THE APPLICATION OF CORRELATION THEORY

TO

CERTAIN PROBLEMS OF EDUCATION

BY

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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THE APPLICATION OF CORRELATION THEORY TO CERTAIN PROBLEMS OF EDUCATION

INTRODUCTION

Within the past ten or fifteen years, there has been a tendency among those contributing to psychological and educational literature to attempt to measure abilities of students and to study educational facts by statistical methods. If there are abilities that can be measured, this method offers a hopeful outlook in regard to settling certain questions such as those of the correlation of abilities and the transfer of training. There are various degrees of merit to these statistical studies, with results and interpretations varying from those entirely justifiable to those that seem wholly unwarranted. It is the purpose of this paper to report on an examination of the literature in which mathematical statistics has been utilized in regard to the nature of the methods used.

ABSTRACTS AND CRITICISMS OF ARTICLES

In an article on "A Study in Formal Discipline," the method is to be criticized because data are given for but one class of twenty-four students while in fact it is claimed that conclusions are drawn from ten classes. The author's conclusions can not be checked from the data given. In order to attempt to prove his point, namely, that students able in mathematical reasoning are not even generally able in practical reasoning and law, the grades were arranged in vertical columns, lines were drawn emphasizing cases that were unlike and disregarding cases which were alike. His result is due to

over-emphasis of cases favorable to the author's argument.

The errors of this article are discussed in an article "On the Correlation of the Marks of Students in Mathematics and in Law." Dr. H. L. Rietz obtained the data from Dartmouth College which Mr. Lewis used, and from these data he found that there exists a significant positive correlation coefficient between the marks of students in mathematics and in law. The result which Mr. Lewis obtained was equivalent to a negative correlation coefficient. Hence, Mr. Lewis drew incorrect conclusions from his data. In the article by Dr. Rietz, the data in the form of correlation tables are given for the years 1897 (the year cited as the example in Mr. Lewis' paper), 1898, 1899, 1900, 1901, and a combined table for the years 1897-1901. The mean grade in mathematics, the mean grade in law, the correlation coefficient to three decimal places and the probable error are given for each table. The data given enable one to check the author's results. He does not draw the lines of regression, but explains this omission as follows: "While the number of variates used in the tables for separate years is too small to warrant the calculation of the means of arrays, we find from the combined table, except near the extremes, that the means of arrays lie fairly near the line of regression. We can therefore predict fairly well from the correlation coefficient the average grades of students in law for assigned grades in mathematics."2

One of the most careful pieces of work on the marks of students is an article on "Correlation of Efficiency in Mathematics and Efficiency in Other Subjects."3 The grades of some 1200 students of the University of Illinois were examined to determine the correlation between efficiency of students in mathematics and their efficiency in foreign languages and in natural science.

2. Loc. cit., p.89.
The data are given in correlation tables and the lines of regression are drawn. The correlation coefficients of 0.48 and 0.44 obtained are higher than those obtained by Pearson for the correlation between the statures of father and son. The correlation coefficients for mathematics-foreign languages are given when grades at 50 per cent are included and when grades at 50 per cent are excluded.

A form of procedure for computing the correlation coefficient is given in tabular form. The question of probable error is also considered. Mr. C. N. Moore has said in regard to the size of the correlation coefficients obtained by the authors of this article, "In view of the standing of one of the writers as an expert on statistical subjects this conclusion deserves especial consideration." ¹

In an article by Thorndike and Woodworth, "On the Influence of Improvement in One Mental Function upon the Efficiency of Other Functions,"² people were tested upon their ability to estimate areas. The subjects were aided to the extent that they were allowed to ascertain the real area after each judgment. Table I gives the average area before training, the average area after training and, for the training series, the average error at the end of training, stated in square centimeters. In Table II, the ratio of error after training to error before training is given. Tables I and II give data for six subjects only. The idea in this paper is to determine whether the difference found in judging magnitudes is such as would be expected from chance can be tested by comparing the actual differences between the average errors with the probable differences as computed from the probability curve. However, sufficient data are not given for this purpose. The authors give curves for three individuals in which the ordinates represent the mean square error of judgment of areas of ten to one hundred square centimeters. No

² Psychological Review. 1901. Vol. 8, pp.247-261; 384-395; 553-564.
correlation coefficients are computed although the authors speak of the desirability of correlation in the beginning of the article.

The article, "Correlation between Ability to Think and Ability to Remember with Special Reference to United States History," signed "Educational Statistician, State Board of Education, Madison, Wis.," was read at a meeting of the National Association of Directors of Educational Research and so it should be a carefully prepared article. The author states, "The percentages of correct answers were converted into units of variability, assuming a normal distribution of ability in history for each of the highest grades of the elementary school. The unit chosen was the probable error." Later he says, "The statistical assumptions need concern us very little at this time." However, it does not seem justifiable, except perhaps for rough approximations, to assume a normal distribution when there are several probability curves, any one of which the distribution of grades might follow. The unit chosen does not affect the form of distribution. He computes the Pearson correlation coefficient for the relation between thought and memory in children, but he gives no data. This makes his result lose a great deal of its value for there is no way of checking his results, and often conclusions which the writer has overlooked may be drawn from the data. He also computes the coefficient by the method of unlike signed pairs, securing a coefficient seven-hundredths higher and with a smaller probable error. He maintains that there are decided limitations to the use of correlation coefficient and that the regression coefficient much better indicates the extent to which achievement in remembering historical facts may serve as an index of achievement in judgment or thought about them. He gives the regression coefficients for information ability on thought ability and for thought ability on information ability, but gives no data or correlation tables. He then gives the regression equation.

