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A NONSTATIONARY STOCHASTIC MODEL FOR STRONG-MOTION EARTHQUAKES

Mohammad Amin, Ph.D.
Department of Civil Engineering
University of Illinois, 1966

By examining a number of strong-motion earthquake records registered at the West Coast of the United States, it is concluded that over their significant duration the accelerograms are not stationary stochastic processes as has been assumed in the majority of the studies made thus far for the purpose of probabilistic approach to aseismic design of structures. Use has been made of nonstationary Gaussian filtered shot noise processes to obtain a more realistic stochastic model for strong-motion accelerograms. Only a linear-second-order-filter has been considered. The parameters of the model are determined from the earthquake records considered herein. Several pseudo-earthquakes are generated from the model on a digital computer. The pseudo-earthquakes bear resemblance to the earthquake records both in their appearance and in their average effects on the linear-single-degree-of-freedom systems.

The problems associated with the reduction of accelerograms into digital form have also been considered. Results are presented to show the nature of processing errors and their engineering significance.
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1. INTRODUCTION

1.1 Object and Scope

In the past, studies towards the establishment of a rational basis for design of structures to resist the destructive action of earthquakes have been made using two distinctly different approaches: deterministic and probabilistic. The deterministic approach has provided much valuable information regarding the behavior of structures during earthquakes. However, the well-known differences in the detailed characteristics of different accelerograms, and the unpredictability of future earthquakes make it clear that a more rational aseismic design of structures can be possible only if based on the results obtained from a probabilistic approach.

The primary objectives of this investigation are to study a number of strong-motion accelerograms registered at the West Coast of the U.S. for the purpose of classifying them from the point of view of stochastic process theory, and to select a random process to represent the records from which pseudo-earthquakes may be generated using a digital computer. The problem of obtaining an adequate statistical description for the earthquake records which is consistent with the known facts about the records has been the subject of much interest, and constitutes a basic step in the probabilistic approach to aseismic design. To accomplish the above objectives it was necessary to reduce a number of accelerograms into digital form. Therefore, it became necessary to consider the problems associated with the reduction of records and their engineering significance as a secondary objective.
A brief review of the available tools in the stochastic process theory to accomplish the primary objectives of this study is presented in Chapter 2. In Chapter 3 the problems associated with processing of accelerograms and their engineering significance are discussed. Also, in this chapter eight strong-motion accelerograms recorded to date at the West Coast of the U.S. are examined for the fundamental assumption of stationarity. In Chapter 4, the selection of a nonstationary filtered shot noise process\(^{(1)}\)\(^{\star}\) for modeling earthquake motions of the type considered in this study is described. Chapter 5 includes some of the results obtained for the pseudo-earthquakes generated from the stochastic model and their comparison with the real earthquake records.

Of the many possible choices of filters only one, namely, a second-order-linear filter has been considered. Also, it must be emphasized that records considered here have been registered on firm ground at moderate distances from the epicenter and no attempt has been made to consider effects of soil conditions of the sites.

1.2 Brief Review of Related Works

The advances in the probabilistic approach to aseismic design of structures and the aspects of this problem that need major attention have been summarized elsewhere.\(^{(2)}\) It can be seen from this survey that although earthquakes are distinctly transient in nature, most of the studies made thus far are based on the assumption that earthquake accelerations can be represented by stationary random processes. Several investigators have

\(^{\star}\) Numbers in parentheses refer to the references on Pages 72 and 73.
used white noise models for such studies. In order to consider processes with finite variance and with some degree of correlation between the closely spaced ordinates, properties which most physical phenomena possess, filtered white noise processes have been considered. Two types of filters that are commonly used are the single degree of freedom systems and a low pass filter. Also, by studying temporal autocorrelation functions of 5 accelerograms, an expression has been suggested for the autocorrelation function of earthquakes, this leads to the same form of the power spectral density which is obtainable from the filtered white noise of a single degree of freedom system.

A visual inspection of accelerograms indicates that over a record duration of 25 seconds or so, the statistics of the record, (variance for example), will vary with time. Inasmuch as a probabilistic approach to aseismic design must consider the probability of failure over the duration of the record, it appears that the nonstationary aspects of earthquake motions must be considered.

In two cases, earthquakes have been modeled by nonstationary random processes. In one case, the product of a particular stationary process and a time dependent envelope was used. This approach appears to be reasonable, but no specific expression for the envelope is given. In the other case, the nonstationary random function is used, in which the $\phi_i$ are a set of independent random phase angles, $a_i$ and $\alpha_i$ are deterministic constants. In one form of the model, the $\omega_i$'s are deterministic constants while in another form, these are treated as
independent random variables. Because \( n \) is usually taken to be 20, the task of relating the constants \( a_i \) and \( \alpha_i \) of this model to the statistics of earthquake records appears to require an extensive parametric study.

Finally, the use of filtered Poisson processes with several types of weighing functions has been suggested,\(^{14}\) but detailed study of the feasibility of the specific models has not been made.

The stochastic model considered herein is a nonstationary process and involves a limited number of parameters that can be related to a minimum number of statistics from the earthquake records. Also, it may be considered as a filtered Poisson process.

1.3 Acknowledgement

This investigation was conducted in the Department of Civil Engineering as part of the research program on probabilistic aspects of structural mechanics and dynamics. The author wishes to express his grateful appreciation to his advisors Dr. N. M. Newmark, Professor and Head of the Civil Engineering Department, and Dr. A. H.-S. Ang, Professor of Civil Engineering for their helpful suggestions and guidance during the course of this investigation.

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The author was introduced to the subject of stochastic structural dynamics by Dr. Y. K. Lin, Professor of Aeronautical and Astronautical Engineering at the University of Illinois. His instructive comments are gratefully appreciated.
Thanks are also due to the staff of the Digital Computer Laboratory of the University of Illinois where most of the numerical work was performed.

1.4 Notation

The symbols used in this investigation are listed below:

\[ A, a = \text{constants} \]
\[ A_o = p^2 U_o = \text{pseudo-acceleration} \]
\[ B = \text{equivalent bandwidth of a member function of a stationary random process, cps.} \]
\[ b = \text{constant, sec.} \]
\[ c = \text{constant, sec.}^{-1} \]
\[ C_i, (i=1,...,n) = \text{coefficients of the polynomial expression assumed for the true baseline of acceleration trace} \]
\[ \text{COV}_Y(t_1, t_2) = \text{covariance function of } Y(t) \]
\[ d_o = \text{constant, in.} \]
\[ E[ ] = \text{mathematical expectation of the random quantity in the bracket} \]
\[ F_Y(y, t) = \text{first order probability distribution function of } Y(t) \]
\[ F_{Y_1,...,Y_n}(y_1,...,y_n; t_1,...,t_n) = \text{nth order probability distribution function of } Y(t) \]
\[ F = \frac{\omega}{2\pi} = \text{undamped frequency of } L_{02}, \text{ cps} \]
\[ F_S(t) = \text{nonstationary filtered shot noise process} \]
\[ f_Y(y, t) = \text{first order probability density function of } Y(t) \]
\[ f_{Y_1,...,Y_n}(y_1,...,y_n; t_1,...,t_n) = \text{nth order probability density function of } Y(t) \]
\[ f = \text{frequency, cps} \]
\[ f_0 = \text{undamped natural frequency of a linear-single-degree-of-freedom system} \]
\( k \quad G_{y}(\omega) \quad = \text{power spectral density of } k_{y}(t) \)

\( G_{\text{max}} \quad = \text{maximum of } k_{G_{y}}(\omega) \)

\( g \quad = \text{gravitational acceleration} = 386.4 \text{ in/sec}^{2} \)

\( H(\omega) \quad = \text{complex frequency response function of } L \)

\( h(t) \quad = \text{impulse response function of } L \)

\( I(t) \quad = \text{intensity function of nonstationary shot noise process} \)

\( I_{o} \quad = \text{constant, } \frac{g^{2}}{\text{sec}^{3}} \)

\( i, j, k \quad = \text{subscripts or superscripts} \)

\( j \quad = \sqrt{-1} \)

\( L, L_{k}, L_{k1}, L_{k2} \quad = \text{ordinary-linear-differential-operators with constant coefficients} \)

\( k_{k}(k=0,1,...n) \quad = \text{coefficient of } L_{k1}; \quad k=0 \text{ denotes a single operator, sec}^{-1} \)

\( m_{y}(t) \quad = \text{mean value function of } Y(t) \)

\( \text{m}_{y} \quad = \text{temporal mean value of } k_{y}(t) \)

\( N(t) \quad = \text{Poisson counting process} \)

\( n_{t} \quad = \text{deterministic counting process} \)

\( P[ \quad ] \quad = \text{probability of the event in the bracket} \)

\( p = 2\pi f_{o} \quad = \text{undamped circular frequency of linear-single-degree-of-freedom system, sec}^{-1} \)

\( Q(t) \quad = \text{random input of } L \)

\( q(t) \quad = \text{deterministic input of } L \)

\( q(\omega) \quad = \text{Fourier transform of } q(t) \)

\( R_{Y}(t_{1}, t_{2}) \quad = \text{autocorrelation function of } Y(t) \)

\( R_{c}(t), R_{s}(t) \quad = \text{random functions} \)

\( k_{r_{y}} \quad = \text{temporal mean square value of } k_{y}(t) \)

\( r_{k} \quad = \text{temporal mean square value of the } k\text{th segment of a member function} \)
\( S_Y(\omega) \) = power spectral density of stationary \( Y(t) \); 
\( -\infty < \omega < \infty \)

\( S(t), S_1(t), S_2(t) \) = nonstationary shot noise processes

\( s \) = time lag

\( T \) = undamped natural period of linear-single-degree-of-freedom system

\( T_k \) = independent random time points

\( t \) = time

\( U_o \) = maximum spring deformation of linear-single-degree-of-freedom system

\( V_o = pU_o \) = pseudo-velocity

\( V_Y(t) \) = variance function of \( Y(t) \)

\( V_{\tilde{F}_k} \) = variance of \( \tilde{F}_k \)

\( \hat{V}_{\tilde{F}_k} \) = estimate of \( V_{\tilde{F}_k} \)

\( \nu_o \) = constant, in./sec.

\( W_Y(f) = 4\pi S_Y(\omega) \) = experimental power spectral density of stationary process \( Y(t) \)

\( X_k (k=1, \ldots, n) \) = independent random variables

\( x_1, x_2 \) = constants, sec.

\( Y(t) \) = an arbitrary random process; also, output of \( L \)

\( \dot{Y}(t) \) = mean square derivative of \( Y(t) \)

\( k_Y(t) \) = \( k \)th member function of \( Y(t) \)

\( y(t) \) = deterministic output of \( L \), also, adjusted ground displacement

\( \tilde{y}(\omega) \) = Fourier transform of \( y(t) \)

\( y_o(t) \) = unadjusted ground displacement

\( \alpha \) = confidence level; also a constant

\( \beta \) = fraction of critical coefficient of damping of linear-single-degree-of-freedom system
\( \beta_i (i=1, \ldots, n) = \) definite integrals required in the application of baseline adjustment

\( \gamma_k (k=0,1, \ldots, n) = \) damping factor of \( L_{k2} \); \( k=0 \) denotes a single operator

\( \Delta Q = \) small change in the arbitrary quantity \( Q \)

\( \delta(t) = \) Dirac delta function

\( \varepsilon^2 = \frac{V_{r_k}}{V_Y} = \) normalized variance of \( r_k \)

\( \hat{\varepsilon}^2 = \) estimate of \( \varepsilon^2 \)

\( \eta = |\Delta \beta_i| \ (i=1,2,3) \)

\( \rho_Y (t_1, t_2) = \) normalized covariance of \( Y(t) \)

\( \sigma_Y (t) = \sqrt{V_Y (t)} = \) standard elevation of \( \dot{Y}(t) \)

\( \tau = \) time lag

\( \tau, \tau_1, \tau_2 = \) variables of integration

\( k_y (\tau) = \) temporal autocorrelation function of \( Y(t) \)

\( \chi^2 (n) = \) chi-square with \( n \) degrees of freedom

\( \omega_k (k=0,1, \ldots, n) = \) undamped circular frequency of \( L_{k2} \); \( k=0 \) denotes a single operator, sec.^{-1}

\( \omega = \) 2\( \pi \) f

\( \omega_d = \omega \sqrt{1-\gamma_0^2} \)

\( <> = \) temporal average of the enclosed function
2. FUNDAMENTALS OF RANDOM VIBRATION

2.1 Introduction

When a dynamical system is acted on by an excitation that can be described by deterministic means, i.e., analytically or graphically, the vibration problem is solved by deterministic approach and the problem is that of deterministic vibration. Often situations are encountered where the excitation varies from observation to observation regardless of the care exercised to control the known causes of the excitation. To cope with such situations the excitation must be prescribed as a random or a stochastic process, and the vibration problem is termed that of random vibration.

The theory of random processes, which can be viewed as the dynamic counterpart of the classical probability theory, and its application to random vibration has been well developed, (see Refs. 15 and 16 for example). The purpose of this chapter is to summarize a few items from this field that are used in this study.

2.2 Random Processes

2.2.1 Characterization of Random Processes

A deterministic function of an independent variable t, say \( y(t) \), disregarding the multi-valued case, has a definite value at a given \( t \). No such definite prediction can be made about the value of a random process \( Y(t) \) when \( t \) is given. For each \( t \), say \( t = t_i \), there corresponds a large
number, perhaps infinite, of possible values of \( Y(t_i) \), i.e., \( Y(t_i) \) is a
random variable. While the deterministic function \( y(t) \) can be depicted
by a single curve, there is a collection of curves that correspond to
various realizations of the random process \( Y(t) \). The totality of possible
records depicting outcomes of \( Y(t) \) is referred to as the ensemble of
member functions of \( Y(t) \) or simply ensemble of \( Y(t) \). An ensemble of a
hypothetical random process is depicted in Fig. 1. Superscripts are used
to identify various member functions. Thus \( ^3y(t) \) is the third member
function of \( Y(t) \).

If we write \( Y_i \) for the random variable specifying the value of
the random process \( Y(t) \) at \( t = t_i \), i.e., \( Y_i = Y(t_i) \), for analytical purposes
the random process \( Y(t) \) is considered as the family of random variables \( Y_i \),
\( i = 1, 2, \ldots \). The probability law of \( Y(t) \) is specified by giving the joint
probability law of the family of random variables \( Y_i \). It should be remarked
that the \( Y_i \)'s may be discrete or continuous random variables and the
parameter \( t \) in the description of \( Y(t) \) may also be discrete or continuous.

The probability law of a family of random variables can be
specified by the joint probability distribution function or the joint
characteristic function. The various order probability distribution
functions are defined as follows:

\[
F_Y(y,t) = P[Y \leq y \text{ at } t]
\]

\[
F_{Y_1, Y_2}(y_1, y_2; t_1, t_2) = P[Y_1 \leq y_1 \text{ at } t = t_1 \text{ and } Y_2 \leq y_2 \text{ at } t = t_2]
\]

\[
F_{Y_1, Y_2, \ldots, Y_n}(y_1, y_2, \ldots, y_n; t_1, t_2, \ldots, t_n) = P[Y_1 \leq y_1 \text{ at } t = t_1, \nonumber
\]

\[
Y_2 \leq y_2 \text{ at } t = t_2, \ldots, \text{ and } Y_n \leq y_n \text{ at } t = t_n]
\]
Where $F_{Y_1}, F_{Y_1,Y_2},$ and $F_{Y_1,Y_2,...,Y_n}$ are respectively, the first, second and \(n\)th order probability distribution functions; the $y$'s are range variables, and $P$ denotes the probability of the event within the brackets. In the case of continuous random variables probability densities may be used to specify the probability law. The second order density $f_{Y_1,Y_2}(y_1,y_2;t_1,t_2)$, for example, is related to the corresponding joint distribution by the relation

$$f_{Y_1,Y_2}(y_1,y_2;t_1,t_2) = \frac{\partial^2 F_{Y_1,Y_2}(y_1,y_2;t_1,t_2)}{\partial y_1 \partial y_2}$$  

and has the following interpretation

$$f_{Y_1,Y_2}(y_1,y_2;t_1,t_2)dy_1dy_2 = P[Y_1 < y_1 \leq y_1 + dy_1 \text{ at } t = t_1 \text{ and } Y_2 < y_2 \leq y_2 + dy_2 \text{ at } t = t_2]$$  

The complete specification of the probability law of a random process is not an easy task and can be accomplished for only a limited number of random processes. Sometimes useful but incomplete information is provided by specifying the important statistics of the process; for instance the first and second moments.

