SYSTEMATIC TREATMENT OF UNCERTAINTY IN CONSEQUENCE-BASED RISK MANAGEMENT OF SEISMIC REGIONAL LOSSES

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A Report of the Mid-America Earthquake Center

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The Mid-America Earthquake (MAE) center aims to treat various uncertainties inherent in its Consequence-based Risk Management (CRM) in a systematic manner. In order to achieve this goal, a Task Group on Interdisciplinary Coordination (TGIC) of the MAE center develops a probabilistic framework to estimate the uncertainty in social and economic losses in a region caused by seismic hazard. This document presents the probabilistic framework under development with a numerical example. The total direct loss of an inventory of three buildings is estimated with its uncertainty quantified. We incorporate the uncertainties in the intensity of a scenario earthquake, inventory identification, performance of structural/non-structural components, content loss, liquefaction hazard, and damage states. Examples on the use of a probabilistic hazard map in regional loss estimation and on the direct loss of a bridge inventory are currently under development.

I. Inventory data and scenario seismic hazard

For simplicity, this example considers the total loss of three building inventory items in the Memphis test bed region. Table 1 lists the structural and occupancy types of the inventory items, the fundamental periods (T_e) of the structures, the mean (λ_{S_a}) and standard deviation (β_{S_a}) of the natural logarithm of the spectral acceleration (S_a) at each inventory location, and their assessed structural values (M). URM denotes unreinforced masonry building.

Table 1. Example data and scenario hazard

No.	Structural type	Occupancy type	T_e (sec)	$\ln S_a$		M (US \$)
				λ_{S_a}	β_{S_a}	
1	Concrete	Industrial	0.95	-1.710	0.887	136,400
2	URM	Commercial	0.60	-1.463	0.827	415,393
3	URM	Industrial	0.60	-1.514	0.840	811,346

II. Structural damage

II-1. Structural damage fragility and limit-state exceedance probability

The fragility $P(LS_i \mid S_a)$ is defined as the conditional probability that a certain type of structure will exceed the prescribed limit state LS_i for a given spectral acceleration S_a. The fragilities developed by the MAE center can be described as

$$P(LS_i \mid S_a) = \Phi\left(\frac{\ln S_a - \lambda_i}{\beta_i}\right) \quad (1)$$

where $\Phi(\cdot)$ is the cumulative density function (CDF) of the standard normal distribution, and λ_i and β_i are the fragility parameters for the *i*-th limit state of a given structural type. This form of fragility is being internally referred as "Type I."

There also exist MAE center fragilities described in terms of drift (Wen et al. 2004).

$$P(LS_i \mid S_a) = \Phi\left(-\frac{\lambda_C^i - \lambda_{D|S_a}}{\sqrt{\beta_C^2 + \beta_{D|S_a}^2 + \beta_M^2}} \right) \tag{2}$$

where λ_C^i denotes the natural logarithm of the median drift capacity for the *i*-th limit state, $\lambda_{D|S_a}$ is the natural logarithm of the median drift demand determined from a fitted power law equation (Cornell et al. 2002) for a given spectral acceleration, and β_C, $\beta_{D|S_a}$ and β_M are the standard deviation of the natural logarithm of the capacity, demand and model error, respectively. When the power law is defined as $D = a_1 (S_a)^{a_2}$, the parameters of the Type I fragilities are

$$\lambda_i = \frac{(\lambda_C^i - \ln a_1)}{a_2} \tag{3a}$$

$$\beta_i = \frac{\sqrt{\beta_C^2 + \beta_{D|S_a}^2 + \beta_M^2}}{a_2} \tag{3b}$$

The exceedance probability for an unknown spectral acceleration is derived as

$$P(LS_i) = \Phi\left(\frac{\lambda_{S_a} - \lambda_i}{\sqrt{\beta_i^2 + \beta_{S_a}^2}} \right) \tag{4a}$$

$$P(LS_i) = \Phi\left(-\frac{\lambda_C^i - \lambda_{D|S_a = m_{S_a}}}{\sqrt{\beta_C^2 + \beta_{D|S_a}^2 + a_2^2 \beta_{S_a}^2 + \beta_M^2}} \right) \tag{4b}$$

where $m_{S_a} = e^{\lambda_{S_a}}$ is the median of the spectral acceleration.

Table 2 lists the fragility parameters for the three limit states considered; Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP). The exceedance probabilities $P(LS_i)$ computed by Eq. (4a) are also listed.

Table 2. Fragility parameters and limit state exceedance probabilities (structural damage)

Inventory items		Limit states, LS_i		
		IO	LS	CP
1 Concrete Bracci (3-story)	λ_i	-1.991	-1.523	-1.175
	β_i	0.509	0.392	0.425
	$P(LS_i)$	0.608	0.423	0.293
2 URM Wen (2-story)	λ_i	-1.890	-1.200	-0.693
	β_i	0.300	0.300	0.330
	$P(LS_i)$	0.686	0.383	0.194
3 URM Wen (2-story)	λ_i	-1.890	-1.200	-0.693
	β_i	0.300	0.300	0.330
	$P(LS_i)$	0.663	0.362	0.182

