A DETERMINATION OF THE ABSOLUTE COEFFICIENT OF VISCOSITY OF AIR

BY

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY MARGARET KATE DAWSON

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E. H. Williams

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Committee on Final Examination*

*Required for doctor's degree but not for master's
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I HISTORICAL AND THEORETICAL

The viscosity of a gas may be defined as that property whereby it resists the relative motion of its parts. When such relative motion is produced by some force acting upon the gas, it dies away in a short interval of time after the force is removed.

The subject of the viscosity of gases was studied but little before 1665 and was first considered only as a correction in work with the pendulum where great accuracy was required. In 1857, however, Clausius published the memoir in which he gave the beginnings of the modern kinetic theory of gases and, four years later, Maxwell, connecting what he called the three principal diffusions, diffusion of heat, of matter and of momentum (or viscosity), with the idea of the mean free path of the molecule, stated that the viscosity ought to be independent of the pressure of the gas. Since this statement was a direct consequence of the assumptions of the kinetic theory, it immediately became a matter of theoretical interest to determine experimentally whether or not the viscosity did change with change of pressure. Some of the most important of this early work was done by O.E. Meyer, who not only made determinations of the coefficient of viscosity himself but also collected data of earlier investigators and studied it from the new viewpoint. An excellent historical sketch of the work that has been done in the study of viscosity is given by M. Brillouin.¹

According to the kinetic theory, if a gas is flowing in a definite direction, the viscosity or internal friction consists in a transfer, effected by the heat motion of the molecules, of the directed motion from the faster moving layers to those moving more slowly.

¹ Brillouin, Leçons sur la Viscosité des Liquides et des Gaz, Vol. II, Book III.
and finally a transformation of the motion so carried over into undirected heat motion.

Newton formulated the general law of viscous resistance which states that when a liquid or a gas flows in such a way that there is relative motion between different layers, the tangential stress acting across a surface such as CD (Fig. 1), tending to equalize the velocities, is proportional to the gradient of the velocity. The coefficient of viscosity is then defined as the ratio of the tangential stress to the velocity gradient.

An expression for the value of the coefficient of viscosity may be obtained in the following way. Assume a gas flowing over a horizontal surface AB and let the distance of any point above the surface be \( x \). Also let the velocity \( v \), which is zero next to the surface AB, vary in such a way that it is always directly proportional to the distance \( x \) and numerically equal to it. Under these conditions the velocity gradient is obviously equal to unity.

The velocity \( v \) is vanishingly small in comparison with the heat motion of the gas and will exert no appreciable effect upon the number of molecules which pass from one layer to another in a given interval of time. Consider the motion across the layer CD at a distance \( x \) above AB. If \( N \) is the number of molecules per unit volume of the gas and \( G \) the mean value of the speed, the number of particles passing across CD per unit time is

\[ \frac{dN}{dt} \]

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\[ \frac{dN}{dt} \]

passing across unit area of CD in one direction per unit time will be, according to the kinetic theory, $1/6 \, N \, G$. The particles begin their path toward this layer from different depths but on the average they come from a distance equal to $L$, the mean free path, that is, from a layer $x-L$ above AB. Their mean forward velocity is then given by

$$v = x-L$$

and the momentum carried over by each molecule is

$$mv' = m(x-L)$$

in which $m$ is the mass of the molecule. The total momentum carried across unit area per unit time is then

$$1/6 \, N \, m \, G \, (x-L).$$

At the same time there is a transfer of momentum in the opposite direction across unit area of CD equal to

$$1/6 \, N \, m \, G \, (x-L).$$

The layers above CD, therefore, lose in unit time the quantity of momentum

$$1/6 \, N \, m \, G \, (x+L) - 1/6 \, N \, m \, G \, (x-L) = 1/3 \, N \, m \, G \, L.$$ 

This quantity is the tangential stress across the surface AB and since, under the given conditions, the velocity gradient is equal to unity, the coefficient of viscosity is

$$\gamma = 1/3 \, N \, m \, G \, L.$$ 

Substituting the value for the mean free path given by Clausius, namely,

$$L = 3/4 \, \lambda^3/\pi s^2,$$

in which $\lambda^3$ is the volume occupied by a single molecule, and $s$ the radius of the sphere of action of the molecule, the expression for $\gamma$ becomes
\[ \eta = N\lambda^3 \frac{mG}{4\pi a^2}. \]

Since \( N\lambda^3 = 1 \),

\[ \eta = \frac{mG}{4\pi a^2}. \]

This expression contains no term dependent upon the pressure of the gas and its development thus gives the proof of the law formulated by Maxwell that the viscosity of a gas is independent of the pressure.

