Cover page

Title: **Matrix-based System Reliability Analysis of Urban Infrastructure Networks: A Case Study of MLGW Natural Gas Network**

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MATRIX-BASED SYSTEM RELIABILITY ANALYSIS OF URBAN INFRASTRUCTURE NETWORKS: A CASE STUDY OF MLGW NATURAL GAS NETWORK

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ABSTRACT

Urban infrastructure systems such as utility networks for electricity-, water-, sewage- and gas-services, transportation systems and telecommunication networks are critical backbones of modern societies. However, these systems are often susceptible to natural and man-made hazards. Structural damages of components in these infrastructure networks not only disrupt residential and commercial activities, but also impair post-disaster response and recovery efforts, resulting in substantial socio-economic losses. Therefore, estimating the reliability of these infrastructure systems is vital to urban stakeholders such as utility companies, urban planners, and policy makers as well as to residents and business owners. Evaluation of the performance and connectivity of such urban infrastructure systems is complex in nature due to the large number of network components, complex network topology and component/system interdependencies. Due to this complexity, network reliability analysis is often performed by repeated network analyses for random samples of hazard scenarios and component status, which prevents rapid risk assessment and near-real-time risk-informed decision making. In this paper, an analytical, i.e. non-sampling-based network reliability analysis method is proposed for urban infrastructure systems. First, a review of previous research on network reliability analysis is given, followed by a brief summary of the recursive decomposition algorithm (RDA) and the matrix-based system reliability (MSR) method that constitute the proposed network reliability analysis methodology. As a case study, the proposed methodology is applied to a Memphis Light, Gas and Water (MLGW) natural gas network of Shelby County of Tennessee. Based on seismic hazard maps from the Mid-America Earthquake Center's risk management software *MAEviz* and the topological characteristics of an MLGW gas network, the reliability of the network components are computed by use of a geographic information system, *ArcGIS*. All the disjoint cut sets and link sets of the simplified MLGW gas network are efficiently identified by use of the RDA. The MSR method, which can account for statistical dependence between components and incomplete information, is employed to evaluate the connectivity reliability of the gas network in an efficient manner. The results of the system reliability analysis are presented for earthquake scenarios with different magnitudes. By integrating the MSR method with an advanced network analysis algorithm such as the RDA, we can perform network reliability analysis for a complex infrastructure system without random samplings. This approach will enable us to perform risk analysis and various statistical inferences rapidly. Based on the results of the case study, further research is in progress to (1) account for the statistical dependence between seismic intensities at adjacent network components, (2) estimate the average downtime and socio-economic losses, (3) account for the interdependency between different infrastructure systems, and (4) provide useful information for decision making on disaster planning, response, recovery, and mitigation using the proposed methodology.

1 INTRODUCTION

Urbana infrastructure systems such as power, water, sewage, telecommunication and transportation networks are critical backbones of modern societies. In the past 20 years, the disruptions of various lifeline systems and equipments induced by man-made and natural hazards have caused severe socio-economic consequences around the world (Abraham et al., 2004; Chang et al., 1996; Wu et al., 2006). It also has been learned that lifeline systems play a critical role in disaster planning, mitigation, response and recovery (Chang et al., 2001). With the fast development of the world economy and higher security requirements, the safety of lifeline systems are of greater concern these days. Therefore, estimating the reliability of these infrastructure systems is vital to urban stakeholders such as utility companies, urban planners, and policy makers as well as to residents and business owners. Evaluation of the performance and connectivity of such urban infrastructure systems is complex in nature due to the large number of network components, complex network topology, and component/system interdependency. Due to this complexity, network reliability analysis is often performed by use of random samplings of hazard scenarios and component status, which prevents rapid risk assessment and decision makings during or after a hazardous event. This paper proposes a new non-sampling-based system reliability method for lifeline networks and demonstrates it through a case study of a Memphis Light, Gas and Water (MLGW) natural gas network. After a brief summary of previous research on network reliability analysis of civil infrastructure systems, this paper proposes a new network reliability analysis methodology based on the matrix-based system reliability (MSR) analysis method (Kang et al., 2007; Song and Kang, 2007). It employs an advanced network algorithm such as the recursive decomposition algorithm (RDA; Li and He, 2002). The results and observations from the case study are summarized, and the identified future research topics are also presented.

