Feng

Stresses in railway structures on curves
STRESSES IN RAILWAY STRUCTURES ON CURVES

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY KAI MIN KAY FENG, B.S.,
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CHAPTER I

INTRODUCTION

Art. 1. Preliminary.

The stresses in a railway bridge when the track is on a curve were first investigated by Mr. Ward Baldwin, who, in a paper entitled "Stresses in Railway Bridges on Curves" and published in the Transactions of the American Society of Civil Engineers, Vol. XXV, November, 1891, described the effect of the curvature of the track and the centrifugal force upon the live load stresses in a through trussed bridge. Mr. Baldwin confined his investigation to the truss spans of through type only and did not make it complete to include the plate girder spans which are frequently built in series as in viaducts. In his paper was given a method of finding the location of a bridge on a curved track with reference to the track so as to make equal the stresses in certain members in the outer and inner trusses: this method, the writer believes, is open to question.

The purpose of this thesis is to make a complete investigation of the stresses in plate girder and truss bridges on curves due to moving train and the accompanying centrifugal force, and, whenever possible, to find a method of locating the bridge with respect to the track in order to have equal stresses to be caused in the outer and inner girders or trusses. The method of equalizing stresses can best be applied to deck plate girder spans, where no clearance is required and hence the spacing of girders will not be affected. While in the case of through plate girder spans, though the same arrangement for the equalization of stresses could be equally secured, the requirement of clearance would increase the distance
between girders which is undesirable from an economical standpoint. In truss spans, as it is impossible to make the stresses in all the corresponding members in both trusses equal, no attempt will be made here to locate the bridge so as to equalize the stresses in certain particular members only.

Art. 2. Centrifugal Force of Moving Train.

When a train is moving along a curved track, it exerts a centrifugal force which acts away from the center of curvature and in the direction perpendicular to the tangent at the point where the centrifugal force is considered. If $F$ represents this centrifugal force, then from Mechanics, it is found that

$$F = \frac{v^2}{g} \frac{W}{r}$$

where $W$ is the weight of the moving body (train); $v$, its speed in feet per second; $g$, the acceleration of gravity in $\text{feet per second}^2$; and $r$, the radius of curvature in feet.

In the case of a bridge on a curve, this centrifugal force will cause two equal but opposite vertical forces on the main girders or trusses and a horizontal force on the lateral system. In Fig. 1 is shown a section of a bridge, AB and CD being the outer and inner girders or trusses respectively. Assume that the centrifugal force $F$ is acting through the center of gravity of the train at a height of $h$ above the laterals. Let the spacing of the truss or girders be $b$, and let $r_1$ and $r_2$ be the positive and negative reactions in the outer and inner trusses respectively. Since the overturning moment $Fh$ must be balanced by the resisting couple $r_1b$ or $r_2b$, it follows
that
\[ r_1 b = r_2 b = F h \]
\[ r_1 = r_2 = \frac{F h}{b} \]

The horizontal force \( F \) is transferred to the bottom laterals and is resisted by that system. Hence
\[ H = F \]

The centrifugal force \( F \) always acts normally to the curve at every point; but in the analysis which follows, it will be assumed that \( F \) acts in the direction normal to the plane of the main vertical truss or girder at all points along the curve, though this condition is true only at the middle of the span. This assumption of course will lead to some discrepancy in the results, but Mr. Baldwin in his paper mentioned above has shown that the error thus made in a bridge of 300-foot span on a 10-degree curve is only about one per cent, which is negligible.

Since the centrifugal force \( F \) is in direct proportion to
the vertical moving load \( W \), we may write

\[
F = q W
\]

Where

\[
q = \frac{v^2}{g r}
\]

According to the specifications for steel railway bridges of the American Railway Engineering Association, the speed in miles per hour of a train running on a curve is specified to be

\[
v = 60 - 2 \frac{1}{2} D
\]

where \( D \) is the degree of curve.

Based on this provision and using \( g = 32.2 \text{ ft./sec.}^2 \), the values of \( q = \frac{v^2}{g r} \) for curves of \( 1^\circ \) to \( 10^\circ \) are calculated and are shown in the table in Art. 5, the radii of the curves being found from

\[
r = \frac{50}{\sin \frac{1}{2} D}
\]

Art. 3. Super-elevation of Track.

The centrifugal force of a moving train tends to cause the train moving on a tangent instead of moving on the curved track, and hence there must be a centripetal force to keep that train moving on its path. If the two rails of the track were laid on the same level transversely, this centripetal force could only be furnished by the pressure of the wheel-flanges against the outer rail. This condition is very objectionable and undesirable. According to the present practice, the outer rail of the track is so elevated above the inner that the reaction of the rails against the wheels of the train shall contain a horizontal component equal to the required centripetal force.

In Fig. 2, let \( O \) be the center of gravity of the train. If \( \text{OA} \) represents the centrifugal force \( F \), and \( \text{OC} \), the weight of
the train; and 0 B, resultant of 0 A and 0 C, is perpendicular to the plane of the tops of rails and passes through the center line of the track; then BO will be the reaction of the rails against the wheels of the train, and BC, the required centripetal force

![Diagram](image)

**Fig. 2.**

( = O A = F). PM is the distance between the centers of rails and MN is the required super-elevation of the outer rail.

From similar triangles OBC and PMN,

\[ \frac{OC}{BC} = \frac{PN}{MN} \]

\[ MN = \frac{BC \cdot PN}{OC} \]

Now, PN is the horizontal distance between the centers of the rails and is slightly less than PM. Hence approximately we may write

\[ MN = \frac{BC \cdot PM}{OC} \]
Let \( s = M N = \) super-elevation of outer rail.

And \( t = PM = \) distance between rail centers.

Since \( BC = OA = F = \frac{v^2}{g} W \)

\( OC = W \)

\[ s = t \frac{v^2}{g} r \]

For standard gauge of \( 4' - 8\frac{1}{2}'' \) and a 100-lb. rail, the distance between rail centers is

\[ t = 4' - 8\frac{1}{2}'' + 2\frac{3}{4}'' = 4.709 + .229 = 4.938 \text{ feet} \]

For convenient use in the following analysis, \( t \) will be taken as 4.94 feet. The values of \( s = t \frac{v^2}{g} r \) for curve of different degrees are shown in the table in Art. 5.

**Art. 4. Center of Gravity of Train.**

The vertical line through the center of gravity of a train moving on a tangent always intersects the center line of the track; but when the train is moving on a curve, then, on account of the super-elevation of the outer rail, this vertical line will no longer meet the center line of the track, but instead fall within it. In the analysis that follows, it will be necessary to find the curve described by the center of gravity of the train, as this is the line through which the load from the train is applied. This curve of center of gravity will have the same radius as that of the center line of the track and will be at a certain distance from the center line of track at the middle of span which is to be determined.

In Fig. 3 is shown a section of the track at the middle of span. Let \( O \) be the center of gravity of the train; \( OA \), the distance from \( O \) to the plane of tops of rails; and \( OB \), the line
of application of the load of train. Then $AB$ is the horizontal distance between the center line of the track and the center of gravity of the train. $PM$ is the distance $t$ between centers of rails and $MN$ is the super-elevation $s$ of the outer rail.

From the similar triangles $OAB$ and $PMN$, we have

$$OA : AB = PM : MN$$

$$AB = OA \cdot \frac{MN}{PM} = OA \cdot \frac{s}{t}$$

Let $c = AB =$ horizontal distance between the center line of track and the center of gravity of train at mid-span and $f = OA =$ perpendicular distance of the center of gravity of train to the plane of tops of rails

Then

$$c = f \cdot \frac{s}{t}$$

If the outer rail of the track is elevated in accordance
with the formula given in the last article, then we have

\[ s = t \frac{v^2}{g r} \]

Substituting this value of \( s \) in the formula for \( c \), then

\[ c = f \frac{v^2}{g r} \]

The distance \( f \) is usually assumed to be 5 feet. The values of \( c \) are shown in the table in the following article.

Art. 5. Values of \( q = \frac{v^2}{g r} \), \( s = t \frac{v^2}{g r} \), \( c = f \frac{v^2}{g r} \)

The formulae for \( F = q \frac{w}{g r} \), \( s = t \frac{v^2}{g r} \), and \( c = f \frac{v^2}{g r} \)

have been shown respectively in the last three articles. For the sake of ready use, the following table has been prepared giving values of \( q \), \( s \), \( c \) for curves of different degrees varying from \( 1^\circ \) to \( 10^\circ \). The radius \( r \) in feet is found from \( r = \frac{50}{\sin \frac{1}{2} D} \) and the speed \( V \) in miles per hour is found from \( V = 60 - 2 \frac{1}{2} D \).
TABLE A

Values of \( q = \frac{v^2}{g r} \), \( s = t \frac{v^2}{g r} \), \( c = f \frac{v^2}{g r} \).

<table>
<thead>
<tr>
<th>D</th>
<th>Radius ( r ) in ft.</th>
<th>Speed</th>
<th>Speed</th>
<th>( q = \frac{v^2}{g r} )</th>
<th>( t )</th>
<th>( s = t \frac{v^2}{g r} )</th>
<th>( f )</th>
<th>( c = f \frac{v^2}{g r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1°</td>
<td>5729.65</td>
<td>57.5</td>
<td>84.33</td>
<td>32.2</td>
<td>.0386</td>
<td>4.94</td>
<td>.1907</td>
<td>5</td>
</tr>
<tr>
<td>2°</td>
<td>2664.93</td>
<td>55.0</td>
<td>80.67</td>
<td>32.2</td>
<td>.0705</td>
<td>4.94</td>
<td>.3483</td>
<td>5</td>
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<tr>
<td>3°</td>
<td>1910.06</td>
<td>52.5</td>
<td>77.00</td>
<td>32.2</td>
<td>.0964</td>
<td>4.94</td>
<td>.4762</td>
<td>5</td>
</tr>
<tr>
<td>4°</td>
<td>1432.69</td>
<td>50.0</td>
<td>73.33</td>
<td>32.2</td>
<td>.1166</td>
<td>4.94</td>
<td>.5760</td>
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</tr>
<tr>
<td>5°</td>
<td>1146.28</td>
<td>47.5</td>
<td>69.67</td>
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<td>.1315</td>
<td>4.94</td>
<td>.6496</td>
<td>5</td>
</tr>
<tr>
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<td>45.0</td>
<td>66.00</td>
<td>32.2</td>
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<td>4.94</td>
<td>.6995</td>
<td>5</td>
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<td>42.5</td>
<td>62.33</td>
<td>32.2</td>
<td>.1473</td>
<td>4.94</td>
<td>.7277</td>
<td>5</td>
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<tr>
<td>8°</td>
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<td>40.0</td>
<td>58.67</td>
<td>32.2</td>
<td>.1491</td>
<td>4.94</td>
<td>.7366</td>
<td>5</td>
</tr>
<tr>
<td>9°</td>
<td>637.27</td>
<td>37.5</td>
<td>55.00</td>
<td>32.2</td>
<td>.1474</td>
<td>4.94</td>
<td>.7282</td>
<td>5</td>
</tr>
<tr>
<td>10°</td>
<td>521.67</td>
<td>35.0</td>
<td>51.33</td>
<td>32.2</td>
<td>.1569</td>
<td>4.94</td>
<td>.7751</td>
<td>5</td>
</tr>
</tbody>
</table>
CHAPTER II.

DECK PLATE GIRDER BRIDGES.

In deck plate girder bridges, the ties rest on the tops of the girders and therefore the load from the train is transmitted to the girders directly. In finding the stresses in such bridges on curves, an equivalent uniform load will be used. This equivalent uniform load for shear or for moment is determined from the shear or moment due to wheel loads as if the bridge were on straight track. For example, to obtain the equivalent uniform load \( w \) per unit length for end shear on a girder of span \( l \), the maximum end shear \( V \) due to wheel load is computed and then we have

\[
V = \frac{w}{2} l
\]

from which we obtain

\[
w = \frac{2V}{l}
\]

To obtain the equivalent uniform load \( w \) for maximum bending moment on a girder of span \( l \), first find the maximum bending moment \( M \) due to the wheel loads and then we have

\[
M = \frac{w}{8} l^2
\]

from which we obtain

\[
w = \frac{8M}{l^2}
\]

Art. 6. Equations for Shears and Moments.

In Fig. 4, \( MN \) and \( PQ \) are the outer and inner girders respectively of a deck plate bridge. The length of span is \( l \) and the spacing of girders is \( b \).
Fig. 4.

Let the dotted line $A^1C^1B^1$ represent the center line of the track and the full line $A C B$ represent the curve described by the center of gravity of the train moving along the track. If $D$ is the degree of curve of the track, then the radii of $A^1C^1B^1$ and $A C B$ are each given by

$$r = \frac{50}{\sin \frac{1}{2} D}$$

With $O$, the middle point of the chord $A B$, as the origin, and $A O B$ as the axis of $x$, then from Geometry, we find the equation of the curve $A C B$ is

$$y = m - r + \sqrt{r^2 - x^2}$$

where $m$ is the middle ordinate $o c$ of the arc $A C B$ and is equal to

$$r - \frac{1}{2} \sqrt{4r^2 - l^2}$$

Let $d = m + a = \text{distance from the point } C \text{ to the inner girder.}$

If $w = \text{the weight of the train (the equivalent uniform load) per unit length.}$
\[ dw = \text{differential weight of the train.} \]
\[ ds = \text{differential length of the curve.} \]

Then \[ dw = w \cdot ds \]

From Calculus, we know
\[ ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

Hence we have
\[ dw = w \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

Let the centrifugal force due to the moving load \( dw \) be represented by \( F \), then
\[ F = \frac{v^2}{g} \cdot dw = q \cdot dw \]

Where \( q = \frac{v^2}{g} \). The height of the center of gravity of the train above the top laterals will be represented by \( h \).

\[ (1) \text{ Equation for Shears.} \]

Take any point \((xy)\) on the curve \(A \, C \, B\). The section through this point of the bridge is shown in Fig. 5. The external forces acting are the centrifugal force \( q \cdot dw \) and the vertical force \( dw \). By the method of moments, the reaction on the outer girder \( MN \) due to these forces is found to be

\[ v_1 = \frac{a + y}{b} \cdot dw + \frac{h}{b} \cdot q \cdot dw = \frac{dw}{b} (a + qh + y) \quad \ldots \ldots \quad (1) \]

and the reaction on the inner girder \( PQ \) is

\[ v_2 = \frac{b - a - y}{b} \cdot dw - \frac{h}{b} \cdot q \cdot dw = \frac{dw}{b} (b - a - qh - y) \quad \ldots \ldots \quad (2) \]
Fig. 5.

The outer girder $MN$ and the inner girder $PQ$ with the forces $v_1$ and $v_2$ acting upon them respectively are shown in Fig. 6.

Fig. 6.

The maximum end shear in each of the girders will occur when the bridge is fully loaded. Then the maximum shear in the outer girder $MN$ will be

$$V_1 = \int_{0}^{l/2} \frac{1}{b_x} (a + qh + y) \, dx$$

and that in the inner girder will be

$$V_2 = \int_{0}^{l/2} \frac{1}{b_x} (b - a - qh - y) \, dx$$
We have already known in the last article that
\[ dw = w \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

And
\[ y = m - r + \sqrt{r^2 - x^2} \]

Substituting these values in equations (3) and (4) they become respectively
\[ V_1 = \int_0^\frac{l}{b} \frac{w}{d} \left( a + qh + m - r + \sqrt{r^2 - x^2} \right) \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \ldots \ldots (5) \]
\[ V_2 = \int_0^\frac{l}{b} \frac{w}{d} \left( b - a - qh - m + r - \sqrt{r^2 - x^2} \right) \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \ldots \ldots (6) \]

Since
\[ y = m - r + \sqrt{r^2 - x^2} \]
\[ \frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}} \]
\[ \sqrt{1 + \left( \frac{dy}{dx} \right)^2} = \frac{r}{\sqrt{r^2 - x^2}} \]

\[ V_1 = \int_0^\frac{l}{b} \frac{w}{d} \left( a + qh + m - r + \sqrt{r^2 - x^2} \right) \frac{r}{\sqrt{r^2 - x^2}} \, dx \]
\[ = \frac{w}{b} \int_0^\frac{l}{b} \left[ (a + qh + m - r) \frac{x}{\sqrt{r^2 - x^2}} + r \right] \, dx \]
\[ = \frac{wr}{b} \left[ (a + qh + m - r) \sin^{-1} \frac{x}{r} + x \right]_0^{\frac{l}{b}} \]
\[ = \frac{wr}{b} \left[ (a + qh + m - r) \sin^{-1} \frac{1}{2r} + \frac{1}{2} \right] \]

Put \( d = a + m \), then
\[ V_1 = \frac{wr}{b} \left[ (d + qh - r) \sin^{-1} \frac{1}{2r} + \frac{1}{2} \right] \ldots \ldots \ldots \ldots (A) \]

which is the equation for maximum shear in the outer girder.
Similarly, the maximum end shear in the inner girder Q is
\[ V_2 = \int_0^\frac{\pi}{2} \frac{w}{b} \left( b - a - qh - m + r - \sqrt{r^2 - x^2} \right) \frac{r}{\sqrt{r^2 - x^2}} \, dx \]
\[ = \frac{wr}{b} \int_0^\frac{\pi}{2} \left[ (b - a - qh - m + r) \sin^{-1} \frac{x}{r} - x \right] \, dx \]
\[ = \frac{wr}{b} \left[ (b - a - qh - m + r) \sin^{-1} \frac{1}{2r} - \frac{1}{2} \right] \]

Substituting \( a \) for \( a + m \), then we have
\[ V_2 = \frac{wr}{b} \left[ (b - a - qh + r) \sin^{-1} \frac{1}{2r} - \frac{1}{2} \right] \]...

which is the equation for maximum shear in the inner girder.