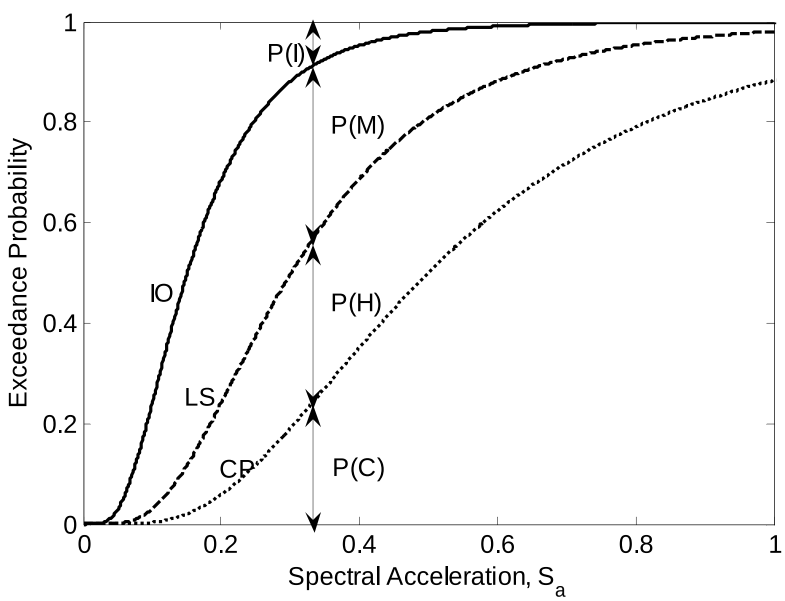

Figure 1. Computing probabilities of damage states

II-2. Probability of structural damage states by ground shaking

Bai et al. (2006) proposed four distinct states for structural damages by ground shaking: Insignificant (I), Moderate (M), Heavy (H), and Complete (C). As illustrated in Figure 1, we can compute the probabilities of the four damage states from the limit-state exceedance probabilities as follows.

$$P(I) = 1 - P(IO) \tag{5a}$$

$$P(M) = P(IO) - P(LS) \quad (5b)$$

$$P(H) = P(LS) - P(CP) \quad (5c)$$

$$P(C) = P(CP) \quad (5d)$$

We compute the damage state probabilities from the limit-state exceedance probabilities reported in Table 2 by using Eq. (5) and report in Table 3.

Table 3. Probabilities of structural damage states

Inventory items	Probability of damage states			
	I	M	H	C
1 (Concrete)	0.392	0.185	0.130	0.293
2 (URM1)	0.314	0.304	0.189	0.194
3 (URM2)	0.337	0.301	0.181	0.182

II-3. Consideration of structural damages caused by ground failure

A structure can be damaged not only by ground shaking, but also by ground failure such as soil liquefaction. If we use four states (I, M, H and C) for the damages by ground failure as well, and assume structural damage by ground shaking and that by ground failures are statistically independent of each other, the probability that a structure will exceed a certain damage state either by ground shaking or ground failure is obtained as

$$P_{COMB}[DS \geq I] = 1 \quad (6a)$$

$$P_{COMB}[DS \geq M] = P_{GS}[DS \geq M] + P_{GF}[DS \geq M] - P_{GS}[DS \geq M] \cdot P_{GF}[DS \geq M] \quad (6b)$$

$$P_{COMB}[DS \geq H] = P_{GS}[DS \geq H] + P_{GF}[DS \geq H] - P_{GS}[DS \geq H] \cdot P_{GF}[DS \geq H] \quad (6c)$$

$$P_{COMB}[DS \geq C] = P_{GS}[DS \geq C] + P_{GF}[DS \geq C] - P_{GS}[DS \geq C] \cdot P_{GF}[DS \geq C] \quad (6d)$$

where $P_{COMB}[DS \geq X]$ denotes the probability that a structure will exceed a damage state X either by ground failure or ground shaking, and P_{GS} and P_{GF} denote the probabilities of exceedance by ground shaking and ground failure, respectively. Then, the combined probabilities of damage states are computed as

$$P_{COMB}[DS = I] = 1 - P_{COMB}[DS \geq M] \quad (7a)$$

$$P_{COMB}[DS = M] = P_{COMB}[DS \geq M] - P_{COMB}[DS \geq H] \quad (7b)$$

$$P_{COMB}[DS = H] = P_{COMB}[DS \geq H] - P_{COMB}[DS \geq C] \quad (7c)$$

$$P_{COMB}\left[DS=C\right]=P_{COMB}\left[DS\geq C\right] \tag{7d}$$

In this example, the probability of "Complete" ground failure, $P_{GF}[DS\geq C]=$ $P_{GF}[DS=C]$ is defined as the probability that the liquefaction potential index (LPI) is greater than 15. An algorithm has been developed and documented within the MAE Center to evaluate this probability, $P(LPI>15)$. The proposed algorithm evaluates the complete ground failure probabilities of the three buildings in the example as 1.51% (Concrete), 1.96% (URM1) and 1.93% (URM2), respectively. In this example, we also assume that a ground failure either causes Complete (C) or Insignificant (I) damages only. Therefore, $P_{GF}[DS\geq M]$ and $P_{GF}[DS\geq H]$ are also the same as $P(LPI>15)$. Combining these ground failure probabilities with the probabilities of structural damages caused by ground shaking (Table 3) by Eqs. (6) and (7), the combined probabilities of structural damages are obtained and reported in Table 4.

Table 4. Probabilities of structural damage after liquefaction hazard is considered

Inventory items	Combined probability of damage states			
	I	M	H	C
1 (Concrete)	0.386	0.182	0.128	0.304
2 (URM1)	0.308	0.298	0.185	0.209
3 (URM2)	0.330	0.295	0.177	0.197

II-4. Mean and standard deviation of damage ratio

The damage ratios of inventory items are critical inputs to social and economic loss models. Bai et al. (2006) proposed a probabilistic model for the structural damage ratios to account for the uncertainty in structural damages. They assume that a structure is subjected to one of the four damage states (I, M, H and C) with the probabilities computed by Eq. (7). For a given damage state, the damage ratio follows the beta distribution with a prescribed range. The mean of the Beta distribution is assumed to be at the midpoint of the range while the standard deviation is given as one-third of the length of the range. Table 5 shows the proposed range, mean and standard deviation of Beta distribution for each damage state.

