Distribution of heating in
Synchronous converter armatures
DISTRIBUTION OF HEATING IN SYNCHRONOUS CONVERTER ARMATURES

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PREFACE.
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As the Science of Electrical Engineering advances, there is, perhaps, no electrical machine which demands more attention than the Synchronous Converter. Its applicability extends into practically every department of the Science and its uses are many and varied. And yet, so rapid has been the development of this machine, and the extension of its usefulness, that a great many of its characteristics are unknown to the Engineering world at large, even though they affect to a great extent the efficient application of the machine. It is due to lack of knowledge of some of the most important characteristics of the machine, that a great many engineers hesitate to use synchronous converters, and that, when they at last install the machines, they are at loss to intelligently interpret the seemingly eccentric behavior of their electrical systems. As a consequence, the converter is more of a mystery than of an aid to electrical engineers in general.

One of the most important of these characteristics is that of the variation and distribution of the heating in the armature coils with varying conditions of operation. It is due to this unusual variation, that many interesting and very practical phenomena take place. And it is upon the increase and decrease of this heating for certain conditions, that the fitness of a given machine for given conditions, is determined. Since the building of a machine for any certain installation, and its efficient operation after it is installed depend very largely upon this heating characteristic, an accurate knowledge of the behavior of the converter under given
conditions is valuable both to the designing and to the operating engineer.

It is the purpose of this paper to show the effect of varying conditions of operation, such as those due to leading or lagging currents of varying percent power factor, upon the variation and distribution of heating in the armature coils of converters of different phase ratings; to place this variation upon a comparative basis; and to discuss certain practical issues arising from the investigation.

Several articles were consulted in the preparation of this paper, references to which will be made in the discussion. Chief among these were: "Armature Heating in Synchronous Converters", by Dr. Steinmetz; "Rotary Converters", by Dr. E.J. Berg; and "Railway Converters", by E.J. Woodbridge. Ideas from all these sources are comprehended in this paper, and the results taken as being representative of the present development of Synchronous Converters.

May 27, 1910.

Eugene Stuart High.
DISCUSSION.
The tendency of the development of the generation of electrical power is toward the advancement of alternating current. This is due, for most part, to the present idea of the centralization of generating stations and the consequent long distance transmission of the power to the centers of distribution. It is due also, to the rapid increase in electrical railway interests, which demand a constant supply of power to the line at more frequent intervals than it is possible to construct separate generating stations. In many instances, it is necessary to transmit the power considerable distances, and since the necessarily low generated voltages of direct current systems, and the extremely high line drop when any large amount of power is transmitted, are prohibitive factors where direct current is desired, it is necessary to generate alternating voltages, raise the voltage by transformers, reduce it at the end of the line, and procure direct current by the use of some system of conversion. This is the tendency of present day development.

The system of conversion of alternating current to direct current, which is most economical and practical, and hence, most generally used, is that procured by the use of synchronous converters. Now, alternating voltage is generated at from single phase to six phase, with possible practical exclusion of five phase. Also, although it is possible to use six phase converters on three phase systems and vice versa, it is most general practice to use a given
phase converter with a line of the same phase rating. At all events, all phases from one to six are being taken up in converter construction, and in an investigation of converter operation, it is necessary to consider machines of practically all the above mentioned phases.

In the case of a machine having any one of the generally used phase ratings, the conditions under which it may operate may be extremely varied in different installations; but, owing to the general construction of the machine, the one condition which affects its operation to the greatest extent is that of use on heavily reactive transmission lines or other systems of like nature. All the other varying conditions, such as a changing load or an unstable line, will, of course, affect the operation of the machine after it is installed; but the factor which determines the converter for a given set of conditions, and which determines whether or not machines on hand may be used to meet new conditions, is that of the power factor of the system upon which the machine is to operate. Consequently, in an investigation of a given converter, it is extremely necessary to determine its action under varying conditions of power factor.

Now, synchronous converters of a given type of construction, no matter of what phase rating they may happen to be, will all have the same characteristics under varying conditions of power factor; but all of these characteristics will not be of the same relative magnitude or importance. It is due to this fact that machines of relatively higher economy under certain conditions, will become vastly less efficient than other uneconomical machines under different conditions. Hence the mystery of the operation of synchronous converters.

This change in applicability of given machines with change in the power factor of the systems with which they are to be
used, is due to variation of heating in the coils of the converter armatures. This variation has greatest effect upon the rating of the machines for differing conditions of operation and is the cause, for the most part, of such phenomena as the uneven over-heating in the armature coils, the breakdown of machines when operated far under the manufacturer's rating, and so on. All of these are, in reality, due to the ignorance of the operating engineer, who does not appreciate the effect of the heavily reactive system upon the heating in the machine under his charge, and consequently endeavors to use a machine which is not at all applicable to the conditions.

This variation of armature heating follows a very definite law as the power factor of the system changes; and it is probable that a discussion of this law and a representation of its application will be of interest both to operating and to designing engineers, who have anything to do with synchronous converters. The following theory is that upon which the investigation is usually made and is readily interpreted as the symbolical statement of the law which is followed by the variation of armature heating. A knowledge of the essential principles of the construction and operation of a synchronous converter is pre-supposed in the development of this theory.

The armature winding of the present day converter is a continuous winding of either the simplex lap or wave style. The multiplex windings are not well adapted for use in converters due to the complex conditions arising from the increased number of leads to the collector rings and to the commutator segments. There are then, for the direct current in the armature, two paths when the wave winding is used, or a number of paths equal to the number of poles in the field when the lap winding is used. The method of
connecting the commutator segments corresponds to that used in any direct current generator of the usual type; connections to each segment being made from between the corresponding coils on the armature. The collector rings are connected each to a certain part of the armature in such a way that the points of connection will separate the armature winding into as many parts as there are collector rings. Thus, we have,

Single phase, two rings, two equal parts of the winding;
Three phase, three rings, three equal parts of the winding;
Quarter phase, four rings, four equal parts of the winding;
Six phase, six rings, six equal parts of the winding;
And so on.