2. Loc. cit., p.443.
3. " " 
formed by using the regression coefficient. The author could have treated his subject much more adequately by giving his data in the form of correlation tables and drawing the lines of regression so that the reader might check the computed results, and determine from the form of the regression curve the interpretation to be placed on the correlation coefficient.

In an article on "The Grading of Students," Mr. Meyer criticizes the conclusion of Professor W. S. Hall that grades of students when tabulated yield results which conform to the binomial curve. The author thinks students can not be graded in such a mechanical way. Then he proceeds to outline a method for grading. He says, "It seems plausible to start from the assumption that the combined mental and moral ability which we want to measure is distributed among different people in accordance with the probability curve." Since there are several probability curves, it is perhaps an unwarranted assumption that such complex things as grades should follow the normal probability curve. He gives data for classes in the University of Missouri where the students in each class are divided into three groups, 50 per cent medium, 25 per cent superior, and 25 per cent inferior.

The article entitled "On the Significance of the Teacher's Appreciation of General Intelligence" is a very thorough statistical treatment of the subject. Seventeen tables are given in which the data in the form of correlation tables are exhibited. The relationship of clothes and intelligence in school children is shown graphically. By calculating the standard deviations of the arrays and the standard deviation of the whole population, the regression curve is found to be very closely linear. The correlations are computed for

(1) standard and age, (2) standard and intelligence, (3) standard and order in examination, (4) standard and percentage of marks, (5) standard and clothing, (6) age and intelligence, (7) age and order in examination, (8) age and percentage of marks, (9) age and clothing, (10) intelligence and order in examination, (11) intelligence and percentage of marks, (12) intelligence and clothing, (13) order in examination and percentage of marks, (14) order in examination and clothing, (15) percentage of marks and clothing, (16) school and intelligence, and (17) school and clothing. The correlations (1), (3), (4), (6), (9), (10), (11), (14), (15), were found by the correlation ratio method; (2), (5), (12), (16), and (17) were found by mean square contingency; (7), (8), and (13) were found by the product-moment method. The conclusion reached is that the teacher's judgment of general intelligence will give at least an estimate of the examination value of his pupil, and he believed it to be of even more importance.

Karl Pearson has written a paper "On the Value of the Teacher's Opinion of the General Intelligence of School Children." Adequate data are given in the form of correlation tables. He computes correlations by correlation ratios and by multiple correlation. His conclusion is that the teacher's estimation of general capacity does mean something and that it has a very direct and practical value when properly registered and handled.

In the paper, "On the Correlation of Mental and Physical Tests," the author states that his data were obtained from Columbia University. He computes correlation coefficients for tests in Quickness and Accuracy, Memory, Physical Tests, and Class Standing. There are forty-two coefficients computed in all. For comparison purposes, he obtains a correlation of .66 for weight and height of students. However, he gives no correlation tables or data. There is therefore no published information that enables us to know how closely

2. Wissler, Clark, Columbia Univ. contrib. to Ed. No. 9.
his data would give linear regression. We shall comment further on the desirability of knowing that we have linear regression in the interpretation of the correlation coefficient.

"Changes in the Age of College Graduation" is an article giving convincing data. The calculations are based on 20,000 cases and include the graduates of eleven different colleges. The results are given for decades. He favors the use of the median age, but the arithmetic mean of ages is also given. The ages are furthermore represented graphically in histograms. His data show that a larger percentage have graduated under twenty-three in the later decades than in the earlier decades.

In a paper on "Correlation between the Oral and Written Work of Pupils in the Fundamentals of Addition," by Mr. Earle E. Wilson, he gives absolutely no data, but he gives the mean, median, average deviation, standard deviation, probable error, coefficient of variability, and coefficient of correlation. (Pearson formula), possible error of \( r \) for both written and oral tests.

In a paper on "Physical vs Mental Ability," the same author drew his conclusions from the records of one hundred twenty-eight boys. He gives a tabulated distribution of the boys: (1) athletic ability distribution, (2) scholarship distribution, but he did not give correlation tables. He gives the following measures and the formula used for each one for physical tests and scholastic standing: mean, \( \frac{\sum x}{n} \), median \( \frac{n+1}{2} \), average deviation \( \frac{\sum d}{n} \), standard deviation \( \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n}} \), probable error \( (0.6745 \sigma) \), coefficient of variability \( \frac{\sigma}{\mu} \), coefficient of correlation (Pearson's) \( \frac{\sum xy}{n \sigma_1 \sigma_2} \), and probable error of correlation \( (0.6745 \sqrt{\frac{1 - r^2}{n}}) \).

