CONVERSE

An Experimental Determination
Of the Surface Tension of Liquid Air

Physics
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1909
AN EXPERIMENTAL DETERMINATION OF THE SURFACE TENSION OF LIQUID AIR

BY

EDWARD CHAPMAN CONVERSE

A. B. University of Illinois, 1904

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF ARTS

IN PHYSICS

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1909
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

EDWARD CHAPMAN CONVERSE

ENTITLED AN EXPERIMENTAL DETERMINATION OF THE SURFACE TENSION OF

LIQUID AIR

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Arts in Physics

Recommendation concurred in:

Committee on Final Examination

Head of Department
The surface tension of liquid air has not been accurately determined for the freshly made liquid. The values given are for the liquid after it has stood for some time and, since the nitrogen evaporates more rapidly than the oxygen, do not represent the surface tension when freshly made, but of a mixture of liquid oxygen and liquid nitrogen. Estimates based on these figures are inaccurate and only general in character, being between nine and ten dynes per centimeter. It is the purpose of this investigation to determine the value more definitely by readings taken as soon as possible after liquefying and noting the change of the surface tension with time. The data is then plotted upon time-surface tension axes and the curve extrapolated to the surface tension axis thus obtaining a value for zero time or when freshly made.

Historical.

Surface tension may be determined in various ways. One of the oldest ways is the Capillary Tube Method. Capillary tubes are dipped into the liquid which rises in the bore, if the liquid wets the tube. The elevation of the liquid surface and the radius of the tube being measured, the surface tension is calculated by the following formula,

\[ T = \frac{\pi rhg}{2} \]

where \( T \) is the surface tension, \( r \), the radius of the tube, \( h \), the elevation of the liquid, \( d \), the density, and \( g \), the factor of gravity. This is very convenient and valuable for rough work or

| Watson- A TEXT BOOK OF PHYSICS. p. 185. |
as a check method but is not very accurate. The best experimenters do not get concordent results, thus, the values given for the surface tension of water at room temperature by this method vary from 73.3 to 77.3 dynes per centimeter.

Hall, in the Philosophical Magazine for Nov. 1893, describes several newer and more accurate methods. He measures, with a balance, the pull due to the surface tension as a glass frame, shaped Fig. 1, is lifted from the liquid. The liquid clings to the upper bar and forms a double film, one part on each side. Then the surface tension is roughly equal to the tension, as measured by the balance, divided by twice the length of the frame.

The frame must be raised several centimeters above the surface of the liquid before the true films form. It was found that the force required to raise the frame increased with the distance, up to a certain point, then decreased again. The pull at this point is known as the maximum weight and serves as a basis for another method of measuring surface tension. This method can be used to advantage upon liquids whose films are too delicate to hold long enough for direct measurement of their tension. The excess of weight over the tension of films is due chiefly to the weight of the liquid held between them. Corrections must be applied for this and for buoyancy, pull upon the sides of the frame due to surface tension, etc. Otherwise the method is the same as above.

Hall developed a formula for this method but it is too long and complicated for use.
Foley improved upon Hall's work by substituting mica frames for glass. He also developed a simpler formula. The advantages of mica frames over those of glass will point out the requisites of the frames:

1. Mica frames can be made very thin thus reducing the correction due to enclosed volume of water, or other liquid used, to a minimum.

2. Mica frames are easier to make and use.

3. The edges are straight and the corners are sharp, rendering the length and thickness of the frame more certain.

4. To correct for the volume of enclosed liquid the density need not be accurately known. This is very important in liquid air work since the density is variable.

5. The correction varies as the thickness of the frame and for rough work no correction is needed if thin frames are used. Then the surface tension is equal to the maximum weight divided by twice the length of the frame.

The formula developed, known as the mica frame formula, is

$$T = \frac{w}{2(1 - t)} + \frac{d l^2 t^2}{4(1-t)^2} - \frac{lt}{4(1-t)^2} \sqrt{\frac{l^2 t^2 + 4w(l-t)d}{4}}$$

Where; $T$ is the surface tension.

- $w$, the net maximum weight.
- $l$, the length of the frame.
- $t$, the thickness of the frame.
- $d$, the density of the liquid.

It will be noticed that several of the factors are constants depending upon the dimensions of the frame and when once calculated the solution for other readings with the same frame is simplified.