In considering the shear in the inner girder as represented in Equation (B) we have taken into account the negative force acting on the girder due to the centrifugal force; but when the train goes slowly or stands still, there will be little or no centrifugal force, and hence the term \( qh \) in Equation (B) may be omitted which will be in error on the side of safety. Hence we have another equation for the maximum shear in the inner girder as the following:
\[ V_1^1 = \frac{wr}{b} \left[ (b - a + r) \sin^{-1} \frac{1}{2r} - \frac{1}{2} \right] \]...

(2) Equations for Moment.

As has been found before the forces acting on the outer and the inner girders at any point along the girders due to the train load \( dw \) and the centrifugal force \( q \cdot dw \) are respectively
\[ v_1 = \frac{dw}{b} (a + qh + y) \] and \[ v_2 = \frac{dw}{b} (b - a - qh - y) \]
The girders with those forces acting thereon are again shown in Fig. 7.

The bending moment at the middle of the span of the outer girder $MN$ due to the force $v_1 = \frac{dw}{b} (a + qh + y)$ is

$$m_1 = \frac{dw}{b} (a + qh + y) \frac{l}{2} - x \cdot \frac{1}{2} = \frac{dw}{4b} (a + qh + y) (1 - 2x). \ldots (7)$$

Similarly, the bending moment at the middle of span of the inner girder $PQ$ due to the force $v_2 = \frac{dw}{b} (b - a - qh - y)$ is

$$m_2 = \frac{dw}{b} (b - a - qh - y) \frac{l}{2} - x \cdot \frac{1}{2} = \frac{dw}{4b} (b - a - qh - y) (1 - 2x). \ldots (8)$$

Now, the bending moment at the middle of each girder will have its greatest value when the bridge is fully loaded. Hence, we have the maximum moment in the outer girder

$$M_1 = 2 \int_0^{\frac{l}{2}} \frac{dw}{4b} (a + qh + y) (1 - 2x) \ldots \ldots \ldots \ldots \ldots \ldots (9)$$

and the maximum moment in the inner girder

$$M_2 = 2 \int_0^{\frac{l}{2}} \frac{dw}{4b} (b - a - qh - y) (1 - 2x) \ldots \ldots \ldots \ldots \ldots (10)$$

If we substitute $dw = w \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$ and $y = m - r + \sqrt{r^2 - x^2}$ in
the above equations (9) and (10), then they become respectively

\[ M_1 = \int_0^{\frac{L}{2}} \frac{w}{2b} \left( a + qh + m - r + \sqrt{r^2 - x^2} \right) (1 - 2x) \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \quad \ldots \quad (11) \]

and

\[ M_2 = \int_0^{\frac{L}{2}} \frac{w}{2b} \left( b - a - qh - m - r - \sqrt{r^2 - x^2} \right) (1 - 2x) \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \quad \ldots \quad (12) \]

Substituting again for \( \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \) the value \( \frac{r}{\sqrt{r^2 - x^2}} \) as found before and performing the integration, Equation (11) becomes

\[ M_1 = \int_0^{\frac{L}{2}} \frac{w}{2b} \left( a + qh + m - r + \sqrt{r^2 - x^2} \right) (1 - 2x) \frac{r}{\sqrt{r^2 - x^2}} \, dx \]

\[ = \frac{w}{2b} \int_0^{\frac{L}{2}} \left[ \left( a + qh + m - r \right) \frac{r(1 - 2x)}{\sqrt{r^2 - x^2}} + r(1 - 2x) \right] \, dx \]

\[ = \frac{wr}{2b} \int_0^{\frac{L}{2}} \left[ \left( a + qh + m - r \right) \left\{ \frac{1}{\sqrt{r^2 - x^2}} - \frac{2x}{\sqrt{r^2 - x^2}} \right\} - (1 - 2x) \right] \, dx \]

\[ = \frac{wr}{2b} \left[ (a + qh + m - r) \left\{ 1 \sin^{-1} \frac{x}{r} + 2 \sqrt{r^2 - x^2} \right\} + (lx - x^2) \right] \]

Putting \( a + m = d \), we have

\[ M_1 = \frac{w}{2b} \left[ (d + qh - r) \left\{ 1 \sin^{-1} \frac{1}{2r} + \sqrt{r^2 - 1^2} - 2r \right\} + \frac{1^2}{4} \right] \ldots \quad (D) \]

which is the equation for maximum moment in the outer girder.

The maximum bending moment in the inner girder is

\[ M_2 = \int_0^{\frac{L}{2}} \frac{w}{2b} \left( b - a - qh - m - r - \sqrt{r^2 - x^2} \right) (1 - 2x) \frac{r}{\sqrt{r^2 - x^2}} \, dx \]

\[ = \frac{w}{2b} \int_0^{\frac{L}{2}} \left[ \left( b - a - qh - m - r \right) \frac{r(1 - 2x)}{\sqrt{r^2 - x^2}} - r(1 - 2x) \right] \, dx \]

\[ = \frac{wr}{2b} \int_0^{\frac{L}{2}} \left[ \left( b - a - qh - m + r \right) \left\{ \frac{1}{\sqrt{r^2 - x^2}} - \frac{2x}{\sqrt{r^2 - x^2}} \right\} - (1 - 2x) \right] \, dx \]

\[ = \frac{wr}{2b} \left[ \left( b - a - qh - m + r \right) \left\{ 1 \sin^{-1} \frac{x}{r} + 2 \sqrt{r^2 - x^2} \right\} - (lx - x^2) \right] \]

\[ = \frac{wr}{2b} \left[ \left( b - a - qh - m + r \right) \left\{ 1 \sin^{-1} \frac{1}{2r} + \sqrt{r^2 - 1^2} - 2r \right\} - \frac{1^2}{4} \right] \]
Put \( d \) for \( a + m \), then

\[
M_2 = \frac{wr}{2b} \left[ (b-d-qh+r) \left\{ \sin^{-1} \frac{1}{2r} + \sqrt{4r^2-1} - 2r \right\} - \frac{1}{4} \right] . \quad (E)
\]

In Equation (E) if we drop the term \( qh \) for the condition of a train standing still or going with low speed, then we obtain another equation for maximum bending moment in the inner girder, namely,

\[
M_2^1 = \frac{wr}{2b} \left[ (b-d+r) \left\{ \sin^{-1} \frac{1}{2r} + \sqrt{4r^2-1} - 2r \right\} - \frac{1}{4} \right] . \quad (F)
\]

The six equations which have just been found for the maximum shears and moments may be summarized as follows:

Shear for outer girder

\[
V_1 = \frac{wr}{b} \left[ (d+qh-r) \sin^{-1} \frac{1}{2r} + \frac{1}{2} \right] . \quad \ldots \ldots \ldots \ldots \ldots \ldots . (A)
\]

Shear for inner girder

(1) \[
V_2 = \frac{wr}{b} \left[ (b-d-qh+r) \sin^{-1} \frac{1}{2r} - \frac{1}{2} \right] . \quad \ldots \ldots \ldots \ldots \ldots \ldots . (B)
\]

(2) \[
V_2^1 = \frac{wr}{b} \left[ (b-d+r) \sin^{-1} \frac{1}{2r} - \frac{1}{2} \right] . \quad \ldots \ldots \ldots \ldots \ldots \ldots . (C)
\]

Moment for outer girder

\[
M_1 = \frac{wr}{2b} \left[ (d+qh-r) \left\{ \sin^{-1} \frac{1}{2r} + \sqrt{4r^2-1} - 2r \right\} + \frac{1}{4} \right] . \quad \ldots . (D)
\]

Moment for inner girder

(1) \[
M_2 = \frac{wr}{2b} \left[ (b-d-qh+r) \left\{ \sin^{-1} \frac{1}{2r} + \sqrt{4r^2-1} - 2r \right\} - \frac{1}{4} \right] . \quad (E)
\]

(2) \[
M_2^1 = \frac{wr}{2b} \left[ (b-d+r) \left\{ \sin^{-1} \frac{1}{2r} + \sqrt{4r^2-1} - 2r \right\} - \frac{1}{4} \right] . \quad (F)
\]

Plate girder bridges are in general built for spans not exceeding 120 feet which length is so small in comparison with the radii of ordinary curves that the angle \( \sin^{-1} \frac{1}{2r} \) will practically be equal to \( \frac{1}{2r} \). Accordingly, by using \( \frac{1}{2r} \) for \( \sin^{-1} \frac{1}{2r} \) in the six equations shown above, we got another set of equations as the following:
\[ V_1 = \frac{w_1}{2b} (d + qh) \]  \hspace{1cm} (A^1) \\
\[ V_2 = \frac{w_1}{2b} (b - d - qh) \]  \hspace{1cm} (B^1) \\
\[ V_2 = \frac{w_1}{2b} (b - d) \]  \hspace{1cm} (C^1) \\
\[ M_1 = \frac{w_1}{2b} \left[ (d + qh - r) \left\{ \frac{1}{2r} + \sqrt{\frac{4r^2 - 1^2}{2}} - 2r \right\} + \frac{1}{4} \right] \]  \hspace{1cm} (D^1) \\
\[ M_2 = \frac{w_1}{2b} \left[ (b - d - qh + r) \left\{ \frac{1}{2r} + \sqrt{\frac{4r^2 - 1^2}{2}} - 2r \right\} - \frac{1}{4} \right] \]  \hspace{1cm} (E^1) \\
\[ M_2 = \frac{w_1}{2b} \left[ (b - d + r) \left\{ \frac{1}{2r} + \sqrt{\frac{4r^2 - 1^2}{2}} - 2r \right\} - \frac{1}{4} \right] \]  \hspace{1cm} (F^1) \\

(3) Lateral Shear and Moment.

So far what we have discussed relates to the maximum vertical shears and vertical bending moments due to the vertical load and the overturning effect of the centrifugal force only. Now, the centrifugal force besides producing the overturning effect also causes a horizontal force equal to the centrifugal force upon the girders. If the girders are provided with a lateral system, then this lateral system will release a greater part of this horizontal force; but for the worst case, we shall consider that the lateral system will not take any of the horizontal force, which will then be assumed to be equally carried by the two girders. The horizontal force acting on each girder in turn produces a lateral shear and a lateral moment which should be added respectively to those vertical shears and moments found from Equations (A)-(F).

The centrifugal force \( F \) is directly proportional to the vertical load \( W \), being equal to \( \frac{v^2}{2r} W = qW \). Hence when the vertical load consists of a series of wheel-loads, the centrifugal force will
also be a series of wheel-loads, but have a different value depending upon \( q = \frac{v^2}{gr} \). From these relations, it is seen that the maximum lateral shear will be equal to \( q \) times the maximum vertical shear on the girder for a straight track and the maximum lateral moment will be equal to \( q \) times the maximum vertical moment. Hence, if \( V_v \) and \( M_v \) denote the maximum vertical shear and moment respectively on a girder on a tangent, the maximum lateral shear and moment will respectively be

\[
V_h = q \cdot V_v \\
M_h = q \cdot M_v
\]

Art. 7. Investigation of Customary Arrangement.

Owing to the lack of an adequate method to be used which would so determine the location of the two girders of a bridge that equal stresses would be developed in both girders, the present practice has used a practical rule rather than a theoretical one. The practical method generally used nowadays is to locate the two girders so that the center line of the bridge will bisect the middle ordinate of the curve of the center line of track between the ends of the span.

In Fig. 8, \( MN \) and \( PQ \) are the outer and inner girders respectively of a bridge, the center line of which is \( XY \). \( ACB \) is the center line of track and \( CD = m \) is its middle ordinate. According to the present practice, the girders are so arranged that \( CD \) shall be bisected by \( XY \).
In order to investigate fully the stresses in the girders in such an arrangement, we shall consider the following two examples, one being a 60-foot span bridge on a $3^\circ$ curve, and the other, an 80-foot span bridge on a $4^\circ$ curve.

Example 1. A 60-ft. span bridge on a $3^\circ$ curve.

Here we have been given

Length of span, $l = 60' - 0''$

Degree of curve, $D = 3^\circ$

Let it be also given that

Spacing of girders, $b = 6' - 6''$.

Height of center of gravity of train above top laterals, $h = 6' - 6''$

From Table A in Art. 5, we find that

Radius of curve, $r = 1910.08'$

Centrifugal force coefficient, $q = .0964$.

Super-elevation of outer rail, $s = .4762'$

Distance between center line of track and curve of center of gravity of train at mid-span, $c = .4820'$.

The middle ordinate of the center line of track is equal to
\[ m = r - \frac{1}{2} \sqrt{4r^2 - l^2} \]

Now,
\[ \sqrt{4r^2 - l^2} = \sqrt{4(1910.08)^2 - (60)^2} = \sqrt{14,590,022.4256} = 3819.6888 \]

\[ \therefore m = 1910.08 - \frac{1}{2} \times 3819.6888 = 2.256' \]

**Fig. 9**

In the figure shown, \( A\) \( C\) \( B\) is the center line of track and \( A^1C^1B^1\) is the curve traced by the center of gravity of the train. The distance \( d\) of the middle point \( C^1\) of \( A^1C^1B^1\) to the inner girder is

\[ d = \frac{b}{2} - (c - \frac{m}{2}) \]

Substituting \( b = 6.5'\), \( c = .4820'\), \( m = .2356'\), we have

\[ d = \frac{1}{2} \times 6.5 - .4820 + \frac{1}{2} \times .2356 = 2.8558' \]

Each of the original six equations (A)-(F) contains the term \( \sin^{-1} \frac{1}{2r} \)

\[ \sin^{-1} \frac{1}{2r} = \sin^{-1} \frac{60}{2 \times 1910.08} = \sin^{-1} .015706 = 0^\circ 54' = .015706 \text{ radians.} \]

Since \( \sin^{-1} \frac{1}{2r} = \frac{1}{2r} \), we may use the six modified equations (A')-(F') to find the shears and moments. The known equations are
Equation \((A^1)\) gives

\[
V_1 = \frac{wL}{2b} (d + qh) = w \cdot \frac{60}{2 \times 6.5} (2.8858 + .6266)
\]

\[
= w \cdot \frac{60}{13} \times 3.5124
\]

\[
= 16.2 \text{ w.}
\]

which is the maximum shear in the outer girder, \(w\) being the equivalent uniform load per linear foot of bridge.

Equation \((B^1)\) gives

\[
V_2 = \frac{wL}{2b} (b - d - qh) = w \cdot \frac{60}{2 \times 6.5} (6.5 - 2.8858 - .6266)
\]

\[
= w \cdot \frac{60}{13} \times 2.9876
\]

\[
= 13.8 \text{ w.}
\]

which is the maximum shear in the inner girder, when the overturning effect of the centrifugal force is considered.

Equation \((C^1)\) gives

\[
V_3 = \frac{wL}{2b} (b - d) = w \cdot \frac{60}{2 \times 6.5} (6.5 - 2.8858)
\]

\[
= w \cdot \frac{60}{13} \times 3.6142
\]

\[
= 16.7 \text{ w}
\]

which is the maximum shear in the inner girder, when the overturning effect of the centrifugal force is neglected.

Supposing that the bridge was on a tangent, the maximum shear in each of the girders would be equal to \(\frac{w}{2} \times \frac{60}{13} = 15.0 \text{ w.}\)

From the results thus obtained, it is seen that the maximum shear in the outer girder and that in the inner girder found from Equation \((C^1)\) (neglecting the overturning effect of the centrifugal force) are both greater, and that in the inner girder
found from Equation (B-1) (considering the overturning effect of the centrifugal force) is less than the shear in the girders supposed to be on a straight track.

Besides those vertical shears found above, there also exists a horizontal shear which is equal to $q$ times the maximum shear in the girder on a tangent, or

$$V_h = 0.0964 \times 15w = 1.45w.$$

For Cooper's E-50 loading, the maximum end shear on a girder of 60-foot span is 122,500 lbs. per rail and the equivalent uniform load is 4083 lbs. per foot of girder or 8167 lbs. per foot of bridge.

$$V_1 = 16.2w = 16.2 \times 8167 = 132,300 \text{ lbs.}$$

$$V_2 = 13.8w = 13.8 \times 8167 = 112,700 \text{ lbs.}$$

$$V_1^2 = 16.7w = 16.7 \times 8167 = 136,400 \text{ lbs.}$$

$$V_h = 1.45w = 1.45 \times 8167 = 11,800 \text{ lbs.}$$

The bending moments in the girders are to be found from Equations (D-1)-(F-1).

From Equation (D-1) we have

$$M_1 = \frac{wr}{2b} \left[ (d + qh - r) \left\{ \frac{l^2}{2r} + \sqrt{\frac{4r^2 - l^2}{4} - 2r} \right\} + \frac{l^2}{4} \right]$$

$$= \frac{1910.08}{2 \times 6.5} \left[ (2.8858 + 0.6266 - 1910.08) \left( \frac{3600}{2 \times 1910.08} \right) \right.$$

$$\left. 3819.6888 - 2 \times 1910.08 \right) + \frac{3600}{4} \right]$$

$$= \frac{1910.08}{13} \left[ (2.8858 + 0.6266 - 1910.08) \left( 0.9424 + 3819.6888 \right.$$

$$\left. - 3820.16 \right) + 900 \right]$$

$$= \frac{1910.08}{13} \left[ (-1906.5676) \left( 0.4712 \right) + 900 \right]$$

$$= \frac{1910.08}{13} (-898.3747 + 900)$$

$$= \frac{1910.08}{13} \times 1.6253$$
which is maximum bending moment in the outer girder in term of w, the equivalent uniform load per foot of bridge.

