an exam preparation tool and the experiment was conducted a few days before the second midterm exam. There were 101 participants in total and each was randomly assigned to one of the three groups. In the end, there were 33 participants in the solution-only group, 33 participants in the immediate-retest group and 36 participants in the delayed-retest group. As compensation for participating in the experiment, subjects were given access to additional practice problems and video solutions which they could use at home.

4.1.2 Material

The six practice problems used in the experiment were from past midterm exams (see Appendix C). They can be considered as difficult problems. The average performance by students who performed less than the mean on their first midterm (same condition for invited students in this experiment) on these six problems in the past is 45% (with no partial credit). Together these six questions covered only half of the topics in the second midterm. These six problems were used in the pre-test
and post-test in free-response form and in the re-test problems in multiple-choice form (for some experiment groups). The diagrams and text between the three practice sets were the same; only the variables’ values were changed. The solution videos for these practice problems emphasized the process needed to solve the problem and avoided showing the final answer formulas. Each video was about four minutes long. In the post-test, there were six additional problems which were follow-up questions to the initial six problems (near-transferred problems).

4.1.3 Procedure

The experiment setting is summarized in figure 4.1. At the beginning, participants were given a pretest consisting of the six problems. They had half an hour to solve these problems. They were told to work through the problems as if they were in a real exam. They were told to move on to the next problem if they got stuck in any problem. All participants work, except a copy of their final answers, were collected at the end of that half hour. No feedback on their performance was given.

Figure 4.1: Schematic of the experiment setting

After the pre-test, subjects watched the video instructions specific to their condition. The instruction video explained how to use the online system and also told them of the upcoming retest problems (in the case of two of the conditions). After the instructional video, participants watched the solution videos for the pretest problems. They could rewind, pause or skip the videos. After watching each video, they were asked to rate their understanding of the video solution. After they
moved on to the next video, they could not go back. All participants had the chance to see each video at two different parts of this portion of the experiment, depending on their test condition.

For participants in the immediate-retest group, after they rated their understanding of the solution video, they had to solve the same problem with different variable values. These questions were multiple choice and students were given feedback immediately. The video solutions were also given at this time as a second chance for these participants group.

The third condition is similar to the previous condition. The solution videos and their retest problems in the delayed-retest group were separated by another solution video or another retest problem. Participants took about seventy-five minutes to finish this part.

After the video solutions portion of the experiment, all participants were provided with new practice problems on other topics to work on for thirty minutes. During this time, cookies and beverages were provided while they worked. They could have a small break if they wanted. The purpose of this activity was to clear the images of the solution video from students’ short term memory.

During the last forty five minutes of the experiment, participants were given the posttest consisting of the same six problems with different variable values and another six follow up problems. Again, they were told to work through the problems as if they were in the real exam. They were told to move on to the next problem if they ever stuck at any problems. At the end of the experiment, all work was collected. No feedback to the posttest was given.

4.1.4 Results

The work on the pretest and posttest from all treatment groups was graded by two graduate students without knowledge of which treatment group the students participated in. The scores were based on the students problem solving processes and explanations. Any scores that the graders disagreed on were later discussed and resolved. The final correlation between the two graders was 0.92.

The average number of problems with full scores on the pretest was about 20% or about one out of six problems correct. This was not a surprise, since the six problems we chose were difficult problems and participants were low-performance students.
After the treatment and half an hour of distraction with another set of practice exam problems on unrelated topics, all participants took the posttest consisting of six zero-transfer problem and six near-transfer problems. All groups showed equally significant gain on both zero-transfer problems ($t_{\text{delayed re-testing}}(df = 5) = 8.9, p < 0.001, t_{\text{immediate re-testing}}(df = 5) = 10.9, p < 0.001, t_{\text{video only}}(df = 5) = 7.7, p < 0.001$) and near-transfer problems ($t_{\text{delayed re-testing}}(df = 5) = 6.1, p < 0.001, t_{\text{immediate re-testing}}(df = 5) = 5.7, p < 0.005, t_{\text{video only}}(df = 5) = 5.5, p < 0.005$). Figure 4.2 shows the learning gain from the treatment for all participants. On average participants performed significantly better on the posttest (both zero-transfer problems and near-transfer problems).
To normalize the learning gain of our participants, the performance on the pretest and posttest in the experiment were compared to the actual performance of these problems in the past (by students who scored less than the mean in the first midterm) (Figure 4.4). The fairest comparison might be with the near-transfer posttests. On average, participants did significantly better on these posttests compared to the actual midterm performance in the past. Although, the performance on the last four problems (2 zero-transfer and 2 near-transfer posttests) was low, this might be due to the time constraint of the experiment.
Due to the time limitation of the experiment, a large portion of participants did not have enough time to start working on the last 5 posttest problems (2 zero-transfer problems and 3 near-transfer problems) (Figure 4.5). For this reason, the scores on the last five posttest problems were lower than the first seven problems. When blank answer sheets were excluded, the performance on the last 5 posttests are comparable to the first 7 posttests (see Figure 4.6).
Figure 4.5: Percent of blank answer sheet for each problem on the posttest
Performance on each problem
(exclude no attempt)

Figure 4.6: Comparison between performance within the experiment (exclude no attempt) and actual midterm performance in the past

Further evidence of the learning gain from these practice exams and video solutions can be found in the students’ performance on the actual midterm exam (Figure 4.7). The participants showed significantly better performance than the non-participants \( t(df = 197) = 3.07, p < 0.05 \) with the effect size of 0.3. Note that the class average on the second midterm exam is about 10% lower than on the first midterm exam. However, the self-selection effect can also contribute to this result.
Figure 4.7: Performance on midterm exam before and after the treatment of the participants and invited non-participants
Further analysis showed that the treatment helped participants mostly on near-transfer exam problems (Figure 4.8). To see the effect of the treatment, we broke down the second midterm problems into 3 categories: near-transferred problems, topic-related problems, and circuit problems. (Near-transferred problems required about the same knowledge as or less than the practice problems in the experiment treatment. Topic-related problems required knowledge beyond the practice problems or required knowledge that was not focused on in the practice problems. All participants received the same practice problems on circuits as an experiment distractor.) Participants outperformed non-participants on these near-transferred problems ($t(df = 175) = 3.63, p < 0.0005$).

4.2 Experiment 4: Discussion

This experiment demonstrated that students, (including weaker ones) can learn how to solve hard physics problems by watching worked out solution videos. They can also retain the knowledge after watching different solution videos for an hour and working on other practice problems on different topics for half an hour. The learning gain is more than mere memorized formulas since participants can reproduce the problem solving processes. Furthermore, they were able to solve the transfer problems. The evidence of the learning gain here is firm and the video solutions in this experiment have shown to be effective.
This large learning gain is different from our results in the distributed practice experiment, where no learning gain could be found from any treatment group including the group taking practice exams in a classroom with a proctor. One possible difference between these two experiments was in the situation of the learning activity. When the learning materials (online solution videos) were provided to participants to use any time, the learning gains were not found. In contrast (as in the present study), when participants had to focus on the learning material during the experiment session, the learning gain was comparatively larger. Thus, encouraging students to pay more attention while using the learning materials (solution videos) might be more effective for learning.
Chapter 5

The missing piece in exam preparation tool: Motivation and goal during the learning activity

In all of the previous experiments, we focused on improving the exam preparation tool through manipulating the practice exams and the types of feedback provided. We created this tool to be a platform for formative assessment as well as an effective learning resource. Working on practice exams can provide students with a sense of their readiness for the real exam [70]. Any struggles on practice problems that students encounter can make them pay attention to their knowledge gaps and also prepare them to learn from subsequent learning resources [10]. Providing worked out solutions to students at the right moment can be an effective learning tool [47, 11, 92]. However, despite the utility of the exam preparation tool, the learning gain is limited by the individual effort made by the students [110] or caused by the learning task [111]. Comparing the experiments investigated in this thesis, learning from solution videos during the experiment is more effective than providing students with the videos to use freely at home. It is quite clear that different situations impose different kinds of motivation and learning goals onto students. Thus, in order for the exam preparation tool to help students effectively, students’ motivation and learning goals must be taken into account.

The purpose of practice exams is primarily to prepare for an examination. Thus, their utility as learning tools is limited to certain kinds of problems. What practice exams can do is optimize learning gain when over-confidence and false self-assessment are the cause of the problem. A few studies have shown a correlation between confidence and performance. One of my studies showed that low-performance students were over-confident in predicting their exam performance which agrees with the findings of Bol and Hacker [50] and Rebello [51]. In another experiment related to learning, Dunlosky and Rawson [112] found a negative correlation between over-confidence during the learning phase and score on final retention. These findings might suggest a vicious cycle: low performance leads to over-confidence, over-confidence leads to poor learning, and poor learning leads to low performance. One of the reasons for over-confidence is flawed self-assessment. Dunning, Heath and Suls [113] found that flawed self-assessment happens often because people do not take what they do not know into account. If this is the case for students over-confidence, practice exams as a formative assessment can help by telling students what they should, but do not, know and break them out of the vicious cycle.

Aside from telling students what they need to learn, practice exams can also prepare students to learn from the subsequent learning resources. Students can be guided to particular parts of the solution by the parts they struggle on in the practice problems. A similar idea was originally
introduced by Schwartz and Bransford [10]. They showed that they can improve students readiness to learn by pre-exposing students with contrasting cases to analyze. These contrasting cases help students generate differentiated knowledge structures, which in turn help them to understand the subsequent learning resources deeper, i.e. create a “time for telling.” Schwartz and Martin [11] later showed that this activity of analyzing contrasting cases is even more effective than tell-and-practice in preparing students for the subsequent learning resources. Unlike analyzing contrasting cases to prepare students to learn new materials, practice exams create a “time for telling” for students (during exam preparation) by guiding them to the parts of the solutions that they need to pay attention to.

Right after we create the right “time for telling,” it is important to provide effective learning resources, i.e. the worked out solution videos. Thus, solution feedback might be a key component for learning during this stage. Schwartz and Martin [11] showed that without subsequent learning resources, the “time for telling” from analyzing contrasting cases showed no effect on learning. The effect from working on practice exams might be similar. In fact, Epstein et al [61] showed that without feedback there is no improvement in performance. In paired problem experiment (chapter 1), we showed that students improved more with worked out solutions rather than showing them only the correct final answers. It is not a surprise since worked examples have shown to be an effective learning tool [47, 92].

Based on our results from the paired problem experiment, we expected that improving the quality of the solutions would lead to increased learning gains. However, in a follow-up experiment (Chapter 2), participants who used practice exams with one-on-one tutoring session, which was hypothesized to be the best kind of feedback, did not outperform other participants who used other feedback mechanisms. Thus, solution feedback might be limited in the degree to which it can improve learning.

Another surprise came from the experiment with the retesting condition (chapter 4), where participants worked on the practice exams and watched the worked out video solutions during the experiment session. The average learning gain from this experiment was much larger than any previous experiment. In the first experiment with the online exam preparation tool, the immediate gain was about 8%. In the second experiment with various feedback systems, the gain on the posttest was about 10% (20% after two sessions). In the distributed experiment, there was no measurement for learning gain since the practice exams changed from week to week. These three experiments provided worked-out solutions for students to study on their own. In the last experiment with the retesting condition, the worked-out solutions were provided during the experiment session and the gain on the posttest was about 60% on zero-transferred problems and 40% on near-transferred problems. This large learning gain was also seen in another experiment with learning activities which took place within the experiment session (mastery experiment). This suggested that the students motivation and goal during the use of the learning tool might be an important element for learning.

Motivation and goals from the current practice exam and video solution feedback promote
some learning. However, we believe the impact is limited because it lacks or does not optimize the students motivation especially during the learning period. From the tutor experiments (chapter 2) where no differences between experimental groups were found, we can conclude that those conditions might not motivate students to put more effort or attention while watching the solution video. In fact, by having tutoring sessions, students spend less time preparing for the exam on their own. This is counter intuitive to us as instructors but it might be just reasonable from students point of view.

Aguilar, Walton and Wieman [114] recently raised the concern of physics instructions from students perspectives. Regardless of the instructors good intentions, individual beliefs and perceptions on physics ability and on many other aspects of the classroom can significantly impact students motivation, goals, and their learning success. For example, students who believe that ability in physics is something they were born with are more likely to avoid challenges after encountering difficult problems [115]. Race and gender can also play a negative roll for learning and result in learning gaps. Despite how well-design the learning tool (in term of the contents) is from instructors perspective, how student perceives it is equally important.

Looking back to our exam preparation tool, it is unclear how students see the tool. We hope that they see it as an opportunity to learn from their mistakes with the help of the solution videos. However, it is possible that they use the tool only to check their readiness for the exam, i.e. they might have a specific performance goal in mind. If the later assumption is true, the practice exams are more likely to hurt low-performance students rather than helping them [116]. Lower-performance students are more likely to get a low score on practice exams, and thus, they perceive themselves as having low-ability. With a performance goal in mind, they may perceive their mistakes in the practice exam sessions as proof of their low-ability instead of a chance for improvement. This can also cause helplessness behavior [117]–avoiding difficult problems or just quitting,–which in turn prohibits them from learning with the exam preparation tool.

The results from our series of experiments on practice exams suggest that practice exams and video solution feedback are important, but not sufficient for student learning. Indeed, the large learning gain observed in the last experiment, suggests that the students effort caused by the learning activities can be improved; thus increase the learning gain. To do so, motivation and goal on the learning activities from student’s perspective should be taken into account. These are a few possible development: 1) Incorporate this learning activity in to the course with extra credit, 2) emphasis the learning from the video solution feedback by providing students similar practice problems after watching the video solution and 3) organize practice problems into set of skills for students to master. These modifications are intended to increase students effort and to show that their ability in physics can be improved.
References


Appendix A: Similar Paired Problems

A.1 Electric Field

1) Two negative charges, $Q$, are equal in magnitude. They are separated by a distance $d$.
$Q = -\sqrt{2} \mu C$ and $d = 10 \text{ cm}$.

Put a positive test charge at $x = y = z = 0$. Move the charge a small amount ($\delta$) along the positive $y$-axis and release it with zero initial velocity. After releasing the test charge, it

- doesn’t move at all.
- starts to move along the $y$-axis in the negative $y$ direction.
- starts to move towards the charge on the left.

Incorrect. The correct answer is “starts to move along the $y$-axis in the negative $y$ direction.”

2) Three charges are placed in the configuration shown. They are initially pinned in place. If the charges are suddenly unpinned and so free to move, in which direction does $Q_C$ initially accelerate?

- to the right
- to the left
- It does not move.

Incorrect. The correct answer is “to the left”
3) Three charges reside on an equilateral triangle with sides equal to $d$. All three charges are equal in magnitude and sign.

What is the magnitude of the force $F$ on one of the three charges $q$ due to the total electric field from the other two charges $q$ on the corners of the equilateral triangle?

- $F = 810 \text{ N}$
- $F = 177 \text{ N}$
- $F = 1403 \text{ N}$
- $F = 21.1 \text{ N}$
- $F = 135 \text{ N}$

Incorrect. The correct answer is "$F = 1403 \text{ N}$".

4) Consider four charges equally spaced in the $x$-$y$ plane around a circle of radius $r_1$, as shown below.

