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INVESTIGATION OF THE QUANTITATIVE DETERMINATION OF PRECIPITATION BY RADAR

INTERIM REPORT NO. 2

by

E. A. Mueller - A. L. Sims - R. Cataneo

December 1967

ECOM

ATMOSPHERIC SCIENCES LABORATORY
UNITED STATES ARMY ELECTRONICS COMMAND • FORT MONMOUTH, N.J.

Contract DA-28-043 AMC-02071(E)
ILLINOIS STATE WATER SURVEY
at the

University of Illinois
Urbana, Illinois

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TECHNICAL REPORT ECOM-02071-2 December 1967

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1 April 1967 to 30 September 1967

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Prepared by

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ABSTRACT

The results of two years of radar measurement of rainfall amounts over the Kankakee raingage network has shown that the accuracy is not greatly different than the accuracy over the much closer East Central Illinois network. In either case if the standard error of estimate of radar amounts is chosen as a measure of the error, a result of about 0.2 inch for storm total is obtained.

Results of preliminary study of extrapolation of radar rainfall equations to other parts of the world is reported. A method depending upon the percentage of rain days which are thunderstorms and the relative humidity at a level of 0.5 Km has been tentatively chosen.

A survey report on the relationships between rainfall rate and radar reflectivity in the measurement of precipitation shows that the relationships introduce less uncertainty than the errors in present day radar measurement of reflectivity.

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RADAR MEASUREMENT OF
RAIN USING THE KANKAKEE NETWORK

In order to evaluate the usefulness of radar to measure rainfall at more distant ranges, the Kankakee Raingage Network was established near Kankakee, Illinois in the spring of 1966. This network is located at a mean range of 64 nautical miles from the CPS-9 radar near. Champaign. It consists of 16 recording gages spaced in a nearly square grid of 4 rows, each row having 4 gages. The network area is approximately 400 square miles. A more detailed description of this network is contained in Technical Report EC0M-00032-F. The results of the 1966 operations of this network are in Technical Report ECOM-02071-1.

Data

The Kankakee Raingage Network was operated in 1967 during the period from March 22 to. September 28. Fifty-four sets of 24-hour raingage charts were obtained. The charts were changed on the gages twice weekly.

The CPS-9 radar operated to record most of the rain occurring over the network between 0800 and 2400. Step-gain scope photographs were made at intervals of 5 to 8 minutes. A nominal elevation angle of 0 degrees and a range of 80 nautical miles were used.

Twenty-one storms were chosen for analysis on the basis of having at least one gage with 0.05 inch or more of rain, at least 30 minutes of concurrent radar and gage data, and a mean gage indicated network rainfall total of 0.01 inch or more. Also, no analysis was attempted on rainstorms in which rain occurred at less than half of the gages.

The information on the raingage charts was converted to punch cards by the use of an "Auto-trol" chart reader. These cards were processed by a computer program which produced a listing of 15-minute rainfall amounts for each gage. These amounts were then combined to obtain 30-minute amounts for each gage and mean network amounts for each 30-minute period and for each storm.

Radar-Rainfall Analysis

The radar data analysis began with the tracing of the echoes onto paper. The radar film was projected to a scale of 4 mm per mile, making the network an

8-cm square on the tracings. The network area was then drawn on the tracings along with the iso-echo contours.

Using a planimeter, the fraction of the network covered by echo was determined for each gain step in each series. The measurements for each series within each 30-minute period were combined by averaging areas, weighting each area by the time period it represented. This gave a 30-minute average area for each step.

An appropriate rainfall rate was determined for each gain step level from the radar equation, a R-Z relationship, and the radar calibration data. The radar equation derived by Probert-Jones[1] was used. By substituting in this equation the parameters of the CPS-9 radar and a range of 64.2 nautical miles, the radar equation was reduced to the following form:

$$P_r/P_t = 3.06 \times 10^{-20} Z$$

where Z is the radar reflectivity in $mm^6 \, m^{-3}$. The transmitted power, P_t, was measured frequently during the data collection season. Also, measurements of the minimum discernible received power, P_r, were made for each step using a known calibrating signal after each rainfall period. From these calibration measurements, a minimum discernible Z was determined for each step level and each storm date.

To convert Z to rainfall rate, R, the following relationships were used, depending upon the type of rain which was predominant during the storm:

Thunderstorms	$Z = 435R^{1.48}$
Rainshowers	$Z = 370R^{1.31}$
Continuous Rain	$Z = 311R^{1.43}$

These relationships are the regressions of R as a function of Z as determined by Jones[2] from drop-size distributions taken in Illinois.

These rainfall rates were then combined with the 30-minute average areas for each step to give a mean network 30-minute rainfall amount for each 30-minute period, as indicated by radar. The radar 30-minute amounts are plotted against the network gage 30-minute amounts in figure 1. The plot is on a logarithmic scale, and points which had either amount less than 0.0001 inch

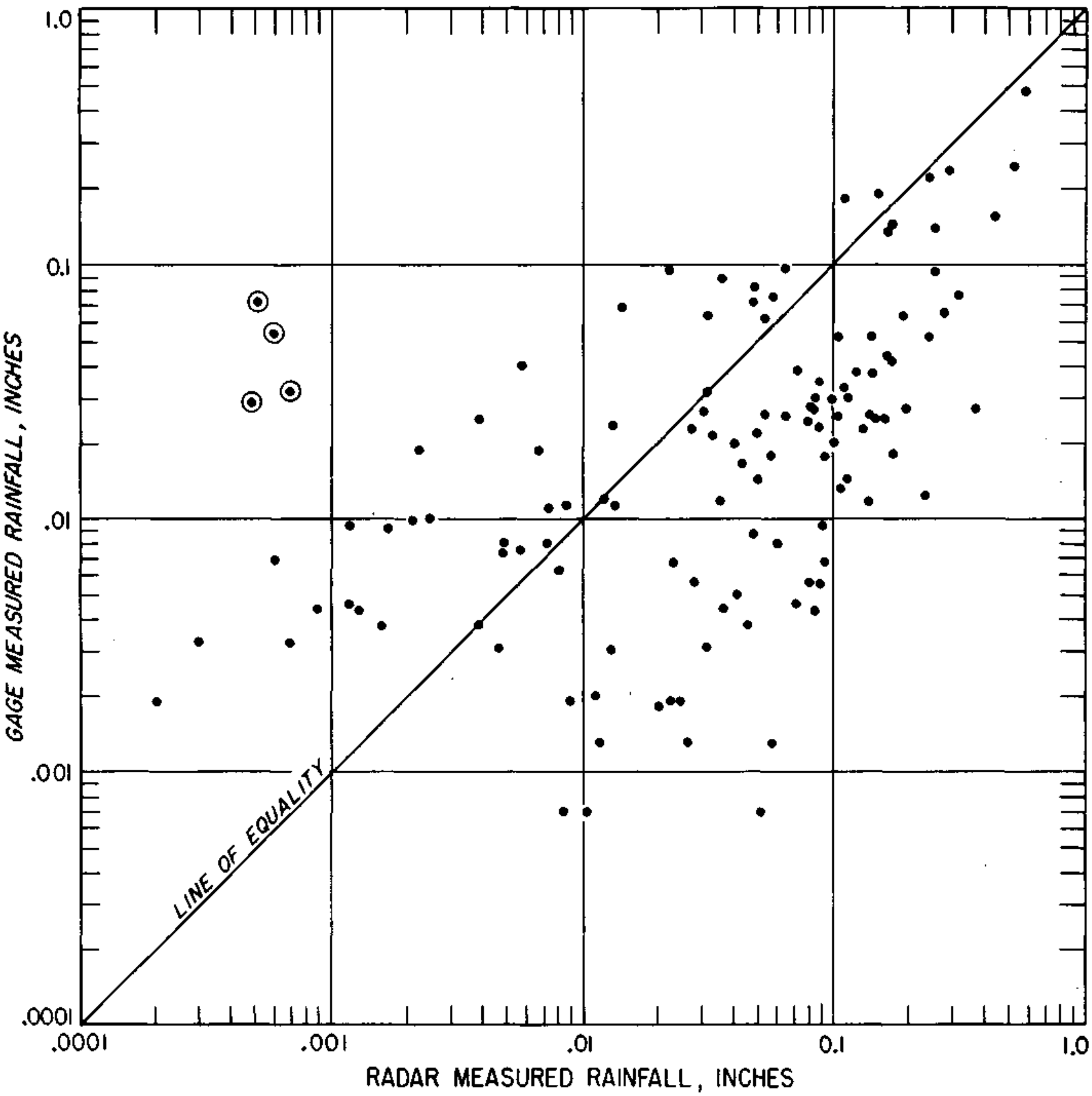

Fig. 1. Radar and raingage measured 30-minute rainfall for the Kankakee Network, 1967. The encircled points represent severe attenuation.

have not been plotted. Data which have a gage amount with no radar amount, and also some points with a radar amount and no indication of rain on the gage charts were not plotted.

The scatter of points on figure 1 is somewhat less than it was in 1966. The linear correlation coefficient has increased from 0.37 to 0.74. The data are also skewed toward the radar measurement being the greater. Sixty-seven percent of the 30-minute periods had radar measured rainfall greater than the gage measured amounts.

Total storm amounts were found by summing all the 30-minute amounts for each storm. These rainfall amounts are presented in table 1 and are plotted in figure 2. The storm totals also show the radar measured amount greater than gage amount in most cases. The season totals also reflect this with the radar total of 10.55 inches being 106 percent higher than the gage total of 5.11 inches.

The data from both years were combined for a linear statistical analysis. Some of the results of this analysis are present in table 2, along with similar data for each separate year. For the 30-minute data, points having either rainfall amount less than .0001 inch have not been included in the analysis.