2.2.2 Important Statistics of Random Processes

Two types of averages can be computed for a random process: ensemble averages and temporal averages. Ensemble averages refer to the mathematical expectations computed across the time axis and will be preceded by the operator $E$. Temporal averages are averages computed along the time axis for individual member functions and will be denoted with the
symbol \langle \rangle. Because they are associated with individual member functions, temporal averages do not in general yield useful information about an arbitrary random process. However, they are useful for a restricted class of random processes to be discussed later.

Of the many ensemble averages that can be defined, some of the averages associated with the first and second order probability densities play an important role. If \( g(Y) \) denotes a known function of the random process \( Y(t) \), \( E[g(Y)] \) is given by

\[
E[g(Y)] = \int_{-\infty}^{\infty} g(y)f_Y(y,t)\,dy \tag{2.4}
\]

In general \( E[g(Y)] \) depends on \( t \). If \( n \) number of representative member functions of \( Y(t) \) are available, \( E[g(Y)] \) can be computed from (see Fig. 1)

\[
E[g(Y)] = \frac{1}{n} \sum_{k=1}^{n} g[y(t)] \tag{2.5}
\]

The important statistics associated with the first probability density are obtained when \( g(Y) \) assumes specific forms. The mean value function is defined by

\[
m_Y(t) = E[Y(t)], \tag{2.6}
\]

the mean square of \( Y(t) \) is obtained when \( g(Y) = Y^2 \); and the variance function \( V_Y(t) \) is obtained by letting \( g(Y) = [Y - m_Y]^2 \). Mean, mean square, and variance are related by the relation

\[
V_Y(t) = E[Y^2(t)] - m_Y^2(t) \tag{2.7}
\]

The positive square root of the variance is called the standard deviation,
and will be denoted by $\sigma_Y(t)$, while the positive square root of $E[Y^2(t)]$ is known as the "root-mean-square" of the process.

The mean value function of a stochastic process may be regarded as an average function such that the various realizations of the process are grouped around it and oscillate in its neighborhood. The variance function is a measure of departures from the mean value function. For normal processes, from the knowledge of mean and variance the exact probability of any instantaneous departure from the mean can be computed. For processes that are not normal use of the well known Chebyshev's inequality establishes a crude estimate for this probability, i.e.,

$$P[|Y(t) - m_Y(t)| > \alpha \sigma_Y(t)] \leq \frac{1}{\alpha^2} \quad (2.8)$$

where $\alpha$ is a positive constant.

Now consider two functions $g_1(Y_1)$ and $g_2(Y_2)$; here $Y_1$ and $Y_2$ are two random variables $Y_1(t_1)$ and $Y_2(t_2)$ obtained from the random process $Y(t)$ at two time instants $t_1$ and $t_2$. $E[g_1(Y_1) \cdot g_2(Y_2)]$ is defined as:

$$E[g_1(Y_1) \cdot g_2(Y_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_1(y_1)g_2(y_2)f_{Y_1,Y_2}(y_1,y_2; t_1,t_2)dy_1dy_2 \quad (2.9)$$

In the general case this expectation is a function of both $t_1$ and $t_2$ and for sufficiently large representative samples of $Y(t)$ it can be computed from the relation. (See Fig. 1).

$$E[g_1(Y_1) \cdot g_2(Y_2)] = \frac{1}{n} \sum_{k=1}^{n} g_1[^kY(t_1)][^kY(t_2)] \quad (2.10)$$

When $g_1(Y_1) = Y_1$ and $g_2(Y_2) = Y_2$, the above expectation is referred to as autocorrelation function of the process $Y(t)$ and denoted...
by \( R_Y(t_1, t_2) \). The covariance function or covariance kernel of \( Y(t) \), \( \text{CoV}_Y(t_1, t_2) \), refers to the same expectation when \( g_1(Y_1) = Y_1 - m_Y(t_1) \) and \( g_2(Y_2) = Y_2 - m_Y(t_2) \). It can readily be verified that

\[
\text{CoV}_Y(t_1, t_2) = R_Y(t_1, t_2) - m_Y(t_1)m_Y(t_2),
\]

(2.11)

\[
E[Y^2(t)] = R_Y(t, t),
\]

and

\[
V_Y(t) = \text{CoV}_Y(t, t).
\]

The normalized covariance is defined as,

\[
\rho_Y(t_1, t_2) = \frac{\text{CoV}_Y(t_1, t_2)}{\sigma_Y(t_1)\sigma_Y(t_2)}
\]

(2.12)

Note that \( R_Y \), \( \text{CoV}_Y \), and \( \rho_Y \) are all symmetric functions of \( t_1 \), and \( t_2 \). Also, it can be shown that \(-1 \leq \rho \leq 1\).

The covariance function establishes a measure of "coherence" between \( Y_1(t_1) \) and \( Y_2(t_2) \), i.e., values of \( Y(t) \) at two different times. Because it is the average product of deviations of \( Y_1 \) and \( Y_2 \) from their respective means, when \( \text{CoV}_Y(t_1, t_2) \) is large and positive (\( \rho \approx 1 \)), one can conclude that \( Y_1 \) and \( Y_2 \) tend to be large together and small together. When the \( \text{CoV}_Y(t_1, t_2) \) is large numerically but negative (\( \rho \approx -1 \)), \( Y_1 \) tends to be large when \( Y_2 \) is small. When \( \text{CoV}_Y(t_1, t_2) \) is zero, positive products appear about as often as negative products on the average. Therefore, when \( \text{CoV}_Y(t_1, t_2) = 0 \), \( Y(t) \) is said to be an uncorrelated process.

For Gaussian (normal) processes the knowledge of the mean value and covariance functions is sufficient to describe the probability law of
the random process completely. Also, from the covariance of \( Y(t) \), the covariance of the random process \( \dot{Y}(t) \) can be determined, where dot denotes differentiation with respect to \( t \) in the mean-square sense.

\[
\text{CoV}_Y(t_1, t_2) = \frac{\partial^2 \text{CoV}_Y(t_1, t_2)}{\partial t_2 \partial t_1} \quad (2.13)
\]

This information is useful in view of the following generalization of the Chebyshev's inequality

\[
P[|Y(t) - m_Y(t)| > \alpha \text{ for all } t \text{ in } a \leq t \leq b] \leq \frac{1}{2\sigma^2} \left[ V_Y(a) + V_Y(b) + 2 \int_a^b \sigma_Y(t) \sigma_Y(t) dt \right] \quad (2.14)
\]

Where \( V_Y(t) = \text{CoV}_Y(t, t) \). In words, Eq. (2.14) provides an upper bound to the probability that \( Y(t) \) will cross a preassigned value about its mean value function in the time interval \( a \leq t \leq b \).

In contrast to the ensemble averages which represent the average properties of all the member functions, the temporal averages refer to the quantities computed for individual member functions; these averages do not contain useful statistical information except for a special class of random processes as will be noted below.

Summarized below are several commonly discussed temporal averages. With reference to the kth member function, \( Y(t) \), the temporal mean value is defined as.

\[
\overline{k}_Y = \langle k_Y(t) \rangle = \lim_{t_d \to \infty} \frac{1}{t_d} \int_{-t_d/2}^{t_d/2} k_Y(t) dt \quad (2.15)
\]
The temporal mean square is defined as:

\[ k_{\Delta y} = \langle [k_y(t)]^2 \rangle = \lim_{t_d \to \infty} \frac{1}{t_d} \int_{-t_d/2}^{t_d/2} [k_y(t)]^2 dt \] (2.16)

and the temporal autocorrelation is given by

\[ k_{\Delta y}(\tau) = \langle [k_y(t)k_y(t+\tau)] \rangle = \lim_{t_d \to \infty} \frac{1}{t_d} \int_{-t_d/2}^{t_d/2} k_y(t)k_y(t+\tau) dt \] (2.17)

Clearly

\[ k_{\Delta y}(0) = k_{\Delta y} \]

2.2.3 Nonstationary, Stationary, and Ergodic Random Processes

Random processes are known as: nonstationary and stationary, depending on whether or not their probability laws are functions of actual time instants. If the probability law is invariant under shifts of time scale, say by \( a \), the process is said to be a stationary process (in the strict sense). Otherwise, it is a nonstationary process. In terms of probability densities, strict stationarity requires that

\[ f_y(y, t+a) = f_y(y, t) \]

\[ f_{y_1, y_2}(y_1, y_2; t_1+a, t_2+a) = f_{y_1, y_2}(y_1, y_2; t_1, t_2) \] (2.18)

\[ f_{y_1, y_2, y_3}(y_1, y_2, y_3; t_1+a, t_2+a, t_3+a) = f_{y_1, y_2, y_3}(y_1, y_2, y_3; t_1, t_2, t_3) \]
Because of this invariance under time shifts one concludes that for strictly stationary random processes the first order density is independent of \( t \) and the higher densities depend only on the lags, i.e., \( t_{i+1} - t_i \), between the time instants considered. If only the first two relations are satisfied the random process is said to be weakly stationary or stationary in the wide sense. In the case of normal processes, however, wide sense stationarity implies stationarity in the strict sense.

Intuitively a stationary process must be produced by a random mechanism which does not change as the time progresses; stationary records must have no definite beginnings and endings. In reality, however, every real process must start and end. For some problems the nonstationary effects associated with the beginning and ending of the record can be neglected if their durations are negligible in comparison with the duration of the stationary part of the record. Then, the process may be assumed to be stationary. If this condition is not met the assumption of stationarity should, strictly speaking, be abandoned.

An important step in modeling a physical phenomenon by a random process is to classify the phenomenon as stationary or nonstationary. Stationarity in the strict sense cannot usually be established from a limited number of available records. A test for wide sense stationarity is summarized in the sequel.

For a stationary random process the ensemble averages associated with the first probability density are constants and those associated with the second probability density are a function only of the time lag between the time instants considered. Letting \( t_2 = t_1 + \tau \), then

\[
R_y(t_1, t_2) = R_y(t_1, t_1 + \tau) = R_y(\tau)
\]

(2.19)
and

\[ \text{CoV}_Y(t_1, t_2) = \text{CoV}_Y(t_1, t_1+\tau) = \text{CoV}_Y(\tau) \quad (2.20) \]

Also, because the autocorrelation function is a symmetric function of its arguments, one concludes that \( R_Y(\tau) = R(-\tau) \), i.e., \( R(\tau) \) is an even function. It can also be shown that

\[ R_Y(\omega) \geq | R_Y(\tau) | \quad (2.21) \]

For stationary processes another statistic, known as the power spectral density is defined by the Wiener-Khintchine relations

\[ S_Y(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_Y(\tau)e^{-j\omega\tau} \, d\tau \quad (2.21) \]

\[ R_Y(\tau) = \int_{-\infty}^{\infty} S_Y(\omega)e^{j\omega\tau} \, d\omega \]

Thus, the spectral density and the autocorrelation function of a stationary process constitute a Fourier transform pair. From the second of Eqs. (2.21) we have

\[ R_Y(\omega) = \mathbb{E}[Y^2(t)] = \int_{-\infty}^{\infty} S_Y(\omega) \, d\omega \quad (2.22) \]

Therefore, spectral density represents the contribution to mean square value from the frequency band \( d\omega \) centered at \( \omega \). It may be remarked in passing that similar procedures are available for the frequency decomposition of the autocorrelation of nonstationary processes. (18)

In the definition of \( S_Y(\omega) \) both negative and positive frequencies are included. It can be shown that \( S_Y(\omega) \) is a non-negative even function,
and for experimental purposes it is convenient to use frequency in units of cycles per seconds a function \( W_Y(f) = 4\pi S_Y(\omega) \) is used. From Eq. (2.22) it follows that

\[
R_Y(\omega) = E[Y^2(t)] = \int_0^\infty W_Y(f) df
\]

(2.23)

where

\[
f = \frac{\omega}{2\pi}
\]

Taking the expectation of both sides of Eqs. (2.15) through (2.17) one can conclude that for stationary processes

\[
E(k_{\bar{m}_Y}) = m_Y
\]

\[
E(k_{\bar{T}_Y}) = E[Y^2(t)]
\]

(2.24)

and

\[
E(k_{\phi_\tau}(\tau)) = R_\tau(\tau)
\]

In words, Eq. (2.24) states that for a stationary process \( Y(t) \) the expected values of the temporal averages of the member functions are equal to the corresponding ensemble averages of the random process. In statistical terminology, the temporal averages are unbiased estimates of the corresponding ensemble averages for stationary processes. This relationship also exists between the ensemble spectral density \( S_Y(\omega) \) and the member spectral density \( k_{G_Y}(\omega) \) which is defined by

\[
k_{G_Y}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k_{\phi_\tau}(\tau)e^{j\omega \tau} d\tau
\]

(2.25)
\[ k_{\phi_y}(\tau) = \int_{-\infty}^{\infty} k_{G_y}(\omega) e^{j\omega\tau} d\omega \quad (2.25) \]

Ergodic processes are those stationary processes for which the temporal averages (not the means of temporal averages) are identical with the corresponding ensemble averages. Therefore, for an ergodic process any one member function is sufficiently representative of the other member functions. In this sense, ergodic processes constitute a sub-class of stationary processes.

2.3 Random Output of Linear Filters

A time invariant linear filter is an operator that converts an input to an output and satisfies two conditions: the output corresponding to the superposition of two inputs is the superposition of the corresponding individual outputs; the only effect of delaying an input by a fixed time is a delay in the output by the same time. In this thesis, the word filter will be restricted to mean an ordinary-linear-differential-operator with constant coefficients of the type

\[ L = \sum_{k=0}^{n} L_k \quad (2.26) \]

Where the \( L_k \)'s are distinct and each has one of the two forms

\[ L_{k1} = \frac{d}{dt} + \xi_k \quad (2.27) \]

or

\[ L_{k2} = \frac{d^2}{dt^2} + 2\gamma_k\omega_k \frac{d}{dt} + \omega_k^2 \quad (2.28) \]

\( \xi_k, \gamma_k, \omega_k \) are positive constants and \( 0 < \gamma_k < 1 \). With this definition of \( L_k \) the roots of the characteristic equation of the operator \( L \) will be
negative real or complex conjugate pairs with negative real parts. Consequently, the output to an input in the past will decay with time and will have oscillatory characteristics.

