ENHANCED AUTOMATIC GENERATION CONTROL WITH UNCERTAINTY

BY

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DISSE TATION

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Abstract

Maintaining reliability is a key aspect in power system operations. One process that helps in achieving this goal is automatic generation control (AGC), which is responsible for restoring the system frequency to the nominal value, and the real power interchange between balancing authority (BA) areas to the scheduled values. In this dissertation, we present the limitations of current AGC system implementations, and propose modifications in their design in order to increase their efficiency.

The AGC system goal has become more challenging due to the radical transformations occurring in the structure and functionality of power systems. These transformations are enabled by the integration of new technologies, such as advanced communication and power electronics devices, and the deepening penetration of renewable resources. For example, renewable-based generation is highly variable and intermittent, and might undermine the objective of AGC systems. A framework that may be used to quantify the effects of various uncertainty sources, such as load variations, renewable-based generation, and noise in communication channels, on the system characteristics is presented in this dissertation. To this end, we develop a method to analytically propagate the uncertainty from the aforementioned sources to the system frequency and area control error (ACE), and obtain expressions that approximate their probability distribution functions. We make use of the proposed framework and derive probabilistic expressions of the frequency performance criteria, developed by the North American Electric Reliability Corporation (NERC). Such expressions may be used to determine the limiting values of uncertainty that the system may withstand.
Our studies show that some advances are necessary in AGC system implementations, due to changes in power systems, such as the deregulation of the power industry and the integration of new technologies. The basic concept of AGC systems that is used by the BA areas has not changed severely over the past years. We aim in proposing AGC system modifications that are realistic and implementable in real large-scale systems. The high complexity of power systems is an obstacle when performing several processes related to reliability. In order to overcome such issues, we propose a systematic reduction of the synchronous generator model with low computational effort. In addition, we use the derived reduced model to describe a BA area dynamic behavior by including only the BA area variables. We use the developed models to design adaptive AGC systems, with self-tuning gain techniques, that decrease the unnecessary regulation and reduce the associated costs, since they take into account the actual system conditions in the determination of the control gains.

Furthermore, each BA area implements its own AGC system. However, if all the BA areas were operated as one single BA area, then the regulation amounts as well as the associated costs would be less. Operating separately and locally, individual BA areas are obliged to purchase more expensive ancillary services to accommodate the variability and uncertainty from high penetration of renewable-based resources. Thus, some level of coordination between BA areas is favorable for all entities. We propose a coordination scheme between BA areas that would decrease the regulation amounts and costs. Our approach is inspired from trying to mimic the AGC system, in the scenario where all areas are assumed to be one single BA area. To this end, we use the individual ACEs of each BA area to approximate the ACE in the scenario where all BA areas are assumed to be a single BA area. Then, we allocate the approximated ACE to the individual AGC systems proportionally to their size. Next, we mimic the AGC allocation for the entire area without the need for exchanging cost information between the BA areas. To this end, we develop a distributed algorithm that provides the same solution as the centralized AGC allocation, with the total mismatch of regulation
being the only information exchanged between BA areas.

Moreover, the AGC dispatch in many independent system operators (ISOs) is determined through a market mechanism, as mandated by the restructuring of power systems. However, we investigate the possibility of using the economic signals from the real-time markets (RTMs) instead of having AGC markets for the AGC dispatch. To do so, we start out by giving the formulation of the economic dispatch (ED) process, which is used to clear the RTM, and use it to obtain appropriate economic signals. We also discuss that the quality of the AGC service provided is affected by the ramping characteristics of the regulating units chosen to participate in AGC. We propose a systematic method for the AGC dispatch taking into account the economic signals from the ED process as well as the quality of the AGC service provided.

The proposed ideas are illustrated through several test systems. We choose small systems to provide insights into the proposed methodologies and large-scale systems to demonstrate their scalability.
To my family
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<tr>
<td>ACE</td>
<td>Area Control Error</td>
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<tr>
<td>ADI</td>
<td>Area Control Error Diversity Interchange</td>
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<td>AFRC</td>
<td>Actual Frequency Response Characteristic</td>
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<td>AGC</td>
<td>Automatic Generation Control</td>
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<td>BA</td>
<td>Balancing Authority</td>
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<td>CAISO</td>
<td>California ISO</td>
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<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<td>DAE</td>
<td>Differential Algebraic Equation</td>
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<td>DAM</td>
<td>Day-Ahead Market</td>
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<td>ED</td>
<td>Economic Dispatch</td>
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<td>EOC</td>
<td>Effective Offer of Commitment</td>
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<td>FERC</td>
<td>Federal Energy Regulatory Commission</td>
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<td>ISO</td>
<td>Independent System Operator</td>
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<td>ISONE</td>
<td>ISO New England</td>
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<td>LFC</td>
<td>Load Frequency Control</td>
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<td>LMP</td>
<td>Locational Marginal Price</td>
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<td>MISO</td>
<td>Midwest ISO</td>
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<td>NERC</td>
<td>North American Electric Reliability Corporation</td>
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<td>NYISO</td>
<td>New York ISO</td>
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<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
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<td>Acronym</td>
<td>Description</td>
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<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
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<td>PDF</td>
<td>Probability Distribution Function</td>
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<td>PMU</td>
<td>Phasor Measurement Unit</td>
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<td>PST</td>
<td>Power System Toolbox</td>
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<tr>
<td>RTM</td>
<td>Real-Time Market</td>
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<tr>
<td>SCADA</td>
<td>Supervisory Control And Data Acquisition</td>
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<tr>
<td>SDE</td>
<td>Stochastic Differential Equation</td>
</tr>
<tr>
<td>SEWBLS</td>
<td>Sliding Exponentially Weighted Window Blockwise Least Squares</td>
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<td>SMA</td>
<td>Selective Modal Analysis</td>
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<tr>
<td>SMIB</td>
<td>Single-Machine Infinite-Bus</td>
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<td>UC</td>
<td>Unit Commitment</td>
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<td>WECC</td>
<td>Western Electricity Coordination Council</td>
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CHAPTER 1
INTRODUCTION

In this chapter the stage is set for the work presented in this dissertation. Our research interests lie in the automatic generation control (AGC) system and the challenges it is facing due to the changes in power systems structure. We start by discussing the motivation for, and the background behind, our research so as to allow the reader to better understand the nature of the problems considered and the solutions we have developed. Also, a brief description of the current state-of-the-art in the field of AGC is provided. We then summarize the scope and the contribution of this work, and outline the contents of the rest of the dissertation.

1.1 Overview of Load Frequency Control

Power system operations include challenging problems due to the large-scale nature of the interconnected power systems, the nonlinear characteristics in the system, the extensive data acquisition needs and the salient characteristics of power systems. In this complex setting there are two overriding objectives in power system operations: reliability and economics. Power system operations include several control processes across different time scales, as shown in Figs. 1.1-1.2, which exchange information to meet these objectives. The goal of the unit commitment (UC) is to determine the minimum cost strategies for the start-up and shutdown of generation units to supply the forecasted load. The UC process determines the schedule of the hourly start-up and shutdown of units and as a result which generating units are used to supply the forecasted load for a given period, in a manner consistent with the generation equipment limitations and
operating policies [1], [2], [3], [4]. Typical generation equipment limitations are
the minimum (maximum) output, minimum up (down) time and ramping rates.
Commitment is determined a day-ahead of real time to allow slower thermal units
to be started if need be. The UC process is usually performed once per day to
account for the large, slow changes in demand, which are predictable to a certain
extent, and is partitioned into hourly intervals. Next, the economic dispatch (ED)
serves to allocate the total generation among the committed units, determined by
the UC, so as to minimize the costs of serving the system load subject to physical

Figure 1.1: Key stages in power systems operations.

Figure 1.2: Timescale of power system operations.
The ED process determines the generation outputs of the committed units. This process may be formulated in various ways. For example, the ED process could optimize some cost function subject to the power balance and the generators output lower and upper constraints. Transmission losses may also be included in the power balance constraint through some sensitivity coefficients. However, such a formulation neglects the representation of the active and reactive flow, and voltage constraints. In contrast, the optimal power flow (OPF) formulation includes such constraints [7]. Furthermore, it ensures the power balance at each node. The model used for the power system in the OPF may be based in either an AC or DC formulation, and we have ACOPF or DCOPF respectively.

The ED process is performed every hour to meet the day-ahead forecasted load and every 5-10 minutes to meet the minute-ahead forecasted load [8], [9], [10], [11]. However, the power grid requires that generation and load closely balance moment by moment. The day-ahead forecast does not match the actual demand, as depicted in Fig. 1.3. As a result frequent adjustments to the output of generators are necessary due to generation outages, line outages, intermittent generation or just load demand fluctuations. The deviations needed are small and occur on fast time scales of seconds. Thus, it is computationally infeasible to solve large centralized dispatch problems to manage such deviations. These adjustments are driven by several processes that consist load frequency control, as shown in Fig. 1.4,
which is considered to be one of the most important aspects of ancillary services. The balance of generation and load may be judged by measuring the system frequency. A non-nominal frequency in the system results in a lower quality of the delivered electrical energy. Many of the devices that are connected to the system work best at nominal frequency. Furthermore, too low frequencies (lower than \( \approx 57 \text{ – } 58 \text{ Hz} \)) may lead to a complete power system collapse. Once a generating unit is tripped or a block of load is added to the system, the power mismatch is initially compensated by an extraction of kinetic energy from the system inertial storage, which causes a declining system frequency. In primary control, the control task of priority is to bring the frequency back to acceptable values. If the frequency is increasing, more power is being generated than used, and all the machines in the system are accelerating. If the system frequency is decreasing, more load is on the system than the instantaneous generation can provide, and all generators are slowing down. This is part of the primary control. Primary control reestablishes balance between load and generation, but the system frequency differs from the nominal frequency because of the “droop” of generators. Consequently line flows differ from the scheduled values.
We use the AGC system in order to maintain the frequency at the nominal value and the power net interchange between balancing authority (BA) areas at the schedules values. Power systems are divided into several balancing authority (BA) areas that are responsible for maintaining (i) load-interchange-generation balance within the BA area, and (ii) the interconnection frequency as close as possible to its nominal value at all times. To ensure that reliability obligations of each BA area, within the interconnection, are maintained, the North American Electric Reliability Corporation (NERC) has introduced various frequency performance criteria, such as CPS1, CPS2 and BAAL [12]. When a BA area fails to comply with these standards, it is penalized. Thus, underperforming of a BA area and potential detrimental impacts on system reliability are avoided. Most BA areas implement tie-line bias control, and the AGC command is driven by the value of the area control error (ACE), which includes two terms: (i) the deviation of the sum of tie line flows between the BA area and other BA areas from the scheduled values, and (ii) the BA area obligation to support frequency. The second term depends on the frequency bias factor, which is unique for each BA area, and ideally reflects the BA area natural response. The objective of the AGC system is to make ACE zero. The AGC control is conducted every two-four seconds [13]. After some minutes, tertiary control determines the generation loading based on economic criteria subject to physical constraints. All these control mechanisms are implemented in a market environment with various rules at each independent system operator (ISO).

With the restructuring of power systems, all these control processes are part of the so-called electricity markets. Typically, electricity markets are held at different points in time ranging from a year to five-minutes ahead (see, e.g., [8], [9]), and trade the identical MWh commodity. However, prices may differ since system information and market conditions available at the time the MWh commodity is cleared are different for each market. Real-time markets (RTMs) are cleared at a higher frequency, typically every five minutes, than the hourly day-ahead markets (DAMs), which are cleared once a day. The imperfect information on
the real-time system conditions for the next day, is reflected in the outcomes of the 24 DAMs. Both markets are cleared using the same OPF tool. A difference between these markets is that while the demand may be price-responsive in the DAMs, it is typically, fixed in the RTMs. Such a market design with different lead times and clearing frequencies is commonly referred to as a multi-settlement system [10], [11].

The restructuring of power systems mandates that the AGC system is incorporated in the market environment. The classical load frequency control based on ACE is difficult to be implemented in a deregulated power system environment. The load change in the BA area causes frequency change, and all governors respond to this change instantaneously, whether or not they are selected for AGC. Moreover, which generators are actually chosen to participate in the AGC system affects the quality and cost of the ancillary service provided. The generators have different characteristics in terms of ramping rates that affect the quality of AGC service provided. The Federal Energy Regulatory Commission (FERC) has issued Order No. 755 that describes several characteristics that the AGC allocation as well as the AGC payments must meet [14]. In North America, regulation reserve markets for AGC with fully dispatchable regulation power capacity within 5 min are available. The AGC markets determine the regulation cost, which is equal to the bid of the marginal resource providing the ancillary service. Regulation costs also include the generators opportunity costs, from forgoing the energy market or other ancillary services markets. In addition, regulation bids typically include an additional cost component to cover the reduction in plant efficiency that thermal generators experience to enable fast and controlled regulation response. They may also include additional maintenance costs from the increased damage of operating in this mode. Some ISOs co-optimize all of the generator bids, both for energy and regulation markets. Such a process guarantees at the same time each generator the maximum profit, and minimizes the overall cost of procuring energy and ancillary services. Each generator submits its energy cost and operating limitations, and the market clearing process schedules each generator for the optimal
mix of energy and ancillary service production each hour [15].

1.2 Review of the State of Art in Automatic Generation Control Systems

A thorough literature review of research in AGC’s basic concepts is presented in [16]. The authors describe AGC systems based on DC or AC power flow formulation, optimal, centralized, decentralized and adaptive control. An area in the AGC systems that gains a lot of attention is the effects that the new technologies applied to the grid have on the frequency control. The analysis of the effects of small wind turbines output on the load frequency control process is studied in [17]. Various wind turbine output scenarios, based on actual data, are combined with system load variations to test the effectiveness of present AGC control system implementations. A new integrated control system of a wind farm based on two control levels, a supervisory active and reactive system control and a machine control system, which ensures that set points at the wind turbine level are reached, is given in [18]. The authors claim that such a control system provides improved performance of the system and a better grid integration of the wind energy without significant extra costs. An extended-term dynamic simulation to quantify AGC performance in power systems with smart grids is given in [19]. In [20], the authors formulate the frequency regulation problem by viewing the future electric energy systems as a general dynamical system driven by disturbances and propose a modified AGC system that better responds to fast disturbances. The analysis of the system behavior in the case an attacker gains access to the AGC signal and injects undesirable inputs to the system is studied in [21]; in this work, the authors propose the design of an optimal control strategy to destabilize a two-area power system in the case such a cyber attack occurs. In [22], the authors propose a framework to study the impact of small-signal stochastic power injections, which are modeled as Markov chains, on power system dynamics. To this end, they obtain approximate expressions for moments of algebraic and dynamic
Some research is focused on the design of AGC systems that will increase their performance by taking into account the challenges introduced by the “smarter” grid. Several papers are dedicated to developing reduced-order models for system components that might be used in the design of AGC systems. For example, in [23] a method is proposed for reducing the state matrices of a linear system by keeping the dominant eigenvalues and eigenvectors as the original system. A similar approach is given in [24], where the authors use selective modal analysis to construct a simplified model. Eigenanalysis is a preferred approach for developing simplified models as discussed in [25]. In [26], the classical approach of modeling the BA area dynamics is presented. Such models may be used in the design of AGC systems. In [27], an AGC system is proposed based on optimal control theory by taking into consideration a set of frequency response requirements the system must satisfy. A flexible AGC algorithm that includes flat frequency, flat tie-line and tie-line bias control is presented in [28]. The authors also include in their formulation an approximate economic dispatch algorithm that makes use of a predetermined table of the economic loading of units. They pinpoint that the use of the actual frequency response characteristic in the calculation of ACE is important. A description of the AGC system role and its limitations is given in [13]. The authors also mention why a good approximation of the frequency bias factor in the ACE calculation is the actual frequency response characteristic. The role of the ACE in the AGC system and why it is often filtered is discussed in [29]. In [30], a modified AGC two-area system is presented. The system takes into account the effect of bilateral contracts on system dynamics and obtains optimal parameters for the control system using a gradient Newton algorithm. In [31], the authors propose a stochastic optimal relaxed AGC system by taking into account NERC’s control performance standards, which leads to significant reduction in the generator movements for thermal plants and reduces control cost by regulating the relaxation factors online. A presentation of the current basics of calculating frequency bias factors, showing their limitations and proposing a
new method for sizing them is given in [32]. In [33], the authors discuss issues related to very short-term load prediction, security economic dispatch, variable generation management, and adaptive AGC unit tuning, to make the AGC system more efficient.

Another approach in increasing the efficiency of AGC systems is the coordination of the AGC systems of neighboring BA areas. NERC has proposed that BA areas coordinate in some extent, by proposing the area control error diversity interchange (ADI). The ADI was introduced by [34], and is the pooling of individual ACEs to take advantage of the ACE diversity, i.e., sign differences associated with the momentary generation-load imbalances of each BA area. By pooling ACE the participants are able to reduce the control burden on individual BA areas, the unnecessary generator control movement, the sensitivity to resources with potentially volatile output, and to realize improvements in frequency performance criteria [34], [35]. BA area coordination helps them meet their renewable integration objectives. The amount of required balancing reserves, and regulation reserve ramping requirements can be reduced through BA area coordination.

In [36], the authors describe the control system applied in the Italian transmission network, and with several cases studies indicate the benefits of coordinated regulation. The authors in [37], propose a decentralized load frequency control system, the goal of which is to obtain robust PI controllers in a multi-area power system. An AGC system for an interconnected power system taking advantage of hydrothermal generation characteristics, is proposed in [38]. In [39], a modification of the ACE diversity interchange (ADI) program is presented, by taking into account the transmission constraints with the use of sensitivity factors. An automatic optimal control system is presented in [40], which optimizes the active and reactive power coordinately to improve the efficiency of the control system.

Moreover, the relationship of the AGC system and the electricity markets is important among ISOs, and has been studied in a great extent. The most common type of AGC market is flat rate, because of its simplicity, however, in this case the response quality of the participating generating units is not taken into
account [41, pp. 84-86]. The AGC payments, according to FERC Order No. 755 [14], include consideration for capacity set aside to provide the regulation service and the energy that the resource injects into the system. These payments also cover the opportunity costs from foregone sales of electricity. A survey of the frequency control ancillary services in power systems from various parts of the world by focusing on the economic features is given in [42]. ISO New England (ISONE) makes payments for frequency regulation service to reflect the amount of work performed by a resource by taking into account the absolute amount of energy injected and withdrawn, which is referred to as a “mileage” payment. In ISONE, the fastest units are chosen among the ones cleared in the regulation market [43]. California ISO (CAISO), New York ISO (NYISO), Midwest ISO (MISO) and PJM pay a capacity payment to all resources that clear the frequency regulation market, and then net the amount of regulation up and regulation down provided by these resources [14]. In [44], the objectives of ancillary services market are mentioned and it is shown that they may not be designed independently of other market structures, such as energy markets. The authors in [45], propose a decentralized AGC system taking into account the competitive environment in regulation. In [15], a summary of ancillary service market designs is presented and possible modifications to current designs, which accommodate for the deepening penetration of renewable resources, are discussed.

1.3 Scope and Contributions

This work is focused on two main topics (i) to identify challenges and quantify their effects in the performance of AGC systems, (ii) to propose alternative designs that increase the efficiency of AGC systems using the available new technologies, and smoothly integrate the AGC system in electricity market environment. We dedicate the following paragraphs to discuss further each of the two points.

(i) The AGC system goal to restore system frequency and real power inter-
change to the desired values has become more challenging due to the radical transformations occurring in the structure and functionality of power systems. These transformations are enabled by the integration of new technologies, such as advanced communication and power electronics devices, and the deepening penetration of renewable resources. These new technologies, however, raise new challenges in the reliable operation of power systems [46]. For example, wind generation is not only intermittent and highly variable, it also introduces an additional source of uncertainty to power system operations.

The combination of uncertainty from load variations, renewable-based generation, and noise in communication channels, however, may affect the AGC system performance; thus hindering the overall system reliability. We propose a framework to evaluate the effects of uncertainty in load variations, renewable-based generation and noise in communication channels, and to assess the AGC system performance. We use the framework to calculate several moments and approximate the probability distribution function of system variables, such as frequency. We also obtain probabilistic expressions of the frequency performance criteria — CPS1, CPS2 and BAAL — based on the framework we developed. We use these probabilistic expressions and inspect for which scenarios of uncertainty the reliability metrics are met. Thus, we may find limiting values of uncertainty that the system may withstand. Part of this work is presented in [47], [48].

(ii) The basic concept of AGC systems that is used by the BA areas has not changed severely over the past years. Some advances have been made in control concepts, due to changes in power systems such as deregulation of power industry and use of renewable-based generation. Investigations that have been carried out reveal that the AGC system, used by BA areas, results in relatively large overshoots and transient frequency deviation. The main focus is to propose improvements in the design of the AGC system that may have an actual implementation, i.e., they are simple and use information
available from phasor measurement units (PMUs) or supervisory control and data acquisition (SCADA). Approaches such as optimal control need the availability of all state variables, which is difficult to achieve. We aim in designing an AGC system that is realistic and implementable in real large-scale systems. We take advantage of the available measurements of the system frequency, the area control error and the total generation and propose adaptive AGC control systems that increase the efficiency, since they take into consideration the actual system conditions in the determination of the control gains.

In addition, one other way to increase the AGC system efficiency is to obtain some level of coordination between BA areas. We propose a coordination methodology between the BA areas by mimicking the behavior of the AGC system in a scenario where the whole interconnected system is assumed to be operated by a single BA area. In order to do so, we use the individual ACEs of each BA area to approximate the ACE in the scenario where all BA areas are assumed to be a single BA area. Then, we allocate the approximated ACE to the individual AGC systems proportionally to their size. Next, we mimic the AGC allocation for the entire area without the need for exchanging cost information between the BA areas. To this end, we develop a distributed algorithm that provides the same solution as the centralized AGC allocation, with the total mismatch of regulation being the only information exchanged between BA areas.

Moreover, we investigate the possibility of using the economic signals of the RTMs to allocate the AGC signal, without formulating a separate AGC market mechanism. To this end, we present a systematic method of allocating the AGC signal among the generators by taking into consideration the ramping characteristics of each generator, as well as economic criteria defined by the ED process, which is used to clear the RTM. We compare the proposed method with other two methods currently used in industry. The
proposed approach meets the guidelines, as specified in Order No. 755. In this part, we summarize some of the results from our work in [49], [50], [51].

1.4 Outline of the Dissertation

This dissertation is structured in several chapters. An overview of each chapter, their topics and main contributions is given here.

**Chapter 2** We present the power system modeling that is the basis for all subsequent chapters. More specifically, we provide three models, the full model and the classical model, with and without the governor dynamics, for the description of the behavior of synchronous generators. We also model the dynamic behavior of renewable-based generation. In addition, we include network equations and the AGC system dynamics for both continuous time and discrete.

**Chapter 3** This chapter proposes a framework that may be used to quantify the effects of uncertainty in load variations, renewable-based generation, and noise in communication channels, in the AGC system. To this end, we model the power system dynamics, the network and the AGC system dynamic behavior, as well as the various uncertainty sources. We develop a unified stochastic differential equation model, propagate the uncertainty from the aforementioned sources to the system frequency and ACE, and approximate their probability distribution functions. We make use of this framework and obtain probabilistic expressions of the frequency performance criteria, developed by the NERC. Such expressions may be used to determine the limiting values of uncertainty that the system may withstand. The proposed ideas are illustrated through a two-machine four-bus system, the three-machine nine-bus WECC system and a 48-machine 140-bus system.

**Chapter 4** The high complexity of power systems is an obstacle when performing several processes related to reliability. A number of approaches have been
proposed to reduce the system variables by keeping certain levels of accuracy. In this chapter, we propose a systematic reduction of the synchronous generator model with low computational effort, by using the selective modal analysis method. In addition, we use the derived reduced model to describe a BA area dynamic behavior by including only the BA area variables. There are several applications where these models are useful. In this chapter, we use the developed models to design adaptive AGC systems that decrease the unnecessary regulation and reduce the associated costs. To this end, we estimate online the actual frequency response characteristic of a BA area, and use that value in the calculation of the ACE. As a result, the ACE shows the exact number of MW needed to restore the system frequency and the real power interchange to the desired values. We demonstrate the proposed ideas with a single-machine infinite-bus, the three-machine nine-bus WECC, and a 48-machine 140-bus systems.

Chapter 5 Maintaining the demand-generation balance has become more challenging when an interconnected power grid is operated locally and separately by an individual BA area. In this chapter, we propose a coordination between BA areas that would decrease the regulation amount needed as well as the associated costs. Our approach is inspired from trying to mimic the AGC system, if all areas were considered to be one entire BA area. In order to do so, we use the individual ACEs of each BA area to approximate the ACE in the scenario where all BA areas are assumed to be a single BA area. Then, we allocate the approximated ACE to the individual AGC systems proportionally to their size. Next, we mimic the AGC allocation for the entire area without the need for exchanging cost information between the BA areas. To this end, we develop a distributed algorithm that provides the same solution as the centralized AGC allocation, with the total mismatch of regulation being the only information exchanged between BA areas. Furthermore, we describe the ADI methodology, which is the current
method of BA area coordination. We demonstrate the proposed ideas with
the three-machine nine-bus WECC, and compare our method with other
three methods to demonstrate its capabilities.

Chapter 6 In this chapter, we propose an alternative approach to the AGC
markets. We investigate the possibility of using economic signals from the
RTMs to select which units participate in regulation. RTMs are usually
cleared using the ED process. To this end, we formulate the ED process
and gain insights into the economic characteristics of the generating units
from the RTMs. We value the quality of AGC service by taking into consid-
eration the ramping constraints of the generating units. More specifically,
we include in our formulation a parameter that quantifies the importance of
using fast regulating units, based on the net load variability of each system.
Then, we propose a systematic way to determine, in real time, the power al-
located to each generator participating in AGC. The proposed methodology
is illustrated in the three-machine nine-bus WECC system and is compared
with other allocation methods, which are also presented in this chapter.

Chapter 7 In this chapter, a summary of the findings of this work and research
avenues for future work is presented.
Part I
Analysis of Automatic
Generation Control Systems
In this chapter, we review the modeling aspects that we make detailed use throughout the dissertation. We start out with presenting three models describing the dynamics of a synchronous generator. We also provide the dynamical system that describes the behavior of wind-based generation. We then give the network equations and three models for the AGC system. We consider a power system with the set of $N$ nodes $\mathcal{N} = \{1, \ldots, N\}$, with the slack bus at node 1, and the set of $L$ lines $\mathcal{L} = \{\ell_1, \ldots, \ell_L\}$. The set of generating units is $\mathcal{I} = \{1, 2, \ldots, I\}$.

2.1 Synchronous Generators

In this section, we present three models that describe the synchronous generator dynamic behavior. In particular, we describe the full nine-state model, and the classical model with and without the governor.

2.1.1 Full Nine-State Synchronous Generator Model

We use a nine-state machine model as described in [52, p. 140] for the representation of the synchronous generator dynamics. The two-axis nine-state synchronous machine dynamic circuit is depicted in Fig. 2.1. For the $i^{th}$ synchronous machine, the nine states are: the field flux linkage $E_{q_i}'$, the damper winding flux linkage $E_{d_i}'$, the rotor electrical angular position $\delta_i$, the rotor electrical angular velocity $\omega_i$, the scaled field voltage $E_{fd_i}$, the stabilizer feedback variable $R_{fi}$, the scaled output of the amplifier $V_{R_i}$, the scaled mechanical torque to the shaft $T_{Mi}$, and
The parameters in (2.1)-(2.9) describe the machine characteristics and their definitions may be found in [52]. We define as $P_{Ci}$ the AGC command signal that the system operator sends to each generator $i$ participating in regulation. We also denote by $\omega_s$ the synchronous speed, $I_{di}$ ($I_{qi}$) the d-axis (q-axis) component of the stator current, $\theta_i$ the voltage angle, and $V_i$ the voltage magnitude at bus $i$.

