“HOW WILL I KNOW MY STUDENTS ARE SUCCESSFUL?”
EXAMINING THE CONCEPTIONS OF SUCCESS HELD BY PRE-SERVICE SECONDARY
MATHEMATICS TEACHERS IN AN EQUITY-ORIENTED PROFESSIONAL
DEVELOPMENT PROGRAM

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Abstract

This study investigates the conceptions of success in mathematics that pre-service teachers hold for marginalized students. Participants are two white, male pre-service secondary mathematics teachers who intend to work in high needs schools and are participating in an equity-oriented professional learning program at a large Midwestern university. Gaining an understanding of how pre-service teachers’ views of success in mathematics evolve over the course of their two-year sojourn in the program can help build understanding of how teacher educators can better support pre-service teachers to broaden their conceptions of success in mathematics. To that end, this study addresses the following questions: What conceptions of success do pre-service mathematics teachers who intend to teach in high needs schools hold for marginalized students? Do their conceptions of success evolve as they participate in an equity-oriented professional learning community? If so, how do their conceptions of success change?

I employ a case study method along with a three-dimensional conceptual framework for considering success: academic achievement, mathematical power, and critical consciousness. One participant’s experiences in the program highlight the potential value of uncertainty for those entering the teaching profession. The second participant’s experiences connect to Schoenfeld’s (2011) Resources, Orientations, and Goals framework, highlighting the tensions in daily decision-making in which pre-service teachers engage when their own orientations are not in alignment with those of their mentor teachers. Findings suggest that conceptions of success may be situational rather than developmental. I discuss limitations as well as implications for teacher education practice and further research.
# Table of Contents

Chapter 1 Introduction ................................................................. 1

Chapter 2 Literature Review .......................................................... 9

Chapter 3 Methodology .................................................................. 32

Chapter 4 Heath’s Case .................................................................. 49

Chapter 5 Raphael’s Case ............................................................... 71

Chapter 6 Conclusions and Implications .......................................... 95

References .................................................................................. 110

Appendix A Inventory of Ideas/Feelings ......................................... 129

Appendix B Dimensions of Equity Instrument ................................ 130

Appendix C Program Structural Model ......................................... 133

Appendix D Program Overview ...................................................... 134

Appendix E Codebook .................................................................. 137
Chapter 1

Introduction

In the pursuit of equity in mathematics education, beliefs about the purpose of schooling shape assumptions about what students should know and be able to do (e.g., Fullan, 2003; Mirra & Morrell, 2011). For example, after the American Revolution, one of the purposes of schooling in the United States was to develop a literate citizenry capable of resisting oppression and supporting the development of the new Republic (Fraser, 2001). If we continue to embrace this ideal as one of the intentions of schooling, we must ask what evidence we can seek to indicate the accomplishment of this goal. Mirra and Morrell (2011) note that current public policy has led to indicators of student success being narrowed to standardized test scores. But performing well on standardized assessments does not necessarily translate to the critical citizenry and resistance to tyranny to which we aspire, thus making the elevation of standardized test scores an inappropriate indicator for the accomplishment of this particular goal.

Individuals and groups continue to weigh in on the goals that we should be targeting and the indicators that we should seek to identify success in mathematics education. As previously noted, public policy dictates that standardized assessment outcomes be used to indicate students success (Mirra & Morrell, 2011). That is, students are deemed successful if they are able to perform well on state and national tests. Many seek to help students become mathematically literate in order to gain “full access to the political and economic institutions of this society” (Silva, Moses, Rivers, & Johnson, 1990, p. 377) and to pursue their personal goals (Hochschild & Scovronick, 2003). At the school level, some campuses and districts now target social justice goals. For example, the motto of Community School for Social Justice in the South Bronx is “Erase injustice. Write your own future” (Community School for Social Justice, n.d.). The school
community seeks to develop in students the desire and ability to address issues that impact them and their communities. Parents also express opinions about what they want of their children’s schools, as a group of Latin@ parents of middle school children at a public school in Chicago demonstrated when they expressed a desire for their children to use math to help recognize and resist oppression in their lives (Gutstein, 2006b).

Within this context, teacher educators are responsible for guiding pre-service teachers to develop some manner by which they hold themselves accountable for student learning. What does the cacophony about the purposes of mathematics education mean to teachers who are being prepared at this time? That is, in this evolving landscape of increased standardized testing, Common Core State Standards, and pressure to close the achievement gap while addressing issues of identity and power, what sense are they making of the goals they will hold for their students? Toward that end, in this paper, I report on the goals that two pre-service secondary mathematics teachers (PSMTs) who seek to serve in high needs schools hold for their future students. I examine how their conceptions of success evolve and the issues that arise for each as they participate in an equity-oriented professional learning program.

Positioning Myself

My interest in the development of PSMTs’ definitions of success comes from years spent in public schools, both as a student and a teacher. I have a vivid memory of sitting in a desk next to the window in my sixth grade math class while taking a quiz. We were asked to multiply and divide fractions in the context of word problems. I had no idea how to differentiate between a

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1 I have adopted the term Latin@ from Gutiérrez (2010), who writes: “I use the @ sign to indicate both an “a” and “o” ending (Latina and Latino). The presence of both an ‘a’ and ‘o’ ending decenters the patriarchal nature of the Spanish language where is is customary for groups of males (Latinos) and females (Latinas) to be written in the form that denotes only males (Latinos). The term is written Latin@ with the ‘a’ and ‘o’ intertwined, as opposed to Latina/Latino, as a sign of solidarity with individuals who identify as lesbian, gay, bisexual, transgender, questioning, and queer (LGBTQ).”
multiplication problem and a division problem. I did, however, remember that we covered multiplication first, followed by division the following week, so I presumed that the problems on the top half of the page were multiplication while those that followed were division. From there I simply manipulated the fractions presented in the word problems. When my teacher returned our papers the following day, I received a grade of A on that quiz, but it was not until I was an adult that I actually understood the content that I was quizzed on when I was eleven years old. While this is my most vivid memory from my middle school mathematics classes, it is indicative of my public schooling experiences after elementary school.

I was one of only nine or ten black\textsuperscript{2} students in the honors track at my high school. These numbers are even more stark in light of the fact that I graduated in a class of close to 400 students that was about 45% black, 45% white, 5% Asian American, and 5% Latin@. There were five tracks at my school—honors, accelerated, regular, basic, and practical—and black and students dominated the lower two levels. If our teachers noticed the disproportionate placement of students in our tracked system, they never shared this with their students. More embarrassingly, I did not think to question it. I simply found a seat near the other one or two black students in the class, sat quietly, and copied what was on the overhead projector.

My parents always encouraged me to go to college, but my senior English teacher was the only school official to reinforce this message. It was not until late in my senior year that I learned that many of the white students who were also ranked near the top of our class were encouraged and guided through the college application process by our guidance counselors.

\textsuperscript{2} I use the term black, as opposed to African American, as an expression of solidarity with all individuals of African descent living in the United States, regardless of their country of origin or nationality. I also recognize that, regardless of country of origin, black students in the United States are racialized in similar ways, making the distinction between African American and other black students irrelevant in the context of this research.
I attended an HBCU (Historically Black College or University) in the District of Columbia, where I enrolled in a calculus course. The class met for three hours every Saturday morning, and it was always filled to capacity. The instructor and his wife, who came to class with him each week, were very involved with students both in and out of the class, feeding students who were hungry and offering a room in their home to those who lost their housing. They told us repeatedly that there were people in the world—including previous teachers—who thought we, as black students, were unintelligent and incapable of learning high-level mathematics. And they told us that those people were wrong. Again, this was a message that I received from my parents while growing up, but it was the first time that I heard it from a school official.

Years later, I re-entered the classroom as an elementary and middle school mathematics teacher in Houston’s historic East End. The majority of my students were Latin@ and came from low-income households. Many were recent immigrants to the United States and a few were in the early stages of learning the English language. Our principal segregated the students who were immigrants, children of immigrants, or spoke English as a second language because “they’re different” (C. Galaviz, personal communication, September 2001). They were placed in classes with less experienced teachers and held to lower standards by the school. To illustrate, we were required to administer a practice state standardized test every Friday. Target passing rates and results for each class were posted in the front office each Monday. Target passing rates for the classes that were predominately English learners were 50% while targets for the other classes were 70%.

As a teacher, my intention was to do for my students what most of my teachers had failed to do for me. Consequently, we worked together to develop conceptual understanding instead of
memorizing procedures. We also worked on test-taking skills in order to be prepared to jump the multiple systemic hurdles one must clear in order to pursue postsecondary education. I encouraged all of them to consider attending college so as to have more options in their professional lives and greater financial security. Most importantly to me, we discussed the bias and prejudice that people might hold against them due to their ethnicity, socioeconomic status, first language, or immigration status. And I let all of my students know, in no uncertain terms, that they were capable of learning anything that they encountered in a mathematics classroom.

After eight years as a teacher, I realized that I needed to learn more in order to continue to grow professionally; however, I was not sure what I needed to know or how to learn it. This led me to graduate school, where I learned of different frameworks for defining student success in mathematics (which I will describe later). With exposure to different ideas, I began to reflect on my experiences as a teacher and to question the goals that I set for my students. For example, instead of preparing my students to navigate the formal schooling system, perhaps I could have encouraged them to challenge the nature of the system instead. Why, in our bilingual neighborhood, were students who came to school knowing Spanish required to learn English, whereas students who came to school knowing English were not required to learn Spanish? Why were requests for out-of-state field trips denied to our campus while students in schools in other neighborhoods within the same district traveled to Disney World in Florida and Space Camp in Alabama? What’s more, how I could combine the exploration of these questions with the mathematics that my students and I were learning in the classroom was something that I had yet to learn—and am still learning.

It took me years to arrive at a point in my professional trajectory where I questioned the very nature of the system within which I worked—and when I reflect on my years as a teacher, I
cannot help but feel that I did my students a disservice. So I began to wonder what it would take to get PSMTs to contemplate these ideas before they entered the classroom. What would it take to help them consider broader notions of success for their future students? I was a black woman processing these new definitions of success while in graduate school after eight years as a teacher. Could learning about other definitions of success impact pre-service teachers in the same way if they did not share in some way the marginalized status of the students they would teach? Could other experiences besides learning about definitions of success impact their goals for students? These were some questions that drove me to want to understand further the evolution of pre-service mathematics teachers’ conceptions of success for students.

**This Research**

As previously mentioned, my research addresses the development of goals and indicators for student success in mathematics. The “apprenticeship of observation” (Lortie, 1975) leads many pre-service teachers to believe that they understand the process of schooling. Consequently, they may intend to design and lead their own classrooms in the same manner in which they themselves were educated without ever questioning the appropriateness of such a plan. Kuglemass (2000) asserts that effective teachers must have the “courage to create schools that look very different from those we have now” (p. 193). If pre-service teachers’ operating definitions of success are the same as the ones held by their teachers, our schools will look very much the same as those we have now, and will continue to marginalize particular subgroups of students.

In the current milieu of high stakes education and with a multitude of voices weighing in on the purposes and goals of education, if teachers are unable to articulate appropriate student success indicators, they may simply adopt the goals of those around them, which may or may not
be appropriate for their students. If we do not understand the sense that PSMTs are making of the term “success,” we will be ill-equipped to develop appropriate and meaningful professional learning experiences for them. To that end, I investigated the goals that two PSMTs held for their students and how these goals evolved as the pre-service teachers progressed through an equity-oriented teacher education program. Gaining an understanding of how their views of success in mathematics evolved over the course of their two-year sojourn in the program can help build understanding of how teacher educators can better support pre-service teachers to broaden their conceptions of success and, ideally, teach toward that end.

Outline of the Chapters

Chapter two is a review of relevant literature. I draw on theory that addresses models for equity in mathematics education, which served as a proxy as I developed a working definition for success. I also discuss the types of learning experiences that researchers have designed for educators to encourage teaching for equity and social justice. I then present my research questions and conceptual model.

Chapter three outlines the methods used in this study. I begin by explaining my rationale for using case study methods. Next, I describe the context of the professional learning program and participants. Then I discuss my sources of data and collection methods. I conclude the chapter with a description of my analysis of data.

Chapters four and five detail the individual cases of two of the participants in the professional learning program. For each participant, I provide a brief description of his background before presenting his trajectory on my conceptual model. I conclude each chapter with a discussion of important issues that arose for each participant.
I complete the dissertation report in chapter six by discussing the tensions the participants experienced and how they connect to the literature. I review the goals and findings of this study and its significance to the field. Implications for teacher education practice and further research are discussed.
Chapter 2

Literature Review

I draw primarily from two bodies of literature: (a) theoretical frameworks that seek to provide language to research and work toward equity in mathematics education and (b) research on transformative professional learning experiences for PSMTs. Transformative learning experiences are those that serve to diminish deficit thinking, (Haycock, 2001; Johnson & Fargo, 2010; Kose & Lim, 2011; Valencia, 1997; Valenzuela, 1999); encourage teachers to recognize the political nature of schooling (Ahlquist, 2001; Gutstein, 2000; Weissglass, 2000); and lead to teaching for equity, diversity, and social justice (Johnson & Fargo, 2010; Kose & Lim, 2011; Zeichner, 1993). These literatures contribute to a conceptual model for investigating the development of PSMTs’ conceptions of success as they participate in an equity-oriented professional learning program.

Frameworks for Considering Student Success in Mathematics

Beneath many aspects of school reform lie articulated and unarticulated beliefs about the purposes of schooling. These purposes include increasing the number of options available for students in the current social structure (e.g., Gutstein, 2007), preparing students to become critical citizens in a democracy (e.g., Banks, 2001; Dewey, 1916; hooks, 1994), and enhancing the variety of thought and the potential contributions of diverse groups of students to mathematics and mathematics-related fields (e.g., Gutiérrez, 2007). They also impact the goals that teachers and schools set for their students, and as scholars continue to explore how socio-cultural and socio-political issues play out in mathematics education, research is beginning to unearth the changes that arise when schools—particularly those that serve marginalized students (defined here as black, Latin@, low income, and English language learning students)—have
goals larger than simply closing the achievement gap (e.g., Atweh, Forgasz, & Nebres, 2001; Lerman, 2000). Issues such as voice, identity, power, personal agency, and social justice are becoming more central in the discussion (e.g., Gutiérrez, 2010; Gutstein, 2003; 2006a; 2007; Gutstein & Peterson, 2005; Malloy, 2002; Osler, 2007). Given these turns in mathematics education, we might ask: How do pre-service mathematics teachers who intend to work in high needs schools define success for their students?

Here I discuss some of the lenses through which educators currently view success in mathematics education: academic achievement, mathematical power, and critical consciousness. This is by no means an exhaustive list, and in reality many scholars and educators look to a combination of indicators, which I also discuss.

**Academic Achievement**

There is no shortage of literature documenting that an academic achievement gap in mathematics persists in the United States between black, Latin@, English language learning, and low-income K-12 students on one hand and their white, Asian American, and higher income peers on the other hand (e.g., Anyon, 1981; Chiu & Khoo, 2005; Lubienski & Crockett, 2007; National Research Council, 2001; Perl, 1973). Consequently, one goal of schooling is for students from different races, socioeconomic groups, and linguistic backgrounds to achieve similar outcomes related to mathematics achievement (e.g., Allexsaht-Snider & Hart, 2001). That is, one would not be able to predict outcomes for students based on race/ethnicity, socioeconomic status, or first language (Gutiérrez, 2002). Outcomes here include grades; standardized assessments; advancement to high level mathematics courses in secondary school; college admittance, persistence, and completion; time taken to complete degree programs; and
the pursuit of mathematics-related careers (e.g., Arbona & Novy, 1990; Conley, 2005; Gutiérrez, 2007; Trent, 2008).

College-bound students must navigate a series of standardized assessments (e.g., state tests, Advanced Placement exams, and college entrance exams). Achievement on these and other assessments is a precursor to college admission. For many teachers, then, their students are deemed successful when they demonstrate that they are able to negotiate the content, language, and format of such tests. In other words, they must know how to “play the game” (Gutiérrez, 2009b, p. 5) of school mathematics and gain access to higher levels of schooling. The goal is to help marginalized students understand the traditional or dominant mathematics taught in schools, allowing them to develop the cultural capital to take full advantage of the economic opportunities in society (Gutiérrez, 2007; Philip, 2011).

The mastery of dominant mathematics can help prepare students to participate in a global economy (Loomis, Rodriguez, & Tillman, 2008; Wang, Lin, Spalding, Klecka, & Odell, 2011; Zhao, 2010). Mathematics-related fields tend to have higher status, greater stability, and greater earnings (both starting and long-term) than other areas, with students who majored in math-related fields earning up to 35% more than their peers (Pascarella & Terenzini, 2005). A higher percentage of engineering majors were working in their fields of study a year after graduation than were business majors each year since 1976. Also, average salary for engineering majors was $2,000 to $6,900 more than that of business majors (National Center for Education Statistics, 2009).

Advancing to higher-level mathematics courses in high school has advantages as well. For example, if students take and pass an Advanced Placement calculus or statistics exam, they may be able to opt out of taking the same course while in college. This translates directly into
savings on tuition, as the student is required to take fewer courses than her peers who did not earn college credit while still in high school. Further, taking calculus in high school can be seen as a measure of intelligence. To illustrate, in her study of high school mathematics departments whose practices contributed to advancing students of color to high level courses, Gutiérrez (2007) notes that black and Latin@ students from low-income communities who enrolled in a local prestigious university were faced with negative biases by their middle class peers. The attitude with which they were received changed when students from low SES backgrounds played “the calculus card” (Gutiérrez, 2007, p. 48). That is, perceptions of the black and Latin@ students improved when others learned that they had taken and performed well in calculus while still in high school.

Focusing on the achievement gap has further advantages. It is a way to examine trends that extend beyond a single classroom, school, or community, thereby influencing research agendas and public policy. For example, both Head Start and The No Child Left Behind Act of 2001 were developed as a result of examinations of achievement data (Hess & Petrilli, 2006; Lubienski, 2008). However, identical performance on the same assessments does not constitute justice. That is, if some students are forced to assimilate in order to produce similar results, equity has not been achieved (Gutiérrez, 2008; Lubienski, 2008). Ideally, all students—those who have been marginalized and those who have not—can be successful in mathematics by learning in an environment that empowers them by building on their cultural and linguistic identities (Civil, 1994; Gutiérrez, 2009b; Moll, Amanti, Neff, & Gonzalez, 1992). Further, focusing solely on eliminating the gaps does not necessarily address the complex question of why the gaps exist, and creates the risk of developing archaic notions about differences in achievement (Boaler, 2002; Gutiérrez, 2008; Haycock, 2001; Stinson, 2006). For example,
Bourdieu (1986) asserts that schooling reproduces the social power structure of a hierarchical society by favoring students who enter school with a specific set of knowledge. This knowledge or capital, which Bourdieu equates to “power” (p. 243), is that of the upper and middle classes. Yosso (2005) argues that researchers have used Bourdieu’s theory to explain the academic achievement gap as a result of the cultural deficits of people of color. This deficit view “takes the position that minority students and families are at fault for poor academic performance because: (a) students enter school without the normative cultural knowledge and skills; and (b) parents neither value nor support their child’s education” (Yosso, 2005, p. 75). She presents a “community cultural wealth” (Yosso, 2005, p. 70) model that instead recognizes the assets that children of color bring with them to school. With the recognition of community cultural wealth, instead of designing schools with a focus on developing the cultural capital that they perceive to be lacking in students of color, educators design schools with the intention of capitalizing on students’ own unique assets of aspirational, linguistic, familial, social, navigational, and resistant capital (Yosso, 2005).

The traditional achievement perspective is also limited because it holds the achievement of white, higher SES students as the standard for success despite the fact that their levels of achievement may not represent ideal mathematics performance (Gutiérrez, 2008; Lubienski & Crockett, 2007). If we only seek to close the achievement gap, we are, in essence, saying that those at the top of the achievement scale do not need to improve. It also implies that the models we have for understanding white and middle class student achievement are appropriate for making policy for other groups (e.g., black, Latin@, and low income students) that do not fare as well on standardized achievement tests.
Mathematical Power

Another frame through which educators view success for their students is through the development of mathematical power, which can be described along four dimensions: conceptual understanding, procedural fluency, strategic mathematical thinking, and productive disposition (Baroody, 1998; English, 2002; Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000; Van de Walle, Karp, & Bay-Williams, 2012).

Conceptual understanding is a deep sense of mathematics as a web of interconnected ideas. Procedural fluency is the ability to carry out procedures efficiently (quickly and accurately), appropriately, and flexibly. It represents a mastery of basic skills that is tied to conceptual understanding. Routine expertise is the ability to accomplish familiar tasks, but not a new or different task. Procedural fluency as defined here represents adaptive expertise—knowledge learned in a meaningful fashion, which can be applied to new situations (Baroody, 1998; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Van de Walle et al., 2012).