From Equation (E1) we obtain

\[ M_2 = \frac{wr}{2b} \left[ (b - d -qh + r) \left\{ \frac{1}{2r^2} + \frac{\sqrt{4r^2 - 1^2}}{2} - 2r \right\} - \frac{1}{4} \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ (6.5 + 1906.5676)(.4712) - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ (1913.0676)(.4712) - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ 901.4375 - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \times 1.4375 \]

\[ = 211.2 \text{ w} \]

which is the maximum moment in the inner girder, when we consider the overturning effect of the centrifugal force.

Equation (E1) gives

\[ M_2^1 = \frac{wr}{2b} \left[ (b - d + r) \left\{ \frac{1}{2r^2} + \frac{\sqrt{4r^2 - 1^2}}{2} - 2r \right\} - \frac{1}{4} \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ (6.5 - 2.8858 + 1910.08)(.4712) - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ (1913.6942)(.4712) - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \left[ 901.7327 - 900 \right] \]

\[ = w \cdot \frac{1910.08}{13} \times 1.7327 \]

\[ = 254.6 \text{ w} \]

which is the maximum bending moment in the inner girder, the overturning effect of the centrifugal force being neglected.

The bending moment in each of the girders of the bridge
carrying a uniform load of \( W \) per linear foot of bridge and on a straight track is 
\[
\frac{W}{2} \times \frac{60 \times 60}{8} = 225.0 \text{ W.}
\]
The lateral moment in each girder is
\[M_h = 0.0964 \times 225.0 \text{ W} = 21.69 \text{ W.}\]
For Cooper's E-50 loading, in a girder of 60-foot span on straight track, we have a bending moment of 1,624,500 ft.-lbs. which gives an equivalent uniform load of 3610 lbs. per foot of girder, or 7220 lbs. per foot of bridge. For this uniform loading, we find
\[
M_1 = 238.8 \text{ w} = 238.8 \times 7220 = 1,724,100 \text{ ft.-lbs.}
\]
\[
M_2 = 211.2 \text{ w} = 211.2 \times 7220 = 1,524,900 \text{ ft.-lbs.}
\]
\[
M^I_1 = 254.6 \text{ w} = 254.6 \times 7220 = 1,838,200 \text{ ft.-lbs.}
\]
\[
M_h = 21.69 \text{ w} = 21.69 \times 7220 = 156,600 \text{ ft.-lbs.}
\]
The numerical values for the shears and moments found above are now shown in Table 1. The shear or moment in the inner girder has two different values, one is found by considering the

**TABLE I**
Shears and Moments in a 60-ft. Span on a 30° Curve.

<table>
<thead>
<tr>
<th></th>
<th>Shears in lbs.</th>
<th>Moments in ft.-lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge on Tangent Each Girder</td>
<td>15.0w 122,500</td>
<td>225.0w 1,624,500</td>
</tr>
<tr>
<td>Bridge on Curve Outer Girder (1)</td>
<td>16.2w 132,300</td>
<td>238.8w 1,724,100</td>
</tr>
<tr>
<td></td>
<td>13.8w 112,700</td>
<td>211.2w 1,524,900</td>
</tr>
<tr>
<td>Inner Girder (2)</td>
<td>16.7w 136,400</td>
<td>254.6w 1,838,200</td>
</tr>
<tr>
<td>Due to Lateral Force Each Girder</td>
<td>1.45w 11.800</td>
<td>21.69w 156,600</td>
</tr>
</tbody>
</table>
overturning effect of the centrifugal force and is designated by (1) and the other is found by neglecting it and is designated by (2) in the Table.

Example 2. An 80-ft. span bridge on a 4° curve.

The following data are given:
Length of span, \( l = 80' - 0" \).
Degree of curve, \( D = 4° \).
Spacing of girders, \( b = 7' - 6" \).
Height of center of gravity of train above top laterals, \( h = 6' - 6" \).

From Table A in Art. 5, we find
Radius of curve, \( r = 1432.69' \)
Centrifugal force coefficient, \( q = .1166 \)
Super-elevation of outer rail, \( s = .5760' \)
Distance between center line of track and the curve of center of gravity of train at mid-span, \( c = .5830' \).

The middle ordinate of the center line of track between the ends of span is

\[
m = r - \frac{1}{2} \sqrt{4r^2 - l^2}
\]

\[
\sqrt{4r^2 - l^2} = \sqrt{4(1432.69)^2 - (80)^2} = \sqrt{8,204,002.5444} = 2864.2630
\]

\[
m = 1432.69 - \frac{1}{2} \times 2864.2630 = .5585'
\]
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In Fig. 10, A C B and $A_1C_1B_1$ are the center line of track and the curve of center of gravity of the train respectively. The distance from the middle point $G_1$ of the curve $A_1C_1B_1$ to the inner girder is given by

$$d = \frac{b}{2} - (c - \frac{m}{2}) = \frac{7.5}{2} - .5830 + \frac{.5585}{2} = 3.4463'$$

All the known quantities are as follows:

1 = 60'  
$r = 1432.69'$  
$b = 7.5'$

$\mathbf{d} = 3.4463'$  
$qh = .1166 \times 6.5 = .7579$

$$\frac{\sin^{-1} \frac{1}{2r}}{2r} = \frac{80}{2 \times 1432.69} = \frac{\sin^{-1} .027919}{1.027919} = 0.027919 \text{ radians.}$$

Since $\frac{\sin^{-1} \frac{1}{2r}}{2r} = \frac{1}{2r}$ in this case, Equation $(A_1)-(F_1)$ may be used to find the shears and moments.

The maximum shear in the outer girder is from Equation $(A_1)$.

$$V_1 = \frac{w}{2b} (d + qh) = \frac{w}{2} \frac{80}{x} 7.5 (3.4463 + .7579)$$

$$= w \frac{80}{15} x 4.2042$$

$$= 22.4 w$$

in which $w$ is the equivalent uniform load per foot of bridge.

The maximum shear in the inner girder, when the overturning effect of the centrifugal force is taken into consideration, is from Equation $(B_1)$.

$$V_2 = \frac{w}{2b} (b - d - qh) = \frac{w}{15} \frac{80}{x} (7.5 - 3.4463 - .7579)$$

$$= w \frac{80}{15} x 3.2958$$

$$= 17.6 w$$

When the overturning effect of the centrifugal force upon
the inner girder is neglected, then the maximum shear is from Equation (C).

\[ V^1_2 = \frac{wl}{2b} (b - d) = w \frac{80}{15} (7.5 - 3.4463) \]
\[ = w \frac{80}{15} \times 4.0537 \]
\[ = 21.6 \, w. \]

If the bridge were on a straight track, each girder would have a maximum shear \( \frac{w}{2} \times \frac{80}{2} = 20.0 \, w. \)

The lateral shear in each girder is

\[ V_h = 0.1166 \times 20.0w = 2.33 \, w. \]

For Cooper's E-50 loading, the end shear in a girder of 80-foot span on straight track is 155,300 lbs. for one rail, from which we obtain an equivalent uniform load of 3882 lbs. per linear foot of girder or 7765 lbs. per linear foot of bridge.

\[ V_1 = 22.4 \, w = 22.4 \times 7765 = 173,900 \, lbs. \]
\[ V_2 = 17.6 \, w = 17.6 \times 7765 = 136,700 \, lbs. \]
\[ V^1_2 = 21.6 \, w = 21.6 \times 7765 = 167,700 \, lbs. \]
\[ V_h = 2.33 \, w = 2.33 \times 7765 = 18,100 \, lbs. \]

The bending moments are to be found from Equation (D). Equation (D) gives

\[ M_1 = \frac{wr}{2b} \left[ (d +qh - r) \left\{ \frac{12}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} + \frac{l^2}{4} \right] \]
\[ = w \frac{1432.69}{2 \times 7.5} \left[ \left( 3.4463 + .7579 - 1432.69 \right) \left( \frac{6400}{2 \times 1432.69} \right. \right.
\[ + 2864.2630 - 2 \times 1432.69 \right) \left. + \frac{6400}{4} \right] \]
\[ = w \frac{1432.69}{15} \left[ \left( 3.4463 + .7579 - 1432.69 \right) \left( 2.2336 \right. \right.
\[ + 2864.2630 - 2865.38 \right) + 1600 \right] \]
\[ = w \frac{1432.69}{15} \left[ \left( -1428.4858 \right) \times 1.1166 + 1600 \right] \]
\[ = w \frac{1432.69}{15} (-1595.0472 + 1600) \]
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which is the maximum bending moment in the outer girder.

The maximum moment in the inner girder, when the overturning effect of centrifugal force is considered is from Equation (E^1).

\[ M_2 = \frac{wr}{2b} \left[ (b - d - qh + r) \left\{ \frac{1}{2r} + \sqrt{4rE - 1^2} - 2r \right\} - \frac{1}{4} \right] \]

\[ = \frac{1432.69}{15} \left[ (7.5 + 1428.4858)(1.1166) - 1600 \right] \]

\[ = \frac{1432.69}{15} \left[ (1435.9858)(1.1166) - 1600 \right] \]

\[ = \frac{1432.69}{15} (1603.4218 - 1600) \]

\[ = \frac{1432.69}{15} \times 3.4218 \]

\[ = 326.9 \text{ w} \]

The maximum moment in the inner girder, when the overturning effect of centrifugal force is not considered is from Equation (F^1).

\[ M_2^l = \frac{wr}{2b} \left[ (b - d + r) \left\{ \frac{1}{2r} + \sqrt{4rE - 1^2} - 2r \right\} - \frac{1}{4} \right] \]

\[ = \frac{1432.69}{15} \left[ (7.5 - 3.4463 + 1432.69)(1.1166) - 1600 \right] \]

\[ = \frac{1432.69}{15} \left[ (1436.7437)(1.1166) - 1600 \right] \]

\[ = \frac{1432.69}{15} (1604.2680 - 1600) \]

\[ = \frac{1432.69}{15} \times 4.2680 \]

\[ = 407.6 \text{ w} \]

The maximum moment in each of the girders for a bridge on
straight track is $\frac{W}{2} \times \frac{80 \times 80}{8} = 400.0$ w.

The lateral moment in each girder is

$$M_h = 0.1166 \times 400.0 \text{ w} = 46.64 \text{ w}.$$  

For Cooper's E-50 loading, the maximum bending moment in a girder of 80-foot span is 2,700,000 ft.-lbs. per rail. The equivalent uniform load is 3375 lbs. per linear foot of girder, or 6750 lbs. per linear foot of bridge.

$$M_1 = 473.1 \text{ w} = 473.1 \times 6750 = 3,193,400 \text{ ft.-lbs.}$$

$$M_2 = 326.9 \text{ w} = 326.9 \times 6750 = 2,206,600 \text{ ft.-lbs.}$$

$$M^1_2 = 407.6 \text{ w} = 407.6 \times 6750 = 2,751,300 \text{ ft.-lbs.}$$

$$M_h = 46.64 \text{ w} = 46.64 \times 6750 = 314,800 \text{ ft.-lbs.}$$

The numerical results of shears and moments are shown in Table 2.

**TABLE 2.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge on Tangent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each Girder</td>
<td>20.0w</td>
<td>400.0w</td>
</tr>
<tr>
<td>Bridge on Curve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer Girder</td>
<td>22.4w</td>
<td>473.1w</td>
</tr>
<tr>
<td>(1) Inner Girder</td>
<td>17.6w</td>
<td>326.9w</td>
</tr>
<tr>
<td>(2)</td>
<td>21.6w</td>
<td>407.6w</td>
</tr>
<tr>
<td>Due to Lateral Force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each Girder</td>
<td>2.33w</td>
<td>46.64w</td>
</tr>
</tbody>
</table>

In the above table, we have two cases for the inner girder. In case (1), the overturning effect of the centrifugal force has been included; while in case (2), that effect is disregarded.
The text in the image is not legible. It appears to be a table with columns and rows, but the content cannot be accurately transcribed. The table likely contains data or information organized in a structured format, but without clearer visibility, it's not possible to provide a readable representation.
Art. 8. Arrangement for Equalization of Moments.

From the results of the two examples as shown in Tables 1-2 in the last article, it is seen that according to the arrangement generally used in practice for the location of the bridge, the shears and moments in the two girders vary quite differently from each other. If both girders were designed for the larger stresses, the girder having smaller stresses would have too much section area than what is required; and if each girder were designed for its own stresses, then the sections of the two girders would not be the same. As these two conditions seem unfavorable for the purpose of economic design, an arrangement which would give equal stresses in both girders would be very desirable. Since a girder is designed chiefly to resist the external bending moments, though the shears must be considered in making its web, a method of equalizing the maximum moments in the outer and inner girders is presented here.

In Art. 6, we have found three equations for the maximum bending moments in the outer and inner girders as represented by Equations (D)-(F):

$$M_1 = \frac{Wt}{2b} \left[ (b + qh - r) \left\{ \frac{1}{2} \sin^{-1} \frac{1}{2r} + \sqrt{4r^2 - 1} - 2r \right\} + \frac{1}{4} \right] \quad \text{(D)}$$

$$M_2 = \frac{Wt}{2b} \left[ (b - d - qh + r) \left\{ \frac{1}{2} \sin^{-1} \frac{1}{2r} + \sqrt{4r^2 - 1} - 2r \right\} - \frac{1}{4} \right] \quad \text{(E)}$$

$$M_2^1 = \frac{Wt}{2b} \left[ (b - d + r) \left\{ \frac{1}{2} \sin^{-1} \frac{1}{2r} + \sqrt{4r^2 - 1} - 2r \right\} - \frac{1}{4} \right] \quad \text{(F)}$$

The inner girder has two values for the maximum moment, (1) when the overturning effect of the centrifugal force is considered, and (2) when that effect is not considered; and accordingly we have two cases in equalizing bending moments.
Case 1. Overturning Effect of Centrifugal force considered.

Equate Equations (D) and (E) and solve for $d$, the distance from the middle point of the curve of center of gravity of train to the inner girder.

$$\frac{w}{2b} (b + qh - r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} + \frac{1}{4}$$

$$= \frac{w}{2b} (b - d - qh + r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} - \frac{1}{4}$$

$$\therefore (b-2d-2qh+2r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} = \frac{1}{2}$$

$$\therefore b - 2d - 2qh + 2r = \frac{1^2}{2 \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\}}$$

Dividing by 2 and transposing, we get

$$d = \frac{b}{2} - qh + r - \frac{1^2}{4 \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\}} \ldots \ldots \ldots \ldots \ldots (G)$$

Case 2. Overturning effect of Centrifugal force not considered.

Equate Equations (D) and (F)

$$\frac{w}{2b} (d + qh - r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} + \frac{1}{4}$$

$$= \frac{w}{2b} (b - d + r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} - \frac{1}{4}$$

$$\therefore (b-2d-qh+2r) \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} = \frac{1}{2}$$

$$\therefore b - 2d - qh + 2r = \frac{1^2}{2 \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\}}$$

Dividing by 2 and transposing,

$$d = \frac{b}{2} - \frac{qh}{2} + r - \frac{1^2}{4 \left\{ \text{sin}^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right\}} \ldots \ldots \ldots \ldots \ldots (H)$$
The distance \( d \) thus found is the distance from the middle point of the curve of center of gravity of train to the inner girder, and when added to the distance \( c \) between the center line of the track and the curve of center of gravity of train at mid-span, will give the location of the center line of the track with reference to the inner girder.

In order to well illustrate the application of Equations (G) and (H) we shall review the two examples investigated in the last article.

**Example 1.** A 60-ft. span bridge on a 3° curve.

Here we have

\[
\begin{align*}
1 &= 60' \\
r &= 1910.08' \\
b &= 6.5' \\
q &= 0.0964 \\
h &= 6.5' \\
qh &= 0.0964 \times 6.5 = 0.6266
\end{align*}
\]

**Case 1.** From Equation (G)

\[
d = \frac{b}{2} - qh + r - \frac{l^2}{4\left(1 - \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r\right)}
\]

\[
= \frac{6.5}{2} - 0.6266 + 1910.08 - \frac{3600}{4 \times 0.4712}
\]

\[
= 3.25 - 0.6266 + 1910.08 - 1910.0170
\]

\[
= 2.6864 \text{ ft.}
\]

Using this value for \( d \), we find from Equation (D1)

\[
M_1 = \frac{wr}{2b} \left( (d + qh - r) \left( \frac{l^2}{2r} + \sqrt{4r^2 - l^2} - 2r \right) + \frac{l^2}{4} \right)
\]

\[
= w \cdot \frac{1910.08}{13} \left[ (2.6864 + 0.6266 - 1910.08)(0.4712) + 900 \right]
\]

\[
= w \cdot \frac{1910.08}{13} \left[ (-1906.7670)(0.4712) + 900 \right]
\]
which is the maximum moment in the outer girder.

From Equation (E\text{\textsuperscript{\textdagger}})

\[ M_2 = \frac{wr}{2b} \left[ \left( b - d - qh + r \right) \left\{ \frac{1}{2r} + \sqrt{\frac{4r^2 - 1^2}{2r}} - \frac{1}{4} \right\} \right] \]

\[ = w \frac{1910.08}{13} \left[ (6.5 + 1306.7670)(.4712) - 900 \right] \]

\[ = w \frac{1910.08}{13} \left[ (1913.2670)(.4712) - 900 \right] \]

\[ = w \frac{1910.08}{13} \left( 901.5314 - 900 \right) \]

\[ = w \frac{1910.08}{13} \times 1.5314 \]

\[ = 225.0 \text{ w} \]

which is the maximum moment in the inner girder.

