COMPARISON OF GIRDERLESS WITH
BEAM AND GIRDER REINFORCED
CONCRETE FLOORS

BY

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I recommend that the thesis prepared under my supervision by JULIAN CLYDE BANNISTER entitled Comparison of Girderless with Beam and Girder Reinforced Concrete Floors be approved as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

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COMPARISON OF GIRDERLESS WITH BEAM AND GIRDER FLOORS.

Introduction.

In this thesis it is proposed to discuss three of the methods now in use for designing girderless reinforced concrete floors, and to point out as far as possible the advantages or defects of each as evidenced by observations and experiments. Also to make a comparative design, using the ordinary beam and girder construction, with regard to economy, safety, ease of erection, accuracy of design, and fire resisting properties. This design will be of a floor of the same size and for the same loading as in a building recently erected at Minneapolis, Minn. for the Weber and Deere Company, which is a fair example of the type of construction under consideration.

General Discussion.

Girderless reinforced concrete floors are the most important recent development in building construction. While as yet this type is in its infancy, the advance that is being made in its development and the many examples of it in evidence throughout the country make it a subject of considerable importance to builders and designers. The many advantages which it offers over the beam and girder type in the matter of economy and head room, make it especially desirable to the occupant, while its simplicity of design and ease of erection appeals to the contractor and the designer.

The most serious disadvantage which is urged against it is the
lack of exact knowledge concerning the internal stresses occurring in
the slabs. This knowledge is not necessary for safe design, as we
shall attempt to show in our discussion. However, there is always the
question of the minimum amount of material which may properly be used
with perfect safety and hence it is extremely desirable to know exactly
the stresses which occur in this type of floor. Floors designed from
the empirical formulas in general use, when tested, give results suffi-
ciently strong to guarantee a floor to stand a test load of double
the design load without undue deflection or damage to the construction.

A study of the various methods of design in general use reveals
the fact that in every case the analysis is based upon the theory of the
flexure or bearing power of flat, homogeneous plates supported in var-
ious ways, and with some modifications to make the design conform to
concrete as the working material instead of steel, the material consid-
ered in all flat plate theories. This uniformity of acceptance of a
basic principle might lead us to expect a similarity in the working out
of results, but such is by no means the case. The actual designs of
this construction differ radically in their treatment and in the amount
of steel reinforcement used, due to variance of opinion as to how the
plate should be considered supported, and also as to the manner of load-
ing for maximum stresses. Owing to this marked difference of opinion
among the experts, and also to the lack of knowledge of the stresses in
slabs supported in the manner hereinafter described, no exact method
of analysis may be stated. Nor can this be attempted until there have
been more tests of the kind undertaken and successfully carried out by
Mr. Arthur R. Lord assisted by Prof. H. C. Moore and Prof. A. N. Talbot,
all of the University of Illinois Experiment Station.
This test was one made on the before mentioned Weber and Deere Building and may be regarded as one of the most complete and authoritative tests yet made of this type of construction. A detailed description of the method of making the test will not be undertaken here. Suffice it to say that every precaution known to modern experimenters was used to secure accurate results.

In the discussion of the three methods such computations, according to each method, will be made as will serve for comparison; and each design will be made for the design load and according to the general dimensions of the Weber and Deere Building. In this way we may hope to arrive at conclusions which should indicate the relative merits of the systems in so far as the results of a single experiment may be relied upon. As before stated, until more tests of this kind have been made, no definite rules and formulas can be stated, yet the thoroughness and care with which this experiment was performed will warrant a comparison from which we should be able to make valuable deductions.

Description and Results of Experiment.

The building tested was an eleven story and basement warehouse under construction at Minneapolis, Minn. The design is by the Concrete Steel Products Co. of Chicago and was constructed by the Leonard Construction Co. of the same city.

The floor was designed for a uniform live load of 225 lbs. per sq. ft. and the details of the reinforcement are as shown in Fig. 1. The actual amount of this reinforcement in the four steel bands amounts to 7.80 sq. ins. The dimensions of the panels are 18 ft. 6 ins. by 19 ft. 1 in. A 1:2:4 mixture was used, the slab thickness being 9 5/16
FIG. 9.—FALSE WORK FOR INSTRUMENTS AND OBSERVERS.
ins. with an effective depth of 6 ins.