Strategic mathematical thinking involves competence in the processes of mathematics problem-solving and adaptive thinking. It entails developing a sense of inquiry and the ability to attack and solve varied types of problems (Baroody, 1998; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Van de Walle et al., 2012). A productive mathematics disposition is evidenced by a student’s belief that she can learn mathematics along with a desire to do so. Recognition of one’s capabilities in mathematics is an important aspect of the mathematics “character” for which the Common Core State Standards call (Daro, 2011; Gutiérrez & Irving, 2012). It is important because a person’s feelings about her ability to accomplish a task impact whether she will attempt it and how much effort she will apply (Bandura, 1977; Bandura,
Barbaranelli, Caprara, & Pastorelli, 2001). More specifically, belief in one’s ability to do well is highly correlated with achievement in mathematics (Pietsch, Walker & Chapman, 2003).

Mathematical power as a goal is not without its limitations. For example, scholars have argued that people developed mathematics as a tool to describe and interact with the world around us (Gutiérrez & Irving, 2012). As a result, the way mathematics developed may be related to the culture of different peoples in different environments (D’Ambrosio, 2006). When mathematical power is cited as a goal in isolation, it may perpetuate the view of mathematics as an apolitical content area rather than a socially constructed and ever evolving discipline (de Freitas, 2008; Gutiérrez & Dixon-Román, 2011).

**Critical Consciousness**

As previously mentioned, schools are broadening their notions of the types of goals they have for their students beyond pure academics. For example, in Chicago, Infinity High School’s vision is to provide “the space and opportunity for students to empower themselves to become critical thinkers who are culturally and socially responsible” (Infinity Math, Science and Technology High School, n.d.). The mathematics vision for grades K-12 in the Amherst Public Schools in Massachusetts states, “Students will understand that mathematics is both a tool and a way of thinking that can be used to address problems in the world” (Amherst Regional Public Schools, 2010). One aspect of being able to address world problems in a socially responsible manner is the development of personal agency.

Personal agency is the awareness of the consequences of one’s actions and the ability to understand and control one’s actions for a given purpose (e.g., Darber, Baltodano, & Torres, 2003; hooks, 1994; Vallacher & Wegner, 1989; Wagner, 2007; Walter & Gerson, 2007; Zimmerman & Cleary, 2006). With regard to mathematics, personal agency may be viewed as
using mathematics for a given purpose and having control over what that purpose might be (Gutstein, 2006a). Gutstein (2006a) describes the development of students’ personal agency in mathematics as an interrelated process of “using mathematics and going beyond mathematics” (p. 72).

“Using mathematics” (Gutstein, 2006a, p. 72) entails analyzing real or realistic situations—preferably those that are relevant to students’ lives—mathematically to help students understand data. For example, students may examine the impact of gentrification in their neighborhood (Gutstein, 2006a) or differential standardized testing outcomes (Osler, 2007). By engaging in such activities, students can learn mathematics through the examination of social and economic issues. Exploring problems that impact their world can also serve as motivation to encourage students who may have lost interest in mathematics (Malloy, 2002).

“Going beyond mathematics” (Gutstein, 2006a, p. 72) refers to students exploring the multifaceted issues behind social injustices, forming their own positions relative to these issues, and possibly engaging in action to address them. For example, Gutstein (2006a) describes how his middle school students examined the relative size of continents on different types of world maps. Students were disturbed when they discovered that the maps used in most of their classrooms distorted relative sizes on the globe (for example, shifting the equator so that the Southern hemisphere, populated primarily by people of color, appears smaller than the Northern hemisphere). As a result, students began to question the accuracy and legitimacy of their school-taught knowledge, which they had previously accepted without question.

Beauboeuf-Lafontant (1999) asserts that effective teachers of marginalized students “recogniz[e] the existence of oppression in their students’ lives and [seek] … to encourage children to understand and undermine their subordination” (p. 702). Similarly, an important goal
of teaching math for social justice “is that students themselves are ultimately part of the solution
to injustice” (Gutstein, 2003, p. 39). Freire (1970) notes that conscientzação (or developing
sociopolitical consciousness) is the process by which students, as empowered individuals,
achieve a deepening awareness of the social realities that shape their lives and discover their own
capacities to re-create them. To illustrate, Gutstein (2006a) helped his middle school students
examine the impact of gentrification in their neighborhood. Students viewed the building of high
cost housing in their low-income neighborhood as an injustice and participated in community
activities that opposed the proposed plans. In another example, Strawhun (Turner & Strawhun,
2005) supported her sixth grade class of predominately black and Latin@ students as they
collected and analyzed data related to the overcrowding in their school relative to the wealthier
magnet school housed one floor below in the same building. Students presented their findings to
the board of education, contributing to public discourse surrounding the issue. In both of these
examples, teachers reported that simultaneously studying the mathematics and participating in a
local community struggle developed students’ feelings of personal agency. Students began to
believe that they could effect change in their lives.

One could expect, then, that mathematics teachers interested in issues of social injustice
may count among their goals that their students recognize how mathematics can be a powerful
tool in helping them serve their communities, look beyond outcomes of social injustice and look
to analyzing causes, and have personal agency regarding their ability to address issues of social
injustice in their communities supported by a sense of their own identities. Ideally, meeting these
goals will empower students to “change the game” (Gutiérrez, 2009b, p. 5) rather than simply
playing it.
Teaching mathematics with the goal of developing critical consciousness in students is not without its challenges. One difficulty is striking an appropriate balance between teaching rigorous mathematics and addressing authentic social justice issues. Teachers may find it difficult to accomplish both in a single lesson or unit (e.g., Bartell, 2006; Gregson, 2011; Gutstein, 2006a). For example, in Bartell’s (2010) study of teachers who attempted to teach mathematics for social justice for the first time, students did not move beyond practicing previously learned mathematics to expand their mathematical knowledge into new areas. Additionally, social justice issues were sometimes oversimplified. In attempting to teach mathematics for social justice, both ends were compromised. Further, Freedman (2007) warns against implementing critical pedagogy in a manner that is undemocratic. This may happen because, despite a teacher’s intentions, she is in a position of power in the classroom. Freedman suggests guidelines to help teachers avoid this situation, including helping students view opposing perspectives on issues of social injustice.

**Multiple Indicators of Success**

While I have categorized different views of success in mathematics, many educators embrace a more nuanced, multidimensional view of success. Even those who support traditional academic goals question the limitations of a purely dominant or conventional definition of success (e.g., Conley, 2005; S. Turner, 2008). For example, Lubienski (Lubienski & Gutiérrez, 2008), who advocates continued examinations of the achievement gap, also supports research that examines the causes of the gap and addressing injustices in the current system of schooling. Further, Turner (2008), who reviews data on college enrollment and completions rate outcomes, notes, “Surely ‘success in college’ is much richer and more complicated than simply receiving a degree, ideally in four years, and the measure should capture the breadth and depth of academic
accomplishment as well as broader personal growth” (p. 119). One aspect of personal growth is the development of personal agency, which, as previously noted, is an important component of the development of critical consciousness.

Because of my own philosophical assumption that true success is achieved when aspects of all three lenses—academic achievement, mathematical power, and critical consciousness—are addressed, I believe that the task for teacher educators is to encourage pre-service mathematics teachers to consider broad notions of success. Here I discuss some of the methods that scholars have employed in teacher education to achieve that end.

**Transformative Professional Learning Experiences**

In the current age of high stakes testing, the majority of pre-service teachers who are planning to work with marginalized students already embrace the idea of academic achievement and closing the achievement gap (e.g., de Freitas, 2008; Mirra & Morrell, 2011; Wilson, 2007). Many also already subscribe to the notion of the development of mathematical power, either as a means to close the achievement gap or as an end in itself. The question is what can we, as teacher educators, do to broaden their notions of what success looks like beyond those definitions? In my work, I seek to help PSMTs consider a multidimensional perspective of success that includes the development of critical consciousness as its goal (e.g., de Freitas, 2008). Instead of viewing schooling as merely a means to subvert systems of marginalization, I seek to help them recognize and challenge the role that schooling itself plays in said marginalization (Bourdieu, 1986; de Freitas, 2008; Diversity in Mathematics Education Center for Learning and Teaching, 2007; Mirra & Morrell, 2011).

As previously mentioned, transformative learning experiences are those that serve to diminish deficit thinking, (Haycock, 2001; Johnson & Fargo, 2010; Kose & Lim, 2011; Valencia
1997; Valenzuela 1999); encourage teachers to recognize the political nature of schooling (e.g., Ahlquist, 2001; Gutstein, 2000; Weissglass, 2000); and lead to teaching for equity, diversity, and social justice (Johnson & Fargo, 2010; Kose & Lim, 2011; Zeichner, 1993). Scholars continue to research productive transformative learning models to help PSMTs develop the dispositions they need in order to broaden their goals for marginalized students.

de Freitas (2008) notes that a significant obstacle to PSMTs’ development is the “mastery identity” (p. 44) with which many enter their programs. They view their own accomplishments in school mathematics as a matter of natural capability or merit. It is simply who they are. Such a perspective prevents them from recognizing the impact that politics and culture have on mathematics education and the role that both played in their own achievements. de Freitas (2008) sought to “trouble” their “mastery identity” (p. 44) in order to help them recognize both the sociocultural factors that contributed to their success and the political nature of their future careers as mathematics teachers. She posits that the recognition of the role that inequitable external factors play in education is crucial if mathematics teachers are to consider addressing them in their own classrooms. That is, in order for teachers to consider teaching with a goal of developing critical consciousness, they must first recognize inequity in schooling by accepting the fact that society gave them certain advantages in schooling not afforded to others. Identity, then, must be addressed in any effort to support pre-service teachers to broaden their perspectives to include critical indicators for school success. This can be a disconcerting process, leading to resistance on the part of those who have been privileged by the current schooling system as their identities are threatened (Boler & Zembylas, 2003; de Freitas, 2008; Gutstein, 2000; Rodriguez & Kitchen, 2005).
Research suggests that a pre-service teacher will not question her own “mastery identity” unless prompted to do so (de Freitas, 2008; hooks, 2000; Winks, 1999). Isolated coursework and add-on approaches have been found to have little impact on broadening teachers’ conceptions of self and, by extension, student success (Wideen, Mayer-Smith, & Moon, 1998). Here I discuss some of the transformative learning experiences that teacher educators have designed to that end.

A. Ball (2009) proposes that professional learning programs for mathematics educators include structured opportunities for reflection in generative ways. Along this vein, scholars have prompted pre-service teachers to reflect on their personal experiences with school mathematics; their own conceptions of students, mathematics, teaching, and learning; and the ways that their actions as teachers can contribute to the marginalization of subgroups of students (e.g., de Freitas, 2008; Grumet, 2010; Zeichner, 1993).

Despite the intentions of teacher educators, research suggests that these types of reflections have had mixed results (K. Gutiérrez & Vossoughi, 2010; Richardson, 1996; Wang et al., 2011). For example, in their study of 35 teacher candidates from two different university programs, Schussler, Stooksberry, and Brecaw (2010) found that pre-service teachers did not know themselves well enough to thoroughly identify and articulate their own conceptions, making it difficult to question said conceptions. Alternatively, de Freitas (2008) found that self-study narratives had positive outcomes in her study of preparing PSMTs to teach for diversity. A self-study narrative is a form of auto-ethnography. While the purpose of traditional ethnography is “to gain access to other minds and other ways of life so as to represent what it is like to be a differently situated human being” (Shweder, 1996, p. 17), auto-ethnography is an attempt to critically examine one’s own positioning (de Freitas, 2008). de Freitas asked participants to
reflect on their own experiences in math, paying particular attention to moments that elicited strong emotional responses. She found that the self-narratives worked particularly well in “helping the pre-service teachers understand the intersections between their experiences in school mathematics and the cultural framing of those experiences” (de Freitas, 2008, p. 50). These studies suggest that pre-service teachers may benefit from being scaffolded in their reflection processes because it is difficult for them to be self-critical. de Freitas suggests that self-study narratives may be a critical component of this scaffolded, reflection process.

Mirra and Morrell (2011) lament the “neoliberal approach” (p. 408) to teacher education, which they describe as passive training with a focus on the mechanics of teaching to the exclusion of the dispositions, stances, and beliefs they believe to be important to the profession. They argue that a teacher’s role is not as a distributor of knowledge, but rather as a civic agent, and they contend that effective teacher learning toward this end—toward having teachers develop a civic agent identity—is “collective, productive, and active” (Mirra and Morrell, 2011, p. 413). In their case study examination of teachers who engaged with marginalized students around issues of importance to their communities, Mirra and Morrell (2011) assert that participation had a positive effect on teachers’ collective, productive, and engaged identities, which was in turn reflected in their practice. The authors report that the teachers developed dialogic rather than hierarchical relationships with students and developed a professional community among themselves. They also note the development of teachers’ personal agency, in which the teachers began to “challenge the ways that they are positioned and to take responsibility for taking the profession in new directions” (Mirra and Morrell, 2011, p. 417). They report that teachers developed a deeper sense of commitment to the communities they served and came to believe that a relationship between the school and the community can be
beneficial for schools. Although the teachers who participated in the program were not all mathematics teachers, this study suggests that engaging teachers and students in dialogic relationships outside of the traditional classroom may be beneficial for broadening teachers’ conceptions of the roles they play in students’ education.

Mirra and Morrell (2011) further postulate that participating in their project encouraged teachers to develop communities of practice with a shared mission and purpose, which many scholars believe is an ideal environment for educators to refocus their practice (e.g., Darling-Hammond, Hammerness, Grossman, Rust, & Shulman, 2005). What do we believe happens in a community that allows people to learn more deeply than if they were to work on their own? As Bartell (2006) relates, the idea behind community models is to “draw upon situated, socio-cultural perspectives of teacher learning” (p. 1) that center learning within teachers’ interactions with each other as they participate in their work. The focus moves from the individual to the community’s development of a shared knowledge base. Consequently, learning is influenced not only by teachers’ personal backgrounds and beliefs, but also by their participation in the professional development community (Bartell, 2006; Garmon, 2004; Lave & Wenger, 1991; Stein, Silver, & Smith, 1998). Garmon (2004) arrived to a similar conclusion, attributing the broadened conceptions of a teacher candidate in part to her membership in a supportive community. He asserts that pre-service teachers need appropriate support group experiences to promote growth and “provide feelings of acceptance, caring, safety, and support” as they push each other to grapple with previously unexplored beliefs (Garmon, 2004, p. 209). He cites a supportive community as a critical factor in developing greater awareness of self and others.

Reading research literature has remained a consistent component of teacher education. In an effort to problematize the assumption that mathematics is a neutral field, teacher educators
have had their pre-service teachers read articles about white privilege, language, social justice (Gutiérrez, 2009a), gender (Frankenstein, 1990), the funds of knowledge with which children enter the classroom (Civil, 1994; González, Andrade, Civil, & Moll, 2001; Moll et al., 1992), the politics of mathematics education (Frankenstein, 1994), and the role that each of these plays in mathematics (Jilk, 2006; Moschkovich, 2002; Zevenbergen, 2000). Further, scholars have assigned readings designed to prepare teachers for taking on the task of addressing these issues, such as those that highlight the lived experiences and successes of marginalized students (Gutiérrez, 2009a; Stinson, 2010), identity construction, and capitalizing on students’ cultural identities (Jilk, 2006; Yosso, 2005). After reading such articles, scholars have documented PSMTs making comments such as, “I’m not one for social justice” (de Freitas, 2008, p. 45) and “Math is math period! How can a link be made between math and culture?” (Leonard & Jackson Dantley, 2005, p. 98). Alternatively, in their comparison of the impact of two professional development programs, Kose and Lim (2011) found that reading about how to teach for social justice (e.g., Brantlinger, 2005; Turner & Strawhun, 2005) had a positive impact on teachers’ stated beliefs related to the practice. It appears that having pre-service teachers read articles may serve as a mechanism for changes in perspective, but is not sufficient as a standalone practice (de Freitas, 2008).

Much has been written about the importance of teachers’ content knowledge (e.g., Grossman, Schoenfeld, & Lee, 2005; Ma, 1999; Shulman, 1986). Many mathematics teachers were taught by rote instruction, limiting their own understanding of the field (Ball, 1988). Further, schools that serve marginalized students are more likely than their counterparts to employ teachers who do not hold a degree in their teaching field (Darling-Hammond, Barnett, Haselkorn, & Fideler, 1999; Grossman & Schoenfeld, 2005). Consequently, many teachers’
content knowledge and pedagogical content knowledge are limited (e.g., Grossman & Schoenfeld, 2005; Shulman, 1986). Research suggests that educators who have a strong understanding of mathematics content are more likely to implement effective instructional practices, such as connecting instruction with students’ lives, fostering students’ confidence in their mathematical abilities, and presenting content in a manner that makes it accessible for all learners (Baroody, 1998; Grossman & Schoenfeld, 2005; Moschkovich, 2007; National Council of Teachers of Mathematics, 2000).

By engaging in math activities that encourage them to investigate rather than memorize, teachers as learners can deepen their own understanding of mathematics concepts (D. Ball, 1988). I posit that as teacher candidates relearn mathematics in a way that makes more sense to them and begin to understand the connections within mathematics, they can also be inspired to consider the question of the very nature of mathematics and mathematics education (e.g., Lockhart, 2008).

As previously noted, Garmon (2005) asserts that a secure environment in which participants feel safe to reflect on their beliefs and to discuss issues of diversity in an authentic manner facilitates their personal growth. One-on-one mentoring can provide such a safe space to challenge and be challenged without judgment (Kram & Isabella, 1985). The mentor role manifests itself differently for different individuals: listening, questioning, pushing back, providing information, or coaching (Shea, 1992). Of primary importance is that the mentor encourages reflection and alternative ways of thinking in a supportive environment (Mott, 2002).

**Research Questions**

This study addresses the following questions: What conceptions of success do pre-service mathematics teachers who intend to teach in high needs schools hold for their students? Do their
conceptions of success evolve as they participate in an equity-oriented professional learning community? If so, how do their conceptions of success change?

**Conceptual Model**

Scholars have offered different theoretical frameworks for considering equity, learning, and success in mathematics. For example, Gutiérrez’ (2009b) dimensions for thinking about learning touch on and expand upon each of the success categories I have discussed: academic achievement, mathematical power, and critical consciousness. Her framework consists of four interrelated dimensions: access, achievement, identity, and power. Access examines the resources available to students, such as technology and qualified teachers. Achievement is related to mainstream standards for measuring student success, such as grades, standardized test scores, and rates of participation in mathematics courses. Identity is correlated with how students view themselves while doing mathematics. Power relates to students’ sense of personal agency to enact social transformations both within classrooms and in broader society.

Access and Achievement can be thought of as comprising the dominant axis, preparing students to participate economically in society and privileging a status quo. … Identity and Power make up the critical axis. The critical axis, where identity can be seen as a precursor to power, ensures that students’ frames of reference and resources are acknowledged in ways that help build critical citizens so that they may change the game.

All four dimensions are necessary if we are to have true equity. (Gutiérrez, 2009b, p. 6)

In Gutiérrez’ model, success is represented in the intersection of the dominant and critical axes. Academic achievement and the development of mathematical power (on the dominant axis) can contribute to the development of critical consciousness (the critical axis), just as engaging in
social justice mathematics can contribute to academic achievement and the development of mathematical power.

Of note in this multidimensional view of equity is the recognition that we move to broader notions of learning than the simple constructs of access and achievement, which measure student success along conventional lines such as the achievement gap. Additionally, identity, on the critical axis, can be an important aspect of the learning process (e.g., Jilk, 2006; 2009; Ladson-Billings, 1995a; Moll et al., 1992). For example, having a positive sense of cultural heritage and learning about the accomplishments of one’s people lead to a positive sense of ethnic identity, which in turn provides a buffer against negative social experiences such as racism and is positively associated with academic achievement and personal agency (e.g., Gutstein, 2006a; Ladson-Billings, 1995a; Phinney, 1990).

Gutstein (2006a) offers a theoretical framework for teaching mathematics for social justice that includes both mathematics goals and social justice goals.