In 1901, Knipp, by above method made determinations of the variation of the surface tension of liquid air with time. He plotted the data upon time-surface tension axes and by extrapolation of the curve estimated the surface tension when freshly made as between nine and ten dynes per centimeter. However as the freshest liquid he had was over an hour old, the point of the intersection of the curve with the surface tension axis is very uncertain. About the same time Grunmach determined the surface tension by the method of ripples. He analyzed each sample and the one nearest to liquid air that he used contained about half liquid oxygen. He makes no estimate as to the value when freshly made. Thus it will be seen that the value of the surface tension of liquid air proper has not been accurately determined, the values given being general. It is desirable to have it better defined. At the suggestion of and under the direction of Dr. Knipp I have attempted to do so. The special aim being to obtain values for shorter time and thus locate the point of intersection of the curve with the surface tension axis more exactly. By taking the average of several determinations a good value ought to be obtained. The upper part of the curve varies with the con-

\(^1\)C. T. Knipp - Density and Surface Tension of Liquid Air.


ditions as discussed below, approaching the value of the surface tension of liquid oxygen and need not be determined.

**Preliminary Work.**

Considerable time was spent experimenting with capillary tubes and the maximum weight method upon water. A number of mechanical difficulties were encountered. Most of the balances at hand had not amplitude enough to dip the frame into the liquid and raise it high enough to reach the maximum weight position. One, a Bunge balance, was satisfactory in this respect but was not very sensitive. As the release lever was turned the pans were dipped about 2 millimeters and all raised about 5 millimeters giving ample variation. Some time was required to secure a balance. The pull varied with time due to draining down of adhering water and usually the films broke before adjustment could be made. Then the frame was dipped again and weights further adjusted until a balance was obtained, if considerable time elapsed this weight would be less than if little time passed during the weighing, due to the draining down of the water. Thus, it is evident, time is an important factor. The afterweight should be taken at once after the gross weight, while there is the same amount of adhered water upon the frame as when the maximum pull was recorded. The legs should dip into the liquid the same distance as before to correct for suction and buoyancy upon them. The difference of above weights is the pull due to the maximum weight of the surface tension.

Since the surface tension of liquid air varies with time and considerable time is necessary to secure equilibrium with an ordi-
nary balance, it is evident that a more rapid method is desirable.
In 1901, Knipp employed the sucking action of a solenoid to measure the drag due to surface tension upon a platinum vane when determining the surface tension of water above 100 degrees C. He obtained very satisfactory results. The solenoid is also used commercially as a lifting agent. It was decided to adapt it to this work. The current to hold the frame with films attached at maximum weight position and the current to hold it in the same place after the films are broken being known, the difference is the current necessary to overcome the maximum pull of the surface tension. By calibration this can be reduced to weight equivalents and the results calculated by the mica frame formula.

Description of Apparatus.
To facilitate cutting the mica frames, Foley made a die. Having decided to use mica frames a similar die was made. It consists of two similar pieces of steel whose flat faces fit accurately together.

\[ \text{Face} \quad \text{End} \quad \text{Top} \]

DIAGRAM of APPARATUS.

a, Solenoid coil.
b, Supporting block.
c, Core.
d, Leveling base.
e, Shelf.
f, Framework on b.
g, Storage battery.
h, Ammeter.
i, Millivoltmeter.
S, Shunt to V.
m, Micrometer.
n, Leveling base.
p, Pier.
q, Low tin resistance.
r, High tin resistance.
t, Tapper.
u, Dwarf bulb.
w, Mica frame.
See Fig. 3 above. The pieces were cut to shape upon a milling machine by Mr. Strong, the Physics mechanician. The faces were ground flat with emery dust upon plate glass in order that they might fit perfectly. The outer edges were filed true with silk files. Two pins extend from the face of one half and fit closely into holes in the opposite half. These serve to hold the parts in place while cutting and to punch holes in the mica frame for the attachment of the suspension. The use of the die also tends to prevent checking and splitting back of the mica. The mica, split to desired thinness, is placed between the faces and the pins punched thru. The die is then held in the hand or clamped in a vice and a knife edge passed around it thus leaving a frame of the same shape and size as the die. The sides of the frame were slightly roughened with fine emery cloth to insure clinging of the liquid to it.