The scatter of data as indicated by the standard errors of estimate is less for 1967 than for 1966, particularly for the storm total rainfall. Since no systematic reason for this tendency is known, it is assumed that it is a random difference between the storms occurring in the 2 years. The results for the combined data will then be taken as the appropriate measure of the ability to measure rain with radar at this range of 64 nautical miles.

The linear regression lines for the combined data are

$$R_G = 0.123 + 0.324\ R_R$$

for storm total rainfall, and

$$R_G = 0.0148 + 0.341\ R_R$$

for 30-minute rainfall, where R_G is the gage indicated amount and R_R is the radar indicated amount.

There are several possible reasons for the scatter observed and for the bias in the data toward a larger radar indication of rainfall. Attenuation

TABLE 1

TOTAL STORM RAINFALL AMOUNTS FOR THE KANKAKEE NETWORK, 1967

Storm Date	Total Time in Analysis (hrs)	Gage Amount (in)	Radar* Amount (in)	Radar/Gage Ratio	Radar-Gage	Storm Type
5/15	1.0	0.013	0.026	2.05	+0.013	RW
8/19	1.5	0.036	0.067	1.86	+0.031	RW
8/18 late	2.0	0.057	0.35	6.16	+0.293	RW
8/18 early	1.5	0.077	0.080	1.04	+0.003	TRW
5/18	2.0	0.097	0.22	2.25	+0.123	TRW
6/24 late	3.0	0.11	0.56	5.06	+0.45	RW
8/8	6.5	0.12	0.094	0.79	-0.026	R
5/28 late	2.5	0.13	0.14	1.06	+0.01	TRW
6/9	3.5	0.16	0.40	2.53	+0.24	RW
7/26	6.0	0.17	0.69	4.13	+0.52	TRW
5/7	1.5	0.19	0.056	0.30	-0.134	R
6/7	8.5	0.19	1.13	6.05	+0.94	RW
6/24 early	2.0	0.21	0.48	2.30	+0.27	RW
5/5	4.0	0.22	0.87	3.97	+0.65	R
5/28 early	2.0	0.22	0.41	1.84	+0.19	TRW
6/10	3.0	0.27	0.16	0.60	-0.11	RW
7/18	3.0	0.29	0.0023	0.0078	-0.2877	TRW
6/21	4.5	0.29	0.21	0.70	-0.08	RW
5/10	2.0	0.58	0.64	1.11	+0.08	TRW
6/16	9.5	0.67	2.42	3.63	+1.75	TRW
7/23	3.0	1.01	1.54	1.53	+0.43	TRW
Totals	72.5	5.11	10.55	2.06	5.44	

*Based upon previous antenna measurements which have not been verified since the antenna change.

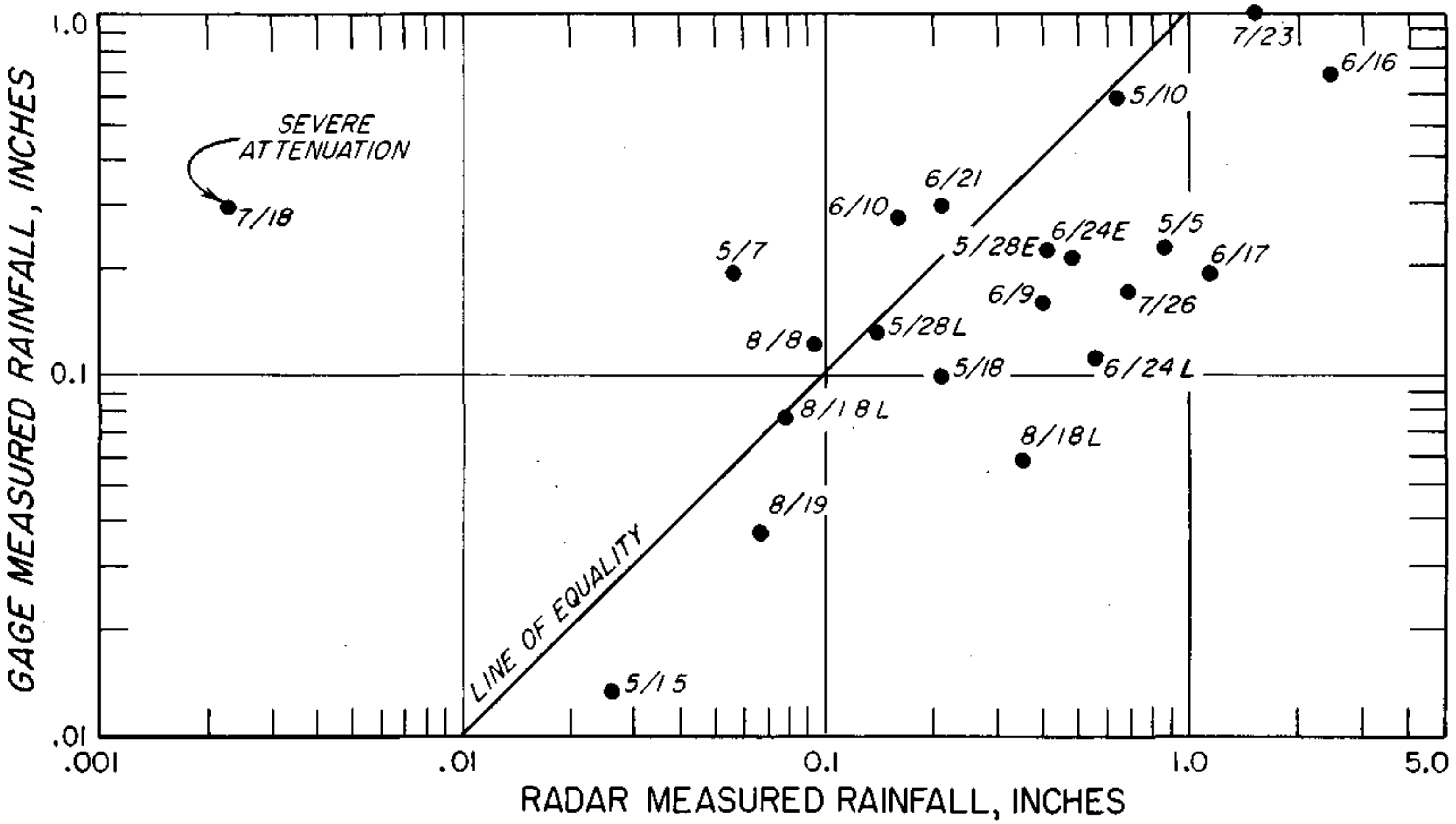

Fig. 2. Radar and gage measured total storm rainfall for the Kankakee Network, 1967. (Where two storms occurred on the same day, the earlier one is indicated by an E, the later by an L.)

contributes to the scatter by significantly reducing the radar measurement in some cases. The 18 July 1967 storm is an example of this. Although attenuation is involved to varying degrees for any range with 3-cm radar, this effect can be more pronounced at long ranges. The longer path length to the area of interest increases the probability that attenuating rainfall will be encountered. Also, due to the 1/R2 reduction of returned signal, there is an increased likelihood that distant echoes will be completely attenuated, thereby removing any possibility of attenuation correction.

TABLE 2

SOME STATISTICAL PARAMETERS FOR THE
RADAR-GAGE RAINFALL DATA, KANKAKEE NETWORK, 1966-1967

	30-Minute Rainfall			Mean Storm Total Rainfall		
	1966	1967	Combined	1966	1967	Combined
N, no. of data points	124	126	250	12	21	33
Network Mean radar rainfall (in)	0.0413	0.0831	0.0624	0.446	0.501	0.481
Network Mean gage rainfall (in)	0.0332	0.0389	0.0361	0.342	0.243	0.279
Correlation coefficient	0.37	0.72	0.60	0.62	0.71	0.63
Standard error of estimate (in)	0.048	0.043	0.046	0.26	0.16	0.21

The top of the radar beam over the Kankakee Network was nearly 10,500 feet above mean sea level. Since variations in the index of refraction of the air may shift the beam in the vertical, the exact extent of the beam is uncertain. During the 1967 operations the freezing level remained at or below 10,000 feet MSL for all the storms until May 28. For these storms, the radar beam would have included some solid precipitation particles over the network. The melting zone which produces high reflectivity would also be in the beam for these storms. In general, it is expected that for cases where the radar was looking at sampling volumes near or above the freezing level, the radar was sampling precipitation quite different from that occurring at the ground and also quite different from the rains from which the R-Z relationships were derived. Even when the freezing level is high, hail may give a different reflectivity than would be predicted from the Z-R equations used. Hail was detected at the ground on June 9, 10, and 16 and July 23, and small hail was undoubtedly present aloft on other days.

Another problem noted in this study is that of accurately measuring the mean network rainfall with the gages. Although the gages were spaced only 5 miles apart, frequently the areas of the more significant rates had dimensions smaller than the gage spacing. When this happened, the probability increased that the heavier rain would miss the gages or occur briefly at only a few gages. The radar, with its range resolution of 0.5 mile and azimuth resolution at the network of 1.3 miles, can measure the areal distribution of the rain to a finer scale than can be done with a network of gages at a 5-mile spacing. This would tend to give a gage mean network amount less than that measured by the radar, since the small intense areas are those most likely to be missed by the gages. This may well be a major cause of the apparent over measurement by radar.

In addition the differences in time scales undoubtedly accounts for the greater scatter of 30 minute amounts than of the storm totals. The general bias of the radar in 1967 may well be the result of an error in the radar calibrations between 1966 and 1967. If an error of about 4 db in any of the radar parameters occurred, the bias would be removed. Between 1966 and 1967 the entire pedestal section was exchanged. The pedestal from the CPS-9 formerly at Massachusetts Institute of Technology was installed in place of the former pedestal. The new pedestal has a hydraulic drive system which is far superior to the drive system which had been in use. It is quite possible that the antenna gain of the new antenna as installed is slightly greater (say 2 db) than the former antenna. If this were so and since the gain enters as the square, the entire bias could be accounted for. Time has not permitted antenna pattern measurements to be made. Therefore, all of these results should be considered preliminary and subject to change.

