RAPID RISK ASSESSMENT AND DECISION SUPPORT FOR URBAN INFRASTRUCTURE NETWORKS BY MATRIX-BASED SYSTEM RELIABILITY METHOD

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Abstract

Risk assessment of urban infrastructure networks under natural and man-made hazards is often performed by repeated computational simulations based on random samples of hazard intensities and corresponding component status. This sampling-based approach may prevent near-real-time application of probabilistic methods to hazard mitigation of infrastructure networks and risk-informed decision support. This paper proposes a non-sampling-based, multi-scale network reliability analysis approach based on the recently proposed matrix-based system reliability method. This approach accounts for the spatial correlation of hazard intensities and makes use of the disjoint cut sets or link sets identified by a state-of-the-art network analysis algorithm. The paper demonstrates the proposed approach through a case study of a Memphis Light, Gas and Water natural gas network under seismic hazard.

Introduction

Urban infrastructure networks for transportation, telecommunication, and utility services of electricity, water, sewage and gas are critical backbones of modern societies. Natural and man-made hazards cause structural damages of network components, which may disrupt not only residential and commercial activities but also post-disaster responses and recovery efforts, resulting in substantial socio-economic losses. Therefore, risk assessment of these infrastructure systems and risk-informed decision support are vital to utility companies, urban planners, and policy makers, residents and business owners. Evaluation of the performance and connectivity of such urban infrastructure systems is complex in nature due to a large number of network components, network topology and component/system interdependencies. Thus, risk assessment is often performed by repeated computational simulations based on random samples of hazard intensities and corresponding component status, which often prevents rapid risk assessment and near-real-time, risk-informed decision making for hazard mitigation.

As an attempt to identify opportunities and challenges in near-real-time application of probabilistic methods to hazard mitigation of urban infrastructure networks, this paper proposes a non-sampling-based, multi-scale network reliability analysis methodology based on the matrix-based system reliability (MSR) method (Song and Kang, 2007). The proposed method is demonstrated through a case study of a gas network under earthquake hazard scenarios. The probabilities that service nodes are disconnected from the source nodes are computed by a multi-scale system reliability analysis approach that can account for spatial correlation of seismic demands. For efficient network connectivity analysis, disjoint cut sets or link sets are identified by a state-of-the-art network analysis algorithm. We examine the accuracy and efficiency of the proposed method by comparison with Monte Carlo simulations. The paper also introduces future research topics to further develop the proposed approach for rapid risk assessment and risk-based decision support for lifeline networks.

Matrix-based System Reliability Method

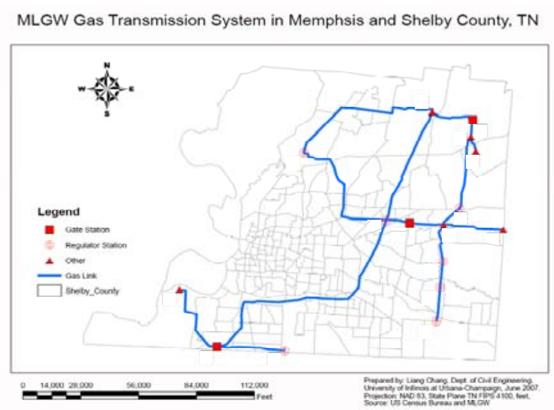
The failure of a complex engineering system is often described by a logical function of component events such as the occurrences of failure modes or the failures of physical components or subsystems. The sample space of n component events with s_i distinct states, $i = 1,...,n$, can be subdivided into $m = \Pi_{i=1}^{n} s_i$ mutually exclusive and collectively exhaustive (MECE) events. The probability of any general system event can be described by the inner product of two vectors:

$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p} \tag{1}$$

in which $\mathbf{c}$ is the "event vector" whose element is 1 if its corresponding MECE event is included in the target system event E_{sys}, and 0 otherwise; and $\mathbf{p}$ is the "probability vector" that contains the probabilities of the MECE events. Matrix-based procedures have been proposed to construct both vectors efficiently (Song and Kang, 2007). This matrix-based formulation also enables us to obtain various importance measures of components, probabilities of components/systems updated by available information or assumptions, and the probabilities of various system events, all through efficient matrix calculations (Kang *et al*., to appear). The method has been further developed to account for statistical dependence between component events and to compute parameter sensitivities of probabilities of general system events (Song and Kang, to appear). Thus, this non-sampling-based methodology can provide a new analysis framework that helps assess the risk of urban infrastructure networks rapidly and obtain various measures useful for risk-informed decision support.