In addition to the specified dynamics, we also have a set of algebraic equations.
The $i^{th}$ synchronous generator algebraic equations are

$$V_i e^{j\theta_i} + (R_{s_i} + jX_{d_i}')(I_{d_i} + jI_{q_i}) e^{j(\delta_i - \frac{\pi}{2})}$$

$$- \left[ E_{d_i}' + (X_{q_i}' - X_{d_i}')I_{q_i} + jE_{q_i}' \right] e^{j(\delta_i - \frac{\pi}{2})} = 0 . \quad (2.10)$$

### 2.1.2 Classical Model with Governor Dynamics

We use the nine-state model, given in Section 2.1.1, to obtain the classical model. One simplifying assumption in the case of the classical model is to consider $X_{d_i}' = X_{q_i}'$, $X_{d_i} = X_{q_i}$, and $R_{s_i} = 0$. Then (2.10) simplifies to

$$V_i e^{j\theta_i} + jX_{d_i}'(I_{d_i} + jI_{q_i}) e^{j(\delta_i - \frac{\pi}{2})} - \left( E_{d_i}' + jE_{q_i}' \right) e^{j(\delta_i - \frac{\pi}{2})} = 0 .$$

In addition, we may categorize the synchronous generator dynamics into slow and fast. The states $E_{d_i}'$, $E_{fd}$, and $V_{R_i}$, are considered to be part of the fast dynamics and the rest are considered to be slow. Only, the states $\delta_i$, and $\omega_i$ are represented in the classical model, and $\delta_i$, $\omega_i$, and $P_{SV_i}$ in the classical model including the governor dynamics. To this end, we set the time constants of the fast dynamics equal to zero and those of the slow, which are not represented in the model, to infinity. Thus, we have $T_{q_{v_i}}' = 0$, $T_{A_i} = 0$, $T_{E_i} = 0$, and $T_{d_{o_i}}' \rightarrow \infty$, $T_{F_i} \rightarrow \infty$, $T_{CH_i} \rightarrow \infty$ for the classical model including the governor dynamics. We have that $E_{q_i}'$ is constant and is equal to $E_{q_i}'0$, which we denote by $E_i$. The machine dynamics for the classical model including the governor dynamics are given by

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s, \quad (2.11)$$

$$\frac{2H_i d\omega_i}{\omega_s dt} = P_{SV_i} - E_iI_{q_i} - D_i(\omega_i - \omega_s), \quad (2.12)$$

$$T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV_i} + P_{C_i} - \frac{1}{R_{D_i}}(\omega_i - 1) . \quad (2.13)$$

It is easy to show that $P_{G_i} = Re \left( (I_{d_i} - jI_{q_i}) e^{-j(\delta_i - \frac{\pi}{2})} jE_i e^{-j\delta_i} \right) = E_i I_{q_i} = \frac{E_i V_i \sin(\delta_i - \theta_i)}{X_{d_i}}$.

In the same rationale, we may calculate the reactive power, which is equal to
\[ Q_{G_i} = -\frac{V^2}{X_{d_i}} + \frac{E_{Vi} \cos(\delta_i - \theta_i)}{X_{d_i}} \]. The set of differential equations that describe the \( i^{th} \) synchronous generator dynamics are

\[
\frac{d\delta_i}{dt} = \omega_i - \omega_s, \quad (2.14)
\]

\[
\frac{2H_i d\omega_i}{\omega_s dt} = P_{SV_i} - \frac{E_{Vi} \sin(\delta_i - \theta_i)}{X'_{d_i}} - D_i(\omega_i - \omega_s), \quad (2.15)
\]

\[
T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV_i} + P_{Ci} - \frac{1}{R_{D_i}} \left( \frac{\omega_i}{\omega_s} - 1 \right). \quad (2.16)
\]

The model given in (2.14)-(2.16) is known as the classical model including the governor dynamics.

### 2.1.3 Classical Model

We use the same assumptions made in Section 2.1.2, and furthermore that \( T_{SV_i} = \infty \). Then \( P_{SV_i} = \text{constant} = P_{SV_i0} \) and the classical model is given by

\[
\frac{d\delta_i}{dt} = \omega_i - \omega_s, \quad (2.17)
\]

\[
\frac{2H_i d\omega_i}{\omega_s dt} = P_{SV_i0} - \frac{E_{Vi} \sin(\delta_i - \theta_i)}{X'_{d_i}} - D_i(\omega_i - \omega_s). \quad (2.18)
\]

The classical synchronous machine dynamic circuit is depicted in Fig. 2.2.

We illustrate the differences between the full and the classical model with a four-bus test system, as depicted in Fig. 2.3a, which contains two synchronous generating units in buses 1 and 4, one wind generation unit in bus 2 and load in bus 3. The machine, network, and load parameter values for this example are listed

![Classical synchronous machine dynamic circuit](image)
here: the system MVA base is 100; the synchronous speed, $\omega_s = 377 \text{ rad/s}$; the machines shaft inertia constants, $H_1 = 23.64$ and $H_4 = 6.4$; the machines damping coefficients $D_1 = 0.0125$, $D_4 = 0.0068$, the machine impedances, $X_{d1} = 0.146$, $X_{q1} = 0.8958$, $X'_{d1} = 0.0608$, $X'_{q1} = 0.1198$, $X_{d4} = 0.0969$ and $X'_{q4} = 0.1969$, $X_{q4} = 0.0969$ and $X_{q4} = 0.8645$; the governor droops $R_{D1} = R_{D4} = 0.05$; and the parameters $T'_{do1} = 8.96$, $T'_{do4} = 6.0$, $T'_{qo1} = 0.31$, $T'_{qo4} = 0.535$, $T_{SV1} = T_{SV4} = 2$, $T_{F1} = T_{F4} = 0.35$, $K_{F1} = K_{F4} = 0.063$, $T_{E1} = T_{E4} = 0.314$, $K_{E1} = K_{E4} = 1$, $T_{A1} = T_{A4} = 0.2$, and $K_{A1} = K_{A4} = 20$. The network impedances between bus $i$ and $j$ are denoted by $Z_{ij}$, so we have: $Z_{12} = 0.0101 + j0.0504$, $Z_{14} = Z_{23} = 0.0074 + j0.0372$, $Z_{34} = 0.0127 + j0.0636$. We solve the power flow equations and the machine algebraic equations such that the wind generation in bus 2 is $P_{W2} = 0.298$, the synchronous generator output in bus 4 is $P_{G4} = 0.5$, the load in bus 3 is $P_{L3} + jQ_{L3} = 2.5 + j1.25$, the voltage magnitude in bus 1 is $V_1 = 1$ and
bus 4 is $V_4 = 1.02$, with generator in bus 1 as the slack bus. We modify the load in
bus 3 as follows $P_{L_3} = 2.7$ and plot the rotor electrical angular speed of generator
1 in Fig. 2.3b. We consider the nine-state model as reference and notice that
as we make further simplifications we lose accuracy in the representation of the
actual system behavior. However, there are cases depending on the application,
where a simplified model is more appropriate and its accuracy is acceptable for
the timescales of interest.

2.2 Wind-Based Generation

Several models, up to 10 states, may be used to describe the dynamic behavior of
wind-based generation. We assume a first order dynamical model, which yields
an accurate relationship between the wind speed and the real power generated
by a collection of wind turbines (see, e.g., [53], [54]), for the purposes of this
dissertation. We denote by $P_{W_i}$ the active generation output of a wind resource,
and by $Q_{W_i}$ the reactive wind generation output at bus $i$. We assume that $Q_{W_i} = 0$
for all wind-based generation. The dynamic behavior of a wind resource at bus $i$
is given by

$$\dot{P}_{W_i} = \varrho_{1_i} P_{W_i} + \varrho_{2_i} v_i + \varrho_{3_i},$$

(2.19)

where $v_i$ is some average wind speed at bus $i$, and $\varrho_{1_i}$, $\varrho_{2_i}$ and $\varrho_{3_i}$ are parame-
ters that depend on the wind-based generation characteristics. Sufficiently small
variations around a system nominal trajectory ($v_i^*, P_{W_i}^*, Q_{W_i}^* = 0$) may be approx-
imated by

$$\Delta \dot{P}_{W_i} = \varrho_{1_i} \Delta P_{W_i} + \varrho_{2_i} \Delta v_i,$$

(2.20)

where $\Delta v_i$ is the variation in the wind speed at bus $i$. 

2.3 Network

Let $P_{Gi}$ and $P_{Wi}$ represent the real power generated from the synchronous generator and wind resource at bus $i$, and let $P_{Li}$ represent the real power load at bus $i$. Further, let $Q_{Gi}$ and $Q_{Li}$ denote the reactive power supplied by the synchronous generator and demanded by the load at bus $i$, respectively. Then, we model the network using the standard nonlinear power flow formulation (see, e.g., [52]), and for the $i$th bus, we have that:

$$P_{Gi} + P_{Wi} - P_{Li} = \sum_{k=1}^{n} V_i V_k (G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)), \quad (2.21)$$

$$Q_{Gi} - Q_{Li} = \sum_{k=1}^{n} V_i V_k (G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)), \quad (2.22)$$

where $G_{ik} + jB_{ik}$ is the $(i, k)$ entry of the network admittance matrix and for each model for the synchronous generator we have

Nine-State Model

$$P_{Gi} = I_{di} V_i \sin(\delta_i - \theta_i) + I_{qi} V_i \cos(\delta_i - \theta_i), \quad (2.23)$$

$$Q_{Gi} = I_{di} V_i \cos(\delta_i - \theta_i) - I_{qi} V_i \sin(\delta_i - \theta_i). \quad (2.24)$$

Classical Model with and without the Governor Dynamics

$$P_{Gi} = E_i I_{qi} = \frac{E_i V_i \sin(\delta_i - \theta_i)}{X'_{di}}, \quad (2.25)$$

$$Q_{Gi} = -\frac{V_i^2}{X'_{di}} + \frac{E_i V_i \cos(\delta_i - \theta_i)}{X'_{di}}. \quad (2.26)$$

2.4 Automatic Generation Control System

Unlike in traditional AGC modeling [1], we explicitly consider the network; this way, we are capturing the effect that the network has on the overall closed-loop
system dynamic behavior. We assume that there are \( M \) BA areas within an interconnected system and define \( \mathcal{A} = \{1, \ldots, M\} \). For each \( m \in \mathcal{A} \), we denote by \( \mathcal{A}_m \subset \mathcal{A} \) the set of BA areas that have transmission lines connected to BA area \( m \), and \( \mathcal{G}_m \) the set of generators in BA area \( m \). Also, we denote the actual power interchange out of BA area \( m \) to \( m' \) as \( P_{mm'} \) and the actual frequency of BA area \( m \) as \( f_m \). Then, we have

\[
P_{mm'} = \sum_{l \in \mathcal{B}_{mm'}, l' \in \mathcal{B}_{m'm}} V_l V_{l'} \left( G_{ll'} \cos(\theta_l - \theta_{l'}) + B_{ll'} \sin(\theta_l - \theta_{l'}) \right),
\]

where \( \mathcal{B}_{mm'} (\mathcal{B}_{m'm}) \) is the set of nodes in BA area \( m (m') \) with tie lines to nodes in BA area \( m' (m) \). The actual frequency of BA area \( m \) may be defined in various ways. We select two definitions for \( f_m \)

\[
f_m = \sum_{i \in \mathcal{B}_m} \gamma_{1i} \left( f_{nom} + \frac{1}{2\pi} \frac{d\theta_i}{dt} \right),
\]

\[
f_m = \sum_{i \in \mathcal{G}_m} \gamma_{2i} \left( f_{nom} + \frac{1}{2\pi} \Delta\omega_i \right),
\]

where \( \mathcal{B}_m \) is the set of buses in BA area \( m \), \( f_{nom} \) is the nominal system frequency, each \( \gamma_{1i}, i \in \mathcal{B}_m \) (\( \gamma_{2i}, i \in \mathcal{G}_m \)), represents some weighting factor and \( \sum_{i \in \mathcal{B}_m} \gamma_{1i} = 1 \) (\( \sum_{i \in \mathcal{G}_m} \gamma_{2i} = 1 \)), \( \Delta\omega_i \) the deviation of the rotor electrical angular speed \( \omega_i \) from the synchronous speed \( \omega_s \). Then, the ACE for BA area \( m \) is given by

\[
ACE_m = \sum_{m' \in \mathcal{A}_m} (P_{mm'} - P_{mm'_{sch}}) - b_m (f_m - f_{nom}),
\]

\[
\sum_{m' \in \mathcal{A}_m} \Delta P_{mm'} - b_m \Delta f_m,
\]

where \( b_m \) is the frequency bias factor for BA area \( m \), which is negative, and \( P_{mm'_{sch}} \) is the scheduled real power interchange out of BA area \( m \) to \( m' \), \( \Delta P_{mm'} = P_{mm'} - P_{mm'_{sch}} \), and \( \Delta f_m = f_m - f_{nom} \). Thus, when either \( \Delta f_m > 0 \) or \( P_{mm'} > P_{mm'_{sch}} \) the BA area \( m \) is over generating. The ACE is positive and shows that the generation
needs to be reduced. The value of the frequency bias factor $b_m$ is the mathematical expression of the net change in a BA area’s net actual interchange for a change in interconnection frequency. It is a fundamental reliability service provided by a combination of governor and load response.

Define a new state for BA area $m$ in the system, $z_m$, which, at steady state, is the total power generated in the BA area $m$. Following a combination of the models presented in [55, p. 237] and [1, pp. 352-355], it can be shown that the evolution of $z_m$ is given by

$$\dot{z}_m = -z_m - \eta_1 \frac{dACE_m}{dt} - \eta_2 ACE_m + \sum_{i \in G_m} P_{Gi}, \quad (2.32)$$

where $\eta_1, \eta_2$ system dependent control gains. Note $z_m$, the total generation in BA area $m$, decreases if $ACE_m$ is positive, i.e., $f_m$ is greater than the nominal frequency or the real power interchange is greater than that scheduled.

We also provide a modification of (2.32), which is a continuous time function, and introduce the equivalent discrete function. Let $z_m[k] := z_m(kh)$, where $h$ takes values between two and four seconds and $k$ is an integer. Then, the AGC system dynamic behavior is given by:

$$z_m[k + 1] = z_m[k] + h(-z_m[k] - \eta_1 (ACE_m[k] - ACE_m[k - 1]))$$

$$-\eta_2 ACE_m[k] + \sum_{i \in G_m} P_{Gi}). \quad (2.33)$$

A simpler AGC system is just an integral control, which is given by

$$\dot{z}_m = -ACE_m. \quad (2.34)$$

There are cases where such an AGC system is useful, as shown later in Chapter 4. Even in this simpler case, the physical interpretation of $z_m$ remains the same.

The ISOs actually follow a discrete AGC system when they are implementing load frequency control. The continuous time equation is used in some cases in
this dissertation because it allows us to take advantage of various mathematical tools.

Each generator \( i \in \mathcal{G}_m \) participates in the AGC system with \( P_{C_i} = \phi_i(z_m) \) for \( i \in \mathcal{G}_m \), where \( \phi_i(\cdot) \) is some function. One way to define \( \phi_i(\cdot) \) is through participation factors. Each generator \( i \) in BA area \( m \) participates in AGC by a participation factor \( \kappa_i^m \). So, the generator \( i \) AGC command output is \( P_{C_i} \), which is determined by

\[
P_{C_i} = P_{ED_i} + \kappa_i^m(z_m - \sum_{j \in \mathcal{G}_m} P_{ED_j}),
\]

(2.35)

where \( P_{ED_i} \) is the economic dispatch signal for generator \( i \). We can see from (2.35) that \( z_m = \sum_{i \in \mathcal{G}_m} P_{C_i} \). We denote by \( \kappa_i^m \) the participation factor of generator \( i \)

(a) One-line diagram of the three-machine nine-bus WECC power system.

(b) Block diagram of the AGC system for the WECC system.

Figure 2.4: AGC system example.
in the AGC system, with $\sum_{i \in \mathcal{G}_m} \kappa_i^m = 1$, $\forall m \in \mathcal{M}$. There are various ways to determine the AGC participation factors $\kappa_i^m$; for instance we may use economic criteria or take into account unit ramping characteristics [19], [49], [56].

We design the block diagram of the AGC system, given in (2.32) for $\eta_1 = 0$ and $\eta_2 = 1$, in Fig. 2.4b, of the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system, which is depicted in Fig. 2.4a. The machine, network and load parameter values may be found in [52]. We only have one BA area, so $M=1$, $\mathcal{G} = \{1, 2, 3\}$, $\mathcal{B} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and based on the block diagram we get: $\dot{z} = -z - ACE + \sum_{i \in \mathcal{G}} P_{G_i}$, since $z = \sum_{i \in \mathcal{G}} P_{C_i}$. We go through a simple example to explain how the AGC system works. If there is a decrease in the load, then $z > \sum_{i \in \mathcal{B}} P_{L_i}$. To this end, we need the AGC system to decrease $z$, so the total generation at the new steady state is smaller and equal to the new total load, and the system frequency is equal to the nominal. The total
load $\sum_{i \in B} P_{Li}$ is equal to the total generation after a few seconds due to primary control, so $\sum_{i \in B} P_{Li} = \sum_{i \in G} P_{Gi}$, as seen in Fig. 2.5b. Since $z > \sum_{i \in B} P_{Li}$, then $z > \sum_{i \in G} P_{Gi}$. In addition, since $z > \sum_{i \in G} P_{Gi}$, then $\Delta f_m > 0$, and $ACE > 0$. So, $z - \sum_{i \in G} P_{Gi} > 0$ and $ACE > 0$, and as we can see from the block diagram, the negative signs after the integrator make $z$ to decrease as expected. Then, the frequency is restored to the nominal value. We plot the ACE, the AGC system state $z$, and the total generation $\sum_{i \in G} P_{Gi}$ in Fig. 2.5, after a decrease in load by 0.2 pu.

Now, let us go through another simple example. We modify the load at bus 5 by 0.45 pu. We keep the AGC system off and turn it on at time $t = 150$ s. As depicted in Fig. 2.6a, the frequency and the ACE converge to non-nominal and non-zero values respectively, due to primary control. However, in order to restore the system frequency to the nominal value, we need the AGC system. We notice that
after the AGC system is turned on, the ACE converges to zero and the frequency to the nominal value. In Fig. 2.6b, the evolution of the generators output is depicted, for participation factors: $\kappa_1 = 0.520$, $\kappa_2 = 0.2618$, and $\kappa_3 = 0.2362$.

Another important issue in AGC systems is the quality of service provided. This is influenced by the ramping rates of the generators participating in regulation. To this end, we depict in Fig. 2.7, the frequency evolution for two different cases. For each case we modify the participation factor $\kappa_i$ of each generator, as seen in Fig. 2.7. Generator 3 is faster than generator 2, which is faster than generator 1. We notice that when the participation factor of the fastest generator is set to $\kappa_3 = 0.8$, then the frequency converges to zero faster than when it is set to $\kappa_3 = 0.1$. Thus, the quality of AGC service provided is better when the ramping rates of the regulating units are high.
CHAPTER 3

EFFECTS OF UNCERTAINTY ON FREQUENCY CONTROL

We devote this chapter to describe the challenges faced by the AGC systems due to the technological changes in the electric grid. More specifically, we discuss how the uncertainty from load variations, renewable-based generation and noise in communication channels affect the performance of the AGC system. To this end, we develop a framework that captures the uncertainty arising from the aforementioned sources, and propagates it in the system variables, such as frequency. Furthermore, we present the frequency performance criteria, and obtain their probabilistic expressions to find limiting values of uncertainty that a system may withstand.

3.1 Introduction

The AGC system goal to maintain reliability is becoming more challenging due to the radical transformations occurring in the structure and functionality of power systems. For example, wind-based generation is not only intermittent and highly variable, it also introduces an additional source of uncertainty to power system operations. As a consequence, ISOs must schedule adequate electric supply from traditional generators on AGC to manage larger net system load variations caused by increased levels of wind-based generation. The AGC system accepts measurements of the real power interchange between BA areas, the area frequency and the generators output as inputs from field devices and processes them to obtain the output control signals, i.e., generator control commands. The combination of uncertainty from load variations, renewable-based generation and noise in com-
munication channels, however, may affect the AGC system performance, thus hindering the overall system reliability. Furthermore, the communication networks on which AGC operation relies, may lead to increased vulnerabilities and risk for cyber attacks [57]. For example, a cyber attack may mask itself as noise in communication channels and influence the AGC system performance. Thus, there exists a need to study the combined effect of load variation uncertainty, renewable-based generation uncertainty, and noise in communication channels on the function of the AGC system.

The functionality of the AGC system is quantified by performance frequency criteria set by NERC. More specifically, CPS1 is a statistical measure of ACE variability and assesses the impact of individual ACEs on interconnection frequency variations over a 12-month sliding window using one-minute average compliance factors. CPS2 is a monthly measure that a BA area must report to NERC and is a statistical measure of the ACE magnitude; it is calculated by averaging the ACE for each 10-minute period within a month. CPS2 is the percentage calculated by dividing the number of averages that are less than the BA area CPS2 limit by the total number of averages. A monthly CPS2 score of 90% or more is considered acceptable [13]. BAAL is designed to replace the CPS2 standard; therefore, no controversy is expected from interaction of local frequency-based controls with the CPS2 requirements. The BAAL standard is expected to relax the area regulation needs and reduce the regulation burden on resources providing regulation service [12].

Given the changes in structure that power systems are undergoing, there is a need to investigate if the current AGC system implementations are sufficient for meeting their objectives and to determine their limitations. To this end, this chapter proposes a framework to evaluate the effects of uncertainty sources, such as load variations, renewable-based generation and noise in the communication channels, and to assess AGC system performance. The framework includes models for power system dynamics, the AGC system dynamic behavior, and the aforementioned uncertainty sources, as described in Section 3.2. We use the framework to
calculate several moments and approximate the probability distribution function of system variables, such as frequency. We also present the frequency performance criteria — CPS1, CPS2 and BAAL — in Section 3.3 and obtain their probabilistic expressions based on the framework we developed in Section 3.4. To this end, we may explore if reliability criteria are met under different scenarios for each of the uncertainty sources. For example, we may investigate whether or not the functionality provided by current AGC system implementations is appropriate for dealing with high levels of renewable-based generation combined with noise in communication channels. In Section 3.5, we summarize the results presented.

3.2 Power System Dynamics Stochastic Model

In this section, we present the non-linear and linearized power system model including the AGC system dynamics. In addition, we provide the modeling of the uncertainty sources to develop the power system dynamic stochastic model. We, then, use the generator model to approximate the probability distribution functions of algebraic and dynamic system variables.

For the timescales of interest we choose a nine-state model for the synchronous generators, as described in Section 2.1.1. More specifically, for the \(i^{th}\) synchronous machine, the nine states are: the field flux linkage \(E'_{q_i}\), the damper winding flux linkage \(E'_{d_i}\), the rotor electrical angular position \(\delta_i\), the rotor electrical angular velocity \(\omega_i\), the scaled field voltage \(E_{fd_i}\), the stabilizer feedback variable \(R_{fi}\), the scaled output of the amplifier \(V_{R_i}\), the scaled mechanical torque to the shaft \(T_{Mi}\), and the steam valve position \(P_{SV_i}\). We define as \(P_{Ci}\) the AGC command signal that the system operator sends to each generator \(i\) participating in regulation. We also denote by \(I_{dq_i}\) (\(I_{q_i}\)) the d-axis (q-axis) component of the stator current, \(\theta_i\) the voltage angle, and \(V_i\) the voltage magnitude at bus \(i\). Let \(P_{Gi}\) and \(P_{Wi}\) represent the real power generated from the synchronous generator and wind resource at bus \(i\), and let \(P_{Li}\) (\(Q_{Li}\)) represent the active (reactive) power load at bus \(i\). Consider a network with \(N\) nodes and \(I\) generators, which is divided into \(M\) BA areas.
We define \( x = [x_1^T, \ldots, x_I^T]^T \), where \( x_i = [E_{q_i}^e, E_{d_i}^e, \delta_i, \omega_i, E_{fd_i}, R_{fi}, V_{R_i}, T_{Mi}, P_{SV_i}]^T \) and \( u = [P_{C_1}, \ldots, P_{C_I}]^T \), as given in (2.35). In addition, the vector of machine algebraic variables by \( \tilde{y} = [\tilde{y}_1^T, \ldots, \tilde{y}_I^T]^T \), with \( \tilde{y}_i = [I_{d_i}, I_{q_i}]^T \), the vector of network variables \( y = [y_1^T, \ldots, y_N^T]^T \), with \( y_i = [\theta_i, V_i]^T \). The vector of active (reactive) loads is denoted by \( P_L = [P_{L_1}, \ldots, P_{L_N}]^T \) (\( Q_L = [Q_{L_1}, \ldots, Q_{L_N}]^T \)), and the vector of wind generation by \( P_W = [P_{W_1}, \ldots, P_{W_N}]^T \). The dynamic behavior of wind-based resources is given in (2.19). We denote by \( v = [v_1, \ldots, v_N]^T \) the vector of wind speeds. We denote the vector of AGC states by \( z = [z_1, \ldots, z_M] \). The AGC system is described in Section 2.4, where we use (2.29) for the expression of the system frequency, and \( \eta_1 = 0, \eta_2 = 1 \), for the AGC system in (2.32). The network equations are given in Section 2.3. Then, the system dynamic behavior including the AGC system is described by

\[
\dot{x} = f(x, y, \tilde{y}, u),
\]
\[
\dot{z} = h(x, y, \tilde{y}, z),
\]
\[
\dot{P}_W = q(P_W, v),
\]
\[
u = k(z),
\]
\[0 = g_1(x, y, \tilde{y}),\]
\[0 = g_2(x, y, \tilde{y}, P_L, Q_L, P_W),\]

where \( g_1(\cdot) \) and \( g_2(\cdot) \) represent the machine algebraic and network equations, respectively (see, e.g., [52]). The functions \( f, h, q, k, g_1, \) and \( g_2 \) are continuously differentiable with respect to their arguments.

### 3.2.1 Small-Signal Model

In the non-linear differential algebraic equation (DAE) model described in (3.1)-(3.6) we consider three sources of uncertainty arising from load variations, wind-based generation and noise in communication channels. For the timescales of interest we assume that the disturbances due to the uncertainty sources introduce
a small error and therefore we may linearize the system along a nominal trajectory 
\((x^*, y^*, \tilde{y}^*, u^*, z^*, P^*_W, v^*, P^*_L, Q^*_L)\). Sufficiently small variations around the system nominal trajectory may be approximated by

\[
\Delta \dot{x} = A_1(t) \Delta x + A_2(t) \Delta y + A_3(t) \Delta \tilde{y} + B_1(t) \Delta u, \quad (3.7)
\]
\[
\Delta \dot{z} = A_4(t) \Delta x + A_5(t) \Delta y + A_6(t) \Delta \tilde{y} + A_7(t) \Delta z, \quad (3.8)
\]
\[
\Delta \dot{P}_W = \varrho_1(t) \Delta P_W + \varrho_2(t) \Delta v, \quad (3.9)
\]
\[
\Delta u = B_2(t) \Delta z, \quad (3.10)
\]
\[
0 = C_1(t) \Delta x + C_2(t) \Delta y + C_3(t) \Delta \tilde{y}, \quad (3.11)
\]
\[
0 = C_4(t) \Delta x + C_5(t) \Delta y + C_6(t) \Delta \tilde{y} + D_1(t) \Delta P_L + D_2(t) \Delta Q_L + D_3(t) \Delta P_W, \quad (3.12)
\]

where the matrices \(A_1(t), A_2(t), A_3(t), A_4(t), A_5(t), A_6(t), A_7(t), B_1(t), B_2(t), C_1(t), C_2(t), C_3(t), C_4(t), C_5(t), C_6(t), D_1(t), D_2(t)\) and \(D_3(t)\), and the vectors \(\varrho_1(t), \varrho_2(t)\) are defined appropriately and evaluated along the nominal trajectory as the partial derivatives of the functions \(f, h, q, k, g_1, g_2\) in (3.1)–(3.6) (see, e.g., [52], [58]). In our formulation, we consider \(\Delta Q_L = 0\), so we ignore the term \(D_2(t) \Delta Q_L\) in (3.12). We assume the nominal trajectory is well behaved and admits the invertible Jacobians \(C_3(t)\) and \(C_6(t) C_3^{-1}(t) C_1(t) - C_5(t)\).