The three social justice pedagogical goals are (1) reading the world with mathematics, (2) writing the world with mathematics, and (3) developing positive cultural and social identities. The three mathematical pedagogical goals are (1) reading the mathematical world, (2) succeeding academically in the traditional sense, and (3) changing one’s orientation to mathematics. (Gutstein, 2006a, p. 24)

Put simply, “reading the world” means understanding the social, political, and historical realities in one’s environment and society. “Writing the world” means being able to use mathematics to make changes in said society. Gutstein, a college professor and researcher who taught a middle school mathematics class in a Chicago public school for two years, used a combination of reform-based curricula (80-85% of the time) and social justice projects (15-20%
of the time) with his middle school students. Reflecting on her experiences as a student in Gutstein’s class, Maria notes that learning mathematics this way was more interesting because it related to her life:

What made this experience different than other classrooms was a number of factors. First, the issues were applicable to real life, and many were personally relevant to us at more than one level. As low-income, Latino, immigrant children, some of the issues were directly linked to our own neighborhood, while others were issues of social justice on a global level. (Gutstein, 2006a, p. 167)

Students who have developed “positive social and cultural identities” can demonstrate “cultural competence” (Ladson-Billings, 1995b, p. 476). That is, they are able to stay connected to their own linguistic, cultural, and social norms while navigating the dominant culture. “Reading the mathematical word” (Gutstein, 2006a, p. 24) is synonymous with the development of mathematical power. “Succeeding academically in the traditional sense” (Gutstein, 2006a, p. 24) aligns with Gutiérrez’ (2009b) achievement dimension. The goal of “changing one’s orientation to mathematics” means “that students fundamentally change their orientation toward mathematics from seeing it as a series of disconnected, rote rules to be memorized and regurgitated, to a powerful and relevant tool for understanding complicated, real-world phenomena” (Gutstein, 2006a, p. 30).

There are aspects of both Gutiérrez’ and Gutstein’s frameworks that I would either not include or consider differently. As I see it, equity in mathematics is a broader concept than success indicators in mathematics. For example, as indicated in Gutiérrez’ equity framework, access relates to “opportunities to learn” (Oakes, 1990). That includes: “quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom
environment that invites participation, and infrastructure for learning outside of class hours” (Gutiérrez, 2009b, p. 5). While this is an important consideration in targeting equity in mathematics education, it does not serve to indicate success for an individual student or a group of students. For example, a student cannot be deemed successful in an Algebra class on the basis of the curriculum being rigorous. Further, while there is significant overlap between the framework presented by Gutstein and the categories I identify for considering success, I do not include a “change” in students’ orientation to mathematics. It presumes that students enter with a view of mathematics as skills: a collection of unrelated basic facts with fixed rules, formulas, and procedures to be learned through memorizing by rote (Baroody, 1998). While this may be the case, I do not presume it to be true in all students. Consequently, I prefer to consider the possibility that students may enter the classroom with an assortment of dispositions toward mathematics.

Thus, I borrow from both Gutiérrez (2009b) and Gutstein (2006a) in framing conceptions of success in mathematics. Figure 1 provides a conceptual model for considering the trajectory of a pre-service mathematics teacher as she progresses through the program. Each circle of the Venn diagram represents one of the overarching categories that I previously identified: academic achievement, mathematical power, and critical consciousness. There is overlap between each two categories and among the three, representing the fact that in reality many teachers look for a variety of indicators of success in their students.
In the following example illustrates the possible trajectory of an individual as she participates in the professional learning program. I anticipate that the majority of pre-service teachers will enter the program with a mastery identity. As her identity is troubled through participation in the professional learning community, a pre-service teacher may move from a focus on one category to another, or an overlap of two or three.

The trajectory is not straight to highlight the fact that I do not anticipate a linear progression. In this example, she begins with a focus on academic achievement (the star) and then adds the goal of mathematical power (marker 2) before moving completely into that.
category (marker 3). Marker 4 is outside the diagram to indicate a time of confusion and indecision. She is uncertain as to what goals she wants to work toward with her students. At the end of the time period illustrated here, she has settled in the middle of the Venn diagram, considering all three categories. The lines connecting the markers do not indicate knowledge of how a participant moved from one point to the next; they indicate sequence only.

My purpose is to present and apply a heuristic that can be useful when considering conceptions of success; the model provides a visual representation of the data. The model is limited in that it does not illustrate the nature of a participant’s statements nor does it demonstrate the depth of the evidence for each marker. In chapters 4 and 5, I address these concerns as I unpack each of the participant’s narratives.
Chapter 3

Methodology

The methods used in this study reflect: (a) my identity as a public school-educated teacher who seeks to contribute to equity in public schools and (b) my understanding that centering learning around individuals’ interactions with each other as they participate in the pursuit of a collective goal allows people to learn more deeply than if they were to engage in similar work alone (e.g., Hawley & Valli, 1999; Little, 2002; Miller, 2001). Rather than trying to remain distant from my participants as a sign of objectivity, I served as a participant observer, contributing lessons gleaned from my experiences as a public school teacher to the community’s knowledge base (Bogdan & Biklen, 2003; Lather, 1986). I worked directly with participants, facilitating discussions, providing one-on-one mentoring, giving feedback on mathematics activities, moderating online discussion forums, supporting the job search, and otherwise sharing my expertise.

I have chosen a case study approach for this research because I am interested in the changes that occur in PSMTs’ conceptions before they enter the classroom. That is, I am more concerned with the individual cases and less with the variables surrounding them. As Schwandt (2001) notes, “in case study, the case is at center stage, not variables” (p. 23). Case study methods are particularly useful “when the inquirer seeks answers to how” a phenomenon occurs, “when the object of study is a contemporary phenomenon in a real-life context, when boundaries between the phenomenon and the context are not clear, and when it is desirable to use multiple sources of evidence” (Schwandt, 2001, p. 23). The phenomenon here is the evolution of participants’ conceptions, and I am interested in how that evolution occurs. As this study is
embedded within the context of a professional development model, it lends itself to a case study approach.

I do not seek to claim generalizability for the broad population, as that is not the purpose of a case study; however, there is still much that can be learned from this type of research. According to Stake (1995),

[S]ingle cases are not as strong a base for generalizing to a population of cases as other research designs. But people can learn much that is general from single cases. They do that partly because they are familiar with other cases and they add this one in, thus making a slightly new group from which to generalize, a new opportunity to modify old generalizations. (p. 85)

Thus, through case studies, we deepen and broaden our understanding of the complexities of the human experience.

In the following sections I describe the participants of this study. I then describe the context of the professional development program and the rationale for each component of the program, several of which served as data sources for this research. I continue by explaining how the data were collected. I conclude this chapter with a description of how I analyzed the data.

**Participants**

My study is situated within a larger professional development and research project (Gutiérrez, Irving, & Gerardo, in preparation). I was a research assistant in that project and, as previously noted, was heavily involved in supporting participants in the project. Participants in the project matriculated as members of one of four cohorts. The first cohort, composed of three participants (hereafter referred to as scholars or PSMTs interchangeably), entered the equity-based teacher education program in the fall of 2009 and graduated in May 2011. The second
cohort of four scholars began in 2010 and graduated in 2012. The third cohort of seven scholars entered the program in 2011 and graduated in 2013. The fourth and final cohort of six scholars entered the program in 2012 and graduated in 2014. The present study is concerned with the second cohort.

Scholars were able to opt out of the research portion of the project. Because the research project was closely tied to scholars’ funding and academic standing, their consent forms (whether they consent or decline) were sealed in envelopes. Members of the research team did not know which scholars were and were not a part of the research project until each scholar had completed the program and graduated from the university. Individuals who were members of cohorts 3 and 4 had not graduated at the time I began this study and consequently are not included here. I was with cohort 1 for their second year in the program, and was not involved in the program development or data collection during their first year. The two years I actively participated in the design and implementation of the program coincided with the two years that the four members of cohort 2 were in the program. I explore the findings related to two members of that cohort: Heath and Raphael. As their mentor, I spent a significant amount of time one-on-one with each of them over the course of their two-year sojourns in the equity-based teacher education program providing me with the opportunity to come to know them more closely than the other scholars. Consequently, these two participants provided the richest data for the research questions addressed in this paper. Each of them gave consent to use his data in this study. I will provide more detailed information related to Heath and Raphael in chapters four and five, respectively.
Context

Here I describe our model for preparing secondary teachers to engage marginalized students in creative and rigorous mathematics. Our equity-oriented teacher education program is a national loan forgiveness program for pre-service secondary mathematics teachers who seek to work in high needs schools. Each scholar was accepted into the regular teacher education program at a large Midwestern university, either as an undergraduate with a mathematics major and a secondary education minor or as a graduate student pursuing a Master’s degree with certification in secondary education.

Each scholar received a scholarship in the amount of $10,000 per year. Undergraduates were eligible to receive the scholarship for two years while graduate students were eligible for one year. Each scholar signed a contract committing to work in a high needs school for two years for every year she or he received funding. That is, undergraduate scholars committed to working in a high needs school for at least four years, and graduate students committed to serving in a high needs school for at least two years. According to the National Science Foundation (NSF), which funded the scholarship program, a high needs school is defined as one with a high percentage of students from families with incomes below the poverty line, a high percentage of secondary teachers in content areas for which they are not prepared to teach, and/or a high teacher turnover (National Science Foundation, 2006). The terms high percentage and high teacher turnover were subject to interpretation. In line with other national loan forgiveness programs for students preparing to teach in areas of need, we used 40 percent or higher in defining a high percentage of students from families that qualify for free or reduced lunch.

Our model was based on what we believe we know about engaging pre-service teachers in transformative professional learning experiences, those that help PSMTs recognize the
political nature of schooling and lead to teaching for equity, diversity, and social justice. Within a supportive community (Bartell, 2006; Garmon, 2004; Lave & Wenger, 1991; Stein, Silver, & Smith, 1998) we sought to “trouble their mastery identity” (de Freitas, 2008; hooks, 2000; Winks, 1999) and help them begin to develop a civic agent identity (Mirra and Morrell, 2011), provided structured opportunities for reflection (e.g., A. Ball, 2009; de Freitas, 2008; Grumet, 2010; Zeichner, 1993) and secure mentoring environments (Garmon, 2005; Kram & Isabella, 1985), and facilitated learning experiences around research literature (e.g., Civil, 1994; Frankenstein, 1990; 1994; Gutiérrez, 2009a; González et al., 2001; Jilk, 2006; Moll et al., 1992; Moschkovich, 2002; Zevenbergen, 2000) and mathematics (D. Ball, 1988).

**Bi-Weekly Seminar**

One goal of our professional development program was for scholars to work together as they became increasingly reflective practitioners and deepened their understanding of the complexities of teaching (Garmon, 2005; Gutiérrez, 2009a; National Staff Development Council, 2001). Twice a month, scholars participated in a seminar in which we engaged them in activities and discussions designed to broaden their knowledge and conceptions. Scholars attended the three-hour seminar eight times per semester over the course of their two years in the program, for a total of 96 hours. While the agenda of each meeting was different, based on the current needs and desires of the scholars, there were activities in which we regularly engaged the scholars. Here I describe those activities.

**In My Shoes.** Our partner teacher (whom I will describe), a scholar, or a member of the research team presented a challenging scenario that she or he had faced in a secondary school setting. The presenter (most often in the role of teacher) explained what happened without sharing how she or he responded or how the situation was resolved (if it was). The scholars then
asked probing questions and discussed possible responses as well as the potential complications with any given approach they might try. After suggesting responses, the scholars then rehearsed the responses with one person in the role of the individual who presented the situation and others representing other stakeholders. Finally, the original presenter shared the actual outcome—how she or he responded and what happened subsequently. An example of a scenario that was explored in seminar follows. A scholar, as a student teacher, was helping students with their work in class. Several students raised their hands. The scholar walked by one student to help another. The student who was bypassed said loudly, “She isn’t helping me because I’m black!” Questions from scholars included asking about the racial make up of the class, the race of the student who was helped first, why the scholar went to one student first, how the scholar perceived the student’s comment about racism (i.e., was it a serious complaint or stated in jest), the level of trust and respect that existed in the classroom, and whether or not other students heard the accusation. Scholars then suggested and debated the pros and cons of possible teacher responses, which involved a mixture of and fell on a spectrum between: ignoring or addressing the statement, speaking to the student privately or in front of the entire class, making light of the comment or taking it seriously, and responding angrily or calmly. The goal of In My Shoes was not to present scholars with a scripted response for every possible situation they would confront in the classroom, but to help them recognize the complexities of day-to-day life when working with students and be prepared to address the unexpected (Britzman, 2003; Edwards, Gilroy, & Hartley, 2002).

**Mathematics to Change Kids’ Minds.** One source of In My Shoes situations was our partner teacher, Philip. Along with other members of the professional learning community, Philip also engaged the scholars in rigorous mathematics activities, demonstrating the types of
problems and activities—those that are challenging to university students—in which he engaged his own high school students. These activities provided scholars with opportunities to develop deeper understandings of mathematics, see the interconnections between disparate areas of “school” mathematics, learn different ways of engaging students in interesting mathematics lessons, and develop a realistic picture of the rigorous mathematics that students are capable of doing. Seminar was also a forum for scholars to share the activities they developed for the middle schools students who participated in an after-school math club (which I will describe), rehearsing the types of questions and hints they would present to students.

**Readings and media.** While *In My Shoes* and *Mathematics to Change Kids’ Minds* were structured around the scholars’ perspectives, we also brought in others’ points of view. Several times a semester, scholars viewed and discussed videos and video blogs related to diversity, schooling, and mathematics. The group watched documentaries (e.g., *Color of Fear*, *Waiting for Superman*, and *Papers*) and TED (Technology, Entertainment, Design) Talks (e.g., Ken Robinson Says Schools Kill Creativity and Arthur Benjamin’s Formula for Changing Math Education) suggested by members of the research team as well as other scholars. Scholars were also asked to read a variety of materials, including research and opinion pieces. Examples of readings include Gutstein’s (2006) *Reading and writing the world with mathematics: Toward a pedagogy for social justice*, Lipman’s (2003) *High stakes education: Inequality, globalization, and urban school reform*, Pollock’s (2008) *Everyday anti-racism*, and Lockhart’s (2008) *A mathematician’s lament*. Media were selected to encourage scholars to think about things they may not have considered before; challenge previously held beliefs about students, mathematics, and schooling; and deepen their awareness of the complexities of teaching.
**Discussions.** At the heart of each seminar meeting were the discussions that served to facilitate scholars’ personal growth and provided the impetus to help them develop dispositions that were productive towards serving diverse groups of students. As such, we developed seminar to be a secure environment in which scholars were supported to take risks, to reflect on their beliefs, and to discuss issues of diversity in a candid manner. The instructor and graduate research assistants refrained from lecturing or running the seminar in an authoritative way. Instead, we generally introduced a topic and then facilitated scholars’ discussions. We might have pushed for further explanation or asked a clarifying question, but rarely did we interject with our own opinions on an issue. We also emphasized the idea that there was no single right answer to the issues we discussed, and that all members of the community—scholars, graduate research assistants, and the instructor—were encouraged to challenge each other’s views. There was rarely a clear closure to the discussions, and scholars frequently continued to discuss the issues at hand outside of seminar.

**Online Forums**

Prior to seminar discussions, scholars were often given a prompt related to the topic for the day and asked to post their initial thoughts in online forums. The nature of each forum allowed scholars to both express their thoughts and respond to the thoughts of others, engaging in a written conversation. Scholars were asked to post such things as their reactions to videos and readings and their reflections on their experiences with the after school math club and professional development sessions.

**Mentoring**

Although seminar discussions and professional development activities were useful spaces for scholars to be exposed to new things and grow, we developed mentoring sessions that
allowed us to check in with scholars individually and to pay particular attention to their needs. As previously noted, research suggests that a secure environment in which participants feel safe to reflect on their beliefs and to discuss issues of diversity in an authentic manner facilitates their personal growth (Garmon, 2005). One-on-one mentoring can provide such a safe space to challenge and be challenged without judgment (Kram & Isabella, 1985). Consequently, each scholar was provided with a mentor who was a member of the research team. Each mentor (a professor and two graduate students) had taught mathematics in a secondary school serving marginalized students. Each scholar met with her or his mentor for approximately one hour twice a month. The research team met prior to each set of mentoring sessions to identify topics for probing based on what occurred in seminar—what was or was not said, body language, etc. We sought to understand how scholars felt about the structure (e.g., How did you feel about another scholar facilitating? Why do you think people did or did not participate? Would you be willing to lead a session?) as well as the content (e.g., Did the session make you think of things in a different way? Is there anything you want to push back on? Do you have any lingering questions?). During mentoring sessions, we also asked them to reflect on professional development experiences and their field placements (e.g., What did you walk away from the session with? What have you noticed in your placement? How do you think it might apply to anything you might teach in your future classroom? What kind of math do you think your students could do? Which students?). Mentoring sessions were also a place for scholars to voice thoughts and ask questions that they may not have brought up in whole group environments. For example, some scholars worked in jobs that relate to education (e.g., teaching assistants in math courses, resident assistants in housing complexes on campus) that they liked to discuss, or they had an idea for a future seminar session that they thought we should try to incorporate.
Math Club

A frequent topic of both seminar and mentoring sessions was our program-sponsored, after-school mathematics club held at the local public library. The library was near a middle school and served as an after-school gathering place for students who were waiting for their parents. The club met once a week for two hours and sought to meet the needs of students who had not been positioned in society as mathematically capable. Scholars developed, prepared and facilitated math stations (challenging mathematics activities presented in engaging formats such as games and puzzles), recruited middle school students who were in the library to come participate, and supported returning participants in facilitating activities with their peers. The math club provided scholars with the opportunity to interact with real students and rehearse teaching moves. What can I do/say if a student is struggling with the math? Doesn’t want to participate? Doesn’t get along with or want to work with others? Isn’t interested in what I have to say? Is overzealous with his peers? How do I capitalize on students’ interests, knowledge, and skills? How do I find out what their interests, knowledge, and skills are?

Professional Development

Scholars participated in two additional professional development structures: Summer Boot Camp held in August of each year, and PD Saturdays, held once a semester.

Summer Boot Camp. Boot Camp was comprised of two days of intensive activities designed to support many of the goals addressed in other components of the model. We engaged in team building to facilitate the development of community, rehearsing to prepare scholars for the unexpected occurrences in the day-to-day life of the classroom, and mathematics activities to deepen scholars understanding of the content.
**PD Saturdays.** Once each semester, scholars participated in additional professional development activities in a nearby major city. In the fall, we visited our partner teacher’s school site to see his classroom and meet his high school students. The visit provided existence proof that it was possible to engage high school students in rigorous and creative mathematics. Scholars also had an opportunity to gain some insight into students’ perspectives of their mathematics education. In the spring, scholars attend the Metropolitan Mathematics Club conference. The conference provided a venue for scholars to learn from other teachers and become part of a larger community of mathematics teachers.

**Model Teacher**

The model teacher with whom we had an ongoing relationship was a graduate of the regular teacher education program at our university. He was a National Board certified high school mathematics teacher with 10 years of experience who was engaging his students in the creative and rigorous mathematics that we encouraged our scholars to do with their own students. As previously mentioned, he engaged our scholars in mathematics activities and presents In My Shoes scenarios, and scholars visited his campus once a year to observe his classes and interact with his students.

**Data Sources and Collection**

The present study examined secondary data collected in the *Mathematics, Marginalized Youth, and Creative Insubordination: A Model for Preparing Teachers to Reclaim the Profession* project. Data related to this research were collected from August 2010 to May 2012 from several sources and using a variety of methods to gain a broader sense of scholars’ knowledge and dispositions and to reduce the risk of systematic bias if data were to be collected from a single source (Maxwell, 2005). These data sources were selected because they provided the best
opportunities for individual scholars to express their views and opinions related to conceptions of success.

**Inventory of Ideas/Feelings**

The inventory of ideas and feelings was a set of prompts given to scholars immediately before their intake interview (see Appendix A). The prompts included:

- When I think about teaching/leading in a high needs school, I get excited about…
- When I think about teaching/leading in a high needs school, I wonder…
- The concerns I have are…
- When I think about teaching students who may be different from me in terms of how they are racialized, gendered, positioned with respect to class or language, etc., I feel…
- The things I possess that I think I will most need to prepare to be an excellent mathematics teacher for marginalized students are…

We provided each scholar with the sheet of prompts and then gave her or him ten to fifteen minutes to respond. There were no right answers. We used the scholar’s responses as talking points for the subsequent interview.

**Dimensions of Equity**

Scholars were asked to complete the dimensions of equity *How Would You Classify It?* instrument based on Gutiérrez’ (2006) framework (see Appendix B). The first part of the sheet listed ten different school-related scenarios and asked the scholar to classify each scenario as related to access, achievement, identity, or power. Scholars were further asked to indicate whether they believed each scenario was a positive indicator of the category or a violation of the category. The example with which they were provided read: *Parents are invited to the school to*
learn how to support their student with the new IMP curriculum that was recently adopted. (+ Access)

Recruitment Statement

In the fall of 2010, we asked the scholars, What would you say to someone if they asked you to explain and describe this program? Each scholar wrote a short statement, one or two sentences, which we collected and used for recruitment purposes online and on a bulletin board on campus.