In deterministic problems the output of L, Y(t), to any input, q(t), is usually obtained using either impulse response function, h(t), or frequency response function, H(ω), which are associated with the filter L. Impulse response is the solution to the equation

\[ L[h(t)] = δ(t) \]  \hspace{1cm} (2.29)

subject to the zero initial conditions at t = 0, i.e.,

\[ h(0) = \frac{dh(0)}{dt} = ... \frac{d^{m−1}h(0)}{dt} = 0 \]  \hspace{1cm} (2.30)

where δ(t) is the Dirac delta function and m denotes the highest order of the derivatives in L. Obviously,

\[ h(t) = 0, \text{ for } t < 0 \]  \hspace{1cm} (2.31)

Frequency response function, H(ω), is determined as the steady-state solution to the equation,

\[ L[y(t)] = e^{jωt} \]  \hspace{1cm} (2.32)

that is, the input is q(t) = e^{jωt}

With this information the solution to an aperiodic input q(t) in Eq. (2.32) subject to zero initial conditions at t = t₀ is obtained formally as

\[ y(t) = \int_{t₀}^{t} h(t−τ)q(τ)dτ \]  \hspace{1cm} (2.33)

or

\[ \tilde{y}(ω) = H(ω)\tilde{q}(ω) \]  \hspace{1cm} (2.34)
where bar over a quantity denotes Fourier transformation. It can be shown that \( h(t) \) and \( H(\omega) \) are a Fourier transform pair, i.e.,

\[
H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt
\]

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega
\]

The impulse response and frequency response functions for the first order filter, Eq. (2.27), and the second order filter, Eq. (2.28), are given by Eqs. (2.36) and (2.37), respectively.

\[
h(t) = e^{-\frac{\omega_o}{2} t}, \quad t \geq 0
\]
\[
= 0, \quad t < 0
\]

\[
H(\omega) = \frac{\frac{1}{\omega_o}}{\omega_o^2 + j\omega}
\]

\[
h(t) = \frac{1}{\omega_d} e^{-\gamma \omega_o t} \sin \omega_d t, \quad t \geq 0
\]
\[
= 0, \quad t < 0
\]

\[
H(\omega) = \frac{1}{\omega_o^2 - \omega^2 + 2j\gamma \omega_o \omega}
\]

where

\[
\omega_d = \omega_o \sqrt{1 - \gamma^2}
\]

When the input \( Q(t) \) is a random process, the output \( Y(t) \) may be studied in the frequency domain using \( H(\omega) \) or in the time domain using \( h(t) \). In the following presentation, the results of time domain analysis will be emphasized; very little will be said about the analysis in the
frequency domain.

Assuming that integration and expectation operations may be interchanged, from Eq. (2.33), one concludes

\[ m_Y(t) = \int_0^t h(t-x)dx = -\int th'dt \]  

(2.39)

Therefore, for zero initial conditions, the expected value of the output is determined from the knowledge of the mean square function of the input. If \( m_Q(t) = 0 \), \( m_Y(t) \) is also zero. Because

\[ Y(t_1)Y(t_2) = \int_0^t \int_0^t h(t_1 - \tau_1)h(t_2 - \tau_2)h(t_1 + \tau_1)Q(\tau_1)Q(\tau_2) d\tau_2 d\tau_1 \]

(2.40)

the autocorrelation function of \( Y(t) \) is

\[ R_Y(t_1, t_2) = \int_0^t \int_0^t h(t_1 - \tau_1)h(t_2 - \tau_2)R_Q(\tau_1, \tau_2) d\tau_2 d\tau_1 \]  

(2.41)

Combining Eqs. (2.39) and (2.40) yields

\[ \text{CoV}_Y(t_1, t_2) = \int_0^t \int_0^t h(t_1 - \tau_1)h(t_2 - \tau_2)\text{CoV}_Y(\tau_1, \tau_2) d\tau_2 d\tau_1 \]

(2.41)

On letting \( t_1 = t = t \) in Eqs. (2.40) and (2.41) the expressions for mean square and variance functions of \( Y(t) \) are obtained, respectively, as follows:

\[ E[Y^2(t)] = \int_0^t \int_0^t h(t - \tau_1)h(t_2 - \tau_2)h(t_1 + \tau_1)Q(\tau_1)Q(\tau_2) d\tau_2 d\tau_1 \]  

(2.42)
and

\[ V_Y(t) = \int_{t_0}^{t} \int_{t_0}^{t} h(t-\tau_1)h(t-\tau_2) \text{CoV}_Q(\tau_1, \tau_2) \, d\tau_2 \, d\tau_1 \]  

(2.43)

From the foregoing it is clear that the filter output, \( Y(t) \) is always a nonstationary random process. However, if the operating time is long, the response to a stationary input \( Q(t) \) will tend to a stationary process as \( t \to \infty \), and in this case a simple relationship exists between the input power spectral density and the power spectral density of the stationary output. This relation is

\[ S_{Y_\text{stationary}}(\omega) = |H(\omega)|^2 S_Q(\omega) \]  

(2.44)

As an illustration of some of the concepts summarized in this Section, consider a first order filter. Let \( Q(t) \) be a random process with mean zero. Therefore, \( R_Q(t_1, t_2) = \text{CoV}_Q(t_1, t_2) \). Also let,

\[ R_Q(t_1, t_2) = R_Q(t_2-t_1) = A\delta(t_2-t_1), \quad t_2 \geq t_1 \]  

(2.45)

Where \( A \) is a constant, and consequently \( Q(t) \) is a stationary process which is commonly known as white noise. Such a process has a constant power spectral density at all frequencies.

Let us consider the filter output under three conditions:

(a) \( t_0 = -\infty \), then

\[ \text{CoV}_Y(t_1, t_2) = A \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} e^{-\rho_0(t_1-\tau_1)} e^{-\rho_0(t_2-\tau_2)} \delta(\tau_2-\tau_1) \, d\tau_2 \, d\tau_1 \]  

(2.46)
Therefore, \(Y(t)\) is stationary when the stationary input operates for a long time. The resulting variance is

\[
V_Y = \text{CoV}_Y(0) = \frac{A}{2\lambda_o} \tag{2.47}
\]

(b) \(t_o = 0\), then

\[
\text{CoV}_Y(t_1, t_2) = A \int_0^{t_1} \int_0^{t_2} e^{-\lambda_o (t_1 - \tau_1)} e^{-\lambda_o (t_2 - \tau_2)} \delta(\tau_2 - \tau_1) d\tau_2 d\tau_1
\]

\[
= \frac{A}{2\lambda_o} \left[ e^{-\lambda_o (t_2 - t_1)} - e^{-\lambda_o (t_2 + t_1)} \right] \tag{2.48}
\]

Clearly \(Y(t)\) is nonstationary. Letting \(t_2 = t_1 + \tau\)

\[
\text{CoV}_Y(t_1, t_1 + \tau) = \frac{A}{2\lambda_o} \left[ e^{-\lambda_o \tau} - e^{-\lambda_o (2t_1 + \tau)} \right]
\]

Now as \(t_1 \to \infty\) the limit of this expression is

\[
\text{CoV}_Y(\tau) = \frac{A}{2\lambda_o} e^{-\lambda_o \tau}
\]

demonstrating that \(Y(t)\) becomes stationary after a long time. Also, from Eq. (2.48),

\[
V_Y(t) = \frac{A}{2\lambda_o} \left[ 1 - e^{-2\lambda_o t} \right] \tag{2.49}
\]
(c) \( t_0 = 0 \), but \( Q(t) \) ceases after \( t = t_d \). In this case, \( Q(t) \) is nonstationary and the variance of \( Y(t) \) is given by.

\[
V_Y(t) = \frac{A}{2\ell_o} \left[ 1 - e^{-2\ell_o t} \right], \quad t \leq t_d
\]

\[
= V_Y(t_d) e^{-2\ell_o (t - t_d)}, \quad t \geq t_d
\]

\( V_Y(t) \) for the above three cases is depicted in Fig. 2. Note the pronounced effect of nonstationarity on the output variance caused by removal of the input in case c after \( t = t_d \).

\( V_Y(t) \) for a second order filter, Eq. (2.28), for the inputs as those considered in the three cases above exhibit the same general trend as in Fig. 2 with some superimposed oscillations. The output to a stationary input approaches a stationary process after some time, the rate at which the stationarity condition is approached depends on \( \gamma_j \). When \( \gamma_j = 0 \), the variance is always increasing and never approaches a stationary value.

### 2.4 A Test for Stationarity

In modeling random phenomena by stochastic processes it is important to determine from the experimental results whether the records are stationary or nonstationary before suitable models are selected for their descriptions. The importance of this classification becomes apparent when it is noted that the statistics of a nonstationary process can depart considerably from a comparable stationary process and the extreme probabilities which are usually of interest are known to be sensitive to the variations in the statistics. To clarify this point
further consider the following situation.

Assume that the random process \( Y(t) \) is the output of a suitably chosen filter. Also let the process be a mean zero process. From Eq. (2.14) one concludes that for deterministically known initial conditions at \( t = 0 \), the upper bound to the probability that \( Y(t) \) will leave a preassigned region in the interval \( 0 \leq t \leq t_d \) is given by

\[
P[|Y(t)| > \alpha, \ 0 \leq t \leq t_d] \leq \frac{1}{2\alpha^2} \left[ V_Y(t_d) + 2 \int_0^{t_d} \sigma_Y(t) \sigma_Y(t) dt \right]
\]

From the comments made about the behavior of \( V_Y(t) \) at the end of Section 2.3, (see also Fig. 2), it is readily noted that the right-hand side of the above equation will be quite different for cases a and c or b and c.

In general a large number of experimental records are needed to establish the weak stationarity of a random process. The knowledge of weak stationarity supplemented by the assumption of normality insures the strict stationarity of the process. Strict stationarity cannot usually be established from the records. For this reason, by stationarity is meant stationarity only in the weak sense.

In Refs. 17 and 18 a method has been suggested to examine an ensemble of experimental records for stationarity. The procedure, summarized in Fig. 3, is as follows:

First, at a given level of significance, each member record is tested for self-stationarity. A self-stationary record is defined as one in which the temporal mean, mean square, and autocorrelation function in any one sub-interval of the record agrees, within prescribed limits, with all the other sub-intervals of the same record. If not all the member records are self-stationary, then, the process may or may not be
stationary but it definitely is not ergodic. The stationarity of such an ensemble should be established by performing ensemble averaging and noting whether the results can be considered invariant to shifts in the time scale. On the other hand, if all the records are self-stationary, then, the process will be stationary. In addition, if the same stationary results are obtained from all the member functions, the process will be ergodic.

Now returning to the problem of determining the self-stationarity of individual member records, let \( \{ y(t), 0 \leq t \leq t_d \} \) be a measured record of the process \( \{ Y(t), -\infty < t < \infty \} \). It is hypothesized that \( Y(t) \) is stationary with mean zero and variance \( V_Y \). The interval \( (0, t_d) \) is subdivided into \( n \) equal sub-intervals as shown in Fig. 4, where for convenience the superscript \( i \) is omitted. The temporal mean squares are evaluated for sub-interval, \( k \), yielding a collection of \( n \) mean square values \( \bar{r}_k \). The \( \bar{r}_k \)'s will be different, however, if the record is self-stationary the variation among them will be small. To obtain an index for this variation, it is further assumed that the process is approximately normal. Under this assumption it can be shown that the quantity

\[
\epsilon^2 = \frac{\overline{r}_k}{V_Y} = \frac{1}{B t_d}
\]  

(2.51)

Where \( B \) is the equivalent bandwidth of the record and can be approximated by,

\[
B = \frac{V_Y}{G_{\text{max}}}
\]

(2.52)

where \( G_{\text{max}} \) is the maximum value of the record spectral density, \( G_Y(\omega) \).

Now if \( \overline{r}_k \) is the variance estimated for the random variables \( \bar{r}_k \) from \( n \) observations, an estimate of \( \epsilon^2 \) can be obtained from:
Because \( \bar{\epsilon}^2 \) is an estimate of the variance,

\[
\frac{\bar{\epsilon}^2}{\epsilon^2} \text{ is distributed as } \frac{\chi^2_{(n-1)}}{n}
\]

Where \( \chi^2_{(n-1)} \) denotes chi-square distribution with \( (n-1) \) degrees of freedom.

In summary, the application of the test for self-stationarity at \( \alpha \) level of significance consists of the following steps:

1. \( G_y(\omega) \) for the record of duration \( t_d \) is estimated and using Eqs. (2.51) and (2.52) \( \epsilon^2 \) is determined.
2. The record is subdivided, and \( \bar{r}_k, k = 1, 2, \ldots, n \), are determined.
3. From the information in (b), \( \bar{\epsilon}^2 \) is evaluated using Eq. (2.53). Here \( V_y \) is unknown; it can be estimated by the best available value, namely, the arithmetic mean of \( \bar{r}_k \).
4. Using \( (1-\alpha) \) percentiles of the \( \chi^2_{(n-1)} \) a region of acceptance for the ratio \( \frac{\bar{\epsilon}^2}{\epsilon^2} \) is determined. If the computed \( \frac{\bar{\epsilon}^2}{\epsilon^2} \) is greater than this limit, with probability \( (1-\alpha) \) the record is considered to be self-nonstationary in the interval \( (0, t_d) \).
3. A STUDY OF STRONG-MOTION ACCELEROMETERS FOR STATIONARITY AND ASSOCIATED PROBLEMS

3.1 General

To arrive at a realistic stochastic model for earthquakes it is necessary to have a number of earthquake records available for the purpose of classifying the phenomenon and making a comparison of the mean features of the generated member functions from the model to the real earthquake records. In the majority of past studies, earthquakes have been assumed to be stationary random processes without an examination of the records for stationarity.

For engineering studies the most reliable quantity recorded during earthquakes is the time history of the ground acceleration. The ground velocity, displacement, and structural responses are then integrated from the accelerograms using either analog or digital computers. When using digital computers the records must, of course, be digitized first.

In studying earthquake records, the two types of errors associated with the use of accelerograms must be recognized: errors that occur when the record is registered, and errors that occur in processing the records for digital calculation. To the first category belong the instrument errors and the effect of the structure in which the instrument is housed. The second group of errors occur when the accelerogram is digitized and in the determination of a set of acceptable ground velocity and displacement diagrams corresponding to the digitized record. The determination of the velocity and displacement is complicated because
of the unknown position of the zero acceleration line on the accelerogram and because motion is already in progress when the instrument starts reading, since a level of excitation is needed to trigger the accelerometer.

It is known that results obtained by different people for the same record vary. This variation in the computed quantities has been of significance in itself. Because it is necessary to process a number of records, a study of the expected variation in the results due to processing of the records is therefore, worthy of investigation.

It is the purpose of this chapter to summarize the study made on the nature of differences to be expected in the ground motion and response spectra caused by different methods of processing the same accelerogram, and to demonstrate that the assumption of stationarity of earthquake records is of questionable validity; consequently, in modeling earthquakes nonstationarity should be considered.

3.2 Reduction of Records and Associated Problems

To study the effect of processing accelerograms, N21°E component of the Taft, California record of 7/21/52 was used. This accelerogram was selected because it is one of the clearer strong-motion accelerograms recorded to date.

From the negative made of the original accelerogram four contact prints were obtained. The time scale and the starting point of the record were marked on these duplicate prints and these were distributed among four people at the following schools: University of Illinois; California Institute of Technology; University of California, Berkeley;
and University of Michigan. Each person digitized approximately the first 60 seconds of his print independently. The acceleration null line was determined visually in each case by the individual operator and no baseline adjustment was applied. The accelerogram was assumed to be piecewise linear, and the coordinates of the points of definition of the record were read. The data was put on IBM cards and sent to the other persons. Each operator used his best judgment to define the accelerogram by a series of straight lines.

At Illinois the record was digitized by the writer using a Benson-Lehner-X-Y coordinate reader. Because of the limitation on the range of the X-Y reader, the entire record could not be digitized in a single setting. It was necessary to shift the record and realign it as carefully as possible to align with the temporary baseline of the preceding segment. This shifting of the record occurred at about 30 seconds.

Prior to the reduction of the total record and in order to obtain a measure of the reliability of the readings obtained by the X-Y reader, the first 15 seconds of the record was digitized by machine and visually with the aid of an engineer's scale. The resulting velocity diagrams are compared in Fig. 5. Since the significant features of the two diagrams are essentially the same, the X-Y reader was used to digitize the accelerogram because of the degree of flexibility it provides in digitizing the records.