The purpose of the article "Mentality of Nations" was to compare the States of the Union and of different countries as to education and diffusion of knowledge, and to determine what relation, if any, intellectual conditions may have to patho-social and other conditions in those countries. "Mentality" is used to mean diffusion of education, knowledge, or information throughout the population as a whole. The statistics are for the year 1908. The data given are (1) mentality in districts of the United States, (2) mentality in states of the United States, (3) mentality in ten countries of the world, (4) percentage book production for each subject, (5) sociological conditions for ten countries including births, deaths, number still born, marriages, emigrants; (6) patho-social conditions (crime in general, murder or suicide, theft, all offenses and crimes, number of insane in institutions, number of paupers, number of suicides, number of illegitimate births, number of divorces). In this paper, the conclusions are drawn by mere inspection of the data.

In a critical paper by Charles W. Moore, "On Correlation and Disciplinary Values," he states that the scientific basis of the anti-disciplinarians seems on careful examination to be largely imaginary. He discusses the interpretation to be given to the correlation coefficient. In discussing the interpretation of fractional values, he gives examples of Pearson's correlation of 0.51 for correlation in stature in collateral inheritance, Wissler's correlation coefficients from 0.51 to 0.75, with the exception of French and rhetoric which was 0.30; Spearman's correlations for four subjects, Classics, French, English, and Mathematics from 0.64 to 0.83; and the correlation coefficients of 0.48 and 0.44 obtained by Rietz and Shade which he especially commends. However, the anti-disciplinarians frequently refer to correlation coefficients in the range 0.4 to 0.5 as corresponding to a low degree of relationship between

the variables involved. He gives four correlation tables for algebra-algebra (boys), algebra-algebra (girls), geometry-English (boys), geometry-English (girls). He gives the correlation coefficients and their probable errors obtained from the above mentioned data. He does not draw the lines of regression.

In an article on "A Comparison of Elementary and High School Grades,"¹ the data were from the Iowa City School records. Only such records were included as show the completion of the last four years of the elementary school and the first two years of high school. The author found the correlation between the average elementary school grade and the average high school grade to be 0.71 by the Pearson formula. From this fact he concluded that those best in the elementary school are best in high school. He found that the correlations between specific subjects were also quite high, although they were usually less than between the general averages. His correlation coefficients between the same subjects in different schools, say between grade history and high school history, were not markedly higher and many were not so high as those between different subjects either in the same school or in different schools. He did not give correlation tables or data.

Mr. W. T. Foster in "Should Students Study"² tabulates the percentage of those attaining distinction in later life with the number who received first, second, third, fourth, and pass-degree honors. He draws conclusions from mere inspection of the data. He says, "A similar correlation is found between the degree of success of undergraduates at Oxford and their subsequent distinction as clergymen."³ He does not use the term correlation in the sense of mathematical correlation.

Caroline Burke's article on "The Collecting Instinct"⁴ gives data for the number of collections different children have acquired. The average

number of collections to a child is computed. Conclusions are drawn by mere
inspection of the data.

In the article, "Early Interests: Their Permanence and Relation
to Abilities," it is stated that the data are from three hundred forty students,
but no data are given. Coefficients of correlation are given for correlation
of elementary interest with high school interest, $r = 0.85$; of elementary
interest with college interest, $r = 0.66$; and of high school interest with
college interest, $r = 0.79$. The article lacks weight because no data are given
with which to check the conclusions.

Irving King and Morris Adelstein have also written on "The Perma-
nence of Interests and their Relation to Abilities." Their data are from
one hundred forty University of Iowa students, but the data are not exhibited.
Coefficients of correlation are computed. The number of students involved
is rather small to make the results conclusive.

In a paper on "Correlation between Reading Tests and General
Ability," the author says, "The value of a test depends upon the accuracy with
which it measures some specific ability and the extent to which it gives clearly
uniform results under similar conditions." In education the experimenter
is never quite sure of identical conditions. Dr. King of the State University
of Iowa set out to find whether there was correlation between the Kansas silent-
reading tests and other subjects. Median grades by sexes are given for the
ninth, tenth, eleventh and twelfth grades of the Iowa City High School. The
number of students tested is not given. The results are given in the form:
students averaging E in school work made a median grade of 42.0 in the reading
test, students averaging C made a median grade of 41.3, those averaging M made

a median grade of 25.0, those averaging P made a median grade of 21.3, those averaging F made a median grade of 14.0. The coefficients of co-ordination (Spearman Foot-rule formula) for freshmen engineers between the Kansas silent-reading tests and hard-opposites tests are found to be \( R = 0.18 \); between Kansas silent-reading tests and scholastic rank, \( R = 0.12 \); and between hard-opposites tests and scholastic rank, \( R = 0.42 \). Although no data or correlation tables are given, the author says, "It will be seen from the above that the results of the hard-opposites tests are a much better measure of the sort of ability that is expressed in class ranks than is the Kansas silent-reading test."\(^1\) This conclusion should be supported by data in order to be convincing. Other coefficients of correlation are given for Junior and Senior liberal arts' students, but no data is given for them. The author states that these Spearman coefficients of coordination may be transmitted into approximate coefficients of correlation by multiplying each by the factor 1.5. This rule is likely to give only a rough approximation to the value of the correlation coefficient.

