OBSERVATIONS AND NUMERICAL SIMULATIONS OF ELECTRIFIED, MEDIUM-SCALE TRAVELING IONOSPHERIC DISTURBANCES (MSTIDS) IN THE NIGHTTIME, MID-LATITUDE IONOSPHERE

BY

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DISSERTATION

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This work will present an advancement of the physical understanding of medium-scale traveling ionospheric disturbances (MSTIDs). This will be accomplished primarily through an observational study and with the aid of numerical simulations. MSTIDs are instabilities in the ionosphere characterized by vertical displacements in electron density with wavefronts at an angle to the magnetic meridian. The instabilities typically occur at night near mid-latitudes and are electrified. The polarization electric fields within the structure cause the displacement of electron density to form under an unstable configuration.

Previous work on MSTIDs has provided the groundwork for the current study of the physical processes that generate the instabilities. A theoretical description provides a basic understanding of how the instabilities develop in the ionosphere, and includes the important parameters that impact the growth rate of MSTIDs. Using 630.0-nm airglow imaging cameras, a climatological study is conducted to establish long-term trends of the instabilities at two longitudinal sectors not previously studied. The low-latitude extent of MSTIDs is also investigated from the observational study.

The numerical simulation work utilizes the SAMI3 (Sami3 is Another Model of the Ionosphere) model, which captures the fundamental physics of the ionosphere. The model simulates a “wedge” region of the ionosphere for the self-consistent development of MSTIDs. Once MSTIDs are generated in the model, synthetic observations are calculated and compared to observational data found in the literature. In addition, simulation case studies serve to isolate parameters that influence the growth of MSTIDs in SAMI3, gaining further physical insight into their development. Finally, future research directions are provided that utilize the results from the current work.
To my family: past, present, and future
I would first like to thank my advisor, Professor Jonathan Makela, for his continued guidance, support, and direction for my research. I have learned a wealth of information from him over the past few years, not just on upper atmosphere studies but also how to think as a researcher. This methodology of thinking will be an immeasurable skill that I will have for the rest of my life.

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<th>Description</th>
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<tr>
<td>CASI</td>
<td>Cornell All-Sky Imager</td>
</tr>
<tr>
<td>CCMC</td>
<td>Community Coordinated Modeling Center</td>
</tr>
<tr>
<td>CNFI</td>
<td>Cornell Narrow-Field Imager</td>
</tr>
<tr>
<td>CTIO</td>
<td>Cerro Tololo Inter-American Observatory</td>
</tr>
<tr>
<td>DE-2</td>
<td>Dynamics Explorer 2 (satellite)</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>Es</td>
<td>Sporadic E</td>
</tr>
<tr>
<td>ESF</td>
<td>Equatorial Spread-F</td>
</tr>
<tr>
<td>EVP</td>
<td>Error Vector Propagation</td>
</tr>
<tr>
<td>GITM</td>
<td>Global Ionosphere Thermosphere Model</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HSI</td>
<td>Horizontally Stratified Ionosphere</td>
</tr>
<tr>
<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
</tr>
<tr>
<td>ISR</td>
<td>Incoherent Scatter Radar</td>
</tr>
<tr>
<td>MPI</td>
<td>Message Passage Interface</td>
</tr>
<tr>
<td>MSTID</td>
<td>Medium-Scale Traveling Ionospheric Disturbance</td>
</tr>
<tr>
<td>NCAR</td>
<td>National Center for Atmospheric Research</td>
</tr>
<tr>
<td>NRL</td>
<td>Naval Research Laboratory</td>
</tr>
<tr>
<td>PDM</td>
<td>Partial Donor Method</td>
</tr>
<tr>
<td>PICASSO</td>
<td>Portable Ionospheric Camera and Small-Scale Observatory</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>R</td>
<td>Rayleigh $(1 , R = 10^6 , \text{ph/cm}^2/\text{s})$</td>
</tr>
<tr>
<td>RTI</td>
<td>Rayleigh-Taylor instability</td>
</tr>
<tr>
<td>SAMI</td>
<td>Sami Is Another Model of the Ionosphere</td>
</tr>
<tr>
<td>SEVP</td>
<td>Stabilized Error Vector Propagation</td>
</tr>
<tr>
<td>SFU</td>
<td>Solar Flux Unit $(1 , \text{SFU} = 10^{-22} , \text{W/m}^2/\text{Hz})$</td>
</tr>
<tr>
<td>SOR</td>
<td>Successive Over Relaxation</td>
</tr>
<tr>
<td>TEC</td>
<td>Total Electron Content</td>
</tr>
<tr>
<td>TECU</td>
<td>TEC Units</td>
</tr>
<tr>
<td>TGCM</td>
<td>Thermospheric General Circulation Model</td>
</tr>
<tr>
<td>TIEGCM</td>
<td>Thermosphere Ionosphere General Circulation Model</td>
</tr>
<tr>
<td>TIME-GCM</td>
<td>Thermosphere Ionosphere Mesosphere Electrodynamics General Circulation Model</td>
</tr>
<tr>
<td>VER</td>
<td>Volume Emission Rate</td>
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- $AG_{630.0}$ 630.0-nm airglow emission (integrated in altitude)
- $\theta$ Angle between $\mathbf{E}'$ and magnetic east or latitude
- $\alpha$ Angle between $\mathbf{k}$ and magnetic east or PDM parameter
- $N_A$ Avogadro’s constant
- $\sigma_0$ Background value
- $\nu$ Collision frequency
- $\sigma_H$ Conductivity (Hall)
- $\Sigma_H$ Conductivity (Hall), integrated along the magnetic field line
- $\sigma_P$ Conductivity (Pedersen)
- $\Sigma_P$ Conductivity (Pedersen), integrated along the magnetic field line
- $\sigma_0$ Conductivity (specific)
- $J$ Current density
- $\delta$ Declination angle or perturbation
\( n \)  Density

\( D \)  Dip angle (also known as inclination angle)

\( q \)  Dipole coordinate across the magnetic field line

\( p \)  Dipole coordinate along the magnetic field line

\( \phi \)  Dipole coordinate in the longitudinal direction

\( f \)  Distribution function

\( e \)  Electron species

\( j \)  Electron or ion species, or cell index

\( e \)  Electron charge

\( q \)  Electron charge (signed)

\( N \)  Electron density, Integrated along the magnetic field line

\( E \)  Electric field

\( F \)  Flux

\( \omega \)  Frequency (angular)

\( f \)  Frequency (ordinary)

\( g \)  Gravity acceleration

\( \gamma \)  Growth rate

\( n_g \)  Group refractive index

\( v_g \)  Group velocity

\( \Omega \)  Gyrofrequency

\( \kappa \)  Gyrofrequency to collision frequency ratio or Boltzmann’s constant

\( Q \)  Heat flux

\( h \)  Height

\( j \)  Imaginary unit (engineering notation)

\( i \)  Imaginary unit (physics notation)

\( i \)  Ion species or cell index

\( \Phi \)  Ionospheric electric potential
\( \mathcal{L} \)  Loss term

\( \mathbf{B} \)  Magnetic field

\( m \)  Mass

\( \rho \)  Mass density or pseudorange

\( A \)  Mean molecular weight

\( b \)  Mobility

\( n \)  Neutral species

\( \mathbf{U} \)  Neutral wind velocity

\( u \)  Neutral wind (zonal direction, positive eastward)

\( v \)  Neutral wind (meridional direction, positive northward)

\( \parallel \)  Parallel with respect to \( \mathbf{B} \)

\( \mu_0 \)  Permeability (vacuum)

\( \epsilon_0 \)  Permittivity (vacuum)

\( \perp \)  Perpendicular with respect to \( \mathbf{B} \)

\( \tilde{\cdot} \)  Perturbation value

\( \mathbf{P} \)  Pressure

\( \mathcal{P} \)  Production term

\( r \)  Radius

\( R_E \)  Radius of the Earth

\( H_n \)  Scale height (Neutrals)

\( c \)  Speed of light

\( T \)  Temperature

\( t \)  Time

\( \mathbf{V} \)  Velocity

\( V_{630.0} \)  Volume emission rate (of the 630.0-nm airglow emission)

\( \lambda \)  Wavelength or Debye length

\( k \)  Wavevector
CHAPTER 1
INTRODUCTION

This work contributes to the understanding of nighttime, electrified, medium-scale traveling ionospheric disturbances (MSTIDs) through the combination of both observational and numerical simulation techniques. MSTIDs are instabilities that develop and travel in the ionosphere, a region of Earth’s upper atmosphere. Before we begin our discussion of this work, it is necessary to describe the fundamental characteristics of the ionosphere, including the constituents in the region and the fundamental physics that govern the ionosphere. This chapter will provide the necessary background material for an in-depth study of MSTIDs, in addition to describing the motivation and contribution of this work.

1.1 Constituents in the Upper Atmosphere

The focus of this work is primarily on Earth’s ionosphere, which can be described as a conducting shell of free electrons located between approximately 85- and 600-km altitude. In addition to free electrons, this region of the atmosphere has several neutral and ionized constituents present.

For example, Figure 1.1 plots profiles of neutral densities and temperatures in the ionosphere, and shows the exponential decay of the neutrals as a function of increasing altitude. From a neutral temperature viewpoint, the region is known as the thermosphere, and the neutral temperatures are larger compared to the mesosphere region below. Figure 1.1 shows that above approximately 150 km, the neutral temperature is relatively constant in this region.

This layer of the upper atmosphere can be considered a boundary region between Earth’s atmosphere, below, and interplanetary space, above. As a boundary layer, the region can be influenced by forces from its neighbor-
Figure 1.1: Example daytime profiles of the neutral constituents between 85 and 1000 km (left), and also an example profile of the neutral temperatures (right). The NO profile is obtained from the SAMI2 model (Huba et al., 2000). The remaining values are obtained from the MSIS climatological model.

ionizing regions. For example, UV and EUV radiation from the Sun partially ionizes the neutral constituents and generate free electrons. The charged constituents embedded in the atmosphere are known as a plasma. Figure 1.2 plots an example daytime profile of several ionized species. If we assume charge neutrality, a topic that will be discussed in Section 1.2.1, the sum of ion density constituents is the electron density:

\[ n_e = \sum_i n_i, \quad (1.1) \]

where \( n_i \) is the density of the \( i \)th ion constituent. The electron density profile is also plotted in Figure 1.2. Here, the \( n_e \) peak is near 300-km altitude. Above approximately 200 km, the electron density matches the \( O^+ \) profile quite well, which is a result of the \( O^+ \) density dominating this region. Between approximately 200 and 500 km, \( O^+ \) can be a few orders of magnitude larger than the other ion constituents in this region. Also, note the ion temperature is similar to the neutral temperature (\( T_i \approx T_n \)) for altitudes less than about 300 km. This approximation is particularly valid during the evening time period (not shown).
Figure 1.2: Similar to Figure 1.1, except the plasma constituents and temperature are displayed. The values are obtained from the IRI climatological model.

The properties of the ionosphere can change as a function of time. For example, as the Sun sets each day, the photoionization source is no longer available to ionize the neutral species, and as a result, the plasma will recombine back to neutral. Figure 1.3 compares an example daytime and nighttime profile of \( n_e \), in addition to the plasma temperatures, \( T_i \). At some altitudes, such as \( \sim 150 \) km, the electron density can decrease by several orders of magnitude during the transition from day to night. Two peaks in electron density are shown in Figure 1.3, with a local maximum just above 100 km, and a global maximum of \( n_e \) near 300 km. The regions near the peaks are known as the E- and F-regions, respectively, of the ionosphere.

1.2 Physics in the Ionosphere

In this section, we describe important physical concepts related to the ionosphere. The work in this dissertation will often refer back to these fundamental concepts. For example, the theoretical work describing the development of MSTIDs is dependent on knowledge of the conductivity and ion velocity in the ionosphere. Furthermore, the numerical simulations in this dissertation will model the governing equations that describe the physics in the upper
atmosphere, and it is important to understand these relations. This section will describe the governing physical equations and conductivity in the ionosphere.

1.2.1 Governing Equations

The first governing equation is the ion continuity equation. Following Huba et al. (2000) and assuming no production or loss, the equation can be written as:

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i \mathbf{V}_i) = 0.$$  \hspace{1cm} (1.2)

Here, $n_i$ is the ion density, and $\mathbf{V}_i$ is its velocity. Equation 1.2 states that the density must be conserved in the system. That is, the time rate of change for the number of particles must be accounted for by the velocities of the particles.

Next, the ion and electron momentum equations are given as (Huba et al., 2000):

Figure 1.3: Example profiles showing the daytime and nighttime profile of the plasma density (left) and ion temperatures (right). Similar to Figure 1.2, the values are obtained from the IRI climatological model.
\[
\begin{align*}
\frac{\partial V_i}{\partial t} + V_i \cdot \nabla V_i &= -\frac{1}{\rho_i} \nabla P_i + \frac{e}{m_i} (E + V_i \times B) + g - \nu_{in}(V_i - U) \\
&\quad - \sum_{i' \neq i} \nu_{ii'}(V_i - V_{i'}). \quad (1.3a)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial V_e}{\partial t} + V_e \cdot \nabla V_e &= -\frac{1}{\rho_e} \nabla P_e + \frac{e}{m_e} (E + V_e \times B) + g - \nu_{en}(V_e - U). \quad (1.3b)
\end{align*}
\]

Here, \( \rho \) is mass density, \( P \) is pressure, \( e \) is the electron charge, \( m \) is mass, and \( g \) is the acceleration of gravity. \( E \) is the electric field. The term \( \nu \) is the collision frequency. In general, \( \nu_{\alpha\beta} \) refers to the “\( \alpha-\beta \) collision frequency”. That is, \( \nu_{in}, \nu_{en}, \) and \( \nu_{ii'} \) represent the ion-neutral, electron-neutral, and ion-ion collision frequencies, respectively. Equations 1.3a and 1.3b conserve momentum in the system. The momentum generated by the forces caused by pressure, gravity, electric and magnetic fields, etc., must sum to zero with respect to each constituent (both the ions and electrons).

The final equation is the divergence-free condition:

\[
\nabla \cdot J = 0. \quad (1.4)
\]

This equation is based on the charge neutrality assumption and can be derived from Maxwell’s equations. A brief sketch of the derivation follows. First, the divergence of Ampere’s law is calculated:

\[
\begin{align*}
\nabla \cdot \left( \nabla \times B = \mu_0 \epsilon_0 \frac{\partial}{\partial t} E + \mu_0 J \right) \\
0 &= \epsilon_0 \frac{\partial}{\partial t} (\nabla \cdot E) + \nabla \cdot J. \quad (1.5)
\end{align*}
\]

Gauss’ law is used to substitute \( \nabla \cdot E = \rho_e/\epsilon_0 \):

\[
\begin{align*}
0 &= \epsilon_0 \frac{\partial}{\partial t} (\nabla \cdot E) + \nabla \cdot J \\
0 &= \epsilon_0 \frac{\partial}{\partial t} (\rho_e/\epsilon_0) + \nabla \cdot J \\
0 &= \frac{\partial}{\partial t} \rho_e + \nabla \cdot J. \quad (1.6)
\end{align*}
\]
Next, the equation is integrated over a volume, $V$, as exemplified in Figure 1.4, and Gauss’ theorem is used to transform the volume integral into a surface integral:

$$\int_V \int \int dV \left\{ \nabla \cdot \mathbf{J} = -\frac{\partial}{\partial t} \rho_e \right\}$$

$$\int_V \int \nabla \cdot \mathbf{J} dV = -\frac{\partial}{\partial t} \int_V \rho_e dV$$

$$\int_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \int_V \rho_e dV$$

$$\approx 0. \quad (1.7)$$

Here we have imposed the charge neutrality assumption to approximate the time rate of charge density in volume $V$ to be 0. Finally, by taking the limit of $V$ as it approaches 0, we have $\nabla \cdot \mathbf{J} = 0$:

$$\lim_{V \to 0} \left\{ \int_S \frac{\mathbf{J} \cdot d\mathbf{S}}{|V|} \right\} = 0$$

$$\nabla \cdot \mathbf{J} = 0 \quad (1.8)$$

Fundamentally, Equation 1.8 is a result of charge neutrality in the ionosphere. That is, sources or sinks of current density cannot occur under Equation 1.8, due to the constraint that the number of ions must equal the number of electrons:
\[ n_e \approx \sum_i n_i. \quad (1.9) \]

This equation states that the electron density is approximately the sum over each \( i \)th ion density constituent.

These equations will be used extensively within the numerical simulation framework for modeling the development of MSTIDs in the nighttime, mid-latitude ionosphere. For example, the momentum equation (Equation 1.3a) is used to calculate ion velocities in the model, and the continuity equation (Equation 1.2) describes the transport of the plasma within the simulation space. Also, the divergence free condition is invoked to solve for the potential self-consistently within the numerical model. The application of these equations in the numerical model will be further discussed in Chapter 5.

1.2.2 Conductivity

Conductivity is a measure of material’s ability to conduct an electric current. With respect to the upper atmosphere, conductivity affects the development of electric fields and how current flows in the region, and is therefore an important concept for understanding ionospheric electrodynamics. An interesting property of the ionosphere is that the conductivity is anisotropic, meaning that the conductivity is directionally dependent and therefore must be described by a tensor as opposed to a scalar value. This is a result of the plasma being embedded in Earth’s geomagnetic field, \( \mathbf{B} \), and subject to Lorentz forces. The current density, \( \mathbf{J} \), can be written as a tensor product between the conductivity and electric field:

\[ \mathbf{J} = \mathbf{\sigma} \cdot \mathbf{E}. \quad (1.10) \]

Without loss of generality, \( \hat{x} \) and \( \hat{y} \) are perpendicular to the magnetic field line direction, and \( \hat{z} \) points along the magnetic field line. The conductivity tensor can be expanded as:

\[ \mathbf{\sigma} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}. \quad (1.11) \]
Table 1.1: Terms used in conductivity expressions (Equations 1.12 – 1.14).

Note: \( j \) can refer either to an ion particle (\( i \)) or electron particle (\( e \)).

<table>
<thead>
<tr>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n ) Electron Density</td>
<td>(# electrons) / m(^3)</td>
</tr>
<tr>
<td>( q_j ) Fundamental Signed Charge</td>
<td>C</td>
</tr>
<tr>
<td>( e ) Absolute Value of Fundamental Charge (( =</td>
<td>q_j</td>
</tr>
<tr>
<td>( b_j ) Mobility (( = \frac{q_j}{m_j \nu_{jn}} ))</td>
<td>C \cdot \text{sec} / kg</td>
</tr>
<tr>
<td>( \kappa_j ) Ratio of Gyrofrequency to Collision Frequency (( = \frac{q_j B}{m_j \nu_{jn}} ))</td>
<td>C \cdot \text{T} \cdot \text{sec} / kg</td>
</tr>
<tr>
<td>( B ) Magnetic Field Strength</td>
<td>T</td>
</tr>
<tr>
<td>( m_j ) Mass of Constituent ( j )</td>
<td>kg</td>
</tr>
<tr>
<td>( \nu_{jn} ) Collision Frequency Between Constituent ( j ) and Neutrals</td>
<td>(# collisions) / s</td>
</tr>
</tbody>
</table>

Here, \( \sigma_P \), \( \sigma_H \), and \( \sigma_0 \) are the Pedersen, Hall, and specific conductivities, respectively. The conductivities are functions of parameters in the ionosphere and can be written as (Kelley, 2009, Section 2.2):

\[
\sigma_P = n e \left[ b_i \left( \frac{1}{1 + \kappa_i^2} \right) - b_e \left( \frac{1}{1 + \kappa_e^2} \right) \right] \quad \text{(1.12)}
\]

\[
\sigma_H = \frac{n e}{B} \left[ \frac{\kappa_e^2}{1 + \kappa_e^2} - \frac{\kappa_i^2}{1 + \kappa_i^2} \right] \quad \text{(1.13)}
\]

\[
\sigma_0 = n e (b_i - b_e) \quad \text{(1.14)}
\]

Table 1.1 details the quantities in the conductivity equations. Here, we see the influence of the geomagnetic field strength (\( B \)), and also the coupling to the neutral parameters through the ion-neutral collision frequency (\( \nu_{in} \)).

Equation 1.10 can also be written as:

\[
\mathbf{J} = \sigma_P \mathbf{E}_\perp - \sigma_H \mathbf{E}_\perp \times \hat{z} + \sigma_0 \mathbf{E}_\parallel. \quad \text{(1.15)}
\]

Here, \( \hat{z} \) is in the direction of the magnetic field line and the symbols \( \perp \) and \( \parallel \) are with respect to the magnetic field line, \( \mathbf{B} \). It should be noted that
Table 1.2: Conductivities in the ionosphere and their associated directions.

<table>
<thead>
<tr>
<th>Type of Conductivity</th>
<th>Direction with respect to B and E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_P$ Pedersen</td>
<td>$\perp B, \parallel E_\perp$</td>
</tr>
<tr>
<td>$\sigma_H$ Hall</td>
<td>$\perp B, \perp E$</td>
</tr>
<tr>
<td>$\sigma_0$ Specific</td>
<td>$\parallel B$</td>
</tr>
</tbody>
</table>

Equation 1.15 can be modified to include the neutral wind, $U$, to obtain the effective electric field, $E' = E + U \times B$. In this way, the current density equation is expanded as:

$$J = \vec{\sigma} \cdot E'$$
$$= \vec{\sigma} \cdot (E + U \times B)$$
$$= \sigma_0 E_\parallel + \sigma_P (E_\perp + U_\perp \times B) - \sigma_H (E_\perp + U_\perp \times B) \times \hat{z}. \quad (1.16)$$

Inspection of Equation 1.15 reveals that the Pedersen, Hall, and specific conductivities refer to directions with respect to the electric and magnetic fields. That is, $\sigma_P$ is the conductivity in the direction perpendicular to the magnetic field line and parallel to the perpendicular component of the electric field ($\perp B, \parallel E_\perp$). $\sigma_H$ is in the direction perpendicular to the magnetic field line and perpendicular to the electric field ($\perp B, \perp E$). To complete the tensor, $\sigma_0$ is the conductivity in the direction parallel to the magnetic field ($\parallel B$). The conductivity descriptions are summarized in Table 1.2, which lists each conductivity term and their associated directions. Figure 1.5 provides a pictorial view of $\vec{\sigma}$ and the relationship of each conductivity component to the electric and magnetic field directions.

We can use the climatological models described in Section 1.3 to provide example profiles of the conductivities. Specifically, MSIS, IRI, and IGRF are used to calculate the neutral, plasma, and magnetic terms, respectively, in Equations 1.12-1.14 to obtain the conductivity values. Figure 1.6 shows example daytime profiles for $\sigma_P$, $\sigma_H$, and $\sigma_0$, in addition to a nighttime Pedersen conductivity profile. Note that the conductivity in the direction along the magnetic field, $\sigma_0$, is several orders of magnitude larger than the Pedersen and Hall conductivities (which have been scaled in the figure). As a result, electric fields can map along the magnetic field line direction. This concept will be further discussed in the Section 1.4.
Figure 1.5: Diagrams showing the relationship between the components of the tensor conductivity and the electric and magnetic fields. After http://wdc.kugi.kyoto-u.ac.jp/ionocond/exp/icexp.html.

Figure 1.6: An example conductivity profile. Note the scaling of $\sigma_P$ and $\sigma_H$ and the relatively large conductivity of $\sigma_0$. After Kelley (2009, Section 2.2).
Finally, we note that the magnetic field line integration of the conductivity is often used in ionospheric studies. For example, the integrated Pedersen conductivity, $\Sigma_P$, is defined as:

$$\Sigma_P = \int_{\text{field line}} \sigma_P \ dz,$$

where $z$ is along the field line direction.

1.3 Climatological Models

As exemplified in Figure 1.3, the electron density profile can be greatly altered over the course of a few hours. It is challenging to develop a single, simple model to describe parameters in the upper atmosphere due in part to the dynamical nature in this region. For example, seasonal and yearly (i.e., 11-year solar cycle) variations, as well as spatial variations, are present in the system. Additionally, the coupling between ion and neutral parameters, which are subject to fluid dynamics and electromagnetics, add to the complexity of parameter estimation. One type of methodology to overcome this challenge is through the development and use of climatological models. These models are based on historical, measured data and use statistical techniques to provide an estimate of an upper atmosphere parameter as a function of location and time. Although they are not exact values, they are representative of a parameter for a given latitude, longitude, altitude, hour, season, and year.

There are several climatological models available to the upper atmosphere research community. Table 1.3 lists a few examples and the parameters that each model provides. The models are commonly written in the Fortran programming language. Appendix A describes the pyglow coding project that uses Python wrapper functions for the climatological models, which enables the models to be used in the high level language of Python. The pyglow package was developed as a part of this dissertation and was utilized in various aspects of the work. For example, Figures 1.1-1.3 were plotted using the Python wrappers for the MSIS and IRI climatological models to obtain the neutral and plasma parameters, respectively.

Climatological models are commonly used in conjunction with physics-
Table 1.3: A listing of example climatological models that are frequently used in the upper atmosphere research community.

<table>
<thead>
<tr>
<th>Climatological Model</th>
<th>Description</th>
<th>Terms Modeled</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRI</td>
<td>International Reference Ionosphere</td>
<td>Plasma</td>
<td>Bilitza and Reinisch (2008)</td>
</tr>
<tr>
<td>HWM93/07</td>
<td>Horizontal Wind Model 1993 / 2007</td>
<td>Neutral Wind</td>
<td>Hedin et al. (1996)</td>
</tr>
<tr>
<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
<td>Magnetic Field</td>
<td>Finlay et al. (2010)</td>
</tr>
</tbody>
</table>
based, numerical simulations. For example, within a numerical simulation, climatological models can be used to initialize quantities (e.g., electron density). In addition, instead of solving parameters self-consistently (e.g., neutral winds), a numerical model can call climatological models to calculate these values.

1.4 Coupled System

In general, the upper atmosphere can be considered a complex, non-closed, and coupled system. In this section, we will provide a few examples of the coupling in the ionosphere, including the mapping of electric fields across the magnetic field direction and the interactions of the neutral species with the plasma.

In Section 1.2.2, we noted that the conductivity along the magnetic field line direction, $\sigma_0$, is orders of magnitude larger than both the Pedersen and Hall conductivities (i.e., Figure 1.6). Consequently, the geomagnetic field lines can be considered equipotentials, and large-scale (i.e., 10s of km), electric fields can be efficiently communicated thousands of kilometers along the field line direction (Farley, 1959, 1960).

This effect can be described by starting with the current density relation from Equation 1.15, substituting the scalar potential, $\mathbf{E} = -\nabla \Phi$, and enforcing that $\nabla \cdot \mathbf{J} = 0$ (Section 1.2.1):

$$
\mathbf{J} = \sigma_P \mathbf{E}_\perp - \sigma_H \mathbf{E}_\perp \times \hat{z} + \sigma_0 \mathbf{E}_\parallel
$$

$$
\nabla \cdot \mathbf{J} = \nabla \cdot \left( -\sigma_P \nabla \Phi + \sigma_H (\nabla \Phi \times \hat{z}) - \sigma_0 \nabla \Phi \right) = 0
$$

$$
- \sigma_P \nabla_\perp^2 \Phi + \sigma_H \left[ \frac{\partial^2}{\partial x \partial y} \Phi - \frac{\partial^2}{\partial y \partial x} \Phi \right] - \sigma_0 \frac{\partial^2}{\partial z^2} \Phi = 0
$$

$$
\sigma_P \nabla_\perp^2 \Phi + \sigma_0 \frac{\partial^2}{\partial z^2} \Phi = 0
$$

$$
\frac{\partial^2}{\partial x^2} \Phi + \frac{\partial^2}{\partial y^2} \Phi + \frac{\sigma_0}{\sigma_P} \frac{\partial^2}{\partial z^2} \Phi = 0 \quad (1.18)
$$

Again, $\hat{z}$ is in the field line direction, with $\hat{x}$ and $\hat{y}$ both being perpendicular to $\mathbf{B}$. 

13
Now, we change variables with:

\[ dx' = dx \]
\[ dy' = dy \]
\[ dz' = \sqrt{\frac{\sigma_p}{\sigma_0}} \, dz. \]  

(1.19)

Substitution of the prime variables into Equation 1.18 recovers the Laplacian, \((\nabla')^2 \Phi = 0\). In the primed coordinate system, the Laplacian describes a scenario with isotropic conductivity.

Given that \(\sigma_0 >> \sigma_P\) (e.g., Figure 1.6), \(dz'\) is relatively small compared to \(dx'\) and \(dy'\). Effectively, the primed system condenses the coordinate direction along \(B\). Therefore, the solution to the Laplacian in the un-primed coordinate system has \(\Phi\) approximately constant along the magnetic field direction. If \(\sqrt{\sigma_0/\sigma_P}\) were infinite, \(\Phi\) would be exactly constant along this direction. For example, \(\sqrt{\sigma_0/\sigma_P}\) can be greater than \(10^3\) above 300-km altitude (Kelley, 2009, Section 2.4). In this way, perpendicular electric fields in the ionosphere can be mapped along the magnetic field line direction.

Another way to describe the coupling of electric fields along \(B\) is based on inspection of Equation 1.18. Given that \(\sigma_0/\sigma_P\) is large, a slight derivative in the electric field along the magnetic field line direction \((E_z = \partial \Phi/\partial z)\) will result in a large magnitude of the third term in Equation 1.18. Therefore, in order to satisfy Equation 1.18, the derivatives of the electric fields in the perpendicular direction must be large to cancel the \(\frac{\sigma_0}{\sigma_P} \frac{\partial^2}{\partial z^2} \Phi\) term, which may not be physically possible in the ionosphere. Therefore, we require small changes of \(\Phi\) in the \(\hat{z}\) direction to balance the perpendicular electric fields derived from Equation 1.18, and this translates to an approximately constant value of \(\Phi\) in the magnetic field line direction.

Another example of coupling in the ionosphere is collisions between the plasma and neutral species. Figures 1.1 and 1.2 show that the neutral constituents are several orders of magnitude larger than the plasma. The ion-neutral collision frequency, \(\nu_{in}\), describes the number of collisions the ions have with the neutrals, and can be approximated as (Kelley, 2009, Section 2.2):
Figure 1.7: Example profiles of the ion-neutral collision frequency, $\nu_{in}$, during the day and night.

$$\nu_{in} = 2.6 \times 10^{-9} (n_n + n_e) A^{-1/2}. \quad (1.20)$$

Here, $A$ is the mean molecular weight given as:

$$A = \frac{\rho_n}{n_n} N_A. \quad (1.21)$$

In this equation, $\rho_n [g/cm^3]$ is the mass density of the neutrals, and $N_A$ is Avogadro’s constant ($N_A = 6.022 \times 10^{23}$ [items/mol]).

Figure 1.7 shows that the ion-neutral collision frequency is relatively high in the E-region ionosphere (approximately 100 km in altitude), where the large population of neutrals can collide with the ions (e.g., Schunk and Nagy (2000), Chapter 4). This coupling of the neutrals to the plasma through collisions can greatly influence the conductivity (e.g., Equations 1.12-1.14), mobility, and other properties of the plasma.

In addition to the neutral constituents coupling into the plasma, the neutral wind can also affect the electrical properties of the ionosphere. As shown in Equation 1.16, the effective electric field can be written in terms of the neutral wind, $U$. As a result of maintaining divergence free current densities in the ionosphere, polarization $\mathbf{E} \times \mathbf{B}$ drifts develop to transport plasma. One example of this effect is known as the F-region dynamo (Heelis, 2004; Kelley,
2009, Section 3.2), in which the perpendicular neutral wind, along with the Pedersen conductivity in the F-region, causes electric fields to develop in the post-sunset, equatorial ionosphere.

