Reactance of Alternators

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# Reactance of Alternators

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Modern developments in the field of electrical engineering have had a marked effect upon all the apparatus required in generating, transmitting and utilizing electric energy. Probably this has been most notable in the increase in size of systems of distribution due both to the greater territory which can be covered by means of modern transmission lines operating at high voltages and to the increased demand for power in territories already connected. Directly incidental to this change is the enlargement of the individual unit in an effort to secure higher efficiency and to avoid a multiplicity of machines.

The natural result of larger systems and units and of increased voltages, currents and amounts of power is to bring into prominence problems which were before considered insignificant and were neglected but which now require most careful attention and accurate solution. Prominent among these is the entire problem of the generation of electric energy, the operation of changing the kinetic energy supplied at the shaft to the electric energy delivered to the line. The problem itself is an old one but requires the best of up-to-date practice and ideas if proper provision is to be made for the present-day conditions of operation in synchronism with other alternating current machinery, of maintaining the close regulation required by modern apparatus, and of adequate
protection against probable surges attending sudden changes in the character or distribution of the load. This problem of adapting alternators for use under the above conditions requires first a study of the normal electric and magnetic forces within that machine, or rather of the effect of the design upon those forces. The several conditions of normal operation under load, of operation under short-circuit, of hunting, of operation as a rotary condenser, etc., should then be considered, the object being to ascertain what features of design give best results in each case and in all cases.

This thesis is a first analysis of the forces under normal conditions and as such, as will later appear from definitions and explanations, is necessarily a detailed study of the synchronous reactance of the alternator. Since no hard-and-fast rules can be laid down in most cases, an effort has been made to present methods of calculation or test rather than specific results in formulas. The conclusions drawn in the theory have been checked to a considerable extent by tests performed by the author or by seniors and graduate students in the Electrical Engineering Department of the University of Illinois, prominent among those assisting in the tests being, W. F. Harshman, G. P. Sawyer, D. L. Smith and C. E. Weeks, all seniors with the Class of '11. Acknowledgment is also due the several members of the faculty who were ever ready with valuable advice and assistance.
II - STATEMENT OF PROBLEM

General

It is improbable that any student not familiar with the standard construction of the alternator and with the fundamental principles involved, will be interested in this thesis. Such knowledge on the part of the reader will therefore be assumed and attention directed at once to the so-called synchronous impedance of the armature, the impedance which causes the variation in terminal voltage between no load and full load (vector difference), thus affecting the regulation of the machine.

Impedance being a function of the frequency and, in this case, frequency being a function of the speed, it is necessary to specify at what speed the impedance is to be obtained. Synchronous speed is standard practice in this case and the impedance thus is termed "synchronous impedance". In like manner we have the "synchronous reactance" as the reactance component of the synchronous impedance. In all cases where either impedance or reactance of the armature are referred to in this thesis, those at synchronous speed are intended.

A very general and satisfactory definition of impedance is that it is the factor by which to multiply the current in amperes flowing through a circuit to get the voltage consumed by resistance, self-induction and capacity.
in that circuit. To facilitate in the study of voltages and currents in their component parts, impedance is considered to have two components, resistance and reactance, resistance being the factor by which to multiply the current to get the component of the drop in phase with the current, and reactance being the factor by which to multiply the current to get the component of the drop in quadrature therewith. The resistance is thus determined by the energy conditions and the reactance by the wattless conditions of the circuit. Where direct current is used the energy loss is represented by the heat loss in the wires, a quantity dependent upon the quality and dimensions of the wires and the rate of flow of current or the quantity in a given time. In alternating current circuits there must be added to this any hysteresis and eddy current losses in the medium surrounding the conductor and including the same, and the so-called skin effect in the conductor itself. These effects are so minimized in the design of the alternator armature however as to be inappreciable except, in some cases, while the circuit is in a transient state, and as this thesis is a study only of steady conditions, the entire subject of resistance will be dismissed.

Reactance

By definition, reactance produces a voltage drop in quadrature to the current, this drop being that to over-
come a self-induced E. M. F. where capacity may be neglected as is here possible. When a current passes through a conductor or coil, for there must always be a return circuit, a magneto-motive-force is produced in time-phase with the current. If this force acts upon a path which contains a sufficiently large airgap, it will set up a magnetic field which will vary in time phase with the force, and the field will produce an E. M. F. proportional to its rate of change.

Since we can refer to a constant phase relation only where the variation is harmonic, in this case we must deal with equivalent sine waves and the E. M. F. induced will be in quadrature and lagging behind the current which indirectly produces it; likewise, the E. M. F. necessary to be impressed to overcome this induced voltage must be in quadrature to the current and leading it. Where there are several turns in the coil, the several magnetic and electric forces will be in phase, being produced by the same current, and the constants of the winding,—the factor for turns in the m.m.f. and that for self inductance in the E. M. F.—increasing to give the correct results.

Synchronous Reactance of Alternators

That the above described phenomena will take place in the case of the alternator armature is evident, for current is there flowing in conductors and local fields will be set up around the conductors. In Figs. 1 and 2,
Magnetic Fields in an Alternator Armature

--- Exciting Field, --- = Armature Field, = Resultant Field
possible paths for these magnetic fields in a single phase armature are shown for two conditions of powerfactor. The bands on the armature represent the coils in its winding. In Fig. 1, the power factor is unity and the armature field will be seen to be in a purely local path, passing through the shoes of the poles but not cutting the exciting turns. Its effect upon the main or exciting field is merely to distort or shift it. In Fig. 2, the fields are shown when the load is purely inductive, the current lagging by 90°. In this case there are two paths for each coil, one bridging the gap between the pole-tips and the other extending through the poles and yoke. The former path will have a much greater reluctance than when it was as shown in Fig. 1, and the constants will not be the same. The latter path, in threading the poles, is in direct opposition to the exciting field and reduces that flux but does not distort nor shift it. Had the current an angle of lead of 90° with reference to the induced E. M. F. the armature magnetomotive force would be reversed and the exciting field would be increased rather than decreased.

Separation of Synchronous Reactance

While the real phenomenon, in any case, is essentially one of self-induction, the analysis of the problem is simplified if the purely local fields be separated from those threading through the poles, the reactance due to the former being termed that of self inductance and of the latter
that of armature reaction. Whether the drop due to reaction be considered as voltage necessary to overcome an induced E. M. F. or as that lost because of a decreased exciting field will have no effect upon its final magnitude or phase relation. Regarding the amount, as much voltage will be consumed in overcoming an induced E. M. F. due to the cutting of a given number of lines in a given time as would be lost from the machine voltage were the exciting field to lose that many lines and the coil thus fail to cut them.

The phase relation in the case of armature reaction, may be determined by an inspection of Fig. 2. When the current lags, there is a decrease in the generated E. M. F. Then the voltage due to the armature field, being $180^\circ$ out of phase with this generated E. M. F., lags behind the current by $90^\circ$ or the current leads the E. M. F. it indirectly produces, by this amount. When the current leads the generated E. M. F., there is an increase in the same; that due to the armature field being in phase with this E. M. F. must again, by hypothesis, lag behind the current responsible for it by an angle of $90^\circ$.