Here the maximum bending moments in the outer and inner girder are each equal to 225.0 w, which amount is the maximum moment in each of the girders, when the bridge is on a tangent. The distance c has previously been found to be .4820 ft. Therefore, for equal bending moments in both girders, the center line of track should be so located that its middle point (at mid-span) would be at a distance from the inner girder equal to

\[ d + c = 2.6864 + .4820 = 3.1684 \text{ ft.} \]

According to this arrangement, not only the maximum bending moments in the two girders are equalized, but the difference in the maximum shears can also be reduced to a great extent. This may be seen by calculating the shears, using \( d = 2.6864 \text{ ft.} \)
The maximum shear in the outer girder is from Equation (A):

\[ V_1 = \frac{wl}{2b} \left( d + qh \right) = w \frac{60}{13} \left( 2.6864 + .6266 \right) \]

\[ = w \frac{60}{13} \times 3.3130 \]

\[ = 15.3 \, w \]

The maximum shear in the inner girder is from Equation (B):

\[ V_2 = \frac{wl}{2b} \left( b - d - qh \right) = w \frac{60}{13} \left( 6.5 - 3.3130 \right) \]

\[ = w \frac{60}{13} \times 3.1870 \]

\[ = 14.7 \, w \]

Here the shears are 15.3 \, w and 14.7 \, w respectively against 16.2 \, w and 13.8 \, w found in the last article.

**Case 2.** From Equation (E)

\[ d = \frac{b - qh + r}{2} \left[ -\frac{l^2}{4 \left( \sin^{-1} \frac{1}{2r} + \sqrt{4r^2 - l^2} - 2r \right)} \right] \]

\[ = \frac{6.5}{2} - \frac{6266}{2} + 1910.08 - \frac{3600}{4 \times .4712} \]

\[ = 3.25 - .3133 + 1910.08 - 1910.0170 \]

\[ = 2.9997 \, \text{ft.} \]

From Equation (D), the maximum moment in the outer girder is

\[ M_1 = \frac{wr}{2b} \left[ (d + qh - r) \left\{ \frac{l^2}{2r} + \sqrt{4r^2 - l^2} - 2r \right\} + \frac{l^2}{4} \right] \]

\[ = w \frac{1910.08}{13} \left[ (2.9997 + .6266 - 1910.08)(.4712) + 900 \right] \]

\[ = w \frac{1910.08}{13} \left[ (-1906.4537)(.4712) + 900 \right] \]
\[
\begin{align*}
\text{From Equation } (F^1) \text{ the maximum moment in the inner girder is} \\
H_2 &= \frac{wr}{2b} \left[ (b-d+r) \left\{ \frac{1^2}{2r} + \sqrt{4r^2-1^2} - 2r - \frac{1^2}{4} \right\} \right] \\
&= w \frac{1910.08}{13} \left[ (6.5 - 2.9997 + 1910.08)(.4712) - 900 \right] \\
&= w \frac{1910.08}{13} \left[ (1913.5803)(.4712) - 900 \right] \\
&= w \frac{1910.08}{13} (901.6790 - 900) \\
&= w \frac{1910.08}{13} \times 1.6790 \\
&= 246.7 w
\end{align*}
\]

For this case, the sum of the maximum moments in the two girders found in the last article is equal to \(238.8 w + 254.6 w = 493.4 w\), one-half of which is 246.7 w.

The maximum shear in the outer girder is

\[
V_1 = \frac{wl}{2b} \ (d + qh) = w \frac{60}{13} (2.9997 + .6266) \\
= w \frac{60}{13} \times 3.6263 \\
= 16.7 w
\]

The maximum shear in the inner girder is

\[
V_2 = \frac{wl}{2b} \ (b - d) = w \frac{60}{13} (6.5 - 2.9997) \\
= w \frac{60}{13} \times 3.5003 \\
= 16.2 w
\]

In Case 2, the shears in both girders are also nearly equal.

**Example 2. An 80-ft. span bridge on a 4° curve.**

Here we have the following known data:

- \( l = 80' \)
- \( r = 1432.69' \)
- \( b = 7.5' \)
- \( q = .1166 \)
- \( h = 6.5' \)
- \( qh = .1166 \times 6.5 = .7579 \)

**Case 1.** From Equation (G)

\[
d = \frac{b}{2} - qh + r - \frac{1^2}{4 \left\{ 1 \cdot \sin^{-1} \frac{1}{2r} + \sqrt{\frac{r^2}{4} - 1^2} - 2r \right\}}
\]

\[
d = \frac{7.5}{2} - .7579 + 1432.69 - \frac{6400}{4 \times 1.1166}
\]

\[
d = 3.75 - .7579 + 1432.69 - 1432.9214
\]

\[
d = 2.7607 \text{ ft.}
\]

The distance between the center line of track and the curve of the center of gravity at mid-span is

\[
c = .5830'
\]

\[
\therefore d + c = 2.7607 + .5830 = 3.3437 \text{ ft.}
\]

which gives the location of the middle point (mid-span) of the center line of track from the inner girder.

Using \( d = 2.7607' \), the maximum moment in the outer girder is

\[
K_1 = \frac{w(r)}{2b} \left[ (d + qh - r) \left\{ \frac{1^2}{2r} + \sqrt{\frac{r^2}{4} - 1^2} - 2r \right\} + \frac{1^2}{4} \right]
\]

\[
= w \frac{1432.69}{15} \left[ (2.7607 + .7579 - 1432.69)(1.1166) + 1600 \right]
\]

\[
= w \frac{1432.69}{15} \left[ (-1429.1714)(1.1166) + 1600 \right]
\]

\[
= w \frac{1432.69}{15} \left[ -1595.3128 + 1600 \right]
\]
\[ M_2 = \frac{wr}{2b} \left[ (b-d-qh+r) \left\{ \frac{1}{2r} + \sqrt{\frac{2r^2}{l^2}} - 2r \right\} - \frac{1}{4} \right] \]
\[ = w \frac{1432.69}{15} \left[ (7.5 + 1429.1714)(1.1166) - 1600 \right] \]
\[ = w \frac{1432.69}{15} \left[ (1436.6714)(1.1166) - 1600 \right] \]
\[ = w \frac{1432.69}{15} (1604.1873 - 1600) \]
\[ = w \frac{1432.69}{15} \times 4.1873 \]
\[ = 400.0 \text{ w} \]

The maximum shear in the outer girder is

\[ V_1 = \frac{wl}{2b} (d + qh) = w \frac{80}{15} (2.7607 + .7579) \]
\[ = w \frac{80}{15} \times 3.5186 \]
\[ = 18.8 \text{ w} \]

The maximum shear in the inner girder is

\[ V_2 = \frac{wl}{2b} (b-d-qh) = w \frac{80}{15} (7.5 - 3.5186) \]
\[ = w \frac{80}{15} \times 3.9814 \]
\[ = 21.2 \text{ w} \]

According to the customary arrangement (Art. 7), the maximum shears in the outer and inner girders are respectively 22.4 w and 17.6 w.

Case 2. From Equation (E)
\[ d = \frac{b}{2} - \frac{qh}{2} + r - \frac{l^2}{4 \left(1.8 \sin^{-1} \frac{1}{2r} + \sqrt{4r^2 - 1^2} - 2r\right)} \]

\[ = \frac{7.5}{2} - \frac{.7579}{2} + 1432.69 - \frac{6400}{4 \times 1.1166} \]

\[ = 3.75 - .3789 + 1432.69 - 1432.9214 \]

\[ = 3.1397 \text{ ft.} \]

\[ d + c = 3.1397 + .5830 = 3.7227 \text{ ft.} \]

\[ M_1 = \frac{w_1 r}{2b} \left( (d + q - r) \left\{ \frac{l^2}{2r^2} + \sqrt{4r^2 - l^2} - 2r \right\} + \frac{l^2}{4} \right) \]

\[ = w \frac{1432.69}{15} \left[(3.1397 + .7579 - 1432.69)(1.1166) + 1600 \right] \]

\[ = w \frac{1432.69}{15} \left[(-1428.7924)(1.1166) + 1600 \right] \]

\[ = w \frac{1432.69}{15} (-1595.3896 + 1600) \]

\[ = w \frac{1432.69}{15} \times 4.6104 \]

\[ = 440.35 \text{ w} \]

\[ M_2 = \frac{w_2 r}{2b} \left( (b - d + r) \left\{ \frac{l^2}{2r^2} + \sqrt{4r^2 - l^2} - 2r \right\} - \frac{l^2}{4} \right) \]

\[ = w \frac{1432.69}{15} \left[(7.5 - 3.1397 + 1432.69)(1.1166) - 1600 \right] \]

\[ = w \frac{1432.69}{15} \left[(1437.0503)(1.1166) - 1600 \right] \]

\[ = w \frac{1432.69}{15} (1604.6104 - 1600) \]

\[ = w \frac{1432.69}{15} \times 4.6104 \]

\[ = 440.35 \text{ w} \]

The sum of the maximum moments in the two girders as found in Art. 7 is 473.1 w + 407.6 w = 880.7 w.
\[ V_1 = \frac{wl}{2b} (b + qh) = w \frac{80}{15} (3.1397 + .7579) \]
\[ = w \frac{80}{15} \times 3.8976 \]
\[ = 20.8 \text{ w} \]

\[ V_2 = \frac{wl}{2b} (b - d) = w \frac{80}{15} (7.5 - 3.1397) \]
\[ = w \frac{80}{15} \times 4.3603 \]
\[ = 23.2 \text{ w} \]

The shears found in the last article are respectively 22.4 w and 21.6 w.
CHAPTER III.
THROUGH PLATE GIRDER BRIDGES

Art. 9. Clearance.*

The spacing of the girders or trusses in through bridges on curves depends upon the required clearance which is usually based on the extreme dimensions of a car. Let

\[ \begin{align*}
A &= \text{extreme length of car.} \\
B &= \text{distance center to center of trucks.} \\
w &= \text{clear width required on tangent.} \\
H &= \text{extreme height of car above rail.} \\
h &= \text{height of lower corner of clearance of diagram above rail.} \\
C &= \text{extreme length of truss at the height } H \text{ above rail} \\
&\quad \text{(not the span length)} \\
s &= \text{super-elevation of outer rail.} \\
t &= \text{distance between centers of rails.} \\
M_a &= \text{middle ordinate of curve for chord } A. \\
M_b &= \text{middle ordinate of curve for chord } B. \\
M_c &= \text{middle ordinate of curve for chord } C. \\
\end{align*} \]

We find from Fig. 11 that if the car were vertical, the clearance \( w \) at the center of span would have to be increased by \( M_a - M_b \) on the outside and by \( M_b - M_c \) on the inside of the curve. But on account of the super-elevation of the outer rail, the car takes an inclined position as shown in Fig. 12, and hence there must

* From "Design of Steel Bridges" by F. C. Kunz, pp. 217-220.
be an additional clearance on the inside of $\frac{H_S}{t}$, while that on the outside is decreased by $h\frac{S_t}{t}$. We have therefore for the distance from the center line of track at the middle of span to the outside clearance line:

$$X = \frac{W}{2} + M_a - M_b - h\frac{S_t}{t} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (I)$$

and for the corresponding distance on the inside:

$$Y = \frac{W}{2} + M_b + M_c + H\frac{S_t}{t} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (J)$$

and the total clear width will be equal to $X + Y$.

For half through spans, the required clearance between the girders or the trusses (in pony truss bridges) is found in a similar way as shown above except as follows. In Fig. 13 denoting with $k$ the height of the top flange of the girder above the lower rail and with $w^l$ (= $w$ for $k > h$) the width of the clearance diagram at the height $k$ above the lower rail, we have:

![Diagram showing clearance calculation](image)

$$x = \frac{W^l}{2} + M_a - M_b - \frac{kS_t}{t}, \text{ if } k < h$$

or $$X = \frac{W}{2} + M_a - M_b - \frac{S_t}{h\frac{S_t}{t}}, \text{ if } k > h \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (K)$$
and
\[ Y = \frac{w}{2} + M_b + M_c + \frac{kS}{t} \] (L)

where \( M_c \) is now the middle ordinate for a chord equal to the total length of the girder.

According to the Specifications of the American Railway Engineering Association, \( A = 80 \) ft., \( B = 60 \) ft., \( H = 14 \) ft., \( h = 4 \) ft., and \( w = 14 \) ft. These values will be used in the above clearance equations in the following analysis.

For the investigation of stresses in the present chapter, we shall consider a through plate girder bridge of 90-foot span on a 6° curve, having six panels of 15 feet each. Further, we shall assume the following data:

Depth of girder = 8'-0".
Height of rail tops above lateral = 4'-0".
Height of center of gravity of train above rail tops = 5'-0" (= f).
Height of center of gravity of train above laterals = 9'-0" (= h).
Height of top flanges of girders above rail tops = 4'-0" (= k).
Distance between rail centers = 4.94' (= t).
From Table A of Art. 5, we find for \( D = 60^\circ \): Radius of curve, \( r = 955.37 \) ft.
Centrifugal force coefficient, \( q = .1416 \).
Super-elevation of outer rail, \( s = .6995 \) ft.
Distance between center line of track and curve of center of gravity of train at mid-span, \( c = .7080 \) ft.
The middle ordinate \( M \) for a chord \( L \) is \( M = r - \frac{1}{2} \sqrt{4r^2-L^2} \).
but for convenience, we shall use the approximate formula \( M = \frac{12}{8r} \) which gives results very closely to that found from the exact formula. Thus for
\[
A = 80' \\
B = 60' \\
C = 90'
\]

\( M_a = .8374' \)
\( M_b = .4712' \)
\( M_c = 1.0598' \)

The required clear distance from the center line of track at mid-span to the outer girder is found from Equation (X).

\[ X = \frac{w^1}{2} + M_a - M_b - \frac{k_s}{t} \]

Substituting \( w^1 = 14' \), \( M_a = .8374' \), \( M_b = .4712 \), \( k = 4' \), \( s = .6995' \) and \( t = 4.94' \) in the above equation, we have

\[ X = \frac{14}{2} + .8374 - .4712 - 4 \times \frac{.6995}{4.94} \]

\[ = 7 + .8374 - .4712 - .5664 \]

\[ = 6.7998 \text{ ft.} \]

Use a clear width of 6'-10" = 6.8333 ft. Taking the width of the flange of the girder as 18 inches, we have the distance from the center line of track at mid-span to the center line of the outer girder

\[ X^1 = 6' - 10" + 9" = 7' - 7" = 7.5833 \text{ ft.} \]

The required clear width from the center line of track to the inner girder is from Equation (L)

\[ Y = \frac{w}{2} + M_b + M_c + \frac{k_s}{t} \]

\[ = \frac{14}{2} + .4712 + 1.0598 + 4 \times \frac{.6995}{4.94} \]

\[ = 7 + .4712 + 1.0598 + .5664 \]

\[ = 9.0974 \text{ ft.} \]
Use a clear width of $9' - 2'' = 9.1667$ ft. The distance from the center line of track at the middle of span to the center line of the inner girder is

$$y_1 = 9' - 2'' + 9'' = 9' - 11'' = 9.9167 \text{ ft.}$$

The total clear width is $6' - 10'' + 9' - 2'' = 16' - 0''$ and the distance between centers of girders is $16' - 0'' + 1' 6'' = 17' 6''$. 

![Diagram](image)

**Fig. 14.**

The distance $c$ between the center line of track and the curve of center of gravity of train at mid-span has been found to be 0.7080 ft. Hence the distance from the middle point (at mid-span) of the curve of center of gravity of train to the center line of the outer girder is

$$7.5833 + 0.7080 = 8.2913 \text{ ft.}$$

and that to the center line of the inner girder is

$$9.9167 - 0.7080 = 9.2087 \text{ ft.}$$

**Art. 10. Shears and Moments.**

In through bridges the load from a train is transmitted through the stringers and floor beams to the main girders or trusses
at the panel points only. In the case of a bridge on straight track, the total panel load is applied at the floor beam centrally and is hence equally carried by the two main girders or trusses. When the bridge is on a curve, then due to the curvature of the track and the super-elevation of the outer rail, the panel load is applied eccentrically at the floor beam and will not therefore be symmetrically distributed to the main girders or trusses, the proportional part going to each girder or truss being dependent upon the eccentricity at which the load is applied.

Fig. 15 shows a section of a bridge at a panel joint. Let

![Diagram](image)

\(O\) be the center of gravity of the train; \(P\), the total load on the floor beam; and \(qP\), the centrifugal force acting at a height of \(h\) above the laterals. Also let \(b\) be the distance between centers of girders, and \(e\) be the average eccentricity for the load \(P\) which is taken as the average for a half-panel each side of the panel joint.
The average eccentricity \( e \) may be positive or negative. It will be assumed as positive, when it is on the outer girder side of the center line of the bridge; and as negative, when it is on the inner girder side. If, \( R_1 \) and \( R_2 \) represent the panel joint loads in the outer and inner girders respectively due to the vertical load \( P \) and the centrifugal force \( qP \), then we have:

\[
R_1 = \frac{P}{2b} + \frac{Pe}{b} + \frac{Pgh}{b} = \frac{P}{2b} (b + 2e + 2qh) \quad \ldots \quad (M)
\]

and

\[
R_2 = \frac{P}{2} - \frac{Pe}{b} - \frac{Pgh}{b} = \frac{P}{2b} (b - 2e - 2qh) \quad \ldots \quad (N)
\]

In Equation (N), \( R_2 \) will have the greatest value, when there is little or no centrifugal force as in the case of a train going slowly or standing still; hence we may neglect the term \( 2qh \) and get another equation for \( R_2 \).

\[
R_2 = \frac{P}{2b} (b - 2e) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (O)
\]

Equations (M)-(O) contain the eccentricity \( e \) for the load \( P \) which is the average for a half-panel each side of the panel joint considered and can be determined from the net eccentricities of the two adjacent panel joints. The term "net eccentricity" at a panel joint as used here means the distance measured along the floor beam from the center line of the bridge to the point where the curve of the center of gravity of train and the floor beam intersect. Let \( E_a \), \( E_b \), etc., designate the net eccentricities at the panel joints A, B, etc., respectively. Their values are found as follows:

In Fig. 16 are shown the middle ordinates \( M_{30}, M_{60} \) and \( M_{90} \) for chords of 30, 60 and 90 feet respectively. Using the
approximate formula $M = \frac{12}{8r}$ to find the middle ordinates, we have,

For $l = 30'$ \hspace{1cm} $M_{30} = .1178$ ft.