As a boundary layer between Earth’s atmosphere, below, and space, above, the ionosphere is subject to forces from these two regions. For example, waves and momentum are carried to the ionosphere region from below in the form of thermal tides. Gravity waves (e.g., Hines, 1960) generated in the terrestrial atmosphere can be transported to the upper atmosphere to interact with the ionosphere. In addition, solar radiation and energetic particles from the sun provide forcing from above, and interplanetary magnetic fields affect the region. The superposition of all coupling factors results in a unique and complex system to study.

1.5 Dynamic System

In Section 1.1, we first observed an example of ionospheric dynamics by comparing daytime and nighttime profiles of the electron density, $n_e$. The decrease in nighttime density is primarily caused by the removal of the production source. That is, as the Sun sets, its energy can no longer supply ionization in the upper atmosphere. Also, solar storms can greatly change the properties of the plasma on relatively short time scales (e.g., Shen et al., 1976). In this section, we will describe examples of ionospheric dynamics, including solar cycle variations and instabilities in the ionosphere.

There are several other time scales that affect the ionosphere. For example, seasonal configurations of the neutral wind, through neutral and electrical coupling described in Section 1.4, can change the distribution of plasma density. Also, the Sun’s irradiance changes on an 11-year cycle which alters production levels of plasma in the ionosphere. A common proxy for the Sun’s radiation output is the F10.7 solar index, which is a measurement of the incoming radio flux at 2800 MHz, corresponding to a wavelength of 10.7 cm. This value is measured in solar flux units (SFU), defined as $1 \text{ SFU} = 10^{-22} \text{ W/m}^2/\text{Hz}$. Figure 1.8 plots the F10.7 value of the recent, full solar cycle. Also plotted in Figure 1.8 is the F10.7A value, which is the 81-day moving average of the F10.7 value. In this figure, the largest solar flux values define solar maximum, occurring between approximately 2000 and
Figure 1.8: The F10.7 and F10.7A values over an 11-year solar cycle.

2002. Solar minimum occurs during the beginning and end of the dates in Figure 1.8 (about 1996/1997 and 2007, respectively), corresponding to a F10.7 value of approximately 75 SFU.

Instabilities in the ionosphere are another example of system dynamics. For example, at low-latitudes, where the magnetic field is nearly parallel to Earth’s surface, unstable, large-scale (rising 100s of km in altitude) “plumes” of density depletions can develop in the nighttime F-region (approximately between 200-500-km altitude). Figure 1.9 depicts this instability, commonly referred to as equatorial spread-F (ESF), developing over Jicamarca, Peru. The backscatter power of 3-m irregularities in plasma density is plotted as a function of altitude and time. These data provide an example of the dynamics in the ionosphere with depletions moving several hundred kilometers in altitude in a few hours.

E-region irregularities can also be present in the equatorial region (e.g., Farley, 2009). These instabilities are primarily driven by the equatorial electrojet current in the E-region, a region of large conductivity and sharp electron density gradients with respect to altitude. The plasma instabilities generate plasma waves which can be observed through radar remote sensing techniques. Similar to ESF, E-region instabilities are primarily field-aligned.

Instabilities are not limited to form solely at the equatorial region. Another type of instability, known as medium-scale traveling ionospheric disturbances (MSTIDs), are often observed in the mid-latitude ionosphere. The instabil-
Figure 1.9: Example radar data observing Equatorial Spread F (ESF). Reprinted from The Earth’s Ionosphere, Volume 96, Second Edition, Michael C. Kelley, Equatorial Plasma Instabilities and Mesospheric Turbulence, Page 132, Copyright (2009), with permission from Elsevier.

...instabilities are characterized by raised and lowered bands of electron density with respect to altitude and have wavefronts at an angle to the magnetic field line. They typically have wavelengths of 50-500 km and travel at velocities of 100 m/s (Garcia et al., 2000). Figure 1.10 shows an example image sequence of a MSTID propagating in the mid-latitude ionosphere. The raised and lowered bands of electron density are shown as light and dark bands in images taken of the 630.0-nm airglow layer (occurring at an altitude of approximately 250 km) (Shiokawa et al., 2003a).

The general features and governing physics of MSTIDs are inherently different from those of ESF. However, both instabilities share the property that they develop as a result of maintaining divergence free current densities in the ionosphere. MSTIDs have also been observed to propagate to low-latitudes and seed the development of ESF (Miller et al., 2009). Given its potential influence on ESF, it is important to understand the physical mechanisms involved for the development of MSTIDs. Although the basic theory for MSTIDs is accepted, additional research is needed to understand the details of how they develop, including the important physical parameters that impact the generation of the instabilities.
1.6 Chemistry: 630.0-nm Airglow Emission

Natural airglow emissions occur in the upper atmosphere through a variety of chemical mechanisms at a wide-range of altitudes. Although there are several emission lines, we focus on the 630.0-nm airglow emission because it occurs around 250-km altitude (near the F-region, an area of interest for ionospheric dynamics) and also due to its relative intensity (i.e., it can be measured by optical instrumentation). As a result, the 630.0-nm airglow emission is commonly used as a tracer for understanding ionospheric dynamics. For example, Chapter 4 details a climatological study of MSTID occurrences in the Central Pacific and South American sectors utilizing 630.0-nm filtered CCD imagers to observe MSTIDs in the ionosphere. Here we give a brief overview of how this emission is modeled and calculated, following the description provided by Link and Cogger (1988, 1989).

This emission is primarily the result of a three-step process. First, the dense $O^+$ near 300-km altitude (e.g., Figure 1.2) charge exchanges with a neutral $O_2$ molecule at a rate, $k_1$:

$$O^+ + O_2 \stackrel{k_1}{\rightarrow} O_2^+ + O.$$  \hspace{1cm} (1.22)

Second, the $O_2^+$ molecule dissociatively recombines with an electron at a rate, $\beta_{1D}$:
\( O_2^+ + e^- \xrightarrow{\beta_1 D} 2O(^3P,^1D,^1S). \)  

(1.23)

The third step involves the \( O(^1D) \) state, which has a lifetime of 110 s. At the end of the lifetime, \( O(^1D) \) will drop to a lower level of excitation at a rate, \( A_{1D} \), and a photon will be emitted with a wavelength \( \lambda = 630.0 \text{ nm} \):

\[ O(^1D) \xrightarrow{A_{1D}} O(^3P) + h\nu(\lambda = 630.0 \text{ nm}). \]  

(1.24)

However, during its lifetime, the \( O(^1D) \) state may react with other major constituents in the region and be lost without emitting a photon. These are competing effects and will reduce the volume emission rate of the 630.0-nm airglow emission:

\[ O(^1D) + N_2 \xrightarrow{k_3} O(^3P) + N_2 \]  

(1.25)

\[ O(^1D) + O_2 \xrightarrow{k_4} O(^3P) + O_2 \]  

(1.26)

\[ O(^1D) + e^- \xrightarrow{k_5} O(^3P) + e^- \]  

(1.27)

Combining the effects of chemical reactions in the ionosphere, the volume emission rate (VER), \( V_{630.0} \), can be calculated as (Link and Cogger, 1988, 1989):

\[ V_{630.0} = \frac{0.76\beta_1 Dk_1[O^+][O_2]}{1 + (k_3[N_2] + k_4[O_2] + k_5[e^-])/A_{1D}} \text{ [ph/cm}^3/\text{s]} \].  

(1.28)

Here, the terms in brackets represent densities in units of cm\(^{-3}\). This process is summarized in Table 1.4, which provides example values of the rates from Link and Cogger (1988, 1989) that were derived from laboratory experiments.

With the aid of the climatological models described in Section 1.3, Figure 1.11 plots example profiles of the \( n_e \) density (\( \approx [O^+] \) near 300-km altitude, see Figure 1.2) and the neutral \( O_2 \) and \( N_2 \) densities, along with an example profile of the calculated \( V_{630.0} \) from Equation 1.28. Notice that in Figure 1.11, the peak 630.0-nm airglow emission is at approximately 250-km altitude, which is below the \( n_e \) peak altitude at about 300-km altitude. This is a result of the competing effects between the peak \( n_e \) at 300 km and the neutral densities exponentially increasing with decreasing altitude. The “sweet-spot” between these competing effects is at 250 km, as represented by the peak VER.
Table 1.4: 630.0-nm airglow emission process described in Section 1.6.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Description</th>
<th>Modeled Value (Link and Cogger, 1988)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O^+ + O_2 \rightarrow k_1 \rightarrow O^+_1 + O_2 )</td>
<td>Charge exchange</td>
<td>( k_1 = 3.23 \times 10^{-12} e^{(3.72 \times 10^3 T_e^{-1.87} \text{ cm}^3/\text{s})} )</td>
</tr>
<tr>
<td>( O_2^+ + e^- \rightarrow k_2 \rightarrow O(3P) + O(3P) )</td>
<td>Dissociate recombination</td>
<td>( k_2 = 7.45 \times 10^{-3} \text{ [cm}^3/\text{s}] )</td>
</tr>
<tr>
<td>( O(1D) \rightarrow k_3 \rightarrow O(3P) ) + h\nu</td>
<td>Spontaneous emission</td>
<td>( k_3 = 2.0 \times 10^{-11} e^{(111.8 \text{ cm}^3/\text{s})} )</td>
</tr>
<tr>
<td>( O(1D) + N_2 \rightarrow k_4 \rightarrow O(3P) + O_2 )</td>
<td>Loss mechanism</td>
<td>( k_4 = 2.9 \times 10^{-11} \text{ [cm}^3/\text{s}] )</td>
</tr>
<tr>
<td>( O(1D) + e^- \rightarrow k_5 \rightarrow O(3P) + e^- )</td>
<td>Loss mechanism</td>
<td>( k_5 = 1.6 \times 10^{-12} T_e \text{ [cm}^3/\text{s}] )</td>
</tr>
</tbody>
</table>
The neutral density is relatively constant throughout the night. However, the plasma density in the ionosphere can be dynamic. Note that a change in the layer height of $n_e$ will also change the magnitude of $V_{630.0}$. For example, if the electron density layer decreases in altitude, the magnitude of $V_{630.0}$ will increase as a result of the higher charge exchanges between $O^+$ and $O_2$ (Equation 1.22) at lower altitudes (recall that the neutral density exponentially increases with decreasing altitude). However, a magnitude change of $n_e$ will also change the magnitude of the 630.0-nm emission. Therefore, there is an ambiguity with a magnitude change of $V_{630.0}$, as it could be the result of a change in the magnitude of $n_e$, and/or a change in altitude of the $n_e$ layer. This is important to keep in mind during our study of MSTIDs, which are characterized by height-band changes (in altitude) of electron density.

1.7 Total Electron Content (TEC)

The ionosphere is a dispersive medium, meaning that the refractive index is a function of an electromagnetic (EM) wave’s frequency. As a result, the behavior of two EM waves will vary if their frequencies are different. The GPS constellation, along with dual-frequency GPS receivers, can take
advantage of the dispersive nature of the ionosphere in order to measure the
total electron content (TEC), which is defined as the integration of electron
density, \( n_e \), along a spatial path length. TEC is commonly given in “TEC units” (TECU), and can be calculated as:

\[
\text{TEC} = \int_{h_0}^{h_0+h} n_e \, dh \cdot \frac{1 \text{ TECU}}{10^{16} \text{ electrons/m}^2} \text{ [TECU].} \tag{1.29}
\]

Here, the integration is in the \( h \) direction, which is the line-of-sight distance between the GPS receiver and a GPS satellite.

Similar to the naturally occurring airglow emission, the total electron content (TEC) can be a useful metric for understanding the dynamics of the ionosphere. In this section, we will briefly summarize how the TEC is measured from GPS receivers. We follow the derivation found in Kudeki (2010, Section 2.4) and Misra and Enge (2006, Section 5.3.2).

The group velocity, \( v_g \), describes the speed of the envelope of a wave. The information encoded in a GPS signal travels at the group velocity:

\[
v_g = \frac{c}{n_g(h)}, \tag{1.30}
\]

where \( n_g(h) \) is the group refractive index, defined as:

\[
n_g(h) = \frac{1}{\sqrt{1 - \frac{f_p^2(h)}{f^2}}}. \tag{1.31}
\]

Here, \( f \) is the frequency of the EM signal, and \( f_p \) is the plasma frequency.

In a short time, \( dt \), an EM wave will move a short distance, \( dh \), and from Equation 1.30, \( dt \) is calculated as:

\[
dt = \frac{n_g(h)}{c} \, dh. \tag{1.32}
\]

Substituting \( n_g(h) \) and integrating both sides with respect to height in the above equation results in:

\[
t = \int_{h_0}^{h_0+h} \frac{1}{\frac{c}{\sqrt{1 - \frac{f_p^2(h')}{f^2}}}} \, dh'. \tag{1.33}
\]

This can be interpreted as the time it takes for a wave with frequency \( f \) traveling a distance \( h \) in a medium that is described by its group refractive
index, \( n_e(h) \).

Now, we introduce a few assumptions in order to produce a meaningful analytical solution. The square of the plasma frequency can be approximated as \( f_p^2(h) \approx 80.6n_e(h) \). In the ionosphere, the frequencies of GPS signals (\( f_1 = 1575.42 \) MHz and \( f_2 = 1227.60 \) MHz) are much larger than the plasma frequency, which is on the order of 10 MHz: \( f >> f_p(h) \). Therefore, \( f_p(h)/f << 1 \). Using the fact that \( (1 + a)^p \approx 1 + pa \) for \( |a| << 1 \), we can write:

\[
t = \frac{1}{c} \int_{h_0}^{h_0+h} \left[ 1 + \frac{1}{2} \left( \frac{f_p(h')}{f} \right)^2 dh' \right]
= \frac{1}{c} \left[ h + \int_{h_0}^{h_0+h} \frac{1}{2} \left( \frac{f_p(h')}{f} \right)^2 dh' \right]
= \frac{1}{c} \left[ h + \frac{40.3}{f^2} \int_{h_0}^{h_0+h} n_e(h') dh' \right]
= \frac{1}{c} \left[ h + \frac{40.3}{f^2} \text{TEC} \right].
\]

(1.34)

Again, \( t \) can be interpreted as the time it takes for the signal envelope to travel through the ionosphere. The distance that the envelope signal travels, known as the pseudorange, \( \rho \), is defined as:

\[
\rho \equiv ct
= h + \frac{40.3}{f^2} \text{TEC}.
\]

(1.35)

The TEC can be solved for by the measurements obtained from a dual frequency GPS receiver. This type of receiver measures the L1 and L2 signals from the satellites in the GPS constellation, corresponding to frequencies of \( f_1 = 1575.42 \) MHz and \( f_2 = 1227.60 \) MHz, respectively. The pseudorange for each frequency is given as:
Finally, the TEC is solved for by subtracting $\rho_1$ from $\rho_2$ and rearranging:

$$\rho_2 - \rho_1 = \left(\frac{40.3}{f_2^2} - \frac{40.3}{f_1^2}\right) \text{TEC}$$

$$\text{TEC} = \frac{\rho_2 - \rho_1}{f_2^2 - f_1^2} = \frac{c(t_2 - t_1)}{f_2^2 - f_1^2}.$$  \hspace{1cm} (1.37)

Here, $t_2 - t_1$ is the phase difference between the two envelope signals. Therefore, one can find the TEC if the delay between the signals is measured.

Finally, we note that knowledge of the TEC can add a correction to the pseudorange to obtain an approximate distance between the receiver and GPS satellite:

$$h \approx \rho_1 - \frac{40.3}{f_1^2} \text{TEC}$$

$$= ct_1 - \frac{40.3}{f_1^2} \text{TEC.}.$$  \hspace{1cm} (1.38)

This approximation is due to the assumptions made in deriving Equation 1.34, mainly that $f >> f_p$. The travel time, $t$, can be measured within the GPS framework.

1.8 Motivation and Contribution

The primary goal of this work is to advance knowledge and the physical understanding of electrified MSTIDs in the nighttime ionosphere. This goal will be accomplished by a climatological study and through the use of advanced numerical modeling. In this way, we can better understand the physical
mechanisms that influence the development of the instabilities.

An observational database of MSTIDs from the Central Pacific and South American sectors will be developed by cataloging occurrences of the instability from these locations. The data will be analyzed from imagers located in Hawaii and Chile, and over six years of data are presented in order to establish an occurrence rate database. Occurrence rate trends will be compared between the two longitudinal sectors. The field-of-view of the instrumentation used in our study extends near the magnetic equator, and as a result the propagation extent of MSTIDs toward low-latitudes will be quantitatively measured and studied.

In addition, we will model the development of MSTIDs within a numerical simulation framework. The theory for the development of MSTIDs has been established by linearizing terms in the equations describing the structures’ growth. However, during the process of linearizing the equations, higher-order terms are excluded. Although this is sufficient to understand the basic development of MSTIDs, the higher-order terms may be important to describe MSTIDs as they grow and develop in the ionosphere. It is possible that a closed, analytical solution including the higher order terms does not exist. Therefore, the numerical model SAMI3 (Sami3 is Another Model of the Ionosphere) is used to study MSTIDs.

SAMI3 solves the fundamental, physics-based equations for a longitudinal “wedge” of the ionosphere with a grid encompassing both the Northern and Southern hemispheres. Therefore, we can simulate a MSTID developing at mid-latitudes and observe electrical conjugate effects within the model. To our knowledge, this is the first numerical simulation study of MSTIDs with a full 3D grid encompassing both the mid- and low-latitudes, allowing for coupling and electrical conjugate effects to be observed within a numerical model.

We also develop synthetic observations within the model, enabling us to make comparisons of observational data found in the literature with the SAMI3 results. The synthetic measurements will include TEC, 630.0-nm airglow emission, and electron density and drift profiles. The synthetic observations will provide additional support to confirm a MSTID is developed within the numerical model.

With the establishment of MSTIDs within SAMI3, we can then use the model as a tool to further elucidate the physics in the development of the
instability at mid-latitudes. For example, we conduct sensitivity studies with
the neutral wind parameters and measure their impact on the growth rate
of MSTIDs. The solar conditions are modified within the model to under-
stand its influence on the development of MSTIDs. As a MSTID approaches
the equator, it passes through the equatorial anomaly, a region of enhanced
electron density. This region will be modeled within the MSTID numerical
simulation. Given that the instability is subjected to enhanced electron den-
sity, its growth can be studied in order to further investigate the MSTID
equatorward limitation hypothesized by 
Shiokawa et al. (2002). Finally, the
instability is seeded at different latitudes in the model to understand mag-
netic dip angle dependence. The numerical studies will aid in advancing our
physical understanding of the generation and development of MSTIDs.

The following chapters in this dissertation are organized as follows. Chap-
ter 2 provides a literature review, and will discuss previous observational,
theoretical, and numerical simulation studies on MSTIDs. Chapter 3 de-
scribes the theory of the Perkins instability, which is commonly used to de-
scribe the development of MSTIDs. It will be helpful to carefully detail the
equations associated with the instability in order to understand how MSTIDs
can be generated within the numerical model. Next, Chapter 4 will present
the results from the climatology study conducted in the Central Pacific and
South American sectors. The study will provide occurrence rate statistics of
MSTIDs over a six year period, and include a discussion on the instability
traveling toward low latitudes.

We will then shift the work toward the simulation framework. Chapter 5
describes the SAMI3 model and its associated numerical schemes, providing
a basic understanding of how the upper atmosphere can be numerically mod-
eled based on the fundamental, physics-based equations described in Section
1.2.1. Chapter 6 includes the results of using SAMI3 for the self-consistent
description of MSTIDs, and consequently Chapter 7 details synthetic obser-
vations and numerical case studies of a MSTID as it is developed within the
model. Finally, we will summarize and conclude this work in Chapter 8, and
also provide future research directions.
CHAPTER 2

LITERATURE REVIEW OF PREVIOUS MSTID STUDIES

In order to provide context for the current work, this chapter will review past studies of MSTIDs. Previous work on MSTIDs have been conducted through observations, theory, and numerical simulations. The following sections provide a review on each methodology, and outline the information that is currently known in the respective area of study.

2.1 Experimental Observations of MSTIDs

One of the first observational studies of MSTIDs was conducted by Behnke (1979). In his study at the Arecibo Observatory, which is considered to be a mid-latitude site, the incoherent scatter radar (ISR) beam was rotated at a constant 15° zenith angle to measure the nighttime F-layer height in 6° azimuthal increments. The measurements indicated a disturbance of the F-peak with a height-band variation of about 50 km. Behnke (1979) calculated that the instabilities had wavefronts that were aligned from NW-SE and traveled in the SW direction with a wavelength of about 150 km. Figure 2.1 depicts observations from the Behnke (1979) study, displaying schematic representations of the F-layer height measurements. In this figure, the hatched and unhatched regions represent an increase and decrease in the F-layer height, respectively.

The disturbances measured by Behnke (1979) were characteristically different compared to previous instabilities observed in the ionosphere. For example, observations of equatorial spread F (ESF), which primarily occur near the geomagnetic equator, are characterized by a large depletion of electron density (i.e., a “bubble”), are aligned with the magnetic field line, and typically travel eastward. However, the observations by Behnke (1979) had variations in the F-peak height as opposed to depletions, and were organized
into bands at an angle to the magnetic meridian that propagated southwest.

Additional radar experiments have been used to understand the instability in more detail. The phased-array MU radar in Japan has measured MSTID signatures from 50-MHz coherent backscatter in the F-region and recorded 3-m irregularity patches moving westward (Fukao et al., 1991). Similar to the Behnke (1979) study, the irregularities were characterized by banded structures of raised and lowered (with respect to altitude) electron density. Furthermore, the Fukao et al. (1991) study showed that the patches were accompanied by ion drifts of 100-200 m/s in the outward direction from the radar, which cause height variations in the F-peak.

Different facets of the instabilities can also be observed through a wide variety of instrumentation. The Dynamics Explorer 2 (DE-2) satellite has made in situ measurements of the electric fields internal to MSTIDs at mid-latitudes in the F-region, mainly between 300- and 400-km altitude (Saito et al., 1995). The amplitudes of the electric fields were a few mV/m and were measured in conjugate hemispheres, typically occurring after midnight. The results from the studied showed that the instabilities were electrified, and via $\mathbf{E} \times \mathbf{B}$ drifts, the electric fields were the source of the ion drifts which subsequently displaced the electron density bands in altitude.
A significant result from the Saito et al. (1995) study was that it led to the theory that the electric field internal to a MSTID maps to the conjugate hemisphere, thus producing the signature of the instability there. As discussed in Section 1.4, the relatively large conductivity along the magnetic field line direction, which approximates $B$ as an equipotential field line, results in the electric field mapping effect. This concept is outlined in Figure 2.2, and shows a schematic track of the DE-2 satellite making electric field measurements in both hemispheres. Conjugate electric fields are produced in the ionosphere in both the Northern and Southern hemispheres. One can think of the F-region electric fields as completing the circuit created by field-aligned current densities, and as being required in order to maintain divergence free current densities ($\nabla \cdot J = 0$).

Imaging techniques have also been employed as an additional tool to study MSTIDs. Optically filtered, cooled CCD imaging systems can observe the 630.0-nm “red-line” emission, described in Section 1.6, occurring below the F-peak near 250-km altitude. The imaging data collected by the instruments can be used as a tracer for F-region dynamics, and thus can resolve
MSTID properties. *Garcia et al.* (2000) used 630.0-nm airglow imaging and observed enhanced and depleted structures, representative of lowered and raised electron density, respectively, organized into bands at an angle to the magnetic meridian. Figure 2.3 reproduces a result from the *Garcia et al.* (2000) study and displays variations in the intensity of the 630.0-nm airglow emission. Note that the general features shown in the airglow imaging are similar to the schematic developed by *Behnke* (1979) in Figure 2.1, mainly that structures have wavefronts that are not aligned with the magnetic field line. *Garcia et al.* (2000) were able to measure the velocity of the structures, and the compass plot in Figure 2.4 shows a summary of the propagation velocities from their study. A majority of the MSTIDs observed from their study had speeds of about 100 m/s and were propagating southwestward in the Northern Hemisphere.

The airglow imaging technique has been used to investigate the mapping of internal MSTID electric fields with imagers located in two magnetic conjugate locations (*Otsuka*, 2004). The imaging data showed simultaneous MSTIDs in each hemisphere, confirming the theory that the structures are electrified and the electric field maps along the magnetic field line, producing the signature of the MSTID in the conjugate hemisphere. Figure 2.5 shows the geomagnetic conjugate observations using 630.0-nm airglow imaging. The MSTID signature is apparent in both hemispheres and is mirrored.
Observational data has shown that MSTIDs propagate toward the equator, and researchers have investigated the ability of the instabilities to reach low-latitudes primarily through experimental studies. Shiokawa et al. (2002) has proposed an equatorward limit of MSTIDs to be approximately 18° magnetic latitude. They argued that in the equatorial anomaly region (which is near the proposed cutoff), the enhanced electron density increases the time constant for ion drag. As a result, gravity waves may be inhibited to reach the F-region, and therefore not be able to seed the development of MSTIDs. However, MSTIDs have been observed at low latitudes, suggesting that this topic needs to be revisited.

Dual-frequency GPS receivers, capable of measuring the TEC (i.e., Section 1.7), are commonly used to study MSTIDs. The MSTID signature will correspond to fluctuations of TEC values due to the GPS measurement integrating through the height-band variations of electron density, which are indicative of the instability. Kotake et al. (2006) used GPS receivers in Japan, Europe, United States, Australia, and South America in 1998, 2000, and 2001 and found high occurrences of nighttime MSTIDs during solar minimum. A dense network of GPS receivers can generate 2D spatial TEC maps of MSTIDs, and Kotake et al. (2007) used this technique to measure southwestward propa-
Figure 2.5: Geomagnetic conjugate observations of MSTIDs observed in (a) Sata, Japan, and (b) Darwin, Australia. Reprinted from Otsuka (2004) by permission of John Wiley and Sons. Copyright 2004 by the American Geophysical Union.
Figure 2.6: Occurrence rates of MSTIDs measured in Japan during 2002 as a function of time and day of year. Reprinted from Otsuka et al. (2011) by permission of Springer.

Secchi disk imaging of MSTIDs over Southern California. Tsugawa et al. (2007) developed TEC fluctuation maps of MSTIDs using a network of GPS receivers in North America. They found that the wavefronts of MSTIDs can extend about 2,000 km along the wavefront direction. Otsuka et al. (2011) used a similar technique in Japan to measure TEC perturbations (on the order of 0.5 TECU) and generated occurrence rate maps of MSTIDs in 2002. Figure 2.6 displays a result from the Otsuka et al. (2011) study and shows high occurrences of nighttime MSTIDs near midnight during the June solstice. Overall, observations of MSTIDs utilizing TEC data offer insight on their spatial, temporal, and occurrence rate properties.

Typical parameters and properties of MSTIDs can be established by conducting long-term, climatological studies. For example, climatological measurements using airglow imaging over the Japanese sector from 1998-2000 show that MSTIDs primarily have a wavelength of 100–300 km and velocities ranging between 50–100 m/s, corresponding to 630.0-nm airglow fluctuations on the order of 5-15% (Shiokawa et al., 2003a). In their study, Shiokawa et al. (2003a) observed MSTIDs occurring primarily in the solstice time periods, with a major maximum occurrence peak near the June solstice and a minor peak near the December solstice. In Brazil, seven years of airglow imaging data were analyzed and found 28 MSTID events that occurred mainly during low solar activity and near the June solstice time frame (Candido et al.,...
Coincident observations have been leveraged to further support established MSTID properties. For example, the signature of MSTIDs has been shown in data from 630.0-nm airglow imagers coexisting with TEC derived from a network of dual-frequency GPS receivers \((Saito et al., 2001)\). The banded perturbations in the airglow imaging data were coincident to the change in TEC. The observations by \textit{Saito et al.} (2001) also showed that the MSTID perturbations intensified as they traveled southwest in both types of observations. Coincident airglow imaging, satellite ion-drift measurements, and neutral wind estimates from Fabry-Perot interferometers have been used to verify the role of the polarization electric field in the development of MSTIDs \((Shiokawa et al., 2003b)\). That is, the polarization electric fields produce perturbation ion drifts, generating the vertical displacements of electron density that are characteristic of the instability.

In the past 50 years, observational data has proved to be a valuable method to understand MSTID characteristics. Key properties of MSTIDs have emerged as a result of observational work found in the literature, and can be summarized as follows:

- The instabilities can be described as vertical displacements of plasma density, organized into banded structures at an angle to the magnetic meridian.

- MSTIDs primarily occur at mid-latitudes, where the magnetic field forms a finite dip angle, \(D\), with respect to the horizontal plane.

- The wavefronts are typically aligned NW to SE in the Northern Hemisphere and SW to NE in the Southern Hemisphere, and propagate westward and equatorward.

- MSTIDs are electrified, and the electric field internal to the instability, through \(\mathbf{E} \times \mathbf{B}\) drifts, play a role in the formation of the F-layer height changes.

- The electrified nature of MSTIDs, along with the large conductivity along the magnetic field direction, results in the signature of the instability in the conjugate hemisphere.
The signature of MSTIDs manifests itself in airglow imaging and TEC data, and these observational techniques can be used for understanding properties of the instability, such as wavelength, velocity, and occurrence rate.

A natural motive that follows from the observational work is understanding how and why MSTIDs develop in the nighttime ionosphere. Several questions are prevalent, such as:

- Why are the structures oriented the way they are?
- What are the underlying factors that cause them to develop?
- Which parameters drive the development of the instabilities?
- Why are the instabilities electrified?

In order to answer these questions, a theoretical framework is necessary to investigate the physics of the instability development. Chapter 3 will detail the theory of MSTIDs, offering insight on the processes that are involved to generate the instabilities. The next section will outline the relevant work that has been conducted to better understand MSTIDS from a theoretical perspective.

2.2 Historical Developments on the Theory of MSTIDs

The theoretical work can be traced back to Perkins (1973), who investigated the stability of the mid-latitude, nighttime F-layer. He found that a stable configuration for the F-layer was a result of a force balance between the effects of gravity and electric fields. In his analysis, Perkins (1973) determined that the balance could be upset and unstable electric field modes would develop if there existed a north-south component of the electric field.

In his derivation, Perkins (1973) started with the fundamental physics equations (i.e., Section 1.2.1) and introduced linear perturbations in the terms in order to find unstable modes. In order to make the equations tractable, he assumed equipotential field lines (i.e., Section 1.4) and integrated the equations along the magnetic field line direction. Thus, the equations were reduced from three to two spatial dimensions. For his pioneering
work at describing the stability of the nighttime, mid-latitude, F-region ionosphere, the instability that can generate MSTIDs is often referred to as the “Perkins instability.”

Huang et al. (1994) expanded on Perkins’ work by including nonlinear terms in the analysis and studied the initiation of the instability by gravity waves propagating from below in the lower thermosphere. The concept of gravity wave interaction was further explored (Miller, 1996; Kelley and Miller, 1997), proposing that a neutral atmosphere disturbance could impact plasma in the ionosphere, in effect seeding the development of MSTIDs. Also, the mapping of electric fields along the magnetic field lines from the conjugate F-region and/or sporadic E (Es) layers has also been suggested (Huang et al., 1994), providing another example of potential coupling effects that could influence the instability.

Hamza (1999) revisited the formulation of the instability derived by Perkins (1973) to include the neutral wind, $U$, and horizontal gradients in the background electron density. In addition to a northward electric field providing the destabilizing mechanism for the instability (for a Northern Hemisphere configuration), an eastward neutral wind also contributes to MSTID growth via the effective electric field, $E' = E_0 + U \times B$. As we will see in both the observational and simulation chapters of this work, the neutral wind in the F-region has an important influence on the generation of modes consistent with the Perkins instability.

Although the Perkins instability can explain unstable height-band variations at mid-latitudes, the theory suffers from a sign discrepancy in the real part of the wave frequency, $\omega_{Re}$, compared to observations. Along with the wavevector, $k$, $\omega_{Re}$ describes the propagation velocity of the instability. For a Northern Hemisphere configuration near the pre-midnight timeframe, the neutral wind is typically in the southeast direction. Under this scenario, the Perkins theory predicts the development of an instability with a wavevector $k$ in the northeast quadrant, and that it should travel in the northeast direction. However, MSTIDs are often observed to propagate southwestward, which is inconsistent with the theory.