Calculate the magnitude of the force on test charge $Q$ due to the other three charges.

- $F = 16.2 \text{ N}$
- $F = 9.9 \text{ N}$
- $F = 18.2 \text{ N}$
- $F = 21.6 \text{ N}$
- $F = 37.8 \text{ N}$

Incorrect. The correct answer is "$F = 9.9 \text{ N}$".
5) A particle of mass $m$ and charge $q$ moves with velocity $v$ around charge $-Q$, which is fixed at the origin. An external uniform electric field $E_{\text{ext}}$ points in the $x$-direction and can be turned on and off. Initially $E_{\text{ext}} = 0$ and the particle moves in uniform circular motion around charge $-Q$. (Neglect all gravitational effects.)

For $E_{\text{ext}} = 0$, calculate the magnitude of the velocity of the particle.

- $|v| = 36.7 \text{ m/s}$
- $|v| = 26.0 \text{ m/s}$
- $|v| = 82.2 \text{ m/s}$

Incorrect. The correct answer is $|v| = 26.0 \text{ m/s}$.

6) In a classical model of the hydrogen atom, an electron (charge $-1.6 \times 10^{-19} \text{ C}$ and mass $9.11 \times 10^{-31} \text{ kg}$) orbits around a heavy, fixed proton (charge $+1.6 \times 10^{-19} \text{ C}$). In the atom's ground state, the radius of the electron's circular orbit is roughly $0.5 \times 10^{-10} \text{ m}$. What is the electron's speed $v$?

- $v = 1.59 \times 10^4 \text{ m/s}$
- $v = 5.25 \times 10^4 \text{ m/s}$
- $v = 7.43 \times 10^4 \text{ m/s}$
- $v = 2.25 \times 10^6 \text{ m/s}$
- $v = 3.18 \times 10^6 \text{ m/s}$

Incorrect. The correct answer is $v = 2.25 \times 10^6 \text{ m/s}$.

7) Eight charges are equally spaced on the $x-y$ plane around a circle of radius $r$. Find the direction of the electric field at the origin in terms of the angle, $\theta$, between the electric field vector and the positive $x$-axis.

- $\theta = 0^\circ$
- $\theta = 90^\circ$
- $\theta = 225^\circ$
- $\theta = 270^\circ$
- $\theta = 315^\circ$

Incorrect. The correct answer is $\theta = 270^\circ$.

8) Four charges are placed at the corners of a square of side $2a$. Let $a = 3 \text{ m}$ and $q = 2 \mu \text{C}$.

At point $B$, located at $(x,y) = (a,0)$, what is the direction of the electric field, $E_B$?

- $E_B$ is pointing towards positive $x$.
- $E_B$ is pointing towards negative $x$.
- $E_B$ is pointing towards positive $y$.
- $E_B$ is pointing towards negative $y$.
- $E_B$ is zero.

Incorrect. The correct answer is "$E_B$ is pointing towards positive $y$."
9) Three point charges are placed in the xy-plane at the positions shown in the figure below, and fixed in place. The point \( \text{M} \) is located at \((x,y) = (d,d)\), also in the xy-plane.

Calculate the \( x \)-component of the electric field at the point \( \text{M} \).

- \( E_x = -2.1 \times 10^7 \text{ N/C} \)
- \( E_x = -3.6 \times 10^8 \text{ N/C} \)
- \( E_x = +6.2 \times 10^4 \text{ N/C} \)
- \( E_x = -1.8 \times 10^7 \text{ N/C} \)
- \( E_x = -2.5 \times 10^7 \text{ N/C} \)

Incorrect. The correct answer is \( E_x = +6.2 \times 10^4 \text{ N/C} \).

10) A positive point charge \(+q\) is placed on the y-axis at \( y = +3 \text{ cm} \). A negative point charge \(-2q\) is placed at the origin.

Calculate the \( y \)-component of the electric field at the point \( \text{X} \) marked on the figure (i.e., on the x-axis at \( x = +3 \text{ cm} \)).

- \( E_y = +1.02 \times 10^6 \text{ N/C} \)
- \( E_y = -1.86 \times 10^6 \text{ N/C} \)
- \( E_y = -1.70 \times 10^7 \text{ N/C} \)
- \( E_y = -2.40 \times 10^7 \text{ N/C} \)
- \( E_y = -3.39 \times 10^7 \text{ N/C} \)

Incorrect. The correct answer is \( E_y = -1.70 \times 10^7 \text{ N/C} \).
11)
A negative charge \(-q\) is placed in two different positions inside a cylinder, as depicted at right.

Compare the magnitude of the total electric flux \(\Phi_{\text{cylinder}}\) through the cylinder in the two cases shown:

- \(\Phi_{\text{cylinder}}\) is larger in Case 1 than in Case 2.
- \(\Phi_{\text{cylinder}}\) is smaller in Case 1 than in Case 2.
- \(\Phi_{\text{cylinder}}\) is the same in both cases.

Correct.

12)
Compare the magnitude of the electric flux \(\Phi_{\text{bottom}}\) passing through the shaded bottom face of the cylinder in the two cases.

- \(\Phi_{\text{bottom}}\) is larger in Case 1 than in Case 2.
- \(\Phi_{\text{bottom}}\) is smaller in Case 1 than in Case 2.
- \(\Phi_{\text{bottom}}\) is the same in both cases.

Incorrect. The correct answer is "\(\Phi_{\text{bottom}}\) is larger in Case 1 than in Case 2."

In Figure A, a positive charge \(+Q\) is placed at the center of a cube. In Figure B, this same positive charge is moved upwards, closer to the upper surface of the cube.

Consider the total electric flux \(\Phi_{\text{cube}}\) passing outward through the cube:

- \(\Phi_{\text{cube}}\) in Figure A > \(\Phi_{\text{cube}}\) in Figure B.
- \(\Phi_{\text{cube}}\) in Figure A < \(\Phi_{\text{cube}}\) in Figure B.
- \(\Phi_{\text{cube}}\) is the same in both figures.

Correct.

14)
Consider the electric flux \(\Phi_{\text{bottom}}\) passing outward through the shaded bottom face of the cube:

- \(\Phi_{\text{bottom}}\) in Figure A > \(\Phi_{\text{bottom}}\) in Figure B.
- \(\Phi_{\text{bottom}}\) in Figure A < \(\Phi_{\text{bottom}}\) in Figure B.
- \(\Phi_{\text{bottom}}\) is the same in both figures.
A.2 Gauss’s Law

1)
A charged sphere of radius \( R_1 \) and total charge \( Q \) is placed at the center of a hollow spherical conducting shell (inner radius \( R_2 \), outer radius \( R_3 \)) which has a net charge \( Q_{\text{shell}} \).

\[
Q = 6 \times 10^9 \text{C}
\]
\[
Q_{\text{shell}} = 4 \times 10^9 \text{C}
\]
\[
R_1 = 0.1 \text{ cm}
\]
\[
R_2 = 1.0 \text{ cm}
\]
\[
R_3 = 2.0 \text{ cm}
\]
(Note: the figure is not drawn to scale)

Find the magnitude of the electric field, \( E \), outside of the spherical shell at a distance of \( R_4 = 5 \text{ cm} \) from its center.

\( E(R_4) = 0 \)

\( E(R_4) = 7200 \text{ N/C} \)

\( E(R_4) = 14400 \text{ N/C} \)

Incorrect. The correct answer is "\( E(R_4) = 7200 \text{ N/C} \)"

2)
Determine the resulting charge density \( \sigma \) on the \textit{inner} surface of the conducting spherical shell.

\( \sigma = 4.8 \text{ \mu C/cm}^2 \)

\( \sigma = -4.8 \text{ \mu C/cm}^2 \)

\( \sigma = 9.5 \text{ \mu C/cm}^2 \)

\( \sigma = -9.5 \text{ \mu C/cm}^2 \)

\( \sigma = 83 \text{ C/m}^2 \)

Incorrect. The correct answer is "\( \sigma = -4.8 \text{ \mu C/cm}^2 \)"

3)
A metal sphere of radius \( a \) is centered on the origin, and carries a total charge \( Q_{\text{sphere}} \). Surrounding this sphere is a spherical metal shell of inner radius \( b \) and outer radius \( c \). This shell is also centered on the origin, and carries a total charge \( Q_{\text{shell}} \).

Find the magnitude \( |E| \) of the electric field at a radius of \( 8 \text{ m} \) from the origin.

\( |E| = 2.8 \times 10^2 \text{ N/C} \)

\( |E| = 4.2 \times 10^2 \text{ N/C} \)

\( |E| = 7.0 \times 10^2 \text{ N/C} \)

\( |E| = 2.3 \times 10^2 \text{ N/C} \)

\( |E| = 3.4 \times 10^1 \text{ N/C} \)

Incorrect. The correct answer is "\( |E| = 4.2 \times 10^2 \text{ N/C} \)"

4)
What is the total charge \( Q_{\text{inner}} \) on the \textit{inner} surface of the spherical shell?

\( Q_{\text{inner}} = 0 \text{ \mu C} \)

\( Q_{\text{inner}} = -2 \text{ \mu C} \)

\( Q_{\text{inner}} = +5 \text{ \mu C} \)

Correct.
5) A solid insulating long cylinder, concentric with the y-axis, has a net charge \( q = 50 \, \mu \text{C} \) over a length \( L = 5 \, \text{m} \), and a uniform distribution of charge throughout its volume from \( r = 0 \) to \( r = r_0 = 2 \, \text{cm} \). The cylinder length \( L >> r_0 \) is large compared to \( r_0 \) thus will be treated as an infinitely long cylinder. Note: The diagram is not drawn to scale.

What is the magnitude of the electric field \( E \) at \( r = r_0 \)?

\[ \begin{align*}
\text{A} & \text{ B} & \text{ C} \\
\bigcirc E & = 4.5 \times 10^5 \, \text{N/C} \\
\bigcirc E & = 1.1 \times 10^6 \, \text{N/C} \\
\bigcirc E & = 9.0 \times 10^6 \, \text{N/C} \\
\end{align*} \]

Correct.

6) A long cylinder of radius \( a \) and length \( L \) is made of insulating material and is coaxial with the z-axis. Surrounding it, and also coaxial with the z-axis, is a cylindrical shell made of metal; it is of the same length \( L \), has inner radius \( b \) and outer radius \( c \). (Both cylinders are so long compared with their radii that they may be considered of infinite length.) Line-charge densities of \( \lambda_{\text{metal}} \) and \( \lambda_{\text{metal}} \) are placed on the inner cylinder and outer shell respectively. The values of all parameters are given in the figure.

Calculate the magnitude of the electric field \( E \) at the location \( x = 3 \, \text{m} \) on the positive x-axis.

\[ \begin{align*}
\text{A} & \text{ B} & \text{ C} \\
\bigcirc E & = 5.72 \, \text{N/C} \\
\bigcirc E & = 17.3 \, \text{N/C} \\
\bigcirc E & = 28.8 \, \text{N/C} \\
\bigcirc E & = 48.0 \, \text{N/C} \\
\bigcirc E & = 144 \, \text{N/C} \\
\end{align*} \]

Incorrect. The correct answer is \( |E| = 17.3 \, \text{N/C} \).

7) Calculate the magnitude of the surface charge density \( \sigma_b \) on the inner surface of the cylindrical metal shell (at \( r = b \)).

\[ \begin{align*}
\text{A} & \text{ B} & \text{ C} \\
\bigcirc |\sigma| & = 2.29 \times 10^{11} \, \text{C/m}^2 \\
\bigcirc |\sigma| & = 7.23 \times 10^{11} \, \text{C/m}^2 \\
\bigcirc |\sigma| & = 1.16 \times 10^{10} \, \text{C/m}^2 \\
\end{align*} \]

Correct.
5) A long metal (perfectly conducting) cylinder of radius $a$ and length $L$ is concentric with the $z$-axis. Surrounding it, and also concentric with the $z$-axis, is a cylindrical shell made of insulating material. It is of the same length $L$, has inner radius $b$ and outer radius $c$. (Both cylinders are so long compared with their radii that they may be considered of infinite length.) A total negative charge $Q_{\text{metal}}$ is placed on the inner metal cylinder, while a total positive charge $Q_{\text{outer}}$ is uniformly distributed over the volume of the outer insulating cylinder. The values of all parameters are given in the figure.

Calculate the magnitude of the electric field $E$ at a radius of 5 cm from the $z$-axis.

* $|E| = 2.19 \times 10^4 \text{ N/C}$
* $|E| = 7.20 \times 10^4 \text{ N/C}$
* $|E| = 5.40 \times 10^3 \text{ N/C}$
* $|E| = 1.08 \times 10^3 \text{ N/C}$
* $|E| = 1.80 \times 10^3 \text{ N/C}$

Incorrect. The correct answer is $|E| = 7.20 \times 10^4 \text{ N/C}$

9) Calculate the magnitude of the electric field $E$ at a radius of 7 cm from the $z$-axis.

* $|E| = 4.55 \times 10^4 \text{ N/C}$
* $|E| = 9.85 \times 10^4 \text{ N/C}$
* $|E| = 1.05 \times 10^5 \text{ N/C}$
* $|E| = 5.76 \times 10^5 \text{ N/C}$
* $|E| = 1.30 \times 10^7 \text{ N/C}$

10) What is the surface charge density $\sigma_{\text{metal}}$ on the outer surface of the metal cylinder?

* $\sigma_{\text{metal}} = -1.77 \times 10^3 \text{ C/m}^2$
* $\sigma_{\text{metal}} = -7.07 \times 10^2 \text{ C/m}^2$
* $\sigma_{\text{metal}} = -1.06 \times 10^5 \text{ C/m}^2$

Correct.
11) Consider three flat slabs of identical dimensions; their area \( A = 3 \text{ m}^2 \) is so large compared to their 2 cm thickness that they may be considered of infinite area for purposes of calculation. The figure below shows how they are positioned. Slabs \( a \) and \( b \) are made of glass (an excellent insulator), while the middle slab is made of copper (an excellent conductor). The copper slab is uncharged. However, the two glass slabs \( a \) and \( b \) are given total charges \( Q_a \) and \( Q_b \) respectively, distributed uniformly throughout their volumes. The values of all parameters are given in the figure.

Calculate the \( x \)-component of electric field at the position \( x = -3.5 \text{ cm} \) on the \( x \)-axis.

\[
\begin{align*}
E_x (x = -3.5 \text{ cm}) &= +18.4 \text{ kN/C} \\
E_x (x = -3.5 \text{ cm}) &= +47.1 \text{ N/C} \\
E_x (x = -3.5 \text{ cm}) &= -84.7 \text{ N/C} \\
E_x (x = -3.5 \text{ cm}) &= -33.0 \times 10^3 \text{ N/C} \\
E_x (x = -3.5 \text{ cm}) &= +179 \times 10^3 \text{ N/C}
\end{align*}
\]

Incorrect. The correct answer is \( E_x (x = -3.5 \text{ cm}) = -84.7 \text{ N/C} \)

12) Calculate the total charge \( Q_x \) which resides on the right-hand face of the copper slab (i.e. at \( x = 0 \)).

\[
\begin{align*}
Q_x &= -2.25 \text{ nC} \\
Q_a &= -7.00 \text{ nC} \\
Q_b &= -9.50 \text{ nC}
\end{align*}
\]

Incorrect. The correct answer is \( Q_x = -2.25 \text{ nC} \)

Two thin infinite planes of insulating material with charge densities \( \sigma_1 = 5 \text{ \mu C/m}^2 \) and \( \sigma_2 = -10 \text{ \mu C/m}^2 \) are directly adjacent to each other at the origin. At \( x = -12 \text{ cm} \) to \( -10 \text{ cm} \) an infinite conducting plane of thickness 2 cm has a net charge density \( \sigma_3 = +5 \text{ \mu C/m}^2 \). All planes are perpendicular to the \( x \)-axis.