The method is shown in Figure I, which illustrates the essential parts of a three-phase connection.

Due to the fact that, for each phase, the armature connections are constant for any machine, it follows that a definite ratio exists between the alternating current voltage and the direct current voltage. This ratio is determined as follows:

Suppose an armature to be rotating in a field produced by "k" poles, and that it has "s" turns per pole; suppose also, that an effective alternating voltage of "e" volts is generated in each turn due to the field excitation and to the revolution of the armature. The total turns upon the armature are "ks". Now, if "n" is the number of collector rings, there are "n" divisions in the armature winding, and also, there are \( \frac{ks}{n} \) turns per division or per phase, since "n" is also the phase rating in the case of any polyphase machine except quarter phase. The voltage per phase is the vector sum of the voltages in the separate turns of each phase, as is indicated in Figure 2, where \( E_4 \), the voltage for each of the three
phases indicated, is the vector sum of the voltages, \( "e", "e" \), generated in each phase. The numerical sum of these voltages per phase is \( \frac{KSE}{\pi} \), which follows directly. Since the turns are arranged about the circumference, \( E_a \), the phase voltage, is represented by a vector chord, subtending an arc of length equal to \( \frac{KSE}{\pi} \), where \( "e" \) is the vector voltage in each turn. This arc equals \( \frac{2\pi}{\pi} \) radians. Then the length of the chord follows directly, and

\[
E_a = \frac{2KSE}{\pi} \sin \frac{\pi}{\pi}
\]

\[
= \frac{KSE}{\pi} \sin \frac{\pi}{\pi}
\]

where \( E_a \) is the effective value of alternating voltage per phase.

Now, in any converter, the direct voltage is equal to the maximum value of the single phase alternating voltage. Then

\[
E_d = \frac{12KSE}{\pi} \sin 90^\circ
\]

\[
= \frac{12KSE}{\pi}
\]

Then the ratio,

\[
\frac{E_a}{E_d} = \frac{\frac{KSE}{\pi} \sin \frac{\pi}{\pi}}{\frac{12KSE}{\pi}}
\]

\[
= \frac{\frac{1}{12} \sin \frac{\pi}{\pi}}{\frac{1}{12} \sin \frac{\pi}{\pi}}
\]

for all phases, where "n" is, as noted above, the number of collector rings.

The current ratio in each phase, however, is much different, and more factors enter into its calculation. It has been found that the alternating voltage has the value,

\[
E_a = \frac{E_d}{2} \sin \frac{\pi}{\pi} \text{ current}
\]

Due to the vector displacement of the alternating in the different phases in the armature, the line current on the alternating side, has the value,

\[
I_a = 2I \sin \frac{\pi}{\pi}
\]


where \( I \) is the alternating current in each phase. The total alternating power input will be
\[ P_a = E_a I_a \]
\[ = \frac{E_a I_a}{2 \pi \sin \frac{\pi}{4} H} \]

Now, assuming a direct current power output equal to the alternating current power input, that is, assuming zero losses in the machine,

\[ P_a = P_d \]
\[ P_d = E_0 I_0 \]

where \( I_0 \) is the direct line current.

Then

\[ E_0 I_0 = \frac{\pi E_a I_a}{2 \pi \sin \frac{\pi}{4} H} \]
\[ = \frac{\pi E_0 I_a}{2 \pi \sin \frac{\pi}{4} H} \]
\[ = \frac{\pi E_0 I_0}{2 \pi} \]

Then the ratio,

\[ \frac{I_a}{I_0} = \frac{2 \sqrt{2}}{\pi} \]

which is seen to be the ratio of the alternating line current to the direct line current. Now,

\[ I_a = 2\pi \sin \frac{\pi}{4} \]
\[ I_0 = 2 I_0 \]

where \( I_0 \) is the direct current in each circuit of a two circuit winding. Then

\[ \frac{2 \pi \sin \frac{\pi}{4}}{2 I_0} = \frac{2 \sqrt{2}}{\pi} \]

and

\[ \frac{I}{I_0} = \frac{2 \sqrt{2}}{\pi \sin \frac{\pi}{4}} \]

which is seen to be the ratio of the effective alternating current in the phase winding to the direct current in that winding. Then the maximum alternating current in the winding is

\[ I_m = \frac{4 I_0}{\pi \sin \frac{\pi}{4}} \]
In Figure 5, an armature is represented as rotating in a field produced by two poles, and is shown in an instantaneous position. Let \( \alpha \) be the angle in electrical degrees, between the center of a phase winding and the plane of commutation, which last is assumed neutral. Let \( \phi \) be the angle of lag between the alternating current and the counter-generated voltage in any given turn. Let \( \beta \) be the angular displacement of any other coil in the section from the center of the phase section of winding. (A.I.E.E. Vol.27, No.2)

Now, considering the instantaneous value of the alternating current in the coil displaced by angle \( \phi \) from the center coil of the phase section. When this coil passes a point, displaced by an angle \( \frac{\pi}{2} \) (electrical) from the plane of commutation, the alternating voltage will, at that instant be a maximum. The alternating current at the same instant will be,

\[
I_s = I_m \cos \phi
\]

and

\[
I_{\text{max}} = \frac{I_0}{\cos \phi}
\]

As the coil in question moves from the point displaced by angle \( \frac{\pi}{2} \) from the plane of commutation, the generated voltage will vary as the cosine of the angle of displacement. That is,

\[
E = E_m \cos (\frac{\pi}{2} - \alpha - \phi)
\]

Now, since the resistance of the coil is constant, the variation of the current follows the same law, and

\[
I_s = I_m \cos (\frac{\pi}{2} - \alpha - \phi)
\]

Then

\[
I_s = \frac{N I_c \cos (\frac{\pi}{2} - \alpha - \phi)}{n \sin \frac{\pi}{4}}
\]

where \( I_s \) is the instantaneous value of alternating current in the coil which is displaced by the angle \( \phi \) from the center of the phase section. Then

\[
I_s' = \frac{N I_c \cos (\frac{\pi}{2} - \alpha - \phi)}{n \sin \frac{\pi}{4} \cos \phi}
\]
which is the instantaneous value of the alternating current in the center coil of the phase section.