The MSTID could be subject to secondary effects that cause the instability to travel in the observed direction. Kelley and Makela (2001) used a simple dipole model to describe the possibility of a polarization electric field ($E_p$) along the wavefront direction, as shown in Figure 2.7. The polarization...
electric field is developed as a result of an uplift in electron density in the banded region. That is, a rising slab of electron density will cause a reduction in the integrated Pedersen conductivity, which in turn will generate $E_p$ in order to maintain divergence free current densities ($\nabla \cdot J = 0$). Under this scenario, $E_p \times B$ drifts would propagate the MSTID structure southwest, which is consistent with the observed direction. However, current modeling developments have not shown this mechanism to be prevalent in numerical simulations of MSTIDs.

An improved explanation of the equations derived by Perkins (1973) was provided by Zhou and Mathews (2006), in which they attributed the source of the instability to $E \times B$ drifts. The derivation of the instability by physical arguments offers additional insight on MSTIDs and highlights the important parameters and conditions that are required for the development of the instability. Under this type of analysis, the Perkins instability was also studied with the neutral wind present (Zhou and Mathews, 2006). An overview of this technique and analysis is provided in Section 3.4.

Finally, coupling aspects from sporadic E (Es) layers have also been thoroughly studied (e.g., Tsunoda and Cosgrove, 2001; Cosgrove, 2004; Tsunoda, 2006). The relatively small theoretical growth rate, on the order of $10^{-4}$ [e-folds/s], does not align with experimental observations of MSTIDs.
In the Es coupling theory, large polarization electric fields from Es layers can couple along the magnetic field lines. As a result, the feedback coupling process provides an increased growth rate to the development of MSTIDs in the F-region.

The progress for the theoretical description of MSTIDs has steadily increased alongside experimental observations. The theory describes many factors influencing the development of MSTIDs, including nonlinearities, coupling aspects, and parameters such as the neutral wind. A detailed discussion on the theory of MSTIDs, including an analysis of the instability, will follow in Chapter 3.

2.3 Development of MSTIDs in Numerical Simulations

In addition to the observational and theoretical work, numerical simulations have been used to study MSTIDs. Numerical simulations can serve as a tool to test theory. Also, model results can be analyzed against observations to verify the behavior of MSTIDs. Compared to the methods used to obtain observational data, a numerical framework offers fewer limitations for recording state parameters associated with the instabilities. As a result, data from numerical simulations provide additional insight on MSTID properties. In this section, advancements found in the literature with respect to numerical modeling of MSTIDs are discussed.

Early numerical simulation work of MSTIDs can be divided into two phases. The first phase of modeling focused on the high-level equations associated with the Perkins instability. These equations describe the time evolution of the electrostatic potential ($\Phi$), and the magnetic field line integration of both the Pedersen conductivity ($\Sigma$) and electron density ($N$).

That is, Equations 13, 14, and 15 of Perkins (1973) were modeled with parameter inputs obtained from a combination of climatological models and observations. The work from the first phase is referred to as “2D modeling” because the equations are integrated along the field line direction, resulting in equations describing the parameters in the 2D plane of the magnetic field.

The second phase of modeling approached the simulation of MSTIDs from a lower-level, physics-based point of view. Simulations involved three-dimensional modeling of the ionosphere, providing a grid space analogous to reality.
MSTIDs were generated self-consistently by invoking the three fundamental equations of momentum conservation, ion continuity, and divergence-free current densities, as discussed in Section 1.2.1.

Although both phases of simulations describe the development of MSTIDs, the physics-based approach of the second phase is advantageous for several reasons. For example, the second phase of modeling can take advantage of the full, 3D spatial grid for the calculation of synthetic observations in the model. The simulation work from the first phase describes the time evolution of integrated quantities pertaining to MSTIDs at mid-latitudes, and therefore is somewhat limited in calculating only field-line integrated quantities. In addition, simulations from the second phase are not restricted to model linear perturbations as the fundamental, physics-based equations are used.

From the first phase of modeling, the earliest work was completed in the form of a Naval Research Laboratory (NRL) memo report shortly after the theory was developed by Perkins (1973) (Scannapieco et al., 1975). Unfortunately, it was never published in a peer-reviewed journal to receive greater visibility, but Scannapieco et al. (1975) set out to numerically simulate the equations developed by Perkins (1973). They showed the nonlinear evolution of the equations and noted that $E \times B$ drifts were associated with the instability.

The next record of MSTID simulations occurred in the mid-1990s (Miller, 1996). The 2D modeling work considered the time evolution of the Perkins equations when perturbed by gravity waves and found good agreement between the simulated and theoretical growth rate. A pseudo-spectral method has been used in numerical case studies to investigate the evolution of the structure with initial conductivity perturbations (Zhou et al., 2005, 2006). The studies noted that $E \times B$ drifts within the instability could be the driver for the propagation of the structures.

As the simulations became more sophisticated, the work of numerical studies of MSTIDs moved into the second phase with three-dimensional simulations using the fundamental, physics-based equations of the ionosphere. Significant progress in this area was obtained with the first three-dimensional simulations generating a MSTID (Yokoyama et al., 2008). In their work, Yokoyama et al. (2008) considered a spatial “box” region in the mid-latitude ionosphere and, from an initial random seeding, successfully generated the
directional bands of lowered and raised electron density (with respect to altitude) associated with a MSTID. Figure 2.8 plots a time series of the percent change in local density perturbation at 280 km altitude from the study by Yokoyama et al. (2008). Although the magnitudes of the perturbations are small, dominant modes are established in the system that are indicative of MSTIDs and are consistent with observations summarized in Section 2.1.

In the numerical work of Yokoyama et al. (2008), only the electrostatic calculations within an isolated “box” region were implemented and coupling aspects were not considered. Studies progressed by investigating the E-region coupling effects for enhancing the growth rate of MSTIDs (Yokoyama and Hysell, 2010). It was also found that with an equatorward neutral wind in the E-region, along with coupling effects, the instability propagated in the
observed westward and equatorward direction (Yokoyama et al., 2009). The conjugate appearance of MSTIDs with spatial “box” regions in the Northern and Southern hemispheres has also been investigated, with an implicit electric field communicated between the regions by the assumption of equipotential field lines (Yokoyama, 2014). The studies indicated that the neutral wind configuration in the conjugate hemisphere can affect the growth of MSTIDs. Also, numerical studies have shown scale-size dependence for the coupling between Es and F-layers and the subsequent growth of MSTIDs (Yokoyama, 2013).

The current modeling work is beginning to understand the physical development of the instabilities in considerate detail. However, additional work is needed to relate experimental observations to data produced within the simulations for a complete and self-consistent description of MSTIDs by each experimental technique. Ideally, although it may be computationally expensive, the simulations would not be constrained to a gridded “box” region in a hemisphere in order for the full coupling of MSTIDs to be modeled and studied.

2.4 Summary

For continued advancement in MSTID research, it is important to have an understanding of the previous work that has been conducted on the topic. This chapter summarized the observational, theoretical, and numerical simulation aspects of studies found in the literature. In Section 2.1, an overview of the observational work of MSTIDs was provided. The studies were accomplished through a variety of instrumentation, including by radar, satellite, airglow imaging cameras, and dual-frequency GPS receivers. Section 2.2 discussed the theory, which has progressed since the initial work by Perkins (1973). The theoretical developments have detailed the underlying processes that are involved to generate the instability.

Finally, numerical simulations have also been used to study MSTIDs, and Section 2.3 described two methodologies. The first included simulating the high-level equations associated with the Perkins instability, while the second method simulates the fundamental, physics-based equations for numerically modeling MSTIDs. The simulation studies in this dissertation follow from the
second method and simulate the low-level equations with the SAMI3 model. In this way, we are able to investigate MSTIDs developing in a 3D spatial grid, and can utilize the model to gain physical insight on their growth.

In the next chapter, in order to further understand the development of MSTIDs, an in-depth discussion on the theoretical work will be provided. The derivation of the equations associated with the instability will be outlined. The knowledge will be leveraged to explain MSTID observations in Chapter 4 and also to construct the procedure for numerically generating the instabilities in Chapter 6.
In this chapter, we will discuss the basic formulation for the theory of MSTID development in the nighttime, mid-latitude ionosphere. First, the coordinate system will be defined. One direction in the coordinate system will be along the geomagnetic field line, which is advantageous to simplify the dimensionality of the governing equations. A discussion on the linear stability analysis is provided, which is a general procedure for deriving instability growth rates. The original MSTID theoretical formulation by Perkins (1973) is presented, including the full derivation in Appendix C. This careful analysis provides a deeper understanding of the instability from the fundamental equations.

Next, the interactions between the governing equations that subsequently describe the development of the instability are presented in a flow chart, which further aids the reader in understanding the instability. The physical equations form the foundation for this analysis technique. Finally, a physical explanation for the instability is discussed, offering yet another way of interpreting MSTIDs, and details the parameters that affect the instability development. The theoretical information from this chapter will be leveraged for the development of MSTIDs within a numerical simulation framework. In addition, the results from the numerical simulations should follow from the theoretical descriptions provided in this chapter.

3.1 Coordinate System

In order to begin deriving the equations that describe MSTID theory, a coordinate system must be selected for the analysis. It is instructive to choose a coordinate system that provides an advantage for solving equations easily, and one that leads to a concise description of the instability. We use the fact that due to the high conductivity along the magnetic field line,
the ionospheric potential solution is constant along the \( \mathbf{B} \) direction. As a result, the governing equations can be integrated along the magnetic field line direction, thus simplifying the problem from three to two spatial dimensions and reducing the complexity of the equations.

A Cartesian coordinate system is selected to describe mid-latitude dynamics in the ionosphere, with the coordinate \( \hat{z} \) along the magnetic field line direction. Magnetic east is described by \( \hat{y} \). To complete the coordinate system, \( \hat{x} \) points radially outward from the magnetic field lines and follows the right-hand rule. In the Northern Hemisphere, this is commonly referred to as “perpendicular and north”, because it is perpendicular to the field line and also points in the northern direction.

Figure 3.1 gives an example of the coordinate system that is based on the direction of the geomagnetic field line. The IGRF climatological model was used to develop realistic magnetic field lines for this plot and displays apex altitudes between approximately 100 and 3,000 km. The right-hand side of Figure 3.1 is a close-up view of the red box region from the left-hand plot and highlights the coordinate system. Here we see that \( \hat{z} \) points along the magnetic field line, and each \((x, y)\) pair describes a particular magnetic field line.

In the following sections, this coordinate system will be used for deriving the theoretical formulation of MSTIDs. Before developing the theoretical framework, a short discussion on the linear stability analysis is provided.
This analysis is a key technique that is used to derive the description of the instability.

3.2 Linear Stability Analysis

In the theoretical work for MSTIDs, a procedure known as a linear stability analysis is conducted to derive the growth rate of the instability. This analysis involves linearizing partial differential equations (i.e., the fundamental, physics-based equations in the ionosphere) in order to make solutions tractable. To begin, a perturbation value is introduced into the quantity, $\psi(x,t)$:

$$\psi(x,t) = \psi_0(x,t) + \tilde{\psi}(x,t). \quad (3.1)$$

Here, $\psi_0(x,t)$ is the unperturbed, “background” quantity. The perturbed quantity, $\tilde{\psi}(x,t)$, has the form:

$$\tilde{\psi}(x,t) \propto e^{j(\omega t - k \cdot x)} = e^{\gamma t} e^{j(\omega_{Re} t - k \cdot x)} \quad (3.2)$$

Where we have invoked a complex $\omega$ (Appendix B) to obtain the expression as a function of the growth rate, $\gamma$.

Due to the properties of the exponential function, derivative operations on the perturbation quantity can be transformed as:

$$\partial/\partial t \rightarrow \gamma + j\omega_{Re}$$
$$\nabla \rightarrow -j\mathbf{k} \quad (3.3)$$

As a result, the equations are linearized and can result in a closed, analytical solution.

As shown in Equation 3.2, the perturbation quantity can be described by a wavevector, $\mathbf{k}$, and its associated growth rate, $\gamma$. Typically, the procedure for the linear stability analysis assumes a perturbation of the form in Equation 3.2. Then, once the relations are linearized with the aid of Equation 3.3, a
solution for $\gamma$ is found. In general, the relation for $\gamma$ will be a function of the wavevector, $k$. One can think of the equation for $\gamma(k)$ as describing how fast a particular $k$ mode develops. For $\gamma(k) > 0$, the particular wavevector will be unstable and grow, while for $\gamma(k) < 0$, the wavevector will dampen (and therefore will be stable).

3.3 Original Derivation \textit{(Perkins, 1973)}

This section will outline the original theoretical developments of MSTIDs researched by \textit{Perkins} (1973). In his work, Perkins begins with the governing physical equations for the mid-latitude ionosphere. Simplifying assumptions are introduced in order to make the equations manageable. Then, a linear perturbation is applied and the modes corresponding to unstable growth are solved for.

To begin, three fundamental equations are invoked: the continuity, momentum and divergence-free current density relations (Equations 1.2 - 1.4). Knowledge of the nighttime, mid-latitude ionosphere characteristics can be used to simplify the equations. Following, \textit{Perkins} (1973), the following assumptions are made:

1. The wave frequency of interest, $\omega$, has time-scales much smaller than the motions of the ions and electrons (i.e., $\nu_{in}$). Therefore, the inertial terms on the left-hand side of Equation 1.3a are approximated as 0.

2. As discussed in Section 1.2.1, the plasma can be considered quasi-neutral, and thus the electron density is approximately equivalent to the summation of the ion density constituents: $n_e \approx \sum_i n_i$.

3. The current density is divergence-free, $\nabla \cdot J = 0$, as justified in Section 1.2.1.

4. The electron mass is much smaller than the ion mass. Therefore, compared to the ion-electron collision frequency, $\nu_{ie}$, the ion-neutral collision frequency, $\nu_{in}$, dominates in the ion momentum equation.

5. Compared to other forces in the momentum equation, the force due to gravity, $g$, applied to the electrons is small and is therefore neglected.
6. The temperatures in the ionosphere can be considered isothermal, and remain relatively constant as a function of time. Therefore, the pressure term in the momentum equation can be simplified to $P = n_i k_B T$. In addition, the ion and electron temperature are approximately equivalent, $T_i \approx T_e$.

Under these assumptions, the governing equations can be simplified to:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}_i) = 0$$  \hspace{1cm} (3.4)

$$0 = -k_B T \nabla n + ne(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) + nm_i g + nm_i \nu_{in}(\mathbf{U} - \mathbf{V}_i)$$  \hspace{1cm} (3.5a)

$$0 = -k_B T \nabla n - ne(\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) + nm_e \nu_{en}(\mathbf{U} - \mathbf{V}_e) + nm_e \nu_{ei}(\mathbf{V}_i - \mathbf{V}_e)$$  \hspace{1cm} (3.5b)

$$\nabla \cdot \mathbf{J} = 0$$

$$\nabla \cdot en(\mathbf{V}_i - \mathbf{V}_e) = 0$$  \hspace{1cm} (3.6)

From here, Perkins (1973) manipulates the coupled equations to describe the dynamics in the mid-latitude ionosphere. Due to the complexity involved in the derivation, the full procedure is provided in Appendix C. The steps can be summarized as:

1. From the momentum equation, the ion and electron velocities, $\mathbf{V}_i$ and $\mathbf{V}_e$, are derived.

2. The expressions for the velocities are used to solve for the current density, $\mathbf{J}$, and then the divergence-free current density relation is invoked, $\nabla \cdot en(\mathbf{V}_i - \mathbf{V}_e) = 0$, from Equation 3.6.

3. The divergence free current density and continuity equations are integrated along the magnetic field line direction. This simplifies the equations to two spatial dimensions.

4. A stability analysis is conducted to find the steady-state of the ionosphere. That is, the $\partial(\cdot)/\partial t$ terms are set to 0.
5. A linear perturbation is introduced into the system of equations and an expression for the growth rate is obtained. Positive growth rates correspond to unstable modes that develop in the nighttime, mid-latitude ionosphere.

The outcome from the analysis in Appendix C gives the growth rate for the instabilities as:

\[
\gamma = \frac{cE_0 \cos D}{BH_n} \sin \alpha \sin(\theta - \alpha) \text{ [e-folds/s]}. \tag{3.7}
\]

Here, \( \theta \) is the angle between \( E_0 \) and magnetic east, and \( \alpha \) is the angle between \( k \) and magnetic east. Note that in this particular derivation, the neutral wind, \( U \), was not included. The reader is referred to Appendix C for the full derivation that details these steps.

3.4 Alternative Derivation Sketch

In order to provide further context for the theoretical work, the instability can also be derived from an alternative approach that is based on physical arguments (Zhou and Mathews, 2006). This approach will offer additional insight into the development of MSTIDs. For example, the neutral wind term is included in this analysis technique and a more thorough description of the main drivers of the instability is revealed.

To begin, similar to the derivation by Perkins (1973), the three fundamental equations are invoked and the equations are integrated along the magnetic field line to reduce spatial dimensionality and complexity. Table 3.1 lists the quantities that will be used for this description. In order to remain consistent with the previous work, CGS units are used in the beginning stages of the derivation. Figure 3.2 displays the flowchart of equations used in the alternative derivation.

Starting on the upper right-hand side of Figure 3.2, the equation from the divergence free condition is integrated along the magnetic field line (Perkins, 1973). Now the equations are two-dimensional in the plane of the magnetic field. Then, as described in Section 3.2, a perturbation is applied to both the potential (\( \Phi \)) and integrated Pedersen conductivity (\( \Sigma \)). Finally, the
<table>
<thead>
<tr>
<th>Description</th>
<th>Units (CGS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$  Electron Density</td>
<td>number/cm$^3$</td>
</tr>
<tr>
<td>$n_i$ Ion Density</td>
<td>number/cm$^3$</td>
</tr>
<tr>
<td>$V_i$ Ion Velocity</td>
<td>cm/s</td>
</tr>
<tr>
<td>$V_e$ Electron Velocity</td>
<td>cm/s</td>
</tr>
<tr>
<td>$\nu_{in}$ Ion–Neutral Collision Frequency</td>
<td>collisions/s</td>
</tr>
<tr>
<td>$T$  Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$\Omega_i$ Gyrofrequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>$m_i$ Ion Mass</td>
<td>g</td>
</tr>
<tr>
<td>$g$  Acceleration Due to Gravity</td>
<td>cm/s$^2$</td>
</tr>
<tr>
<td>$c$  Speed of Light</td>
<td>cm/s</td>
</tr>
<tr>
<td>$q$  Electric Charge</td>
<td>esu</td>
</tr>
<tr>
<td>$H_n$ Neutral Scale Height</td>
<td>cm</td>
</tr>
<tr>
<td>$D$  Magnetic Dip Angle</td>
<td>$^\circ$</td>
</tr>
<tr>
<td>$U$  Neutral Wind</td>
<td>cm/s</td>
</tr>
<tr>
<td>$\Phi$ Electric Potential</td>
<td>statV</td>
</tr>
<tr>
<td>$E$  Electric Field</td>
<td>statV/cm</td>
</tr>
<tr>
<td>$J$  Current Density</td>
<td>esu/cm$^2$s</td>
</tr>
<tr>
<td>$B$  Magnetic Inductance</td>
<td>G</td>
</tr>
<tr>
<td>$\Sigma$ Integrated Conductivity</td>
<td>cm/s</td>
</tr>
<tr>
<td>$N$  Integrated Electron Density</td>
<td>$1/cm^2$</td>
</tr>
<tr>
<td>$k$  Perturbation Wavevector</td>
<td>$1/cm$</td>
</tr>
</tbody>
</table>
Momentum equation

\[ 0 = -2T \nabla n + ne \left( \frac{V_j \times B}{c} \right) - ne \nabla \Phi + nm_i g - m_i n V_j \nu_{in} \]

Divergence free condition

\[ \nabla \cdot J = 0 \]

\[ \nabla \cdot \left[ \kappa n (V_i - V_e) \right] = 0 \]

Ion continuity equation

\[ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i V_i) = 0 \]

\[ \left. \int_{\text{field line}} \right. \frac{\partial \Sigma}{\partial t} \frac{\text{d}z}{dz} + \nabla \cdot \Sigma \left( g \times \frac{\vec{\Sigma}}{\Omega_i} - \frac{\nabla \Phi \times \vec{z}}{B} \right) = \frac{\kappa e q N \sin^2 D}{\Omega_i BH_n} + \Sigma \frac{\partial \Phi}{\partial y} \frac{\cos D}{BH_n} + \Sigma U_z \sin D \frac{\partial}{H_n} \]

\[ \Phi_0 + \Phi \]

\[ \Sigma_0 + \tilde{\Sigma} \]

\[ \tilde{E}_{\text{pseudo}} = \frac{1}{\Sigma_0} \left( -E_{0y} + U_z \frac{B \tan D}{c} \right) \]

\[ \tilde{E}_y = -k_y \frac{\Sigma}{k^2 \Sigma_0} \left[ k \cdot E_0 + \frac{B}{c} k \cdot (U_\perp \times \vec{z}) \right] \]

\[ E_y + E_{eq} = E_{0y} + \tilde{E}_y - (E_{0y} + \tilde{E}_{\text{pseudo}}) = \tilde{E}_y - \tilde{E}_{\text{pseudo}} \]

Figure 3.2: Equation flow chart for the development of MSTIDs. After Perkins (1973); Zhou and Mathews (2006).
perturbation component of the eastern electric field, \( \tilde{E}_y \), is derived. One can think of this electric field as driving an \( \mathbf{E} \times \mathbf{B} \) drift in the \( \hat{x} \) direction.

Next, moving to the left-hand side of Figure 3.2, the ion continuity equation is used to derive the steady-state motion of the plasma. Again, the equation is integrated along the field line. To arrive at a steady-state configuration of the ion density peak height, the \( \partial(\cdot)/\partial t \) terms are set to 0. Given that the layer is moved away from its equilibrium, \( \nu_{in} \) (the ion-neutral collision frequency) is changed (\( \nu_{in} \rightarrow \nu_{in} + \tilde{\nu}_{in} \)). This action is balanced by adding a pseudo-electric field to the left-hand side of the equation: \( E_{0y} \rightarrow E_{0y} + E_{\text{pseudo}} \). Next, the expression for \( E_{\text{pseudo}} \) is isolated by removing the higher order terms.

From the steady state equation, the \( \nu_{in} \) term can be represented as an equivalent electric field. This electric field is written as \( E_{eq} = -(E_{0y} + \tilde{E}_{\text{pseudo}}) \). Now, given a downward perturbation in the height layer, the local Pedersen conductivity will increase due to its dependence on \( \nu_{in} \) (that is, there are more neutrals to collide with at lower altitudes). As a result, the perturbation of the integrated Pedersen conductivity will be positive, \( \tilde{\Sigma} > 0 \). If \( E_y + E_{eq} > 0 \), the superposition of the electric fields \( E_y \) and \( E_{eq} \), through an \( \mathbf{E} \times \mathbf{B} \) drift, will move the layer back up and restore it to equilibrium. Conversely, given an upward perturbation in the height layer (\( \tilde{\Sigma} < 0 \)), if \( E_y + E_{eq} < 0 \), the decreased total electric fields will restore the height layer back downward. It can be shown that both scenarios of a decaying perturbation lead to (Zhou and Mathews, 2006):

\[
E_{0y} - \frac{k_y}{k^2} \mathbf{k} \cdot \left( \mathbf{E}_0 + \mathbf{U}_\perp \times \frac{\mathbf{B}}{c} \right) - U_z \frac{B \tan D}{c} > 0 \tag{3.8}
\]

Reversing the sign of the inequality leads to the condition for instability growth:

\[
E_{0y} - \frac{k_y}{k^2} \mathbf{k} \cdot \left( \mathbf{E}_0 + \mathbf{U}_\perp \times \frac{\mathbf{B}}{c} \right) - U_z \frac{B \tan D}{c} < 0 \tag{3.9}
\]

As discussed in Appendix B, given that the perturbation is proportional to \( e^{j(\omega t - \mathbf{k} \cdot \mathbf{x})} \), the imaginary part of \( \omega \) is the growth rate, \( \gamma \). The growth rate gives a quantitative measure of how quickly the instability develops. Using the requirements for an unstable scenario from Equation 3.9, the growth rate of the instability can be derived by considering the motion of the layer due to \( \mathbf{E} \times \mathbf{B} \) drifts (Zhou and Mathews, 2006, converted to SI units):
\[ \gamma = \frac{\cos D}{BH_n} \left[ -E_{0y} + \frac{k_y}{k^2} \mathbf{k} \cdot (\mathbf{E}_0 + \mathbf{U}_\perp \times \mathbf{B}) \right] + \frac{\sin D}{H_n} U_z \text{ [e-folds/s].} \quad (3.10) \]

If \( \mathbf{U} = 0 \), Equation 3.10 can be reduced to the formulation found in Equation 3.7.

The theoretical growth rate of MSTIDs is analyzed with the aid of Equation 3.10. Table 3.2 lists example values for Equation 3.10 that will be used for this investigation. The values in the table are primarily from climatological models (i.e., HWM93 and IGRF) and give a representative value of a quantity in the average sense. Also, a dip angle of \( 45^\circ \) is selected, corresponding to a mid-latitude region.

Figure 3.3 plots \( \gamma \) as a function of wavevector, \( \mathbf{k} = k_y \hat{y} + k_x \hat{x} \), and displays only positive growth rates. The wavevectors with the largest growth rates correspond to a \( \mathbf{k} \) in the first quadrant (given that the growth rate is real-valued, the spectrum is mirrored against the \( k_y = -k_x \) line). These modes result in NW-SE bands for a Northern Hemisphere configuration. The theory for MSTIDs is able to explain the alignment of the bands at an angle to the magnetic meridian, as commonly observed from experimental studies.

It can be shown that \( \gamma \) maximizes for a \( \mathbf{k} \) that lies halfway between \( \mathbf{E}' = \mathbf{E}_0 + \mathbf{U}_\perp \times \mathbf{B} \) and magnetic east (i.e., at \( \theta/2 \)) (Garcia et al., 2000). These bounds are plotted in Figure 3.3 as black dashed lines, and the growth maximizes at \( \alpha = \theta/2 \) (blue solid line). The theoretical description in Equation 3.10 does not have scale size dependence, but including second-order nonlinear terms in the analysis has been shown to increase the growth for

<table>
<thead>
<tr>
<th>Table 3.2: Representative values used for the analysis of the MSTID growth rate equation (Equation 3.10).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>( H_n )</td>
</tr>
<tr>
<td>( U_x )</td>
</tr>
<tr>
<td>( U_y )</td>
</tr>
<tr>
<td>( U_z )</td>
</tr>
<tr>
<td>( E_x )</td>
</tr>
<tr>
<td>( E_y )</td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( D )</td>
</tr>
</tbody>
</table>
Figure 3.3: The theoretical growth rate, Equation 3.10, as a function of $k_y$ and $k_x$. Only positive growth rates are shown. Also, the black dashed lines represent angles at $\mathbf{E}_0 + \mathbf{U}_\perp \times \mathbf{B} (\theta)$ and magnetic east, and the blue solid line is at $\theta/2$. 
Figure 3.4: The theoretical growth rate, Equation 3.10, as a function of $|U_\perp|$. In addition to the values listed in Table 3.2, a wavevector of $k = 2\pi/57.95 \hat{y} [\text{km}^{-1}] + 2\pi/98.92 \hat{x} [\text{km}^{-1}]$ is used. The angle of $U_\perp = U_y \hat{y} + U_x \hat{x}$ from Table 3.2 is constant in the growth rate calculations.

larger wavelengths due to spatial harmonics (Huang et al., 1994).

Notice that the theoretical growth rate in Figure 3.3 is on the order of $10^{-4}$ [e-folds/s], which corresponds to an e-fold approximately every 2.75 hours. The theoretical calculations show that it takes several hours for one e-fold of growth, which is in contrast to experimental observations recording the instability developing in a relatively short time span. However, changing the parameters in Table 3.2 can produce a larger theoretical growth rate. For example, Figure 3.4 plots $\gamma$ as a function of $|U_\perp|$, and shows that $\gamma \propto |U_\perp|$. A neutral wind magnitude of 200 m/s, which is considered to be very large, can result in a much quicker growth rate on the order of $10^{-3}$ [e-folds/s], or an e-fold about every 15 minutes.

As previously mentioned, it has been hypothesized that MSTIDs may be restricted to develop and propagate in the low latitude regions (e.g., Shiokawa et al., 2002). This may be explained theoretically by recasting the growth rate equation in terms of $\sin^2 \theta$. First, the growth rate is written in terms of angles between $E_\theta + U_\perp \times B$ and magnetic east ($\theta$) as well as the angle
between k and magnetic east (α). Also, Σ and N can be lumped together in terms of the plasma density-weighted average value of the ion-neutral collision frequency: \( \langle \nu_{in} \rangle = \Sigma \Omega B/eN \). Neglecting the second term of Equation 3.10 and by writing the equilibrium condition of a height layer as \( |E_0 + U_\perp \times B| \cos \theta = Neg \sin^2 D/\Sigma \Omega \cos D \), Equation 3.10 can be recast as (Perkins, 1973; Makela and Otsuka, 2011):

\[
\gamma = \frac{g \sin^2 D \sin(\theta - \alpha) \sin \alpha}{\langle \nu_{in} \rangle H_n \cos \theta} \text{ [e-folds/s]}. \tag{3.11}
\]

In this form, \( \gamma \) is proportional to \( \sin^2 D \) as well as \( \langle \nu_{in} \rangle^{-1} \).

### 3.5 Physical Explanation

Alternatively, the instability can be derived from physical arguments. Polarization electric fields develop to maintain current density continuity (i.e., \( \nabla \cdot J = 0 \)), and the associated \( E \times B \) forces create the rising and falling bands of electron density with respect to altitude, which can be unstable.

Figure 3.5 depicts this scenario. In this example, an initial perturbation with the wave vector k has perturbed the mid-latitude ionosphere in the Northern Hemisphere, lowering and raising Regions 1 and 2 in altitude, respectively. Region 0 is labeled as the unperturbed area and includes background values of the integrated Pedersen conductivity (\( \Sigma_0 \)). In this configuration, there exists an effective electric field, \( E' \), in the upper-right plane. This electric field could be comprised of a background electric field, \( E_0 \), and a \( U \times B \) term. Here, we assume that the neutral wind, \( U \), lies in the lower-right plane and \( U \times B \) is dominant over the background electric field. This is a reasonable assumption for the nighttime ionosphere in the Northern Hemisphere (Makela and Otsuka, 2011).

Given that \( E' \) traverses discontinuities of integrated Pedersen conductivity in the raised and lowered bands, current continuity must be maintained in the direction normal to the slabs: \( J_{0n} = J_{1n} = J_{2n} \). Additionally, the boundary conditions require the tangential electric fields across the interfaces must be the same, \( \hat{n} \times (E_a - E_b) = 0 \). Given the requirement that \( E_{0t} = E_{1t} = E_{2t} \) and noting \( \Sigma_1 > \Sigma_0 > \Sigma_2 \), then \( J_{1t} > J_{0t} > J_{2t} \), as depicted in Figure 3.5.

In Region 2, the eastern electric field is larger compared to the reference
Region 0. This region will be raised higher in altitude via an $\mathbf{E} \times \mathbf{B}$ drift and $\Sigma_2$ will decrease more. A positive feedback loop is created and this cycle will continue, making Region 2 unstable. Likewise, Region 1 now has a smaller eastern electric field and, relative to Region 0, the band decreases in altitude ($\Sigma_1$ increases more), and the cycle repeats.