Table 5. Probabilistic model for structural damage ratio (Bai et al. 2006)

Damage states, DS_i	Range of Beta distribution (%)	Mean of damage ratio, $\mu_{D\|DS_i}$ (%)	Standard deviation of damage ratio, $\sigma_{D\|DS_i}$ (%)
1: Insignificant	[0, 1]	0.5	0.333
2: Moderate	[1, 30]	15.5	9.67
3: Heavy	[30, 80]	55	16.7
4: Complete	[80, 100]	90	6.67

The mean and variance of the damage ratio (D) of an inventory item are computed by

$$\mu_D = \sum_{i=1}^{4} [P(DS_i) \cdot \mu_{D|DS_i}] \tag{8a}$$

$$\begin{aligned}
\sigma_D^2 &= \mathrm{E}[D^2] - \mu_D^2 \\
&= \sum_{i=1}^{4} \left\{ P(DS_i) \cdot E[D^2 \mid DS_i] \right\} - \mu_D^2 \qquad (8b) \\
&= \sum_{i=1}^{4} \left[P(DS_i) \cdot (\sigma_{D|DS_i}^2 + \mu_{D|DS_i}^2) \right] - \mu_D^2
\end{aligned}$$

where $P(DS_i)$, $i = 1, \ldots, 4$ denotes the combined probabilities of the *i*-th damage state such as those shown in Table 4, and $\mu_{D|DS_i}$ and $\sigma_{D|DS_i}$ are the conditional mean and standard deviation of Beta distribution given DS_i damage state, shown in Table 5. The means and variances of the damage ratios of the three inventory items in this example are computed by Eq. (8) and listed in Table 6.

Table 6. Mean and variance of structural damage ratios

Inventory	Mean, μ_D	Variance, σ_D^2
1: Concrete	0.374	0.156
2: URM1	0.338	0.127
3: URM2	0.323	0.125

III. Non-structural damage

III-1. Probabilistic models for non-structural damage states

In order to estimate the probabilities of non-structural damage states, we adopt the HAZUS non-structural fragility curves developed for four limit-states: Slight, Moderate, Extensive and Complete. As illustrated in Figure 2, five damage states, None (N), Slight (S), Moderate (M), Extensive (E) and Complete (C) are derived from the four limit-states. To be consistent with the probabilistic model on the structural damage, we combine the damage states N and S and name it Insignificant (I). The other HAZUS damage states M, E and C are renamed to Moderate (M), Heavy (H) and Complete (C), respectively.

For each damage state, HAZUS assigns a deterministic damage ratio. Consider the damage ratios given in Figure 3a. If a non-structural component is in Moderate state, for example, the damage ratio is assumed to be ‘b’ exactly. To be consistent with the beta-distribution-based probabilistic model proposed for structural damage, we introduce fourranges of non-structural damage states whose boundaries are midpoints between the HAZUS damage ratio values (See

Figure 3b). Then, we assume that the mean of the damage ratio in each interval is at its midpoint and the standard deviation is one third of the interval length. There exist two types of non-structural damages: acceleration-sensitive and drift-sensitive. Tables 7 and 8 show the probabilistic models obtained by the aforementioned procedure.

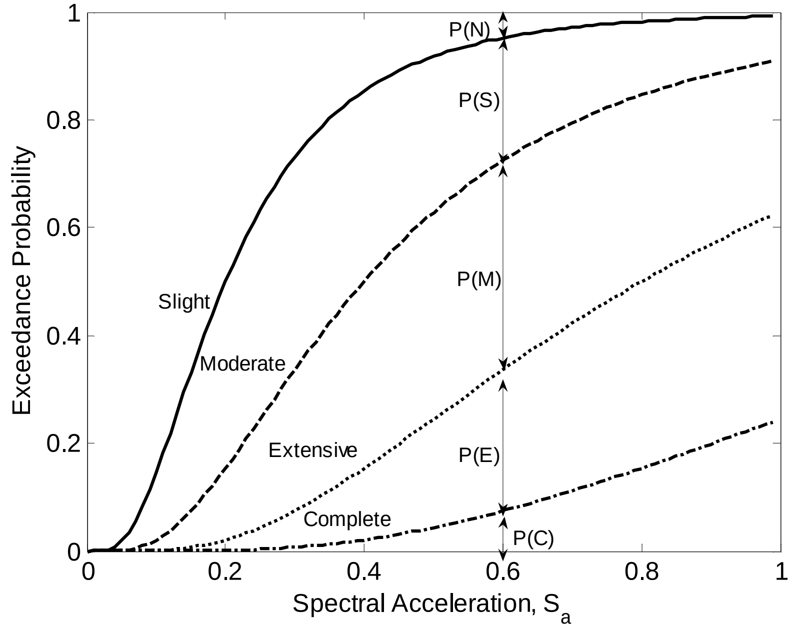

Figure 2. Acceleration-sensitive non-structural fragility curves (HAZUS)