The floor tested was the fourth from the ground and conditions during its construction were such as to give an unfavorable showing of strength. Owing to a failure in the supply of concrete during the pouring of this floor an unusual number of bulkhead separations occur which would tend to decrease the strength. Also the test was made only forty days after pouring, a time so short as not to allow the concrete to be thoroughly cured. Again, in measuring the stress in the steel or concrete the higher value was given whenever a choice was possible; and, in computing the steel stresses from the measured deformation the modulus was taken at 1,375,000 lbs. per sq. in. instead of a lower value, say 1,500,000 lbs. per sq. in. which would probably have been a more just value for concrete of the age of that tested. Table I shows the stresses found in different parts of the slab. The test continued over a period of six days and the maximum load put upon the floor was 550 lbs. per sq. ft.

Fig. 2 represents the loaded portion of the floor (all the panels shown were loaded except the one marked) and shows the position of minute cracks as they appeared at different points. The dotted lines refer to cracks which appeared on the under side of the slabs.

Mr. Lord sums up his conclusions from the test in the following way: "The test gives certain well-defined indications."

"It indicates positively that moments are much greater at the support than at the center of the span.

It indicates, by the position of the cracks, the section for which moments should be calculated at the support.

It indicates that the stress at the center of the span is much
### TABLE OF STRESSES

<table>
<thead>
<tr>
<th></th>
<th>Design 225 lbs per sq.ft</th>
<th>Maximum Load 350 lbs per sq.ft</th>
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<td></td>
<td>L. L.</td>
<td>D. L.</td>
<td>Total</td>
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<td><strong>Reinforcement</strong></td>
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<td>Over Head Diagonal Band Maximum Average</td>
<td>13 800</td>
<td>6 900</td>
<td>20 700</td>
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<td></td>
<td>11 000</td>
<td>5 500</td>
<td>16 500</td>
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<td>5 000</td>
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<td></td>
<td>9 000</td>
<td>4 500</td>
<td>13 500</td>
</tr>
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<td>2 400</td>
<td>1 200</td>
<td>3 600</td>
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<td></td>
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<tr>
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<td>2 800</td>
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<td>4 200</td>
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<td></td>
<td>2 500</td>
<td>1 300</td>
<td>3 800</td>
</tr>
<tr>
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<td>2 300</td>
<td>6 900</td>
</tr>
<tr>
<td></td>
<td>3 800</td>
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<tr>
<td></td>
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<td>234</td>
<td>700</td>
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</tbody>
</table>

*Table I.*
lower than the computed stress at this point would lead us to expect. It indicates that bulkheads act to increase deflections and stresses.

The test shows that the design of this particular building is well balanced and that the stresses are safe. It does indicate, however, that serious questions may be entertained as to the stresses in many buildings of this type where higher percentages of steel have been used and very little additional strength provided at the support.

It indicates that the steel at the center receives its maximum stress for the condition of one panel only loaded."

With this brief description of results and general outline of conclusions we will pass on to a consideration of the three methods of design. Such supplementary statements of results or conclusions from this experiment will be made throughout this paper as occasion may demand.

The Turner System.

Mr. C. A. F. Turner, a Civil Engineer of Minneapolis, Minn. is perhaps the most extensive user of this type of construction in the country; in fact Mr. Turner claims to be the inventor of the system and has several patents pending which he hopes will give him exclusive rights to the use of his methods of design. Fig. 5 illustrates the method of reinforcement used by Mr. Turner. In principle it is the same as is used in all buildings of this type, consisting of two diagonal bands in each panel, and two bands perpendicular to each other and in the line of the columns. The red portions indicate where only one band of steel rods exist in a thickness of the slab and the dotted lines
Fig. 3.
show what Mr. Turner assumes to be the "lines of weakest section". They occur where we would naturally expect them, i.e., bisecting the red areas between the columns where there is less steel to resist the stress than in any other point in the slab. This point is further established by the appearance of cracks in the floor on the under side, approximately along these lines in the test of the Weber and Deere Building.

Mr. Turner does not stop here, however, but goes on to state that this is the line of maximum moment in the panel. Mr. Lord's experiment demonstrates conclusively, however, that the maximum moment is at the edge of the support. Thus, at the very outset, in Mr. Turner's theory, we are confronted by a flat contradiction to conclusive evidence. But let us look farther. To counterbalance this seeming deficiency Mr. Turner uses an increased steel area at the supports, furnished by the column rods which are bent out radially and which will certainly take a radial stress; also the concentric rings which take circumferential stress. It would seem then that we must determine whether the resisting moment at the support, for several panels loaded, (the condition of loading for maximum moment at the support), or the resisting moment at the center or line of weakest section for one panel loaded gives the least factor of safety and judge from the result whether Mr. Turner is warranted in basing his design upon the moment at the center. If the resisting moment at the center does give the least factor of safety then we may conclude that, provided a proper value for the moment is used, Mr. Turner's method of procedure is correct.