There are some cases in which it appears that appreciable liquid water is stored aloft for lengthy times. On one case in 1966, the radar indicated a mean network rainfall of 0.11 inch during a 1-1/2 hour period during which no rain was detected on the gage charts. Another case on June 1, 1967 had radar echoes over the network when the radar antenna was at an angle of 3 or 4 degrees for much of the day; however, no echoes were observed at 0 degrees and no rain was recorded by the gages. It would appear that on this day precipitation sized particles were present aloft without falling to the ground.

In summary the accuracy of this technique for measuring rainfall at a range of 60 miles is best represented by the standard error of estimate. For the 1967 data this value is 0.16. This may be interpreted that a measurement of a storm total by the radar will be within ± 0.16 of an inch of the storm total by the raingage network 68% of the time. For this to be statistically valid the homoscedasticity of the data points should be examined. Unfortunately there are insufficient data points to perform a rigorous test of homoscedasticity. Reference to table 1 gives the impression that the values of the arithmetic differences are more stable with amount than the ratios and so at least it is more meaningful to quote the standard error of estimate from a linear regression than for a logarithmic regression. Some impression of homoscedasticity can be obtained by noting the arithmetic deviations do not tend to be particularly larger at the high or low end of the table.

These results compare favorably with previous results using the same technique over the East Central Illinois network of 30 miles range where a standard error of estimate of 0.15 inches was obtained. Thus, it would appear that the expected decrease in accuracy at the longer ranges was not found. Most probably the errors inherent in the technique for reducing the radar data mask the errors due to range. Additionally, it should be noted that a number of very light rains have been discarded from this analysis but they were included in the analysis of the East Central Illinois network. With these lighter rains at the distant ranges a poorer correlation would have resulted.

WORLD-WIDE R-Z RELATIONSHIPS

Introduction

One of the requirements of the present contract is to present the raindrop size data, collected at nine locations in the world, in such a form so that it can be used by radar meteorologists in the field to determine amounts and rates of precipitation. The use of a recognized climatological classification as a means for presenting guides to radar observational techniques was suggested for this study. The results to date of this investigation are presented.

Raindrop data taken with the drop camera which have been used to determine R-Z relationships for nine different locations throughout the world is an excellent source of data for a climatological study. Since the climates sampled were quite varied, it was hoped that they would be a representative cross-section of many other areas of the world. If this is indeed correct, then an extrapolation of these R-Z relationships to other areas of similar "drop-spectra climates" would be appropriate. Before this was attempted, however, it was necessary to determine the following: 1) the factors that affect changes in the overall R-Z equations: 2) the availability of data related to these factors.

It was first believed that perhaps one of the standard schemes of climatic classification, Köppens,[4] for example, would divide the world into regions of similar drop-size spectra, hence similar R-Z relationships. However, this approach was not satisfactory, since most of the methods used for classifying climates are based on temperature and precipitation amounts in various combinations, and drop-size distribution variations are not dependent, to any appreciable degree, upon these parameters. For example, according to Köppen, both Island Beach, New Jersey and Champaign, Illinois are classified as a Cfa climate, which is a warm, temperate, rainy climate without a dry season and with a hot summer. The R-Z equations for these areas are: $Z = 256\ R^{1.41}$ for New Jersey, and $Z = 372\ R^{1.47}$ for Illinois, which suggest a substantial difference in the drop-size spectra for rains of similar rainfall rates, so apparently the factors affecting the drop-size distributions are not the ones used in Köppen's classification method.

Procedures

In an attempt to find the appropriate parameters, many were eliminated simply because of their limited availability. For example, the concentration of condensation nuclei was considered as a parameter, but sufficient data were not available. The field was narrowed down to three possible variables: 1) the mean percent of the annual number of rain days that are thunderstorm days, 2) the mean annual relative humidity at 0.5 Km (above 1600 ft) above ground level, and 3) the mean annual freezing level above ground.

The first variable was considered because previous analyses of the drop data collected from the nine locations indicated the presence of larger drops in thunderstorm rains, hence larger coefficients in the R-Z equations. Therefore, it followed that the greater the percentage of rain days that were thunderstorm days, the larger would be the coefficient of the overall R-Z relationship for a particular area (when the other variables remained constant).

The second was investigated because the relative humidity between cloud base and ground affects the amount of evaporation of the raindrops as they fall from cloud base to the ground. The effects of evaporation on smaller drops is more pronounced than on larger ones, resulting in larger coefficients in the R-Z equation when compared to a situation where no evaporation is occurring on the raindrops.[5]

The third parameter was considered because it appeared reasonable to assume that the height of the freezing level when compared to the average cloud base height would be an indication of the amount of cloud growth that occurred above and below the freezing level. If the mechanisms responsible for changes in drop-size distribution were dependent on how much of the precipitation formed by the ice or water process, then the third independent variable would have a direct bearing on the R-Z equation.

Analysis of Data

In order to evaluate the importance of these three factors, the corresponding data for the nine sampled locations were obtained. The nine locations are: Miami, Florida; Island Beach, New Jersey; Franklin, North Carolina; Champaign, Illinois; Corvallis, Oregon; Woody Island, Alaska; Majuro, Marshall Islands; Bogor, Indonesia; and Flagstaff, Arizona. Since R-Z relationships have been established for these areas, some measure of the effectiveness of the three variables can be achieved. The multiple correlation coefficients as well as the individual correlation coefficients of both A and b, the coefficient and exponent, respectively, in the R-Z equation, and the standard error of estimate of A and b are shown in table 3. The correlation between the first two independent variables and A was good; however, freezing level was poorly correlated. This was further demonstrated by the fact that the standard error of estimate (S.E.E.) decreases when freezing level is not included; the removal of an independent variable usually results in an increase in the S.E.E. The multiple correlation remains essentially unchanged, which also is an indication of the lack of effectiveness of freezing level as one of the independent variables. Because of this, it was decided not to use freezing level in the analysis. The S.E.E. of A (55.3) means that an error of less than ±55.3 units on A can be expected 68% of the time; the S.E.E. of b is .087. With the nine points (locations) used, and with six degrees of freedom, a correlation of .906 (observed sample multiple correlation on A), is significant at the 99 percent significance level.

Using the regression coefficients calculated from the analysis, a family of lines can be generated for different values of relative humidity, using A as ordinate and thunderstorm days as abscissa (figure 3); this allows a determination of A for any combination of thunderstorm days and relative humidity. The same approach is used to determine b values (figure 4). The regression equations for A and b are: $A = 1.3716(D) - 4.7015(R) + 571$, and $b = -.0002580(D) - .004437(R) + 1.7759$ where D = mean annual percentage of rain days which are thunderstorm days, R = mean annual relative humidity at 0.5 Km above ground level.

In order to demonstrate the relative effectiveness of the above method, the data associated with the two independent variables for the original nine locations were used to determine the A and b values from figures 3 and 4. These values were then compared in table 4 with the actual R-Z relationships established for these areas. The percent thunderstorm days and relative humidity data used for the nine locations were obtained from nearby areas when not available for the actual location. The coefficients of the predicted relationships are within 19% (< 1 db in Z) in 8 of the 9 cases. The only case outside of this value was Franklin, North Carolina where the difference was 44% (< 2 db in Z). The exponents of the predicted relationships can be similarly examined at a particular rainfall rate. If a rain rate of 20 mm/hr is chosen, 8 of the 9 predicted relationships are within 1.8 db in Z. North Carolina has a 3.2 db difference. North Carolina was the poorest fit location which may be indicative of one of the limitations of

TABLE 3

CORRELATION COEFFICIENTS AND STANDARD ERRORS OF ESTIMATE OF THE R-Z REGRESSION COEFFICIENTS WITH CLIMATIC PARAMETERS

	Correlation Coefficients		
$Z = AR^b$	Mean Annual Percent Rain Days that are Thunderstorm Days (1)	Mean Annual Relative Humidity at 0.5 Km above Ground (2)	Mean Annual Freezing Level in ft above Ground (3)
A	.59	-.80	-.30
b	.06	-.70	-.52

	Multiple Correlation with Variables (1), (2), & (3)	Multiple Correlation with Variables (1) & (2)	Standard Error of Estimate with Variables (1), (2), & (3)	Standard Error of Estimate with Variables (1) & (2)
A	.91	.91	59.4	55.3
b	.70	.70	.094	.087

