ROLL RELATED RETURN IN THE
S&P GSCI EXCESS RETURN INDEX

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THESIS

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ABSTRACT

Standard & Poor’s Goldman Sachs Commodity Index™ (S&P GSCI) is the largest tradable commodity index fund in the world with more than $80 billion in S&P GSCI-related investments. Investors have been led to believe that investing in the S&P GSCI during periods of rising commodity prices will be profitable. However, the return performance of the S&P GSCI rarely equals the price change of its underlying spot commodities. This thesis examines the historical excess returns of S&P GSCI futures holdings from 2007 to 2013, duplicating the official S&P GSCI trading methods, and finds that S&P GSCI excess returns differ from returns on corresponding investments in commodity futures due to the interaction between term structure effects and futures returns.
To All Faculty Members in the UIUC ACE Department
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1. INTRODUCTION

1.1 Background

Commodity index funds have grown in popularity since they were introduced in the early 1990s. According to the Commodity Futures Trading Commission (CFTC), in 2013 more than $260 billion was invested globally in long-only commodity index funds (CFTC 2013). Investors can gain exposure to returns from commodity indexes through over-the-counter (OTC) contracts with swap dealers, or they can buy investment funds whose returns are linked to a specific commodity index, including exchange-traded funds (ETFs) and exchange-traded notes (ETNs) (Sanders & Irwin 2012). Among the growing number of long-only commodity indexes, the largest one is the Standard & Poor’s Goldman Sachs Commodity Index™ (S&P GSCI). Approximately $80 billion is invested in the S&P GSCI and its related subindexes (Standard & Poor’s 2013a).

Unlike equity indexes such as the Standard & Poor’s 500 index (S&P 500) and the Dow Jones Industrial Average Index (DJI), which hold common stocks, the S&P GSCI contains only commodity futures contracts (Standard & Poor’s 2007, page 4). Futures contracts expire and cannot be held indefinitely. Prior to expiration, each sooner-to-expire futures contract must be sold and replaced with a later-to-expire futures contract in the same commodity. The S&P GSCI portfolio turnover rate is much higher than equity indexes although the S&P GSCI makes no changes to its underlying commodity categories.
Investors have been led to believe that investing in the S&P GSCI during periods of rising commodity prices will be profitable (Weinschenk 2013). However, there has been a divergence between the S&P GSCI cumulative excess returns and spot price changes since the S&P GSCI became tradable in the early 1990s. From 1991 to 2013, the annualized excess return of the S&P GSCI seldom outperformed the annualized spot price changes of its underlying commodities. For instance, the S&P GSCI ER Index, which measures the cumulative excess returns of the S&P GSCI, had a 5-year return of only 18.73% from 2009 to 2013, a period when the S&P GSCI Spot Index increased by more than 77% (Figure 1). Standard & Poor’s (2013b, page 8), which publishes the S&P GSCI, attributes the asymmetry of performance between the S&P GSCI Spot Index and the ER Index to term structure effect. Term structure effect indicates the price difference between outgoing futures and incoming futures at contract replacement, so adding the cumulative term structure effect to the returns of the S&P GSCI Spot Index provides a measure of return for the S&P GSCI ER Index. Burton and Karsh (2009) also give the same explanation.

1.2 Problems and Current Solutions

Using term structure effect to explain the divergence between returns on the S&P GSCI and its underlying spot commodities is a common practice. Since the price of each commodity’s outgoing and incoming futures on the contract replacement date will

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1 The S&P GSCI excess return is different from the excess return in the equity market. We will discuss this difference in the “S&P GSCI Section” below.
2 Spot price in the S&P GSCI means the price of the S&P GSCI futures holdings, not the cash price.
3 Term structure is classified as contango or backwardation. Under contango, the price of an outgoing futures is less than the price of an incoming futures at contract replacement. The opposite term structure is backwardation, where the price of an outgoing futures is higher than the price of an incoming futures at contract replacement (See Figure 2).
usually be different, some investors incorrectly believe term structure can directly influence commodity futures investment performance (Philips 2008 and Hard Assets Investor 2007). This belief suggests that investors’ gain or loss will be influenced by the price difference between two different futures contracts.

The concept of term structure effect was criticized by Burton and Karsh (2009). Although both outgoing and incoming futures contracts represent the same commodity, they suggest that the outgoing and incoming futures contracts should be treated as two different assets. Under this view, simply replacing an old asset with a new asset within a portfolio will not change the net value of this portfolio, and any difference in the price of two different assets does not result in an investable return. To demonstrate why term structure effect does not indicate an actual return, Burton and Karsh applied traditional equity trading methods by using the entire funds received from selling the outgoing futures contract to purchase the incoming futures contract, and conclude that term structure effect cannot directly produce returns to S&P GSCI investors.\(^4\)

When investors evaluate the return performance of an index portfolio, they normally assume that the entire funds received from the sale of Asset A can be used in the purchase of Asset B. However, the S&P GSCI uses a different reinvestment procedure. In the S&P GSCI, the same number of futures contracts must be sold and bought for each commodity within each year (CME 2005, Standard & Poor’s 2007, and

\(^4\) However, Burton and Karsh did not follow the correct S&P GSCI index procedures to re-measure index return performance after correcting for roll return. S&P GSCI sets the quantity weight (number of contracts) constant for each commodity for each year, so the same number of futures contracts are sold and bought for the outgoing and incoming contract months, respectively. This is different from an equity index, which can (and does) sell and buy different numbers of shares for the differently priced outgoing and incoming assets, respectively. I will discuss this issue in Chapter 3.
Goldman Sachs). Since the prices of the outgoing and incoming futures contracts are likely different, the calculation of returns can be quite complex, and the relationship between the outgoing and incoming futures prices becomes a factor in that calculation. If the entire funds from selling old futures are used to purchase new futures at the contract replacement date, then the quantity weight of the individual commodity futures will no longer be constant within a year.

Apart from Burton and Karsh (2009), others including Shemilt and Unsal (2004) and Johnson and Sharenow (2013) also explored the concept of term structure effect in commodity futures. Shemilt and Unsal show how term structure causes the change in spot prices to not equal the returns of commodity futures investment in the long run. Johnson and Sharenow stated that term structure effect can create returns to commodity investors if and only if prices are the same on both the outgoing futures and the incoming futures at contract replacement. However, neither Shemilt and Unsal nor Johnson and Sharenow provide a clear statement to measure futures returns at contract replacement using the S&P GSCI trading method.

1.3 Objectives

A number of researchers have examined the relationship between futures term structure and excess returns of long-only positions in individual commodity futures across time to explain excess returns of the S&P GSCI. Studies by Nash and Smyk (2003), Feldman and Till (2006), Erb and Harvey (2006), and Gorton, Hayashi, and Rouwenhorst (2007) support the strong relationship between term structure and futures excess returns, while Sanders and Irwin (2012) and Bessembinder et al. (2012) find
evidence of independency between term structure and futures returns in the long run. All of these studies are limited in scope to whether or not term structure will affect futures returns, and do not explore the calculation procedure of the S&P GSCI excess return or how variations in excess returns of individual futures contracts can affect excess returns on the S&P GSCI.

The objective of this thesis is to test if commodity futures term structure can explain the entire divergence between the S&P GSCI Spot Index and ER Index in the long run. In other words, I want to explore if contract replacement will bring side returns\(^5\), either positive or negative, to the S&P GSCI investment. If the divergence is significantly different from the cumulative term structure effect described by Standard & Poor’s, then side returns should be available within the S&P GSCI. My results include some information about the relationship between futures term structure and the S&P GSCI excess return, but this thesis will not test the efficiency of futures term structure in predicting the excess returns of individual commodity futures. For simplicity, the effects of transaction costs, index fund management fees, and taxes are excluded from this study since the S&P GSCI does not include these effects in its indexes.

### 1.4 Methodology and Data

This thesis examines the period from January 5, 2007 to January 7, 2014. There are two reasons to select this period. First, the S&P GSCI has maintained the same 24

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\(^5\) Side return in the S&P GSCI means the return that can only be received from the S&P GSCI investment, and cannot be received from individual futures investments by duplicating the S&P GSCI trading method.
commodities in the index during this period⁶. Tracking investment returns of the same 24 commodities across years will be more consistent than tracking returns of different commodities in each year. Second, during this period commodity prices climbed to record levels, collapsed following the U.S. financial crisis, and then recovered. These large fluctuations provide a range of market conditions to test the hypothesis of this thesis.

I examine in detail the return generation and measurement process within the S&P GSCI and use a daily flow-of-funds procedure during contract replacement periods. In these periods, actual daily profits and losses are calculated to investigate if any non-term structure factors will occur to explain the gap between index returns and commodity price changes. Simultaneously testing 24 commodities can be difficult, so I first examine four individual commodity futures — NYMEX crude oil, NYMEX natural gas, CBOT corn, and CME live cattle — and their impacts on the returns of the S&P GSCI Crude Oil Subindex, the S&P GSCI Natural Gas Subindex, the S&P GSCI Corn Subindex, and the S&P GSCI Live Cattle Subindex, respectively. There are several reasons for selecting these particular individual commodities. First, these commodities have the largest dollar weights in the energy sector, agricultural sector, and livestock sector respectively in the S&P GSCI index, and experience large price fluctuations each year. Second, together these four commodities account for more than 42% of the dollar weight in the S&P GSCI (Standard & Poor’s 2013b, page 44). Third, NYMEX crude oil and NYMEX natural gas futures undergo replacement each month, and the frequency in replacing these futures will be helpful to provide the maximum number of individual tests. Corn is a storable commodity with an annual production cycle, so the price difference will be largest at the

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⁶ The commodity categories in the S&P GSCI may be changed depending on the S&P GSCI policy. However, the S&P GSCI makes no changes for these commodity categories from 2007 to 2013.
transition from old crop contract to new crop contract, and a large price difference between two futures at replacement time will be useful to test whether term structure effects contribute to index price divergence. Live cattle is a non-storable commodity with a continuous production cycle, so it does not have a stable term structure. After these four individual commodities have been tested, I will extend this approach to the full 24-commodity index.

The 24 commodity futures in the S&P GSCI are traded at the Chicago Board of Trade (CBOT), Chicago Mercantile Exchange (CME), New York Mercantile Exchange (NYMEX), Kansas City Board of Trade (KCBT), Intercontinental Exchange (ICE), and London Metal Exchange (LME). The daily settlement prices for all except the LME commodity futures are collected from Barchart Advanced Commodity Service, the LME commodity futures prices are obtained from Thomson Reuters, and the daily settlement values for the S&P GSCI Spot Index, ER Index, and subindexes are provided by Standard & Poor’s.

1.5 Overview

This thesis has the following structure. Chapter 2 contains a review of relevant literature associated with commodity futures, futures term structure, roll return, the influences of term structure on commodity futures returns, and the divergence between the S&P GSCI cumulative investment returns and spot price changes across time. Chapter 3 contains a detailed overview of the S&P GSCI index structure, trading strategy, return components, and related investment products. Chapter 4 introduces the daily flow of funds model, empirical procedures, and data used in this study. Chapter 5 presents the
test results, derivation of equations, and discussion. Finally, Chapter 6 provides a summary and conclusion.
2. LITERATURE REVIEW

2.1 Introduction

This chapter reviews publications and investment reports that are relevant to this thesis. The first section describes the background of the commodity futures market. The second section discusses the theory of commodity futures term structure. The third section indicates the current debate on the influences of term structure on commodity futures returns. The roll return misconception and risk premium argument are included. The last section presents some fund managers’ explanations for the asymmetry between the S&P GSCI cumulative investment returns and spot price changes across time.

