The Design and Test of a New Type of Telephone Repeating Coil

Electrical Engineering

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THE DESIGN AND TEST OF A NEW TYPE OF TELEPHONE REPEATING COIL

BY

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THESIS
Submitted in Partial Fulfillment of the Requirements for the
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IN

THE GRADUATE SCHOOL
OF THE
UNIVERSITY OF ILLINOIS
1915
A Cozy Place in the Snow for
The Thinnest Snowman

Snow I

The snowman's heart - a fine frozen heart

Parson's Noke

Merry Christmas

[Handwritten note at the bottom]
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Hubert Michael Turner

ENTITLED The Design and Test of a New Type of Telephone Repeating Coil

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Electrical Engineering

Morgan Brooks
In Charge of Thesis

Head of Department

Recommendation concurred in:

Committee on Final Examination*

*Required for doctor's degree but not for master's.
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I INTRODUCTION.

The usual type of telephone repeating coil consists essentially of two separate coils of insulated wire wound upon an iron core. One of these coils, called the primary, receives energy from one circuit and the other, called the secondary, delivers energy to a second circuit. This type of repeating coil depends for its action upon the magnetizing effect of the primary current upon the iron core. It may be stated that repeating coils for local communication have been developed to a high state of perfection but for long-distance telephone transmission, e.g., of from 500 to 4000 miles, there seems to be room for improvement.

It has been said that, in general, any important change in design or type must be justified by commercial or engineering considerations, such as lower cost, greater economy, or improved performance. With these things in mind the design of a coil without iron was undertaken. The object of this investigation was to ascertain by experiment whether or not the non-ferric repeating coil could be made commercially superior to the regular type.

Function of repeating coils.

The chief function of repeating coils is to keep the different sections of telephone lines physically separate. Such circuits provide a means by which three conversations may be
carried on at the same time over two pairs of wires. The third conversation is made possible by use of what is known as the phantom circuit. This circuit is obtained by connecting the leads from the third telephone to the mid-points of the repeating coils in the metallic circuit, as shown in Fig. 1, thereby using the two wires of each metallic circuit as one side of the phantom. Such a phantom circuit is in operation on the transcontinental line between New York and San Francisco.

**Fig. 1.**

In addition to the three conversations it is possible to send simultaneously and without interference as many as eight telegraph messages. Such a quadruplex system is used between New York and Chicago.

The repeating coil also makes possible the use of the common battery system.

Discussion of regular type.

Repeating coils may be divided into two classes, the ring-through coil which repeats both ringing and talking currents,
and the talk-through coil which repeats voice currents only. It has been found that ring-through coils, in order to transmit the energy of the low frequency ringing currents, require a large amount of iron. In the talk-through coils iron is also used but the amount is much less, the reason being that the talking efficiency of coils with large iron cores is low.

The presence of iron in repeating coils is objectionable for the following reasons: first, energy is dissipated in the core by hysteresis and eddy current losses and thus the efficiency is reduced; second, the wave is distorted due to the cyclical variation of the permeability and the reaction of the eddy flux.

The iron being situated in a magnetic field of rapidly varying value, it is evident that energy is consumed in reversing the magnetism twice in each cycle and also by the eddy current losses.

Even more harmful than this loss of energy is wave distortion. The rise and fall of current is modified by the magnetic effect of eddy currents, and the effect of hysteresis in the core is to distort the wave still further. The overtones or harmonics which give character to the voice and by which one is able to recognize a friend speaking over the telephone, are modified and the quality of the received tones are impaired by this distortion.

Dr. Lupin in discussing his famous telephone loading coil made the following statement:

"It should be observed here as a warning that unless inductance coils with iron cores are constructed in such a way as to keep down the magnetization, hysteresis, Joule heat current losses and the distortion of the current by varying value of
magnetic permeability at each cycle of magnetization will work disastrously." The point is that Dr. Pupin recognized distortion as a possibility although apparently he did not investigate the matter further.

Mr. L. C. Helwig, Rensselaer Polytechnic Institute, in discussing the results of some rather elaborate experiments on repeating coils of the regular type with the core removed and divided into four equal parts so that 1/4, 1/2, 3/4, or all of it might be used, has the following to say:

"The great discrepancies between the current values at any time obtained from the theoretical equation and those obtained from the oscillograph curves, can only be explained on the theory that the iron core itself was acting as a relatively good conducting secondary circuit.