3.2.1 Baseline Adjustment

Because of the uncertainty in the initial conditions of the ground motion and in the location of the true zero acceleration line, it is a common knowledge that the computed ground velocity and displacement
from any digitized accelerogram will not be the true ground motions. The velocity diagram obtained from the reduction of the accelerogram performed at Illinois is presented in Fig. 6. Since the motion during an earthquake is a to and fro motion, the velocity diagram in Fig. 6 is clearly not correct. The corresponding displacement diagram is of the same nature as the velocity curve with a maximum value of 1937 in. It is of interest to note that such a terminal maximum displacement could be caused by a constant acceleration shift of 0.0027 g, g denotes the gravitational acceleration, which corresponds to a distance of about 0.5 mm. on the accelerogram.

In the past, three procedures have been used to adjust the baseline of a digitized record to obtain an acceptable set of ground velocity and displacement diagrams. A constant acceleration shift has been applied to obtain zero terminal velocity. This approach is questionable because the point at which the reduction of the acceleration trace ends need not coincide with the point at which the motion terminates. A series of constant acceleration shifts have been applied to make the velocity diagram oscillate about its null line. In addition to the fact that constant intermediate acceleration shifts cannot be justified physically, no two persons working independently are apt to get the same results using this procedure. A third method has been to assume that the true acceleration null line has the equation of a second degree parabola and to determine the constants of the parabola by minimizing the computed square error in the velocity. This approach was first used in Ref. 19 Although it also is arbitrary, it is more acceptable from a statistical viewpoint in that it corresponds to the well-known method of least square.
Perhaps, the best procedure for baseline adjustment is to examine the unadjusted velocity and displacement curves and on the basis of this to arrive at a baseline adjustment in each individual case. On the other hand, there is merit in a procedure which can be applied directly with a minimum amount of personal judgment. In this sense the latter procedure appears to be the best method among those described above. The assumption of polynomials of higher degree than linear such as a parabola for the equation of baseline can be made plausible only by noting that for the earthquake records considered thus far the unadjusted velocity curves do exhibit a curved trend. This curved trend has sometimes been attributed to the warping of the paper.

Method of Least Square for Baseline Adjustment — Using least square method a polynomial expression of degree $n$ is determined for the true zero acceleration line as follows. Let $y(t)$ denote the ground displacement. The subscript $o$ will be used to indicate the "as-read" or unadjusted values; dots denote differentiation with respect to time. Let it be desired to adjust the particular segment of the record from $t = t_o$ to $t = t_d$. The adjusted values of ground acceleration, velocity, and displacement are obtained from the unadjusted values by the equations

$$y(t) = y_o(t) - \sum_{j=1}^{n} c_j (t-t_o)^{j-1}, \quad t_o < t \leq t_d \quad (3.1)$$

$$= y_o(t) - c_1, \quad t = t_o$$

$$\dot{y}(t) = y_o(t) - \sum_{j=1}^{n} \frac{1}{j} c_j (t-t_o)^{j-1} \quad (3.2)$$
and
\[ y(t) = y_0(t) - \sum_{j=1}^{n} \frac{1}{j(j+1)} C_j (t-t_0)^{j+1} - v_o (t-t_0) - d_o \]  
(3.3)

where
\[ v_o = \dot{y}_0(t_o) - \dot{y}(t_o) \]
\[ d_o = y_0(t_o) - y(t_o) \]

The adjustment coefficients \( C_j \) are determined by minimizing the integral
\[ \int_{t_0}^{t_d} \dot{y}^2(t) dt \]. Therefore,

\[ \frac{\partial}{\partial C_i} \int_{t_0}^{t_d} \dot{y}^2(t) dt = 0, \quad i = 1, 2, \ldots, n \]  
(3.4)

Application of Eqs. (3.4) leads to the following set of simultaneous linear equations for the coefficients \( C_j \).

\[ \sum_{j=1}^{n} C_j \frac{1}{j(j+1)} (t_d-t_0)^{j+1} = \beta_i - \frac{v_o}{(i+1)(t_d-t_0)}, \quad i = 1, 2, \ldots, n \]  
(3.5)

where,
\[ \beta_i = \frac{1}{(t_d-t_0)^{i+2}} \int_{t_0}^{t_d} \dot{y}_o(t)(t-t_0)^i dt \]  
(3.6)

For the special case when a second degree parabola is used for the total length of the record, i.e., \( t_o = 0 \) and \( t_d = \) duration of record, and if the initial conditions are all zero, Eqs. (3.5) and (3.6) become

\[
\begin{bmatrix}
1/3 & 1/8 & 1/15 \\
1/4 & 1/10 & 1/18 \\
1/5 & 1/12 & 1/21 \\
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 t_d \\
C_3 t_d^2 \\
\end{bmatrix}
= 
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3 \\
\end{bmatrix}
\]  
(3.7)
and

$$\beta_i = \frac{1}{\tau_{i+2}^d} \int_{t_0}^{t_d} y(t) t^{i} dt$$  \hspace{1cm} (3.8)$$

The solution of Eqs. (3.7) is

$$\begin{bmatrix}
C_1 \\
C_2 t_d \\
C_3 t_d^2
\end{bmatrix} =
\begin{bmatrix}
300 & -900 & 630 \\
-1800 & 5760 & -4200 \\
1890 & -6300 & 4725
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}$$ \hspace{1cm} (3.9)$$

3.2.2 Sensitivity of Computed Ground Motion to Inaccuracies in Evaluation

of $\beta_i$

A study of Eq. (3.5) reveals that the adjustment coefficients $C_i$ and consequently the computed ground displacement diagram are extremely sensitive to possible inaccuracies in the evaluation of values of $\beta_i$.

To demonstrate the degree of this sensitivity, consider the particular case of the parabolic baseline adjustment. Let errors of magnitude $\eta$ be introduced in the evaluation of the integrals, $\beta_i$. Assume further that errors have the follows signs: $(\Delta \beta_1, \Delta \beta_2, \Delta \beta_3) = \eta$, $(1, -1, 1)$. The corresponding changes in the values of $C_i$ as determined from Eq. (3.9) are: $(\Delta C_1, \Delta (C_2 t_d), \Delta (C_3 t_d^2)) = \eta$, $(1830, -11760, 12915)$. These values of $\Delta C$ cause the following maximum changes in the computed ground velocity and displacement.

$$\Delta y_{\text{max}} = 165 \ t_d \eta, \quad \text{at} \quad \frac{t}{t_d} = 0.199$$

$$\Delta y_{\text{max}} = 51.1 \ t_d^2 \eta, \quad \text{at} \quad \frac{t}{t_d} = 0.480$$ \hspace{1cm} (3.10)$$

For a duration of 60 secs. and a value of $\eta = 5 \times 10^{-8} g$, the corresponding changes computed from Eqs. (3.10) are $0.192$ in./sec., and $3.56$ in.,
respectively. It is, therefore, clear that small computational errors in the values of $\beta_i$ produce sizeable changes in the computed ground displacement.

In the two tables given below are summarized four cases in each of which the errors of magnitude $\eta$ were assumed for the values of $\beta_i$. The four cases considered cover all possible combinations of signs for errors of equal magnitude that may occur in the case of parabolic baseline adjustment.

<table>
<thead>
<tr>
<th>Value of $i$</th>
<th>Assumed $\Delta \beta_i \times 1/\eta$</th>
<th>Corresponding $\Delta(C_i t_d^{i-1}) \times 1/\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>1</td>
<td>1 1 -1 -1</td>
<td>30 1830 1230 570</td>
</tr>
<tr>
<td>2</td>
<td>1 -1 -1 -1</td>
<td>-240 -11760 -8160 -3360</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 -1</td>
<td>315 12915 9135 3465</td>
</tr>
</tbody>
</table>

for $t_d = 60$ secs., $\eta = 5 \times 10^{-8} \text{in/sec.}$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta y_{\text{max}} / \eta \times 1/\eta$</th>
<th>$\Delta y_{\text{max}} / \eta \times 1/\eta$</th>
<th>$\Delta y_{\text{max}} / \eta \times 1/\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.23</td>
<td>3.81</td>
<td>4.030</td>
</tr>
<tr>
<td>2</td>
<td>831</td>
<td>812</td>
<td>3.56</td>
</tr>
<tr>
<td>3</td>
<td>559</td>
<td>528</td>
<td>2.22</td>
</tr>
<tr>
<td>4</td>
<td>257</td>
<td>232</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Note particularly that the signs of errors play an important role in the resulting changes of ground velocity and displacement. The changes caused by the errors of the same sign, case 1, are negligible in comparison to those resulting from the critical combination of signs in case 2.

In view of the sensitivity of the adjusted displacements to the possible inaccuracies in the evaluation of $\beta_i$, special care was exercised.
in computing the integrals that define $\beta_i$. For the 60 secs. of the record, digitized at Illinois, values of $\beta_i$ and $(\Delta y)_{max}$ are summarized in the table below using four schemes of integration.

**Scheme 1.** The integrals were evaluated by the application of Simpson's rule taking account of the fact that the unadjusted velocity diagram consists of parabolic segments.

**Scheme 2.** The integrals were first integrated by parts, and the resulting integrals, involving the unadjusted ground displacement instead of the unadjusted ground velocity, were evaluated by application of Simpson's rule. Since the displacement curve is smoother than the associated velocity curve, the results computed in this case may be expected to be more accurate than those obtained in Scheme 1 using the same time interval of integration.

**Schemes 3 and 4.** In these schemes, the values of the integrals between consecutive points of definition of the acceleration diagrams were evaluated by direct integration taking account of the parabolic variation of the unadjusted velocity diagram. These two schemes differed only in the manner in which the various terms in the integrated expressions were grouped, but can otherwise be considered as mathematically "exact". Scheme 3 involves small differences between numbers of nearly equal magnitude and would be expected to be less accurate than scheme 4 for which the grouping of terms is preferable. The detailed expressions used are given in Appendix A.
values of \(10^6 \times \beta_i/g\)

<table>
<thead>
<tr>
<th>(i)</th>
<th>Scheme 1 (\Delta t = .02)</th>
<th>Scheme 2 (\Delta t = .2)</th>
<th>Scheme 3 (\Delta t = .1)</th>
<th>Scheme 4 (\Delta t = .01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>949.471</td>
<td>949.462</td>
<td>949.463</td>
<td>949.466</td>
</tr>
<tr>
<td>2</td>
<td>721.956</td>
<td>721.947</td>
<td>721.948</td>
<td>721.950</td>
</tr>
<tr>
<td>3</td>
<td>583.220</td>
<td>583.213</td>
<td>583.213</td>
<td>583.215</td>
</tr>
</tbody>
</table>

values of \((\Delta y)_{max}\) in in., assuming Scheme 4 as exact

<table>
<thead>
<tr>
<th>(\Delta y) (\times 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0047</td>
</tr>
<tr>
<td>.0347</td>
</tr>
<tr>
<td>.0134</td>
</tr>
<tr>
<td>.00072</td>
</tr>
<tr>
<td>.1655</td>
</tr>
</tbody>
</table>

It is comforting to note that \((\Delta y)_{max}\) in all cases considered here are negligible. However, poor grouping of terms in the computation of \(\beta_i\) can lead to sizeable errors, of the order of 3 in. or so, in the computed ground displacements using the same type of polynomial for the baseline.

In Fig. 7 are presented the ground displacement curves computed from the Cal. Tech. data using the same record duration and a parabolic expression for the baseline. The difference between the two computed displacement curves is caused by errors in the evaluation of \(\beta_i\). The curve shown by solid line corresponds to the results obtained by the four schemes of integration considered above. The curve shown by dashed lines corresponds to a slightly poor grouping of terms in the expressions used to evaluate \(\beta_i\).

3.2.3 Baseline Adjustments Considered

The unadjusted velocity diagram computed for the Illinois reading of the record is presented in Fig. 6. It is clear from Fig. 6 that some type of baseline adjustment is necessary. It is also noted that the slope of the general trend of the velocity curve changes at about
Concerning this change of slope, it may be recalled that the record had to be shifted and reset on the machine at about 30 secs. because of the limited range of the X-Y reader. Although special care was exercised to align the record, some deviation from the temporary baseline of the previous segment was inevitable. This deviation is most likely the cause of the change in the general trend of the unadjusted velocity.

To obtain, by using simplest possible baseline adjustment, a first order approximation to what may be called a "balanced" record the following constant acceleration shifts determined from the trend of the unadjusted velocity curve were applied to the record

\[-0.0026g, \ 0 \leq t \leq 30.23\]

and

\[-0.0034g, \ t \geq 30.23\]

The resulting baseline for the velocity diagram is shown in Fig. 6 by dotted lines. The velocity and displacement diagrams obtained after the application of this simple baseline adjustment are presented in Fig. 8. The velocity diagram in Fig. 8 is not far from being balanced, however, the displacement diagram requires further adjustment. This is to be expected because the constant acceleration shifts described above were arrived at by visual examination of the velocity curve. It is interesting to note the type of adjustment that is possible even by such a crude procedure.

The least square procedure was used to apply several types of baseline adjustments to the Illinois reading of the record and to the modified record described above. The duration of the record considered
in all of these computations was 60.65 sec. To facilitate the presentation of results, the following designations will be used to identify various adjusted records. I1 refers to the unadjusted record reduced at Illinois. I1M denotes the modified record presented in Fig. 8. Letters L, P, and C following I1 or I1M identify the type of baseline adjustment applied to those records by the least square procedure. L denotes a linear, P a parabolic, and C a cubic baseline adjustment. Thus, I1MP designates an adjusted record obtained from I1M by the application of a parabolic baseline adjustment to that record.

The designations for all the adjusted records are summarized in the following table.

<table>
<thead>
<tr>
<th>Record</th>
<th>Baseline Adjustment</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unmodified Version of Illinois Reading No. 1</td>
<td>None</td>
<td>I1</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>I1L</td>
</tr>
<tr>
<td></td>
<td>parabolic</td>
<td>I1P</td>
</tr>
<tr>
<td></td>
<td>cubic</td>
<td>I1C</td>
</tr>
<tr>
<td>Illinois Reading No. 1 modified and shown in Fig. 8</td>
<td>See Fig. 6</td>
<td>I1M</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>I1ML</td>
</tr>
<tr>
<td></td>
<td>Parabolic</td>
<td>I1MP</td>
</tr>
<tr>
<td>*Unmodified version of Illinois Reading No. 2</td>
<td>cubic</td>
<td>I2C</td>
</tr>
</tbody>
</table>

* For this reading computations were based on a record duration of 60 seconds.

3.2.4 Computation of Spectral Values

The deformation spectra to be presented in this thesis were obtained by integrating the equation of motion of the linear single-degree-of-freedom system using an iterative scheme as described in Ref. 20. The effect of time interval of integration on the accuracy of the computed
maximum spring deformation, $U_0$, and the associated times, $t_m$ is summarized in Table 1. The time of interval of integration used to obtain the results were such that $f_o \Delta t \leq 1/20$.

One step in the computation need be commented on. In the response computations, the ground acceleration is assumed to have a linear variation between the consecutive data points for the adjusted records. A comparison of the ground displacements computed in this manner and by taking account of the actual variation in various adjusted records indicated that the computed results were in good agreement. For this reason the assumption of linear variation of the ground acceleration was used in all the response computation.

3.3 **Nature and Significance of Variation in Results Computed from Processed Accelerograms**

There are two sources of discrepancy when a given accelerogram is prepared as a digital input: reading of the acceleration trace, and the type of baseline adjustment. Because these discrepancies are inevitable, it is of some interest to demonstrate the order of magnitude of variations in the ground motion as a result of these errors. The significance of these variations will be evaluated by observing the manner in which the deformation spectrum of a simple linear oscillator is affected by different digital inputs obtained from the same original record.