### 3.2.2 Stochastic Differential Equation Model

We model the variation in the wind speed as a stochastic process:

\[
d \Delta v_i = a_i \Delta v_i \, dt + b_i \, dW^3_i, \quad (3.13)
\]

where \(W^3_i\) is a Wiener process and \(a_i, b_i\) are parameters constructed based on a priori knowledge of the wind speed probability distribution [59]. For the load variations, we follow a similar approach as for the wind generation [60]. To this end, we model the load variations as a stochastic process driven by a Wiener
process

\[ d\Delta P_{L_i} = \nu_1 \Delta P_{L_i} dt + \nu_2 dW_t^2. \]  

(3.14)

Potential noise in communication channels may cause uncertainty in measurements of \( \Delta P_{mm'}, \Delta f_m \) and \( \Delta P_{G_i} \), which are used as feedback inputs for AGC. Let \( \Gamma \) be the vector containing all the \( \Delta P_{mm'}, \Delta f_m \), and \( \Delta P_{G_i} \). We denote the measurements of \( \Gamma \) as \( \hat{\Gamma} \),

\[ \hat{\Gamma} = \Gamma + \eta, \]  

(3.15)

where \( \eta \) represents the measurement noise, modeled as Gaussian white noise. The area control error as well as the AGC system is affected by \( \eta \) as may be seen in (2.30) and (2.32). Including this additional source of uncertainty in (3.8), we obtain

\[ \Delta \dot{z} = A_4(t) \Delta x + A_5(t) \Delta y + A_6(t) \Delta \tilde{y} + A_7(t) \Delta z + A_8(t) \eta. \]  

(3.16)

In (3.11) and (3.12), since \( C_3(t) \) and \( C_6(t)C_3^{-1}(t)C_1(t) - C_5(t) \) are invertible, we can solve for \( \Delta y \) and \( \Delta \tilde{y} \). We substitute \( \Delta y, \Delta \tilde{y}, \) and \( \Delta u \) in (3.7), (3.9) and (3.16), and obtain the following ordinary differential equation (ODE) model:

\[ dX_t = AX_t dt + BdW_t, \]  

(3.17)

where \( X_t = [\Delta x, \Delta z, \Delta P_L, \Delta P_W, \Delta v]^T \), \( A, B \) as defined in the Appendix A, and \( dW_t = [dW_t^1, dW_t^2, dW_t^3]^T \).

There are cases where we wish to represent the deepening penetration of wind generation and the increased level of variability in the output. We assume that the wind penetration is now \( \hat{P}_{W_i}' = \xi_i P_{W_i} \). To this end, we use (2.20) and model the variation in the wind generation as

\[ \Delta \hat{P}_{W_i} = \varrho_1 \Delta P_{W_i} + \xi_i \varrho_2 \Delta v_i. \]  

(3.18)

We have \( P_{W_i}' = \xi_i P_{W_i} \rightarrow \Delta P_{W_i}' = \xi_i \Delta P_{W_i} \), since the nominal point around which
we linearize is now $P_{Wi}^*=\xi_i P_{Wi}^*$. We now need to modify only one entry in the $B$ matrix to represent the deepening penetration of renewable-based generation.

3.2.3 Infinitesimal Generator

The overall model, described in (3.17), is used to study the impact of the uncertainty sources on the system performance. To this end, we use the generator of the process $X_t$ to calculate the statistics of the states of interest. Specifically, given a twice continuously differentiable function $\psi$, the generator of the process $X_t$ is defined as (see, e.g., [61]):

\[
(L\psi)(x,t) := \frac{\partial \psi(x,t)}{\partial t} + \frac{\partial \psi(x,t)}{\partial x}Ax + \frac{1}{2}\text{Tr}\left(B \frac{\partial^2 \psi(x)}{\partial x^2}B^T\right). \tag{3.19}
\]

The evolution of the expected value of $\psi(x)$ is governed by Dynkin’s formula (see, e.g., [61]):

\[
\frac{dE[\psi(X(t))]}{dt} = E[(L\psi)(X(t))], \tag{3.20}
\]

where $E[\cdot]$ is the expectation operator. By properly choosing function $\psi$, we may obtain ODEs that yield the desired moments of the dynamic/algebraic states (e.g., the expected values and variances of the load voltages and area frequencies). Therefore, we may study the effect of the uncertainty in wind-based generation, load variations and noise in communication channels on the system frequency and the area control error, which are used in the frequency performance criteria. For example, we may use (3.19) and (3.20) to obtain a formula for the evolution of the first and second moments of the system states:

\[
\frac{dE[X_t]}{dt} = A E[X_t], \tag{3.21}
\]
\[
\frac{d\Sigma(t)}{dt} = AS(t) + \Sigma(t)A^T + BB^T, \tag{3.22}
\]
\[
E[X_tX_t^T] = \Sigma(t) + E[X_t]E[X_t]^T. \tag{3.23}
\]
3.2.4 Numerical Examples

We illustrate the proposed framework with a four-bus test system, as depicted in Fig. 3.1, which contains two synchronous generating units in buses 1 and 4, one wind generation unit in bus 2 and load in bus 3. For this example we do not consider load variations. The machine, network, and load parameter values for this example are listed here: the system MVA base is 100; the synchronous speed, \( \omega_s = 377 \text{ rad/s} \); the machines shaft inertia constants, \( H_1 = 23.64 \) and \( H_4 = 6.4 \); the machines damping coefficients \( D_1 = 0.0125 \) and \( D_4 = 0.0068 \); the machine impedances, \( X_{d1} = 0.146 \), \( X_{d4} = 0.8958 \), \( X_{q1} = 0.0969 \) and \( X_{q4} = 0.8645 \); the governor droops \( R_{D1} = R_{D4} = 0.05 \); and the parameters \( T_{do1} = 8.96 \), \( T_{do4} = 6.0 \), \( T_{qo1} = 0.31 \), \( T_{qo4} = 0.535 \), \( T_{SV1} = T_{SV4} = 2 \), \( T_{F1} = T_{F4} = 0.35 \), \( K_{F1} = K_{F4} = 0.063 \), \( T_{E1} = T_{E4} = 0.314 \), \( K_{E1} = K_{E4} = 1 \), \( T_{A1} = T_{A4} = 0.2 \), and \( K_{A1} = K_{A4} = 20 \). The network impedances between bus \( i \) and \( j \) are denoted by \( Z_{ij} \), so we have:

\[
Z_{12} = 0.0101 + j0.0504, \quad Z_{14} = Z_{23} = 0.0074 + j0.0372, \quad Z_{34} = 0.0127 + j0.0636.
\]

For simplicity, we consider one BA area for the system \( (M = 1) \) and choose the frequency bias factor to be \( b_1 = -0.1 \text{ MW/Hz} \). The AGC participation factors for each generator are \( \kappa_1 = \frac{2}{3} \) and \( \kappa_4 = \frac{1}{3} \). We solve the power flow equations and the machine algebraic equations such that the wind generation in bus 2 is \( P_{W2} = 0.298 \), the synchronous generator output in bus 4 is \( P_{G4} = 0.5 \), the load in bus 3 is \( P_{L3} + jQ_{L3} = 2.5 + j1.25 \), the voltage magnitude in bus 1 is \( V_1 = 1 \) and in bus 4 is \( V_4 = 1.02 \), with generator in bus 1 as the slack bus. We linearize the system of non-linear equations around the nominal point determined by solving
the algebraic equations. The variation of the wind generation output in bus 2 is $\Delta P_{W_2}$, and its evolution is described by

$$
\Delta \dot{P}_{W_2} = -0.1585 \Delta P_{W_2} + 0.0118 \Delta v_2,
$$

(3.24)

where the variation in the wind speed $\Delta v_2$ is described by the stochastic process $d\Delta v_2 = -6.2697 \Delta v_2 dt + 10.9571 dW_t$.

We model potential noise in communication channels as a white noise process which we denote by $\eta$. Then, from (2.31) the ACE becomes

$$
ACE = 0.1(\Delta f + \eta).
$$

(3.25)

In order to investigate the effectiveness of the AGC system, we choose to calculate the mean value and higher-order moments of the frequency deviation as
Figure 3.3: Case (ii): Noise in communication channels.

given in (2.29) by assigning equal weights to each bus, i.e., $\gamma_{2i} = \frac{1}{2}$, $i = 1, 4$, so we have

$$\Delta f = \frac{1}{4\pi} (\Delta \omega_1 + \Delta \omega_4).$$

(3.26)

To this end, the frequency deviation may be expressed as a linear combination of the system states

$$\Delta f = CX_t,$$

(3.27)

where $C = \frac{1}{2\pi} [\frac{1}{2} 0 0 \frac{1}{2}] [0_{4x4} 1 0_{4x7} 1 0_{4x5} 0_{4x13}]$. Then $E[\Delta f] = CE[X_t]$ and $E[\Delta f^2] = CEE[X_tX_t^T]C^T$.

We run three test cases in which we consider uncertainty in (i) the wind generation output, (ii) noise in communication channels and (iii) a combination of both. We validate the accuracy of the proposed framework by comparing the results with averaged Monte Carlo simulations, using the non-linear model given in
Figure 3.4: Case (iii): Uncertainty in wind-based generation and noise in communication channels.

(3.1)-(3.6). Figures 3.2-3.4 depict the evolution of the mean value and the second moment of the frequency variation for the aforementioned three cases. The results obtained with the proposed framework are superimposed on those obtained by averaging the results of 1000 Monte Carlo simulations. The results provided by the analytical method, i.e., Dynkin’s formula, match those obtained by averaging the results of repeated simulations. In this case, the AGC system meets its objective, since the mean value of the frequency variation $E[\Delta f]$ converges to zero and the second order moment $E[\Delta f^2]$ for all cases converges to a small value with magnitude of $10^{-6}$. We may use the limits of $E[\Delta f^2]$ and obtain an approximation for the standard deviation of $\Delta f$.

In case (iii) this approximation is higher than that in cases (i) and (ii), since $\lim_{t \to \infty} E[\Delta f^2]_{(i)} = 3.04 \times 10^{-7}$, $\lim_{t \to \infty} E[\Delta f^2]_{(ii)} = 3.51 \times 10^{-6}$ and $\lim_{t \to \infty}$
$\mathbb{E}[\Delta f^2]_{(iii)} = 4.53 \times 10^{-6}$, as depicted in Figs. 3.2b, 3.3b and 3.4b. This is expected since the combination of uncertainty in both wind-based generation and noise in communication channels is reflected in the AGC performance, thus higher variations of frequency are observed. However, the resulting variations in frequency are still sufficiently small that they lie within acceptable limits for the system reliability. Moreover, we notice that noise in communication channels has a greater effect on the frequency deviation than wind-based generation. This is due to the wind turbine characteristics, as well as the features of the wind speed data. However, it is possible to choose another case for which the opposite effect may be observed.

Finally, we increase the wind penetration from the initial value $P_{W_2} = 0.298$ to $P'_{W_2} = \xi P_{W_2}$, where the parameter $\xi$ belongs in $[1, 6]$ and is modified in increments of 0.5. Then, we investigate the impacts on the second moment of $\Delta f$. In order to represent the deepening penetration of wind generation and the increased level of variability in the output, we model the variation in the wind generation as

$$\Delta \dot{P}_{W_2} = -0.1585 \Delta P_{W_2} + 0.0118 \xi \Delta v_2.$$ (3.28)

We have $P'_{W_2} = \xi P_{W_2} \rightarrow \Delta P'_{W_2} = \xi \Delta P_{W_2}$, since the nominal point around which we linearize is now $P'_{W_2}^\star = \xi P_{W_2}^\star$. We observe that the second moment of $\Delta f$ is higher as we increase the wind penetration levels, as shown in Fig. 3.5.
3.3 Frequency Performance Criteria

NERC has established the CPS1, CPS2 and BAAL criteria to measure whether or not system frequency is maintained within certain limits. More specifically, for BA area $m$, CPS1 is given by

$$\sum_{i \in \mathcal{T}_1} \langle ACE \rangle_{1m_i} \langle \Delta f \rangle_{1m_i} \leq -b_m \epsilon_{1m_i}^2,$$

(3.29)

where $\langle \cdot \rangle_{1m_i}$ denotes the $i$th average over a one-minute period for BA area $m$ of each variable respectively, $\mathcal{T}_1$ is the set of time instants for which we have measurements for the one-minute averages of $\Delta f_m$ and $ACE_m$ over a one year period, $|\mathcal{T}_1|$ is the cardinality of the set $\mathcal{T}_1$, and $\epsilon_{1m}$ is a constant unique for each BA area $m$. The CPS2 is designed to limit the BA area unscheduled power flows. To this end, the CPS2 is given by

$$\langle ACE \rangle_{10m_i} \leq L_{10m_i},$$

(3.30)

$$1 - \frac{\text{number of violations of (3.30)}}{|\mathcal{T}_2|} \geq 0.9,$$

(3.31)

where $\langle \cdot \rangle_{10m_i}$ denotes the $i$th average over a 10-minute period for BA area $m$ of the area control error, $\mathcal{T}_2$ is the set of time instants for which we have measurements for the 10-minute averages of $ACE_m$ over a one month period, $|\mathcal{T}_2|$ is the cardinality of the set $\mathcal{T}_2$, and $L_{10m}$ a constant specific for each BA area $m$. The BAAL criterion, which will replace the CPS2 criterion, may be formulated as follows

$$BAAL_{low}(f_m) = -b_m \frac{(f_{low} - f_{nom})^2}{f_m - f_{nom}},$$

(3.32)

$$BAAL_{high}(f_m) = -b_m \frac{(f_{high} - f_{nom})^2}{f_m - f_{nom}}.$$  

(3.33)

For each violation, the BAAL standard allows a BA area to have its ACE outside the BAAL limits for a certain time, which is 30 min. The BAAL relaxes the regulation limits compared to CPS2 as shown in Fig. 3.6. For small frequency
deviations from the nominal value, the BAAL limit is larger than that enforced from CPS2. However, for large frequency deviations the opposite occurs, i.e., the BAAL criterion enforces tighter limits. Such a criterion is better in the sense that larger frequency deviations are penalized more than smaller ones.

The frequency criteria are expressed in MW-Hz units. There is a rationale behind such a choice. As seen in Fig. 3.7, there are cases where ACE is negative and the frequency deviation from the nominal value $\Delta f$ is positive, and their product is negative. A negative ACE means the BA area is exporting less than the scheduled value. Positive frequency deviation means that the entire interconnection is over-generating. So the BA area with ACE positive is helping the interconnection to restore its frequency to the nominal value. Similar arguments may be made for the opposite case. However, when $ACE$, $\Delta f$, and $ACE \cdot \Delta f$ are all positive, then the BA area is exporting more than scheduled, and the interconnection is over-generating. Thus, the BA area is hurting the interconnection.

Figure 3.6: BAAL criterion.
3.4 Probabilistic Expression of Frequency Performance Criteria

We use the framework developed in Section 3.2 to obtain probabilistic expressions of the three frequency performance criteria. To this end, we express the area control error of BA area $m\ ACE_m$ and the deviation of the area frequency from
the nominal value $\Delta f_m$ as a function of the system states $X_t$. We linearize (2.29) and (2.30) along the nominal trajectory, and obtain

$$\Delta f_m = \Phi_{1m} X_t,$$

(3.34)

$$ACE_m = \Phi_{2m} X_t.$$  

(3.35)

We wish to obtain the probability distribution functions (pdfs) of $ACE_m$ and $\Delta f_m$. Since, the overall model given in (3.17) is driven by a Wiener process, then the system states $X_t$ follow a normal distribution [62]. Thus, only the first and second moments are needed, to obtain the pdfs of $X_t$. Both $ACE_m$ and $\Delta f_m$ are linear combinations of $X_t$, so they also follow a normal distribution. We use (3.20) and obtain the first and second moments of $ACE_m$ and $\Delta f_m$ by appropriately selecting the function $\psi(\cdot)$. For example, if we choose $\psi(X_t) = \Phi_{2m} X_t$, then we may determine the mean value of $ACE_m(t)$. The first moment of $ACE_m(t)$ is zero, since in this case $\psi(x)$ is time invariant and linear with respect to $X_t$, so $\frac{\partial^2 \psi(x)}{\partial x^2}$ is zero. In order to determine the second moment of $ACE_m(t)$, we set $\psi(X_t) = \Phi_{2m} X_t X_t^T \Phi_{2m}^T$ in (3.20). To this end, the random variable $ACE_m(t)$ follows a Gaussian distribution with zero mean and variance $\sigma_{ACE}^2$.

We notice that the variables included in (3.29)-(3.30) are time averages of either $ACE_m$ or $\Delta f_m$, so we need to determine the pdfs of those variables, given that we have the pdfs of $ACE_m$ and $\Delta f_m$. We show the derivation for the vector of random variables $X_t$, since both $ACE_m$ and $\Delta f_m$ are linear combinations of $X_t$. The vector of random variables $X_t$ follows a Gaussian distribution with zero mean and covariance matrix $\Sigma_X$, which is determined from Dunkin’s formula.

\[ \mathcal{I} = \{t_j, j = 1, \ldots, J\} \]

\[ \mathcal{T} = [t_s + (i - 1)L, t_s + iL], i = 1, \ldots, K \]

Figure 3.8: Time interval.
using (3.22) and (3.23). Let us assume a time interval \([t_s, t_e]\), as seen in Fig. 3.8, we choose a window length \(L\) and define the subinterval \(\mathcal{T}^i = [t_s + (i-1)L, t_s + iL]\), \(i = 1, \ldots, K\), where \(K = \frac{t_e - t_s}{L}\). For each subinterval, we define the measurement subset \(\mathcal{M}^i = \{t_j, j = 1, \ldots, J\}\). For the one-minute average, with \(L = 1\) min, we have the average random variable \(\langle X \rangle_1\), and for the 10-minute average, with \(L = 10\) min, we have \(\langle X \rangle_{10}\). We now have that

\[
\langle X \rangle_{L_i} = \frac{1}{J} \sum_{t_j \in \mathcal{M}^i} X(t_j), i = 1, \ldots, K.
\]

(3.36)

In order to determine the pdf of the \(L\)-minute averages of the \(X_t\), we use the central limit theorem for dependent variables [63]. The cardinality \(J\) of each \(\mathcal{M}^i\) is sufficiently large so as to permit the application of the central limit theorem. To this end, we have that \(\langle X \rangle_{L_i}\) follows a Gaussian distribution with zero mean and covariance matrix \(\Sigma_L\), which is given by

\[
\Sigma_L = \mathbb{E}[X(t_1)X^T(t_1)] + 2\mathbb{E}[X(t_1)X^T(t_2)] + \cdots + 2\mathbb{E}[X(t_1)X^T(t_J)].
\]

(3.37)

We notice that \(\Sigma_L\) is a matrix whose elements consist of combinations of the elements of the matrices \(\mathbb{E}[X(t_i)X^T(t_i)]\) and \(\mathbb{E}[X(t_i)X^T(t_j)]\), for \(i, j = 1, \ldots, J\). We know that the value of \(\mathbb{E}[X(t_i)X^T(t_i)]\) is \(\Sigma_X\). In order to determine the values of \(\mathbb{E}[X(t_i)X^T(t_j)]\), for \(i, j = 1, \ldots, J\) with \(i \neq j\), we use the fact that \(X_t\) is a stationary process. It may be shown that the system given in (3.17) is a wide-sense stationary process [64]. So, we have that \(\mathbb{E}[X(t_i)X^T(t_j)] = \mathbb{E}[X(t_i - t_j)X^T(t_i - t_j)]\), and we use the fact that \(\mathbb{E}[X(t_i - t_j)X^T(t_i - t_j)] = e^{A(t_j - t_i)}\mathbb{E}[X(t_i)X^T(t_i)] = e^{A(t_j - t_i)}\Sigma_X\), for \(t_i < t_j\) [62]. We use this procedure and obtain the pdfs of \(\langle ACE \rangle_{1m_i}, \langle \Delta f \rangle_{1m_i}\), and \(\langle ACE \rangle_{10m_i}\). For example, \(\langle ACE \rangle_{1m_i}\) follows a Gaussian distribution with zero mean and variance \(\sigma^2_{\langle ACE \rangle_{1m_i}}\), which is equal to \(\Phi_2^m \Sigma_1 \Phi^T_2\), where \(\Sigma_1\) is the covariance matrix of the random variable \(\langle X \rangle_{11}\).

Furthermore, we assume that the elements of the discrete time stochastic process \(\{\langle X \rangle_{L_i}, i = 1, \ldots, N\}\) are independent and identically distributed random
variables, thus ergodic. So the statistical properties (such as its mean and variance) of the process may be deduced from a single, sufficiently long realization. To this end, the CPS1 is equivalent to

\[ \Phi_{2m} E[(\langle X \rangle_1 \langle X \rangle_1^T)] \Phi_{1m}^T < -b_m \epsilon_m^2, \tag{3.38} \]

where \( E[\langle X \rangle_1 \langle X \rangle_1^T] = \Sigma_1 \). As for CPS2 we define the variable

\[ \Upsilon_i = \begin{cases} 1, & |\langle ACE \rangle_{10m_i}| < L_{10m} \\ 0, & \text{otherwise} \end{cases}, \quad \text{for } i = 1, \ldots, K. \tag{3.39} \]

So the CPS2 may be written as:

\[ E[\Upsilon_i] = \Pr\{|\langle ACE \rangle_{10m_i}| < L_{10m}\} = \Pr\{\langle ACE \rangle_{10m_i} < L_{10m}\} \]
\[ - \Pr\{\langle ACE \rangle_{10m_i} < -L_{10m}\} \geq 0.9, \tag{3.40} \]

which may be easily calculated since \( \langle ACE \rangle_{10m_i} \) follows a Gaussian distribution, with known mean and variance values.

For the BAAL criterion, we wish that

\[ -b_m (f_{\text{low}} - f_{\text{nom}})^2 \leq \frac{\sum_{t \in \mathcal{T}_3} ACE_m(t) \Delta f_m(t)}{|\mathcal{T}_3|} \leq -b_m (f_{\text{high}} - f_{\text{nom}})^2, \]

where \( \mathcal{T}_3 \) is the set of time instants for which we have measurements for \( ACE_m(t) \) and \( \Delta f_m(t) \) for a 30 minute period. However, we may express \( ACE_m(t) \) and \( \Delta f_m(t) \) as a function of \( X_t \). We assume that the statistical properties (such as its mean and variance) of the process may be deduced from a single, sufficiently long realization. So equivalently, we have

\[ \frac{\sum_{t \in \mathcal{T}_3} ACE_m(t) \Delta f_m(t)}{|\mathcal{T}_3|} = E[ACE_m(t) \Delta f_m(t)] = \Phi_{2m} E[X_t X_t^T] \Phi_{1m}^T. \]

The BAAL criterion may be expressed as follows

\[ b_m^2 (f_{\text{low}} - f_{\text{nom}})^2 \leq \Phi_{2m} E[X_t X_t^T] \Phi_{1m}^T \leq b_m^2 (f_{\text{high}} - f_{\text{nom}})^2. \tag{3.41} \]

We validate the results given in this section through a series of numerical examples.
3.4.1 Numerical Illustrations

We present several case studies to demonstrate the capabilities of the proposed methodology. We use a small system and the three-machine nine-bus system, to provide insights into the results presented. We demonstrate that the pdfs calculated by using Dynkin’s formula as well as the pdfs for the one-minute and for 10-minute average of the system variables match the results we obtain via Monte Carlo simulations of the non-linear system given in (3.1)-(3.6). To this end, the probabilistic expression of the frequency regulation criteria provides a good approximation of the actual frequency regulation measures. Furthermore, we include a larger system to show that there are no computational limitations in the proposed method.

Three-Machine Nine-Bus Power System

We illustrate the proposed methodology with the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system model, which is depicted in Fig. 3.9; this model contains three synchronous generating units in buses 1, 2 and 3, and load in buses 5, 6 and 8. We consider a wind-based generation at bus 6. The machine, network and load parameter values may be found in [52, pp. 170-172]. For simplicity, we consider one BA area for the system \((M = 1)\) and choose the frequency bias factor to be \(b = -1.1517 \text{ pu/Hz}\). The AGC

![Figure 3.9: One-line diagram of the three-machine nine-bus WECC power system.](image-url)
participation factors for each generator are \( \kappa_1 = 0.28 \), \( \kappa_2 = 0.47 \) and \( \kappa_3 = 0.25 \). Unless otherwise noted, all quantities in the numerical results section are expressed in per unit (pu) with respect to 100 MVA as base power. We solve the power flow equations and the machine algebraic equations such that the wind generation in bus 6 is \( P_{W_6} = 0.298 \), the load in bus 5 is \( P_{L_5} + jQ_{L_5} = 1.25 + j0.50 \), in bus 6 is \( P_{L_6} + jQ_{L_6} = 0.90 + j0.30 \) and in bus 8 is \( P_{L_8} + jQ_{L_8} = 1.50 + j0.35 \). We consider the generator in bus 1 as the slack bus. We linearize the system of non-linear equations around the nominal point determined by solving the algebraic equations. The noise in communication channels is modeled as a Gaussian distribution with zero mean and variance 0.01. The load variation is given by

\[
d\Delta P_{L_i} = -2 \cdot 10^{-6} \Delta P_{L_i} dt + 5 \cdot 10^{-3} dW_i^2, \quad \text{for } i = 5, 6, 8. \tag{3.42}
\]

The variation of the wind generation output in bus 6 is \( \Delta P_{W_6} \) and its evolution is described by

\[
\Delta \dot{P}_{W_6} = -0.1585 \Delta P_{W_6} + 0.0118 \Delta v_6, \tag{3.43}
\]

where the variation in the wind speed \( \Delta v_6 \) is described by the stochastic process

\[
d\Delta v_6 = -2.65 \cdot 10^{-4} \Delta v_6 dt + 1.62 \cdot 10^{-2} dW_3^3. \]

We use the Euler-Maruyama method to obtain paths of the stochastic differential equations (see, e.g., [65]). We use Dynkin’s formula as given in (3.20), with \( \psi(X_t) = \Phi_2 X_t \) and \( \psi(X_t) = \Phi_2 X_t X_t^T \Phi_2^T \), to calculate the mean value and the second moment of the area control error, respectively, as depicted in Fig. 3.10. The results obtained with the proposed framework are superimposed on those calculated by averaging the results of 1000 Monte Carlo simulations. The results provided by the analytical method, i.e., Dynkin’s formula, provide a good approximation compared to those obtained by averaging the results of repeated simulations. We notice that in this case, the AGC system meets its objective, since the mean value of the area control error \( \mathbb{E}[ACE] \) converges to zero and the second-order moment \( \mathbb{E}[ACE^2] \) converges to a small value with magnitude of \( 10^{-4} \). Since we know the first and second moment of the random variable \( ACE \), we may determine its pdf. To this
end $ACE$ follows a Gaussian distribution with zero mean and variance $4.211 \cdot 10^{-4}$.

We use the data from repetitive Monte Carlo simulations, to derive an empirical cdf of $ACE(t)$ and compare it to the cdf from the analytical approach, as depicted in Fig. 3.11a. We notice that the analytical method provides a bigger standard deviation for $ACE$ than that calculated based on the Monte Carlo simulations.

In the proposed framework the linearized model, given in (3.7)-(3.12), is used, whereas, in the Monte Carlo simulations the DAE model, given in (3.1)-(3.6), is used; that is the reason for the discrepancy in the results, as shown in Fig. 3.11a. To this end, the effects of uncertainty sources on the system are magnified with the analytical approach, which may lead to more conservative actions from the system operators. However, the analytical method provides a good approximation, and has the advantage of faster computations.

We now investigate the effects on the area control error of deepening renewable-
based generation, as described in (3.18). More specifically, we increase the wind penetration from the initial value $P_{W6} = 0.298$ to $P'_{W6} = \xi P_{W6}$, where the parameter $\xi$ belongs in $[1, 6]$ and is modified in increments of 0.5. Then, we investigate the impacts on the second moment of $ACE$. We observe that the second moment of the area control error is higher as we increase the wind penetration levels, as shown in Fig. 3.12. This result was expected, since renewable-based generation introduces variability and uncertainty to the system, which is reflected in the area control error.