It is interesting to note that, while the distortion or shifting effect on the exciting field is due to the local or self-inductance flux and is therefore not a part of armature reaction, still this effect must be considered when determining the proper phase relations in the machine, the generated E. M. F. being dependent upon the distorted field.
The Problem and Method of Attack

In practice it is not customary to find any of the conditions of power factor just considered but rather a combination of them. The path for the self-inductive field is usually found only partly under the pole and the effect of armature reaction is usually divided between distortion and direct action with or against the exciting field, the proportion in each case depending upon the phase displacement. If desired, the problem may be treated in this complex state. It will be simplified greatly, however, if the current be separated into its in phase and quadrature components, the ultimate effect of each determined, and these effects combined into a final solution. This method is followed in this thesis, the object of which is to discuss in detail the several conditions influencing the effect of the current flowing in the alternator armature upon the characteristics of the machine.
III - THE REACTANCE OF SELF INDUCTANCE

Local Magnetic Fields

It has been shown that a part of the synchronous impedance is due to a local field of magnetic lines surrounding the conductors in the armature winding but not cutting the exciting field turns and this part has been termed the self inductive reactance. The important function of this local field demands its analysis as the first essential to an understanding of its effect and of this reactance.

The voltage induced by a magnetic field depends upon the rate of change of the lines. The rate of change in a path of constant permeability is that of the current responsible for the field and the number of the lines is determined by the reluctance of the path and the ampere turns impressed. In the case at hand, the presence of the large air gap in the path--large in percent of the total length of the path--provides a constant permeability of unity. The effective turns may be obtained from the winding data. But the reluctance is not so readily disposed of.

The Reactance of a Single Slot

Figure 3 shows a representative armature slot with dimensions. Since the permeability of the iron is many times that of the air, the length of the path for any line may be assumed to be the length of the air traversed by
that line. To further facilitate in the study of the path, it can be divided into several branches, occupied by the fluxes $\varphi_1$, $\varphi_2$, $\varphi_3$ and $\varphi_4$ as shown in Fig. 4. From the law of the magnetic circuit,

$$\varphi = \frac{F}{R} \quad (1)$$

where $\varphi$ is the flux in lines, $F$ is the magneto motive force in Gilberths and $R$ is the reluctance.

![Fig. 3.](image)

![Fig. 4.](image)

By definition

$$R = \frac{l''}{a^2}$$

where $l''$ is the length of the path in centimeters, $a$ is the area in square centimeters and $\mu$ is the permeability which in the case of air as here used is unity. In as much as dimensions of slots where found on blue prints are usually in inches, the ratio $\frac{l''}{a}$ must be divided by 2.54 to make it
applicable to this condition. Then

\[ R = \frac{1'}{2.54 a'} \]  \hspace{1cm} (2)

where \(1'\) is in inches and \(a'\), in square inches.

By derivation

\[ F = .4 \pi n i \]

where \(i\) is the current flowing and \(n\) is the number of effective turns, an effective turn being that combination of turns used to carry the total armature current and may be one or more wires. Since the maximum value of the flux is the one considered in the derivation of magnetic formulae, the maximum M. M. F. as produced by the maximum current, \(I\), should be used, or

\[ F = .4 \pi n I \]  \hspace{1cm} (3)

Substituting (2) and (3) into (1), the value of the maximum flux is

\[ \Phi = \frac{.4 \pi n I \cdot 2.54 a'}{l'} = 3.2 \frac{a'nI}{l'} \]  \hspace{1cm} (4)

The values of \(\Phi_1, \Phi_2, \Phi_3\) and \(\Phi_4\) may now be obtained by substitution of dimensions into equation (4). In obtaining \(a'\), unit length of armature may be assumed. The values resulting are

\[ \Phi_1 = \frac{3.2 NI A}{M}, \quad \Phi_2 = \frac{3.2 NI H}{C}, \quad \Phi_3 = \frac{3.2 NI D}{E} \]
and \( \Phi_4 = \frac{3.2NI}{EB} \) \( \quad (5) \)

The method for determining the last expression, that for \( \Phi_4 \), is suggested in Fig. 3. At the distance \( x \) from the bottom of the slot, the M.M.F. is \( \frac{x}{E} \) times the total M.M.F.; the area is \( \Delta x \) and the length is \( B \). Thus the element of flux

\[ \Delta \Phi = \frac{3.2NIx(\Delta x)}{EB} \]

This flux encloses \( \frac{x}{E} \) \( N \) turns. Since \( \Phi_4 \) is to be combined with \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) and the effect of the total upon the winding is to be determined, an equivalent value must be obtained for \( \Phi_4 \) which will have the same effect upon the entire coil as the several parts, \( \Delta \Phi \), have upon their respective parts of the coil. \( \Delta \Phi \) as found encloses \( \frac{x}{E} \) \( N \) turns, an equivalent flux to have the same effect upon \( N \) turns would be \( \frac{x}{E} (\Delta \Phi) \), this value being \( \Delta \Phi_4 \), or a differential element of that equivalent flux \( \Phi_4 \). Then

\[ \Delta \Phi_4 = \frac{x}{E} (\Delta \Phi) = \frac{3.2NIx^2(\Delta x)}{E^2B} \]

Passing to the differential and integrating between the limits of \( \Phi_4 = 0 \) where \( x = 0 \) and \( \Phi_4 = \Phi_B \) where \( x = E \),

\[ \int_0^{\Phi_B} d \Phi_4 = \frac{3.2NI}{E^2B} \int_0^E x^2 dx = \frac{3.2NI}{E^2B} \frac{E^3}{3} = \frac{3.2NI}{3B} \]
Having the values of flux in the several paths as given in (5) and the effective length of the armature in inches as 1, the total flux is the sum of the several fluxes multiplied by 1, or

\[ \Phi = (\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4) \times 1 = 3.2 N I l g \tag{6} \]

where \( g \) is the constant \[ \left[ \frac{A}{M} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3B} \right] \]

Since equivalent sine waves are being considered, the fundamental equation for induced voltage may be used, and

\[ E_{\text{effective}} = \frac{\sqrt{2} \pi f N \Phi}{10^8} \tag{7} \]

where \( E \) is the effective voltage per slot, \( f \) is the frequency in cycles per second and \( N \) is the number of effective conductors per slot. The reactance per slot may therefore be determined from (6) and (7) as follows:

\[ x = \frac{\sqrt{2} \pi f N 3.2 N I l g}{10^8} \tag{8} \]

The Single Slot -- End Connections

This value takes no account of the end connections and should therefore be increased, the amount varying in
different machines. A rough inspection of the average machine will show that the mean effective length of the magnetic circuit surrounding the end connections is about ten times that where the conductor is in the slot, and from this an approximate constant by which to increase the reactance per slot is found to be one tenth times the proportional lengths of end connection to effective slot. If $l_e$ is the length of the end connections in inches and $k_e$ is the constant above discussed as 0.1, the value of $x$, from (8), becomes,

$$x = \frac{6.4 \pi f N^2 l g}{10^8} (1 + \frac{l_e}{1} - k_e) \quad \text{(9)}$$