2 NETWORK RELIABILITY ANALYSIS FOR CIVIL INFRASTRUCTURE SYSTEMS

The uncertainty in the performance of lifeline networks arises not only from the components (e.g. pipelines or substations) and the configuration of a network, but also from the environment such as the magnitude of an earthquake and the maximum sustained wind speed. The reliability of a network can be measured in three perspectives (Li, 2005): (1) structural component reliability; (2) connectivity reliability between node pairs; and (3) performance reliability, e.g., maintaining minimum water head/pressure. This paper focuses on analytical, i.e. non-sampling-based reliability analysis on the connectivity. The basic idea of analytical network reliability analysis is to convert a complex network to the combination of simple sub-networks such as parallel or series systems, and then compute the network reliability from the probabilities of the sub-systems. Kroft (1967) first used a shortest path algorithm to compute network reliability. Panoussis (1974) and Taleb-Agha (1975 and 1977) proposed to compute general network reliability by converting complex lifeline networks to “series systems in parallel” (SSP) networks. However, finding the shortest paths is not always an easy task, especially for large-scale networks. Aggarwal and Misra (1975) proposed disjoint shortest path algorithm. Later, researchers (Dotson and Gobien, 1979; Yoo and Deo, 1988; Torrieri, 1994; Li and He, 2002) improved this algorithm to estimate the exact reliability of large-scale complex networks. A full probability analytic algorithm (Wu and Sha, 1998) and ordered binary decision diagram (OBDD) algorithm (Kuo et. al, 1999) are also capable of finding exact network reliability, but neither of them is able to handle large-scale networks. The RDA (Li and He, 2002) is efficient in identifying cut sets or link sets for a wide class of networks, but it is not able to account for the statistical dependence between the demands or capacities of network components. Recently, Kang et al. (2007) proposed a matrix-based system reliability (MSR) method, which can estimate the reliability of general systems through efficient matrix-calculations with the statistical dependence considered. This paper proposes to use the MSR method in conjunction with a network analysis algorithm such as the RDA.

3 PROPOSED METHODOLOGY

The MSR method subdivides the sample space of component events with s_i distinct states, $i = 1,...,n,$ into $m = \Pi_{i=1}^{n} s_i$ mutually exclusive and collectively exhaustive (MECE) events. The probability of any general system event is then described by the inner product of two vectors:

$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p} \tag{1}$$

where $\mathbf{c}$ is the "event vector" whose element is 1 if its corresponding MECE event is included in the target system E_{sys} event, and 0 otherwise; and $\mathbf{p}$ is the "probability vector" that contains the probabilities of all the MECE events. Efficient matrix-based procedures were proposed to efficiently obtain these vectors by use of matrix computer language such as Matlab® (Kang et al., 2007). The MSR method has the following merits over existing system reliability methods. First, the probability of a system event is always calculated by a simple matrix multiplication as in Equation 1 regardless of the complexity of the system event definition. Second, the MSR method separates the tasks of identification of system event $(\mathbf{c})$ and computation of probability calculations $(\mathbf{p})$, which allows for an easy integration with other computation modules, e.g. geographic information system (GIS) or network analysis algorithms. Moreover, the matrix-based procedures proposed along with the method help obtain $\mathbf{c}$ and $\mathbf{p}$ vectors efficiently. Third, even if one has incomplete information on the component failure probabilities and/or their statistical dependence, the matrix-based framework still enables us to obtain the narrowest possible bounds on any general system event (Song and Der Kiureghian, 2003) based on the available information. Fourth, once $P(E_{sys})$ is obtained, one can calculate the probabilities of other system events of interest, conditional probabilities and component importance measures (Song and Der Kiureghian, 2005) without additional probability calculations.

A drawback of the MSR method is that the size of the vectors increases exponentially with the number of component events. This may be a critical issue in case a network with a large number of components is considered. However, this can be overcome by a multi-scale approach (Der Kiureghian and Song 2007) or subdivide the system event into multiple disjoint link sets or cut sets constituted by smaller number of components. When disjoint cut sets or link sets, $S_i,\ i = 1,...,N_{set}$ are identified, the system failure probability or reliability is computed by summing up the results of the MSR analyses of individual sets, i.e.

$$P(E_{sys}) = \sum_{i=1}^{N_{set}} P(S_i) = \sum_{i=1}^{N_{set}} \mathbf{c}_i^{\mathrm{T}}\mathbf{p}_i \tag{2}$$

where $\mathbf{c}_i$ and $\mathbf{p}_i$ are the event and probability vectors of the *i*-th disjoint cut set or link set. These disjoint sets can be efficiently identified by making use of an advanced network algorithm such as RDA (Li and He, 2002).