"From a study of these curves, I am convinced that the value and effect of the eddy currents produced in the iron wire core, are very much greater than is generally supposed, and that their effect upon the secondary current wave form in the telephone transformer should not be neglected."

Iron does not increase the permeability sufficiently to justify its use. The engineer, who is accustomed to dealing with high magnetic saturation say of from 8000 to 15 000 lines per square centimeter, where \( \mu \) may be as high as 5000, is likely to overestimate the value of iron as a means of increasing the flux. Professor Ewing in his book on "Magnetic Induction in Iron and Other Metals" (pages 119 and 128) states that for extremely weak magnetizing forces the value of \( \mu \) may fall below 100.

A few examples showing the actual value of \( \mu \) in the case of
iron-cored coils used for telephone purposes may be of interest. In one of Dr. Pupin's loading coils, where 80 feet of No. 12 wire was wound in two layers of 48 turns each as shown in Fig. 2, the inductance obtained, according to Dr. Pupin, was 0.042 henries.

In other words, this is equivalent to an effective permeability of 56. In another coil of 1600 feet of No. 6 wire with an iron core the inductance is 0.4 henries. The same wire wound into prescribed shape for maximum inductance gives 0.062, or an equivalent permeability of only 3. Western Electric 25-A repeating coil, in which 1000 feet of No. 26 wire is used in each winding, has an inductance of approximately 0.75 henries. An inductance of 0.12 henries could have been obtained without the use of iron. Thus it is seen that the equivalent permeability is only 0.25.

If it is necessary to keep down the magnetization, as Dr. Pupin has suggested, the very object for which the iron is used is defeated.
II THE DESIGN.

From the discussion which has preceded it would evidently be advantageous to eliminate the core loss and wave distortion. The best way to accomplish this result, and the method here proposed, is to design a coil without iron.

Since it is the object of these coils to transfer energy from one circuit to another it is desirable to make the inductance large and the resistance small, for with a given frequency, the relative positions of the primary and secondary remaining the same, the magnetic field should be strong and the dissipating factor low. The foregoing principle was used in this design.

In conformity with general practice the primary and secondary had the same number of turns. The weight of the repeating coil was kept within reasonable limits. No changes whatever in the subscriber's station were contemplated.

Since the calculations involved were those applying to coils without iron, in which the inductance is a function of the shape of the coil, reference is here made to the University of Illinois Engineering Experiment Station Bulletin No. 55 on "Inductance of Coils" by Professor Morgan Brooks and the writer of this thesis.

In the bulletin referred to it was shown that, regardless of the size of the conductor, the shape or relative dimensions of the coil for producing the maximum inductance from a given
length of wire was the same. The prescribed shape for maximum inductance is shown in Fig. 3, where

\[ a : b : c = 1.5 : 1.2 : 1.0 \]

Reference is made to equation 17, page 27 of the above-mentioned bulletin, which for convenience is given here.

\[ L_m = \frac{(0.609 \times 10^{-9} \times Cm^{5/3})}{D^{2/3}} \]  \hspace{1cm} (1)

where \( L_m \) is the maximum inductance in henries, and where \( Cm \), the length of the conductor, and \( D \), the overall diameter of the wire, including the insulation, are expressed in centimeters. It will be observed that the maximum inductance is dependent upon the ratio of length to outside diameter of the conductor, and independent of weight so long as this ratio is constant.
According to the fundamental principle previously given, the inductance of the primary and secondary windings must be large in comparison with their respective resistances. Since the resistance of a given size of wire varies directly with its length and the inductance varies as the \( 5/3 \) power it is evident that the ratio of inductance to resistance varies as the \( 2/5 \) power of the length. In other words it was necessary to use a considerable length of wire in order to make this ratio large.

The usual form of the equation for resistance is

\[
R = \frac{(10.8 \times l)}{d^2},
\]

where \( l \) is the length of the conductor in feet, and \( d \) is the diameter of the bare wire in mils (0.001 inch).

If \( l \) and \( d \) are changed to centimeters

\[
R = \frac{10.8 \,(Cm/30.48)}{(1000 \,d)^2 / 0.45} = 2.566 \times 10^{-6} \, \frac{Cm}{d^2} \quad (2)
\]

The ratio of \( L \) to \( R \) is obtained as follows:

\[
\frac{L}{R} = \left( \frac{0.609 \times 10^{-9} \times Cm^{5/3}}{(2.466 \times 10^{-5} \times Cm)} \right) / \frac{D^{2/3}}{d^2} = 2.665 \times 10^{-4} \, \frac{d^2 \, Cm^{2/3}}{D^{2/3}} \quad (3)
\]

Here the ratio of inductance to resistance is given in terms of the dimensions of the wire used.