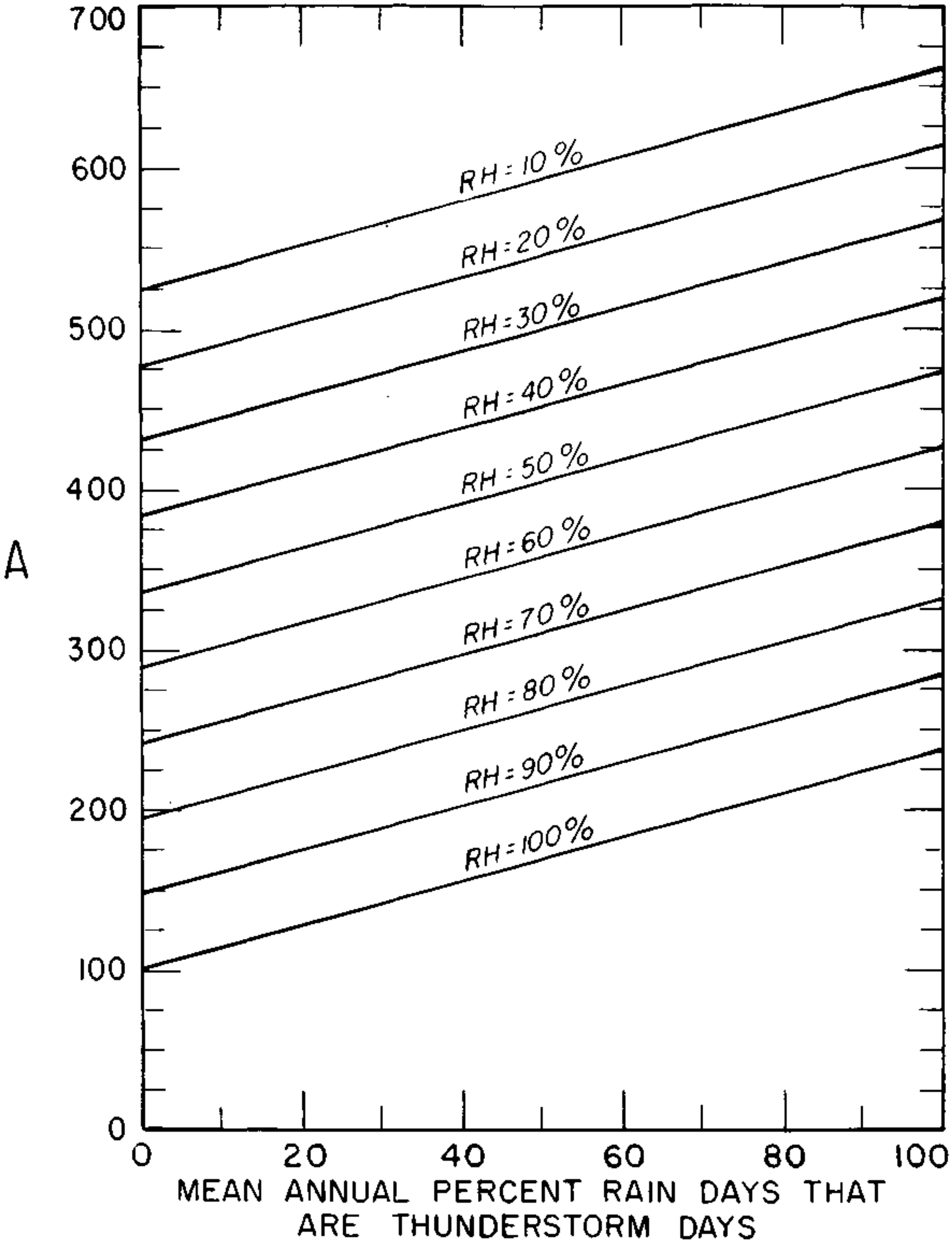

Fig. 3. Graphical method for determination of A, the coefficient of the R-Z relationship.

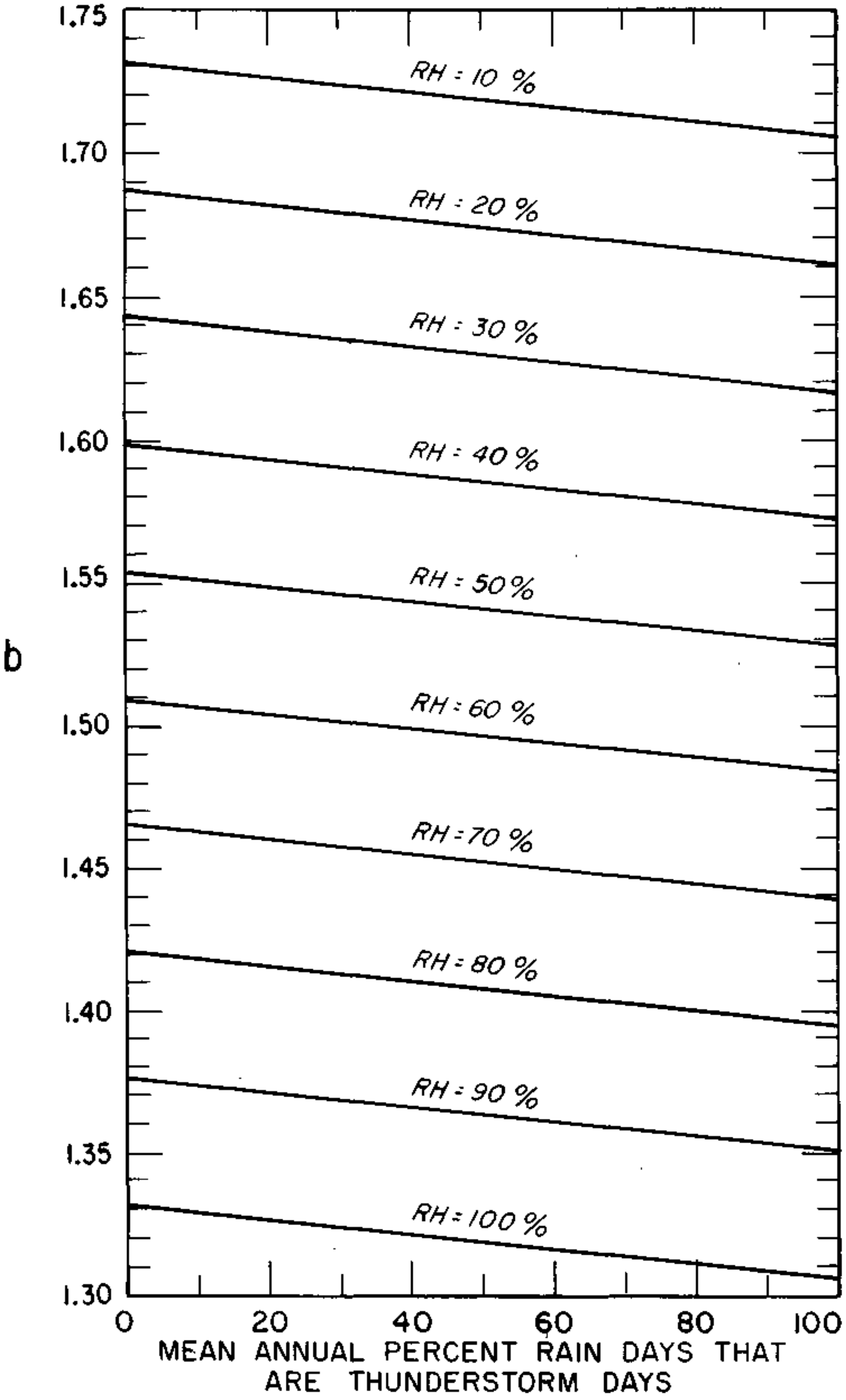

Fig. 4. Graphical method for determination of b, the exponent of the R-Z relationship.

the method. The site of the data collection in North Carolina was in a mountainous area and at an elevation of 4460 feet MSL. The local conditions due to the mountains and height might influence the drop-size distributions. Thus, any location which possesses unusual features such as mountains may be subjected to considerably more error than the more common flatter areas. Another possible cause of the large discrepancy between the predicted and actual R-Z equation in North Carolina may be due to the difference in the mean percent of rain days that were thunderstorm days, which was used in the prediction, as compared to that which actually occurred during the data collection period (41 percent for the former and 36 percent for the latter).

Discussion

In the above procedure, the result is a single R-Z equation for a particular area. This is also a limitation since, as was demonstrated in previous reports,[2] the R-Z relationships are subject to wide variations for different storms, so using an average R-Z for all cases introduces some error. In some areas of the world, using one equation for all rains would not be prohibitive, because the synoptic condition producing the rain doesn't vary greatly and drop-size distributions tend to vary with different synoptic conditions. For example, in Flagstaff, Arizona in summer and in Bogor, Indonesia, virtually all of the rains are produced by air mass orographic thundershowers. Unfortunately the amount of drop-size data which are available with knowledge of storm types are limited to three locations, so analysis is not deemed wise.

TABLE 4

COMPARISON OF ACTUAL AND PREDICTED R-Z EQUATIONS FOR THE NINE SAMPLED LOCATIONS

Location	Mean Annual Percent of Rain Days that are Thunderstorm Days *	Mean Annual Relative Humidity-at 0.5 Km above Ground **	Actual R-Z Equation	Predicted R-Z Equation
Miami). Florida	55	75	$Z = 286R^{1.43}$	$Z = 293R^{1.43}$
Island Beach, New Jersey	20	66	$Z = 256R^{1.41}$	$Z = 288R^{1.48}$
Franklin, North Carolina	41	62	$Z = 234R^{1.39}$	$Z = 337R^{1.49}$
Champaign, Illinois	43	64	$Z = 372R^{1.47}$	$Z = 328R^{1.48}$
Corvallis, Oregon	4	55	$Z = 301R^{1.64}$	$Z = 318R^{1.53}$
Woody Island, Alaska	0.5	73	$Z = 267R^{1.54}$	$Z = 227R^{1.45}$
Majuro, Marshall Islands	6	85	$Z = 221R^{1.32}$	$Z = 180R^{1.40}$
Bogor, Indonesia	100	85	$Z = 305R^{1.44}$	$Z = 307R^{1.37}$
Flagstaff, Arizona	90	30	$Z = 593R^{1.61}$	$Z = 553R^{1.62}$

* Obtained from Weather Bureau normals over period 1921-1950

** Obtained from "Upper Air Climatology of the United States" - Part I Weather Bureau Technical Paper No. 32; data based on radiosonde observations taken at 0300 GMT during period 1946-1955

RADAR OPERATIONS

The CPS-9 radar was operated for 1112 hours during the spring and summer. No major maintenance difficulties were experienced. During the past period a new "programmer" was installed on the radar. This programmer contains auxiliary equipment which controls the operation of the radar and the performance of the data collection. It allows an operator to select a sequence of frames (each frame is a particular combination of display range, receiver gain, and antenna elevation angle) most suited to his data collection requirements.