RDA recursively decomposes the network into sub-graphs until there exist no paths between the source and terminal nodes in all sub-graphs. When paths between source and terminal are found in sub-graphs, they are disjoint link sets and thus contribute to the network reliability; for those sub-graphs containing no paths, they are disjoint cut sets that contribute to the system failure probability. RDA is applicable to all types of networks regardless of their size or topology.

After the Boolean descriptions of the disjoint cut sets and link sets are identified, the corresponding event vectors $\mathbf{c}_i$'s are obtained by use of the following matrix-based procedures (Kang et al., 2007):

$$\begin{aligned} \mathbf{c}^{\bar{E}} &= \mathbf{1} - \mathbf{c}^{E} \\ \mathbf{c}^{E_1 \cdots E_n} &= \mathbf{c}^{E_1} .* \mathbf{c}^{E_2} .* \cdots .* \mathbf{c}^{E_n} \\ \mathbf{c}^{E_1 \cup \cdots \cup E_n} &= \mathbf{1} - (\mathbf{1} - \mathbf{c}^{E_1}) .* (\mathbf{1} - \mathbf{c}^{E_2}) .* \cdots .* (\mathbf{1} - \mathbf{c}^{E_n}) \end{aligned} \tag{3}$$

where **1** denotes a vector of 1's; and "**.***" is the Matlab® operator for element-by-element multiplication.

When component events are statistically independent of each other, the probability vectors $\mathbf{p}_i$'s are constructed based on the failure probabilities of the lifeline network elements by the following recursive matrix-based procedure:

$$\mathbf{p}_i^{[1]} = \begin{bmatrix} P_1 & 1-P_1 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{p}_i^{[j]} = \begin{bmatrix} \mathbf{p}_i^{[j-1]} \cdot P_j \\ \mathbf{p}_i^{[j-1]} \cdot (1-P_j) \end{bmatrix} \quad \text{for } j = 2,3,\ldots,n_i \tag{4}$$

where n_i is the number of the components in the *i*-th cut set or link set, P_j is the failure probability or reliability of the *j*-th component in the set; and $\mathbf{p}_i = \mathbf{p}_i^{[n_i]}$. The probability vectors can be obtained even in case the components have statistical dependence (Kang et al., 2007).

As seen in Equation 4, the failure probabilities or reliabilities of the network elements, P_j need to be obtained. For lifeline systems subjected to seismic hazard, the failure probabilities of network elements are evaluated based on seismic intensities measures such as peak ground acceleration (PGA), peak ground velocity (PGV), or permanent ground deformation (PGD). According to the Guidelines by American Lifeline Alliance (ALA, 2001), the seismic damages of gas pipelines are affected by the earthquake intensity, soil condition, pipeline material (e.g. cast iron), type of connector, etc. A regional risk assessment software *HAZUS-MH* (FEMA, 2003) uses a PGV-based damage model for gas pipelines in which the rate of repairs (per unit kilometer) is given as $\lambda \cong 0.0001 \times PGV^{2.25}$ where PGV is given in cm/s. In the case study of this paper, we compute the repair rates of the pipeline elements using this PGV-based damage model in conjunction with the PGV values by the Mid-America Earthquake Center's risk management software *MAEViz*, which generates PGV values at each location for a scenario earthquake (Fernandez and Rix, 2006). The repair rate of each link is then calculated using *ArcGIS*'s Spatial Analyst extension.

In the case study of this paper, the failures along a pipeline are modeled as a non-homogeneous Poisson process. Further research efforts are currently being made to account for the statistical dependence between the failures at adjacent locations by use of theories of stochastic processes. The failure probability of the *j*-th line element is given as

$$P_j = 1 - e^{-\int_0^{L_j} \lambda_j(l)\cdot dl} \cong 1 - e^{-\sum_{k=1}^{N_j} \lambda_j^k \Delta L_k} \tag{5}$$

where $\lambda_j(\cdot)$ and L_j are the repair rate function and the total length of the *j*-th pipeline, respectively; λ_j^k and ΔL_k are the representative repair rate and the length of the *k*-th segment; and N_j is the number of the discretized segments for the *j*-th pipeline.