We have shown that the proposed framework provides a good approximation of the system behavior, as validated via Monte Carlo simulations of the non-linear system. In order to calculate the values for the frequency performance criteria, we need to determine the pdfs of the one-minute and 10-minute averages of the system variables. In this case study, we consider only one BA area, thus the
frequency deviation and the area control error are proportional to each other, i.e., $ACE = -b \Delta f$. To this end, the vectors $\Phi_1$ and $\Phi_2$, are related with $\Phi_1 = \frac{1}{b} \Phi_2$. The frequency performance criteria may be expressed as a function of the characteristics of the $ACE$, one-minute average of $ACE$, $\langle ACE \rangle_1$, and the 10-minute average of $ACE$, $\langle ACE \rangle_{10}$, if we substitute $\Phi_1$ in the equations of Section 3.4 with $\Phi_1 = \frac{1}{b} \Phi_2$. We first need calculate the correlation of random variables $ACE(t_i)$ and $ACE(t_j)$ for some $i, j$ with $t_j > t_i$, i.e., $\mathbb{E}[ACE(t_i)ACE(t_j)] = \Phi_2 \mathbb{E}[X_{t_i}X_{t_i}^T]e^{A(t_j-t_i)}\Phi_2^T$. We depict the correlation for $ACE(t_0 = 0)$ with $ACE(t)$, for $t > 0$ in Fig. 3.11b. We notice that the correlation of random variables $ACE(t_1)$ and $ACE(t_k)$ drops significantly for $|t_1 - t_k| > 300$ s. In contrast with the one-dimensional case where the exponential term decays with all elements positive, in the matrix case due to the eigenvalues the direction is modified and we have negative correlation between the random variables. We use the central limit theorem for dependent variables and find that the random variable $\langle ACE(t) \rangle_1$, the one-minute average of the area control error, follows a Gaussian distribution with zero mean and variance $8.27 \cdot 10^{-6}$, and that $\langle ACE(t) \rangle_{10}$, the 10-minute average of the area control error, follows a Gaussian distribution with zero mean and variance $1.15 \cdot 10^{-6}$. We use the data from repetitive Monte Carlo simulations, calculate the one-minute and 10-minute averages, and derive empirical cdfs of $\langle ACE(t) \rangle_1$ and $\langle ACE(t) \rangle_{10}$, which we compare to the cdfs of the Gaussian distributions from the analytical approach, as shown in Figs. 3.13a and 3.13b. As in the case for the random variable $ACE$, the standard deviations for the one-minute and 10-minute

![Figure 3.12: Deepening wind penetration.](image-url)
averages are higher with the analytical approach. This is due to the fact that the error introduced in the random variable \( ACE(t) \) is propagated to \( \langle ACE(t) \rangle_1 \) and \( \langle ACE(t) \rangle_{10} \).

Based on the analysis in Section 3.4, we calculate the values of the frequency performance criteria. CPS1 criterion is equal to
\[
\frac{1}{\delta_E} \mathbb{E}[\langle ACE(t) \rangle_1^2] = \frac{1}{1.1517} \cdot 8.27 \cdot 10^{-6} = 7.179 \cdot 10^{-6}.
\]
We use the Monte Carlo simulations and calculate the CPS1 based on (3.29). Thus, we have that CPS1 is \( 6.799 \cdot 10^{-6} \). As for CPS2, we modify the value of \( L_{10} \) and show the sensitivity of the proposed method with respect to \( L_{10} \), as depicted in Fig. 3.14a. We notice that the proposed framework shows that CPS2 is violated in cases where it was not violated as established through the Monte Carlo simulations. However, the values of \( L_{10} \) corresponding to such cases were very small, and for larger values of \( L_{10} \) the results from the analytical approach and the Monte Carlo simulations are close and agree that no
violations are present. For the BAAL criterion, we need to calculate the values of $\mathbb{E}[ACE \Delta f]$. We know that $ACE = -b \Delta f$, thus $\mathbb{E}[ACE \Delta f] = \frac{\mathbb{E}[ACE^2]}{-b}$. We compare the value obtained from the analytical method with the results from the Monte Carlo simulations for $\sum_{t \in \mathcal{T}} ACE(t) \Delta f(t)$, where $\mathcal{T}$ corresponds to a 30-minute period. We depict the results in Fig. 3.14b. The probabilistic expression of the frequency performance criteria provides a good approximation of those calculated based on simulations. The analytical method magnifies the effects of the considered uncertainty sources, however its advantage is computational efficiency, which makes the introduced error acceptable. In order to quantify the effects of uncertainty sources on the system performance we need to run simulations for an entire year in the case of CPS1. In contrast, by using the proposed framework and the probabilistic expression of the frequency performance criteria, we only need to solve a system of ordinary differential equations.
Next, we demonstrate the scalability of the proposed methodology for large power systems. In particular, we examine the IEEE 48-machine test system, which consists of 140 buses and 233 lines [66]. To implement our analysis method, we use the MATLAB-based Power Systems Toolbox (PST) [67], and add the AGC system model described in (2.32) to it. We use the proposed framework to calculate the values for the frequency performance criteria and compare them with the results obtained via Monte Carlo simulations. We consider three sources of uncertainty — load variation, renewable-based generation, and noise in communication channels. Load variation and renewable-based generation are modeled as Wiener processes without scaling and drift, and noise in communication channels as white noise. We consider the entire system as one BA area, and we set the frequency bias factor

![Figure 3.15: Area control error characteristics.](image)

(a) Cumulative distribution function of $ACE(t)$.

(b) Correlation of the area control error.
for the AGC system at $-1$ pu. The AGC participation factors are set proportional to the inertia constants of the generators. We obtain the linear model with the help of the PST, and determine the matrices $A$ and $B$ in (3.17). We use Dynkin’s formula to obtain the first and second moment of $ACE$ and approximate its cdf. In Fig. 3.15a, we compare the cdf obtained with Dynkin’s formula, with the empirical cdf of the ACE determined by numerous Monte Carlo simulations of the non-linear system. We also depict the correlation for $ACE(t_0 = 0)$ with $ACE(t)$, for $t > 0$ in Fig. 3.15b. We may use this information and approximate the pdfs of the time averages of $ACE$ used in the frequency performance criteria, and thus determine their values. For instance, the value of CPS1 is $0.751 \cdot 10^{-8}$ calculated with the proposed framework and $1.711 \cdot 10^{-8}$ via simulations, respectively. We notice that the proposed framework provides a good approximation as also established in the smaller test system.

3.5 Summary

In this chapter, we developed a framework that was used for studying the impact on AGC system performance of uncertainty that arises from load variations, renewable-based generation and noise in communication channels. Through the case studies, we showed that Dynkin’s formula provides a good approximation of the system actual state, as validated via Monte Carlo simulations. We also demonstrated that our model captures the higher uncertainty caused by the deepening penetration of renewable resources. The proposed methodology may be used to detect, in a timely manner, the existence of a cyber attack, by computing the system frequency statistics and comparing them with those of the wind-based generation and communication channel noise. In order to find the limiting values of uncertainty that the system may withstand and maintain the desired reliability levels, we use the developed framework to obtain probabilistic expressions of the frequency performance criteria, and investigate the needs for new designs in AGC systems due to the changes in the electric grid.
Part II
Proposed Automatic Generation Control System Designs
We devote this chapter to the detailed analysis of alternative AGC system designs. We start out with the motivation of the presented work, we propose a reduced model for the synchronous generator dynamics that may be used to derive a model describing a BA area dynamic behavior. We utilize the latter to design adaptive AGC systems. We make use of available measurements of the system frequency, the area control error and the total generation, to online estimate the control gains in the AGC systems and increase their efficiency.

4.1 Introduction

The appropriate level necessary to describe power system components, e.g., synchronous generators, is determined by the type of phenomena that need to be studied. For example, simplified models, such as the classical model, may be used in studies where the focus is on slow varying transients. The general idea behind the simplified models is to approximate the behavior of selected dynamics, without having to explicitly solve the differential equations, by means of various integral manifolds. In the same vein, the reduced-order modeling techniques may be used in the BA area level to describe the BA area dynamic behavior.

Such simplified models may be used in the design of new AGC systems, since the slower varying transients are sufficient for describing the BA area dynamic behavior. There is a need for new AGC system designs since investigations reveal that current implementations result in relatively large overshoots and transient frequency deviations [68]. Most BA areas implement tie-line bias control, and the
AGC command is driven by the value of the ACE, which includes the deviation of the sum of tie line flows between the BA areas from the scheduled values and their obligation to support frequency. In order to prevent the AGC system from “fighting” the area natural response, the BA area obligation to support frequency is added in the ACE calculation. This term includes the frequency bias factor, which in the ideal case reflects the actual frequency response characteristic (AFRC) of the BA area. The AFRC is the change in frequency that occurs for a change in load-generation balance in an interconnection. The change of frequency is influenced by the natural load response to frequency and the governor response, which is determined by the generators droop settings. The amount of frequency decline from a lost generator or a change in load varies based on the time of day, the season as well as by interconnection. In the case where the frequency bias factors are equal to the AFRCs, if there is an external disturbance the ACE for the BA areas does not change, besides the ACE of the BA area where the disturbance occurred. Thus, the AGC systems of the BA areas do not “fight” their AFRC. In cases where the system frequency remains below acceptable values, then other BA areas provide support to restore the frequency to the nominal value. The frequency bias factor used in the ACE calculation is a negative number, i.e., the BA area output increases as frequency declines and is expressed in MW/0.1Hz. The curve that describes the relationship between changes in frequency and in the real power flow of a BA area tie line is depicted in Fig. 4.1. As it may be seen in Fig. 4.1, the relationship is non-linear. However, when the ACE is calculated by the various BA areas, a linear approximation is used and the slope of the line is set to a value equal to $b_m$. The simplest way to calculate the frequency bias factor, which is adopted by most ISOs, is the 1% of peak load method and stays constant throughout the year. In most cases, the 1% load method is greater than the BA area natural response and causes over-regulation in the BA area. Since over-regulation increases the regulation cost, BA areas are interested in improving the operations of generating control at low cost and maintaining control performance at satisfactory levels. Therefore, a method that estimates the AFRC, which in
turn is used in the ACE calculation increases the efficiency of the AGC system. Simplified power system models may be used to derive a relationship between system variables that we have measurements of and the AFRC; then, we may estimate the value of the AFRC.

Eigenanalysis is a commonly used approach for developing simplified models [25]; however, when performing eigenanalysis, the resulting reduced model is linear and sometimes it is difficult to interpret the equations physically. In this chapter, we keep the non-linearity of the synchronous generator model and substitute a portion of the differential equations with algebraic equations. We achieve this by choosing a submatrix of the linearized system and calculate the eigenvalues, by using the selective modal analysis (SMA) method [24]. The states of the reduced model are three: (i) the rotor electrical angular position, (ii) the rotor electrical angular velocity, and (iii) the mechanical power. The advantages of the reduced model are that it is simpler than the full model, approximates the system behavior in satisfactory levels, provides better accuracy compared to the classical model, and has lower computational burden compared to other reduced models, since only the eigenvalues of a submatrix are needed. We use the proposed reduced model to derive a set of differential equations that describe a BA area dynamics. We wish that this set of equations depends only on BA area vari-

Figure 4.1: Relationship between frequency deviation and load variation for each BA area.
ables. To this end, we use optimization techniques and define the BA area droop and damping coefficients. The derived model approximates better the BA area behavior than other BA area dynamic modeling, where the droop and damping coefficients are the summation of each generator droop and damping values [69]. We compared the proposed models with others, such as the full nine-state model, and validated that they provided a good approximation of the actual system state.

We use the BA area model to design adaptive AGC systems. Such a model is sufficient to model the system dynamics for AGC implementation, since the output command of the latter is the total generation needed in the BA area to restore the system frequency and the net interchange between BA areas to the desired values. In this regard, models that do not consider each generator states, but the BA area variables are sufficient. We modify the AGC system design and include an adaptive proportional controller with self-tuning gain technique that reflects the system AFRC. To implement such an adaptive controller, we need to online estimate the AFRC by using the BA model and deriving a relationship between the AFRC, the system frequency, the area control error and the total generation. In this chapter, we showed that the proposed estimation technique provides a good approximation of the AFRC and that when used in the ACE calculation the system frequency converges faster to the nominal value.

The remainder of this chapter consists of five additional sections. In Section 4.2, we describe the SMA methodology and derive the reduced-order model for the synchronous generator. We use this model in Section 4.3 to construct the model describing the BA area dynamic behavior. Once, we know how the BA area is behaving as a whole, we may derive a relationship between the ACE, the frequency deviation, the total generation and the AFRC at each time instant. In Section 4.4, we use this relationship and two different models for the AGC system to estimate the AFRC. We use a sliding exponentially weighted window blockwise least-squares algorithm, since there are available measurements of the ACE, the frequency and the total generation, and design the adaptive AGC systems. In Section 4.5, we summarize the proposed ideas, and the results presented.
4.2 Proposed Reduced-Order Generator Model

In this section, we describe the selective modal analysis method, and derive the proposed reduced-order synchronous model. We also present some numerical examples of the proposed reduced model.

4.2.1 Selective Modal Analysis Methodology

The SMA method is a framework for simplifying linear time-invariant models of dynamic systems in the form of \( \dot{x} = Ax \), with \( x \in \mathbb{R}^n \). This linear system contains several modes, that are determined by the eigenvalues of the matrix \( A \). Furthermore, there are associations between the states \( x \) and the natural modes of the system matrix \( A \), which describe the “dynamic pattern of behavior” of the system. We may determine such associations with the participation matrix, whose \((i, j)\)th element is defined as

\[
P_{ij} = \frac{|v_{ij}| u_{ij}|}{\sum_{\forall j} |v_{ij}| u_{ij}|},
\]

where \( v_{ij} \) (\( u_{ij} \)) is the \((i, j)\) element of the right (left) eigenvector. Each row corresponds to a state variable and each column to a mode. The sum of all rows of the participation factors is equal to 1. This means that all the states participate in some extent to each mode. A linear dynamic system contains many dynamic patterns of behavior that may be termed relevant or non-relevant for some intended application. Let us assume that there are \( h \) modes of interest. By using participation factors, \( n_1 \) states, denoted by \( r \), are found to be related to the modes of interest. Consequently, there are \( n_2 = n - n_1 \) states, denoted by \( \zeta \), not related to the relevant modes, where \( n \) is the number of total states. Note that \( n_1 \geq h \), i.e., there is at least one state associated with each mode. We partition the original system to

\[
\begin{bmatrix}
\dot{r} \\
\dot{\zeta}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
r \\
\zeta
\end{bmatrix}.
\]
A first approximation of the dynamic behavior of the relevant states would be \( \dot{r} = A_{11}r \). However, we wish to include the effects of the less relevant dynamics in the reduced system. Ideally, we wish to replace the less relevant portion of the model by some static model in such a form that the original \( h \) modes are not affected by the substitution. To this end, we use the transfer function \( H(s) = A_{12}(I - sA_{22})^{-1}A_{21} \), where \( I \) is an \( n_2 \times n_2 \) identity matrix. Let us consider the simple case, where \( h = 1 \) and the associated eigenvalue \( \lambda \), thus we have

\[
\Xi v = H(\lambda)v \Rightarrow \Xi = H(\lambda),
\]

(4.3)

where \( v \) is the right eigenvector that corresponds to mode \( h \) and the reduced-order model that keeps the dynamic behavior of the mode of interest, and incorporates the dynamic behavior of the less relevant states is

\[
\dot{r} = (A_{11} + \Xi)r.
\]

(4.4)

An interesting property of this choice is that when we replace the Laplace operator \( s \) in the less relevant states \( \zeta \), we “freeze” the less relevant portions for \( s = \lambda \). This may be extended for \( 1 \leq h \leq n_1 \), and we may determine the matrix \( \Xi \) by

\[
\Xi v_i = H(\lambda_i)v_i, \forall i = 1, \ldots, h.
\]

(4.5)

The previous approach is true when the eigenvalues are real distinct numbers. There is a variation of the algorithm when there exists a conjugate pair of eigenvalues. Let us assume that there exists \( \lambda_{j_1} = \lambda_{j_2}^* \). Then, we have

\[
\Xi \text{Re}\{v_{j_i}\} = \text{Re}\{H(\lambda_{j_i})v_{j_i}\},
\]

(4.6)

\[
\Xi \text{Im}\{v_{j_i}\} = \text{Im}\{H(\lambda_{j_i})v_{j_i}\}.
\]

(4.7)

We presented a brief summary of the SMA method that will be used in the Section 4.2.2. In particular, we demonstrated how the matrix \( \Xi \) is defined in
order to develop the reduced model in the form

\[ \dot{r} = (A_{11} + \Xi)r. \]

(4.8)

Further details may be found in [24].

4.2.2 Reduced-Order Synchronous Generator Model

We use the nine-state model, given in (2.1)-(2.9), to obtain a reduced three-state model with \( \delta_i, \omega_i, \) and \( P_{SV,i} \). We wish to keep (2.3), (2.4), and (2.9), and substitute the remaining differential equations with algebraic ones. To do so, we use concepts from the SMA, which is a method for simplifying complicated linear systems, as described in Section 4.2.1. We denote \( x_i = [r_i^T, \zeta_i^T]^T \), with \( r_i = [\delta_i, \omega_i, P_{SV,i}]^T \) and \( \zeta_i = [E_{q,i}', E_{d,i}', E_{f,i}, R_{f,i}, V_{R,i}, T_{Mi}]^T \), \( y_i = [V_i, \theta_i]^T \), \( \tilde{y}_i = [I_d,i, I_q,i]^T \), and \( u_i = P_{Ci} \).

We may linearize the system described in (2.1)-(2.10) along a nominal trajectory \((x_i^\star, y_i^\star, \tilde{y}_i^\star, u_i^\star)\). Sufficiently small variations around the system nominal trajectory may be approximated by

\[
\Delta \dot{x}_i = A_{11} \Delta x_i + A_{21} \Delta y_i + A_{31} \Delta \tilde{y}_i + B_i \Delta u_i, \quad (4.9)
\]

\[
0 = C_{11} \Delta x_i + C_{21} \Delta y_i + C_{31} \Delta \tilde{y}_i. \quad (4.10)
\]

where the matrices \( A_{11}, A_{21}, A_{31}, B_i, C_{11}, C_{21}, \) and \( C_{31} \) are defined appropriately and evaluated along the nominal trajectory as the partial derivatives of the functions given in (2.1)-(2.10). We assume the nominal trajectory is well behaved and admits the invertible Jacobian \( C_{31} \). As long as \( C_{31} \) is invertible, we can solve for \( \Delta \tilde{y}_i \). We substitute \( \Delta \tilde{y}_i \) in (4.9) and obtain \( \Delta \dot{x}_i = A_i \Delta x_i + D_i \Delta y_i + B_i \Delta u_i \), with \( A_i = A_{11} - A_{31} C_{31}^{-1} C_{11} \), and \( D_i = A_{21} - A_{31} C_{31}^{-1} C_{21} \). We now divide the states \( x_i \) into relevant \( r_i \) and less relevant states \( \zeta_i \), and rewrite the system into partitioned
The concept of SMA is to approximate the behavior of the relevant states $\Delta r_i$ with a set of differential equations that contain only $\Delta r_i$ and $\Delta y_i$, i.e., to substitute the differential equations of $\Delta \zeta_i$ with a set of algebraic equations. We do so by “freezing” the less relevant states, with the help of eigenanalysis methods. More specifically, we select the three natural modes which define the dynamic pattern of interest. Those are the two complex eigenvalues, where the relative participation of $\Delta \delta_i$ and $\Delta \omega_i$ is high, and the real eigenvalue, where the contribution of $\Delta P_{SV_i}$ is high. One way to calculate the three eigenvalues is by using the entire matrix $A_i$. However, such an approach increases the computational burden in large-scale systems. That is why we choose the submatrix $A_{11,i}$ to determine the values of the three modes. It has been shown in [70], that this matrix yields good approximations to the frequencies of the swing modes. Moreover, the machine stator reactances occur in combination with larger network reactances, which reinforces the statement that the eigenanalysis of $A_{11,i}$ provides a reasonably good approximation. To this end, we solve (4.11) with respect to $\Delta \zeta_i$, and we have

$$
\Delta \zeta_i = (sI - A_{22,i})^{-1} A_{21,i} \Delta r_i + (sI - A_{22,i})^{-1} D_{2,i} \Delta y_i + (sI - A_{22,i})^{-1} B_{2,i} \Delta u_i,
$$

where $I$ is the identity matrix. However, as it may be seen from (2.1)-(2.10), the value of $B_{2,i}$ is zero. So, we have

$$
\Delta \zeta_i = (sI - A_{22,i})^{-1} A_{21,i} \Delta r_i + (sI - A_{22,i})^{-1} D_{2,i} \Delta y_i.
$$

We fix the Laplace operator $s$ to the values corresponding to the three eigenvalues and obtain a set of linear equations $\Delta \zeta_i = A_{\zeta,i} \Delta r_i + D_{\zeta,i} \Delta y_i$. In particular, $A_{\zeta,i}$ satisfies the property $A_{\zeta,i} v_j = Z_i(\lambda_j) v_j$, for $j = 1, \ldots, 3$, where $Z_i(s) = (sI - A_{22,i})^{-1} A_{21,i}$, $\lambda_j$ the eigenvalue corresponding to mode $j$, and $v_j$ the right eigenvector of mode $j$. In the case of a conjugate pair of complex eigenvalues the equation is slightly different, details may be found in [24] or in Section 4.2.1. For simplicity, we set $D_{\zeta,i} = D_{2,i}$.\textsuperscript{65}
We notice that in (2.4) the state $T_{M_i}$ appears. We wish to substitute it with a function of the relevant states $r_i$. We write (2.8) as $(sT_{CH_i} + 1)T_{M_i} = P_{SV_i}$. If we set the Laplace operator $s = 0$, then $T_{M_i} = P_{SV_i}$. The eigenvalues chosen are close to the imaginary axes, since they are related to the more “unstable” modes. Thus, we may argue that a fairly good approximation of $T_{M_i}$ is $P_{SV_i}$. However, when we calculate the value of $T_{M_i}$, we use the linear approximation as determined by the elements of the matrix $A_{\zeta_i}$.

To sum up, we use the small $3 \times 3$ matrix $A_{11i}$ to calculate the desired eigenvalues and determine the values of $A_{\zeta_i}$ and $D_{\zeta_i}$. The overall reduced-order model is now given by

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s,$$  \hspace{1cm} (4.12)\[ \frac{2H_i d\omega_i}{\omega_s dt} = P_{SV_i} - P_{G_i} - D_i(\omega_i - \omega_s), \hspace{1cm} (4.13) \]

$$T_{SV_i}\frac{dP_{SV_i}}{dt} = -P_{SV_i} + P_{C_i} - \frac{1}{R_{D_i}}(\omega_i - 1), \hspace{1cm} (4.14)$$

$$\zeta_i = \zeta_i^* + A_{\zeta_i}\Delta r_i + D_{\zeta_i}\Delta y_i, \hspace{1cm} (4.15)$$

where $\zeta_i^*$ is the value of $\zeta_i$ at the nominal trajectory $(x_i^*, y_i^*, \tilde{y}_i^*, u_i^*)$, with the machine algebraic equations given in (2.10). We denote by $P_{G_i} = E_{d_i}I_{d_i} + E_{q_i}I_{q_i} + (X_{q_i} - X_{d_i}')I_{q_i}I_{d_i}$.

The relationship of the proposed model with the classical model is also studied. In the classical model several assumptions are made; those are: $X_{d_i} = X_{q_i}', R_{S_i} = 0, X_{q_i} = X_{q_i}', T_{qoi} = 0, T_{dqi} \rightarrow \infty, T_{F_i} \rightarrow \infty, T_{A_i} \rightarrow \infty, T_{E_i} \rightarrow \infty, T_{CH_i} = 0$, and $T_{SV_i} \rightarrow \infty$, as described in Section 2.1.3. The classical model is closely related to setting the Laplace operator $s = 0$ for the fast dynamics and $s \rightarrow \infty$ for the slow dynamics. Thus, the approximation of the full model is better in the case of the proposed reduced model, where values that describe the dynamic pattern of interest are used in the Laplace operator.
4.2.3 Numerical Results

We illustrate the differences between the proposed reduced model, the full model and the classical model, with and without the governor dynamics, with a single-machine infinite-bus (SMIB) test system, as depicted in Fig. 4.2, which contains one synchronous generating unit and load in bus 1. The voltage at bus 2 is fixed at 1\(\angle 0\). The machine, network, and load parameter values for this example are listed here: the system MVA base is 100; the synchronous speed, \(\omega_s = 377\) rad/s; the machine shaft inertia constant, \(H = 23.64\); the machine damping coefficient, \(D = 0.0125\); the machine impedances, \(X_d = 0.146\), \(X'_d = 0.0608\), \(X_q' = 0.1969\), and \(X_q = 0.8645\); the governor droop \(R_D = 0.05\); and the parameters \(T''_{do} = 8.96\), \(T'_{qo} = 0.31\), \(T_{SV} = 2\), \(T_F = 0.35\), \(K_F = 0.063\), \(T_E = 0.314\), \(K_E = 1\), \(T_A = 0.2\), and \(K_A = 20\). The network impedance between bus 1 and 2 is \(X_i = 0.5\). We solve the power flow equations and the machine algebraic equations such that the synchronous generator output in bus 1 is \(P_{G_1} = 0.8\), the load in bus 1 is \(P_{L_1} + jQ_{L_1} = 1 + j0.5\), the voltage magnitude in bus 1 is \(V_1 = 0.871\).

We choose the SMIB system, since the network impedances are comparable with the machine stator reactances, and thus the proposed reduced method yields the worst results with this system. We demonstrate that even in this “worst” case scenario the proposed method provides a very good approximation of the system behavior. We modify the load in bus 1 as follows \(P_{L_1} = 1.3\) and plot the rotor electrical angular velocity of generator 1 in Fig. 4.3. We consider the nine-state model as reference and notice that as we make further simplifications we lose accuracy in the representation of the actual system behavior. The proposed model

\[
\begin{align*}
V_1\angle \theta_1 & \quad jX_i \\
\downarrow & \\
V_2\angle \theta_2 & \quad \downarrow \\
& \quad P_{L_1} + jQ_{L_1}
\end{align*}
\]

Figure 4.2: One-line diagram of a single-machine infinite-bus power system.
is very close in terms of damping and frequency of oscillations with the full model, in contrast with the classical model. We may explain this fact by linearizing the four models. The associated eigenvalues where $\Delta \delta$, $\Delta \omega$, and $\Delta P_{SV}$ have the largest participation, determine the damping, and the frequency of oscillations. The proposed reduced model keeps the three eigenvalues that are the more “unstable”, and that is why it describes the system behavior with good accuracy. Whereas, the eigenvalues of the classical model do not match the unstable ones of the full system. More specifically, we show the values of the aforementioned eigenvalues in Table 4.1.

### 4.3 Balancing Authority Area Dynamics

In this section, we derive the BA area model based on the reduced-order model for the synchronous generator that is only area variable dependent. We also provide

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>Reduced model</th>
<th>Classical model</th>
<th>Classical model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.2151 \pm j2.9256$</td>
<td>$-0.2556 \pm j3.0082$</td>
<td>$-0.0542 \pm j3.5146$</td>
<td>$-0.0500 \pm j3.4849$</td>
</tr>
<tr>
<td></td>
<td>$-0.5156$</td>
<td>$-0.5163$</td>
<td>$-0.4916$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
some numerical results to validate the proposed model.

### 4.3.1 Balancing Authority Area Model

For each generator $i$, we use the proposed reduced model, as described in (4.12)-(4.15). We define $\Delta \omega_i = \omega_i - \omega_s$, $M_i = \frac{2H_i}{\omega_s}$, and $\tilde{R}_D = R_D \omega_s$, so we have

$$\frac{d\delta_i}{dt} = \Delta \omega_i, \quad (4.16)$$

$$M_i \frac{d\Delta \omega_i}{dt} = P_{SV_i} - P_{G_i} - D_i \Delta \omega_i, \quad (4.17)$$

$$T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV_i} + P_{C_i} - \frac{1}{\tilde{R}_D} \Delta \omega_i, \quad (4.18)$$

with the algebraic equations (2.10) and (4.15).