Mentoring Session Transcripts

Most mentoring sessions were recorded with an audio recorder. I also took notes during each session. On rare occasions, the recorder failed. In those cases, I relied on my notes when analyzing data.

Field Notes

The members of the research team kept ongoing field notes. We met once a week to reflect, exchange notes, and prepare learning experiences for the scholars.

E-Portfolios

In addition to the data sources from our program, I also analyzed portions of participants’ e-portfolios, which they prepared as part of the regular teacher education program: the philosophy of teaching statement and the impact on student learning statement.

Philosophy of teaching statement. A philosophy of teaching statement is a reflective account of an educator’s beliefs and values as they relate to teaching and learning. A teaching statement may include one’s goals for student learning as well as preferred methods for teaching and assessment. It may also include a statement about how one creates a classroom environment that values and capitalizes on diversity of language, thought, ways of learning, community
resources, and other assets with which children enter the classroom (Kaplan, O'Neal, Meizlish, Carillo, Kardia, 2012). Heath and Raphael both included references to their goals for student learning in their teaching statements.

**Impact on student learning statement.** An impact on student learning statement provides an opportunity for teachers to reflect on the effect of their instruction on student learning. A pre-service teacher develops a design for instruction and assessment, including a method for data collection (e.g., pre- and post test, student questionnaire) aligned with learning objectives. The pre-service teacher then performs an analysis of student learning using the collected data. Finally the pre-service teacher reflects on the outcomes. He evaluates his performance as a teacher and perhaps identifies future action for improved practice and professional growth (Idaho State Board of Education, 2011; Southern University at New Orleans College of Education, 2005). Heath and Raphael each reflected on the learning of two students: a boy and a girl. Both scholars analyzed standard assessments (e.g., quizzes) and provided these as artifacts to support their reflections.

**Data Analysis**

I developed initial ideas about analytic organizational codes by referring to the conceptual frame for success that I brought to this research (Maxwell, 2005). I developed a codebook in which I listed these codes and their meanings (see Appendix E). The five framing categories for the codes were Academic Achievement, Mathematical Power, Critical Consciousness, Teacher’s Identity, and Transformative Professional Learning Experiences. I developed descriptive codes related to the literature within each category. For example, the code *AA-ap* was used to identify instances in which references were made to Advanced Placement courses, either explicitly or implicitly.
Data analysis occurred in several ongoing phases. In phase 1, I listened to the recordings of each of Heath’s mentoring sessions and transcribed selected portions. I transcribed selectively because the larger project included much more than the questions specific to this research (Gutiérrez et al., 2013), a limitation of my study. The idea of how PSMTs view success was embedded in the overall framework of our program model, but it was not the primary focus. The portions of the recordings that I elected to transcribe were those that addressed the concept of success in mathematics, either directly or tangentially; however, their statements were embedded in particular contexts, which were different throughout the study. Gibbs (2007) warns that transcription “introduces issues of accuracy, fidelity and interpretation” (p. 11), and selective transcription in particular can cause the researcher to “miss out the setting, context, body language and general ‘feel’ of the session” (p.11). To address this concern, I also attached my field notes to each transcript. This served to help me remember details about mentoring sessions, such as the inflection in the participant’s voice and the amount of time that he paused between statements and, consequently, aided in my interpretation process. After transcribing Heath’s mentoring sessions, I compiled all of the textual data into a single, chronological file with line numbers added to aid in the organizational process (Schwandt, 2001). I then coded Heath’s data using the codes that I developed initially. The coded passages ranged from a single sentence to several paragraphs, depending on how long Heath discussed the topic of success (Foote et al, 2013).

While coding Heath’s data, I made additions and changes to the original codes as I deemed necessary. For example, I began with the code CC-agency: students’ personal agency; using mathematics for a given purpose and having control over what that purpose might be; recognition of power to effect change. In the process of reviewing Heath’s data, I found that an
additional dimension was needed to describe students’ sense of power. In a mentoring session we discussed a seminar conversation in which the scholars had been asked to define one of the four dimensions of Gutiérrez’ Equity Framework. Heath noted,

David did power. His description was very much only looking at the stuff outside the classroom or later in life. I always thought of power as in the classroom. Who has the power in the classroom? I would add David’s definition, not replace mine with it. (H300-H303)³

Raphael made a similar statement:

I always thought of it as, I think, how [the principal investigator] does. It influences what goes on in the classroom and who decides what happens. Some of the scholars talked about power based on taking a lot of math classes so that you can gain power in the real world beyond school. I hadn’t thought about it in that way before. It’s true, but I didn’t think about it that way. I don’t so much disagree. I agree that more education leads to more power in general in life, but I think in this case it’s more about the math classroom. Who influences what happens in the math class. Typically in the past the teacher always decides. (R1359-1R367)

After reviewing these sections of the data, I decided to differentiate between power related to what happens outside the classroom (CC-agency) and power that is related to what happens inside the classroom (CC-classpow). This is an important distinction as the classroom teacher has a more direct impact on what happens inside of the classroom than outside, and this may influence the instructional decisions that he or she makes. I made several passes of the data, refining codes with each pass.

³ Line numbers refer to the single document into which each participant’s textual data were compiled. “H#” refers line numbers in Heath’s textual data; “R#” refers to line numbers in Raphael’s textual data.
After coding Heath’s data, I organized it into a qualitative data table as “a convenient way to display text from across the whole dataset in a way that makes systematic comparisons easier” (Gibbs, 2007, p. 78). The data table included a chronological recording of the context in which data were collected, the topics discussed during different components of the professional development program, and the codes that I applied to Heath’s data. I then mapped my findings for Heath onto my conceptual model. I gained a complete view of his placement on the model to develop a sense of what I might learn from Heath’s trajectory and drew what I believe to be warranted conclusions, highlighting shifts in thinking as well as tensions in the consideration of different notions of success (Schwandt, 2001). I recorded initial thoughts related to Heath’s trajectory in his case study report.

In phase 2, I repeated the same analysis process for Raphael, concluding with initial thoughts related to his trajectory in his case study report. As I reviewed Raphael’s data, I further revised my codebook. In phase 3, I returned to Heath’s data in light of the new codes developed while examining Raphael’s case. I made additions to Heath’s coded data and revisions to his trajectory. I chose to examine the participants’ data consecutively rather than concurrently because I wanted to initially identify their individual issues before considering similarities and overlaps between the cases, which I did in phase 4 of data analysis.
Chapter 4

Heath’s Case

Heath is a white man from a small town in a Midwestern state where the majority of his classmates were from similar middle-class backgrounds. He is the child of a kindergarten teacher and a businessman. Heath is a self-described “music, artsy person” (H197) who plays the piano, trumpet and French horn. He did not enter college with the intention of becoming a mathematics teacher, applying to the engineering program instead. He soon realized that he was interested in “actually making a difference in someone’s life” (H4) and subsequently switched from engineering to education.

Heath frequently reflected on his own secondary schooling as he considered his experiences in college. He lamented the emphasis that was placed on procedures and routine expertise rather than adaptive expertise. He was also frustrated in college level mathematics classes as he discovered that he lacked the conceptual understanding around many mathematics topics that he felt his peers all seemed to have. Heath also noted a lack of technology in his secondary classrooms, which he felt would have been helpful based on what he was experiencing in college. Heath described himself as curious and motivated, noting that during high school he often tried to teach himself the “whys” (H89) that he felt he was not learning in the classroom.

Heath’s Trajectory

There were twelve points during Heath’s two years in the program at which I identified references to student success related to my conceptual model. I present those twelve points here.

Semester 1: Fall 2010

Heath’s intake interview took place in October 2010. At that time, when asked about his interpretation of a high needs school, he described it as “one in which its students don’t meet
expectations of state, a school that’s lacking in achievement” (H10-H12). This response suggests that he views a successful school as one in which the students meet the expectations of the state. Because state expectations are defined, in part, along the lines of test scores, Heath seemed to be viewing a failing school as one in which students have low test scores. One could extrapolate from this, then, that he viewed a successful school and successful students as those whose test scores met or exceed the state’s standards—a measure of academic achievement.

In the same conversation, when asked what things (knowledge, qualities, experiences, etc.) he possessed that he thought would help him become an excellent mathematics teacher for marginalized students, he stated, “Love for math. I don’t consider myself the most knowledgeable person in the math field but I have a high respect for it and want to share that with my students and make math relevant” (H14-H17). I interpreted this as a reference to a productive disposition. One might argue that questioning mathematics—rather than deferring to it or having respect for it—is more indicative of a productive disposition. I do not believe that respecting mathematics and questioning it are mutually exclusive. For example, respect for mathematics might include recognizing its inherent beauty and usefulness, which does not require questioning it. Further, Heath’s response about his love of mathematics being an asset coupled with the recognition that he does not view himself as “the most knowledgeable person” when it comes to the subject implied that one’s disposition towards mathematics (e.g., having high respect for it), rather than simply skills or ability, was important in learning one’s practice. This reference to a productive disposition was an aspect of mathematical power.
The diagram in figure 3 represents Heath’s conceptual model in Fall 2010, his first semester as a scholar in our program. The star-shaped marker (marker 1) in figure 3 represents Heath’s initial position (October 2010) on the conceptual model – the intersection between academic achievement and mathematical power.

**Semester 2: Spring 2011**

In January 2011, Heath completed the *How Would You Classify It?* dimensions of equity instrument (Gutiérrez, 2006). The first part of the sheet listed ten different school-related scenarios and asked the scholar to classify each scenario as related to access, achievement, identity, or power. They were further asked to indicate whether they believed each scenario was a positive indicator of the category or a violation of the category. Although these were hypothetical situations, they provided some insight into Heath’s point of view regarding positive and negative situations.

Heath’s responses suggested that he supported the development of students’ personal agency, an indicator for critical consciousness. For example, one scenario reads, *Mr. Taylor’s students learn how to use geometry to solve a problem about the school’s redistricting policy that threatens to bus families to other schools. They present their “better” solution to the school*
board. In presenting their solution to the school board, Heath stated that the “students are taking control of the situation and experiencing power,” (H44-H45) indicating that this was a positive indicator for power. Another example was presented in the second part of the worksheet, which asked scholars to draft three original scenarios and categorize them in the same manner as those that were given. Heath drafted an example to model a positive representation of power: “A math teacher introduces a new concept by letting the class lead the discussion and ask questions” (H54-H55). When presented with a scenario in which 30 percent of a math/science magnet high school’s student population goes on to get jobs in math-based fields (e.g., engineering, physics, mathematics), Heath noted that students at the magnet school were fortunate in that “they are able to identify with others who also have a legit interest in mathematics” (H35-H36). Further, given the situation, Instead of just high test scores, Principal Malik values and rewards teachers for supporting students to develop a positive disposition towards mathematics (thinking of themselves as “math people”), Heath stated, “Being in a club where everyone has the same shirt creates a bond and community between the members and gives them something to identify with” (H30-H32). One might expect a pre-service teacher to appreciate the value of a supportive community; however, as noted previously, many view mathematics as having little or no relationship to culture. Heath’s responses suggest that he places importance on the development of students’ identities as members of a mathematical community, as the identity developed relates to students as “math people.”

When presented with a scenario of students in a school practicing standardized assessment-type problems and subsequently making AYP (Adequate Yearly Progress) under NCLB (No Child Left Behind), Heath remarked that this practice had a negative impact on both “their mathematical knowledge” and their “interest in math” (H49). As aforementioned,
mathematical power includes the development of conceptual understanding (a deep sense of mathematics as a web of interconnected ideas), procedural fluency (the ability to carry out procedures efficiently, appropriately, and flexibly), strategic thinking (competence in the processes of mathematics problem-solving and adaptive thinking), and productive disposition (a student’s belief that she can learn mathematics along with a desire to do so). Instead of focusing on the fact that students performed well on the standardized assessment – evidence of academic achievement – Heath’s response was focused on mathematical power. He noticed and commented on the possibility that the process of practicing standardized assessments, rather than engaging in other types of learning experiences, could have a negative impact on students’ “mathematical knowledge” (conceptual understanding) and their “interest in math” (productive disposition) (H49).

Heath asserted that having students repeatedly solve the same type of problem was a violation of achievement because, “Students are simply doing exercises instead of doing actual problems where they have to figure out what is being asked and then do it” (H51-H52). Heath seemed to be placing importance on strategic thinking, which, as previously mentioned, involves competence in the processes of mathematics problem-solving and adaptive thinking—developing a sense of inquiry and the ability to attack and solve varied types of problems (Baroody, 1998; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Van de Walle et al., 2012).

In the context of this activity, Heath observed that students taking Advanced Placement classes was important (H58). This is of note because he had made multiple references to mathematical power and critical consciousness. With this positive reference, one can infer that he had not completely jettisoned the idea that aspects of academic achievement were also important.
One of Heath’s first mentoring sessions took place on February 11, 2011. During that conversation, he made multiple references to mathematical power. Specifically, he noted his desire for conceptual understanding for himself and his future students after being in situations in which he felt lost mathematically. For example, when discussing one of the courses he was taking at the time—Math for Math Teachers—he stated, “I feel like my high school education in math was very. Not. Good” (H72). He continued,

I’m still behind on some of the concepts of things. I feel like all my classes were just like, here are the rules. Here are the shortcuts. And so I just, like, missed all that [conceptual] stuff, which I don’t want to happen when I teach. (H78-H81)

He further suggested that he wanted his students to engage deeply in mathematics, investigating to ask and understand not just how to do problems, but also why things work.

There’s lots of people out there who feel like they never got to ask whys or they never, they never really were supported to understand things conceptually or to, like you’re saying, you know to dig deeper. (H89-H91)

Perhaps due to the lack of an emphasis on conceptual understanding in his own secondary classes, Heath credited his own productive disposition with helping him learn mathematics, an indication of mastery identity (de Freitas, 2008). As noted previously, a productive mathematics disposition is evidenced by a student’s belief that he can learn mathematics along with a desire to do so. Recognition of one’s capabilities in mathematics is important because a person’s feelings about his ability to accomplish a task impact whether he will attempt it and how much effort he will apply (Bandura, 1977; Bandura, Barbaranelli, Caprara, & Pastorelli, 2001). Heath put forth extra effort in learning mathematics, as evidenced when he says,
I mean, when I was in high school I would always, like, I’d have to read the book at home and then, like, I had to know why. Like, I was always that person who had to know why this happened. And in class we never really talked about that. So I had to learn that on my own. (H84-H87)

He hoped to support the development of this disposition in his own students, stating that he wanted to make his mathematics classes “more interesting than what I was given” (H128-H129).

The following month (March 11, 2011), I explicitly asked Heath, How does a teacher know his students are successful? What will it look like? His response presented a mixture of references to grades and disposition as well as a state of confusion. Regarding grades, he identified grades as an indicator of success, but noted limitations in relying completely on them as a source of information. He stated,

Well I know you don’t want to just completely base it on grades because I know it’s easy to, I feel like sometimes it’s easy to get a good grade but not know anything. Um, but I also feel like you can get an idea from that. Like, if everyone’s failing then obviously something’s going wrong. Um, so I feel like you can, you can start by looking at grades. (H210-H214)

Here, Heath observed the limitations of a focus on grades (an indicator of academic achievement) in noting, as researchers have, that a student does not necessarily have to demonstrate conceptual understanding in order to receive high scores (e.g., McNeil & Valenzuela, 2000). However, Heath also noted that grades can reveal trends in students’ data (e.g., Hess & Petrilli, 2006; Lubienski, 2008), which may provide insight into where instruction or assessment may need to be revisited.
Heath identified a productive disposition as an indicator of success, noting that recognizing the importance of the subject area influences how well students retain what they have learned. He continued by noting that he was not sure how to measure such an indicator, stating,

And then I feel like, at least for me, from my experience as a student, sometimes, yeah I can do great on tests and I can understand everything that’s going on, but then I just flush it all out because I don’t think I’m ever going to use it again and it’s not important. So to be able to, to make your subject actually interesting and relevant to their lives where they want to keep it, I don’t know how you evaluate it. So the answer is, I don’t know. (H219-H224)

Heath’s response indicated that he valued helping students develop an interest in learning mathematics (productive disposition), but it also suggested that he was not sure what that would look like in a classroom. That is, productive disposition could be a goal; however, the evidence to seek to measuring the attainment of that goal was elusive.

Figure 4. Heath’s conceptual model in Spring 2011.

The diagram in figure 4 represents Heath’s trajectory on the conceptual model in Spring 2011, his second semester as a scholar in our program. Marker 2 in the center of the diagram at
the intersection of academic achievement, mathematical power, and critical consciousness represents Heath’s position on the conceptual model in January 2011. The following month (February 2011) his responses indicated a focus on mathematical power (marker 3). During a mentoring session in March 2011, Heath referred to academic achievement and mathematical power as indicators of success in mathematics (marker 4). As indicated by marker 5, he also expressed confusion and uncertainty, something we refer to as Nepantla in the program – “the uncomfortable space where there is no solid ground, that has no official recognition,” (Gutiérrez, 2012, p. 35).

**Semester 3: Fall 2011**

During the first mentoring session of Fall 2011 (September 21, 2011), Heath and I discussed the middle school math club that the scholars facilitated at the local public library. At that point, Heath had not attended a regular club meeting; however, he was present at a recent session during which members of the research team sought input from the middle school participants. Heath expressed appreciation for the fact that the students were asked for their input, noting, “I like how [the facilitators] asked students how to make [math club] better,” (H236) suggesting that he embraced the idea of students having a voice in how their experiences are shaped – an aspect of critical consciousness. Scholars generally consider critical consciousness to be about students being aware of injustices in society and being able to critique the world around them (e.g., Gutstein, 2006a). Here, Heath was beginning to include references about personal agency in math club, but not necessarily in students’ day-to-day lives.

The following month (October 12, 2011) our conversation focused on a site visit to our model teacher’s school. Philip’s high school students were positive about their experience in his class, which supported the development of productive dispositions toward mathematics;
however, Heath noted that the students were members of the math club, and stated, “It would be interesting to hear from someone for whom math is not their favorite subject and doesn’t like the style of the class” (H289-H290). He recognized that the enjoyment students feel for Philip’s mathematics classroom might not be universal and was concerned about those whose preferences were not being addressed. “How do you get kids interested who aren’t really interested” (H295). His responses suggested that he recognized the importance of a productive disposition for all students.

Further discussion included Heath’s fall placement. Heath was critical of his cooperating teacher’s instructional style, noting a lack of the development of conceptual understanding.

The teacher gives students fill-in-the-blank lecture notes. I don’t like that. That’s what he does all the time. No investigation or discovery learning. He’s literally just saying, if two lines are parallel and they are cut by a transversal, dot dot dot. I don’t feel like you learn deeply from that. (H268-H271)

Heath went on to talk about teaching a lesson the following day. The expectation was that the cooperating teacher would prepare the lesson for Heath to implement. As Heath was not happy with his cooperating teacher’s instructional style, he stated that he intended to add more opportunities for students to communicate and have more voice in the class. In planning this, he reflected on his experience visiting Philip’s classroom, remarking that something he appreciated about the class was the fact that the discussion was student-led, indicating an appreciation for students’ agency in the classroom.

Later that month (October 28, 2011) we discussed Gutiérrez’ (2009) dimensions of equity framework in seminar, which I believe served as the catalyst for an expansion of Heath’s conceptions of critical consciousness. To illustrate, when I asked Heath to reflect on the
conversation, he noted that David, another scholar, described power as “very much only looking at stuff outside the classroom or later in life” (H300-H301). Heath continued, “I always thought of power as in the classroom. Who has the power in the classroom? I would add it to David’s definition, not replace it” (H301-H302). The reference to critical consciousness as personal agency that Heath discussed earlier in the semester was limited to math club, a proxy for the classroom. Here he expanded that definition to include students’ experiences both inside and outside of the classroom.

During this same conversation, Heath discussed a lesson on solving systems of linear equations that he taught in his cooperating teacher’s classroom. While Heath wanted to have students construct the lines, the coop insisted that they simply solve the problem algebraically. Heath expressed concern that students would not understand the meaning underlying the mathematics if they only manipulated the symbols. “They understood how to do it, but I don’t know if they understood why they were doing it” (H325-H326). In other words, the students understood the series of steps involved when solving a system of equations algebraically, but they did not understand why the process worked or how it indicated the correct solution within a particular context. I interpreted his statement to imply a concern for conceptual understanding. Heath presented another lesson the following week. His concern in this class was that his students appeared uninterested. His words, “I was afraid I was losing people” (H335) indicated a concern for students’ dispositions.