The deformation spectrum is a plot against the undamped natural frequency, $f_o$, of the maximum spring deformation $U_0$ experienced by a linear-single-degree-of-freedom system acted upon by a given excitation. The quantity $U_0$ is also expressed in terms of pseudo-velocity $V_o = pU_o$. 
and pseudo-acceleration $A_0 = p^2 U_0$, in which $p$ is the circular frequency. $U_0$, $V_0$, and $A_0$ are all measures of spring deformation, and using a log-log plot when $V_0$ is plotted on the vertical scale and $f_0$ on the horizontal scale the lines sloping at $45^\circ$ up to the right are lines of constant $U_0$ and those, sloping at $45^\circ$ up to the left are the lines of constant $A_0$, see Fig. 38 for example.

3.3.1 Effect of Baseline Adjustment on the Ground Motion

The adjustment coefficients $C_i$ depend on the record duration $(t_d - t_0)$ and the degree $n$ of the polynomial assumed for the baseline. Because for the computation of velocity $C_i$'s are multiplied by $(t-t_0)^1$ and for the displacement by $(t-t_0)^{i+1}$ and since the factor $(t-t_0)$ is usually a large number, it is expected that the changes in the velocity diagram resulting from small changes in $C_i$ will be significantly smaller than the corresponding changes in the displacement diagram. Also, the coefficients $C_i$ are so small that when plotted, the adjusted acceleration diagram appears to be essentially identical to the unadjusted one. For this reason, only the unadjusted acceleration diagram plotted from the digitized record II is presented in Fig. 9.

The adjusted velocity and displacement curves obtained from the accelerogram using different types of baseline adjustment and record durations are presented in Figs. 8, 10, 11, and 12. The ground motions in Fig. 12 were obtained from II by adjusting the record in two segments of about 30 secs. each, and using the final motion of the first segment as initial input to the second segment. From Figs. 8 and 12 it is noted that the velocity diagrams obtained by using two entirely different baseline adjustments are essentially the same. The adjusted displacement
curves, however, are quite sensitive to the type of baseline adjustment used, see Figs. 10 and 12. Parts a and b of Fig. 11 show the variation in the computed displacement curves when several values of record duration are considered in the computation. Again the associated velocity diagrams were essentially the same and are not presented.

In Figs. 10 and 11c it is noted that for the record durations considered, 60.65 for Fig. 10 and 30.23 for Fig. 11c, the displacement curves of IIP, IIC and IIMP, IIMC appear to have converged to a final value. For these record durations, therefore, there is no reason to consider a polynomial of higher degree than second. However, because the displacement curves of IIP, in Fig. 11b, for example, vary considerably with the record duration, the converged curves cannot, obviously, be considered as the true displacement of the ground.

3.3.2 Effect of Reading Errors on the Ground Motion

Reading errors refer to the errors that are introduced when one assumes that the accelerogram is piecewise linear and attempts to read the coordinates of the points of the definition of the record. They are caused by the manner in which the accelerogram is approximated by straight lines by different individuals, the thickness of the lines in the record, and the human errors in reading the coordinates of the points. Since every person digitizing the record is believed to have performed his task as carefully as possible, the errors are assumed to be inevitable.

As an example of the difference in the manner of approximating the acceleration record by a series of straight lines, it is noted that the number of data points used for the first 60 secs. of the record were 696, 748, 811, and 818 for reductions at Berkeley, Michigan, Cal. Tech.,
and Illinois. The velocity and displacement diagrams obtained from these four reductions of the record are presented in Fig. 13 and the top two sets of Fig. 14. A parabolic baseline adjustment was applied to all records.

Disregarding minor local differences the velocity diagrams in Fig. 13 are comparable. There is a more pronounced difference among the displacement curves, however. As judged by the duration and total amplitudes of the pulses in the displacement curves shown in Fig. 14, the differences between the displacement curves due to reading errors are of an entirely different character than the systematic differences due to baseline adjustment shown in Fig. 10. This is to be expected because the reading errors are of a more random type.

To see the type of reading errors that is introduced by the same person, the accelerogram was reduced at Illinois for a second time by the writer. One adjusted displacement curve obtained from this record, I2C, is compared with the displacement diagram IIP of the first Illinois reduction in the lower portion of Fig. 14. A cubic expression for baseline was used in this case because the record was reset on the machine at the 30 and 45 sec. points when it was being digitized. The appreciable differences between the displacement curves of this figure is a further evidence of the inevitably of reading errors.

3.3.3 Effect of Baseline Adjustment on the Deformation Spectra

The deformation spectra corresponding to the unadjusted record I1 and one adjusted version thereof, I1ML, are compared in Fig. 15 for several values of the damping factor β. The sizeable difference in the response of low frequency systems for the two inputs is to be expected.
For, if the baseline shift is considered as a step acceleration of $\Delta y$, the corresponding change in the pseudo-velocity $V_0$ of a linear undamped system will be $\frac{\Delta V}{\pi f}$. Consequently, for low frequency systems, the change in the pseudo-velocity caused by baseline adjustment is higher.

To demonstrate the effect of the type of baseline adjustment on the deformation spectra, the scatter bands obtained from the comparison of spectra of IIL, IIP, IIM, and IIML are presented in Fig. 16. From Figs. 15 and 16 it is clear that although some type of baseline adjustment is essential, the type of baseline adjustment used is not important. Also, the minor differences resulting from different methods of baseline adjustments is restricted to low frequency systems. This is a consequence of the fact that only displacement diagrams are sensitive to the type of adjustment.

3.3.4 Effect of Reading Errors on the Deformation Spectra

The scatter bands to the deformation spectra obtained by comparing the spectra of the records reduced at Berkeley, Cal. Tech., Illinois, and Michigan are shown in Fig. 17. All the records were adjusted using a parabolic baseline adjustment and a common duration of 60 secs.

The difference in responses of systems noted here are without doubt more significant than the differences associated with considering different types of baseline adjustment. Also, disregarding the case of $\beta = 0$ where the response is influenced by the resonance effect produced by small high frequency pulses in the acceleration trace, the scatter in the results is smaller for high frequency systems. This may be attributed to the errors which are introduced in reading the ordinates and abscissas of the acceleration trace. The high frequency region of the deformation
spectrum is acceleration sensitive, therefore, only the errors introduced in ordinates of the actual acceleration traces affect the results of a high frequency system. The rest of the spectrum is sensitive to the details of the velocity and displacement diagram, therefore the errors introduced in the abscissa as well as those in the ordinates of the acceleration trace affect the results.

The reading errors discussed above are the errors that were introduced by different operators reducing the record. While some records may conceivably contain more "inaccuracies" than others, the results should be considered as descriptive of discrepancies that may be expected due to readings. Since one operator may introduce consistent errors such as tendency to read the upper or lower edges of the thick lines in the accelerogram, it is of interest to compare the spectra corresponding to the inputs obtained by the same person reading the accelerogram twice. Figure 18 provides such a comparison, where spectra of I1P and I2C are presented. The differences noted here for systems with $f > 1$ cps is much less than those noted in Fig. 17 for the readings of four different operators.

3.4 Examination of Strong-Motion Earthquake Records for Stationarity

3.4.1 Records Considered

During an earthquake the initial disturbances after a chaotic start at the focus undergo countless reflections and refractions before they are registered at a recording station usually in the form of ground acceleration. For this reason the accelerograms have the appearance of random functions. The statistical nature of earthquakes have
long been recognized and several attempts have been made in the past to
idealize them as random processes of one type or another. Although the
accelerograms appear to be nonstationary, with a few exceptions, in all
the related studies they have been modeled by stationary random pro­
cesses such as white noise and processes with a power spectral density
which is a rational function of $\omega$.

In order to place random vibration analysis of earthquake
effects on structures on a more realistic basis it is necessary to deter­
mine whether the phenomenon should be considered as a stationary or non­
stationary process. For this purpose, ideally, a large descriptive
ensemble of the strong-motion records registered at one station is needed.
Such information is not available at this time. The only possible
approach, therefore, is to draw conclusions from the strong-motion records
registered at a larger region such as the West Coast of the U.S. for which
a few records are now available. Even in this case conclusive statistical
evidence of nonstationary cannot be obtained. Nevertheless, in the
remaining sections of this chapter several results will be presented
which are believed to show that earthquakes are nonstationary processes.

Selected for examination are two horizontal components of four
of the strongest ground motions registered at the West Coast of the U.S.
to date. The records were digitized as described in Section 3.2. Using
the least square procedure a parabolic baseline adjustment was applied
in each case. The record duration and maximum values of acceleration,
velocity, and displacements are given in Table 2. The ground motions for
two of the records considered are shown in Figs. 9 and 19.

Strong-motion accelerograms are essentially of two types: some
records contain one or two short series of major pulses. Examples of
this type are Ferndale and Eureka, California records of 12/12/54. In other cases the record is composed of a number of high intensity pulses. The ground motions selected for study are of this latter type. They have another factor in common, in that they all have been registered at moderate epicentral distances (45 miles or less). Therefore, in the absences of a better choice they were grouped together and their stationarity or nonstationarity was examined.

3.4.2 Application of Test for Stationarity

The application of the test for stationarity described in Section 2.4 requires the segmentally computed temporal mean square values and the record power spectral density. Because the accelerogram is assumed to have a linear variation between the data points, the mean square values over each interval were evaluated by squaring and integrating the piecewise linear function over this interval. The record spectral densities were determined using the procedure recommended in Ref. 21.

The estimated power spectral density is for a record $y(t)$ digitized at equally spaced intervals. The accelerograms were not digitized at equal intervals. Because the minimum time step, $\Delta t$, of the records was 0.02 sec., the ordinates at .02 sec. intervals of the piecewise linear function were determined, and then the power spectral density of each accelerogram was evaluated. The maximum lag in the computation of autocorrelation from which the power spectral density is determined was 5% to 10% of the record duration. These are commonly recommended values for maximum lag in the computation of the autocorrelation. The computed record power spectral densities for two accelerograms are shown in Fig. 20.
3.4.3 Results of Test for Stationarity and Conclusions

To motivate some of the conclusions to be drawn from the results presented in this section, it is noted that the significant segment of an earthquake record is not restricted only to the segment containing the high intensity acceleration pulses. For example, significant velocity pulses occur after 30 seconds in the N21°E components of the Taft record, shown in Fig. 9. In this region the acceleration trace has very low intensity acceleration pulses. To provide another evidence for this observation the maximum spring deformation for systems with 5% damping subjected to each of the records in Table 2 were obtained and the associated times of maximum deformations were noted. From this information the spectra for, t_m, were prepared and are presented in Figs. 21 and 22. For each system the quantity t_m indicates the significant record duration. It is to be noted that t_m is strongly frequency dependent and that for 6 of the systems with 0.3 ≤ f ≤ 5 cps in Fig. 21a, 9 in Fig. 21b, 9 in Fig. 22a and 11 in Fig. 22b, the quantity t_m is greater than 15 seconds. For systems with values of damping less than 5% considered here the values of t_m are even closer to the end of the record than those shown in the above mentioned figures.

From the preceding remarks it is clear that when modeling accelerograms with a random process the records with durations greater than 15 seconds must be considered. For this reason it was decided to consider records of 25 sec. duration. 25 sec. was selected because two of the records were not available for more than 25 sec. The records were then tested for self-stationarity over a period of 25 secs. For each record, the segmental mean square values were computed over 1 sec. and 2 sec. intervals. Typical results are presented in Figs. 23
through 25. It is clear that the mean square values over the intervals are subject to considerable variation. The results of test for self-stationarity applied to the 8 records are summarized in Table 3. All of the records fail the test for self-stationarity and are, therefore, not self-stationary.

An ensemble of self-nonstationary records may still be a stationary process, as the fluctuation in various records may balance each other in the process of ensemble averaging. A visual examination of the acceleration traces in Figs. 9 and 19 indicates that this can not be the case over a period of 25 seconds for the records considered. Because although the fluctuations may balance each other in the intense segment of the records, after 15 seconds or longer the acceleration traces become less intense in all records and decreases with time.

To obtain a possible trend for nonstationary variance of earthquakes which is necessary for the stochastic model of earthquakes to be discussed in Chapter 4, the variance of the eight records was evaluated by ensemble averaging. All the records were scaled to a maximum ground velocity of 10 in./sec. and the variances were computed at instants which are 0.1 sec. apart. The results are summarized in Fig. 26. Because of the small size of the sample the results vary greatly and no statistical inference can be made. Nevertheless a definite trend is established. It is clear from Fig. 26 that the variance has an increasing trend over about the first 2 seconds, and decreases after about 15 seconds indicating again that over a duration of 25 seconds the records can not be considered stationary. Accordingly, earthquakes can not be modeled by a stationary process of a specified power spectral density as it has been done in a majority of the studies made thus far. A nonstationary model is required.
4. FILTERED NONSTATIONARY GAUSSIAN SHOT NOISE AS MODEL FOR STRONG-MOTION EARTHQUAKES

4.1 General

Under suitable conditions a stationary random process of known power spectral density can be represented by passing a white noise process through a linear filter. Recently, it has been suggested that the important statistics of a nonstationary process can be simulated by filtering a nonstationary shot noise, which is the nonstationary counterpart of a white noise. It is the purpose of this chapter to model earthquakes by a filtered nonstationary shot noise process and to describe a means of generating member functions of the mathematical model on a digital computer.

4.2 Nonstationary Shot Noise

A random process $S(t)$ is called a nonstationary shot noise if its mean value and covariance functions satisfy the following equations:

$$m_S(t) = 0$$

$$\text{CoV}_S(t_1, t_2) = I(t_1) \delta(t_2 - t_1), \quad t_2 \geq t_1$$

$I(t_1)$ is a positive continuous function. In other words, a nonstationary shot noise is a process with zero mean, infinite variance, and completely uncorrelated ordinates. When $I(t_1)$ is a constant, the process will be a stationary process and in this case it is referred to as white noise. Equation (4.1) does not completely specify the probability law of $S(t)$. If in addition to Eq. (4.1) $S(t)$ is also specified to be a Gaussian
process, then, the complete description of the probability law of \( S(t) \) is given by Eq. (4.1).

Because the correspondence between the random processes and their statistics is not one-to-one, many random processes can be found to satisfy Eq. (4.1). A commonly used example of a shot noise is the following special type of filtered Poisson process:

\[
S_1(t) = \sum_{k=1}^{N(t)} X_k \delta(t-t_k) \tag{4.2}
\]

where the \( X_k \)'s are mutually independent random variables and are identically distributed as a random variable \( X \); \( N(t) \) is a simple Poisson counting process; and \( T_k \) is a random variable for the time at which an impulse of magnitude \( X_k \) occurs in the interval \((0,t]\). Using the properties of a filtered Poisson process, \((5)\) it can be shown that if the underlying Poisson process \( N(t) \) is nonhomogenous, then, \( S_1(t) \) is a nonstationary shot noise; and if \( N(t) \) is homogenous \( S_1(t) \) is a white noise process.

In order to generate member functions of \( S_1(t) \) on a digital computer, three sets of random variables \( N(t), Y_k, \) and \( T_k \) are required. A more convenient procedure for generating member functions of a shot noise on a digital computer is to consider a sequence of equally spaced impulses as follows:

\[
S_2(t) = \sum_{k=1}^{n_t} X_k \delta(t-t_k) \tag{4.3}
\]

where the \( X_k \)'s are mutually independent random variable (not identically distributed) such that

\[
E(X_k) = 0 \tag{4.4}
\]
\[ V(X_k) = I(t_k) \Delta t, \quad (4.4) \]

\( \Delta t \) is the uniform small time interval between the impulses; \( t_k \) denotes the time at which the \( k \)th impulse occurs; and \( n_t \) is a specified number of impulses in the interval \((0, t]\), (see Fig. 27).