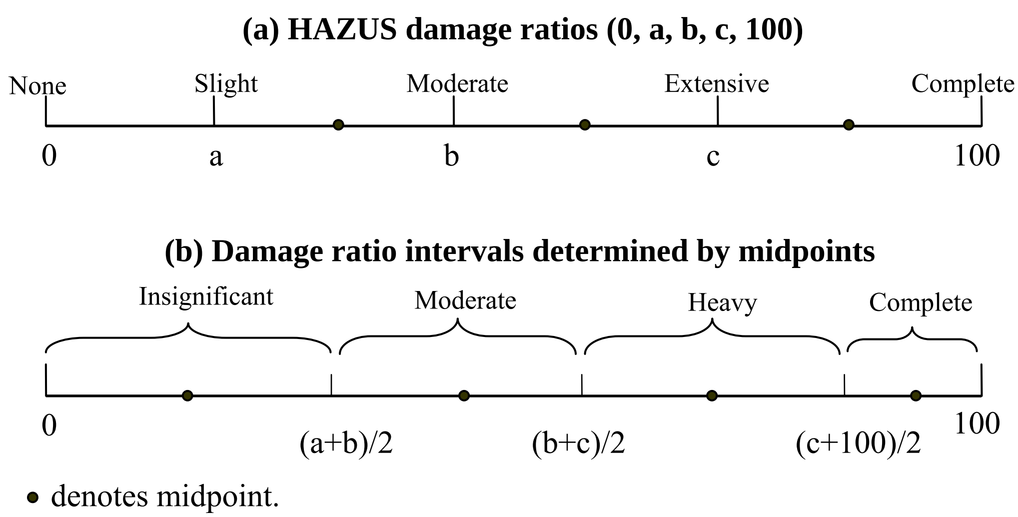

Figure 3. Probabilistic model for non-structural damage ratios

Table 7. Probabilistic model for acceleration-sensitive non-structural damage ratio

Damage states, DS_i	Range of Beta distribution (%)	Mean of damage ratio, $\mu_{D\|DS_i}$ (%)	Standard deviation of damage ratio, $\sigma_{D\|DS_i}$ (%)
1: Insignificant	[0, 6]	3.0	2.0
2: Moderate	[6, 20]	13.0	4.67
3: Heavy	[20, 65]	42.5	15.0
4: Complete	[65, 100]	82.5	11.7

Table 8. Probabilistic model for drift-sensitive non-structural damage ratio

Damage states, DS_i	Range of Beta distribution (%)	Mean of damage ratio, $\mu_{D\|DS_i}$ (%)	Standard deviation of damage ratio, $\sigma_{D\|DS_i}$ (%)
1: Insignificant	[0, 6]	3.0	2.0
2: Moderate	[6, 30]	18.0	8.0
3: Heavy	[30, 75]	52.5	15.0
4: Complete	[75, 100]	87.5	8.3

III-2. Acceleration-sensitive non-structural damage

HAZUS acceleration-sensitive non-structural fragilities are given in terms of spectral accelerations. By combining the uncertainties of spectral acceleration by Eq. (4a), we can compute the exceedance probabilities $P(LS_i)$ for acceleration-sensitive non-structural damage. Table 9 shows the HAZUS fragility parameters and the computed exceedance probabilities.

Table 9. Fragility parameters and limit state exceedance probabilities (acceleration-sensitive non-structural damage)

Inventory items		Limit states, LS_i		
		Moderate (M)	Extensive (E)	Complete (C)
1 Concrete	λ_i	–0.9162	–0.2231	0.47
	β_i	0.68	0.68	0.68
	$P(LS_i)$	0.239	0.0917	0.0256
2 URM	λ_i	–0.9162	–0.2231	0.47
	β_i	0.65	0.65	0.65
	$P(LS_i)$	0.302	0.119	0.033
3 URM	λ_i	–0.9162	–0.2231	0.47
	β_i	0.65	0.65	0.65
	$P(LS_i)$	0.287	0.112	0.0309

The probabilities of the four damage states are then computed by

$$P(I) = 1 - P(Moderate) \tag{9a}$$

$$P(M) = P(Moderate) - P(Extensive) \tag{9b}$$

$$P(H) = P(Extensive) - P(Complete) \tag{9c}$$

$$P(C) = P(Complete) \tag{9d}$$

Table 10 shows the computed probabilities of the damage states.

Table 10. Probabilities of acceleration-sensitive non-structural damage states

Inventory items	Probability of damage states			
	I	M	H	C
1 (Concrete)	0.761	0.147	0.066	0.026
2 (URM1)	0.698	0.182	0.086	0.033
3 (URM2)	0.713	0.175	0.081	0.031

Non-structural damages caused by ground failure are taken into account by the procedure in Eqs. (6) and (7). Table 11 shows the probabilities after liquefaction hazard is considered.

Table 11. Probabilities of acceleration-sensitive non-structural damage after combining liquefaction hazard

Inventory items	Combined probability of damage states			
	I	M	H	C
1 (Concrete)	0.750	0.145	0.065	0.040
2 (URM1)	0.685	0.179	0.085	0.052
3 (URM2)	0.700	0.171	0.080	0.050

The means and variances of the damage ratios are computed by Eq. (8) and listed in Table 12.

Table 12. Mean and variance of acceleration-sensitive non-structural damage ratios

Inventory	Mean, μ_D	Variance, σ_D^2
1: Concrete	0.102	0.035
2: URM1	0.123	0.043
3: URM2	0.118	0.041

III-3. Drift-sensitive non-structural damage

The HAZUS fragility curves for drift-sensitive non-structural damage are given in terms of spectral displacement instead of spectral acceleration. As shown in Table 1, the uncertainties in the seismic intensity are quantified in terms of spectral acceleration. Hence, we derive the mean and variance of the logarithm of the spectral displacement from those of spectral acceleration. When the units of spectral acceleration and displacement are the gravity acceleration (g) and inches, respectively, the spectral displacement (S_d) is described in terms of the spectral acceleration by

$$S_d = 9.8 S_a T_e^2 \tag{10}$$

where T_e is the fundamental period of the structure shown in Table 1. Then, the mean and variance of the natural logarithms of the spectral displacement are computed as

$$\lambda_{S_d} = \lambda_{S_a} + \ln(9.8 T_e^2) \tag{11a}$$

$$\beta_{S_d}^2 = \beta_{S_a}^2 \tag{11b}$$

Table 13 shows the results of the conversion.