Consulting the table of stresses (Table I) we note that the stresses at the center of the span are much lower than those at the support, that in fact they are considerably more than 50 percent lower,
Again, from the theory of fixed continuous beams we know that the stress at the center of the span is one half that at the supports; hence it is only natural to suppose that the same would be true of the continuous slab supported on four sides. The fact that it is actually less than half in the case of our experiment is undoubtedly due to the effect of the slab action in reducing the stress. Thus, it would appear that if we are to design our floor for the moment at the center we must choose a value for the moment which will be the maximum at this point and this occurs, as we know, when only one panel is loaded. It is unfortunate that Mr. Lord in his test did not load a single panel to determine the maximum stress at the center as this would enable us to determine the ratio of the center moment to that at the support, and aid materially in forming a just opinion of the worth of Mr. Turner's theory. However, again referring to the table of stresses we note that in the outer panels, which more nearly approach the condition of one panel loaded, the average stress recorded is 9900 lbs. as compared with 5200 and 7900 lbs. for the centrally located panels, showing an increase of nearly 50 percent in the average. Evidently this stress in the outer panels was for the load not applied on two of the four adjacent panels to the one in which the stress occurred. Hence we may say that the stress would probably increase again as much or 100 percent in all were none of the adjacent panels loaded. This, of course, is only an approximation, but the increase would be at least the amount stated which would make the stress at the center, for a single panel loaded, about, say 12,000 lbs. Evidently the smaller we take this stress the smaller will be the ratio of the center moments to the moment at the support and hence if we compute our steel area for the center and multiply it by the
inverted ratio for the steel area at the support we are erring on the side of safety by assuming a low stress at the center.

Now comparing this stress of 12,000 lbs. with the stress at the support we note that it is just one-half the latter which is given as 24,000 lbs. Thus, it appears that twice as much steel is required at the support as in the center of the panel. In other words, if the steel at the support, as used by Mr. Turner in his construction, has twice the area of that at the center, we may assume that his design is safe, provided a proper value for this moment is used.

Using Mr. Turner's value for the bending moment at this center section of \( M = \frac{WL}{20} \), where \( W \) is the total panel load and \( L \) the distance between supports, we have, using the design load,

\[
M = \frac{121,000 \times 12 \times 12}{50} = 554,000 \text{ in. lbs.}
\]

\[
A_s = \frac{554,000}{0.36 \times 3 \times 15000} = 6.20 \text{ sq. ins.}
\]

It will be noted that Mr. Turner uses a value of \( f_s = 15000 \) as compared with the other designers who use 16000 for the allowable unit stress on the steel. Using the higher value then of \( f_s \) we find that the corresponding value of the moment is \( \frac{WL}{40} \).

He further recommends the use of 1 1/4 in. round rods for vertical reinforcement in the columns, and as these are what form the bent out radial reinforcement at the support, their area must be added to that found above in fixing the total area at this point. Eight of these rods are bent out by Mr. Turner in forming his "mushroom head", thus giving an additional area of 9.32 sq. ins. Beside this there are the two concentric rings of the same size as the column rods having an area of 2.45 sq. ins. making a total of 11.77 sq. ins. of steel reinforce-
ment at the support. Thus, reasoning that twice the reinforcement is needed at the support as at the center of the span, we would say that 12.40 sq. ins. is required at the support, whereas the actual area is 13.47 sq. ins., showing an excess of 6.07 sq. ins. That this excess is probably needed in Mr. Turner's design, we shall now attempt to show.

Mr. Turner has written a book called "Concrete Steel Construction" in which he sets forth the details of his design. Nowhere in this book, to the writer's knowledge, does Mr. Turner specify the size of capitals to be used with his floors. Furthermore, the writer knows that Mr. Turner has designed buildings in which no capitals were used. Observations have shown that this condition tends greatly to increase the stress over the support to a marked extent. This is probably due to two causes: First, the large increase in steel area at the support causes an increase in the steel percentage which is beyond the limit of good practice. This would tend to cause an uneven distribution of stresses in both the concrete and steel, while if capitals of sufficient size were used the concrete area would be correspondingly increased sufficiently to take the compression due to negative moment at the support. Second, the shearing area is greatly increased by the use of capitals and where none are used there would be great danger of having an insufficient area, especially where thin slabs prevail. Hence we may feel justified in criticizing Mr. Turner's design in this respect.