Figure 3.6 details this process, from both current density (top plots) and electric field (bottom plots) descriptions with the boundary conditions enforced. For example, the current densities normal to each slab are equivalent ($J_{0n} = J_{1n} = J_{2n}$). Using the relation $\mathbf{J} = \Sigma \mathbf{E}$ and the requirement that tangential electric fields must be equal, then $J_{1t} > J_{0t} > J_{2t}$. From the current density descriptions, the electric field values are calculated in the bottom plots of Figure 3.6 and display the $x$ and $y$ components. Here, we see the eastward electric field in the raised band is larger compared to the background value, and through $\mathbf{E} \times \mathbf{B}$ drifts, will raise the layer in altitude in an unstable manner. Conversely, the electric field in the lowered region is smaller with respect to the background electric field. As a result, the layer will lower further. The unstable feedback process will continue in each band.

This explanation is advantageous to highlight that the instability is created and sustained as a result of maintaining divergence free current densities.
Figure 3.6: Depiction of the normal ($n$) and tangential ($t$) current densities with respect to the perturbation wavefront (top row), as well as the magnetic east ($y$) and perpendicular and north ($x$) electric fields (bottom row). A background region is displayed (first column), in addition to a perturbed lowered region (second column) and raised region (third column), each with respect to altitude.
Therefore, for simulating MSTIDs, it is critical that the numerical framework models $\nabla \cdot \mathbf{J} = 0$. Without this description, the polarization electric fields are not created or sustained, and thus the instabilities do not form. It is important to keep this in mind when analyzing the numerical model for simulating MSTIDs in Chapter 5.

3.6 Conclusion

In this chapter, an overview of MSTID theory and development has been presented. The original derivation by Perkins (1973) started with the governing physical equations, and under a few simplifying assumptions of the region, described the stability of the nighttime, mid-latitude ionosphere. Then, using a linear stability analysis, a perturbation was applied to the equilibrium condition. As a result, the unstable modes can be calculated using the growth rate relation.

This chapter offered several different viewpoints toward the theory of the instability, including an alternative derivation sketch that used $\mathbf{E} \times \mathbf{B}$ forces to describe the development of MSTIDs. The theoretical growth rate was analyzed as a function of $k$ and showed that the largest modes corresponded to wavefronts at an angle to the magnetic meridian, which is commonly observed in experimental studies. However, the theoretical growth rate is small and on the order of $10^{-4}$ [e-folds/s], which does not coincide with observations.

In addition, the instability can be heuristically derived by enforcing divergence free current densities and boundary conditions on the electric fields as the F-layer is initially perturbed in altitude. As a result, an unstable feedback mechanism will occur, creating further vertical displacements of electron density. This method stressed the importance of maintaining $\nabla \cdot \mathbf{J} = 0$ in the ionosphere for the development of the instabilities.

In the next chapter, a climatological study of MSTID occurrences is presented with the aid of 630.0-nm CCD imagers from two longitudinal sectors. The long-term study will provide further insight into the characteristics of MSTIDs, and this knowledge will be leveraged to simulate the instability within a numerical model.
CHAPTER 4

CLIMATOLOGICAL STUDY OF MSTIDS
FROM OBSERVATIONS

In this chapter\textsuperscript{1}, an observational study of MSTID occurrences is conducted. First, we introduce the study and the instrumentation used to observe occurrences of MSTIDs in both the Central Pacific and South American sectors. The methodology to develop occurrence rate data is discussed, and the results from the statistical analysis are presented. Various aspects of MSTID occurrences are described, including seasonal and solar cycle dependence, a longitudinal comparison, the neutral wind impact on the growth rate, and propagation into low geomagnetic latitudes. In addition, the data are compared with previous studies found in the literature and the general trends of occurrences are found to be consistent. Knowledge of MSTID properties can be leveraged to study instabilities within a numerical simulation framework.

4.1 Introduction

In addition to theory, properties of MSTIDs can be understood through the results of climatological studies. Long-term datasets are used to observe seasonal and solar cycle trends, as well as measuring the wavelength and propagation velocity of MSTIDs. For example, previous climatological studies have shown MSTIDs with wavelengths on the order of 50–500 km typically traveling westward and equatorward at velocities of approximately 100 m/s (Garcia et al., 2000). MSTIDs developed in numerical simulations (i.e., Chapter 6) should match the descriptions provided by climatological studies.

As discussed in Section 2.1, a variety of methodologies have been used to conduct climatological studies, including satellite \textit{in situ} measurements (e.g., Saito et al., 1995), TEC data collected by GPS receiver networks (e.g., Otsuka

\begin{footnotesize}
\textsuperscript{1}This chapter is based on the work published in Duly et al. (2013).
\end{footnotesize}
et al., 2011), and imaging of the 630.0-nm airglow emission as described in Section 1.6 (e.g., Garcia et al., 2000; Shiokawa et al., 2003a; Candido et al., 2008; Martinis et al., 2010; Fukushima et al., 2012). The airglow imaging technique provides a few advantages compared to the other two methods. The 630.0-nm imagers collect 2D spatial information that spans hundreds of km in each horizontal direction, and can also have a time cadence of about a few minutes during the nighttime operation of the imagers. Although satellite in situ measurements provide a direct measurement of a parameter, they do not provide 2D spatial information. TEC measurements require several GPS receivers to obtain substantial information in the horizontal plane, as the measurement by each receiver is analogous to one pixel recorded in an airglow imager. Airglow imaging does have its drawbacks compared to the other two methodologies, mainly that the instrumentation must be operated at night under clear sky conditions. Given that we are interested in MSTIDs developing after sunset, a long-term study should include a large number of clear nights for the analysis of instability properties.

We collect MSTID occurrence rate data with the aid of 630.0-nm filtered images from three instruments located in the Central Pacific and South American sectors. These data are gathered from September 2006 through December 2012. The analysis of the results from the long-term study of MSTIDs fill the data gap between the previously studied Japanese and Brazilian sectors (Shiokawa et al., 2003a; Martinis et al., 2010). Two of the instruments used for this study have fields of view covering low geomagnetic latitudes. Therefore, we are able to measure the low-latitude extent of MSTIDs as they propagate equatorward. Establishing the climatological database near low-latitudes will aid in determining the effectiveness of MSTIDs as a seeding mechanism for ESF.

4.2 Instrumentation

For the climatological study, we collect data from the Cornell All-Sky Imager (CASI) and the Cornell Narrow-Field Imager (CNFI) which are co-located at Haleakala, Hawaii (20.71°N, 203.74°E). Data are also recorded by the Portable Ionospheric Camera and Small-Scale Observatory (PICASSO) imager located at the Cerro Tololo Inter-American Observatory (CTIO) in Chile.
(−30.1°N, 289.19°E). Each camera system operates at night and includes controlling software that cycles the instrument through various filters, including one isolating the 630.0-nm emission. Other filters are used to observe additional optical emissions, but are not considered in this study. Also, the imaging systems periodically record dark images of the CCD to characterize and reduce the dark noise of the instrument.

The CASI and CNFI instruments, each with different viewing geometries, are located atop the Haleakala Volcano in Maui, Hawaii. Both instruments employ a 1024 × 1024 CCD array that is binned by a factor of 2 for an effective 512 × 512 resolution. In addition, each CCD is cooled to −40°C to reduce thermal noise, and the CCD records 90-s exposure times. Previously, CASI and CNFI have been used to study equatorial plasma bubbles (e.g., Kelley et al., 2002a; Makela et al., 2004). For the current study, we select data from nights beginning in September, 2006, to coincide with the observations at CTIO.

CASI has a 180° field of view, centered on zenith. On the other hand, CNFI has a 47° field of view and looks south with an azimuth angle of 188.0° at an elevation angle of 17.9°. As originally proposed by Tinsley (1982), this viewing geometry looks along the magnetic field lines in the airglow layer, enabling an optimal viewing configuration for the detection of ESF (see Figure 1 of Kelley et al. (2002b)). Although field-aligned irregularities such as ESF are not of primary interest in this study, the viewing configuration of CNFI enables MSTID detection for regions equatorward of Hawaii at low dip angles. This viewing configuration gives higher spatial resolutions for lower elevation angles than the CASI system, allowing for clearer detection of MSTIDs at lower latitudes.

Turning to the South American sector, PICASSO is located near La Serena, Chile. PICASSO has a 2184 × 1472 CCD array, binned by a factor of 3 to improve signal to noise ratio. The field of view of PICASSO is 80° × 60° with an azimuth of about 356° at an elevation angle of about 9°, covering approximately 18° of geomagnetic latitude at an assumed emission altitude of 250 km. The imaging system also records 90-s exposures of the 630.0-nm emission and is thermally cooled to −30°C. Analogous to CNFI at Haleakala, PICASSO looks equatorward at low geomagnetic dip angles. PICASSO began recording measurements of the nighttime ionosphere in September 2006 and has been previously used to observe MSTIDs at low geomagnetic lati-
Figure 4.1: The viewing geometry of CASI, CNFI, and PICASSO. Also overlaid are magnetic dip angles at 250 km computed from the IGRF model. Reprinted from Duly et al. (2013).

4.3 Methodology and Data Presentation

To formulate the climatology, we observe the image sequence recorded from each imager and hand-classify each night’s images with labels of “usable,”
Figure 4.2: An example of a MSTID propagating southwest over Haleakala, Hawaii and recorded by CASI and CNFI. The images are spaced approximately 20 minutes apart. Also overlaid are magnetic dip angles at 250 km. Reprinted from Duly et al. (2013).

“MSTID,” and “cloudy/bad data.” The subset of “MSTID” labels were included in the set of “usable” labels, as usable nights consisted of clear skies with the possibility of observing ionospheric structure. That is, a night with label “MSTID” was also labeled “usable.” “Cloudy / bad data” corresponded to cloudy sky conditions, light contamination in the images (i.e., when the moon was up), and/or occasional instrument malfunction. MSTIDs were verified against false positives from sequences of the 557.7-nm and 777.4-nm filtered images. The instability is not typically observed in these bands.

An example image sequence of MSTIDs observed over Hawaii is displayed in Figure 4.2. The images are 20 minutes apart and show the characteristic light and dark bands in the NW-SE orientation. The MSTID is propagating southward and equatorward in the fields of view of both CASI and CNFI.

Then, the data labels are binned across various metrics to understand the climatological trends of MSTID occurrences. First, data are binned against the December solstice months, June solstice months, and equinox months. We define the December solstice months as November, December, January, and February; June solstice months as May, June, July, and August; and finally equinox months as March, April, September, October. This division provides a reasonable representation of each season. Also, the data are binned by month (for all years) to study long-term seasonal occurrence trends. In addition, the data labels are binned as a function of the associated 81-day averaged solar flux value (F10.7A) (see Section 1.5) to study solar cycle
dependence. Figure 4.3 shows the F10.7 and F10.7A values for the duration of the study, with solar conditions ranging from solar minimum (F10.7A ≈ 75 SFU) to moderate (F10.7A ≈ 145 SFU). Overall, data collected from September 2006 through December 2012 resulted in 820 and 1331 usable nights for CASI and CNFI, respectively, while data from PICASSO totaled 939 usable nights.

Figures 4.4, 4.5, and 4.6 provide the summary statistics for CASI, CNFI, and PICASSO, respectively. In each plot, the left-hand ordinate corresponds to the raw counts for the usable nights (wide red bars) and MSTIDs (thin cyan bars). The right-hand ordinate displays the corresponding occurrence rate (blue dashed line), which is the ratio of MSTID counts to usable night counts. For each figure, the three left-hand plots display the data binned by the seasonal time period, while the top-right plot bins the data by each month for all years. Finally, the lower-right plot bins the data against the associated F10.7A value. In the next section, we provide a detailed discussion of the results from Figures 4.4-4.6, including comparisons of MSTID observations found in the literature.
Figure 4.4: Summary statistics for the CASI instrument. Reprinted from Daly et al. (2013).
Figure 4.5: Summary statistics for the CNFI instrument. Reprinted from Daly et al. (2013).
Figure 4.6: Summary statistics for the PICASSO instrument. Reprinted from *Duly et al.* (2013).
4.4 Discussion

Table 4.1 provides the summary of the occurrence rate data, including the total number of usable nights and MSTID counts for each instrument throughout the period of study. Overall, CASI recorded the highest number of MSTID occurrences, with almost half of the observable nights indicating a MSTID in the sequence of images from the camera system. CASI and PICASSO recorded fewer MSTIDs during the study, with an occurrence rate of 7.0% and 4.7%, respectively.

There could be several explanations for the relatively low occurrence rates recorded from CNFI and PICASSO. As mentioned, CNFI and PICASSO look equatorward with a low elevation angle. The low elevation look directions of CNFI and PICASSO could integrate through multiple MSTID wavefronts, making detection from the imaging data difficult. Also, given that there is a $\sin^2 D$ dependence for the growth rate of MSTIDs (Equation 3.11) (Perkins, 1973; Hamza, 1999), the growth rate could be smaller for structures developing in the low geomagnetic latitude as viewed by CNFI and PICASSO.

Finally, a MSTID could be limited to propagate as it travels toward the equator. For example, MSTIDs viewed by CASI may not travel far enough south to appear in the field of view of CNFI. For the structures that are able to propagate to be observed by CNFI, the MSTIDs are often observed from CASI’s field of view about 30 - 90 minutes prior. This makes MSTIDs observed by CNFI an approximate subset of MSTIDs observed by CASI. The growth rate, propagation, and/or geometrical observational effects may inhibit observations and/or development of MSTIDs at low geomagnetic latitudes.

In the next sections, we discuss various occurrence rate properties of MSTIDs that are revealed from this study. Seasonal trends will be established with

<table>
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<th></th>
<th>CASI</th>
<th>CNFI</th>
<th>PICASSO</th>
</tr>
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<tbody>
<tr>
<td>Total Usable Nights</td>
<td>820</td>
<td>1331</td>
<td>939</td>
</tr>
<tr>
<td>Total MSTID Counts</td>
<td>401</td>
<td>93</td>
<td>44</td>
</tr>
<tr>
<td>Occurrence Rate</td>
<td>48.9%</td>
<td>7.0%</td>
<td>4.7%</td>
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the data binned by the two solstices and equinox time frames. Next, the high solstice occurrence rates are attributed to the neutral wind impact on the linear MSTID growth rate. The solar cycle dependence on the occurrence rate is investigated, as well as a comparison of the occurrence rate between longitudinal sectors. We discuss the low-latitude extent of MSTIDs observed by CNFI and PICASSO. Finally, the established climatological properties are compared to studies found in the literature.

4.4.1 Solstice and Equinox Trends

The left-hand plots of Figures 4.4–4.6 can be used to establish seasonal trends of MSTID occurrence rates from this study. During the December solstice, CASI recorded relatively large occurrence rates ranging between 35% and 72% during the study. Comparatively, CNFI and PICASSO recorded lower rates, which could possibly be explained by the factors discussed in the previous section. For the December solstice, CNFI recorded a maximum 30.7% occurrence rate (23 MSTID observations) during 2008, and PICASSO recorded a maximum occurrence rate of about 12% (5-7 MSTID observations) in both 2008 and 2009.

CASI recorded the highest occurrence rates during the June solstice, with rates ranging between 40% and 78%. The maximum rate recorded by CNFI in the June solstice was in 2010, with a 10.1% occurrence rate (11 MSTID observations). The low number of available nights for PICASSO may have biased the occurrence rate in 2009, but there were no more than 3 MSTID observations for a given year during the June solstice timeframes.

Similar trends occurred in the equinox time period as well, with CASI observing the largest number of occurrence rates compared to the other two instruments. CASI recorded an occurrence rate as large as 41.5% (22 MSTID counts) during the equinox time period. On the other hand, CNFI and PICASSO recorded relatively smaller rates, and CNFI recorded a maximum of 3 MSTID counts during this time period, with a maximum occurrence rate of 4.5% in 2009. The maximum occurrence rate for the equinox time period for PICASSO was in 2009, corresponding to a 12.5% occurrence rate. However, similar to the June solstice, the low number of usable data from that time period may bias the occurrence rate for the equinox time frame.
PICASSO had its largest number of MSTID observations in 2008 with 5 counts during the June solstice.

The solstice and equinox trends of the MSTID occurrence rates are summarized in the top-right plots of Figures 4.4–4.6. The data are plotted for all years as a function of month, offering another view of MSTID occurrences during the period of study. Although the occurrence rates vary in magnitude between each instrument, the data show that there are occurrence rate peaks recorded by each imager near the June and December solstices. It is difficult to determine the major and minor maximum of the occurrence peaks as statistical biases may influence the results. For example, there appears to be an occurrence rate maximum near the December solstice for the results obtained from CNFI, but there were also fewer observations during this time frame. However, each instrument recorded a relatively large number of MSTID occurrences during the solstices.

4.4.2 Solstice Occurrence Rates: Neutral Wind Impact

To investigate the occurrence rate seasonality, we study the contribution of the neutral wind to the maximum growth rate for MSTIDs. From the growth rate equation (Equation 3.10), the neutral wind vector, $\mathbf{U}$, plays a large role for the development of MSTIDs. Given that $\mathbf{U}$ is highly dependent on season (both in amplitude and direction), it is a viable candidate to influence the seasonal occurrence of MSTIDs. Following Garcia et al. (2000), the neutral wind contribution to the maximum growth rate in Equation 3.10 can be expressed in the form:

$$\gamma_{\text{max}} \propto u \sin D \sin \delta + u \cos \delta - v \sin \delta - v \sin D \cos \delta,$$

where $D$ is the magnetic dip angle, $\delta$ is the magnetic declination angle (which describes the magnetic field geometry with respect to the true north direction), and $u$ and $v$ are the zonal (positive eastward) and meridional (positive northward) neutral wind, respectively. Equation 4.1 is plotted for Haleakala (top) and the magnetic conjugate point of Haleakala (bottom) as a function of local time and day of year in Figure 4.7. The magnetic quantities given in Equation 4.1 are obtained from IGRF, while the neutral wind values are from the climatological Horizontal Wind Model (HWM93) (Hedin et al., 1996). It
is important to note that these values are climatological values and represent the general seasonalities of the neutral wind. However, they are sufficient to determine the local time and seasonal trends associated with Equation 4.1.

At Haleakala, results shown in Figure 4.7 indicate that the neutral wind provides a large contribution to the generation of MSTIDs near the December solstice. However, at the conjugate point of Haleakala, the neutral wind contribution to the growth rate is largest near the June solstice. Therefore, it is possible that the MSTIDs observed during the December solstice at Haleakala are due to the conducive neutral winds there, while MSTIDs observed near the June solstice are due to MSTIDs being generated in the southern hemisphere and efficiently mapping to be observed at Haleakala.
(i.e., Section 1.4). As noted in Section 2.1, the conjugacy of MSTIDs has been observed before with *in situ* MSTID polarization electric field measurements (*Saito et al.*, 1995), and also with 630.0-nm airglow imaging systems (*Otsuka*, 2004). The conjugacy effect will be further explored through the numerical simulation of MSTIDs in Chapter 7.

4.4.3 Solar Cycle Dependence

Figures 4.4–4.6 also provide information on the solar cycle dependence of MSTIDs. The data show that MSTID occurrences coincide with low F10.7A values, representative of solar minimum. For example, from a possible 182 usable nights with a F10.7A value less than 70 SFU, CASI recorded MSTIDs on 105 nights, as shown in the summary statistics in Figure 4.4. In addition, the majority of MSTID observations recorded by CNFI and PICASSO coincided with a F10.7A value smaller than 90 SFU.

One possible explanation could be the effect of the ion-neutral collision frequency on the theoretical growth rate. This term affects the initial steady-state height of the F-layer, which in turn influences how fast the instability develops. Equation 3.11 shows that the Perkins growth rate is inversely proportional to $\langle \nu_{in} \rangle$, the plasma density-weighted average value of the ion-neutral collision frequency (*Perkins*, 1973; *Kelley and Fukao*, 1991), defined as:

$$
\langle \nu_{in} \rangle = \frac{\int n_e(z)\nu_{in}(z) \, dz}{\int n_e(z) \, dz}.
$$

(4.2)

Here, the integration is along the magnetic field line direction, $z$.

During solar minimum (low F10.7A values), plasma density is relatively low in value and $\langle \nu_{in} \rangle$ is decreased (i.e., there are less ion-neutral collisions in the F-region). Therefore, the theoretical growth rate for MSTIDs is larger. For example, using climatological values and Equation 1.20, on December 21st at 23:00 LT at Arecibo (18.37°N, −66.62°E), $\langle \nu_{in} \rangle = 0.27$ [collisions/s] for the year 2001 (solar max). However, for the year 2008 (solar min), $\langle \nu_{in} \rangle = 0.08$ [collisions/s], which is over three times smaller compared to the solar maximum value. The dataset from the current study agrees with this description of the prevalence of MSTIDs during low F10.7A values.
4.4.4 Longitudinal Comparison

The coincident observational period between the Central Pacific and South American sectors enables a study of the longitudinal dependence of MSTID occurrence rates. For this investigation, occurrence rate data are compared between CNFI and PICASSO, as both of these instruments have a field of view toward the equator and at low geomagnetic dip angles. As a result, any major effects on the occurrence rates which are caused by geometrical and/or dip angle dependence are reduced.

Comparing Figures 4.5 and 4.6, the general trends of MSTID occurrences seem to be consistent between the data collected by CNFI and PICASSO. That is, high occurrence rates are prevalent in the solstices and during low solar flux conditions. However, as summarized in Table 4.1, PICASSO recorded lower occurrence rates compared with CNFI. For example, from the monthly binned data (top right of Figures 4.5 and 4.6), CNFI recorded a 22.4% and 11.3% occurrence rate for January and July, respectively, while PICASSO recorded lower rates of 10.5% and 8.9%, respectively. Slight discrepancies between each instrument’s viewing geometry may be responsible for the differences in occurrence rates. Although both instruments look equatorward, as shown in Figure 4.1 the field of view for PICASSO covers lower absolute geomagnetic dip angles (approximately $-20^\circ$ to $0^\circ$) compared to CNFI (approximately $15^\circ$ to $30^\circ$). The occurrences of MSTIDs may be subject to limitations at lower dip angles as previously discussed. That is, the growth rate, propagation of the instabilities, and geometrical observational effects may impact the number of occurrences recorded by PICASSO, and could explain the discrepancies between the rates compared with the measurements at CNFI.

4.4.5 Low-Latitude Extent

Although relatively few compared to CASI, the MSTIDs observed from CNFI and PICASSO are significant because they occur at low geomagnetic dip angles. Figures 4.8 and 4.9 display the latitudinal extent of MSTIDs observed by CNFI and PICASSO, respectively. For each instrument, a MSTID was tracked as it traveled equatorward, and the location of the last observable sighting of the structure was recorded. These data are denoted with a white
As shown from the Figures 4.8–4.9, MSTIDs observed from this study can propagate near the geomagnetic equator, to dip angles as low as \( \sim 14^\circ \). The average dip angle extent recorded by CNFI was 24.75°, compared to 18.33° recorded by PICASSO. It should be noted that PICASSO recorded about half as many observations compared to CNFI, and also each instrument had different viewing limitations, which could perhaps bias the statistics. Also, the distribution from CNFI appears to be slightly skewed to the right, toward higher dip angle values.

Previous studies have suggested an equatorward propagation limit of MSTIDs to be around 18° geomagnetic latitude \((Shiokawa et al., 2002)\), which corresponds to a dip angle of approximately 39° at 250 km altitude. \(Shiokawa et al. (2002)\) attributed the limitation to the increase of electron density at lower latitudes (i.e., near the equatorial anomaly region). The increase of electron density also increases ion drag, which could inhibit the propagation of gravity waves to potentially seed the development of MSTIDs in the F-region. However, the results from the current study show that all of the dip angles recorded by CNFI and PICASSO were equatorward of this hypothesized limitation. The data from the climatology study suggest that although MSTIDs
may not begin their initial development at low geomagnetic latitudes, they are able to propagate there. In Chapter 7, we will further explore the idea of the low-latitude extent of MSTIDs through the use of numerical modeling.

4.4.6 Comparison to Previous Studies

Here we present a detailed comparison of the occurrence rate data from the current study to studies found in the literature. Previous studies have also found large MSTID occurrence rates during the solstices. For example, Shiokawa et al. (2003a) used airglow imagers to measure MSTIDs properties in the Japanese sector from October 1998 through September 2000. In their study, Shiokawa et al. (2003a) measured a major maximum in the peak occurrence rates of about 55% in the June solstice at Rikubetsu, Japan (43.5°N, 143.8°E, dip= 57.4°). Compared to the data obtained from CASI, 64%-72% for June and July, this rate is slightly lower (compare Figure 4.4 with Figure 4 of Shiokawa et al. (2003a)). Also, in Shigaraki, Japan, (34.8°N, 136.1°E, dip= 48.5°), which is south of Rikubetsu, Shiokawa et al. (2003a) found MSTID occurrences rates of about 45% near the June solstice, which is lower compared to the rates measured in the current study.

Near the December solstice, 18% and 12% occurrence rates were recorded for Rikubetsu and Shigaraki, respectively, compared to a 56% occurrence
rate recorded by CASI for the same season. The different solar conditions may be responsible for the discrepancies in occurrence rates between the two studies. For the Shiokawa et al. (2003a) study, F10.7A values ranged from \( \sim 125 \text{ SFU} \) to \( \sim 200 \text{ SFU} \) (March 1999 and June 2000, respectively), representative of a time period closer to solar maximum, whereas the current study solar conditions were more moderate, ranging from \( \sim 70 \text{ SFU} \) to \( \sim 145 \text{ SFU} \) (November 2008 and October 2011, respectively). However, the general trends of maximum occurrences during the solstices remain consistent between the two studies.

As discussed previously, the PICASSO imager recorded the fewest number of MSTID occurrences, with a 4.7% occurrence rate for the period of study. Martinis et al. (2006) performed a climatological study of MSTIDs in El Leoncito, Argentina \((-31.8^\circ \text{N}, 290.7^\circ \text{E}, \text{dip} = -31.5^\circ)\), which is about 230 km southeast of Cerro Tololo, Chile, using an all-sky imager (analogous to CASI’s field of view) that observed low geomagnetic dip angles. The study by Martinis et al. (2006) was performed primarily during solar maximum between 2000 and 2005, with F10.7A values reaching \( \sim 220 \text{ SFU} \) in October 2011. In total, Martinis et al. (2006) recorded a 5% occurrence rate at El Leoncito, which is comparable to the results obtained from PICASSO.

However, due to the solar cycle dependence on MSTID growth, a lower occurrence rate might be expected for the Martinis et al. (2006) study due to the solar conditions. The low-latitude, equatorward viewing geometry of PICASSO, compared with the all-sky imager used in Martinis et al. (2006), could explain the similar occurrence rates derived from the two studies under different solar conditions. Recall that at Haleakala, MSTIDs observed by CASI were not always subsequently observed by CNFI, in part due to the field of view for CNFI covering low geomagnetic dip angles. Similarly, in South America, MSTIDs south of PICASSO that are traveling toward its field of view may be inhibited to be observed as a result of the growth rate, propagation, and/or geometrical observation effects. Therefore, under similar solar conditions, PICASSO could record lower occurrence rates of MSTIDs compared with an all-sky imager in the same region. As a result, the varying solar conditions between the two studies could offset the effect of different viewing geometries in the respective instruments.
4.5 Conclusion

In this chapter, we have presented a climatological study of MSTIDs from the Central Pacific and South American sectors from September 2006 through December 2012. Airglow imaging camera systems observing the 630.0-nm emission were used to develop a climatological database of MSTID occurrences. From the data, seasonal and monthly trends were established, with a peak of occurrence rates occurring during the solstices. The data were also binned against solar flux value, relating occurrence rates to solar conditions. This study measured large MSTID occurrences primarily during low solar conditions. The neutral wind configuration in the F-layer was found to be an influential factor in the MSTID growth rate, and could explain the seasonality of their occurrences.

These data supplement the existing climatological databases, specifically for years within the recent deep solar minimum and for two geographical sectors where long-term observations had not been previously conducted. In addition, we will use the results obtained from this study to drive simulation studies of MSTIDs. For example, the neutral wind influence on MSTID growth can be numerically simulated, as well as investigating the possible MSTID propagation limitations. Before presenting the numerical simulation aspect of this work, it is important to discuss the model that will be used for the numerical studies. In the next chapter, the SAMI3 (Sami3 Is Another Model of the Ionosphere) is analyzed.
CHAPTER 5
NUMERICAL MODELING OF EARTH’S IONOSPHERE WITH SAMI3

In this chapter, numerical modeling of Earth’s ionosphere is introduced as a tool for studying upper atmosphere dynamics. First, a broad overview of ionospheric modeling is provided, and SAMI3 (Sami3 Is Another Model of the Ionosphere) is discussed. SAMI3 uses a dipole coordinate system, which is advantageous to break down the governing equations that describe the physics parallel and perpendicular to the magnetic field line. Next, we present the equations modeled in SAMI3, including the self-consistent potential solver. The equations are based on the fundamental, governing equations discussed in Section 1.2.1. The code is parallelized with the message passing interfacing (MPI) standardization. The model divides the multiple processors into Master and Worker tasks, and a typical time step in SAMI3 is sketched out to portray the interactions between the tasks as the simulation is executed. Finally, we highlight the numerical schemes in SAMI3, including a high-order perpendicular transport scheme. The SAMI3 model will be used in subsequent chapters for numerical investigations of MSTIDs.

5.1 Models of the Ionosphere

Physics-based numerical modeling of the ionosphere can be leveraged as a tool to understand physical processes of the ionosphere. Several groups have developed numerical models to study the ionosphere, with variations between the models dependent on a particular focus of study.

A comprehensive listing of upper atmosphere models can be found in <em>STEP: Handbook of Ionospheric Models</em> (Schunk, 1996). Here, a few examples of physics-based ionospheric models commonly used within the research community are provided<sup>1</sup>. For example, the Global Ionosphere Thermosphere

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<sup>1</sup> These models can be requested to run at the Community Coordinated Modeling Center
Model (GITM) (Ridley et al., 2006) portrays ionospheric dynamics near the auroral zone. The Thermospheric General Circulation Models (TGCMs), developed at the National Center for Atmospheric Research (NCAR), simulate the upper atmosphere on a global scale. Specifically, the Thermosphere Ionosphere General Circulation Model (TIEGCM) (e.g., Richmond et al., 1992) is a first-principle, global model that computes the electrodynamics of the ionosphere self-consistently (in the low- and mid-latitude regions). The GCM models are not limited to ionospheric electrodynamics. For example, the Thermosphere Ionosphere Mesosphere Electrodynamic General Circulation Model (TIME-GCM) includes physical processes from the mesosphere and upper stratosphere (Roble and Ridley, 1994).

However, these models suffer from deficiencies with respect to numerical simulations of MSTIDs. The TGCM models, for example, have an upper boundary of about 600 km and are unable to capture inter-hemispheric connections, which are important to study the conjugate nature of MSTIDs. Also, the finest grid resolution for the TGCM models is $2.5^\circ \times 2.5^\circ$ (about $300 \times 300$ km), which is too large to capture wave dynamics on the order of 100 km (i.e., the wavelength of MSTIDs).

In this work, SAMI3 (Sami3 is Another Model of the Ionosphere) is selected to investigate the development of MSTIDs within a numerical framework. The next sections will provide an overview of SAMI3, outline the equations modeled, and describe the dipole coordinate system within the model. Also, the numerical schemes that are implemented in SAMI3 will be discussed.

### 5.2 The SAMI3 Numerical Model

In the current study, we extend the previous work of numerical modeling by simulating MSTIDs with the SAMI3 (Sami3 is Another Model of the Ionosphere) model. As outlined in the literature review of MSTID numerical modeling (Section 2.3), we follow the second phase of modeling by considering the fundamental, physics-based equations in a three-dimensional simulation space.