The power factor of the coil in this case may be expressed as the \( \cos \tan^{-1} (X/R) \) or the \( \cos \tan^{-1} (2\pi fL/R) \).
For voice currents $2\pi f$ is usually taken as 5000. This mean value may be obtained experimentally by Haupt's method. As shown in Fig. 4 a resistance is connected in shunt with the telephone and gradually increased until the signals are just audible. A condenser is then put in place of the resistance and its capacity varied until the signals are again just audible. He assumes that the impedance of the shunt path is the same in the two cases. Therefore $R$ is equal to $1/2\pi fC$ from which $f$ may be determined.

\[
\frac{X}{R} = 5000 \quad L/R = 1.33 \frac{d^2}{Cm^2/3} / \frac{D^2/3}{}. \tag{4}
\]

For the benefit of those who prefer the English system of units these equations will be given expressing the length of the conductor, $Ft$, in feet and the diameter of the bare wire, $d$, and the overall diameter, $D$, in inches.

\[
L_m = 97.3 \times 10^{-9} \times \frac{Ft^{5/3}}{D^{2/3}}. \tag{5}
\]

\[
R = 10.8 \times 10^{-6} \frac{Ft}{\Omega}. \tag{6}
\]

\[
L/R = 0.009 \frac{d^2}{Ft^{2/3}} / \frac{D^2/3}{}. \tag{7}
\]

\[
X/R = 45.06 \frac{d^2}{Ft^{2/3}} / \frac{D^2/3}{}. \tag{8}
\]
For convenience of reference and to facilitate calculation curves will be plotted showing the variation of $L$, $L/R$, and $X/R$ with length of conductor (see Figs. 5, 6, 7, 8 pages 11, 12, 13, 14). On logarithmic cross section paper the curves representing these equations become straight lines which are determined by one point and the slope of the line which is the exponent of the variable.

A sample calculation will be given showing how $L$, $R$, $L/R$, and $X/R$ are obtained from equations (5), (6), (7), and (8) for 10000 feet of No. 20 enameled wire where $d = 0.032$ and $D = 0.034$ inches (see Table I).

TABLE I.  
Enameled wire. 

<table>
<thead>
<tr>
<th>No.</th>
<th>Ohms per 1000 ft.</th>
<th>$d$ in.</th>
<th>$D$ in.</th>
<th>Rounds per 1000 feet</th>
<th>Turns per in.</th>
<th>Turns per sq. in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>3.32</td>
<td>0.064</td>
<td>0.067</td>
<td>12.68</td>
<td>14</td>
<td>322</td>
</tr>
<tr>
<td>16</td>
<td>4.01</td>
<td>0.054</td>
<td>0.054</td>
<td>7.97</td>
<td>16</td>
<td>360</td>
</tr>
<tr>
<td>18</td>
<td>6.37</td>
<td>0.040</td>
<td>0.040</td>
<td>5.61</td>
<td>23</td>
<td>567</td>
</tr>
<tr>
<td>20</td>
<td>10.14</td>
<td>0.033</td>
<td>0.033</td>
<td>3.14</td>
<td>29</td>
<td>865</td>
</tr>
<tr>
<td>22</td>
<td>10.12</td>
<td>0.033</td>
<td>0.033</td>
<td>3.12</td>
<td>29</td>
<td>865</td>
</tr>
<tr>
<td>24</td>
<td>20.68</td>
<td>0.020</td>
<td>0.020</td>
<td>1.83</td>
<td>45</td>
<td>3000</td>
</tr>
<tr>
<td>26</td>
<td>40.75</td>
<td>0.016</td>
<td>0.016</td>
<td>0.777</td>
<td>57</td>
<td>5560</td>
</tr>
<tr>
<td>28</td>
<td>54.79</td>
<td>0.014</td>
<td>0.014</td>
<td>0.485</td>
<td>71</td>
<td>5100</td>
</tr>
<tr>
<td>30</td>
<td>103.00</td>
<td>0.010</td>
<td>0.012</td>
<td>0.305</td>
<td>88</td>
<td>7530</td>
</tr>
</tbody>
</table>
Fig. 6.
\[ \log L = \log 97.5 - 9 \log 10 + \frac{5}{3} \log 10000 - 2/3 \log 0.034 \]
\[ = 1.9861 - 9.0000 + 6.6667 - 1.0410 \]
\[ = 0.6858 \]