Now, the resultant current is the numerical difference between the values of direct current and alternating current in the coil under consideration. Then

\[ I_r = I_0 \left( 1 - \frac{4 \sin \left( \frac{x+\phi}{2} \right)}{n \sin \frac{\pi}{m} \cos \phi} \right) \]

and \( I_r \) is the resultant instantaneous current. If \( I_0 \) is the direct line current, then

\[ I_r = \frac{I_0}{2 \pi} \]

in an armature with an \( 2m \)-circuit winding, where "\( m \)" is the number of pairs of poles in the case of a lap wound armature, and is constant and equal to unity in the wave wound machine.

Suppose the resistance of the coil is unity. Now, since the heating in the coil varies directly as the product of the resistance into the square of the resultant current, then

\[ KH = \frac{I_r^2}{2} \]

where "\( r \)" is the resistance of the coil. Then, if "\( r \)" is equal to unity,

\[ KH = \frac{I_0^2}{4 \pi^2} \left[ 1 - \frac{4 \sin \left( \frac{x+\phi}{2} \right)}{n \sin \frac{\pi}{m} \cos \phi} \right]^2 \]

and

\[ KH = \frac{I_0^2}{4 \pi^2} \left[ 2 \pi - \frac{2 \pi \sin \left( \frac{x+\phi}{2} \right)}{n \sin \frac{\pi}{m} \cos \phi} + \frac{4 \pi \sin^2 \left( \frac{x+\phi}{2} \right)}{n^2 \sin \frac{\pi}{m} \cos^2 \phi} \right] \times 4 \]

where \( KH \) represents the heating in the center coil in any position from \( \phi = 0 \) to \( \phi = \pi \) in electrical units of angular measurement. Then,

\[ KH = \frac{I_0^2}{m^2} \left[ 2 \pi - \frac{2 \pi \sin \left( \frac{x+\phi}{2} \right)}{n \sin \frac{\pi}{m} \cos \phi} + \frac{4 \pi \sin^2 \left( \frac{x+\phi}{2} \right)}{n^2 \sin \frac{\pi}{m} \cos^2 \phi} \right] \]

Now, suppose the machine is operated as a direct current generator, delivering \( I_0 \) amperes to the direct current line. The
heating in each coil of any section is then proportional to \( \frac{I^2}{m^2} \)

Then the ratio of the heating in the coil under consideration when the machine is run as a converter, to that when the same direct line current is mechanically generated in the armature, is

\[
y = \frac{I^2}{m^2}
\]

and

\[
y = \left[2 - \frac{2 \sin (x + \varphi)}{m \sin \frac{x}{m} \cos \varphi} + \frac{y \sin^2 (x + \varphi)}{m^2 \sin^2 \frac{x}{m} \cos^2 \varphi} \right]
\]

Then the corresponding ratio of the average heating for any coil distant \( \beta \) degrees from the center of the phase section is

\[
y' = \frac{1}{\pi} \int_{\beta}^{\pi + \beta} y \, dx
\]

Then

\[
y' = \frac{1}{\pi} \int_{\beta}^{\pi + \beta} \left[2 - \frac{2 \sin (x + \varphi)}{m \sin \frac{x}{m} \cos \varphi} + \frac{y \sin^2 (x + \varphi)}{m^2 \sin^2 \frac{x}{m} \cos^2 \varphi} \right] \, dx
\]

which by easy stages of integration, reduces to the equation,

\[
y' = \left[2 - \frac{y}{\pi m \sin \beta} (\cos \beta - \sin \beta \tan \varphi) + \frac{2 \sec^2 \varphi}{m^2 \sin^2 \beta} \right]
\]

where \( y' \) is the average heating ratio for any coil distant \( \beta \) degrees from the center; \( n \) is the number of collector rings; \( \varphi \) is the angle of lag; and \( -\varphi \) would be the angle of lead.

It is easily seen that to determine the average heating ratio for any phase section, and consequently for the armature for balanced loading, it is only necessary to find the mean heating ordinate over the total angular space occupied by the phase winding. This is done as follows:

\[
y_{av} = \frac{1}{2\pi} \int_{-\pi}^{\pi} y' \, d\beta
\]
\[ Y_{av} = \frac{n}{2\pi} \int \frac{\sqrt{2} \sin \frac{n\pi}{H}}{m \sin \frac{n\pi}{H}} (\cos \beta - \sin \beta \tan \phi) + \frac{2 \sec^2 \phi}{m^2 \sin^2 \frac{n\pi}{H}} \frac{1}{d\beta} \]

which reduces to the equation,

\[ Y_{av} = \left[ -155 + \frac{2}{m^2 \sin^2 \frac{n\pi}{H}} (1 + \tan^2 \phi) \right] \]

and \( Y_{av} \) is the average heating ratio for the total armature at balanced loading.