Because of the first of Eq. (4.4):

\[ m_S(t) = 0. \]

And if \( t_2 \geq t_1 \)

\[ \text{CoV}_{S_2}(t_1, t_2) = E \left[ \sum_{k=1}^{n_{t_1}} X_k \delta(t_1-t_k) \sum_{j=1}^{n_{t_2}} X_j \delta(t_2-t_j) \right] \]

\[ = E \left[ \sum_{k=1}^{n_{t_1}} X_k^2 \delta(t_1-t_k) \delta(t_2-t_k) \right. \]

\[ + \sum_{k \neq j}^{n_{t_1}} \sum_{j=1}^{n_{t_1}} X_k X_j \delta(t_1-t_k) \delta(t_2-t_j) \]

\[ + \sum_{k=1}^{n_{t_1}} \sum_{j=n_{t_1}+1}^{n_{t_2}} X_k X_j \delta(t_1-t_k) \delta(t_2-t_j) \] \]

\[ = I(t_1) \delta(t_2-t_1) \]

This last result is obtained by virtue of the independence of \( X_k \) and \( X_j \) and Eq. (4.4), and the fact that \( \int I(\tau) \delta(t_1-\tau) \delta(t_2-\tau) d\tau = I(t_1) \delta(t_2-t_1) \).

Thus, the member functions generated from Eq. (4.3) are from a non-stationary shot noise process.
4.3 Filtered Nonstationary Shot Noise

A nonstationary shot noise and its stationary counterpart, a white noise, are processes that possess violent fluctuations in short intervals of time. This is the consequence of the infinite variance and lack of correlation between the values of the process at closely spaced time instants. It is difficult to imagine any physical process in nature that behaves in this manner. However, from these processes, other random processes having finite variance and correlation can be obtained by passing shot noise or white noise through linear filters. The resulting processes are referred to as nonstationary filtered shot noise and filtered white noise, respectively.

The mean, covariance, and variance of a filtered shot noise are obtained by substituting Eq. (4.1) into Eqs. (2.39), (2.41) and (2.43) respectively. Assuming zero initial conditions at \( t = 0 \) and letting \( F_S(t) \) denote a filtered shot noise process, then the statistics of the filtered process are,

\[
\begin{align*}
\mu_{F_S}(t) &= 0 \\
\text{Cov}_{F_S}(t_1, t_2) &= \int_0^{t_1} I(\tau_1) h(t_1-\tau_1) h(t_2-\tau_1) d\tau_1, \quad t_2 \geq t_1 \\
\sigma^2_{F_S}(t) &= \int_0^{t} I(\tau_1) h^2(t-\tau_1) d\tau_1
\end{align*}
\]

(4.5)

If the underlying shot noise is Gaussian, the filtered process, \( F_S(t) \), will also be Gaussian and is called a filtered nonstationary Gaussian shot noise. In this case, Eq. (4.5) specifies the probability law of \( F_S(t) \) completely.

The covariance and variance of a filtered shot noise resulting from a second-order filter are as follows: for \( t_2 \geq t_1 \) and \( 0 \leq t_1 \leq x_1 \).
\[
\text{CoV}_{FS}(t_1, t_2) = \frac{1}{4\omega_0^3} \left\{ e^{-\gamma_0^\omega (t_2-t_1)} \right\} \left\{ \frac{1}{\gamma_0^\omega} \left( \frac{t_1}{x_1} \right)^2 - \frac{\omega_0}{\omega_d} \left( \frac{1+\gamma_0^\omega - 2\gamma_0^\omega}{\gamma_0^\omega \omega_d x_1^2} \right) \left( \frac{t_1}{x_1} \right) \right\} \\
+ \frac{1}{2(\omega_d x_1)^2} \left( \frac{1+3\gamma_0^\omega + 4\gamma_0^\omega^6}{\gamma_0^3} \right) \cos\omega_d(t_2-t_1) + \frac{\omega_0}{\omega_d} \left[ \left( \frac{t_1}{x_1} \right)^2 \right] \\
- \left( \frac{2\gamma_0^\omega}{\omega_d x_1^3} \right) \left( \frac{t_1}{x_1} \right) + \frac{4\gamma_0^\omega - 1}{2(\omega_d x_1)^2} \right] \sin\omega_d(t_2-t_1) \right\} \frac{e^{-\gamma_0^\omega_0 (t_2+t_1)}}{2(\omega_d x_1)^2}.
\]

for \( x_1 \leq t_1 \leq x_2 \)

\[
\text{CoV}_{FS}(t_1, t_2) = \frac{1}{4\omega_0^3} \left\{ e^{-\gamma_0^\omega_0 (t_2-t_1)} \right\} \left\{ \frac{1}{\gamma_0^\omega} \cos\omega_d(t_2-t_1) + \frac{\omega_0}{\omega_d} \sin\omega_d(t_2-t_1) \right\} \\
+ \frac{e^{-\gamma_0^\omega_0 (t_1+t_2)} \left( \frac{1+2\gamma_0^\omega x_1}{\gamma_0^3} \right) \cos\omega_d(t_2-t_1) - \left[ 2(\omega_d x_1) \right] \left( 1-2\gamma_0^2 \right) + \gamma_0^\omega (4\gamma_0^2 - 3) \right] - \left[ 4\gamma_0^\omega \omega_d x_1 - \left( \frac{\omega_0}{\omega_d} \right) (4\gamma_0^2 - 1) \right] \sin\omega_d(t_2+t_1-2x_1) \right\} \\
+ \frac{e^{-\gamma_0^\omega_0 (t_1+t_2)}}{2(\omega_d x_1)^2} \left( \text{same as for } 0 \leq t_1 \leq x_1 \right)
\]

for \( t_1 \geq x_2 \).
\[
\text{CoV}_F(t_1, t_2) = \frac{1}{4\omega_o^3} \left\{ -c(t_1-x_2)+\gamma_o\omega_o(t_2-t_1) \right\} \left\{ 2\left( \frac{\omega_o}{\omega_d} \right)^2 \frac{1}{2\gamma_o-\xi} - \frac{2\gamma_o-\xi}{4(1-\gamma_o\xi)+\xi^2} \cos\omega_d(t_2-t_1) + \frac{4\left( \frac{\omega_o}{\omega_d} \right)}{4(1-\gamma_o\xi)+\xi^2} \sin\omega_d(t_2-t_1) \right\} \\
+ e^{-\gamma_o\omega_o(t_1+t_2-2x_2)} \left\{ \left( \frac{\omega_o}{\omega_d} \right)^2 \left( \frac{1}{\gamma_o} - \frac{2}{2\gamma_o-\xi} \cos\omega_d(t_2-t_1) \right) \right. \\
+ \left. \left( \frac{\omega_o}{\omega_d} \right) \left[ \frac{2(2\gamma_o-\xi)}{4(1-\gamma_o\xi)+\xi^2} - \gamma_o \right] \cos\omega_d(t_1+t_2-2x_2) \right\} \\
+ \left( \frac{\omega_o}{\omega_d} \right) \left[ \frac{4}{4(1-\gamma_o\xi)+\xi^2} \right] \sin\omega_d(t_1+t_2-2x_2) \\
- \gamma_o\omega_o(t_1+t_2-2x_1) \\
+ e^{-\gamma_o\omega_o(t_1+t_2)} \left( \frac{3}{2(\omega_d x_1)^2} \right) \left( \text{same as for } x_1 \leq t_1 \leq x_2 \right) \\
+ \frac{e^{-\gamma_o\omega_o(t_1+t_2)}}{2(\omega_d x_1)^2} \left( \text{same as for } 0 \leq t_1 \leq x_1 \right) \right\} \quad (4.6)
\]

for \(0 \leq t \leq x_1\)

\[
V_F(t) = \frac{1}{4\omega_o^3} \left\{ \frac{1}{\gamma_o} \left( \frac{t}{x_1} \right)^2 - \left( \frac{\omega_o}{\omega_d} \right) \left( \frac{1+\gamma_o^2-2\gamma_o^4}{\gamma_o^2\omega_d x_1} \right) \left( \frac{t}{x_1} \right) \right\} \\
+ \frac{1}{2(\omega_d x_1)^2} \left( \frac{1+\gamma_o^4-4\gamma_o^6}{\gamma_o^3} \right) + \frac{-2\gamma_o\omega_o t}{2(\omega_d x_1)^2} \left[ -\frac{1}{\gamma_o} + \gamma_o(4\gamma_o^2-3)\cos2\omega_dt \right. \\
- \left. \left( \frac{\omega_d}{\omega_o} \right)(4\gamma_o^2-1)\sin2\omega_dt \right] \right\}
\]
for $x_1 \leq t \leq x_2$

$$V_{FS}(t) = \frac{I_o}{4\omega_o^3} \left\{ \frac{1}{\gamma_o} + \frac{-2\gamma_o \omega_o (t-x_1)}{2(\omega_d x_1)^2} \left\{ \frac{1-2\gamma_o \omega_o x_1}{\gamma_o^3} - \left[ \frac{2\omega_o x_1 (1-2\gamma_o^2)}{\gamma_o^4} \right. \right. \right. \\
\left. \left. \left. \gamma_o (4\gamma_o^2-3) \right] \right. \right. \right\} + \frac{e}{2(\omega_d x_1)^2} \right\} \cos 2\omega_d (t-x_1) - \left[ 4\gamma_o \omega_d x_1 - \left( \frac{\omega_d}{\omega_o} \right) (4\gamma_o^2-1) \right] \sin 2\omega_d (t-x_1) \right\}$$

for $t > x_2$

$$V_{FS}(t) = \frac{I_o}{4\omega_o^3} \left\{ e^{-c(t-x_2)} \left\{ 2 \left( \frac{\omega_o}{\omega_d} \right) \left[ \frac{1}{2\gamma_o-\xi} - \frac{2\gamma_o-\xi}{4(1-\gamma_o \theta+\xi^2)} \right] \right\} \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r
where
\[
I(t) = \begin{cases} 
I_0 \left(\frac{t}{x_1}\right)^2, & 0 \leq t \leq x_1 \\
I_0, & x_1 \leq t \leq x_2 \\
I_0 e^{-ct}, & t \geq x_2
\end{cases}
\] (4.8)

\(I_0, x_1, x_2,\) and \(c\) are positive constants, and \(\delta = \frac{c}{\omega_0}\). \(V_F(t)\) and \(I(t)\) for a specific choice of the parameters are illustrated in Fig. 29, which shows that the variance function of the filter output is of the same shape as \(I(t)\). As pointed out in Section 2.1.3 and shown in Fig. 2 case b, the variance of a filtered white noise increases monotonically with \(t\) initially and tends to a stationary value; therefore, filtered white noise processes cannot be used to simulate a process whose variance increases initially and then decreases at later times, a trend which is expected of earthquakes (see Section 3.4.3). Also, for \(I(t)\) in the range \(x_1 \leq t \leq x_2\), the \(\text{CoV}_{F_S}(t_1, t_2)\) tends to a stationary value as can be observed from Eq. (4.6). Therefore, in this range the filtered process is stationary. This observation will influence the selection of the parameters of the filter in Section 4.4.2.

4.4 Filtered Nonstationary Gaussian Shot Noise Model

The filtered nonstationary shot noise can be used to simulate any nonstationary process with desired variance and covariance functions. The function \(I(t)\) is selected so as to give the desired variance, while
the filter is selected so as to yield the desired covariance function. If
the variance and covariance functions are known in every detail, as
suggested in Ref. 1, a trial and error procedure may have to be used to
obtain the details of both functions satisfactorily. In the case of
earthquakes none of these functions are known exactly; however, some
crude estimates can be obtained from the available records.

Before discussing the detailed choices of \( I(t) \) and the filter,
it should be remarked that a filtered shot noise is a zero-mean process;
the same is expected to be true of earthquake motions at some distance
from the fault. Since the observed value at a given time is essentially
the sum of many independent effects each of which may be positive or
negative. The assumption of zero mean is not necessary when dealing with
linear systems; the non-zero mean can be subtracted from the process and
treated separately.

4.4.1 Choice of \( I(t) \)

Although the variance computed from the eight accelerograms,
Fig. 26, can not be considered to be a sound statistical estimate of the
true variance, a general trend for the variance function of accelerograms
can be inferred from this figure. During the first one or two seconds,
the variance increases. After about 15 seconds, the figure has a decaying
trend. During the interval from 2 to 15 seconds, it is difficult to
estimate the time variation of the variance from this figure. However, it
may be reasonable on the basis of Fig. 26 to assume that the variance
will have a stationary value. As pointed out earlier, the function \( I(t) \)
must be chosen such as to have the same shape as the variance function;
the following parameters were chosen for \( I(t) \) as given by Fig. 29.
The parameter $I_o$ of Eq. (4.8) is a measure of earthquake intensity and cannot be selected solely from a crude estimate of the variance such as that of Fig. 26. In Chapter 5, a value for $I_o$ will be chosen from a comparison of the average response spectra of the generated member functions from the model, and of those of real accelerograms. The decaying trend after 15 seconds, corresponding to $c = 0.18$, is sketched in Fig. 26.

4.4.2 Choice of Filter

Before discussing the manner in which a filter $L$ is determined for earthquakes, it is instructive to consider two filtered nonstationary shot noise processes $Y_1(t)$ and $Y_2(t)$ obtained from $S_2(t)$, Eq. (4.3), by passing this process through a first order and a second order filter respectively. Here, $Y(t)$ denotes random ground acceleration.

Using Eqs. (2.27) and (2.28)

$$\frac{d}{dt} Y_1(t) + \omega_0 Y_1(t) = S_2(t)$$

and

$$\frac{d^2}{dt^2} Y_2(t) + 2\gamma \omega_0 \frac{d}{dt} Y_2(t) + \omega_0^2 Y_2(t) = S_2(t)$$

From Eqs. (2.33), (2.36), (2.37), and (4.3) the solutions to Eqs. (4.9) and (4.10) subject to zero initial conditions at $t = 0$ are given by

$$Y_1(t) = \sum_{i=1}^{n_t} X_i \frac{\beta o}{e^\beta o(t-t_i)}$$
\[ Y'_2(t) = \frac{1}{\omega_d} \sum_{i=1}^{n_t} x_i e^{j \omega_0 (t-t_i)} \sin \omega_d (t-t_i) \] (4.12)

The corresponding ground velocities are

\[ \dot{V}_1(t) = \frac{1}{\beta_0} \sum_{i=1}^{n_t} x_i [1-e^{j \omega_0 (t-t_i)}] \] (4.13)

\[ \dot{V}_2(t) = \frac{1}{\omega_0^2} \sum_{i=1}^{n_t} x_i \left\{ 1 - \frac{e^{j \omega_0 (t-t_i)}}{\omega_d} [\gamma_0 \omega_0 \sin \omega_d (t-t_i)] + \omega_d \cos \omega_d (t-t_i) \right\} \] (4.14)

The elementary pulses which are superimposed to obtain the accelerations and velocities for both of the above cases are shown in Fig. 28. Note that the elementary pulse shape associated with \( Y'_2(t) \) oscillates about the t-axis and those for the other three quantities are always on one side of this axis.

Because the elementary pulses for \( \dot{V}_1(t) \) are always on one side of the axis, in a particular realization of this process, the member function behaves somewhat like the path taken by a particle performing a random walk. It is well known that the number of returns to the origin, or the number of zero crossings of the particle path is rather infrequent. Therefore, all member functions of \( \dot{V}_1(t) \) will stay on one side of the t-axis for long intervals of time. Similar comments are applicable to \( \dot{V}_1(t) \) and \( \dot{V}_2(t) \) because their elementary pulses do not oscillate about the t-axis. This property is observed when the velocity diagrams were computed from the generated member functions of \( \dot{V}_2(t) \), Fig. 35.
Since a drift in the ground velocity can be removed by a small shift in the baseline of the acceleration trace, while a significant change in the acceleration diagram is needed to remove a drift in the acceleration trace itself, it was concluded that a first order filter could not be used effectively for earthquake motions. The acceleration diagrams generated by using a second order filter do not have a drift, therefore, second order filters can be more effectively used to simulate earthquake-type disturbances.