Three important methods have been used for investigating the viscosity of gases, the oscillation of a vessel surrounded by the gas, the constant deflection method and the flow of gases through capillary tubes. In the very early work, the method of oscillation was used a great deal and various modifications were made in order to secure greater accuracy. In its simplest form, a circular disc, suspended by a wire, is set in oscillation in its own plane and the amplitude of the vibration decreases because of the friction of the medium in which it oscillates. This method, which had been used for determining the coefficient of viscosity of a liquid, was modified by Meyer when applied to the study of gases. Three coaxial discs were used instead of one and were arranged so that they could be oscillated either as three or a single disc. Maxwell made a further improvement by placing between the three movable discs four fixed discs, equally distant from one another. With this arrangement the effect of the viscosity was magnified and, in addition, the final formula gave the viscosity coefficient directly instead of the square root of the coefficient as in Meyer's formula. The first consideration in this work was to find whether or not the viscosity was

independent of the pressure and with Maxwell's apparatus very consistent results were obtained over a considerable range of pressure. The objection to this method lies in the fact that the differential equations involved can not be exactly integrated and the formula must be regarded as an empirical one which is only approximately correct.

In the constant deflection method, two concentric metal cylinders are arranged so that the outer of the two rotates with constant velocity about the inner. The viscosity of the air in the space between the two produces a drag upon the inner cylinder and causes it to be deflected. This method has been used with great success in recent work at the University of Chicago and is well adapted for absolute measurements of the coefficient of viscosity. The value obtained by Lachlan Gilchrist in 1913 was, \( \gamma = 1710.8 \times 10^{-7} \) at 0° Centigrade, and E.L. Harrington, having made further refinements in the apparatus, obtained in 1916 the result \( \gamma = 1707.9 \times 10^{-7} \) at 0° Centigrade.

The third method, that of the flow of gases through capillary tubes, was first used by Graham. Although the formula for finding the coefficient of viscosity by this method had not yet been developed, he found that the ratio of the time of flow of different gases under the same conditions is the same for tubes of various lengths and for different pressures. And he computed relative coefficients of viscosity which he called transpiration coefficients.

Poiseuille's law for the flow of liquids through capillary tubes, which assumes that the density does not vary along the length

of the tube, can not be used for gases. However, a similar formula, taking into account the change of density due to the varying pressure, was developed by Meyer and is of the form

$$\gamma = \frac{\pi r^4 (p_1^2 - p_2^2) t}{16 V l p_2}$$

in which $\gamma$ is the coefficient of viscosity, $r$ the radius of the capillary bore, $l$ the length of the tube, $p_1$ the pressure at the entrance of the tube, and $p_2$ the pressure at the exit, and $V$ the volume of gas which flows out of the tube in the time $t$. With the aid of this formula Meyer was able to calculate $\gamma$ from Graham's data. The gas used by Graham was compressed in a vessel connected with the capillary used and the pressure decreased as the gas flowed out, the decrease being a function of the time. Meyer developed the formula for the condition where $p_1$ changed very slowly and $p_2$ was constant using the relation

$$\frac{dV}{l} = \frac{\pi}{2 p_2} \left( \frac{p_1^2 - p_2^2}{r^4} \right) \frac{dt}{\gamma l}$$

in which $dt$ is a time interval so short that the ordinary formula may be used for that interval. He also corrected for temperature changes due to the expansion of the gas as the pressure changed along the tube.

A simpler development of Poiseuille's law, in the modified form applying to gases, is given by Poynting and Thomson. Although the density $d$ of the gas changes along the length of the tube the product of the density and velocity is constant, since equal masses pass each cross section in the same time, or we may say that $pV$ is independent

of \( z \), where \( z \) is a length measured along the axis of the tube and \( p \) is the pressure of the gas. Since \( v \) varies inversely as \( p \), the variation of \( v \) will cause relative motion between different parts of the gas at the same distance from the axis of the tube and there will be viscous forces at right angles to the ones considered in the development of the formula. These may be neglected, however, since the gradient of velocity across the tube is negligible in comparison with that along its length.