\( L = 4.5 \) henries.

\[ \log R = \log 10.6 - 6 \log 10 + \log 10000 - 3 \log 0.034 \]
\[ = 1.0334 - 6.0000 + 4.0000 - 5.0103 \]
\[ = 2.0051 \]

\( R = 105 \) ohms (this checks with the value in tables).

\[ \log \left( \frac{L}{R} \right) = \log .009 + 2 \log .032 + \left( \frac{2}{5} \right) \log 10000 \]
\[ - \left( \frac{2}{5} \right) \log 0.034 \]
\[ = 5.9845 + 5.0103 + 2.6667 - 1.0410 \]
\[ = 12.6102 \]

\( \frac{L}{R} = 0.041 \)

\[ \log \left( \frac{X}{R} \right) = \log 45.06 + 2 \log .034 + \left( \frac{2}{5} \right) \log 10000 \]
\[ - \left( \frac{2}{5} \right) \log 0.034 \]
\[ = 1.6858 + 5.0103 + 2.6667 - 1.0410 \]
\[ = 2.3098 \]

\( \frac{X}{R} = 0.04 \).

Power factor = \( \cos \tan^{-1} \left( \frac{X}{R} \right) \)
\[ = \cos 89^\circ 45' \]
\[ = 0 \) approximately. \]
As a first approximation it was assumed that a good air-cored coil should have the resistance and inductance and therefore the same power factor as a good iron-cored coil, say the L. L. 25-A.

The object of this preliminary design was two-fold: first, to determine whether the amount of copper required was prohibitive; second, to ascertain whether the performance of the coil justified further refinements in design.

If the average inductance of L. L. 25-A is taken as 0.75 henries and the average copper resistance as 40 ohms per coil, the ratio of reactance to resistance is 94, where \( B^2T = 5000 \). Since inductance depends upon the compactness of the winding it is desirable to use thin insulation such as enamel.

The coils of the prescribed shape for maximum inductance might be placed end to end to form the repeating coil or one might be placed inside the other but in either case there would be considerable leakage flux between the coils, as will be more fully discussed later. The leakage flux can be materially reduced by winding the primary and the secondary at the same time so that they are contiguous throughout their length as indicated in Fig. 9, page 24.

Since the equations and the curves given apply accurately only to closely wound coils of the prescribed shape for maximum inductance allowance must be made in the value of \( X/R \), on account of the decrease of inductance due to the spaced winding (see Fig. 9). If instead of 94, 110 is used for \( X/R \) it will compensate for this decrease and also make some allowance for imperfections in winding.
Now by referring to the curve sheet on page 14 it will be seen that 1000 feet of No. 14, 1540 feet of No. 16, 2500 feet of No. 18, 6700 feet of No. 20, or 10,500 feet of No. 24 enameled wire will give the required value of X/R. But X/R = 110 is not the only condition that must be fulfilled. R must not greatly exceed 40 ohms and L must approximate 0.88 henries (increased from 0.75 on account of spaced winding). Table II gives values of R and L for sizes and lengths indicated which will make X/R = 110.

**Table II.**

<table>
<thead>
<tr>
<th>No.</th>
<th>Length in feet</th>
<th>Weight in pounds</th>
<th>Ohms per 1000 ft.</th>
<th>R</th>
<th>Lm</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>1000</td>
<td>13.68</td>
<td>2.58</td>
<td>2.55</td>
<td>0.06</td>
</tr>
<tr>
<td>16</td>
<td>1540</td>
<td>13.60</td>
<td>4.01</td>
<td>6.20</td>
<td>0.13</td>
</tr>
<tr>
<td>18</td>
<td>2500</td>
<td>13.50</td>
<td>8.57</td>
<td>13.80</td>
<td>0.26</td>
</tr>
<tr>
<td>20</td>
<td>4000</td>
<td>12.96</td>
<td>10.14</td>
<td>40.06</td>
<td>0.95</td>
</tr>
<tr>
<td>22</td>
<td>6700</td>
<td>13.40</td>
<td>16.18</td>
<td>108.00</td>
<td>2.34</td>
</tr>
<tr>
<td>24</td>
<td>10500</td>
<td>13.90</td>
<td>25.63</td>
<td>256.30</td>
<td>6.24</td>
</tr>
</tbody>
</table>

The results given under Lm are the maximum values obtainable from size and length of wire specified as taken from curves on pages 11 and 14. These values will not be realized on account of the spaced winding. It will be noted that in these approximate values there is no appreciable difference in the weight of wire required to produce the given value of X/R, since the increased length of the smaller sizes compensates for the greater weight per unit of the larger sizes.
It is evident, therefore, that to meet approximately the conditions specified, that is, \( R = 40 \), \( L = 0.88 \), and \( X/R = 110 \), it was necessary to use 4000 feet of No. 20 enameled wire in the primary and a like amount in the secondary.