For each BA area $m \in \mathcal{A}$, we define

$$\delta^m = \sum_{i \in \mathcal{G}_m} M_i \delta_i, \quad \Delta \omega^m = \sum_{i \in \mathcal{G}_m} M_i \Delta \omega_i,$$

$$P^m_{SV} = \sum_{i \in \mathcal{G}_m} P_{SV_i}, \quad P^m_G = \sum_{i \in \mathcal{G}_m} P_{G_i},$$

$$z_m = \sum_{i \in \mathcal{G}_m} P_{C_i}, \quad M^m = \sum_{i \in \mathcal{G}_m} M_i.$$

From (2.35) we may obtain that by definition $z_m = \sum_{i \in \mathcal{G}_m} P_{C_i}$. We add (4.16)-(4.18) for all generators in $\mathcal{G}_m$ to obtain

$$\frac{d\delta^m}{dt} = \Delta \omega^m, \quad (4.19)$$

$$M^m \frac{d\Delta \omega^m}{dt} = P^m_{SV} - P^m_G - \sum_{i \in \mathcal{G}_m} D_i \Delta \omega_i, \quad (4.20)$$

$$\sum_{i \in \mathcal{G}_m} T_{SV_i} \frac{dP_{SV_i}}{dt} = -P^m_{SV} + z_m - \sum_{i \in \mathcal{G}_m} \frac{1}{\tilde{R}_D} \Delta \omega_i. \quad (4.21)$$

We modify (4.20)-(4.21) by using the definitions for the BA area variable $\Delta \omega^m$,
and obtain

\[
M^m \frac{d\Delta \omega^m}{dt} = P_{SV}^m - P_G^m - \sum_{i \in \mathcal{G}_m} \frac{D_i}{M_i} M^m \Delta \omega^m + \sum_{i \in \mathcal{G}_m} \frac{D_i}{M_i} \sum_{j \in \mathcal{G}_m, i \neq j} M_j \Delta \omega_j,
\]

(4.22)

\[
\sum_{i \in \mathcal{G}_m} T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV}^m + z_m - \sum_{i \in \mathcal{G}_m} \frac{1}{R_{D_i} M_i} M^m \Delta \omega^m + \sum_{i \in \mathcal{G}_m} \frac{1}{R_{D_i} M_i} \sum_{j \in \mathcal{G}_m, i \neq j} M_j \Delta \omega_j,
\]

(4.23)

which we can rewrite as

\[
M^m \frac{d\Delta \omega^m}{dt} = P_{SV}^m - P_G^m - \sum_{i \in \mathcal{G}_m} \frac{D_i}{M_i} M^m \Delta \omega^m + \sum_{i \in \mathcal{G}_m} \left( \sum_{j \in \mathcal{G}_m, i \neq j} \frac{D_j}{M_j} \right) M_i \Delta \omega_i,
\]

(4.24)

\[
\sum_{i \in \mathcal{G}_m} T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV}^m + z_m - \sum_{i \in \mathcal{G}_m} \frac{1}{R_{D_i} M_i} M^m \Delta \omega^m + \sum_{i \in \mathcal{G}_m} \left( \sum_{j \in \mathcal{G}_m, i \neq j} \frac{1}{R_{D_j} M_j} \right) M_i \Delta \omega_i.
\]

(4.25)

We wish to substitute the terms referring to each synchronous generating unit with a BA area variable, to do so from (4.24)-(4.25) we wish that \( \sum_{j \in \mathcal{G}_m, i \neq j} \frac{D_j}{M_j} = \sum_{j_2 \in \mathcal{G}_m} \frac{D_j}{M_{j_2}}, \sum_{j_1 \in \mathcal{G}_m} \frac{1}{R_{D_j} M_{j_1}} = \sum_{j_2 \in \mathcal{G}_m} \frac{1}{R_{D_{j_1}} M_{j_2}} \) and \( T_{SV_{j_1}} = T_{SV_{j_2}} \) for all \( i, j_1, j_2 \in \mathcal{G}_m \). We may rewrite the equations as

\[
\frac{D_i}{M_i} = c_1, \text{constant}, \forall i \in \mathcal{G}_m,
\]

(4.26)

\[
\frac{1}{R_{D_i} M_i} = c_2, \text{constant}, \forall i \in \mathcal{G}_m,
\]

(4.27)

\[
T_{SV_i} = c_3, \text{constant}, \forall i \in \mathcal{G}_m.
\]

(4.28)

We may interpret (4.26), as a desire to have uniform damping across the system.
Then, we would have

\[ M^m \frac{d\Delta \omega^m}{dt} = P_{SV}^m - P_G^m - c_1 M^m \Delta \omega^m, \quad (4.29) \]

\[ c_3 \frac{dP_{SV}^m}{dt} = -P_{SV}^m + z_m - c_2 M^m \Delta \omega^m. \quad (4.30) \]

In order to determine the parameters \(c_1\), \(c_2\) and \(c_3\), we wish to minimize the Euclidean norm of the errors of the ratios given in (4.26)-(4.28). However, there is a constraint connecting \(c_1\) and \(c_2\). The deviations of the rotor angular speeds from the nominal value are the same for each generator \(i \in \mathcal{G}_m\) and the BA area. We use the reduced-order model for each generator \(i\), given in (4.16)-(4.18), and the Laplace transformation, to derive that

\[ s^2 T_{SV_i} + s(M_i \Delta \omega_i + P_{Gi} T_{SV_i} + D_i T_{SV_i} \Delta \omega_i) - P_{Ci} \]

\[ + \frac{1}{R_{Di}} \Delta \omega_i + P_{Gi} + D_i \Delta \omega_i = 0. \quad (4.31) \]

When \(t \to \infty\) then \(s \to 0\), and we have that

\[ \left( \frac{1}{R_{Di}} + D_i \right) \Delta \omega_i = P_{Ci} - P_{Gi} = -(P_{Gi}(t) - P_{Gi}(0)), \quad (4.32) \]

since \(P_{Ci}(t) = P_{Gi}(0)\), when the AGC system is not included. We use (4.29), (4.30) and in a similar way we find the relationship that holds for \(t \to \infty\) for the BA area \(m\). Since, \(\Delta \omega(t) = \Delta \omega(t) = \Delta \omega^m(t), \forall i \in \mathcal{G}_m\) as \(t \to \infty\), we have

\[ \frac{P_{Gi}(t) - P_{Gi}(0)}{\frac{1}{R_{Di}} + D_i} = \frac{P_G^m(t) - P_G^m(0)}{(c_1 + c_2)M^m}, \forall i \in \mathcal{G}_m, t \to \infty. \quad (4.33) \]

Since \(\sum_{i \in \mathcal{G}_m} (P_{Gi}(t) - P_{Gi}(0)) = P_G^m(t) - P_G^m(0)\), then by using (4.33) we may derive that

\[ \sum_{i \in \mathcal{G}_m} \left( \frac{1}{R_{Di}} + D_i \right) = (c_1 + c_2)M^m. \quad (4.34) \]
The optimization problems may now be written as

$$\min_{i \in \mathcal{G}_m} \left( c_1 - \frac{D_i}{M_i} \right)^2 + \sum_{i \in \mathcal{G}_m} \left( c_2 - \frac{1}{M_i R_D} \right)^2$$

s. t. $$\sum_{i \in \mathcal{G}_m} \left( \frac{1}{R_D} + D_i \right) = (c_1 + c_2) M^m,$$

$$\min_{i \in \mathcal{G}_m} \left( c_3 - T_{SV_i} \right)^2.$$

(4.35)

(4.36)

By solving the optimization problems we obtain the solutions

$$D^m = c_1 M^m = \frac{1}{2|\mathcal{G}_m|} \sum_{i \in \mathcal{G}_m} M^m_i (D_i - \frac{1}{R_D}) + \frac{1}{2} \sum_{i \in \mathcal{G}_m} \left( \frac{1}{R_D} + D_i \right),$$

(4.37)

$$\frac{1}{R_D} = c_2 M^m = \frac{1}{2} \sum_{i \in \mathcal{G}_m} \left( \frac{1}{R_D} + D_i \right) - \frac{1}{2|\mathcal{G}_m|} \sum_{i \in \mathcal{G}_m} M^m_i (D_i - \frac{1}{R_D}),$$

(4.38)

$$T_{SV}^m = c_3 = \sum_{i \in \mathcal{G}_m} T_{SV_i} / |\mathcal{G}_m|,$$

(4.39)

where $|\mathcal{G}_m|$ the cardinality of the set $\mathcal{G}_m$.

We may describe the BA area dynamic behavior by

$$\frac{d\delta^m}{dt} = \Delta \omega^m,$$

(4.40)

$$M^m \frac{d \Delta \omega^m}{dt} = P_{SV}^m - P_G^m - D^m \Delta \omega^m,$$

(4.41)

$$T_{SV}^m \frac{d P_{SV}^m}{dt} = -P_{SV}^m + z_m - \frac{1}{R_D^m} \Delta \omega^m,$$

(4.42)

where $P_G^m = \sum_{i \in \mathcal{G}_m} \sum_{k=1}^N V_i V_k \{ G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k) \} + P_L$, with $P_L^m$ the BA area $m$ total load.
4.3.2 Numerical Results

We illustrate the proposed model for the BA area dynamics with the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system, which is depicted in Fig. 4.4, contains three synchronous generating units in buses 1, 2 and 3, and load in buses 5, 6 and 8. The machine, network and load parameter values may be found in [52]. We consider the system as one BA area and show how good approximation is provided with the proposed model, given in (4.40)-(4.42), which we refer to as method (i). We compare our method, with a similar model by setting \( D_m = \sum_{i \in \mathcal{G}_m} D_i, \frac{1}{R_D} = \sum_{i \in \mathcal{G}_m} \frac{1}{R_{D_i}}, \) and \( T_{SV} = \frac{\sum_{i \in \mathcal{G}_m} T_{SV_i}}{|\mathcal{G}_m|}, \) which is commonly found in literature [26]. This model is referred to as method (ii). The benchmark model is the full model described in (2.1)-(2.10),

![Figure 4.4: One-line diagram of the three-machine nine-bus WECC power system.](image)

![Figure 4.5: Speed of center of inertia with the three methods.](image)
which we use to calculate the speed of center of inertia, and refer to it as method (iii). In Fig. 4.5, we depict the speed of center of inertia calculated by the three methods. In this example, we do not include the AGC system, that is my \( \omega \) does not converge to the synchronous speed. We notice that the approximation of the BA area dynamics with method (i) is better than that of method (ii), in terms of magnitude of oscillations and time to reach steady state. However, both methods deviate from the method (iii), which is the reference method. The reason is that in both methods (i) and (ii), there are no individual states for each generator, and we only consider the BA area states.

4.4 Adaptive Automatic Generation Control Systems

In this section, we derive the relationship connecting the AFRC and the system variables, which is used to design adaptive AGC systems. We present two different options for an adaptive AGC system, based on which AGC model we are using. In Section 4.4.2, we use the model given in (2.32), and develop the adaptive AGC system I, and in Section 4.4.3, the AGC model given in (2.34), and develop adaptive AGC system II.

4.4.1 AFRC Determination

We now use (4.40)-(4.42), which represent a BA area dynamics, to calculate its AFRC. The AFRC of BA area \( m \) is equal to \( \beta_m = -2\pi(\frac{1}{R_{D_{m}}} + D^m) = -2\pi \sum_{i \in G_m} \left( \frac{1}{R_{D_{i}}} + D_{i} \right) \). In Fig. 4.6, we depict the speed of center of inertia for a system where an AGC system is not present, and a 1 pu load increase occurs. We notice that due to primary control the system frequency converges to a value different than nominal. In particular, we may approximate the deviation \( \Delta \omega \) from the synchronous speed \( \omega_s \), with \( \Delta \omega = \frac{\Delta P_L}{2\pi \beta} \), where \( \Delta P_L \) is the change in load.

In the case where we set the frequency bias factor equal to the AFRC we have
non-interactive control and it is fair in the sense that the area, in which the load disturbance has occurred, is the only one that reacts to restore the frequency and net tie flow to the desired values. More specifically, we show that under some assumptions the optimal value for the frequency bias factor is the AFRC. We define 

\[ P_G^m = P_{G_0}^m + \Delta P_L^m + \Delta P_{\text{losses}}^m, \]

where \( P_{G_0}^m \) denotes the total generation of BA area \( m \) in steady state. In the same rationale, we have \( z_m = z_{m_0} + \Delta z_m \). At steady state the following relationship holds: \( z_{m_0} = P_{G_0}^m \). As in (4.31), we may obtain a similar relationship for the entire BA area \( m \) with \( s = 0 \), by using (4.40)-(4.42). We denote \( \Delta \omega^m = 2\pi \Delta f_m \), with \( \Delta f_m = f_m - f_{\text{nom}} \), and at time \( t = k \), we have

\[ \beta_m \Delta f_m(k) + \Delta z_m(k) - \Delta P_L^m - \Delta P_{\text{losses}}^m = 0. \]  

(4.43)

The AGC system is discrete in reality, and usually exercised every one second. To this end, for one BA area, we rewrite the AGC system given in (2.34) by using (2.30), and Euler’s method: 

\[ \Delta z(k+1) - \Delta z(k) = b \Delta f(k). \]

So, \( \Delta z(k) = b \sum_{i=0}^{k-1} \Delta f(i) \). We use (4.43) and the aforementioned AGC algorithm, and for \( \Delta f(0) = \frac{\Delta P_L + \Delta P_{\text{losses}}}{\beta} \), and \( \Delta z(0) = 0 \), we see that \( \Delta f(1) = 0 \) when \( b = \beta \). In such a case, the ACE is corrected in only one control period [28]. Numerical results of this claim may be found in [50].

We have that \( \Delta f_m = f_m - f_{\text{nom}} \) and \( \frac{d\Delta \omega^m}{dt} = 2\pi \frac{d\Delta f_m}{dt} \). Then, we combine (4.41) and (4.42) into one equation using the Laplace transformation, and ignore the

![Figure 4.6: AFRC of a power system.](image-url)
second-order terms since they are negligible due to the system inertia. Thus, we have that

\[ s2\pi(M^m + D^mT_{SV}^m)\Delta f_m - \beta_m\Delta f_m = z_m - (1 + sT_{SV}^m)P_G^m. \quad (4.44) \]

We use (4.44) to describe a BA area dynamics and we use it in the subsequent section to derive a relationship between the ACE, the frequency and the total generation of the BA area.

4.4.2 Adaptive Automatic Generation Control System I

In order to estimate the AFRC, we use (4.44) in combination with (2.32), with \( \eta_1 = 0 \) and \( \eta_2 = \eta \), to obtain

\[ -\eta_mACE_m = sK_m\Delta f_m - (1 + s)\beta_m\Delta f_m + s(1 + T_{SV}^m)P_G^m, \quad (4.45) \]

where \( K_m = 2\pi(M^m + D^mT_{SV}^m) \). We now express \( \Delta f_m \) as

\[ \Delta f_m = s\frac{1}{2}\sum_{i \in G_m} \gamma_1_i \theta_i, \]

with \( \gamma_1_i = \frac{M_i}{M_m} \), and substitute in (4.45). We also use the moving average definition

\[ \langle ACE(k) \rangle_m = \frac{1}{T} \int_{t_{\text{start}(k)}}^{t_{\text{end}(k)}} ACE_m(t) dt, \] with \( t_{\text{end}(k)} = kT \), and \( t_{\text{start}(k)} = (k - 1)T \).

For every step \( k \), referring to time instant \( t = t_{\text{end}(k)} = kT \),

\[ \beta_m(k) = \frac{\eta_mT\langle ACE(k) \rangle_m + \left( P_G^m(t_{\text{end}(k)}) - P_G^m(t_{\text{start}(k)}) \right)(1 + T_{SV}^m) + K_m \left( \Delta f_m(t_{\text{end}(k)}) - \Delta f_m(t_{\text{start}(k)}) \right)}{\Delta f_m(t_{\text{end}(k)}) - \Delta f_m(t_{\text{start}(k)}) + \sum_{i \in G_m} \gamma_1_i \left( \theta_i(t_{\text{end}(k)}) - \theta_i(t_{\text{start}(k)}) \right)}, \quad (4.46) \]

since we have measurements for all the necessary values. We use the sliding exponentially weighted window blockwise least squares (SEWBLIS) algorithm for the online estimation of the frequency bias factor for BA area \( m \beta_m \) [71].

In order to formulate our problem we introduce the following variables:

\[ \chi(k) = \phi(k)\beta_m(k), \quad (4.47) \]
\[ w(k) = \chi(k) + v(k), \quad (4.48) \]
where $\chi(k)$ is the system output, $w(k)$ is the measured output and $v(k)$ is a zero-mean white Gaussian sequence that accounts for measurements noise and modeling errors. We have that $\phi(k)$ is the denominator, and $\chi(k)$ is the nominator of (4.46), respectively. The SEWBLS solution is

$$
\hat{\beta}_m(k) = \left[(\phi_{k-L+1}^k)^T \Lambda_{k-L+1}^k \phi_{k-L+1}^k\right]^{-1} \left[(\phi_{k-L+1}^k)^T \Lambda_{k-L+1}^k w_{k-L+1}^k\right],
$$

(4.49)

where $\phi_{k-L+1}^k = [\phi(k-L+1), \phi(k-L+2), \ldots, \phi(k)]^T$, and $\Lambda_{k-L+1}^k$ is an $L \times L$ diagonal matrix with diagonal elements being the forgetting factors, $\lambda^{L-1}, \lambda^{L-2}, \ldots, \lambda^0$. The values of $\lambda$ vary from 0 to 1. After several tests, we concluded that a time period of $T = 1$ min and a window length of $L = 30$ min provides good results in terms of convergence speed and accuracy.

We present several case studies to demonstrate the capabilities of the proposed methodology. We use a small system, the three-machine nine-bus system, to present the insights we gained into the AGC system. We demonstrate that setting the frequency bias factor equal to the AFRC is beneficial for the interconnection. We also show that when using the estimation of the AFRC as the frequency bias factor, the system frequency has smaller oscillations than when using other values. Furthermore, we include a larger system to show that there are no computational limitations in the proposed method.

Three-Machine Nine-Bus Power System

We illustrate the proposed methodology with the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system model, which is depicted in Fig. 4.7; this model contains three synchronous generating units in buses 1, 2 and 3, and load in buses 5, 6 and 8. The machine, network and load parameter values may be found in [52]. As depicted in Fig. 4.7, we consider two BA areas $\mathcal{A} = \{1, 2\}$. We increase the load at bus 5 by 0.15 pu. In order to keep the tie line power at the scheduled value, the generation must be increased.
Figure 4.7: One-line diagram of the three-machine nine-bus WECC power system with two BA areas.

![One-line diagram]

Figure 4.8: ACE of the two BA areas.

(a) Area control error of area 1, $ACE_1$, where the load change $\Delta P_L = 0.15$ pu occurred.

![ACE_1 graph]

(b) Area control error of BA area 2, $ACE_2$.

![ACE_2 graph]

in BA area 1. However, the frequency deviates from the nominal value in both BA areas. BA area 1 exports to BA area 2 which results in $ACE_1 < 0$, since it is under-generating; and $ACE_2 > 0$, since BA area 2 is importing less than...
suggested, as depicted in Figs. 4.8a-4.8b. We implement the AGC system by modifying the frequency bias factors $b_1$ and $b_2$ among three values: (A-i) the AFRC, (A-ii) 0.5AFRC, and (A-iii) 1.5AFRC respectively. The AFRC for BA area 1 is $\beta_1 = -0.7394$ pu/Hz and for BA area 2 $\beta_2 = -0.4121$ pu/Hz. We notice in Fig. 4.8a, in case (A-i), where $b_1 = \beta_1$, the $ACE_1$ converges quicker to zero and has lower in magnitude oscillations. The reason is that the value of $ACE$ represents how many MW are needed to restore the BA area frequency, instead of a smaller or bigger value in the case of 0.5$\beta_1$ and 1.5$\beta_1$, respectively. In addition, in Fig. 4.8b, we may see that $ACE_2$ is smallest in the case where $b_2 = \beta_2$, and therefore, the BA area participates the least in regulation.

We now consider the system as one BA area and modify the system load as a stochastic differential equation: $dX_t = aX_t + \zeta W_t$, with $a = -5 \cdot 10^{-6}$ and $\zeta = 0.01$, and $W_t$ is a Wiener process, as described in [72]. The online estimation
of the AFRC $\beta$ with $\lambda = 0.95$ in the SEWBLS for a period of 6 hours is given in Fig. 4.9. We notice that the algorithm provides a good approximation of the AFRC, which in this case is $\beta = -1.152$ pu/Hz, since the maximum relative absolute error observed is 18.5%. In Fig. 4.10, we depict the system ACE, when using (A-iv) the online estimation $b = \hat{\beta} = -1.241$ pu/Hz for hour 3, (A-v) the AFRC $b = \beta$, and (A-vi) $b = -1.7$ pu/Hz for the period of seven minutes. We may see that case (A-v) provides the best results, since the best choice for the frequency bias factor is the AFRC. We notice that case (A-iv) is very close to case (A-v), as desired. The reason is that the estimation is very close to the AFRC. Case (A-vi) has the largest deviation from case (A-v), and presents the biggest oscillations. We also note that for this time period the maximum absolute value of ACE is for case (A-iv) 0.24 pu, for case (A-v) 0.23 pu, and for case (A-vi) 0.34 pu, which further supports the fact the the use of the online estimation $\hat{\beta}$ in $b$ is a good practice.

48-Machine 140-Bus Power System

Next, we demonstrate the scalability of the proposed methodology to the online estimation of the AFRC for large power systems. In particular, we examine the IEEE 48-machine test system, which consists of 140 buses and 233 lines [66]. To implement our analysis method, we use the MATLAB-based Power Systems
Toolbox (PST) [67], and add the AGC system model described in (2.32) and (2.35) to it. We use the proposed algorithm to estimate the AFRC and use it in the calculation of the ACE. We modify the system load in a similar way as in the three-machine nine-bus system. In Fig. 4.11, the system frequency is plotted for the period of 7 minutes by using: (B-i) the estimated AFRC $b = \hat{\beta} = -5381$ MW/Hz, (B-ii) the ARFC $b = \beta = -5475$ MW/Hz, and (B-iii) the value $b = -16424$ MW/Hz in the ACE calculation. One can see that the proposed method yields good results and the system frequency is close to the nominal. In addition, the maximum deviation of the system frequency from the nominal value is 0.05 Hz for case (B-i), 0.06 Hz for case (B-ii), and 0.08 Hz for case (B-iii).

4.4.3 Adaptive Automatic Generation Control System II

In order to estimate the AFRC, we use (4.44) in combination with (2.34), neglect the second-order terms, to obtain

$$ACE_m = s\beta_m \Delta f_m - sP^m_G.$$  \hspace{1cm} (4.50)

With the introduction of the phasor measurement units (PMUs), $h$ is very small, and thus, we may approximate the derivatives in (4.50) with good accuracy. For every step $k$, referring to time instant $t = kh$, where $h$ is the time step between the available measurements, we have

$$\beta_m(k) = \frac{ACE_m(k) + \frac{P^m_{G}(k)-P^m_{G}(k-1)}{h}}{\Delta f_m(k)-\Delta f_m(k-1)}.$$  \hspace{1cm} (4.51)

The sliding exponentially weighted window blockwise least squares (SEWBLBS) algorithm is used for the online estimation of the frequency bias factor for BA area $m \beta_m$ [71].
In order to formulate our problem we introduce the following variable:

\[ \chi(k) = \phi(k)\beta_m(k), \quad (4.52) \]
\[ w(k) = \chi(k) + v(k), \quad (4.53) \]

where \( \chi(k) \) is the system output, \( w(k) \) is the measured output and \( v(k) \) is a zero-mean white Gaussian sequence that accounts for measurements noise and modeling errors. We have that \( \phi(k) \) is the denominator, and \( \chi(k) \) is the nominator of (4.51), respectively. The SEWBLS solution is

\[ \hat{\beta}_m(k) = \left[ (\phi_{k-L+1}^k)^T \Lambda_{k-L+1}^k \phi_{k-L+1}^k \right]^{-1} \left[ (\phi_{k-L+1}^k)^T \Lambda_{k-L+1}^k w_{k-L+1}^k \right], \quad (4.54) \]

where \( \phi_{k-L+1}^k = [\phi(k-L+1), \phi(k-L+2), \ldots, \phi(k)]^T \), and \( \Lambda_{k-L+1}^k \) is an \( L \times L \) diagonal matrix with diagonal elements being the forgetting factors, \( \lambda^{L-1}, \lambda^{L-2}, \ldots, \lambda^0 \). The values of \( \lambda \) vary from 0 to 1. After several tests, we concluded that a window length of \( L = 10 \) min provides good results in terms of convergence speed and accuracy.

Three-Machine Nine-Bus Power System

We use this system to demonstrate that the proposed algorithm for the online estimation of the AFRC provides a good approximation. To this end, we modify
Figure 4.13: ACE with three different frequency bias factors.

the system load as a stochastic differential equation: \( dX_t = aX_t + \zeta W_t \), with \( a = -5 \cdot 10^{-6} \) and \( \zeta = 0.01 \), and \( W_t \) is a Wiener process, as described in [72]. At time, \( t = 30 \) min, the unit commitment changes, and generator 3 no longer participates in the system. The generators AGC participation is at first: \( \kappa_1 = 0.24, \kappa_2 = 0.50, \) and \( \kappa_3 = 0.26 \), and after the unit commitment changes: \( \kappa_1 = 0.50, \) and \( \kappa_2 = 0.50. \)

The online estimation of the AFRC \( \beta \) with \( \lambda = 0.95 \) in the SEWBLS for a period of 70 min is given in Fig. 4.12. We notice that the algorithm provides a good approximation of the AFRC, which in this case is \( \beta = -1.152 \) pu/Hz, for the first 30 min and \(-0.7881 \) pu/Hz for the subsequent minutes. The maximum relative absolute error observed is 27.5%. We notice that the proposed method captures the event of the change of the set of generators, and the estimation of the AFRC changes accordingly. In Fig. 4.13, we depict the system ACE, when using the online estimation \( b = \hat{\beta} = -1.158 \) pu/Hz for the 20th min, the AFRC \( b = \beta \), and a fixed value \( b = -1.7 \) pu/Hz for the period of one minute. We may see that the case where \( b = \beta \) provides the best results, since the best choice for the frequency bias factor is the AFRC. We notice that when \( b = \hat{\beta} \) is very close to ideal case, as desired. The reason is that the estimation is very close to the AFRC. The case when \( b = -1.7 \) pu/Hz presents the biggest oscillations. We also note that for this time period the maximum absolute value of ACE for the three cases are: 0.2360 pu, 0.1138 pu, and 0.3475 pu respectively, which further supports the fact the the use of the online estimation \( \hat{\beta} \) in \( b \) is a good practice.
48-Machine 140-Bus Power System

Next, we demonstrate the scalability of the proposed methodology to the online estimation of the AFRC for large power systems. In particular, we examine the IEEE 48-machine test system, which consists of 140 buses and 233 lines [66]. To implement our analysis method, we use the MATLAB-based Power Systems Toolbox (PST) [67], and add the AGC system model described in (2.34)-(2.35) to it. The AGC signal is allocated to the generators with a ratio proportional to their inertia constant. We use the proposed algorithm to estimate the AFRC and use it in the calculation of the ACE. We modify the system load in a similar way as in the three-machine nine-bus system. In Fig. 4.14, the ACE is plotted for the period of 1 minute by using: the estimated AFRC $b = \hat{\beta} = -5278$ MW/Hz, the ARFC $b = \beta = -5475$ MW/Hz, and the value $b = -15833$ MW/Hz in its calculation. One can see that the proposed method yields good results and the ACE is close to zero. In addition, the maximum deviation of the ACE is 4.91 pu, 4.30 pu, and 14.25 pu, respectively.