SAMI3 has its origins in a two-dimensional model named SAMI2 (Huba et al., 2008). SAMI2 was developed at the Naval Research Laboratory (NRL) (CCMC), at http://ccmc.gsfc.nasa.gov/.
Figure 5.1: An example slice of electron density ($n_e$) calculated in SAMI2 near 23:30 LT.

as a tool to study the ionosphere, and was spun off from a mid-latitude ionosphere model developed in the mid-1970s at the NRL (Oran et al., 1974). The SAMI2 model solves the ion and momentum equations for seven ion-species ($\text{H}^+$, $\text{He}^+$, $\text{N}^+$, $\text{O}^+$, $\text{N}_2^+$, $\text{NO}^+$, and $\text{O}_2^+$) in a longitudinal “slice” of the ionosphere for altitudes ranging from approximately 100 km to over 1000 km. As will be explained in Section 5.3, a dipole coordinate system is used for the grid spacing with one direction along the field line. An example slice of the electron density calculated in SAMI2 is displayed in Figure 5.1. This slice depicts the plasma density before local midnight with a peak in the F-region near 300-km altitude.

SAMI3 is similar to SAMI2, except it includes grid spacing in the longitudinal direction and a potential solver, in effect modeling the ionosphere on a global scale. In the original SAMI2 code, the electric field is calculated from a climatological (e.g., Fejer, 1993), analytical, or data-driven model. However, SAMI3 solves for the potential self-consistently within the model. Also, the equations were updated to solve the ion and momentum equations in three dimensions. There is also a version of the code that models a “wedge” of the ionosphere and is commonly termed “SAMI3/ESF” for its primary use in Equatorial Spread F (ESF) studies. For the current study, we will use the wedge model of SAMI3 for the numerical simulations as we are primarily interested in MSTIDs developing within a region spanning a few degrees in longitude. In addition, the wedge model of SAMI3 provides a large
spatial domain extending to mid-latitudes in both hemispheres, enabling an investigation of MSTIDs developing in the conjugate hemisphere.

One of the goals for SAMI3 was to numerically model the development of ESF (Huba et al., 2008). With the potential solver in SAMI3, the electric fields are solved self-consistently to generate the large-scale polarization electric fields required to drive density depletions. Using SAMI3, various numerical simulations have been conducted to study the development of ESF, including the effects of zonal neutral winds (Huba et al., 2009), meridional neutral winds (Krall et al., 2009; Huba and Krall, 2013), and gravity waves (Krall et al., 2013a,b).

Even though the original SAMI3 code was primarily intended to study low-latitude instabilities, the model can be used to study the development of MSTIDs at mid-latitudes. In fact, SAMI3 has several advantages for the current numerical studies of MSTIDs. Similar to the development of ESF, polarization electric fields play an important role in the development of MSTIDs because an $E_p \times B$ drift drives the plasma motion to form the raised and lowered bands. Therefore, the current studies should produce polarization electric fields necessary for the development of MSTIDs, similar to the configuration found in Figure 3.5.

In addition, the electrostatic calculation in the model allows for conjugate effects of MSTIDs to be studied. Conjugate observations of MSTIDs have shown that the structures are mirrored along the geomagnetic equator and develop in conjugate hemispheres (Otsuka, 2004). In this work, the conjugacy effect can be numerically studied with SAMI3’s grid covering both the Northern and Southern hemispheres. Along with the sophisticated equation solvers, the large 3D spatial domain of SAMI3 enables us to extend the previous numerical work to explore the development of MSTIDs based on fundamental, physics-based equations. To our knowledge, these are the first numerical studies that use a full magnetic field grid to investigate the conjugacy of MSTIDs.

5.3 Dipole Coordinate System

SAMI3 uses a dipole coordinate system for its grid. The primary advantage for this type of grid, as compared to a Cartesian coordinate system, is
that the equations can be broken down into calculations both parallel and perpendicular to \( B \). This decoupling is a result of the presence of Earth’s magnetic field line, and simplifies the implementation of the equations within the model. For example, due to the “freezing-in” condition of plasma within the magnetic flux tubes (Dungey, 1958; Hines, 1964; Rishbeth and Garriott, 1969, Section 7.53), the velocity perpendicular to the magnetic field line can be simplified as:

\[
V_{i,\perp} = \frac{E \times B}{B^2}.
\] (5.1)

However, for the velocity perpendicular to the magnetic field line, \( V_{i,\parallel} \), Equation 1.3a must be used, which includes the inertia, pressure, gravity, and collision terms in addition to the electric field force.

In SAMI2/3, the conversion from geographic to dipole \((q, p, \phi)\) coordinates is a three-step process and includes the following conversions (Huba et al., 2000):

Spherical Geographic \(\Rightarrow\) Spherical Titled \(\Rightarrow\) Spherical Eccentric \(\Rightarrow\) Dipole

The first two conversions are described in Huba et al. (2000, Appendix A). The dipole coordinate system can be described in terms of the spherical eccentric system:

\[
q = \frac{R_E^2}{r_e^2} \cos \theta_e.
\] (5.2)

\[
p = \frac{r_e}{R_E \sin^2 \theta_e}.
\] (5.3)

\[
\phi = \phi_e.
\] (5.4)

Here, \( R_E \) is the radius of the Earth (\( \approx 6378.1 \) km), \( \theta_e \) and \( \phi_e \) are the eccentric latitude and longitude, respectively. \( r_e \) is the radial distance from the center point of the spherical eccentric coordinate system and is equivalent to the summation of the altitude and Earth’s radius \((r_e = h_e + R_E)\).

The inverse transformation, going from dipole to spherical eccentric, can be accomplished by first solving for \( r_e \) in the following equation:
\[ q^2 \left( \frac{r_e}{R_E} \right)^4 + \frac{1}{p} \left( \frac{r_e}{R_E} \right) - 1 = 0. \]  

(5.5)

Instead of finding an analytical expression for \( r_e \) in Equation 5.5, the non-linear equation can be solved numerically with Newton’s method (e.g., Section 5.6.2 Heath, 1996).

Once \( r_e \) is solved for, \( \theta_e \) can be found either from Equation 5.2 or 5.3. For example, isolating \( \theta_e \) in Equation 5.2:

\[ \theta_e = \cos^{-1} \left( \frac{r_e^2}{q R_E^2} \right). \]  

(5.6)

Figure 5.2 plots values for the dipole coordinate system, with three example \( p \) values. The three field lines are plotted with a constant \( p \) value and the \( q \) is ranged. From this figure, we see that the \( p \) value represents the peak apex height of a field line in Earth radii (also called the L-shell). It should be noted that the latitudes are with respect to the eccentric latitude, \( \theta_e \), and not the geographic latitude.

Figure 5.3 plots several constant \( p \) values as a function of eccentric latitude and altitude. The right-hand figure displays a zoomed version of the left-hand figure, showing that \( q \) is along the magnetic field line and a constant \( p \) value represents a particular field line. This figure is similar to the IGRF field lines plotted in Figure 3.1. The grid system in SAMI3 can be considered an approximation to the IGRF magnetic field line model. Also, the coordinate system in SAMI3 is analogous to the coordinate system used for the theoretical development of MSTIDs described in Section 3.1, with one coordinate in the direction of the magnetic field line. Finally, we note that the dimensional variable, \( s \), is commonly used in SAMI3 for the coordinate along the magnetic field line, which is simply defined as \( s_i = R_E q_i \).

5.4 Equations Modeled

SAMI3 models the coupled, dynamical evolution of the fundamental, physics-based equations relating to Earth’s upper atmosphere. These equations closely follow the governing equations described in Section 1.2.1, and thus model the realistic physics in the ionosphere. The full explanation for the equations are found in Huba et al. (2000).
Figure 5.2: Three example field lines using the dipole coordinate system. Each field line can be represented by a $p$ value. The eccentric latitude is used for the calculations.

Figure 5.3: Several field lines plotted as a function of eccentric latitude, $\theta_e$, and altitude (left). A zoomed-in version (right) shows that $q$ is along the field line direction, and $p$ represents a magnetic field line.
The first equation modeled in SAMI3 is the ion-continuity equation:

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i V_i) = P_i - L_i n_i.$$  

(5.7)

Here, $P$ and $L$ are production and loss terms, respectively, for each ion constituent, $i$, and each have their own appropriate model involving photoionization, radiative recombination, and chemistry. Equation 5.7 is responsible for advancing the simulation in time. The continuity equation is only required for the ions, as the electron density can be found by assuming charge neutrality. Therefore, the electron density is simply the sum of ion densities: $n_e = \sum_i n_i$.

This momentum equation is used to solve for ion and electron velocities in SAMI3. The ion momentum equation is reproduced here from Equation 1.3a:

$$\frac{\partial V_i}{\partial t} + V_i \cdot \nabla V_i = -\frac{1}{\rho_i} \nabla P_i + \frac{e}{m_i} (E + V_i \times B) + g - \nu_{in}(V_i - U) - \sum_{j \neq i} \nu_{ij}(V_i - V_j).$$  

(5.8)

Similarly, the electron momentum equation follows from Equation 1.3b:

$$\frac{\partial V_e}{\partial t} + V_e \cdot \nabla V_e = -\frac{1}{\rho_e} \nabla P_e + \frac{e}{m_e} (E + V_e \times B) + g - \nu_{en}(V_e - U)$$

$$0 \approx -\frac{1}{n_e m_e} \nabla P_e + \frac{e}{m_e} (E + V_e \times B).$$  

(5.9)

A few simplifying assumptions have been made for the electron momentum equation. The electron inertia term (left-hand side of Equation 5.9) is approximated as zero because of the small electron mass relative to the ion mass. Also, the force due to gravity is small, again compared to its effect on the ions. Finally, assume that the electron collision frequency is much smaller than the electron gyro frequency, $\nu_e << \Omega_e$, (i.e., the electrons are highly magnetized), due to the small electron mass compared to the ion mass. As a result, the $\nu_{en}$ term can be approximated as zero in Equation 5.9.

The ion and electron temperatures are also modeled in SAMI3. The ion temperature equation is given as:
∂T_i \over \partial t + V_i \cdot \nabla T_i + \frac{2}{3} T_i \nabla \cdot V_i + \frac{2}{3} n_{i}k \nabla \cdot Q_i = Q_{in} + Q_{ii} + Q_{ie}, \quad (5.10)

where \( Q_{in} \), \( Q_{ii} \), and \( Q_{ie} \) are the heating terms due to the ion-neutral, ion-ion, and ion-electron collisions, respectively. The heat flux, \( Q_i \), is given as \( Q_i = -\kappa_i \nabla T_i \), with \( \kappa_i \) as the thermal conductivity. Equation 5.10 is modeled after Banks and Kockarts (1973); Millward et al. (1996), and details are further discussed in Huba et al. (2000).

The electron temperature equation is given as:

\[ \frac{\partial T_e}{\partial t} - \frac{2}{3} n_e k_b^2 \frac{\partial}{\partial s} \kappa_e \frac{\partial T_e}{\partial s} = Q_{en} + Q_{ei} + Q_{phe}, \quad (5.11) \]

where \( Q_{en} \), \( Q_{ei} \), and \( Q_{phe} \) are heating terms due to electron-neutral collisions, electron-ion collisions, and photoelectron heating. \( \kappa_e \) is the parallel electron thermal conductivity. Again, the reader is referred to Huba et al. (2000) for details on the temperature equations that are used within the numerical model.

Compared to SAMI2, SAMI3 contains a self-consistent potential solver for modeling the potential in the ionosphere. The relevant equation is the divergence-free current density, \( \nabla \cdot \mathbf{J} = 0 \). This is implemented in SAMI3 by first writing the current density as:

\[ \mathbf{J} = \sigma_P \left( \mathbf{E} + \mathbf{U} \times \mathbf{B} + \frac{\mathbf{g} \times \mathbf{B}}{\nu_{in}} \right) + \sigma_H \hat{b} \times \left( \mathbf{E} + \mathbf{U} \times \mathbf{B} + \frac{\mathbf{g} \times \mathbf{B}}{\nu_{in}} \right), \quad (5.12) \]

where \( \sigma_P \) and \( \sigma_H \) are the Pedersen and Hall conductivities, respectively, \( \mathbf{U} \) is the neutral wind, \( \mathbf{B} \) is the magnetic field, and \( \hat{b} \) is a unit vector in the direction of the magnetic field. The polarization electric field, \( \mathbf{E} \), is the unknown variable and is colored in red.

Next, we invoke the divergence free condition, \( \nabla \cdot \mathbf{J} = 0 \) and assume equipotential field lines by integrating terms along the field line. By writing \( \mathbf{E} \) in terms of the unknown scalar potential, \( \Phi \) (that is, \( \mathbf{E} = -\nabla \Phi \)), the divergence-free condition results in the form:
\[
\int_{\text{field line}} \nabla \cdot \left( \sigma_p (-\nabla \Phi + \mathbf{U} \times \mathbf{B} + \frac{\mathbf{g} \times \mathbf{B}}{\nu_{in}}) \right) \\
+ \sigma_H \hat{b} \times \left( -\nabla \Phi + \mathbf{U} \times \mathbf{B} + \frac{\mathbf{g} \times \mathbf{B}}{\nu_{in}} \right) \, dz = 0. \tag{5.13}
\]

From here, the equation is transformed into a 2D, scalar elliptical partial differential equation. Appendix D provides the details of how the equation is derived, and is summarized as:

\[
\nabla \cdot \mathbf{J} = 0
\]

\[
\frac{\partial}{\partial p} \Sigma_{pp} \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial \phi} \Sigma_{p\phi} \frac{\partial}{\partial \phi} \Phi - \frac{\partial}{\partial \phi} \Sigma_H \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial p} \Sigma_H \Phi = \frac{\partial}{\partial p} F_{\nu V} + \frac{\partial}{\partial \phi} F_{\phi V} - \frac{\partial}{\partial \phi} F_{\phi g} + \frac{\partial}{\partial p} F_{pg}. \tag{5.14}
\]

### 5.5 Implementation in Fortran and MPI Parallelization

SAM3 is written in the Fortran programming language and utilizes Message Passing Interface (MPI) parallelization. It is approximately 10,000 lines of code. For the current work, we use the “wedge” version of SAM3, and this model is parallelized geometrically. That is, each worker task operates on a longitudinal sub-wedge from the whole wedge space. In this way, there is a “Master” task and several “Worker” tasks used to run the model. The number of total processors equals the master task plus the summation of worker tasks. For example, in this work, we commonly use 12 processors, with one processor dedicated to the master task and 11 processors for the worker tasks.

Figure 5.4 provides an overview of how the SAM3 model is executed. Several subroutine steps are omitted from this simple representation, and the flowchart highlights the plasma transport and potential solve routines within the model for a typical time step. The model begins by reading the namelist file, which outlines several input parameters for the simulation,
such as time, solar conditions, and how often the data is written out. Then, the plasma parameters are loaded in, with each longitudinal slice in SAMI3 initialized from a previous SAMI2 run. The output variables, used to read out variables as the model steps through time, are also initialized.

At this point, the program breaks into two sections: one for the Master task and one for the Worker tasks. The Worker tasks are responsible for simulating the ionosphere within their sub-wedge, while the Master task solves for the potential and uses data from each Worker task for the calculation. For a Worker task, the neutral constituents are first initialized using the MSIS climatological model. A Worker transports plasma along the magnetic field line and across the magnetic field line, handling each transport routine in series. Plasma is transported along the magnetic field line (Equation 5.7) using the velocity obtained from momentum equation (Equation 5.8). Then, each Worker task sends data to the Master task, including electron densities and parameters used in the potential calculation (Equation 5.14, Appendix D).

Once all the data are gathered from each Worker task, the Master task solves for the potential, $\Phi$, and distributes the solution to each respective Worker task. If necessary, the Master task outputs the simulation data, and this process continues, waiting for each Worker task to send its information to solve for the potential in the next time step.

Once the workers have received the potential from the current time step, the electric field is calculated as $\mathbf{E} = -\nabla \Phi$. The plasma is transported across the magnetic field line via a $\mathbf{E} \times \mathbf{B}$ drift (Equation 5.1). Afterward, the new time step is calculated from the Courant condition, the neutrals are updated by calling MSIS (if necessary). The process repeats for the next time step.

5.6 Numerical Schemes in SAMI3

This section will outline a few numerical schemes in SAMI3, which describe the implementation of the governing physics equations into numerical code. The schemes presented here highlight the numerical implementation related to plasma transport and solving for the electrostatic potential within the model. The descriptions of the schemes listed here follow from Huba et al. (2000); Huba and Joyce (2014).
Figure 5.4: Flowchart of time-sequence of a SAMI3 numerical simulation, highlighting the subroutines relevant for plasma transport and the potential solve. The code uses MPI parallelization, with the Worker tasks operating on a sub-wedge of the model.
One of the key concepts for the plasma transport in SAMI3 is breaking down the transport into two steps: plasma transport along the magnetic field line and plasma transport across the magnetic field lines. This is a consequence of the plasma in the ionosphere having a larger magnetic force \((\mathbf{J} \times \mathbf{B})\) compared to the hydrodynamic force \((\nabla \cdot \mathbf{P})\). As a result, plasma transport across the magnetic field line is dominated by the \(\mathbf{E} \times \mathbf{B}\) drift (Equation 5.1).

A time-splitting technique is used to handle transport along and across the magnetic field line. First, plasma is advected along the magnetic field line and advances to the intermediate time step, \(t^*\):

\[
t_0 + \Delta t \rightarrow t^* \quad \text{(Parallel transport)}. \tag{5.15}
\]

Then, using the results from this advection step, plasma is advected in the perpendicular direction via \(\mathbf{E} \times \mathbf{B}\) drifts using the same time step, completing the time step to \(t_1\):

\[
t^* + \Delta t \rightarrow t_1 \quad \text{(Perpendicular transport)}. \tag{5.16}
\]

The next two sections detail each of these transport processes.

### 5.6.1 Parallel Transport

For the transport along the magnetic field line, the velocity obtained from Equation 5.8 is used to advect the plasma in Equation 5.7 in the \(s\) direction. Here, we assume that the ion velocity along the magnetic field line, \(V_{is}\), has been solved by Equation 5.8 (\(V_{is}\) is solved using a similar numerical technique described here). First, write the \(s\) component of the ion continuity equation following the curvilinear factors found in Appendix D.1:

\[
\frac{\partial}{\partial t} n_i + \frac{1}{h_q} \frac{\partial}{\partial q} (n_i V_{is}) = \mathcal{P}_i - \mathcal{L}_i n_i
\]

\[
\downarrow
\]

\[
\frac{\partial}{\partial t} n_i + b_s^2 \frac{\partial}{\partial s} \left( \frac{n_i V_{is}}{b_s} \right) = \mathcal{P}_i - \mathcal{L}_i n_i. \tag{5.17}
\]
Next, Equation 5.17 is discretized and finite differenced. In the next expression, the superscripts represent the time-index of the quantity and the subscript, $j$, represent the spatial index. The subscript $i$, indicative of the ion species, is dropped for clarity of finite differencing terms. A central differencing formula (e.g., Jin, 2011) is used to obtain second-order accuracy.

$$\frac{n_j^{t+\Delta t} - n_j^t}{\Delta t} + (b_s)_j \frac{(nV)^{t+\Delta t}_{j+1/2} - (nV)^{t+\Delta t}_{j-1/2}}{\Delta s} = P_j^t - L_j^t n_j^{t+\Delta t}.$$ (5.18)

Here, $\Delta t$ is the time step and $\Delta s = (s_{j+1} - s_{j-1})/2$. Equation 5.18 is an implicit scheme. That is, the values of $n$ at the next time step, $n^{t+\Delta t}$, depend on its neighbors solution at $t + \Delta t$. In other words, one cannot explicitly write out the solution for $n_j^{t+\Delta t}$. In practice, a system of linear equations is developed and the solution to $n_j^{t+\Delta t}$ (for all points of $j$) is found by a matrix solve.

However, in SAMI2/SAMI3, the velocity term, $V$, is evaluated at the current time step, $t$, as opposed to the next time step, $t + \Delta t$. This is advantageous to model the subsonic plasma in the ionosphere (Huba et al., 2000). The drawback is that small time steps must be used. The Courant condition for solving for the next time step ($\Delta t < \Delta s V$) is dependent only on the ion velocity, as opposed to the summation of ion velocity and sound speed. As a result, $\Delta t$ must be relatively small.

Incorporating the evaluation of $V$ at $t$ in Equation 5.18 results in a semi-implicit scheme:

$$\frac{n_j^{t+\Delta t} - n_j^t}{\Delta t} + (b_s)_j \frac{(n^{t+\Delta t} V^t)_{j+1/2} - (n^{t+\Delta t} V^t)_{j-1/2}}{\Delta s} = P_j^t - L_j^t n_j^{t+\Delta t}.$$ (5.19)

In this scheme, the $n^{t+\Delta t} V^t$ quantities are not available at the half grid points, $j - 1/2$ and $j + 1/2$. Therefore, a donor cell method is used to solve for the quantities at these grid points. In this technique, we define the left and right velocities as:
\[ V_l = \frac{V_{j-1} + V_j}{2}. \]  
\[ V_r = \frac{V_j + V_{j+1}}{2}. \]  
(5.20)  
(5.21)

Also, it is instructive to introduce flux terms, which are defined as:

\[ F_{j+1/2} = (n^{t+\Delta t}V^t)_{j+1/2} \]  
(5.22)  
\[ F_{j-1/2} = (n^{t+\Delta t}V^t)_{j-1/2} \]  
(5.23)

Substituting this into Equation 5.19, we have:

\[ \frac{n_{j+\Delta t}^t - n_j^t}{\Delta t} + (b_s)_j \frac{F_{j+1/2} - F_{j-1/2}}{\Delta s} = \mathcal{P}_j^t - \mathcal{L}_j n_{j+\Delta t}^t. \]  
(5.24)

The values for the flux are dependent on the directions of \( V_l \) and \( V_r \). There are four cases, and each case is listed in Table 5.1 and illustrated in Figure 5.5. Here we see that the donor cell method selects density to be transported based on the sign of the velocity at the mid-points. For example, if the velocity is moving from left-to-right at the midpoint \( j - 1/2 \) (\( V_l > 0 \)), then the density in the left cell \( (n_{j-1}) \) will be used for flux calculation, \( F_{j-1/2} = V_l n_{j-1} \). Likewise, if the velocity is moving right-to-left at the midpoint \( j - 1/2 \) (\( V_l < 0 \)), density at the \( j \) point would be used and the flux would be written as \( F_{j-1/2} = V_l n_j \). Similarly, this concept is applied to the flux at the other midpoint, \( j + 1/2 \).

Substituting \( F_{j+1/2} - F_{j-1/2} = a_0 n_{j-1}^{t+\Delta t} + b_0 n_j^{t+\Delta t} + c_0 n_{j+1}^{t+\Delta t} \) into Equation 5.24, we have:

\[ \frac{n_{j+\Delta t}^t - n_j^t}{\Delta t} + (b_s)_j \frac{a_0 n_{j-1}^{t+\Delta t} + b_0 n_j^{t+\Delta t} + c_0 n_{j+1}^{t+\Delta t}}{\Delta s} = \mathcal{P}_j^t - \mathcal{L}_j n_{j+\Delta t}^t. \]  
(5.25)

The quantities, \( a_0, b_0, \) and \( c_0 \) are based on the various cases (i.e., the sign of \( V_l \) and \( V_r \)), and are derived from the right-hand column of Table 5.1:
Figure 5.5: Fluxes at the mid-points on the grid used within the semi-implicit transport scheme.

Table 5.1: Fluxes at the half grid points using the donor cell method.

<table>
<thead>
<tr>
<th>Case</th>
<th>Velocity sign</th>
<th>Fluxes</th>
<th>Flux difference $(F_{j+1/2} - F_{j-1/2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$V_r &gt; 0$</td>
<td>$F_{j+1/2} = V_r n_j$</td>
<td>$-V_l n_{j-1} + V_r n_j$</td>
</tr>
<tr>
<td></td>
<td>$V_l &gt; 0$</td>
<td>$F_{j-1/2} = V_l n_{j-1}$</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>$V_r &lt; 0$</td>
<td>$F_{j+1/2} = V_r n_{j+1}$</td>
<td>$-V_l n_j + V_r n_{j+1}$</td>
</tr>
<tr>
<td></td>
<td>$V_l &lt; 0$</td>
<td>$F_{j-1/2} = V_l n_j$</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>$V_r &gt; 0$</td>
<td>$F_{j+1/2} = V_r n_j$</td>
<td>$(V_r - V_l) n_j$</td>
</tr>
<tr>
<td></td>
<td>$V_l &lt; 0$</td>
<td>$F_{j-1/2} = V_l n_j$</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>$V_r &lt; 0$</td>
<td>$F_{j+1/2} = V_r n_{j+1}$</td>
<td>$-V_l n_{j-1} + V_r n_{j+1}$</td>
</tr>
<tr>
<td></td>
<td>$V_l &gt; 0$</td>
<td>$F_{j-1/2} = V_l n_{j-1}$</td>
<td></td>
</tr>
</tbody>
</table>
\[ a_0 = \begin{cases} 
-V_t, & \text{if } V_r > 0 \text{ and } V_t > 0 \text{ (Case 1)} \\
0, & \text{if } V_r < 0 \text{ and } V_t < 0 \text{ (Case 2)} \\
0, & \text{if } V_r > 0 \text{ and } V_t < 0 \text{ (Case 3)} \\
-V_t, & \text{if } V_r < 0 \text{ and } V_t > 0 \text{ (Case 4)} 
\end{cases} \] (5.26)

\[ b_0 = \begin{cases} 
V_r, & \text{if } V_r > 0 \text{ and } V_t > 0 \text{ (Case 1)} \\
-V_t, & \text{if } V_r < 0 \text{ and } V_t < 0 \text{ (Case 2)} \\
V_r - V_t, & \text{if } V_r > 0 \text{ and } V_t < 0 \text{ (Case 3)} \\
0, & \text{if } V_r < 0 \text{ and } V_t > 0 \text{ (Case 4)} 
\end{cases} \] (5.27)

\[ c_0 = \begin{cases} 
0, & \text{if } V_r > 0 \text{ and } V_t > 0 \text{ (Case 1)} \\
V_r, & \text{if } V_r < 0 \text{ and } V_t < 0 \text{ (Case 2)} \\
0, & \text{if } V_r > 0 \text{ and } V_t < 0 \text{ (Case 3)} \\
V_r, & \text{if } V_r < 0 \text{ and } V_t > 0 \text{ (Case 4)} 
\end{cases} \] (5.28)

From Equation 5.25, grouping-like terms results in:

\[
A \frac{(b_s)_{j} a_0}{\Delta s_j} n_{j-1}^{t+\Delta t} + B \left( \frac{1}{\Delta t} + \frac{(b_s)_{j} b_0}{\Delta s_j} + L_j^t \right) n_j^{t+\Delta t} + C \frac{(b_s)_{j} c_0}{\Delta s_j} n_{j+1}^{t+\Delta t} = \frac{\Delta t}{n_j^t} + P_j^t. \] (5.29)

Equation 5.29 can be written for each point, j, along the magnetic field line, in effect developing a tri-diagonal system of linear equations. The matrix equation is solved and the density at the next time step is found for each j point. Finally, we should note that the boundary conditions at the edge of the field line have \( n_j^{t+\Delta t} = P_j^t/L_j \).
5.6.2 Perpendicular Transport

The perpendicular transport scheme follows a similar procedure as the parallel transport, but uses a higher order, partial donor method to transport plasma across field lines. In the simulation study of MSTIDs, it is important to have a non-diffusive scheme to numerically sustain the instability. This section follows the description provided by Huba (2003, Chapter 3).

We begin with the continuity equation, written as:

\[
\frac{\partial}{\partial t} n + \alpha \nabla \cdot n \mathbf{V} = 0. \quad (5.30)
\]

The continuity equation is set to zero, as the production and loss “source terms” on the right-hand side are accounted for with the parallel transport. Again, we have dropped the subscript \(i\), representing an ion constituent, for simplicity. The \(\alpha\) term represents the curvilinear factor in taking the divergence in the \(p\) or \(\phi\) direction. For the purposes of this discussion, we will neglect this term for simplicity to focus on numerical schemes. One can also think of this term being lumped into the \(\mathbf{V}\) term.

The finite volume method is used for this equation. First, Equation 5.30 is integrated over the cell volume, \(d^3x\).

\[
\iiint \frac{\partial}{\partial t} n \, d^3x = -\iiint \nabla \cdot n \mathbf{V} \, d^3x \quad \frac{\partial}{\partial t} n_c = -\iiint (n \mathbf{V})^{t+\Delta t/2} \cdot \hat{n} \, d^2x. \quad (5.31)
\]

Here we define a conserved quantity, which is simply the number of electrons, as \(n_c = \iiint n \, d^3x\). Also, Gauss’ law is used to transform the volume integral to a surface integral on the right-hand side. The flux at each interface is defined as:

\[
\mathbf{F}^{t+\Delta t} = (n \mathbf{V})^{t+\Delta t/2}. \quad (5.32)
\]

Finite differencing is used to calculate \(n_c\) at the next time step:
Figure 5.6: Due to Gauss’ law in Equation 5.31, the flux at the interfaces of the cell (i.e., at \( j + 1/2 \)) are needed for the numerical transport scheme.

\[
\frac{n_{c}^{t+\Delta t} - n_{c}^{t}}{\Delta t} = - \iint F^{t+\Delta t/2} \cdot \hat{n} \, d^2x
\]
\[
n_{c}^{t+\Delta t} = n_{c}^{t} - \Delta t \iint F^{t+\Delta t/2} \cdot \hat{n} \, d^2x. \tag{5.33}
\]

Once the conserved value is calculated, the density is found by \( n_{t+\Delta t} = n_{t+\Delta t}^{c}/V \), where \( V \) is the cell volume.

Equation 5.33 is the core equation used to advance the density forward in time. The flux calculation is at the half time step, and a second-order Adams-Bashforth time stepping procedure is used to calculate a quantity at \( t + \Delta t/2 \):

\[
A^{t+\Delta t/2} = A^{t} + \frac{\Delta t}{2\Delta t_{0}} (A^{t} - A^{t-\Delta t_{0}}). \tag{5.34}
\]

Also, due to flux calculation at the edge of the cells, we need a methodology to calculate the parameters values at the half grid points similar to the scheme for transport along the magnetic field lines. In one-dimension, this is shown in Figure 5.6.

The flux can be decomposed into a left (L) and right (R) state value:
\[ F_{j+1/2} = F^L_{j+1/2} + F^R_{j+1/2} = (nV)^L_{j+1/2} + (nV)^R_{j+1/2}. \] (5.35)

The left state (L) represents the flux corresponding to density transported from \( j \) to \( j + 1 \), while the right state (R) corresponds to density transported from \( j + 1 \) to \( j \).

The density at \( j + 1/2 \) can be obtained by using a fourth-order interpolation scheme. Following Huba (2003), this can be derived by first writing the conserved quantity, \( G_c \), as:

\[ G_c(x) = \int_{-\infty}^{x} G_c(s) \, ds. \] (5.36)

We can obtain the quantity \( G_c(x) \) by simply differentiating \( G_c(x) \):

\[ G_c(x) = \frac{\partial G_c(x)}{\partial x}. \] (5.37)

The derivative term is discretized with a standard fourth-order finite differencing scheme:

\[ \left( \frac{\partial G_c(x)}{\partial x} \right)_{j+1/2} \approx \frac{1}{12\Delta x} \left( G_{c,j-3/2} - 8G_{c,j-1/2} + 8G_{c,j+3/2} - G_{c,j+5/2} \right). \] (5.38)

Plugging in the definition for \( G_c \), we recognize:

\[ \left( \frac{\partial G_c(x)}{\partial x} \right)_{j+1/2} \approx \frac{1}{12\Delta x} \left[ \frac{G_{c,j-3/2} - G_{c,j+5/2}}{-\Delta x(g_{c,j-1} + 7g_{c,j} + g_{c,j+1} + g_{c,j+2})} + 8 \left( \frac{G_{c,j+3/2} - G_{c,j-1/2}}{8\Delta x(g_{c,j} + g_{c,j+1})} \right) \right] \]

\[ = \frac{1}{12} \left[ -g_{c,j-1} + 7g_{c,j} + 7g_{c,j+1} - g_{c,j+2} \right] \]

\[ = \frac{7}{12} (g_{c,j} + g_{c,j+1}) - \frac{1}{12} (g_{c,j-1} + g_{c,j+2}). \] (5.39)

In summary, the conserved density (i.e., the number of particles), can be written at the half grid point, \( j + 1/2 \), as:
\[ n_{\text{HO},j+1/2} = \frac{7}{12} (n_{c,j} + n_{c,j+1}) - \frac{1}{12} (n_{c,j-1} + n_{c,j+2}). \] (5.40)

Here, the subscript “\( \text{HO} \)” represents a “high-order” value of the quantity. Finally, we recognize that the density can be calculated as
\[ n_{j+1/2} = n_{\text{HO},j+1/2}/V, \]
where \( V \) is the volume of the cell, as before.