2.2 Background of Commodity Futures Market

Commodity futures are standardized forward contracts that can be used to represent a specific quantity and quality of cash commodity at a specific future date and price. The futures prices reflect the expected cash prices at the time when futures expire based on Hieronymus’s (1977) theory that the future price will generally converge to the cash price on the delivery date. For storable commodities\(^7\) within the same production cycle\(^8\), the futures prices incorporate the current cash prices, carrying charges, and

\(^7\) For non-storable commodities, “futures prices in different contract months are largely independent of one another, and are determined solely by expected supply and demand conditions in respective months.” In addition, non-storable commodities do not have carrying charges. (Peterson & Choi 2014)

\(^8\) Futures prices in different production cycles cannot be compared because there is no carrying charge for product which has not been produced.
Convenience yield\(^9\) (Kaldor 1939). Consequently, futures prices of storable commodities tend to be highly correlated with the cash prices of the underlying commodity. For investment purposes, individuals can invest directly in cash commodities, or they can take long positions on commodity futures contracts. It can be costly to store and maintain the quality of storable cash commodities, and there are numerous non-storable cash commodities in which it would be impossible to invest directly, so investors may instead select commodity futures rather than cash commodities. The S&P GSCI was designed to simplify the process of investing in a diversified portfolio of commodity futures (Standard & Poor’s 2007, page 4).

### 2.3 Theory behind Commodity Futures Term Structure

Term structure in commodity futures refers to the price differences among futures contracts with different expiration dates, and is commonly classified as either contango or backwardation. For contango, the price of a sooner-to-expire futures contract is less than the price of a later-to-expire futures contract. For backwardation, on the contrary, the price of a sooner-to-expire futures contract is greater than the price of a later-to-expire futures contract (See Figure 2).

For storable commodities, a contango term structure reflects the carrying charge of the underlying cash commodities when they are produced within the same production cycle (Working 1948, page 1). This carrying charge is composed of storage (warehouse) cost, insurance payment, and interest expense, and they are fully included in the price of a

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\(^9\) Conveniency yield is a negative component of commodity futures prices, which is in the opposite direction of the carrying charge and cash price. It indicates that market forecasts commodity cash price in later days will go lower.
futures contract. If there is no inventory shortage, then the price of an earlier expiration month futures contract will be less than the price of a later expiration month future contract\textsuperscript{10}. There is an economic upper bound for storable commodity futures in contango. If the price difference between two different commodity futures exceeds the carrying charge, then speculators will have a profitable arbitrage opportunity to go short in the deferred month futures and go long in the nearby month futures\textsuperscript{11}. The market will force the price difference to narrow (Peterson & Choi 2014).

A backwardation term structure reflects convenience yield (Kaldor 1939). When the quantity demanded of a storable commodity within the same production cycle is high relative to the quantity supplied, the price of an earlier expiration month futures contract can be higher than the price of its later expiration month futures contract. This price structure encourages immediate sales relative to deferred sales by providing a negative return to storage, and discourages immediate purchases relative to deferred purchase. Unlike contango, which has an economic upper bound, “there is no economic limit on the strength of backwardation imposed by low inventory. No one can move a quantity of commodity from later months to the present” to obtain a profitable arbitrage opportunity from backwardation (Geman & Smith 2012, page 6). When the convenience yield exceeds the carrying charge, the price of an earlier expiration month futures contract will be greater than the price of a later expiration month futures contract and result in a backwardation term structure.

\textsuperscript{10} If inventory shortage occurs, the price of an earlier expiration month future contract might be greater than the price of a later expiration month future, which is defined by backwardation.

\textsuperscript{11} This arbitrage opportunity will not be available for non-storable commodities because these commodities cannot be carried over time.
2.4 Backwardation Term Structure & Futures Return

The relationship between backwardation term structure and commodity futures returns was studied by Working. Working (1933) develops the theory of storage, and shows that inventory scarcity will raise the price of the nearby month futures more than the deferred month futures, and thus cause an inverse carrying charge. Working (1948, page 28) thinks the higher price of the nearby month futures only reflects information about current inventory scarcity, not the prediction of price change in later days. Extending from Working’s theory of storage, futures buyers will receive a profit from backwardation if the condition of inventory scarcity can be maintained from the current month to later months, and drive up the prices of deferred month futures. However, this condition may not always occur.

2.4a Roll Return

In the S&P GSCI, roll return is the price difference between the spot month future contract and a later expiration month future contract at the time when the index replaces its holdings. The amount of roll return is determined by commodity futures term structure. Investors treat roll return as positive for backwardation and negative for contango because backwardation allows investors to purchase futures contract with a cheaper price and vice versa. Roll return is a hypothetical return because rolling futures contracts is the same as replacing assets, which will not produce any gain and loss directly (Sanders & Irwin 2012). Johnson and Sharenow (2013) stated that roll return could be received by investors if and only if the price of the spot month futures could always stay the same.

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12 Inverse carrying charge is the same as backwardation. The term “backwardation” was not used by the public at the time when Working developed the theory of storage.
across time. However, some investors still believe roll return can directly influence commodity futures investment performance.

From 2007 to 2013, investors including Hard Assets Investor (2007), Philips (2008), and Van Eck Global (2013) rely on roll return to explain the S&P GSCI return performance. Hard Assets Investor treats positive roll return as a benefit to commodity futures returns because backwardation allows investors to replace an expensive futures contract with a cheaper futures contract. Philips attributes the high return performance of the S&P GSCI from 1983 to 1996 to positive roll return caused by a backwardated term structure, and predicts the return performance of the S&P GSCI in later years would depend on whether or not roll return could remain positive. Van Eck Global emphasizes that rolling futures contracts at contango will be harmful to the return performance of the S&P GSCI. It suggests buying commodity futures which go farther out on the forward commodity curve, and replace a small portion of contracts each day to mitigate the negative roll return caused by contango.

The claims for roll return made by Hard Assets Investor, Philips, and Van Eck Global are in conflict with the performance of the NYMEX crude oil market in 2007 and 2009. The NYMEX crude oil futures experienced contango term structure in both 2007 and 2009, but the S&P GSCI Crude Oil Subindex still received a net return of 51.97% in 2007 and 16.58% in 2009 (See Table 1). Roll return might be useful to forecast futures return performance in a specific time period, but it cannot determine the current gain and loss of a futures investment.
2.4b Term Structure as a Driver of Investment Returns

A number of researchers have examined the relationship between futures term structure and excess returns of long-only positions in individual commodity futures across time to explain excess returns of the S&P GSCI. Studies by Nash and Smyk (2003), Feldman and Till (2006), Erb and Harvey (2006), and Gorton, Hayashi, and Rouwenhorst (2007) support the strong relationship between term structure and futures excess returns, while Sanders and Irwin (2012) and Bessembinder et al. (2012) find evidence of independency between term structure and futures returns in the long run. All of these studies are limited in scope to whether or not term structure will affect futures returns, and do not explore the calculation procedure of the S&P GSCI excess return or how variations in excess returns of individual futures contracts can affect excess returns on the S&P GSCI.

Nash and Smyk (2003) analyze each of the individual commodity futures that are included in the GSCI portfolio, from 1983 to 2002. They find a commodity futures contract that stays at backwardation for a longer time period than other commodities, will generally receive a higher return performance than futures contracts of other commodities (See Figure 3). Erb and Harvey (2006, page 93) track the excess return of the 26-commodity GSCI portfolio from 1992 to 2004. They find the annualized excess return of the 26-commodity GSCI portfolio under backwardation was 11.25%, but -5.01% under contango (See Figure 4). Feldman and Till (2006, page 12) also find a strong relationship

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13 The S&P GSCI was called GSCI before 2007 when Goldman Sachs transferred the index to Standard & Poor’s.
14 Excess return in the S&P GSCI means pure return from commodity futures investment, not the return above the T-bill rate. It will be introduced in chapter 3.
between backwardation and futures returns for soybeans, corn, and wheat from 1950 to 2000. Gorton, Hayashi, and Rouwenhorst (2007, page 57) develop a model based on Working’s (1948) theory of storage to analyze the relationship between commodity inventory levels and commodity futures return performance from 1969 to 2006. They find the expected returns of general commodity futures increase when physical inventory decreases. Meanwhile, the magnitude of backwardation in storable commodities will go up at an increasing rate when the inventory level goes down. Gorton, Hayashi, and Rouwenhorst’s findings imply that the relationship between futures returns and term structure is probably present for storable commodities.

In contrast to the research results above, Sanders and Irwin (2012) conclude that the return performance of a long-only commodity portfolio is not directly determined by market term structure. They investigate the poor return performance of the S&P GSCI from 2006 to 2011. In addition, they examine 20 individual commodity futures returns from 1951 to 2010, and find the average return performance of long-only individual commodity futures in the long run is not statistically different from zero regardless of what term structure these futures experience. Bessembinder et al. (2012, page 36) investigate the return performance of NYMEX crude oil futures from 1990 to 2006 at a time when there is a contango term structure. They find the price difference between two futures contracts at the time when contract replacement occurs does not indicate futures returns. The contango term structure of NYMEX crude oil reflects information about carrying charges not investment loss, and the return performance of NYMEX crude oil futures depends on the price change of the underlying cash crude oil.
Existing literature is limited to whether or not term structure or roll return will affect futures returns. Empirical analysis to date does not explore the calculation procedure of the S&P GSCI return as a way to show investors how variations in the returns of individual futures contracts can affect returns on the S&P GSCI.

2.5 The Divergence between the S&P GSCI Excess Returns and Spot Price Changes

There has been divergence between the S&P GSCI cumulative excess returns and spot price changes since the S&P GSCI became tradable in the early 1990s. From 1991 to 2013, the annualized excess return of the S&P GSCI seldom outperformed the annualized spot price changes (See Figure 1). The S&P GSCI Excess Return Index, which measures the cumulative return performance of the S&P GSCI, had a 5-year-return of only 18.73% from 2009 to 2013, a period when the prices of the S&P GSCI spot commodities increased by more than 77%. Standard & Poor’s (2013b, page 8) which publishes the S&P GSCI declared that the excess return represents term structure effect plus spot price change, so adding the cumulative term structure effect to the S&P GSCI spot price changes provides a measure of the S&P GSCI excess return. Both Shemilt and Unsal (2004) and Burton and Karsh (2009) also give the same explanation.

Goldman Sachs, in a presentation by Shemilt and Unsal (2004), show how term structure causes the change in commodity spot price to not equal the excess return of commodity futures in the long run. The example Shemilt and Unsal use is NYMEX crude oil in backwardation. If the NYMEX crude oil spot month future price is $40 per barrel, and the second month NYMEX crude oil future price is only $38, then replacing the futures contract from the spot month with the second month will lead investors to receive
a higher excess return than the spot price change. Because the price of the second month NYMEX crude oil futures will be used as the spot price by the S&P GSCI after the current spot month futures is replaced, purchasing the second month futures $2 cheaper than the current spot price helps excess returns to outperform the S&P GSCI spot price changes by $2. An increase in the NYMEX crude oil spot price by $n after one month will bring investors $n+2 profit, which is $2 higher than the NYMEX crude oil spot price change. On the other side, if the NYMEX crude oil spot price goes down by $n one month later, then investors’ loss will be $n-2, which is $2 smaller than the NYMEX crude oil spot price change. When replacing futures contracts under backwardation, futures investment returns will always outperform the change in spot prices, and vice versa.

Burton and Karsh (2009) analyze the S&P GSCI excess return calculation procedure at contract replacement (Figure 5). They assume the S&P GSCI owns 100 front month futures contracts of a specific commodity at a price of $110 per contract, and to replace all of these contracts with deferred month futures at a price of $143 per contract. Because the total amount of funds that the S&P GSCI can collect from the sale of 100 front month futures is $110*100 = $11,000, Burton and Karsh claim this amount of funds restricts the index to purchase only $11,000/$143 = 76.9 deferred month futures contracts. Using this method, the contract replacement procedure does not produce any gain or loss because the total fund balance after the replacement is still $11,000. According to Burton and Karsh, the S&P GSCI Spot Index only reflects the price information of its futures contracts and not returns available to investors. Therefore, changing the price from $110 to $143 at contract replacement does not affect investment
returns. The only factor that will influence the index return performance is the price change of these 76.9 deferred month futures contracts after they become index holdings. From this, they conclude that “Whenever a commodity exhibits a contango curve, futures excess returns will underperform the spot price changes, while the opposite is true when the curve is backwardated. However, the outperformance of the excess returns versus spot price changes in a backwardation market does not represent a profit, which is the same as the underperformance in a contango environment does not represent a loss.” Consequently, the divergence between the S&P GSCI Spot Index and ER Index is due to the cumulative term structure effect.