A solenoid 10 cm. long, consisting of 4 layers of about 75 turns each of No. 20 silk covered copper wire, was wound upon a glass tube the ends of which were flared to hold the wire in place. The outside diameter of the solenoid was 1.4 cm. It was slipped into a rubber tube for protection and mounted in the center of a block of wood 4x4x1 inches, being held in place by a set screw. See Fig. 4. This allowed considerable adjustment vertically. The block was mounted upon a small leveling base which was clamped to a heavy ring stand as shown in Fig. 4. A brass frame was fastened to the top of the block for support and guidance of the core. The core consisted of No. 18 soft iron wire, small enough to be saturated by the magnetic field of the coil. It was annealed and straightened by tying a weight to one
end, the other to a support and heating red hot with a Bunsen flame. The flame was played over the wire to prevent rapid cooling. Pieces about 220 cm. long were cut and laid aside for use. A piece of small copper wire was soldered to one end, passed thru a smooth hole in the frame work upon the block and a globule melted upon the other end. This served to support the system when no current was flowing. No guide was placed at the bottom of the solenoid as the diameter of the tube was only about a half larger than the core and so served as a guide. The dimensions of the core when completed were; total length 15.3 cm.; length iron, 10.3 cm.; length copper, 4.5 cm.; diameter of iron, 1.15 mm. See Fig. 5. To attach the part of the system below the core the lower end was flattened and a small hole drilled thru. A wire double hook was hooked into this and the thread running to Fig.5. the mica frame tied to the hook. Several kinds of suspensions were used but silk floss served the purpose best. The ends of a piece of the fibre about a foot long were tied to the frame. To the middle of this a single thread passed up to the hook. The double part extended out of the Dewar bulb for purposes of leveling.

As there is always a tendency for bodies to cling together slightly, the core adhered to the tube surrounding it. It requires a small force to overcome this and if the effect is not eliminated the current to lift the system will be uncertain and an unsteady motion will result. It was overcome by placing a tapper, consisting of an electric bell with gong removed, so as to tap upon the leveling base under the supporting block. This
kept the core in slight vibration and in an equilibrium position so that it responded to the magnetic forces without hindrance. The tapper was clamped to the same ring stand as the leveling base. The current to operate it was obtained from a large storage cell. To control the upward movement of the core a micrometer screw was attached to the ringstand so that its tip, over which a flat cap had been placed, was directly above the knob of the core guide. The ringstand was clamped to the top of a pier. A movable shelf was attached to the side of the pier.

Upon this was placed a large leveling base for support and adjustment of height. The distance from the core to the liquid surface was about 44 cm. The entire system was thus attached to the same pier and was free from vibrations of the floor.

The lifting current, 3 to 4 amperes, was drawn from a battery of 6 to 8 storage cells and controlled by two tin frame resistances, one for coarse and one for fine adjustment. The parts of the second were first disconnected and then joined as shown in Fig. 6. It will be seen that the clips are moved toward a a double path is provided and the resistance is therefore lessened a small amount. Moving the clips toward b has the opposite effect. This gave a delicate variation of current and was easy to manipulate. An ammeter was included for rough reading of the current but for accurate determination a Weston Direct Reading Semi-portable Laboratory Standard Voltmeter was used.

As the liquid air plant is owned by the College of Engineering it was convenient to use and liquid air could be obtained at any time and in any quantity. The compressor was made by the Norfolk
Iron Works. The liquifier is a Hanson liquefier and works at a pressure of 2000 to 3000 pounds per square inch. Moisture and carbon dioxide are removed by lime and potassium hydroxide, rendering filtering unnecessary.

Calibration of Apparatus.

The voltmeter has three coils, one for 150 volts, one for 15 volts, and one for 3 volts. The last was used. Its resistance is 495.06 ohms, hence the current it will carry is \(3 + 495.06\) or \(.006059\) amperes from ohms law. Therefore, since a current of 3 to 4 amperes was used it was necessary to shunt the voltmeter. Since the current in a shunt circuit is inversely proportional to the resistance, we have, if we let \(x\) = the shunt resistance and allow a maximum of 7 amperes,

\[
\frac{x}{495.06} = \frac{.006}{7-.006}, \quad x = \text{about} \quad .42 \text{ Ohms.}
\]

There was at hand a coil of No. 14 insulated manganin wire whose resistance was .227 ohms per meter. On account of heating a double wire had to be used. Hence to get .42 ohms resistance a double length of about 380 cm. was used. This was wound bifilar and the ends soldered to heavy copper rods which were clamped firmly between two strips of wood about a foot long as shown in Fig. 7. The other ends of the rods were amalgamated and dipped into mercury cups. From these cups connections were made to the rest of the apparatus. The resistance was measured by the Carey-Foster method and found to be .424 ohms. The current for a reading of three, is, as above, letting \(x\) be the total current,
\[
\frac{0.424}{495.06} = \frac{0.006059}{x} - 0.006059 \quad x = 7.0762.
\]

Since when this current flows in the circuit the reading is 3 it follows that if we divide 7.0762 by 3 we will get the factors for converting the voltmeter readings into amperes, which is 2.3584. However, since the readings are proportional to the current it is not necessary to change them to amperes if the calibration is properly done, i.e., if the value of the scale divisions is found in grams pull. The apparatus was so calibrated (see below) and throughout the work the readings were used directly. The coil was immersed in an oil bath for constant temperature. Since the wire is manganin, small changes in temperature will not produce appreciable changes in the resistance.