There is one caveat for using the fourth-order interpolation scheme for the density at the half grid point in Equation 5.40. The interpolation technique may create spurious modes near sharp discontinuities and unphysical numerical diffusion. Therefore, a partial donor method (PDM) is used to monitor and prevent numerical diffusion in the transport scheme.

The PDM can be derived from reasoning about the maximum amount of density change that can occur within a cell after a time step. In this derivation, we assume the velocity is moving from left to right, and therefore we derive the left (L) state in Equation 5.35. This is represented in Figure 5.7. Suppose there is some value, \( n_{\text{PDM},j+1/2} \), that can be used to represent the density at the grid point \( j \). The maximum change of density at point \( j \) is \( (n_{j-1} - n_j)\Delta x \). That is, all of the density at \( j \) is replaced by the density in the previous cell. Using \( n_{\text{PDM},j+1/2} \), the total change in cell \( j \) is \( (n_{j-1} - n_{\text{PDM},j+1/2})V\Delta t \). Equating these two terms will result in an expression for \( n_{\text{PDM},j+1/2} \):

\[
(n_{j-1} - n_{\text{PDM},j+1/2})V\Delta t = (n_{j-1} - n_j)\Delta x
\]

\[
(n_{j-1} - n_{\text{PDM},j+1/2}) \frac{V\Delta t}{\Delta x} = n_{j-1} - n_j
\]

\[
\alpha n_{j-1} - n_{j-1} + n_j = \alpha n_{\text{PDM},j+1/2}
\]

\[
(\alpha - 1)n_{j-1} + n_j = \alpha n_{\text{PDM},j+1/2}
\]

\[
n_{\text{PDM},j+1/2} = \frac{1}{\alpha} [n_j + (\alpha - 1)n_{j-1}] . \quad (5.41)
\]

Here, we have defined the parameter \( \alpha = V\Delta t/\Delta x \), which in practice is selected as \( 0 < \alpha \leq 1 \). If \( \alpha = 1 \), the full donor cell is implemented, which is not advantageous due to high numerical diffusion, and therefore it is common to choose \( \alpha < 1 \).
Similarly, the PDM value associated with the right-hand state (R) can be derived as:

\[
(n_{j+1} - n_{PDM,j+1/2})V \Delta t = (n_{j+1} - n_j)\Delta x = \alpha n_{PDM,j+1/2} = n_{j+1} - n_j + n_j
\]

\[
n_{PDM,j+1/2} = \frac{1}{\alpha} [n_j + (\alpha - 1)n_{j+1}].
\]

(5.42)

Figure 5.7 provides an example of the values of \(n_{PDM,j+1/2}\) under a constant positive velocity. We should note that \(n_{PDM,j+1/2}\) is the minimum physical value of density that can be selected for the half-grid point \(j + 1/2\). If it were any lower, an unphysical amount of density may be in cell \(j\) after advection. For example, with \(\alpha = 1\), \(n_{PDM,j+1/2} = n_j\), and if the value actually used for \(j + 1/2\) were smaller than this value, there would be density remaining in the cell, which may contribute to an unrealistic state. This threshold can be adjusted via the \(\alpha\) parameter.

Several options for the value at the half-grid point, \(j + 1/2\) are available to use: \(n_j\), \(n_{HO,j+1/2}\), and \(n_{PDM,j+1/2}\). Ideally we would choose the high-order interpolated value, \(n_{HO,j+1/2}\), but as previously discussed, if this value is less than \(n_{PDM,j+1/2}\), then unphysical density can occur after the time step at the \(j\) grid point.

Therefore, if \(n_{PDM,j+1/2} < n_{HO,j+1/2}\), then we use the \(n_{HO,j+1/2}\) value. On the other hand, if \(n_{PDM,j+1/2} > n_{HO,j+1/2}\), then the \(n_{PDM,j+1/2}\) value is used. Again, this threshold is based on the \(\alpha\) parameter, which dictates the amount of numerical diffusion. For example, in the simulation work of MSTIDs in the following chapters, we set \(\alpha = 1/6\).

5.6.3 Potential Solve

Finally, we note the solver used in the model to calculate the electrostatic potential. This equation was derived in Appendix D, and is reproduced here as:
\[
\frac{\partial}{\partial p} \Sigma_{pp} \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial \phi} \Sigma_{pp} \frac{\partial}{\partial \phi} \Phi - \frac{\partial}{\partial \phi} \Sigma_H \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial p} \Sigma_H \frac{\partial}{\partial \phi} \Phi
\]

\[
= \frac{\partial}{\partial p} F_{pV} + \frac{\partial}{\partial \phi} F_{pV} - \frac{\partial}{\partial \phi} F_{\phi g} + \frac{\partial}{\partial p} F_{p g}.
\] (5.43)

The numerical solver for Equation 5.43 is described as follows. First, the equation is broken down using a standard finite difference method to discretize the equation. An example 5-point stencil is shown in Figure 5.8. The \( \phi \) direction is indexed as \( i \), while the \( p \) direction is indexed as \( j \). The cross terms (i.e., Hall terms with \( i \pm 1, j \pm 1 \)), are lumped together as a source term and iterations of the solution are processed for convergence [Huba, personal communication].

Next, by using the 5-point stencil, the discretized equation for point \((i, j)\) has the following form:
Figure 5.8: A 5-point stencil used for finite differencing Equation 5.43.

\[
a_1(i, j)\Phi(i-1, j) + a_2(i, j)\Phi(i, j-1) + a_3(i, j)\Phi(i, j) + a_4(i, j)\Phi(i, j+1) + a_5(i, j)\Phi(i+1, j) = S(i, j).
\]

(5.44)

The \(a(i, j)\) terms include the coefficients from discretizing the potential equation. \(S(i, j)\) includes the source terms, and also the Hall terms that are used for the iteration technique.

Equation 5.44 can be written for each \((i, j)\) point and a system of linear equations is obtained of the form \(A\Phi = b\). The boundary conditions in SAMI3 are periodic in the \(\phi\) direction, and have Neumann boundary conditions in the \(p\) direction. Solving the matrix equation, \(A\Phi = b\), by direct inversion is costly \((O(n^3))\), where \(n\) is the dimension of the matrix \(A\) and is the total number of unknown \(\Phi\) points in the simulation space). Iterative techniques can be used to solve the matrix equation, and include a class of techniques such as successive over relaxation (SOR) or multigrid methods. However, in SAMI3, a direct method known as the stabilized error vector propagation (SEVP) method is used to solve Equation 5.44.

SEVP is based on the error vector propagation (EVP) method. The basic idea for EVP is recognizing that in Equation 5.44, \(\Phi(i, j+1)\) can be written as a function of the remaining terms. That is, knowing the previous two rows
(j and j − 1), we can advance the solution of Φ at j + 1 for each i.

Once the solution has reached the last row (where the solution is known due to the boundary condition), the error from the last row is related to the beginning row. Then, each row is corrected as the rows are advanced. A few iterations are applied to resolve round-off errors from the initial correction from the final to first row.

The EVP technique works well but is limited to a few number of rows (∼ 12) (Madala, 1978). For a larger number of rows (which is necessary for solving the potential in SAMI3), a variant of the EVP technique is used, in which the relatively large (i, j) space is broken down into blocks along j. In effect, this stabilizes the EVP technique, hence the term “stabilized error vector propagation” (SEVP). Details are described in Madala (1978) and Sashegyi and Madala (1989). The key point is that a solution for Φ is obtained from Equation 5.43.

5.7 Conclusion

The SAMI3 numerical model is a powerful tool for studying the ionosphere. In the following chapters, we will numerically simulate MSTIDs within SAMI3. To begin, we show MSTIDs are self-consistently developed in the model. Once we establish that MSTIDs are generated in SAMI3, we can calculate synthetic observations within the model and compare to observational data. Also, we conduct several case studies to further investigate the development of MSTIDs in the nighttime, mid-latitude ionosphere.
CHAPTER 6

SELF-CONSISTENT GENERATION OF MSTIDS WITHIN THE SAMI3 NUMERICAL MODEL

In this chapter\(^1\), we use the SAMI3 numerical model described in the prior chapter to investigate the self-consistent generation of MSTIDs. To begin, the context of this work is provided with respect to the previous work found in the literature (as detailed in Section 2.3), in addition to the advantages for using the SAMI3 numerical model in the current work. Next, the input parameters are discussed, and are justified by observations and studies of MSTIDs. The simulation results are presented, including two cases: a random perturbation and a perturbation with a prescribed wavevector, \( \mathbf{k} \). This chapter illustrates that MSTIDs can be developed in the SAMI3 model.

6.1 Introduction

As discussed in Section 2.3, researchers have used numerical modeling for MSTID studies. In the current work, we follow from the second phase of MSTID numerical modeling by using the fundamental equations, as opposed to derived parameters from the first phase of modeling. In this way, we are able to simulate MSTIDs based on physics from the governing equations, as opposed to quantities derived through various assumptions (e.g., constant neutral wind, zero spatial gradients in the field line integrated quantities, etc.).

The three-dimensional modeling space is advantageous to calculate synthetic observations that are based on numerical integration in the altitudinal direction (e.g., TEC and integrated 630.0-nm airglow emission), providing further insight of the instabilities as they are developed in the model. The two-dimensional simulations described in Section 2.3 model the magnetic field-line integrated quantities. Therefore, it would be difficult to produce

\(^{1}\)This chapter is based on the work published in Duly et al. (2014).
synthetic measurements that mimic commonly used aeronomy instrumentation, such as dual-frequency GPS receivers and airglow imaging cameras, as these instruments typically measure quantities that are not magnetic field-line integrated.

There are additional advantages for using the SAMI3 model with MSTID studies. The wedge model for SAMI3 uses a full magnetic field-line grid (e.g., Figures 5.1 and 5.3). As a result, we are able to investigate the signature of a MSTID in the conjugate hemisphere, which has been observed experimentally (Otsuka, 2004). These are the first numerical simulation results of MSTIDs that use a full magnetic flux tube grid. Also, SAMI3 solves for the ionospheric potential, $\Phi$ (Section 5.4), which is crucial for the self-consistent generation of MSTIDs due to the requirement that $\nabla \cdot J = 0$ must be maintained in order for the instability to develop (Section 3.5). Following Zhou and Mathews (2006) and Figure 3.2, the perturbation potential can be written as (converting to SI units):

$$\tilde{\Phi} \approx \frac{i\tilde{\Sigma}_P}{k^2\Sigma_{P,0}} [k \cdot (E_0 + U \times B)]. \quad (6.1)$$

Here, “0” and “~” represent the background and perturbed quantities, respectively. Recall that $\Sigma_P$ is the Pedersen conductivity integrated along the magnetic field line (Equation 1.17). By solving the ionospheric potential self-consistently in SAMI3, we are able to model the perturbation potential from Equation 6.1. Under the scenario discussed in Section 3.5, the Perkins instability theory dictates that unstable rising and falling bands (with respect to altitude) of electron density will be produced. In this way, MSTIDs should be able to develop in SAMI3.

### 6.2 Input Parameters and Perturbations

This section will discuss the input parameters and perturbations that are used for simulating MSTIDs in SAMI3. The longitude of the simulation is centered at 0°E and spans approximately 4° in longitude (therefore, local time and universal time are the same). The grid configuration has a relatively high-resolution, with $(n_z, n_f, n_l) = (301, 402, 99)$. Here, $n_z$ refers to the number of points along the field line ($s$ direction), $n_f$ refers to the number...
of field lines in altitude for each longitude point ($p$ direction), and finally $n_1$ corresponds to the number of points in the longitudinal (or zonal) direction ($\phi$ direction). Compared to previous simulations, the grid is relatively dense in order to resolve MSTIDs at mid-latitudes. At mid-latitudes, the approximate grid distances for the $p$, $s$, and $\phi$ directions are 5.7 km, 7.6 km, and 4.1 km, respectively, at 250-km altitude. 11 worker tasks are used in addition to the single master task for the MPI execution, and a 4-s time step advances the simulation in time.

Parameters in SAMI3 are initialized with values from a nominal SAMI2 run. That is, SAMI2 is run for 48 hours and the results are used as input for SAMI3. Transients are removed from the system during the initial SAMI2 run. The SAMI3 simulation begins at 21:30 LT for December 21st, 2008, based on the climatological study of MSTIDs indicating large MSTID counts during this timeframe (Chapter 4). In addition, solar minimum conditions are modeled with $F10.7 = F10.7A = 75$ SFU, again based on the observations and theory of MSTIDs occurring during solar minimum.

The climatological model HWM93 is used for the neutral wind values within SAMI3. The neutral wind geometry for this time and season sets up an effective electric field, $E' = E_0 + U \times B$, in the upper-right altitudinal plane for the Northern Hemisphere (i.e., pointing NE), which follows from the configuration given in Figure 3.5 for the development of unstable NW-SE banded structures (Makela and Otsuka, 2011). Therefore, the simulation should produce NW-SE bands of raised and lowered electron density for a Northern Hemisphere configuration.

SAMI3 is executed for one hour in order for the F-layer to reach a steady-state height. After this period, a perturbation is introduced into the system, and two types of perturbations are used: a random perturbation and a perturbation with a prescribed wavevector, $k$. The results from the two cases of perturbation are presented individually.

The theory for the Perkins instability assumes that the Pedersen conductivity is initially perturbed. Therefore, for the first case, a random array is generated for the field lines of size $n_f \times n_1$ (i.e., the total number of field lines in the simulation), and these values are spatially smoothed in order to maintain numerical stability (Miller, 1996). This smoothing process is implemented by low-pass filtering the perturbation array such that half of the spectrum is retained. The values in the array are random, but high frequency
modes are removed. Then, the values from the array are used to perturb the
Northern Hemisphere local Pedersen conductivity on each field line for five
minutes. The perturbation region affects the field lines that are between
$27^\circ$-30$^\circ$N at 250-km altitude for all longitudes.

The second type of perturbation redistributes the density along a field line
by a wavevector, $k$. This perturbation has a similar effect of modifying $\Sigma_P$, but could represent the initial effects of a MSTID seeding mechanism, such
as a gravity wave or a sporadic E (Es) layer. The $k$ value selected is based
on previous observations and is characteristic of a typical MSTID. After the
perturbation is applied in the model for each case, the simulation is executed
for 55 minutes in order for transients to leave the system.

6.3 Simulation Results

In this section, we present the results for the self-consistent generation of
MSTIDs in SAMI3. The results are calculated using a control run in which
no perturbation is applied within the model. In this way, the effects of the
MSTID can be isolated in SAMI3.

The change in the integrated Pedersen conductivity, $\Sigma_P$ (Equation 1.17),
will be used throughout the presentation of the results, which is defined as:

$$\Delta \Sigma_P = \frac{\Sigma_{P, \text{pert}} - \Sigma_{P, \text{control}}}{\Sigma_{P, \text{control}}} \times 100 \, [\%]. \quad (6.2)$$

The $\Delta \Sigma_P$ parameter can be used to represent MSTID dynamics. For exam-
ple, Equation 6.1 shows that the perturbed integrated Pedersen conductivity,
$\tilde{\Sigma}_P$ (which is equivalent to $\Sigma_{P, \text{pert}} - \Sigma_{P, \text{control}}$), is proportional to the per-
turbed ionospheric potential calculation. As discussed in Chapter 3, the
Perkins instability describes the conditions when a perturbation in $\Sigma_P$ is unus-
able, and the developed perturbed potential, via $E \times B$ drifts, subsequently
result in vertical displacements of electron density that are a signature of
MSTIDs.

Table 6.1 lists parameters relevant for theoretical calculations of MSTIDs
(e.g., the linear growth rate) that will be used throughout the discussion.
The parameters are based on input values (i.e., a perturbation wave vector),
climatological values used in SAMI3 (i.e., HWM93 neutral winds), and values
calculated within the model (i.e., electric and magnetic field terms). In the next two sections, the results from each perturbation case are presented.

6.3.1 Case 1: Random Perturbation

In the first case, the local Pedersen conductivity is randomly perturbed for five minutes in the Northern Hemisphere. Figure 6.1 displays $\Delta \Sigma_P$ in 15-minute increments, beginning with a total 0.07% change in $\Sigma_P$ (calculated from integrating the perturbed, local Pedersen conductivity) after the transients have left the system. The $\Delta \Sigma_P$ values are mapped to 250-km altitude in the Northern Hemisphere, although they could be represented at any point along the magnetic field line. Figure 6.2 plots the power spectral density (PSD) of $\Delta \Sigma_P$ at 45 minutes (i.e., the last frame from Figure 6.1). The PSD plot shows that the dominant $k$ modes for $\Delta \Sigma_P$ are in the first and third quadrants, corresponding to the NW-SE banded structures in Figure 6.2.

The initial case study is a proof of concept that MSTIDs are able to self-consistently generate in SAMI3, given that the Perkins instability theory assumes an initial perturbation in the conductivity at mid-latitudes. Recall that in the theoretical formulation for MSTIDs (Section 3.4), the $k$ modes with the largest growth rate lie halfway between $E' = E_0 + U \times B$ and magnetic east ($\hat{y}$). From Table 6.1, the angle of $E'$ with respect to magnetic east is calculated to be $\theta = 61.1^{\circ}$. This bound is plotted in Figure 6.2 along with $\theta = 0^{\circ}$ (i.e., magnetic east). As shown, a majority of the PSD lies within these two bounds, and the dominant $k$ modes agree well with the theoretical description. Given that the initial perturbation is random, the Perkins instability can be confidently attributed to the developed modes.

Previous studies have investigated the development of MSTIDs by random perturbation, both from numerical simulation and theoretical aspects. For example, from the first phase of modeling with the Perkins derived equations, studies of the resultant PSD by random perturbations found dominant modes between $E'$ and magnetic east (Scannapieco et al., 1975; Zhou et al., 2005; Miller, 1996, Chapter 3), consistent with the results of the current study. From the second phase of modeling, Yokoyama et al. (2008) simulated MSTID development in a 3D spatial box region in the Northern Hemisphere and $k$ modes between $E'$ and magnetic east were also established. In addi-
Table 6.1: Quantities used for theoretical calculations of MSTIDs for comparison with model output, including the linear growth rate equation (Equation 6.3).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Value for $\gamma$ Calculation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_n$</td>
<td>Neutral scale height</td>
<td>40 km</td>
<td><em>Perkins</em> (1973)</td>
</tr>
<tr>
<td>$k_x$</td>
<td>Perturbation wave vector</td>
<td>0.04038 km$^{-1}$</td>
<td>Case 2 input perturbation</td>
</tr>
<tr>
<td>$k_y$</td>
<td>Perturbation wave vector</td>
<td>0.04813 km$^{-1}$</td>
<td>Case 2 input perturbation</td>
</tr>
<tr>
<td>$U_x$</td>
<td>Neutral wind</td>
<td>$-43.85$ m/s</td>
<td>SAMI3 / HWM93</td>
</tr>
<tr>
<td>$U_y$</td>
<td>Neutral wind</td>
<td>131.55 m/s</td>
<td>SAMI3 / HWM93</td>
</tr>
<tr>
<td>$U_z$</td>
<td>Neutral wind</td>
<td>$-40.92$ m/s</td>
<td>SAMI3 / HWM93</td>
</tr>
<tr>
<td>$E_x$</td>
<td>Electric field</td>
<td>$-1.48$ mV/m</td>
<td>SAMI3 (Equation 5.13)</td>
</tr>
<tr>
<td>$E_y$</td>
<td>Electric field</td>
<td>$9.71 \times 10^{-3}$ mV/m</td>
<td>SAMI3 (Equation 5.13)</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic field strength</td>
<td>$2.86 \times 10^{-5}$ T</td>
<td>SAMI3</td>
</tr>
<tr>
<td>$D$</td>
<td>Magnetic dip angle</td>
<td>$46.72^\circ$</td>
<td>SAMI3</td>
</tr>
</tbody>
</table>
Figure 6.1: Beginning with an initial random perturbation (Case 1), the time sequence of $\Delta \Sigma_P$ is plotted in 15-minute increments for the first 45 minutes of the SAMI3 simulation. The values of $\Delta \Sigma_P$ are mapped to 250-km altitude. Reprinted from Duly et al. (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.
6.3.2 Case 2: Specified $k$ Perturbation

Instead of artificially perturbing $\sigma_p$, the initial $\Delta \Sigma_p$ can also be accomplished by redistributing the electron density along the magnetic field lines. In the second case of perturbation, the redistribution of $n_e$ is prescribed by a $k$ having $\lambda = 100$ km at an angle $\alpha = 40^\circ$ with respect to magnetic east at 250-km altitude in the Northern Hemisphere. The amplitude of the perturbation is approximately 30 km in the magnetic field line direction. The corresponding $k_x$ and $k_y$ values are listed in Table 6.1. Analogous to the previous case, only the parameters in the Northern Hemisphere are initially affected.

Similar to Figure 6.1, Figure 6.3 plots the time sequence of $\Delta \Sigma_p$ in 15-minute increments for 45 minutes. An initial $\Delta \Sigma_p = 1.3\%$ is present in

---

Figure 6.2: The power spectral density (PSD) of the last frame from Figure 6.1. The theoretical bounds of the Perkins instability are also displayed (dashed lines). Reprinted from Daly et al. (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.

Yokoyama et al. (2008) calculated a $\Delta \Sigma P = 0.1\%$ in their numerical simulations (i.e., Figure 2.8), which is similar to the results presented here. Theoretical analysis of the PSD of MSTIDs (Garcia et al., 2000; Makela and Otsuka, 2011) are also consistent with the results from SAMI3.
Similar to Figure 6.1, but with the specified $k$ perturbation (Case 2) in SAMI3. The value is larger compared to Case 1 but was required in order to obtain significant perturbation values in the synthetic measurements (presented and discussed in the next chapter). Figure 6.3 shows that the value of $\Delta \Sigma_P$ increases as a function of time.

Using a specific mode for the perturbation enables the growth of the mode to be isolated for comparisons against the theoretical growth rate. Figure 6.4 displays the maximum $\Delta \Sigma_P$ calculated as a function of time. The theoretical value calculated from the linear growth rate equation is also plotted as $\Sigma_P(t) = \Sigma_P(t = t_0)e^{\gamma t}$. The growth rate equation is reproduced from Equation 3.10 as (Zhou and Mathews, 2006):

$$\gamma = \frac{\cos D}{BH_n} \left[ -E_{by} + \frac{k_y}{k_z^2} k \cdot (E_0 + U \times B) \right] + \frac{U_z \sin D}{H_n} \left[ \text{e-folds/s} \right]. \tag{6.3}$$

The terms listed in Table 6.1 are used in Equation 6.3 and the growth rate is
Figure 6.4: The growth of the MSTID as a function of time for the Case 2 perturbation from the SAMI3 model (solid blue). Also shown are the theoretical values calculated from the linear growth rate (dashed red). Reprinted from Duly et al. (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.

calculated to be $\gamma = 3.687 \times 10^{-4}$ [e-folds/s], which is an e-fold about every 45 minutes.

Figure 6.4 shows that the growth of the MSTID in SAMI3 is larger than the linear theory for the first 30 minutes, at which point the values saturate in the model. Although the quantities are different, the growth in the model is the same order of magnitude as the theoretical calculation. Differences between these values may be the result of the neutral wind influence from the conjugate hemisphere (e.g., Yokoyama, 2014). Recall that the growth rate equation is derived by assuming a constant $U$ in the generative F-region hemisphere only. However, the (non-constant) neutral winds provided by HWM93 in both F-regions influence the potential calculation, and as a result may affect the growth in the model.
6.4 Discussion

Previous numerical simulations have also observed saturation in the growth of MSTIDs. For example, Yokoyama et al. (2008) calculated saturation at approximately 60-90 minutes in their modeling work. Discrepancies between the models may be responsible for the different saturation times. The model used by Yokoyama et al. (2008) only considers a box region in the Northern Hemisphere, while SAMI3 has a full flux tube grid. Also, the dynamo electric field in the $p$ direction is modeled in SAMI3 (i.e., $E_p$), while the dynamo is not modeled in the work by Yokoyama et al. (2008) (Yokoyama, personal communication). The dynamo in the zonal direction, $E_\phi$, is not modeled in SAMI3 due to the periodic boundary conditions in the $\phi$ direction (i.e., there is no large-scale, background potential drop in the longitudinal direction in SAMI3). Differences in the models such as these may be responsible for various saturation times.

Although the growth of MSTIDs is self-consistent in SAMI3, one outstanding issue is that the structures do not propagate westward and equatorward, which is commonly observed in MSTID experimental studies. The theory for MSTIDs gives the real component of the instabilities as (Garcia et al., 2000):

$$\omega_{Re} = \frac{k \cdot (E_0 \times B)}{B^2}.$$  \hspace{1cm} (6.4)

Equation 6.4 is used to analyze both the phase velocity and group velocity of a MSTID. First, the phase velocity is calculated as:

$$v_p = \frac{\hat{k} \cdot \omega_{Re}}{|k|} = \frac{\hat{k} \cdot (E_0 \times B)}{B^2}. \hspace{1cm} (6.5)$$

The phase velocity is along the $k$ direction and its magnitude is the component of the $E_0 \times B$ drift in the direction of $k$. This agrees with the SAMI3 numerical results. The motion of a constant phase plane is simply the $E_0 \times B$ drift in the direction of the wavefronts.

The group velocity is used to investigate the motion of the packet, or envelope, of the MSTID wave, which is defined as:
\[ v_g = \nabla_k \omega_{Re} \]
\[ = \frac{\partial \omega_{Re}}{\partial k_x} \hat{x} + \frac{\partial \omega_{Re}}{\partial k_y} \hat{y} \]
\[ = \left( \frac{\mathbf{E}_0 \times \mathbf{B}}{B^2} \right) x \hat{x} + \left( \frac{\mathbf{E}_0 \times \mathbf{B}}{B^2} \right) y \hat{y} \]
\[ = \frac{\mathbf{E}_0 \times \mathbf{B}}{B^2}. \quad (6.6) \]

Therefore, the group velocity is simply the \( \mathbf{E}_0 \times \mathbf{B} \) drift. This is observed in the SAMI3 numerical results, as the MSTID structure propagates with the background \( \mathbf{E}_0 \times \mathbf{B} \) drift.

From both the phase and group velocity perspective, the SAMI3 numerical results agree with the linear theory of MSTIDs. The model results are in disagreement with experimental observations of MSTIDs primarily propagating in the westward and equatorward direction. It should be noted that due to \( \gamma \propto k \cdot (\mathbf{E}_0 + \mathbf{U} \times \mathbf{B}) \), an appropriate \( \mathbf{E}_0 \) and \( \mathbf{U} \) can result in both positive growth of a MSTID and a phase velocity in the observed direction. Figure 6.5 provides this scenario as an example. Due to the angle between \( \mathbf{E}_0 \times \mathbf{B} \) and \( \mathbf{k} \) being greater than 90°, the phase propagation is negative for the \( \mathbf{k} \) in the first quadrant (i.e., a southwest phase propagation). However, the growth rate is positive due to the angle between \( \mathbf{E}' = \mathbf{E}_0 + \mathbf{U} \times \mathbf{B} \) and \( \mathbf{k} \) being less than 90°. This suggests that under an appropriate \( \mathbf{E}_0 \) and \( \mathbf{U} \), MSTIDs could develop and propagate in the SAMI3 numerical model that agree with observations.

Previous numerical studies have also encountered the limitation of the structures propagating in the observed direction (Yokoyama et al., 2008). Subsequent numerical modeling studies included a sporadic E (Es) layer (Yokoyama and Hysell, 2010), which resulted in an increased MSTID growth rate and enabled the structures to propagate in the direction consistent with observations. In the current study, although the perturbation modeled the initial effects of a Es layer, the full, dynamic coupling of a Es layer was not included, which could explain the lack of equatorward and westward propagation.

Kelley (2011) has hypothesized that MSTIDs originate as gravity waves in the auroral zone, which are common near this region. Then, Joule heat-
Figure 6.5: Example scenario of $E_0$, $U$, and $k$ which results in positive growth and a southwest phase velocity.
ing dampens out every mode not consistent with the Perkins instability as the structures propagate toward the equator. This theory uses the Perkins instability primarily as a filtering mechanism rather than a generative mechanism for MSTIDs. The proposed theory by Kelley (2011) could also explain why the original theory of MSTIDs and model results are inconsistent with observations. That is, the structures begin as atmospheric waves that travel equatorward, as opposed to being initially generated at mid-latitudes.

6.5 Conclusion

In this chapter, we have used the SAMI3 numerical model to show the self-consistent generation of MSTIDs. Starting with an initial local conductivity perturbation, dominant \( k \) modes are established between \( E' = E_0 + U \times B \) and magnetic east, which is consistent with the theoretical description of MSTIDs and previous numerical simulation work. Also, the linear growth rate predicted by the Perkins instability agrees well with the model results for about 30 minutes, after which the instability growth saturates in the model.

In the next chapter, we take advantage of the self-consistent generation of MSTIDs in SAMI3 for synthetic observations of the instability and compare the results against observational data found in the literature. Given that these are the first numerical results that use a full magnetic flux tube grid for the development of MSTIDs, the SAMI3 model will be used to investigate its conjugate nature. Also, parameters that affect the growth of the instability in the model will be modified, and these investigations will serve as case studies for isolating key parameters involved for the generation of MSTIDs. These studies take aim to further elucidate the physics of MSTID development in the nighttime ionosphere through the use of numerical simulations.
This chapter will present synthetic observations of MSTIDs in the SAMI3 model, in addition to case studies investigating parameters that affect the development of the instability\(^1\). The work leverages the self-consistent modeling of MSTIDs in SAMI3 from the previous chapter. The synthetic observations mimic instrumentation that is commonly employed to study MSTIDs, and the calculations will be compared to observational data to verify the characteristics and development of the instability in the model. The numerical case studies will identify key parameters that affect MSTID growth, in part based on the theoretical growth rate equation. That is, neutral wind scaling effects, solar conditions (which affect the neutral scale height, \(H_n\)), the influence of the equatorial anomaly, and dip angle dependence will be investigated for their impact on MSTID growth in SAMI3.

### 7.1 Synthetic Observations

In this section, the results from the previous chapter of the self-consistent generation of MSTIDs will be used to calculate synthetic observations. Specifically, we calculate the integrated total electron content (TEC), integrated 630.0-nm airglow emission, in addition to \(\mathbf{E} \times \mathbf{B}\) drifts and electron density, which are measured by dual-frequency GPS receivers, airglow imaging cameras, and incoherent scatter radars (ISRs), respectively. A comparison of synthetic observations to experimental observations found in the literature will aid a discussion of the numerical modeling of MSTIDs within SAMI3.

The Case 2 perturbation from Section 6.3.2 will be used for the synthetic results presented here. Recall that this perturbation investigated the growth of a specific \(k\) mode within SAMI3 by initially redistributing the density

\(^1\)Portions of this chapter are based on the work published in Duly et al. (2014).
along the magnetic field lines in the Northern Hemisphere. As discussed in Section 6.3.2, the growth rate of this mode is similar to the theoretical value, and the growth saturates at approximately 30 minutes. The results presented are displayed at 45 minutes into the simulation, once the MSTID has been fully developed in the model.