There are two possibilities that may be considered in using second-order filters: a single second-order filter, or a number of them connected in cascade. Since it is obviously simpler to conjure a single filter, and since its use provides a preliminary step for the other case, only a single second-order filter has been considered in this investigation.

For a second order filter, the ranges of values for the filter frequency \( F = \omega_0 / 2\pi \) and \( \gamma_0 \) were determined as follows: it was assumed that the accelerograms may be considered as realizations of a stationary random process in the strong phase portion of the records, i.e., \( 2 \leq t \leq 15 \) sec. Also, by virtue of Eq. (2.24), for a stationary process temporal autocorrelation function provides an estimate of the ensemble autocorrelation function. Therefore, the temporal autocorrelation functions for the 8 accelerograms were evaluated over a period of \( 2 \leq t \leq 15 \) sec. using the procedure described in Ref. 21. The normalized autocorrelations, \( k_\phi(\tau) / k_\phi(0) \), for all the records are presented in Figs. 30 and 31. For time lags, \( \tau \), less than 0.1 or 0.12 sec. these figures are comparable. For greater values of \( \tau \) no common trend can be observed.
It was noted in Section 4.3 that for the I(t) chosen, the covariance of the filtered shot noise becomes nearly stationary when I(t) is a constant, i.e., \( x_1 \leq t \leq x_2 \). The normalized autocorrelation in this case is

\[
\frac{R_{FS}(\tau)}{R_{FS}(0)} = \rho_{FS}(\tau) = e^{\gamma_o \omega_o \tau} \left[ \cos \omega_o \tau + \frac{\gamma_o}{\sqrt{1 - \gamma_o^2}} \sin \omega_o \tau \right]
\] (4.15)

Values obtained from Eq. (4.15) for several choices of \( \gamma_o \) and \( F = \omega_o/2\pi \) are depicted in Figs. 32 through 34. From a comparison of Figs. 32 through 34 with Figs. 30 and 31 it can be seen that a second-order filter with parameters

\[ 4 \leq F \leq 5 \text{ cps} \] (4.16)

\[ 0.4 \leq \gamma_o \leq 0.6 \]

is a reasonable model for simulating strong-motion earthquakes of the type considered in this study.

4.4.3 Generation of Member Functions

There are two objectives to be served by generating member functions from a proposed stochastic model for earthquakes. First, the generated member functions can be used to obtain a Monte Carlo solution for those random vibration problems that are not amenable to analytic treatment such as many non-linear systems and first passage problems. Second, the similarities between the member functions and the actual earthquake records will provide a qualitative measure of the reasonableness of the proposed model.
The generation of member functions from the proposed model requires the generation of the member functions of $S_2(t)$, i.e., a sequence of impulses of random magnitude $X_i$ arranged on the time axis with $t_i - t_{i-1} = \Delta t$, and the computation of filter response to this sequence of impulses. $X_i$'s were generated as a sequence of independent normal random numbers with zero mean and $V(X_i) = I(t_i)\cdot \Delta t$, using $I_o = 1$ in Eq. (4.6). For the final choice of $I_o$ the results are multiplied by $\sqrt{I_o}$. The assumption of normality in the case of real earthquakes is supported intuitively by the fact that, at any time an observed record is the sum of a large number of reflected and refracted waves resulting from the disturbances at the focus.

To compute the filter response, Eq. (4.12) is written in a slightly different form to obtain a recursive relation for the filter response. Using a trigonometric expansion, write

$$Y_2(t) = \frac{1}{\omega_d} [R_c(t)\sin\omega_d t - R_s(t)\cos\omega_d t] \quad (4.17)$$

where

$$\begin{cases}
R_c(t) \\
R_s(t)
\end{cases} = \sum_{i=1}^{n_t} X_i e^{\gamma \omega_o (t-t_i)} \begin{cases}
\cos\omega_d t_i \\
\sin\omega_d t_i
\end{cases} \quad (4.18)$$

The functions $R_c(t)$ and $R_s(t)$ can be computed recursively as follows. If these quantities are known at time $t$ the same quantities at time $t+s$ are given by,

$$\begin{cases}
R_c(t+s) \\
R_s(t+s)
\end{cases} = \sum_{i=1}^{n_{t+s}} X_i e^{\gamma \omega_o (t+s-t_i)} \begin{cases}
\cos\omega_d t_i \\
\sin\omega_d t_i
\end{cases}$$
The value of $y_2(t+s)$ are then computed by substituting from Eq. (4.19) into (4.17). The integration procedure described above was compared with results of an alternative procedure which makes use of piecewise linear forcing function with ordinates $\frac{X_i}{\Delta t}$ at $t = t_i$. The resulting ground accelerations and velocities were essentially identical.

The pseudo-earthquakes for which results are presented in Chapter 5 were generated using the above procedure with $\Delta t = .005$ sec. and $F = 5$ cps. The filter output was punched out at intervals of .02 sec. This was taken greater than $\Delta t$ for the sake of economy in the computation of the ground velocities, displacements, and the structural responses. In all of these computations the acceleration traces were assumed to vary linearly between the points of definition. To remove the velocity drifts of the type shown in Fig. 35, three segmental baseline adjustments were applied to the acceleration trace using the procedure described in Section 3.2.1.
5. DISCUSSION OF RESULTS FOR PSEUDO-EARTHQUAKES

5.1 General

From a pragmatic standpoint, the goodness of a stochastic model when used to idealize a random phenomenon must be judged by comparing the important statistics of the model such as mean, variance, and covariance functions with the corresponding functions estimated from a sufficient number of records. Because of the insufficient number of earthquake records available, no quantitative estimates of the above functions are possible. Hence, the goodness of a model can only be inferred from a comparison between the appearance and average response of structures subject to excitations in the form of member functions generated from the model and the gross average response from real earthquakes. In this chapter the ground motions and response spectra for linear systems corresponding to member functions generated from the filtered nonstationary Gaussian shot noise model are presented, and a comparison is made between the average deformation spectra of the pseudo-earthquakes and the smoothed velocity spectra of real earthquakes. On the strength of this comparison and on the fact that the statistics of the model can be chosen to agree with the general trend of the corresponding statistics estimated from real earthquake records, the proposed filtered nonstationary Gaussian shot noise process is a satisfactory model describing strong-motion earthquakes.

5.2 Ground Motions

Eight pseudo earthquakes were obtained from the proposed model
using the procedure described in Section 4.4.3. The ground motion corresponding to a pseudo-earthquake is shown in Fig. 36. The ground motions of pseudo-earthquakes are not too dissimilar to those of real earthquakes; examples of the incomplete acceleration loops, half-cycle velocity, and somewhat periodic displacement pulses exhibited in these motions can be found in the real earthquakes as the visual comparison of Figs. 9 and 19 with Fig. 36 demonstrates.

Consider the ratio \( \frac{\ddot{y}_m \dot{y}_m}{\gamma_m^2} \) in which \( \ddot{y}_m \), \( \dot{y}_m \), and \( \gamma_m \) denote the peak values of the ground acceleration, velocity, and displacement respectively. This ratio is unity for a simple harmonic motion. In the case of the 8 earthquakes it has the values 6, 8, 4.7, 13, 5.9, 12.2, 14.5, and 12.9. The same ratio for the pseudo earthquakes is 9.6, 5.5, 6.2, 5.9, 5.0, 10.2, 13.7, and 9.6. The range of variation of this quantity in the pseudo-earthquakes is seen to be comparable to those of the real earthquakes.

### 5.3 Deformation Spectra

The deformation spectra for a pseudo-earthquake are shown in Fig. 37. For purposes of comparison the deformation spectra for a real earthquake obtained from the tabulated results in Ref. 23, are shown in Fig. 38. In a logarithmic plot the deformation spectra of real earthquakes is roughly trapezoidal in shape, approaching the peak values of the ground displacement and acceleration for low and high frequency systems respectively. The corresponding spectra from pseudo-earthquakes are similar in many respects. For a large sequence of impulses an undamped spectrum is nearly independent of the frequency, and the same is also true of the damped spectra, except at high frequencies the damped spectra fall off
exponentially (on arithmetic plot). It is, therefore, seen that the operation of filtering the impulses produces records where deformation spectra resemble those of real earthquakes more closely.

In Fig. 39, the deformation spectra of three pseudo-earthquakes are compared for two values of damping factor \( \beta \). Here, the three pseudo-earthquakes were generated from the same sequence of random numbers, but the filter frequency of the model, \( F \), was taken as 4, 4.5, and 5 cps, respectively. These values of \( F \) are representative of the range established for this parameter from the comparison of the covariance functions of the model and the record covariance functions of the real earthquakes, Eq. (4.16). Because for the same \( \beta \), the spectra of the three inputs are not significantly different, the comparisons to be made for the eight pseudo-earthquakes with \( F = 5 \) cps and the real earthquakes will also be applicable for other values of \( F \) in the range from 4 to 5 cps.

In the field of earthquake engineering the average effect of past earthquakes on structures is sometimes represented by smoothed velocity spectra. These spectra are obtained by first making the area under the undamped spectra from \( T = 0.1 \) to \( T = 2.5 \) sec. the same to normalize the records, then, the normalized spectra are averaged at several frequencies. Finally, a smoothed curve is drawn through the results for each value of the damping factor. For the eight earthquake records considered in this study, the smoothed velocity spectra have been presented elsewhere, see for example Ref. 25. These curves are depicted in Fig. 41 by solid lines. In Fig. 40 the average spectra for four earthquakes (one component of each of the four quakes considered in this study) are shown by dashed lines. It is seen that the average spectra oscillate about the smoothed velocity spectra.
The average spectra of the pseudo-earthquakes are compared with
the smoothed spectra of the real earthquakes in Fig. 41. The general
agreement demonstrated in this figure is considered satisfactory. Because
of Fig. 39, it can be seen that the use of a filter frequency, \( F \), of 4
cps in place of 5 cps to generate the pseudo-earthquakes would have im-
proved the agreement in Fig. 41 very slightly.

To obtain the agreement in Fig. 41, the value of \( I_0 \) in Eq. (4.8)
turns out to be \( 252 \, \text{g}^2/\text{sec}^3 \). In the region corresponding to the strong
phase duration of the earthquake records, this value of \( I_0 \) makes the
standard deviation of the earthquake records .065g and .09g for values of
\( F = 5 \) and 4 cps, respectively. Considering that in the intense accelera-
tion region of the earthquakes modeled in this study, the peak values vary
from 0.16g to 0.31g, these values of the standard deviation appear to be
reasonable. Also, in order to normalize the pseudo-earthquake spectra it
was necessary to multiply the individual spectra by the factors of 0.460,
0.520, 0.420, 0.405, 0.425, 0.410, 0.370, and 0.396. The corresponding
factors that were used to obtain the results in Fig. 40 for the real earth-
quakes were 0.327, 0.453, 0.479, and 0.646. Because the normalization
factors are obtained by equating the area under the undamped spectra to
the same constant, the fact that the values of these factors for pseudo-
earthquakes are comparable to those of real earthquakes is a further
indication that the pseudo-earthquakes, on the average, have the same
effects on the structures as the real earthquakes.

5.4 Conclusions and Recommendations for Future Work

From the study of the nonstationary Gaussian filtered shot noise
model considered in this thesis, the following conclusions emerge.
1. A filtered shot noise process is shown to be a satisfactory model for describing strong-motion earthquakes. Its member functions bear resemblance to real earthquake records, both in visual appearance and in the average effects on structures. In addition, the model has the important advantage of having statistics which are shown to be qualitatively similar to those of real earthquakes.

2. The use of a single second order filter limits the number of parameters that can be varied to obtain a closer agreement in the covariance function and the deformation spectra of the pseudo-earthquakes and those of the real earthquakes. A combination of at least two second order filters connected in cascade should be considered.

3. When considering a filtered shot noise process, in essence, a filtered Poisson process is obtained; the weighing function of the underlying filtered Poisson process is the impulse response function of the filter. Other types of weighing functions, including one with a full cycle ground velocity pulse should also be considered.

4. Extension of this work should include an investigation to determine a means for computing the probability of structural safety of a single-degree-of-freedom system to filtered shot noise excitations. A Monte Carlo type of solution is of course applicable. However, when the system is linear, and by assuming that the filter output, system response, and their first derivatives are both Markovian and Gaussian (also by discretizing the state space), it may be possible to obtain a numerical solution to the problem. Results have been obtained for a white noise input to a simple system \(^{26}\) using this approach.
REFERENCES


APPENDIX A

SCHEMES 3 AND 4 FOR EVALUATION OF $\beta_1$

The quantities $\beta_1$ are defined by:

$$\beta_1 = \frac{1}{(t_d-t_o)^{i+2}} \int_{t_o}^{t_d} (t-t_o)^i \dot{\gamma}_o(t) dt, \quad i = 1, 2, \ldots$$

Assuming the unadjusted acceleration trace is piecewise linear,

$$\dot{\gamma}_o(t) = \dot{\gamma}_{k-1} + \ddot{\gamma}_{k-1}(t-t_{k-1}) + \frac{1}{2} s_k (t-t_{k-1})^2, \quad t_{k-1} \leq t \leq t_k \quad (A.1)$$

where

$$\dot{\gamma}_{k-1} = \dot{\gamma}_o(t_{k-1})$$

$$\ddot{\gamma}_{k-1} = \ddot{\gamma}_o(t_{k-1})$$

$$s_k = \frac{\ddot{\gamma}_k - \ddot{\gamma}_{k-1}}{t_k - t_{k-1}}$$

Therefore,

$$\beta_1 = \frac{1}{(t_d-t_o)^{i+2}} \sum_{k=1}^{n} \int_{t_o}^{t_k} \chi(\tau) d\tau + \int_{t_o}^{t_{k-1}} \tau \chi(\tau) d\tau + \frac{1}{2} s_k \int_{t_o}^{t_{k-1}} \tau^2 \chi(\tau) d\tau \quad (A.3)$$

where

$$\chi(\tau) = (\tau + t_{k-1} - t_o)^i$$

Schemes 3 and 4 differ in the manner in which integrals involving $\chi(\tau)$ are evaluated.