Another reason sometimes advanced for the use of large capitals is that the stresses are not transferred to the outer rods of the beams passing over the support unless the supporting area is large. A brief consideration of a series of tests recently made on wide beams by Prof. Talbot will further the discussion at this point. These beams were 56
ins. wide, 4 ft. 10 ins. long and 5 ins. deep to the steel. Thus, the ratio of depth to width is 1/12 or somewhat in excess of the ratio of depth to steel to width of band in the flat slab. The beams were supported across their full width at the ends and were supported at the third points in some cases for their full width, in others for one-half their width and in still others for one-fifth their width. These beams were made and tested in duplicate at approximately sixty days. The average load carried by beams supported over full width was 15,770 lbs., those supported over half width carried an average of 16,000 lbs., and those supported over fifth width carried an average of 15,500 lbs. From this we see that for the beams supported over one-half their width there was no falling off in the amount of load carried and for those supported over one-fifth their width only a small decrease. In the case of the beam one-half supported then we may assume that all the steel was equally stressed.

So in the case of our flat slabs, if we consider that part of the slab equal to the width of the steel bands as wide beams supported over a fraction of their width, we have an analogous condition and may reason accordingly. It would then seem, if we want all the steel in the band to be effective, that we should support the band over one-half its width. As the band in Mr. Turner's design is a fixed proportion of the span \( \frac{7}{10}L \) to \( \frac{1}{2}L \) we may say that the width of the capital should be at least \( 0.2 L \).

It will now be desirable to compare the stress in the steel as obtained by substitution in Mr. Turner's formula and the actual stress as observed in the test. Obviously the comparison must be with the stress observed at the center of the span for a single panel loaded as
this is the condition for which our value of the moment was taken. Also, in our computations we must use the actual steel area rather than that obtained by the Turner method. Using the live load of the test of 550 lbs. per sq. ft. we have

\[ m = \frac{165,600 \times 13.06 \times 12}{50} = 757,000 \text{ in. lbs.} \]

\[ f_s = \frac{757,000}{.36 \times 8 \times 7.30} = 14,500 \text{ lbs. per sq. in.} \]

The stress, as before computed in this article for the above condition, is 12,000 lbs. per sq. in. This is a favorable comparison and shows approximately that the value of the moment used is within the limit of safety.

Mr. Turner has been very successful in the use of the "Turner System" as many satisfactory occupancies demonstrate; he deserves a great deal of credit for being the first to put forth a design which was such a direct departure from anything previously undertaken.

As stated before, we are unable to state definitely whether the value assumed in this system for the moment is low or not and with this reservation and the one criticism noted we feel warranted in concluding that Mr. Turner’s design is safe, economical and within the limits of good practice.

**The McMillan System.**

Mr. A. B. McMillan of the Aberthaw Construction Company, Boston, has had occasion to design and construct several buildings of this type and his method will now be considered.

His design is based upon general deductions from Grashof’s analysis of the stresses in a steel plate. These deductions are taken from two formulas of the Grashof analysis for the stress in plates sup-
ported in different ways. The first, \( f = \frac{5Wl^2}{4T^2} \) is for plates supported at two edges and continuous, being similar to a wife beam. The second, 
\[ f = \frac{WL^2}{4.27T^2} \]
is for a plate supported at rows of points forming squares of side equal to \( L \), \( T \) being the thickness of the plate. This last, it will be noted, is similar to the girderless floor. He points out that the plate supported on points is 3.2 times as strong as the other, then, since this is true, he reasons that the bending moment in the first case is 3.2 times as great as in the second. Now the bending moment for the first case would be \( \frac{1}{6}WL \), hence for the second it would be \( \frac{1}{6}WL \times \frac{1}{25.6} \) or \( \frac{WL}{25.6} \) from which he takes the bending moment for his design of \( \frac{WL}{25} \), \( W \) being the total load in this case and \( L \) the panel length.

In reasoning this way, Mr. McMillan assumes that Grashof's analysis is correct, but the writer has ascertained that the value of this analysis of the second case referred to above is not clearly established or of unquestioned exactness. However, the values given by Grashof are probably very close approximations, and hence McMillan may be justified in assuming this much, but immediately the question arises as to the applicability of these ratios from Grashof, derived for a homogeneous plate in which Poison's ratio plays an important part, to a concrete slab. In other words, is Mr. McMillan justified in saying that because a homogeneous plate, supported at its four corners is 3.2 times as strong as one supported on two edges, the same will be true of a concrete slab with different manner of transferring stresses, etc? It would seem that unless this statement were borne out by supplementary tests and data we would not be warranted in making the assumption. However, Mr. McMillan's success with this type of building does warrant our
making further investigation of his method, so we will proceed.