Table 13. Conversion from spectral acceleration to spectral displacement

Inventory items	λ_{S_a}	$\lambda_{S_d} = \lambda_{S_a} + \ln(9.8T_e^2)$	$\beta_{S_d}^2 = \beta_{S_a}^2$
1: Concrete	-1.710	0.470	0.887
2: URM1	-1.463	-0.202	0.827
3: URM2	-1.514	-0.253	0.840

This conversion allows us to follow all the procedures developed for the acceleration-sensitive non-structural damages. Tables 14-17 show the results of the computations.

IV. Contents Loss

HAZUS uses the acceleration-sensitive non-structural fragilities to determine the states of contents loss. For each content loss state, a deterministic loss ratio is assigned. Table 18 shows a probabilistic model proposed for the content loss ratios to be consistent with the models for structural/non-structural damage ratios. Table 19 shows the means and variances of the content loss ratios of the buildings in this example.

Table 14. Fragility parameters and limit state exceedance probabilities (drift-sensitive non-structural damage)

Inventory items		Limit states, LS_i		
		Moderate (M)	Extensive (E)	Complete (C)
1 Concrete	λ_i	0.3646	1.5041	2.1972
	β_i	0.98	0.93	1.03
	$P(LS_i)$	0.532	0.211	0.102
2 URM	λ_i	0.0770	1.2179	1.9095
	β_i	1.23	1.23	1.03
	$P(LS_i)$	0.425	0.169	0.055
3 URM	λ_i	0.0770	1.2179	1.9095
	β_i	1.23	1.23	1.03
	$P(LS_i)$	0.412	0.162	0.052

Table 15. Probabilities of drift-sensitive non-structural damage states

Inventory items	Probability of damage states			
	I	M	H	C
1 (Concrete)	0.468	0.321	0.109	0.102
2 (URM1)	0.575	0.256	0.114	0.055
3 (URM2)	0.588	0.251	0.110	0.052

Table 16. Probabilities of drift-sensitive non-structural damage considering liquefaction

Inventory items	Combined probability of damage states			
	I	M	H	C
1 (Concrete)	0.461	0.316	0.107	0.116
2 (URM1)	0.564	0.251	0.112	0.074
3 (URM2)	0.576	0.246	0.108	0.070

Table 17. Mean and variance of drift-sensitive non-structural damage ratios

Inventory	Mean, μ_D	Variance, σ_D^2
1: Concrete	0.228	0.082
2: URM1	0.185	0.066
3: URM2	0.180	0.065

Table 18. Probabilistic model for content loss ratio

Damage states, DS_i	Range of Beta distribution (%)	Mean of damage ratio, $\mu_{D\|DS_i}$ (%)	Standard deviation of damage ratio, $\sigma_{D\|DS_i}$ (%)
1: Insignificant	[0, 3]	1.5	1.0
2: Moderate	[3, 15]	9.0	4.0
3: Heavy	[15, 37.5]	26.25	7.5
4: Complete	[37.5, 50]	43.75	4.17

Table 19. Mean and variance of the contents loss ratios

Inventory	Mean, μ_D	Variance, σ_D^2
1: Concrete	0.059	0.011
2: URM	0.071	0.013
3: URM	0.069	0.013

V. Consideration of inventory uncertainty

There exist uncertain errors in identifying the structural types of inventory items by remote sensing. For example, a concrete building could be mistakenly classified into the URM building category. We may assume a probability of accurate identification, denoted by p_{id}, to account for this uncertainty. This means there is $(1 - p_{id})$ probability that the structure belongs to any of the *other* structural types in the inventory. Then, the mean of the damage ratio is adjusted as

$$\mu_{\tilde{D}} = p_{id}\mu_D + (1 - p_{id})\mu_{D_r} \tag{12}$$

where $\mu_{\tilde{D}}$ is the mean damage ratio with the inventory uncertainty considered, μ_D is the mean damage ratio based on the identified structure type such as those reported in Tables 6, 12, 17 and 19, and μ_{D_r} is the mean damage ratio for unknown structural type. The latter, denoted as "representative" mean damage ratio, is estimated as the weighted average of the mean damage ratios based on the other identified structural types except for the originally predicted structure type, that is,

$$\mu_{D_r} = \frac{1}{N}\sum_{j=1}^{N_{id}} n_j \cdot \mu_D^j \tag{13}$$

where N_{id} is the number of representative structure types identified, i.e., the total number of structure types minus 1; n_j is the number of the inventory items identified as the *j*-th representative structural type, $j = 1,\ldots,N_{id}$; $N = \sum_j^{N_{id}} n_j$ is the total number of representative inventory items (excluding the originally predicted structures); and μ_D^j is the mean damage ratio estimated based on the *j*-th representative structural type at the given site. Note that for this example, the hazard will be assumed constant with respect to structure type. In fact, each structure type can have its own period, and a calculation should be performed for each period to estimate an appropriate spectral acceleration. When the hazard is *transformed* from spectral acceleration to spectral displacement for drift-sensitive damage estimation, the period *is* used.