Shemilt and Unsal (2004) show a brief explanation of the divergence between the NYMEX crude oil excess return and spot price changes, but they do not provide a clear equation to measure futures returns at contract replacement by using the S&P GSCI trading method. Notice that Burton and Karsh’s (2009) model provides an equation to measure the S&P GSCI excess returns, but their equation may not tell the actual return performance of the S&P GSCI because this model is different from what the S&P GSCI actually uses. In the S&P GSCI, the same number of futures contracts must be sold and bought for each commodity within each year (CME 2005, Standard & Poor’s 2007, and Goldman Sachs) as opposed to investing the total funds received from the sale of contract A to purchase contract B. Two different trading models would be expected to receive two different returns. Therefore, it is necessary to focus on how the S&P GSCI excess return is measured and why it is different from spot price changes.
2.6 Summary

This chapter reviews relevant academic research and investment advice on the commodity futures market, futures term structure, roll return, the influences of term structure on commodity futures returns, and the divergence between the S&P GSCI cumulative excess returns and spot price changes across time. An understanding of the futures market and futures term structure is important because the research objective of this thesis is focused on the investment returns of the S&P GSCI, a commodity index which only invests in futures contracts. Both roll return and the divergence between the S&P GSCI cumulative excess returns and spot price changes are originated from futures term structure at contract replacement. However, whether or not the roll return can explain the entire divergence between the S&P GSCI cumulative excess returns and spot price changes remains to be solved. Much of the S&P GSCI-related literature is limited to the ability of term structure to forecast excess returns of commodity futures investments, and does not explore how the S&P GSCI excess return is calculated, or how variations in the excess returns of individual futures contracts can affect excess returns on the S&P GSCI. For example, Burton and Karsh do not explain the actual return performance of the S&P GSCI, in part because their model is different from what the S&P GSCI actually uses. This thesis will analyze how the S&P GSCI replaces its futures holdings and measures its excess returns.
3. S&P GSCI INVESTMENT

3.1 Introduction

This chapter highlights the differences between the S&P GSCI and traditional equity indexes such as the S&P 500. The purpose of this chapter is to help readers gain a better understanding of the S&P GSCI and three of its return measurements, which are spot return, excess return (ER), and total return (TR). These differences and descriptions will become important as I examine the behavior of the S&P GSCI ER Index and its relationship to the S&P GSCI Spot Index. Topics discussed include the S&P GSCI index structure, trading strategy, return components, and related investment products.

3.2 Index Structure and Trading Strategy

The S&P GSCI represents a static long-only investment in various commodity futures. Since the beginning of 2007, it has held long positions in futures contracts for the same 24 commodities. For diversification purposes and to make the S&P GSCI representative of the world commodity markets, the 24 commodities selected by the index come from six sectors: six energy products traded on NYMEX, five industrial metals traded in LME, eight agricultural products traded on CBOT, KCBT, and ICE, three livestock products traded on CME, and two precious metals traded on NYMEX (Goldman Sachs). The quantity weights of the 24-commodity futures in the S&P GSCI portfolio are determined by these commodities’ average world production quantities in the last five years. Investors can mimic the S&P GSCI investment strategy by setting the
quantity ratio of each commodity futures the same as the official S&P GSCI (CME 2007, page 25). For instance, if the latest five-year average annualized world production quantity for NYMEX crude oil is 0.5 billion barrels and for CBOT corn is 12.5 billion bushels, one NYMEX crude oil futures contract contains 1000 barrels of crude oil and one CBOT corn futures contract contains 5000 bushels of corn, then the futures quantity ratio between NYMEX crude oil and CBOT corn in the S&P GSCI will be $0.5 \text{ billion} / 1000 : 12.5 \text{ billion} / 5000 = 1:5$. Regardless of the size of the investment, in order to mimic the S&P GSCI portfolio in the example above, investors have to keep the quantity ratio, 1:5, constant between NYMEX crude oil futures and CBOT corn futures within an entire calendar year when investing in these two futures.

In addition to the diversified 24-commodity index portfolio, the S&P GSCI also has subindexes that track each of its individual commodity futures as well as various combinations of the 24 commodities. For instance, the S&P GSCI Natural Gas Subindex reflects the performance of the natural gas futures contract traded on NYMEX. Because each S&P GSCI individual commodity subindex holds only one specific commodity, investors can mimic these subindexes by simply investing in the same commodity futures as these subindexes.\textsuperscript{15}

The S&P GSCI component replacement procedure is different from equity indexes like the S&P 500. The S&P 500 assumes the entire dollar amount from the sale of Asset A will be used in the purchase of Asset B. However, in the S&P GSCI, the number of futures contracts of each commodity is held constant for the entire year in

\textsuperscript{15} Futures quantity ratio does not need to be considered here because each individual commodity futures makes up 100% of its corresponding subindex.
order to keep the quantity ratio of each commodity futures constant, and is rebalanced once a year in the beginning of the 5th business day of each January\textsuperscript{16} based on the underlying commodities’ world production data (CME 2005, Standard & Poor’s 2007, Goldman Sachs). In each month within a year, the contract replacement requires the same number of futures contracts to be sold and bought for each commodity in order to keep the index composition constant on a quantity basis\textsuperscript{17}. During pre-established contract replacement periods, 20\% of the total number of contracts of a sooner-to-expire commodity futures contract will be sold and the same number of contracts of a later-to-expire contract will be bought each day from the 5\textsuperscript{th} business day to the 9\textsuperscript{th} business day of the month. Then, on the 10\textsuperscript{th} business day of that month, all sooner-to-expire futures contracts have been replaced with the same number of later-to-expire contracts. Therefore, the S&P GSCI is quantity weighted, unlike the S&P 500 which is capitalization weighted.

3.3 Three Returns in the S&P GSCI

Futures contracts, unlike common stocks, have an expiration date and therefore cannot be held indefinitely. Prior to expiration, each sooner-to-expire futures contract must be sold and replaced with a later-to-expire futures contract in the same commodity. Since the prices of the outgoing and incoming futures contracts are likely different but the S&P GSCI sets the quantity weight constant for each of its commodity futures within an entire year, the S&P GSCI spot return and excess return described below will be different. The S&P GSCI uses three indexes to reflect its spot price change and return performance.

\textsuperscript{16} The 5\textsuperscript{th} business day of each January is the transition date between the old calendar year and new calendar year. It is the only date that the S&P GSCI rebalances the quantity weights of the 24 commodity holdings (Standard & Poor’s 2013b, page 39).

\textsuperscript{17} Notice that the contract replacement method actually used by the S&P GSCI differs completely from the way that Burton & Karsh (2009) use to calculate S&P GSCI excess returns.
They are the S&P GSCI Spot Index, the S&P GSCI Excess Return Index (ER Index), and the S&P GSCI Total Return Index (TR Index). All three indexes have a same base value of 100 on January 2nd, 1970 (Standard & Poor’s 2013b, page 9). The terms spot return, excess return, and total return are used in other investment products, but the meanings of these three returns for the S&P GSCI are quite different. The following sections will describe in detail each of these three returns, and contrast them with other investment products.

3.3a S&P GSCI Spot Return

The S&P GSCI Spot index uses the daily percentage change of its futures holdings’ prices to indicate spot return (Standard & Poor’s 2013b, page 8). The calculation procedure for the S&P GSCI daily spot return on the contract replacement date simply replaces the outgoing futures prices with the incoming futures prices without any adjustments to the composition of the index. As a result, the spot index can only indicate the price changes of its futures holdings, and cannot be used to reflect the return performance that the S&P GSCI investor can receive. For instance, when a $110 January NYMEX crude oil futures contract is replaced with a $100 February NYMEX crude oil futures contract in the S&P GSCI, the price level of NYMEX crude oil futures in the index will decrease from $110 to $100, which indicates a (100-110)/110 = -9.09% spot return on the crude oil portion of the index. However, this $10 price decrease or -9.09% spot return does not represent an actual loss to investors because the S&P GSCI spot index is assumed to be un-investable (Standard & Poor’s 2007, page 3). In contrast, a similar price change between Asset A and Asset B in an investable equity index like the S&P 500 will have no impact on the index return because the S&P 500 will make an
adjustment to its index when replacing components. Therefore, the S&P GSCI Spot Index is intended to be used as a barometer of commodity price level, and not as an investment vehicle.

3.3b S&P GSCI Excess Return

The S&P GSCI ER index measures the return performance of the S&P GSCI (Standard & Poor’s 2012b, page 8). An important point is that excess return in the context of the S&P GSCI means the pure return from investing in commodity futures contracts, not the return above the T-bill rate (Standard & Poor’s 2007, page 3). In the equity market, excess return means the difference between capital gain or loss and the T-bill return. The S&P GSCI excess return is comparable to the capital gain or loss of an equity investment. According to the Standard & Poor’s (2012, page 40), the daily percentage change of S&P GSCI ER Index is calculated from the ratio between the dollar amount that the index gains or loses on each trading day and the dollar amount invested. The S&P GSCI ER index is then compounded.

For instance, assume the S&P GSCI Crude Oil ER Subindex value is 100 at the end of day 1 with a $100 February NYMEX crude oil contract as its asset holding. If the NYMEX crude oil contract goes up to $105 at the end day 2, then the daily percentage change for the S&P GSCI Crude Oil ER Subindex from day 1 to day 2 will be \((105-100)/100 = 5\%\), and the ER index value at the end of day 2 will be \(100\times(1+5\%) = 105\). If the price of the NYMEX crude oil contract goes down by $2 to $103 at the end of day 3, then the daily percentage change for the S&P GSCI Crude Oil ER Subindex from day 2
to day 3 will be \(-\frac{2}{105} = -1.904\%\), and the ER index value at the end of day 3 will be
\[100 \times (1 + 5\%) \times (1 - 1.904\%) = 103.\]

**3.3c S&P GSCI Total Return**

S&P GSCI total return measures the excess return plus the return from a 3-month T-bill. The S&P GSCI assumes investors have two equal funds for investment. One of the funds is invested in the S&P GSCI, and another fund is invested in 3-month T-bills

*(Standard & Poor’s 2013b, page 41)*. Suppose that an investor has $200 with $100 invested in the S&P GSCI, and $100 invested in T-bills. If the daily excess return of the S&P GSCI is 0.1%, and the daily return from the T-bill is 0.01%, then the daily total return indicated by the S&P GSCI TR Index will be 0.11%. The S&P GSCI total return is a hypothetical return that cannot be realized by investors. This thesis will not be concerned with the total return index, and the total return index is mentioned here only for the sake of completeness.

**3.4 Related Investment Products**

Investors can gain exposure to excess returns from the S&P GSCI through over-the-counter (OTC) contracts with swap dealers, or they can buy investment funds such as exchange traded fund (ETFs) and exchange traded notes (ETNs) whose returns are linked to the S&P GSCI and its subindexes *(Goldman Sachs)*. iShare and iPath developed ETNs to track excess returns of the S&P GSCI, S&P GSCI Crude Oil Subindex, and S&P GSCI Natural Gas Subindex. VelocityShares created ETNs to track excess returns of the S&P GSCI Crude Oil Subindex, Natural Gas Subindex, Gold Subindex, and Silver Subindex.
In addition to OTC contracts and investment funds, Chicago Mercantile Exchange (CME) has an S&P GSCI futures contract that allows investors to make short term investments in the S&P GSCI. The underlying index expires each month so that the commodity futures included in the index futures are consistent and avoid any influences from the monthly contract replacement. CME also has an S&P GSCI Excess Return Index futures contract. The S&P GSCI Excess Return Index futures include the influence from monthly contract replacement to provide investors with the opportunity to make long term investments. However, the trading volume of S&P GSCI-related futures traded at CME is much smaller compared to the OTC market and investment funds market that I introduced above.