It is now desired to find the variation of the permissible direct current rating with power factor. When \( \phi = 0 \), which is the condition under which the rating is made,

\[ Y_{av}^0 = \left[ -155 + \frac{2}{m^2 \sin^2 \frac{n\pi}{H}} \right] \]

Now, if as before, \( I_0 \) is the direct line current; and if \( m \) is unity; the actual heating \( \Sigma \) is proportional to

\[ \Sigma = I_0^2 \left[ -155 + \frac{2}{m^2 \sin^2 \frac{n\pi}{H}} \right] \]

When \( \phi \) varies,

\[ \Sigma = I_0^2 Y_{av} \]

Then let

\[ I_0^2 Y_{av} = Y_{av}^0 \]

Then

\[ I = \left[ \frac{Y_{av}^0}{Y_{av}} \right]^{\frac{1}{2}} \]

\[ = \left[ \frac{-155 + \frac{2}{m^2 \sin^2 \frac{n\pi}{H}}}{\left( -155 + \frac{2}{m^2 \sin^2 \frac{n\pi}{H}} (1 + \tan^2 \phi) \right) \} \right]^{\frac{1}{2}} \]

and

\[ I = \left[ Y_{av}^0 \left( Y_{av} + \frac{2 \tan \phi}{m^2 \sin^2 \frac{n\pi}{H}} \right) \right]^{-\frac{1}{2}} \]

from which equation, the relation of rating to power factor may be determined.
From the equations for $y'$ and $y_{av}$, it is possible to determine the curves showing the relation of the heating ratio to change in power factor and to variation in phase. From a general inspection of the equation for $y_{av}$, it is seen that for any constant value of $\phi$, the heating ratio approaches a minimum as the number of collector rings increases. If the equation is investigated for a minimum value of $y_{av}$, the following results are obtained:

$$
\frac{d(y_{av})}{d(n^{-1})} = \frac{4(1+\tan^2\phi)\sin^2\frac{\pi}{n} - 4\pi(1+\tan^2\phi)n^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}}{\sin^2\frac{\pi}{n}} = 0
$$

Then

$$
\sin\frac{\pi}{n} = \frac{\pi}{n} \cos\frac{\pi}{n}
$$

$$
\tan\frac{\pi}{n} = \frac{\pi}{n}
$$

$$
\sec\pi n^{-1} = \pm 1
$$

$$
\cos\pi n^{-1} = 1
$$

$$
\pi n^{-1} = 0
$$

$$
n = \infty
$$

and the minimum value of the heating ratio (average) occurs with infinite phase, which is, of course, never practically obtained or even desired.

Also, the following determination may be made:

$$
y_{av} = \frac{2(1+\tan^2\phi)}{n^2\sin^2\frac{\pi}{n}} - 1.55
$$

For any constant percent power factor,

$$
D = 2(1+\tan^2\phi), \quad k = -1.55
$$

Then

$$
y_{av} = \frac{D}{n^2\sin^2\frac{\pi}{n}} - k
$$

from which the curves of the variation of $y_{av}$ with "n" may be determined.

The evaluation of the above equations for the different phases to be investigated, is the result of the direct substitution.
of the value of "n", corresponding to any phase for which the heating ratios are desired. The results are definite, from inspection, for any definite value of "n".

For the value $n = \infty$, however, the expression containing $\sin \frac{\pi}{n}$ is an indeterminate of the form $\frac{0}{0}$, and must be evaluated by a scheme of differentiation. It is desired first, to evaluate

$$\sqrt{r} = n^2 \sin^2 \frac{\pi r}{n}$$

This may be thrown into the form, with "n" varying, of

$$\sqrt{r} = \frac{\sin^2 \pi n}{r}$$

The first differentials with respect to $n^{-1}$ of both numerator and denominator, are then taken separately, and

$$\frac{d}{dn^{-1}} \left( \sin^2 \pi n \right) = 2 \pi \sin \left( \pi n^{-1} \cos \pi n^{-1} \right)$$

(Townsend and Goodenough, p. 342)

$$\frac{d \left( \sin^2 \pi n^{-1} \right)}{dn^{-1}} = 2 \pi n$$

Then

$$\sqrt{r} = \frac{\pi \sin \pi n^{-1} \cos \pi n^{-1}}{n^{-1}}$$

which evaluated for $n = \infty$ results again in $\frac{0}{0}$, and is indeterminate.

The second derivatives are then taken and

$$\frac{d}{dn^{-1}} \left( \pi \sin \pi n^{-1} \cos \pi n^{-1} \right) = -\pi^2 \left( \sin^2 \pi n^{-1} \cos^2 \pi n^{-1} \right)$$

also

$$\frac{d}{dn^{-1}} \frac{n^{-1}}{n^{-2}} = 1$$

Then

$$\sqrt{r} = -\pi^2 \left( \sin^2 \pi n^{-1} - \cos^2 \pi n^{-1} \right)$$

and this equation, evaluated for $n = \infty$, is

$$\sqrt{r} = \pi^2 \cos^2 (0) = \pi^2$$

So that

$$\sqrt{r} = n^2 \sin^2 \frac{\pi r}{n} \left|_{n = \infty} \right. = \pi^2$$
In a similar manner, the equation
\[ J = N \sin \frac{\pi}{n} \]
may be evaluated for \( n = \infty \). As follows:

Then
\[ J = N \sin \frac{\pi}{n} \]

and
\[ \frac{d (\sin \pi \pi - 1)}{d (n - 1)} = \pi \cos \pi \pi \]

and this equation, evaluated for \( n = \infty \), is
\[ J = \pi \cos \theta = \pi \]

So that
\[ J = N \sin \frac{\pi}{n} \bigg|_{n = \infty} = \pi \]

A slight investigation of the ratio equations shows that the ratio values for leading current, that is, when \( \theta \) is negative, are exactly symmetrical with those for positive values of \( \theta \), about the line \( \beta = 0 \), in the case of each phase rating. The average ratios are, of course, the same for any given power factor and phase rating, whether \( \theta \) is positive or negative, due to the presence of the \( 10 \pi \) factor. For this reason, no consideration is given the leading power factor values, since with the same percent power factor, there is no change in the results, and no difference in distribution, except an actual symmetrical shift of the ratio values about the line \( \beta = 0 \), in each phase section.