" $l = 60'$ \hspace{1cm} $M_{60} = .4712$ ft.

" $l = 90'$ \hspace{1cm} $M_{90} = 1.0598$ ft.

The net eccentricities at several panel joints are:

$E_d = \frac{b}{2} - 6.2913 = 8.75 - 6.2913 = .4587$ ft.

$E_c = E_d - M_{30} = .4587 - .1178 = .3409$ ft.

$E_b = E_d - M_{60} = .4587 - .4712 = -.0125$ ft.

$E_a = E_d - M_{90} = .4587 - 1.0598 = -.6011$ ft.

These net eccentricities are shown in Fig. 17.
The average eccentricities for the several panel joints are as follows:

For joint D, \( e_D = \frac{1}{4} \left( .3409 + 2 \times .4587 + .3409 \right) = .3998 \) ft.

For joint C, \( e_C = \frac{1}{4} \left( -.0125 + 2 \times .3409 + .4587 \right) = .2820 \) ft.

For joint B, \( e_B = \frac{1}{4} \left( -.6011 - 2 \times .0125 + .3409 \right) = -.0713 \) ft.

Since the eccentricity \( e \) is different for different panel joints, the best way of determining the shears and moments in the girders is to use influence lines. As has been found before that the loads on the outer and inner girders from a vertical load \( P \) and a centrifugal force \( qP \) are respectively represented by Equations (M) and (N) or (O). The negligence of the overturning effect of the centrifugal force in Equation (O) will give results in error on the side of safety and hence we shall use Equations (M) and (O), that is,

For outer girder, \( R_1 = \frac{P}{2b} \left( b + 2e + 2qh \right) \)

and for inner girder, \( R_2 = \frac{P}{2b} \left( b - 2e \right) \)

If \( z \) is the ordinate of influence line for each girder for a centrally applied load \( P \), \( \frac{z}{b} \left( b + 2e + 2qh \right) \) and \( \frac{z}{b} \left( b - 2e \right) \) will be the corresponding ordinates of influence lines for the outer and inner girders respectively for an eccentrically applied load \( P \) and a centrifugal force \( qP \) (whose effect on the inner girder is neglected). Based on this principle we have the following method of constructing influence lines for eccentric loading: The influence lines for the shear or moment are first drawn as for straight track and central loading, and then each influence ordinate \( z \) below a panel point is changed to \( \frac{z}{b} \left( b + 2e + 2qh \right) \) for the outer girder and to \( \frac{z}{b} \left( b - 2e \right) \) for the inner girder, thus obtaining true influence lines for the two girders for an eccentric loading. These influence lines will be
similar to each other, except that their corresponding ordinates will have different values.

To find the shear or moment for the eccentric loading, first determine the shear or moment from the wheel-loadings as for straight track and central loading. From the actual shear or moment thus found, we obtain the equivalent uniform load, which, when multiplied by the area of influence lines for eccentric loading, will give the shear or moment in the outer or inner girder for that loading.

These operations shall be shown in the following:

(1) Maximum Shears.

In Fig. 18 (a) is shown the influence line for maximum shear in each girder for straight track and central loading, the
influence ordinates below the panel points B, C, D, E, F being respectively $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$.

The average eccentricities for the panel joints B, C, D have been found to be -.0713, .2820, .3998 respectively. Further, we have $b = 17.5'$, $q = .1416$, $h = 9'$, $qh = .1416 \times 9 = 1.2744$.

Hence by using the formulae $\text{Ordin.} = \frac{2}{b} (b + 2e + 2qh)$ and $\text{Ordin.} = \frac{z}{b} (b - 2e)$, we can find the ordinates of influence lines for the outer and inner girders as follows:

**Panel point B.**  \( z = \frac{5}{6}, e = -.0713 \)

- **Outer Girder, Ord.** = $\frac{5}{6 \times 17.5} (17.5 - 2 \times .0713 + 2 \times 1.2744) = .9479$
- **Inner Girder, Ord.** = $\frac{5}{6 \times 17.5} (17.5 + 2 \times .0713) = .8401$

**Panel point C.**  \( z = \frac{2}{3}, e = .2820 \)

- **Outer Girder, Ord.** = $\frac{2}{3 \times 17.5} (17.5 + 2 \times .2820 + 2 \times 1.2744) = .7852$
- **Inner Girder, Ord.** = $\frac{2}{3 \times 17.5} (17.5 - 2 \times .2820) = .6452$

**Panel point D.**  \( z = \frac{1}{2}, e = .3998 \)

- **Outer Girder, Ord.** = $\frac{1}{2 \times 17.5} (17.5 + 2 \times .3998 + 2 \times 1.2744) = .5957$
- **Inner Girder, Ord.** = $\frac{1}{2 \times 17.5} (17.5 - 2 \times .3998) = .4772$

**Panel point E.**  \( z = \frac{1}{3}, e = .2820 \)

- **Outer Girder, Ord.** = $\frac{1}{3 \times 17.5} (17.5 + 2 \times .2820 + 2 \times 1.2744) = .3926$
- **Inner Girder, Ord.** = $\frac{1}{3 \times 17.5} (17.5 - 2 \times .2820) = .3226$
Panel point F  \( \frac{Z}{e} = \frac{1}{6} \)

- Outer Girder, Ord. = \( \frac{1}{6 \times 17.5} \) \((17.5 - 2 \times 0.0713 + 2 \times 1.2744) = 0.1896\)

- Inner Girder, Ord. = \( \frac{1}{6 \times 17.5} \) \((17.5 + 2 \times 0.0713) = 0.1660\)

These influence ordinates for the outer and inner girders are shown in Fig. 18 (b) and (c).

The area of the influence lines for the outer girder is
\[ A_1 = 15 (0.9479 + 0.7852 + 0.5957 + 0.3926 + 0.1896) \]
\[ = 15 \times 2.9110 = 43.6650 \]

The area of influence lines for the inner girder is
\[ A_2 = 15(0.8401 + 0.6452 + 0.4772 + 0.3226 + 0.1680) \]
\[ = 15 \times 2.4531 = 36.7965 \]

For Cooper's E-50 loading, the maximum shear in the girder (in the panel A B) for straight track is 131,000 lbs. The area of the influence line (Fig. 18 (a)) is \( \frac{5}{6} \times \frac{90}{2} = 37.5 \). Hence the equivalent uniform load per linear foot of girder is \( 131,000/37.5 = 3490 \) lbs.

The maximum shear in the outer girder is
\[ V_1 = 43.6650 \times 3490 = 152,500 \text{ lbs.} \]

and that in the inner girder is
\[ V_2 = 36.7965 \times 3490 = 126,500 \text{ lbs.} \]

The horizontal shear \( (= 2 \times 0.1416 \times 131,000 = 37,100 \text{ lbs.}) \) produced by the centrifugal force will be wholly taken by the diagonals of the lateral system and will hence not affect the girders.

\[ (2) \text{ Maximum Moments.} \]

The influence line for maximum bending moment in a girder for straight track and central loading is shown in Fig. 19 (a).
The influence ordinates below the panel points B (F), C (E), and D are respectively 7.5, 15 and 22.5.

The calculations for the influence ordinates for the outer and inner girders are as follows:

**Panel points B and F.**

\[ z = 7.5 \quad e = -0.0713 \]

**Outer Girder: Ord.**

\[ \frac{7.5}{17.5} (17.5 - 2 \times 0.0713 + 2 \times 1.2744) = 8.5312 \]

**Inner Girder: Ord.**

\[ \frac{7.5}{17.5} (17.5 + 2 \times 0.0713) = 7.5611 \]

**Panel points C and E**

\[ z = 15 \quad e = 0.2820 \]

**Outer Girder: Ord.**

\[ \frac{15}{17.5} (17.5 + 2 \times 0.2820 + 2 \times 1.2744) = 17.6681 \]

**Inner Girder: Ord.**

\[ \frac{15}{17.5} (17.5 - 2 \times 0.2820) = 14.5166 \]

**Panel point D**

\[ z = 22.5 \quad e = 0.3998 \]

**Outer Girder: Ord.**

\[ \frac{22.5}{17.5} (17.5 + 2 \times 0.3998 + 2 \times 1.2744) = 26.8051 \]

**Inner Girder: Ord.**

\[ \frac{22.5}{17.5} (17.5 - 2 \times 0.3998) = 21.4719 \]

The influence ordinates thus found are shown in Fig. 19 (b) and (c).

The area of the influence lines for the outer girder (Fig. 19 (b)) is

\[ A = 15 (2 \times 8.5312 + 2 \times 17.6681 + 26.8051) \]

\[ = 15 \times 79.2037 = 1188.0555 \]
and that for the inner girder (Fig. 19 (c)) is

\[
A_2 = 15 \left( 2 \times 7.5611 + 2 \times 14.5166 + 21.4719 \right)
= 15 \times 65.6273 = 984.4095
\]

For Cooper's E-50 loading, the maximum bending moment in the girder for straight track is 3,337,000 ft.-lbs. The area of influence line (Fig. 19 (a)) is 22.5 \times \frac{90}{2} = 1012.5. The equivalent uniform load is therefore equal to \(\frac{3,337,000}{1012.5} = 3295\) lbs. per foot of girder.
For curved track, the bending moment in the outer girder is
\[ M_1 = 1188.0555 \times 3295 = 3,914,600 \text{ ft.}-\text{lbs.} \]
The bending moment in the inner girder is
\[ M_2 = 984.4095 \times 3295 = 3,243,600 \text{ ft.}-\text{lbs.} \]
Since the bottom flanges of the two girders constitute the chord members for the lateral system, they are subjected to the direct stresses produced by the centrifugal force. The maximum bending moment at the middle point of either girder due to the centrifugal force (\( = q \times \text{vertical load} \)) is equal to \( q \times \text{total vertical bending moment in the two girders for straight track, or} \)
\[ M_h = 2 \times .1416 \times 3,337,000 = 945,000 \text{ ft.}-\text{lbs.} \]
The distance between the center of the two girders is 17.5 ft. Hence the direct stress in the bottom flange of either girder is 945,000/17.5 = 54,000 lbs. and is tensile for the outer girder and compressive for the inner.

The shears, bending moments and direct stresses are summarized in Table 3.

Table 3.
Shears, Moments and Direct Stresses in a Through Plate Girder Bridge of 90-ft. Span on a 6° Curve.

<table>
<thead>
<tr>
<th></th>
<th>Shears in lbs.</th>
<th>Moments in ft.-lbs.</th>
<th>Direct Stresses in Bottom flange in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge on Tangent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each Girder</td>
<td>131,000</td>
<td>3,337,000</td>
<td>0</td>
</tr>
<tr>
<td>Bridge on Curve</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer Girder</td>
<td>152,500</td>
<td>3,914,600</td>
<td>+ 54,000</td>
</tr>
<tr>
<td>Inner Girder</td>
<td>128,500</td>
<td>3,243,600</td>
<td>- 54,000</td>
</tr>
</tbody>
</table>
Art. 11. Arrangement for Equalization of Moments.

From Table 3 shown above, it is noticed that both the shear and moment in the outer girder are still much greater than those in the inner, though the overturning effect of the centrifugal force upon the inner girder has been disregarded. Since the spacing of the two girders is fixed by the required clearance, it is doubtful whether an arrangement for equalizing bending moments would be applicable as the distance between girders would then be increased. But for a theoretical treatment we shall try to determine the location of the girders with reference to the track so as to have equal bending moments in the two girders.

In the last article in finding the moments, the influence lines and the same equivalent uniform load were used. It is then obvious that if the areas of influence lines for moments for both girders are made equal, the bending moments will be equal.
In Fig. 20, let $E_a$, $E_b$, $E_c$ and $E_d$ be the net eccentricities at the panel joints A, B, C and D respectively and $M_{90}$, $M_{60}$ and $M_{30}$ be the middle ordinates for chords of 30, 60, and 90 feet. We have

$$
E_a = E_d - M_{90} = Ed - 1.0598
$$

$$
E_b = E_d - M_{60} = Ed - .4712
$$

$$
E_c = E_d - M_{30} = Ed - .1178
$$

If $e_b$, $e_c$, $e_d$ represent the average eccentricities for the panel joints B, C, D respectively, then we have

$$
e_b = \frac{1}{4}[E_d - 1.0598 + 2(E_d - .4712) + (E_d - .1178)] = Ed - .5300
$$

$$
e_c = \frac{1}{4}[E_d - .4712 + 2(E_d - .1178) + Ed] = E_d - .1767
$$

$$
e_d = \frac{1}{4}[2(E_d - .1178) + 2Ed] = E_d - .0589
$$

The influence ordinates for the outer and inner girders will be found from $\text{Ordin.} = \frac{Z}{b} (b + 2e + 2qh)$ and $\text{Ordin.} = \frac{Z}{b} (b - 2e)$ respectively, $Z$ being the influence ordinates for straight track.

**Panel point B.**

$Z = 7.5 \quad e_b = E_d - .5300$, $qh = 1.2744$

Outer Girder: Ord. = $\frac{7.5}{b} \left[ b + 2(E_d - .5300) + 2 \times 1.2744 \right] = \frac{7.5}{b} (b + 2E_d + 1.4868)$.

Inner Girder: Ord. = $\frac{7.5}{b} \left[ b - 2(E_d - .5300) \right] = \frac{7.5}{b} (b - 2E_d + 1.0600)$

**Panel point C**

$Z = 15 \quad e_c = E_d - .1767$

Outer Girder: Ord. = $\frac{15}{b} \left[ b + 2(E_d - .1767) + 2 \times 1.2744 \right] = \frac{15}{b} (b + 2E_d + 2.1954)$

Inner Girder: Ord. = $\frac{15}{b} \left[ b - 2(E_d - .1767) \right] = \frac{15}{b} (b - 2E_d + .3534)$
Panel point D

\[ z = 22.5 \quad ed = Ed - 0.0589 \]

Outer Girder: Ord. = \[ \frac{22.5}{b} \left[ b + 2(Ed - 0.0589) + 2 \times 1.2744 \right] = \frac{22.5}{b} (b + 2Ed + 2.4310) \]

Inner Girder: Ord. = \[ \frac{22.5}{b} \left[ b - 2(Ed - 0.0589) \right] = \frac{22.5}{b} (b - 2Ed + 0.1178) \]

The area of influence lines for each girder is equal to the sum of the influence ordinates multiplied by the panel length. Hence the influence area for the outer girder is

\[ A_1 = 15 \left[ 2 \times \frac{7.5}{b} (b + 2Ed + 1.4888) + 2 \times \frac{15}{b} (b + 2Ed + 2.1954) \right. \]
\[ + \frac{22.5}{b} (b + 2Ed + 2.4310) \]
\[ = 15 \times \frac{7.5}{b} (9b + 18Ed + 19.0522). \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1) \]

The influence area for the inner girder is

\[ A_2 = 15 \left[ 2 \times \frac{7.5}{b} (b - 2Ed + 1.0600) + 2 \times \frac{15}{b} (b - 2Ed + 0.5534) + \frac{22.5}{b} (b - 2Ed + 0.1178) \right] \]
\[ = 15 \times \frac{7.5}{b} (9b - 18Ed + 3.8870). \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2) \]

Equating equations (1) and (2), we get

\[ 15 \times \frac{7.5}{b} (9b + 18Ed + 19.0522) = 15 \times \frac{7.5}{b} (9b - 18Ed + 3.8870) \]
\[ 36Ed = -19.0522 + 3.8870 = -15.1652 \]
\[ Ed = -0.4213 \text{ ft.} \]

which is the required net eccentricity at the panel point D (mid-span) for the equalization of bending moments in both girders.

The distance c between the center line of track and the curve of center of gravity of train at mid-span is .7080 ft. Hence the distance from the center line of track at the middle of the span to the center line of bridge is .7080 - .4213 = .2867 feet.
The minimum distance from the center line of track at mid-span to the center line of the inner girder is 9.8474 feet, the required clear distance being 9.0974 ft. and the width of the flange of the girder being taken as 18 inches. Therefore, the minimum distance between the center lines of the bridge and the inner girder is 9.8474 - .2867 = 9.5607 feet. Make the distance between center lines of girders equal to 19' - 3", or \( b = 19.25 \) ft., \( \frac{b}{2} = 9.6250 \) ft. The spacing of girders here is changed from 17.5 ft. to 19.25 ft., the increase being 1.75 ft.

Using \( E_d = -.4213 \), the net eccentricities at other panel joints are found as follows:

Panel joint A, \( E_a = E_d - M_{90} = -.4213 - 1.0598 = -1.4811 \) ft.

Panel joint B, \( E_b = E_d - M_{60} = -.4213 - .4712 = -.8925 \) ft.

Panel joint C, \( E_c = E_d - M_{30} = -.4213 - .1178 = -.5391 \) ft.

These net eccentricities are shown in Fig. 21.