**Scheme 3** - Evaluating the integrals involving $\chi(\tau)$ directly and rearranging terms, the following expression for $\beta_1$ is obtained.
\[ \beta_i = \frac{-1}{(i+1)(t_d-t_o)^{i+2}} \sum_{k=1}^{n} (t_k-t_o)^{i+1} \left[ \frac{s_k}{(i+2)(i+3)} (t_k-t_o)^2 - \frac{y_k}{i+2}(t_k-t_o) \right] \]

\[ - (t_{k-1}-t_o)^{i+1} \left[ \frac{s_k}{(i+2)(i+3)} (t_{k-1}-t_o)^2 - \frac{y_{k-1}}{i+2}(t_{k-1}-t_o) \right] \] (A.4)

**Scheme 4 - Write**

\[ X(\tau) = (\tau+t_{k-1}-t_o)^i = (i!) \sum_{m=0}^{i} \frac{\tau^{i-m}(t_{k-1}-t_o)^m}{m! (i-m)!} \]

Substituting in Eq. (A.3), the following expression for \( \beta_i \) is obtained:

\[ \beta_i = \frac{1!}{(t_d-t_o)^{i+2}} \sum_{k=1}^{n} e_{1k} y_{k-1} (t_k-t_{k-1}) + e_{2k} y_{k-1} (t_k-t_{k-1})^2 \]

\[ + \frac{e_{3k}}{2} (y_{k-1}^2 - y_{k-1}) (t_k-t_{k-1})^2 \] (A.5)

in which

\[ e_{1k} = \sum_{m=0}^{i} \frac{(t_{k-1}-t_o)^m(t_k-t_{k-1})^{i-m}}{m! (i-m)! (i+1-m)!} \]

\[ e_{2k} = \sum_{m=0}^{i} \frac{(t_{k-1}-t_o)^m(t_k-t_{k-1})^{i-m}}{m! (i-m)! (i+2-m)!} \]

\[ e_{3k} = \sum_{m=0}^{i} \frac{(t_{k-1}-t_o)^m(t_k-t_{k-1})^{i-m}}{m! (i-m)! (i+3-m)!} \]
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</table>

The time intervals of integration used to obtain the results presented in this thesis were such that \((\Delta t) : f_o \leq 20\)

**TABLE 1.** EFFECT OF TIME INTERVAL OF INTEGRATION, \(\Delta t\), ON THE ACCURACY OF MAXIMUM SPRING DEFORMATION, \(U\), AND THE ASSOCIATED TIMES, \(t_m\) - ELASTIC SYSTEMS SUBJECTED TO RECORD I1ML
<table>
<thead>
<tr>
<th>Record</th>
<th>Component</th>
<th>$t_d$</th>
<th>$\ddot{y}_{\text{max}}$</th>
<th>$t_{\text{max}}$</th>
<th>$\ddot{y}_{\text{min}}$</th>
<th>$t_{\text{min}}$</th>
<th>$\ddot{y}_{\text{max}}$</th>
<th>$t_{\text{max}}$</th>
<th>$\ddot{y}_{\text{min}}$</th>
<th>$t_{\text{min}}$</th>
<th>$\ddot{y}_{\text{max}}$</th>
<th>$t_{\text{max}}$</th>
<th>$\ddot{y}_{\text{min}}$</th>
<th>$t_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC1</td>
<td>NS</td>
<td>29.16</td>
<td>0.292</td>
<td>2.20</td>
<td>-0.312</td>
<td>2.02</td>
<td>10.71</td>
<td>5.46</td>
<td>-12.80</td>
<td>1.52</td>
<td>1.09</td>
<td>6.92</td>
<td>-8.24</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>29.82</td>
<td>0.219</td>
<td>24.82</td>
<td>-0.157</td>
<td>11.02</td>
<td>12.48</td>
<td>2.12</td>
<td>-10.46</td>
<td>11.18</td>
<td>14.69</td>
<td>8.84</td>
<td>-1.34</td>
<td>0.90</td>
</tr>
<tr>
<td>EC2</td>
<td>NS</td>
<td>25.08</td>
<td>0.165</td>
<td>3.28</td>
<td>-0.257</td>
<td>3.20</td>
<td>5.31</td>
<td>2.76</td>
<td>-10.00</td>
<td>3.28</td>
<td>4.66</td>
<td>3.20</td>
<td>-2.79</td>
<td>10.12</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>25.06</td>
<td>0.182</td>
<td>15.04</td>
<td>-0.142</td>
<td>15.26</td>
<td>7.44</td>
<td>5.06</td>
<td>-8.14</td>
<td>2.52</td>
<td>4.39</td>
<td>8.78</td>
<td>12.04</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>S69E</td>
<td>30.00</td>
<td>0.157</td>
<td>6.58</td>
<td>-0.146</td>
<td>3.80</td>
<td>7.72</td>
<td>6.80</td>
<td>-4.77</td>
<td>2.70</td>
<td>11.96</td>
<td>14.80</td>
<td>-6.7</td>
<td>3.20</td>
</tr>
<tr>
<td>OL</td>
<td>SB0W</td>
<td>30.26</td>
<td>0.301</td>
<td>19.50</td>
<td>-0.189</td>
<td>19.68</td>
<td>6.29</td>
<td>8.48</td>
<td>-7.24</td>
<td>18.72</td>
<td>3.67</td>
<td>15.58</td>
<td>-6.48</td>
<td>19.50</td>
</tr>
<tr>
<td></td>
<td>S10E</td>
<td>30.48</td>
<td>0.147</td>
<td>11.12</td>
<td>-0.189</td>
<td>10.88</td>
<td>6.20</td>
<td>19.88</td>
<td>-6.31</td>
<td>8.66</td>
<td>7.03</td>
<td>20.12</td>
<td>-2.46</td>
<td>11.36</td>
</tr>
</tbody>
</table>

$\ddot{y}_{\text{max}}$, $t_{\text{max}}$, $\ddot{y}_{\text{min}}$, $t_{\text{min}}$, $\ddot{y}_{\text{max}}$, $t_{\text{max}}$, $\ddot{y}_{\text{min}}$, $t_{\text{min}}$.

EC1 = ElCentro 5/18/40, EC2 = ElCentro 12/30/34,
TA = Taft 7/21/52, OL = Olympia 4/13/49

* record duration used for parabolic baseline adjustment

** computed by assuming adjusted acceleration trace is piecewise linear

**TABLE 2. RECORD CONSIDERED IN THE TEST FOR STATIONARITY**
TABLE 3. RESULTS OF TEST FOR SELF-STATIONARITY APPLIED TO EARTHQUAKE RECORDS OVER A COMMON DURATION, \( t_d = 25 \) SEC.

<table>
<thead>
<tr>
<th>Record</th>
<th>Comp.</th>
<th>B (cps)</th>
<th>( \bar{e}^2 ) n=20</th>
<th>( \bar{e}^2 ) n=10</th>
<th>( e^2 ) n=20</th>
<th>( e^2 ) n=10</th>
<th>Ratio ( \bar{e}^2/e^2 ) n=20</th>
<th>( \bar{e}^2/e^2 ) n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC1</td>
<td>NS</td>
<td>5.94</td>
<td>1.71</td>
<td>.841</td>
<td>.135</td>
<td>.067</td>
<td>12.7</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>5.09</td>
<td>.571</td>
<td>.255</td>
<td>.157</td>
<td>.079</td>
<td>3.6</td>
<td>3.2</td>
</tr>
<tr>
<td>EC2</td>
<td>NS</td>
<td>4.91</td>
<td>2.12</td>
<td>1.42</td>
<td>.163</td>
<td>.081</td>
<td>13.0</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>6.04</td>
<td>.584</td>
<td>.402</td>
<td>.132</td>
<td>.066</td>
<td>4.4</td>
<td>6.1</td>
</tr>
<tr>
<td>TA</td>
<td>N21E</td>
<td>4.13</td>
<td>.903</td>
<td>.660</td>
<td>.194</td>
<td>.097</td>
<td>4.6</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>S69E</td>
<td>5.18</td>
<td>.884</td>
<td>.709</td>
<td>.154</td>
<td>.077</td>
<td>5.7</td>
<td>9.2</td>
</tr>
<tr>
<td>OL</td>
<td>S80W</td>
<td>7.73</td>
<td>.535</td>
<td>.465</td>
<td>.103</td>
<td>.052</td>
<td>5.2</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>S10E</td>
<td>8.89</td>
<td>.519</td>
<td>.379</td>
<td>.090</td>
<td>.045</td>
<td>5.8</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Upper limit of \( \bar{e}^2/e^2 \) for self-stationarity
at 5% confidence level
See Ref. 17.

\( \bar{e}^2 \): Mean square value of \( e^2 \)

EC1 = ElCentro 5/18/40, EC2 = ElCentro 12/30/34,
TA = Taft 7/21/52, and OL = Olympia 4/13/49

\( n \) = number of segmentally computed mean square values, \( r_k \).
FIG. 1 ENSEMBLE OF AN ARBITRARY RANDOM PROCESS $y(t)$
ONLY THREE OF THE MANY MEMBERS ARE SHOWN

FIG. 2 VARIANCE OF THE OUTPUT OF LINEAR FIRST ORDER FILTER TO WHITE NOISE AND NONSTATIONARY EXCITATIONS
FIG. 3 DIAGRAMATIC SUMMARY OF THE TEST OF AN ENSEMBLE OF RECORDS FOR STATIONARITY

\[ \bar{r}_k = \frac{1}{t_{d_k} - t_{d_{k-1}}} \int_{t_{d_{k-1}}}^{t_{d_k}} y(t)^2 \, dt, \quad t_{d_k} - t_{d_{k-1}} = \text{a common value} \]

FIG. 4 ILLUSTRATION OF THE SEGMENTAL MEAN SQUARE VALUES, \( \bar{r}_k \)
FIG. 5 UNADJUSTED VELOCITY DIAGRAMS FOR FOUR READINGS OF RECORD.
Base Line For Modified Version IIM

FIG. 6 UNADJUSTED VELOCITY DIAGRAM, — RECORD II.
FIG. 7 EFFECT OF INACCURACIES IN EVALUATION OF $\beta_i$ ON THE COMPUTED GROUND DISPLACEMENT—SAME DIGITAL INPUT.

- Solid line: Corresponds to Four Schemes of Integration Discussed on Page 38
- Dashed line: Corresponds to a Poor Grouping of Terms in Expression for Computation of $\beta_i$
**FIG. 8 ADJUSTED VELOCITY AND DISPLACEMENT DIAGRAMS — RECORD IIM.**
FIG. 9 GROUND ACCELERATION AND VELOCITY DIAGRAMS.
Fig. 10 Effect of baseline adjustment on displacement diagrams of records IIM and II; $t_d = 60.65$ sec.
FIG. II EFFECT OF RECORD DURATION, $t_d$, AND TYPE OF ADJUSTMENT ON DISPLACEMENT DIAGRAM.
FIG. 12 VELOCITY AND DISPLACEMENT DIAGRAMS FOR 11--SEGMENTALLY ADJUSTED.
FIG. 13 VELOCITY DIAGRAMS FOR FOUR INDEPENDENT READINGS OF ACCELEROMETER. — ALL RECORDS PARABOLICALLY ADJUSTED, $t_d = 60.00$ secs.
FIG. 14 DISPLACEMENT DIAGRAMS FOR FIVE INDEPENDENT READINGS OF ACCELEROMETER; $t_d = 60.00$ secs.
FIG. 15 COMPARISON OF DEFORMATION SPECTRA FOR UNADJUSTED AND ADJUSTED RECORDS.
FIG. 16 EFFECT OF TYPE OF BASE LINE ADJUSTMENT ON DEFORMATION SPECTRA.

Records III, IIIP, IIM, IIIM; $t_d = 60.65$ sec.

Damping Factor,
$\beta = 0$
$\beta = 0.02$
$\beta = 0.1$
$\beta = 1.00$

Pseudo-Velocity, $V_0$, in./sec.
Undamped Natural Frequency, $f_0$, cps
All Records Parabolically Adjusted; $t_d = 60.00$ sec.

FIG. 17 EFFECT OF READING ERRORS ON DEFORMATION SPECTRA

---

RECORD READ BY FOUR DIFFERENT PERSONS.
FIG. 18 EFFECT OF READING ERRORS ON DEFORMATION SPECTRA—RECORD READ TWICE BY SAME PERSON.
FIG. 19 GROUND MOTION FOR S.80W. COMPONENT OF OLYMPIA WASHINGTON RECORD OF 4/13/49.
Record Durations Considered = 250 sec.
FIG. 21 SPECTRA FOR TIME OF MAXIMUM DEFORMATION, $t_m$, ELASTIC SYSTEMS WITH $\beta = .05$
FIG. 22 SPECTRA FOR TIME OF MAXIMUM DEFORMATION, $t_m$, ELASTIC SYSTEMS WITH $\beta = .05$
FIG. 23 SEGMENTALLY COMPUTED MEAN SQUARE VALUES, $\bar{r}_k$
N.S. COMP. OF EL CENTRO 5/18/40

Time, sec.

$\bar{r}_k$ for $\bar{y}(t)/\bar{y}_{\text{max.}}$
FIG. 24 SEGMENTALLY COMPUTED MEAN SQUARE VALUES, $\bar{r}_k$
N.S. COMP. OF EL CENTRO 12/30/34
FIG. 25 SEGMENTALLY COMPUTED MEAN SQUARE VALUES, $\bar{r}_k$
SIOE COMP. AND OLYMPIA 4/13/49
FIG. 26 VARIANCE COMPUTED BY ENSEMBLE AVERAGING FROM EIGHT ACCELEROGRAMS
RECORDS NORMALIZED TO THE SAME $\dot{y}_m$
$I(t) = \frac{1}{S(t)} \left( \int S_2(t)dt \right)$

**FIG. 27** SHOT NOISE PROCESS DEFINED BY EQ (4.3)

**FIG. 28** ELEMENTARY PULSES FOR FILTERED SHOT NOISE PROCESS OBTAINED FROM FIRST ORDER AND SECOND ORDER FILTERS
FIG. 29 VARIANCE OF NONSTATIONARY FILTERED SHOT NOISE—SECOND ORDER—LINEAR FILTER

\[ F = \frac{\omega}{2\pi} = 5 \text{ cps} \]
\[ \gamma = 0.5 \]
FIG. 30  NORMALIZED TEMPORAL AUTOCORRELATION FUNCTIONS FOR EL CENTRO RECORDS

El Centro Records
of 5/18/40

\( k_{\phi_y}(\tau)/k_{\phi_y}(0) \)

Interval Considered =
\( 2 \leq \tau \leq 15 \) sec.

El Centro Records
of 12/30/34

\( k_{\phi_y}(\tau)/k_{\phi_y}(0) \)

Interval Considered =
\( 2 \leq \tau \leq 15 \) sec.
NORMALIZED TEMPORAL AUTOCORRELATION FUNCTIONS FOR TAFT AND OLYMPIA RECORDS

FIG. 31

Interval Considered = 2 ≤ t ≤ 15 sec.

Taft Records of 7/21/52

Olympia Records of 4/13/49
FIG. 32 NORMALIZED AUTOCORRELATION OF FILTERED SHOT NOISE IN THE STATIONARY RANGE, Eq.(4-15), $F = 3$ cps
FIG. 33 NORMALIZED AUTOCORRELATION OF FILTERED SHOT NOISE IN THE STATIONARY RANGE, Eq. (4-15), $F = 4$ cps
FIG. 34 NORMALIZED AUTOCORRELATION OF FILTERED SHOT NOISE IN THE STATIONARY RANGE, Eq. (4-15), $F = 5$ cps
FIG. 35 UNADJUSTED VELOCITY DIAGRAM FOR PSEUDO-EARTHQUAKE NO. 2.
FIG. 36 GROUND MOTION FOR PSEUDO-EARTHQUAKE NO. 2
FIG. 38 DEFORMATION SPECTRA FOR S.80W. COMPONENT OF OLYMPIA, WASHINGTON RECORD OF 4/13/49.
FIG. 39 COMPARISON OF DEFORMATION SPECTRA FOR THREE PSEUDO-EARTHQUAKES. THE SAME SEQUENCE OF RANDOM NUMBERS—FILTER FREQUENCY VARIES
**FIG. 40** COMPARISON OF AVERAGE DEFORMATION SPECTRA OF FOUR EARTHQUAKES AND SMOOTHED VELOCITY SPECTRA OF REAL EARTHQUAKES.

**FIG. 41** COMPARISON OF AVERAGE DEFORMATION SPECTRA OF PSEUDO-EARTHQUAKES AND SMOOTHED VELOCITY SPECTRA OF REAL EARTHQUAKES.
VITA

Mohammad Amin was born on January 8, 1936 in Tabriz, Iran. He graduated from Phirooz Bahran High School, Teheran, Iran in 1954. He was enrolled in the Worcester Polytechnic Institute, Worcester, Massachusetts in September, 1955 and obtained the degree of Bachelor of Science in Civil Engineering with High Distinction from that Institute in June, 1959. A graduate fellowship enabled him to earn the degree of Master of Science in Civil Engineering from the University of Illinois in June, 1960. Appointments as a research assistant and associate at the University of Illinois have made it possible for him to pursue his studies toward the degree of Doctor of Philosophy and to gain some valuable research experience. His other technical experience includes structural drafting during summer employments from 1956 to 1959.

He is a member of the Seismological Society of America, Associate Member of American Society of Civil Engineers, and Sigma Xi; member of Tau Beta Pi and Phi Kappa Phi honorary societies, and a member of Sigma Alpha Epsilon fraternity.