During a mentoring session the following month (November 9, 2011), Heath and I discussed the activity he was preparing for the after-school math club. He was in the process of making his activity more rigorous. He noted that after interacting with students at school, visiting Philip’s classroom, and participating in the math club, he realized that, “the math I thought was
too complicated wasn’t because it was too hard for students. It was that I didn’t fully understand it myself” (H405-H406). He realized that students had a greater capacity for understanding mathematics than the level for which he originally gave them credit. That is, he recognized that students were capable of learning mathematics with understanding, a precursor to teaching with that end in mind.

Figure 5. Heath’s conceptual model in Fall 2011.

The diagram in figure 5 represents Heath’s trajectory on the conceptual model in Fall 2011, his third semester as a program scholar. Marker 6 represents Heath’s position on the conceptual model on September 21, 2011. The following month (October 12, 2011) Heath’s responses indicated a focus on both critical consciousness and mathematical power (marker 7). Marker 8 indicates that his position remained unchanged later the same month (October 28, 2011). Marker 9 represents Heath’s position on the model on November 9, 2011.

Semester 4: Spring 2012

Heath’s student teaching took place at Oak Hill High School during the spring of 2012. He worked with two cooperating teachers: Mr. Anderson and Mr. Baldwin. Mr. Anderson had been at Oak Hill for nine years. Heath described him as a thorough planner who provided constructive feedback after his observations, noting, “He’s not just like, that was bad or that was
good” (H529-H530). Mr. Baldwin was in his fifth year of teaching at the school. Heath observed that Mr. Baldwin was stricter than Mr. Anderson,

because he’s with freshmen and has to keep limits and boundaries noticeable. [For example,] I was walking around and one group wasn’t where they should be in their work. So I was, when they finished, one girl, she kidded with me and said something like, ha, we did it. I thought she was just kidding, but Baldwin said it was disrespectful and made her come back and apologize. I think he wants to make sure they see me as an authoritative figure. (H534-H539)

While they had different teaching styles, both Mr. Anderson and Mr. Baldwin encouraged Heath to try new things and experiment with his instructional practices, noting that they would be there to help him if something failed. Heath related, “My coops are encouraging me to experiment and push myself while they are here to catch me if I make mistakes” (H521-H522).

Heath taught three classes while at Oak Hill: Geometry, Algebra II, and Algebra Relearn. As Heath described his classes, “Accelerated geometry is probably 95% white. Algebra 2 is about 50% white and 50% other. In Algebra Relearn there are two white students and then three that are Hispanic and then everyone else is black” (H567-H569).

The Relearn class, with 10 to 15 students, was for those who had previously failed a mathematics course, many of whom needed to earn this credit to graduate. In the early fall (February 12, 2012), Heath was concerned that the students were not interested enough to put in full effort to learn the material. Heath stated that each student needed individual prompting “to open a book or they won’t do it” (H567). An exception was a female student named Dora, who, Heath reported, developed a more productive disposition throughout the semester. He reported, “One thing I’m proud of is Dora. At first, when I came in she was shy and wouldn’t work. I’m
building a relationship, and she knows that I can help her, and she’s asking me questions” (H574-H576). Heath further noted that Dora began to understand mathematical procedures, saying, “The other day she was getting elimination and substitution, and we were both really excited” (H566-H567).

Heath’s Algebra II class, a regular level course with sophomores and juniors, was using new textbooks, which were designed to encourage the development of conceptual understanding through investigation and problem solving. Heath appreciated the change in materials. The new texts also encouraged students’ independence from the teacher, group work, and communication. The lessons were designed to be student-centered rather than teacher-centered, placing more control in students’ hands. This was a different instructional approach than that to which the students at Oak Hill were accustomed, typically having had classes that were teacher-centered and skills-based.

Later the same month (February 26, 2012), Heath’s students performed poorly on a quiz, and in assessing their work Heath decided that they did not have the prerequisite conceptual knowledge, which concerned him. We discussed how he could check for understanding during lessons and govern himself accordingly. He used formative assessments to guide his instruction, and was willing to shift directions if he believed it would benefit his students.

I’ve done exit slips and quick checks and formative assessments, things that aren’t for a grade. One of my coops was saying that one of my biggest strengths is my ability to change a lesson and not stress out about it. I know that if we keep pushing through they won’t learn anything and it’s going to be terrible. (H688-H691)

Heath also made multiple references to students’ dispositions toward mathematics. For example, the Algebra class was using new books. Heath stated that his students “hate it. Hate. It”
He planned to ask them for feedback on what they did and did not like about the new text. He hypothesized that the students disliked the requisite group work and homework, to which they were unaccustomed, and that “They probably also don’t like that there’s a lot of explaining and a lot of sentence questions” (H664-H665). When I asked how he might respond to student opposition, he stated, “I’ll say that if they have to work through a problem they’ll have a better understanding of it” (H660-H661), an indication of mathematical power and the first Common Core State Standard for Mathematical Practice: Make sense of problems and persevere in solving them. To encourage students to complete their homework, he planned to appeal to their desire to learn by “stressing that it’s going to feel like you’re not learning anything if you don’t do the homework” (H674-H674). Here, I believe that Heath was relying on students’ desire to learn to encourage them to do their homework.

Heath completed his e-portfolio in the latter half of the spring semester. Two components of his e-portfolio, the Impact on Student Learning statement and the Philosophy of Education statement, included references to academic achievement and mathematical power. His Impact on Student Learning statement highlighted his experiences with two high school students while student teaching. Heath reported both students’ improvement in the form of rising grades. For example, one student’s “scores have gone from a F to a B+” (H440) while Heath taught the course.

Heath continued to discuss the same focus students, citing references to mathematical power. Of one of the students, he noted, “When I first met this student, I saw a lot of potential in him, but could tell that he was easily discouraged” (H409-H410). One could extrapolate from Heath’s statement that he recognized the lack of a productive disposition can impede one’s growth in mathematics, which is supported by research that suggests that belief in one’s ability to
do well (an element of productive disposition) is highly correlated with achievement in mathematics (Pietsch, Walker & Chapman, 2003).

Procedural fluency is tied to conceptual understanding. Heath appeared to realize this when he said,

During our after school tutoring, we discussed the different methods for solving equations, both graphically and algebraically. It was quickly realized that he had a hole [sic] in his prior knowledge when it came to working with absolute value and square roots. I spent time going over what these operations did and how they worked. (H411-H414)

Conceptual understanding also seemed to be important to him, as evidenced when he said,

On the day she came in for extra help, no other students were in the classroom and I took this opportunity to work one-on-one with her and figure out what misconceptions she possessed and get her all the help she needed while I could. (H425-H427)

His statements suggested that Heath was concerned with conceptual understanding and productive disposition, both of which are components of mathematical power.

*Figure 6.* Heath’s conceptual model in Spring 2012.
The diagram in figure 6 represents Heath’s trajectory on the conceptual model in Spring 2012, his fourth and final semester as a program scholar. Marker 10 in the overlap between mathematical power and critical consciousness represents Heath’s position on the conceptual map in the early part of his final semester (February 12, 2012). Later that same month (February 26, 2012), his response indicated a focus on the development of mathematical power (marker 11). Marker 12, in the overlap between academic achievement and mathematical power, is in the same portion of the conceptual map in which Heath began at the beginning of his sojourn as a program scholar.

**Heath’s Development**

Toward the end of Heath’s first academic year as a scholar, he was positioned outside of the conceptual model that I brought to this research. As previously mentioned, I explicitly asked Heath, *How does a teacher know his students are successful? What will it look like?* At other times throughout his tenure as a scholar, Heath spoke confidently about goals for his future students. For example, he planned to focus on the development of conceptual understanding because he viewed his own secondary mathematics education as highly procedural. At this point in the spring of 2011, however, he seemed less sure. He mentioned potential indicators, then immediately followed each by describing its limitations. To illustrate, as previously mentioned, Heath suggested grades as an indicator of success, followed by the statement, “I feel like sometimes it’s easy to get a good grade but not know anything” (H211-H212). He continued by noting the value of grades in that they can reveal trends in students’ learning. Further, when trying to identify specific indicators for a productive mathematics disposition as evidenced by a student’s desire to learn, he noted that doing well in class is not necessarily correlated with wanting to learn mathematics and viewing it as useful. So, while at other times he spoke
confidently about goals for student success, this time, after pausing, he said with a wry chuckle and hands thrown in the air, “So the answer is, I don’t know” (H224).

Gutiérrez (2013) asserts, “clarity and a stance on teaching that maintains solidarity with and commitment to one’s students” (p. 11) are necessary to be an effective teacher of marginalized students. As it relates to this research, that means clearly identifying the evidence a teacher will seek to signal success for his students. For example, if academic achievement is the goal, one might expect Heath to consider grades; standardized assessments; advancement to high level mathematics courses in secondary school; college admittance, persistence, and completion; time taken to complete degree programs; and the pursuit of mathematics-related careers (e.g., Arbona & Novy, 1990; Conley, 2005; Gutiérrez, 2007; Trent, 2008). Heath’s uncertainty represented what I believed was no more than a lack of clarity. Being neither here nor there, Nepantla was a departure from the trajectory of Heath’s development.

After completing Heath’s model and viewing the overall picture, my perspective changed. Not only was Heath’s position outside the model a part of his evolution, but it was also a pivotal point in his development—an unexpected finding of the present research. Here, I offer two related reasons drawn from the literature as well as from my own experience as a K-16 educator: (a) teaching is itself an inherently uncertain enterprise and (b) uncertainty is necessary for development.

A growing body of literature recognizes that teaching is an enterprise fraught with uncertainty (e.g., Helsing, 2007; Elmore, 2004; Floden & Clark, 1988; Lortie, 1975). Floden and Clark (1988) tease apart some of the different types of uncertainty involved with teaching: What do students know? What effect will particular instructional strategies have on student learning? What content should be taught? Adding to their list of questions, but more germane to this
research, I ask, “What evidence should teachers seek to demonstrate students’ development?” Adding to the uncertainty is the fact that such questions are not asked in isolation, but in combination with each other against an ever-changing backdrop. The same question may have a different answer at a different point in time or with a different student. Further, answering one question frequently raises another. These questions are particularly difficult for those new to the field of teaching, as they have little experiential knowledge upon which to draw (Floden & Clark, 1988; Wassermann, 1999).

Researchers further note that although ambiguity is a recognized aspect of the teaching profession (e.g., Lampert, 1985), teacher education programs have traditionally focused on the mechanics of what teachers should know and be able to do rather than how to face, embrace, and manage uncertainty (e.g., Floden & Clark, 1988; Helsing, 2007). For example, Wassermann (1999) describes her teacher education program as one that attempted to instill the “correct answers” (p. 466) in order to prepare her for the profession. Once she entered the classroom, she realized that her preparation did not properly equip her for “a profession in which there are few, if any, clear cut answers, a profession riddled with ambiguity and moral dilemmas that would make Solomon weep” (p. 466). In her study on teacher development, Britzman (2007) notes the lament of a novice teacher, who when asked about her teacher education program stated simply, “They didn’t prepare me for the uncertainty” (p. 8). Although she understood some of the mechanics of teaching, she was unprepared for the reality, for which there is rarely a formulaic answer.

In light of the uncertain nature of teaching, perhaps PSMTs need something in addition to best practices, instructional methods, and assessment techniques. Perhaps they can benefit from an environment in which they learn to grapple with questions that are not so easily answered.
(Grumet, 2014; Lampert, 1985). Such is the environment that we strived to create in the equity-oriented professional learning program (Gutiérrez, Irving, & Gerardo, in preparation). Heath’s pondering of the direct question of success indicators, ending with a smile and his hands thrown in the air, suggested that in that moment he became aware of the fact that potentially easy answers, such as focusing on grades, may not be the right answers.

I believe that Heath’s position outside of the model at the end of his second semester was a pivotal point in his development. Shortly thereafter, his summer began. His return in the fall marked a notable shift in focus. He appeared to embrace ideals that were broader in scope than those with which he was previously aligned. Gutiérrez (2012) draws upon the work of Gloria Anzaldúa when she asserts that such a position of uncertainty has the potential to contribute to PSMTs’ development. This space—Nepantla—is “the place that we birth new perspectives on reality” (Gutiérrez, 2012, p. 35). I posit that Heath was in a state of Nepantla and that his time of uncertainty did, in fact, have great value.

Anzaldúa and Keating (2009) describe Nepantla as a space where “[t]ransformations occur” (p. 243). As it relates to this research, such a transformation would be indicated by the contemplation of ideas heretofore unconsidered or to movement in emphasis from one criterion to another criterion depending on context and circumstances. Scholars note that the priorities that teachers hold are shaped, in part, by their own experiences (e.g., Dahlgren & Chiriac, 2009; Nesport, 1987; Rimm-Kaufman & Sawyer, 2004). Like many of his peers who planned to teach in schools serving marginalized students, Heath embraced the idea of academic achievement (e.g., de Freitas, 2008; Mirra & Morrell, 2011; Wilson, 2007). Previously, I stated that Heath reflectively considered his own secondary mathematics experiences as he thought about his future students, particularly noting that he felt underprepared relative to his peers when he came
to college. Given this experience, an early focus on conceptual understanding—mathematical power—as a means to academic achievement was not surprising.

While mathematical power remained a relatively consistent priority throughout the remainder of Heath’s two years, immediately following his positioning outside of the conceptual model, he began to make references to indicators that suggested consideration of critical consciousness as a goal. That is, after being in a place of uncertainty, Heath developed a new perspective, further supporting the idea that he was in *Nepantla*.

As stated earlier, I seek to help pre-service mathematics teachers consider a multidimensional perspective of success that includes the development of critical consciousness as its goal. This is not a linear model, going from a simple view to a complex view of success. Britzman (2007) notes that development is not about moving from unawareness to cognizance, asserting that development is a disjointed rather than linear process. Heath’s priorities continued to shift, eventually returning to a focus on academic achievement and mathematical power. One might view this as regression—that he ended in the same place that he began; however, I would argue that, although he considered similar goals, he was actually in a very different place developmentally. That is, although he cited references to academic achievement and mathematical power both at the beginning and end of his time as an undergraduate scholar, the quality of the references he made was different. To illustrate, in his intake interview (Fall 2010) Heath stated that a high needs school was one in which the state’s expectations were not met, while in the Impact on Student Learning Statement (Spring 2012) he sought to help prepare each student to enter and succeed in the field of her or his choice. Of note is that the earlier reference to academic achievement used a measure external to students, their school, and their community while the later reference centers students. Further, earlier the reference to mathematical power
was to productive disposition. One could extrapolate from his statements that, as some scholars posit, he thought that hard work alone is what leads to success, overlooking structural inequities in society (e.g., de Freitas, 2008). However, he later referred to mathematical power in terms of procedural fluency grounded in conceptual understanding. That is, his references to academic achievement and mathematical power were broader later than earlier.

Heath’s trajectory as a whole represents growth; that his views evolved in organic and varied ways, is an indication of development. It is not important that Heath’s conceptions of success remain static, because priorities shift in the classroom necessarily (Rimm-Kaufman & Sawyer, 2004). That is, it was not important for Heath to be placed in the center of the model and remain there because a teacher’s goals shift depending on the situation. Mid-spring, for example, is a time when many states administer high-stakes standardized assessments. The results of these assessments impact individual students as well as schools and districts. Consequently, a teacher may consider test preparation (academic achievement) to be an important focus in March and April. I believe that the ability to shift one’s goals is a strength rather than a weakness. If a teacher does not adjust goals as necessary, he or she is ignoring reality. Further, he or she must intentionally and strategically adjust, rather than simply respond to the latest external push. Heath’s evolutionary experience may helped prepare him for continued change and growth. Heath’s development, one might argue, was facilitated not by the introduction of an answer as to what evidence he should seek to identify student success, but in the posing of the question itself.
Chapter 5

Raphael’s Case

Raphael is a white man from the Washington, DC area. He attended school in Maryland until his family moved to South Africa during his sophomore year of high school. Raphael noted that living in the African nation was enlightening because he experienced being the racial minority for the first time in his life. Raphael’s family returned to the United States after a year in South Africa. He graduated from high school in a Midwestern college town.

When reflecting on his own secondary education, Raphael was critical of the experience. He said that his instruction was primarily procedural without much reference to the underlying concepts. For example, when discussing probability instruction, he said, “Combinations was always just multiply. Why? Because that’s what we do. I don’t like that kind of thing—formulas.” (R1398-R1401). Consequently, Raphael stated that he planned to help his future students develop conceptual understanding.

Raphael began college as a computer science major. He was unsatisfied because it was “not as problem solving as I expected” (R785) and viewed his fellow computer science people as “not people I would want to, like, hang out with” (R792-793). He enjoyed mathematics and physics, citing them as his favorite subjects, because, “Math makes sense. I think logically. I like being able to find the answer to something, and then done” (R2340-2341). He considered changing his major to engineering, but had similar concerns about the field. He finally selected the field of mathematics education because he loved the content and appreciated the social nature of teaching.
Raphael’s Trajectory

There were thirteen points during Raphael’s time as a program scholar at which I identified references to student success related to the conceptual model that I brought to this research. I present those thirteen points here.

Semester 1: Fall 2010

Raphael’s intake interview took place in October 2010. At that time he reflected on his experience tutoring in a local high school. He noted that he struggled with students—how to get them to learn and listen in class (R2342). When asked about the concerns he had about teaching in a high needs school, Raphael commented, “Can I motivate kids to learn?” (R2343). Raphael’s responses suggested a concern with students’ desire to learn mathematics—as aspect of a productive disposition. When asked why he was interested in teaching mathematics (as opposed to another content area), Raphael replied, “Math makes sense. I think logically” (R2340). I interpreted his statement that “math makes sense” as a recognition of the connections underlying mathematical concepts (conceptual understanding). Further, logic is a component of strategic mathematical thinking. Taken as a whole, I interpreted Raphael’s statements to imply an interest in mathematical power. Productive disposition, conceptual understanding, and strategic mathematical thinking are all components of mathematical power.
Figure 7. Raphael’s conceptual model in Fall 2010.

The diagram in figure 7 represents Raphael’s conceptual model in Fall 2010, his first semester as a program scholar. The star-shaped marker (marker 1) represents Raphael’s initial position on the conceptual model—mathematical power.

Semester 2: Spring 2011

Raphael completed the dimensions of equity How Would You Classify It? instrument in January 2011 (Gutiérrez, 2006). As previously mentioned, the first part of the sheet listed school-related scenarios. Each scholar was asked to classify the scenarios as positively or negatively related to Gutiérrez’ (2006) access, achievement, identity, and power framework. Despite being hypothetical situations, they provided some insight into scholars’ perspectives.

Raphael’s responses on the dimensions of equity instrument suggested concern with students’ sense of identity as doers and knowers of mathematics. To illustrate, given the scenario, Homeworks are of the form: Here’s a problem; now do 30 just like it, Raphael noted that this could have a negative impact on students’ identities because “Doing problems like this one make students not care about math, and become ‘not math people’” (R78-R79). If one defines “math people” as those who feel that they can and want to do mathematics, Raphael’s explanation can be categorized as productive disposition, a component of mathematical power.
Another example from the equity instrument read, *The school creates a “mathlete club” where students do creative and challenging math problems after school. The principal pays to have T-shirts printed for them.* Raphael indicated that this would have a positive impact on students’ identities. He wrote, “The T-shirts makes [sic] the achievement personal for them, allowing it to become a part of their identity” (R86-R87). When presented with the scenario in which students use geometry to address the school board about redistricting and busing (*Mr. Taylor’s students learn how to use geometry to solve a problem about the school’s redistricting policy that threatens to bus families to other schools. They present their “better” solution to the school board.*), Raphael also noted that this is a positive indicator for identity, writing, “Students are able to apply math to something that they can strongly identify with. Math becomes personal for them” (R90-R92). Here Raphael’s reference to identity was indicative of personal agency (a component of critical consciousness), as students use mathematics for a given purpose and have control over what that purpose might be, similar to how mathematics was used by students in Gutstein’s (2006a) study.

When presented with the scenario, *Instead of just high test scores, Principal Malik values and rewards teachers for supporting students to develop a positive disposition towards mathematics (thinking of themselves as “math people”)*, Raphael categorized it as a positive example of achievement, explaining, “People are better at math once they’ve convinced themselves that they are good at it. This will ultimately lead to higher achievement” (R70-R72). His response suggests that, rather than a goal in and of itself, identity seems to be a means to the more important end—achievement. In another example, *Students practice problems like the ones that will appear on standardized tests. Later that year, their school makes AYP under NCLB,* Raphael wrote that this was also an example of positive achievement, explaining, “Students
make yearly progress; a significant achievement” (R75-R76). Further, in a situation in which 30 percent of a math/science magnet high school’s student population goes on to get jobs in math-based fields (e.g., engineering, physics, mathematics), he indicated that this was an example of negative achievement, stating that 30% seemed like a low percentage of students given that the school is a math/science magnet school. Entering STEM-related fields is an indicator for academic achievement (R82-83).