4.5 Summary

In this chapter, we developed various models to describe a power system behavior that may be used for various applications. More specifically, we propose a reduced-order generator model that it is simpler than the full model but approximates the
system behavior in satisfactory levels, provides better accuracy compared to the classical model, and has lower computational burden compared to other reduction methods. Subsequently, we used the reduced model to develop a BA area model that only depends on the BA area variables, such as speed of center of inertia. We demonstrated in the numerical results section that these models provide a good approximation of the system state compared to the full model, which is considered as reference. We use the developed models, and in particular the BA area model to design two adaptive AGC systems. To this end, we express the AFRC as a function of the BA area variables that we have measurements of. Then, we used the SEWBLs algorithm to estimate the AFRC and modify the control gain of the AGC systems. We showed that in both cases the use of the AFRC gives better results in the frequency regulation, in terms of the magnitude of the oscillations and the time the frequency converges to the nominal value. Furthermore, we showed that the proposed methods give a good approximation to the AFRC.
We devote this chapter to propose a coordination between BA areas that would decrease the regulation amount needed as well as the associated costs. Our approach is inspired from trying to mimic the AGC system, if all areas were considered to be one entire BA area. To this end, we modify the ACE, which is fed into the AGC system of each BA area, and determine the AGC dispatch based on a distributed algorithm that detects the least cost generators with the only information exchanged, between BA areas, being the mismatch of the total regulation needed.

5.1 Introduction

Reliability is a very important issue in power systems, since generation and load must remain balanced at all time. Significant imbalances might cause large interconnection frequency deviations. The interconnection is divided into several entities, which are called BA areas, which are responsible for maintaining load-interchange-generation balance within a BA area, and for supporting the interconnection frequency in real time. Each BA area implements appropriate control systems, such as AGC to achieve these goals. The performance of an AGC system of a BA area is evaluated by the frequency performance criteria — CPS1, CPS2, and BAAL — defined by NERC. Besides the quality of AGC systems determined by the aforementioned criteria, another regulatory issue is the payments to the units participating in ancillary services. According to FERC Order No. 755, frequency regulation service must provide compensation based on (i) the actual service provided, including a capacity payment that includes the marginal unit
opportunity costs and (ii) a payment for performance that reflects the quantity of frequency regulation service provided by a resource when the resource is accurately following the dispatch signal. A simultaneous (Vickrey) auction, which is implemented by ISONE, provides truthful bidding incentives, minimizes total cost of regulation, and is non-discriminatory, since identical resources receive the same payment. Thus, it satisfies the requirements mandated by FERC Order No. 755.

When the regulation amount and the cost are determined in a BA area level, the individual AGC systems might cause overregulation, i.e., if all the BA areas were operated as one, then the regulation amounts as well as the associated costs would be less. The generation reserves are expensive and may be limited, thus a method that would reduce the regulation amount needed, would be beneficial for all BA areas. In addition, due to the high penetration of renewable resources, which are highly variable and intermittent, the BA area role is more challenging. Operating separately and locally, individual BA areas are obliged to purchase more expensive ancillary services to accommodate the variability and uncertainty from high penetration of renewable-based generation. To this end, some level of coordination between BA areas is favorable for all entities. NERC has proposed that BA areas collaborate in some extent, by proposing the area control error diversity interchange (ADI) methodology. The ADI was introduced by [34], and is the pooling of individual ACEs to take advantage of the ACE diversity, i.e., sign differences associated with the momentary generation-load imbalances of each BA area. By pooling ACE the participants will be able to reduce the control burden on individual BA areas, the unnecessary generator control movement, the sensitivity to resources with potentially volatile output, and to realize improvements in frequency performance criteria [34], [35]. BA area coordination will help the BA areas meet their renewable integration objectives. The amount of required balancing reserves and regulation reserve ramping requirements can be reduced through BA area coordination and the control performance indices are improved.

In this chapter, we propose a methodology that imitates the centralized solution
of the AGC system and dispatch, as if all BA areas were one. In order to do so, we need to approximate the ACE of the entire area, as a whole, from the ACE of the individual BA areas. Then, we allocate it appropriately to the individual AGC systems proportionally to their size. The next step is to imitate the AGC dispatch of the entire area, without the exchange of any cost information between the BA areas. To this end, we develop a distributed algorithm that provides the same solution as the centralized AGC dispatch, with exchange of limited information. In particular, each BA area only knows its own cost functions and the only information exchanged is the total mismatch of regulation. In Section 5.2, we present the mathematical modeling of the AGC market formulation, based on a Vickrey auction, in Section 5.3 the AGC dispatch, and in Section 5.4 possible BA area coordinations methods, such as the ADI methodology. Next, in Section 5.5, we describe our proposed approach, which is divided into two subsections: (i) the adjusted ACE determination of each BA area, and (ii) the derivation of the distributed algorithm that solves the centralized AGC dispatch. In Section 5.6, we demonstrate the proposed method on the three-machine nine-bus WECC system, and we summarize the results in Section 5.7.

5.2 Automatic Generation Control Market Formulation

We formulate the AGC market mechanism for BA area $m$. Resource $i$ submits its bid that consists of a capacity offer and effective offer for commitment (EOC) $\gamma_i^m$, a service offer $\sigma_i^m$, a range, from which we calculate the capacity offered $c_i^m$, and the automatic response rate $\zeta_i^m$. The resource selection is done every hour in order to minimize expected cost subject to operational constraints. The operational constraints include the capacity requirement $r_c^m$, the service requirement $r_s^m$, and the response time requirement $r_t^m$. The settlement, i.e., the Vickrey payment, is calculated (i) ex-ante: determining the cost savings of each resource to the system, and (ii) ex-post: using the capacity cost including EOC and the service cost for the actual regulation amount offered. Resources are accepted all-or-nothing. Let us
assume we have the set of $S^m$ sellers $\mathcal{S}^m = \{1, \ldots, S^m\}$. The decision variables of
the problem are $\delta_i^m$, which are binary, and show if bid $i$ is selected, i.e., if $\delta_i^m = 1$,
then resource $i$ in BA area $m$ is selected for regulation, and $a_i^m$ is the amount of
service offered. The hour $h$ resource selection optimization statement is

$$\begin{align*}
\min & \quad \sum_{i \in \mathcal{S}^m} \delta_i^m c_i^m \gamma_i^m + \sum_{i \in \mathcal{S}^m} a_i^m \sigma_i^m \\
\text{s. t.} & \quad \sum_{i \in \mathcal{S}^m} \delta_i^m c_i^m \geq r_c^m \\
& \quad \sum_{i \in \mathcal{S}^m} a_i^m = r_s^m \\
& \quad \frac{a_i^m}{c_i^m} \leq r_i^m \quad \forall i \in \mathcal{S}^m \\
& \quad \delta_i^m = 1 \iff a_i > 0 \quad \forall i \in \mathcal{S}^m \\
& \quad 0 \leq a_i^m \leq c_i^m \quad \forall i \in \mathcal{S}^m.
\end{align*}$$

The solution of (5.1) provides us with $\delta_i^m, a_i^m$, for $i \in \mathcal{S}^m$. Let us assume that
we have the set of cleared sellers $\tilde{\mathcal{S}}^m = \{1, \ldots, \tilde{S}^m\}$, i.e. $\delta_i^m = 1$ for $i \in \tilde{\mathcal{S}}^m$, and
$\delta_i^m = 0$, for $i \notin \tilde{\mathcal{S}}^m$, then the total cost is $\eta_m = \sum_{i \in \mathcal{S}^m}(c_i^m \gamma_i^m + a_i^m \sigma_i^m)$. The ex-
ante cost savings of resource $i$, $\pi_i^m$, to the system are calculated by solving (5.1),
without resource $i$, and determining the new total cost $\eta_m - i$. If the set of sellers
cleared is now $\tilde{\mathcal{S}}^m$, and the associated amounts $a_j^m$ for each cleared resource $j$,
then we have $\eta_m - i = \sum_{j \in \tilde{\mathcal{S}}^m}(c_j^m \gamma_j^m + a_j^m \sigma_j^m)$. To this end, $\pi_i^m = \eta_m - i - \eta_m$. The ex-
ante costs for BA area $m$ are

$$\pi_{\text{reg, ante}}^m = \sum_{i \in \tilde{\mathcal{S}}^m} \pi_i^m.$$  (5.2)

The ex-post costs are calculated by the actual regulation amount offered by
resource $i$, determined by the AGC dispatch, which is described in Section 5.3.
Table 5.1: Bids of resources A, B, and C.

<table>
<thead>
<tr>
<th>Resource Number</th>
<th>Regulation Capacity $c_i$ (MW)</th>
<th>Capacity + EOC Service Offer $\gamma_i$ ($/MW$)</th>
<th>Service Offer $\sigma_i$ ($/MW$)</th>
<th>Response Rate $\zeta_i$ (MW/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

5.2.1 Numerical Illustration

We go through a simple example of the AGC market mechanism. Let us assume that we have three resources participating in regulation, A, B, and C. The bids of each resource are given in Table 5.1. The capacity requirement is $r_c = 10$ MW, the service requirement $r_s = 10$ MW, and the response time requirement is $r_t = 10$ min. We solve the optimization problem given in (5.1), and find the total cost $\eta = $45 with resources A and B to be selected to participate in regulation. Thus, $\tilde{\mathcal{S}} = \{A, B\}$. Resource A provides $a^*_A = 5$ MW of capacity, since it is the least cost generator but reaches its capacity limit. Then, resource B provides the remaining $a^*_B = 5$ MW. The service is provided within 5 min, since resource A provides its regulation in 5 min and resource B in 2.5 min. In order to determine the ex-ante costs, we solve (5.1), first without resource A and then without resource B to determine the respective total costs. When resource A is not included in the market, the total cost is $\eta^{-A} = $50, since only resource B is chosen. When resource B is not included in the market, then $\eta^{-B} = $80, since both resources A and C are selected. Resources A and C each provide 5 MW. Thus, $\pi_A = \eta - \eta^{-A} = $5, and $\pi_B = \eta - \eta^{-B} = $35. The ex-ante payments are $c_{\text{reg, ante}} = \sum_{i \in \tilde{\mathcal{S}}} \pi_i = 5 + 35 = $40.

Now, we investigate what happens if the capacity and the service offers are negatively correlated. The bid of each resource is shown in Table 5.2, and we have the same operational constraints. We solve the optimization problem given in (5.1), and find the total cost $\eta' = $50 with only resource B to be selected to participate in regulation, since it meets alone all the operational requirements and...
Table 5.2: Bids of resources A, B, and C, with negatively correlated service and capacity offers.

<table>
<thead>
<tr>
<th>Resource Number</th>
<th>Regulation Capacity $c_i$ (MW)</th>
<th>Capacity + EOC Offer $\gamma_i$ ($$/MW$)</th>
<th>Service Offer $\sigma_i$ ($$/MW$$)</th>
<th>Response Rate $\zeta_i$ (MW/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>5</td>
<td>1</td>
<td>5</td>
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<tr>
<td>$B$</td>
<td>10</td>
<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>$C$</td>
<td>15</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

minimizes cost; thus, $\mathcal{S}' = \{ B \}$. The total cost without considering resource B is $\eta' - B = 55$, where only resource C participates in regulation. To this end, the cost savings to the system of resource B are $\pi'_B = 5$. The ex-ante payments in this case are $c_{\text{reg, ante}} = \sum_{i \in \mathcal{S}} \pi_i = 5$.

5.3 Automatic Generation Control Dispatch Formulation

The AGC dispatch for BA area $m$ is implemented throughout the hour, and its objective is to dispatch the cleared resources, determined by the AGC market, in order to minimize cost subject to response time and other operational constraints. We denote by $\rho^m_i$ the response time requirement for the AGC dispatch. The decision variables are the amounts of regulation $a^m_i$ of all cleared resources, i.e., $i \in \mathcal{S}'$. We have

$$\min \sum_{i \in \mathcal{S}'} a^m_i \sigma^m_i$$

s. t.

$$\sum_{i \in \mathcal{S}'} a^m_i = \left| z_m - \sum_{j \in \mathcal{G}_m} P_{ED_j} \right|$$

$$\frac{a^m_i}{\zeta^m_i} \leq \rho^m_i \quad \forall i \in \mathcal{S}'$$

$$0 \leq a^m_i \leq c^m_i \quad \forall i \in \mathcal{S}'$$

The regulation amount needed is set to $\left| z_m - \sum_{j \in \mathcal{G}_m} P_{ED_j} \right|$, since the AGC system
determines the total generation needed to restore frequency and the real power interchange to the desired values, and \( \sum_{j \in \mathcal{G}_m} P_{ED_j} \) is the total generation of BA area \( m \) following the signals of the most recent economic dispatch process. Thus, the difference in these two quantities is the regulation amount needed. The results of (5.3), determine the optimal regulation quantity \( a^m_i \) for \( i \in \mathcal{J}_m \). We may calculate the participation factors as follows \( \kappa^m_i = \frac{a^m_i}{z_m - \sum_{j \in \mathcal{G}_m} P_{ED_j}} \). The ex-post costs for BA area \( m \) are

\[
c_{\text{reg, post}}^m = \sum_{i \in \mathcal{J}_m} \left( a^m_i \sigma^m_i + c^m_i \gamma^m_i \right). \tag{5.4}
\]

5.3.1 Numerical Illustration
We use the service offers given in Table 5.1, where only resource A and B are cleared in the AGC market, so \( \mathcal{J} = \{A, B\} \). For a time instant, let us assume that \( |z - \sum_{j \in \mathcal{G}_m} P_{ED_j}| = 9 \text{ MW} \), and \( \rho_t = 10 \text{ min} \). Then, \( a^*_A = 5 \text{ MW} \), and \( a^*_B = 4 \text{ MW} \). To this end, we may determine the participation factors to be \( \kappa_A = \frac{5}{9} = 0.556 \), and \( \kappa_B = \frac{4}{9} = 0.444 \). We notice that \( \sum_{i \in \{A,B\}} \kappa_i = 1 \), as expected. The ex-post costs in this case are \( c_{\text{reg, post}} = \sum_{i \in \mathcal{J}} \left( a^m_i \sigma^m_i + c^m_i \gamma^m_i \right) = 5 \cdot 1 + 5 \cdot 1 + 4 \cdot 3 + 10 \cdot 2 = 42 \).

5.4 Possible Balancing Authority Area Coordination Methods
In this section, we present some possible BA area coordination methods. In particular, we describe the ADI methodology currently used by many BA areas, and the constrained fair-splitting dispatch scheme to determine the adjusted ACE of each BA area.
5.4.1 Area Control Error Diversity Interchange Methodology

We denote the summation of the individual ACEs by $ADI$, which is equal to

$$ADI = \sum_{m \in \mathcal{A}} ACE_m.$$  \hspace{1cm} (5.5)

Depending on the sign of the ADI the BA areas that belong in $\mathcal{A}$ are assigned into either the majority or the minority group. More specifically, if $ADI > 0$, then BA area $m$ with $ACE_m > 0$ is part of the majority group $\mathcal{A}^M$, and if $ACE_m' < 0$, then BA area $m'$ belongs to the minority group $\mathcal{A}^\mu$. A mathematical formulation of such a criterion is

$$m \in \begin{cases} 
\text{the majority group } \mathcal{A}^M, & \text{if } ACE_m \cdot ADI > 0 \\
\text{the minority group } \mathcal{A}^\mu, & \text{if } ACE_m \cdot ADI < 0 
\end{cases}.$$  \hspace{1cm} (5.6)

For every BA area $m$ in $\mathcal{A}^\mu$, the adjusted ACE is $ACE_m^a = 0$. We denote by $ADI^\mu = \sum_{m \in \mathcal{A}^\mu} ACE_m$, i.e., the sum of ACEs of the BA areas that belong in the minority group. For the BA areas that belong in the majority group, we use the equal allocation method to determine the adjusted ACEs. To this end, we have

$$ACE_m^a = ACE_m + \frac{ADI^\mu}{|\mathcal{A}^M|},$$  \hspace{1cm} (5.7)

where $|\mathcal{A}^M|$ is the cardinality of the set $\mathcal{A}^M$. However, the ADI adjustment must not change the sign of the ACE. To this end, we have the following condition

$$\text{if } \frac{|ADI^\mu|}{|\mathcal{A}^M|} > |ACE_m|, \text{ then } ACE_m^a = 0.$$  \hspace{1cm} (5.8)

We denote by $\overline{\mathcal{A}}^M$ the set of BA areas that satisfy the condition given in (5.8). In such a case, the remaining amount is redistributed to the other generators in the majority group as follows

$$ACE_m^a = ACE_m + \frac{ADI^\mu + \sum_{i \in \overline{\mathcal{A}}^M} ACE_i}{|\mathcal{A}^M| - |\overline{\mathcal{A}}^M|}, \quad m \in \mathcal{A}^M - \overline{\mathcal{A}}^M.$$  \hspace{1cm} (5.9)
5.4.2 Numerical Illustration

Let us go through a simple example to describe the current process of ADI. Let us assume that we have four interconnected BA areas: $A_1$, $A_2$, $A_3$ and $A_4$. We present in Table 5.3, the calculation of the ADI between those areas. The ADI is equal to, $ADI = 14$ MW. So, areas $A_3$ and $A_4$ are part of the majority group $\mathcal{M}$, and $A_1$, $A_2$ are part of the minority group $\mathcal{M}$. We set the ACE of the BA areas in the minority group equal to zero, as seen in Table 5.3. We have that $ADI = \sum_{m \in \mathcal{M}} ACE_m = \sum_{m \in \{A_1, A_2\}} ACE_m = -86$ MW. For the BA areas in Table 5.3: The adjusted ACE calculation between four BA areas with the ADI methodology.

<table>
<thead>
<tr>
<th>BA Area</th>
<th>Raw ACE [MW]</th>
<th>ADI Adjustment [MW]</th>
<th>Adjusted ACE [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$-46$</td>
<td>$+46$</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$-40$</td>
<td>$+40$</td>
<td>0</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$+60$</td>
<td>$-46$</td>
<td>$+14$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>$+40$</td>
<td>$-40$</td>
<td>0</td>
</tr>
</tbody>
</table>

the majority group we have

$$ACE_{A_4}^a = ACE_{A_4} + \frac{ADI}{|\mathcal{M}|} = 40 - \frac{86}{2} = -3. \quad (5.10)$$

However, the ADI method does not permit to change the sign of the ACE of any BA area. To this end, we set $ACE_{A_4}^a = 0$ MW, based on (5.8), and $\mathcal{M} = \{A_4\}$. Thus, $\mathcal{M} - \mathcal{M} = \{A_3\}$ and we use (5.9) to calculate the modified ACE according for BA area $A_3$. So, we have

$$ACE_{A_3}^a = ACE_m + \frac{ADI + \sum_{i \in \mathcal{M}} ACE_i}{|\mathcal{M}| - |\mathcal{M}|} = 60 + \frac{-86 + 40}{2 - 1} = 14$$. \quad (5.11)
5.4.3 Fair-Splitting Dispatch for Adjusted ACE

There are other ways of modifying the ACEs of the BA areas. For example, we may use the constrained fair-splitting dispatch scheme to determine the adjusted ACE. The objective function of the optimization problem of the fair-splitting dispatch is identically zero, if the feasible set is nonempty, or $\infty$, if the feasible set is empty. The feasibility problem is, thus, to determine whether the constraints are consistent, and if so, find a point that satisfies them. We denote by $\underline{x}_i$ the lower adjusted ACE limit and by $\overline{x}_i$ the upper adjusted ACE limit for BA area $i$. For example, if $ACE_i > 0$, then $\underline{x}_i = 0$ and $\overline{x}_i = ACE_i$. If $ACE_j < 0$, then $\underline{x}_j = ACE_j$ and $\overline{x}_i = 0$. We wish to determine the adjusted ACE for each BA area $i$, $x_i$ which lies between $[\underline{x}_i, \overline{x}_i]$. We define $ADI$ the summation of the raw ACEs of all BA areas, i.e., $ADI = \sum_{m \in A} ACE_m$. To this end, we may write our problem as

$$\begin{align*}
\text{find } & \quad x_i \\
\text{s. t. } & \quad \sum_{i \in A} x_i = ADI \\
& \quad \underline{x}_i \leq x_i \leq \overline{x}_i, \forall i \in A.
\end{align*}$$

(5.12)

The solution to this problem is

$$x_i = \underline{x}_i + \frac{ADI - \sum_{i \in A} x_i}{\sum_{i \in A} (\overline{x}_i - \underline{x}_i)} (\overline{x}_i - x_i).$$

(5.13)

We use this method to calculate the values of ADI using the fair-splitting dispatch algorithm, as shown in Table 5.4.

However, as we can easily see the BA areas are still providing more regulation than needed, even if it is less than what they would have to provide initially. We are motivated from such a solution, to distribute the ADI in a way that all the BA areas have the same sign, and try to mimic the behavior of a BA area in the scenario where all BA areas are assumed be a single BA area. Such an approach
Table 5.4: The adjusted ACE calculation between four BA areas determined with the fair-splitting dispatch.

<table>
<thead>
<tr>
<th>BA Area</th>
<th>Raw ACE [MW]</th>
<th>Adjusted ACE [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>-46</td>
<td>-21.27</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-40</td>
<td>-18.49</td>
</tr>
<tr>
<td>$A_3$</td>
<td>60</td>
<td>32.26</td>
</tr>
<tr>
<td>$A_4$</td>
<td>40</td>
<td>21.50</td>
</tr>
</tbody>
</table>

is given in the Section 5.5.

5.5 Proposed Coordination Scheme among Balancing Authority Areas

In this section, we describe the proposed methodology of BA coordination. We first determine the adjusted ACE that is fed into each AGC system, and then develop a distributed algorithm to solve the centralized AGC dispatch problem. When considering the entire area as a whole we do not include the sub- or superscript $m$ in our notation.

5.5.1 Adjusted ACE Determination

Ideally, we would like to implement the AGC system in one BA area, which would be the union of the individual BA areas. In such a case, from (2.31), the ACE
where $\Delta P_{\text{losses}_{mm'}} = P_{\text{losses}_{mm'}} - P_{\text{losses}_{mm'}}_{\text{sch}}$, i.e., the difference in the losses between BA areas $m$ and $m'$ when the interchange is other than the scheduled. The statement (5.14) is based on the fact that $\Delta P_{mm'} + \Delta P_{m'm} = \Delta P_{\text{losses}_{mm'}}$, for each $m, m' \in \mathcal{A}$, if BA areas $m$ and $m'$ are connected. If the frequency bias factor is equal to the actual frequency response characteristic (AFRC), then $b = \sum_{m \in \mathcal{A}} b_m$. BA areas ideally set the frequency bias factor equal to the AFRC, or to a close value. Therefore, we may argue that $b \approx \sum_{m \in \mathcal{A}} b_m$. Moreover, in order for (5.14) to hold we make the assumption that $\Delta f = \Delta f_m$, for $\forall m \in \mathcal{A}$, which is reasonable since the system is interconnected, and thus operated under the same frequency.

The AGC system for one BA from (2.32), with $\eta_1 = 0$ and $\eta_2 = 1$, is as follows

$$
\frac{dz}{dt} = -z - ACE + P_G 
\approx -z - ADI + P_G + \Delta P_{\text{losses}} 
\approx -z - ADI + P_G,
$$

(5.15)

where $P_G = \sum_{m \in \mathcal{A}} P_G^m$, $P_G^m = \sum_{i \in G_m} P_{Gi}$, $\Delta P_{\text{losses}} = \sum_{m \in \mathcal{A}} \sum_{m' \geq m} \Delta P_{\text{losses}_{mm'}}$.

To this end, we may approximate the AGC system in each BA area $m$, by neglecting the loss term with

$$
\frac{dz_m}{dt} = -z_m - \frac{b_m}{\sum_{m \in \mathcal{A}} b_m} ADI + P_G^m, \forall m \in \mathcal{A}.
$$

(5.16)
(a) One-line diagram of the three-machine nine-bus WECC power system.

(b) The AGC state $z$ for the BA areas considered as one, and separately.

Figure 5.1: Demonstration of ACE determination.

If we sum up (5.16) for all $m \in \mathcal{A}$, we obtain (5.15). Thus, $ACE_m^a = \frac{b_m}{\sum_{m \in \mathcal{A}} b_m} ADI$.

To this end, each BA area, by modifying its ACE, contributes to some extent to mimic the behavior of the AGC system of the entire area as one BA area. The coefficient $\frac{b_m}{\sum_{m \in \mathcal{A}} b_m}$ is used so that the BA areas participate in the ADI according to their size.

Numerical Example

In order to demonstrate the proposed determination of $ACE_m$, we choose the three-machine nine-bus system depicted in Fig. 5.1a, which consists of two BA areas $\mathcal{A} = \{1, 2\}$. We model the AGC system, as if the two BA areas were one and determine the value of $z$. Then, we use the proposed algorithm to determine the
ACE and feed it in the AGC system, \( z_1 \) and \( z_2 \), of each BA area respectively. We sum up the values of \( z_1 \) and \( z_2 \), and compare it with \( z \), as depicted in Fig. 5.1b. We notice that we approximate very well the behavior of the AGC system of one area, since \( z_1 + z_2 \), matches the value of \( z \). In this simple example, when we consider the system as one BA area, we have that the real power interchanges between BA areas 1 and 2 are \( P_{12} = 0.3097 \), and \( P_{21} = -0.2938 \). When we consider two areas, we have \( P_{12} = 0.3098 \), and \( P_{21} = -0.2939 \). In this case, \( \Delta P_{\text{losses}_{12}} = 0 \), that is why the two signals in Fig. 5.1b match exactly.

5.5.2 Distributed Algorithm of Automatic Generation Control Dispatch

Another issue arising from such a method is how each \( z_m \) is allocated among the regulating units in the BA areas. In the case where we consider all the BA areas as one, we would solve (5.3) for one area. We may modify the formulation of (5.3), as follows

\[
\min \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{F}_m} a^m_i \sigma^m_i
\]

s. t. \( \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{F}_m} a^m_i = b \) \( \forall m \in \mathcal{A}, i \in \mathcal{F}_m \),

where \( b = |z - \sum_{m \in \mathcal{A}} \sum_{j \in \mathcal{G}_m} P_{ED_j}| \), \( \rho_t = \min\{\rho_i^m : m \in \mathcal{A}\} \), and \( \overline{\rho}_i^m = \min\{\zeta_i^m, \rho_i, \epsilon_i^m\}, \forall m \in \mathcal{A}, i \in \mathcal{F}_m \). We introduce barrier functions to represent the inequality constraints (see, e.g., [73]). We now have the equivalent problem

\[
\min \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{F}_m} \left( a^m_i \sigma^m_i - \mu_n \log(a^m_i) - \mu_n \log(\epsilon_i^m - a^m_i) \right)
\]

s. t. \( \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{F}_m} a^m_i = b \longleftrightarrow \lambda ,\)
where $\mu_n$ is a positive sequence, with $\mu_n \to 0$, as $n \to \infty$, e.g., $\mu_n = \frac{1}{10^n}$. The dual variable $\lambda$ denotes the marginal cost of providing one more MW of regulation. We denote by $a = \{a^m_i : \forall m \in \mathcal{A}, i \in \mathcal{J}^m\}$, and we have the Lagrangian of (5.18) $L(a, \lambda) = \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} \left( a_i^m \sigma_i^m - \mu_n \log(a_i^m) - \mu_n \log(\pi_i^m - a_i^m) \right) + \lambda (\sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} a_i^m - b)$. According to [74, p. 243], we may add the term $\frac{1}{2} (\sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} a_i^m - b)^2$ in the Lagrangian function and solve an equivalent problem. From [75], the dynamics of the saddle point are given by

$$a_{[k+1]} = a_{[k]} - \gamma L_a(a, \lambda),$$

$$\lambda_{[k+1]} = \lambda_{[k]} + \gamma L_\lambda(a, \lambda), \quad (5.19)$$

where $\gamma > 0$ is the stepsize and $L(\cdot)$ is the partial derivative of $L$ with respect to the argument in the subscript. The only restriction in the initial conditions is that $0 < a_i^m[0] < \bar{a}_i^m$ for all $m \in \mathcal{A}, \ i \in \mathcal{J}^m$. It is shown in [75] that the values of $a_{[k]}$ and $\lambda_{[k]}$ converge to the optimal values, i.e., $a \to a^*$, and $\lambda \to \lambda^*$. The proposed distributed algorithm that gives the same solution as the centralized one, given in (5.17), is

$$a_{i[k+1]} = \begin{cases} 0 & , a_{i[k]} \leq \bar{a}_i^m \\ a_{i[k]} & , a_{i[k]} > \bar{a}_i^m \\ a_{i[k]} - \gamma (\sigma_i + \lambda_{[k]} + (\sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} a_i^m - b) - \mu_n \frac{1}{\pi_{i-1}} + \mu_n \frac{1}{\pi_{i-1}}), & \text{otherwise} \end{cases}, \quad (5.21)$$

$$\lambda_{[k+1]} = \lambda_{[k]} + \gamma (\sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} a_i^m - b). \quad (5.22)$$

So far, we have established that the total ex-post regulation costs $c_{\text{reg, post}}$ are minimized, and are equal to

$$c_{\text{reg, post}} = \sum_{m \in \mathcal{A}} \sum_{i \in \mathcal{J}^m} \left( a_i^m \sigma_i^m + c_i^m \gamma_i^m \right). \quad (5.21)$$

The total cost is $c_{\text{reg}} = c_{\text{reg, post}} + \sum_{m \in \mathcal{A}} c_{\text{reg, ante}}^m$. However, another issue is how much each BA area contributes to the cost. One way of doing so, is to allocate it
Figure 5.2: Evolution of the amounts $a_i$ for case (i).