7.1.1 Data presentation

Recall from Section 1.7 that dual-frequency GPS receivers are able to calculate the TEC by the phase delay between two measured GPS signals. In SAMI3, we can numerically integrate the density in altitude to calculate the TEC:

\[
\text{TEC} = \int_{85 \text{ km}}^{800 \text{ km}} n_e \, dh \cdot \frac{1 \text{ TECU}}{10^{16} \text{ electrons/m}^2} \cdot \frac{10^{16} \text{ electrons/m}^2}{1 \text{ TECU}}.
\] (7.1)

This equation is based on the general expression from Equation 1.29 with \( h_0 = 85 \text{ km} \) and \( h = 715 \text{ km} \). This altitude range captures the majority of electron density in SAMI3 (e.g., Figure 5.1).

Figure 7.1 presents the TEC values as a function of latitude and longitude for both the Northern (top row) and Southern (bottom row) hemispheres. Each pixel in this figure represents one synthetic measurement of the TEC by a GPS receiver. The values from the perturbation run (left column) are displayed in addition to the percent change with respect to the control run in which no perturbation was applied (right column). The percent change is shown to isolate the perturbation due to the MSTID within SAMI3. Notice that even though the initial perturbation was applied in the Northern Hemisphere, the signature of the MSTID is prevalent in the Southern Hemisphere as well. This feature will be further discussed in the subsequent section.

The integrated 630.0-nm airglow emission is also calculated within SAMI3, which could represent the measurements taken by airglow imaging cameras, such as the ones used for the climatological study of MSTID occurrences in Chapter 4. In SAMI3, the volume emission rate (VER), \( V_{630.0} \), is calculated following the model presented in Section 1.6 (Link and Cogger, 1988, 1989), and similar to the TEC calculation, the VER is numerically integrated in altitude:
Figure 7.1: The total electron content (TEC) calculated within the SAMI3 model for the self-consistent generation of MSTIDs. The Northern and Southern hemispheres are displayed (top and bottom rows, respectively). The perturbation run is shown (left column) in addition to the percent change with respect to the control run (right column). Reprinted from Duly et al. (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.
Figure 7.2: Similar to Figure 7.1, but the integrated 630.0-nm airglow emission is displayed. Reprinted from *Duly et al.* (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.

\[
\text{AG}_{630.0} = \int_{85 \text{ km}}^{800 \text{ km}} \frac{\text{Equation 1.28}}{\text{V}_{630.0}} \, dh \cdot \frac{1 \text{ R}}{10^6 \text{ ph/cm}^2/\text{s}} \text{ [R].} \quad (7.2)
\]

Figure 7.2 displays the results from this synthetic observations. Analogous to Figure 7.1, each point in the figure represents one synthetic pixel measurement. Also, the airglow perturbation is prevalent in the Southern Hemisphere, which can be considered a signature of a MSTID there.

Next, we present synthetic results of eastern \( \mathbf{E} \times \mathbf{B} \) drift and electron density, representative of measurements taken by an ISR. These data are presented as a function of altitude and longitude, which mimic several altitudinal scans at each longitude by the radar system. Figure 7.3 shows the eastern component of the \( \mathbf{E} \times \mathbf{B} \) drift at 28°N latitude, from the perturbation run (left) and also the absolute change with respect to the control run (right). The absolute change, as opposed to the percent change, was selected to represent the perturbation due to the large altitudinal gradients of the drift. Similarly, profiles of the electron density are displayed in Figure 7.4.
Figure 7.3: Altitudinal profiles of the eastern $E \times B$ drift as a function of altitude and longitude at 28°N latitude. The results from the perturbation run are displayed (left), in addition to the absolute change with respect to the control run (right). Reprinted from *Duly et al.* (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.

Figure 7.4: Similar to Figure 7.3, but the electron density, $n_e$, is displayed. Reprinted from *Duly et al.* (2014) by permission of John Wiley and Sons. Copyright 2014 by the American Geophysical Union.
7.1.2 Discussion

From Figure 7.1, there is about a 1% change in the TEC value due to the MSTID developed in SAMI3. This agrees with previous studies of MSTIDs using dual frequency GPS receivers in the Japanese, European, and American sectors that measured TEC fluctuations on the order of 1% (Kotake et al., 2006). However, additional studies have found MSTIDs to have fluctuations of about 1 TECU (Ogawa et al., 2002), which is larger than the current modeling results. The background TEC in the Ogawa et al. (2002) study was ∼ 10 TECU, which is about an order of magnitude larger than the background TEC of the current study. The low background TEC values are a result of simulating solar minimum in SAMI3 (F10.7 = 75 SFU). A case study in the next section will investigate the solar cycle dependence on the generation of MSTIDs, which affect the background values such as TEC.

Figure 7.2 displays the integrated 630.0-nm airglow emission, and here we see the MSTID signature as a perturbation in airglow intensity. As discussed in Section 1.6, an altitudinal change in the F-layer will result in a change in airglow intensity, and as the MSTID is developed in SAMI3, the $\mathbf{E} \times \mathbf{B}$ forces result in the height layer change. MSTID climatological studies have found 630.0-nm airglow intensity perturbations to be from 5-15% (Shiokawa et al., 2003a; Ogawa et al., 2002), and the current study agrees with the lower end of this range.

In both Figures 7.1 and 7.2, it is apparent that the MSTID signature is prevalent in the Southern Hemisphere, even though the initial perturbation occurred in the Northern Hemisphere. The perturbation values in the Southern (load) Hemisphere are smaller compared to the Northern (generative) Hemisphere, suggesting that the load hemisphere produces a smaller perturbation compared to the generative hemisphere. Previous studies have investigated the conjugacy effect. For example, airglow imaging experiments have observed conjugate MSTIDs (Otsuka, 2004) (see also Figure 2.5), and the features presented from the current modeling study agree well with conjugate observations. The conjugacy is a result of the high conductivity along the magnetic field line, which is appropriately modeled in SAMI3. That is, the electric field from the electrostatic potential ($\mathbf{E}_\perp = -\nabla_\perp \Phi$) is equivalent for all values along the magnetic field line. Therefore, as the electric field associated with the MSTID develops in the Northern Hemisphere due to the
Perkins instability, the electric field produces a similar effect in the Southern Hemisphere to displace electron density, which in turn affects the TEC and 630.0-nm airglow emission in the respective hemispheres. These are the first numerical modeling results that use the full-flux tube grid of SAMI3 to show the conjugacy of MSTIDs.

Figure 7.3 shows the effects of a MSTID on the eastern $E \times B$ drift value. The results show about a $\pm 2$ m/s variation in the eastern $E \times B$ drift. Previous experimental studies have used ISRs to measure perturbation $E \times B$ drift values of MSTIDs (Kelley et al., 2000). Although the modeling results match the observations qualitatively, the observations by Kelley et al. (2000) measured perturbations in the drifts of about an order of magnitude larger compared to the current modeling results, suggesting that other mechanisms may play a role to produce a larger perturbation.

The electron density profiles in Figure 7.4 show the effects of the perturbation drift within the SAMI3 model. Perturbations in the drift calculation cause electron density to be moved across the magnetic field lines, resulting in the enhancements and depletions, respectively, depicted in Figure 7.4. The altitude of the peak electron density, $h_m F_2$, has variations of about $\pm 10$ km in the model at 28°N latitude. Again, similar to the eastern $E \times B$ drift perturbations, the results presented here are qualitatively consistent with previous modeling results, but larger perturbations have been measured experimentally (Behnke, 1979; Kelley et al., 2002a; Kelley, 2009, Figure 6.25).

### 7.2 Case Studies

The previous simulation work has established that MSTIDs can be generated within the SAMI3 numerical model and that the basic descriptions of MSTIDs from experimental observations agree well with the synthetic observations derived from the model. Now, we can take advantage of the model with simulation case studies to investigate the mechanisms that influence MSTID growth. The case studies are designed to isolate the effect of a particular parameter (i.e., $U$, $H_n$, $n_e$, $D$) in SAMI3 on the MSTID. In this way, we are able to understand how various conditions impact the development of the instability.

The results from four case studies are presented. The first case study scales
the HWM93 neutral wind values by a constant factor. Next, SAMI3 is run for solar maximum conditions to investigate solar cycle dependences. Another case study simulates a MSTID approaching the equatorial anomaly, a region of enhanced electron density. Finally, the last case study investigates the magnetic dip angle effects on the growth of MSTIDs. For each case study, the model is perturbed using the prescribed wavevector perturbation from Section 6.3.2, which initially redistributes the electron density along the field line in the Northern Hemisphere to introduce a change in the integrated Pedersen conductivity. The ΔΣ_p parameter is plotted as a function of time, similar to Figure 6.4, to succinctly represent the growth of MSTIDs within the model.

7.2.1 Neutral wind scaling effects

The neutral wind plays a vital role in the growth rate for MSTIDs (see Equation 6.3). Recall that the neutral wind, through the effective electric field, \( E' = E_0 + U \times B \), polarizes the banded structures of vertically raised and lowered electron density, and this unstable scenario causes the development of MSTIDs (i.e., Figure 3.5). Therefore, it should be expected that as the \( U \) parameter is changed, the resulting growth of the MSTID is also affected. For this case study, SAMI3 is executed with a 20% decrease and increase in the neutral wind values calculated by HWM93, presented as “0.8 \( \times \) \( U \)” and “1.2 \( \times \) \( U \),” respectively, and compared against the nominal case, “1.0 \( \times \) \( U \).”

Figure 7.5 shows the growth for each case. The increased neutral wind from the “1.2 \( \times \) \( U \)” case results in a faster growth (i.e., a steeper slope) of approximately \( 1.9 \times 10^{-3} \) [e-folds/s] (estimated from the initial growth) compared to the nominal case of about \( 1.6 \times 10^{-3} \) [e-folds/s], an increase of about 19%. A decrease in the neutral wind results in a slower growth, approximated from Figure 7.5 as \( 1.4 \times 10^{-3} \) [e-folds/s] (about 13% less than the nominal case). From Equation 6.3, the theoretical growth rate is a function of both \( E_0 \) and \( U \). The background electric field, \( E_0 \), calculated from the potential solution in Equation 5.13, approximately scales with the neutral wind value. As a result, the theoretical growth rate is approximately scaled by the neutral wind factor imposed in this case study. The modeling results from Figure 7.5 agree with this description.
Figure 7.5: Case study of MSTID growth by scaling the neutral wind, $U$. Scaling factors of 0.8 (dotted red boxed) and 1.2 (dashed green triangles) are used to compare against the nominal case (solid blue circles).

Figure 7.5 shows that the saturation levels are proportional to the scaling factor of the neutral wind. However, the times that the levels saturate appear to be consistent, with each saturation case occurring at approximately 30 minutes. The numerical modeling of MSTIDs enables this effect to be studied, which is not described by the linear theory. Overall, the neutral wind has a significant effect on the growth of MSTIDs, and the parameterization of neutral winds, either through climatological modeling or experimental measurements, is crucial to describe the development of MSTIDs.

7.2.2 Solar cycle effects

As discussed in the climatological study (Section 4.4.3), large occurrence rates of MSTIDs were recorded during solar minimum conditions. This case study will investigate the development of MSTIDs under different solar flux values, representative of various solar conditions. In the theoretical linear growth rate, solar cycle dependence is represented by the neutral scale height term, $H_n$. Following the derivation of the Perkins instability (Equation C.14), the neutral scale height can be defined as:
Figure 7.6: Example profiles of $\nu_{in}$ for both solar maximum and solar minimum conditions. $\nu_{in}$ is calculated by Equation 1.20 and the upper atmospheric parameters are obtained using climatological models.

$$\frac{1}{H_n} = -\frac{1}{\nu_{in}} \frac{\partial \nu_{in}}{\partial h}. \quad (7.3)$$

The ion-neutral collision frequency, $\nu_{in}$, is dependent on solar condition, and $h$ is in the vertical direction. Figure 7.6 plots two example profiles of $\nu_{in}$, for both solar maximum and solar minimum years (2001 and 2008, respectively). From these profiles, $H_n$ can be calculated using central differencing for the numerical differentiation of $\partial \nu_{in}/\partial h$ in Equation 7.3. For the solar maximum case, $H_n = 58.7$ km, while for the solar minimum case, $H_n = 38.6$ km (both measured at 250 km altitude). The growth rate is inversely proportional to the neutral scale height, and as a result is larger during solar minimum. These characteristics should be reflected in the numerical simulations.

Figure 7.7 plots the results of MSTID growth in SAMI3, for both the solar maximum case ($F_{10.7} = 175$ SFU) and the nominal, solar minimum case ($F_{10.7} = 75$ SFU). For the solar maximum case, the initial growth rate is smaller compared to the solar minimum case, as predicted by the linear theory. However, the MSTID in the solar maximum run saturates at a later time compared to the solar minimum run, and the $\Delta \Sigma_P$ value overtakes the value from the solar minimum case at around 35 minutes.
The modeling results from this case study agree with the description provided by the linear theory, but the saturation levels, which are not predicted by the theory, appear to vary between solar conditions. The simulation results suggest that although there is slower MSTID growth during solar maximum, the amplitude of $\Delta \Sigma_P$ reaches a higher value. However, for the solar maximum case, the background airglow intensity values are increased and the percent change in airglow intensity compared to the background values is $\sim 3.2\%$ (not shown), which is smaller compared to $\sim 6.0\%$ from the solar minimum case (Figure 7.2). This suggests that MSTIDs may be difficult to detect in airglow instrumentation during solar maximum conditions.

It should be noted that the neutral wind configuration (i.e., magnitude and direction) may be different between solar conditions. As investigated in the previous case study, the neutral wind can greatly influence the growth of the instability. For the current case study, only the effects of solar conditions are isolated through the $H_n$ term, and therefore used the same HWM93 values for both cases. Also, the seed mechanisms for MSTIDs, either by gravity waves, sporadic E layers, or another mechanism, may also vary between solar conditions, both in the strength and occurrence rate of the seed.
7.2.3 Modeling the equatorial anomaly region

The climatological study detailed in Chapter 4 noted that MSTIDs could reach dip angles as small as $|D| = 14^\circ$. In addition, previous experimental studies have observed MSTIDs traveling to low-latitudes and providing the seed mechanism for ESF (Miller et al., 2009). In order to further investigate the viability of MSTIDs reaching low-latitudes and potentially seeding ESF, the next case study models MSTIDs as they approach the equatorial anomaly, a region of enhanced electron density. The equatorial anomaly region lies in the path of a MSTID as it travels equatorward, and this case study will investigate the behavior of instability growth subject to the increased density of the region.

As noted in Section 6.3.2, the MSTIDs developed in SAMI3 move eastward along with the $\mathbf{E} \times \mathbf{B}$ drift. Although the direction agrees with the theoretical description, it is in contrast to experimental observations, which have commonly recorded MSTIDs traveling westward and toward the equator. Therefore, to model the instability as it approaches the equatorial anomaly region, the density is linearly increased in the model near the instability by a factor of two from $t = 0$ minutes to $t = 30$ minutes. This method is advantageous to isolate only the effects of an increased $n_e$ as a function of time, as opposed to other factors such as low dip angles, which will be investigated in the next case study.

Figure 7.8 plots the results from the case study, showing both the nominal case and the simulation with the MSTID subject to an increased electron density. As shown from the results, the enhanced electron density causes the growth in the model to decrease with respect to the nominal run. An increased electron density will increase the background integrated Pedersen conductivity, $\Sigma_{P,0}$. Equation 6.1 shows that the perturbation potential, $\tilde{\Phi}$, is inversely proportional to the background Pedersen conductivity, $\Sigma_{P,0}$. Therefore, an increase in $\Sigma_{P,0}$, due to the equatorial anomaly region, will produce a smaller perturbation potential. As a result, the electric fields that drive the instability will be smaller, and consequently the instability growth rate should decrease, which is consistent with the modeling results presented here.

This decreased growth rate due to the density enhancement can also be due to the neutral scale height term, $H_n$, as shown in Equation 6.3. That is, an increased $n_e$ results in a larger ion-neutral collision frequency, which increases
Figure 7.8: MSTID growth in SAMI3 for the nominal case (solid blue circles) and subject to increased electron density (dashed green triangles).

$H_n$, and as a result decreases the growth rate. This effect is analogous to the results from the solar conditions case study, in which solar maximum conditions increased the electron density in a similar manner.

The modeling results presented show that although the equatorial anomaly region may slow the growth of MSTIDs, it does not completely inhibit the instability from developing (i.e., the growth rate remains positive subject to the increased electron density). Previous work has proposed an equatorward limitation of MSTIDs to be around $|D| = 39^\circ$, attributing the equatorial anomaly region playing a role to inhibit the growth of MSTIDs (Shiokawa et al., 2002). In their work, Shiokawa et al. (2002) noted that ion drag is larger in regions of increased electron density, and thus gravity waves, a possible seeding mechanism for MSTIDs, could be inhibited to initiate the development of MSTIDs. However, the modeling results presented here indicate that although the seeding mechanism may not be available in the equatorial anomaly, the region does not prevent MSTIDs from developing as they pass through the enhanced $n_e$. In addition, the current modeling results support the previous observations of MSTIDs traveling to low-latitudes (Miller et al., 2009; Duly et al., 2013).
7.2.4 Small dip angle effects

The final case study investigates the effects of generating MSTIDs at a smaller magnetic dip angle, $D$. The nominal case has $D = 46.72^\circ$ (Table 6.1), and here a MSTID is also generated at $D = 40.93^\circ$ for comparison. This value is chosen close to the nominal case in order to isolate the effects of the magnetic dip angle only. At a given altitude, the neutral wind configuration and background electron density are a function of $D$, and the location at $D = 40.93^\circ$ was selected to mitigate the effects of large changes in these parameters compared to the region at $D = 46.72^\circ$.

Figure 7.9 plots the nominal $D = 46.72^\circ$ case in addition to a MSTID generated at $D = 40.93^\circ$. The simulation results show an increased growth rate for the smaller dip angle case. In order to investigate the theoretical growth rate against a change in dip angle, Equation 6.3 is broken down into two terms:

$$\gamma = \cos D \left[ -E_{0y} + \frac{k_y}{k^2} k \cdot (E_0 + U \times B) \right] + \frac{U_z \sin D}{H_n} \ [\text{e-folds/s}]. \ (7.4)$$

Written in this form, Term 1 is proportional to $\cos D$ while Term 2 is proportional to $\sin D$.

Using the values from Table 6.1, Figure 7.10 plots Term 1, Term 2, and their summation (i.e., the theoretical $\gamma$) as a function of $D$. Both terms and their summation increase as the dip angle decreases (Term 2 has this property because $U_z$ is negative). The modeling results agree with the theoretical growth rate description. It is important to note that the values in Figure 7.10 were calculated by changing $D$ only, and that the remaining terms in Table 6.1 were otherwise constant in the calculations. Although it may not be realistic to assume that the neutral wind pattern (and other parameters) at $|D| << 46.72^\circ$ is equivalent that to mid-latitudes, the theoretical calculations in Figure 7.10 provide insight on isolating the effects of the magnetic dip angle only.

Intuitively, MSTIDs can be thought of as the result of the mechanisms associated with unstable perturbation electric field modes developing in the nighttime, mid-latitude ionosphere. Given a perturbation in the electric
Figure 7.9: Comparisons of the growth of MSTIDs with different magnetic dip angles, $D$. The nominal case is shown with $D = 46.72^\circ$ (solid blue), as well as $D = 40.93^\circ$ (dashed green).

Figure 7.10: The theoretical growth rate plotted as a function of $D$ (solid black). Portions of the growth rate are also shown (see Equation 7.4), including the terms proportional to $\cos D$ (Term 1, dashed red) and proportional to $\sin D$ (Term 2, dotted blue). Equation 7.4 is used for the calculation, and the remaining terms in this equation are taken from Table 6.1.
field, the resultant \( \mathbf{E} \times \mathbf{B} \) drifts perpendicular to the magnetic field line cause a height displacement, \( \Delta h \), in electron density. The resultant height change is proportional to \( \cos D \). For example, at the magnetic equator \( (D = 0^\circ, \cos D = 1) \), the perturbation \( \mathbf{E} \times \mathbf{B} \) drift is completely in the vertical direction. This is in contrast to the other extreme with vertical magnetic field lines \( (D = 90^\circ, \cos D = 0) \), in which a perturbation \( \mathbf{E} \times \mathbf{B} \) drift is entirely in the horizontal direction, which would not enable the instability to develop. Therefore, at \( D = 40.93^\circ \), the mechanism that creates the instability has a larger effect compared to at \( D = 46.72^\circ \), and the modeling results agree with this description.

It should be noted that for a given initial redistribution of electron density along the magnetic field line (i.e., the initial perturbation), the height change of electron density, which is proportional to \( \Delta \Sigma_p \), is also proportional to \( \sin D \). In other words, at the magnetic equator, a change of density along the magnetic field lines does not produce a vertical height change of density. Therefore, there are two competing effects: the effectiveness of the seeding mechanisms as a function of \( \sin D \) in addition to the \( \cos D \) dependence due to the theoretical growth rate. This is summarized in Figure 7.11, in which the growth rate term from Figure 7.10 is penalized by a \( \sin D \), “seeding effectiveness” factor.

Previous work has shown that the Perkins instability growth rate is proportional to \( \cos D \) (Perkins, 1973; Huang et al., 1994; Hamza, 1999; Garcia et al., 2000; Zhou and Matheus, 2006; Makela and Otsuka, 2011). However, researchers have also stated that the theoretical growth rate is proportional to \( \sin^2 D \) (Perkins, 1973; Makela and Otsuka, 2011). The derivation to transform the growth rate equation to be proportional to \( \sin^2 D \) involves the following substitution (e.g., Makela and Otsuka, 2011):

\[
\frac{|\mathbf{E}'| \cos D \cos \theta}{B} = g \frac{\sin^2 D}{\langle \nu_{\infty} \rangle} \quad (7.5)
\]

Equation 7.5 is derived by considering steady-state forces in the mid-latitude ionosphere. That is, the effects of the electric field, through the upward component of \( \mathbf{E} \times \mathbf{B} \) drifts (left-hand side of Equation 7.5), must be balanced by downward gravitational and diffusion forces (right-hand side of Equation 7.5). Using the growth rate equation with the electric field description (i.e., Equation 6.3), which is proportional to \( \cos D \), Equation 7.5
Figure 7.11: Similar to Figure 7.10, but also shows the theoretical growth rate with a \( \sin D \), seeding effectiveness factor.

is used to substitute the gravity and ion-neutral collision frequency terms. From this substitution, Equation 3.11 can be derived and the growth rate is proportional to \( \sin^2 D \).

However, it may be problematic from a theoretical standpoint to use this substitution. First, the substitution of a steady-state condition into the growth rate equation may not be appropriate as the growth rate equation models the dynamic growth of the instability. Additionally, given that the effective electric field plays a fundamental role in the development of MSTIDs (Sections 3.4, 3.5, 7.2.1, Equation 6.1), using the growth rate equation with the gravity and ion-neutral collision frequency description may mask out the main driver of the instability. Furthermore, the \( \langle \nu_{in} \rangle \) term is a function of \( E' \), and not vice-versa. That is, given that \( E' \) is calculated by the potential equation and the neutral winds, a \( \langle \nu_{in} \rangle \) is selected (which is a surrogate for layer height) that will balance downward diffusion and electric fields driving plasma upward in steady-state. Therefore, the growth rate description from Equation 6.3, which is proportional to \( \cos D \), may provide a more complete description of MSTID dynamics.
7.3 Conclusion

This chapter has demonstrated the utility of SAMI3 for investigating the conditions conducive to MSTID development. Synthetic observations of MSTIDs were calculated in the model that provided TEC, 630.0-nm airglow emissions, and profiles of both eastern $E \times B$ drifts and $n_e$. The results showed perturbations in both the TEC and airglow emission, and height-layer changes in the F-peak. Furthermore, the MSTID signature was prevalent in the conjugate hemisphere. These qualitative descriptions of the synthetic observations match the experimental observations. Further work is needed to add a Es-layer in SAMI3, which may increase the growth rate and produce larger perturbation values in the synthetic observations. As a result, the quantitative descriptions of MSTIDs might become more consistent with observations.

Also, the model was used for case studies of MSTID development. The studies highlighted that the magnitude of the neutral wind plays an important role in the growth of the instability. During solar maximum conditions with all other parameters equal, the model results showed that the instability could develop within SAMI3 at a slower rate compared to solar minimum conditions. However, the remaining parameters (e.g., the neutral wind) may be different under various solar conditions, which can greatly influence the growth of MSTIDs.

The growth of MSTIDs was investigated as the instability was subject to an increased electron density, mimicking a MSTID approaching the equatorial anomaly region. Again, the model results showed that although the growth rate was decreased, the instability was able to develop. The dip angle effects of the growth were also studied, and a decrease in $D$ resulted in faster growth in the model. It was argued that the electric field description of the growth rate (Equation 6.3), which is primarily proportional to $\cos D$, describes the fundamental mechanism that generates the instability.
CHAPTER 8

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This work has advanced the understanding of MSTIDs using both observational and numerical simulation techniques. The theoretical work on MSTIDs, originally discussed by Perkins (1973), provides a basic insight as to how the instabilities are generated in the nighttime, mid-latitude ionosphere. The linear theory also outlines the major parameters that affect the instability growth, and describes the important physical concepts that must be addressed in order to self-consistently generate the instabilities within a numerical framework. For example, given appropriate background conditions, maintaining divergence free current densities ($\nabla \cdot J = 0$) in the ionosphere is crucial to develop and sustain MSTIDs from both a theoretical and numerical perspective.

The climatological study in this work established MSTID occurrences from over six years of 630.0-nm airglow imaging data at two longitudinal sectors. The data were in good agreement with the basic theory, and large occurrences of MSTIDs were recorded during solar minimum conditions throughout the solstice time periods. It was found that the neutral wind, either from the local or conjugate hemisphere, may play a substantial role in the development of the instabilities. Higher occurrence rates of MSTIDs were observed at larger magnetic dip angles relative to observations near the geomagnetic equator. However, there were several instances of the instability recorded at dip angles as low as $|D| = 14^\circ$, suggesting that MSTIDs are not restricted from propagating to the low latitudes.

The SAMI3 wedge model was introduced as a tool for studying the upper atmosphere, and serves as a powerful numerical simulation framework for investigating the physical processes involved for the self-consistent generation of MSTIDs. First, initial simulations showed that the physics within the model are able to capture the appropriate mechanisms that produce the instabilities. A random perturbation resulted in modes consistent with the
Perkins instability (i.e., k modes between $E' = E_0 + U \times B$ and magnetic east), and a perturbation with a prescribed k created growth that was slightly faster than the theoretically predicted value, but was on the same order of magnitude of $\sim 10^{-4}$ [e-folds/s].

With the establishment of MSTIDs modeled in SAMI3, synthetic observations were calculated, including the TEC and vertically integrated 630.0-nm airglow emission. These calculations are representative of experimental data recorded by dual-frequency GPS receivers and airglow imagers that observe MSTIDs. In addition, profiles of the electron density and eastern $E \times B$ drifts, representative of measurements taken by an ISR, were provided. With an initial perturbation occurring in the Northern Hemisphere, the signature of the instability also appeared in the conjugate, Southern Hemisphere. This effect has been previously observed, and these are the first self-consistent modeling results, using a numerical framework such as SAMI3, to show the signature of a MSTID in the conjugate hemisphere. The general features of the synthetic observations matched studies found in the literature, although quantitatively the values were smaller compared to experimental results.

Finally, case studies of MSTID development in SAMI3 were conducted. The results show that the growth rate of MSTIDs approximately scales with the magnitude of the neutral wind. A case study during solar maximum conditions showed a decrease in growth rate, $\gamma$ (as predicted by the linear theory), but given an initial perturbation, MSTIDs are still established within the model. A MSTID approaching the equatorial anomaly was also modeled by increasing the electron density within the simulation. As the instability was subject to the enhanced density, the growth rate decreased but remained positive, and thus the instability was able to develop. This case study found that MSTIDs may be able to pass through the equatorial anomaly region without being severely damped by the increase in $n_e$. The last case study simulated a MSTID at a smaller geomagnetic dip angle, $D$, and an increased growth rate was recorded. Given that the remaining parameters are held constant (including the initial perturbation), the model results suggest that $\gamma \propto \cos D$.

This study has investigated the Perkins instability as the generative mechanism for the development of MSTIDs in the mid-latitude, nighttime ionosphere. It should be emphasized that the growth of the instability, as predicted by the linear theory, is slow, on the order of $10^{-4}$ [e-folds/s], and the
SAMI3 numerical model was used to verify the growth of MSTIDs through numerical simulations. Although there is small growth of the instabilities, this work suggests that the physics described by the Perkins instability does indeed play a role in the development of MSTIDs. However, the full generative mechanism is not yet completely understood.

Although this work has significantly advanced the knowledge of the physical processes that are involved for the generation of MSTIDs, there remain open questions on their properties. A few potential research topics that could be addressed in future studies that utilize the results established in the current work are as follows:

- **Using experimentally derived or first-principle model neutral winds in SAMI3 in conjunction with airglow imaging observations.** The current studies have shown that the neutral winds play an important role in the generation of MSTIDs. In the numerical simulation work, the neutral winds were taken from the climatological HWM93 model, but it would be beneficial to measure the neutral winds in the F-region (ideally from each hemisphere) in conjunction with airglow imagers recording the development of MSTIDs. Then, the measured neutral winds could be input into SAMI3 and with an initial perturbation, the resultant synthetic observations of the MSTID could be compared with the results from the airglow imaging data. In addition, the neutral winds could be calculated from a first-principles model, such as by the TIEGCM and/or the TIME-GCM models, providing a more realistic neutral wind configuration for the development of MSTIDs. These exercises would further emphasize the importance of a robust model of the neutral winds for understanding upper atmosphere dynamics.

- **Implementation of a sporadic E (Es) layer in SAMI3.** As noted in the numerical simulation work, previous studies have found that the addition of a Es layer in a numerical model can enhance the growth rate of MSTIDs and propagate the structure westward and equatorward (Yokoyama and Hysell, 2010). This could be done by including a simple time-dependent model of a metallic ion in SAMI3, such as Fe+ or Mg+, between an altitude range of about 100-120 km. With the addition of a Es layer in SAMI3, the magnitude of the perturbations in the synthetic
measurements may better align with experimental observations. Given
that the developed MSTIDs propagate equatorward, the full equator-
ward extent of the simulation space in SAMI3 will enable investigations
of MSTIDs traveling to low-latitudes, which was observed in the cli-
matological study from the current work. Furthermore, the coupling
to low-latitude instabilities, which has been previously observed exper-
imentally (Miller et al., 2009), can also be explored within the SAMI3
numerical model.

• Investigating the seeding mechanisms for the self-consistent
generation of MSTIDs. In the current numerical simulation work,
the seeding mechanism was directly imposed within the SAMI3 model.
Instead of an artificial perturbation, it would be interesting to couple a
gravity wave model with SAMI3 in order to investigate the effectiveness
of the seeding mechanism for generating MSTIDs. Various seeding
mechanisms (e.g., by Es layers) could also be implemented within the
MSTID/SAMI3 framework created in the current work. In this way,
a more complete description of how MSTIDs are generated could be
developed.

• Model gravity wave propagation from high-latitude regions.
In order to investigate the Joule heating filtering mechanism proposed
by Kelley (2011), gravity waves beginning in the high latitudes can
be modeled in SAMI3 as they propagate equatorward. As a result,
the extent the Perkins instability plays in interacting with these waves
could be studied. This would lead to further insight as to how MSTIDs
are developed in the nighttime ionosphere through understanding how
the Perkins instability may filter waves as opposed to strictly generating
them.
Pyglow is a collection of wrappers for climatological models commonly used in the upper atmosphere community. In this appendix, we give a basic outline of the implementation of this package, and discuss its strengths. The overall goal of pyglow is to call climatological models in the highly flexible, high-level Python programming language.

At the time of this writing, the source code for pyglow is available at https://github.com/timduly4/pyglow. The code is open-sourced, encouraging modification and improvements to the code base.