The programmer can be divided into six subassemblies: power supply, timing network, program-camera control, receiver gain control, display range control, and antenna elevation control as shown in figure 5.

The power supply provides DC power for the logic circuits and the camera operation. It contains three regulated supplies that drive solid-state logic and two unregulated supplies that drive relay logic. AC power is supplied from the radar set.

Besides power from the radar set, the programmer requires a signal that indicates the end of one azimuthal scan and the beginning of the next. This switching point is manually adjustable with the cursor of the console PPI. The programmer receives this signal and generates a sequence of timing pulses that are sent to the program-camera control.

The program-camera control operates in three modes: single-program mode, two-program mode, or a timed mode. The difference between the three modes is in the switching that takes place at the end of a program.

The control panel has four patch boards: receiver gain, antenna elevation, display range, and skip. Each of the patch boards has 33 hubs for each program. Each hub represents a frame number, the first is named "0" and the last "32". In the single-program mode, the operator chooses one of the two programs and patches a sequence of frames in that program. This chosen sequence is then repeated indefinitely.

Two-program operation is the same as single-program operation, except that the programmer alternates between two different sequences. All the capabilities of single-program operation are available in two-program operation, plus a combined total of 64 photographed frames and two non-photographed frames.

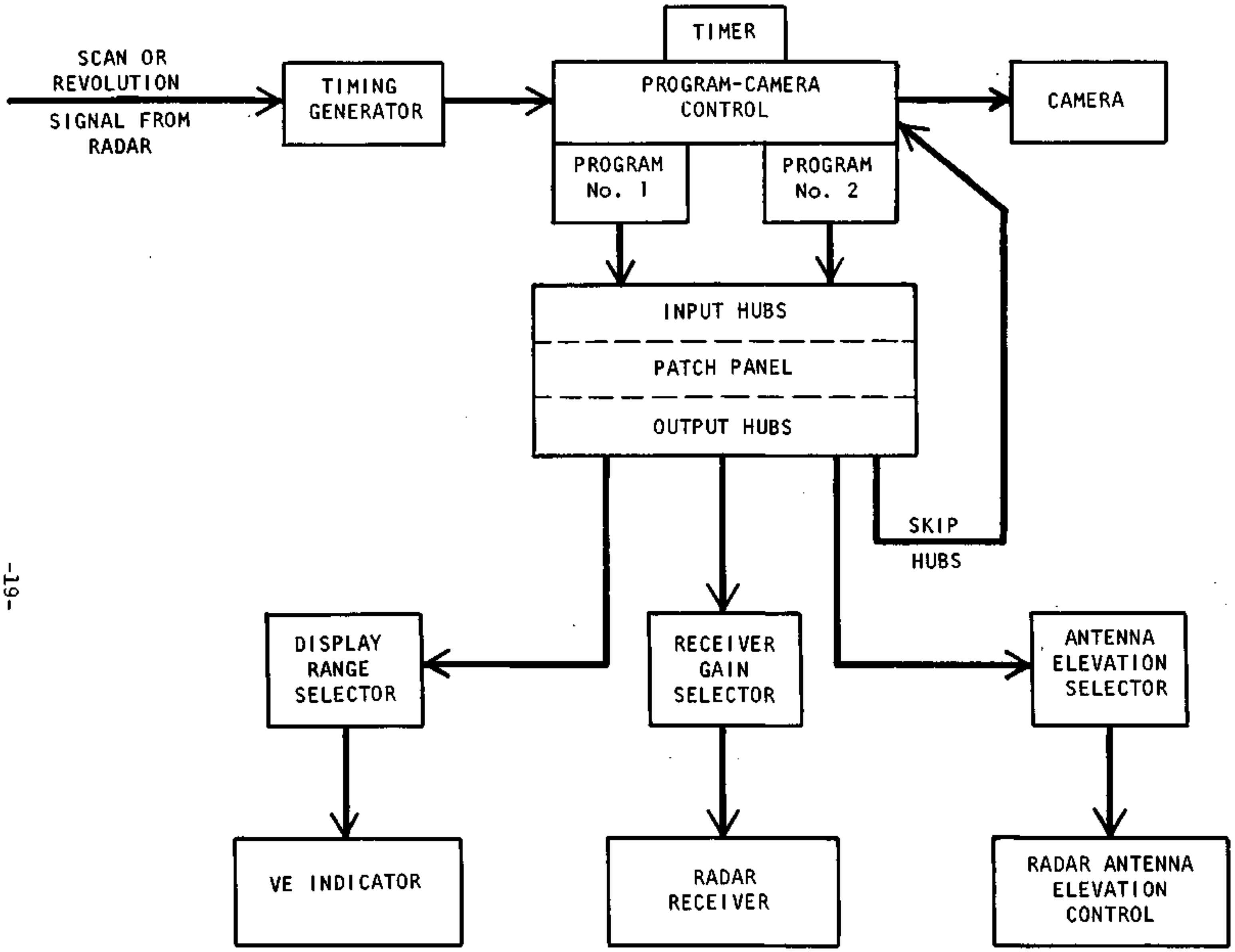

Fig. 5. Block diagram of the programmer for the CPS-9 radar.

One or both programs can be used in the timed mode. Single-program timed operation is used for surveillance when small amounts of data are collected every "M" minutes. "M" can be any multiple of two minutes between 1 and 11, or 2 to 22 minutes.

The two-program timed mode is used when one program is a normal continuous data collection program and the other program is a priority timed program. In this mode, the timed program runs at the beginning of a period M set by the operator, and the other program runs during the part of the period not used by the timed program.

Any receiver gain (15 levels), display range (50, 100, 250, 400 n.m.), and antenna elevation (0, 1, 2-25°) can be chosen for a given frame. Since the antenna elevation drive on the antenna pedestal is slower than the electronic display range, and receiver gain controls, the antenna elevation is varied at the slowest rate. Other than this, there are no restrictions on the frame combinations possible or on the frame sequences. The frame variables change only if a change is "patched on" one of the boards. If two adjacent frames are "patched" identically, or the second is not patched, no variable will change.

To enable rapid program changes, a skip ability was incorporated into the program control. Any frame patched into the skip board is passed over and the next frame is activated. By setting up a program covering an intense weather situation, a less severe situation can be observed and recorded by skipping the frames that are not needed. The end of the program is also variable by moving one patch cord. Single program operation provides 32 photographed frames and one non-photographed frame ("0").

The range and gain selectors are rotary switches driven by SCR's (silicon-controlled rectifiers). The elevation selector is a synchro-transmitter driven by a reversible motor. The synchro is positioned by an indexed plate and detent. The frame selectors are rotary switches driven by SCR's.

This programmer has been installed and operates well. It provides great flexibility in the type of data and in the operation of the radar in a prescribed manner. It has also been designed with a view toward adding an automatic signal processor and recorder other than the camera, which will be accomplished later.

SUMMARY AND CONCLUSIONS

The two years of data over the Kankakee network has indicated that the measurement accuracy of the radar has not been reduced at the greater range of the network. It is felt that this is more of an indictment of the analysis procedures than of the inherent ability of the radar. Thus it is believed that if the radar analysis could have been achieved with higher precision not only would the size of the standard error of estimate been different for the two networks but in addition they would both be smaller. With modern technology of signal processing available a better estimate of the radar average return power is possible and smaller errors should result.

The precision possible using the step gain PPI photographs and manual tracing, planimetering, and integration of rates is found to be slightly less than 0.2 inches of rain for the storm total. This value is larger by about 4 times than that which would be expected from considerations of the variability of the radar rainfall relationship from drop size data.

A method for extrapolating radar rainfall relationships based on the percentage of rain days which are thunderstorm days and the relative humidity at 0.5 Km elevation has been determined. This appears to be the most promising means of extrapolating these data to other areas of the earth.

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APPENDIX A

SURVEY OF RELATIONSHIPS BETWEEN RAINFALL RATE AND RADAR REFLECTIVITY IN THE MEASUREMENT OF PRECIPITATION*

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ABSTRACT

Basic to any measurement of rainfall amount is the relationship between the radar cross section or reflectivity and the rainfall rate. Numerous investigations of this relationship have been made in the last two decades from both a direct measurement of the radar reflectivity and the rainfall amount, as well as indirect measurements of the raindrop size spectra. Calculations of the radar reflectivity and rainfall rate from these spectra can be made and the relationships determined. Both methods are discussed in this paper and a summary of the relationships presented.

These relationships show differences in excess of 500% in rainfall rate at the same reflectivity. These large differences are primarily associated with differences in geographic locality. In addition, there are smaller differences on the order of 150% that can be attributed to different types of rain or different synoptic conditions.

Some data are available which are indicative of the differences in the relationship on a given day, depending upon the location within the storm which is sampled. This is briefly described and in only one case out of 15 is there a significant difference.

Estimates of the effects of evaporation, accretion, and coalescence on the relationship are made and show some of the reasons for the differences in the relationships noted at different geographical locations.

The accuracy of the relationships is investigated with attention directed to the evaluation of total storm amounts. It is shown that, in general, the relationships introduce less uncertainty than the uncertainty in obtaining a radar measurement of the reflectivity.