Effect of Position of Slot Relative to Pole

This calculation is also limited to the slot when it is between the poles, i.e. when $M$ as shown is entirely in the air space. Discretion must therefore be used in determining the numerical values of the dimensions, choosing $M$ as the double air gap when the slot is under the pole, etc. In practice the inductance between the poles is sometimes multiplied by 1.5 to give the inductance under the pole, but this cannot always be relied upon. For accurate work the only safe method to pursue will be to make the complete calculations.
Effect of Adjoining Idle Teeth

The derivation of $x$ is further limited in that it involves but one slot, a condition never met in practice, although one slot per pole and phase is not an unusual winding. Naturally in such a case, to determine the self inductance of any coil - the inductive effect upon that coil due to any current flowing in the coil - the adjoining phase windings need not be considered and the paths through the adjoining teeth should be dealt with in the same manner as those already discussed. While these paths necessarily have longer air gaps, still the pole shoes are likely to assist in keeping down their reluctance and it is surprising what a proportionally large effect they may have on the results. Referring to the check test; on the two phase alternator the self-inductive reactance due to the flux linking through the adjoining phase was found to equal that due to the flux in the more local paths. In this case there was no mutual effect between phases, for, as will later be demonstrated, this effect is not present in a two phase machine.

Self Inductance for More than One Slot

When the adjoining slots contain conductors from the same phase winding as shown in Fig. 5, the conductors may be considered as parts of a coil, similar to a solenoid. Since the currents in A and B (See Fig. 5) are in phase the
lines of force which tend to pass around the individual slots will neutralize each other between the slots and form the combined field about the total phase winding. The ampere-turns and hence the M.M.F. is now doubled but the effective lengths of the several paths (excepting (a)) are also doubled, these lengths being those of the air gaps. The components of flux will thus remain unaltered and the reactance per slot will be just the same as before. Furthermore, this will be true whatever the number of slots. The reactance of the winding, therefore, when the coils in the several slots are in series, will appear to be that of one slot multiplied by the number of slots.

This reasoning, however, neglects the flux in the path shown at (a) Fig. 5, the flux already referred to as that in nearby teeth when the field is permitted to enclose more than the one slot. Since the length of this path has not been changed the increased M.M.F. will increase the number of lines following it and, due to this greater flux, the induced E. M. F. for the same current and turns, and therefore the reactance, will be increased. With the slot under the pole, the path (a) will be seen to be an important
one. Also with an increase in the number of slots per pole and phase, the lines in the path (a) will be seen to vary because of the increased M.M.F. and also, probably, because of a variation in the reluctance, there being a greater chance for the path to include parts of the poles when extended around a long coil or winding. Other constants, determined by the number of slots per pole and phase and the way their conductors are connected, must therefore be introduced into the expression for \( x \).

Thus, letting \( s \) be the number of slots in series, \( k_s \) the constant discussed above for \( s \) slots and \( k_t \) the constant to include the effect of adjoining idle teeth, from (9)

\[
x = \frac{6.4 \pi f N^2 l g s k_s k_t}{10^8} \left( 1 + \frac{1}{1 - k_e} \right)
\]

Mutual Induction in the Polyphase Armature

The question of mutual induction between phases has merely been suggested. To study this effect in any one phase winding due to the current flowing in the other phases, let

* See page 27 for a special case involving the increase in \( x \) as applied in equation (10).
\( n = \) the phase index,
\( \varphi_M = \) all the flux due to any one phase threading the teeth in the other phases.
\( e_M = \) the self-induced voltage due to \( \varphi_M \).
\( \varphi_{m1-2-3-etc.} = \) the flux threading the teeth in only one phase winding due to the turns in one other phase, as designated by the subscript, or a part of \( \varphi_M \).
\( e_{1-2-3 etc.} = \) the voltage mutually induced in the one phase by the several fluxes \( \varphi_{m1-2-3-etc.} \).
\( \varphi_m = \varphi_{m1} + \varphi_{m2} + \varphi_{m3} + \ldots = \) the total mutual flux threading one phase due to all the other phases.
and \( e_m = e_1 + e_2 + e_3 + \ldots = \) the total voltage mutually induced by \( \varphi_m \).

Also, for uniformity, choose a balanced system and a round rotor field structure, so that, the magneto motive forces and reluctances being the same,

\[
\begin{align*}
\varphi_{m1} &= \varphi_{m2} = \varphi_{m3} \text{ etc. in magnitude} \\
\text{and } e_1 &= e_2 = e_3 \text{ etc. in magnitude}
\end{align*}
\]

Designate the phase windings as \( I_1, I_2, I_3 \) etc., and let \( I_1 \) be the one under study.

Any flux due to \( I_1 \) enters the armature at one side and leaves at the other. Since we are considering but one half of the conductors of \( I_1 \) — the other half being at the
next pole — the entire flux due to the conductors under consideration will thread the teeth included in ninety electrical degrees. (See Fig. 6.) The angle subtended by one phase is $\frac{180}{n}$, then the total flux from $I_1$ threading teeth outside of $I_1$, or the total mutual flux due to $I_1$, will be included within the angle $\frac{180 - \frac{180}{n}}{2} = 180 \frac{n - 1}{2n}$. Thus the mutual flux cutting one phase is to the total mutual flux, due to one phase, as

$$\frac{\Phi_{m1}}{\Phi_M} = \frac{\frac{180}{n}}{\frac{180}{n} - \frac{1}{2}} \text{ and } \frac{\Phi_{m1}}{\Phi_M} = \frac{2}{n - 1} \Phi_M \quad (12)$$

Since the voltages induced in $I_1$ by mutual from $I_2$, $I_3$ etc. are $e_2$, $e_3$ etc., it is only necessary to add them vectorially to obtain the total $e$ of mutual induction. Also since the currents in $I_2$, $I_3$ etc. have definite phase relations with $I_1$, their induced voltages will bear that phase relation to the self induced voltage in $I_1$, the voltage used to determine the reactance of self induction. $e_1$ should therefore be used as the reference vector.

If $I_2$ adjoins $I_1$ and leads it, $e_2$ leads $e_1$ by the
angle $\frac{180^\circ}{n}$ and, vectorially, $e_2 = e_1 \cos \frac{180^\circ}{n} - j e_1 \sin \frac{180^\circ}{n}$. Since $e_2 = e_1$, $e_2 = e_1 \left[ \cos \frac{180^\circ}{n} - j \sin \frac{180^\circ}{n} \right]$ (13)

If $I_3$ adjoins $I_1$ and lags behind it, $e_3$ lags behind $e_1$ by the angle $\frac{180^\circ}{n}$ and

$$e_3 = e_3 \cos \frac{180^\circ}{n} + j e_3 \sin \frac{180^\circ}{n} = e_1 \left[ \cos \frac{180^\circ}{n} + j \sin \frac{180^\circ}{n} \right].$$ (14)

If $I_4$ adjoins $I_2$ and leads it, $e_4$ leads $e_1$ by the angle $2 \frac{180^\circ}{n}$ and

$$e_4 = e_1 \left[ \cos \left( 2 \frac{180^\circ}{n} \right) - j \sin \left( 2 \frac{180^\circ}{n} \right) \right].$$ (15)

From the equations for $e_3$ and $e_4$, if $I_5$ adjoins $I_3$ and lags behind it,

$$e_5 = e_1 \left[ \cos \left( 2 \frac{180^\circ}{n} \right) + j \sin \left( 2 \frac{180^\circ}{n} \right) \right]$$ (16)

and so on.