Case Study: MLGW Gas Network

This paper proposes a non-sampling-based, multi-scale network reliability analysis framework and demonstrates it through a case study of a Memphis Light, Gas and Water (MLGW) natural gas network of Shelby County of Tennessee under earthquake hazard scenarios. Figure 1 shows a simplified model of the gas transmission network (modified from Chang *et al*. 1996). MLGW receives the purchased gas at the gate stations (■) that are considered supply facilities or source nodes for the given gas network. The gas transmission pipelines (▬) transmit gas through networks while regulator stations (●) and other service facilities (▲) are considered demand or service nodes. Figure 2 shows its directed graph model for network analysis. The arrows indicate the directivity of the gas flow, from source to customers, and from pipelines with high pressure to those with low pressure (Bowker, 2007). A subjunctive source node, *Node 12* is added to the graph to facilitate finding the multi-source network's node-pair connectivity. The pipelines are represented by virtual "link" nodes in the graph. As a result, the simplified network is characterized by a 37-node and 40-arc network.



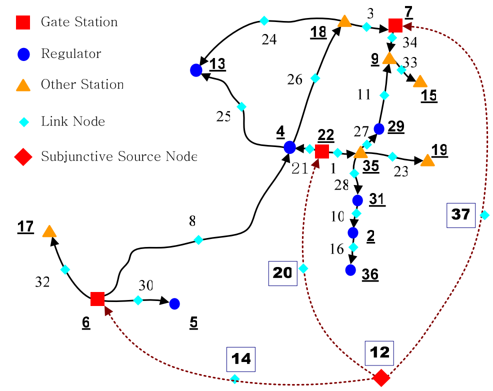


Figure 1. MLGW gas transmission network **Figure 2. Directed graph model**

The case study uses a set of scenario earthquakes with the epicenter N35.54º W-90.43º at Blytheville, AR. The corresponding hazard maps are created by use of the Mid-America Earthquake Center's risk management software *MAEviz* to find the attenuated peak ground velocities (PGV) for the pipeline segments and peak ground accelerations (PGA) for the stations at the nodes.

Multi-Scale Network Reliability Analysis

As shown in Adachi and Ellingwood (2007), the failure probability of a line-type network element can be underestimated if it is modeled by a small number of segments. Thus, a large number of components may be required for accurate estimates, which leads to an overwhelmingly complex system reliability analysis problem especially when the spatial correlation is considered. In order to overcome this challenge, Der Kiureghian and Song (2008) proposed a multi-scale system reliability analysis approach that considers subsets of the components of a system as "super-components" and performs small-size system reliability analyses at multiple levels. For the given case study, we first perform MSR analysis for each gas transmission pipeline based on the failure probabilities of constituent segments. These lower-scale system reliability analyses are followed by additional MSR analyses to obtain the probabilities that service nodes are disconnected from the source nodes. These higher-scale analyses make use of the failure probabilities of the pipelines computed by the lower-scale analyses.