What is the magnitude of the electric field \( E \) at \( x = -6 \text{ cm} \)?

\[
\begin{align*}
E &= 0.0 \text{ N/C} \\
E &= 5.7 \times 10^5 \text{ N/C} \\
E &= 1.8 \times 10^6 \text{ N/C} \\
E &= 2.9 \times 10^6 \text{ N/C} \\
E &= 8.5 \times 10^6 \text{ N/C}
\end{align*}
\]

Incorrect. The correct answer is \( E = 5.7 \times 10^5 \text{ N/C} \)

14) Calculate the charge density \( \sigma_{3L} \) on the left surface of the conducting plane.

\[
\begin{align*}
\sigma_{3L} &= 2 \text{ \mu C/m}^2 \\
\sigma_{3L} &= -2 \text{ \mu C/m}^2 \\
\sigma_{3L} &= 0 \text{ \mu C/m}^2 \\
\sigma_{3L} &= -5 \text{ \mu C/m}^2 \\
\sigma_{3L} &= 5 \text{ \mu C/m}^2
\end{align*}
\]

Incorrect. The correct answer is \( \sigma_{3L} = 0 \text{ \mu C/m}^2 \)
A.3 Electric Potential

1) Two point charges $q_1$ and $q_2$ are placed on the $y$-axis at positions $y = +a$ and $y = -a$ respectively, as shown in the figure at right. Take the reference potential to be zero at infinity ($V_0 = 0$).

Which one of the following statements is true about the $V_0$, the electric potential at the origin relative to the potential at infinity?

- $V_0 < 0$
- $V_0 = 0$
- $V_0 > 0$

Correct.

2) Consider two cylindrical, infinitely long charged rods with linear charge densities of $\pm \lambda$ where $\lambda > 0$ that lie perpendicular to the plane of the paper. Points A, B, C and D lie in a plane perpendicular to the charged rods. The locations of the points are indicated in the figure in terms of dimensions $a$ and $h$. (The dashed lines form a rectangular grid.)

Which choice gives the correct relationship between the potential at points B, C, and D?

- $V_B > V_C > V_D$
- $V_B = V_C = V_D$
- $V_B < V_C < V_D$

Incorrect. The correct answer is "$V_B > V_C > V_D"."
3) A metal sphere of radius $a$ is centered on the origin, and carries a total charge $Q_{sphere}$. Surrounding this sphere is a spherical metal shell of inner radius $b$ and outer radius $c$. This shell is also centered on the origin, and carries a total charge $Q_{shell}$.

Find the potential difference $V_a - V_c$ between the surface of the metal sphere ($r = a$) and the outer surface of the metal shell ($r = c$).

- $V_a - V_b = +20 \text{ kV}$
- $V_a - V_c = +15 \text{ kV}$
- $V_b - V_c = 0$
- $V_a - V_c = -15 \text{ kV}$
- $V_b - V_c = -20 \text{ kV}$

Incorrect. The correct answer is $V_a - V_c = -15 \text{ kV}$

4) A charged sphere of radius $R_1$ and total charge $Q$ is placed at the center of a hollow spherical conducting shell (inner radius $R_2$, outer radius $R_3$) which has a net charge $Q_{shell}$.

- $Q = 6 \times 10^{-9} \text{ C}$
- $Q_{shell} = -4 \times 10^{-9} \text{ C}$
- $R_1 = 0.1 \text{ cm}$
- $R_2 = 1.0 \text{ cm}$
- $R_3 = 2.0 \text{ cm}$

(Note: the figure is not drawn to scale)

Find the potential, $V$, inside the conducting spherical shell, at a distance of $R_3 = 1.8$ cm from its center. Assume that the potential at infinity is zero.

- $V(R_3) = -1000 \text{ V}$
- $V(R_2) = +1000 \text{ V}$
- $V(R_3) = +900 \text{ V}$
- $V(R_2) = +1500 \text{ V}$
- $V(R_3) = 0 \text{ V}$

Incorrect. The correct answer is $V(R_3) = +900 \text{ V}$
5)

Two point charges \( q_1 \) and \( q_2 \) are placed on the \( y \)-axis at positions \( y = +a \) and \( y = -a \) respectively, as shown in the figure at right. Take the reference potential to be zero at infinity (\( V_\infty = 0 \)). A third point charge \( q_3 \) is now brought in from infinity to the position \( x = +b \), as shown in the figure at right.

If the signs of all three charges were reversed (while keeping their magnitudes the same), the amount of work required to assemble the collection would stay the same.

\[ \text{True} \]

\[ \text{False} \]

Incorrect. The correct answer is "True".

6)

Three point charges are placed on the \( z \)-axis as illustrated in the figure. The charges \( Q \) are located above and below the origin at a distance \( a \) on the \( z \)-axis. The charge \( q \) is located at the origin.

If the charges \( Q \) and \( q \) are equal in magnitude and opposite in sign, then the potential energy stored in this charge collection (or the work necessary to assemble the charges from infinity) is

\[ \begin{align*}
\text{negative}. \\
\text{zero}. \\
\text{positive}.
\end{align*} \]

Incorrect. The correct answer is "negative."

7)

A positive point charge \( +q \) is placed on the \( y \)-axis at \( y = +a \). A negative point charge \( -2q \) is placed at the origin.

Calculate the magnitude of the work required to bring an additional point charge \( +q \) from infinity to the point \( X \). (The other two charges are held in position during this procedure.)

\[ \begin{align*}
|W| &= 2.98 \text{ J} \\
|W| &= 3.44 \text{ J} \\
|W| &= 5.76 \text{ J} \\
|W| &= 6.91 \text{ J} \\
|W| &= 8.94 \text{ J}
\end{align*} \]

Correct.

8)

Two point charges \( q_1 \) and \( q_2 \) are placed on the \( y \)-axis at positions \( y = +a \) and \( y = -a \) respectively, as shown in the figure at right. Take the reference potential to be zero at infinity (\( V_\infty = 0 \)).

A third point charge with charge \( q_3 = -12 \mu C \) placed at \( x = b \) on the \( x \)-axis. Calculate the total amount of work \( W \) required to assemble all three charges from infinity.

\[ \begin{align*}
W &= -27.0 \text{ J} \\
W &= -13.6 \text{ J} \\
W &= -11.2 \text{ J} \\
W &= -3.6 \text{ J} \\
W &= +15.5 \text{ J}
\end{align*} \]

Incorrect. The correct answer is \( W = -13.6 \) J.
9)

Three point charges are placed in the xy-plane at the positions shown in the figure below, and fixed in place. The point M is located at \((x,y) = (d,0)\), also in the xy-plane.

If a charge \(Q' = 2 \, \mu C\) having mass \(m = 0.2 \, \text{kg}\) is placed at the point M and then released (with the other three charges remaining fixed in place), what is the ultimate velocity of the charge \(Q'\) as it approaches infinity?

- \(v = 1.1 \, \text{m/s}\)
- \(v = 2.7 \, \text{m/s}\)
- \(v = 4.2 \, \text{m/s}\)
- \(v = 5.0 \, \text{m/s}\)
- \(v = 6.5 \, \text{m/s}\)

Incorrect. The correct answer is "\(v = 1.1 \, \text{m/s}\". 

10)

Four charges sit at the corners of a rectangle (height = 2h, width = 2w) as shown. A test particle having charge \(q = -3 \, \mu C\) and mass \(m = 7 \, \text{grams}\) is dropped from rest at \((x,y,z) = (0,0,0)\) and follows a straight-line trajectory down the z-axis through the origin.

Find the magnitude of the particle's velocity as it passes through the origin. Neglect all gravitational effects.

- \(|v| = 0 \, \text{m/s}\)
- \(|v| = 18.6 \, \text{m/s}\)
- \(|v| = 37.1 \, \text{m/s}\)
- \(|v| = 64.3 \, \text{m/s}\)
- \(|v| = 248.4 \, \text{m/s}\)

Incorrect. The correct answer is "\(|v| = 37.1 \, \text{m/s}\". 

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A.4 Capacitor

1) You are asked to build a capacitor from two circular metal plates of radius $3\text{ cm}$, separated by an air gap of width $d$. You must design your capacitor so that it will store a charge of $80\text{ pC}$ when connected to a $9\text{-volt}$ battery. What gap-spacing $d$ should you use? (You may assume that the plates are very large compared to the gap).

- $d = 0.52 \text{ mm}$
- $d = 0.90 \text{ mm}$
- $d = 1.2 \text{ mm}$
- $d = 2.8 \text{ mm}$
- $d = 3.5 \text{ mm}$

Incorrect. The correct answer is "$d = 2.8 \text{ mm}$".

2) A parallel plate capacitor, such as found in computer monitors, is made of two strips of aluminum foil with a plate area of $0.75\text{ m}^2$. The plates are separated by a layer of polyethylene $20\text{ \mu m}$ thick (dielectric constant $\kappa = 2.3$). Suppose a potential difference of $10\text{ V}$ is applied to this capacitor.

What is the magnitude of the charge $Q$ on each plate?

- $Q = 2.3 \text{ \mu C}$
- $Q = 0.76 \text{ \mu C}$
- $Q = 7.6 \text{ \mu C}$
- $Q = 23 \text{ \mu C}$
- $Q = 76 \text{ \mu C}$

Incorrect. The correct answer is "$Q = 7.6 \text{ \mu C}$".
3) Six identical capacitors of capacitance 5 pF are connected to a 9-volt battery as shown in the circuit diagram.

Calculate the total energy $U_{\text{total}}$ stored in all six capacitors.

- $U_{\text{total}} = 3.38 \times 10^{-11} \text{ J}$
- $U_{\text{total}} = 1.35 \times 10^{-10} \text{ J}$
- $U_{\text{total}} = 2.03 \times 10^{-10} \text{ J}$
- $U_{\text{total}} = 3.04 \times 10^{-10} \text{ J}$
- $U_{\text{total}} = 2.22 \times 10^{-9} \text{ J}$

Incorrect. The correct answer is $U_{\text{total}} = 1.35 \times 10^{-10} \text{ J}$.

4) What is the total energy stored, $W$, in the three capacitors in the system?

- $W = (1/3) \, C \, V^2$
- $W = (3/4) \, C \, V^2$
- $W = (1/5) \, C \, V^2$
- $W = (2/3) \, C \, V^2$
- $W = (5/4) \, C \, V^2$

Incorrect. The correct answer is $W = (1/3) \, C \, V^2$. 

Each $C = 5 \text{ pF}$
5) Three uncharged capacitors are connected to a battery as shown in the figure below. The battery voltage and capacitances are given on the right side of the circuit. Calculate the voltage drop, $V_3$, across the capacitor $C_3$.

- $V_3 = 2$ V
- $V_3 = 6$ V
- $V_3 = 9$ V

Incorrect. The correct answer is "$V_3 = 6$ V".

6) Four capacitors are connected to a battery in the manner shown at right. All capacitances are given in the table next to the figure, but the voltage $V$ delivered by the battery is unknown. What is known is that, when connected to the network, the capacitor $C_1$ acquires a charge of magnitude $|Q_1| = 15$ nC = $15 \times 10^{-9}$ C on each of its plates.

What is the charge $Q_4$ on capacitor $C_4$?

- $Q_4 = 7.5$ nC
- $Q_4 = 15$ nC
- $Q_4 = 30$ nC

Incorrect. The correct answer is "$Q_4 = 7.5$ nC".

A.5 Similarity Level

Paired problems that showed a large improvement. The algebraic answer to the two problems is identical, so it would be possible to plug the numbers from one problem into the answer for the other.

A charged sphere of radius \( R_1 \) and total charge \( Q \) is placed at the center of a hollow spherical conducting shell (inner radius \( R_2 \), outer radius \( R_3 \)) which has a net charge \( Q_{\text{shell}} \).

Find the magnitude of the electric field, \( E \), outside of the spherical shell at a distance of \( R_4 = 5 \text{ cm} \) from its center.

a. \( E(R_4) = 0 \)
b. \( E(R_4) = 7200 \text{ N/C} \)
c. \( E(R_4) = 14400 \text{ N/C} \)

\[
\begin{align*}
Q &= 6 \times 10^{-9} \text{ C} \\
Q_{\text{shell}} &= -4.0 \times 10^{-6} \text{ C} \\
R_1 &= 0.1 \text{ cm} \\
R_2 &= 1.0 \text{ cm} \\
R_3 &= 2.0 \text{ cm}
\end{align*}
\]
(Note: the figure is not drawn to scale)

A metal sphere of radius \( a \) is centered on the origin, and carries a total charge \( Q_{\text{sphere}} \). Surrounding this sphere is a spherical metal shell of inner radius \( b \) and outer radius \( c \). This shell is also centered on the origin, and carries a total charge \( Q_{\text{shell}} \).

\[
\begin{align*}
a &= 1.5 \text{ m} \\
b &= 3.0 \text{ m} \\
c &= 4.5 \text{ m} \\
Q_{\text{sphere}} &= -5 \text{ \mu C} \\
Q_{\text{shell}} &= +2 \text{ \mu C}
\end{align*}
\]

Find the magnitude \( |E| \) of the electric field at a radius of \( 8 \text{ m} \) from the origin.

a. \( |E| = 2.8 \times 10^2 \text{ N/C} \)
b. \( |E| = 4.2 \times 10^2 \text{ N/C} \)
c. \( |E| = 7.0 \times 10^2 \text{ N/C} \)
d. \( |E| = 2.3 \times 10^3 \text{ N/C} \)
e. \( |E| = 3.4 \times 10^3 \text{ N/C} \)

Figure A.1: Similar paired problems with identical solution steps
Solution

To find the electric field in this spherically symmetric problem we can use:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q_{\text{enc}}}{r^2}
\]

The enclosed charge includes both the sphere and shell:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{(4 \text{ nC} + 6 \text{ nC})}{(0.05 \text{ m})^2} = 7200 \text{ N/C}
\]

(B)

To find the electric field in this spherically symmetric problem we can use:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q_{\text{enc}}}{r^2}
\]

The enclosed charge includes both the sphere and shell:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{(-5 \mu\text{C} + 2 \mu\text{C})}{(8 \text{ m})^2} = 4.2 \times 10^3 \text{ N/C}
\]

(B)
Paired Problems that showed negative improvement. Perhaps because if one uses the algebraic solution for one problem, it does not give the correct answer for the second problem.

Consider three flat slabs of identical dimensions: their area \( A = 3 \, \text{m}^2 \) is so large compared to their 2 cm thickness that they may be considered of infinite area for purposes of calculation. The figure below shows how they are positioned. Slabs \( a \) and \( b \) are made of glass (an excellent insulator), while the middle slab is made of copper (an excellent conductor). The copper slab is uncharged. However, the two glass slabs \( a \) and \( b \) are given total charges \( Q_a \) and \( Q_b \) respectively, distributed uniformly throughout their volumes. The values of all parameters are given in the figure.