![Fig. 21.](image)

The average eccentricities are:

For joint B, \( e_b = \frac{1}{4} \left( -1.4811 - 2 \times .8925 - .5391 \right) = -.9513 \) ft.

For joint C, \( e_c = \frac{1}{4} \left( -.8925 - 2 \times .5391 - .4213 \right) = -.5980 \) ft.

For joint D, \( e_d = \frac{1}{4} \left( -.5391 - 2 \times .4213 - .5391 \right) = -.4802 \) ft.
Having determined the average eccentricities, the influence ordinates for shears and moments can be easily found from Ordin. = \( Z/(b + 2e + 2qh) \) or Ordin. = \( Z/(b - 2e) \), according to whether they are for the outer or for the inner girder.

(1) Maximum Moments.

Panel points B and F: \( Z = 7.5, \ e = -.9513, \ b = 19.25, \ qh = 1.2744 \)

Outer Girder: Ord. = \( \frac{7.5}{19.25} \) \((19.25 - 2 \times .9513 + 2 \times 1.2744) = 7.7518 \)

Inner Girder: Ord. = \( \frac{7.5}{19.25} \) \((19.25 + 2 \times .9513) = 8.2413 \).

Panel points C and E: \( Z = 15, \ e = -.5980 \)

Outer Girder: Ord. = \( \frac{15}{19.25} \) \((19.25 - 2 \times .5980 + 2 \times 1.2744) = 16.0541 \).

Inner Girder: Ord. = \( \frac{15}{19.25} \) \((19.25 + 2 \times .5980) = 15.9319 \).

Panel point D: \( Z = 22.5, \ e = -.4802 \)

Outer Girder: Ord. = \( \frac{22.5}{19.25} \) \((19.25 - 2 \times .4802 + 2 \times 1.2744) = 24.3566 \).

Inner Girder: Ord. = \( \frac{22.5}{19.25} \) \((19.25 + 2 \times .4802) = 23.6225 \).

The influence lines are shown in Fig. 22.

The area of influence lines for the outer girder (Fig. 22 (b)) is

\[ A_1 = 15 \times (2 \times 7.7518 + 2 \times 16.0541 + 24.3566) = 15 \times 71.9684 = 1079.5260 \]

and that for the inner girder (Fig. 22 (c)) is

\[ A_2 = 15 \times (2 \times 8.2413 + 2 \times 15.9319 + 23.6225) = 15 \times 71.9689 = 1079.5335 \]
If we use an equivalent uniform load of 3295 lbs. per foot of girder, then each girder will have a bending moment of $1079.5 \times 3295 = 3,557,000$ ft.-lbs.

The direct stress in the bottom flanges of the girders is $945,000 / 19.25 = 49,000$ lbs., being in tension for the outer and in compression for the inner girder.
(2) Maximum Shears.

The influence ordinates for shears are computed as follows:

**Panel point B**

\[ Z = \frac{5}{6} \quad e = -0.9513 \]

Outer Girder: Ord. \[ = \frac{5}{6 \times 19.25} (19.25 - 2 \times 0.9513 + 2 \times 1.2744) = 0.8613 \]

Inner Girder: Ord. \[ = \frac{5}{6 \times 19.25} (19.25 + 2 \times 0.9513) = 0.9157 \]

**Panel point C**

\[ Z = \frac{2}{3} \quad e = -0.5980 \]

Outer Girder: Ord. \[ = \frac{2}{3 \times 19.25} (19.25 - 2 \times 0.5980 + 2 \times 1.2744) = 0.7133 \]

Inner Girder: Ord. \[ = \frac{2}{3 \times 19.25} (19.25 + 2 \times 0.5980) = 0.7081 \]

**Panel point D**

\[ Z = \frac{1}{2} \quad e = -0.4802 \]

Outer Girder: Ord. \[ = \frac{1}{2 \times 19.25} (19.25 - 2 \times 0.4802 + 2 \times 1.2744) = 0.5413 \]

Inner Girder: Ord. \[ = \frac{1}{2 \times 19.25} (19.25 + 2 \times 0.4802) = 0.5249 \]

**Panel point E**

\[ Z = \frac{1}{3} \quad e = -0.5980 \]

Outer Girder: Ord. \[ = \frac{1}{3 \times 19.25} (19.25 - 2 \times 0.5980 + 2 \times 1.2744) = 0.3567 \]

Inner Girder: Ord. \[ = \frac{1}{3 \times 19.25} (19.25 + 2 \times 0.5980) = 0.3540 \]

**Panel point F**

\[ Z = \frac{1}{6} \quad e = -0.9513 \]

Outer Girder: Ord. \[ = \frac{1}{6 \times 19.25} (19.25 - 2 \times 0.9513 + 2 \times 1.2744) = 0.1723 \]

Inner Girder: Ord. \[ = \frac{1}{6 \times 19.25} (19.25 + 2 \times 0.9513) = 0.1931 \]
The influence lines are shown in Fig. 23, (a) being for girders on straight track, and (b) and (c) being for the outer and inner girders on curved track.

The area of influence lines for the outer girder is

\[
A_1 = 15 \left( .6613 + .7133 + .5413 + .3567 + .1723 \right) \\
= 15 \times 2.6449 = 39.6735
\]

and that for the inner girder is

\[
A_2 = 15 \left( .9157 + .7061 + .5249 + .3540 + .1931 \right) \\
= 15 \times 2.6958 = 40.4370.
\]
Using an equivalent uniform load of 3490 lbs. per foot of girder, we have the maximum shear in the outer girder

\[ V_1 = 39.6735 \times 3490 = 138,600 \text{ lbs.} \]

and the maximum shear in the inner girder is

\[ V_2 = 40.4370 \times 3490 = 141,200 \text{ lbs.} \]

The shears and moments found above are now tabulated in Table 4.

**TABLE 4.**

Shears, Moments and Direct Stresses
in a Through Plate Girder Bridge of 90-ft. Span on a 6° Curve.

<table>
<thead>
<tr>
<th></th>
<th>Shears in lbs.</th>
<th>Moments in ft. lbs.</th>
<th>Direct Stresses in Bottom Flange in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge on Tangent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each Girder</td>
<td>131,000</td>
<td>3,337,000</td>
<td>0</td>
</tr>
<tr>
<td>Bridge on Curve</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer Girder</td>
<td>138,600</td>
<td>3,557,000</td>
<td>+ 49,000</td>
</tr>
<tr>
<td>Inner Girder</td>
<td>141,200</td>
<td>3,557,000</td>
<td>- 49,000</td>
</tr>
</tbody>
</table>

By looking at Table 4, we find that not only the maximum bending moments in both girders are made equal, but the maximum shears are also nearly equal, which fact is very desirable so far as the girders only are concerned. But in securing such an arrangement the spacing between the center lines of girders must be at least 19.25 feet instead of 17.5 feet which is the required minimum spacing. This change of spacing of girders would no doubt increase the floor beams and laterals in sections and lengths, the cost of which might offset the saving derived from having the two girders being made of the same section.
<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>Row 2</td>
<td>Row 3</td>
</tr>
<tr>
<td>Value 1</td>
<td>Value 2</td>
<td>Value 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above shows the data for various categories. The values in Column 1 range from 1 to 10, while those in Column 2 are from 20 to 50. Column 3 contains a mix of letters and numbers.
CHAPTER IV.

TRUSSED BRIDGES


The formulae for determining the required clear width for through bridges have been found in Art. 9 of the last chapter; that is,

\[ X = \frac{W}{2} + M_a - M_b - \frac{h_s}{t} \]  
and  
\[ Y = \frac{W}{2} + M_b - M_c + \frac{h_s}{t} \]

in which \( X \) and \( Y \) are the clear distances from the center line of track at the middle of the span to the outer and inner trusses respectively. The total clear width is \( X + Y \).

Fig. 24.
For the present chapter, let us assume a through truss bridge of 200-foot span on a 5° curve. Also let the panel length be 25 feet, the height of truss be 34 feet, and the height of the center of gravity of trains be 6.5 feet above the top laterals of the stringers and 10 feet above the bottom laterals of the trusses. Fig. 24 shows the type of truss and the general dimensions.

The other data are taken from Table A of Art. 5 as follows:

Radius of curve, \( r = 1146.28 \) ft.

Centrifugal force coefficient, \( q = 0.1315 \)

Super-elevation of outer rail, \( s = 0.6496 \) ft.

Distance between center line of track and curve of center of gravity of train, \( c = 0.6575 \) ft.

The middle ordinates for chords A, B, C are found from the approximate formula \( M = \frac{12}{5r} \), thus

For \( A = 80' \) \( M_A = 0.6979 \) ft.

" \( B = 60' \) \( M_B = 0.3926 \) ft.

" \( C = 175' \) \( M_C = 3.3396 \) ft.

Further, we have \( w = 14', h = 4', \) and \( H = 14'. \) Hence from Equation (I) we find

\[
X = \frac{14}{2} + 0.6979 - 0.3926 - 4 \times \frac{0.6496}{4.94} \\
= 7 + 0.6979 - 0.3926 - 0.5260 \\
= 6.7793 \text{ ft.}
\]

and from Equation (J) we find

\[
Y = \frac{14}{2} + 0.3926 + 3.3396 + 14 + \frac{0.6496}{4.94} \\
= 7 + 0.3926 + 3.3396 + 1.8410 \\
= 12.5732 \text{ ft.}
\]

These are the minimum required clear distances. Assuming the width of each truss as 28 inches, the required distances from
the center line of track at mid-span to the center lines of the outer and inner girders are respectively

\[ X^1 = 6.7793 + 1.6667 = 7.9460 \text{ ft. using } 8'-0" = 8.00 \text{ ft.} \]

and

\[ Y^1 = 12.5732 + 1.1667 = 13.7399 \text{ ft. using } 13'-9" = 13.75 \text{ ft.} \]

The distance between center lines of trusses is therefore equal to \( 8.00 + 13.75 = 21.75 \text{ ft.} \)

The distance \( c \) is \( .6575 \text{ ft.} \) Hence the distance from the curve of the center of gravity of train at mid-span to the center line of the outer truss is

\[ 8.0000 + .6575 = 8.6575 \text{ ft.} \]

and that to the center line of the inner truss is

\[ 13.7500 - .6575 = 13.0925 \text{ ft.} \]

**Art. 13. Stresses in Main Trusses.**

Since the bridge under investigation has parallel-chord trusses, the stresses in all the truss members can be found from the shears in the panels or from the bending moments at the panel points. Like what has been done in the last chapter, influence lines will be employed to find the shears or moments in the trusses and the influence ordinates will be computed from Ordin. \( = \frac{Z}{b}(b + 2e + 2qh) \) for the outer and Ordin. \( = \frac{Z}{b}(b - 2e) \) for the inner trusses, the overturning effect of the centrifugal force having been disregarded.

In order to compute the influence ordinates for shears, moments or end reactions, we must first determine the average eccentricities for the several panel joints. Let \( E_e, E_d, E_c, \) etc., be the net eccentricities for the panel joints \( E, D, C, \) etc., respectively. From Fig. 25, we find

\[ E_d = E_e - M50, \quad E_c = E_e - M100, \text{ etc.} \]
The middle ordinates $M_{50}$, $M_{100}$, etc., will be found from

$$M = \frac{12}{8r}$$

(approximately). Thus

When $l = 50'$

$M_{50} = 0.2726$ ft.

" $l = 100'$

$M_{100} = 1.0905$ ft.

" $l = 150'$

$M_{150} = 2.4536$ ft.

" $l = 200'$

$M_{200} = 4.3619$ ft.

The net eccentricities are:

At $E$, $E_e = \frac{b}{r} - 8.6575 = \frac{1}{2} \times 21.75 - 8.6575 = 2.2175$ ft.

" D. $E_d = E_e - M_{50} = 2.2175 - 0.2726 = 1.9449$ ft.

" C. $E_c = E_e - M_{100} = 2.2175 - 1.0905 = 1.1270$ ft.

" B. $E_b = E_e - M_{150} = 2.2175 - 2.4536 = -0.2361$ ft.

" A. $E_a = E_e - M_{200} = 2.2175 - 4.3619 = -2.1444$ ft.

Fig. 26 shows these net eccentricities.
The average eccentricities for the several panel joints are taken as the average for a half-panel each side of the joints and are computed as follows:

For E, \( e_e = \frac{1}{4} (1.9449 + 2 \times 2.2175 + 1.9449) = 2.0812 \text{ ft.} \)

" D, \( e_d = \frac{1}{4} (1.1270 + 2 \times 1.9449 + 2.2175) = 1.8086 \text{ ft.} \)

" C, \( e_c = \frac{1}{4} (-.2361 + 2 \times 1.1270 + 1.9449) = .9907 \text{ ft.} \)

" B, \( e_b = \frac{1}{4} (-2.1444 - 2 \times .2361 + 1.1270) = -.3724 \text{ ft.} \)

" A, \( e_a = \frac{1}{4} (-2.1444 - .2361) = -1.1903 \text{ ft.} \)

Having determined the average eccentricities for the different panel joints, we may proceed to find the ordinates of the influence lines for the end reactions, shears and moments for the outer and inner trusses by the formulae

\[ \text{Ordin.} = \frac{Z}{b} (b + 2e + 2qh) \]  
and \( \text{Ordin.} = \frac{Z}{b} (b - 2e) \) respectively. In all cases, \( b = 21.75', \) \( q = .1315, \) \( h = 10', \) and \( 2qh = 2 \times .1315 \times 10 = 2.6300. \)

\(1\) End Reactions.

In Fig. 27 (a) is shown the influence line for the end reaction of each truss for straight track. The influence lines for the outer and inner trusses for curved track are shown in (b) and (c) respectively. The influence ordinates for Fig. 27 (b) and (c) are computed as follows:

Panel point A:

\[ Z = 1 \quad e = -1.1903 \]

Outer Truss: Ord. = \( \frac{1}{21.75} (21.75 - 2 \times 1.1903 + 2.6300) = 1.0115 \)

Inner Truss: Ord. = \( \frac{1}{21.75} (21.75 + 2 \times 1.1903) = 1.1095. \)

Panel point B:

\[ Z = \frac{7}{8} \quad e = -.3724 \]

Outer Truss: Ord. = \( \frac{7}{8 \times 21.75} (21.75 - 2 \times .3724 + 2.6300) = .9508 \)

Inner Truss: Ord. = \( \frac{7}{8 \times 21.75} (21.75 + 2 \times .3724) = .9050. \)
Fig. 27
Influence Lines for Reactions
Panel point C: 

\[ Z = \frac{3}{4} \quad e = .9907 \]

Outer Truss: Ord. \[= \frac{3}{4x21.75}(21.75 + 2 \times .9907 + 2.6300) = .9090 \]

Inner Truss: Ord. \[= \frac{3}{4x21.75}(21.75 - 2 \times .9907) = .6817 \]

Panel point D: 

\[ Z = \frac{5}{8} \quad e = 1.8086 \]

Outer Truss: Ord. \[= \frac{5}{4x21.75}(21.75 + 2 \times 1.8086 + 2.6300) = .8045 \]

Inner Truss: Ord. \[= \frac{5}{4x21.75}(21.75 - 2 \times 1.8086) = .5211 \]

Panel point E: 

\[ Z = \frac{1}{2} \quad e = 2.0812 \]

Outer Truss: Ord. \[= \frac{1}{2x21.75}(21.75 + 2 \times 2.0812 + 2.6300) = .6561 \]

Inner Truss: Ord. \[= \frac{1}{2x21.75}(21.75 - 2 \times 2.0812) = .4043 \]

Panel point F: 

\[ Z = \frac{3}{8} \quad e = 1.8086 \]

Outer Truss: Ord. \[= \frac{3}{8x21.75}(21.75 + 2 \times 1.8086 + 2.6300) = .4827 \]

Inner Truss: Ord. \[= \frac{3}{8x21.75}(21.75 - 2 \times 1.8086) = .3126 \]

Panel point G: 

\[ Z = \frac{1}{4} \quad e = .9907 \]

Outer Truss: Ord. \[= \frac{1}{4x21.75}(21.75 + 2 \times .9907 + 2.6300) = .3030 \]

Inner Truss: Ord. \[= \frac{1}{4x21.75}(21.75 - 2 \times .9907) = .2272 \]

Panel point H: 

\[ Z = \frac{1}{8} \quad e = -.3724 \]

Outer Truss: Ord. \[= \frac{1}{8x21.75}(21.75 - 2 \times .3724 + 2.6300) = .1536 \]

Inner Truss: Ord. \[= \frac{1}{8x21.75}(21.75 + 2 \times .3724) = .1293 \]
The area of the influence line for the end reactions of each truss for straight track is 100.00 and for Cooper's E-50 loading the maximum reaction is 326,300 lbs. Hence, the equivalent uniform load is 3265 lbs. per foot of truss.

Now, the areas of the influence lines for the outer truss (Fig. 27 (b)) and for the inner truss (Fig. 27 (c)) are respectively 118.69 and 93.40. Therefore, the maximum end reaction for the outer truss is

\[ R_1 = 118.69 \times 3265 = 387,500 \text{ lbs.} \]

and that for the inner truss is

\[ R_2 = 93.40 \times 3265 = 305,000 \text{ lbs.} \]

The reactions are shown in Table 5.

<table>
<thead>
<tr>
<th>TABLE 5.</th>
<th>Maximum End Reactions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactions</td>
<td>Influence Areas</td>
</tr>
<tr>
<td>in lbs.</td>
<td>in lbs.</td>
</tr>
<tr>
<td>Bridge on Tangent</td>
<td>326,300</td>
</tr>
<tr>
<td>Each Truss</td>
<td></td>
</tr>
<tr>
<td>Bridge on Curve Outer Truss</td>
<td>387,500</td>
</tr>
<tr>
<td>Bridge on Curve Inner Truss</td>
<td>305,000</td>
</tr>
</tbody>
</table>

(2) Maximum Shears.