For the floor in question, Mr. McMillan's computations would be as follows:

\[ M = \frac{121,000 \times 19.08 \times 12}{25} = 1,105,000 \text{ in. lbs. and} \]

\[ A_s = \frac{1,105,000}{.86 \times 8 \times 16000} = 10.05 \text{ sq. ins.} \]

This value of the steel area, it will be noted, is considerably in excess of that actually used in the bands while Turner's method gave a lesser area than that actually used. When we consider that the value of the moment used by Mr. McMillan is, as he specifically states, the maximum value in the panel, which we know occurs at the edge of the capital, we would suppose that the area as computed above would be the area at the support. Instead, however, Mr. McMillan adds to this area the same amount of steel that Mr. Turner adds at the support. Furthermore, the value of the maximum stress as determined by Grashof is for the stress in the center of the panel and not at the support, as Mr. McMillan says. From these two facts we are forced to conclude that the value of the moment \( \frac{\bar{M}}{25} \) as used by Mr. McMillan is the value of the moment at the center and not at the support. This is the effect of his treatment at any rate and it will be so considered in the further discussion of the subject.

For the test load applied of 550 lbs. per sq. ft., \( \bar{M} \) would equal 165,000 lbs. and

\[ M = \frac{165,000 \times 19.08 \times 12}{25} = 1,515,000 \text{ in. lbs.} \]

and for this \( M \) and the actual steel area of 7.80 sq. ins. the computed stress would be

\[ f_s = \frac{1,515,000}{.86 \times 8 \times 7.80} = 28,100 \text{ lbs. per sq. in.} \]
Comparing this value, which must be that of the stress at the center as before noted, we see that it is over twice the actual stress of 12,000 lbs. per sq. in. a seeming indication that the value of the computed moment is too great.

Mr. McMillan evidently recognizes the value of large capitals in this construction as he gives a formula for computation of capital diameter. The value of the diameter is made dependent upon the slab thickness, the allowable shear and the panel load and is as follows:

\[ d = \frac{W}{TV}, \quad \text{W being the total panel load, T the slab thickness, and V the allowable shear.} \]

Substituting our values in this formula we have for the diameter of capital,

\[ d = \frac{121,000}{9.2 \times 30} = 52 \frac{1}{2} \text{ ins.} \]

as compared with the 54 in. capitals of the building. This makes the diameter of the capitals .25 L, well within the minimum limit before mentioned of .20 L. Mr. McMillan uses the same value for width of the steel reinforcing band as used by Mr. Turner - \( \frac{7}{16} \) L.

From the foregoing discussion it is evident that although the McMillan system appears faulty in some minor details, yet it certainly gives a safe design. Mr. McMillan's attitude is a conservative one. He seems to feel that the subject, being one concerning which we have so little exact knowledge, should be treated cautiously. He has been successful with his construction which is confined mostly to the east.

The Concrete Steel Products Co. System.

Of the theory of this system very little can be said. The engineers of the company have apparently chosen a coefficient for WL both at the support and at the center of the span. This coefficient is appar-
antly based upon experiments, no attempt being made to establish theory in support of it. It will be noted that the value taken for the moment \(\left(\frac{WL}{2}\right)\) at the center, is approximately a mean between those used by Mr. Turner and Mr. McMillan.

From the description given of the building it is evident that most of the details of construction conform to the rules of good practice as enumerated in the discussion of the other two systems. The manner of reinforcement above the column heads differs in one respect; instead of using concentric rings, somewhat smaller rods are used and bent in the form of squares, these resting upon the radial rods as shown in Fig. 1. The capitals are of a diameter equal to .24L.