The failure probabilities of the nodes are also obtained. The PGA data of *MAEviz* are first imported into *ArcGIS*, and the failure probabilities of the nodes are obtained by use of the fragility curves of compressor station by *HAZUS-MH*. Since *HAZUS-MH* does not have the fragility curves for gate stations, we assume the gate station owns the same fragility property as compressor stations in the case study. It is also assumed that a node loses its connectivity when it is in either "extensive" or "complete" damage state. The probability of exceeding "extensive" damage state is given by

$$P(exceeds\, extensive \mid PGA = a) = \Phi\left(\frac{\ln a - \ln 0.77}{0.65}\right) \tag{6}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

4 CASE STUDY: MLGW NATURAL GAS NETWORK

Earthquake impacts on urban natural gas system may cause not only outage of gas services, but also post-disaster fires, which may result in serious consequences, e.g. the 1923 Great Kanto Earthquake. The area of this case study, Shelby County of Tennessee, including the city of Memphis, is a major urban area with a population more than 911,438 (US Census 2006 data). Memphis Light, Gas, and Water Division (MLGW) provides utility services to more than 420,000 customers (MLGW, 2007) in this area. The natural gas facilities operated by MLGW cover 750,000 square mile service territory (Benson, 1992) through more than 3,500 miles of mains and 2,200 miles of service lines (Chang et al., 1996). For the purpose of demonstrating the proposed methodology without undue complexity, we use a simplified gas transmission network model shown in Figure 1 (Chang et al., 1996), which consists of 16 nodes (gas stations and regulator stations) and 17 links (transmission lines). The gate stations, where MLGW receives the purchased gas are considered supply facilities or source nodes for the given gas network. The gas transmission mains are considered distributing elements while regulator stations and other service facility considered demand nodes. Although the network used in this research only contains 33 elements, a multi-scale approach (Der Kiureghian and Song, 2007) can be used to assess the system reliability by aggregating or disaggregating service areas into equivalent service nodes.

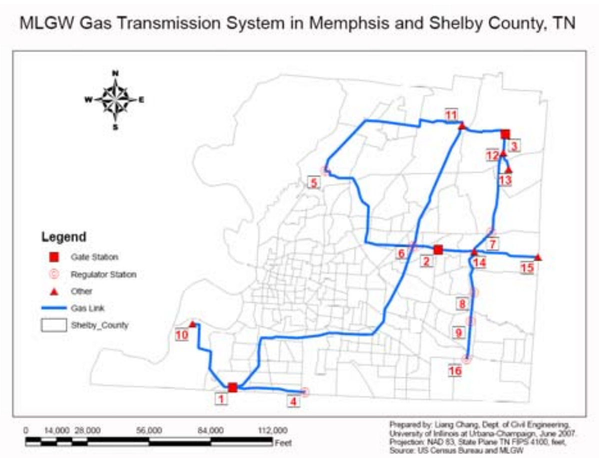

Figure 1. MLGW gas network in Shelby County, TN (modified from Chang et al., 1996).

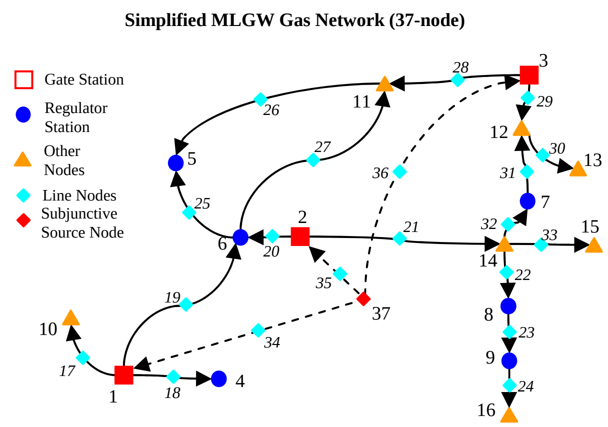

Figure 2. Directed graph model of MLGW gas network.