The influence lines for shears in the trusses for straight track and for curved track are shown in Figs. 28-30, and their influence ordinates are the same as those for the end reactions, except that some of them are of opposite signs. The computations will not be repeated here.
Fig. 28
Influence Lines for Shears in Each Truss for Straight Track
Fig. 27
Influence Lines for Shears in Outer Truss for Curved Track
Shear in AB

Shear in BC, GH

Shear in CD, FG

Shear in DE, EF

Fig. 30
Influence Lines for Shears in Inner Truss for Curved Track
Table 6 gives the maximum shears in the several panels in each truss for straight track found from Cooper's E-50 loading. The areas of influence lines are found from Fig. 28. The equivalent uniform load for shear in a panel is equal to the maximum shear in that panel divided by the corresponding influence area.

The maximum shears in the outer and inner trusses for curved track are shown in Table 7. They are found by multiplying the influence areas by the corresponding equivalent uniform loads shown in Table 6. Thus, for example, the area of influence lines for shear in panel A B of the outer truss is 106.05 and from Table 6, the equivalent uniform load for shear in A B is 3100 lbs. per foot. Hence, the shear in A B of the outer truss is equal to 106.05 × 3100 = 328,800 lbs.

**TABLE 6**
Shears in Each Truss for Straight Track.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Shears in lbs.</th>
<th>Influence Areas</th>
<th>Equivalent Uniform Load in ft./lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B</td>
<td>271,500</td>
<td>87.50</td>
<td>3100</td>
</tr>
<tr>
<td>B C</td>
<td>204,900</td>
<td>64.28</td>
<td>3190</td>
</tr>
<tr>
<td>C D</td>
<td>147,500</td>
<td>44.64</td>
<td>3305</td>
</tr>
<tr>
<td>D E</td>
<td>98,000</td>
<td>28.57</td>
<td>3430</td>
</tr>
<tr>
<td>E F</td>
<td>56,400</td>
<td>16.07</td>
<td>3510</td>
</tr>
<tr>
<td>F G</td>
<td>26,900</td>
<td>7.14</td>
<td>3770</td>
</tr>
<tr>
<td>G H</td>
<td>6,850</td>
<td>1.78</td>
<td>3850</td>
</tr>
</tbody>
</table>

(3) Maximum Moments.

In Figs. 31-33 are shown the influence lines for the
Fig. 31
Influence Lines for Moments in Each Truss for Straight Track.
Fig. 32
Influence Lines for Moments
in Outer Truss for Curved Track
Fig. 33
Influence Lines for Moments in Inner Truss for Curved Track
TABLE 7.

Shears in Both Trusses for Curved Track.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Equivalent Uniform Loads in lbs./ft.</th>
<th>Outer Truss Influence Areas</th>
<th>Shears in lbs.</th>
<th>Inner Truss Influence Areas</th>
<th>Shears in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B</td>
<td>3100</td>
<td>106.05</td>
<td>328,800</td>
<td>79.53</td>
<td>246,500</td>
</tr>
<tr>
<td>B C</td>
<td>3190</td>
<td>80.82</td>
<td>257,300</td>
<td>55.54</td>
<td>177,200</td>
</tr>
<tr>
<td>C D</td>
<td>3305</td>
<td>56.82</td>
<td>187,800</td>
<td>37.88</td>
<td>125,200</td>
</tr>
<tr>
<td>D E</td>
<td>3430</td>
<td>35.96</td>
<td>123,300</td>
<td>24.63</td>
<td>84,500</td>
</tr>
<tr>
<td>E F</td>
<td>3510</td>
<td>19.56</td>
<td>68,700</td>
<td>14.52</td>
<td>51,000</td>
</tr>
<tr>
<td>F G</td>
<td>3770</td>
<td>8.21</td>
<td>31,000</td>
<td>6.94</td>
<td>26,200</td>
</tr>
<tr>
<td>G H</td>
<td>3850</td>
<td>1.92</td>
<td>7,390</td>
<td>1.88</td>
<td>7,240</td>
</tr>
</tbody>
</table>

Bending moments at the panel points in the trusses for straight track and for curved track. The influence ordinates for the outer and inner trusses for curved track are found in the same manner as those for reactions or shears from the formulae $\text{Ordin.} = \frac{2}{b}(b + 2e + 2qh)$ and $\text{Ordin.} = \frac{2}{b}(b - 2e)$ respectively. Their calculations will not be shown here.

The maximum bending moments at different panel points in each truss for straight track from Cooper's E-50 loading are given in Table 8 with the influence areas and equivalent uniform loads. The influence areas and the bending moments for the outer and inner trusses for curved track are given in Table 9.

(4) Stresses.

**Top and Bottom Chords.** The stress in a chord member is equal to the bending moment at the opposite panel point divided by the height of the truss. For instance, the stress in the top chord
TABLE 8.

Moments in Each Truss for Straight Track.

<table>
<thead>
<tr>
<th>Panel Point</th>
<th>Moments in ft.-lbs.</th>
<th>Influence Areas</th>
<th>Equivalent Uniform Loads in lbs./ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>6,787,000</td>
<td>2187.5</td>
<td>3100</td>
</tr>
<tr>
<td>C</td>
<td>11,244,000</td>
<td>3750.0</td>
<td>3000</td>
</tr>
<tr>
<td>D</td>
<td>14,010,000</td>
<td>4687.5</td>
<td>2990</td>
</tr>
<tr>
<td>E</td>
<td>14,820,000</td>
<td>5000.0</td>
<td>2985</td>
</tr>
</tbody>
</table>

TABLE 9.

Moments in Both Trusses for Curved Track.

<table>
<thead>
<tr>
<th>Panel Point</th>
<th>Equivalent Uniform Loads in lbs./ft.</th>
<th>Outer Truss Influence Areas</th>
<th>Moments in ft.-lbs.</th>
<th>Inner Truss Influence Areas</th>
<th>Moments in ft.-lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>3100</td>
<td>2551.29</td>
<td>8,219,000</td>
<td>1988.22</td>
<td>6,164,000</td>
</tr>
<tr>
<td>C</td>
<td>3000</td>
<td>4623.41</td>
<td>13,870,000</td>
<td>3330.03</td>
<td>9,990,000</td>
</tr>
<tr>
<td>D</td>
<td>2990</td>
<td>5838.02</td>
<td>17,456,000</td>
<td>4103.78</td>
<td>12,270,000</td>
</tr>
<tr>
<td>E</td>
<td>2985</td>
<td>6248.11</td>
<td>18,651,000</td>
<td>4356.48</td>
<td>13,004,000</td>
</tr>
</tbody>
</table>

members bd is equal to the bending moment at the panel point C divided by the height of the truss which is 34 feet. The bending moment at C for straight track is 11,244,000 ft.-lbs. Hence the stress in bd is $\frac{11,244,000}{34} = 330,700$ lbs. in compression. The stress in df = $M_{at \Delta} \div 34$, in AC = $M_{at B} \div 34$, and in CE = $M_{at D} \div 34$.

End Posts and Diagonals. The stress in an inclined member is equal to the shear in the corresponding panel multiplied by the ratio of the length of the member to the height of the truss. The length of the diagonal is 42.2 ft. and the ratio of the length of the
member to the height of the truss is therefore \( \frac{42.2}{34} \). For example, the shear in the panel AB for straight track is 271,500 lbs. and the stress in the end post AB is then equal to 271,500 \( \times \frac{42.2}{34} \) = 336,900 lbs. in compression. The stresses in other inclined members are found in like manner.

**Hangers.** When a bridge is on a tangent with central loading, the stress in a hanger is equal to the maximum floor beam load. But if the bridge is on a curve, then the stress in the hanger in the outer truss will be \( \frac{P}{b} (b + 2e + 2qh) \) and that in the inner truss will be \( \frac{P}{b} (b - 2e) \), where \( P \) is the floor beam load (one rail). For Cooper's E-50 loading, the maximum floor beam load for one rail is 94,600 lbs. Hence the stress in each hanger for straight track is 94,600 lbs.

For curved track, the stresses in the hangers will be found as follows:

**Hanger b B**  
\( b = 21.75 \),  \( e = -0.3724 \),  \( 2qh = 2.6300 \)

Outer Truss: Stress = \( \frac{94,600}{21.75} (21.75 - 2 \times 0.3724 + 2.6300) = 102,800 \) lbs.

Inner Truss: Stress = \( \frac{94,600}{21.75} (21.75 + 2 \times 0.3724) = 97,800 \) lbs.

**Hanger d D**  
\( e = 1.8086 \)

Outer Truss: Stress = \( \frac{94,600}{21.75} (21.75 + 2 \times 1.8086 + 2.6300) = 121,800 \) lbs.

Inner Truss: Stress = \( \frac{94,600}{21.75} (21.75 - 2 \times 1.8086) = 78,800 \) lbs.

The stresses in all the members in the trusses are shown in Table 10.

It has been shown before that the centrifugal force causes a horizontal force which is to be carried by the lateral system. For simplicity, we shall assume that the horizontal force is applied at the panel points of the outer truss only. The assumption as to the distribution of the horizontal force between the panel points of the outer and inner trusses has no effect upon the stresses in the chords and diagonals, except those in the struts (the floor-beams). Fig. 34 shows the lateral system and the forces acting.
According to the usual assumption, the diagonals will be considered to take tensions only. In Fig. 35 (a) and (b) are shown the diagonals which are in action for maximum moments at the panel points and maximum shears in the panels respectively for a train going from right to left.

Since the centrifugal force at any point is equal to \( q \) times the total vertical load at that point, it follows that the shears in the panels and the moments at the panel points in the lateral truss due to the horizontal force (= centrifugal force) are
respectively equal to $2q$ times the shears and moments in one vertical truss (for straight track and central loading) due to the vertical load. The shears and moments in the vertical truss for straight track have already been found and shown in Table 6 and Table 8 respectively. The shears and moments in the lateral truss will be shown in the following two tables. The centrifugal force coefficient $q$ is .1315, and $2q = .2630$

**TABLE 11.**

Shears in Lateral Truss.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Shears in Vertical Truss in lbs.</th>
<th>$2q$</th>
<th>Shears in Lateral Truss in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>271,500</td>
<td>.2630</td>
<td>71,400</td>
</tr>
<tr>
<td>BC</td>
<td>204,900</td>
<td>.2630</td>
<td>53,900</td>
</tr>
<tr>
<td>CD</td>
<td>147,500</td>
<td>.2630</td>
<td>38,800</td>
</tr>
<tr>
<td>DE</td>
<td>98,000</td>
<td>.2630</td>
<td>25,800</td>
</tr>
<tr>
<td>EF</td>
<td>56,400</td>
<td>.2630</td>
<td>14,800</td>
</tr>
<tr>
<td>FG</td>
<td>26,900</td>
<td>.2630</td>
<td>7,100</td>
</tr>
<tr>
<td>GH</td>
<td>6,850</td>
<td>.2630</td>
<td>1,800</td>
</tr>
</tbody>
</table>

**TABLE 12.**

Moments in Lateral Truss.

<table>
<thead>
<tr>
<th>Panel Point</th>
<th>Moments in Vertical Truss in lbs.</th>
<th>$2q$</th>
<th>Moments in Lateral Truss in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>6,787,000</td>
<td>.2630</td>
<td>1,785,000</td>
</tr>
<tr>
<td>C</td>
<td>11,244,000</td>
<td>.2630</td>
<td>2,957,000</td>
</tr>
<tr>
<td>D</td>
<td>14,010,000</td>
<td>.2630</td>
<td>3,685,000</td>
</tr>
<tr>
<td>E</td>
<td>14,820,000</td>
<td>.2630</td>
<td>3,898,000</td>
</tr>
</tbody>
</table>
The stresses in the chords are found by dividing the bending moments by the distance between the center lines of trusses which is 21.75 feet. The stresses in the struts (floor beams) are equal to the shears and those in the diagonals are equal to the shears multiplied by the ratio \( \frac{33.1}{21.75} \), 33.1 ft. being the length of the diagonals. The stresses thus found are shown in Table 13 and Table 14.

**TABLE 13.**

Stresses in Chord Members.

<table>
<thead>
<tr>
<th>Outer Chord Member</th>
<th>Stresses in lbs.</th>
<th>Inner Chord Member</th>
<th>Stresses in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B</td>
<td>0</td>
<td>A\textsuperscript{1}B\textsuperscript{1}</td>
<td>- 82,100</td>
</tr>
<tr>
<td>B C</td>
<td>+ 82,100</td>
<td>B\textsuperscript{1}C\textsuperscript{1}</td>
<td>-136,000</td>
</tr>
<tr>
<td>C D</td>
<td>+136,000</td>
<td>C\textsuperscript{1}D\textsuperscript{1}</td>
<td>-169,400</td>
</tr>
<tr>
<td>D E</td>
<td>+169,400</td>
<td>D\textsuperscript{1}E\textsuperscript{1}</td>
<td>-179,200</td>
</tr>
</tbody>
</table>

**TABLE 14.**

Stresses in Web Members.

<table>
<thead>
<tr>
<th>Diagonals Member</th>
<th>Stresses in lbs.</th>
<th>Struts (F B.) Member</th>
<th>Stresses in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A\textsuperscript{1}B</td>
<td>+108,600</td>
<td>A A\textsuperscript{1}</td>
<td>-71,400</td>
</tr>
<tr>
<td>B\textsuperscript{1}C</td>
<td>+82,200</td>
<td>B B\textsuperscript{1}</td>
<td>-53,900</td>
</tr>
<tr>
<td>C\textsuperscript{1}D</td>
<td>+59,100</td>
<td>C C\textsuperscript{1}</td>
<td>-38,800</td>
</tr>
<tr>
<td>D\textsuperscript{1}E</td>
<td>+39,400</td>
<td>D D\textsuperscript{1}</td>
<td>-25,800</td>
</tr>
<tr>
<td>E\textsuperscript{1}F</td>
<td>+22,600</td>
<td>E E\textsuperscript{1}</td>
<td>-14,800</td>
</tr>
<tr>
<td>F\textsuperscript{1}G</td>
<td>+10,800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G\textsuperscript{1}H</td>
<td>+ 2,750</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column 1</td>
<td>Column 2</td>
<td>Column 3</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Value A</td>
<td>Value B</td>
<td>Value C</td>
<td></td>
</tr>
<tr>
<td>Value D</td>
<td>Value E</td>
<td>Value F</td>
<td></td>
</tr>
<tr>
<td>Value G</td>
<td>Value H</td>
<td>Value I</td>
<td></td>
</tr>
<tr>
<td>Value J</td>
<td>Value K</td>
<td>Value L</td>
<td></td>
</tr>
<tr>
<td>Value M</td>
<td>Value N</td>
<td>Value O</td>
<td></td>
</tr>
<tr>
<td>Value P</td>
<td>Value Q</td>
<td>Value R</td>
<td></td>
</tr>
<tr>
<td>Value S</td>
<td>Value T</td>
<td>Value U</td>
<td></td>
</tr>
<tr>
<td>Value V</td>
<td>Value W</td>
<td>Value X</td>
<td></td>
</tr>
<tr>
<td>Value Y</td>
<td>Value Z</td>
<td>Value AA</td>
<td></td>
</tr>
</tbody>
</table>

**Table Data**

- **Column 1**: Description of data
- **Column 2**: Additional details
- **Column 3**: Further information
Art.15. **Stresses in Stringers.**

In Chapter II we have discussed the shears and moments in deck plate girder bridges and have given several equations, which, when modified, may be used to determine the shears and moments in stringers. In the case of deck plate girders as shown in Chapter II, the center line of track is symmetrical about the middle of the span which condition does not exist in stringers in a through truss bridge. Since the span of stringers is usually about 25 feet, and the variation of eccentricities of the load applied to the stringers is not very large, we may take the average eccentricity of the panel as the eccentricity of the load for the stringers and consider it to be constant throughout the span of the stringers. Having obtained the eccentricity (average for a panel) for the applying load, the shear or moment in the outer stringer will be \( \frac{P}{b} (b + 2e + 2qh) \) and that in the inner stringer will be \( \frac{P}{b} (b - 2e) \), where \( P \) is the shear or moment in each stringer for straight track and central loading, \( b \) is the spacing of the stringers and \( h \) is the height of the center of gravity of the trains above the top laterals of the stringers. The effect of the overturning moment of the centrifugal force upon the inner stringer has been neglected. Using the average eccentricity of the panel, this method gives very close results for shear and moment and hence the derivation of exact formula will be unwarranted.

There are two ways by which stringers in a bridge are usually arranged. Firstly, for short spans and flat curves, the stringers are so arranged that the center lines of all pairs of stringers will be coincident with the center line of the bridge. Secondly, for spans and sharp curves, the stringers in each panel
are so located as to conform more or less with the curve of the track. In the second case, though the center lines of the pairs of stringers will in general not coincide with the center line of the bridge, they must be parallel to each other.

**Case 1. Axes of Stringers and Bridge Coincide.**

In this case the axis of each pair of stringers is coincident with the center line of the bridge as shown in Fig. 36, the spacing of stringers being assumed to be 7.5 ft.

![Fig. 36](image)

**Fig. 36.**

Fig. 37 shows the net eccentricities at the panel points as found in Art. 13.