![Figure A.2: Similar paired problems with non-identical solution steps](image)

9. Calculate the total charge \( Q_R \) which resides on the right-hand face of the copper slab (i.e. at \( x = 0 \)).

   a. \( Q_R = -2.25 \, \text{nC} \)
   b. \( Q_R = -7.00 \, \text{nC} \)
   c. \( Q_R = -9.50 \, \text{nC} \)

Two thin infinite planes of insulating material with charge densities \( \sigma_1 = 5 \, \mu\text{C/m}^2 \) and \( \sigma_2 = -10 \, \mu\text{C/m}^2 \) are directly adjacent to each other at the origin. At \( x = -12 \, \text{cm} \) to \(-10 \, \text{cm} \) an infinite conducting plane of thickness 2 cm has a net charge density \( \sigma_3 = +5 \, \mu\text{C/m}^2 \). All planes are perpendicular to the x axis.

4. Calculate the charge density, \( \sigma_{3L} \), on the left surface of the conducting plane.

   a. \( \sigma_{3L} = 2 \, \mu\text{C/m}^2 \)
   b. \( \sigma_{3L} = -2 \, \mu\text{C/m}^2 \)
   c. \( \sigma_{3L} = 0 \, \mu\text{C/m}^2 \)
   d. \( \sigma_{3L} = -5 \, \mu\text{C/m}^2 \)
   e. \( \sigma_{3L} = 5 \, \mu\text{C/m}^2 \)

---

Figure A.2: Similar paired problems with non-identical solution steps
Solution

Draw a Gaussian surface around the right surface of the conductor with one side in the conductor:

The electric field in the conductor is zero, and the electric field to the right of the conductor is (same as above):

\[
E = \frac{\sigma_a}{2\varepsilon_0} - \frac{\sigma_b}{2\varepsilon_0} = \left(\frac{2.5 \text{ nC}}{3 \text{ m}^2}\right) - \left(\frac{7 \text{ nC}}{3 \text{ m}^2}\right) = -84.7 \text{ N/C}
\]

Finally, from Gauss’s Law we can find the surface charge density:

\[
\Phi_x = \frac{Q_{net}}{\varepsilon_0}
\]

\[
0 + E \cdot A = \frac{\sigma_x \cdot A}{\varepsilon_0}
\]

\[-84.7 \text{ N/C} = \frac{\sigma_x}{\varepsilon_0}
\]

\[
\sigma_x = 0.75 \text{ nC/m}^2
\]

And then the charge is calculated by multiplying by the area.

(A)
Draw a Gaussian surface around the left surface of the conductor with one side in the conductor:

\[ \sigma_3 \]

\[ \sigma_1 = 5 \, \mu C/m^2 \]
\[ \sigma_2 = -10 \, \mu C/m^2 \]
\[ \sigma_3 = 5 \, \mu C/m^2 \]

\[ x = -12 \text{ cm} \]
\[ x = -10 \text{ cm} \]

conductor

insulators

The electric field in the conductor is zero, and the electric field to the left of the conductor is:

\[ E = \frac{-10 \, \mu C/m^2 + 5 \, \mu C/m^2 + 5 \, \mu C/m^2}{2\varepsilon_0} = 0 \, N/C \]

Finally, from Gauss’s Law we can find the surface charge density:

\[ \Phi_x = \frac{Q_{ext}}{\varepsilon_0} \]
\[ 0 = \frac{\sigma_{3L}}{\varepsilon_0} \]
\[ \sigma_{3L} = 0 \, \mu C/m^2 \]

(C)
### A.6 Performance Table

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| Average    | 58.8% | 63.5% | 66.0% |

*Paired problems with exact final formula*
Appendix B: Tutor Experiment

B.1 Practice problems and solutions used in the experiment
Practice exam A

1. A 6 kg box is pulled across a horizontal floor by a rope. The tension in the rope is \( T = 5 \) N. Consider a time interval during which the velocity of the box increases from 0.8 m/s to 1.5 m/s. Suppose now that there is friction between the box and the floor. The box now has to be pulled for 2 m to increase its speed from 0.8 m/s to 1.5 m/s (still with tension \( T = 5 \)N). How much work is done on the box by friction?

\[ \begin{align*}
\text{a.} & \quad -5.17 \text{ J} \\
\text{b.} & \quad -4.83 \text{ J} \\
\text{c.} & \quad +4.83 \text{ J} \\
\text{d.} & \quad +5.17 \text{ J} \\
\text{e.} & \quad \text{None of the above or more information needed}
\end{align*} \]

With confidence: 100% 75% 50% 25% 0%

2. In lab 4 a cart of mass \( M = 700 \text{ g} \) is attached to a spring with force constant \( k = 3 \text{ N/m} \) and suspended on a frictionless incline plane that makes a 20° angle with respect to the horizontal. With the spring in the unstretched position, the cart is released from rest at \( x = 0 \). What is the maximum extension of the spring when the cart is at its lowest point on the incline?

\[ \begin{align*}
\text{a.} & \quad 0.8 \text{ m} \\
\text{b.} & \quad 1.6 \text{ m} \\
\text{c.} & \quad 2.3 \text{ m} \\
\text{d.} & \quad 4.6 \text{ m} \\
\text{e.} & \quad \text{None of the above or more information needed}
\end{align*} \]

With confidence: 100% 75% 50% 25% 0%
3. A massless spring of spring constant $k = 300 \text{ N/m}$ hangs vertically in the earth's gravitational field. Its relaxed length is 1.2 m. A 1 kg mass is attached to the spring. Suppose now that the mass is pulled down from the equilibrium position a distance of 0.1 m and released. What is the speed of the mass when it returns to the equilibrium position?

a. 0.29 m/s  
b. 1.73 m/s  
c. 2.45 m/s  
d. 8.66 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

4. A comet of mass $10^9 \text{ kg}$ is observed at a distance from the sun of $8 \times 10^{11} \text{ m}$ (mass of sun = $2 \times 10^{30} \text{ kg}$) at a speed of 17000 m/s. Assuming no forces on it other than the sun's gravity, how fast will it be going when it is a distance of $2.25 \times 10^{11} \text{ m}$ from the sun? (The universal gravitational constant is $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$)

a. 23000 m/s  
b. 23700 m/s  
c. 24300 m/s  
d. 33800 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
5. Two identical blocks initially have the same velocity $V$ at the bottom of two ramps. The first ramp inclined at a shallower angle ($\theta_1$) with respect to the horizontal than the second ramp ($\theta_2$). In both cases there is the same (non-zero) coefficient of kinetic friction between the blocks and the ramps. The maximum heights reached by the blocks are $h_1$ and $h_2$ respectively. Which statement is correct concerning the maximum heights reached by the blocks?

a) $h_2 = h_1$  
   b) $h_2 > h_1$  
   c) $h_2 < h_1$  
   d) More information needed

Please explain:

With confidence: 100%  75%  50%  25%  0%

6. A cart on a track enters the bottom of a frictionless loop-the-loop at with a velocity $V$. What is the maximum radius, $R$, the loop can have such that the cart does not fall off at the top?

a. $R = V^2/5g$  
   b. $R = V^2/4g$  
   c. $R = V^2/2g$  
   d. $R = V^2/g$  
   e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%
Identical constant forces push two blocks A and B over identical horizontal surfaces for identical periods of time. The masses are initially at rest. The mass of A is twice the mass of B.

7. Which block ends up with the biggest momentum?

   a) Block A       b) Block B

   c) Both blocks end up with the same momentum.   d) More information needed

   Please explain:

   ____________________________________________________________

   With confidence:  100%  75%  50%  25%  0%

8. Which block ends up with the biggest kinetic energy?

   a) Block A       b) Block B

   c) Both blocks end up with the same momentum.   d) More information needed

   Please explain:

   ____________________________________________________________

   With confidence:  100%  75%  50%  25%  0%
9. An egg of mass \( m = 0.15 \) kg with initial velocity \( v = 3 \) m/s are incident on a trampoline. The egg makes a complete elastic collision with the trampoline. If the average force on the egg during the collision with the trampoline is 15 N, what is the interaction time, \( t_f \)?
   a. 0.01 s  
   b. 0.03 s  
   c. 0.06 s  
   d. 0.20 s  
   e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

10. An artillery shell of mass 20 kg is fired from a rail car which is initially at rest on a horizontal frictionless track. The combined mass of the car and cannon is 2000 kg. As viewed by someone on the ground the shell moves with an initial speed of 300 m/s at an angle of 27° above the horizontal and the rail car recoils to the right. Relative to the ground, what is the speed of the rail car after the shell is fired?

   a. 2.67 m/s  
   b. 26.7 m/s  
   c. 267 m/s  
   d. 300 m/s  
   e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
11. A 1000 kg horse trailer with frictionless wheels is sitting in a level parking lot. The trailer is 4 m long, and its center of mass is at its center. Its passenger, a 500 kg horse, breaks free from its stall at one end of the trailer and walks to the center. How far does the trailer move relative to the ground? Treat the horse as a point particle. The mass of the trailer above does not include the 500 kg horse.

a. 2/3 m  
b. 4/3 m  
c. 1 m  
d. 2 m  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

12. A 12 kg block moves in the x-direction at 28 m/s, and a 24 kg block moves in the y-direction at 8.0 m/s. Find the magnitude of the velocity of their center of mass.

\[
\begin{align*}
12 \text{ kg} & \quad 28 \text{ m/s} \\
\quad & \quad \uparrow 8.0 \text{ m/s} \\
24 \text{ kg} & \\
\end{align*}
\]

a. 10.7 m/s  
b. 14.6 m/s  
c. 29.1 m/s  
d. 36.0 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
13. A block of mass $M_1$ slides on a horizontal, frictionless surface with a velocity of $V_1$. It collides with an ideal, massless spring which is attached to a block of mass $M_2$ which is initially at rest. The spring has a spring constant of $k = 20 \text{ N/m}$. At the instant the spring is maximally compressed, both masses will be traveling with a common velocity, $V$, of?

![Collision Diagram]

a. $V = \frac{M_1 M_2 V_1}{M_1 + M_2}$
b. $V = \frac{M_1 V_1}{M_1 + M_2}$
c. $V = \frac{(M_2 - M_1) V_1}{M_1 + M_2}$
d. $V = \frac{(M_1 + M_2) V_1}{M_2 - M_1}$
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

14. A ball of clay of mass $m = 0.5 \text{ kg}$ strikes a block of mass $M = 8.0 \text{ kg}$ which slides on a frictionless table as it compresses a spring with spring constant $k = 60 \text{ N/m}$. The initial speed of the ball of clay is $v = 12 \text{ m/s}$. The spring is initially at its relaxed length. What is the maximum compression of the spring, $d$, after the collision (the clay sticks to the block)?

![Collision Diagram]

a. 0.266 m  
Frictionless Table
b. 0.532 m  
c. 1.06 m  
d. 1.10 m  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
15. An explosion occurs which splits a bomb, initially at rest in outer space, into a chunk of mass $M_1$ and a chunk of mass $M_2$. The ratio of their kinetic energies after the explosion is given by $KE_1 / KE_2 = ?$

a. $1$

b. $\frac{M_1}{M_2}$

c. $\frac{M_2}{M_1}$

d. $\frac{M_1 - M_2}{M_1 + M_2}$

e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Solution A

1. $W_f = -5.17$ J

2. $x_{\text{max}} = 1.6$ m

3. $v_{\text{equilibrium}} = 1.73$ m/s

4. $v_f = 33800$ m/s

5. b) $h_2 > h_1$

6. $R = \frac{v^2}{5g}$

Work and Conservation of Energy

7. c) Both blocks end up with the same momentum

8. b) Block B ends up with the biggest kinetic energy

9. $t_f = 0.06$ s

Problems on Impulse

10. $v_{x, \text{car+cannon}} = 2.67$ m/s

11. $\Delta x = 2/3$ m

Center of mass & cons of momentum

12. $|v_{\text{cm}}| = 10.7$ m/s

13. $V = \frac{M_1v_1}{M_1+M_2}$

14. $d_{\text{max}} = 0.266$ m

15. $\frac{KE_1}{KE_2} = \frac{M_2}{M_1}$

Combination
1. Using Work-Kinetic Energy Theorem ($\Delta K = W_{\text{net}}$), where net work consists of work done by friction ($W_f$) and work done by the rope tension ($W_T$), we have:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_T + W_f$$

$$\frac{1}{2}(6 \text{ kg})(1.5 \frac{m}{s})^2 - \frac{1}{2}(6 \text{ kg})(0.8 \frac{m}{s})^2 = (5 \text{ N}) \cdot (2 \text{ m}) + W_f$$

$$W_f = -5.17 \text{ J}$$

2. In this problem, the energy is conserved. When the spring is maximally extended, the kinetic energy is zero and all the change in gravitational potential energy is stored in the spring.

$$KE_i + PE_{g,i} + PE_{spring,i} = KE_f + PE_{g,f} + PE_{spring,f}$$

$$KE_i = KE_f = 0$$

$$\Delta PE_g = -mg(sin 20 \cdot x)$$

$$\Delta PE_{spring} = \frac{1}{2}kx^2$$

Thus,

$$\Delta PE_g + \Delta PE_{spring} = 0$$

$$\frac{1}{2}kx^2 - mg(sin 20 \cdot x) = 0$$

$$x = \frac{2mg \sin 20}{k} = \frac{2 \cdot (0.7 \text{ kg}) \cdot (9.8 \frac{m}{s^2}) \cdot \sin 20}{(3 \text{ N/m})} = 1.56 \text{ m}$$

3. In this problem, the energy is conserved. The kinetic energy at the equilibrium position is equal to the work done by the spring force and the gravitational force from the releasing point to the equilibrium.

Since $mg = kx_{\text{released}}$ at the equilibrium, total horizontal force is equal to $kx_{\text{from equilibrium}}$.

$$KE_{\text{equilibrium}} = \int_{-0.1}^{0} kx_{\text{from equilibrium}} \, dx_{\text{from equilibrium}} = \frac{1}{2}k(0.1 \text{ m})^2$$

$$\frac{1}{2}mv_{\text{equilibrium}}^2 = \frac{1}{2}k(0.1 \text{ m})^2$$

$$v_{\text{equilibrium}} = 1.73 \text{ m/s}$$
4. In this problem, the energy is conserved.

\[ PE_i + KE_i = PE_f + KE_f \]

We can consider that \( M_{\text{sun}} \gg m_{\text{comet}} \) and that all change in kinetic energy is from the comet and that gravitational energy is zero at infinity.

\[ -\frac{GM_{\text{sun}}m_{\text{comet}}}{R_i} + \frac{1}{2}m_{\text{comet}}v_i^2 = -\frac{GM_{\text{sun}}m_{\text{comet}}}{R_f} + \frac{1}{2}m_{\text{comet}}v_f^2 \]

\[ v_f = 33800 \text{ m/s} \]

5. If the incline is frictionless, then both masses can move up the incline to the same height. However, since it is not frictionless and the friction on the incline depends on the angle, \( f = \mu mg \cos \theta \), both masses won’t move up to the same height.