As has been stated, a large part of the early study of the viscosity of gases was for the purpose of showing that it was independent of the pressure. Maxwell's law was verified by many independent investigations but failed to hold for either very low or very high pressures. In the latter case the assumption made in the development of the equation

\[ \eta = \frac{mg}{4w^2}, \]

that the curved portions of the path traced by the molecule during actual encounters with other molecules is negligible in comparison with the straight path between encounters, is no longer true. Kundt and Warburg\(^{10}\) worked with low pressures and found deviations from the law as soon as the pressure fell below 1/60 of an atmosphere but they attributed these changes to an increase in the external friction, that is, the friction caused by the slip of the gas over the walls of the tube. In the case of liquids the layer next to the wall of the tube is stationary and the first investigators thought this to be true of gases also but this was found to be a false assumption. The coefficient of slip \( \theta \) is defined as the ratio of \( \eta \) to \( \varepsilon \), the coefficient of external friction. The presence of this slip along the wall of the tube causes an apparent decrease in the internal friction and

\(^{10}\) Kundt and Warburg, Pogg. Annalen, CLV, pp 337, 525.
since \( \theta \) was found to vary inversely as the pressure the deviations from the law of Maxwell become pronounced at low pressures. Meyer\(^{11}\) found \( \theta \) to be very nearly equal to the mean free path. Kundt and Warburg concluded that the internal friction was really constant down to pressures so low that they could no longer be measured with great accuracy. At a much higher rarefaction there is a sudden drop in the value of the internal viscosity.

From the kinetic theory, the coefficient of viscosity of gases should increase with increase of temperature and it was first thought that it should be directly proportional to the absolute temperature. Experiment showed that viscosity of gases does increase with rising temperature but the actual law of the increase was difficult to determine. Various attempts were made to develop an empirical equation of the form

\[
\eta = \eta_0 (1 + \alpha \theta)^n
\]

in which \( \alpha \) is the coefficient of expansion of the gas and \( \theta \) the temperature on the Centigrade scale. However, \( n \) was not found to be constant but varied not only with the gas used but also with the temperature.

The most satisfactory formulation of the rate of increase of viscosity with temperature was given by Sutherland\(^{12}\) in 1893. In his derivation he took account of the molecular forces which cause collisions between molecules that otherwise would not collide. Taking account of these forces, the formula becomes,

\[
\frac{\eta}{\eta_0} = \left( \frac{T}{273} \right)^{1/2} \frac{1 + \frac{C}{273}}{1 + \frac{C}{T}}
\]


\(12\). Sutherland, Phil. Mag., Series 5, Vol. 36, p 507.
T is the absolute temperature and C a constant which is determined experimentally. From the data obtained by Holman, who made careful measurements of the variation of the viscosity with temperature, Sutherland found the value of C to be 113. In 1909 Fisher made a most careful investigation in which he redetermined the value of the constant C, obtaining the result, C = 124.

A very large part of the investigation of the viscosity of gases has been for the purpose of finding the laws of change with temperature, the effect of changes in pressure and the relation between the coefficients of viscosity of different gases. The laws of the viscosity of mixtures of two gases have been developed theoretically and some work has been done with vapors, including a study of the effect due to dissociation.

The determination of the value of the elementary electric charge by Millikan involved knowing the value of the coefficient of viscosity of air and led to its accurate redetermination both by the constant deflection and capillary tube method. I.M. Rapp, in 1913, made a careful study of the errors which had hitherto prevented the obtaining of reliable values with the latter method and obtained values which agreed within one tenth of one per cent with the best results obtained by the constant deflection method.

13. Holman, Phil. Mag., Series 5, Vol. XXI.
II SELECTION AND CALIBRATION OF TUBES

The purpose of this investigation was to develop a suitable experiment in order that the coefficient of viscosity of air might be determined in the laboratory. The capillary tube method was chosen and, in most respects, the method of Rapp was followed.

A large part of the difficulty in using the capillary tube method for absolute determinations of viscosity has arisen because of inaccuracy in determining the radius of the capillary bore and in the selection of imperfect tubes. It is not sufficient to obtain an average value of the radius from the length and mass of a column of mercury filling the capillary and, in this work, the tubes were calibrated by the method developed by Fisher\textsuperscript{15} in 1909. A smaller percentage of error is found in the use of tubes of comparatively large bore. The method is strictly accurate only when the bore is circular in cross section. Jena glass tubing seems to best fulfill this condition but since it was impossible to obtain supplies of Jena glass the first tube was selected from a limited number of Jena glass capillaries and the second tube from a supply of glass made by the Corning Glass Company. Both of these showed a slight ellipticity in cross section when tested with a micrometer microscope. The small error due to this ellipticity made the results obtained for the radius slightly too high.

The tubes were cleaned by drawing through them solutions of sodium bichromate in sulphuric acid, potassium hydroxide, nitric acid and distilled water. Absolute alcohol and dry air were forced through them to remove the moisture.