Lower-scale MSR Analysis: Failure Probabilities of Pipelines and Stations

First, we subdivide each pipeline into segments and describe the failure of the pipeline as the union of the segment failure events. Assuming that the failures within each of n segments follow a homogeneous Poisson process, the failure probability of the i-th segment, $i = 1,\dots,n$ is computed as

$$P_i = 1 - \exp(-\nu_i \cdot \Delta l_i) \qquad (2)$$

where ν_i is the average failure occurrence rate (per unit length) along the segment and Δl_i is the length of the segment. The case study adopts a PGV-based damage model in *HAZUS-MH* (FEMA, 2003), i.e. $\nu_i = (0.0001) \cdot PGV_i^{2.25}$ (in per unit kilometer) in which PGV_i is the uncertain PGV (in cm/s) at the midpoint of the *i*-th segment, modeled as

$$PGV_i = \overline{PGV_i} \cdot \varepsilon_i \,, \quad i = 1, ..., n \tag{3}$$

where $\overline{PGV_i}$ denotes the attenuated PGV at the midpoint of the *i*-th segment from the *MAEviz* hazard map, and ε_i is a lognormal random variable with the parameters $\lambda_i = \ln \bar{\varepsilon}_i$ and $\zeta_i = [\ln(1 + \delta_{\varepsilon_i}^2)]^{0.5}$ in which $\bar{\varepsilon}_i$ and δ_{ε_i} respectively denote the median and coefficient of variation (c.o.v.) of ε_i. The median and c.o.v. of PGV_i are derived as $\overline{PGV_i} \cdot \bar{\varepsilon}_i$ and δ_{ε_i}, respectively. The case study assumes $\bar{\varepsilon}_i = 1$ and $\delta_{\varepsilon_i} = 0.6$. The spatial correlation between PGVs at the *i*-th and *j*-th segments is described by a correlation coefficient model of their natural logarithms

$$\rho_{\ln PGV_i, \ln PGV_j} = \rho_{\ln \varepsilon_i, \ln \varepsilon_j} = \exp(- \| \mathbf{x}_i - \mathbf{x}_j \| / L_{corr}) \tag{4}$$

where ρ denotes the correlation coefficient, $\mathbf{x}_i$ and $\mathbf{x}_j$ respectively denote the geographical coordinates of the *i*-th and *j*-th segments, and L_{corr} is the correlation length representing the strength of the spatial correlation (Wang and Takada, 2005). During the case study, L_{corr} is varied to examine the effect of the spatial correlation on the probabilities of pipelines failures and network disconnections.

For MSR analysis of pipelines with statistically dependent component events (i.e. segment failures), the correlation coefficients between standard normal random variables $Z_i = (\ln \varepsilon_i) / \zeta_i$, $i = 1, ..., n$ are fitted by a generalized Dunnett-Sobel (DS) class correlation coefficient matrix (Song and Kang, to appear). As a result, Z_i is represented as

$$Z_i(\mathbf{S}, U_i) = \sqrt{1 - \sum_{k=1}^{m} r_{ik}^2} \cdot U_i + \sum_{k=1}^{m} r_{ik} S_k \,, \quad i = 1, ..., n \tag{5}$$

where r_{ik} is a coefficient of the generalized DS class correlation model, and U_i, $i = 1, ..., n$ and S_k, $k = 1, ..., m$ are statistically independent standard normal random variables. In particular, S_k's are termed as "common source random variables" (CSRVs) since Z_i's are conditionally independent of each other for given outcomes of S_k's. From total probability theorem, the system failure probability in equation (1) is computed as

$$P(E_{sys}) = \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s} \tag{6}$$

where $\mathbf{S} = \{S_1, ..., S_m\}$ is a vector of CSRVs, $f_{\mathbf{S}}(\mathbf{s})$ is the joint probability density function of $\mathbf{S}$, and $\mathbf{p}(\mathbf{s})$ is the "probability" vector constructed by use of the conditional component failure probabilities given $\mathbf{S} = \mathbf{s}$. From equations (2), (3), (5) and the PGV-based damage model, the conditional failure probability of the *i*-th segment is

$$P_i(\mathbf{s}) = \mathrm{E}_{U_i}\{1 - \exp[-(0.0001) \cdot \overline{PGV_i}^{2.25} (\varepsilon_i(\mathbf{s}, U_i))^{2.25} \cdot \Delta l_i]\} \tag{7}$$