A large amount of work has been done in determining the constants of solenoids and the relation between the pull and the current used in them by S. P. Thompson, Underhill and others. Underhill tested the relation between the pull and the distance of the core in the coil. The results are shown by the curves of Fig. 8. The different curves represent different current strengths. It will be seen that when the plunger is a distance of about one-third to two-thirds the length of the coil from the end the pull does not vary much with distance for a given current strength but increases with the current. Thus if the end of the core is placed near the middle of the coil, the

lifting power is proportional to the current and does not vary much over a considerable distance. The relation of the pull to the current is shown by Fig. 9 which is also from Underhill. The pull is plotted against the current and a linear relation results except near the origin where the core is not saturated by the magnetic field. It has been shown also by many others that the pull is proportional to the current when the core is saturated. In this work these conditions were in vogue so this was not worked out completely but data was taken to find the position of the core corresponding to the maximum of the curves in Fig. 8 and the upper end placed just below this to allow a little for distance raised.

To get the weight equivalents of the voltmeter readings the following method was used. Since in making the determinations the current to hold the frame in the maximum weight position and that to hold it in the same position without the films is recorded, the difference is the current necessary to overcome the pull of the surface tension. Therefore it is the value of this difference that is desired. The system was loaded with a certain weight, the current to hold it noted, .1 gm. added and the increase of current to hold the system recorded. This was continued until a gram had been added by tenths. The initial load was varied and the process repeated. While the results were practically constant for increase of current per tenth gram, it was found that the increase was slightly less with heavy loads. During the final calibration the initial load was made equal to the after-weight in liquid air and weight added by .05 gm. intervals up to
Calibration Curve.

Showing gram equivalents for voltmeter readings.
.2 gm. It was not necessary to go further since the pull due to the maximum weight of liquid air is less than .15 gm. It was that the increase per tenth gram in the readings was .076 scale divisions. This factor could have been used but it was thought best and more accurate to plot the data upon a large scale and read the weight values directly from the curve. See data Table I. and curve in Fig. 10 which is a reduced copy.

**TABLE I.**

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Data and Discussion.

The apparatus was put in readiness to use before liquid air was made in order to save time in getting first reading. The mica frames were cleaned with hydrochloric acid and potassium hydroxide, then washed in distilled water and allowed to dry while the liquid air was made. Some of the liquid was run into the Dewar bulb, stirred around and poured out to cool the bulb to lessen the loss by evaporation when the run proper was made. The time at the beginning and end of run was recorded. Since the liquid coming off first is older than that coming off later the mean of the ages
for first and last was taken as the age at the end of run. The
time passing until the reading was taken was added directly to
this to obtain the age at that time.

The desired amount of liquid air being collected, the bulb was
placed in position as shown in Fig. 4, the frame introduced and
leveled by means of the suspension. The current was adjusted with
the high resistance to a value a little below that necessary to
lift the frame out of the liquid. As the current was increased
by changing the low resistance the system was gradually raised up
to the maximum weight point when it suddenly increased it's speed
and usually went high enough to break the films. The value of the
current which just took the system thru the maximum weight point
was recorded as the maximum reading, the current to hold the
system in the position where the sudden start upward took place,
was the afterreading and the difference that proportional to the
maximum weight pull of the surface tension. The value of this in
grams was found from the calibration curve and the surface tension
calculated by the mica frame formula. To control the upward move-
ment the tip of the micrometer was placed just above the position
of the core guide when the upward shoot took place. Readings were
taken at various intervals of time as shown by the data tables.
It was found that a reading could be taken in about a minute,
whereas it took ten to fifteen with a balance. Each reading was
checked two or three times and the average used. This gave a de-
cided advantage over the balance. Since there necessarily errors
in reading and the surface tension varied with time -ten minutes
making considerable difference at the beginning of the series- it
is desirable to check the readings but it could not be done with a
balance. The individual readings seemed to have about the same
accuracy.
TABLE II.

Data for curve A. Bulb 3/4 full.