It has been mentioned that the foregoing theory relates
entirely to conditions of balanced loading upon the different phases. This is the theoretical operating condition, and is the condition upon which most machines are designed and rated. If, however, a polyphase machine must operate under unbalanced load conditions, a very convenient assumption may be made. The machine may be considered as two single phase converters or as a polyphase converter and a single phase converter. Since a single phase converter may be expected to operate fairly satisfactorily, the combination may be expected to operate at an appreciable efficiency, even though the efficiency can not be so high as that of the complete machine on balanced conditions. The effect of the unbalancing upon the heating in the machine does not depend so much upon the amount of unbalancing as it does upon the percent power factor of the unbalanced phases. No attempt will be made to calculate the effect upon the heating ratio of the unbalancing; but such points as are apparent will be taken from the results for balanced loading. (Steinmetz, "Elements", p. 296)

The evaluation of the ratio formulae, and the resulting curves will be found in the Appendix to this discussion.

From these results, it is possible to draw a great many conclusions. However, only a few of the most important will be discussed, in order of their importance. They will be taken up as follows: Copper Losses and Rating, Rating Limit for Economical Construction, and Three Phase versus Six Phase Operation.

A number of interesting conclusions regarding the copper losses in converter armatures may be drawn from the comparison curves. The copper losses vary as the square of the resultant current, and the curves showing the variation of the heating ratio are exactly the same as those which show the ratio of copper losses, when the machine is operated as a converter and as a direct current
It is plain from an investigation of the comparison curves, that, as the percent power factor decreases, the specific heating ratio increases in any machine of a given phase rating. Also, at the same time, the average ratio increases. A glance at the distribution curves shows that, as the percent power factor decreases, the heating, which is a maximum at the winding taps at unity power factor, is increased on one side of the taps to a greater extent than it is on the other side. Due to this increase, the heating is crowded into a few coils on one side of the ring connection, and these coils are greatly over-heated. The maximum heating in the coils close to the taps, increases much more rapidly than the average heating in the phase section.

Now, when a machine is built, its rating is made on the assumption of operation on balanced loading and at 100% power factor. Due to the desire for economical construction, the copper in the armature is not capable of carrying extremely high currents for any length of time. Consequently, when a machine with a given rating is operated at full load on a low power factor, it is more of a mystery why it does not at times burn up, rather than why it occasionally does.

The Rating-Power Factor comparison curve shows some interesting data on this point. For instance, if a three phase machine is operated on 85% power factor, its load should not exceed three-quarters its rated output, if the copper losses are to be kept constant. A six phase machine should not be loaded to exceed 65% of its rating, under the same conditions. Converters are constructed however, so that an overload may be carried; and it is clear, that when a machine is carrying full load on a low power factor, it is
running constantly on its overload capacity.

Because of these points, it is necessary to investigate the conditions under which a machine is to operate, before a machine with proper rating can be chosen. And it is necessary to install a machine rated above the expected load if the power factor varies to any extent from unity.

The Heating-Phase Rating comparison curve shows another interesting point. It has been observed that, as the number of collector rings increases, the average heating for any given power factor decreases and the minimum ratio is found when "n" is infinite. It may be seen from this curve, that, after a point is reached where "n" is equal to six, the variation of ratio from the value for "n" equal to six to that for "n" infinite, is exceedingly small. Due to the fact that additional collector rings and phase sectioning of the armature mean additional cost, while after the six phase is reached, the gain in reduction of heating is very small, it follows that, beyond six phase, it is not economical to construct machines for practical operation. And the loss in economy increases rapidly with increase in the value of "n".

One very valuable conclusion which may be derived from the fore-going investigation is that, as the number of collector rings increases, the permissible rating for the same load increases. This is of most practical importance in the case of three and six phase converter operation. If a three phase machine is operated on 80% power factor, its permissible rating is about 70% of full rating. With a six phase machine under the same conditions, the permissible rating is about 60% of full rating. A glance at the average heating curves shows that for 80% power factor, the average heating for six phase is about 66% of the average heating for three phase under the
same load. It follows, then, that a six phase machine may be rated from 75% to 100% higher than the same machine operated three phase under the same conditions. Thus, a great many three phase converters operating at poor efficiency, could be changed into very efficient machines, by the application of three more collector rings and a little additional wiring. Although this application does not consider the losses in the machine, and the influence upon the heating of hunting, and the change in the intensity of harmonics with change in voltages, it may be considered to be fairly accurate, since the corresponding losses in the six and three phase machines are approximately proportional.  (A.I.F.E., Vol. 27, No. 2)

If the distribution curves in the separate phase sections are considered, it is found that all the curves have the same general shape: those corresponding to the lower values of "n", being farther developed owing to the greater range of the phase angle. In as much as the constant coefficients for each phase change with variation of "n", the curves do not coincide. The distribution follows a transcendental law, which is a constant for all values of "n" between unity and infinity.

It may be observed that, for unity power factor, the heating is a maximum at the phase taps in the winding; and that, as the power factor decreases, the heating is shifted from the bars in the armature on one side of the tap to those on the other side. And it is to be noted that at times, the heating on the latter side is increased 100% to 200% with a corresponding decrease in radiating surface, due to the crowding of the heating losses into a few bars.

An interesting point which may be noted is that, as the power factor decreases, the minimum heating in a phase section shifts from the center along the section toward one of the taps, as the heating increases in the other end of the section. For the
larger values on "n" and $\phi$, this minimum finally reaches the tap, and the average heating in the coil immediately begins to rise more rapidly. This does not have appreciable effect upon the average ratio however, since the rate of increase changes slowly.

In conclusion, it follows that the distribution of the heating in the armature windings of synchronous converters depends upon the application of a definite law. Also, that this distribution has great effect upon the design, rating, and operating of converters. And consequently, an investigation of this law and of the variation of the heating losses in the phase section of an armature winding, results in a clearing up of much of the "mystery", which attends the practical application of converters to modern conditions of operation.
APPENDIX.