In this example, the representative damage ratio is computed as

$$\mu_{D_r} = \mu_D^{\mathrm{URM}} \tag{14a}$$

for inventory item I1, and

$$\mu_{D_r} = \mu_D^{\mathrm{con}} \tag{14b}$$

for inventory item I2 and I3.

where μ_D^{con} and μ_D^{URM} respectively denote the mean damage ratio estimated based on the identifications as concrete and URM buildings. Assuming $p_{id} = 0.85,$ the mean damage ratio is updated as

$$\mu_{\tilde{D}} = 0.85\mu_D + 0.15\mu_{D_r} \tag{15}$$

For the first inventory item identified as a concrete building, the adjusted mean damage ratio is

$$\mu_{\tilde{D}} = 0.85\mu_D^{\mathrm{con}} + 0.15\mu_D^{\mathrm{URM}} \tag{16}$$

The adjusted variance of the damage ratio is

$$\begin{aligned} \sigma_{\tilde{D}}^2 &= \mathrm{E}[\tilde{D}^2] - \mu_{\tilde{D}}^2 \\ &= 0.85[(\mu_D^{\mathrm{con}})^2 + (\sigma_D^{\mathrm{con}})^2] + 0.15[(\mu_D^{\mathrm{URM}})^2 + (\sigma_D^{URM})^2] - \mu_{\tilde{D}}^2 \end{aligned} \tag{17}$$

For the second and third inventory items identified as URM buildings, the adjusted means and variances of the damage ratios are

$$\mu_{\tilde{D}} = 0.15\mu_D^{\mathrm{con}} + 0.85\mu_D^{\mathrm{URM}} \tag{18a}$$

$$\begin{aligned} \sigma_{\tilde{D}}^2 &= \mathrm{E}[\tilde{D}^2] - \mu_{\tilde{D}}^2 \\ &= 0.15[(\mu_D^{\mathrm{con}})^2 + (\sigma_D^{\mathrm{con}})^2] + 0.85[(\mu_D^{\mathrm{URM}})^2 + (\sigma_D^{URM})^2] - \mu_{\tilde{D}}^2 \end{aligned} \tag{18b}$$

Table 20 shows the structural/non-structural damage ratios and the content loss ratios adjusted by Eqs. (16)–(18).

Table 20. Means and variances of damage ratios and content loss ratios adjusted by inventory uncertainty

Inventory buildings		$\mu_{\tilde{D}}$	$\sigma^2_{\tilde{D}}$
1: Concrete	SD	0.359	0.151
	NA	0.102	0.035
	ND	0.218	0.079
	CL	0.059	0.011
2: URM	SD	0.356	0.134
	NA	0.123	0.042
	ND	0.198	0.071
	CL	0.071	0.013
3: URM 2	SD	0.340	0.132
	NA	0.118	0.041
	ND	0.191	0.069
	CL	0.069	0.013

Notation: SD: structural damage, NA: acceleration-sensitive non-structural damage, ND: drift-sensitive non-structural damage, CL: content loss

VI. Loss estimation

As a simple example of social and economic losses caused by seismic hazard, we consider the loss of an inventory item defined by

$$Loss_i = M_i(\alpha_i^{SD}\tilde{D}_i^{SD} + \alpha_i^{NA}\tilde{D}_i^{NA} + \alpha_i^{ND}\tilde{D}_i^{ND} + \alpha_i^{CL}\tilde{D}_i^{CL}) \tag{19}$$

where M_i is the total assessed value of the *i*-th inventory item; α_i^{SD}, α_i^{NA} and α_i^{ND} are the fractions of the values of structural and non-structural (acceleration- and drift-sensitive) components; α_i^{CL} is the ratio of the contents value to the structural assessed value; $\tilde{D}_i^{SD}$, $\tilde{D}_i^{NA}$ and $\tilde{D}_i^{ND}$ are the damage ratios of the *i*-th inventory item adjusted by the inventory uncertainty; and $\tilde{D}_i^{CL}$ is the adjusted content loss ratio. Table 21 shows the fractions of structural and non-structural values of commercial and industrial occupancies defined by HAZUS. In this example, we assume α^{CL} to be 150, 100 and 150% for the inventory items 1, 2 and 3, following the assumption by HAZUS that α^{CL} can be 50, 100 or 150% only.

Table 21. Fraction (%) of structural and non-structural values

Occupancy type	α^{SD}	α^{NA}	α^{ND}
Commercial	29.4	43.1	27.5
Industrial	15.7	72.5	11.8

The total loss of the inventory is obtained by aggregating the losses of the inventory items, that is,

$$Loss = \sum_{i=1}^{N} Loss_i \tag{20}$$

Then mean of the total loss is estimated as

$$\mu_{Loss} = \sum_{i=1}^{N} M_i \left(\alpha_i^{SD} \mu_{\tilde{D}_i^{SD}} + \alpha_i^{NA} \mu_{\tilde{D}_i^{NA}} + \alpha_i^{ND} \mu_{\tilde{D}_i^{ND}} + \alpha_i^{CL} \mu_{\tilde{D}_i^{CL}} \right) \tag{21}$$

Assuming the damage ratios of different inventory items are conditionally independent given a seismic intensity, the variance of the total loss is computed as

$$\sigma_{Loss}^2 = \sum_{i=1}^{N} M_i^2 \left[(\alpha_i^{SD})^2 \sigma_{\tilde{D}_i^{SD}}^2 + (\alpha_i^{NA})^2 \sigma_{\tilde{D}_i^{NA}}^2 + (\alpha_i^{ND})^2 \sigma_{\tilde{D}_i^{ND}}^2 + (\alpha_i^{CL})^2 \sigma_{\tilde{D}_i^{CL}}^2 \right] \tag{22}$$

The coefficient of variation (c.o.v.) of the total loss is

$$\delta_{Loss} = \frac{\sigma_{Loss}}{\mu_{Loss}} \tag{23}$$

The mean, standard deviation and c.o.v. of the total loss of the example inventory are estimated as 0.365 (million US$), 0.208(million US$) and 56.84 %, respectively.