The limiting angle in any case must be that which gives $\pm 90^\circ$ in space on the armature, for a greater angle would have to do with coils which act upon that part of $I_1$ at the next pole. Also, these coils have parts within the $\pm 90^\circ$, acting upon the part of $I_1$ under consideration. Since, under normal conditions, the electrical quantities are in quadrature when the coils are displaced $90^\circ$ in space, the limiting value of $\frac{180^\circ}{n}$ is $90^\circ$. From this it will seem that there is no mutual induction between phases in a two-
phase armature and that a similar condition exists in any machine of even phase index between the phases displaced by 90°, as in a six-phase. (See Fig. 6.) That this is a fact is proven later.

Returning to the equations of the E.M.F. induced and adding (13), (14), (15) and (16), etc.,

\[
e_m = e_1 \left[ \cos \frac{180°}{n} - j \sin \frac{180°}{n} + \cos \frac{180°}{n} + j \sin \frac{180°}{n} + \cos \frac{180°}{n} - j \sin \frac{180°}{n} \right].
\]

It will here be seen that the sine values disappear so that the resultant mutual induced voltage is in phase with the self induced in the winding and also that the equation for this resultant voltage includes a cosine series. Simplified and continued for other phase windings,

\[
e_m = 2 e_1 \left[ \cos \frac{180°}{n} + \cos \left(2 \frac{180°}{n}\right) + \cos \left(3 \frac{180°}{n}\right) + \ldots \right]
\]

etc. - the coefficients in the angles being determined indirectly by the amount of space displacement of the inducing coil, measured in phase angles and being limited by the quadrature position.

Let the cosine series be represented by \( K_c \), then

\[
e_m = 2 K_c e_1
\]

Since \( e_m \) and \( e_M \) are in phase, the total voltage
induced in $I_1$ by that part of its own flux which cuts the other phases and by mutual from the other phases, is

$$e_0 = e_m + e_M. \quad (19)$$

Let $\varphi_p = \text{an equivalent flux threading the path of } \varphi_M,$ to induce $e_0$. Referring again to the formula,

$$e = \frac{\sqrt{2 - \pi f_m n \varphi}}{10^8} = K \varphi,$$

we have

$$e_0 = K \varphi_p, \quad e_m = K \varphi_m, \quad e_M = K \varphi_M. \quad (20)$$

From (18) and (20)

$$K \varphi_m = 2 K_c K \varphi_M$$

or

$$\varphi_m = 2 K_c \varphi_M$$

Now substituting from (12),

$$\varphi_m = 2 K_c \frac{2}{n-1} \varphi_M.$$  

and

$$e_m = K \varphi_m = 2 K K_c \frac{2}{n-1} \varphi_M. \quad (21)$$

Substituting into (19) from (20) & (21),

$$K \varphi_p = 2 K K_c \frac{2}{n-1} \varphi_M + K \varphi_M$$

or

$$\varphi_p = \left( \frac{4 K_c}{n-1} + 1 \right) \varphi_M = K_P \varphi_M \quad (22)$$
Here \( \varphi_p \) is the total equivalent flux in teeth outside of the phase winding, equivalent in effect to the flux of self-induction outside of the phase winding plus the flux of mutual induction cutting the phase winding, \( \varphi_m \) is the flux of self-induction outside the phase winding, and \( K_p \) is the constant by which \( \varphi_m \) is multiplied to obtain \( \varphi_p \). When the reluctances of the several paths are equal and the phases are balanced, as assumed in the above derivation,

\[
K_p = 1 + \frac{4 K_c}{n - 1}
\]

(23)

where \( n \) and \( K_c \) are as used in the development.

<table>
<thead>
<tr>
<th>( n ) = phase index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>15</th>
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<tbody>
<tr>
<td>( K_c )</td>
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<td>0</td>
<td>.5</td>
<td>1.12</td>
<td>1.37</td>
<td>2.66</td>
<td>4.28</td>
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</tr>
<tr>
<td>( \frac{4 K_c}{n - 1} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>.95</td>
<td>1.12</td>
<td>1.10</td>
<td>1.19</td>
<td>1.22</td>
</tr>
<tr>
<td>( K_p = 1 + \frac{4 K_c}{n - 1} )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1.95</td>
<td>2.12</td>
<td>2.10</td>
<td>2.19</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Table I. Development of Mutual Inductance Factors for Polyphase Armatures.

Substitutions into (23) for \( n \) give the results shown in Table 1. More points are taken than necessary to show the consistency of the results. It will be noticed from the table that single phase is indeterminate but since there is but one phase, \( K_c \) must be zero and \( K_p = 1 \). For two-phase, \( K_c \) is zero and \( K_p = 1 \) because the phases are 90° displaced and will have no mutual effect. For all other phases the constant \( K_p \), may be assumed to be 2.0, except in very accurate determinations.
It should be borne in mind that the development and constants as here given apply only for conditions of constant reluctance at all points about the armature and for balanced phases -- currents and power factors. These conditions are easily satisfied in machines of the round rotor type, those where the self-inductance is of great importance. In definite pole machines operating under poor power factor, the constants would be approximately correct, for all of the "mutual paths" would then have much the same amount of air gap. When the winding is under the pole, it might be safe to neglect the constant $K_c$ altogether. Finally, there will be a constant $K_p$ whose value may be approximated in a general way but which, for very accurate work, must be determined for each specific machine. Only one thing is certain, there will be no mutual induction in single phase armatures nor in two-phase armatures when the power factors in the two phases are at all alike.

A Complete Expression for the Reactance

From the value of $K_p$ as given in explaining Equation 22 it will be seen that, to include the effect of mutual induction between phases, we have but to multiply $K_t$ as given for Equation 10 by $K_p$. Thus a complete expression for $X$ is

$$X = \frac{6.4 \pi f N^2 1\ g\ s\ K_s\ K_t\ K_p}{10^8} \left( 1 + \frac{1}{e} K_e \right)$$

(24)
Here

\[ X = \text{reactance in ohms per phase.} \]
\[ f = \text{frequency in cycles per second.} \]
\[ N = \text{effective conductors per slot.} \]
\[ l = \text{effective length of armature in inches.} \]
\[ g = \text{slot path constant} = \left( \frac{A}{M} + \frac{H}{C} + \frac{D}{B} + \frac{E}{3B} \right) \text{ where} \]

A, B etc. are dimensions in inches as indicated in Fig. 4, page 11 except that the slot may be near the pole.

\[ s = \text{slots in series per phase.} \]
\[ K_s = \text{constant for number of slots per pole and phase.} \]
\[ l_e = \text{length of end connections in inches.} \]
\[ K_e = \text{proportionality constant for end connection path.} \]
\[ K_t = \text{constant for idle teeth of adjoining phase winding.} \]
\[ K_p = \text{constant for mutual induction between phases.} \]
IV - THE REACTANCE OF ARMATURE REACTION.