To apply RDA to the MLGW gas network, we first represent the network as a directed graph model shown in Figure 2. The arrows indicate the directivity of the gas flow, from source to customers, and from pipelines with high pressure to those with low pressure (Bowker, 2007). A subjunctive source node, *Node 37* is added to the graph to facilitate finding the multi-source network's node-pair connectivity (Li and He, 2002) and the pipelines are replaced with virtual "link" nodes in the graph. As a result, the simplified network in Figure 1 is represented by a 37-node and 40-arcs network in Figure 2. For example, if we want to evaluate the connectivity between sources and the service area represented by *Node 6* (*source node* = 37 and *terminal node* = 6), RDA identifies four disjoint link sets: {(37, 34, 1, 19, 6), ($\overline{34}$, 2, 6, 20, 35, 37), (1, 2, 6, 20, 34, 35, 37), ($\overline{19}$, 1, 2, 6, 20, 34, 35, 37)} in which the numbers with and without an upper bar indicate the failures and survivals of the corresponding elements, respectively.

Based on the seismic hazard maps from *MAEViz* software as shown in Figure 3, we calculate the repair rates of links (Figure 4), the failure probabilities of links (Equation 5) and the failure probabilities of nodes (Equation 6), all by use of *ArcGIS*. In this case study, we use a set of scenario earthquakes with different magnitudes with the epicenter N35.535° W-90.43° at Blytheville, AR.

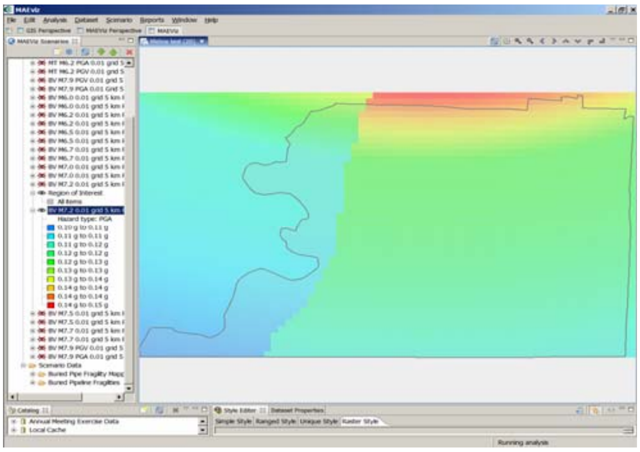

Figure 3. PGA map of Shelby County, TN by *MAEviz*.

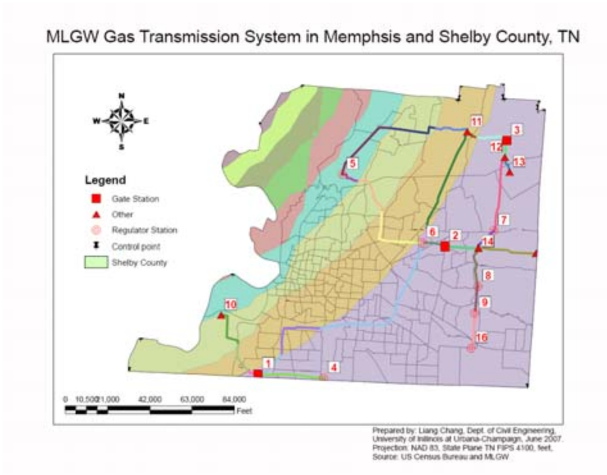

Figure 4. Repair rates of MLGW gas network computed by ArcGIS.

The event (**c**) and probability (**p**) vectors are obtained for each link set or cut set by use of matrix-based procedures in Equations 3 and 4. The probability of outage at each distribution node, i.e. its disconnection from all the sources is computed by Equation 2. Figure 5 shows the reliability and failure probabilities of network nodes as a result of the system reliability analysis. Figure 6 shows the probability of outage at *Node 6* with earthquake magnitude in the earthquake scenarios varied. The results are then imported into *ArcGIS* for displaying spatial distribution of the reliabilities/failure probabilities. Figure 7 shows the system connectivity failure probabilities of the nodes (bar graphs) and the links (colors) for the given earthquake scenario with magnitude M = 7.2.

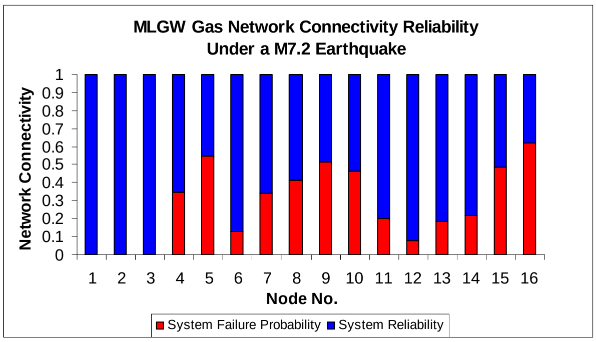

Figure 5. Connectivity reliabilities and failure probabilities of network nodes.