\textsuperscript{15} W.J. Fisher, Phys. Rev., Vol. 28, p 73.
In Fisher's method of calibration the radius is assumed constant over a length $dl$ of the tube. His development is as follows.

Let some mark on the tube at $C$ be taken as a reference point and let the distance from this point to the center of a short mercury thread be $l$. At the point $l$ on the tube the cross section $Q = \frac{V}{\lambda}$ in which $V$ is the volume of the short mercury thread and $\lambda$ the length of an equivalent cylinder of mercury having the same volume as the mercury thread, which in good tubes is in the "form of a frustrum of a right circular cone capped at each end by a spherical segment of one base." The total volume of the tube from zero to $l$ is $v$. Then

$$dv = Qdl = \frac{V}{\lambda} \cdot dl.$$  

If the temperature is kept constant,

$$v = \int_0^l \frac{dl}{\lambda}.$$  

The area of a curve plotted with $y = \frac{1}{\lambda}$, and $x = l$

$$= \int y \, dx = \int_0^l \frac{dl}{\lambda} = \frac{V}{\lambda}.$$  

Instead of finding the volume $V$ of the short thread directly, the total volume of a long thread between two points $l'$ and $l''$ is determined by weighing. This volume is designated as $v''-v'$.

Then

$$V = \int_{l'}^{l''} \frac{1}{\lambda} \, dl.$$
The gas is considered as flowing through a series of short tubes 
\( dl \) in length, and for each of these Poiseuille's law, 
\[
F = \frac{\pi r^4}{16 \rho \eta} (p_1 - p_2)
\]
is assumed true.

\( F \) is the volume transpired per second at the point where the pressure is \( p \). For each short length of radius \( r \) and length \( dl \)
\[
\frac{16 \rho F \eta}{\pi} = r^4 \frac{d(p^2)}{dl}.
\]
Since the left-hand member of the equation is constant,
\[
d(p^2) = \frac{16 \rho F \eta}{\pi} \frac{dl}{r^4}.
\]
Also
\[
\int_0^1 d(p^2) = p_1^2 - p_2^2 = \frac{16 \rho F \eta}{\pi} \int_0^1 \frac{dl}{r^4}
\]
But
\[
Q = \pi r^2 = \frac{V}{\lambda}; \implies \frac{1}{r^4} = \frac{\pi^2 \lambda^2}{V^2}
\]
And thus
\[
p_1^2 - p_2^2 = 16 \rho F \eta \frac{\pi}{V^2} \int_0^1 \lambda^2 \, dl
\]
and
\[
\eta = \frac{V^2 (p_1^2 - p_2^2)}{16 \pi \rho F \int_0^1 \lambda^2 \, dl} = \frac{(v'' - v')^2 (p_1^2 - p_2^2)}{16 \pi \rho \int_0^1 \lambda^2 \, dl \left[ \int_0^1 \frac{dl}{\lambda} \right]^2}
\]

\[
\int_0^1 \lambda^2 \, dl \text{ is found from the area of a curve plotted with } y = \lambda^2 \text{ and } x = 1.
\]
In practice, a scratch was placed on the tube near its middle point and it was placed on the dividing engine above a standard scale. The position of the scratch was read accurately on the scale by means of the microscopes on the dividing engine. The fixed cross hair was then set on that end of the mercury thread which was toward the scratch and the reading of the scale recorded. The distance from the scratch to this point was designated as \( l' \) (Fig. 2). The height of the meniscus \( h_2 \) was found by using the micrometer microscope with a pitch of one tenth of a millimeter. The dividing engine screw was
Curve I  \( x = 1, \quad y = \frac{1}{\lambda} \)

Curve II  \( x = 1, \quad y = \lambda^2 \)
then turned until the fixed cross hair was set on the end of the meniscus \( h_2 \) and the length, \( \lambda_2 \), of the mercury thread from the tip of one meniscus to the other was recorded. The height of the meniscus \( h_2 \) was determined.

The mercury thread was then moved to a new position such that it slightly overlapped its former one and a new set of readings taken. In this way the entire tube was calibrated. The mercury column was about one and a half centimeters in length. From this data the length \( \lambda' \) of the thread without the meniscuses, the distance \( l \) from the scratch to the middle of the mercury column, and \( \lambda \) the length of the equivalent cylinder of mercury, were found. Fisher considered that

\[
\lambda = \lambda' + \frac{1}{2}(h_1 + h_2)
\]

Rapp, by a more accurate mathematical treatment, obtained the value

\[
\lambda = \lambda' + 0.53(h_1 + h_2)
\]

In the calibration of the second tube corrections were made for temperature changes during the course of the calibration.