3.5 Summary

This chapter reviewed detailed information of the S&P GSCI index structure, trading strategy, return components, and related investment products. Based on the information above, I introduce the difference between the S&P GSCI and the S&P 500. The purpose of this chapter is to help readers to get a better understanding of the S&P GSCI. This chapter also helps readers to distinguish the terminology of returns between the S&P GSCI and the S&P 500. Three types of returns for the S&P GSCI have been specified, and the spot return and excess return of the three returns will be used in chapter 4 and chapter 5.
4. METHODOLOGY AND DATA

4.1 Introduction

This chapter outlines the empirical analysis of the divergence between the S&P GSCI ER Index and Spot Index from 2007 to 2013. First, I review the S&P GSCI Spot Index and ER Index, and introduce the method that I use to re-measure the S&P GSCI excess return. Second, I discuss the methodology used to test the hypothesis that term structure effect does not fully account for the divergence between the S&P GSCI ER Index and Spot Index. Last, I describe the data used in this study and summarize this chapter.

4.2 Existing Excess Return Model and Daily Flow of Funds Model

In chapter 2 and chapter 3, I introduced the S&P GSCI Spot Index that represents the price or total dollar value of the S&P GSCI. The value of the S&P GSCI Spot Index expressed in index points is equal to the S&P GSCI total dollar value divided by a constant (Standard & Poor’s 2013b, page 35). This constant is adjusted only on the beginning of 5th business date of each January to keep the spot index unchanged when the new quantity weight of 24 commodity holdings has been used (Standard & Poor’s 2013b, page 33). As a result, the daily percentage changes of the S&P GSCI Spot Index

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18 The Index Total Dollar Value = Futures Price * Average World Production Quantity (Standard & Poor’s 2012, page 32)
19 The 5th business day of each January is the transition date between the old calendar year and new calendar year. It is the only date that the S&P GSCI will rebalance the quantity weight of the 24 commodity holdings (Standard & Poor’s 2013b, page 2).
beyond the 5th business day of each January will be the same as the daily percentage changes of the S&P GSCI total dollar value. The S&P GSCI does not make any adjustment to the total dollar value of the spot index in contract replacement periods, so the price differences between outgoing futures contracts and incoming futures contracts at contract replacement, which I will call term structure effect, are fully included in the S&P GSCI total dollar value. However, the changes of the S&P GSCI total dollar value caused by term structure effects cannot be counted as profits and losses. Profits and losses of trading a futures contract should be calculated by subtracting the purchase price from the sale price of the same contract, not from a different contract.

According to Standard & Poor’s (2013b, page 8), the S&P GSCI ER Index calculates the investment returns of the S&P GSCI excluding the entire term structure effect in the Spot Index. In this thesis, I will develop a daily flow of funds model to test the description of term structure returns provided by Standard & Poor’s. This flow of funds model separates the cumulative term structure effect from the total dollar value of the S&P GSCI, and re-measures the profits and losses of the S&P GSCI.

The daily flow of funds model will trade the same futures contracts and quantities as the official S&P GSCI, which sets the quantity weight for each of the 24-commodity futures holdings constant within each calendar year20 and rebalances the index once a year on the beginning of the 5th business day of each January. In the rest of the year, during pre-established roll periods, 20% of the total number of contracts of a sooner-to-expire commodity futures contract will be sold and the same number of contracts of a

---

20 Calendar Year in this thesis means S&P GSCI Year, which start on the 5th business day of each January and end on the 4th business of next January.
later-to-expire contract will be bought each day from the 5th business day to the 9th business day of the month. Then, on the 10th business day of that month, all sooner-to-expire futures contracts have been replaced with the same number of later-to-expire contracts. In contrast, the excess return model used by Burton and Karsh (2009) changes the quantity of the S&P GSCI futures holdings to a constant dollar-basis, which causes the number of futures contracts of the S&P GSCI to vary within a year. Therefore, the proportions of futures holdings in the excess return model used by Burton and Karsh are different from the proportions of futures holdings in the S&P GSCI.

4.3 Method

Within each calendar year, the daily flow of funds model examines in detail the return generation and measurement process within the S&P GSCI and uses a daily flow-of-funds procedure during contract replacement periods21. In these periods, actual daily profits and losses are measured in dollars rather than percentages to ensure that the term structure effect is excluded from the S&P GSCI investment returns. I will compare the daily investor fund balance22 that is converted from the daily percentage changes of the official S&P GSCI ER Index with the daily investor fund balance calculated from the daily flow of funds model. If the daily investor fund balance calculated by the two methods differ statistically, then the cumulative term structure cannot fully explain the divergence between the S&P GSCI ER Index and Spot Index. I will search for the sources of the divergence between the S&P GSCI ER Index and Spot Index that cannot

21 Detailed information about this daily flow of funds procedure is provided in section 4.3b.
22 The daily investor fund balance on day t is equal to the starting fund that has been invested in the beginning of the S&P GSCI year plus the cumulative daily profits and losses that are earned from the beginning of the S&P GSCI year to the end of day t.
be explained by the cumulative term structure effect. Otherwise, the cumulative term structure effect will account for the entire divergence between the S&P GSCI ER Index and Spot Index.

4.3a The S&P GSCI Total Dollar Value and Measurement Procedure

The S&P GSCI total dollar value in each calendar year is measured independently from other calendar years to avoid the influence from index rebalancing on the 5th business day of each January. In non-rolling periods, I measure the total dollar value of the S&P GSCI by equation (1) used by Standard & Poor’s (2013b, page 32):

\[
V_t = \sum_{i=1}^{24} Q_i \cdot P_{i,t}
\]

where \(V_t\) denotes the total dollar value of the diversified 24-commodity S&P GSCI futures holdings on day \(t\), \(P_{i,t}\) denotes the price of contract \(i\) on day \(t\), and \(Q_i\) denotes the average world production quantity of commodity \(i\) in the index, expressed in terms of futures contracts. The range of \(i\) is from 1 to 24, which represents the 24 commodities used in the S&P GSCI from 2007 to 2013.

In contract replacement periods beyond the 5th business day of each January, Standard and Poor’s (2013b, page 38) measures the total dollar value of the diversified 24-commodity futures holdings by equation (2):

\[
V_t = \sum_{i=1}^{24} [Q_i \cdot (CRW_{1,i,t} \cdot P_{1,i,t} + CRW_{2,i,t} \cdot P_{2,i,t})]
\]

where \(CRW_{1,i,t}\) denotes the quantity roll weight of the outgoing contract \(i\) on day \(t\), and \(CRW_{2,i,t}\) denotes the quantity roll weight of the incoming contract \(i\) on day \(t\). \(CRW_{1,i,t}\) begins with 100% on the 5th business day in the rolling month, and decreases by 20% per
day on the next 4 business days to 0%. CRW2\textsubscript{i,\textit{t}} begins with 0% on the 5\textsuperscript{th} business day in the rolling month, and increases by 20% per day on the next 4 business days to 100%. The summation of CRW1\textsubscript{i,\textit{t}} and CRW2\textsubscript{i,\textit{t}} is always equal to 100%. P1\textsubscript{i,\textit{t}} is the price of the outgoing contract i on day t, and P2\textsubscript{i,\textit{t}} is the price of the incoming contract i on day t. V\textsubscript{\textit{t}} and Q\textsubscript{i} have the same meaning as in equation (1).

The S&P GSCI individual commodity subindexes, such as the S&P GSCI Crude Oil Subindex, hold a single commodity rather than multiple commodities. Without loss of generality, I simplify the analysis by assuming the quantity of futures contract to be 1\textsuperscript{23}, and treat the price of that individual futures contract as the total dollar value of individual commodity subindexes in non-rolling periods by using equation (3):

\begin{equation}
V_{\textit{t}} = P_{\textit{1t}}
\end{equation}

In contract replacement periods beyond the 5\textsuperscript{th} business day of each January, the total dollar value of the S&P GSCI individual commodity subindexes, which assume holding only one contract, will be measured by equation (4):

\begin{equation}
V_{\textit{t}} = \text{CRW1}_{\textsubscript{i,\textit{t}}} \ast P_{\textsubscript{1\textit{t}}} + \text{CRW2}_{\textsubscript{i,\textit{t}}} \ast P_{\textsubscript{2\textit{t}}}
\end{equation}

All of the variables in both equation (3) and equation (4) have the same meanings as the variables in equation (1) and equation (2).

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\textsuperscript{23} For the S&P GSCI individual commodity subindexes, the quantity of futures contract can be ignored because each individual commodity takes 100\% of the quantity position in their subindexes.
4.3b Daily Returns & Investor Fund Balance Calculated by the Daily Flow of Funds Model

The daily profit or loss of the S&P GSCI calculated by the daily flow of funds model is indicated in equation (5):

\[
M_t = (V_t - S_t) - V_{t-1} = F_t - F_{t-1}
\]

\(M_t\) denotes the daily profit or loss of the S&P GSCI at the end of day \(t\). \(V_t\) and \(V_{t-1}\) denotes the total dollar value of the S&P GSCI at the end of day \(t\) and day \(t-1\), respectively. \(S_t\) denotes the term structure effect caused by contract replacement at the end of day \(t\), which will be positive for contango and negative for backwardation. On non-rolling days, \(S_t\) will be zero, and the S&P GSCI daily profit or loss can be measured by taking the difference between \(V_t\) and \(V_{t-1}\) directly.

In contract replacement periods beyond the 5th business day of each January, the daily term structure effect needs to be measured and deducted from the index total dollar value in order to calculate the daily profit or loss. \(F_t\) and \(F_{t-1}\) are investor fund balances in the S&P GSCI on day \(t\) and day \(t-1\) respectively. The daily change in investor fund balance is the same as the daily profit or loss because both measure the daily returns to S&P GSCI investors. The daily term structure effect \(S_t\) is measured by equation (6) for the S&P GSCI:

\[
S_t = 20% * \sum_{i=1}^{24} [Q_i * (P2_{i,t} - P1_{i,t})]
\]

and equation (7) for the individual commodity subindexes:

\[
S_t = 20% * (P2_{i,t} - P1_{i,t})
\]
The daily investor fund balance $F_t$ is measured by equation (8):

$$
(8) \quad F_t = V_t - \sum_{i=1}^{t} S_t
$$

$S_t$ in equations (6) and (7) denotes the term structure effect on day $t$, which is caused by the price difference between the outgoing contract and the incoming contract. $P_{1,t}$ and $P_{2,t}$ denote prices of the outgoing contract and the incoming contract respectively at the end of day $t$. The 20% component means the S&P GSCI replaces 20% of the total number of futures contracts each day, and will complete the replacement procedure in 5 days. $\sum_{i=1}^{t} S_t$ in equation (8) denotes the cumulative term structure effect from the beginning of the calendar year\(^{24}\) to the end of day $t$. After deducting the cumulative term structure effect from the S&P GSCI total dollar value $V_t$ on day $t$, the remaining dollars in the S&P GSCI at the end of day $t$ represents the actual fund balance owned by an S&P GSCI investor, and named as $F_t$.

### 4.3c Daily Investor Fund Balance Converted from the Official S&P GSCI ER Index

To test if the daily cumulative investment returns for the S&P GSCI ER Index is the same as the daily cumulative profits and losses measured by the daily flow of funds model, the S&P GSCI ER Index will be converted to the S&P GSCI investor fund balance by using equation (9):

$$
(9) \quad CF_t = (1 + ER_t) \times CF_{t-1} = V_0 \times \prod_{i=1}^{t}(1 + ER_t)
$$

$CF_t$ denotes the daily investor fund balance at the end of day $t$ converted from the official S&P GSCI ER Index. $ER_t$ is the daily percentage change of the official S&P GSCI ER

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\(^{24}\)At the moment when the S&P GSCI finishes rebalancing its index quantity weight in the beginning of 5\(^{th}\) business day of January, a new S&P GSCI based calendar year starts.
Index from the end of day t-1 to the end of day t. V₀ is the S&P GSCI index total dollar value in the beginning of the 5th business day in January after the S&P GSCI finishes its annual rebalancing process.