Raphael’s first mentoring session took place on February 10, 2011. During that conversation, he referenced success indicators related to academic achievement, as evidenced when I asked him to envision his students as he would like to see them at the end of the academic year and describe what he saw. Raphael stated, “I mean part of it is just straight grades. So they’ve turned everything in and done all the work” (R36-R37). Grades and other traditional forms of assessment are indicators of academic achievement. Further, assessing student success by the amount of work completed and submitted, rather than the content of that work, may be considered an aspect of having students “play the game” (Gutiérrez, 2009b, p. 5) —navigating the current schooling system in order to progress through it. Several minutes later, Raphael confirmed this interpretation when he stated, “I think ultimately play the game, you know, you have to do that first” (R55-R56). Raphael’s emphasis is clearly on helping students negotiate the current schooling system; however, his statement that this must be done “first” could be interpreted as an allusion to helping students “change the game” (Gutiérrez, 2009b, p. 5), or make changes to the schooling system.

As he continued to discuss his image of a student who had successfully completed the school year, Raphael included descriptors related to multiple facets of mathematical power. For example, in a reference to conceptual understanding, Raphael said, “If they could prove at some
point that they’d *learned* the material, that [would be] good” (R41-R42). He continued, saying, “There’s teaching like math puzzles for instance, like one of my favorite things to do. … That was always a big thing for me. And being able to do things like that, that kind of test universal problem solving skills more than exercises or learning algorithms or doing things” (R49-R55). Here, he referenced strategic thinking and a productive disposition (both indicators of mathematical power) as he discussed the types of learning experiences he wanted to provide for his students.

At a subsequent mentoring session, (February 24, 2011) we discussed Raphael’s impressions of the sixth grade classroom in which he was placed. His observations included characteristics that could contribute to or compromise the development of students’ mathematical power. For example, when I asked Raphael to tell me about his students, he volunteered that he was impressed by their mental math skills, saying,

> I guess something that’s been really encouraging is how good these students [are] at mental math. I know mental math is not the focus of, you know, what we want necessarily kids to know, but I think it’s a fairly decent indicator of how they’re doing. Because they had problems where they had to multiply two-digit numbers and divide three-digit numbers by two-digit numbers and that sort of thing, and they weren’t allowed to use calculators. So they’re being forced to do it mentally, and they were significantly better at mental math than most of the college students that I tutor. Some of them are actually better than I am. (R113-R122)

Raphael’s comment clearly implies concern with procedural fluency. However, mental arithmetic of this nature (e.g., multiplying multi-digit numbers) requires conceptual understanding of the meaning of multiplication as well as base ten, place value concepts in order
to decompose and recompose numbers (e.g., Parrish, 2010). To illustrate, $72 \times 16$ can be thought of as $70 \text{ groups of } 10 \ [700] + 70 \text{ groups of } 6 \ [420]$, which sums to 1120; Add to this $2 \text{ groups of } 10 \ [20$, for a running total of 1140] and $2 \text{ groups of } 6 \ [12]$ for a final product of 1152.

Consequently, one can infer that Raphael is also commenting on the underlying concepts that students must master in order to engage in mental arithmetic at this level. Both procedural fluency and conceptual understanding are aspects of mathematical power.

Raphael further stated that he “wasn’t so thrilled” about the posters with mathematical formulas (e.g., area = length $\times$ width) that covered the classroom walls. According to Raphael “it seems like there’s a strong focus on being able to just use formulas and not so much the thought process on coming up with it” (R172-R178). While he acknowledged that the students might have developed the meaning of the formulas before he was placed in their classroom, Raphael’s concern that students may not understand how the formulas were derived indicates that he was considering the development of students’ procedural fluency through conceptual understanding.

He continued, saying, “I think kids would remember it a whole lot easier, kind of deriving it in a sense. So I think I would hope to do that sort of thing more so than formulas that I would either post up or make them memorize” (R228). That is, he believed that students fare better when they develop and understand mathematical formulas rather than having the formulas presented to them.

During Raphael’s third mentoring session (March 8, 2011), he described a card game that he used to introduce his sixth graders to probability. Raphael reported that his cooperating teacher suggested the use of an activity related to gambling, which is how he designed the game. The player draws one card from the deck. If the card is a heart, the player wins seven dollars. If the card is not a heart, the player wins nothing. The player must pay two dollars each time she
draws a card. I asked Raphael if he felt like using a game was a valid method for teaching mathematics. He replied in the affirmative, saying, “I think so. I mean it gets kids interested, which is good” (R362). Students’ interest in mathematical concepts is one component of a productive disposition; consequently, I interpreted Raphael’s assertion as a reference to mathematical power.

During the same conversation, Raphael described another time during which he was responsible for designing and implementing learning experiences about conversions between fractions and decimals for small groups of students. Raphael began his description by explaining that the sixth graders were “still kind of taught in a systematic way of doing things, which I’m generally against that” (R415-R416). A place value chart was posted on the wall; point was written about the decimal and labeled arrows pointed to the tenths, hundredths, and thousandths places. Raphael again expressed concern about rote memorization of formulas, facts, and structures without understanding. He stated,

It’s good that they don’t have to memorize things in some sense because that takes the focus away from memorizing things, but I think it enforces just using formulas and stuff. I think that they were basically taught to look at the number without the [decimal point] and then take away the decimal and then that’s your numerator. And then look back at it with the decimal and then count how many places are to the right of the decimal and then that’s how many zeros are on the bottom. (R422-R428)

In short, students were being taught a procedure got expressing decimals as fractions with little emphasis on meaning. In light of his concerns about students learning procedurally rather than conceptually, I asked Raphael how he taught his lesson. Raphael reported that he tried to use qualitative reasoning and benchmark numbers to help the students determine if their answers
made sense. He explained that one student said 0.35 was equivalent to \( \frac{35}{10} \). Raphael responded by asking the student if 0.35 was more than one or less than one. The student replied that is was less than one. Raphael continued by asking if \( \frac{35}{10} \) was more or less than one. The student responded that the fraction was more than one, but apparently failed to recognize the conflict between this statement and his previous statement. When I asked Raphael if this line of questioning was helpful for his students, he replied, “They seem to get fairly confused with that. So I eventually stopped kind of doing that because it wasn’t, it didn’t seem to be helping” (R451-R453). If, as Raphael described, the students were unaccustomed to employing qualitative reasoning and asking if their answers made sense, one might expect them to initially be confused by the approach (e.g., Van de Walle et al., 2012). While Raphael found the students’ lack of cognitive conflict troubling, he did not seem equipped to address it as a novice teacher.

In the same conversation, I explicitly asked Raphael, *How will you know when your students are successful? Like what will it look like?* He answered, “I guess the biggest indication for success would be students solving things that they haven’t seen before. I think that would be the ultimate goal, to give students things they haven’t seen before, that I haven’t taught them” (R525-R526). When I asked him to elaborate, he shared as an example a video he watched of a middle school class in which the teacher’s goal was for students to figure out how to determine the surface area of a cylinder. He said, “If you just look at the formula \( A = 2\pi r h + 2\pi r^2 \) straight up I think it’s fairly complicated” (R550). Raphael explained that the students were provided with the opportunity to see a cylinder, which they disassembled into a rectangle and two circles. Using what they knew about the area of a circle, the area of a rectangle, the diameter of a circle, and the relationship between the diameter of the circle and the length of the rectangle, the students were able to derive the formula “as a class you know, with pretty limited guidance”
Such an example of adaptive thinking and extending one’s cognitive schema by constructing new knowledge is an example of mathematical power (Kilpatrick, Swafford, & Findell, 2001; Van de Walle et al., 2012).

**Figure 8.** Raphael’s conceptual model in Spring 2011.

The diagram in figure 8 represents Raphael’s conceptual model in Spring 2011, his second semester as a program scholar. Marker 2 in the center of the model represents Raphael’s position on the conceptual model in January 2011. The following month (February 10, 2011), his responses indicated a focus on academic achievement and mathematical power (marker 3). In two subsequent mentoring sessions during the semester (February 24, 2011 and March 8, 2011), Raphael made references to the development of mathematical power (markers 4 and 5).

**Semester 3: Fall 2011**

In the fall of 2011, the scholars visited the school where Philip, our model teacher, worked. During the subsequent mentoring session (October 7, 2011), Raphael stated that he enjoyed the visit and was impressed by the level of mathematics with which the high school students were engaged. Raphael shared that he and his fellow scholars were encouraged to walk around the classroom and interact with the students while they worked on their tasks. He remembered, “We had their worksheets and looked at their work. It was hard stuff that they were
Advanced stuff. We didn’t even know how to do it right off the bat.” (R1215-R1216). His response indicated an appreciation for students’ advancement to higher-level mathematics in high school (academic achievement). Raphael also noted that in their mathematics class, several students expressed appreciation for the rigorous approach by which Philip taught, noting that they felt they were learning more in his class than in previous mathematics courses. Raphael noted that Philip’s students were “working and being successful and they appreciate it and are learning from it” (R1198). His observation highlighted his perception of the productive disposition toward learning mathematics held by the students in Philip’s classroom.

At the following mentoring session (October 22, 2011), Raphael shared a conversation that he had with his students. That semester, he was placed in a small rural town in which high school football was an important community activity. While planning a lesson on statistics, he thought that it would be advantageous to capitalize on students’ interest in the sport. He asked his students how they could convince someone that their school was better than their rival school. They discussed data about the number of football wins for each team over the past ten years. Raphael was pleased that his students were engaged in the discussion, saying, “They got really into it” (R1275). He was also pleasantly surprised with the content of the discussion, citing as an example,

One of the kids brought up that [our school] is more consistent by looking at the line plot, which was kind of cool. [The rival school] has gone to the championship a couple of times, but [our school] is more consistent. … I was pleased with that because I wasn’t expecting that to come up. (R1279)

Raphael’s description highlighted an interest in his students’ engagement with mathematics as they applied it to a relevant and authentic context (an indication of productive dispositions
toward learning mathematics) as well as an understanding of representations of data (an indication of conceptual understanding), which I interpreted as references to mathematical power.

I asked Raphael how he felt the students in his school were doing in general. He replied that it was difficult to tell because this was his first experience observing in a high school, but that he could “look up standardized test scores” (R1300) (an indicator of academic achievement) in order to make comparisons.

At the following session (October 28, 2011), I asked Raphael what types of things he was noticing at his placement school. He stated that he was experiencing “culture shock” (R1427) because the town and school were very small. He continued by observing that his students “don’t have access to higher level math classes like calculus because my coop is the most highly certified teacher there, and she is working on her masters to be able to teach calculus” (R1432-R1435). His comment indicated a concern that students would not have the opportunity to advance to higher-level mathematics courses in high school, an indicator of academic achievement.

Later in the semester (November 17, 2011), we discussed another lesson from his field placement. Raphael presented his students with a problem. He then broke them into groups of three to come up with their own solutions and later share their problem-solving approaches with the class. Raphael stated that the students were more enthusiastic about participating than usual. As an example, he described a girl who was typically afraid to speak in front of the whole class:

This time she was okay going up to the board and writing what she had even though she thought she didn’t have the correct answer. I was pleased about that. … She knew that she wasn’t the only one who didn’t know it, which I think helped. (R1499-R1503)
Raphael’s observation highlighted his interest in making sure that students were engaged and willing to participate, an indicator of mathematical power.

Raphael also shared that he was trying to help his students develop better mental math skills. After calculating the product of 34 and 8 without the aid of any tools, his students were amazed. Then he wrote his thought process out on the board \[ e.g., 34 \times 8 = (30 \times 8) + (4 \times 8) = 240 + 32 = 272 \] and the students were able to “see it’s not that hard.” (R1516). Here Raphael demonstrates interest in procedural fluency developed from conceptual understanding of the meaning of multiplication, the distributive property, and base ten place value (Parrish, 2010).

![Diagram of conceptual model]

**Figure 9.** Raphael’s conceptual model in Fall 2011.

Figure 9 illustrates Raphael’s conceptual model in Fall 2011, his third semester as a program scholar. Marker 6 in the intersection of academic achievement and mathematical power indicates Raphael’s position on the conceptual model on October 7, 2011. Later in the month (October 22, 2011) his responses indicated a continued focus on academic achievement and mathematical power (marker 7). During a mentoring session on October 28, 2011, Raphael referred to academic achievement as an indicator of success in mathematics (marker 8). The following month (November 17, 2011) his focus was on mathematical power (marker 9).
Semester 4: Spring 2012

Raphael’s student teaching took place at Joseph Keels High School during the spring of 2012. Raphael worked with his cooperating teacher, Mr. Caswell, to teach three sections of Algebra I and one section of Geometry. Raphael described Mr. Caswell, saying, “He’s pretty old school. He’s the high school football coach and is the stereotype of a high school football coach who’s also a math teacher” (R1680-R1681). He continued,

I don’t want to say bad things about my coop, but he seems kind of jaded and sort of like he’s been teaching for a while and he’s set in his ways. And if students aren’t learning, it’s not his fault is the general philosophy that I pick up on. He doesn’t seem like he puts too much effort into going above and beyond to help kids out. (R1791-R1795)

Raphael was concerned about engaging in practices that were aligned with his own philosophical stance while maintaining a positive relationship with his cooperating teacher. He said,

I don’t want to come off as this young idealist who’s coming into his class. He makes fun of other teachers saying they’re out to save the world and save every kid. I want to keep a good relationship with my coop and I’m worried about my actions saying that I disagree with him. (R1795-R1799)

Further, the classroom environment, according to Raphael, was “kind of depressing. There’s not a whole lot on the walls. The walls are painted psych ward yellow. It’s very sparse.” (R1718).

In a mentoring session on February 15, 2012, Raphael described how he attempted to combat what he perceived as a negative environment by engaging his students, but found it difficult to do so. He stated, “I keep telling my kids we’re going to have lots of fun today. I’ve been teaching really boring topics, like \((x^2)(x^3)\), add the exponents. How do I make this
interesting?” (R1726-R1729). His statement suggested a concern for students’ dispositions
towards mathematics, an indicator of mathematical power.

The following month (March 14, 2012), Raphael discussed the continued tension he felt
in trying to balance his preferred style of teaching, which was still in development, and his
cooperating teacher’s norms in the classroom. Further, Raphael believed that the students had
become acclimated to a particular type of instruction that would be difficult to counteract. He
explained,

It’s kind of frustrating because things are done so systematically and formulaically in that
class and kids don’t often know or care where things come from. This is what I was
talking about doing things differently. But I didn’t want to challenge my coop in front of
the class. (R1959-R1963)

He continued, using the rules of exponents as an example. His students were taught that with an
expression such as \((x^3)(x^4)\), one simply adds the exponents. Raphael recalled forgetting the
rules in his own mathematics courses; however, he was able to derive the rule by writing out
\((xxx)(xxxx) = xxxxxx = x^7\). He wanted his students to be able to do the same.

Raphael also expressed frustration with the fact that his students would not or could not
solve word problems. He explained,

There were two word problems that a lot of kids didn’t even attempt, which annoyed me.
I think it was Dan Meyer’s TED Talk [Math Class Needs a Makeover] that that’s one of
the signs that you aren’t teaching your kids to think—if they are reluctant to [solve
problems]. (R2203-R2205)

I interpreted Raphael’s frustration with a skills approach to instruction as well as a desire to
engage students in problem-solving as a concern with the development of mathematical power.
Raphael completed his e-portfolio in the latter half of the spring semester. His Impact on Student Learning statement highlighted his experiences with two high school students while student teaching. His statement included references to academic achievement and mathematical power. For example, Raphael stated that one of the students, Mario, was “not being successful in school throughout the 3rd quarter (he finished with a 32.42%)” (R2078-R2079). Here Raphael appeared to use grades as a proxy for success, an indicator for academic achievement. However, as he continued to discuss Mario, Raphael wrote that he was discouraged because Mario “had such large gaps in prior knowledge” (R2088). I interpreted Raphael’s statement as recognition of the interconnectedness of mathematics and the need for students to develop cognitive schemas by building on what they already know, an indication of the development of mathematical power. Further, Raphael reported that during the fourth quarter, Mario began “working hard in math class” (R2099) and appeared motivated to improve, a reference to a productive disposition (mathematical power). Raphael ended his statement with a report that Mario earned scores of 42/44 (95%) and 44/46 (96%) on two quizzes during the fourth quarter. So although Raphael appeared to recognize the importance of conceptual understanding, he reported Mario’s success in terms of traditional scores without reference to actual content.

Raphael’s Philosophy of Education statement—another component of his e-portfolio—included references to mathematical power and critical consciousness. For instance, he wrote,

Too many young people have been told, or simply accepted the lie that they are simply not good at math and are powerless to change that. This is not only a false belief, but it is also a dangerous one. (R2165-R2167)

I interpreted this statement as a reference to both disposition and personal agency. Raphael continued, “Accepting that one is bad at math is also accepting that one does not have the logical
thinking abilities and the problem-solving skills necessary to thrive in society” (R2168-R2170). I believe that this reference to strategic thinking alluded to mathematical power. Raphael asserted that he sought to engage in equitable teaching practice, which meant, in part, that his students “are able to bring their own identity into my classroom and then relate and formulate the mathematics that they learn into a fuller identity that they can take with them beyond my class” (R2173-R2175). I interpreted this reference to identity and the suggestion that students will take what they learn outside the four walls of the classroom as references to critical consciousness.

Figure 10. Raphael’s conceptual model in Spring 2012.

The diagram in figure 10 illustrates Raphael’s conceptual model in Spring 2012, his fourth semester as a program scholar. Marker 10 in mathematical power represents Raphael’s position on the conceptual map in the early part of his final semester (February 15, 2012). The following month (March 14, 2012), his responses indicated a continued focus on the development of mathematical power (marker 11). Marker 12 in the overlap between academic achievement and mathematical power and marker 13 in the overlap between mathematical power and critical consciousness indicate Raphael’s position within the model toward the end of his tenure as a program scholar.
**Raphael’s Development**

My interest in conceptions of success is rooted, in part, in the recognition that a teacher’s conceptions of success impact decisions in his or her practice (e.g., Aguirre & Speer, 2000; Cohen, 1990; Speer, 2008). However, as Raphael’s case highlights, espousing particular goals for student success does not necessarily lead to practice aligned with those goals. Here, I discuss dissonance between Raphael’s expressed goals for students and his instructional decisions.

Raphael demonstrated a tension between his goals and his actions in the classroom. For example, recall the episode in which he was trying to help a group of sixth graders who were procedurally converting fractions to decimals without any understanding of why the procedure worked. Raphael attempted to teach the topic conceptually, trying to help students see why, for example, the equation “\(0.35 = \frac{35}{10}\)” does not make sense. Recall Raphael’s statement regarding the lesson: “They seem to get fairly confused with that. So I eventually stopped kind of doing that because it wasn’t, it didn’t seem to be helping” (R451-R453). As stated before, Raphael seemed to find the incident troubling; however, he allowed students to continue with the procedural approach. I draw on Schoenfeld’s (2011a) Resources, Orientations, and Goals (ROG) framework to explore this issue further. I do not discuss Schoenfeld’s (2011a) ROG framework as a new theoretical framework; rather it is a useful tool in explaining Raphael’s findings.

Schoenfeld (2011a) theorizes that human behavior and decision making “in ‘well-practiced’ domains—teaching in particular—can be understood as a function of their knowledge and resources, goals, and beliefs and orientations” (Schoenfeld, 2011b, p. 457). One’s decisions are based on complementary resources, orientations, and goals; actions follow suit. Schoenfeld (2011b) continues by describing “well-practiced domains” as “those areas of practice in which individuals have had enough time to develop a corpus of knowledge and routines that shape
much of what they do” (p. 457). In Raphael’s case, (a) he was a pre-service teacher as opposed to a novice or experienced teacher, so he had not yet had time “to develop a corpus of knowledge and routines” in the classroom. That is, his resources were limited. And (b) he seemed to have a relatively developed orientation towards teaching and learning mathematics; however, he had multiple goals, some of which conflicted with each other and with his orientation towards teaching mathematics, leading to instructional decisions that were not aligned with his orientation.