In the article, "The Relative Proficiency of University Students in an Elementary Course in Zoology,"\(^2\) the author begins by stating that he has data on the grades of six hundred fifty students who have taken the elementary courses in zoology at the University of Illinois. He expresses the results obtained as curves in which the grades are used as abscissae, and the percentages of students in the various years of work are given as ordinates. None of the data is given with which to check up the correctness of the curves. This gives the reader no opportunity to draw his own conclusions from the author's data. The fact is mentioned that many educators agree that the distribution of grades should conform to the normal distribution curve, but the author does not believe

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this assumption is justifiable in grading. None of the curves he obtained follows the normal distribution curve.

In the book, "The Modern High School," there is given a table on "Correlation of Subjects Taught with the High School Teachers' Specific Preparation for Teaching in Twenty Towns of "A" file." It is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>No Correlation</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>48</td>
<td>52</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Totals</td>
<td>50</td>
<td>68</td>
<td>118</td>
</tr>
</tbody>
</table>

It is not clear what the author means by "correlation" and "no correlation" here. They evidently are not correlation coefficients. The value of this work would be increased by giving an explanation of the sense in which the term correlation is used.

The article on "The Relationships between the Abilities Involved in the Study of the Grammar School Subjects," contains the statement "that the ratings of the individuals of a class follow the normal distribution curves fairly well" and further states, "This is good evidence of the validity of the ratings." This is open to criticism because Pearson has developed twelve types of probability curves and distributions of such complex things as grades are perhaps as likely to follow some other types as to follow the normal probability curve. It is therefore a highly questionable method that bases the validity of the ratings on the fact that the grades follow the normal

2. Loc. cit., p. 146.
4. Loc. cit., p. 11.
distribution curve. In fact, the distributions should be examined critically on this point. In ascertaining the answers to questions like the following: "How far does ability in English imply ability in geography?" he used the Pearson correlation coefficient as the best measure of the relationship. In explaining the interpretation of the correlation coefficient, he makes the following statement: "Thus, a correlation between English and arithmetic would mean that E's (excellent), G's (good), P's (passable), V's (unsatisfactory), and B's (bad) in English would all do equally well in arithmetic." He gives the correlation coefficients found between different subjects for boys, girls, and the average between them. In his "note" the author says that E, G, P, V, and B are assigned the values of positive and negative. x and y in terms of the "probable error as a unit, which they would have if the abilities in question were distributed according to the normal frequency curve." However, the form of the curve is not affected by the unit chosen. The author also says, "The details of the calculation can readily be surmised by those acquainted with statistics, and would not interest others." Now those familiar with statistics would have more faith in the coefficients computed if the data were given with which to check the results and if the lines of regression in the correlation tables were drawn, and those unfamiliar with statistics could gather at least some information from a correlation table were it given. The false assumption is again used in testifying in regard to the grades used: "As pertinent evidence to their validity, it will be noticed that here likewise the ratings follow roughly a normal distribution scheme." This hardly constitutes evidence that they were a satisfactory set of grades.

1. Loc. cit., p. 4.
4. Loc. cit., p. 13. (Note)
In "The Correlations of the Abilities Involved in Secondary School Work," the author compares the school marks of children of the same parentage to find a measure of heredity as a factor in education. He computes correlation coefficients for different subjects between pairs of children of the same parents. He states that his data involved different systems of grading, but the marks could be made commensurate if the three hypotheses are accepted, namely (1) that the marks give the relative positions of the pupils within the group, (2) that the abilities in the school subjects follow the normal type of distribution, and (3) that high school students represent a random picking from the total group of boys and girls. His data are not given. He further expresses the idea that the second hypothesis is almost surely true. This does not seem to be a justifiable contention.

The data for the article, "The Relationships between the Abilities Involved in Secondary School Subjects," was obtained from the marks of students in the New York Regents' Examination. In computing correlation coefficients, only the upper fifty per cent of the grades were used in order to eliminate the inaccuracy due to the fact that failures in the examination were not always recorded. The grades of boys and girls were treated together. He expresses the thought that the mixture does not produce any considerable amount of spurious correlation because the differences between the sexes in the degree and variability of ability are slight. The article lacks data. Furthermore, the influence of selecting only the upper half of the grades would have a marked effect on the correlation coefficient.

"The Inheritance of the Ability to Learn to Spell" is a more carefully prepared article. The data are given which is very commendable in as much as so many writers omit it. The Pearson coefficient of correlation for

brother-brother, sister-sister, and brother-sister relationships is computed for School A and School B. The author states that the distribution only roughly approximates to the normal distribution. The lines of regression which are important in interpreting the correlation coefficient are not drawn.

The purpose of the paper, "Correlation of some Psychological and Educational Measurements, with special attention to the Measurement of Mental Ability,"¹ was to discover the intercorrelations of some recently-developed educational and vocational tests and certain psycho-physical tests. He found that the Cancellation Tests used correlated negatively with all those tests which proved to be good measures of mental ability. A "Table A" is exhibited in which Average Raw Pearson coefficients and Corrected Pearson coefficients are given, but no data are shown.