Figure 3 shows a reproduction of the curves from which the values of the \( \int_1^{l''} \frac{dl}{\lambda} \) and the \( \int_0^{1} \lambda^2 dl \) were obtained.

### III DESCRIPTION OF APPARATUS AND MANIPULATION

The apparatus used in this investigation is illustrated in Fig. 4. A tank, about 60 centimeters in width and depth and 115 centimeters long, was mounted on braces so that it was about 125 centimeters from the floor. The air used was taken from the pressure system in the laboratory and was passed in succession through sulphuric acid, calcium chloride, phosphorus pentoxide and glass wool, in order that it might be free from moisture and dust particles before passing through the capillary tube. The air from the drying system entered the bottle \( L \) from which one tube led to the capillary and another to
...
a second bottle K which was open to the air. By this means the pressure at the inlet end of the capillary tube was kept constant and equal to atmospheric pressure. A layer of phosphorus pentoxide in K kept moisture from entering L from K.

The tank, containing a tube J, the capillary G, and the bottle F, was kept filled with water and during the course of one determination the temperature never changed more than a tenth of a degree and usually much less. The air before entering the capillary passed through the tube J, thus coming to the temperature of the tank. The bottle F was nearly filled with water and a tube E passed from it into a bucket N which was suspended from a screw D and was arranged with an outlet tube. The air in the bottle F was at first at atmospheric pressure but E, when filled with water, acted as a siphon and water from F flowed from the outlet until the difference in pressure between the air in the bottle and atmospheric pressure was just equal to that due to a column of water BC, extending from the outlet tube of the bucket to the level of water in the bottle. The length of this water column was not measured directly since it was just equal to the height of the water column in the manometer tube M. The pressure $p_2$ at the outlet end of the tube was kept constant by means of the following device used by Rapp. As the water flowed through the outlet, $p_2$ increased but was kept constant by lowering the bucket by means of the screw D.

The method of procedure was as follows. When everything was in readiness, the air was allowed to bubble through the sulphuric acid and, passing through the entire drying system, entered the capillary tube through the coil J. The opening of the stopcock placed at A, between the capillary and the bottle, caused air from the tube to pass
into the bottle, forcing water down through the tube E and out of the opening in the bucket. A telescope was focused on the water level in the manometer tube and the level was kept tangent to the cross hair by the lowering of the bucket. At a given instant a weighed beaker was placed under the outlet tube and a stop watch, which had been calibrated with the standard clock of the laboratory, was started. Measurements of the temperatures of the tank and manometer were made with a Centigrade thermometer graduated to tenths of a degree. The manometer was outside of the tank and thus affected by changes in room temperature. Its temperature was determined by taking the mean of readings taken at the bottom and near the top of the water column. After a time, depending upon the pressure and the size of the tube, the beaker was removed and the time recorded. The manometer height was read.

IV METHOD OF CALCULATION AND CORRECTIONS APPLIED

As given above, the formula used for the calculation of the coefficient of viscosity is,

\[ \eta = \frac{(v'' - v')^2(p_1^2 - p_2^2)t}{16p_2^2V \int_0^1 \lambda^2 d\lambda \left[ \int_1^{1/\lambda} \frac{1}{d\lambda} \right]^2} \]  

(1)

\( p_1 \) is equal to the atmospheric pressure and was determined directly from the barometer in the heat laboratory. The correction for temperature was made. The driving pressure \( (p_1 - p_2) \), was found from the height of the water column in the manometer. This was corrected for capillarity, reduced to the height at \( 0^\circ \) Centigrade and then to equivalent centimeters of mercury. From the values of \( p_1 \) and \( p_1 - p_2 \), \( p_2 \) was calculated. All the pressures were reduced to dynes per square centimeter. To find \( V \), the volume of gas flowing out of the capillary
tube, the water in the beaker was weighed, corrected for evaporation, reduced to the weight it would have in vacuo, and its volume calculated from its mass and its density at the temperature of the tank. The values of $v''-v'$ and the integrals in the denominator were determined as explained above. The weighings of the long mercury thread were made on a chemical balance and were reduced to weight in vacuo and corrected for unequal arms of the balance. Since the bucket was lowered during the course of the experiment the volume of a column of water, with cross section that of the tube E and height equal to the distance the bucket was lowered, was added to the volume of water collected in the beaker. Table I gives the calculation of a typical set of data.