![Fig. 37](image)

**Fig. 37.**

The average eccentricities for the several panels are as follows:
For Panel A B: e = \frac{1}{2}(-2.1444 - .2361) = -1.1903 \text{ ft.}

" " B C: e = \frac{1}{2}(-.2361 + 1.1270) = .4455 \text{ ft.}

" " C D: e = \frac{1}{2}(1.1270 + 1.9449) = 1.5360 \text{ ft.}

" " D E: e = \frac{1}{2}(1.9449 + 2.2175) = 2.0812 \text{ ft.}

Let the height h of the center of gravity of train above the top laterals of the stringers be 6.5 ft. The values of the expressions \frac{1}{b}(b + 2e + 2qh) and \frac{1}{b}(b - 2e) for different panels are computed as follows:

Panel A B, \quad e = -1.1903 \quad 2qh = 2 \times .1315 \times 6.5 = 1.7095

\frac{1}{b}(b + 2e + 2qh) = \frac{1}{7.5}(7.5 - 2 \times 1.1903 + 1.7095) = .911

\frac{1}{b}(b - 2e) = \frac{1}{7.5}(7.5 + 2 \times 1.1903) = 1.317

Panel B C \quad e = .4455

\frac{1}{b}(b + 2e + 2qh) = \frac{1}{7.5}(7.5 + 2 \times .4455 + 1.7095) = 1.347

\frac{1}{b}(b - 2e) = \frac{1}{7.5}(7.5 - 2 \times .4455) = .881

Panel C D \quad e = 1.5360

\frac{1}{b}(b + 2e + 2qh) = \frac{1}{7.5}(7.5 + 2 \times 1.5360 + 1.7095) = 1.637

\frac{1}{b}(b - 2e) = \frac{1}{7.5}(7.5 - 2 \times 1.5360) = .590

Panel D E \quad e = 2.0812

\frac{1}{b}(b + 2e + 2qh) = \frac{1}{7.5}(7.5 + 2 \times 2.0812 + 1.7095) = 1.783

\frac{1}{b}(b - 2e) = \frac{1}{7.5}(7.5 - 2 \times 2.0812) = .445
The shears or moments in the outer stringers are equal to \( \frac{P}{b} (b + 2e + 2qh) \) and those in the inner stringers are equal to \( \frac{P}{b} (b-2e) \), where \( P \) is the shear or moment in each stringer for straight track and central loading. For Cooper’s E-50 loading, the maximum shear and maximum moment in a stringer of 25-ft. span are respectively 71,000 lbs. and 381,300 ft.-lbs. The resulting shears and moments are given in Tables 15-16.

**TABLE 15**
Shears in Stringers.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Shears for Straight Track in lbs.</th>
<th>Shears for Curved Track</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Outer Stringers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{b + 2e + 2qh}{b} ) Shears in lbs.</td>
</tr>
<tr>
<td>A B</td>
<td>71,000</td>
<td>.911</td>
</tr>
<tr>
<td>B C</td>
<td>71,000</td>
<td>1.347</td>
</tr>
<tr>
<td>C D</td>
<td>71,000</td>
<td>1.637</td>
</tr>
<tr>
<td>D E</td>
<td>71,000</td>
<td>1.783</td>
</tr>
</tbody>
</table>

**TABLE 16**
Moments in Stringers.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Moments for Straight Track in lbs.</th>
<th>Moments for Curved Track</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Outer Stringer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{b + 2e + 2qh}{b} ) Moments in ft.-lbs.</td>
</tr>
<tr>
<td>A B</td>
<td>381,300</td>
<td>.911</td>
</tr>
<tr>
<td>B C</td>
<td>381,300</td>
<td>1.347</td>
</tr>
<tr>
<td>C D</td>
<td>381,300</td>
<td>1.637</td>
</tr>
<tr>
<td>D E</td>
<td>381,300</td>
<td>1.783</td>
</tr>
<tr>
<td>Year</td>
<td>Product</td>
<td>Sales</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>2020</td>
<td>Product A</td>
<td>5000</td>
</tr>
<tr>
<td>2021</td>
<td>Product B</td>
<td>7000</td>
</tr>
<tr>
<td>2022</td>
<td>Product C</td>
<td>8000</td>
</tr>
</tbody>
</table>

**Table Notes**
- Product A sales increased by 50% in 2021.
- Product B expenses decreased by 25% in 2022.
- Profit from Product C remains consistent across the years.
Besides the vertical shears and moments shown in the above tables, each stringer is also subjected to a lateral shear of \(71,000 \times 0.1315 = 9,340\) lbs. and a lateral bending moment of \(381,300 \times 0.1315 = 50,100\) ft.-lbs.

**Case 2. Axes of Stringers and Bridge do not Coincide.**

In this case the stringers are not in one straight line and the axis of each pair of the stringers does not coincide with the center line of the bridge. The general arrangement is shown in Fig. 38.

![Fig. 38.](image)

If there are no other requirements for the arrangement, each pair of stringers may be so located with reference to the track that the maximum shears and moments in both stringers will be equal. It has been shown that the shear or moment in the outer stringer is equal to \(\frac{2}{b} \left( b + 2e + 2qh \right)\) and that in the inner stringer is equal to \(\frac{2}{b} \left( b - 2e \right)\), in which \(P\) is the shear or moment in each stringer of a bridge on straight track, \(b\) is the spacing of stringers and \(e\) is the average eccentricity of the track (the curve of center of gravity of train) with reference to the axis of the stringers. In order to equalize the shears or moments in both stringers, we must have

\[ b + 2e + 2qh = b - 2e \]
from which we obtain
\[ e = -\frac{1}{2}qh. \]
that is, each pair of stringers should be so located that the track (the curve of the center of gravity of train) will have an average eccentricity of \(-\frac{1}{2}qh\) with reference to the axis of each pair of stringers.

In the example being investigated
\[ q = .1215 \quad h = 6.5 \]
\[ \therefore \quad e = -\frac{1}{2}qh = -\frac{1}{2} \times .1315 \times 6.5 = -.4274 \text{ ft.} \]

Since we have already determined the net eccentricities of the track at the panel points with reference to the center line of the bridge, the location of the stringers in each panel can be easily found. Fig. 39 shows the locations of the axes of the stringers that will give an average eccentricity of \(-.4274\) ft.

\[
\frac{1}{b}(b + 2e + 2qh) = \frac{1}{7.5}(7.5 - 2 \times .4274 + 1.7095) = 1.1140
\]
\[
\frac{1}{b}(b - 2e) = \frac{1}{7.5}(7.5 + 2 \times .4274) = 1.1140
\]
For a bridge on straight track, each stringer has a shear of 71,000 lbs. and a bending moment of 381,300 ft.-lbs. For the present case, the maximum shear will be

\[ 71,000 \times 1.1140 = 79,100 \text{ lbs.} \]

and the maximum moment will be

\[ 381,300 \times 1.1140 = 424,800 \text{ ft.-lbs.} \]

The lateral shear and moment are as before 9340 lbs. and 50,100 ft.-lbs. respectively.

**Art. 16. Stresses in Floor beams.**

In a bridge on straight track with central loading, the maximum shear on a floor beam is equal to the maximum panel load on either the outer or inner truss; and the maximum bending moment on the floor beam is equal to the product of this panel load multiplied by the distance from the center of either truss to the nearer stringer. If the panel lengths are equal, these stresses will be the same for all floor beams.

When the bridge is on a curve, then from Fig. 40 the maximum panel loads on the outer and inner trusses will be respectively

\[ P_1 = \frac{P}{2b} (b + 2e + 2qh) \]

and \[ P_2 = \frac{P}{2b} (b - 2e) \]

where \( P \) is the total panel load on the floor beam; \( b \), the distance between center lines of trusses (taken as being equal to the length of the floor beam); \( e \), the average eccentricity; and \( h \), the height of the center of gravity of the train above the bottom flange of the floor beam (taken the same as that above the bottom lateral system of the bridge).
The average eccentricities for the several panel joints have been found in Art. 13; that is,

For A A', $e = -1.1903$

" B B', $e = -.3724$

" C C', $e = .9907$

" D D', $e = 1.8086$

" E E', $e = 2.0812$

Let $h = 10$ ft., then $2qh = 2 \times .1315 \times 10 = 2.6300$. The values of the expressions $\frac{1}{b}(b + 2e + 2qh)$ and $\frac{1}{b}(b - 2e)$ are computed as follows:

For floor beam A A', $e = -1.1903$

$$\frac{1}{b}(b + 2e + 2qh) = \frac{1}{21.75}(21.75 - 2 \times 1.1903 + 2.6300) = 1.011$$

$$\frac{1}{b}(b - 2e) = \frac{1}{21.75}(21.75 + 2 \times 1.1903) = 1.109$$
For floor beam B \( B_1 \)

\[ \frac{1}{b} (b + 2e + 2qh) = \frac{1}{21.75} (21.75 - 2 \times 0.3724 + 2.6300) = 1.086 \]

\[ \frac{1}{b} (b - 2e) = \frac{1}{21.75} (21.75 + 2 \times 0.3724) = 1.034 \]

For floor beam C \( C_1 \)

\[ \frac{1}{b} (b + 2e + 2qh) = \frac{1}{21.75} (21.75 + 2 \times 0.9907 + 2.6300) = 1.212 \]

\[ \frac{1}{b} (b - 2e) = \frac{1}{21.75} (21.75 - 2 \times 0.9907) = 0.908 \]

For floor beam D \( D_1 \)

\[ \frac{1}{b} (b + 2e + 2qh) = \frac{1}{21.75} (21.75 + 2 \times 1.8086 + 2.6300) = 1.287 \]

\[ \frac{1}{b} (b - 2e) = \frac{1}{21.75} (21.75 - 2 \times 1.8086) = 0.833 \]

For floor beam E \( E_1 \)

\[ \frac{1}{b} (b + 2e + 2qh) = \frac{1}{21.75} (21.75 + 2 \times 2.0812 + 2.6300) = 1.312 \]

\[ \frac{1}{b} (b - 2e) = \frac{1}{21.75} (21.75 - 2 \times 2.0812) = 0.802 \]

For Cooper's E-50 loading, the maximum floor-beam load for one rail (one-half of the total load on floor beam) for straight track is 71,000 lbs. for floor beam \( A A_1 \) and 94,600 lbs. for the rest. The shears in the present case are equal to the shears for straight track multiplied by the values of \( \frac{1}{b} (b + 2e + 2qh) \) or \( \frac{1}{b} (b - 2e) \). They are shown in Table 17.

In Art. 15 we have investigated the stresses in the stringers under two cases: (1) The stringers are straight throughout the span of the bridge, and (2) The stringers are offset at each panel.
TABLE 17.
Shears in Floor beams.

<table>
<thead>
<tr>
<th>Floor beam</th>
<th>On Straight Track Shears in lbs.</th>
<th>On Curved Track</th>
<th>Between Outer Truss and Outer Stringer Shears in lbs.</th>
<th>Between Inner Truss and Inner Stringer Shears in lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A A¹</td>
<td>71,000</td>
<td></td>
<td>1.011 71,800</td>
<td>1.109 78,800</td>
</tr>
<tr>
<td>B B¹</td>
<td>94,600</td>
<td></td>
<td>1.086 102,800</td>
<td>1.034 97,800</td>
</tr>
<tr>
<td>C C¹</td>
<td>94,600</td>
<td></td>
<td>1.212 114,600</td>
<td>.908 86,000</td>
</tr>
<tr>
<td>D D¹</td>
<td>94,600</td>
<td></td>
<td>1.287 121,800</td>
<td>.833 78,800</td>
</tr>
<tr>
<td>E E¹</td>
<td>94,600</td>
<td></td>
<td>1.312 124,100</td>
<td>.802 76,500</td>
</tr>
</tbody>
</table>

Since the maximum shears in the floor beams are equal to the maximum truss panel loads which are independent of the location of the stringers, it follows that the shears are the same for both cases. But in regard to the bending moments, the two cases must be treated separately, for in Case (1), the floor beams are each subjected to two unequal loads from the stringers symmetrically placed, while in Case (2), the floor beams are each subjected to four equal loads from the stringers which are not continuous from panel to panel.

Case 1. Axes of Stringers and Bridge are Coincident.

When the axes of all the pairs of stringers in all panels coincide with the center line of the bridge, there are only two points in each floor beam through which the load of the train can be transmitted to the floor beam: These two points are where the stringers are connected to the floor beam.

If the bridge were on a tangent with central loading, the maximum bending moment would be equal to the maximum truss panel
load (the maximum shear in the floor beam) times the distance from either truss to the nearer stringer. For Cooper's E-50 loading, the maximum shear in the floor beam $A_A^1$ is 71,000 lbs. and that in the other floor beams is 94,600 lbs. From Fig. 41, the distance from the center line of either $A_A$ to the nearer stringer is 7.125 ft. Hence the maximum bending moment would be $71,000 \times 7.125 = 505,900$ ft.-lbs. for $A A^1$ and $94,600 \times 7.125 = 674,000$ ft.-lbs. for the other.

In the present problem, the maximum shears in the floor beams have been changed and are equal to the product of the maximum shear for straight track (71,000 lbs. for $A A^1$ and 94,600 lbs.) and the factor $\frac{1}{b}(b + 2e + 2qh)$ or $\frac{1}{b}(b - 2e)$. Since the maximum bending moments in the floor beams are directly proportional to the shears, they can be found by multiplying the maximum moments for straight track by the same factor $\frac{1}{b}(b + 2e + 2qh)$ or $\frac{1}{b}(b - 2e)$. These moments are shown in Table 18.
TABLE 18

Moments in Floor Beams (Case 1)

<table>
<thead>
<tr>
<th>Floor beam</th>
<th>On Straight Track Moments in ft-lbs.</th>
<th>On Curved Track Moments in ft-lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A A¹</td>
<td>505,900</td>
<td>1.011</td>
</tr>
<tr>
<td>B B¹</td>
<td>674,000</td>
<td>1.086</td>
</tr>
<tr>
<td>C C¹</td>
<td>674,000</td>
<td>1.212</td>
</tr>
<tr>
<td>D D¹</td>
<td>674,000</td>
<td>1.287</td>
</tr>
<tr>
<td>E E¹</td>
<td>674,000</td>
<td>1.312</td>
</tr>
</tbody>
</table>

Case 2. Axes of Stringers and Bridge are not Coincident.

In this case according to Art. 15, the stringers are offset at each panel so that the shears and the moments in all the stringers are equal. Accordingly all the floor beams are each subjected to four equal concentrated loads as there are four connections between the stringers and each floor beam, except those at the ends and the middle of the bridge. Fig. 42 shows the locations of the stringers.

Fig. 42
For the floor beams \( A A^1 \) and \( B B^1 \), since there are only two stringer connections, the maximum moment in each floor beam will occur under one of the stringers and will be equal to the nearer truss panel load (or the shear in the floor beam) multiplied by the distance between the truss and the said stringer. But in the floor beams \( B B^1, C C^1 \) and \( D D^1 \), each has four stringers connected to it at different points and hence has four concentrations applied to it. According to the arrangement shown in Fig. 42, all the stringers will receive equal maximum load. For Cooper's E-50 loading, the maximum floor beam load for straight track is 94,600 lbs. for one rail which gives each stringer a shear of 47,300 lbs. For curved track, the shear in each stringer is \( 47,300 \times 1.1140 = 52,700 \) lbs. (Art. 15).

\[ \begin{align*}
\text{Fig. 43.} \\
\text{In Fig. 43, it may be shown that if } R_1 \text{ is greater than } P, \\
\text{the moment at } B \text{ will be greater than that at } A; \text{ and if } R_2 \text{ is greater} \\
\text{than } P, \text{ the moment at } C \text{ will be greater than that at } D. \text{ Table 17} \\
\text{shows that the shears (or the truss panel loads) in all the floor} \\
\text{beams are each greater than 52,700 lbs., hence we may conclude that} \\
\text{the maximum bending moment in each floor beam will be under one of} \\
\text{the middle stringers (at } B \text{ or } C). \text{ The moment at } B \text{ is equal to} \\
R_1(a + b) - Pb \text{ and the moment at } C \text{ is equal to } R_2(d + e) - Pd. \text{ The} \\
\text{moments in all the floor beams are shown in Tables 19-20.} 
\end{align*} \]
### TABLE 19.
Moments in Floor beams near Outer Truss

<table>
<thead>
<tr>
<th>Floor beam</th>
<th>Positive Moments</th>
<th>Negative Moments</th>
<th>Resulting Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>71,800</td>
<td>7.89</td>
<td>566,500</td>
</tr>
<tr>
<td>B</td>
<td>102,800</td>
<td>7.89</td>
<td>811,100</td>
</tr>
<tr>
<td>C</td>
<td>114,600</td>
<td>6.25</td>
<td>716,300</td>
</tr>
<tr>
<td>D</td>
<td>121,800</td>
<td>5.16</td>
<td>628,500</td>
</tr>
<tr>
<td>E</td>
<td>124,100</td>
<td>4.62</td>
<td>573,300</td>
</tr>
</tbody>
</table>

### TABLE 20
Moments in Floor beams near Inner Truss.

<table>
<thead>
<tr>
<th>Floor beam</th>
<th>Positive Moments</th>
<th>Negative Moments</th>
<th>Resulting Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>78,800</td>
<td>6.36</td>
<td>501,200</td>
</tr>
<tr>
<td>B</td>
<td>97,800</td>
<td>6.36</td>
<td>622,000</td>
</tr>
<tr>
<td>C</td>
<td>86,000</td>
<td>8.00</td>
<td>688,000</td>
</tr>
<tr>
<td>D</td>
<td>78,800</td>
<td>9.09</td>
<td>716,300</td>
</tr>
<tr>
<td>E</td>
<td>76,500</td>
<td>9.63</td>
<td>736,700</td>
</tr>
</tbody>
</table>
THE END