In his paper, "The Spelling Ability of University Students,"² the author gives data for two classes. In finding the relation between general scholarship and spelling ability, he gives a table in which the number of cases for scholarship, 90% or above, 80% to 89%, 79% or below, and the average number of words misspelled in daily work and in examination are given. By observation of the table, he concludes there is a substantial correlation. A table is also given in which the type of error is expressed in percentages.

In the article, "The Relation of Point-Scale Measurements of Intelligence to Educational Performance in College Students,"³ the mean, the per cent of maximum variation, mean variation, and per cent of mean variation for men and women in the statistical results of Point-Scale Examinations are given. The data are not arranged in correlation tables. However, the authors compute correlation coefficients to draw some of their conclusions, and, in one

class conclude by observation that the correlation is positive. Other results are expressed in percentages in tertiles, and in percentages in quartiles.

In the paper, "Standing of Undergraduates and Alumni," previous articles comparing the success of alumni with their scholarship in college are discussed. The author uses histograms to show his data, in which the successive tenths of the class are laid off on the abscissa axis and the percentage of the total number of successful alumni in each tenth of the class is shown on the ordinate axis.

The data in the article, "The Relative Standing in College of Graduates Entering Various Professions," are arranged in tabular form. The college grades are arranged in tertiles, then the per cent in the lowest, middle, and highest tertile is given opposite the ten occupations listed for English, philosophical studies, science (including mathematics, chemistry, physics, engineering, astronomy), social science (including economics, history, government), and foreign languages. Conclusions are drawn from observation of the data without any formal statistical treatment.

The article on "The Permanence of Interests and their Relations to Abilities" is interesting reading, but the statistical treatment of the subject is not convincing. One hundred individuals were asked to judge themselves concerning the order of both their interests and abilities in mathematics, history, literature, science, music, drawing, and other handwork (defined as carpentering, sewing, gardening, cooking, carving, etc.) at three periods, namely during the last three years of the elementary school period, the high school period and the college period. These were to be ranked in order one to seven. These numbers are subtracted to find the difference between elementary interest rank

and high school interest rank. The permanence of interest is then determined from the sum of these seven differences as compared to the sum of the differences if there had been no change (sum, zero), the sum if there had been a maximum change (sum, twenty-four), and the sum if the relative strength of their interests had changed at random (sum, sixteen). In regard to the number 16, a number which represents rank has a pretty large fluctuation in sampling. Indeed it is hard to estimate what this amounts to. The author states, "For the permanence from the elementary-school period to the junior year of college or professional school in my hundred individuals this figure is, on the average 9, three-fifths of the individuals showing sums of from 6 to 12 for column 2 of Table 3."\(^1\)

Table 3\(^2\) which is for one individual is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Difference Between Elementary Interest Rank and High School Interest Rank</th>
<th>Difference Between Elementary Interest Rank and College Interest Rank</th>
<th>Difference Between High School Interest Rank and College Interest Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>History</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Literature</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Science</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Music</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Drawing</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Other hand-work</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Loc. cit., p. 452.
The figure 9 seems to be a very rough approximation. The author then expresses the thought that this average result of 9 can be expressed as a coefficient of correlation equivalent to over .60 and interprets it as meaning six-tenths of perfect resemblance. It is not at all clear how the author passes over from an average result of 9 to a correlation equal to .60. Indeed there are so many pitfalls in correlation theory that it is very doubtful whether this statement can be accepted as correct. At least good statistical practice would demand that the method be shown by which the value .60 can be legitimately obtained from the data. Furthermore, no data for the one hundred individuals are given and we do not know whether the regression was linear. Then, the fact that r is .60 does not mean that it shows six-tenths of perfect resemblance.

The author also states, "A sum of differences of 3 means a resemblance greater than half of perfect resemblance, as the reader expert in the mathematics of probability will realize." That treatment will not stand critical examination.

The sums 12, 10, 8, and 6 are converted into mean coefficients of resemblance or correlation of +.33, +.55, +.71, and +.83 respectively. He finds that the correlation between an individual's order of subjects for interest and his order for ability to be .91. Then he reaches the conclusion that a person's relative interests are an extraordinary accurate symptom of his relative capacities. This conclusion does not have much weight on the basis of his treatment.

In the article, "Examinations, Grades, and Credits," the author says, "In so far as students are graded on the lines of the probability curve, this may measure the attitude of the examiner rather than the distribution of the men in merit." Curves are exhibited showing (1) the distribution of

the stature of women in inches (data from Karl Pearson), (2) the theoretical
distribution of grades, (3) the most convenient distribution in practice, (4)
the distribution of the average grades assigned in five courses, and (5) the
distribution of grades of the College Entrance Examination Board. Both the
histogram and smooth curve for each are drawn. For the curve showing the
distribution of the average grades assigned in five courses, the data are
given for two hundred students expressed in the percentage of students receiving
A, B, C, D, and F for English A, English B, Mathematics A, History A, and
Economics A. Aside from this, he gives no data for the curves. The conclu-
sions are based upon observation of the data and curves.