From the problem situation,

\[ KE_i = f \cdot x + mgh = \mu mg \cos \theta \cdot \frac{h}{\sin \theta} + mgh \]

\[ h = \frac{KE_i}{(\mu mg \cot \theta + mg)} \]

Since \( 0 < \theta_1 < \theta_2 < 90^\circ \), so \( h_1 < h_2 \)

6. The energy is conserved in this problem. The condition that the cart does not fall off at the top is that the required centripetal force is equal to or greater than \( mg \).

\[ F_c = \frac{mv_{\text{top}}^2}{R} \geq mg \]

Use conservation of energy to calculate velocity at the top of the loop,

\[ \frac{1}{2}mV^2 = \frac{1}{2}mv_{\text{top}}^2 + mg(2R) \]

\[ v_{\text{top}} = \sqrt{V^2 - 2g(2R)} \]

To calculate the maximum radius,

\[ \frac{mv_{\text{top}}^2}{R} = mg \]

\[ \frac{m(V^2 - 2g(2R))}{R} = mg \]

\[ R = \frac{V^2}{5g} \]
7.8. In this problem two blocks were pushed by identical forces for identical periods of time. So, both blocks received the same impulse.

\[ \text{Impulse} = \bar{F}_{\text{average}} \Delta t = \Delta \bar{P} \]

Since both blocks are initially at rest, then they receive the same impulse (change of momentum), they end up with the same momentum.

For kinetic energy, since both blocks have the same momentum, the smaller block must have a bigger velocity. These two conditions imply that smaller block must also have a bigger kinetic energy.

\[ m_A = 2m_B \]
\[ m_A V_A = m_B V_B \]
\[ V_A = \frac{1}{2} V_B \]
\[ (m_A V_A)(V_A) = (m_B V_B) \left( \frac{1}{2} V_B \right) \]
\[ 2KE_A = KE_B \]

So block B has bigger kinetic energy than block A.

9. In the complete elastic collision, the egg bounces back with the magnitude of the final velocity equal to the magnitude of the initial velocity. The change of momentum is equal to the impulse.

\[ |\Delta \bar{P}| = |\text{Impulse}| = |\bar{F}_{\text{average}} \Delta t| \]
\[ |(0.15 \text{ kg}) \left( -3 \text{ m/s} \right) - (0.15 \text{ kg}) \left( 3 \text{ m/s} \right) | = (15 \text{ N}) t_f \]
\[ t_f = 0.06 \text{ s} \]

10. In this problem the horizontal momentum is conserved.

\[ \sum P_{x,i} = \sum P_{x,f} \]
\[ 0 = m_{\text{shell}} \cdot v_{x,\text{shell}} + M_{\text{car+cannon}} \cdot v_{x,\text{car+cannon}} \]
\[ 0 = (20 \text{ kg}) \left( -300 \cos 27^\circ \frac{m}{s} \right) + (2000 \text{ kg}) \cdot v_{x,\text{car+cannon}} \]
\[ v_{x,\text{car+cannon}} = 2.67 \text{ m/s} \]
11. In this problem, the system is initially still and there is no external horizontal force. Therefore, there is no change in momentum of the system and the center of mass stays at the same position.

Set the initial center position of the trailer to be zero (x = 0) and initial position of the horse to be at +2 m.

\[
\frac{(1000 \, kg)(0 \, m) + (500 \, kg)(2m)}{(1500 \, kg)} = \frac{(1000 \, kg)x_{final \, trailer \, center} + (500 \, kg)(x_{final \, trailer \, center})}{(1500 \, kg)}
\]

\[
x_{final \, trailer \, center} = \frac{2}{3} \, m
\]

12. Velocity of the center of mass can be calculated as follows.

\[
\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}
\]

\[
\vec{v}_{cm} = \frac{(12 \, kg)(28 \, i \, m/s) + (24 \, kg)(8 \, j \, m/s)}{(12 \, kg) + (24 \, kg)}
\]

\[
\vec{v}_{cm} = 9.3 \, i + 5.3 \, j \, m/s
\]

\[
|\vec{v}_{cm}| = 10.7 \, m/s
\]

13. Since there is no external force, the momentum is conserved at all time during the collision.

\[
\sum \vec{p}_i = \sum \vec{p}_{final \, compressed}
\]

Given that both masses travel with the same velocity when the spring is maximally compressed,

\[
M_1V_1 = (M_1 + M_2)V
\]

\[
V = \frac{M_1V_1}{M_1 + M_2}
\]
14. This is a two-step problem, inelastic collision first then energy transfer. During the collision, the momentum is conserved, but the energy is not conserved. During the energy transfer into the spring, the energy is conserved.

First process, inelastic collision (1→2)

\[ \sum P_1 = \sum P_2 \]

\[ mv = (m + M)V_2 \]

\[ V_2 = \frac{mv}{m + M} = \frac{(0.5 \, \text{kg}) \left(12 \frac{m}{s}\right)}{(0.5 + 8 \, \text{kg})} = 0.71 \frac{m}{s} \]

Second process, energy transfer (2→3)

\[ \sum E_2 = \sum E_3 \]

\[ \frac{1}{2} (m + M)V_2^2 = \frac{1}{2} k d_{max}^2 \]

\[ d_{max} = 0.266 \, m \]

15. Momentum is conserved in this problem.

\[ \sum P_i = \sum P_f \]

\[ 0 = M_1 v_1 + M_2 v_2 \]

\[ \left| \frac{v_1}{v_2} \right| = \frac{M_2}{M_1} \]

Thus,

\[ \frac{KE_1}{KE_2} = \frac{\frac{1}{2} M_1 v_1^2}{\frac{1}{2} M_2 v_2^2} = \frac{M_1 \left(\frac{M_2}{M_1} v_2\right)^2}{M_2 v_2^2} = \frac{M_2}{M_1} \]
Practice exam B

1. A box with mass \( M = 10 \text{ kg} \) is pulled across a floor by a rope. There is friction between the box and the floor. The tension in the rope is \( T = 25 \text{ N} \). Consider an interval during which the box moves a distance of \( \Delta x = 3 \text{ m} \) and its velocity increases from 2 m/s to 3 m/s. How much work \( (W_f) \) is done on the box by friction?

a. +50 J  
b. +25 J  
c. -25 J  
d. -50 J  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

2. A force \( F = 40 \text{ N} \) pushes a block of mass \( m \) up a frictionless incline as shown below. The block moves at a constant speed of 5 m/s until it contacts the relaxed spring which has a spring constant \( k = 200 \text{ N/m} \). At the point where the block touches the spring, the force is removed. By what amount, \( \Delta x \), is the spring compressed when the block comes to rest? [Hint: Relationship between mass (m) and force (F)]

a. 0.20 m  
b. 0.53 m  
c. 0.71 m  
d. 0.89 m  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
3. A young boy of mass $m = 25$ kg sits on a coiled spring that has been compressed to a length 0.4 m shorter than its uncompressed length and then held at this length. Suddenly the spring is released, and the boy flies vertically into the air. He reaches a maximum distance 0.5 m above his initial position. The spring is ideal and massless and we ignore the air friction. Find the spring constant of the spring in this problem.

\begin{align*}
\text{a.} & \quad 310 \text{ N/m} \\
\text{b.} & \quad 610 \text{ N/m} \\
\text{c.} & \quad 770 \text{ N/m} \\
\text{d.} & \quad 1530 \text{ N/m} \\
\text{e.} & \quad \text{None of the above or more information needed}
\end{align*}

With confidence: 100% 75% 50% 25% 0%

4. An asteroid with mass 250 kg is travelling directly toward the earth. When it is 25,000 km from the surface of the earth, it has a speed of 10 km/s. What is its speed when it hits the surface of the earth? (The mass and radius of the earth are $M_E = 5.98 \times 10^{24}$ kg and $R_E = 6,380$ km respectively, and Newton’s gravitational constant is $G = 6.67300 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$. You should ignore any effects due to air resistance and the rotation of the Earth)

\begin{align*}
\text{a.} & \quad 0.6 \text{ km/s} \\
\text{b.} & \quad 11.2 \text{ km/s} \\
\text{c.} & \quad 14.1 \text{ km/s} \\
\text{d.} & \quad 24.3 \text{ km/s} \\
\text{e.} & \quad \text{None of the above or more information needed}
\end{align*}

With confidence: 100% 75% 50% 25% 0%
5. A mass $m$ starts at the top of the ramp on the left with an initial speed $v_0 = 3 \text{ m/s}$ downward. It descends the frictionless ramp on the left, moves across the frictionless plane and then moves up the rough ramp on the right with $\mu_k = 0.3$. You may neglect the size of the mass. If the mass the block were doubled, final height would

a) Increase  
b) Stay the same  
c) Decrease  
d) More information needed 

Please explain ________________________________________________________________

With confidence: 100% 75% 50% 25% 0%

6. A small block having a mass 0.1 kg starts at rest at the top of a frictionless track a height 1.7 m above the horizontal floor. It slides down the track and then around a loop-the-loop having a diameter of 0.6 m. What is the normal force exerted by the track on the small block as it goes around the top of the loop?

a. 1.0 N  
b. 3.6 N  
c. 6.2 N  
d. 7.2 N  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Two carts with masses $m_A < m_B$, move across a horizontal frictionless surface under the influence of identical forces, $F$, which act for the same length of time, $\Delta t$. The carts start from rest.

7. What is the relation between the kinetic energies, $K_A$ and $K_B$ after the time $\Delta t$?
   a) $K_A < K_B$  
   b) $K_A = K_B$
   c) $K_A > K_B$  
   d) More information needed _____________

Please explain
______________________________________________________________

With confidence: 100% 75% 50% 25% 0%

8. What is the relation between the distances moved by the carts, $d_A$ and $d_B$, during the time the forces were applied?
   a) $d_A > d_B$  
   b) $d_A = d_B$
   c) $d_A < d_B$  
   d) More information needed _____________

Please explain
______________________________________________________________

With confidence: 100% 75% 50% 25% 0%
9. A puck of mass 0.2 kg collides with a wall. The puck will explode if hit with a force greater than 30 N. Before the collision the puck's velocity is 10 m/s. After the collision the puck's velocity is -5 m/s. Assume a constant force is applied to the puck during the collision. What is the shortest time interval over which this puck could have been hit such that it did not explode?

a. 0.07 s  
b. 0.1 s  
c. 0.3 s  
d. 0.5 s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

10. A spherical object of mass $M$, initially at rest, explodes into 3 pieces with masses $M/2$, $M/4$ and $M/4$. After the explosion, the pieces move in the $x$-$y$ plane. Suppose the final velocity of the large piece is $v = v \hat{j}$. In this case, after the explosion, the total momentum of the two smaller pieces, $P_{\text{small}}$, is?

a. $-Mv\hat{j}$  
b. $-\frac{Mv\hat{j}}{2}$  
c. $+\frac{Mv\hat{j}}{2}$  
d. $+Mv\hat{j}$  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
11. A fisherman has docked his boat on the shore as shown, but has not tied the boat to the shore. He is initially in the middle of the boat as shown when he starts to walk on the boat towards the shore. The boat is 20 meters long with its center of mass in the middle, and the mass of the boat is the same as the mass of the man. When he reaches the end of the boat (the end towards shore), how far is he from the shoreline? (Assume that there are no horizontal forces applied by the water on the boat.)

a. 0 m
b. 5 m
c. 10 m
d. 20 m
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

12. Two discs are free to move without friction on a horizontal table. The 0.4 kg disc is initially at the position \((x = 0, y = 1.0)\) m, moving with velocity \((v_x = 3.0, v_y = 0)\) m/s. The 0.6 kg disc is initially at \((x = 1.5, y = 0)\) m, moving with velocity \((v_x = 0, v_y = 2.0)\) m/s. The figure above displays the initial conditions for the two discs in the \(x\)-, \(y\)- coordinates. The magnitude of the initial velocity of the center of mass of the two-disc system is?

a. 1.70 m/s
b. 2.40 m/s
c. 3.87 m/s
d. 5.00 m/s
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
13. A 5 kg block slides on a horizontal, frictionless surface with a velocity of 2 m/s. It collides with an ideal, massless spring which is attached to a 15 kg block which is initially at rest. The spring has a spring constant of $k = 50$ N/m. At the instant the spring is maximally compressed, both masses will be traveling with a common velocity of:

a. 0.0 m/s  
b. 0.5 m/s  
c. 1.0 m/s  
d. 2.0 m/s  
e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%

14. A putty ball of mass $M = 0.5$ kg is traveling horizontally at $v = 2$ m/s. (Ignore the effects of gravity). It strikes a block of mass, which is adjacent to a relaxed ideal spring attached to an infinitely massive wall with a spring constant of $k = 4$ N/m. The putty ball sticks to the block. After the collision, the spring is compressed. What is the maximum compression of the spring?

a. 0.1 m  
b. 0.3 m  
c. 0.4 m  
d. 0.7 m  
e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%
15. A box of mass $m$ sliding to the right on a frictionless horizontal surface collides and
sticks to a second box of mass $3m$ which is initially at rest. What is the ratio of the initial
to the final kinetic energies of the system, that is, $\frac{KE_{initial}}{KE_{final}}$?