The computations for steel area and stress will now be undertaken.

\[
M = \frac{121,000 \times 19.08 \times 12}{22} = 340,000 \text{ in. lbs. and}
\]

\[
A_s = \frac{340,000}{.86 \times 8 \times 16000} = 7.64 \text{ sq. ins.}
\]

For the total load (550 lbs. per sq. ft).

\[
M = \frac{165,000 \times 19.08 \times 12}{22} = 1,145,000 \text{ in. lbs.}
\]

and for the given steel area

\[
f_s = \frac{1,145,000}{.86 \times 8 \times 7.80} = 21,500 \text{ lbs. per sq. in.}
\]

Then solving for the area of the steel at the support,

\[
M = \frac{121,000 \times 19.08 \times 12}{12} = 1,849,000 \text{ in. lbs. and}
\]

\[
A_s = \frac{1,849,000}{.86 \times 8 \times 16,000} = 16.79 \text{ sq. ins.}
\]

Also for the test load

\[
f_s = \frac{165,000 \times 19.08 \times 12}{.86 \times 8 \times 18.27} = 50,000 \text{ lbs. per sq. in.}
\]

The stresses, both at the center and at the support, as computed above, are greater than actually occurred. In the case of the center stress the computed stress exceeded the actual stress by some
9000 lbs., indicating again the extremely low stresses which occur at this point. At the support the computed stress exceeded the average actual stress by about 6000 lbs. but the maximum actual stress recorded was 51,000 lbs. per sq. in. or 1000 lbs. greater than the computed stress.

These comparisons are very favorable and indicate an excellent condition of the construction.

Discussion of Basis for Design.

The question now seems to resolve itself into a determination of a proper factor for $WL$, for the moment at the center or at the support or both. It would further seem that if we are to base our design upon only one moment, it were better to determine a value for the moment at the support and then put a lesser amount of steel in the four bands, the ratio of the two steel areas depending upon the amount we determine the reduction in stress at the center to be. Indications are that this reduction would be considerably more than 50 percent, but how much more we are not justified in saying until we have more data upon which to base our opinion. An equivalent method of procedure would be to determine a value of the moment at the support and one for the moment at the center. As has been noted, The Concrete Steel Products Co. use $\frac{WL}{15}$ at the support and $\frac{WL}{25}$ at the center, which means that the moment at the center is only .45 of that at the support. In the Turner system the value of the moment is $\frac{WL}{40}$, in the McMillan system it is $\frac{WL}{25}$. Other buildings have been designed by different designers using values of $\frac{WL}{12}$ to $\frac{WL}{20}$ for the moment at the support and double that amount at the center. With no exceptions these buildings are giving excellent service.
Because of the smaller amount of steel used by Mr. Turner and the consequent less cost of his construction more buildings are being built after his design than of any other. In fairness to the other designers, however, it must be remarked that in certain instances buildings of Mr. Turner's design have been pronounced in an unsatisfactory condition. There is no intimation of unsafety in these statements, which are based mostly upon the appearance of cracks at the supports of the floors in some of Mr. Turner's buildings, simply that the construction is not in first class condition. It is very likely that these cracks are due to insufficient diameter of capital.

It is evident from the foregoing discussion that no definite values for the moments in a girderless floor can be stated. Even though it were practical to state a value for one condition, this value would not hold for other conditions. For instance, the time of year in which the building was to be constructed would govern the value of the moment since if the concrete were to freeze before cured the construction would be weakened.

It is probable that the value assumed by Mr. McMillan gives more steel than is necessary. At the same time we are loath to say that Mr. Turner's construction is too light. It is certain that the latter's floors stand well and probably would be open to no criticism if larger capitals were used. The system of the Concrete Steel Products Co. is conservative, and it would seem that their method of varying the moment value with the conditions is an excellent one.

In the building at Minneapolis, there was an increase of 50 percent in the test load over the design load before cracks could be noticed. Also the maximum deflection noted was insignificant, being
.521 inches for the load of 550 lbs. per sq. ft. Both of these facts are further indications that the design is well balanced and the stresses safe.

Mr. Turner in his book ("Concrete Steel Construction") quotes many tests made on single panels of some of his floors in which the deflections were small. It should be noted that although this condition of loading does not give the maximum stress over the capital it does at the center and also the greatest deflection at the center. The deflections given are in all cases small compared with the heavy test load used.

Enough has now been said to indicate that the three designs, taken as examples of girderless floor construction, give floors which may be depended upon to carry safely their full design load, and we will now turn to a comparison of this type with the beam and girder type.
Design of Beam and Girder Floor.

We will now design one panel of a floor of the same dimensions as the floor of the Weber and Deere Building, by the beam and girder method, using the same design load of 225 lbs. per sq. ft. as was used in the above building. Fig. 4 represents the method of dividing up the panel as well as the final dimensions of the slab and beams. As will be noted the design consists of a flat continuous slab upon cross beams which in turn rest upon the girders which extend between columns and are simply supported at the latter points.