Time at beginning of run 9:30, at end 10:00 a.m.

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TABLE III.

Data for curve B. Bulb 1/4 full.

Time at beginning of run 9:25, at end 9:41 a.m.

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TABLE IV.

Data for curve C. Bulb 2/5 full.

Time at beginning of run 11:02, at end 11:26 a.m.

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### TABLE V.

Data for curve D. Bulb about 3/5 full.

Time at beginning of run 9:50, at end 10:15 a.m.

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<tr>
<td>12:55</td>
<td>172</td>
<td>1.17&quot;</td>
<td>1.063</td>
<td>.107</td>
<td>.1408</td>
<td>11.74</td>
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### TABLE VI.

Data for curve E. Bulb about 1/4 full.

Time at beginning of run 1:41, at end 1:53 p.m.

<table>
<thead>
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<td>1.069</td>
<td>.092</td>
<td>.1217</td>
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<td>.095</td>
<td>.1254</td>
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<td>1.06</td>
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<td>1.064</td>
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<tr>
<td>3:40</td>
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<td>1.168</td>
<td>1.06</td>
<td>.108</td>
<td>.142</td>
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</table>

Length of frame in all of above determinations, 5.684 cm.

Thickness of frame in all of above determinations, .006 cm.
There is considerable variation in the after readings due to frozen moisture clinging to the suspension. This does not affect the data because the difference of the readings is used. It is well known that the composition of liquid air changes with age. The nitrogen evaporates faster than the oxygen, thus the remaining liquid becomes richer and richer in oxygen. Since liquid oxygen has a higher density and surface tension than nitrogen these properties increase with time and tend to approach those of oxygen.

The rate of evaporation is inversely as the protection afforded the bulb. As it was necessary to look thru the bulb to adjust the mica frame to position, an unsilvered one was used. To lower the rate of evaporation and keep the liquid quiet, the bulb was wrapped in haircloth leaving two small windows at right angles to each other, one for light and the other to look thru, this made the protection constant for all the determinations so this factor was dropped.

The rate of evaporation varies also with the volume of the liquid used, the smaller the volume the faster the rate. However if other conditions are the same and the data plotted upon time—surface tension axes the extrapolated curves ought to cut the surface tension axis at about the same point for zero time. Since it is impossible to have all other conditions exactly the same, especially since there is a loss at the liquefier, some deviation is to be expected. However if several different quantities are used it is probable that a good value can be obtained for the surface tension when freshly made.

At the liquefier the liquid comes in direct contact with the atmosphere and a rapid evaporation from the surface results.
Also the bulb and wrappings contain considerable heat. Much of this will be given up and cause a much more rapid rate of evaporation for several minutes at first than later. This would have the same effect as increasing the age. Since that produced later in the run does not suffer so much loss, the effect would be much greater when small than when large quantities are used. This effect would tend to raise the value where the curve cuts the surface tension axis above what it should be and for small quantities the error would be considerable. It was attempted to reduce this loss by attaching a tube to the outlet and extending this into the Dewar bulb, but no difference was noticed.

The data were plotted as above and the effects noted were found. Fig. 11 is a reduced copy of the curves. They cut the surface tension axis between the values of 9.5 and 10 dynes per cm. Due to the effects discussed above we must give more weight to the curves for larger quantities. These also tend to be a little high. Considering these things the value, indicated by the experiment for the surface tension of liquid air when freshly made is, between 9.4 and 9.6 dynes per cm.

Mathematical Considerations.

Dr. Knipp worked with the curves of Fig. 11 and determined the equation of each as closely as possible. The curves are of the general form of m Fig. 12 whose equation is

\[ S = S_0 \left(1 - e^{-kt}\right) \]

where \( S_0 \) is the value of the ordinate of the asymptote, e, the Naperian base, t, the value of the abscissa, and k, a constant of the curve which varies with the curvature, the greater the curvature the higher the value. \( S \) = value of any ordinate
Curve n, Fig. 12, is the complement of m, its equation is \( S = S_o \left( e^{-kt} \right) \). These curves are graphs of functions which vary with time and are used a great deal in Physics. Rutherford uses them in determining the rate of production and of decay of radio-active phenomena. As the data is plotted in Fig. 11, the ordinates represent surface tension. So represents the maximum value of the ordinate and since the surface tension of liquid air approaches that of liquid oxygen, this value, 13.3 dynes per cm. may be used as \( S_o \). It is more difficult to determine the value of \( k \) from the equation of curve m than from the equation of curve n. Since the curves are complementary the value of \( k \) is the same for each, hence we may use the simpler equation, \( S = S_o e^{-kt} \).

