CRIST

Design of a Water Flow Recorder whose Flow is Directly Proportional to the Head

Mechanical Engineering

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1913
DESIGN OF A WATER FLOW RECORDER WHOSE FLOW IS DIRECTLY PROPORTIONAL TO THE HEAD

BY

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Mechanical Engineering

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INTRODUCTION.


Liquid measuring devices may be classified as follows: Volume meters, in which the liquid is measured by filling and emptying tanks or buckets; displacement meters, in which the liquid moves a piston of some sort; weighing meters, in which the mass of the liquid is balanced against counter weights or springs; impact meters, which utilize small paddle wheels or vanes; and finally, flow meters.

Flow meters have the advantage over the other types mentioned, in that the working parts do not necessarily come into contact with the liquid to be measured. This is a decided advantage when measuring boiler feed water, which often contains sludge or scale forming impurities, or when measuring corrosive liquids.

Flow meters are subdivided as follows: Venturi meters and Pitot tubes, in which the velocity head is measured as a pressure at certain points in the conduit; and weirs, in which the head causing the discharge is ascertained by actual measurement of the difference of level between the surface of the water on the upstream side and the crest of the weir.
Wherever it is possible for the flowing water to have a free surface, the use of the weir is preferred: First, because of the great accuracy which is obtainable in the measurement; and secondly, because the weir is continually open for the removal of mud, sludge, etc.

Another advantage of the open weir is that, by measuring the head independently and then computing the flow by a simple formula, the accuracy of the recording instrument can be easily checked at any time.

With Venturi meters it is just the opposite. Expensive and difficult measuring or weighing tests must be made for the calibration of these meters where the liquid is confined in a closed conduit.
CHAPTER 1.
The Design of a New Form of Weir.

Art. 1. Type of Weir.

The object of this thesis is to present the design of a new form of weir, one in which the flow is directly proportional to the first power of the head, and also to give its advantages over the present forms of weirs in use.

A weir embodying this principle was designed and built by Professor O. V. P. Stout, of the University of Nebraska in 1897. The weir was tested but the results of the test were not published.

Art. 2. Theoretical Derivation of the Form of Notch.

The theoretical derivation of the form of weir is as follows: The plane of the notch is vertical and the crest horizontal. The breadth of the notch at any height above the crest is inversely proportional to the square root of that height.

Let \( a \) = breadth of weir at any height \( y \) above the crest.
Let \( A \) = breadth of weir when \( y = 1 \).

Then \( \frac{a}{\sqrt{y}} = A \)

Let \( h \) = head of water on crest, and \( Q \) = the theoretical discharge in cubic feet per second, all linear dimensions being expressed in feet.
Then at any height $y$ above the crest, the area of the elementary strip = $\frac{a}{y}$ $dy$. The head on this elementary area = $(h - y)$. Then the velocity of flow through the area = $V = \sqrt{2g(h-y)}$. The theoretical flow = $dQ = VdA$. or $dQ = \frac{\sqrt{2g(h-y)}}{h} \frac{a}{y} dy$. Integrating between $y = 0, y = h$.

$$Q = \int_0^h \frac{\sqrt{2g(h-y)}}{h} \frac{a}{y} dy = \frac{\pi}{2} a \sqrt{2g} h.$$  

The term $\frac{\pi}{2} a \sqrt{2g}$ is a constant, therefore, $Q$ is directly proportional to $h$.

The curves plotted from the formulae thus obtained are tangent to the $X$ axis at infinity. The following substitution was made in order to make the weir practicable.
If CD, Fig II., is drawn parallel to the crest AB and the perpendiculmrs CE and DF are dropped to the crest from points C and D, then the area CDPE is equal to the sum of the areas ACE and DFB (considering the points A and B at infinity).

To prove this the following steps are necessary:
1. To find the areas ACE and DFB.
2. To prove that their sum is equal to the area CDPE.

To find 1.

The equation of the curve = \( 2x = \frac{a}{\sqrt{y}} \).

Squaring \( 4x^2 = \frac{a^2}{y} \) or \( y = \frac{a^2}{4x^2} \).

\[ A = \int y \, dx = \int \frac{a^2}{4x^2} \, dx. \]

Integrating between the limits \( x = x_1 \) and \( x = \infty \), the expression becomes \( A = \int_{x_1}^{\infty} \frac{a^2}{4x^2} \, dx = \frac{a^2}{4x_1} \), which is equal to the area \( ACE = area \, DFB \).

To prove 2.

Let \( OF = x_1 \).

DF = \( y_1 = \frac{a^2}{4x_1^2} \).

Therefore \( OF \times FD = \frac{a^2}{4x_1} = \frac{1}{2} \) area CDPE.

But from 1. Area \( ACE = \frac{a^2}{4x_1} \).

Therefore area \( ACE = \frac{1}{2} \) area CDPE, or area \( ACE \) plus area \( DFB = area \, CDPE \).