Definition.

On page 8 armature reaction is referred to as the force due to the armature current directly opposing the exciting field but later on the same page it is shown that the leakage flux due to the entire armature coil has a very direct effect upon the exciting field by distorting it and that its cause must be considered as a part of armature reaction because the generated E. M. F. is dependent upon the altered and distorted field. While this is a purely arbitrary distinction, in studying the design of the machine or its stable operation, it is a convenient distinction, for, as will be seen, it combines the effects of the magnetomotive-force of the total armature coil, greatly simplifying the problem of reaction. This conception of armature reaction will here be accepted. As shown in the Appendix the definition of armature reaction as given on page 8 must hold in the study of transient conditions, and the distorting field will then be considered as producing self-inductance. *

The Single Phase Armature.

Let us first consider a single phase winding. Let

* It is for this reason that the distorting field was so considered in developing the reactance of self-inductance on page 17.
t be the number of turns per pole on the armature or one half the number of effective conductors per pole and let \( i \) be the current at any instant. Then the magnetizing force at that instant, in ampere turns, is \( i \, t \). Since \( i \) varies throughout the cycle and since \( i \, t \) represents the total armature M. M. F. with respect to the given pole, this force will vary having zero for its minimum value and \( \sqrt{2} \, I \, t \) for its maximum value, where \( I \) is the effective current in the armature. While this pulsating force presents a large problem in itself, almost impossible of a satisfactory general analysis, a brief discussion will be of interest.

Considering unity power factor with reference to the generated E. M. F., when the coil is in the position A A (Figure 7) the E. M. F. and hence the current have their maximum values and the M. M. F. is at right angles to the coil and in such a direction as to make the trailing tip the stronger. In the positions, B B and C C, the current will be smaller but in the same direction in the coil and the M. M. F. will be correspondingly smaller but in the same direction with respect to the coil. Thus at A. A. the effect of the reaction is maximum and tends to
distort the exciting field, and at B B and C C its effect is less and tends both to distort and to react directly upon the exciting field, except that, if at B B the reaction is positive and decreases the exciting flux, then at C C is negative and increases the same. In the limiting position D D the armature current and hence its M. M. F. is zero and the field has its natural value.

Double Frequency of Single Phase Armature Reaction

It will thus be seen that, while the coil is passing through 180° from D D, its M. M. F. will reverse with reference to the exciting field causing the latter to pulsate at double the natural frequency of the machine. Were the condition one of zero power factor, lead or lag, the average value of the flux would be greater or less but the double frequency would still exist, due to having a complete cycle of pulsation for every half cycle of generated E. M. F.

For the condition of a sine wave of current, which may be approximated, the presence of this double frequency can be shown mathematically as follows:

Let \( a = E_0 \sin \theta \)
\[ i = I_0 \sin (\theta + a) \]

and \( t = \text{number of effective turns} \). Here \( \theta \) is the angular velocity or \( \omega t \) and \( a \) is the angle of lead of the current with respect to the nominal induced E. M. F. If the current
lags, a will be negative. The reaction also varies as \( \cos \theta \), being maximum when the coil is in the position where \( e = 0 \), while the distortion varies as \( \sin \theta \), in the same way. Thus reaction is
\[
 i_t \cos \theta = A_T R
\]
(1)
and distortion is
\[
i_t \sin \theta = A_T D
\]
(2)
Substituting for \( i \) in (1),
\[
 A_T R = I_0 t \sin (\theta + a) \cos \theta = \frac{I_0 t \sin a}{2} + \frac{1}{2} I_0 t \sin (2 \theta + a)
\]
(3)
Also substituting for \( i \) in (2)
\[
 A_T D = I_0 t \sin (\theta + a) \sin \theta = \frac{I_0 t \cos a}{2} - \frac{1}{2} I_0 t \cos (2 \theta + a)
\]
(4)
Equations (3) and (4) will be seen to contain only functions of \( 2 \theta \) and constants, showing the existence of the double frequency.

It is interesting to note that this double frequency cannot produce a second harmonic in the final E. M. F. for the action is the same at each pole and hence the final voltage will be symmetrical in its positive and negative values. It will probably contain instead a large third and smaller fifth and seventh harmonics.

The Polyphase Armature, Concentrated Winding.

The polyphase armature reaction is more simple because it has a constant value both in direction and in magnitude for steady conditions in the system, as can readily
be shown. As in the single phase calculation, let
\[ e_1 = E_{01} \sin \theta_1 \]
\[ i_1 = I_{01} \sin (\theta_1 + \alpha_1) \]
\[ e_2 = E_{02} \sin \theta_2 \]
\[ i_2 = I_{02} \sin (\theta_2 + \alpha_2) \]

etc. and let \( n \) = the phase index and \( t \) = the number of effective turns per pole and phase.

Consider balanced conditions of loading, then
\[ I_{01} = I_{02} = I_0, \quad E_{01} = E_{02} = E_0 \text{ and } \alpha = \alpha_1 = \alpha_2 = \alpha. \]

Developing and substituting as in single phase, letting \( \theta_1 = \theta \),
\[ ATR_1 = I_0 t \sin (\theta + \alpha) \cos \theta = \frac{I_0 t \sin \alpha}{2} + \frac{1}{2} I_0 t \sin(2\theta + \alpha) \]  
(1)

Since the second phase is like the first except in that its coil is displaced by an angle \( \frac{180^\circ}{n} \),
\[ \theta_2 = \theta + \frac{180^\circ}{n} \text{ and } \]
\[ ATR_2 = \frac{I_0 t \sin \alpha}{2} + \frac{1}{2} I_0 t \sin (2\theta + \frac{360^\circ}{n} + \alpha). \]  
(2)

Also
\[ ATR_n = \frac{I_0 t \sin \alpha}{2} + \frac{1}{2} I_0 t \sin (2\theta + \frac{360^\circ}{n} (n - 1) + \alpha), \]  
(3)

Let \( (2\theta + \alpha) = \beta \). Substituting this and adding together the reactions for all the phases,
\[ ATR = n \frac{I_0 t \sin \alpha}{2} + \frac{1}{2} I_0 t \left[ \sin \beta + \sin (\beta + \frac{360^\circ}{n}) + \cdots \right] \]
\[ + \sin (\beta + \frac{360^\circ}{n} (n - 1)) = n \frac{I_0 \cdot t \cdot \sin \alpha}{2} \quad (4) \]

Developing the expression for distortion in the same way,

\[ A_{TD} = n \frac{I_0 \cdot t \cdot \cos \alpha}{2} \quad (5) \]

Substituting \( \sqrt{2} I \) for \( I_0 \),

\[ A_{TR} = \frac{n}{2} \sqrt{2} \cdot I \cdot t \cdot \sin \alpha , \quad \text{and} \]

\[ A_{TD} = \frac{n}{2} \sqrt{2} \cdot I \cdot t \cdot \cos \alpha , \quad (7) \]

while the resultant or total magnetizing force of the armature is

\[ A_T = A_{TR}^2 + A_{TD}^2 = \frac{n}{2} \sqrt{2} \cdot I \cdot t . \quad (8) \]

It will be noticed that, for a given power factor and effective current, the armature reaction, the distorting force and the total magnetizing force are constant in direction and magnitude. For unity power factor the direct reaction is zero, while for zero power factor there is no distorting influence, the effect of the power factor, in all cases, being represented by the function of \( \alpha \) in (6) and (7). The magnetizing force in the two phase armature is the same as the maximum value in the single phase machine, but is constant. For other polyphase machines the simple constant \( \frac{n}{2} \) is easily applied.
Effect of Distribution of Winding.