Throughout the numerical calculation of the investigation, the armature has been considered to be rotating in a field produced by two poles, and thus having two circuits, whether wave or lap wound. The results of the calculation have been tabulated as far as possible, for the most rapid reference; and the curves depend directly upon the tables for their evaluation. The calculation has been made for each of single, quarter, three, six, twelve, and infinite values of phase rating, and for percent values of power factor from 100 to 50, by intervals of 5%, and for values of 5%. The angular displacement of the coil, that is, angle $\beta$, has been taken at intervals of 5 degrees. Since balanced loading is assumed, the heating ratio for but one phase winding on each machine, has been calculated, since the value for the other windings will be exactly similar.

The evaluation of the ratio formulae follows the substitution of the value of "n", corresponding to the phase rating for which the ratios are desired, as before explained. This method of procedure results in the following special equations:

Single Phase. \( n = 2 \)

\[
\begin{align*}
\gamma' &= [0.75 - 0.35 \cos \beta - 0.35 \sin \beta \tan \phi] + 0.5 \tan^2 \phi \\
\gamma_{av} &= [0.75 + 0.5 - 0.35 \tan^2 \phi]
\end{align*}
\]
Three Phase. \( n = 3 \)

\[
Y' = 5.46 - 0.418(\cos \beta - \sin \beta \tan \theta) + 2.96 \tan^2 \theta \\
Y_{av} = 1.41 + 2.96 \tan^2 \theta
\]

Quarter Phase. \( n = 4 \)

\[
Y' = 5.01 - 1.50(\cos \beta - \sin \beta \tan \theta) + 2.51 \tan^2 \theta \\
Y_{av} = 0.96 + 2.51 \tan^2 \theta
\]

Six Phase. \( n = 6 \)

\[
Y' = 4.12 - 1.24(\cos \beta - \sin \beta \tan \theta) + 2.22 \tan^2 \theta \\
Y_{av} = 0.62 + 2.22 \tan^2 \theta
\]

Twelve Phase. \( n = 12 \)

\[
Y' = 4.09 - 1.09(\cos \beta - \sin \beta \tan \theta) + 2.05 \tan^2 \theta \\
Y_{av} = 0.50 + 2.05 \tan^2 \theta
\]

Infinite Phase. \( n = \infty \)

\[
Y' = Y_{av} = 0.475 + 2.02 \tan^2 \theta
\]

The equations for the variation of rating, result as follows:

Single Phase.

\[
I = \left( \frac{3.45}{3.45 + 0.5 \tan^2 \theta} \right)^{1/2}
\]

Three Phase.

\[
I = \left( \frac{1.41}{1.41 + 2.96 \tan^2 \theta} \right)^{1/2}
\]
Quarter Phase.

\[ I = \sqrt{\frac{0.096}{0.096 + 0.251/n^2}} \]

Six Phase.

\[ I = \sqrt{\frac{0.062}{0.062 + 0.222/n^2}} \]

Twelve Phase.

\[ I = \sqrt{\frac{0.050}{0.050 + 0.205/n^2}} \]

Infinite Phase.

\[ I = \sqrt{\frac{0.0475}{0.0475 + 0.202/n^2}} \]

From these equations the following tables were compiled:

Table I.—which gives the values of the heating ratio for single phase, for any coil distant \( \frac{\beta}{2} \) degrees from the center coil, and for any of the calculated percent power factors.

Table II.—which gives the corresponding values for three phase.

Table III.—which gives the corresponding values for quarter phase.

Table IV.—which gives the corresponding values for six phase.

Table V.—which gives the corresponding values for twelve phase.

Table VI.—which gives the corresponding values for infinite phase.

Table VII.—which gives the variation of average heating ratios, for each value of percent power factor, with the number of collector rings.

Table VIII.—which gives the permissible rating for any percent power factor in any of the generally used phases, in percent of the rating at 100% power factor.
Table IX.---which gives the angles of phase difference corresponding to definite values of percent power factor.

Table X.---which gives the values of the maximum heating ratio, for various percent power factors and various values of "n".

Table XI.---which gives the variation of the A.C.-D.C. voltage ratio with the change in phase rating.

It will be observed that in the cases of all values of percent power factor except 5%, the value of the heating ratio in the table is 1000 times the actual ratio. This method avoids the use of decimal points which would necessarily complicate the tables. In the case of the 5% power factor the tables give the actual ratio.

From the first six of these tables, the polar distribution curves were plotted. The method of plotting is as shown in the sample sheet, and the procedure is as follows: The section center is taken on any radial line. From this line, on each side, the angular distance corresponding to half the phase section is laid off, as shown by the numbers, zero degrees to 45 degrees. On each 5 degree radial line, the corresponding heating ratio is laid off, and the curve shown at "a" is drawn. This is the distribution curve. The circle "b", is then drawn at a distance from zero equal to the average ordinate for any section. This is the average curve. On each sheet, at the unit distance from the zero circle, a curve is drawn which represents the equivalent heating due to the direct line current, and is the basis of comparison for the ratio. Due to the symmetry existing between the polar curves for lag and lead angles of constant power factor, but one curve for leading current is shown. This indicates the shift of heating from one side of the winding tap to the other, for shift of $\varphi$ from positive to negative.
The rectangular curves were plotted directly from the tables. The method of procedure is clear from a comparison of the tables and curves.
### Table I.

**Single Phase.**

**Variation of Heating Ratio with Power Factor.**

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Table II.

Three Phase.

Variation of Keating Ratio with Power Factor.