Proportionally to the ACE of each area. To this end, we have

$$c_{\text{reg}}^m = \frac{|ACE_m|}{\sum_{m \in A} |ACE_m|} c_{\text{reg, post}} + c_{\text{reg, ante}}. \quad (5.22)$$

Such an allocation is fair in the sense that BA areas, which have larger ACEs, pay more than those that have smaller ACEs. The ACE ideally represents the MW amount that needs to be provided to restore the system frequency to the nominal value. Therefore, BA areas with a large ACE need to provide a large regulation amount. For the BA areas that procure the biggest regulation amount, they still pay less than what they would pay if no coordination were present. Thus, they have incentive to coordinate with the other BA areas.

Figure 5.3: Evolution of $\lambda$ for case (i).
Numerical Example

We provide several case studies to demonstrate the capabilities of the proposed distributed algorithm. Let us consider three resources $A$, $B$, and $C$, with service offers $\sigma_A = 3 \$/ MW, $\sigma_B = 5 \$/ MW, and $\sigma_C = 8 \$/ MW$. Their capacity limits are $c_A = c_B = c_C = 10$ MW, their response rates are $\zeta_A = \zeta_B = \zeta_C = 3$ MW/min and the response time requirement is $\rho_t = 10$ min. We determine the values of $a_i = \min\{\zeta_i\rho_t, c_i\}$, for $i = A, B, \text{ and } C$. Thus, we have $a_A = a_B = a_C = 10$ MW.

For case (i), we have that $b_{(i)} = 9$ MW, so the optimal solution is $(a^*_A, a^*_B, a^*_C) = (9, 0, 0)$. In Fig. 5.2, we see that the algorithm converges to the optimal solution.

The marginal cost of providing one more MW of regulation is shown by the dual variable $\lambda$, whose value is $-3 \$/ MW, which is the cost of resource $A$. The evolution of $\lambda$ is depicted in Fig. 5.3. Now, we change $b_{(ii)}$ equal to 15 MW. The optimal
solution is \((a_A^*, a_B^*, a_C^*) = (10, 5, 0)\), and in Fig. 5.4, we see that the algorithm converges to the optimal solution. In addition, the marginal cost is now \(\lambda = -5\) \$/MW, as depicted in Fig. 5.5, which is equal to the cost of resource B.

5.6 Numerical Results

In this section, we illustrate the proposed methodology with the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system, which is depicted in Fig. 5.6, it contains three synchronous generating units in buses 1, 2 and 3, and load in buses 5, 6 and 8. The machine, network and load parameter values may be found in [52]. We consider two BA areas, as depicted in Fig. 5.6, \(\mathcal{A} = \{1, 2\}\). Unless otherwise noted, all quantities in the numerical results section are expressed in per unit (pu) with respect to 100 MVA as base power. We present several case studies and calculate the total MW amount of regulation and the total cost by: (Ai) using the proposed methodology, described in Section 5.5, (Aii) considering \(\{1, 2\}\) as one BA area, (Aiii) having no coordination between the BA areas, and (Aiv) using the ADI method, given in Section 5.4.1.

We modify the load at bus 5 that belongs in BA area 2, as follows \(P_{L_5} = P_{L_{50}} -\)
0.15, where $P_{L0}$ is the initial load equal to 1.25 pu. In a similar way, we modify the load at bus 6, which belongs in BA area 1, as $P_{L6} = P_{L0} + 0.17$, with $P_{L0} = 0.9$ pu. The three generators that belong in $\mathcal{A}$ participate in regulation, and their bids are given in Table 5.5. We solve the market clearing mechanism, described in (5.1), for two cases: (Bi) considering $\{1, 2\}$ as one BA area, (Bii) as two separate BA areas, in order to select which resources are used for regulation and their cost savings to the system. For case (Bi) we have the operational constraints: the capacity requirement is $r_c = 30$ MW, the service requirement is $r_s = 30$ MW, and the response time requirement is $r_t = 20$ min. For case (Bii) we have the operational constraints: the capacity requirements are $r_1^c = r_2^c = 20$ MW, the service requirements are $r_1^s = r_2^s = 20$ MW, and the response time requirements are $r_1^t = r_2^t = 20$ min. We provide the results for the case (Bi), and find that resources 1 and 2 are selected and the total cost is $\eta = $90. We calculate the total cost without resource 1 and 2, respectively, to be $\eta^{-1} = $225 and $\eta^{-2} = $165. To this end, the ex-ante payments to resources 1 and 2 are $135$ and $75$, respectively. The cleared resources for case (Bii) are for BA area 1: generators 2, and 3, and for BA area 2, generator 1. We compare the ex-post costs incurred for regulation and the total regulation amount for the four aforementioned methods. The results are given in Table 5.6.

We notice that the optimal solution is provided by method (Aii), i.e., when we consider the entire system as one BA area, since the regulation amount as well as the associated costs are minimum. At the initial steady state the flows in lines (5, 4) and (9, 6), which connect the BA areas 1, and 2, were $-0.4198$ pu and

<table>
<thead>
<tr>
<th>Resource Number</th>
<th>Regulation Capacity (MW)</th>
<th>Capacity Offer ($/MW$)</th>
<th>Service Offer ($/MW$)</th>
<th>Response Rate (MW/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.5: Bids of generators 1, 2, and 3.
Table 5.6: Ex-post cost and regulation amount for the four methodologies.

<table>
<thead>
<tr>
<th>method</th>
<th>(Ai)</th>
<th>(Aii)</th>
<th>(Aiii)</th>
<th>(Aiv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost of area 1</td>
<td>2.1571</td>
<td>–</td>
<td>49.8469</td>
<td>6.1123</td>
</tr>
<tr>
<td>cost of area 2</td>
<td>2.1557</td>
<td>–</td>
<td>20.1061</td>
<td>2.3058</td>
</tr>
<tr>
<td>total ex-post cost</td>
<td>4.3128</td>
<td>4.2308</td>
<td>70.0680</td>
<td>8.4181</td>
</tr>
<tr>
<td>regulation amount</td>
<td>4.1978</td>
<td>4.1908</td>
<td>36.7212</td>
<td>4.2674</td>
</tr>
</tbody>
</table>

0.5776 pu, respectively. So the scheduled power flow between BA areas 1 and 2 is $P_{12_{sch}} = 0.1578$ pu. After the modifications in loads at buses 5, and 6, the real power interchange is for method (Ai) $P_{12}^{(Ai)} = 0.1520$ pu, for (Aii) $P_{12}^{(Aii)} = 0.1521$ pu, for (Aiii) $P_{12}^{(Aiii)} = P_{12_{sch}} = 0.1578$ pu, and for (Aiv) $P_{12}^{(Aiv)} = 0.1610$ pu. So, the AGC commands in (Ai), (Aii), and (Aiv) create similar flows between the two BA areas. That is not achieved in method (Aiii), where no coordination is present. The reason is that the ACE value includes the real power interchange, and the AGC system goal is to make ACE to zero. In methods (Ai) and (Aiv), since the adjusted ACE is determined by an addition of the individual ACEs, such an event is not observed. We also notice that the real power interchange between BA areas 1 and 2 determined by using method (Ai) is closer to that of (Aii), than that of method (Aiv). That is why, method (Ai) provides a smaller amount of regulation than method (Aiv). The reason is that the ACE of each BA area in method (Ai), is built by considering the ACE of the BA areas as a whole, as

![Area control error with methods (Ai), and (Aii).](image_url)
described in detail in Section 5.5.1. We depict in Fig. 5.7 the ACE with method (Aii) and the addition of the individual adjusted ACEs of BA areas 1 and 2, as determined by method (Ai). We notice that they are very close to each other.

We also notice from Table 5.6 that the minimum cost is achieved by using method (Aii). In this case, only generator 1 is used in regulation since the load imbalance is not over 0.2 pu which is the capacity limit of resource 1, and the ramping requirements are met. Also, in method (Ai), only generator 1 is utilized, since the distributed algorithm, given in Section 5.5.2, provides the same results as the centralized market clearing mechanism. For example, at one time instant where the total regulation needed in 2 MW, the participation factors converge to the values: $\kappa_1^2 = 1$, $\kappa_2^1 = 0$, and $\kappa_3^1 = 0$, as depicted in Fig. 5.8. The total cost is distributed among the areas, based on the coefficients presented in Fig. 5.9. However, in method (Aiii), where no coordination is present, all
generators participate in regulation, instead of only the least cost one. In method (Aiv), the ACE has a lower in magnitude value, thus in BA area 1, only generator 2 is needed in regulation. To this end, the entire system only uses generators 1 and 2 for regulation. Generator 2 is more expensive than generator 1, but since the ADI method does not provide the option of exchanging regulation amounts between BA areas, generator 2 is needed to provide regulation.

5.7 Summary

In this chapter, we set the stage to develop the proposed algorithm by modeling the AGC market clearing mechanism and dispatch. We presented some possible BA area coordination methods, such as the ADI methodology, currently used in industry. We then proposed a possible coordination of the BA areas, in order to increase the efficiency, in terms of quality and cost, of the AGC system. To this end, we aim in approximating the solution of a centralized AGC system and market clearing mechanism. More specifically, we approximated the ACE of the entire area as the summation of the individual ACEs and distributed it into each AGC subsystem accordingly. Next, we developed a distributed algorithm that minimizes the cost of regulation, by allocating the AGC command from the cheapest to the most expensive generator sequentially, until all the regulation amount is met. The advantage of using the distributed algorithm is that the BA areas do not need to exchange any cost information. The only element needed is the total mismatch from the desired regulation and that provided. The proposed approach provides a solution close to the optimal, i.e. if all BA areas were under the same jurisdiction, but respects that each BA area wants to keep certain information from other BA areas, and provide the least cost solution for its customers. We demonstrated in Section 5.6 how the proposed method works and compare it with other three methods. We showed that it approximates very closely the optimal solution of considering all BA areas as one. The reasons are: (i) that the adjusted ACE fed to each AGC system is constructed to approximate the behavior of the
ACE of the entire BA area, and (ii) the AGC dispatch is solved distributively and provides the same solution as a centralized algorithm would. In addition, the results provided with the proposed method were better than other methods currently used in industry, as demonstrated in Section 5.6.
In this chapter, we discuss how the AGC system may be included in the market environment. We investigate the possibility of using economic signals from the RTMs instead of formulating a separate AGC market. To this end, we present the formulation of the ED process, which is used to clear the RTM, to obtain appropriate economic signals. We also discuss that the quality of the AGC service provided is affected by the ramping characteristics of the regulating units chosen to participate in the AGC. We propose a systematic method for allocating the AGC signal, i.e., determining the AGC dispatch, by taking into account the economic signals from the ED process as well as the quality of the AGC service provided.

6.1 Introduction

A critical aspect concerning the AGC system is the allocation of the total generation needed among the generators participating in AGC. This allocation is important, since it affects the cost and quality of the service offered. It needs to be simple, since it is conducted every two to four seconds, but also needs to meet certain criteria. The ISO wishes to maximize social welfare, therefore, the total cost for the AGC system should be minimized. However it was found that this method does not acknowledge the greater amount of frequency regulation service being provided by faster-ramping units [14]. The deepening penetration of renewable resources, with high variability, intensifies the need for fast responding units with high ramping rates participating in AGC, that need to be compensated accordingly.
In this chapter, we present a systematic method of allocating the AGC signal among the generators by taking into consideration the ramping characteristics of each generator, as well as economic criteria defined by the ED process, as described in Section 6.2. In Section 6.3, we develop the proposed approach, where we provide the possibility to take into account the quality of service, i.e., how fast the generators respond, in different extents, based on the needs of each particular system. For instance, systems with deep penetration of renewable resources need fast responsive units in the AGC regulation. In Section 6.4, we present two other AGC allocation methods, currently using in industry. In Section 6.5, we provide numerical results of the proposed approach and compare it with the two other methods. We summarize the results in Section 6.6.

6.2 Economic Dispatch Process

In this section, we present the ED process modeling. In particular, we describe the ED process with loss coefficients, and the DCOPF. We consider a power system with $N$ buses indexed by $\mathcal{N} = \{1, \ldots, N\}$ and $L$ lines indexed by $\mathcal{L} = \{\ell_1, \ldots, \ell_L\}$. We denote each line by the ordered pair $\ell = (n, n')$, $n, n' \in \mathcal{N}$, with the real power flow $f_\ell \geq 0$ whenever the flow is from $n$ to $n'$ and $f_\ell < 0$ otherwise. The set of synchronous generating units is indexed by $\mathcal{I} = \{1, 2, \ldots, I\}$.

The ED objective is to minimize the total cost, which is the sum of the costs of the individual units, subject to the essential constraint imposing that the sum of the generators output must be equal to demand. In the ED process, other physical constraints may be included, such as voltage or real power flow constraints, or in the power balance constraints the losses may be taken into consideration. The various available formulations of the ED are a result of what constraints, as well as, which power model (AC or DC) are used. For the ED that is implemented every five minutes, common formulations are the ED with loss coefficients and DC optimal power flow (DCOPF), due to their simplicity and computational efficiency. Next, we formulate the ED with loss coefficients and subsequently the
6.2.1 ED with Loss Coefficients

We denote the total system load by $P_{\text{load}}$, the system losses by $P_{\text{loss}}$, the net interchange by $P_{\text{inter}}$, the output of the $i^{th}$ unit by $P_{G_i}$ and the $i^{th}$ generation cost function by $\hat{c}_i(\cdot)$. The mathematical formulation of the ED with loss coefficients is

$$\min \sum_{i \in \mathcal{J}} \hat{c}_i(P_{G_i})$$

s. t.

$$\sum_{i \in \mathcal{J}} P_{G_i} = P_{\text{load}} + P_{\text{loss}} + P_{\text{inter}} \quad \leftrightarrow \quad \sigma$$

$$P_{G_i} \geq P_{G_i}^{m}, \quad \forall \ i \in \mathcal{J} \quad \leftrightarrow \quad \eta_i^m$$

$$P_{G_i} \leq P_{G_i}^{M}, \quad \forall \ i \in \mathcal{J} \quad \leftrightarrow \quad \eta_i^M,$$

where $P_{G_i}^{m}$ ($P_{G_i}^{M}$) are the lower (upper) permissible limits of the real power generation at bus $i$ and $\sigma, \eta_i^m$ and $\eta_i^M$ are the Lagrangian multipliers or dual variables associated with the corresponding constraints of the problem. Define the vectors $\eta^m = [\eta_1^m, \ldots, \eta_I^m]^T$ and $\eta^M = [\eta_1^M, \ldots, \eta_I^M]^T$. The calculation of the system losses makes the problem more complicated. We express the system losses as a function of the generators output by using the so-called $B$-coefficients method [5, pp. 162-182]:

$$P_{\text{loss}} = \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} P_{G_i} B_{ij} P_{G_j} ,$$

where $B_{ij}$ are the loss coefficients considered to be constant under certain assumed conditions. More specifically, we have that

$$B_{ij} = \sum_{\ell \in \mathcal{L}} R_{\ell} \omega^i_{\ell} \omega^j_{\ell} ,$$

where $R_{\ell}$ is the line’s $\ell$ resistance and $\omega^i_{\ell}$ is the line $\ell$ generalized generation distribution factor with respect to an injection/withdrawal at bus $i$ [76].
6.2.2 DCOPF

We assume the network to be lossless. We denote the diagonal branch susceptance matrix by \( B_d \in \mathbb{R}^{L \times L} \) and the branch-to-node incidence matrix for the subset of nodes \( \mathcal{N} \) by \( A \in \mathbb{R}^{L \times N} \). The corresponding nodal susceptance matrix is \( B \in \mathbb{R}^{N \times N} \). Let \( P_L \) be the load at bus \( i \), \( P_I \) be the interchange at bus \( i \) (positive if exporting, negative otherwise), \( f \) be the power flow through line \( \ell \), and \( f^M \) (\( f^m \)) be the limit of the real power flow on the same (opposite) direction of line \( \ell \); and define \( P_G = [P_{G1}, \ldots, P_{GN}]^T \), \( P_L = [P_{L1}, \ldots, P_{LN}]^T \), \( P_I = [P_{I1}, \ldots, P_{IN}]^T \), \( f = [f_{\ell_1}, \ldots, f_{\ell_L}]^T \), \( f^M = [f^M_{\ell_1}, \ldots, f^M_{\ell_L}]^T \) and \( f^m = [f^m_{\ell_1}, \ldots, f^m_{\ell_L}]^T \). The optimization problem describing the ED process is

\[
\begin{align*}
\min \quad & \sum_{i \in \mathcal{I}} \hat{c}_i(P_{Gi}) \\
\text{s. t.} \quad & P_G - P_L - P_I = B \theta \quad \leftrightarrow \lambda \\
& f = B_d A \theta \leq f^M \quad \leftrightarrow \mu^M \\
& -f \leq f^m \quad \leftrightarrow \mu^m \\
& P_{Gi} \geq P^m_{Gi}, \quad \forall i \in \mathcal{I} \quad \leftrightarrow \eta^m_i \\
& P_{Gi} \leq P^M_{Gi}, \quad \forall i \in \mathcal{I} \quad \leftrightarrow \eta^M_i,
\end{align*}
\]

where \( \theta_i \) is the voltage phase angle at bus \( i \) and \( \theta \) is the corresponding vector; and \( \lambda, \mu^M, \mu^m, \eta^m_i \) and \( \eta^M_i \) are the dual variables associated with the corresponding constraints of the problem. The vector of dual variables associated with the power balance constraints is known as the vector of locational marginal prices (LMPs) and is denoted by \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N]^T \).

In both optimization problems in (6.1) and (6.4), the dual variables of the equality constraints may be interpreted as the cost needed to satisfy the constraint. As for an inequality constraint, if it is not binding the dual variable is zero. If the dual variable is not zero, i.e., if the constraint is binding, then the dual variable

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may be interpreted as the benefit associated with relieving the constraint. The ED, as defined by the solution of either (6.1) or (6.4), provides, usually every five minutes, the optimal output of each generator, which we denote by $P_{ED_i} = P^*_{G_i}$.

### 6.3 Proposed Automatic Generation Control Dispatch

In this section, we develop the proposed AGC dispatch. More specifically, we determine the marginal cost of a generator to change its output by a small amount. We use this value in the proposed AGC allocation method. In addition, we explain how the importance of fast-responsive units is taken into account in our formulation.

When a disturbance occurs, the system behaves as described in (2.1)-(2.10), (2.21)-(2.22), (2.27)-(2.30), and (2.33). In the discrete AGC system given in (2.33), we use $\eta_1 = 0$, and $\eta_2 = 1$. The generators participating in AGC modify their output so that the generation meets the load at all time. Each generator $i$ in BA area $m$ participates in the AGC by a function $\phi_i(z_m)$. In particular, we have

$$P_{C_i} = P_{ED_i} + \kappa^m_i(z_m - \sum_{j \in G_m} P_{ED_j}), \quad (6.5)$$

where $\kappa^m_i$ is the participation factor of generator $i$ in the AGC system, with $\sum_{i \in G_m} \kappa^m_i = 1$, $\forall m \in \mathcal{A}$. We propose a systematic way of determining the AGC participation factors $\kappa^m_i$ by using economic criteria (defined by the ED process) and by taking into account unit ramping characteristics.

#### 6.3.1 Marginal Generator Cost

We wish to specify the marginal cost for each generator of serving 1 MW of load. To this end, we use the mathematical formulations of the ED process and some basic optimization concepts (see, e.g., [73]). Let us consider the optimization
problem

\[
\min f(x) \\
\text{s. t.} \\
g(x) = 0 \quad \leftrightarrow \nu \\
h(x) \leq 0 \quad \leftrightarrow \tau.
\] (6.6)

The Lagrangian function for this optimization problem is defined as

\[
\mathcal{L}(x, \nu, \tau) = f(x) + \nu^T g(x) + \tau^T h(x).
\]

We know that at the optimal point \(x^*\) we have

\[
\frac{\partial L}{\partial x_i} \bigg|_{x^*} = 0, \quad \forall i,
\]

where \(x_i\) is the \(i^{th}\) element of the vector \(x\). We use this fact to derive the marginal cost of a generator to modify its output.

For the ED with loss coefficients, as defined in (6.1), we denote the vector \(y = [P_{G_1}, \ldots, P_{G_N}, \sigma, \eta^M, \eta^m]^T\), and the Lagrangian is given by

\[
\mathcal{L}(y) = \sum_{i \in \mathcal{I}} \hat{c}_i(P_{G_i}) + \sigma \left( P_{\text{load}} + P_{\text{loss}} + P_{\text{inter}} - \sum_{i \in \mathcal{I}} P_{G_i} \right) + \sum_{i \in \mathcal{I}} (P_{G_i} - P_{G_i}^M)\eta_i^M + \sum_{i \in \mathcal{I}} (P_{G_i}^m - P_{G_i})\eta_i^m.
\] (6.7)

At the optimal point we have

\[
\frac{\partial \mathcal{L}}{\partial P_{G_i}} \bigg|_{y^*} = \frac{\partial \hat{c}_i}{\partial P_{G_i}} \bigg|_{y^*} - \sigma^* \left(1 - \frac{\partial P_{\text{loss}}}{\partial P_{G_i}} \bigg|_{y^*}\right) + \eta_i^{M^*} - \eta_i^{m^*} = 0,
\] (6.8)

where \(\frac{\partial P_{\text{loss}}}{\partial P_{G_i}} = 2 \sum_{j \in \mathcal{I}} B_{ij} P_{G_j}\). We denote \(\eta_i^* = \eta_i^{M^*} - \eta_i^{m^*}\), for \(i \in \mathcal{I}\); then, it follows that

\[
\frac{\partial \hat{c}_i}{\partial P_{G_i}} \bigg|_{y^*} = \sigma^* \left(1 - 2 \sum_{j \in \mathcal{I}} B_{ij} P_{G_j}^*\right) - \eta_i^* = \rho_i.
\] (6.9)

For the DCOPF formulation, as defined in (6.4), we denote the vector \(x = \ldots\)
where \( B \) is the \( i \)th row of \( B \), \( B_d \) is the \( i \)th row of \( B_d \). At the optimal point we have

\[
\frac{\partial \mathcal{L}}{\partial P_{G_i}}\bigg|_{x^*} = \frac{\partial \hat{c}_i}{\partial P_{G_i}}\bigg|_{x^*} - \lambda_i^* - \eta_i^* = 0. \tag{6.11}
\]

So we have

\[
\frac{\partial \hat{c}_i}{\partial P_{G_i}}\bigg|_{x^*} = \lambda_i^* - \eta_i^* = \rho_i. \tag{6.12}
\]

We interpret the partial derivates in (6.9) and (6.12) as the incremental costs \( \rho_i \) as generator \( i \) changes its output by a small amount \( \Delta P_{G_i} \).

### 6.3.2 Proposed AGC Dispatch Formulation

Now, we wish to take into account the ramping characteristics of the synchronous generating units. Each unit’s \( i \)th contribution to raise (lower) its output is constrained by its maximum (minimum) ramping capability \( \upsilon_i^+ (\upsilon_i^-) \) and the units upper (lower) power limits. The convention we are using is that \( \upsilon_i^- \) is a negative number. The units for the ramping rates are usually MW/min. We denote the binary variables \( \delta_m^+, \delta_m^- \in \{0, 1\} \) of BA area \( m \in \mathcal{A} \) to reflect if the total generation needed in the AGC system in BA area \( m \), \( z_m \), is positive or negative. More precisely, \( \delta_m^+ = 1 \) and \( \delta_m^- = 0 \) if \( z_m \geq 0 \), and \( \delta_m^+ = 0 \) and \( \delta_m^- = 1 \) if \( z_m < 0 \). For each BA area \( m \in \mathcal{A} \), the allocation of the AGC signal among the generators
is provided by the solution to the following optimization problem

$$\begin{align*}
\min & \sum_{i \in \mathcal{G}_m} \rho_i P_{C_i} - \delta^+ \zeta_m \sum_{i \in \mathcal{G}_m} \nu_i^+ P_{C_i} + \delta^- \zeta_m \sum_{i \in \mathcal{G}_m} \nu_i^- P_{C_i} \\
\text{s. t.} & \sum_{i \in \mathcal{G}_m} P_{C_i} = z_m \\
& P_{C_i} \leq P^M_{G_i}, \quad \forall i \in \mathcal{G}_m \\
& P_{C_i} \geq P^m_{G_i}, \quad \forall i \in \mathcal{G}_m \\
& f^m_{\ell} \leq \sum_{i \in \mathcal{G}_m} \psi^i_{\ell}(P_{C_i} - P_{G_i}) \leq f^M_{\ell}, \quad \forall \ell \in \mathcal{L}_m,
\end{align*}$$

(6.13)

where $\zeta_m$ is a parameter that weights the importance of using fast responsive units and is affected by the system characteristics. We denote by $\psi^i_{\ell}$ the injection shift factor of line $\ell$ with respect to an injection/withdrawal at bus $i$ and by $\mathcal{L}_m$ the set of lines in BA area $m$. The optimization problem in (6.13) determines $P_{C_i}$ for $i \in \mathcal{G}_m$. Thus, we may determine the participation factors $\kappa^m_i$ for area $m$ by

$$\kappa^m_i = \frac{P_{C_i}}{z_m}, \quad \forall i \in \mathcal{G}_m.$$

### 6.3.3 Weighting Parameter $\zeta_m$

The value of the parameter $\zeta_m$ is affected by the system characteristics. For example, systems with deep penetration of renewable resources have high values of $\zeta_m$. Some metrics to quantify the level of renewables in the system are the net load variations and the required ramping capability. A method of calculating the ramping requirements given some confidence level is given in [77]. The authors take into account the renewable-based generation output and load forecast errors to determine the required ramping at a certain time. In a similar rationale, the ramping capability of the system may be calculated as shown in [78]. The ratio of required ramping to the available ramping capability of the system $\varsigma_m$ is used...
as an input to determine the parameter $\zeta_m$. The ratio $\varsigma_m$ provides a good metric of the net load variation, i.e., high values show large net variations and a deep integration of renewable resources. On the other hand, low values demonstrate that the net load variations are low and that there are not a lot of renewable-based resources in the system. In addition, in order to insert a dollar value for each MW to the parameter $\zeta_m$, we use the average incremental costs of all generators

$$\bar{\rho}_m = \frac{\sum_{i \in G_m} \rho_i}{|G_m|},$$

where $|G_m|$ is the cardinality of the set $G_m$. To have comparable values with the first term of the objective value we insert in the $\varsigma_m$ parameter the average ramping rates

$$\bar{v}_m = \frac{\sum_{i \in G_m} (v_i^+ - v_i^-)}{2|G_m|} \quad \text{We define the parameter } \zeta_m \text{ as}$$

$$\zeta_m = \varsigma_m \frac{\bar{\rho}_m}{\bar{v}_m}.$$

### 6.4 Alternative Automatic Generation Control Dispatch Methods

We compare the results of the proposed method, presented in Section 6.3, with other two AGC allocation methods: alternative method $A_1$, where LMPs are used as economic signals to allocate the AGC command to each generator, and alternative method $A_2$, where the ramping characteristics of each generator are taken into consideration, as discussed in [19]. In particular, by using method $A_1$ the participation factors of each generator $i$ are given by

$$k^m_{i,(A1)} = \frac{1}{|G_m| - 1} - \frac{\lambda_i}{(|G_m| - 1) \sum_{j \in G_m} \lambda_j}, \quad (6.14)$$

where $\lambda_i$ is the LMP at bus $i$, and $|G_m|$ is the cardinality of the set $G_m$. Based on alternative method $A_2$, the AGC participation of each generator $i$ is

$$P_{C_i(A2)} = \begin{cases} \min \left( \frac{v_i^+}{\sum_{j \in G_m} v_j^+} z_m, P^M_{G_i} \right), & \text{if } z_m \geq 0 \\ \max \left( \frac{v_i^-}{\sum_{j \in G_m} v_j^-} z_m, P^M_{G_i} \right), & \text{if } z_m < 0 \end{cases} \quad (6.15)$$
Once $P_{C_i(A2)}$ for $i \in G_m$ are determined, the participation factors may be calculated as follows $\kappa_{i(A2)}^m = \frac{P_{C_i(A2)}}{z_m}$, $\forall i \in G_m$.