The S&P GSCI investor fund balance Fᵢ calculated by equation (8) excludes the influence of term structure effects by subtracting \( \sum_{i=1}^{t} S_t \) from the S&P GSCI total dollar value \( V_t \) at the end of day t. However, the investor fund balance CFᵢ in equation (9) is calculated by compounding the daily percentage changes of the official S&P GSCI ER Index. Fᵢ is excluded from term structure effects as indicated in equation (8), but we will wait until later in this thesis to explain how ERᵢ is calculated by the official S&P GSCI ER Index²⁵. If the investor fund balance CFᵢ calculated by equation (9) completely matches Fᵢ calculated by equation (8), then the cumulative term structure effect determines the entire divergence between the S&P GSCI ER Index and Spot Index.

4.4 Data

The data used in this study include the daily settlement index values of the S&P GSCI Spot Indexes and ER Indexes of the diversified 24-commodity S&P GSCI and Subindexes of selected individual commodities. Also used are the daily settlement prices for the futures of the 24 individual commodities used by the S&P GSCI for 2007-2013.

The 24 commodity futures contracts in the S&P GSCI include: Brent Crude Oil, Gasoil, Cocoa, Coffee, Sugar #11, and Cotton #2, all traded at InterContinental Exchange (ICE); WTI Crude Oil, RBOB Gasoline, Heating Oil, Natural Gas, Gold, and Silver, all

²⁵ There is an equation available in the official S&P GSCI Methodology to calculate the daily percentage change of the S&P GSCI ER Index. However, the information required by this equation is unclear.
traded at New York Mercantile Exchange (NYMEX); Corn, Chicago Wheat, and Soybeans, all traded at Chicago Board of Trade (CBOT); Kansas Wheat, traded at Kansas City Board of Trade (KCBT); Live Cattle, Feeder Cattle, and Lean Hogs, all traded at Chicago Mercantile Exchange (CME); Aluminum, Copper, Lead, Nickel, and Zinc, all traded at London Metal Exchange (LME). The daily settlement prices for all except the LME commodity futures are collected from Barchart Advanced Commodity Service, the LME commodity futures prices are obtained from Thomson Reuters, and the daily settlement values for the S&P GSCI Spot Index, ER Index, and subindexes are provided by Standard & Poor’s.

Simultaneously testing 24 commodities can be difficult, so I first examine four individual commodity futures – NYMEX crude oil, NYMEX natural gas, CBOT corn, and CME live cattle – and their impacts on the returns of the S&P GSCI Crude Oil Subindex, the S&P GSCI Natural Gas Subindex, the S&P GSCI Corn Subindex, and the S&P GSCI Live Cattle Subindex, respectively. There are several reasons for selecting these particular individual commodities. First, these commodities have the largest dollar weight in the energy sector, agricultural sector, and livestock sector respectively in the S&P GSCI index, and experience large price fluctuations each year. Second, together these four commodities account for more than 42% of the dollar weight in the S&P GSCI (Standard & Poor’s 2013b, page 44). Third, NYMEX crude oil and NYMEX natural gas futures undergo contract replacement each month, and the frequency in replacing these futures will be helpful to provide the maximum number of individual tests. Corn is storable commodity with an annual production cycle, so the price difference will be largest at the transition from old crop contract to new crop contract, and a large price
difference between two futures at replacement time will be useful to test whether term structure effects contribute to index price divergence. Live cattle is a non-storable commodity with a continuous production cycle, so it does not have a stable term structure. After these four individual commodities have been tested, I will extend this approach to the full 24-commodity index.

The time frame under this study is from January 5, 2007 to January 7, 2014. There are two reasons to select this time period. First, the S&P GSCI maintains the same 24 commodities in the index during this period. Tracking investment returns of the same 24 commodities across years will be more consistent than tracking returns of different commodities in each year. Second, during this period, commodity prices rose to record levels, collapsed following the global financial crisis, and then recovered. These large fluctuations provide a range of market conditions for us to test my hypothesis.

4.5 Summary

This section outlines the empirical methods and data to analyze the divergence between the S&P GSCI ER Index and Spot Index from 2007 to 2013. I reviewed the existing methods to calculate the gains and losses of the S&P GSCI. Next, I described my method and data to test the hypothesis that the difference between the S&P GSCI ER Index and Spot Index does not come solely from term structure. Finally, I introduced the data that I use in this research. I will present the empirical results and explore the causes of the index divergence in chapter 5.
5. RESULTS AND DISCUSSION

5.1 Introduction

This chapter examines the reason for the divergence between the official and calculated values of the S&P GSCI Spot Index and ER Index. First, I describe the method used to test if the divergence can be fully explained by the cumulative term structure effect as Standard and Poor’s indicated in section 2.5. Next, I analyze the test results to show that there are other components in addition to term structure effect that explain the divergence. Then I explain these additional reasons for the divergence between the official S&P GSCI Spot Index and ER Index. Finally, I discuss the impacts and implications of the equation used by the official S&P GSCI to measure daily excess return.

5.2 Hypothesis Testing Procedure

The first step is to use $V_t$ from equations (1) to (4) in section 4.3a to build the S&P GSCI Spot Index and 4 individual commodity subindexes by compounding the daily percentage changes of $V_t$. To confirm that these calculated spot index values are the same as the official values, I compare these calculated spot index values graphically and quantitatively against the corresponding official spot indexes values. Figure 6 through Figure 10 show that my calculated spot index values, indicated by the green curve, and the corresponding official spot index values, indicated by the blue curve, are closely matched with each other. The green curve in each figure completely covers the blue
curve. Table 2 shows that the calculated and official values for the annual spot returns are matched as well. These results confirm that the commodity futures contracts and procedures used in my model are the same as those used in the official S&P GSCI.

My next step is to test if the S&P GSCI investor fund balance $F_t$ calculated by the daily flow of funds model in equation (8) of section 4.3b (i.e., the calculated fund balance at the end of day $t$) matches the investor fund balance $CF_t$ converted from the official S&P GSCI ER Index in equation (9) of section 4.3c (i.e., the official fund balance at the end of day $t$). Both $F_t$ and $CF_t$ are cumulated by the funds that were invested in the S&P GSCI in the beginning of each calendar year plus daily profits or losses generated by the S&P GSCI futures holdings from the beginning of each calendar year to the end of day $t$.

In equation (8), the daily flow of funds model calculates $F_t$ by deducting the cumulative term structure effect $\sum_{i=1}^{t} S_t$ from the S&P GSCI total dollar value $V_t$. If the price difference between the outgoing futures and incoming futures at contract replacement can explain the entire divergence between the official S&P GSCI Spot Index and ER Index, then the $CF_t$ calculated by equation (9) must be the same as the $F_t$ calculated by equation (8). Both the daily flow of funds model and the official S&P GSCI start with the same investment fund balance and trade the same futures contracts. If official excess return $ER_t$ in equation (9) is fully explained by both the cumulative term structure effect $\sum_{i=1}^{t} S_t$ and total dollar value $V_t$ from equation (1) to equation (4), then $F_t$ and $CF_t$ should be the same. If $F_t$ and $CF_t$ are found to be different, then there will be other returns in addition to the cumulative term structure effect and total dollar value to explain the official S&P GSCI ER Index (ER in equation 9). In other words, the cumulative term structure effect will be
insufficient to explain the entire divergence between the official S&P GSCI Spot Index and ER Index.

To avoid any influence from rounding error, which may affect the testing results, I use a ratio-paired t-test rather than a difference-paired t-test to see if \( F_t \) and \( C_F_t \) differ significantly by testing all of their daily values within a year. I take the natural log of the difference between \( F_t \) and \( C_F_t \) to get \( \ln \left( \frac{F_t}{C_F_t} \right) \), and assume that \( \ln \left( \frac{F_t}{C_F_t} \right) \) follows a normal distribution with mean of zero\(^{26}\). If the test result is not statistically different from zero, then \( F_t \) and \( C_F_t \) are equal. Otherwise, I will search for the reasons for the divergence between \( F_t \) and \( C_F_t \).

5.3 Hypothesis Testing Results

This section first tests the differences between the official investor daily fund balance \( C_F_t \) from equation (9) and the calculated investor daily fund balance \( F_t \) from equation (8). \( C_F_t \) and \( F_t \) are used to build official and calculated ER Indexes daily from 2007 to 2013 for the complete 24-commodity index, and for four subindexes: S&P GSCI Crude Oil Subindex, the S&P GSCI Natural Gas Subindex, the S&P GSCI Corn Subindex, and the S&P GSCI Live Cattle Subindex. Then, I compare the official and calculated daily ER index values and their annual excess returns. Any difference between the official and calculated values indicates the insufficiency of the cumulative term structure effect \( \sum_{i=1}^{T} S_t \) in explaining the entire divergence between the official S&P

\(^{26}\) A Wilcoxon signed-rank test, designed for non-normal distributions, also was used to test if \( F_t \) and \( C_F_t \) are matched. Results did not differ from those for the ratio-paired t-test.
GSCI Spot Index and ER Index for the full 24-commodity index, and for the four individual commodity subindexes.

Results from using the ratio-paired t-test to compare official and calculated investor daily fund balance $CF_t$ and $F_t$ are presented in Table 3. Only three cases were found in which $CF_t$ and $F_t$ are not statistically different: for the S&P GSCI in 2010, CME live cattle in 2012, and NYMEX crude oil in 2013. All other pairs of $CF_t$ and $F_t$ are found to be statistically different, and provide strong evidence that excess returns consist of more than just term structure effects.

In addition, annualized excess returns\textsuperscript{27} calculated by the daily flow of funds model and the official S&P GSCI ER Index for all selected samples from 2007 to 2013 are divergent (Table 4). Although the ratio-paired t-test did not detect the daily difference between $F_t$ and $CF_t$ for the S&P GSCI in 2010, CME live cattle in 2012, and NYMEX crude oil in 2013, the difference of annualized excess returns between the daily flow of funds model and the official S&P GSCI ER Index model suggests the presence of daily differences between $F_t$ and $CF_t$.

Next, I calculate daily Spot Indexes and ER Indexes for the seven years from 2007 to 2013 based on the index total dollar value $V_t$ from equations (1) to (4) in section 4.3a and the daily fund balances $F_t$ in equation (8). I then compare the seven-year return performance of each calculated Spot Index and ER Index with the corresponding official Spot Index and ER Index. Results are presented in Table 5, and show that the calculated and official spot returns are matched for the 24-commodity S&P GSCI and for each of

\textsuperscript{27} Annualized excess returns of the daily flow of funds model and the official S&P GSCI are the annual percentage changes of $F_t$ and $CF_t$, respectively.
the four individual commodity subindexes. However, none of the calculated ER Indexes have the same excess returns as the official ER Indexes. NYMEX crude oil, which accounts for more than 33% of the total dollar weight of the S&P GSCI, had a seven-year cumulative loss of 22.49% for the calculated Crude Oil ER Subindex compared to a loss of 36.87% for the official Crude Oil ER Subindex (Table 5 and Figure 11). CBOT corn, which provides approximately 4% of the dollar weight of the S&P GSCI, had a seven-year cumulative loss of 18.01% for the calculated Corn ER Subindex compared to a loss of 21.21% for the official Corn ER Subindex (Table 5 and Figure 12).