The coil as wound had the following dimensions: radius core 1.75 inches, thickness of winding 1.5 inches, length of winding 6 inches.

The total weight of copper used was about 25 pounds. Thus the amount of copper involved is not prohibitive and the cost of manufacturing compares favorably with that of the regular iron-cored repeating coil. The performance of the coil, which will be more fully explained under 71220, was very satisfactory.

**Final design.**

From results obtained experimentally with the coil made in accordance with the preliminary design it was believed that some further refinement of design was worth while. In the preliminary coil the primary and secondary, as individual coils, were not wound in the most advantageous manner and in consequence thereof more wire was used than otherwise would have been necessary.

Instead of using the self-inductance of the coil as the determining factor mutual inductance should be used, for it is upon the rate of change of the mutual magnetic flux that the transfer of energy from the primary to secondary depends. Assuming that the coefficient of coupling, \( k \), for coils with iron core equal to 0.9 (see Test 2), the mutual inductance may be
calculated as follows:

\[ M = kL = 0.9 \times 0.75 = 0.675 \text{ henries.} \]

The conditions that must be satisfied in this design are:

- \( M \) must be equal to or greater than 0.675 henries,
- \( L \), the true inductance of each coil, must be equal to or greater than 0.75 henries,
- \( R \) must not exceed one-half the effective resistance of 25-A which is approximately 200 ohms.

The value of \( k \) used in these calculations was 0.75. It was further assumed that the true inductance, \( L \), of a coil wound as specified was 0.75 of the maximum inductance, \( L_m \), obtainable from a given length of wire. Therefore, since the required length of wire was taken from curves of maximum inductance, \( L \) was increased accordingly in order to determine the proper length of wire. The factor 0.75 which is only approximate may be obtained more accurately by use of equation 1. Assuming that \( C_m = 1 \) and \( L_m = 1 \) with the primary and secondary in series, the inductance of primary or secondary alone may be determined by substituting \( C_m = 0.5 \) in the equation, other factors remaining unchanged. Therefore half the length of wire wound in the prescribed shape has an inductance of .315. But since inductance varies as the square of the number of turns occupying a given space its value would be reduced to .25. \[ .25 / .315 = .794. \]

\[ M = kL, \]
\[ L = k/M = 0.675 / 0.75 = 0.9 \text{ henries}, \]
\[ L = 0.75 L_m \]
\[ L_m = 0.9 / 0.75 = 1.2 \text{ henries}, \]
\[ X/R = 5000 \text{ L/R} = 6000/R, \]
therefore \( X/R \) falls between 60 and 100, the higher value being obtained when \( R = 40 \).

By referring to Figs. 5 and 6 the lengths of wire, of the various sizes, are found that will give the required inductance \( (L_m = 1.2 \text{ henries}) \), and from Fig. 6 the corresponding values of \( X/R \). The resistance, \( R \), and the total weight of the primary and secondary are calculated from data in Table I, page 10. All of this information is tabulated in Table III.

### Table III.

<table>
<thead>
<tr>
<th>No.</th>
<th>Length</th>
<th>R</th>
<th>( X/R )</th>
<th>Weight of Pri. and Sec. in Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>6000</td>
<td>15</td>
<td>400</td>
<td>132</td>
</tr>
<tr>
<td>18</td>
<td>5500</td>
<td>22</td>
<td>175</td>
<td>68</td>
</tr>
<tr>
<td>18</td>
<td>5000</td>
<td>32</td>
<td>187</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>4050</td>
<td>47</td>
<td>147</td>
<td>20</td>
</tr>
<tr>
<td>22</td>
<td>4500</td>
<td>69</td>
<td>67</td>
<td>17</td>
</tr>
<tr>
<td>24</td>
<td>5500</td>
<td>97</td>
<td>62</td>
<td>9.4</td>
</tr>
<tr>
<td>26</td>
<td>5300</td>
<td>145</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Obviously, the conditions are satisfied by 4500 feet of No. 20, 4500 feet of No. 22, and 5000 feet of No. 24 wire. Taking into consideration the resistance, the ratio of \( X/R \), and the weight, it seemed best to use 4500 feet of No. 22.