The cost and quality of ACG service provided are different based on which allocation method is used. There are several similarities and differences between the three methods. The proposed method and alternative method A1 take into account economic signals, i.e., LMPs. However, the more appropriate economic signal is not the marginal cost of providing another MW at a bus $i$, i.e., $\lambda_i$, but the marginal cost of modifying the output of a generator, which is affected by both the LMP at the bus where the generator is located and the output level of the generator. With alternative method A2 the fastest units are chosen. The proposed method values the necessity of fast ramping units with the use of the weighting parameter $\zeta_m$ as described earlier in Section 6.3.

6.5 Numerical Results

We illustrate the proposed methodology with the standard three-machine nine-bus Western Electricity Coordination Council (WECC) power system, which is depicted in Fig. 6.1, it contains three synchronous generating units in buses 1, 2 and 3, and load in buses 5, 6 and 8. The machine, network and load parameter values may be found in [52].
We consider one BA area for the WECC power system. As a result the ACE is only a function of the frequency deviation. We choose the frequency bias factor to be $b = -0.1$ MW/Hz. We formulate the ED process with the DCOPF, as described in (6.4). The ED process is implemented every 5 minutes. The quantities in this section are expressed in per unit (pu) with respect to a 100 MVA base, unless stated otherwise. The load profile is as follows: $P_{L5} + jQ_{L5} = 1.25 + j0.50$, $P_{L6} + jQ_{L6} = 0.9 + j0.30$, and $P_{L8} + jQ_{L8} = 1.00 + j0.35$. The real power flow limits for all lines in the same (opposite) direction are 1 pu ($-1$ pu). The cost functions for the three generators are (units are in $$/MW): $\hat{c}_1(P_{G1}) = 0.025P_{G1}^2 + 10P_{G1} + 100$, $\hat{c}_2(P_{G2}) = 0.012P_{G2}^2 + 20P_{G2} + 120$ and $\hat{c}_3(P_{G3}) = 0.010P_{G3}^2 + 13P_{G3} + 150$. The minimum (maximum) output in pu for each generator are: $0 \leq P_{G1} \leq 1.2$, $0 \leq P_{G2} \leq 2$ and $0 \leq P_{G3} \leq 1.5$. The ramping characteristics for each unit in MW/min are: $v_1^+ = 3$, $v_2^+ = 2$, $v_3^+ = 1$ and $v_1^- = -3$, $v_2^- = -2$ and $v_3^- = -1$.

In the initial steady state, there is no congestion in the system, thus the uniform LMP for the system is 20.01 $$/MW. The synchronous generators in buses 1 and 3 are at their upper limits. The dual variables associated with the upper limits for the two generators are $\eta_1^M = 9.95 $$/MW and $\eta_3^M = 6.98 $$/MW. The timeframe of the simulations is described as follows: $t = 0$ s a disturbance occurs, $t = 60$ s the ED sends new signals to the generators and the AGC system is implemented every 2 s. In the first case, we modify the load in bus 5 as follows $P_{L5} = 1.7$ pu. In this case, the results of the updated ED process, show that congestion arises in the system and the LMPs at each node are $\lambda_1 = 24.87$, $\lambda_2 = 20.02$, $\lambda_3 = 13.03$, $\lambda_5 = 29.02$, $\lambda_6 = 15.17$ and $\lambda_8 = 22.85$ in $$/MW. We have 6 LMPs because in the DCOPF formulation buses 1 $\equiv 4$, 2 $\equiv 7$ and 3 $\equiv 9$, since they are connected by transformers.

The modification of the load causes a mismatch between generation and demand, and a deviation from the nominal frequency. We use three methods to allocate the AGC signal to restore the frequency to the nominal value: (i) our proposed method, (ii) alternative method A1 and (iii) alternative method A2.
We compare the costs and the quality of AGC service for each method. For the WECC system we choose the value of $\zeta$ to be 2 $\text{$/min/MW}^2$. The calculated values of the marginal cost in $\$/MW for each generator are $\rho_1 = 10.06$, $\rho_2 = 20.02$ and $\rho_3 = 13.03$. We would expect that the participation factor for generator 1 would be the largest; however, since we also consider the network constraints, we end up with $\kappa_1 = 0.3710$, $\kappa_2 = 0.1653$ and $\kappa_3 = 0.4637$ at first. The participation factors after the updated ED process are: $\kappa_1 = 0.3220$, $\kappa_2 = 0.3012$ and $\kappa_3 = 0.3768$. Since, at first the LMPs are equal at all nodes, in method A1, we have that $\kappa_{i(\text{A1})} = \frac{1}{3}$, for $i = 1, 2, 3$. Then, when the ED signal is updated and there is congestion in the system the participation factors become $\kappa_{1(\text{A1})} = 0.2853$, $\kappa_{2(\text{A1})} = 0.3272$ and $\kappa_{3(\text{A1})} = 0.3876$. For method A2, we have constant participation factors for the considered period of time, which are equal to $\kappa_{1(\text{A2})} = 0.5$, $\kappa_{2(\text{A2})} = \frac{1}{3}$ and $\kappa_{3(\text{A2})} = \frac{1}{6}$.

The system frequency is depicted in Fig. 6.2. We notice that the AGC system serves its purpose, i.e., restores the frequency to its nominal value, with all three methods. The associated total cost for AGC service in $\$ for each method are: $c = 55.3738$, $c_{(\text{A1})} = 55.3543$ and $c_{(\text{A2})} = 56.7635$ for the considered time period $[0, 100]$ sec. The minimum cost is achieved by using method A1, as was expected, however in this case the quality of service (ramping characteristics) is not taken into account. In method A2, the cost is high but the fastest unit is mostly used.
to meet the AGC demands. In Fig. 6.3a, we depict the cost for AGC service offered from generator 1 for all three methods. We only plot the cost until 60 s, because after the new signals are sent from the ED, the participation of the units as well as the associated costs are small. Generator 1 has the highest ramp rate in the system. Thus, as we can see from the graph the cost associated with A2 is the highest. The lowest cost is observed with A1, since the participation of generators based on A1 is uniform and does not consider the ramp rates. The proposed method provides a balance between the two as shown in Fig. 6.3a. A modification of the parameter $\zeta$ gives more significance to the cost or the quality of the AGC service. The participation of generator 1 in AGC is depicted in Fig. 6.3b. We notice that after the new signal from the ED at $t = 60$ s, the AGC signals of
all methods are similar and have small values.

In Fig. 6.4a, we depict the cost of AGC associated with generator 2. Both A1 and A2 assign a participation factor of $\frac{1}{3}$, thus the costs associated with A1 and A2 are identical. The proposed method utilizes generator 2 in a lower extent, since the marginal cost $\rho_2$ is the highest and the ramp rate of the generator is 2 MW/min, which is in between the ramp rates of the other two generators. In Fig. 6.4b, we depict the AGC signal to generator 2 $P_{C_2}$. Method A1 uniformly allocates the AGC signal among the generators, until the ED signal is updated and the LMP at bus 2 becomes 20.02 $$/MW, which is higher than the LMP at bus 3, therefore the participation factor becomes $\kappa_{2(A1)} = 0.3272 < \frac{1}{3}$ and the participation of generator 3 is greater, with $\kappa_{3(A1)} = 0.3876$. The LMP at bus 1

![Graph](image)

(a) Cost associated with AGC service for generator 2, with the three methods.

![Graph](image)

(b) The AGC signal for generator 2 $P_{C_2}$, with the three methods.

Figure 6.4: AGC service and associated cost provided by generator 2.
is $\lambda_1 = 24.87$ $$/\text{MW}, which is greater than the LMP at bus 2 $\lambda_2 = 20.02$ $$/\text{MW}$, thus $\kappa_{1(A1)} = 0.2853 < \kappa_{2(A1)}$. However, method A1 neglects the economic signals $\eta_i^m$ and $\eta_i^M$ associated with the lower and upper limit constraints for each generator $i$. Even if the LMP at bus 2 is smaller than that of bus 1, the associated benefit of relieving the constraint associated with the upper limit of generator 1 is $\eta_1^M = 14.81$ $$/\text{MW}$. Thus, the marginal cost of generator 1 is 10.06 $$/\text{MW} which is smaller than that of generator 2, which is 20.02 $$/\text{MW}. That is why the participation factor of our proposed method for generator 2 $\kappa_2 = 0.3012$ is smaller than that of method A1: $\kappa_{2(A1)} = 0.3272$ and $\kappa_1 = 0.3220 > \kappa_{1(A1)}$. Generator 2 has $v_2^+ = 2$, therefore method A2 assigns a participation factor of $\frac{1}{3}$ to generator 2.

We present another case by modifying the system, in order to demonstrate the capabilities of the proposed method, where the generator cost functions are not overlapping and the system is not congested. We increase the line flow limits to 3 pu and the generators limits to 5 pu. We now select non intersecting cost functions (units are in $$/\text{MW})$:

$$
\hat{c}_1(P_{G_1}) = 0.010P_{G_1}^2 + 10P_{G_1} + 100, \quad \hat{c}_2(P_{G_2}) = 0.014P_{G_2}^2 + 15P_{G_2} + 125 \quad \text{and} \quad \hat{c}_3(P_{G_3}) = 0.025P_{G_3}^2 + 20P_{G_3} + 160. 
$$

The ramping characteristics for each unit in MW/min are: $v_1^+ = 1$, $v_2^+ = 2$, $v_3^+ = 3$ and $v_1^- = -1$, $v_2^- = -2$ and $v_3^- = -3$. Since the generators limits are much higher than the total load, only the least cost unit is dispatched. In this case, we have $P_{G_1} = 3.3$ pu and $P_{G_2} = P_{G_3} = 0$. The system LMP is 10.06 $$/\text{MW}. A modification in the load occurs at time $t = 0$ s and we have $P_L = 1.7$ pu. In this case study we vary the parameter $\zeta$ to illustrate the modifications in the AGC signal among the generators. The parameters used for the determination of $\zeta$ are $\bar{\rho} = 15.02$ $$/\text{MW}$ and $\bar{v} = 2$ MW/min for this particular system. The reason $\bar{\rho}$ is higher than the LMP is that generators 1 and 2 are at their lower limits. Then we modify the ratio $\zeta$, i.e., we modify the variability of the net load. The values of $\zeta$, for which the AGC allocations are depicted in Figs. 6.5a-6.5c, are 0.1, 0.5 and 0.8 respectively. We notice that as we increase the value of $\zeta$ the more expensive but faster ramping units are used in regulation. For small values of $\zeta$ only the cheapest generator, i.e., generator 1, participates in the AGC system, as seen in Fig. 6.5a. Once, we
increase the value of $\varsigma$, we notice that the other two more expensive generators participate in the AGC system, as is depicted in Fig. 6.5b. When, the value of $\varsigma$ exceeds a certain value, that is 0.8 in this particular system, only the fastest generator is used in the AGC system, as it may be seen in Fig. 6.5c. We notice
in all figures that once the ED sends the new signal, at \( t = 60 \) s, the entire load is met by generator 1 and the outputs of the other two generators are set to zero.

Now, we fix the value of \( \varsigma \) to 0.5 and compare the results of the proposed method with the two other methods. As it may be seen from Fig. 6.6a, methods A1, A2 assign equal participation of generator 2 in the AGC system equal to \( \frac{1}{3} \). For \( \varsigma = 0.5 \), the proposed method assigns a higher participation equal to 0.48, since the generator provides a good balance between the cost and the ramp rate. Generator 2 is more expensive than generator 1 but cheaper than generator 3. In addition, its ramp rate is 2 MW/min, which is in between the ramp rates of the other two generators. The cost associated with the AGC service offered by generator 2 is shown in Fig. 6.6b.

![Graph](attachment:image.png)

(a) Participation of generator 2 in the AGC system, with the three methods.

![Graph](attachment:image.png)

(b) Cost associated with AGC service for generator 2, with the three methods.

Figure 6.6: AGC service and associated cost provided by generator 2, with non-overlapping cost functions.
6.6 Summary

In this chapter, we investigated the possibility of not using AGC markets for the AGC dispatch. To this end, we presented a systematic method of allocating the AGC signal among the generators by taking into consideration the quality of the AGC service as well as economic criteria. In our modeling approach, we included the ED process, we represented the power system dynamics and incorporated network and other physical constraints. We used the information from the ED process to determine the marginal cost of increasing/decreasing a generator output. We took into account the quality of service, i.e., how fast the generators respond, by including in the objective function a parameter that quantifies the importance of using fast responsive units in AGC regulation. In the numerical studies, we compared the cost as well as the quality of AGC service among three different allocation methods and illustrated that the proposed methodology provides a good balance between cost and quality of AGC service offered. Furthermore, we modified the value of parameter $\zeta_m$ and see its effect on the AGC allocation.
CHAPTER 7
CONCLUSIONS

This chapter is devoted to summarize the results presented throughout the dissertation and to discuss future avenues of research.

7.1 Summary

In this dissertation we analyzed the AGC system, discussed challenges that it is facing, and suggested possible solutions. The radical changes in the electric grid in terms of technological improvements, such as renewable generation and smart grid devices, increase the challenges of the AGC system to maintain reliability. In addition, the load-generation balance becomes harder when an interconnected power grid is operated locally and separately by each individual BA.

We divided the dissertation into two parts. The first is focused in the analysis of AGC systems and in the identification of the challenges it is facing. We demonstrated that there are certain limitations in the current AGC system implementations. That is our motivation for the second part of the dissertation, where modifications for AGC system designs are proposed, in order to increase its performance, taking advantage of the technological improvements of the power grid and the restructuring of power systems.

More specifically, in Chapter 2, we presented the basics for power system modeling along with the current AGC system used by ISOs and discussed how it serves its purpose. In Chapter 3, we proposed a methodology for studying the impact on the AGC system performance of uncertainty that arises from load variations, renewable-based generation and noise in communication channels. Through the
case studies, we showed that the proposed framework provides a good approximation of the system actual state, as validated via Monte Carlo simulations. We also demonstrated that our model captures the higher uncertainty caused by the deepening penetration of renewable-based generation. The proposed methodology may be used to detect, in a timely manner, the existence of a cyber attack, by computing the system frequency statistics and comparing them with those of load variations, wind-based generation, and noise in communication channels. Furthermore, we may use it to determine which buses are more critical if noise is inserted in their measurements. We also wish to find the limiting values of uncertainty that the system may withstand and maintain the desired reliability levels. To this end, we introduced a probabilistic expression of the frequency performance criteria which is used to quantify the limiting amounts of renewable-based generation or potential noise in the communication channels that the system may tolerate.

In Chapter 4, we developed various models to describe a power system behavior that may be used for numerous applications. More specifically, we proposed a reduced-order generator model that it is simpler than the full model, approximates the system behavior in satisfactory levels, provides better accuracy compared to the classical model including the governor dynamics, and has lower computational burden compared to other reduction methods. Subsequently, we used the reduced model to develop a BA area model that only depends on the BA area variables, such as speed of center of inertia and total mechanical power. We demonstrated in the numerical results section that these models provide a good approximation of the system state compared to the full model, which is considered as reference. We used the developed models, and in particular the BA area model, to design two adaptive AGC systems. To this end, we expressed the AFRC as a function of the BA area variables that we have measurements of. Then, we used the SEWBLS algorithm to estimate the AFRC and modify the control gain of the AGC systems. We showed that in both cases the use of the AFRC gives better results in the frequency regulation, in terms of the magnitude of the oscillations and the time the frequency converges to the nominal value. Furthermore, we
showed that the proposed methods give a good approximation of the AFRC.

In Chapter 5, we proposed a possible coordination of the BA areas, in order to increase the efficiency, in terms of quality and cost, of the AGC system. To this end, we set the stage to develop the proposed algorithm by modeling the AGC market clearing mechanism and dispatch. We aimed in approximating the solution of a centralized AGC system and market clearing mechanism. More specifically, we approximated the ACE of the entire area as the summation of the individual ACEs and distributed it into each AGC subsystem accordingly. Next, we developed a distributed algorithm that minimizes the cost of regulation, by allocating the AGC command from the cheapest to the most expensive generator sequentially, until all the regulation amount is met. The advantage of using the distributed algorithm is that the BA areas do not need to exchange any cost information. The only element needed is the total mismatch from the desired regulation and that provided. The proposed approach provides a solution close to the optimal, i.e. if all BA areas were under the same jurisdiction, but respects that each BA area wants to keep certain information from other BA areas, and provide the least cost solution for its customers. We demonstrate how the proposed method works and compare it with other three methods. We showed that it approximates very closely the optimal solution of considering all BA areas as one. The reasons are: (i) that the adjusted ACE fed to each AGC system is constructed to approximate the behavior of the ACE of the entire BA area, and (ii) the AGC dispatch is solved distributively and provides the same solution as a centralized algorithm would. In addition, we demonstrated that the results provided with the proposed method are better than other methods currently used in industry.

In Chapter 6, we examined the possibility of determining the AGC dispatch without the design of an AGC market. To this end, we discussed the importance of the AGC allocation in terms of cost and quality of service, and presented a systematic method of allocating the AGC signal among the generators by taking into consideration both elements — cost and quality. The systematic method consisted of an optimization problem, whose solution provided the participation
factors of regulating units. We used economic signals from the RTM, which is cleared with the ED process. So, in our modeling approach, we included the ED process, we represented the power system dynamics and incorporated network and other physical constraints. We used the information from the ED process to determine the marginal cost of increasing/decreasing a generator output. We took into account the quality of service, i.e., how fast the generators respond, by including in the objective function of the optimization problem constructed, a parameter that quantifies the importance of using fast responsive units in AGC regulation. In the numerical studies, we compared the cost as well as the quality of AGC service among three different allocation methods and illustrated that the proposed methodology provides a good balance between cost and quality of AGC service offered.

7.2 Future Work

7.2.1 Balancing Authority Area Coordination

There are several extensions of the proposed work. One natural thought is that if BA areas coordinate then the regulation requirements at each BA area could decrease. We plan on developing a method that calculates the necessary regulation requirement, to meet reliability criteria, taking into account the level of coordination between BA areas. However, there are some potential problems that emerge when BA areas cooperate. In particular, each BA area schedules how much capacity will be allowed to pass through it at given times of the day. However, there are cases where this amount is greater or lower than the scheduled. This unscheduled energy accumulation is referred to as inadvertent interchange. BA areas coordination might exacerbate the inadvertent interchange effect and there might be cases where BA areas depend on other BA areas in the interconnection for meeting their demand or interchange obligations. One possible extension is to include in our formulation interface limits between BA areas, so that the inadvert-
tent flows are within these limits and do not create congestion problems between BA areas.

In addition, due to the structure of the AGC dispatch, the BA area coordination may be set up as a game, where we find the Nash equilibrium. The participants have no incentive of deviating from the Nash equilibrium. Each BA area has its own individual payoff function, which includes its own cost function and another term which expresses the payments that it requests in case it is providing aid to other BA areas or the payments it has to make when it is being helped by others. More specifically, this is a constrained n-person game, since the constraints for each player, as well as the payoff functions, depend on the strategy of all players. The payoff functions are concave, thus we have a concave game, where a unique equilibrium point exists. In [79], a dynamic model for non-equilibrium situations is proposed, which consists of a set of differential equations that specify the rate of change of each player’s strategy. To this end, we may find the equilibrium point for such a game, and determine each player’s strategy, i.e., we determine how the BA areas may coordinate where all of them have no incentive of deviating from that state.

7.2.2 Automatic Generation Control in the Market Environment

The AGC allocation between the regulating units has been influenced by the restructuring of electricity markets. To this end, the ISOs have established various market designs and rules to allocate the AGC command and pay the respective players by taking into account the guidelines set by FERC Order No. 755. More specifically, the entities that provide regulation service, such as water, steam and combustion turbines, demand response resources, and storage devices, should be paid based on the capacity set aside, the net energy that they inject into the system and the absolute amount of energy withdrawn/injected. These payments should also cover costs for operation, maintenance and loss of potential revenue. FERC Order No. 755 also mandates that there is a uniform price for frequency
regulation capacity. California ISO (CAISO) provides regulation up and down AGC products. Each unit participating in AGC has a capacity requirement and a mileage requirement, which include the ramping capability. In addition, they include in their clearing mechanism the preferred operating point (POP), to which we wish to return as soon as possible, since it represents the most economic operation of the grid. ISO New England (ISONE) gives compensation based on the actual service provided, a capacity payment that includes the marginal unit’s opportunity costs and payment for performance that reflects the quantity of frequency regulation service provided. To this end, ISONE formulated the AGC market as a Vickrey auction whose key properties are that the resources have incentives to bid at true costs, the total cost of regulation is minimized and that identical resources receive the same payment. The regulating resources submit their regulation offer which includes: high limit, low limit, capacity offer price, mileage offer price, automatic response rate and availability for regulation for each hour in the operating day. ISONE allocates the AGC signal among the cleared regulation units from the Vickrey auction giving priority to the fast responsive units. PJM calculates a performance score for each regulating unit, based on the accuracy, the delay and the precision of following the AGC command. This performance score affects which units are chosen as well as their payments.

We may propose a way of selecting the regulating units and how to allocate the AGC command by taking into account several key characteristics that affect the quality as well as the cost of the AGC service provided. We may compare our proposed method with the aforementioned procedures of the various ISOs. The key idea is to construct an optimization problem whose results will provide the AGC dispatch signals to each generator. The objective of the optimization problem is to minimize cost subject to ramping constraints, taking into account short-term load and wind forecast and performance monitoring. We may take into consideration the performance index, which is calculated as the cross correlation of the AGC signal and the actual generators output. We may also consider the losses, since the total regulation amount is affected by the losses variation $\Delta P_{losses}$. 
Let us assume that at time $t_0$ a change in the load $\Delta P_L$ occurs, then the AGC system sends commands to the generators to modify their output to meet the load. The frequency is restored to nominal at time $t_{\text{end}}$, where $z = \sum_{i \in G} P_{ED_i} + \Delta P_L + \Delta P_{\text{losses}}$. From (2.35) for $t = t_{\text{end}}$ we have

$$P_{C_i}(t_{\text{end}}) = P_{ED_i}(t_{\text{end}}) + \kappa_i(\Delta P_L + \Delta P_{\text{losses}}),$$

where $\Delta P_L$ is the same for all allocation methods, however $\Delta P_{\text{losses}}$ depends on where the power is injected. We approximate the system losses with the B-loss coefficients method and include them in our optimization problem. Furthermore, we include in our formulation the fact that we wish minimum switching direction of the AGC command for each generator. We may use stochastic optimal programming since we take into account the wind and load uncertainty, and our decisions at time $t$ consider the system characteristics at time $t' > t$. We also plan on determining the regulation requirements based on the ACE values, instead of some fixed MW value depending on the hour of the day, as actually performed by the ISOs.
REFERENCES


APPENDIX A

The vectors for the uncertainty models of noise in communications channels (3.16), load variations (3.14), and renewable-based generation (3.13) are defined as

\[ \varrho_1 = [\varrho_1, \ldots, \varrho_N]^T, \quad (A.1) \]
\[ \varrho_2 = [\varrho_2, \ldots, \varrho_N]^T, \quad (A.2) \]
\[ a = [a_1, \ldots, a_N]^T, \quad (A.3) \]
\[ b = [b_1, \ldots, b_N]^T, \quad (A.4) \]
\[ \nu_1 = [\nu_1, \ldots, \nu_N]^T, \quad (A.5) \]
\[ \nu_2 = [\nu_2, \ldots, \nu_N]^T. \quad (A.6) \]

The matrices for the ordinary differential set of equations in (3.17) are defined as

\[ A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{bmatrix}, \quad (A.7) \]

\[ B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \\ B_{41} & B_{42} & B_{43} \\ B_{51} & B_{52} & B_{53} \end{bmatrix}, \quad (A.8) \]
with

\[
A_{11} = A_1 + A_2(C_6C_3^{-1}C_2 - C_5)^{-1}(C_4 - C_6C_3^{-1}C_1) - \\
A_3\{C_3^{-1}C_1 + C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}(C_4 - C_6C_3^{-1}C_1)\},
\]

\[
A_{12} = B_3B_2,
\]

\[
A_{13} = A_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_1 - A_3C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_1,
\]

\[
A_{14} = A_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_3 - A_3C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_3,
\]

\[
A_{15} = 0_{9I-1\times N},
\]

\[
A_{21} = A_4 - A_6(C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}(C_4 - C_6C_3^{-1}C_1)
\]

\[
+ C_3^{-1}C_1) + A_5(C_6C_3^{-1}C_2 - C_5)^{-1}(C_4 - C_6C_3^{-1}C_1),
\]

\[
A_{22} = A_7,
\]

\[
A_{23} = A_5(C_6C_3^{-1}C_2 - C_5)^{-1}D_1 - A_6C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_1,
\]

\[
A_{24} = A_5(C_6C_3^{-1}C_2 - C_5)^{-1}D_3 - A_6C_3^{-1}C_2(C_6C_3^{-1}C_2 - C_5)^{-1}D_3,
\]

\[
A_{25} = 0_{M\times N},
\]

\[
A_{31} = 0_{N\times 9I-1},
\]

\[
A_{32} = 0_{N\times M},
\]

\[
A_{33} = diag(\nu_1),
\]

\[
A_{34} = 0_{N\times N},
\]

\[
A_{35} = 0_{N\times N},
\]

\[
A_{41} = 0_{N\times 9I-1},
\]

\[
A_{42} = 0_{N\times M},
\]

\[
A_{43} = 0_{N\times N},
\]

\[
A_{44} = diag(\varrho_1),
\]

\[
A_{45} = diag(\varrho_2),
\]

\[
A_{51} = 0_{N\times 9I-1},
\]

\[
A_{52} = 0_{N\times M},
\]

\[
A_{53} = 0_{N\times N},
\]

\[
A_{54} = 0_{N\times N},
\]

\[
A_{55} = diag(a),
\]

\[
B_{11} = B_{12} = B_{13} = 0_{9I-1\times 1},
\]

\[
B_{21} = A_8, 
\]

\[
B_{22} = B_{23} = 0_{M\times 1},
\]

\[
B_{31} = B_{33} = 0_{N\times 1},
\]

\[
B_{23} = \nu_2, 
\]

\[
B_{41} = B_{42} = B_{43} = 0_{N\times 1},
\]

\[
B_{51} = B_{52} = 0_{N\times 1}, 
\]

\[
B_{53} = diag(b).
\]