In contrast, the seven-year cumulative losses for NYMEX natural gas and CME live cattle are under-reported. Natural gas had a seven-year cumulative loss of 95.54% for the calculated ER Subindex compared to a loss of 93.98% for the official ER Subindex (Table 5 and Figure 14), and live cattle had a seven-year cumulative loss of 39.77% for the calculated ER Subindex compared to a loss of 35.42% for the official ER Subindex (Table 5 and Figure 15). Since the combined dollar weight of NYMEX natural gas and CME live cattle in the S&P GSCI is less than 6%, the under-reporting of losses by the Natural Gas ER Subindex and the Live Cattle ER Subindex is not large enough to offset the over-reporting of losses by the Crude Oil ER Subindex and the Corn ER Subindex. The calculated S&P GSCI ER Index had a seven-year cumulative loss of 12.41% compared to a loss of 16.53% for the official S&P GSCI ER Index (Table 5 and Figure 13).

In section 2.5, Standard & Poor’s uses term structure effect to explain the divergence between the official S&P GSCI Spot Index and ER Index, but results generated by the daily flow of funds model do not support this explanation. The daily
flow of funds model trades the same futures contracts with the same quantities as the official S&P GSCI. It yields the same spot returns but different excess returns (Table 5).

By definition from equation (8), the difference between the calculated spot index and the calculated ER index is term structure effect. Because the calculated and official Spot Index values are identical, but the calculated and official ER Index values differ substantially. The difference between the official Spot Index and the official ER index consist of more than just the term structure effect.

5.4 Sources of the Gap between the Official ER Index and Calculated ER Index

In section 5.3, test results imply that divergence between the official daily fund balance CF\textsubscript{t} and the calculated daily fund balance F\textsubscript{t} is the reason for the gap between the official S&P GSCI ER Index and the calculated ER Index. These test results indicate that the term structure effect cannot fully explain the difference between the official S&P GSCI Spot Index and ER Index. In this section, I will analyze in detail the missing component that explains this difference.

5.4a Daily Excess Return Equation in the Official S&P GSCI Methodology

The official S&P GSCI Methodology (Standard & Poor’s 2013b, page 40) defines the S&P GSCI daily excess return rate\textsuperscript{28} as equation (10):

\begin{equation}
(10) \text{Official ER rate on day } t = \frac{\text{Total Dollars Obtained on day } t \text{ from day } t-1's \text{ Investment}}{\text{Total Dollars Invested on day } t-1} - 1
\end{equation}

However, the definitions of “total dollars invested” and “total dollars obtained” in equation (10) are unclear. The official S&P GSCI description does not specify whether

\textsuperscript{28} The S&P GSCI ER Index is compounded by the S&P GSCI daily excess return rate.
the “total dollars invested” on day t-1 is the daily fund balance $F_{t-1}$ in equation (8) or the total dollar value $V_{t-1}$ from equation (1) to (4). Similarly, it is unclear whether the “total dollars obtained” on day t is the daily fund balance $F_t$ or the total dollar value $V_t$. We know that $CF_t$ in equation (9) is defined as a function of the daily percentage change of the official ER Index value from day t-1 to day t. Therefore, the daily percentage change of the official S&P GSCI ER Index value is a reverse function of $CF_t$, and the “total dollar invested” in equation (10) is $CF_t$. Tests in section 5.3 show that the official daily fund balance $CF_t$ and the calculated daily fund balance $F_t$ are statistically different. Therefore, the calculated daily fund balance $F_t$ is not the “total dollars invested” in equation (10).

In order to test whether the “total dollars invested” in equation (10) is the total dollar value $V_{t-1}$, I assume the “total dollars invested” on day t-1 is the S&P GSCI total dollar value $V_{t-1}$ in section 4.3a, and assume the difference between the “total dollars obtained” on day t and the “total dollars invested” on day t-1 is the daily profit or loss $M_t$ in equation (5). I use equation (11) to calculate the expected daily percentage change of the S&P GSCI ER Index:

\[
(11) \text{ Expected Daily Percentage Change of ER Index } = \frac{M_t}{V_{t-1}}
\]

where $M_t$ is the daily profit or loss of the S&P GSCI at the end of day t measured by equation (5), and $V_{t-1}$ is the S&P GSCI index total dollar value at the end of day t-1 that I introduced in section 4.3a. I compound the expected daily percentage change that I calculated in equation (11) to rebuild the S&P GSCI ER Index and the four individual commodity ER Subindexes in each calendar year, and then measure their expected
annualized excess return rates from 2007 to 2013. All of these expected annualized excess return rates are compared with the annualized excess return rates reported from the official S&P GSCI ER Index and four individual commodity ER Subindexes. The expected annualized excess return rates are the same as the official annualized excess return rates with only negligible differences in a few cases (Table 6).

Based on the result presented in Table 6, the daily percentage changes of the official S&P GSCI ER Index and ER Subindexes are calculated by using the S&P GSCI total dollar value $V_{t-1}$ at the end of day $t-1$ as expressed in equation (11), rather than the daily fund balance $F_{t-1}$. This calculation approach explains the divergence between the official S&P GSCI ER Index and the calculated ER Index.

5.4b Shortcomings and Implications of the Official S&P GSCI Excess Return

In the equity market, the daily percentage change of the official S&P 500 index value on any given day is calculated by dividing the profit or loss received by the S&P 500 at the end of day $t$ and the S&P 500 index total dollar value at the end of day $t-1$, which is similar to equation (11). However, using this equation to measure the daily percentage change of index values is suitable for the S&P 500 but not for the S&P GSCI ER Index. In section 3.2, I discussed the differences between the component replacement procedure used by the S&P 500 and the S&P GSCI. The S&P 500 assumes the entire dollar amount received from the sale of Asset A will be used in the purchase of Asset B. As a result, the S&P 500 index total dollar value is always equal to the S&P 500 investor fund balance. Calculating the daily percentage change of the S&P 500 index value by dividing the daily profit or loss by the index total dollar value on the previous day is the
same as dividing the S&P 500 daily profit or loss by the investor fund balance on the previous day.

In the S&P GSCI, the number of contracts of each individual commodity is held constant for the entire year, and is rebalanced annually on the 4th business day of January as described in section 3.2. The S&P GSCI total dollar value $V_t$ measured in section 4.3a is different from the S&P GSCI investor fund balance $F_t$ as measured by daily flow of funds model in equation (8). The S&P GSCI ER Index is designed to measure investor return performance. Therefore, a better measure of daily return performance is to calculate the daily percentage change of the S&P GSCI ER Index by dividing the S&P GSCI daily profit or loss by the investor fund balance $F_{t-1}$ on the previous day, rather than by the index total dollar value $V_{t-1}$ on the previous day. Notice that, $F_{t-1}$ and $V_{t-1}$ will be different whenever contract replacement occurs, so the difference between $F_{t-1}$ and $V_{t-1}$ is the cumulative price difference of the outgoing futures contracts and incoming futures contracts as described in section 4.3b. But this cumulative price difference cannot be treated as funds available to investors, and thus the daily excess return rate measured by the official S&P GSCI methodology cannot precisely measure the return performance of S&P GSCI investors.

Figure 16 illustrates how the official S&P GSCI Corn ER Subindex has a different excess return measurement process from the calculated Corn ER Subindex, and consequently over-reports losses to investors. For simplicity, I assume that the S&P GSCI Corn Subindex invests in one bushel of corn in the corn futures contract, and the contract replacement period is a single day. Also recall that, the quantities of each futures contract are fixed within any calendar year. Suppose in the contract replacement period, the Corn
Subindex holds a $8 per bushel March position without any leverage, and replaces this $8 March position with a $7 per bushel May position. In effect, the March position has been replaced with the same quantity of May position. One day later, the price of the May position goes down by $1 and is now worth only $6 per bushel. The actual loss received by the Corn Subindex investor is only $1, and the actual daily rate of return is \((-\frac{1}{8})\) = -12.5%.

However, if the daily excess return rate from Day 1 to Day 2 is calculated using equation (11) in section 5.4b, the daily excess return rate from day 1 to day 2 as reported by the official S&P GSCI Corn ER Subindex will be \((-\frac{1}{7})\) = -14.28%. Furthermore, the starting fund that was invested in the S&P GSCI Corn Subindex on Day 1 is $8, so investors will lose \(8\times(-\frac{1}{7}) = -1.14\) from the Corn Subindex investment at the end of Day-2 when using the official method to measure excess returns. This is $0.14 more than the $1 actual loss, and this extra $0.14 loss explains why the official Corn ER Subindex underperforms the Calculated Corn ER Subindex based on the daily flow of funds model in Figure 12.

5.4c Equation Analysis of Divergence between Official ER and Calculated ER

As shown in section 5.2, the calculated and official Spot indexes are effectively identical, and the difference between the calculated Spot Index and calculated ER Index is equal to the cumulative term structure. Therefore, the difference between the calculated ER Index and official ER Index is equal to the additional component that explains the divergence between the official Spot and ER Index in addition to the cumulative term.
structure. In this section, I explicitly analyze the additional component by comparing the calculated ER Index equation with the official S&P GSCI ER Index equation.

I take the natural log of the difference between the compounded official daily excess return rate and the compounded daily excess return rate calculated by daily flow of funds model in each calendar year, shown by equation (12):

\[
(12) \ln \left[ \prod_{i=1}^{t} \left( \frac{F_{t-1} + M_{t}}{F_{t-1}} \right) \right] - \ln \left[ \prod_{i=1}^{t} \left( \frac{F_{t-1} + M_{t} + \sum_{i=1}^{t-1} S_{t-1}}{F_{t-1} + \sum_{i=1}^{t-1} S_{t-1}} \right) \right] = \sum_{i=1}^{t} \ln \left( \frac{F_{t-1} + M_{t}}{F_{t-1}} \right) - \sum_{i=1}^{t} \ln \left( \frac{F_{t-1} + M_{t} + \sum_{i=1}^{t-1} S_{t-1}}{F_{t-1} + \sum_{i=1}^{t-1} S_{t-1}} \right) = \sum_{i=1}^{t} \ln \left( \frac{F_{t-1} + M_{t} + \sum_{i=1}^{t-1} S_{t-1}}{F_{t-1} + \sum_{i=1}^{t-1} S_{t-1}} \right) \]

where \( F_{t-1} \) is the S&P GSCI investor fund balance at the end of day \( t-1 \) calculated by equation (8). \( \sum_{i=1}^{t-1} S_{t-1} \) is the cumulative daily term structure effect from the beginning of the year to the end of day \( t-1 \) calculated by equations (6) and (7) in section 4.3b. \( M_{t} \) is the daily profit or loss in dollars received by the S&P GSCI calculated by equation (5). \( \prod_{i=1}^{t} \left( \frac{F_{t-1} + M_{t}}{F_{t-1}} \right) \) is the compounded daily excess return used by the daily flow of funds model, and \( \prod_{i=1}^{t} \left( \frac{F_{t-1} + M_{t} + \sum_{i=1}^{t-1} S_{t-1}}{F_{t-1} + \sum_{i=1}^{t-1} S_{t-1}} \right) \) is the compounded daily excess return used by the official S&P GSCI from the beginning of the year to the end of day \( t \) derived from equation (11). The summation of \( F_{t-1} \) and \( \sum_{i=1}^{t-1} S_{t-1} \) is \( V_{t-1} \), which introduced in equation (8).
Because the ER Indexes are compounded by daily excess return rates, using the natural log of the difference between the compounded daily excess return rates decomposes the daily difference in investment returns between the official S&P GSCI ER Index and the ER Index calculated by the daily flow of funds model. The final step in equation (12) indicates that if the futures holdings of the S&P GSCI experience a profit (i.e., \( M_t \) is positive) on day \( t \), and if the cumulative term structure effect \( \sum_{i=1}^{t-1} S_{t-1} \) is positive (i.e., contango exists) from the beginning of the year to the end of day \( t-1 \), then the official S&P GSCI ER Index will under-report profits. Conversely, if the futures holdings of the S&P GSCI experience loss (i.e., \( M_t \) is negative) on day \( t \), and if the cumulative term structure effect \( \sum_{i=1}^{t-1} S_{t-1} \) is negative (i.e., backwardation exists) from the beginning of the year to the end of day \( t-1 \), then the official S&P GSCI ER Index will over-report losses. As long as \( M_t \) and \( \sum_{i=1}^{t-1} S_{t-1} \) are in the same direction, either both positive or both negative on each day, then the daily excess return of the official S&P GSCI ER Index will always be lower than the calculated ER Index. In this case, S&P GSCI ER Index fund providers will get consistent profits by under-reporting profits or over-reporting losses to their investors. However, \( M_t \) and \( \sum_{i=1}^{t-1} S_{t-1} \) may not always stay in the same direction. The different direction between \( M_t \) and \( \sum_{i=1}^{t-1} S_{t-1} \) will cause S&P GSCI fund providers to receive consistent losses by over-reporting profits or under-reporting losses to their investors.