![Diagram of two boxes colliding](image)

a. 1
b. 3
c. 4
d. 16
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Solution B

1. $W_f = -50 \text{ J}$
2. $x_{\text{max}} = 0.71 \text{ m}$
3. $k = 1530 \text{ N/m}$
4. $v_f = 14 \text{ km/s}$  \hspace{1cm} \text{Work & Conservation of Energy}
5. b) Stay the same
6. $N = 6.2 \text{ N}$
7. c) $K_A > K_B$
8. a) $d_A > d_B$  \hspace{1cm} \text{Impulse}
9. $t_{\text{shortest}} = 0.1 \text{ s}$

10. $P_{\text{small}} = -\frac{Mv}{2} \hat{j}$

11. The fisherman is 5 m away from the shoreline.

12. $|v_{\text{cm}}| = 1.7 \text{ m/s}$  \hspace{1cm} \text{Center of mass & cons of momentum}
13. $V = 0.5 \text{ m/s}$

14. $d_{\text{max}} = 0.41 \text{ m}$

15. $\frac{KE_i}{KE_f} = 4$  \hspace{1cm} \text{Combination}
1. Using Work-Kinetic Energy Theorem ($\Delta K = W_{\text{net}}$), where net work consists of work done by friction ($W_f$) and work done by the rope tension ($W_T$), we have:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_T + W_f$$

$$\frac{1}{2}(10 \text{ kg}) \left( \frac{m}{s} \right)^2 - \frac{1}{2}(10 \text{ kg}) \left( \frac{m}{s} \right)^2 = (25 \text{ N}) \cdot (3 \text{ m}) + W_f$$

$$W_f = -50 J$$

2. In this problem, the energy is conserved. When the spring is maximally compressed, the kinetic energy is zero and all the change in kinetic energy is transferred into gravitational potential energy and in the spring.

$$KE_i + PE_{g,i} + PE_{spring,i} = KE_f + PE_{g,f} + PE_{spring,f}$$

$$KE_i = \frac{1}{2}mv_i^2 \quad \text{and} \quad KE_f = 0 \quad \text{and} \quad PE_{spring,i} = 0$$

$$\Delta PE_g = mg \sin 40 \cdot x$$

Given in the problem, we know that

$$mg \sin 40 = 40 N$$

Hence,

$$m = \frac{(40 N)}{g \sin 40} = 6.35 \text{ kg}$$

Thus,

$$KE_i = \Delta PE_g + PE_{spring,f}$$

$$\frac{1}{2}mv_i^2 = mg \sin 40 \cdot x + \frac{1}{2}kx^2$$

Solving the quadratic equation, we have

$$x = 0.71 \text{ m or } -1.11 \text{ m}$$

From the problem situation, the answer has to be positive and the answer is $x = 0.71 \text{ m}$.
3. In this problem, the energy is conserved. All potential energy in the spring gets transferred into the gravitational potential energy. That is

\[ \frac{1}{2}kx_{relax}^2 = mgh \]

\[ \frac{1}{2}k(0.4 \text{ m})^2 = (25 \text{ kg})(9.8 \text{ m/s}^{-2})(0.5 \text{ m}) \]

\[ k = 1530 \text{ N/m} \]

4. In this problem, the energy is conserved.

\[ PE_i + KE_i = PE_f + KE_f \]

We can consider that \( M_{\text{earth}} \gg m_{\text{asteroid}} \) and that all change in kinetic energy is from the asteroid and that gravitational energy is zero at infinity.

\[ -\frac{GM_{\text{earth}}m_{\text{asteroid}}}{R_{\text{earth}} + R_{\text{from surface}}} + \frac{1}{2}m_{\text{asteroid}}v_i^2 = -\frac{GM_{\text{earth}}m_{\text{asteroid}}}{R_{\text{earth}}} + \frac{1}{2}m_{\text{asteroid}}v_f^2 \]

\[ v_f = 14 \text{ km/s} \]

5. Let’s consider the work and energy of the mass initially and when it stops on the right incline.

\[ E_{\text{init}} = E_{\text{final}} + W_{\text{friction}} \]

\[ mgh_i + \frac{1}{2}mv_0^2 = mgh_f + \mu mg \cos \theta \cdot \frac{h_f}{\sin \theta} \]

\[ h_f = \frac{mgh_i + \frac{1}{2}mv_0^2}{mg + \mu mg \cot \theta} = \frac{gh_i + \frac{1}{2}v_0^2}{g + \mu g \cot \theta} \]

Since \( h_i \) is not a function of mass, if the mass of the block were doubled, the final height would stay the same.

6. The energy is conserved in this problem. Use conservation of energy to calculate velocity at the top of the loop,

\[ mgh_i = \frac{1}{2}mv_{\text{top}}^2 + mgh_{\text{top}} \]

\[ v_{\text{top}} = \sqrt{2g(h_i - h_{\text{top}})} \]

If the block can reach the top, we know that the centripetal force is equal to mg plus normal force.
7. & 8. In this problem two blocks were pushed by identical forces for identical periods of time. So, both blocks received the same impulse.

\[ \text{Impulse} = \vec{F}_{\text{average}} \Delta t = \Delta \vec{P} \]

Since both blocks are initially at rest, then they receive the same impulse (change of momentum), they end up with the same momentum.

For kinetic energy, since both blocks have the same momentum, the smaller block must have a bigger velocity. These two conditions imply that smaller block must also have a bigger kinetic energy.

\[
m_A < m_B \\
m_A V_A = m_B V_B \\
V_A > V_B \\
(m_A V_A)(V_A) > (m_B V_B)(V_B) \\
KE_A > KE_B
\]

So block A has bigger kinetic energy than block B.

Also, since block A is faster than block B, block A moves further than block B.

\[ d_A > d_B \]

9. The change of momentum is equal to the impulse, given that force cannot be greater than 15 N.

\[
\left| m \vec{v}_f - m \vec{v}_i \right| = | \Delta \vec{P} | = | \text{Impulse} | = | \vec{F}_{\text{average}} \Delta t | \\
\left| (0.2 \text{ kg}) \left( -5 \frac{m}{s} \right) - (0.2 \text{ kg}) \left( 10 \frac{m}{s} \right) \right| = (30 \text{ N}) t_{\text{shortest}}
\]

\[ t_{\text{shortest}} = 0.1 \text{ s} \]
10. In this problem the momentum is conserved.

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ 0 = \frac{M}{2} \cdot v_f + \vec{p}_{\text{small}} \]

\[ \vec{p}_{\text{small}} = -\frac{Mv}{2} \hat{j} \]

11. In this problem, the system is initially still and there is no external horizontal force. Therefore, there is no change in momentum of the system and the center of mass stays at the same position.

Set the shoreline to be zero \((x = 0)\).

\[ cm_{\text{initial}} = cm_{\text{final}} \]

\[ \frac{M(10 \text{ m}) + M(10 \text{ m})}{2M} = \frac{Mx_{\text{fisherman,final}} + M(x_{\text{fisherman,final}} + 10 \text{ m})}{2M} \]

\[ x_{\text{fisherman,final}} = 5 \text{ m} \]

12. Velocity of the center of mass can be calculated as follows.

\[ v_{cm} = \sum \frac{m_i \vec{v}_i}{\sum m_i} \]

\[ v_{cm} = \frac{(0.4 \text{ kg})(3\hat{i} \text{ m/s}) + (0.6 \text{ kg})(2\hat{j} \text{ m/s})}{(0.4 \text{ kg}) + (0.6 \text{ kg})} \]

\[ v_{cm} = 1.2\hat{i} + 1.2\hat{j} \text{ m/s} \]

\[ |v_{cm}| = 1.7 \text{ m/s} \]

13. Since there is no external force, the momentum is conserved at all time during the collision.

\[ \sum \vec{p}_i = \sum \vec{p}_{\text{max,compressed}} \]

Given that both masses travel with the same velocity when the spring is maximally compressed,

\[ M_1V_1 = (M_1 + M_2)V \]

\[ (5 \text{ kg})(2 \text{ m/s}) = (5 \text{ kg} + 15 \text{ kg})V \]

\[ V = 0.5 \text{ m/s} \]
14. This is a two-step problem, inelastic collision first then energy transfer. During the collision, the momentum is conserved, but the energy is not conserved. During the energy transfer into the spring, the energy is conserved.

First process, inelastic collision (1\(\rightarrow\)2)

\[
\sum P_1 = \sum P_2
\]

\[
Mv = (M + 2M)V_2
\]

\[
V_2 = \frac{Mv}{(M + 2M)} = \frac{(0.5 \text{ kg})(2 \text{ m/s})}{(0.5 + 1 \text{ kg})} = \frac{2m}{3 \text{ s}}
\]

Second process, energy transfer (2\(\rightarrow\)3)

\[
\sum E_2 = \sum E_3
\]

\[
\frac{1}{2}(M + 2M)V_2^2 = \frac{1}{2}kd_{max}^2
\]

\[
d_{max} = 0.41 \text{ m}
\]

15. Momentum is conserved in this problem.

\[
\sum P_i = \sum P_f
\]

\[
m_1v_{1,i} = (m_1 + m_2)v_f
\]

\[
v_f = \frac{m_1v_{1,i}}{(m_1 + m_2)}
\]

Thus,

\[
\frac{KE_i}{KE_f} = \frac{\frac{1}{2}m_1v_{1,i}^2}{\frac{1}{2}(m_1 + m_2)v_f^2} = \frac{m_1v_{1,i}^2}{(m_1 + m_2)(\frac{m_1v_{1,i}}{(m_1 + m_2)})^2} = \frac{m_1}{m_1 + m_2} = 4.00
\]
Practice exam C

1. A 6 kg box is pulled across a rough floor by a rope. There is friction between the box and the floor. The tension in the rope is \( T = 5 \) N. Consider a time interval during which the box moves a distance of 2 m, and its velocity decreases from 1.5 m/s to 0.8 m/s. Calculate the work done on the box by friction?
   a. 14.83 J
   b. 5.17 J
   c. -4.83 J
   d. -14.83 J
   e. None of the above or more information needed

   With confidence: 100% 75% 50% 25% 0%

2. A small, 1 kg mass is attached to a spring on a frictionless inclined plane, as shown. The spring has a spring constant of 10 N/m. The mass sits at the lowest point on the incline when the spring is in its equilibrium position (i.e. the point at which the spring force balances \( mg \sin \theta \)). The spring is compressed 0.25 m from its equilibrium position, and the mass is released. It slides down the incline, becomes detached from the spring at the lowest point on the incline, and then slides across a frictionless horizontal surface. You may neglect the size of the mass. After sliding for some distance, the mass encounters a horizontal spring, identical to the spring on the incline, which is attached to an immovable wall. The mass compresses the spring until it momentarily comes to rest. The spring is compressed an amount?
   a. 0.125 m
   b. 0.25 m
   c. 0.55 m
   d. 0.74 m
   e. None of the above or more information needed

   With confidence: 100% 75% 50% 25% 0%
3. A block of mass \( m_1 = 12.5 \) kg hangs from the ceiling on an ideal, massless spring with spring constant \( k = 300 \) N/m. With the block hanging on the spring, the total length of the spring is \( L = 3.5 \) m. When a second block with a mass of \( m_2 = 46 \) kg is tied to the first with a massless string, the spring stretches an additional \( h_0 = 1.5 \) m. The string is cut so that mass \( m_2 \) falls away. What is the maximum velocity of mass \( m_1 \)?

a. 4.96 m/s  
b. 7.35 m/s  
c. 10.39 m/s  
d. 17.49 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

4. A space capsule of mass \( m \) is launched from the surface of the earth with a speed \( V_E = 1.12 \times 10^4 \) m/s. The radius of the earth is \( R_E = 6.37 \times 10^6 \) m. The mass of the earth is \( M_E = 5.97 \times 10^{24} \) kg and Newton's gravitational constant is \( G = 6.67 \times 10^{-11} \) m \(^3\) kg \(^{-1}\) s \(^{-2}\). Neglect air resistance. Determine the speed of the capsule \( V_h \) at a distance from the center of the earth of \( h = 100 \) \( R_E \)

a. 790 m/s  
b. 1290 m/s  
c. 11100 m/s  
d. 15800 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
5. A block $m_1$ with mass 7 kg moves up an inclined plane with an initial velocity $v_0 = 4.7 \text{ m/s}$. The inclined plane is at an angle of $\theta = 45^\circ$ from the horizontal. The coefficient of kinetic friction between the block and the incline is 0.25. If the initial velocity is doubled, the speed when it has traveled $D = 1$ meter up the incline has also doubled?

a) True  
b) False  
c) Need more information ____________

Please explain: ________________________________

With confidence: 100% 75% 50% 25% 0%

6. A small block of mass $M = 300 \text{ kg}$ begins at height $h$ above the ground. It slides down a frictionless surface and encounters a loop of radius $R = 5 \text{ m}$, as shown in the figure. What is the minimum initial height of the block so that it does not derail as it goes through the loop?

a. 2.5 m  
b. 10.0 m  
c. 12.5 m  
d. 15.0 m  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Two blocks of mass $m_A$ and $m_B$ are placed side by side on a frictionless table. At time $t_0$ both blocks are at rest and a constant force of the same magnitude is applied to each of the blocks. Block A has a smaller mass than block B ($m_A < m_B$).

7. How do the momenta of the two blocks compare 5 seconds after $t_0$?
   a) $p_A < p_B$
   b) $p_A > p_B$
   c) $p_A = p_B$
   d) Need more information

   Please explain:
   
   With confidence: 100% 75% 50% 25% 0%

8. How do the kinetic energies of the two blocks compare 5 seconds after $t_0$?
   a) $K_A < K_B$
   b) $K_A > K_B$
   c) $K_A = K_B$
   d) Need more information

   Please explain:
   
   With confidence: 100% 75% 50% 25% 0%
9. A baseball of mass \( m = 0.14 \text{ kg} \) is pitched to a batter at a speed of \( v_i = 40 \text{ m/s} \) (90 mph). The batter hits the ball back on the same trajectory (assume that the ball moves only in the horizontal plane; ignore gravity); the ball leaves the bat with speed \( v_f = 70 \text{ m/s} \). If the average force of the bat on the ball is \( F = 9000 \text{ N} \), what is the length of time, \( \Delta t \), that the bat exerts this average force?

a. 0.00047 s  
b. 0.0011 s  
c. 0.0017 s  
d. 0.012 s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

10. Two 2 kg balls are traveling at 30° angles to the x-axis, as shown. Each ball has a speed of 3 m/s. They collide and stick together. What is the final velocity, \( V \), in m/s of the two-ball system after the collision (\( i \) and \( j \) are unit vectors in the \( x \) and \( y \) directions, respectively)?

\[ V = ? \]

a. 2.6 m/s  
b. 4.1 m/s  
c. 5.1 m/s  
d. 6.0 m/s  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
11. A uniform railroad flatcar of mass $M = 1500$ kg with length $L = 20$ m is at rest with a man of mass $m = 120$ kg standing on one end of the flatcar. The man begins running to the left. The flatcar slides without friction on the ground. When he has reached the other end of the flatcar, how far has the flatcar moved relative to the ground?

a. 0.00 m  
b. 1.48 m  
c. 1.60 m  
d. 20.00 m  
e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%

12. A puck of mass $m_1 = 3$ kg moves in the $x$-direction with velocity $\vec{v}_1 = 15\hat{i}$ m/s and another puck $m_2 = 4$ kg moves in the negative $y$-direction with velocity $\vec{v}_2 = -10\hat{j}$ m/s. What is the magnitude of the velocity of the center-of-mass of this system? (units are m/s)

a. 0.7 m/s  
b. 5.0 m/s  
c. 8.6 m/s  
d. 18.0 m/s  
e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%
13. A glider of mass \( m_1 \) slides on a frictionless track with initial velocity \( v_{1i} = 2 \text{ m/s} \). It hits a stationary glider of mass \( m_2 \). A spring attached to the first glider compresses and relaxes during the collision so that mechanical energy is conserved and the collision is elastic. The velocity of the center of mass of the system is \( V_{CM} = 0.3 \text{ m/s} \). At the instant the spring is maximally compressed, both masses will be traveling with a common velocity. Find the velocity of mass \( m_1 \) when the spring is maximally compressed?

a. 0.0 m/s  

b. 0.3 m/s  

c. 0.8 m/s  

d. 2.0 m/s  

e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%

14. Mass \( m_1 \) is initially moving with speed \( v_1 \) on a frictionless surface. Mass \( m_2 \) is initially at rest. Mass \( m_1 \) collides and sticks to mass \( m_2 \). They move together until they compress a spring of stiffness \( k \). What is the maximum compression of the spring in terms of \( m_1 , m_2 \) and \( v_1 \)?

a. \( \frac{m_1 v_1}{\sqrt{k(m_1+m_2)}} \)

b. \( v_1 \sqrt{\frac{m_1}{k}} \)

c. \( v_1 \sqrt{\frac{m_1 m_2}{k(m_1+m_2)}} \)

d. \( v_1 \sqrt{\frac{m_1-m_2}{k}} \)

e. None of the above or more information needed

With confidence: 100%  75%  50%  25%  0%
15. Two blocks with masses $m_1 = 1.2$ kg and $m_2 = 3.1$ kg on a frictionless surface collide head-on. The initial speed of block 1 is $v_{1,i} = 7$ m/s and block 2 is initially at rest. After the collision, both blocks did not stick together and the final speed of block 1 is $v_{1,f} = 1$ m/s. What is the ratio of the initial and final kinetic energies, $K_i/K_f$?