The coils making up the winding of the armature per pole and phase may be placed in the same plane, i.e. the winding may be concentrated. Since the M. M. F. is perpendicular to the plane of the coil producing it, in this case the several magnetizing forces will add up directly and no correction need be applied in using the formulae as developed. In most machines, however, the winding is distributed and the coils do not produce magnetizing forces in phase. If the winding subtends 180 electrical degrees, the M. M. F. due to the last coil will be 180° out of phase with that due to the first, and the phase relation in any other coils will be determined by the angle subtended. Thus, as shown in Fig. 8, the relation existing between the vector and numerical sums of the several magneto-motive forces in the coils subtending the angle $\varphi'$ is practically that between the chord and arc subtending the same angle. This is readily shown to be $2 \varphi' \sin \frac{\varphi'}{2}$ divided by $\frac{2 \pi r \varphi'}{360}$ or $\frac{360}{\varphi' \pi} \sin \frac{\varphi'}{2}$

Correction for this condition of distribution may then be made by multiplying by the above expression, which may be called $k_d$, the distribution factor, and
\[ AT = k_d \frac{n}{2} \sqrt{2} I t, \]
\[ AT_R = k_d \frac{n}{2} \sqrt{2} I t \sin \alpha, \]
\[ AT_D = k_d \frac{n}{2} \sqrt{2} I t \cos \alpha. \]

The Reactance of Armature Reactions and Regulation by the Optimistic Method

If the reactance in ohms and the drop it produces are desired they may be determined, almost by inspection, from the magnetization curve of the machine. The ampere turns producing distortion are in quadrature with the impressed field turns and therefore cannot be in quadrature with those producing the resultant field. Their direct effect, however, is not great and may be approximated by considering the cross ampere turns as acting at right angles to the other magnetizing forces. Thus, in Fig. 9 (next page) we have the resultant ampere turns \( F \) and the armature reaction \( AT_R \) in phase and the distorting ampere turns \( AT_D \) in quadrature, the total magnetizing force, \( F_0 \), being that impressed by the exciting field. Fig. 10 (next page) shows a magnetization curve where \( F_0 \) at no load produces \( e_o \). Under load the resultant force is \( F \) and produces \( e_i \). The E.M.F. of armature reaction is \( e_a \) where
\[ e_a = e_0 - e_i. \]

If \( x_r \) is the equivalent reactance of armature reaction

\[ x_r = \frac{e_a}{I_a}. \]

It should be noted that \( e_0 \) and \( e_i \) are here assumed to be in phase, an assumption not warranted by fact.

The above principle is the one made use of in determining the regulation of alternators by the optimistic method, so-named because the approximations give too good a value of regulation in the result.

**Fig. 9.**

**Fig. 10.**

Regulation by the Pessimistic Method

Since the alternator is not a shunt machine, it need not be worked at a high density to insure good regulation, a large air gap usually being made to perform that duty while the losses are kept down by using densities
at or below the knee of the saturation curve. Thus, at least within the normal working limits of the machine, a reasonably constant reluctance may be assumed for the various densities and the electro-motive forces may be substituted directly for the magneto-motive forces in the previous discussion. This is the principle made use of in the so-called pessimistic method for determining regulation.

While neither method just described gives absolutely accurate results, the latter or pessimistic is the more satisfactory and upon it are based the tests for regulation as made by manufacturers and engineers. This is particularly fortunate for without a straight line saturation curve, mathematics would apply much less readily to the problem. As it is, a simple vector and complex quantity analysis is possible giving reasonably accurate results, especially when the machine has definite poles so that a large reluctance opposes the distorting turns.

For very accurate results it is necessary to consider the magnetic and electric forces more nearly in their exact relation.
V--ANALYSIS BY VECTORS AND THE COMPLEX NUMBER.

Preliminary Discussion

In this study, the terminal E. M. F. will be considered as the reference vector. The induced voltage will be that actually induced in the armature, being the vector sum of the resistance and self-inductive reactance drops and the terminal E. M. F. The current (where this assists in the development) will also be considered as separated into energy and quadrature components. Thus the induced voltage is

\[ e_1 = e + i_1 r + i_2 x - j (i_1 x - i_2 r) \]  \hspace{1cm} (1)

where

- \( e_1 \) = induced voltage,
- \( e \) = terminal voltage,
- \( i_1 \) = energy component of current,
- \( i_2 \) = quadrature component of current,
- \( r \) = resistance of armature,
- \( x \) = armature reactance of self-inductance.

Care must be taken to choose the correct value of \( x \) in substituting, due to its variation for different positions of the conductor with reference to the poles.

The current \( i_2 \) may have a positive or a negative value. In Fig. 11 it is shown positive and \( \alpha_1 \) is an angle of lag.
In Fig. 12 the relation between $e_1$, $e$ and $I$ is assumed as shown in Fig. 11. The resultant M. M. F. of field and armature windings, $F$, produces a flux $\phi$ in phase with it, both being $90^\circ$ ahead of the induced voltage. As suggested, the components of $F$ are the magneto-motive-forces of the field and armature, and these may be represented directly in the diagram if they have proportional shares in producing the flux. This is true where the magnetic reluctance is constant in all possible paths for the field, a condition found only in those machines having a round rotor type of field pole construction. Here, also, the self-inductance is constant at all points on the armature. Considering this type of machine, the armature magnetizing force is $k_d \frac{n}{2} \sqrt{2} I t$ (See page 34) or $M I$ where $M = k_d \frac{n}{2} \sqrt{2} t$, and this force is in phase with $I$. The vector difference between $F$ and $M I$ is $F_o$, the force to be supplied by the exciting field.

Were $F_o$ permitted to act unhindered it would produce an E. M. F. $e_o$, frequently termed the nominal induced E. M. F. and the vector difference between $e_o$ and $e_1$ is the drop due to the reactance of armature reaction, before referred to as $e_a$. The angle $\alpha$, used in developing the expressions for armature reaction and distortion, is also shown in Fig. 12 as that between $I$ and $e_o$; it will be noticed that the distorting action due to the armature current is $M I \cos \alpha$ and the direct reaction is $M I \sin \alpha$, check-
ing previous equations in this regard. This is more clearly demonstrated by dividing the force $F_0 F$ (M I) into its component parts, $F_0 G$ making $180^\circ$ with $F_0$ and $G F$ in quadrature to $F_0$.