Schoenfeld (2011a) defines a person’s resources in terms of knowledge: “the information that he or she has potentially available to bring to bear in order to solve problems, achieve goals, or perform other such tasks” (p. 25). The art of effective teaching requires an enormous body of knowledge related to diverse learners, pedagogy, content, pedagogical content, communities, and politics from which one can draw. It also requires staying abreast of current research and theory in the field of education. The National Council of Teachers of Mathematics (2014) draws on research to suggest eight teaching practices aligned with its principles for school mathematics and designed to facilitate meaningful learning. The list includes practices aligned with the development of mathematical power, such as “Build procedural fluency from conceptual understanding” and “Use and connect mathematical representations” (NCTM, 2014, p. 10). Drawing on one of the practices, in particular, may have been helpful for Raphael and his students in the example cited above: “Support productive struggle in learning mathematics” (NCTM, 2014, p. 10).

Productive struggle entails an “effort to make sense of mathematics, to figure something out that is not immediately apparent” (Hiebert & Grouws, 2007, p. 287), a process that allows students to develop deeper understandings of mathematics than when the teacher jumps in at the
first sign of trouble to help students by explaining how to solve a problem. Hiebert & Wearne (2003) assert, “all students need to struggle with challenging problems if they are to learn mathematics deeply” (p. 6). An important aspect of the process is the teacher’s actions in support of students to make sure that they are learning and do not reach an unproductive point of frustration. Raphael’s students appeared confused by the sense-making approach to understanding why 0.35 was not equivalent to \( \frac{35}{10} \). According to Raphael, his students were unaccustomed to employing qualitative reasoning and considering whether or not their answers make sense. Consequently, one might expect them to be confused initially by the approach (Reinhart, 2000). Rather than abandoning sense-making, as Raphael reported doing, he could have allowed his students the opportunity to engage in productive struggle. Such a conversation could have been supported with tools such as decimal squares and fraction manipulatives, making the relationship between the two forms visible (NCTM, 2014).

Raphael also might have selected a high-level task that required sense-making rather than simple computation, incorporating multiple representations and requiring significant cognitive effort (Smith & Stein, 1998). Further, he might have asked probing questions that unveiled students’ thought processes rather than focusing on the task from his own perspective (NCTM, 2014). However, from the data gathered, Raphael was not familiar enough with these strategies to apply them in context. Instead, he fell back to the procedural approach, leaving his students in much the same place that they were in before the lesson.

Although Raphael’s resources were limited, he appeared to have a clearly developed orientation towards teaching and learning mathematics; the majority of his references to student success in mathematics related to the development of mathematical power. Further, those references were multidimensional. That is, rather than repeatedly mentioning the same aspect of
mathematical power, over time Raphael referred to all four components: conceptual understanding, procedural fluency, strategic thinking, and productive disposition. As previously mentioned, conceptual understanding is a deep sense of mathematics as a web of interconnected ideas, and procedural fluency entails skills rooted in conceptual understanding. Strategic thinking involves the ability to approach and solve varied types of problems, and productive disposition includes the desire to solve said problems (Baroody, 1998; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Van de Walle et al., 2012).

Given a view of success that incorporates these parameters, one can infer that Raphael did not view mathematics as a set of isolated skills, basic facts, and fixed procedures unrelated to real life. A teacher who subscribed to that perspective could be expected to view learning as what Freire (1970) called “the banking concept of education”—a process of memorization with the teacher’s role being to teach for skill mastery by “stuffing stuff into the stuffee” (A. Baroody, personal communication, August 27, 2007). That is, students would be expected to internalize prescribed content and procedures as presented by the teacher (Freire, 1970). Raphael’s statements related to mathematical power suggest that he views mathematics as a dynamic field—a network of concepts as well as a way of thinking. For example, recall that when describing his vision of a student at the end of the school year, Raphael stated that he would like for his students “to do things like that, that kind of test universal problem solving skills” (R52-R53). He also stated, “the biggest indication for success would be students solving things they haven’t seen before, that I haven’t taught them” (R525-R526). A teacher who viewed mathematics this way could be expected to encourage understanding the conceptual basis for facts, procedures, rules, and formulas as well as developing problem-solving strategies and reasoning ability (NCTM, 2000). From this perspective, the teacher’s role would be to encourage
student participation in constructing understandings and developing procedures. The focus of instruction would be student-centered mathematical inquiry as an end as well as a means to developing conceptual understanding of content (Reys, Lindquist, Lambdun, & Smith, 2012).

Although Raphael appeared to value student-centered instruction, he had multiple goals as a student teacher, some of which seemed at odds with each other. To wit: he wanted to employ reform-oriented mathematics practices to encourage his students to enjoy solving problems in order to learn mathematics conceptually; this goal was in tension with his desire to maintain positive relationships with cooperating teachers who enacted traditional practices. While still in the teacher education program and not in charge of his own classroom, Raphael was faced with the decision either to employ mathematics instructional practices in which he believed with the possibility of alienating his supervising cooperating teacher, or to ensure alignment with his mentor teacher’s philosophical approach and continue the traditional sit-and-get approach to instruction. In the words of Lampert (1985), both presented as “equally undesirable alternatives” (p. 179), and, from Raphael’s perspective, they were mutually exclusive.

Raphael’s case highlights the power differential between student teachers and their cooperating teachers, an issue that has been well documented (e.g., Anderson, 2007). The pedagogical approaches taught in teacher education programs do not always match the approaches implemented by cooperating teachers. Because student teachers are dependent on the developmental reports of their cooperating teachers in order to complete their programs, they may infer (implicitly or explicitly, legitimately or not) that they must teach in the same manner as the cooperating teacher. The result can be a lack of personal and professional agency for the student teacher, making it difficult for him or her to incorporate and begin to develop proficiency in a different style of teaching than that of the cooperating teacher (Beck & Kosnick, 2002). For
example, Barrows (1979) found that, in an effort to maintain a positive relationship with cooperating teachers, student teachers were more likely “to imitate, not experiment, to conform and not challenge, and to accept and not question” (p. 25). Such was the case for Raphael. Although he never reported being explicitly told by his cooperating teachers that he must adopt the same teaching methods and styles, he was also never reported being encouraged to try to develop his own professional styles. Although Raphael’s orientation and goals seemed to pull him in opposing directions, his lack of resources left him better equipped to follow the road that his mentor teachers would approve and the goal that was more likely to be accomplished given the constraints and deeply engrained norms of the classrooms in which he was observing and participating.

Research suggests that supporting educators to become more aware of their ROG framework may have a positive impact on their decision-making (Thomas & Yoon, 2014). Schoenfeld’s theory suggests that the primary influence on Raphael’s decision was the constraint of available resources, suggesting that a teacher (even a pre-service teacher) with similar goals and orientations but more resources and experience may have chosen a different route (Thomas & Yoon, 2014). Teacher educators may be able to help teachers negotiate such tensions by supporting them to become explicitly aware of their own ROG framework and the tensions therein (Schoenfeld, 2011a; Thomas & Yoon, 2014).

Viewed in isolation, Raphael’s conceptual model might seem to represent little growth. The majority of his trajectory represents a focus on mathematical power with little digression away from that framework, even when considering other ideas related to academic achievement and critical consciousness. However, one must consider the context. Recall that Raphael spent considerable amounts of time in placements with teachers whose orientations toward learning
and teaching mathematics conflicted with his. The mentor teachers favored direct, skills-based instruction while Raphael wanted to engage his students in problem solving to develop deeper understandings of mathematics. Although Raphael’s decisions in the classroom sometimes reflected the style of his cooperating teachers rather than his own, he never rejected his own philosophical stance. One might argue that maintaining his own ideals in an environment that rejected them may have been evidence that Raphael was positioning himself to accomplish one of the goals of the larger project of which this research was a part: to reclaim teaching as a profession and teachers as professionals (Gutiérrez et al., 2013).
Chapter 6
Conclusions and Implications

The equity-based teacher professional development model reflects, in part, what we believe we know about engaging PSMTs in experiences that will have an effect on their perspectives related to teaching and learning mathematics. Originally, I thought that I would focus on the correlation between PSMTs conceptions of success and their learning experiences in the program. I soon realized that identifying a simple correlation between activities in the program and evidence of success indicators was not meaningful, as participants’ identities as well as the contexts in which they are placed mediate their conceptions of success for students. Any professional development model will interact with participants’ individual identities and personal histories.

Both Heath’s and Raphael’s conceptions of success evolved as they participated in the equity-oriented professional learning program. Were their evolutions transformative? Schoenfeld (2011) uses the term orientations to describe the lenses through which we view the world around us. As noted, educators who have a “mastery identity” (de Freitas, 2008, p. 44) view their own accomplishments in school mathematics as a matter of merit rather than the result of various factors—many of which have nothing to do with their own work ethic. Both Heath and Raphael exhibited some traits of mastery identity. For example, Heath described himself as curious and motivated and attributed his success in high school to having taught himself the whys that he felt he was not learning in the classroom. At the beginning of his time in the program, one of Raphael’s expressed concerns about teaching in a high needs school was related to motivating students to learn, implying that this was all that was necessary for them to succeed. Over the course of their two years in the program, both participants exhibited some recognition of the
inequities in the schools in which they were placed. For example, Heath observed that almost all of the students in his Accelerated Geometry class were white while the Algebra Relearn class was dominated by black and Latin@ students. Raphael expressed concern about the fact that one of the classrooms in which he was placed lacked technology and that the teacher seemed unqualified to teach the course. Both of these statements were made when I asked them each to tell me about their classes. I did not specifically ask about what concerned them or what inequities they observed. I found encouraging the fact that both brought forth the issues on their own.

Recall that transformative learning experiences are those that address “mastery identity” (de Freitas, 2008, p. 44); serve to diminish deficit thinking, (Haycock, 2001; Johnson & Fargo, 2010; Kose & Lim, 2011; Valencia, 1997; Valenzuela, 1999); encourage teachers to recognize the political nature of schooling (Ahlquist, 2001; Gutstein, 2000; Weissglass, 2000); and lead to teaching for equity, diversity, and social justice (Johnson & Fargo, 2010; Kose & Lim, 2011; Zeichner, 1993). I believe that transformative experiences will encourage PSMTs to consider varied conceptions of success. However, conceptions may be situated rather than absolute or developmental (e.g., Yeh & Barasalou, 2006). Further analysis suggests that Heath’s and Raphael’s conceptions of success (the indicators that they discussed) may have been not only developmental, but situational. Because the purpose of our model was to prepare PSMTs to teach in schools that served students who had been traditionally marginalized in society, many of the experiences that we developed were designed to encourage the scholars to recognize the inequities inherent in our current schooling systems. For example, several times during a semester scholars read articles and watched videos related to issues of diversity, schooling, and mathematics in order to challenge previously held beliefs and deepen their awareness of the
complexities of teaching. Consequently, they may have been more likely to refer to critical consciousness when discussing seminar conversations, readings, and other components of the program. To illustrate, Heath referred to critical consciousness while discussing the after school Math Club, expressing an appreciation for students’ voices being heard. Recall that the club provided scholars with the opportunity to interact with real students and rehearse teaching moves. As with Mirra and Morrell’s (2011) study, Heath’s observation of students outside of the traditional classroom may have helped broaden his conception of the role of teachers; his appreciation for students’ voices being heard regarding how Math Club was run was an example of a dialogic rather than hierarchical relationship. Similarly, Heath expressed an appreciation for student-led discussion in our model teacher’s classroom. Raphael’s references to critical consciousness were also couched in programmatic components. He expressed a desire for students to bring their own identities into the classroom in his Philosophy of Education; however, this desire was not evidenced in his other statements during our mentoring sessions. Raphael’s other references to critical consciousness were embedded in his responses on the Dimensions of Equity instrument.

When considering the program model components, the participants were more likely to refer to issues of critical consciousness. However, providing further evidence for the conjecture that their conceptions of success may have been situational rather than developmental, both participants frequently made references to mathematical power as it related to their students in the field. For example, Heath shared that he encouraged his students to complete their homework by reminding them that it would help them understand the content better. Raphael criticized his cooperating teacher’s style of teaching because it involved direct instruction with rote memorization rather than the development of conceptual understanding. Further, both
participants made references to mathematical power in their Impact on Student Learning statements. All PSMTs in the teacher education program are asked to draft these statements. Heath and Raphael knew that the state’s educator licensing board would view these statements and that the board required concrete evidence of student learning, typically in the form of test scores and grades. Consequently, Heath and Raphael were obligated to make references to academic achievement in their statements; they were not obligated to make references to mathematical power, but did so anyway. The focus on mathematical power as it related to their actual students may have been a result of both participants’ reflections on their own secondary schooling experiences. As white men from middle-class backgrounds, neither participant sighted memories of being marginalized, so issues of critical consciousness during their own adolescences may not have been personally meaningful for them. Both, however, were critical of the procedural nature in which they were taught mathematics and stated that they wanted something different for their own students. Consequently, although critical consciousness was a common theme in the context of program-related structures, mathematical power became the ideal as it related to actual students. That is, the scholars’ personal identities and histories, rather than the program model, appeared to have the greatest impact in the schooling context.

While I cannot definitively state that a particular component of the program model caused changes in participants’ conceptions of success, the participants themselves shared what they felt to be the most powerful learning experiences. Pivotal for Heath were the sense of community and structured support. A professional community can also serve as a foundation on which to develop a shared knowledge base, influenced by participation in the community (Bartell, 2006; Garmon, 2004; Lave & Wenger, 1991; Stein, Silver, & Smith, 1998). Heath’s statement was aligned with Garmon’s (2004) assertion that pre-service teachers need appropriate
support group experiences to promote growth and “provide feelings of acceptance, caring, safety, and support” (p. 209) as they grapple with previously unexplored beliefs. One-on-one mentoring also served to provide such a safe space for Heath to challenge and be challenged without judgment (Kram & Isabella, 1985), encouraging reflection and alternative ways of thinking in a supportive environment (Mott, 2002). As expressed by Raphael, his most pivotal experiences in the program involved interacting with students in our model teacher’s school and in the math club. He noted that these experiences provided existence proof for the level of work of which students were actually capable, raising his expectations for student work. He also cited the way that adults in both environments communicated with students, appreciating what Mirra and Morrell (2011) observed as dialogic rather than hierarchical relationships.

**Limitations**

The research questions addressed in this paper are a small part of a much larger project (Gutiérrez et al., 2013). The idea of how PSMTs view success was embedded in the overall framework of the program model, but it was not the primary focus of the program. Further, I did not develop an instrument with which to measure or identify participants’ conceptions of success. Participants were also unaware of the focus of my own branch of the research project, so their responses frequently digressed into other areas. As a result, I was able to identify only twelve points for Heath and thirteen points for Raphael at which I felt I could discuss their conceptions of success. Further, I only directly asked participants about their conceptions of success one time because I did not want them to tell me what they thought I wanted to hear. In addition, my own research questions developed while working with the participants, so much of the data that were collected during Heath and Raphael’s first few months in the program were not useful for the questions herein. Also, because I was not present with Heath and Raphael in
their school sites, I relied on their self-reporting to learn about the school environments in which they were placed.

In order to be confident about my interpretations, I followed Stake’s (2010) recommendation to “look again and again, several times” (p. 123). That is, I made several passes of the data in both of the present cases. However, member checking (having Heath and Raphael each read their individual cases to determine if I have accurately represented their thoughts and feelings as they relate to conceptions of success for students) may have created a greater degree of confidence (Stake, 2010). This study is further limited in that I served as a participant observer (Bogdan & Biklen, 2003). Because I worked directly with participants over the course of two years, I developed collegial relationships with them, perhaps coloring my perspectives of their statements.

**Implications**

This study explores the conceptions of success held by two PSMTs in an equity-oriented professional development program. Both participants entered the equity-oriented professional learning program with the intention of preparing to teach creative and rigorous mathematics in schools with marginalized students, an enterprise that I believe necessitates embracing multiple perspectives of success. As they participated in the program, their trajectories differed. The differences were not surprising; they were two individuals who brought different experiences to the program and were placed in different environments for field placements. Our model must necessarily intersect with other aspects of PSMTs’ professional preparation. What is notable are the *types* of issues that arose in each case. Although I do not seek to claim generalizability, as Stake (1995) recommends, I hope to help the reader create a “slightly new group of cases from which to generalize” (p. 85) while considering issues in teacher education. That is, I believe that
the participants’ stories raise questions about the kinds of issues stakeholders may wish to consider relative to teacher preparation. Here I discuss the potential implications of this research.

**Implications for Teacher Education Practice**

Teacher education programs seek to help pre-service teachers develop the dispositions necessary to become effective teachers for all students. The development of said dispositions is sometimes viewed as a linear process—something that can be measured and evaluated as a static entity at a particular moment in time. PSMTs’ conceptions of success are also a part of the dispositions necessary to have a positive impact on student learning; however, this study suggests that we should consider not just the static picture, but also the dynamic process through which PSMTs evolve and the situational nature of the evolution. That is, it is important to recognize that PSMTs’ goals for students may change depending on the context in which they are placed and the audience with whom they are interacting.

Preparing teachers is incredibly important work. There are myriad theories about how best to prepare secondary mathematics teachers for the classroom. Stakeholders at the national, state, and local level seek to implement policies designed to close the achievement gap, provide equal access to economic opportunities, and develop conceptual understanding in mathematics. Given the milieu, how do we best prepare teachers to reclaim the profession as they redefine their roles in schools, classrooms, and students’ lives? While I cannot generalize for all PSMTs, the recommendations I make here for teacher education are based on the experiences of Heath and Raphael.

Because identity has a powerful influence on how we view and interact with the world around us, it must be considered in the development of PSMTs’ learning experiences. Teacher educators should help PSMTs explicitly articulate the goals that they have for students and why
they hold those goals. What are their influences? I believe that Weinstein (1989) said it quite eloquently:

> It is almost cliché that teachers must understand their students and adapt instruction to students’ needs. Yet, as teacher educators, we rarely practice what we preach. We have paid scant attention to our own students’ past experiences or to their implicit theories of teaching and of learning to teach. (p. 53)

Encouraging PSMTs to explicitly state their goals for student learning can have the dual benefit of bringing biases and misconceptions to the foreground in order to be addressed and highlighting the diversity of thought and opinion that can exist within a professional community.

Raphael’s case highlights the fact that PSMTs may suppress their own goals for student learning when they compete with more pressing goals. Recall, for example, that Raphael was faced with the decision either to employ mathematics instructional practices that were aligned with his beliefs but could alienate him from his supervising cooperating teacher, or to ensure alignment with his mentor teacher’s philosophical approach and continue the traditional sit-and-get approach to instruction that he had criticized. PSMTs are particularly vulnerable because they do not yet have the cache of resources that more experienced teachers have. Teacher educators can help PSMTs be more aware of their own Resources, Orientations, and Goals frameworks (Schoenfeld, 2010) and how the combination influences their decision-making. Increased self-awareness may help PSMTs develop more solid footing as they negotiate their professional relationships with state level evaluators, cooperating teachers, peers in teacher education courses, and instructors in the teacher education program, among others (e.g, Thomas & Yoon, 2014). For example, in his case study of a novice mathematics teacher, Gregg (1995) explored the challenges she faced when she entered a mathematics department with a particular set of beliefs.
and contradictory practices around teaching and learning mathematics. Her acculturation into the department included “coercion” (Gregg, 1995, p. 448) from other faculty members to adopt the same beliefs and employ the same practices. With more awareness, PSMTs can consciously decide which aspects, if any, of their own frameworks they are willing to subordinate, weighing the benefits and costs and examining the outcomes. They can also begin to develop strategies to ensure the alignment of their practice with their beliefs. For example, when my current PSMTs plan a lesson, I have them add a column in the procedures section of their lesson plan titled “Purpose.” They explicitly connect each task or action to a particular learning theory, specific knowledge of their students, or their own beliefs about teaching and learning. My goal is for them to be mindful of their motivations for how they are planning learning experiences for their students. Further, such self-examination can help uncover the implicit reasons behind particular orientations. For example, a PSMT may not be aware that he holds different expectations for students based on gender or race until asked to explicitly address the question of goals for one’s students. Teacher educators can also seek to establish relationships with teachers whose values are aligned with those of the larger program. Because cooperating teachers have a certain amount of power over their student teachers and, being in the K-12 classrooms, have more credibility about the day-to-day realities of classroom teaching in the current milieu, they can have an enormous impact on the professional development of student teachers. Placing student teachers with mentor teachers who hold antiquated beliefs about mathematics, teaching, and learning is counterproductive.

Heath’s case underscores the suggestion that, in light of the uncertain nature of teaching (e.g., Helsing, 2007; Elmore, 2004; Floden & Clark, 1988; Lortie, 1975), PSMTs may benefit from an environment in which they must grapple with questions that cannot be easily answered.
Grumet, 2014). Pre-service teachers are traditionally taught the mechanics of the profession (e.g., writing lesson plans and assessing student learning relative to measureable objectives), but, as aforementioned, schooling is a complex enterprise that requires more than knowledge of how to prepare and present a lesson. By failing to directly address uncertainty in teacher education programs, we are implicitly telling PSMTs that there are right and wrong answers for every question. Doing so leaves them completely unprepared for the realities of the profession, setting them up for failure (Britzman, 2007). In addition to teaching the mechanics of the profession, teacher educators can continually bring in questions that are not easily answered so that PSMTs are forced to face and address these questions in the larger context of preparing for the profession. Tasks such as reading case studies or (as in our model) In My Shoes discussions against the backdrop of learning the mechanics of teaching can be helpful in this respect. As occurred with Heath, PSMTs can come to view addressing complex situations as part and parcel of the profession and can avoid some of the feelings of helplessness that some novice teachers report (e.g., Britzman, 2007).