After having determined the size and length of wire the next question was the proper proportions for the winding. The maximum
inductance is obtained from the relative proportions shown in Fig. 3 because the summation of mutual inductance between turns, for this particular shape, is a maximum. It was, therefore, proposed to wind the repeating coil, consisting of primary and secondary, so that the relative proportions would be the same as shown in Fig. 9. It will be noted in the figure that the primary and secondary were each wound in two sections of nearly equal lengths.

By this means the leakage flux was reduced to a minimum. Since the self-inductance is a maximum when the primary and secondary are used as one coil the mutual inductance between the two parts is also a maximum, which is the thing that is wanted.

By referring to Fig. 10b, page 61, of the bulletin previously mentioned, it will be found that for 8000 feet of No. 26 wire the
radius of the core should be 1.9 inches, the thickness of winding 1.9 inches and the length 2.5 inches. These dimensions were used in making the second or final coil.

The coils used in this investigation are shown in the accompanying photographs. A is the preliminary coil, B the final coil, and C the 25-A.
III TESTS.

Since the effective resistance of the regular type of telephone repeating coils, at voice frequencies, is considerable greater than the resistance to direct current it is evident that this increase is due to iron losses. It was suspected that an appreciable part of this loss was due to hysteresis and for this reason comparative tests were made on a number of coils of different makes and with different amounts of iron.

The ballistic galvanometer, or step-by-step, method was used. Due to the extremely low values of magnetizing current, the maximum of which was 0.9 milli-amperes, it was necessary to use an Ayrton shunt which was connected in as shown in Fig. 10.

![Fig. 10](image)

Considerable difficulty was experienced in getting reliable data. Each set of readings was repeated many times.

The results tabulated here are the most accurate that could be obtained with the method used.

In the case of the R. M. 26-A it was possible to obtain only four points, the positive and negative maximum and the residual
in the two directions. For the Stromberg-Carlson coil tested sixteen points were obtained.

<table>
<thead>
<tr>
<th>R</th>
<th>L</th>
<th>I (milli-amp.)</th>
<th>Deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.49</td>
<td>0.91</td>
<td>129.5</td>
</tr>
<tr>
<td>∞</td>
<td>0</td>
<td>0.00</td>
<td>129.6</td>
</tr>
<tr>
<td>50</td>
<td>-1.49</td>
<td>-0.91</td>
<td>128.5</td>
</tr>
<tr>
<td>∞</td>
<td>0</td>
<td>0.00</td>
<td>129.6</td>
</tr>
<tr>
<td>50</td>
<td>1.49</td>
<td>0.91</td>
<td></td>
</tr>
</tbody>
</table>

Maximum amplitude of deflection 1.49.

Stromberg-Carlson

<table>
<thead>
<tr>
<th>I</th>
<th>Deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>1000 250.0</td>
<td>129.6</td>
</tr>
<tr>
<td>2000 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>∞</td>
<td>128.5</td>
</tr>
<tr>
<td>100 50.0</td>
<td>258.0</td>
</tr>
<tr>
<td>1000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>2000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>∞</td>
<td>252.9</td>
</tr>
<tr>
<td>100 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>1000 250.0</td>
<td>129.6</td>
</tr>
<tr>
<td>2000 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>∞</td>
<td>128.5</td>
</tr>
<tr>
<td>100 50.0</td>
<td>258.0</td>
</tr>
<tr>
<td>1000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>2000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>∞</td>
<td>252.9</td>
</tr>
<tr>
<td>100 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>1000 250.0</td>
<td>129.6</td>
</tr>
<tr>
<td>2000 250.0</td>
<td>128.5</td>
</tr>
<tr>
<td>∞</td>
<td>128.5</td>
</tr>
<tr>
<td>100 50.0</td>
<td>258.0</td>
</tr>
<tr>
<td>1000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>2000 50.0</td>
<td>252.9</td>
</tr>
<tr>
<td>∞</td>
<td>252.9</td>
</tr>
</tbody>
</table>
Maximum amplitude of deflection was 333.5. It will be noted that this curve was retraced three times, the object being to get a large number of points and at the same time large deflections so that they could be read quite accurately. Similar curves were taken on Dean and Kellogg coils. The hysteresis loops for the . . . 35-A and the Stromberg-Carlson are given on pages 56 and 57. These coils did not show much hysteresis loss. However, the coil with the largest amount was the least efficient.