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<td>572</td>
<td>727</td>
<td>8236</td>
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<tr>
<td>10</td>
<td>053</td>
<td>098</td>
<td>125</td>
<td>175</td>
<td>212</td>
<td>276</td>
<td>338</td>
<td>415</td>
<td>511</td>
<td>631</td>
<td>795</td>
<td>8308</td>
</tr>
<tr>
<td>15</td>
<td>059</td>
<td>116</td>
<td>164</td>
<td>202</td>
<td>245</td>
<td>312</td>
<td>384</td>
<td>462</td>
<td>563</td>
<td>691</td>
<td>861</td>
<td>8378</td>
</tr>
<tr>
<td>Ave.</td>
<td>050</td>
<td>072</td>
<td>098</td>
<td>128</td>
<td>I57</td>
<td>210</td>
<td>263</td>
<td>329</td>
<td>413</td>
<td>521</td>
<td>668</td>
<td>8165</td>
</tr>
</tbody>
</table>

Table VI.

Infinite Phase.

Variation of Heating Ratio with Power Factor.

<table>
<thead>
<tr>
<th>β</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave.</td>
<td>047</td>
<td>069</td>
<td>094</td>
<td>115</td>
<td>153</td>
<td>205</td>
<td>258</td>
<td>323</td>
<td>405</td>
<td>513</td>
<td>657</td>
<td>8530</td>
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</table>

Table VII.

Comparison of Heating Ratios.

Variation with Phase and Power Factor; 0 to 7; 100% to 50%.

<table>
<thead>
<tr>
<th>n</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
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<tbody>
<tr>
<td>Avg.</td>
<td>047</td>
<td>069</td>
<td>093</td>
<td>124</td>
<td>I53</td>
<td>205</td>
<td>258</td>
<td>323</td>
<td>405</td>
<td>513</td>
<td>657</td>
<td>804</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power Factor Percent Lagging.</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave.</td>
<td>047</td>
<td>069</td>
<td>093</td>
<td>124</td>
<td>I53</td>
<td>205</td>
<td>258</td>
<td>323</td>
<td>405</td>
<td>513</td>
<td>657</td>
<td>804</td>
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Table VIII.

Variation of Permissible Rating with Power Factor.

<table>
<thead>
<tr>
<th>Power Factor</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1000</td>
<td>925</td>
<td>825</td>
<td>754</td>
<td>686</td>
<td>625</td>
<td>571</td>
<td>530</td>
<td>480</td>
<td>427</td>
<td>370</td>
<td>326</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>901</td>
<td>820</td>
<td>745</td>
<td>690</td>
<td>615</td>
<td>560</td>
<td>507</td>
<td>460</td>
<td>414</td>
<td>370</td>
<td>326</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>883</td>
<td>790</td>
<td>707</td>
<td>648</td>
<td>573</td>
<td>518</td>
<td>467</td>
<td>420</td>
<td>377</td>
<td>335</td>
<td>296</td>
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<td>6</td>
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<td>850</td>
<td>736</td>
<td>650</td>
<td>593</td>
<td>522</td>
<td>460</td>
<td>410</td>
<td>369</td>
<td>328</td>
<td>291</td>
<td>256</td>
</tr>
<tr>
<td>∞</td>
<td>1000</td>
<td>828</td>
<td>710</td>
<td>620</td>
<td>560</td>
<td>484</td>
<td>430</td>
<td>385</td>
<td>344</td>
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<td>240</td>
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</table>

Table IX.

Value of Lag Angles for Various Power Factors.

<table>
<thead>
<tr>
<th>Power Factor</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deg.</td>
<td>0</td>
<td>18</td>
<td>25</td>
<td>31</td>
<td>35</td>
<td>41</td>
<td>45</td>
<td>49</td>
<td>53</td>
<td>56</td>
<td>60</td>
<td>87</td>
</tr>
<tr>
<td>Min.</td>
<td>0</td>
<td>12</td>
<td>50</td>
<td>44</td>
<td>54</td>
<td>25</td>
<td>34</td>
<td>27</td>
<td>8</td>
<td>38</td>
<td>0</td>
<td>8</td>
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</table>

Table X.

Variation of Maximum Heating Ratio with Power Factor.

<table>
<thead>
<tr>
<th>Power Factor</th>
<th>100</th>
<th>95</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>75</th>
<th>70</th>
<th>65</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>750</td>
<td>1013</td>
<td>1174</td>
<td>1335</td>
<td>1471</td>
<td>1699</td>
<td>1918</td>
<td>2173</td>
<td>2480</td>
<td>2864</td>
<td>3357</td>
<td>21242</td>
</tr>
<tr>
<td>3</td>
<td>302</td>
<td>473</td>
<td>587</td>
<td>677</td>
<td>758</td>
<td>906</td>
<td>1042</td>
<td>1200</td>
<td>1390</td>
<td>1624</td>
<td>1931</td>
<td>12677</td>
</tr>
<tr>
<td>4</td>
<td>183</td>
<td>315</td>
<td>395</td>
<td>476</td>
<td>544</td>
<td>660</td>
<td>769</td>
<td>898</td>
<td>1052</td>
<td>1243</td>
<td>1495</td>
<td>10667</td>
</tr>
<tr>
<td>6</td>
<td>105</td>
<td>198</td>
<td>259</td>
<td>322</td>
<td>378</td>
<td>466</td>
<td>558</td>
<td>655</td>
<td>780</td>
<td>937</td>
<td>1142</td>
<td>9272</td>
</tr>
<tr>
<td>12</td>
<td>059</td>
<td>116</td>
<td>164</td>
<td>202</td>
<td>243</td>
<td>312</td>
<td>384</td>
<td>462</td>
<td>563</td>
<td>691</td>
<td>861</td>
<td>8165</td>
</tr>
<tr>
<td>∞</td>
<td>047</td>
<td>069</td>
<td>094</td>
<td>115</td>
<td>153</td>
<td>205</td>
<td>253</td>
<td>323</td>
<td>405</td>
<td>513</td>
<td>657</td>
<td>8535</td>
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</table>

Table XI.