Equation (1), when corrected for slip becomes,

$$ \tilde{\eta} = \frac{K(p_2^2-p_1^2)t}{16p_2V} \left(1+\frac{\theta}{r}\right) $$

in which

$$ K = \frac{\int_0^1 (v''-v')^2}{\int_0^1 \frac{1}{\lambda^2} d\lambda \left[\int_1^1 \frac{1}{\lambda^2} d\lambda\right]^2} $$

and is a constant for a given tube, $\theta$ is the coefficient of slip, and $r$ the radius of the tube.

A further correction was necessary due to energy in the stream lines at the ends of the tube. M. Brillouin\textsuperscript{16} and Fisher\textsuperscript{17} have each developed an equation for this correction which give practically identical results, although based on different assumptions. An empirical formula, agreeing with Fisher's theoretical one, was


<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Temperature of manometer</td>
<td>29.1°C</td>
</tr>
<tr>
<td>Density of water at 29.1°C</td>
<td>0.9959466</td>
</tr>
<tr>
<td>Height of water column in manometer</td>
<td>140.32 cm.</td>
</tr>
<tr>
<td>Height of water column corrected for capillarity</td>
<td>139.99 cm.</td>
</tr>
<tr>
<td>Pressure due to water column</td>
<td>139.42 gm.</td>
</tr>
<tr>
<td>Pressure in centimeters of mercury at 0°C</td>
<td>10.254 cm.</td>
</tr>
<tr>
<td>Barometric pressure</td>
<td>74.209 cm.</td>
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<tr>
<td>$P_2$</td>
<td>63.955 cm.</td>
</tr>
<tr>
<td>Temperature of tank</td>
<td>27.55°C</td>
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<tr>
<td>Density of water at 27.55°C</td>
<td>0.9963922</td>
</tr>
<tr>
<td>Weight of water in beaker</td>
<td>1088.53 gm.</td>
</tr>
<tr>
<td>Weight of water corrected for evaporation and reduced to weight in vacuo</td>
<td>1090.63 gm.</td>
</tr>
<tr>
<td>Volume of water</td>
<td>1094.58 cc.</td>
</tr>
<tr>
<td>Volume to be added due to lowering of bucket</td>
<td>1.141 cc.</td>
</tr>
<tr>
<td>Total volume of air which flowed from capillary tube</td>
<td>1095.72 cc.</td>
</tr>
<tr>
<td>Time</td>
<td>2644 sec.</td>
</tr>
<tr>
<td>Volume of long mercury thread</td>
<td>$(.048274)^2$ cc.</td>
</tr>
<tr>
<td>$\int_0^1 \lambda^2 dl.$</td>
<td>153.2547 cc.</td>
</tr>
<tr>
<td>$\int_1^1 \lambda^{-1} dl.$</td>
<td>34.8663</td>
</tr>
<tr>
<td>$\eta_1 = (1.048274)^2 \cdot 10.254 \cdot 13.6 \cdot 280.15 \cdot 2644$</td>
<td>$18\pi \cdot 83.955 \cdot 1095.7 \cdot (153.2547)(34.8663)^2$</td>
</tr>
<tr>
<td>$\eta' = 1847.3$</td>
<td></td>
</tr>
</tbody>
</table>
formulated by Rapp and is of the form

$$\eta = \eta_1 \left[ 1 + \frac{A(p_1-p_2)r^4}{1} \right]$$

In this equation $\eta_1$ is the value of $\eta$ corrected for slip. $A$ is a constant which Rapp found to be

$$A = -1.25 \times 10^5.$$  

Rapp's equation was used in this work.

V DISCUSSION OF THE DATA

Table III gives the results obtained with the first tube $A$ and tables IV, V and VI those obtained with the tube $B$. Data taken under conditions where there was a known source of error is not included. Soon after the second tube was placed in the tank a leak was discovered where the tubes enter the bottle. An extra volume of water was forced through the tube $E$ and the results obtained were low. This was remedied and data was taken with tube $B$ until it was noticed that consistent results were no longer being obtained. The tube was then removed and cleaned before further data was taken. The results, beginning with that of May 24, were obtained after the tube was cleaned. The first set of readings after a tube was placed in the tank gave results that were too high. In each case the water in the tank was considerably below the temperature of the room when the first trial was made and the temperature changed slightly throughout the course of the experiment. This change was so slight as not to account for the high values of $\eta$. It seems more probable that a slight trace of moisture in the tube increased the internal friction and that this moisture was removed as the dry air passed through. The investigation seems to show, however, that the best results were obtained
TABLE II
The Dimensions of the Tubes

<table>
<thead>
<tr>
<th>Tube</th>
<th>Length in Cm.</th>
<th>Radius in cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>61.749</td>
<td>0.0308</td>
</tr>
<tr>
<td>B</td>
<td>65.205</td>
<td>0.017129</td>
</tr>
</tbody>
</table>

In Tables III, IV, V, and VI

\( \eta' \) - value obtained directly from Poiseuille's law.