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Survey of Relationships Between Rainfall Rate and Radar Reflectivity in the Measurement of Precipitation

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Introduction

Basic to any measurement of precipitation by means of a radar is some form of a relationship between radar parameters and rainfall rate. The rainfall rate is a function of the raindrop size distribution. In the United States of America the radar meteorologist has informally adopted a unit called reflectivity and designated the symbol Z to represent this quantity. Some investigators refer to Z as the reflectivity factor. Some confusion has arisen as the radar engineer uses reflectivity to represent a slightly different quantity. The back scattering cross section of an object is defined as the area which intercepts an amount of power in the incident beam which if radiated isotropically would yield a reflected signal strength at the transmitter of the same magnitude as the actual object produces. The radar engineer's definition of reflectivity is the average sum of the radar back scattering cross section per unit volume of space. It can be noted that the dimensions of this quantity is per unit length. The radar meteorologist frequently uses Rayleigh's scattering law and removes the constants of wavelength, and refractive index, leaving a term of diameter of the sphere to the sixth power. If the sum of the diameters to the sixth power of the raindrops per unit volume is multiplied by the constants of wavelength and refractive index, the normal radar engineers reflectivity results. Common usage has been to call the value of $\sum D^6$ the reflectivity, Z. Despite this inconsistency in word usage, we will continue to speak of Z as reflectivity. The common units of Z are mm^6/m^3. Most work has been directed toward the relationship between Z and rainfall rate R.

The R-Z relationships are generally reported in the form

$$Z = AR^b$$

Many investigators have noted a tendency in the data for departures from this relationship. Some confusion has arisen from this relationship because of uncertainty of which variable is treated as independent in the original analysis. For the use of the radar meteorologist, who wishes to predict the rainfall rate from measurement of the radar reflectivity, the reflectivity should be treated as the independent variable. If the rainfall rate is considered the independent variable, the exponent is smaller and the coefficient larger for the same data.

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I. Direct Measurement of the Relationship Between Radar Reflectivity and Rainfall Rate

One method for obtaining a relationship between the radar back scattering cross section and the rainfall rate is to actually measure both simultaneously. This obvious method has been attempted by several groups[1,2,3] with varying degrees of success. There are a number of disadvantages of such a straight forward method. The fact that the radar invariably samples rain aloft, and the raingage samples the rain at the surface is one difficulty in the procedure. Austin[4] attempted to minimize this error by directing the radar beam directly over a raingage located on a high point of ground. The radar antenna was directed as low as possible without any ground return showing at the range of the raingage. Most investigators have attempted to time lag the radar observations to compensate for the time of fall of the raindrops.

A second problem associated with the elevated radar sample is the horizontal drift of the raindrops during their fall from the radar beam location to the ground. In order to reduce these effects a network of raingages has been utilized by some groups so that the drift and time lags could be incorporated in the analysis. These methods certainly tend to increase the confidence of the experiment but there remains considerable doubt whether the corrections for time lag and drift can be completely eliminated by these techniques.

A further disadvantage is the immense discrepancy between the sizes of the samples. Neglecting the vertical extent of the radar beam (this amounts to time-smearing in the raingage) and assuming common radar parameters of 1° horizontal beam width and 1 microsecond pulse width, the area over which the radar samples at 10 km is about $2.6 \cdot 10^4$ m^2. The raingage samples an area on the order of $7 \cdot 10^{-2}$ m^2. As the range increases, the radar area is increased proportionately.

To reduce this difficulty one may use more than one raingage under a radar volume such as was performed by Dimaksyan, Zotimov and Zykov5. They used three networks at ranges of 12, 22, and 32 km with 5, 9, and 12 gages all located within their respective radar areas. This yielded a gage density of one gage per 0.04, 0.045 and 0.05 km^2 respectively. Thus the measurement of the radar Z could be related to the average rainfall rate from the average of at least 5 gages. Unfortunately, the calibration results of this work have been directed towards the calibration of a particular radar in terms of deflection of an A scope. Without specific knowledge of the receiver and detector characteristics, it is not possible to use these results elsewhere. The authors were surprisingly successful where in a later paper[6], they report that "when the radar installation is sufficiently sensitive to rainfall intensity, the estimate of total precipitation in an area will be more accurate than could be obtained from a rainfall measuring network of practically <u>any</u> density".

Doherty[7] performed a unique direct measurement which permitted a high confidence in the measurement of the radar scattering. In this experiment the receiver was separated from the transmitter by 860 m and by making measurements of the direct transmission between antennas, it was possible to eliminate the need for knowing precisely the transmitter power and gain of the receiving and transmitting antenna. His results shown in Table 1 indicate a much lower coefficient than that ordinarily found. He found higher A's as the rainfall rate increased. His Doppler frequency records indicated downdrafts on a number of occasions before the onset of rain. This would account for the low A.

Wilson[8], using data from a 1100 square mile raingage network, obtained Z-R relationships for a number of thunderstorms in Oklahoma. His procedure consisted in obtaining the best relationship, using least squares method, between network average amounts from the radar and network average amounts from raingages. In 4 of the 6 storms analyzed, his Z-R relationships did not depart significantly in terms of his measurement error from the frequently quoted $Z = 200\ R^{1.6}$.

Caton[9] used a Doppler radar in conjunction with a raingage to deduce the drop size spectra. The raingage provided an average water flux at the ground level and the radar provided a frequency power spectrum. The drop size spectra were deduced from these two measurements and the reflectivity and rainfall rate calculated from this spectrum. He found little change of Z (1 db) between the melting level and a $Z = 240\ R^{1.3}$ in rain near the cloud base.

Other investigators[10,11] in USSR and Japan have reported Z-R relation from radar measurements. Results are within the range already shown. These differences which may be due so what to technique or measurement error are also thought to be real. One cannot model a rainstorm for the entire world.

II. The Relationship Between Radar Reflectivity and Rainfall Rate from Measurements of Drop Size Spectra

Many problems associated with direct measurement of radar reflectivity and raingage rainfall rates and comparison of the two results, can be eliminated by direct measurement of the drop size spectra. However, new problems arise. The most serious difficulty with this type of measurement is that the volume in space in which the drops can be sampled is limited to a few cubic meters. The assumption must then be made that these few cubic meters are representative of the 10^5 or 10^6 cubic meters sampled by the radar.

A study by Mueller and Sims[12] indicates that for a sample at ground level, a sample of 44 m^3 is required to estimate the rainfall rate to within 10 percent with 95 percent confidence. It is also demonstrated in the same paper that a smaller volume is adequate to determine the R-Z relationship, if an adequate number of samples is included in the analysis. Thus, in this analysis using 1 m^3 samples, less than 12 percent of the variance of data points around the regression line could be attributed to the sample size.

To determine the rainfall rate from drop size spectra requires knowledge of the velocity of the individual raindrops. There has been a nearly universal acceptance of the terminal velocity, reported by Gunn

TABLE 1

Radar Rainfall Relationships from Direct Measurement

Investigator	Geographical Location	Range of Applicability	$Z = AR^b$ A	$Z = AR^b$ b	Accuracy Estimate (Standard Deviation)	Comments
Doherty, L. H.	Ottawa Canada	TRW	70	1.42	2.5db	
		not TRW	38.4	1.63	1.7db	
		$R < 10$ mm/hr	18.6	2.37	1.6db	
		$R < 20$	25.9	2.02	1.7db	
		$R < 40$	33.9	1.79	1.9db	
		$R < 60$	38.2	1.69	2.0db	
Berjuljew, G. P., Beznis, A. M. and others (9 all total)	Valday USSR		340	1.5		The exponent is assumed equal to 1.5 and the coefficient determined from 2 years of rainfall.
Wilson, J. W.	Norman Okla.	TRW	45	1.43		Extreme low coefficient
		TRW	241	1.45		Extreme large coefficient
		TRW	183	1.18		Extreme low exponent
		TRW	141	1.72		Extreme high exponent
Aoyagi, J.	Tokyo		100	1.4		For diffuse radar echoes

and Kinzer[13] as the velocity of the raindrops. This assumption is probably quite reasonable near the ground as is evidenced by the generally good agreement between the average rainfall rates from drop size spectra and the rates from a raingage. However, at the heights sampled by the radar, it is equally certain that the raindrops are not moving with terminal velocity with respect to ground because of the existence of either updrafts or downdrafts. Vertical pointing Doppler radar measurements have confirmed that the drops are moving with velocities with respect to earth that are different than the stagnant air terminal velocities.

Calculations of the radar scattering from the drop size spectra, assuming spherical drops, can be made by either the Rayleigh scattering assumption or from the more complete Mie scattering, depending primarily on the wavelength of the radar under consideration. Since the majority of work is at a wavelength of 3 cm or longer, the simpler Rayleigh scattering is usually assumed adequate. Rayleigh scattering for 3-cm radiation differs from the Mie scattering by less than 2 db at rainfall rates of 400 mm/hr and the difference is much less at lower rainfall rates. Some spectra measuring techniques (e. g., filter paper) measure a spectrum crossing a flat boundary per unit time, so that for these techniques the velocity of the raindrop enters the calculation of the radar reflectivity instead of the calculation of rainfall rate.

Table 2 is a list of R-Z relationships as determined from drop size spectra from a number of different investigators[2], 14-20 and for different types of rains. Diem's observations are taken at a number of locations and exhibit a low exponent. Most of rain was under 12 mm/hr. It is not known whether he has used R or Z as the independent variable. It appears that he might have used R as the independent variable.

The low coefficient for the orographic rains in Hawaii as first reported by Blanchard and later by Fujiwara appear to be in order. The drops in the Hawaiian upslope rainfall are very numerous, quite small and rainfall rates are low.

Dumoulin and Gogolombles[14] performed an experiment similar to Austin's with a radar directed at low elevation angles over a single raingage. Additionally, they obtained a number of drop size spectra in the vicinity of the raingage for which Z-R relationships are reported in Table 2. Their results from the raingage readings and the radar measurements were compared at identical observation times. In general they found good agreement between the rainfall rates obtained from drop size spectra and the raingages. However, when the radar Z was converted to a rainfall rate by means of the spectra-established Z-R relationship, a difference of at least 2 in rainfall rate remains. Dumoulin and Gogolombles indicated also that the Z-R relationship shows a large variation with time during a storm.