The general expression for the mutual inductance, \( M \), between two identical coils is \( M = L I \), where \( I \) is the coefficient of coupling.

The impedance method was used in determining the coefficient of coupling between sections 1 and 2, 3 and 4, (1,2) and (3,4), for the preliminary coil. Anderson's method was used in determining the coefficient of coupling between sections 1 and 2, 3 and 4, 1 and 3, 2 and 3, (1,3) and (2,4) for the second or final coil. The results are tabulated below.

### Preliminary Coil

<table>
<thead>
<tr>
<th>Section</th>
<th>( L )</th>
<th>( I )</th>
<th>( Z )</th>
<th>( f )</th>
<th>( R )</th>
<th>( L )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62.8</td>
<td>0.652</td>
<td>96.5</td>
<td>63</td>
<td>15.3</td>
<td>.235</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>62.8</td>
<td>0.552</td>
<td>96.6</td>
<td>63</td>
<td>25.5</td>
<td>.331</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>62.8</td>
<td>0.765</td>
<td>62.5</td>
<td>63</td>
<td>22.7</td>
<td>.197</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>63.0</td>
<td>0.852</td>
<td>96.7</td>
<td>64</td>
<td>22.7</td>
<td>.234</td>
<td></td>
</tr>
<tr>
<td>1 + 2</td>
<td>144.3</td>
<td>0.986</td>
<td>563.5</td>
<td>63</td>
<td>50.6</td>
<td>.910</td>
<td>.253</td>
</tr>
<tr>
<td>1 + 2</td>
<td>144.3</td>
<td>0.986</td>
<td>51.6</td>
<td>63</td>
<td>50.6</td>
<td>.0153</td>
<td>.963</td>
</tr>
<tr>
<td>3 + 4</td>
<td>151.4</td>
<td>0.400</td>
<td>356.0</td>
<td>65</td>
<td>49.4</td>
<td>.327</td>
<td>.190</td>
</tr>
<tr>
<td>3 + 4</td>
<td>151.4</td>
<td>0.950</td>
<td>48.6</td>
<td>65</td>
<td>48.4</td>
<td>.646</td>
<td>.190</td>
</tr>
<tr>
<td>1, 2 + 3</td>
<td>135.8</td>
<td>0.111</td>
<td>1232.0</td>
<td>63</td>
<td>96.0</td>
<td>5.1</td>
<td>.67</td>
</tr>
<tr>
<td>1, 2 + 3</td>
<td>135.8</td>
<td>0.347</td>
<td>194.0</td>
<td>63</td>
<td>96.0</td>
<td>.77</td>
<td></td>
</tr>
</tbody>
</table>
The plus sign indicates that the coils are in the same direction and the minus sign indicates that they are opposed. The value of $M$ is found as follows: let $L_1$ and $L_2$ be the inductance of the two coils, $M$ the mutual inductance, $L'_1$ and $L''_1$ the inductances of the two coils in series when they are aiding and in opposition respectively, then

$$L' = L_1 + L_2 + 2M$$
$$L'' = L_1 + L_2 - 2M$$
$$M = (L' - L'') / 4$$

As an example take the last pair of readings.

$$M = (3.1 - .416) / 4 = .67 = \sqrt{L_1 L_2}$$
$$k = .67 / \sqrt{L_1 L_2} = .67 / \sqrt{3.1 \times .416} = .77$$