The purpose of the article, "Spelling Ability - Its Measurement
and Distribution,"¹ is stated to be to derive a scale for the measurement of
spelling ability and to show some of its uses and applications. Complete data
for the first and second preferred lists of words are given. In computing
correlation between grades, the author used Spearman's "Foot-rule" for
Measuring Correlation" and then converted the values obtained into Pearson
coefficients of correlation by using the table in G. K. Whipple's "Manual
of Mental and Physical Tests." Then an attempt is made to justify the use
of the Spearman coefficient by computing the correlations for the same data
by the 'product moment' method \( \rho = \frac{\sum xy}{n \delta_1 \delta_2} \) and by the unlike signs method
\( \tau = \cos \pi v \). The average of the three coefficients for each grade is then
found, and the average of these averages is computed. In comparing the average
of these averages with the average value of the coefficients found by each of
the three methods, the author found that the 'average of the average' was nearer
the value of the average of the Spearman coefficients. The argument in favor
of the Spearman coefficient does not seem justifiable because spurious correla-
tion is likely to enter in when taking so many averages. A good many correlation

coefficients are computed for different schools and grades before the conclusion is drawn that the preferred lists were well selected. Histograms are drawn to show the frequency of each rating in the different grades.

Thus far in this paper, there are offered criticisms of statistical methods that have been used in the study of educational problems. We propose next to explain briefly the theory of what seems to be the most satisfactory methods of studying correlation as it is involved in educational problems, and to illustrate by some applications to grades of students.

THE CORRELATION COEFFICIENT

Two associated classes of variables are said to be correlated when, if values are assigned to one variable, the values of the other variable are such that their mean values are functions of the assigned values. Our problem is to determine with what accuracy we can predict mean values of the associated variable \( y \) from assigned values of a variable \( x \). In other words, do high values of one variable tend to go with high values or with low values of the other variable, or is there no such tendency?

The first important point in correlation theory is the determination of the function which expresses the relation between \( x \) and \( y \), when \( x \) and \( y \) are associated classes of variables. Let \( \bar{y} = g(x) \) be the function which gives the mean value of \( y \) corresponding to a selected \( x \). In this equation, if \( g(x) \) is not zero for all values of \( x \), there is said to be correlation between \( x \) and \( y \). Now suppose we have the following system of associated values: \( (x', y'), (x'', y''), \ldots, (x^n, y^n) \) which are actual measurements.

These data should be arranged in a double entry table called a correlation table, in which the values of \( x \) and \( y \) are arranged along the horizontal and vertical axes respectively. In this table a vertical column under \( x \) is known as an \( x \)-array of \( y \)'s. If correlation exists, it has been found that
the points which are the mean value of each x-array do not lie at random over the field, but arrange themselves more or less in the form of a smooth curve called the "curve of regression" of y on x. It has been found that in a large number of cases this curve is approximately a straight line. Then the means of the x-arrays lie exactly on the line, the regression is said to be truly linear.

We shall consider the case where the curve is approximately a straight line. Hence \( y = \Theta(x) = mx + b \). Since we wish to determine \( m \) and \( b \) so that the y's calculated from the equation will deviate as little as possible from the mean of the observed values, it is necessary that we adopt some criteria as to least deviation. As it is more convenient to deal with deviations from mean values, let \((x_1, y_1) (x_2, y_2) \ldots (x_n, y_n)\) be deviations from the mean, and let the co-ordinate axes be taken through the mean of x and y.

If we adopt the least squares criteria, namely that the sum of the squares of deviations from the mean shall be a minimum, the summation \( \frac{1}{n} \sum_{t=1}^{n} (\hat{y}_t - mx'_t - b)^2 \) must be a minimum. Differentiating this function partially with respect to \( m \) and equating it to zero we obtain

\[
-2 \sum n_t x'_t (\hat{y}_t - mx'_t - b) = 0 \quad (1)
\]

\[
-2 \sum n_t (\hat{y}_t - mx'_t - b) = 0 \quad (2)
\]

Since the origin is taken at the mean, \( \sum n_t \bar{y}_t = 0 \) and \( \sum n_t x'_t = 0 \) and therefore \( b = 0 \) (equation (2).) Equation (1) now becomes

\[
\sum_{t=1}^{n} (\sum y_t) x'_t - m \sum_{t=1}^{n} n_t x'_t = 0
\]

Since \( x' \) and \( y' \) are taken as deviations from the mean, equation (3) may be expressed

\[
\sum_{d=1}^{n} y_d x_d - m \sum_{d=1}^{n} x'^2_d = 0 \quad (4)
\]

If \( \sigma_x \) and \( \sigma_y \) represent the standard deviation of the x system and y system of variates respectively,

\[
\sigma^2_x = \sum_{d=1}^{n} \frac{x'^2_d}{n}
\]

\[ \sigma_y^2 = \frac{\sum_{i=1}^{d=3} y_i^2}{d=3} \]

Substituting these values in equation (4), we find

\[ m = \frac{\sum_{i=1}^{d=3} x_i y_i}{\sum_{i=1}^{d=3} x_i^2} \]

Now we may write the equation of the lines of regression as

\[ y = \frac{\sum x_i y_i \sigma_y}{\sum x_i \sigma_x} x, \text{ or } y = r \frac{\sigma_y}{\sigma_x} x, \]

in which the correlation coefficient, \( r = \frac{\sum x_i y_i}{\sigma_x} \).