In the computations which we shall make the nomenclature will be as follows:

\[ W = \text{total load on any panel or beam.} \]
\[ L = \text{length of beam or panel.} \]
\[ M = \text{maximum bending moment.} \]
\[ f_s = \text{allowable stress on steel (16000 lbs. per sq. in.)} \]
\[ f_c = \text{allowable stress on concrete (600 lbs. per sq. in.)} \]
\[ n = \text{ratio of moduli of steel and concrete - taken as 15.} \]
\[ k = \text{distance from top of beam to neutral axis.} \]
\[ p = \text{proportion of steel area to concrete area.} \]
\[ j = \text{distance from center of gravity of steel to center of gravity of compressive stress.} \]
\[ d = \text{distance from top of beam to center of gravity of steel.} \]
\[ b = \text{breadth of beam.} \]

All formulas used are taken from Turneaure and Maurer's "Principles of Reinforced Concrete Construction". We will not undertake a dis-
discussion of these formulas as it is thought that they are of so general a nature as to be familiar to all who are experienced in reinforced concrete design.

Design of Slab.- Consider a section of slab 12 inches wide and continuous over the beam supports. Assume the dead load as 75 lbs. per sq. ft. Then

\[ W = 6.56 \times 100 = 1906 \text{ lbs.} \]
\[ M = \frac{1906 \times 6.56 \times 12}{10} = 14,500 \text{ in. lbs.} \]

Now we have the formula

\[ K = \frac{n f_c}{f'_e + n f_c}, \text{ from which} \]
\[ K = \frac{15 \times 600}{15000 + 15 \times 600} = 0.56. \]

Also we have \( j = 1 - \frac{1}{2} K \) or \( j = 0.33. \)

Having these values we may now use the formula

\[ bd^2 = \frac{M}{\frac{1}{2} f_c K j} \]

and having taken \( b = 12 \) inches we can solve for \( d \). Thus

\[ d^2 = \frac{14500}{\frac{1}{2} \times 600 \times 0.56 \times 0.33 \times 12} = 12.7 \]
\[ d = 3.6 \text{ inches} \]

or a total depth of say 5 inches. The total concrete area of the section is then found to be \( 5 \times 12 = 60 \) sq. ins.

Now from the formula

\[ p = \frac{1}{2} \frac{f_s}{f'_c (\frac{f_s}{f_c} + 1)} \]

we may determine the percentage of steel area. We find that \( p = 0.68\% \), so that the steel area is \( 60 \times 0.0068 = 0.408 \) sq. ins. The rods that will
come nearest to giving this area are 5/8 inch round rods spaced 5 inches on centers. This we find gives an area of 0.440 sq. ins. per sq. ft. of slab and they will be used in this design.

Design of Beams.— The total load on one of the beams due to live load and to the weight of the slab will be $6.56 \times 13.75 \times 500 = 55,800$ lbs. To this must be added the weight of the beam itself which we will assume to be 500 lbs. per lineal ft. So now the total load on the beam is 

$$ W = 55,800 + (500 \times 13.75) = 45,100 \text{ lbs.} $$

$$ M = \frac{45,100 \times 13.75 \times 12}{10} = 1,015,000 \text{ in. lbs.} $$

Proceeding in the same manner as in the first case we find that a beam 13 inches wide by 27 inches deep, with a steel area of 5.51 sq. ins. is required to resist this moment. 17-1/2 in. round bars give an area of 5.54 sq. ins. and this number will be used in these beams. The weight of this beam will be

$$ \frac{18 \times 27}{144} \times 150 = 500 \text{ lbs. per lineal ft.} $$

Design of Girder.— The girder will be designed as simply supported and the moment figured accordingly. The loads on the girder will be the two concentrated loads of the beams at the one-third points and the weight of the girder. We will assume this latter to be 1100 lbs. per lineal ft. From this we compute the dead load moment to be

$$ M = \frac{1100 \times (19.08)^2 \times 12}{5} = 600,000 \text{ in. lbs.} $$

and the moment due to the concentrated loads will be

$$ M = 45100 \times 6.56 \times 12 = 5442000 \text{ in. lbs.} $$

making the total moment

$$ M = 4,042,000 \text{ in. lbs.} $$
For this moment a beam 24 inches wide by 45 inches deep, with 6.71 sq. ins. of steel is required. 22-5/8 inch round rods will be used in this beam giving a steel area of 6.75 sq. ins.

Evidently this is as far as it will be necessary to carry the design, as we now have the general form of the floor together with the details of beams and steel reinforcement. Our next step will be to compute the weight of steel and volume of concrete in each of the two designs.