Consider Loss Ratio (L_r), which is the total loss normalized by the sum of structural, non-structural and content values in a region, that is,

$$L_r = \frac{Loss}{\sum_{i=1}^{N} M_i (\alpha_i^{SD} + \alpha_i^{NA} + \alpha_i^{ND} + \alpha_i^{CL})} = \frac{Loss}{\sum_{i=1}^{N} M_i (1 + \alpha_i^{CL})} = \frac{Loss}{M_{total}} \tag{24}$$

Then, mean and standard deviation of the loss ratio are μ_{Loss} / M_{total} and $\sigma_{Loss} / M_{total}$, respectively. The c.o.v. of the loss ratio is the same as that of the total loss. For the example inventory, the mean and standard deviation of the loss ratio are estimated as 11.42% and 6.48 %, respectively.

Given the estimated mean and standard deviation, and an assumed distribution type, we can find the probability distribution of the loss ratio. We hereby assume the loss ratio follows the lognormal distribution. The lognormal distribution requires two parameters λ and β, which are the mean and standard deviation of the natural logarithm of the quantity. These parameters are obtained from the estimated mean and standard deviation of the loss ratio as follows.

$$\beta = \sqrt{\ln[1 + (\sigma / \mu)^2]} \tag{25a}$$

$$\lambda = \ln \mu - 0.5\beta^2 \tag{25b}$$

The lognormal parameters of the loss ratio in the example are $\lambda = -2.31$ and $\beta = 0.529$ The probability density function (PDF) of the loss ratio is defined as

$$f_{L_r}(l_r) = \frac{1}{\sqrt{2\pi}\beta l_r} \exp\left[-\frac{1}{2}\left(\frac{\ln l_r - \lambda}{\beta} \right)^2 \right] \tag{26}$$

Figure 4 plots the PDF of the loss ratio of the example inventory.

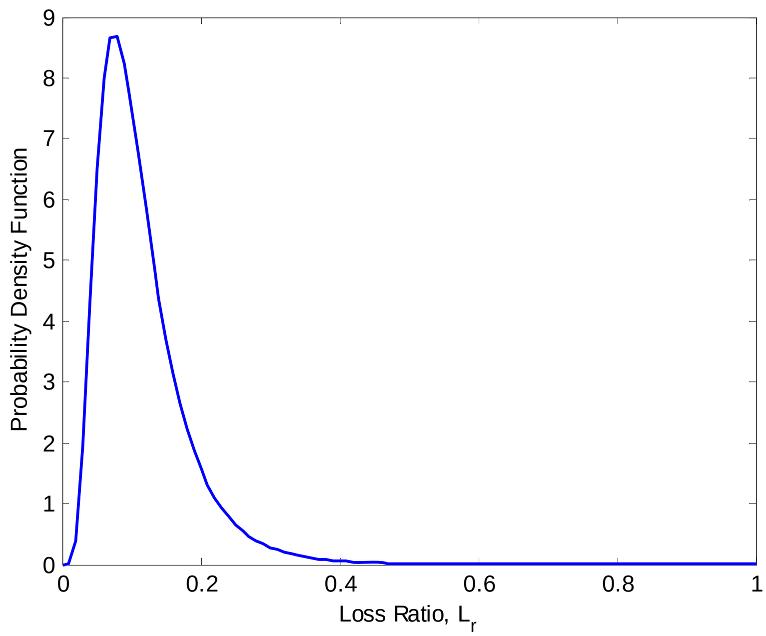

Figure 4. Probability density function of loss ratio

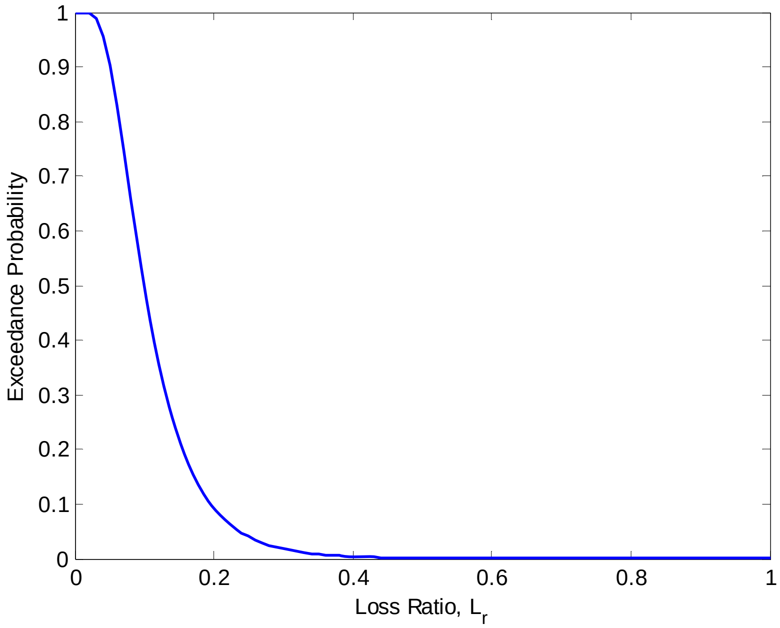

Figure 5. Exceedance probability of loss ratio

We can also estimate the probability that the loss ratio will exceed a certain threshold. This is often referred as complementary cumulative distribution function (CCDF). The CCDF of the lognormal distribution is

$$C_{Lr}(l_r) = 1 - \Phi\left[\frac{\ln(l_r) - \lambda}{\beta}\right] \tag{27}$$

The exceedance probability of the loss ratio is plotted in Figure 5. Table 22 lists the exceedance probabilities at selected thresholds of loss ratio.