In a similar manner, we find the equation of the line of regression of \( x \) on \( y \) to be \( x = r \frac{\sigma_x}{\sigma_y} y \).

The standard deviation of an array is given by the equation

\[ (y - r \frac{\sigma_y}{\sigma_x} x)^2 = \sum y_i^2 - \frac{2 r \sigma_y}{\sigma_x} \sum x_i y_i + \frac{r^2 \sigma_y^2}{\sigma_x^2} \]

Since \( \sigma_y^2 = 2 r^2 \sigma_y^2 + 2 \sigma_y^2 = \frac{\sigma_y^2}{\sigma_x^2} (1 - r^2) \),

\[ (y - r \frac{\sigma_y}{\sigma_x} x)^2 \]

is always positive, the value \( (1 - r^2) \) must be positive. Hence \( r \) is such that \( -1 \leq r \leq +1 \).

Interpretation of \( r \).

If \( r^2 = 1 \), the values of \( y \) computed from assigned values of \( x \) lie on the curve of regression. If \( r = +1 \), the correlation is perfect and positive.
while if \( r = -1 \), the correlation is perfect and negative. If \( r \) is positive, large values of the one variable tend to be associated with large values of the other variable. On the other hand, if \( r \) is negative, large values of one variable tend to go with small values of the other variable. If the variables are functionally independent \( r = 0 \), but the converse is not true.

The existence of correlation does not require that \( r \) have any assigned value such as 0.5. The existence of correlation depends rather on a comparison of the magnitude of \( r \) with its probable error, and on a knowledge of the existence of linear regression. Thus \( r = 0.1 \) with a probable error of 0.0001 means that there is no reasonable doubt as to the existence of correlation.

**THE CORRELATION RATIO**

If the curve of regression is not linear, \( r \) can not be regarded as a satisfactory measure of the amount of correlation. In this case the correlation ratio\(^1 \) \( \rho \) is used. In accurate work it is advisable to compute \( \rho \) as well as \( r \). The quantity \( (\rho^2 - r^2) \) affords a measure of the linearity of the regression.\(^2 \) The correlation ratio is always greater than the correlation coefficient, except when the regression is linear, and in this case \( \rho = r \).

**SUGGESTED METHOD IN CORRELATION STUDIES ON GRADES**

We shall now illustrate the use of these methods as applied to the correlation between the grades of students. The data are grades obtained from the Urbana (Illinois) High School. In the following tables are given the correlation tables for Freshman English with the average of second, third, and fourth-year English; of Freshman Algebra with the average of second, third, and fourth-year English; of Freshman English with Plane Geometry; and of Freshman

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Algebra with Plane Geometry. With these tables the correlation coefficient and its probable error are computed, the lines of regression are drawn, the two correlation ratios for each are computed, and the value \((\gamma^2 - r^2)\) is determined. From these values we have all that is essential for the interpretation of the correlation.

From an examination of \((\gamma^2 - r^2)\) in each of the following tables, it appears that we have linear regression in these cases. The amount of correlation is therefore well described by the correlation coefficient, and we may predict from assigned grades the mean value of associated grades by the use of the correlation coefficient.
TABLE I

Freshman English

<table>
<thead>
<tr>
<th>Class</th>
<th>Average</th>
<th>F</th>
<th>D_a</th>
<th>F_D_a</th>
<th>F_D_a^2</th>
<th>D_a D_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
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<td>1</td>
<td>26</td>
<td>26</td>
<td>676</td>
<td>0.36</td>
</tr>
<tr>
<td>60</td>
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<td>124</td>
<td>248</td>
<td>601</td>
<td>2.10</td>
</tr>
<tr>
<td>65</td>
<td>0.5</td>
<td>3</td>
<td>52</td>
<td>156</td>
<td>784</td>
<td>1.89</td>
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<tr>
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<td>0.3</td>
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<td>2</td>
<td>8</td>
<td>32</td>
<td>1.77</td>
</tr>
<tr>
<td>75</td>
<td>0.1</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>50</td>
<td>0.98</td>
</tr>
<tr>
<td>80</td>
<td>0.0</td>
<td>6</td>
<td>2</td>
<td>12</td>
<td>72</td>
<td>0.27</td>
</tr>
<tr>
<td>85</td>
<td>0.0</td>
<td>7</td>
<td>2</td>
<td>14</td>
<td>98</td>
<td>0.04</td>
</tr>
<tr>
<td>90</td>
<td>0.0</td>
<td>8</td>
<td>2</td>
<td>16</td>
<td>128</td>
<td>0.05</td>
</tr>
<tr>
<td>95</td>
<td>0.0</td>
<td>9</td>
<td>2</td>
<td>18</td>
<td>162</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Correlation coefficient:

\[ r = \frac{.9597}{(1.273)(1.161)} = .649 \pm .016 \]
### Computation in Finding Correlation Ratio of Freshman English and Average of Second, Third, and Four-Year English