I use the existing return performance of both the S&P GSCI Crude Oil ER Subindex and the S&P GSCI Corn ER Subindex from 2007 to 2013 to illustrate my findings in equation (12). For example, NYMEX crude oil futures were in contango in both 2007 and 2009, a long-only position was profitable, and consequently NYMEX
crude oil excess returns in 2007 and 2009 calculated by the daily flow of funds model are higher than the excess returns reported by the official S&P GSCI Crude Oil ER Subindex. For the S&P GSCI Corn ER Subindex, CBOT corn futures were in contango from 2007 to 2011. Except in 2009, when a long-only position would have experienced a substantial loss prior to the first contract replacement period, excess returns calculated by the daily flow of funds model are higher than the excess returns reported by the official S&P GSCI Corn ER Subindex when excess return is positive, and lower when excess return is negative. From 2012 to 2013 when CBOT corn futures were in backwardation, excess returns calculated by the daily flow of funds model are lower than the excess returns reported by the official S&P GSCI Corn ER Subindex when excess return is positive, and higher when excess return is negative (Table 2 and Table 4).

This analysis illustrates why the observed divergence between the official S&P GSCI Spot Index and official ER Index are not fully explained by the cumulative term structure effect alone. As shown by equation (12), the interaction between the daily profit or loss in dollars experienced by the S&P GSCI futures holdings and the cumulative term structure effect must both be taken into account to explain the divergence between the official Spot and ER Indexes.

5.5 Summary

This chapter presents the test results of this thesis. First, I use the daily flow of funds model introduced in Chapter 4 to calculate the Spot and ER Indexes. In this model, the difference between calculated Spot and calculated ER Indexes is fully explained by

29 The exception in 2009 is caused by a major loss for corn futures and occurred before the first contract replacement of the S&P GSCI Corn Subindex in 2009.
the cumulative term structure effect. Next I compare the calculated and official Spot and ER Indexes. While the calculated and official Spot Indexes are found to be identical, the calculated and official ER Indexes are statistically different. Test results indicate that the cumulative term structure effect cannot fully account for the divergence between the official Spot and official ER Indexes. Finally, I derive an equation that explains the divergence between the official ER and calculated ER Indexes. Depending on the signs of profits and the shape of the cumulative term structure (i.e., contango or backwardation), the official S&P GSCI ER Index is found to either underreport actual profits or over-report actual losses generated by S&P GSCI futures holdings from 2007 to 2013.
6. CONCLUSION

6.1 Summary and Review

This thesis analyzes the reason for the divergence between the S&P GSCI Spot Index and ER Index. From 1991 when tradable investment based on the S&P GSCI first became available to investors to the end of 2013, cumulative excess returns have typically lagged cumulative spot returns. The term structure effect, defined here as the difference between a commodity’s outgoing and incoming futures prices when contract replacement occurs, are commonly used to explain this divergence. Meanwhile, existing literature uses only the return performance of individual commodity futures to explain S&P GSCI excess returns, and little research has focused on how the official S&P GSCI excess return is measured. This thesis demonstrates how the term structure effect cannot fully explain the divergence between returns for the S&P GSCI ER Index and Spot Index.

I use a daily flow of funds model to duplicate the official S&P GSCI trading method, and to test the hypothesis that term structure effect fully explains the divergence between the official S&P GSCI Spot Index and ER Index. After a detailed analysis of the excess returns and spot returns of the S&P GSCI and four of its individual commodity futures holdings: NYMEX crude oil, NYMEX natural gas, CBOT corn, and CME live cattle, I find that the cumulative term structure alone does not explain the entire divergence between the official S&P GSCI Spot Index and ER Index. Instead, the interaction between the daily profit or loss in dollars from the S&P GSCI futures holdings and cumulative term structure effect should also be taken into account. Based on the daily
excess return equation used by the official S&P GSCI Methodology, this interaction may result in unexpected profits or losses in addition to returns received from purely investing in individual commodity futures. Depending on my test results from 2007 to 2013, the official S&P GSCI ER Index is found to either under-report actual profits or over-report actual losses. It causes investors to receive lower returns from S&P GSCI index-based investments compared to returns received from directly investing in the same amount of futures contracts held by the S&P GSCI.

6.2 Contribution to Existing Literature

Because of this interaction between the daily profits or losses on the S&P GSCI futures holdings and the cumulative term structure effect, the official S&P GSCI excess return will be less than the actual return performance if those futures holdings experience profits when the cumulative term structure effect is in contango or losses when the cumulative term structure effect is in backwardation. For the four individual commodity futures holdings of the S&P GSCI I examined, directly investing in NYMEX crude oil futures and CBOT corn futures would have generated higher returns than investing in the S&P GSCI Crude Oil ER Subindex and S&P GSCI Corn ER Subindex, respectively, from 2007 to 2013. This occurs because both NYMEX crude oil and CBOT corn were making profits from contango and losses from backwardation during this period. Results were less definitive for NYMEX natural gas and CME live cattle due to the lack of clear term structure effects during the period examined, but nonetheless are consistent with my findings regarding the interaction between profitability and cumulative term structure.
Although a more detailed examination of the relationship between term structure and commodity futures returns falls outside the scope of this study, the limited results presented here cast some doubt on the findings made by researchers such as Nash and Smyk (2003) and Erb and Harvey (2006) that backwardation is more profitable than contango when investing in futures contracts generally, or in the S&P GSCI Index specifically. My thesis focuses on a different time period, and finds that contango is more profitable than backwardation when investing in the S&P GSCI and some commodity futures like NYMEX crude oil and CBOT corn. Combining the results of Nash and Smyk, Erb and Harvey, and my own, it supports the findings of Bessembinder et al. (2012) and Sanders and Irwin (2012) that suggest commodity futures return performance may be independent of commodity futures term structure in the long run.

6.3 Implications

Commodity index funds have grown in popularity since they were introduced in the early 1990s. According to the CFTC, in 2013 more than $260 billion was invested in the long-only commodity index funds globally (CFTC 2013). Among the growing number of long-only commodity indexes, the S&P GSCI is the largest tradable commodity index with GSCI-based investments totaling more than $80 billion (Standard & Poor’s 2013a). As the largest commodity index investment portfolio, any small errors or inconsistencies in the S&P GSCI excess return measurement procedure can generate tremendous losses to its investors. The results of this thesis will be helpful for S&P GSCI investors to better understand how their investment returns are calculated. This study also exposes profitable opportunities for investors to trade between S&P GSCI-related
investment products and individual commodity futures, since both trading methods invest in the same futures contracts but receive different investment returns.
References


http://faculty.fuqua.duke.edu/~charvey/Teaching/BA453_2006/GSCI_Strategic_June_2004.ppt


## Tables

### Table 1. Comparison of Roll Returns and Excess Returns of the S&P GSCI Crude Oil Subindex, 2007 and 2009.

<table>
<thead>
<tr>
<th>Year</th>
<th>Roll Return</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>-16.809%</td>
<td>51.969%</td>
</tr>
<tr>
<td>2009</td>
<td>-77.18%</td>
<td>16.58%</td>
</tr>
</tbody>
</table>

Note: The roll return is calculated by taking the difference between the S&P GSCI Crude Oil Excess Return and Spot Return because the official S&P GSCI Methodology treat excess return to equal spot return plus roll return.

### Table 2. Comparison of Annualized Spot Returns, Official S&P GSCI vs. Calculated Using Daily Flow of Funds Model

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P GSCI Official Spot Return</td>
<td>50.53%</td>
<td>-42.58%</td>
<td>54.47%</td>
<td>14.38%</td>
<td>6.65%</td>
<td>-2.00%</td>
<td>-5.16%</td>
</tr>
<tr>
<td>S&amp;P GSCI Spot Return-Flow of Fund Model</td>
<td>50.53%</td>
<td>-42.58%</td>
<td>54.47%</td>
<td>14.38%</td>
<td>6.65%</td>
<td>-2.00%</td>
<td>-5.16%</td>
</tr>
<tr>
<td>Crude Oil Official Spot Return</td>
<td>68.87%</td>
<td>-55.17%</td>
<td>93.90%</td>
<td>6.92%</td>
<td>14.91%</td>
<td>-8.24%</td>
<td>0.51%</td>
</tr>
<tr>
<td>Crude Oil Spot Return-Flow of Fund Model</td>
<td>68.87%</td>
<td>-55.17%</td>
<td>93.90%</td>
<td>6.92%</td>
<td>14.91%</td>
<td>-8.24%</td>
<td>0.51%</td>
</tr>
<tr>
<td>Natural Gas Official Spot Return</td>
<td>22.88%</td>
<td>-26.33%</td>
<td>4.07%</td>
<td>-22.78%</td>
<td>-30.69%</td>
<td>8.19%</td>
<td>33.46%</td>
</tr>
<tr>
<td>Natural Gas Spot Return-Flow of Fund Model</td>
<td>22.88%</td>
<td>-26.33%</td>
<td>4.07%</td>
<td>-22.77%</td>
<td>-30.69%</td>
<td>8.19%</td>
<td>33.46%</td>
</tr>
<tr>
<td>Corn Official Spot Return</td>
<td>26.61%</td>
<td>-10.67%</td>
<td>0.24%</td>
<td>44.19%</td>
<td>6.89%</td>
<td>6.53%</td>
<td>-37.86%</td>
</tr>
<tr>
<td>Corn Spot Return-Flow of Fund Model</td>
<td>26.61%</td>
<td>-10.67%</td>
<td>0.24%</td>
<td>44.19%</td>
<td>6.89%</td>
<td>6.53%</td>
<td>-37.86%</td>
</tr>
<tr>
<td>Live Cattle Official Spot Return</td>
<td>2.19%</td>
<td>-9.33%</td>
<td>0.17%</td>
<td>24.21%</td>
<td>12.74%</td>
<td>10.53%</td>
<td>2.65%</td>
</tr>
<tr>
<td>Live Cattle Spot Return-Flow of Fund Model</td>
<td>2.19%</td>
<td>-9.33%</td>
<td>0.17%</td>
<td>24.21%</td>
<td>12.74%</td>
<td>10.53%</td>
<td>2.65%</td>
</tr>
</tbody>
</table>
Table 3. Ratio-Paired t-Tests for Daily Values of $F_t$ and $CF_t$

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P GSCI Ratio Paired T-Value</td>
<td>15.29</td>
<td>-8.32</td>
<td>15.91</td>
<td>1.29</td>
<td>6.63</td>
<td>-13.21</td>
<td>-23.20</td>
</tr>
<tr>
<td>S&amp;P GSCI Ratio Paired P-Value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.199</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Crude Oil Ratio Paired P-Value</td>
<td>0.000</td>
<td>0.042</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Natural Gas Ratio Paired T-Value</td>
<td>-11.02</td>
<td>-10.21</td>
<td>-8.19</td>
<td>-7.33</td>
<td>-7.79</td>
<td>3.79</td>
<td>0.56</td>
</tr>
<tr>
<td>Natural Gas Ratio Paired P-Value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Corn Ratio Paired T-Value</td>
<td>-10.44</td>
<td>-7.53</td>
<td>-11.16</td>
<td>11.56</td>
<td>18.82</td>
<td>-16.37</td>
<td>14.39</td>
</tr>
<tr>
<td>Corn Ratio Paired P-Value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Live Cattle Ratio Paired T-Value</td>
<td>2.34</td>
<td>-10.54</td>
<td>-17.67</td>
<td>29.92</td>
<td>-4.31</td>
<td>1.54</td>
<td>-27.36</td>
</tr>
<tr>
<td>Live Cattle Ratio Paired P-Value</td>
<td>0.020</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.125</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: $t = \ln \left( \frac{F_t}{CF_t} \right)$, $H_0: t = 0$; $H_a: t \neq 0$