where $\mathrm{E}_{U_i}\{\cdot\}$ denotes the expectation relative to U_i, and $\varepsilon_i(\mathbf{s}, U_i) = \exp[\zeta_i \cdot Z_i(\mathbf{s}, U_i)]$. Taking advantage of the conditional independence of the segments' failures given $\mathbf{S} = \mathbf{s}$, one can use the matrix-based procedure developed for constructing **p** with statistically independent components (Song and Kang, 2007). Figure 3 shows the failure probability of Pipeline 8 (See Figure 2) obtained by the MSR analysis. It is seen that the assumption of no spatial correlation, i.e. $L_{corr} = 0$ may result in inaccurate estimate of the failure probabilities. Due to the lack of information in this case study, we compute the failure probabilities of the stations at the nodes by use of a PGA-based damage model in *HAZUS-MH* (FEMA, 2003) originally developed for compressor stations. It is also assumed that the station at a node loses its connectivity when it is in either "extensive" or "complete" damage state. Thus, the probability that a station in the network can not transmit gas is computed as $P_j = \Phi[(\ln PGA_j - \ln 0.77)/0.65]$ where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and PGA_j is the attenuated PGA at the *j*-th node from the hazard map of *MAEviz*.

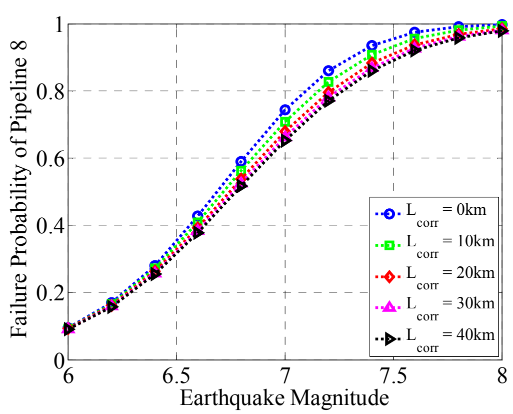


Figure 3. Failure probability of Pipeline 8 by MSR analysis

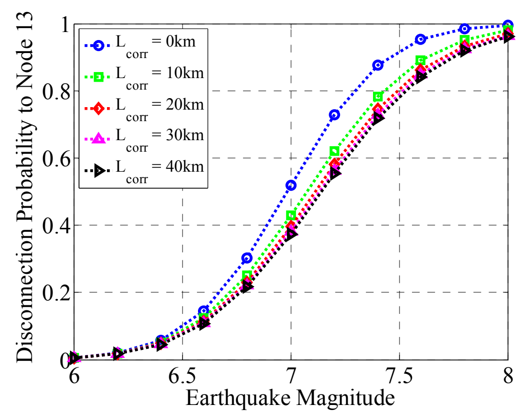


Figure 4. Probability of disconnection of Node 13 by MSR analysis

Higher-scale: Network Connectivity Analysis

The higher-scale system reliability analyses aim to compute the probabilities that the demand nodes are disconnected from the gate stations. This system reliability analysis uses the failure probabilities of the pipelines obtained by the lower-scale system reliability analyses. When the spatial correlation between different pipelines is considered negligible, the components in the system problem are statistically independent, so one can compute the probability of a given node's disconnection by equation (1). The network has 37 components that can be damaged, which results in 2^{37} elements both in **c** and **p**. This size issue can be overcome by describing a disconnection (or connection) event by disjoint cut sets (or disjoint link sets). For example, when disjoint cut sets, C_i, $i = 1, \ldots, N$ are identified, the large system problem is decomposed into N system problems with reduced size:

$$P(E_{sys}) = \sum_{i=1}^{N} P(C_i) = \sum_{i=1}^{N} \mathbf{c}_i^{\mathrm{T}} \mathbf{p}_i \quad (8)$$