<table>
<thead>
<tr>
<th>$x$</th>
<th>$m_x$</th>
<th>$m_y$</th>
<th>$M_y$</th>
<th>$(m_y - M_y)^2$</th>
<th>$(m_y - M_y)^3$</th>
<th>$m_x(m_y - M_y)^2$</th>
</tr>
</thead>
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<tr>
<td>50</td>
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<td>0</td>
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<td>5.72</td>
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<td>32.71</td>
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<td>5</td>
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<td>10.72</td>
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<td>574.55</td>
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<td>5.47</td>
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</tr>
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<td>93.30</td>
<td>7.61</td>
<td>57.91</td>
<td>3474.76</td>
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</table>

\[
0^2 = \frac{10530.42}{588} = 17.9088.
\[
\rho^2 = \frac{0^2}{\varphi^2} = \frac{17.9088}{88.7025} = .5313
\]

\[
\tau = .728
\]

\[
\{ (m^2 - \tau^2) \} = .5313 - .4199 = .1114.
\]

\[
\mu_2 \pm .0173
\]

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m_y$</th>
<th>$M_x$ = $m_x$</th>
<th>$(m_x - M_x)^2$</th>
<th>$(m_x - M_x)^3$</th>
<th>$m_y(m_x - M_x)^2$</th>
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<td>3.62</td>
<td>13.10</td>
<td>2515.20</td>
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<tr>
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<td>161</td>
<td>85.18</td>
<td>1.27</td>
<td>1.61</td>
<td>259.21</td>
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<td>21.52</td>
<td>2603.72</td>
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<td>0</td>
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<td></td>
</tr>
<tr>
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<td>70.10</td>
<td>16.45</td>
<td>272.40</td>
<td>2720.40</td>
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</table>

\[
\rho^2 = \frac{10761.11}{588} = 18.3612
\]

\[
\rho^2 = \frac{0^2}{\varphi^2} = \frac{18.3612}{88.7025} = .4515
\]

\[
\tau = .67
\]

\[
\{ (m^2 - \tau^2) \} = .4515 - .4199 = .0316.
\]

\[
\mu_2 \pm .0096
\]
TABLE II
Freshman Algebra

<table>
<thead>
<tr>
<th>Average of second, third, and fourth-year English</th>
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<th>60</th>
<th>65</th>
<th>70</th>
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<th>80</th>
<th>85</th>
<th>90</th>
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</tr>
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<td>15</td>
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<td></td>
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<td></td>
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<tr>
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<td></td>
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<td>588</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  r &= \frac{0.8123}{(1.143)(1.549)} = 0.458 \pm 0.021 \\
  \gamma_x &= 0.523 \\
  \gamma^2 - r^2 &= 0.274 - 0.2097 = 0.0647 \\
  E_{\gamma x} &= \pm 0.0134 \\
  \gamma_{xy} &= 1.474 \\
  \epsilon &= (\gamma^2_{xy} - r^2) = 0.2251 - 0.2097 = 0.0154 \\
  E_{\gamma_{xy}} &= \pm 0.0068
\end{align*}
\]
### TABLE III

**Plane Geometry**

<table>
<thead>
<tr>
<th>Plane</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>97</td>
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<tr>
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<td>188</td>
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<tr>
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<tr>
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<tr>
<td>47</td>
<td>47</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ r = \frac{79.75}{(1.273)(1.574)} = 0.397 \pm 0.023 \]

\[ \gamma_{xy} = 0.447 \]

\[ \gamma_{yx} - r^2 = 0.175 \]  

\[ \sigma^2 = 0.0134 \]

\[ \sigma_{xy} = 0.0063 \]

\[ \sigma_{yx} = 0.0047 \]

\[ \gamma_{xy} - r^2 = 0.1652 \]  

\[ \sigma^2 = 0.0086 \]

\[ \sigma_{xy} = 0.0050 \]
TABLE IV

Freshman Algebra

<table>
<thead>
<tr>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
<th>100</th>
</tr>
</thead>
<tbody>
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<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

\[
\pi = \frac{1.3012}{(1.577)(1.523)} = 0.541 \pm 0.019
\]

\[
\gamma_{yx} = 0.553
\]

\[
\chi^2 - \lambda^2 = 0.3064 - 0.2926 = 0.0138
\]

\[
\chi_{y \cdot x} = \pm 0.004
\]

\[
\chi_{x \cdot y} = 0.561
\]

\[
\chi^2_{x \cdot y} - \lambda^2 = 0.3157 - 0.2926 = 0.0231
\]

\[
E_{x \cdot y} = \pm 0.0082
\]
In summarizing the methods which educators have used, the following general criticisms should be made. One great difficulty that should be recognized is that in many of the investigations we accept as measurements a set of numbers that perhaps serve at best to put individuals in an order or rank rather than as a measurement of a character. However, assuming that we can accept the data as measurements of characters, we may characterize the statistical methods in the following ways. In some instances, the data are merely inspected, and conclusions are drawn without submitting a tabulation of the data. This method results in many erroneous conclusions which may long pass for scientific results. In other cases, the data are represented by histograms or curves, and the conclusions obtained are based on an inspection of the curves. The use of the figures is nearly always a useful method of picturing correlation, but it can hardly lead to numerical results. Furthermore, there is the method of computing correlation coefficients without exhibiting the data in correlation tables or in tabular form, and without indicating the nature of the lines of regression. Comments on the use of the latter method and its defects occur frequently in this paper.
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