Analysis, Round Rotor Type of Field

A rapid method of analyzing this problem is by the use of the complex number. In studying the armature M. M. F. it will be assumed that $\alpha = \alpha_1$, a not unreasonable approximation when we recall that, even in a very poor machine, the vector difference between $e_o$ and $e$ in percent of $e$, or the percent regulation, is not greater than .05, and the error would be a small percent of this value. Thus, since

$$I = i_1 + j i_2 = I \cos \alpha + j I \sin \alpha, \quad (2)$$

$$A T_R = k_d \frac{n}{2} \sqrt{2} I t \sin \alpha = M I \sin \alpha, \quad (3)$$

and

$$A T_D = k_d \frac{n}{2} \sqrt{2} I t \cos \alpha = M I \cos \alpha. \quad (4)$$

By substitution

$$A T_R = j M i_2 \quad (5)$$

and

$$A T_D = M i_1 \quad (6)$$

also

$$A T = M i_1 + j M i_2 \quad (7)$$

Let $e_i = j n F \quad (8)$
Then, since the saturation curve has been assumed to be a straight line within the working limits,

\[ j n M I = j n M I_1 - n M I_2 \]  \hspace{1cm} (9)

may be considered to be the respective voltages due to armature ampere-turns, or

\[ j n M I = e_a \]  \hspace{1cm} (10)

Since

\[ F = F_0 + M I \]

\[ e_i = e_0 + e_a \]

or

\[ e_o = e_i - e_a \]

Thus, substituting,

\[ e_o = e + i_1 r + i_2 x + n M i_2 - j (i_1 x - i_2 r + n M i_1) \]  \hspace{1cm} (11)

\[ = e + i_1 r + i_2 (x + n M) - j \left[ i_1 (x + n M) - i_2 r \right] \]

\[ = e + i_1 r + i_2 x_o - j (i_1 x_o - i_2 r) \]  \hspace{1cm} (12)

where

\[ x_o = x + n M. \]  \hspace{1cm} (13)

Eq. 12 is of the ordinary form for the total E.M.F. impressed upon a simple series circuit containing resistance and reactance. The reactance \( x_o \) as here given is the total equivalent reactance in the circuit and is therefore the synchronous reactance. The full-load no-load regulation is readily obtained as follows:
Analysis, Definite Pole Machine

In the application of the method here evolved to the definite pole machine, two corrections must be made, first in the choice of \( x \) and second in the constants determining the effect of armature distortion. Methods for determining the reactance of self-inductance are given in detail in previous pages. The use of the complex current makes it necessary to calculate only two values of reactance, that between the poles for \( i_2 \) and that under the poles for \( i_1 \), that between the poles being \( x_2 \) and that under the poles \( x_1 \). For further information regarding these values, refer to their derivation.

As suggested before, the counter E. M. F.'s due to armature reaction and distortion are proportional thereto only when there is a constant reluctance about the armature. When this is not true, as is the case in a definite pole machine, the reluctance for the distorting field being greater, the magnetizing force producing this field must be multiplied by another constant to preserve the desired proportionality. Thus

\[
\Delta T_D = k_o k_d \frac{n}{2} \sqrt{2} i_1 t = M_o i_1
\]

(15)

where \( M_o = k_o M \).
Now the voltage due to distortion is

\[ j n M_0 i_1 \]

and the total voltage of armature reaction is

\[ e_a = j n M_0 i_1 - n M i_2 \]  \hspace{1cm} (16)

Substituting these corrections into the expression for \( e_o \) (Eq. 11)

\[ e_o = e + i_1 r + i_2 x_2 + n M i_2 - j \left( i_1 x_1 - i_2 r + n M_0 i_1 \right) \]

\[ = e + i_1 r + i_2 (x_2 + n M) - j \left[ i_1 (x_1 + n M_0 ) - i_2 r \right] \]  \hspace{1cm} (17)

and the regulation can be obtained as in Eq. 14. In Eq. 17, the value \( x_1 \) is frequently assumed to be \( 1.5 x_2 \) and the value of \( M_0 \) to be \( .5 M \), which constants are found to give fairly accurate results in average machines.
VI--APPLICATION: TEST OF A POLYPHASE ALTERNATOR

General Discussion

The standard open and short circuit tests for synchronous impedance, regulation, etc. are found well explained in texts and handbooks. For this reason, brief discussions will here be given only for those particular tests having as their object the study of the impedance itself or of the separation of the impedance into its components. Such data and curves as are given have been obtained from a 7.5 K.W. definite pole alternator in the Electrical Laboratory at the University of Illinois. Detailed information concerning the machine as well as the methods and results in some of the tests will be found in "A Study of the Reactance of an Alternator" by Harshman and Smith, and in "A Study of Armature Inductance of Alternators" by Sawyer and Weeks, which theses are available at the University Library. The machine was connected polyphase for tests involving armature reaction.

Stationary Impedance Test

Poles Removed or Short-Circuited

Among the standard tests which can be applied to obtain a check upon design calculations is that for stationary impedance. A series of curves are given in Plate I showing the mean results from a large number of tests. These were made with various amounts of distribution
Group I. 6 Coils - 12 slots p.p. (3 poles) (100% Dist.)

Group 2. 3 Coils - 6 slots p.p. (3 poles) (50% Dist.)

Group 3. 1 Coil - 2 slots p.p. (3 poles) (17% Dist.)

Plate I

Showing Slot and Armature Inductance for Various Positions of the Armature Coil.
of armature winding, from 2 slots per pole to 12 slots per pole, the latter being the total number in complete distribution, single phase. For convenience in comparison the inductance was reduced to inductance per slot in milhenries. The angular displacements used as abscissas were taken in each case from a permanent point on the armature, it being impossible to locate the coils under test. Since the winding was divided into sections from one end, the relative position of the curves to the poles are necessarily shifted in proportion to half the angles subtended, making 45° between Groups 1 and 2 and 30° between Groups 2 and 3.

Relative Inductance Between and Under the Poles

The tests with field poles removed naturally give the inductance between poles. With the field short-circuited or excited, which means short-circuited through the exciter armature, the inductance between poles is essentially the same except where the winding is so distributed as to have some turns under the poles whatever its position. The proportionality factor between the inductance under the poles and that between them as shown in Table II, is, for twelve active slots per pole, 1.75, (if .45 be considered the inductance between poles, the factor is 2.35) for six active slots, 2.8, and for two active slots, 2.85. In light of the fact that the customary factor here used is 1.5, these results are interesting. They are easily explained however in that the machine is
small in diameter and has a small air gap, both conditions
tending to exaggerate the increase of flux under the poles;
and more largely in that the effect of armature distortion
here appears in the inductance. It is an interesting co-
incidence that a number of short circuit tests performed up-
on this machine verified the values of reactance of self-
inductance here obtained, also that when conditions stated
above were considered, the design calculations approximated
these results very closely. At the same time the true

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<th>Slots Per Pole</th>
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<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>A Slot Inductance for poles Removed</td>
<td>.45</td>
<td>.45</td>
<td>.28</td>
</tr>
<tr>
<td>B Field Short-Circuited</td>
<td>under poles</td>
<td>1.06</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>between poles</td>
<td>.61</td>
<td>.48</td>
</tr>
<tr>
<td>C Field Excited</td>
<td>under poles</td>
<td>1.00</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>between poles</td>
<td>.60</td>
<td>.45</td>
</tr>
<tr>
<td>D Field Open-Circuited</td>
<td>under poles</td>
<td>1.10</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>between poles</td>
<td>1.25</td>
<td>1.15</td>
</tr>
<tr>
<td>L under poles in B.</td>
<td>1.75</td>
<td>2.8</td>
<td>2.85</td>
</tr>
<tr>
<td>L between poles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II, Inductance per slot in milhenries from stationary impedance tests.

inductance is more accurately obtained when the poles are removed.
The following constants have been given by designers as factors for the number of slots per pole and phase—

<table>
<thead>
<tr>
<th>Slots p.p. &amp; p:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
<td>1.85</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor proposed by designers</td>
<td>1.3</td>
<td>1.5</td>
<td>1.7</td>
<td>1.85</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table III, Inductance Factors for More Than One Slot per Pole and Phase.