\( \eta' \) - \( \eta' \) corrected for slip.

\( \eta \) - \( \eta' \) corrected for end effects.

TABLE III
Data for \( \eta \) Using Tube A

<table>
<thead>
<tr>
<th>Temperature of tank</th>
<th>( p_1-p_2 ) in cm. of Hg.</th>
<th>( \eta \times 10^7 )</th>
<th>( \eta' \times 10^7 )</th>
<th>( \eta \times 10^7 ) at 0°C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.62</td>
<td>5.0075</td>
<td>1857.0</td>
<td>1859.1</td>
<td>1842.0</td>
</tr>
<tr>
<td>18.15</td>
<td>5.1237</td>
<td>1841.45</td>
<td>1843.6</td>
<td>1826.3</td>
</tr>
<tr>
<td>19.80</td>
<td>5.1332</td>
<td>1849.4</td>
<td>1851.5</td>
<td>1834.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
</tbody>
</table>


TABLE IV

Data for \( U \) Using Tube B with High Driving Pressure

<table>
<thead>
<tr>
<th>Date of Trial</th>
<th>Temp. of Tank</th>
<th>( P_1 - P_2 ) in cm. of Hg.</th>
<th>( \gamma \times 10^7 )</th>
<th>( \gamma' \times 10^7 )</th>
<th>( \gamma'' \times 10^7 )</th>
<th>( \gamma''' \times 10^7 ) at 0°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 3</td>
<td>21.9</td>
<td>10.189</td>
<td>1844.6</td>
<td>1842.5</td>
<td>1845.4</td>
<td>1734.3</td>
</tr>
<tr>
<td>May 5</td>
<td>22.35</td>
<td>10.220</td>
<td>1846.3</td>
<td>1850.2</td>
<td>1847.1</td>
<td>1733.9</td>
</tr>
<tr>
<td>May 6</td>
<td>21.75</td>
<td>10.309</td>
<td>1845.8</td>
<td>1849.7</td>
<td>1846.5</td>
<td>1736.1</td>
</tr>
<tr>
<td>May 7</td>
<td>22.66</td>
<td>10.196</td>
<td>1851.8</td>
<td>1855.7</td>
<td>1852.6</td>
<td>1737.8</td>
</tr>
<tr>
<td>May 24</td>
<td>18.33</td>
<td>10.231</td>
<td>1838.7</td>
<td>1842.6</td>
<td>1839.5</td>
<td>1745.7</td>
</tr>
<tr>
<td>May 27</td>
<td>23.52</td>
<td>10.131</td>
<td>1858.1</td>
<td>1862.0</td>
<td>1858.9</td>
<td>1739.9</td>
</tr>
<tr>
<td>May 27</td>
<td>23.46</td>
<td>10.003</td>
<td>1851.4</td>
<td>1855.3</td>
<td>1852.2</td>
<td>1733.5</td>
</tr>
<tr>
<td>May 30</td>
<td>24.20</td>
<td>10.015</td>
<td>1849.0</td>
<td>1852.9</td>
<td>1849.8</td>
<td>1727.8</td>
</tr>
<tr>
<td>May 30</td>
<td>24.25</td>
<td>9.9128</td>
<td>1840.4</td>
<td>1844.3</td>
<td>1841.2</td>
<td>1719.5</td>
</tr>
</tbody>
</table>