TABLE 2

Radar Reflectivity Rainfall Rate Relationships from Drop Size Spectra

	$Z = AR^b$		Standard Error Of Estimate of Log R	
Investigator	A	b		Comments
Marshall, J. S.	220	1.6		Widely accepted and used
Blanchard, D. C.	31	1.71		Orographic Hawaiian rain at cloud base
	16.6	1.55		Orographic Hawaiian rain within the cloud
Fujiwara, M	80	1.38		Orographic Hawaiian rain
Hardy, K. R.	312	1.36		Arizona and Michigan rain with rates greater than 5 mm/hr
Imai (in Japan)	700	1.6		One day of probably warm rain
	300	1.6		One day continuous rain
	200	1.5		Air Mass showers
	80	1.5		Pre-warm front rain
Diem, M	184	1.28		Overall average of different locations
	278	1.30		Entebbe Uganda (tropical)
	240	1.30		Lwin Congo (tropical)
	176	1.18		Palma
	151	1.36		Barza, Italy
	179	1.25		Karlsruhe, Germany spring
	227	1.31		Karlsruhe, Germany summer
	178	1.25		Karlsruhe, Germany fall
	150	1.23		Karlsruhe, Germany winter
	137	1.36		Axel Heiberg Land
Foote, G. B.	520	1.81		Tucson, Arizona
Dumoulin, G., Gogolombles, A	730	1.55		France, Average of all observations, 0.95 correlation coefficient
	255	1.45		
	426	1.5		
Mueller, E. A.	286	1.43	0.198	Florida
	221	1.32	0.170	Marshall Islands
	301	1.64	0.136	Oregon
	311	1.44	0.147	Indonesia
	267	1.54	0.142	Alaska
	230	1.40	0.171	North Carolina
	372	1.47	0.153	Illinois
	593	1.61	0.175	Arizona
	256	1.41	0.163	New Jersey

III. Discussion of the Relationships and their Variability-

It is immediately apparent from examination of Tables 1 and 2 that the constants of the relationships are widely variable. At the extremes one might compare the differences in the Z value at 1 mm/hr between Doherty in Table 1 and Dumoulin's relationships of Table 2. A difference of a factor of 10 exists (10 db). Assuming a measured value of $Z = 10^5$, the difference in rainfall rates calculated from these two relationships would be different by a factor of 5. Thus, differences of at least 500% in rainfall rate exist between different relationships. If one assumes a lower value of Z of 10^2, then, it is only a factor of 3 or 300%. It is probable that some of the differences may be partially due to differences in methods of obtaining the relationships. However, considerable differences exist using the same technique due to topography, geographical variation, rain type, synoptic type, the thermodynamic structure of the atmosphere, evaporation, and to some extent due to coalescence.

Geographical Differences

Results of a study conducted by Diem and the authors over a period of several years are presented in Tables 2 and 3. The Mueller relationships were deduced from drop size spectra obtained from a raindrop camera. This device photographed the raindrops which occurred in a 1 cubic meter volume in a 10 second period. Samples were taken for each minute. From these pictures, the drop size spectra were obtained and the R-Z relationships calculated using a logarithmic least squares fitting technique. The instruments were operated for one year at each of the following locations, Miami, Florida; Majuro, Marshall Islands; Corvallis, Oregon; Bogor, Indonesia; Woody Island, Alaska; Franklin, North Carolina; and Champaign, Illinois. The data from Flagstaff, Arizona represents only a 2 month sample during July and August.

A number of differences between locations can be seen in these data. The two extreme locations are the Marshall Islands and Alaska. The Marshall Island data indicate the highest rainfall rate for a particular radar reflectivity. At a reflectivity of $1.1 \cdot 10^5$ mm6/m^3 in Table 3, nearly 10 times greater rainfall rate is occurring in the Marshall Island climate than in the climate of Alaska. The drop size spectra in the Marshall Islands contain a relatively large number of small droplets which do not yield as much radar return, Z, as the larger but fewer drops in the Alaskan rains.

The climate of Oregon is similar to that of Alaska and thus the relationships are very similar. Florida and Indonesia tend to be nearly the same for low and medium values of the reflectivity, but for the high Z values, Florida has higher rates. This departure at the high rates suggests that different meteorological conditions prevail during high rainfall rate conditions at these two locations.

TABLE 3

Mean Rainfall Rates as a Function of Reflectivity for Different Geographical Locations

Radar Reflectivity mm^6/m^3	Rainfall Rate (mm/hr) Florida	Marshall Islands	Oregon	Indonesia	Alaska	North Carolina
$1.1 \cdot 10^2$	1.0	1.0	0.6	0.5	0.6	0.6
$3.5 \cdot 10^2$	1.0	1.6	1.1	1.1	1.2	1.6
$1.1 \cdot 10^3$	2.5	3.7	2.3	2.4	2.8	3.5
$3.5 \cdot 10^3$	6.3	8.7	5.4	6.0	5.2	7.8
$1.1 \cdot 10^4$	14.5	21.6	9.5	14.4	8.8	17.7
$3.5 \cdot 10^4$	34.8	48.4	18.7	29.5	9.0	38,7
$1.1 \cdot 10^5$	68.5	90.5		65.7	9.2	87.1
$3.5 \cdot 10^5$	167.1			70.0		
$1.1 \cdot 10^6$	247.7			123.8		

TABLE 4

Radar Reflectivity-Rainfall Rate Relations Using Rain Type Stratifications

Location	Rain Type	$Z = aR^b$ a	b	Correlation Coefficient	Standard Error of Estimate	Minutes of Data
Florida	Continuous	322	1.33	0.94	0.187	911
	Showers	250	1.47	0.95	0.185	696
	Thunderstorms	224	1.51	0.94	0.190	902
Marshall Islands	Continuous	226	1.46	0.97	0.184	1491
	Showers	146	1.42	0.92	0.141	952
Oregon	Continuous	295	1.59	0.92	0.133	600
	Showers	327	1.66	0.91	0.135	218
	Thunderstorms	339	1.64	0.95	0.089	82

Differences in the Relationship with Different Rain Types

At some locations, the data were separated into groups according to the rain type classification as reported by the observer operating the camera. The rain types recognized were thunderstorms, rainshowers, and continuous rain. The observers at each location had had some form of weather training and their reports were accepted as filed.

The camera at Franklin, North Carolina was operated on the side of a mountain some 4 miles from the observer's normal duty station. This prevented him from making observations of the rain type occurring at the camera. The observers at Alaska reported continuous rain for nearly all of the data, and their reports of rainshowers were not sufficient to allow meaningful regressions. At the other extreme, nearly all of the data from Indonesia were reported as thunderstorms. At the remaining locations, stratification by rain type was performed and the results of the logarithmic least squares are shown in Table 4.

Since the standard error for this sorting of the data does not decrease appreciably, this stratification does not benefit the user greatly. The more showery a rain becomes the higher the radar reflectivity for medium to high rates. This is indicated by the increase in the size of the exponent from continuous rain through showers to thunderstorms.

Stratification by Synoptic Type

Stratification of data by examining the surface meteorological chart prepared by the U. S. Weather Bureau was attempted. The classification was in accordance with the major disturbance in the area and its relative position to the sampling point. The classification include air mass, pre-cold frontal, cold frontal, post-cold frontal, warm front, overrunning, easterly wave, trough aloft, warm occlusion, cold occlusion, trade wind showers, and intertropical convergence zone. Naturally, not all of these classes were filled at any one location. The data from Indonesia could not be stratified because surface maps were not available.

Table 5 presents the results of the synoptic stratifications for several locations.

Some improvement is suggested in this stratification scheme. The standard errors do reduce somewhat and the correlation coefficients generally are slightly higher. Some reduction in the standard error of estimate might be expected as a result of smaller sample. Confidence limits calculated for the exponent, b, indicate that the chances are remote that these are samples of the same parent population.

Stratification by Thermodynamic Instability

A measure of the instability of the air was investigated to determine whether a significant reduction of the standard error of estimate could

TABLE 5

Radar Reflectivity-Rainfall Rate Relations
Using Synoptic Stratifications

Location	Synoptic Class	$Z = aR^b$ a	 b	Correlation Coefficient	Standard Error of Estimate	Minutes of Data
Florida	Air Mass	323	1.42	0.98	0.180	467
	Pre-Cold Front	280	1.49	0.95	0.188	744
	Cold Front	198	1.54	0.95	0.176	187
	Warm Front	403	1.24	0.96	0.145	341
	Overrunning	302	1.36	0.94	0.165	196
	Easterly Wave	296	1.35	0.97	0.156	536
	Trough Aloft	261	1.43	0.97	0.178	80
	Pre-Cold Occlusion	330	1.66	0.91	0.127	40
Marshall Islands	Easterly Wave	196	1.38	0.95	0.171	1126
	Trade Wind Showers	126	1.47	0.98	0.130	239
	Intertropical Convergence Zone	196	1.38	0.95	0.178	1136
Oregon	Air Mass	322	1.62	0.95	0.094	157
	Post-Cold Front	322	1.70	0.90	0.140	204
	Overrunning	307	1.56	0.92	0.138	352
	Warm Front	295	1.66	0.91	0.143	158
	Warm Occlusion	339	1.48	0.95	0.126	175
	Pre-Warm Occlusion	309	1.92	0.90	0.111	151
	Post-Warm Occlusion	268	1.81	0.88	0.146	320

be obtained. The thermodynamic instability to some extent measures the strength of updrafts and available moisture. Tornado forecasts are based partially on this instability. The vigor of the storm might be reflected in the drop size spectra.