We cannot determine \( k \) directly from the equation as we do not know the value of \( t \), the abscissa. We are working with the upper portion of the curve, the portion above F G H Fig. 12. Since the curve m passes thru 0,0, a constant \( C \) must be added to the age to give the value of the abscissa. As we are working with different portions of the different curves, the value of \( C \) will be different for each curve. The following procedure enables us to calculate \( k \).

Considering two points upon curve n, as \( S_1 \) and \( S_2 \):

\[
S_1 = S_o e^{-kt_1} \\
S_2 = S_o e^{-kt_2}
\]

Solve each for \( S_o \):

\[
S_o = S_1 e^{kt_1} \\
S_o = S_2 e^{kt_2}
\]

Therefore,

\[
S_1 e^{kt_2} = S_2 e^{kt_1}
\]

Whence,

\[
\frac{S_1}{S_2} = \frac{e^{kt_2}}{e^{kt_1}} = e^{k(t_2-t_1)}
\]
Taking logarithms, \( \log S_1 - \log S_2 = K(t_2 - t) \)

\[
K = \frac{\log S_1 - \log S_2}{t_2 - t},
\]

\( t_2 - t \) represents a difference in the abscissas and can be found from the difference of the ages. The values for \( S_2 \) and \( S_1 \) will be 13.3 minus the value of the surface tension at age used. Now turning to curve A, Fig. 11, and taking the difference of ages at 43 and 205 minutes as the difference of \( t \) and \( t_2 \) at those points, we subtract the values of the surface tension from 13.3 for \( S_1 \) and \( S_2 \). We get,

\[
K = \frac{\log 3.21 - \log 1.76}{205 - 43} = 0.00395
\]

The average found by substituting four different sets of values was 0.0038 which was taken as the value of \( k \) for this curve.

Knowing \( k \) we can now calculate \( t \) for any given point on curve A by means of its equation, which is,

\[
S = S_0 (1 - e^{-kt})
\]

Solving for \( t \),

\[
t = \frac{\log S - \log(S_0 - S)}{k}
\]

Using for example 43 minutes as the age we get,

\[
t = \frac{\log 13.3 - \log(13.3 - 10.09)}{0.0038} = 374.
\]

This is the value of the abscissa at that point. Subtracting 43 from 347 gives 331 as the value of \( C \), i.e., the value of \( t \) corresponding to zero age for curve A, Fig. 11. The average obtained for five different points was 335. Now knowing \( k \) and \( C \) we can calculate the value of \( S \) for abscissa equal to 335 or zero age. Making the substitutions, we have,

\[
S = S_0 (1 - e^{-0.00395 \times 335}) = 9.549 \text{ dynes per cm}
\]

This value is very close to the value obtained by extrapolation.
Curves showing the variation of the surface tension of liquid air with time.
This process was repeated for the other curves on Fig. 11 and the results shown in the following table. The values of the surface tension at the end of 30 and 70 minutes were calculated and compared with the corresponding ones from the curves. The calculated values of the surface tension when freshly made agree very well with those obtained by extrapolation. As before, the results are in error due to the initial loss at the liquefier, the values for small quantities being more in error than those for the large. Considering this, the value indicated by the calculations is the same as from extrapolation. It was very gratifying to find that the results checked so well.

TABLE VII.

<table>
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Conclusion

As the value of the surface tension of liquid air had not been definitely determined, this investigation attempted to fix the value more closely. The maximum weight method was used, the frame being lifted by the suction action of a solenoid. The gram values of the current necessary were determined by calibration and the surface tension calculated by the mica frame formula. The results were plotted upon time-surface ten-
sion axes and the curves interpolated to cut the surface tension axis thus obtaining a value for zero time or when freshly made. The results obtained are between 9.5 and 10 dynes per cm. Due to losses at the liquifier, which cannot be accurately corrected for, more weight must be given to the values calculated from the data for the larger quantities, than to the values from the data for the smaller quantities. Taking this into account the values of the surface tension of liquid air when freshly made, as extrapolated by the curves A, C, D, Fig. 11, are between 9.5 and 9.8 dynes per cm.

The exponential equation of each of the curves was determined and the value of the ordinate where the curve intersects the surface tension axis calculated. The results thus obtained for the same curves, Table VII, indicate that the value of the surface tension of liquid air when freshly made lies nearly within the same limits, i.e., between 9.5 and 10 dynes per centimeter.