Mean, omitting value of May 24
1733.6

TABLE V

Data for \( U \) Using Tube B with High Driving Pressure

<table>
<thead>
<tr>
<th>Date of Trial</th>
<th>Temp. of Tank</th>
<th>( P_1 - P_2 ) in cm. of Hg.</th>
<th>( \gamma \times 10^7 )</th>
<th>( \gamma' \times 10^7 )</th>
<th>( \gamma'' \times 10^7 )</th>
<th>( \gamma''' \times 10^7 ) at 0°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2</td>
<td>27.15</td>
<td>10.239</td>
<td>1845.3</td>
<td>1848.2</td>
<td>1846.1</td>
<td>1710.9</td>
</tr>
<tr>
<td>June 2</td>
<td>27.25</td>
<td>10.128</td>
<td>1845.3</td>
<td>1849.2</td>
<td>1846.1</td>
<td>1710.5</td>
</tr>
<tr>
<td>June 3</td>
<td>27.55</td>
<td>10.254</td>
<td>1847.3</td>
<td>1851.2</td>
<td>1848.0</td>
<td>1710.8</td>
</tr>
<tr>
<td>June 3</td>
<td>27.6</td>
<td>10.252</td>
<td>1848.4</td>
<td>1852.3</td>
<td>1849.2</td>
<td>1711.7</td>
</tr>
<tr>
<td>June 4</td>
<td>27.3</td>
<td>10.189</td>
<td>1858.7</td>
<td>1862.6</td>
<td>1859.5</td>
<td>1722.7</td>
</tr>
<tr>
<td>June 4</td>
<td>27.25</td>
<td>10.193</td>
<td>1853.8</td>
<td>1857.7</td>
<td>1854.6</td>
<td>1718.4</td>
</tr>
</tbody>
</table>
### TABLE VI

Data for $\gamma$ Using Tube B with Low Driving Pressures

<table>
<thead>
<tr>
<th>Date of Trial</th>
<th>Temp. of Tank.</th>
<th>$p_{1}-p_{2}$ in cm. of Hg.</th>
<th>$\gamma \times 10^7$</th>
<th>$\gamma' \times 10^7$</th>
<th>$\gamma'' \times 10^7$</th>
<th>$\gamma' \times 10^7$ at $0^\circ$C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 4</td>
<td>26.55</td>
<td>5.656</td>
<td>1859.6</td>
<td>1863.4</td>
<td>1861.7</td>
<td>1728.1</td>
</tr>
<tr>
<td>June 4</td>
<td>26.5</td>
<td>5.615</td>
<td>1859.7</td>
<td>1863.5</td>
<td>1861.8</td>
<td>1732.5</td>
</tr>
<tr>
<td>June 4</td>
<td>26.45</td>
<td>5.406</td>
<td>1860.6</td>
<td>1864.4</td>
<td>1862.7</td>
<td>1729.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Mean</strong> 1730.0</td>
</tr>
</tbody>
</table>
after the water in the tank had come very nearly to room temperature.

When two sets of data were obtained in one day, the second set gave lower values especially if the second set was taken immediately after the first. On June 2nd and 3rd the air was allowed to flow through the tube for more than an hour before the experiment was begun. During this time the bottle J was not left open to the atmosphere. The results obtained on these two days agree with the results obtained by Rapp. On June 4th, data was taken at once without any preliminary flow of air through the tube. The second value obtained on that day was lower than the first and agrees with that obtained under the same conditions on May 30, but is not so low as that obtained on June 2 and 3. If the value of May 24, which was obtained while the temperature of the tank was still considerably below room temperature, is excluded, the values obtained after the tube was cleaned the second time are as a whole lower than those obtained previous to that. These facts seem to point toward the possibility that the high results were due to the presence of a very slight trace of moisture in the tube. Special precautions were taken, when the tube was placed in the tank the second time, to make the connections water tight.

Rapp used transformer oil in the bottle and manometer because it had a low vapor tension and stated that the use of water led to an error due to evaporation effects. Water seemed better adapted for a regular laboratory experiment and an attempt was made to discover the error due to its use and correct for it. It was thought that the presence of water vapor might cause the pressure that was measured to be slightly different from the pressure at the ends of the tube. Mercury manometers, H and I in Fig. 4, were inserted at the ends of the
capillary, one arm of each being open to the air. At the entrance end of the tube no difference between the level of the two mercury columns could be detected. But the difference in pressure obtained from the manometer H at the exit end was always slightly lower than that found from the water manometer. The difference between the two heights of mercury was so small that it could not be read with great accuracy with the cathetometer used, and the exact amount of difference between the readings of the water and mercury manometer could not be determined. Table VI shows that results obtained with low driving pressures were not in agreement with those with the higher pressures. If a correction due to vapor pressure should be applied, it would probably be a larger percentage of the smaller pressure than of the larger one. Before taking the data given in Table VI the air was allowed to flow through the capillary for some time. Earlier data taken with the same pressure but without taking this precaution gave still higher results.

VI CONCLUSION

It would seem, then, that the best results are obtained with this apparatus when the temperature of the tank is approximately that of the room and when dry air has been allowed to pass through the tube for some time before the data is taken. A comparatively large driving pressure gives the best values for the coefficient of viscosity, perhaps due to the fact that a certain correction due to vapor pressure should have been applied.

In conclusion, I wish to express my thanks to Professor E. H. Williams for his assistance in carrying on the investigation.