These results are particularly gratifying when the conditions of the tests and the character of the machine are considered. The values were checked both for single phase and for two phase—in the latter case six slots per pole and phase being the maximum available.

* See Harshman and Smith, p.13.
Poles in Machine, Open-Circuited

The curves obtained for fields unexcited are not interesting except in the application here possible of principles discussed in the theoretical part of this thesis. In Groups 2 and 3, with somewhat concentrated windings, it is evident that the interlinkages when the coils are under the pole have greater effect than the flux threading the poles and yoke when the coils are in the mid position. The following data is taken from Table I:

<table>
<thead>
<tr>
<th>Group</th>
<th>Between poles</th>
<th>Under poles</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25</td>
<td>1.10</td>
<td>1.14</td>
</tr>
<tr>
<td>2</td>
<td>1.15</td>
<td>1.38</td>
<td>.83</td>
</tr>
<tr>
<td>3</td>
<td>.68</td>
<td>.80</td>
<td>.85</td>
</tr>
</tbody>
</table>

The fact that the same ratio holds in these two groups indicates that whatever Group 2 gains by having its end turns under the poles--for the pole pitch is 80 percent--it loses by having the large flux which threads the poles and yoke cut through and not pass around those end turns. In Group 1, however, the factor is much larger for with a completely distributed winding the advantage in having the center of the coils under the poles would disappear while that due to the flux through the poles and yoke would still favor the coil when between the poles.
A saturation curve and a short circuit or synchronous impedance curve provide a ready means for checking data on inductance. The value of the synchronous impedance is obtained directly from the test. To find the components it must be assumed that the current lags 90° behind the voltage in the short circuit test, an assumption readily granted in most cases. The test should also be made with poly-phase connections, if possible, to insure a constant reaction. Thus the entire M. M. F. of the armature is available as armature reaction and may be subtracted from the field M. M.F. for the short-circuited condition to obtain the resultant M. M. F., producing the resultant flux. Application of this value to the saturation curve gives the induced E. M. F., that E. M. F. actually consumed by the passage of the current through the resistance and reactance of self inductance of the armature. Knowing the resistance, the reactance is readily obtained and supplying these terms into the expression for synchronous impedance,

\[ Z_s = \sqrt{r_a^2 + (x + n M)^2} \]

the value of the reactance of armature reaction, \( n M \), may be found.

The values of inductance per slot as obtained by this method * are shown in Table IV with those obtained by

* See Harshman and Smith, p. 17.
the stationary impedance test with poles removed, both cases giving the inductance between the poles. In determining the values from the synchronous impedance test,

<table>
<thead>
<tr>
<th>Slots per Pole &amp; Phase</th>
<th>Inductance per Slot in m. h. Sta. Impedance Method</th>
<th>Syn. Impedance Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.28</td>
<td>.235</td>
</tr>
<tr>
<td>4</td>
<td>.38</td>
<td>.290</td>
</tr>
<tr>
<td>6</td>
<td>.45</td>
<td>.315</td>
</tr>
</tbody>
</table>

Table IV, Inductance Calculated from Stationary and Synchronous Impedance Tests.

the designer's factors for slots per pole, and phase, as given in Table III, page 48 were used, and the uniformity of the results indicates their accuracy. Still all of them are low when compared with the stationary impedance values, this probably being due to an error in using the total current in the armature reaction instead of I sin \( \alpha \), the resistance in this machine being nearly half as great as the self-inductive impedance. The resultant M. M. F. in the test was less than a fourth of the total, hence a small mistake in the reaction would have a large effect upon the results obtained. The results indicate that more careful consideration should be given the resistance and phase relations when this test is applied, if even approximately accurate results are desired.
Calculations from Design Constants

In the calculation from design constants, following roughly the method outlined in this thesis, very consistent and gratifying results were obtained. Sawyer and Weeks present the details in a clear manner with all dimensions and data for this alternator when considered as a two phase machine with six slots per pole and phase (all slots active). Without considering the distorting field and using the factor 1.7 to find the inductance per slot from that for six slots, its value was determined as .213 milhenry. This is less than half of that obtained from the stationary impedance test, but, by considering the effect of armature distortion as was done in that case, it was increased to .443, checking closely the value of .45. Calculations were also made to determine the value under the pole and 1.05 milhenries was found, as compared with 1.06 obtained from stationary impedance, where that test was made with the center of the coil under the short-circuited field. The methods followed were those outlined under "The Reactance of Self-Inductance". Further information regarding the specific data and calculations may be had by referring to the thesis by Sawyer and Weeks.

Summary of Tests

As a summary of the tests, it seems that the
calculation is a safe method for determining the inductance. It should, however, be supplemented by a stationary impedance test with poles removed for the reactance between poles and with the coil under the short-circuited or excited field for the value under the poles. This precaution will remove the possibility, or probability if the student has not had experience in the features of design here presented, of omitting some very important part of the problem. After the values have been determined, care must be taken to use them according to the conditions of operation required, as explained at length in the theoretical discussion.
Appendix---A Suggestion

The matter of the proper design of alternating current apparatus to obtain a high or low synchronous impedance, determined in its component parts, becomes particularly important where there is a possibility of short circuit in an alternator. Here a large armature current at once flows, causing a correspondingly large armature reaction and self-induced or leakage flux. The armature reaction tends to distort and reduce the exciting field while the leakage flux induces an E. M. F. which directly helps to keep the current down.

It is commonly known that, when current starts to flow in a single coil, a magnetic field is established at the same rate as that with which the current starts. When this rate is greatest, therefore, the change in flux is greatest and a large E. M. F. is induced which opposes the flow of the current. It is also known that, should this flux thread another coil, which is part of a closed circuit, the value of the flux at any instant will depend upon the resultant H. M. F. of the two coils, in which case both currents may have a great rate of change and increase to high values without inducing an appreciable E. M. F. in either coil.

Thus in the alternator armature under short circuit the current will be held in check only by the resistance and by the E. M. F. of self induction, that represented by the
reactance of self-inductance. The flux due to the armature reaction threads through the field coils and transfers to them much of the energy which should react against the increase of current in the armature but which now is dissipated in heat and in the magnetic field in the exciter.

In a definite pole machine it will be seen that the leakage flux which serves merely to distort the exciting field and does not cut the field turns cannot be classed here as the effect of armature reaction, but must be included as a part of the flux of self-induction.