Table 22. Probability of exceedance

Loss ratio (%)	Probability of exceedence, % (lognormal distribution)
0	100.00
1	100.00
5	90.24
10	49.43
20	9.27
30	1.83
40	0.42
50	0.11

Based on the estimated uncertainty in the loss ratio, we can predict the loss ratio by an interval with a certain level of confidence. An interval that encloses the true loss ratio with probability $1-\alpha$ (or an interval with 'confidence level' $1-\alpha$) is

$$\left[\exp(\lambda - k_{\alpha/2}\beta), \exp(\lambda + k_{\alpha/2}\beta)\right] \tag{28}$$

where $k_{\alpha/2} = \Phi^{-1}(1-\alpha/2)$. Table 23 shows the coefficient values for selected confidence levels and the corresponding confidence intervals.

Table 23. Confidence intervals on loss ratio

Confidence level, $1-\alpha$ (%)	$k_{\alpha/2}$	Confidence interval (%)
60	0.8416	[6.36, 15.49]
70	1.0364	[5.73, 17.17]
80	1.2816	[5.04, 19.55]
90	1.6449	[4.16, 23.70]
95	1.9600	[3.52, 28.00]
99	2.5758	[2.54, 38.78]

APPENDIX: Data flow chart

Figure 6 illustrates the data flow of the proposed procedure with equation numbers shown.

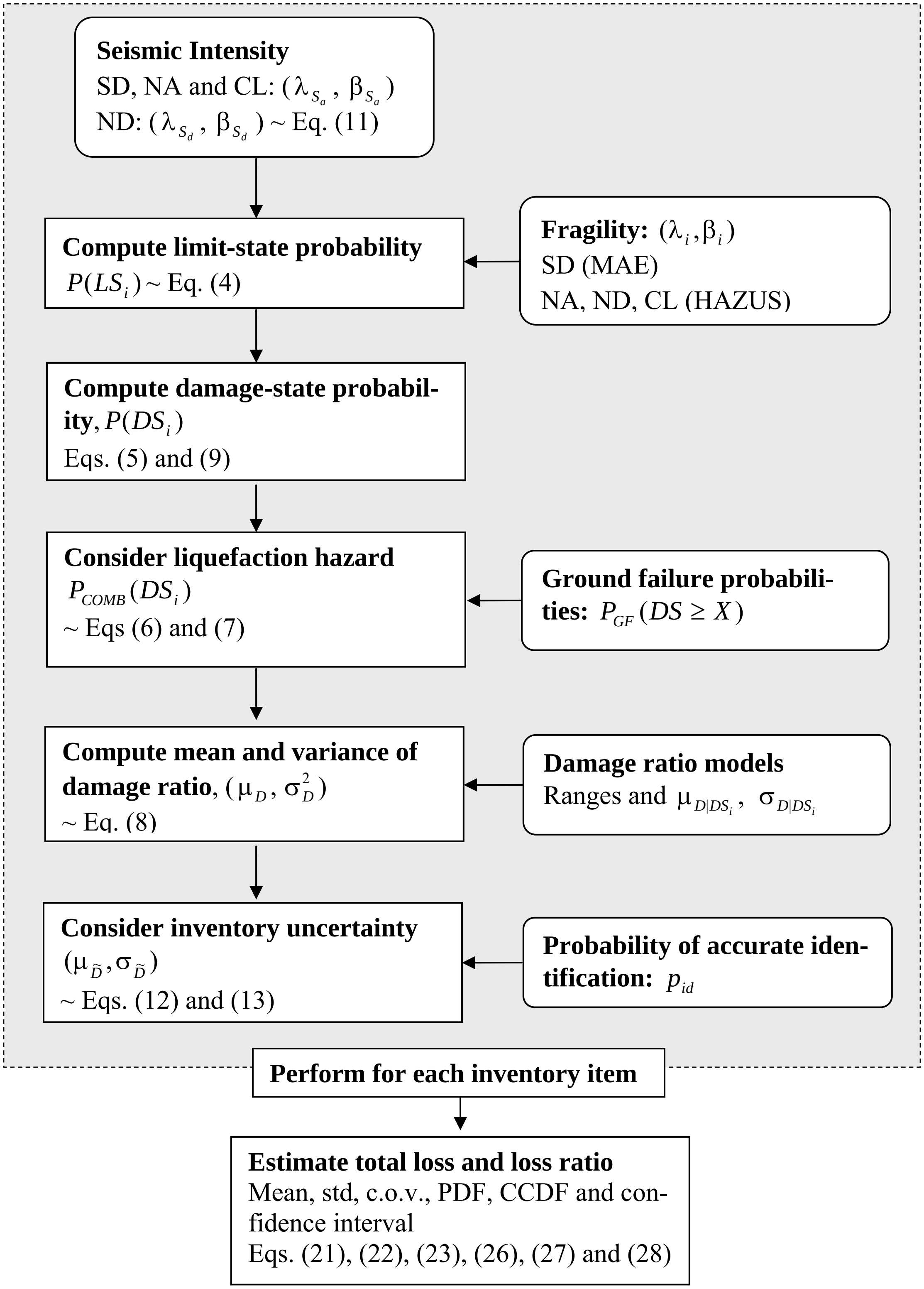

Figure 6. Data flow chart of probabilistic estimation of seismic regional loss