Variation of Voltage Ratio with Phase Rating

<table>
<thead>
<tr>
<th>Values of &quot;n&quot;.</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>12</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>7071</td>
<td>6130</td>
<td>5000</td>
<td>3530</td>
<td>1822</td>
<td>0</td>
</tr>
</tbody>
</table>
CURVES.
Sample Curve.

Distribution of Heating.

Quarter Phase.

"n" 4. P.F. 80%

a—Distribution.

b—Average.

c—Equivalent Heating of Direct Line Current.
Distribution of Heating.

Single Phase.

"n" 2 P.F. 100%

a—Distribution.

Max. .750 Min. .115

b—Average.

.345
Distribution of Heating.

Single Phase.

"n" 2. P.F. 95%

e--Distribution.

Max. I.013 Min. I35

b--Average.

.435
Distribution of Heating.

Single Phase.

"n" 2. P.F. 90%

a—Distribution.

Max. I.174 Min. .161

b—Average.

.462
Distribution of Heating.

Single Phase.

"n" 2.  P.F. 80%

a--Distribution.

Max. I.471  Min. .228

b--Average.

.607
Distribution of Heating.

Single Phase.

"n" 2. P.F. 70%

a--Distribution.

Max. I. 918 Min. .361

b--Average.

.865
Distribution of Heating.
Single Phase.

"n" 2     P.F. 60%

a--Distribution.
Max. 2.480     Min. .578

b--Average.
I.230
Distribution of Heating.

Single Phase.

"n" 2 P.F. 50\%

a—Distribution.

Max. 3.357 Min. .970

b—Average.

I.345
Distribution of Heating.

Three Phase.

"n" 3. P.F. 100%

b—Distribution.

Max. .302 Min. .057

e—Average.

.141
Distribution of Heating.

Three Phase.

"n" 3  P.F. 95%

a—Distribution.

Max. .473  Min. .064

b—Average.

.173
Distribution of Heating.

Three Phase.

"n" 3. P.F. 90%

a—Distribution.

Max. .587 Min. .073

b—Average.

.210
Distribution of Heating.

Three Phase.

"n" 3.  P.F. 80\%

a--Distribution.

Max. 0.758  Min. 0.102

b--Average.

0.296
Distribution of Heating.

Three Phase.

"n" 3.  P.F. 70\(^\circ\)

a—Distribution.

Max. 1.042  Min. .154

b—Average.

.449
Distribution of Heating.

Three Phase.

"n" 3.  P.F. 60%

a—Distribution.

Max. I.390  Min. .256

b—Average.

.665
Distribution of Heating.

Three Phase.

"n" 3. P.F. 50%

a--Distribution.

Max. I. 931 Min. .455

b--Average.

I. 032
Distribution of Heating.
Quarter Phase.

"n" 4. P.F. 100%

b--Distribution.
Max. .183 Min. .051

a--Average.

.096
Distribution of Heating.

Quarter Phase

"n" 4  P.F. 95\%  

a--Distribution.

Max. .315  Min. .055

b--Average.

.123
Distribution of Heating.

Quarter Phase.

"n" 4. P.F. 90%

a—-Distribution.

Max. .395 Min. .059

b—-Average.

.155
Distribution of Heating.
Quarter Phase.

"n" 4. P.F. 80%
a—Distribution.
Max. .544  Min. .077
b—Average.
.227
Distribution of Heating.

Quarter Phase.

"n" 4  P.F. 70.5

a--Distribution.

Max. .769  Min. .119

b--Average.

.357
Distribution of Heating.
Quarter Phase.

"n" 4. P.F. 60%
a--Distribution.

Max. I.052  Min. .204
b--Average.

.54I
Distribution of Heating.

Quarter Phase

"n" 4. P.F. 50%

a—Distribution.

Max. I.495 Min. .387

b—Average.

.854
Distribution of Heating.

Six Phase.

"n" 6. P.F. 100%

a—Distribution
Max. .105 Min. .048

b—Average.
.067
Distribution of Heating.

Six Phase.

"n" 6. P.F. 95%

a--Distribution.

Max. .198 Min. .050

b--Average.

.091
Distribution of Heating.

Six Phase.

"n" 6. P.F. 90%

a—Distribution.

Max. .259 Min. .054

b—Average.

.119
Distribution of Heating.

Six Phase.

"n" 6. P.F. 80\%

a—Distribution.

Max. .378 Min. .064

b—Average.

.183
Distribution of Heating.

Six Phase.

"n" 6. P.F. 70%

a--Distribution.

Max. .552  Min. 120

b--Average.

.298
Distribution of Heating.

Six Phase.

"n" 6. P.F. 60%

a—Distribution.

Max. .780 Min. 216

b—Average.

.460
Distribution of Heating.

Six Phase.

"n" 6       P.F. 50\(^\circ\)

\[ a \]—Distribution.

Max. I,142       Min .404

\[ b \]—Average.

.735
Distribution of Heating.

Twelve Phase.

"n" 12. P.F. 100% & 95%

a c—Distribution.
Max. .059 Min. .046
.I16 .046

b d—Average.
.050 .072
Distribution of Heating.

Twelve Phase.

"n" I2. P.F. -95\%, 90\%, 80\%, 70\%,

a-c-f-g--Distribution.

b-d-e-h--Average.
Distribution of Heating.

Twelve Phase.

"n" I2  P.F. 60%250%

a c--Distribution.

Max. .563  Min. .281
   .861    .493

b d--Average

   .413
   .668
Distribution of Heating.

Infinite Phase.

"n" P.F. varies.

Distribution equals Average.

a---100%  
b---95%  
c---90%  
d---80%  
e---70%  
f---60%  
g---50%  
h---5% (x.010)
GRAPHS SHOWING VARIATION OF AVERAGE HEATING RATIO WITH POWER FACTOR
Graphs showing variation of average heating ratio with phase rating.
Graphs showing variation of maximum heating ratio with power factor.
Graphs showing variation of permissible rating with power factor for same heating.