Referring to Fig. 5 we find the following items for the weight of steel in one panel for the girderless design:

24 - 7/16 in. round rods 19 ft. long @ 0.511 lbs. per ft = 235 lbs.
28 - 7/16 " " 26.9 " " @ 0.511 " " " = 335 "
40 ft. of 1 1/4 in. round rods @ 4.172 " " " = 167 "
52 " 7/8 " " " @ 2.044 " " " = 65 "
24 " 5/8 " " " @ 1.045 " " " = 25 "

Total = 875 "

The volume of concrete is

19.08 x 18.75 x .77 = 273 cu. ft. = 10.20 cu. yds.
(the .77 is the thickness of the slab - 5 5/16 ins).

In the beam and girder design we may itemize the steel as follows:

In the slab:

75 / 5/8 in. round rods 19 ft. long @ 0.576 lbs. per sq. ft. = 555 lb.

In three beams:

51-1/2 in. round rods, 18.75 ft. long @ 0.668 lbs. per sq. ft. = 658 lb.

In one girder:

22-5/8 in. round rods, 19 ft. long @ 1.045 lbs. per sq. ft. = 452 lb.

Total = 1612 lb.
The volume of concrete used in this system is as follows:

In slab:

\[ 18.75 \times 19.08 \times 0.416 = 148.5 \text{ cu. ft.} = 5.51 \text{ cu. yds.} \]

In three beams:

\[ \frac{18 \times 27 \times 18.75}{144} = 189.7 \text{ cu. ft.} = 7.02 \text{ cu. yds.} \]

In one girder:

\[ \frac{24 \times 45}{144} \times 19.08 = 142.9 \text{ cu. ft.} = 5.29 \text{ cu. yds.} \]

Total = 17.82 cu. yds.

Comparing the "Totals" in the above computations we see at a glance the relative economy of materials of the two systems. The beam and girder design requires 34 percent more steel and 75 percent more concrete than the girderless. Further, in regard to economy of materials, it is evident from the nature of the two floors that the cost of forms for constructing the girderless system would be quite an appreciable amount less than for the beam and girder. This would be quite an item when we consider that in most cases an ordinary workman can do nearly all the carpenter work in the floor forms for girderless construction, while expert carpenters and more lumber are required for the other system.

Safety.- The discussion in other parts of this thesis has, we hope, made clear that the girderless floor, if properly designed, is an entirely safe type of construction, and fully as strong as the beam and girder type. In fact it is very reasonable to suppose that the factor of safety afforded by arch and slab action in the former type is even greater than in the latter.

Ease of Erection.- We would naturally expect that a floor of form such as the girderless could be erected with greater ease and speed
than one such as the beam and girder, and experience has proved this supposition to be true. Mr. Turner, who has designed many buildings of both types, is loud in his praise of this particular advantage of the girderless floor. The same has been said by others who have used this type, and so again we find that the girderless floor excels.

Accuracy of Design.—In this particular we must give the beam and girder type precedence. The reason for this is evident from previous discussion, but the writer would add that "accuracy of design" should not be confused with "safety of design" for in the latter particular, as has been stated, the girderless type is dependable.

Fire Resisting Properties.—In this item of comparison we have one of the chief advantages of the girderless floor. In considering this item it is at once apparent that, other things being the same, that construction which exposes the least surface to flames will best resist the disintegrating effects of heat. That construction will also better protect its steel and as this material is most liable to deterioration when exposed to heat it is evident that this is the important consideration. Evidently the girderless floor, with its comparatively small amount of exposed surface is the ideal type and is by all means superior to other types in this respect. Under this head we would also mention the increased ease of fire protection which obtains in this type. The ease with which automatic sprinkler devices may be installed and operated, and also the unobstructed passage way for fire streams afforded by the flat ceilings, are important factors in reducing insurance rates.

Still another advantage which the girderless floor offers might be termed "Desirability of Occupancy". A glance at the accompanying photographs of the interiors of buildings of this construction impresses
Interior Lindeke-Warner building, St. Paul, Minn. Mushroom System.
one with this desirability in three respects: (1) appearance, (2) lighting, and (5) overhead space. The appearance speaks for itself, and manifestly doing away with the beams and girders gives a better lighting in the rooms. The matter of head room is important. We note in our beam and girder design the amount of space taken up by the beams and girders, thus necessitating a greater distance between floors and resulting in an increased height of building. This is evidently very undesirable for a number of reasons and the fact that the girderless floors do away with these gives it a distinct advantage over the other type.

Having thus summed up the relative merits of these two systems we will conclude our remarks having covered the topics outlined in the introduction to this thesis.