a. 1.00  
b. 2.26  
c. 2.58  
d. 3.28  
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Solution C

1. $W_f = -14.83 \text{ J}$
2. $x_{\text{max}} = 0.25 \text{ m}$
3. $v_{\text{equilibrium}} = 7.35 \text{ m/s}$
4. $V_h = 1290 \text{ m/s}$ \quad \textbf{Work & Conservation of Energy}
5. b) False
6. $h_i \geq \frac{5R}{2} = 12.5 \text{ m}$

7. c) $p_A = p_B$
8. b) $K_A > K_B$ \quad \textbf{Problems on Impulse}
9. $\Delta t = 0.0017 \text{ s}$

10. $|V| = 2.56 \text{ m/s}$
11. The flatcar moved 1.48 m to the right

12. $|v_{\text{cm}}| = 8.6 \text{ m/s}$ \quad \textbf{Center of mass & cons of momentum}
13. $V = 0.3 \text{ m/s}$

14. $d_{\text{max}} = \frac{m_2v_1}{\sqrt{k(m_1+m_2)}}$
15. $\frac{K_{E_i}}{K_{E_f}} = 3.28$ \quad \textbf{Combination}
1. Using Work-Kinetic Energy Theorem \((\Delta K = W_{\text{net}})\), where net work consists of work done by friction \((W_f)\) and work done by the rope tension \((W_T)\), we have:

\[
\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = W_T + W_f
\]

\[
\frac{1}{2} (6 \text{ kg}) \left( \frac{0.8}{s} \right)^2 - \frac{1}{2} (6 \text{ kg}) \left( \frac{1.5}{s} \right)^2 = (5 \text{ N}) \cdot (2 \text{ m}) + W_f
\]

\[W_f = -14.83 \text{ J}\]

2. In this problem, the energy is conserved. The gravitational potential energy and the energy stored in the spring on the slope gets transferred into the horizontal spring. Initial and final kinetic energy are zero.

\[PE_{g,i} + PE_{\text{spring, i}} = PE_{g,f} + PE_{\text{spring, f}}\]

Since the equilibrium between tangential forces \((mg \sin \theta = -k \Delta x_{\text{from relaxed}})\) is at the lowest point on the incline, the total tangential force acting on the mass is equal to \(k \Delta x_{\text{from lowest point}}\). This new \(k \Delta x_{\text{from lowest point}}\) represents both tangential gravitational force and spring force. Thus, the total energy initially stored is equal to the work done by the total tangential force on the incline, which is

\[(PE_{g,i} - PE_{g,f}) + PE_{\text{spring, i}} = \frac{1}{2} k \Delta x_{\text{from lowest point}}^2\]

So, when the horizontal spring maximally compressed

\[PE_{\text{spring, f}} = \frac{1}{2} k \Delta x_{\text{horizontal}}^2 = \frac{1}{2} k \Delta x_{\text{from lowest point}}^2\]

Thus,

\[\Delta x_{\text{horizontal}} = \Delta x_{\text{from lowest point}} = 0.25 \text{ m}\]

3. In this problem, the energy is conserved. Energy stored in the spring at 1.5 m below the equilibrium position gets maximally transferred into the kinetic energy at the equilibrium position. That is

\[
\frac{1}{2} m v_{\text{equilibrium}}^2 = \frac{1}{2} k h_0^2
\]

\[v_{\text{equilibrium}} = 7.35 \text{ m/s}\]
4. In this problem, the energy is conserved.

\[ PE_i + KE_i = PE_f + KE_f \]

We can consider that \( M_{\text{earth}} \gg m_{\text{capsule}} \) and that all change in kinetic energy is from the capsule and that gravitational energy is zero at infinity.

\[
- \frac{GM_{\text{earth}}m_{\text{capsule}}}{R_{\text{earth}}} + \frac{1}{2}m_{\text{capsule}}v_E^2 = - \frac{GM_{\text{earth}}m_{\text{capsule}}}{100 * R_{\text{earth}}} + \frac{1}{2}m_{\text{capsule}}v_h^2
\]

\[
- \frac{GM_{\text{earth}}}{R_{\text{earth}}} + \frac{1}{2}v_E^2 = - \frac{GM_{\text{earth}}}{100 * R_{\text{earth}}} + \frac{1}{2}v_h^2
\]

\[ v_h = 1290 \text{ m/s} \]

5. Let's consider the work and energy of the mass initially and when it reaches 1 m.

\[ E_{\text{ini}} = E_{\text{final}} + W_{\text{friction}} \]

\[
\frac{1}{2}mv_0^2 = mgD \sin \theta + \mu mg \cos \theta \cdot D + \frac{1}{2}mv_f^2
\]

\[ v_f = \sqrt{\frac{\frac{1}{2}mv_0^2 - mgD \sin \theta - \mu mg \cos \theta \cdot D}{\frac{1}{2}m}} = \sqrt{\frac{v_0^2 - 2gD \sin \theta - 2\mu g \cos \theta \cdot D}{\frac{1}{2}m}}
\]

Since \( v_f \) is not linearly dependent on initial velocity, \( v_0 \), then if the initial velocity is doubled, the speed when it has traveled \( D = 1 \) meter up the incline has NOT also doubled.

6. The energy is conserved in this problem. The condition that the block does not derail at the top means that the centripetal force at the top is greater than or equal to mg.

\[ mgh_i = \frac{1}{2}mv_{\text{top}}^2 + mgh_{\text{top}} \]

\[ v_{\text{top}} = \sqrt{2g(h_i - (2R))} \]

\[ F_c = \frac{mv_{\text{top}}^2}{R} = \frac{2mg(h_i - (2R))}{R} \]
By the problem condition,

\[ F_c \geq mg \]

\[ \frac{2mg(h_i - (2R))}{R} \geq mg \]

\[ h_i \geq \frac{5R}{2} = 12.5 \text{ m} \]

7. & 8. In this problem two blocks were pushed by identical forces for identical periods of time (5 seconds). So, both blocks received the same impulse.

\[ \text{Impulse} = F_{\text{average}} \Delta t = \Delta \vec{P} \]

Since both blocks are initially at rest, then they receive the same impulse (change of momentum), they end up with the same momentum.

For kinetic energy, since both blocks have the same momentum, the smaller block must have a bigger velocity. These two conditions imply that smaller block must also have a bigger kinetic energy.

\[ m_A < m_B \]

\[ m_A V_A = m_B V_B \]

\[ V_A > V_B \]

\[ \frac{1}{2} (m_A V_A)(V_A) > \frac{1}{2} (m_B V_B)(V_B) \]

\[ KE_A > KE_B \]

So block A has bigger kinetic energy than block B.

9. The change of momentum is equal to the impulse, given that the average force is 9000 N.

\[ |m \vec{v}_f - m \vec{v}_i| = |\Delta \vec{P}| = |\text{Impulse}| = |F_{\text{average}}\Delta t| \]

\[ \left| (0.14 \text{ kg}) \left( -70 \frac{m}{s} \right) - (0.14 \text{ kg}) \left( 40 \frac{m}{s} \right) \right| = (9000 \text{ N})\Delta t \]

\[ t_{\text{shortest}} = 0.0017 \text{ s} \]
10. In this problem the horizontal momentum is conserved.

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$M \left( \frac{3\sqrt{3}}{2} \hat{i} - \frac{3}{2} \hat{j} \right) + M \left( \frac{3\sqrt{3}}{2} \hat{i} + \frac{3}{2} \hat{j} \right) = 2MV$$

$$V = \frac{3\sqrt{3}}{2} \hat{i} = 2.56 \hat{i} \text{ m/s}$$

$$|V| = 2.56 \text{ m/s}$$

11. In this problem, the system is initially still and there is no external horizontal force. Therefore, there is no change in momentum of the system and the center of mass stays at the same position.

Set the initial left-end of the flatcar to be zero ($x = 0$).

$$cm_{initial} = cm_{final}$$

$$\left( \frac{(1500 \text{ kg})(10 \text{ m}) + (120 \text{ kg})(20 \text{ m})}{(1620 \text{ kg})} \right)$$

$$= \frac{(1500 \text{ kg})(x_{left \text{ end, final}} + 10 \text{ m}) + (120 \text{ kg})(x_{left \text{ end, final}})}{(1620 \text{ kg})}$$

$$x_{left \text{ end, final}} = 1.48 \text{ m}$$

The flatcar moved 1.48 m to the right.

12. Velocity of the center of mass can be calculated as follows.

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

$$v_{cm} = \frac{(3 \text{ kg})(15\hat{i} \text{ m/s}) + (4 \text{ kg})(-10\hat{j} \text{ m/s})}{(3 \text{ kg}) + (4 \text{ kg})}$$

$$v_{cm} = 6.4\hat{i} - 5.7\hat{j} \text{ m/s}$$

$$|v_{cm}| = 8.6 \text{ m/s}$$
13. Since there is no external force, the momentum is conserved at all time during the collision. Therefore the velocity of the center of mass stays the same throughout the collision. Given that both masses travel with the same velocity when the spring is maximally compressed, i.e. the velocity of both masses is equal to $V_{cm} = 0.3$ m/s when the spring is maximally compressed.

14. This is a two-step problem, inelastic collision first then energy transfer. During the collision, the momentum is conserved, but the energy is not conserved. During the energy transfer into the spring, the energy is conserved.

First process, inelastic collision (1$\rightarrow$2)

\[ \sum P_1 = \sum P_2 \]

\[ m_1 v_1 = (m_1 + m_2)V_2 \]

\[ V_2 = \frac{m_1 v_1}{(m_1 + m_2)} \]

Second process, energy transfer (2$\rightarrow$3)

\[ \sum E_2 = \sum E_3 \]

\[ \frac{1}{2}(m_1 + m_2)V_2^2 = \frac{1}{2}kd_{max}^2 \]

\[ d_{max} = \frac{m_1 v_1}{\sqrt{k(m_1 + m_2)}} \]

15. Momentum is conserved in this problem.

\[ \sum P_i = \sum P_f \]

\[ m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f} \]

\[ v_{2,f} = \frac{m_1(v_{1,i} - v_{1,f})}{m_2} \]

Thus,

\[ \frac{KE_i}{KE_f} = \frac{\frac{1}{2}m_1v_{1,i}^2}{\frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2} = \frac{m_1v_{1,i}^2}{m_1v_{1,f}^2 + m_2\left(\frac{m_1(v_{1,i} - v_{1,f})}{m_2}\right)^2} = \frac{v_{1,i}^2}{v_{1,f}^2 + \frac{m_1}{m_2}(v_{1,i} - v_{1,f})^2} = 3.28 \]
Customized Homework

1. A 7 kg block is pushed across a rough floor. The block has been pushed with constant 6-N force for 5 m and its speed decreases from 2 m/s to 0.5 m/s. How much work is done on the block by friction? Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. \(W_f = -43.1\) J

2. A 3 kg cart, initially at rest, is sliding down a 30\(^\circ\) frictionless slope. If the cart slides for 5 meter on the slope before it elastically hit with a relaxed spring on the same slope. The spring is fixed with the slope on the other end. The spring constant is 300 N/m. Find the maximum compressed length of the spring by the cart. Show and explain your thinking process in detail for better experience during your next session. \(d_{\text{max}} = 0.75\) m

3. A spring with relaxed length of 1 meter is fixed on the floor vertically. A 3 kg mass is placed on the spring and the spring gets compressed to 0.8 meter in length at equilibrium. If the mass gets compressed 0.2 meter more from equilibrium, calculate the maximum speed of the mass after it gets released. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. \(V_{\text{max}} = 1.4\) m/s

4. A comet of mass 5000 kg travelling at 2000 m/s is 30,000 km away from the surface of the earth. What is its speed when it is at 5000 km away from the surface of the earth? Ignore the air resistance. Mass and radius of the earth are \(M_E = 5.98 \times 10^{24}\) kg and \(R_E = 6380\) km and gravitational constant is \(G = 6.673 \times 10^{-11}\) m\(^3\) kg\(^{-1}\) s\(^{-2}\). Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. \(v_{5000\text{km}} = 7\) km/s
5. A mass, M, on a $45^\circ > \theta > 0$ slope is initially moving at a velocity, V, up the slope. Compare the maximum height of the mass in these conditions. Rank these conditions from lowest to highest maximum height. If possible, show your calculation or reasons of your ranking. If in any conditions, the given information is not enough, please explain why. (e=h<f<g=a=b=c<d, h<i<d)

   a. The mass is on a frictionless $15^\circ$ slope.
   b. The mass is on a frictionless $30^\circ$ slope.
   c. Double the mass, 2M, on a frictionless $15^\circ$ slope.
   d. The mass is initially moving at 2V on a frictionless $15^\circ$ slope.
   e. The mass is on a rough $15^\circ$ slope.
   f. The mass is on a rough $30^\circ$ slope.
   g. The slope is rough and is set to be perfectly vertical.
   h. Double the mass, 2M, on a rough $15^\circ$ slope.
   i. The mass is initially moving at 2V on a rough $15^\circ$ slope.