where $\mathbf{c}_i$ and $\mathbf{p}_i$ are the event and probability vectors of the *i*-th disjoint cut set, $i = 1,...,N$. This reduces the number of components in a system problem significantly. If the size becomes an issue even after the decomposition, one can compute the element of $\mathbf{p}_i$ vector whose corresponding element in $\mathbf{c}_i$ vector is 1, instead of constructing the large-size vectors. The case study uses the recursive decomposition algorithm (Li and He, 2002) to identify the disjoint cut sets or link sets efficiently. The corresponding event vectors $\mathbf{c}_i$ can be obtained by the matrix-based procedure proposed in Song and Kang (2007). The probability vector $\mathbf{p}_i$ is constructed by the matrix procedure proposed for statistically independent components (Song and Kang, 2007) using the component failure probabilities obtained from the lower-scale system reliability analysis.

Figure 4 shows the probability that the regulator station at Node 13 will be disconnected from the gate stations. Significant effect of the spatial correlation on the system failure probabilities is observed. Figure 5 shows that the results by MSR (dotted lines) match those by Monte Carlo simulation (MCS; markers), which is repeated until c.o.v. of the system reliability estimates, i.e. $(1-$ disconnection probability) converges to 3%. The correlation length L_{corr} is 30 km. Figure 6 demonstrates the rapid assessment of the proposed approach by comparing the total CPU times needed for computing system reliability for seven nodes.

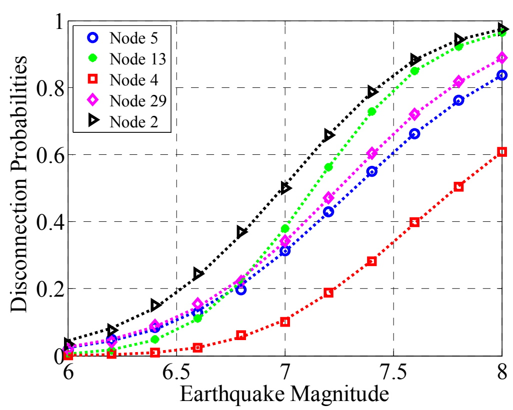


Figure 5. Disconnection probabilities at nodes by MCS and MSR

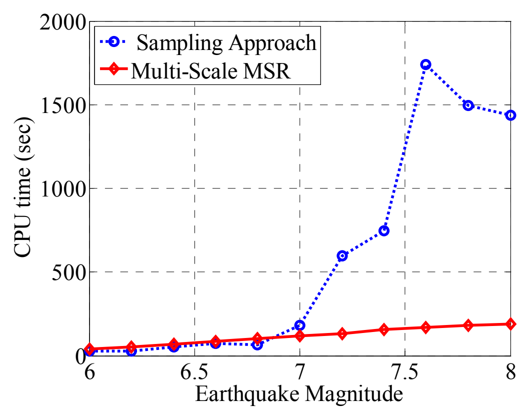


Figure 6. CPU times for system reliability analysis by MSR and MCS

When the spatial correlation between different pipelines is significant, one can perform additional MSR analyses to find out the joint failure probabilities of pipelines. This system reliability problem contains the segments of two pipelines. When the joint failure probability of the *i*-th and *j*-th pipelines is of interest, the system event of interest is $(E_{i,1} \cup ... \cup E_{i,N_i})(E_{j,1} \cup ... \cup E_{j,N_j})$ in which $E_{i,(\cdot)}$ and $E_{j,(\cdot)}$ respectively denote the failures of segments in the *i*-th and *j*-th pipelines. The event vector for this system event is obtained by the matrix-based procedure (Song and Kang, 2007). The computed joint failure probability can represent statistical dependence between the pipelines at the higher-scale network reliability analysis.

Summary and Future Research

A non-sampling-based, multi-scale, matrix-based system reliability analysis framework is proposed for rapid risk assessment of urban infrastructure networks and risk-informed decision support. The case study of an MLGW gas network demonstrates that the proposed method can assess the risk of infrastructure networks efficiently and accurately. In future research, the proposed methodology will be further developed to obtain various decision-support information such as importance measures of components, probabilities updated based on the observed events and parameter sensitivities of system reliability.

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