The concerns for PSMT development highlighted in both Heath’s and Raphael’s cases can also addressed, in part, by interacting with educators who recognize the importance of a clearly articulated ROG framework and the difficulties of maintaining alignment between ideal goals and everyday practice in an environment fraught with uncertainty. As previously mentioned, teaching is “a profession riddled with ambiguity and moral dilemmas” (Wassermann, 1999, p. 466). Recent graduates of the teacher education program, who are both familiar with the philosophy of the program and the realities of teaching in schools that may have different climates, can serve as mentors and coaches, sharing the benefit of their experiences with PSMTs.
For example, recent graduates could participate by sharing In My Shoes moments, as Philip, the program’s model teacher, did with our scholars.

**Implications for Research**

Further research can help address some questions raised in this dissertation. Heath and Raphael are both currently employed as secondary mathematics teachers. Members of the research team have been in touch with Heath and Raphael since they graduated from our university. Preliminary findings suggest that each man has engaged in practices that go beyond the mechanics of teaching. Heath has engaged his students in social justice mathematics, although it is not clear to me to what degree. Raphael participates actively on a social justice committee within his district; I do not know how this has impacted his instruction. The field could benefit from a longitudinal study of Heath’s and Raphael’s experiences. How (if at all) did their conceptions of success continue to evolve once they completed our program and entered their own classrooms? How have their beliefs impacted their practice? What tensions have they experienced between their own views of student success and the ways to support it and the views of those in their current working environments? How are they negotiating those tensions? Upon which professional learning experiences do they still draw or reflect in their day-to-day practice? What advice would they give to teacher educators seeking to develop transformative learning experiences? Why? To address these questions, I would interview each participant again, this time with a specific focus of success in mathematics. In addition to discussing the questions above, we would explore their teaching practices along with the resources they use and how they believe students are or are not benefiting. Further, I would ask them to describe their current school, department, and colleagues. I would be interested to know if they have sought out an informal mentor. If yes, how did they select the individual? For example, is his or her philosophy
of schooling a better alignment than others in the department? Further exploration of Heath’s and Raphael’s cases could shed light on the long term impact of their experiences in the program.

I also have questions and concerns that go beyond these cases and apply to a typical teacher education program. Our program was comprised of a group of participants who, although they received the incentive of scholarship funds, were in the program voluntarily. Further, all of our scholars planned to teach in schools that served schools with marginalized students. For those PSMTs who are part of a regular teacher education program:

- How might PSMTs who do not intend to teach marginalized students respond to the idea of multiple indicators for success, particularly critical consciousness? How might their conceptions of success evolve? How might their trajectories differ from those of the participants in the present study?

- Considering those who do and those who do not go on to teach marginalized students once they complete the program, how are the ideals of academic achievement, mathematical power, and critical consciousness enacted in their practice (if at all)?

A case study approach would be ideal to address these questions because it allows for the exploration of phenomena related to individuals that may otherwise be overshadowed by the group. A research team would be preferable to a single scholar. As with teaching, researchers’ personal identities and histories provide the lenses through which we view data; having different perspectives while analyzing data would contribute to the validity of interpretations. As previously mentioned, the research questions addressed in this paper are a small part of a much larger project (Gutiérrez et al., 2013). The idea of how PSMTs view success was embedded in the overall framework of the program model, but it was not the primary focus of the program.
Further research should center these questions, allowing for a more focused examination and robust understanding of the participants’ conceptions related to student success.

Researchers who want to pursue these questions may wish to develop an instrument and establish a baseline for participants. An instrument designed for this purpose might include several school-based scenarios, each of which presents a dilemma for the teacher. Each scenario would also include three potential courses of action for the teacher to pursue; the courses of action would implicitly reflect the different frameworks for school success. Participants would be asked to articulate what they believe to be the advantages and disadvantages of each course of action before selecting what they feel to be the best course of action given the situation and potential outcomes. Analyzing PSMTs’ responses would provide a better understanding of how clearly they recognize the complexities of decision-making as well as their priorities for students.

After establishing a baseline at the beginning of the program, different versions of the instrument could be administered at regular intervals as PSMTs matriculate through the program. As with Heath’s and Raphael’s cases, researchers should follow participants as they complete the program and enter the profession to develop an understanding of long term outcomes. I would recommend checking in at regular intervals and in a variety of methods in order to be in a better position to triangulate findings. The research team should visit the participants in their school sites, both during and after their time in the program. This will provide a better picture of the environments in which the teacher candidates are placed as well as provide context for how the PSMTs describe their schools and cooperating teachers.

I chose Heath and Raphael’s cases due to proximity and familiarity. I would recommend that researchers develop a greater number of case studies to allow for the possibility of greater generalization (Stake, 1995). Although they had different backgrounds, Heath and Raphael were
both white men; a more diverse case study pool would also be beneficial to provide greater perspective on the ways in which individuals undertake the ideas within the study.

**Implications for This Researcher**

I started this project as a way to examine how PSMTs viewed student success with respect to mathematics. More than just a greater understanding around how pre-service teachers define success, I take away a realization that, as a teacher educator, my own views of my students’ success have evolved. I view my students as successful not when they embrace my own ideals of addressing academic achievement, mathematical power, and critical consciousness, but when they question me about their value. *Why should we be interested in making sure all students are successful in Algebra? Why does it matter if my 7th graders understand place value? Why does the existence of different algorithms for the same operation matter?* These are all questions that I have fielded from my own PSMTs over the last year, and I welcome them. If my students simply accept what I said without question, if they just take up the ideas that I present instead of viewing them with a critical eye and forming their own opinions relative to their own identities and beliefs, then they will not have any real ownership of these ideas. As a result, they are unlikely to endure and be carried with them as professionals.

I currently teach in a teacher education program in a large, public university. As a mathematics education methods instructor, I have the privilege of working with teacher candidates across different programs (Early Childhood, Middle Childhood, Adolescent and Young Adult, and Special Education). In each of the programs, I try to help my teacher candidates recognize the political nature of schooling and the influences that different perspectives have on students’ experiences. I also encourage my teacher candidates to question *everything*—even me. On a recent exit ticket, I asked my students to tell me what they learned
thus far in our class this semester. One student wrote, “Not to just accept what other people say. To form my own ideas and opinions.” That, to me, was success.
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Appendix A

Inventory of Ideas/Feelings

When I think about teaching/leading in a high needs school, I get excited about…

When I think about teaching/leading in a high needs school, I wonder…

The concerns I have are:

When I think about teaching students who may be different from me in terms of how they are racialized, gendered, positioned with respect to class or language, etc., I feel…

The things I possess (knowledge, qualities, experiences, etc.) that I think will help me become an excellent mathematics teacher for marginalized students are:

The things I think I will most need to prepare to be an excellent mathematics teacher for marginalized students are:
Appendix B

Dimensions of Equity Instrument

How Would You Classify It?

*Below are 21 different scenarios. Using your knowledge of the 4 equity dimensions presented in the article by Gutierrez, classify each scenario as primarily about Access, Achievement, Identity, or Power. Place a (+) in front of the numbers you feel are positive indicators of the dimension. Place a (-) in front of the numbers you feel are negative (or violations) of the given dimension. The first one is done for you.*

*Now, on the axes provided, place your numbers on the map closest to the dimension you feel it represents.*

1. Parents are invited to the school to learn how to support their student with the new IMP curriculum that was recently adopted. (+ Access)

2. The math textbooks that students use were published in 1984.

3. Mr. Taylor's students learn how to use geometry to solve a problem about the school's redistricting policy that threatens to bus families to other schools. They present their “better” solution to the school board.

4. Instead of just high test scores, Principal Malik values and rewards teachers for supporting students to develop a positive disposition towards mathematics (thinking of themselves as “math people”).

5. Ms. Applebaum encourages students to guide class discussion. Often students respond to each other, not just to the teacher.

6. Students regularly use graphing calculators to make conjectures about the properties of quadratic functions.

7. Mr. Poetzl encourages his Korean students to use their home language with others who speak it.

8. Students practice problems like the ones that will appear on standardized tests. Later that year, their school makes AYP (Academic Yearly Progress) under NCLB.

9. Ms. Kenziger creates a classroom culture where students bring problems from home (real things that happen to them and their communities) to explore in class.

10. Ms. Gonzalez has a Master's degree in mathematics and is active in NCTM.
11. Families feel Mr. Ank is available to them and makes an effort to understand their culture. When their child is absent for religious holidays, he is not penalized.

12. Isandro comes up with a novel way of doing a problem the teacher has not seen. From then on, students are open to use this new strategy called "Isandro's Grip."

13. Ms. Speciale introduces motion detectors to try to get students to develop a conceptual understanding of functions.

14. Homework assignments are of the form: Here's a problem; now do 30 just like it.

15. A new student enters the school and is quickly informed that her way of “showing all of your work” is not correct in the US, even though her answer is right and she learned it in her math class in the Ukraine.

16. Thirty percent of a math/science magnet high school’s student population goes on to get jobs in math-based fields (e.g., engineering, physics, mathematics).

17. Mr. Cho regularly stays after school to help students who fall behind or don't understand the homework assignment.

18. The school makes a curricular change, moving to a textbook this is only available in English, even though 40 percent of their students speak Spanish at home.

19. The school creates a "mathlete club" where students do creative and challenging math problems after school. The principal pays to have T-shirts printed for them.

20. Teachers complain that the African American families in the school don't show up for parent-teacher conferences and must not value education.

21. Fifty percent of students at Jackson High never move beyond the equivalent of a course in Algebra I before graduating.

When you finish, come up with at least 3 new scenarios that you would classify in different dimensions. For example, you might choose Identity, Access, and Power. Also place these on your map. Write your new scenarios below:

22) 

23) 

24)
Appendix C

Program Structural Model
Appendix D

Program Overview

<table>
<thead>
<tr>
<th>Semester</th>
<th>Structure</th>
<th>Topic(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2010</td>
<td>Online Forum</td>
<td>• Color of Fear</td>
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<td></td>
<td></td>
<td>• Everyday Antiracism</td>
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<td></td>
<td>PD / Model Teacher</td>
<td>• Visit to model teacher’s high school</td>
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<tr>
<td></td>
<td>Seminar</td>
<td>• Introduction &amp; Smyth School Debate</td>
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<td></td>
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<td>• Anti-Racist Teaching, Part 1</td>
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<td></td>
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<td>• Anti-Racist Teaching, Part 2</td>
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<td></td>
<td>• More Mathematics to Change Kids’ Minds</td>
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<tr>
<td></td>
<td></td>
<td>• Racism, Discrimination, Oppression, Stereotyping</td>
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<tr>
<td></td>
<td></td>
<td>• Perspectives on Mathematics Education</td>
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<tr>
<td></td>
<td></td>
<td>• What is Mathematics?</td>
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<tr>
<td></td>
<td></td>
<td>• What would you say to someone who asked you what the Hampton Scholars Program is all about?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Dimensions of Equity Instrument</td>
</tr>
<tr>
<td>Spring 2011</td>
<td>Mentoring</td>
<td>• How is it going overall? How are you feeling about becoming a teacher? About student teaching coming up in January? [Probe for what you feel prepared for; what still worries you?]</td>
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<tr>
<td></td>
<td></td>
<td>• What we’ve observed about you</td>
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<td></td>
<td></td>
<td>• What’s working well about Hampton? [Probe for which sessions seem to have been useful? Probe for what are you learning from any of the things we are doing? Community?]</td>
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<td></td>
<td></td>
<td>• What can be improved? What would you like to see in the future? [Probe for how might we structure things so that they continue to be useful for you in student teaching?]</td>
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<tr>
<td></td>
<td></td>
<td>• Definitions of Success</td>
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<tr>
<td></td>
<td></td>
<td>o How does a teacher know when his students are successful? What will it look like? Push for specific examples. Why?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o How does a teacher of students in a high needs school know when his students are successful? What will it look like? Push for specific examples. Is that the definition that you’ve developed because of where you’re planning to go teach? Why?</td>
</tr>
<tr>
<td>Semester</td>
<td>Structure</td>
<td>Topic(s)</td>
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<td></td>
<td>When your students leave your classroom, what will make you proud?</td>
<td>o Operating definition in placement. How does your school define success/learning for the students in their observation placements?</td>
</tr>
<tr>
<td>Online Forum</td>
<td>• Lockhart’s Lament</td>
<td>• Letter to Lockhart</td>
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<td></td>
<td>• MMC Reflections</td>
<td>• Reading and Writing the World with Mathematics</td>
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<td>• Social Justice Mathematics</td>
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<tr>
<td>PD</td>
<td>• Metropolitan Mathematics Club conference</td>
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<tr>
<td>Seminar</td>
<td>• Assessment</td>
<td>• In Scholars’ Classrooms</td>
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<td></td>
<td>• Social Justice Mathematics</td>
<td>• Tensions around Teaching</td>
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<tr>
<td>Boot Camp</td>
<td>• Humanizing the Mathematics Classroom: Norms, Rituals, &amp; Practice</td>
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<tr>
<td>Math Club</td>
<td>• Site visit to model teacher’s partner school</td>
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<tr>
<td>Mentoring</td>
<td>• Field placement</td>
<td>• Math Club activity</td>
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<tr>
<td>Online Forum</td>
<td>• Colormute</td>
<td>• Math Club Activities</td>
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<td></td>
<td>• Math Club 3-Part Activity with Hints</td>
<td>• Math Club Activity Revisions</td>
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<tr>
<td>Fall 2011</td>
<td>Trip to Whitney Young in Chicago</td>
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<tr>
<td>PD / Mentor Teacher</td>
<td>• Pollack (2004) Colormute: Race dilemmas in an American school</td>
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<td>• Everyday Anti-Racism Negotiating Teaching with Others (and Oneself)</td>
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<td>• Understanding Racialization Practices in Schools</td>
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<tr>
<td>Seminar</td>
<td>• Effective Mathematics Departments that Serve Marginalized Students</td>
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<td></td>
<td>• Mathematics to Changes Kids’ Minds</td>
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<td>• Definitions and Enactments of Equity</td>
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<td>• More Mathematics to Changes Kids’ Minds</td>
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<td></td>
<td>• Culturally Relevant Teaching and Critical Pedagogy</td>
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<tr>
<td>Math Club</td>
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<tr>
<td>Mentoring</td>
<td>• MMC Conference in Chicago</td>
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<tr>
<td>Online Forum</td>
<td>• Letter to Lockhart</td>
<td>• Anti-Racist Teaching</td>
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<td>Semester</td>
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<td>Topic(s)</td>
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</tbody>
</table>
| PD       | • Teaching Math for Social Justice  
|          | • Metropolitan Mathematics Club of Chicago conference  
|          | • Definitions and Practices of Rigorous Mathematics  
|          | • Anti-Racist Teaching in the Era of Standardized Teaching  
|          | • Effective Mathematics Departments that Serve Marginalized Students  
|          | • Teaching Mathematics for Social Justice  
|          | • Interactive Mathematics Program  
|          | • Ken Robinson’s talks  
|          | • Dan Meyer’s talk  
|          | • Lockhart’s Lament  
|          | • Devlin’s Critique of Lockhart’s Lament  
|          | • Walkerdine’s Reasoning in a Postmodern Age  
|          | • Lipman (2004) High Stakes Education  
|          | • Pollack (2008) Everyday Anti-Racism  
|          | • Gutstein  
| Seminar  | • Teaching Math for Social Justice  
|          | • Metropolitan Mathematics Club of Chicago conference  
|          | • Definitions and Practices of Rigorous Mathematics  
|          | • Anti-Racist Teaching in the Era of Standardized Teaching  
|          | • Effective Mathematics Departments that Serve Marginalized Students  
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|          | • Lockhart’s Lament  
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|          | • Walkerdine’s Reasoning in a Postmodern Age  
|          | • Lipman (2004) High Stakes Education  
|          | • Pollack (2008) Everyday Anti-Racism  
|          | • Gutstein  

136
Appendix E

Codebook

Academic Achievement

AA-ap  academic placement courses; advance to higher-level mathematics courses in high school

AA-assets  students’ unique assets; referring to and/or not recognizing cultural and linguistic identities or “community cultural wealth” or capitalizing on students’ own unique assets of aspirational, linguistic, familial, social, navigational, and resistant capital

AA-cause  causes of the academic achievement gap; why gaps exist

AA-econ  economic mobility; develop the cultural capital to take full advantage of the economic opportunities in society

AA-gap  academic achievement gap; goal for students from different races, socioeconomic groups, and linguistic backgrounds to achieve similar outcomes related to mathematics achievement

AA-play  “play the game” of school mathematics; help marginalized students understand the traditional or dominant mathematics taught in schools to “play the game”

AA-testing  standardized testing, grades, and other traditional forms of assessment

AA-trends  trends; examine trends that extend beyond a single classroom, school, or community

Mathematical Power

MP-procedural  procedural fluency; the ability to use basic arithmetic facts efficiently (quickly and accurately), appropriately, and flexibly; adaptive expertise (see Adding it Up definition)

MP-concept  conceptual understanding; deep sense of mathematics as a web of interconnected ideas

MP-prereq  prerequisite; mathematical power not necessarily a precursor to academic achievement

MP-prod  productive disposition; a student’s belief that she can learn mathematics along with a desire to do so
MP-strategic  **strategic thinking**: competence in the processes of mathematics problem-solving and adaptive thinking; developing a sense of inquiry and the ability to attack and solve varied types of problems

MP-stthink  **student thinking**

Critical Consciousness

CC-agency  **students’ personal agency**: using mathematics for a given purpose and having control over what that purpose might be; recognition of power to effect change

CC-beyond  “**Going beyond mathematics**”; students exploring the issues behind social injustices, forming their own positions relative to these issues, and possibly engaging in action to address them

CC-classpow  **classroom power**: students experiencing and exercising power to effect what goes on inside the classroom

CC-studid  **student identity** as a “math person”

CC-using  “**Using mathematics**”; analyzing real or realistic situations mathematically to help students understand data and/or encourage students who may have lost interest in mathematics

Teacher’s Identity

TI-cagentid  **civic agent identity**: dispositions, stances, and beliefs that support a teacher’s role as more than a distributor of knowledge in the classroom; recognize oppression in students’ lives & encourage students to address it; relative to students (as opposed to the system, which is “tpagency”)

TI-comm  **community**: collective, productive, and engaged identity relative to the communities served; commitment to & relationship with communities served

TI-learner  **teacher as learner**: reflections on own learning experiences; lifelong learning

TI-mastery  “**mastery identity**”; failing to recognize inequity in schooling or to acknowledge that society gives some certain advantages in schooling not afforded to others; attributing success to one’s own attributes

TI-mathy  **mathy person**: teacher’s identity as a mathematical person
| TI-rel | relationship with students; dialogic rather than hierarchical relationships |
| TI-tpagency | teacher’s personal agency; challenge the way teachers are positioned; take the profession in new directions; relative to the system (as opposed to “cagentid”); teachers reclaiming the profession and challenging the system; this relates to creative insubordination |
| TI-preparing | preparing for class; thinking through the lesson, scripting potential questions, preparing and completing examples ahead of time |

**Transformative Professional Learning Experiences**

| TPL-cop | communities of practice; shared mission and purpose |
| TPL-differ | different than traditional teacher education program; new ideas put on the table for discussion |
| TPL-lit | literature; reading theoretical and research literature related to white privilege, language, social justice, gender, the funds of knowledge with which children enter the classroom, the politics of mathematics education, and the role that each of these plays in mathematics; also, readings designed to prepare teachers for taking on the task of addressing these issues, such as those that highlight the lived experiences and successes of marginalized students, identity construction, and capitalizing on students’ cultural identities |
| TPL-mentor | mentoring; one-on-one relationship; encouraging reflection and alternative ways of thinking |
| TPL-reflect | reflection; structured opportunities for reflection in generative ways; scaffolded reflection |
| TPL-relearn | relearning mathematics; teachers as learners deepen their own understanding of mathematics and consider the very nature of math and math education |
| TPL-support | support group experiences; environment in which PSMTs feel accepted, cared for, and safe to grapple with their beliefs |
| TPL-uncert | uncertainty; learning to live with and accept uncertainty |