6. A mass, M, with initial velocity, $V_{\text{bottom}}$, at the bottom of the loop complete a vertical circular frictionless loop with radius R. Find the minimum initial velocity, $V_{\text{min}}$, at the bottom of the loop in terms of M, R and g such that the mass complete the loop without slipping. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($V_{\text{bottom, min}} = \sqrt{5gR}$)

7. & 8. Two masses, 2 kg and 5 kg, initially at rest on a frictionless floor are pushed with identical horizontal forces of 10 N for 7 seconds. Find their final momentum and kinetic energy. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($P_{2kg} = P_{5kg} = 70 \text{ kg.m/s, KE}_{2kg} = 1225 \text{ J, KE}_{5kg} = 490 \text{ J}$)

9. A tennis ball of mass 57 g initially moving at 10 m/s gets hit and leaves the racket at 12 m/s in the opposite direction. If the average force from the racket is 300 N, Find the time duration that the racket exerts this average force. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($dt = 0.0042 \text{ s}$)
10 A 65 kg hunter standing on a frictionless ice tried to shoot a bird on the tree. He aimed with the angle of 30° above the horizon. A bullet of mass 5 g was shot at initial velocity of 300 m/s. Find the initial velocity of the hunter after the shooting. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($v_{\text{hunter,after}} = 0.02 \text{ m/s}$)

11 An ant, 0.5 g, is walking on a 25 cm straw, 0.8 g, floating on the water. (Assume that there are no horizontal forces applied by the water on the straw) The ant starts walking from 3 cm to one end to 5 cm to another end. How much does the straw move? Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. (6.5 cm)

12 A 7 kg ball initially at $(x,y)=(1,4)$ is moving in $\hat{x}$-direction with velocity $\vec{v}_{7\text{kg}} = -5\hat{i} \text{ m/s}$. It elastically collides with another 4 kg ball initially at $(x,y)=(-1,0)$ moving in $+\hat{y}$-direction with velocity $\vec{v}_{4\text{kg}} = 10\hat{j} \text{ m/s}$. Find the velocity of the center of mass after the collision. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($v_{\text{cm}} = -\frac{35}{11}\hat{i} + \frac{40}{11}\hat{j}$)

13 A 4 kg block moving at 4 m/s on horizontal frictionless floor collides with an ideal spring attached to another 5 kg block. At the instant the spring is maximally compressed, both masses will be traveling with a common velocity. Find that common velocity. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($v_{\text{common}} = 1.78 \text{ m/s}$)

14 A 13 kg wooden block sitting on a frictionless floor got shot horizontally by a 7 g bullet with velocity of 300 m/s. The block with bullet stuck inside moved to push an ideal spring attached horizontally on a wall. The spring constant is $k = 20 \text{ N/m}$. Find the maximum distance the spring got compressed. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($d = 13 \text{ cm}$)

15 A slippery 7 kg wooden slope was at rest on a frictionless floor. At top of the slope, a 1 kg mass was placed and released from rest. Find the ratio between the kinetic energy of the slope and the kinetic energy of the small mass after they got separated. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session. ($KE_{1\text{kg}} : KE_{7\text{kg}} = 7:1$)
B.2 Similar problems on the practice exams and following midterm

Practice problem A

A 12 kg block moves in the x-direction at 28 m/s, and a 24 kg block moves in the y-direction at 8.0 m/s. Find the magnitude of the velocity of their center of mass.

a. 10.7 m/s  
   b. 14.6 m/s  
   c. 29.1 m/s  
   d. 36.0 m/s  
   e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

Practice problem B

Two discs are free to move without friction on a horizontal table. The 0.4 kg disc is initially at the position (x = 0, y = 1.0) m, moving with velocity (v_x = 3.0, v_y = 0) m/s. The 0.6 kg disc is initially at (x = 1.5, y = 0) m, moving with velocity (v_x = 0, v_y = 2.0) m/s. The figure above displays the initial conditions for the two discs in the x-, y-coordinates. The magnitude of the initial velocity of the center of mass of the two-disc system is?

a. 1.70 m/s  
   b. 2.40 m/s  
   c. 3.87 m/s  
   d. 5.00 m/s  
   e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%
Practice problem C

A puck of mass \( m_1 = 3 \text{ kg} \) moves in the \( x \)-direction with velocity \( \vec{v}_1 = 15\hat{i} \text{ m/s} \) and another puck \( m_2 = 4 \text{ kg} \) moves in the negative \( y \)-direction with velocity \( \vec{v}_2 = -10\hat{j} \text{ m/s} \). What is the magnitude of the velocity of the center-of-mass of this system? (units are m/s)

a. 0.7 m/s 
b. 5.0 m/s 
c. 8.6 m/s 
d. 18.0 m/s 
e. None of the above or more information needed

With confidence: 100% 75% 50% 25% 0%

Homework problem

A 7 kg ball initially at \((x,y) = (1,4)\) is moving in \(-\hat{x}\)-direction with velocity \( \vec{v}_{7\text{kg}} = -5\hat{i} \text{ m/s} \). It elastically collides with another 4 kg ball initially at \((x,y) = (-1,0)\) moving in \(+\hat{y}\)-direction with velocity \( \vec{v}_{4\text{kg}} = 10\hat{j} \text{ m/s} \). Find the velocity of the center of mass after the collision. Draw the situation in the problem. Show and explain your thinking process in detail for better experience during your next session.

\( \text{Answer: } \vec{v}_{\text{cm}} = -\frac{35}{11}\hat{i} + \frac{40}{11}\hat{j} \)
Related midterm problems

The next 2 questions refer to the following situation:
Three masses are positioned on the x-y plane as shown. All distances marked in the diagram are in meters.

![Diagram showing masses and coordinates.]

16. A force of $F = 12$ N in the y-direction is now applied to the 4 kg mass. What is the acceleration, $a$, of the center of mass?

a. $a = 3$ m/s$^2$ in the y-direction
b. $a = 0.75$ m/s$^2$ in the y-direction
c. $a = 1.71$ m/s$^2$ in the y-direction
d. $a = 3$ m/s$^2$ at an angle of 85.9° relative to the x-axis
e. $a = 0.75$ m/s$^2$ at an angle of 85.9° relative to the x-axis

17. For this question consider the original diagram above without a force applied to the system. A new mass having just the right value could be added at position $x = -1$ m, $y = -1$ m, so that the new center of mass of the four-mass system would now be at the origin.

a. True
b. False
Appendix C: Retesting Experiment

C.1 Practice problems used in the experiment
1) The current in the central conducting cylinder with radius $a = 2\, \text{cm}$ flows out of the page. The current in the outer conducting cylindrical shell (inner radius $b = 4\, \text{cm}$ and outer radius $c = 5\, \text{cm}$) flows into the page. The currents are distributed uniformly in each conductor with $J_1 = 2.25\, \text{A/cm}^2$ out of the page, and $J_2 = 1\, \text{A/cm}^2$ into the page.

What is the magnitude of the magnetic field at $r = 4.5\, \text{cm}$?

Please show your work. Please circle your final answer.
2) A tightly wound circular coil with radius \( a = 3 \text{ cm} \) and \( N = 150 \text{ turns} \) lies parallel to the x-y plane. The total resistance of the coil is 5\( \Omega \). A spatially uniform magnetic field extends over the entire region of the coil and points in the +z direction (out of the page). The magnitude of the field varies with time as shown below (the maximum field \( B_0 = 2 \text{ T} \) is obtained at time \( t_2 = 10 \text{ seconds} \)). Neglect the effect of any B fields that might be created in the coil.

What is \( I_1 \), the current induced at time \( t_1 = 5 \text{ seconds} \)? Positive current is defined to be clockwise. (1mA = 10\(^{-3}\)A)

Please show your work. Please circle your final answer.
3) A length of thin copper wire is used to wind a **square** coil that is \( L = 25 \text{ cm} \) on each side. The coil has a single turn. A 9-volt battery is used to drive a current in the wire, whose resistance is measured to be \( 0.02 \Omega \). The coil sits in a uniform magnetic field, \( B = 2.0 \text{T} \). Initially the coil is in the \( x \)-\( z \) plane, as shown below (left).

\[
\begin{align*}
L &= 25 \text{cm} \\
V &= 9 \text{V} \\
R_{\text{wire}} &= 0.02 \Omega \\
B &= 2.0 \text{T}
\end{align*}
\]

How much work is required to rotate the coil about the \( z \)-axis from its initial position where \( \theta_i = 90^\circ \) to its final orientation where \( \theta_f = 35^\circ \), as shown on the right?

Please show your work. Please circle your final answer.
4) A magnetic field, \( B = 0.8 \, \text{T} \), is directed out of the page in a region containing a rectangular up-side-down “U-wire” having width \( W = 0.5 \, \text{m} \), as shown. A resistor of mass \( m \) and resistance \( R = 6 \, \Omega \), which is free to slide without friction on the vertical rails, is released from rest and starts falling in the presence of the earth’s gravitational field, reaching a terminal speed \( v = 3.8 \, \text{m/s} \).

What is the mass of the resistor?
Please show your work. Please circle your final answer.
5) An electron of mass $m$ and charge $q$ is accelerated to the right (in the plane of the page) from rest through a potential difference $V$. The electron then enters middle of the left edge of a region, of height $h$ and width $w$, containing a uniform magnetic field.

How much time, $T$, does the electron spend in the magnetic field region?

Please show your work. Please circle your final answer.

\[ V = 1500 \text{V} \]
\[ B = 0.3 \text{ T out of page} \]
\[ q = -1.6 \times 10^{-19} \text{ C} \]
\[ m = 9.1 \times 10^{-31} \text{ kg} \]
\[ h = 130 \text{ m} \]
\[ w = 100 \text{ m} \]
6) An infinitely long wire carries a current $I_1 = 23$ A. Another wire in the shape of a rectangular loop with sides $a = 0.09$ m and $b = 0.20$ m carries a current $I_2 = 15$ A, and is placed near the infinitely long wire as shown in the figure below. (The side of the loop closest to the wire is a distance $x = 0.01$ m away from it.)

Find the $x$ component of the force on the side BC of the wire, due to the infinitely long wire. Positive is defined to be to the right.

Please show your work. Please circle your final answer.
The current in the central conducting cylinder with radius $a = 1\, \text{cm}$ flows out of the page. The current in the outer conducting cylindrical shell (inner radius $b = 2\, \text{cm}$ and outer radius $c = 3\, \text{cm}$) flows into the page. The currents are distributed uniformly in each conductor with $\mathbf{J}_1 = 2\, \text{A/cm}^2$ out of the page, and $\mathbf{J}_2 = 1.1\, \text{A/cm}^2$ into the page.

What is the magnitude of the magnetic field at $r = 2.4\, \text{cm}$?

Please show your work. Please circle your final answer.
Three situations are shown below. Situation I is the situation shown previously, with $J_1 = 2 \text{ A/cm}^2$ out of the page, and $J_2 = 1.1 \text{ A/cm}^2$ into the page, and the same dimensions for the two wires. Situation II has the same current densities, the radius $a$ is increased to 1.5cm. Situation III has the radius $a$ increased to 1.5cm, and also the current $J_1$ is reversed, so that $J_1 = 2 \text{ A/cm}^2$ into the page.

Compare the magnitudes of the magnetic fields at the blue dot for situations I and II

- $|B_I| > |B_{II}|$
- $|B_I| = |B_{II}|$
- $|B_I| < |B_{II}|$

Please explain your answer:

Compare the magnitudes of the magnetic fields at the blue dot for situations II and III

- $|B_{II}| > |B_{III}|$
- $|B_{II}| = |B_{III}|$
- $|B_{II}| < |B_{III}|$

Please explain your answer:
A tightly wound circular coil with radius $a = 5$ cm and $N = 225$ turns lies parallel to the $x$-$y$ plane. The total resistance of the coil is $3 \Omega$. A spatially uniform magnetic field extends over the entire region of the coil and points in the $+z$ direction (out of the page). The magnitude of the field varies with time as shown below (the maximum field $B_0 = 1.5$ T is obtained at time $t_2 = 18$ seconds). Neglect the effect of any $B$ fields that might be created in the coil.

What is $I_1$, the current induced at time $t_1 = 9$ seconds? Positive current is defined to be clockwise. ($1 \text{mA} = 10^{-3} \text{A}$)

Please show your work. Please circle your final answer.
The same coil (a = 5 cm, N = 225 turns, lies parallel to the x-y plane, and total resistance of the coil is 3Ω) now sits in a new field. A spatially uniform magnetic field extends over the entire region of the coil and points in the +z direction (out of the page). The magnitude of the field varies with time parabolically, as shown below, with k=8T/s². Neglect the effect of any B fields that might be created in the coil.

What is I₁₇, the current induced at time t = 17 seconds? Positive current is defined to be clockwise. Please show your work. Please circle your final answer.
A length of thin copper wire is used to wind a square coil that is \( L = 20 \) cm on each side. The coil has a single turn. A 9-volt battery is used to drive a current in the wire, whose resistance is measured to be 0.1 \( \Omega \). The coil sits in a uniform magnetic field, \( B = 5.0 T \). Initially the coil is in the x-z plane, as shown below (left).

\[
\begin{align*}
\text{L} &= 20 \text{cm} \\
V &= 9 \text{V} \\
R_{\text{wire}} &= 0.1 \Omega \\
B &= 5.0 \text{T}
\end{align*}
\]

How much work is required to rotate the coil about the z-axis from its initial position where \( \theta_i = 90^\circ \) to its final orientation where \( \theta_f = 41^\circ \), as shown on the right?

Please show your work. Please circle your final answer.
A length of thin copper wire is used to wind a square coil that is $L = 20 \text{ cm}$ on each side. The coil has a single turn. A 9-volt battery is used to drive a current in the wire, whose resistance is measured to be $0.1 \, \Omega$. The coil sits in a uniform magnetic field, $B^z = 5.0 \, T \, y^z$. Initially the coil is in the $x$-$z$ plane, as shown below (left).

$L = 20 \text{ cm}$
$V = 9 \text{ V}$
$R_{\text{wire}} = 0.1 \, \Omega$
$B = 5.0 \, T$

Instead of rotating around the $z$-axis like the last problem, now the coil is rotated around the $x$-axis, so that the plane of the plane of the loop makes a $13^\circ$ angle with the $z$-axis. How much work is required to rotate the coil from its initial to final position?

Please show your work. Please circle your final answer.
A magnetic field, $B = 1.8 \, \text{T}$, is directed out of the page in a region containing a rectangular up-side-down “U-wire” having width $W = 1.5 \, \text{m}$, as shown. A resistor of mass $m$ and resistance $R = 4 \, \Omega$, which is free to slide without friction on the vertical rails, is released from rest and starts falling in the presence of the earth’s gravitational field, reaching a terminal speed $v = 0.2 \, \text{m/s}$.

What is the mass of the resistor?

Please show your work. Please circle your final answer.
Now, the magnetic field is doubled in strength to 3.6 T and the direction is reversed, so that it now points into the page as shown below. A new, stronger, heavier resistor is attached to the U-wire, with resistance $R=113 \Omega$, weighing 65 g.

What is the new terminal velocity?

Please show your work. Please circle your final answer.
An electron of mass $m$ and charge $q$ is accelerated to the right (in the plane of the page) from rest through a potential difference $V$. The electron then enters middle of the left edge of a region, of height $h$ and width $w$, containing a uniform magnetic field.

$+V= 300\text{V}$

$B= 1.5 \times 10^{-3} \text{T out of page}$

$q= -1.6 \times 10^{-19} \text{C}$

$m= 9.1 \times 10^{-31} \text{kg}$

$h=2.4\text{m}$

$w=1.8\text{m}$

How much time, $T$, does the electron spend in the magnetic field region?

Please show your work. Please circle your final answer.
An electron of mass $m$ and charge $q$ is accelerated to the right (in the plane of the page) from rest through a potential difference $V$. The electron then enters middle of the left edge of a region, of height $h$ and width $w$, containing a uniform magnetic field.

What is the potential difference $+V$ such that the electron exits the B field region at $x=h/2, y=h/2$, heading in the $+y$ direction?

Please show your work. Please circle your final answer.

\[ +V = ? \]
\[ B = 1.5 \times 10^{-3} \text{T out of page} \]
\[ q = -1.6 \times 10^{-19} \text{ C} \]
\[ m = 9.1 \times 10^{-31} \text{ kg} \]
\[ h = 2.4 \text{ m} \]
\[ w = 1.8 \text{ m} \]
An infinitely long wire carries a current $I_1 = 10$ A. Another wire in the shape of a rectangular loop with sides $a = 0.08$ m and $b = 0.19$ m carries a current $I_2 = 5$ A, and is placed near the infinitely long wire as shown in the figure below. (The side of the loop closest to the wire is a distance $x = 0.02$ m away from it.)

Find the $x$ component of the force on the side BC of the wire, due to the infinitely long wire. Positive is defined to be to the right.

Please show your work. Please circle your final answer.
The current in the infinite straight wire is reversed and doubled, so now $I_{\text{wire}} = 20$ A, pointing down. The square loop is not changed.

Find the $x$ component of the total force on the entire square loop. Positive is defined to be to the right.

Please show your work. Please circle your final answer.