<table>
<thead>
<tr>
<th>Section</th>
<th>Coil Res.</th>
<th>$r$</th>
<th>$S$</th>
<th>$l$</th>
<th>$C_{hl}$</th>
<th>$L$</th>
<th>$M$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.5</td>
<td>1.70</td>
<td>110</td>
<td>11</td>
<td>1000</td>
<td>1</td>
<td>.109</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>26.5</td>
<td>1.70</td>
<td>110</td>
<td>11</td>
<td>1000</td>
<td>1</td>
<td>.177</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>36.6</td>
<td>2.60</td>
<td>110</td>
<td>11</td>
<td>1000</td>
<td>1</td>
<td>.332</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>36.6</td>
<td>2.60</td>
<td>110</td>
<td>11</td>
<td>1000</td>
<td>1</td>
<td>.351</td>
<td></td>
</tr>
<tr>
<td>1 &amp; 5</td>
<td>41.0</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>1</td>
<td>.741</td>
<td></td>
</tr>
<tr>
<td>1 &amp; 3</td>
<td>0</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>.5</td>
<td>.008</td>
<td></td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>7.60</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>1</td>
<td>1.326</td>
<td></td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>0</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>.5</td>
<td>.008</td>
<td></td>
</tr>
<tr>
<td>1 &amp; 5</td>
<td>46.0</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>1</td>
<td>.825</td>
<td></td>
</tr>
<tr>
<td>2 &amp; 4</td>
<td>48.0</td>
<td>1.60</td>
<td>160</td>
<td>16</td>
<td>1000</td>
<td>1</td>
<td>.825</td>
<td></td>
</tr>
<tr>
<td>(1,3) &amp; (3,4) + 13450</td>
<td>220</td>
<td>24</td>
<td>1000</td>
<td>1</td>
<td>5.506</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3) &amp; (2,4) - 0</td>
<td>340</td>
<td>34</td>
<td>1000</td>
<td>.5</td>
<td>.015</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Primary is (1,3) and the secondary is (2,4)
The inductance in this case may be found by substituting the proper values in
\[ L = C \left( r(t + s) + sd \right). \]

The results obtained show that with the coils connected as intended, that is, as shown in Fig. 9, the coefficient of coupling is practically unity, but between the inner and outer sections it drops to about 0.75.

Helwig found that the coefficient of coupling in S.H. coils Nos. 15 and 20 was 0.75 and 0.7 respectively.

The repeating coils were tested for talking efficiency both in the electrical laboratory of the University of Minnesota and on the toll lines of the Tri-State Telephone and Telegraph Company. They were tested both for intensity and quality, and compared with J. L. 25-A as a standard (this coil was selected on account of its universally acknowledged superiority over most other coils). The circuits were so arranged that it was possible to switch readily from one coil to another. The laboratory tests showed that under perfect condition of operation there was no detectable distortion in any case. With a weak 60-cycle disturbance on the line, such as would result from a slight ground on a telephone line in the vicinity of power lines, but not of sufficient magnitude to interfere seriously with talking, the air-cored coil gave round and mellow tones while those of the 25-A were harsh in comparison. Mr. Tallmadge, one of the Post Senior students who was helping on the test, remarked that in case of the air-cored coil the voice sounded like that of a human while with the 25-A it sounded more like a cheap phonograph. Mr. Robertson, who has had consider-
able telephone experience, described the tones with the 25-A as mushy. In each case a variable high resistance was inserted in the line to control the intensity of the signals. In so far as intensity was concerned the 25-A was slightly better than the preliminary coil but no better than if as good as the coil made according to the final design.

When tested out on the toll lines the preliminary coil showed no appreciable change in intensity but the quality was good.

Mr. Seymore, Chief Engineer of the Tri-State Telephone and Telegraph Company, in discussing the tests of this coil on their toll lines, stated that in his opinion the coil was highly efficient.

IV CONCLUSIONS.

The possibilities of the air-cored coil are little appreciated in connection with telephone practice.

High efficiencies at voice frequencies, may be obtained without the use of a prohibitive amount of copper.

The cost of manufacture would usually be less for the air-cored coil, because little hand work would be required.

The testing of non-ferric coils is possible at any available frequency and desired current for it in no way depends upon the fortuitous magnetization of iron.

The theory and calculations are much simplified because the resistance, inductance, and capacity are constant.

The coefficient of coupling is even better than in the regular type of repeating coil.

The iron losses are eliminated.
The electrical efficiency of a repeating coil is a matter of small importance compared with the necessity of reproducing the true wave form. It is by this means and this alone that it is possible to retain in their proper relation the overtones upon which the quality of the voice depends. It is the overtones that give character to the voice; that enables us to recognize a friend at the distant end of a telephone line.

Wave distortion, due to the presence of iron in coils of the usual type, is eliminated by the method herein proposed. While this innovation may introduce new problems I feel confident that it has in a measure solved some of the old ones. Considering all of the points it seems that the air-cored coil may in time be more generally used.

I wish to express my indebtedness to Professor Brooks for first interesting me in the subject of inductance and maintaining that interest by his stimulating suggestions and constructive criticism.

Thanks are also due Mr. Sepmore for his generous cooperation in this work.