A measure of the thermodynamic instability is the amount of energy required to lift a parcel of air from the ground to a prescribed level aloft. If this energy is negative, instability is indicated. In the calculations parcels of air were raised from the surface and from every 50 mb pressure level to 600 mb up to a pressure height of 150 mb. The sum of the energies for each of the parcels is then a measure of the average thermodynamic instability. Radiosonde observations are normally obtained every 12 hours. The nearest radiosonde was used for each storm. The range of instabilities was then divided into groups and logarithmic least square analysis performed on each group. Table 6 contains the result of this analysis. The standard error of estimate is generally larger for this stratification than for either the synoptic type or the rain type stratification. One of the errors which may contribute to this poor stratification is the time separation between the radiosonde ascent and the time of rainfall. Frequently, the upper air conditions change just before the rain occurs. The inadequacies of the radiosonde data along with the loss of accuracy shown by the standard error of estimate preclude the use of this stratification.

Differences in the Relationship Within the Storm

To determine whether different parts of a storm have significantly different relationships,[20] three raindrop cameras were operated simultaneously. These cameras, referred to as site A, B, and C, were 1/2 mile and 1-1/2 miles apart. Table 7 shows the results for three different days. In general, it was noted that the relationships between the cameras did not depart significantly one from another. On only one day of the 8 cases selected for detailed study could a difference be noted. On 10 other days, it was apparent the relationships did not depart greatly from one another. Thus, it would appear that one sampling device to obtain a relationship should be sufficient for any one storm period over an area of four square miles.

Estimates of Environmental Effects on the Relationship

Very few measurements have been made of the effects of evaporation, coalescence, or accretion on the R-Z relationship. Caton[9] calculated the relationship using Doppler radar measurements under conditions of coalescing of raindrops. For a height interval of 975-1125 m a relationship of $Z = 215\ R^{1.30}$ was obtained. At a level of 525-975 m a relationship of $Z = 240\ R^{1.3}$ was found. Atlas21 calculates that coalescence should increase both the coefficient and the exponent. In either case the effect of coalescence between raindrops is considered to be of little importance below the cloud base.

TABLE 6

Radar Reflectivity-Rainfall Rate Relations Using Thermodynamic Instability Stratification

Location	Instability	$Z = aR^b$		Correlation Coefficient	Standard Error of Estimate	Minutes of Data
		a	b			
Florida	1 (highest)	264	1.40	0.97	0.141	136
	2	295	1.36	0.97	0.169	286
	3	307	1.41	0.97	0.150	367
	4	304	1.41	0.96	0.168	416
	5	313	1.39	0.98	0.141	133
	6	206	1.42	0.97	0.105	117
	7	420	1.41	0.97	0.191	161
	8	358	1.31	0.95	0.155	559
	9	352	1.38	0.95	0.146	238
	10 (lowest)	257	1.27	0.96	0.175	167
Marshall Islands	1 (highest)	153	1.38	0.97	0.182	160
	2	207	1.47	0.92	0.241	303
	3	143	1.41	0.97	0.182	356
	4	234	1.36	0.92	0.250	736
	5	172	1.41	0.94	0.227	738
	6	191	1.40	0.96	0.226	76
	7 (lowest)	166	1.46	0.96	0.218	91
Oregon	1 (highest)	237	1.98.	0.86	0.143	32
	2	216	2.01	0.88	0.127	36
	3	217	1.51	0.92	0.136	79
	4	211	1.99	0.86	0.146	369
	5	167	3.05	0.76	0.109	101
	6	232	1.98	0.83	0.160	182
	7	263	1.66	0.88	0.163	99
	8 (lowest)	248	1.90	0.88	0.147	526

TABLE 7

Relationships from Different Locations Within the Storm

$Z = AR^b$

Date	Site	A	b	Comments
15 May 1963	A	476	1.47	Light continuous rain with a maximum rate of 8.6 mm/hr
	B	430	1.45	
	C	406	1.44	
7 June 1963	A	446	1.40	Scattered night time convective activity with showers and thunderstorms
	B	446	1.43	
	C	433	1.43	
31 July 1963	A	575	1.69	Showers and continuous rain along a stationary front. Radar indicate heaviest cell passed over A
	C	298	1.30	

The effect of evaporation can under some conditions produce large differences in the R-Z relationship. Data from Hardy[17] in Flagstaff, Arizona, Foote[22] in Tucson, Arizona as well as data of the authors indicate that the evaporation effects in the dry hot climate of Arizona produce the high coefficients noted in Table 2.

Storm Amounts by Various Equations

One means of demonstrating the importance of using different relationships under different conditions is to evaluate the same storm using different equations. For this study 10 storm periods were selected at random, subject to having at least 45 minutes of data, different synoptic and rain type situations. For these storms a Z value for each minute was available and by using a Z-R relationship, a rainfall rate was calculated for each minute. The total storm rainfall was then calculated. This was performed for the synoptic relationship, the rain type relationship and the standard Marshall Palmer relationship.

It should be realized that this procedure is not strictly valid since the storms chosen for analysis were storms whose data have been incorporated in the determination of the relationships. It would be much better if data which were independent of the analysis could have been obtained. Table 8 shows the result of this analysis for the ten storms at three different locations.

For the Miami storms, the synoptic equation appears to be better than either the rain type or the Marshall Palmer although the differences to rain type are not large. The warm front storm was the poorest fit by the synoptic relationship. In this case the error amounts to 21 percent overestimate of the amount of rainfall. The standard error of estimate for this synoptic relationship is 0.145, which is one of the lower values from the Florida data. A standard error of estimate of this size would indicate that the calculated rate is from 29% lower to 40% higher than the true rate 68% of the time. Fortunately, the scatter around the relationships (measured by the standard error of estimate) is unduly pessimistic in determining the error in total rainfall amounts from storms of relatively long durations. On the other hand, the standard error of estimate provides a more reliable index to determine which type of relationship is best. Again using the Florida data the average standard error of estimate for the synoptic equations is 0.164 and for the rain type equation, is 0.187, thus indicating the superiority in general of the synoptic sorting. As is evidenced in the small sample of 4 storms, the synoptic equation is better than the rain type equations.

Oregon data has an average standard error of estimate of 0.119 for the rain type and 0.134 for the synoptic type. At this location the rain type appears to be a better predicator of the R-Z relationships than the synoptic conditions. Again even with the small sample of 3 storms this appears to be the case.

In the Marshall Islands, both the synoptic equation and the rain type shows a better correspondence between rainfall amounts than the Marshall Palmer equation. Amounts are 30-40% larger than Marshall Palmer and nearly equal to the actual rainfall.

Table 8. Examples of Storm Totals Using Various R-Z Relationships

Location	Synoptic Type	Rain Type	Maximum Rainfall Rate (mm/hr)	Duration (min)	Total Rainfall Actual	 Synoptic Equation	 - Rain Type Equation	 Marshall Palmer
Miami, Fla.	Air Mass	TRW	301	52	65	61	58	45
	Warm Front	R	43	64	14	17	17	13
	Cold Front	TRW	194	63	47	48	49	39
	Easterly Wave	RW	216	126	46	38	32	26
Majuro, Marshall Islands	Intertropical Convergence Zone	RW	45	45	4.6	3.5	3.5	2.4
	Tradewind Shower	RW	65	45	7.8	7.7	7.1	4.3
	Easterly Wave	RW	88	91	17	16	17	10
Corvallis, Oregon	Warm Front	R	6	68	2.3	2.2	2.3	2.8
	Warm Occlusion	R	15	167	11.0	8.9	9.3	11.1
	Cold Front	R 6 RW	11	116	4.8	6.0	5.5	7.5

IV. Summary and Conclusions

The wide differences reported by the many investigators appear to be attributable to the nature of the rain at different locations. Thus although all of the relationships that have been found do undoubtedly contain experimental error, the size of the error with respect to the magnitude of the differences leads to the conclusion that the nature of rainfall is widely variant with location and from day to day at a location.

While discussing the variance or scatter of the relationships, the accuracy which present radar techniques permit should be considered. Theory indicates that the appropriate radar parameter which should be measured to determine rainfall rate is the average return power. This average return power has frequently been measured by photographing a plan position indicator for a number of radar revolutions while the sensitivity of the radar receiver is changed. This technique is commonly called step gain pictures. The accuracy of step gain pictures has been estimated to be as much as ± 5 db. Under ideal conditions of careful calibration with a noise source, the authors feel that the accuracy may be somewhat better (± 3 db) but even so this uncertainty is so great that its use will introduce uncertainty of a magnitude comparable with the uncertainty due to the R-Z relationship. Modern techniques of radar signal processing in either analog or digital integrators allows a reduction in this radar measurement to about ± 1.5 db. At this level it becomes necessary to use different relationships if the best estimate of rainfall rate is to be obtained.

The accuracy of the relationships when considered on a point by point basis is not very good. Thus a standard error of the logarithms of 0.180 indicates that for any one measurement of Z that the indicated rainfall fall rate will lie between 0.66 to 1.52 times the true rate 68% of the time. Fortunately, the instantaneous rainfall rate is usually not as important as the amount of rainfall at the end of the rain. When an average is taken over a number of observations, these limits are materially reduced.

Current attempts are underway to develop a R-Z map for the world, based upon present data, relating the data to other meteorological parameters such as height of freezing level, height of average cloud base, dew point at 0.5 Km, days of rain, etc.

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13. ABSTRACT

The results of two years of radar measurement of rainfall amounts over the Kankakee raingage network has shown that the accuracy is not greatly different than the accuracy over the much closer East Central Illinois network. In either case if the standard error of estimate of radar amounts is chosen as a measure of the error, a result of about 0.2 inch for storm total is obtained.

Results of preliminary study of extrapolation of radar rainfall equations to other parts of the world is reported. A method depending upon the percentage of rain days which are thunderstorms and the relative humidity at a level of 0.5 km has been tentatively chosen.

A survey report on the relationships between rainfall rate and radar reflectivity in the measurement of precipitation shows that the relationships introduce less uncertainty than the errors in present day radar measurement of reflectivity.

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