Table 4. Comparison of Annualized Excess Returns, Official S&P GSCI vs. Calculated Using Daily Flow of Funds Model

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P GSCI Official ER</td>
<td>35.66%</td>
<td>-47.08%</td>
<td>16.45%</td>
<td>3.40%</td>
<td>3.20%</td>
<td>-2.26%</td>
<td>-4.25%</td>
</tr>
<tr>
<td>S&amp;P GSCI ER - Flow of Fund Model</td>
<td>38.65%</td>
<td>-49.09%</td>
<td>23.02%</td>
<td>4.49%</td>
<td>3.25%</td>
<td>-2.34%</td>
<td>-4.26%</td>
</tr>
<tr>
<td>Crude Oil Official ER</td>
<td>51.97%</td>
<td>-57.68%</td>
<td>16.58%</td>
<td>-7.37%</td>
<td>4.80%</td>
<td>-12.70%</td>
<td>-0.65%</td>
</tr>
<tr>
<td>Crude Oil ER - Flow of Fund Model</td>
<td>58.84%</td>
<td>-57.54%</td>
<td>35.11%</td>
<td>-6.80%</td>
<td>5.16%</td>
<td>-12.82%</td>
<td>-0.44%</td>
</tr>
<tr>
<td>Natural Gas Official ER</td>
<td>-20.49%</td>
<td>-37.94%</td>
<td>-53.24%</td>
<td>-41.62%</td>
<td>-43.43%</td>
<td>-27.85%</td>
<td>15.48%</td>
</tr>
<tr>
<td>Natural Gas ER - Flow of Fund Model</td>
<td>-25.50%</td>
<td>-43.67%</td>
<td>-54.33%</td>
<td>-42.25%</td>
<td>-48.28%</td>
<td>-29.92%</td>
<td>17.04%</td>
</tr>
<tr>
<td>Corn Official ER</td>
<td>5.97%</td>
<td>-23.09%</td>
<td>-11.69%</td>
<td>24.02%</td>
<td>5.13%</td>
<td>17.28%</td>
<td>-28.40%</td>
</tr>
<tr>
<td>Corn ER - Flow of Fund Model</td>
<td>8.38%</td>
<td>-27.10%</td>
<td>-11.66%</td>
<td>29.56%</td>
<td>5.37%</td>
<td>14.46%</td>
<td>-24.83%</td>
</tr>
<tr>
<td>Live Cattle Official ER</td>
<td>-7.56%</td>
<td>-26.35%</td>
<td>-9.91%</td>
<td>13.41%</td>
<td>-0.48%</td>
<td>-2.59%</td>
<td>-4.23%</td>
</tr>
<tr>
<td>Live Cattle ER - Flow of Fund Model</td>
<td>-8.10%</td>
<td>-31.01%</td>
<td>-10.51%</td>
<td>14.19%</td>
<td>-0.66%</td>
<td>-2.38%</td>
<td>-4.14%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Official Spot Return</th>
<th>Spot Return-Flow of Funds Model</th>
<th>Official Excess Return</th>
<th>Excess Return-Flow of Funds Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P GSCI</td>
<td>51.40%</td>
<td>51.40%</td>
<td>-16.53%</td>
<td>-12.41%</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>66.25%</td>
<td>66.25%</td>
<td>-36.87%</td>
<td>-22.49%</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>-30.48%</td>
<td>-30.48%</td>
<td>-93.98%</td>
<td>-95.54%</td>
</tr>
<tr>
<td>Corn</td>
<td>15.68%</td>
<td>15.68%</td>
<td>-21.21%</td>
<td>-18.01%</td>
</tr>
<tr>
<td>Live Cattle</td>
<td>47.48%</td>
<td>47.47%</td>
<td>-35.42%</td>
<td>-39.77%</td>
</tr>
</tbody>
</table>

Table 6. Annualized Excess Returns, Official S&P GSCI vs. Expected

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
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<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
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<td>-47.08%</td>
<td>16.45%</td>
<td>3.40%</td>
<td>3.20%</td>
<td>-2.26%</td>
<td>-4.25%</td>
</tr>
<tr>
<td>S&amp;P GSCI ER - Expected</td>
<td>35.74%</td>
<td>-47.06%</td>
<td>16.46%</td>
<td>3.36%</td>
<td>3.30%</td>
<td>-2.24%</td>
<td>-4.27%</td>
</tr>
<tr>
<td>Crude Oil Official ER</td>
<td>51.97%</td>
<td>-57.68%</td>
<td>16.58%</td>
<td>-7.37%</td>
<td>4.80%</td>
<td>-12.70%</td>
<td>-0.65%</td>
</tr>
<tr>
<td>Crude Oil ER - Expected</td>
<td>51.97%</td>
<td>-57.68%</td>
<td>16.58%</td>
<td>-7.37%</td>
<td>4.80%</td>
<td>-12.70%</td>
<td>-0.65%</td>
</tr>
<tr>
<td>Natural Gas Official ER</td>
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<td>-37.94%</td>
<td>-53.24%</td>
<td>-41.62%</td>
<td>-43.43%</td>
<td>-27.85%</td>
<td>15.48%</td>
</tr>
<tr>
<td>Natural Gas ER - Expected</td>
<td>-20.49%</td>
<td>-37.95%</td>
<td>-53.24%</td>
<td>-41.62%</td>
<td>-43.44%</td>
<td>-27.85%</td>
<td>15.50%</td>
</tr>
<tr>
<td>Corn Official ER</td>
<td>5.97%</td>
<td>-23.09%</td>
<td>-11.69%</td>
<td>24.02%</td>
<td>5.13%</td>
<td>17.28%</td>
<td>-28.40%</td>
</tr>
<tr>
<td>Corn ER - Expected</td>
<td>5.97%</td>
<td>-23.11%</td>
<td>-11.69%</td>
<td>24.00%</td>
<td>5.12%</td>
<td>17.28%</td>
<td>-28.36%</td>
</tr>
<tr>
<td>Live Cattle Official ER</td>
<td>-7.56%</td>
<td>-26.35%</td>
<td>-9.91%</td>
<td>13.41%</td>
<td>-0.48%</td>
<td>-2.59%</td>
<td>-4.23%</td>
</tr>
<tr>
<td>Live Cattle ER - Expected</td>
<td>-7.56%</td>
<td>-26.35%</td>
<td>-9.91%</td>
<td>13.41%</td>
<td>-0.48%</td>
<td>-2.59%</td>
<td>-4.23%</td>
</tr>
</tbody>
</table>
Figures

Figure 1. Official S&P GSCI Spot Index vs. Official S&P GSCI ER Index, 1991–2013

![Graph showing the comparison of Official S&P GSCI Spot Index and Official S&P GSCI ER Index from 1991 to 2013. The ER Index is set equal to the S&P GSCI Spot Index at 465.76 on January 8, 1991 to allow comparison of the two indexes.]

Note: ER Index was set equal to S&P GSCI Spot Index at 465.76 on January 8, 1991 to allow comparison of the two indexes.

Figure 2. Futures Term Structure Curve

![Graph showing the futures term structure curve with dates on the x-axis and index value on the y-axis, along with specific values at different dates.]

Source: Burton & Karsh 2009
Figure 3. Annualized Total Return vs. Percentage of Time in Backwardation

Source: Nash and Smyk 2003

Figure 4. Using the Information in the GSCI Term Structure for TAA, July 1992 – May 2004

<table>
<thead>
<tr>
<th>Portfolio Strategy</th>
<th>Compound Annualized Excess Return</th>
<th>Annualized Standard Deviation</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long if GSCI backwarded</td>
<td>11.25%</td>
<td>18.71%</td>
<td>0.60</td>
</tr>
<tr>
<td>Long if GSCI contangoed</td>
<td>-5.01</td>
<td>17.57</td>
<td>-0.29</td>
</tr>
<tr>
<td>Long if GSCI backwarded; short if GSCI contangoed</td>
<td>8.18</td>
<td>18.12</td>
<td>0.45</td>
</tr>
<tr>
<td>Long cash-collateralized GSCI</td>
<td>2.68</td>
<td>18.23</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Source: Erb and Harvey 2006
Figure 5. S&P GSCI Contract Replacement Procedure Used by Burton & Karsh

<table>
<thead>
<tr>
<th>Step</th>
<th>Formula</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy:</td>
<td>Allocation ( t-1 ) / Price of Near Future ( t-1 ) = Number of Contracts purchased</td>
<td>( \frac{$10,000}{$100} = 100 ) contracts</td>
</tr>
<tr>
<td>Sell:</td>
<td>Number of Contracts * Price of Near Future ( t ) = Total Value ( t )</td>
<td>( 100 ) contracts * ( $110 ) = ( $11,000 )</td>
</tr>
<tr>
<td>Buy:</td>
<td>Allocation ( t ) / Price of Deferred Future ( t ) = Number of Contracts purchased</td>
<td>( \frac{$11,000}{$143} = 76.9 ) contracts</td>
</tr>
<tr>
<td>Sell:</td>
<td>Number of Contracts * Price of Deferred Contract ( t+1 ) = Total Value ( t+1 )</td>
<td>( 76.9 ) * ( $157.3 ) = ( $12,100 )</td>
</tr>
<tr>
<td>Excess Return</td>
<td>(Total Value-Initial Allocation) / Initial Allocation = Excess Return</td>
<td>( \frac{$12,100 - $10,000}{$10,000} = 21% )</td>
</tr>
</tbody>
</table>

Source: Burton and Karsh 2009

Figure 6. Official S&P GSCI Crude Oil Spot Subindex vs. Calculated Crude Oil Spot Subindex Using the Daily Flow of Funds Model
Figure 7. Official S&P GSCI Corn Spot Subindex vs. Calculated Corn Spot Subindex Using the Daily Flow of Funds Model

Figure 8. Official S&P GSCI Spot Index vs. Calculated Spot Index Using the Daily Flow of Funds Model
Figure 9. Official S&P GSCI Natural Gas Spot Subindex vs. Calculated Natural Gas Spot Subindex Using the Daily Flow of Funds Model

Figure 10. Official S&P GSCI Live Cattle Spot Subindex vs. Calculated Live Cattle Spot Subindex Using the Daily Flow of Funds Model
Figure 11. Official S&P GSCI Crude Oil ER Subindex vs. Calculated Crude Oil ER Subindex Using the Daily Flow of Funds Model

Figure 12. Official S&P GSCI Corn ER Subindex vs. Calculated Corn ER Subindex Using the Daily Flow of Funds Model
Figure 13. Official S&P GSCI ER Index vs. Calculated ER Index Using the Daily Flow of Funds Model

Figure 14. Official S&P GSCI Natural Gas ER Subindex vs. Calculated Natural Gas ER Subindex Using the Daily Flow of Funds Model
Figure 15. Official S&P GSCI Live Cattle ER Subindex vs. Calculated Live Cattle ER Subindex Using the Daily Flow of Funds Model

Figure 16. Illustration of How the Official S&P GSCI Corn ER Subindex Over-Reports Losses to Investors

**End of Day 1:**

Step 1: Hold **one March contract at $8** without any leverage (before roll)

Fix Quantity Roll

Step 2: Hold **one May contract at $7** (after roll)

May contract goes down by $1 from day 1 to day 2

**End of Day 2:**

Step 3: Hold **one May contract at $6**

Actual Loss from Step 1 to Step 3 = $8 * \( \frac{-1}{8} \) = ($-1)

Starting Fund Balance

Investor Fund Balance on Day 1

The official S&P GSCI Corn ER = $8 * \( \frac{-1}{7} \) = ($-1.1429)

Starting Fund Balance

Index Total Dollar Value on Day 1