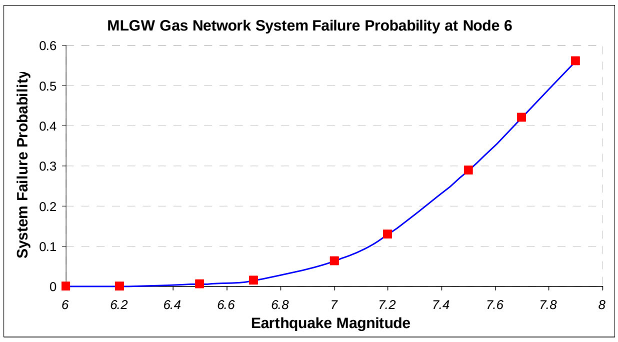

Figure 6. Failure probability at *Node 6* versus earthquake magnitude.

5 CONCLUSIONS

A new approach for network reliability analysis is proposed based on a matrix-based system reliability method. The outage or availability of distribution nodes are identified as disjoint cut sets or link sets by use of an advanced network analysis algorithm such as the recursive decomposition algorithm. The probability of each disjoint link set or cutest is computed by use of matrix-based procedures of the matrix-based system reliability method. The proposed methodology is demonstrated through a case study on the Memphis Light, Gas and Water (MLGW)'s natural gas network in Shelby County, TN, but it can be applied to general lifeline systems for evaluation of their connectivity reliabilities. This approach, in essence, reduces the size and complexity of the large-scale urban infrastructure systems by decomposing the complex system into sub-systems whose reliabilities are easy to evaluate. With this approach, we are able to handle large-scale infrastructure systems and obtain valuable statistical inferences. Based on the results of the case study, further research is in progress to (1) account for the statistical dependence between seismic intensities at adjacent network components, (2) estimate the average downtime and socio-economic losses, (3) account for the interdependency between different infrastructure systems, and (4) provide useful information for decision making on disaster planning, response, recovery, and mitigation using the proposed methodology.

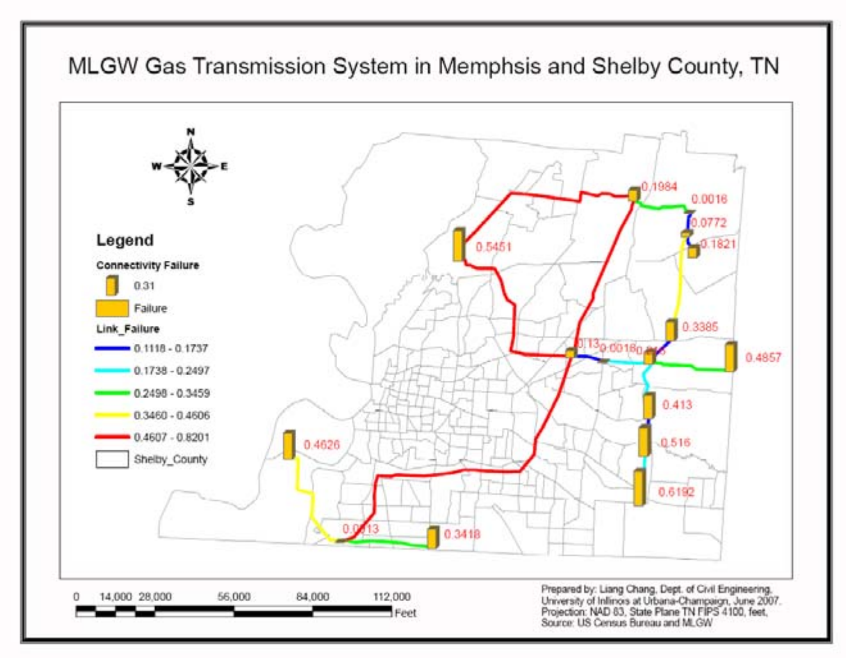

Figure 7. Connectivity failure probabilities of MLGW gas network (*ArcGIS* presentation)

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