In the construction of the weir the areas \( ACE \) and \( DFB \) are replaced by circular orifices whose areas are equal to those of \( ACE \) and \( DFB \) and whose centers of gravity are at the same height.
above the crest as those of the replaced areas.

The method of finding the height of the center of gravity of the replaced area above the crest is given below. Using Fig. II. again:

\[
\bar{y} = \frac{\int \int y \, dx \, dy}{\int \int dx \, dy} = \frac{\int \int y \, dx \, dy}{A}
\]

\[
\bar{y} = \frac{\int_{x=0}^{\infty} \int_{y=0}^{\infty} \frac{a^2 x^2}{2^2} \, dy \, dx}{A} = \frac{\frac{a^4}{16} \int_{x=0}^{\infty} \frac{dx}{x^2}}{A}
\]

\[
\bar{y} = \frac{a^4}{48x^3} = \frac{a^4}{48\pi^3} = \frac{a^2}{12x^2}
\]

\(x\), being the distance OF in Fig. II.

Art. 3. Design of the Experimental Weir.

The weir used in the tests was designed by using the above theory. It was designed for a theoretical capacity of one cubic foot per second at an eight inch head. Thus limiting \(Q\) and \(h\), everything was constant in the formula \(Q = \frac{1}{2} a \sqrt{2g \, h}\) excepting \(a\), which was then easily found.

Using the equation \(X = \frac{a}{1Y}\) the breadth of the weir, \((X)\) at any distance \((Y)\) above the crest could be found. Taking different values of \(Y\) and plotting the values of \(X\) thus found, the outline of the weir was plotted.

The length \(CD\) Fig. II. was made as great as was practicable, thus replacing as little as possible of the theoretical weir by the circular orifices. The limit placed on the width of crest, including the circles, was about twelve inches.
The experimental notch was cut from a piece of three-sixteenths inch boiler plate. Plate I. shows a scale drawing of the notch as designed.
PLATE-1.
DRAWING TO SCALE.
OF
WEIR USED IN TESTS.

Scale $\frac{1}{2}'' = 1''$. 
CHAPTER II.
The Method of Experimenting.

Art. I. Testing Apparatus.

The notch was set on a large tank properly baffled so as to give a smooth water surface both at the hook gage and at the weir.

A hook gage was used in measuring the head on the weir. A calibrated measuring tank, situated below the weir tank, served as a measuring device for the water flowing over the weir.


A constant head was maintained on the weir. The quantity of water flowing through the weir for a certain interval of time was measured in the calibrated tank. By varying the head and proceeding in each case as above, the coefficient of discharge at the various heads was found. All time measurements were accurately taken with a stop watch.

The temperature of the water was not taken into account for the reason that, in finding the coefficient of discharge, all changes in the volume of the water due to a change in the temperature of the water, would cancel out in the calculations.
Art. 3. Results.

The results of the tests are shown in Table 1.

For the weir used in the test, the formula for the actual discharge in cubic feet per second becomes

\[ Q = 0.955 \, h. \]

Art. 4. Conclusion.

The advantages of this form can be enumerated as follows:

1. Since the flow varies as the first power of the head, an error or inaccuracy in the measurement of the head causes only a proportionate error in the computed discharge. In the usual form of weir this error is magnified in the computation.

2. For the reason stated above, accuracy in setting the weir is not so important as with other forms.

3. Its simplicity of use makes it easily understood by others than engineers.

4. Experiments show that it is very accurate at heads above one inch and that the maximum error at very low heads is small.

Figure III shows the variation of the coefficient of discharge with different heads on the weir.
5. It lends itself readily to autographic recording.

Its disadvantages are:

1. Its construction is more complex than that of any other form of weir.

2. It is not applicable to the measurement of large quantities of water, for the form is such that the weir becomes ungainly when the theoretical capacity exceeds two cubic feet per second.

3. The weir is inaccurate at heads lower than one inch.
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<th>Actual Head, Feet</th>
<th>Actual Flow, Gals. per min.</th>
<th>Coefficient of Discharge</th>
<th>Theoretical Flow, Gals. per min.</th>
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CHAPTER III.

Design of a Recording Device for this Type of Notch.

Art. 1. Description.

With this new form of weir the recording device would be very simple as can be seen from Plate II.

A revolving drum B is connected to the stem of a clock A, and it is so arranged that it will make one revolution every twenty-four hours.

A float rod C containing a rack D is connected to a common floating mechanism, the height of which varies with the head on the weir. The rack D meshes with the small pinion E, which is a part of the threaded shaft F. The pencil mechanism G is fastened to the hub H, which acts as a nut on F. Thus as G rises, G is moved to the right a proportionate distance. By means of a pair of small rollers I, which follow along J, the pencil mechanism is held against the drum no matter whether the float rod is moving upward or downward.

This arrangement is much simpler than that required by other weirs. With other forms rectifying devices are necessary to make each ordinate on the drum represent a given quantity of water.