A.1 Introduction

As mentioned in Section 1.3, climatological models are used to calculate upper atmosphere parameters based on measured historical data. That is, instruments measure quantities in the upper atmosphere, and along with mathematical and statistical techniques, a general seasonality is calculated. This is translated into computer programs and can be accessed by a user. Table 1.3 lists a few models that have been developed, along with the respective parameters they provide.

The numerical codes are commonly written in the Fortran programming language, for several reasons. Historically, Fortran is a well established programming language and has been efficiently optimized. Therefore, the execution of the climatological models with Fortran is computationally fast. Also, free compilers are widely available and thus the code for the climatological models can be distributed among researchers.

However, this methodology suffers from a few deficiencies. Fortran can be considered a low-level programming language, which is closer in abstraction to a computer’s instruction set, or architecture, compared to a high-level
Figure A.1: Illustration depicting a comparison of the computer abstraction level for various implementations of climatological models. Pyglow offers an implementation between the two existing implementations, yielding several advantages.

language. This is beneficial for its execution speed, but it can be challenging to develop specific application tools, as programming technical details must be respected when working with the low-level programming language (e.g., antiquated control statements and input/output options).

The Community Coordinated Modeling Center (CCMC) has developed tools to make climatological models more user-friendly. For example, the CCMC hosts an “instant run” website\(^1\) for users to obtain results from climatological models, including IRI, MSIS, and IGRF. This approach can be considered very high-level (i.e., a “black box”), as it abstracts almost all of the computer instruction set out of the process in obtaining the climatological values. The Fortran models are run on the CCMC servers and returned to the user on the online website.

In terms of computer abstraction, the pyglow project lies somewhere in the middle of these two approaches. That is, pyglow is higher-level than the Fortran implementations because it uses the high-level language of Python. However, pyglow is lower-level compared to the CSSC on-line web service because pyglow can be used in scripts within Python to run on demand, thus increasing its flexibility. For example, using the matplotlib plotting package commonly used in Python, plotting the climatological values is straightforward with pyglow. The various levels of complexity are summarized in Figure A.1 and compares the Fortran, CSSC, and pyglow implementations.

\(^1\)http://ccmc.gsfc.nasa.gov/requests/instant_run.php
A.2 Wrapping Fortran subroutines in Python with \texttt{f2py}

Generating wrappers to call Fortran routines from Python is completed with the \texttt{f2py} program\textsuperscript{2}. This tool creates the interface between Fortran and Python. Pyglow uses \texttt{f2py} in order to compile the climatological models used in Python.

In \texttt{f2py}, there are two main steps to develop the interfaces between Fortran and Python. First, a \textit{signature file} is created with \texttt{f2py}. Simply put, \texttt{f2py} scans the targeted Fortran subroutine(s) and automatically generates a file that describes the particular subroutine, including the name of the subroutine and its associated inputs and outputs. It also includes the name of the module that will be called in Python.

The second step includes building the extension module. At this step, a Fortran compiler is required. In pyglow the gfortran compiler is used, based on its wide availability. The signature file guides \texttt{f2py} to create a module that can be executed in Python. At the completion, a shared object file with the extension \texttt{*.so} is created, which is a binary executable file.

Figure A.2 displays a diagram showing the relationships between each part of the pyglow package. As shown, the \texttt{f2py} program (and also gfortran) provides the intermediary between the pyglow Python module and the corresponding Fortran climatological models.

During the installation of pyglow, the first step includes downloading each climatological model from its respective location online. Some models require slight modifications to the source code, and the \texttt{patch} tool is used for these fixes. Next, the signature files are generated, and again the \texttt{patch} tool is used to make minor adjustments to these files. For example, it may be required to explicitly declare the input and output variables in order to compile the shared object file. Finally, the gfortran compiler is used in conjunction with \texttt{f2py} to create the shared object that can be used in Python.

In order for the pyglow module to be used system-wide, the \texttt{setup} tool from the \texttt{numpy} package is used to install pyglow. After the modules have been compiled, \texttt{setup} copies them over to a directory which is included in the Python path. Thus, the associated pyglow modules can be imported in

\textsuperscript{2}\texttt{f2py} is included in the \texttt{numpy} Python package. More information is available at \url{http://cens.ioc.ee/projects/f2py2e/}
Figure A.2: A block diagram representing the relationships between the pyglow Python module, f2py, and each climatological model used in the pyglow package.
the user’s Python scripts anywhere in the system.

A.3 Using the climatological models in pyglow

Now that the climatological models are callable from Python, we need a methodology to call them systematically. Each climatological model has its own unique input variables. For example, IGRF requires the co-latitude, which is defined as $90^\circ - \text{lat}$. Also, MSIS includes a strict requirement for the time year and day-of-year value, in the format YYDDD. In addition, some of the models require values of solar indices (Ap, Kp, F10.7, etc.). To resolve these minor idiosyncrasies, pyglow calculates each input for the respective climatological model on execution.

Furthermore, there are several variables that can be returned from the climatological models, including densities of various constituents (both plasma and neutral), magnetic field terms, and neutral wind values. Therefore, an object-oriented approach was used for organizing the upper atmosphere parameters.

One can recognize that a specific set of parameters are associated with a specific geographical point and time. In pyglow, this is represented in the Point class:

```python
from pyglow.pyglow import Point
pt = Point(dn, lat, lon, alt)
```

Member data of this class will include results from the climatological models. Class functions, or methods, will be used to call the respective models, and the specific input to the climatological model will be calculated (e.g., co-latitude) and/or looked up from a database (i.e., solar indices). For example, to execute IRI:

```python
pt.run_iri() # Method to execute IRI within the Point class
print "ne calculated from IRI is found to be:"
print pt.ne
```

Pyglow contains methods that are able to use the climatological models in Python to calculate derived quantities. For example, Section 1.6 describes a model for the 630.0-nm airglow emission, and includes quantities that can be
calculated from IRI and MSIS. Here, pyglow takes advantage of these models to calculate the volume emission rate (Equation 1.28) as a method within the object.

Code Listing A.1 provides an example usage for calculating a profile of the 630.0-nm volume emission rate, and Figure A.3 displays the plot generated from the code. As shown, this plot can be created with less than 50 lines of code, which is simpler than using previously conventional techniques.

Code A.1: Example script that uses the pyglow Python package.

```python
# Example Python script using pyglow
# and its climatological models
# to plot profile of airglow emission.

from matplotlib.pyplot import *
import numpy as np
from pyglow.pyglow import Point
from datetime import datetime, timedelta

matplotlib.rcParams.update({'font.size': 16})

# Setting lat, lon, and a range of altitudes
lat = 18.37 # Arecibo
lon = -66.62
alts = np.linspace(85, 1000, 100)

# set time (dn)
dn_lt = datetime(2012, 3, 21, 12, 0) + timedelta(hours=12)
tz = np.ceil(lon/15.)
dn_ut = dn_lt - timedelta(hours=tz)

# airglow calculation using pyglow:
ag, ne = [], []
for alt in alts:
    pt = Point(dn_ut, lat, lon, alt)
    pt.run_iri()
    pt.run_msis()
```
In this appendix, we described the pyglow Python package that implements wrapped Fortran climatological models as a module in Python. Using an object-oriented approach, the models can be easily executed, manipulated, and plotted in Python. Furthermore, derived quantities (such as 630.0-nm airglow emission) can be calculated within the pyglow framework.
Using pyglow to generate airglow emission profiles derived from climatological models

Figure A.3: With the aid of pyglow, an example volume emission rate profile can be plotted. The code to generate this plot is found in Code Listing A.1.
In general, the growth rate, $\gamma$, describes and parameterizes how fast an instability develops. In this appendix, a description of the growth rate for a general wave is provided. To begin, start with a simple wave equation:

$$\psi(x, t) = \psi_0 e^{j(\omega t - k \cdot x)}.$$  

(B.1)

Here, the wave $\psi(x, t)$ has an amplitude of $\psi_0$ and spatial wavevector, $k$, which describes the spatial periodicity of the wave and is related to the wavelength by $\lambda = 2\pi/|k|$. The angular frequency, $\omega$, describes the temporal periodicity of the wave and is related to the period by $T = 2\pi/\omega$. If we allow $\omega$ to be complex, that is, $\omega = \omega_{Re} - j\omega_{Im}$, then Equation B.1 can be expressed as:

$$\psi(x, t) = \psi_0 e^{j(\omega_{Re} t - k \cdot x)}$$

$$= \psi_0 e^{j((\omega_{Re} - j\omega_{Im}) t - k \cdot x)}$$

$$= \psi_0 e^{\omega_{Im} t} e^{j(\omega_{Re} t - k \cdot x)}.$$  

(B.2)

The growth rate is defined as $\gamma \equiv \omega_{Im}$. Depending on the sign of $\gamma$, $\psi(x, t)$ exponentially grows or decays as a function of time. The growth rate has units of [e-folds/s]. For example, if $\gamma = 1$ [e-folds/s], then after one second the wave will have grown by a factor of $e \approx 2.718$ (hence the term, “e-fold”). If $\gamma < 0$, then the term $e^{\gamma t}$ decays exponentially.

Figure B.1 provides an example of the normalized growth described by Equation B.2. This plot shows how quickly a wave can grow with growth rates on the order of $10^{-4}$ [e-folds/s]. For example, with $\gamma = 6 \times 10^{-4}$ [e-folds/s], the amplitude of the wave increases by an order of magnitude after one hour.
Figure B.1: Example amplitude growth of a wave as a function of time for various growth rates ($\gamma$).
APPENDIX C

THEORETICAL DERIVATION OF MSTIDS
BY PERKINS (1973)

In this appendix, we will follow the work of Perkins (1973) to derive the equations that describe the stability of the nighttime, mid-latitude ionosphere. We provide an in-depth derivation of the equations from Perkins (1973), including detailed steps that were omitted from the original paper. We begin with the governing physics-based equations and assumptions discussed in Section 3.3. That is, the derivations start with the continuity, momentum, and divergent-free current density descriptions (Equations 3.4, 3.5, and 3.6). CGS units are used to maintain consistency with the original work of Perkins (1973).

C.1 Derivation of Moment Equations

First, we use the coupled system of equations to solve for the ion and electron velocities, which is then used in the continuity and current density equations. Next, the equations are integrated along the magnetic field line to reduce dimensionality, providing an analytical description of the equations in the plane of the magnetic field. In this derivation, the neutral wind term, U, is assumed to be zero.

C.1.1 Calculating $V_i$, $V_e$, and $J_\perp$

To begin, we start with the momentum equation (Equation 3.5). From this equation the ion and electron velocity, $V_i$ and $V_e$, can be derived. The velocities are used to subsequently solve for the current density, which is then used to solve for an electrostatic potential that maintains divergent-free current densities. The ion momentum equation is written as (converted to cgs units):
0 = -2T\nabla n + ne \left( \frac{V_i \times B}{c} \right) - ne\nabla \Phi + nm_i g - m_inV_i\nu_{in}. \quad (C.1)

With respect to the magnetic field, B, the parallel (z) and perpendicular (\perp) velocity components can be derived from the ion momentum equation. They are:

\[ V_{i,z} = g \sin D \nu_{in} \frac{m_i}{m_i \nu_{in}} n \frac{\partial n}{\partial z} \] (C.2)

\[ V_{i,\perp} = -\nabla \Phi \times \hat{z} B - \frac{2Tc}{eB} \cdot \nabla \times \hat{z} + \frac{g \times \hat{z}}{\Omega_i} - \nu_{in} \left( \frac{\nabla \Phi c}{B} + \frac{2Tc}{eB} \frac{\nabla \perp n}{n} - \frac{g_{\perp}}{\Omega_i} \right) \] (C.3)

Also, the electron perpendicular velocity, \( V_{e,\perp} \) will be used later for calculating the current density. From Equation 3.5b, \( V_{e,\perp} \) is calculated to be:

\[ V_{e,\perp} = -\nabla \Phi \times \hat{z} c / B. \] (C.4)

Next, Perkins (1973) invokes the ion continuity equation:

\[ \frac{\partial n}{\partial t} + \nabla (nV_i) = 0. \] (C.5)

Given that \( V_i = V_{i,\perp} + V_{i,z} \hat{z} \), the continuity equation becomes:

\[ \frac{\partial n}{\partial t} + \nabla (nV_{i,\perp}) + \frac{\partial}{\partial z} (nV_{i,z}) = 0. \] (C.6)

Here, we assume that the plasma is highly magnetized and that \( \Omega_i >> \nu_{in} \). That is, the ions will gyrate around the magnetic field line at a faster rate with respect to the frequency of collisions to a neutral constituent. This assumption eliminates the second and fourth term in Equation C.3. With these assumptions, the ion continuity equation is written as:

\[ \frac{\partial n}{\partial t} + \nabla \perp n \left( \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z} c}{B} \right) + \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) = 0. \] (C.7)

Before proceeding, we can use the calculated ion and electron velocities to derive the perpendicular current density:
\[ J_\perp = e n (V_{i,\perp} - V_{e,\perp}) \]

\[
= e n \left( - \nabla \Phi \times \hat{z} / B - \frac{2 T_c}{e B} \frac{\nabla n \times \hat{z}}{n} + \frac{g \times \hat{z}}{\Omega_i} \right.
\]

\[
- \frac{\nu_{in}}{\Omega_i} \left( \frac{\nabla \Phi_c}{B} + \frac{2 T_c}{e B} \frac{\nabla \perp n}{n} - \frac{g \perp}{\Omega_i} \right) + \nabla \Phi \times \hat{z} / B \bigg) \right)
\]

\[
= e n \left( - \frac{2 T_c}{e B} \frac{\nabla n \times \hat{z}}{n} + \frac{g \times \hat{z}}{\Omega_i} - \frac{\nu_{in}}{\Omega_i} \left( \frac{\nabla \Phi_c}{B} + \frac{2 T_c \nabla \perp n}{e B n} - \frac{g \perp}{\Omega_i} \right) \right) \right)
\]

\[
= - \frac{2 T_c}{B} \nabla n \times \hat{z} + n e \frac{g \times \hat{z}}{\Omega_i} - \frac{n e \nu_{in} \nabla \perp \Phi_c}{\Omega_i} - \frac{\nu_{in} 2 T_c \nabla \perp n}{\Omega_i B} + \frac{\nu_{in} g \perp n e}{\Omega_i \Omega_i}.
\]

(Note that Perkins (1973) omits the last \( \Omega_i \) in the current density equation. This is believed to be a typographic error.) The next section will describe integrating these equations along the magnetic field line direction.

### C.1.2 Integrating the divergence free current density equation

With the current density derived, we invoke the divergence free condition and do not allow current density gradients along the field line (\( \frac{\partial}{\partial z} J_z = 0 \):

\[
\nabla \cdot J = 0
\]

\[
\nabla \perp \cdot J_\perp + \frac{\partial}{\partial z} J_z = 0
\]

\[
\nabla \perp \cdot J_\perp = 0
\]

\[
\int dz \left( \nabla \perp \cdot J_\perp = 0 \right)
\]

\[
\nabla \perp \cdot \int J_\perp dz = 0.
\]

(C.9)
\[ \nabla \cdot \int J_\perp \, dz = \nabla \cdot \int \left[ -\frac{2T_c}{B} \nabla n \times \hat{z} + ne \frac{\mathbf{g} \times \hat{z}}{\Omega_i} - ne\nu_{in} \nabla_\perp \Phi_c \frac{\nu_{in} 2T_c \nabla_\perp n}{\Omega_i B} - \frac{\nu_{in} \mathbf{g}_\perp ne}{\Omega_i} \int \right] \, dz = 0. \tag{C.10} \]

Now, we define the integrated electron density, \( N \), and integrated Pedersen conductivity, \( \Sigma_P \), as:

\[ N(x, y) = \int n \, dz. \tag{C.11} \]

\[ \Sigma_P(x, y) = \int \frac{n\nu_{in}ec}{\Omega_i B} \, dz = \int \sigma_P \, dz. \tag{C.12} \]

The first three terms of Equation C.10 are straightforward to integrate along the field line direction. The last two are derived as:

\[ 5\text{th Term} = \int \frac{\nu_{in} n \mathbf{g}_\perp e}{\Omega_i^2} \, dz \]
\[ = \frac{1}{e} \int \frac{\nu_{in} n \mathbf{g}_\perp e m_i c e}{\Omega_i e B} \, dz \]
\[ = \frac{1}{e} \int \sigma_P \mathbf{g}_\perp m_i \, dz \]
\[ = \frac{1}{e} \Sigma_P m_i \mathbf{g}_\perp \]

using \( \mathbf{g}_\perp = -|\mathbf{g}_\perp|\hat{x}, \)

and \( |\mathbf{g}_\perp| = |\mathbf{g}| \cos D \)

\[ \mathbf{g}_\perp = -\hat{x}|\mathbf{g}| \cos D \]
\[ = -\hat{x}\Sigma_P \frac{m_i}{e} g \cos D. \tag{C.13} \]
For the 4th Term, we make use of the vector identity $\nabla_\perp(\nu in n) = n \nabla_\perp(\nu in) + \nu in \nabla_\perp n$. Expanding out $\nabla_\perp(\nu in)$:

$$\nabla_\perp(\nu in) = \frac{\partial \nu in}{\partial x} \hat{x} + \frac{\partial \nu in}{\partial y} \hat{y}$$

$$= \frac{\partial \nu in}{\partial x} \hat{x} \quad \text{(horizontally stratified ionosphere (HSI))}$$

$$= \frac{\partial h}{\partial x} \frac{\partial \nu in}{\partial h} \hat{x}$$

since $h = x \cos D - z \sin D$,

$$\frac{\partial h}{\partial x} = \cos D$$

$$= \cos D \frac{\partial \nu in}{\partial h} \hat{x}$$

defining the neutral scale height as: $\frac{1}{H_n} = -\frac{1}{\nu in} \frac{\partial \nu in}{\partial h}$,

$$\Rightarrow \frac{\partial \nu in}{\partial h} = -\nu in H_n^{-1}$$

$$= -\cos D \nu in H_n^{-1} \hat{x}. \quad (C.14)$$

Applying this result to our vector identity results in:

$$\nu in \nabla_\perp n = \nabla_\perp(\nu in n) + n \cos D \nu in H_n^{-1} \hat{x}. \quad (C.15)$$

Subsequently, applying this result to Term 4:

$$4^{th} \text{ Term} = \int -\frac{2Tc}{B\Omega_i} \nu in \nabla_\perp n \, dz$$

$$= -\int \frac{2Tc}{B\Omega_i} \left[ \nabla_\perp(\nu in n) + n \cos D \nu in H_n^{-1} \hat{x} \right] \, dz$$

$$= -\frac{1}{e} \int \frac{2Tc}{\Omega_i B} \nabla_\perp(\nu in n) \, dz - \frac{1}{e} \int \frac{2Tc}{B\Omega_i} n \cos D \nu in H_n^{-1} \hat{x} \, dz$$

$$= -\frac{1}{e} 2T \nabla_\perp \Sigma_p - \frac{1}{e} 2T \cos D H_n^{-1} \Sigma_p \hat{x}. \quad (C.16)$$

Substituting these relations into the integrated current density results in Equations 11 of Perkins (1973):
\[
\int J_\perp \, dz = \int \left[ -\frac{2Tc}{B} \nabla n \times \hat{z} + ne \frac{g \times \hat{z}}{\Omega_i} - \frac{ne \nu_{in} \nabla \perp \Phi c}{\Omega_i} B \right. \\
\left. - \frac{\nu_{in} 2Tc \nabla \perp n}{\Omega_i B} + \frac{\nu_{in} g \perp ne}{\Omega_i} \right] \, dz
\]

\[
= -\frac{2Tc}{B} \nabla \perp N \times \hat{z} + \frac{Ne g \times \hat{z}}{\Omega_i} - \Sigma_p \nabla \perp \Phi \\
- \frac{1}{e} 2T \nabla \perp \Sigma_p - \frac{1}{e} 2T \cos DH_n \Sigma_p \hat{x} - \Sigma_p \frac{m_i}{e} g \cos D \hat{x} \\
= -\frac{2Tc}{B} \nabla \perp N \times \hat{z} + \frac{Ne g \times \hat{z}}{\Omega_i} - \Sigma_p \nabla \perp \Phi - \frac{2T}{e} \nabla \perp \Sigma_p \\
- \hat{x} \cos D \Sigma_p \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right). \quad (C.17)
\]

Now, we take the divergence of the integrated current density and set it to 0:

\[
0 = \nabla \perp \cdot \int J_\perp \, dz \\
= \nabla \perp \cdot \left[ \underbrace{\frac{2Tc}{B} \nabla \perp N \times \hat{z} - \frac{Ne g \times \hat{z}}{\Omega_i}}_{\text{Term 1}} + \underbrace{\Sigma_p \nabla \perp \Phi}_{\text{Term 2}} + \underbrace{\frac{2T}{e} \nabla \perp \Sigma_p + \hat{x} \cos D \Sigma_p \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right)}_{\text{Term 3}} \right]. \quad (C.18)
\]

We focus on each term from Equation C.18 individually:

\[
\text{Term 1} = \nabla \perp \cdot \left( \frac{2Tc}{B} \nabla \perp N \times \hat{z} \right)
\]

Since \( \nabla \perp N = 0 \) for a HSI,

\[
= 0. \quad (C.19)
\]

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Term 2 = $\nabla_\perp \cdot \left( -\frac{\textbf{e}_g \times \dot{z}}{\Omega_i} \right)$

since $\textbf{g} \times \dot{z} = g \sin(90^\circ - D) \dot{y} = g \cos(-D) \dot{y} = g \cos D \dot{y}$,

$= \nabla_\perp \cdot \left( -\frac{\text{e}_g \cos D}{\Omega_i} \dot{y} \right)$

$= -\frac{\partial N}{\partial y} \frac{g \cos D}{\Omega_i}$. \hspace{1cm} (C.20)

Term 3 = $\nabla_\perp \cdot (\Sigma_P \nabla_\perp \Phi)$. \hspace{1cm} (C.21)

Term 4 = $\nabla_\perp \cdot \left( \frac{2T}{e} \nabla_\perp \Sigma_P \right)$

$= \frac{2T}{e} \nabla^2_\perp \Sigma_P$. \hspace{1cm} (C.22)

Term 5 = $\nabla_\perp \cdot \left( \dot{x} \cos D \Sigma_P \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) \right)$

$= \frac{\partial}{\partial x} \left[ \cos D \Sigma_P \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) \right]$. \hspace{1cm} (C.23)

Plugging these terms back into Equation C.18, we have:

$0 = \nabla_\perp \cdot \int \mathbf{J}_\perp \, dz$

$= \nabla_\perp \cdot (\Sigma_P \nabla_\perp \Phi) - \frac{\partial N \, g \cos D}{\Omega_i} + \frac{2T}{e} \nabla^2_\perp \Sigma_P$

$\quad + \frac{\partial}{\partial y} \left[ \cos D \Sigma_P \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) \right]$}

$= \nabla_\perp \cdot (\Sigma_P \nabla_\perp \Phi) - \frac{\partial N \, g \cos D}{\Omega_i} + \frac{2T}{e} \nabla^2_\perp \Sigma_P + \frac{\partial \Sigma_P}{\partial x} \cos D \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right)$. \hspace{1cm} (C.24)
This equation matches Perkins (1973), Equation 13.

C.1.3 Integrating the continuity equation

We integrated the divergence free equation. Now, the ion continuity equation (Equation C.7) is integrated along the magnetic field line:

\[
0 = \int \left[ \frac{\partial n}{\partial t} + \nabla n \cdot \left( \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z}c}{B} \right) \right. \\
\left. + \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) \right] \, dz. \tag{C.25}
\]

Again, we focus on each term individually:

**Term 1**

\[
\text{Term 1} = \int \frac{\partial n}{\partial t} \, dz = \frac{\partial N}{\partial t}. \tag{C.26}
\]

**Term 2**

\[
\text{Term 2} = \int \nabla n \cdot \left( \frac{g \times \hat{z}}{\Omega_i} \right) \, dz \\
= \int \nabla n \cdot \left( \frac{g \cos D}{\Omega_i} \hat{y} \right) \, dz \\
= \int \frac{\partial n g \cos D}{\partial y} \left( \frac{\Omega_i}{\hat{y}} \right) \, dz \\
= \frac{g}{\Omega_i} \cos D \frac{\partial N}{\partial y}. \tag{C.27}
\]

**Term 3**

\[
\text{Term 3} = \int \nabla n \cdot -\frac{\nabla \Phi \times \hat{z}c}{B} \, dz \\
= -\nabla n \cdot \frac{\nabla \Phi \times \hat{z}c}{B}. \tag{C.28}
\]
Term 4 = \int \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) \, dz \\

In general, \int_a^b \frac{\partial}{\partial z} F(z) = F \bigg|_a^b = F(b) - F(a) \\
= \left[ \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n(z)}{\partial z} \right] \bigg|_{z=a}^{z=b} \\
= 0 \text{ (i.e., no density at ends of field points).} \quad \text{(C.29)}

Plugging these terms back into Equation C.25:

\[ 0 = \int \left[ \frac{\partial n}{\partial t} + \nabla \cdot n \left( \frac{g \times \hat{z}}{\Omega_i} - \nabla \Phi \times \hat{z} \frac{c}{B} \right) \right. \]

\[ + \left. \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) \right] \, dz \]

\[ = \frac{\partial N}{\partial t} + \frac{g}{\Omega_i} \cos D \frac{\partial N}{\partial y} - \nabla \cdot n \frac{\nabla \Phi \times \hat{z} \frac{c}{B}}{B}. \quad \text{(C.30)} \]

This relation matches Perkins (1973), Equation 14.

C.1.4 Integrating the (modified) continuity equation

Next, Perkins (1973) multiples the continuity equation by \( \nu_{in} ec / \Omega_i B = \sigma_p / n \) and again integrates along the magnetic field lines:

\[ 0 = \int \frac{\nu_{in} ec}{\Omega_i B} \left[ \frac{\partial n}{\partial t} + \nabla \cdot n \left( \frac{g \times \hat{z}}{\Omega_i} - \nabla \Phi \times \hat{z} \frac{c}{B} \right) \right. \]

\[ + \left. \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) \right] \, dz. \quad \text{(C.31)} \]

Each term can be calculated as:
\[ \text{Term 1} = \int \frac{\nu_m c \partial n}{\Omega_i B} \frac{\partial n}{\partial t} \, dz = \frac{\partial \Sigma_P}{\partial t}. \quad (C.32) \]

\[ \text{Term 2} = \int \frac{e_i c}{\Omega_i B} \nu_m \nabla \parallel n \cdot \left[ \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z} c}{B} \right] \, dz \]

\[ = \int \frac{e_i c}{\Omega_i B} \left[ \nabla \parallel (\nu_m n) + n \cos D \nu_m H_n^{-1} \hat{x} \right] \cdot \left[ \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z} c}{B} \right] \, dz \]

\[ = \nabla \parallel \Sigma_P \cdot \left[ \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z} c}{B} \right] + \hat{x} \cos \Sigma_P \frac{\partial \Phi}{\partial y} \frac{c}{B} \]

\[ \text{For Term 3, integration by parts is required:} \]

\[ \int_a^b u(x)v'(x) \, dx = u(x)v(x) \bigg|_a^b - \int_a^b u'(x)v(x) \, dx. \quad (C.34) \]
Combining these terms into Equation C.31:

\[
0 = \int \frac{\nu_{in}ec}{\Omega_i B} \left[ \frac{\partial n}{\partial t} + \nabla \cdot \left( \frac{\mathbf{g} \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z}}{B} \right) \right] dz
+ \frac{\partial}{\partial z} \left( \frac{ng \sin D}{\nu_{in}} - \frac{2T}{m_i \nu_{in}} \cdot \frac{\partial n}{\partial z} \right) \]

\[
= \frac{\partial \Sigma_P}{\partial t} + \nabla \Sigma_P \cdot \left[ \frac{\mathbf{g} \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{z}}{B} \right] - \cos D \Sigma_P \frac{\partial \Phi}{\partial y} c - \frac{ec \sin^2 DgN}{\Omega_i B} \frac{H_n}{H_n}.
\]

Finally, rearranging results in Perkins (1973), Equation 15:

\[
(C.37)
\]
\[
\frac{\partial \Sigma_P}{\partial t} + \nabla \perp \Sigma_P \cdot \left[ \mathbf{g} \times \hat{z} - \frac{\nabla \Phi \times \hat{z} c}{B} \right] = \frac{\cos D \Sigma_P}{H_n} \frac{\partial \Phi}{\partial y} \frac{c}{B} + \frac{ec}{\Omega_i B} \frac{\sin^2 D gN}{H_n}.
\] (C.38)

C.2 Stability Analysis

With Equations 13, 14, and 15 of Perkins (1973) verified (Equations C.24, C.30, and C.38, respectively), we can proceed to the stability analysis of the F-layer ionosphere. Two derivations will be presented, including using the moment equations previously derived, and another by invoking force balance in the mid-latitude ionosphere.

C.2.1 Derivation from moment equations (Perkins 1973)

For a horizontally stratified ionosphere, \( \nabla \perp \Sigma_P = \nabla \perp N = 0 \). Therefore, starting with Equation 15 of Perkins (1973),

\[
\frac{\partial \Sigma_P}{\partial t} + \nabla \perp \cdot \left( \mathbf{g} \times \hat{z} - \frac{\nabla \Phi \times \hat{z} c}{B} \right) = \frac{ec \sin^2 D gN}{\Omega_i BH_n} + \frac{\Sigma_P}{\Omega_i} \frac{\partial \Phi}{\partial y} \frac{c \cos D}{BH_n}
\]

\[
\left( \frac{\partial \Sigma_P}{\partial t} = 0 \text{ for steady state, } \nabla \perp \Sigma_P = 0 \right)
\]

\[
\frac{ec \sin^2 D gN_0}{\Omega_i BH_n} + \frac{\Sigma_{P,0}}{\Omega_i} \frac{\partial \Phi}{\partial y} \frac{c \cos D}{BH_n} = 0
\]

\[
\Sigma_{P,0} E_{0y} = \frac{ec \sin^2 D gN_0}{\Omega_i BH_n} \frac{BH_n}{c \cos D} = \frac{N_0 e g \sin^2 D}{\Omega_i \cos D}.
\] (C.39)

C.2.2 Derivation from force balance (Kelley 2007)

We can also derive this equation by force balance arguments for the mid-latitude ionosphere. Through \( \mathbf{E} \times \mathbf{B} \) drifts, the eastward electric field provides an upward force, which is balanced out by the downward force due to gravity.
Velocity due to eastward electric field = \( v_{E \times B} = -\frac{E_{\text{east}}}{B} \hat{z} \). \hspace{1cm} (C.40)

Velocity due to gravity = \( v_g = \frac{g \sin D}{\langle \nu_{in} \rangle} \hat{x} \). \hspace{1cm} (C.41)

Summing the vertical components of each velocity and setting to 0 (i.e., a balance):

\[-\frac{E_{\text{east}}}{B} \cos D + \frac{g \sin D}{\langle \nu_{in} \rangle} \sin D \text{ set} = 0\]

\[\frac{E_{\text{east}}}{B} = \frac{g \sin D}{\langle \nu_{in} \rangle \cos D} \]

\[E_{\text{east}} = \frac{B g \sin^2 D}{c \cos D} \int n(z) dz \frac{e}{\Omega_i} \]

\[= \frac{1}{\Sigma_P} \frac{g \sin^2 D}{\cos D} \frac{N e}{\Omega_i} \]

\[\Sigma_P E_{\text{east}} = \frac{N_0 e g \sin^2 D}{\Omega_i \cos D}. \hspace{1cm} (C.42)\]

Here, we see that this expression matches the previous result.

C.3 Instability Analysis

Following the linear stability analysis in Section 3.2, we perturb both \( \Sigma_P \) and \( \Phi \): That is,

\[\tilde{\Sigma}_P \propto e^{i(\omega t - k \cdot x)} \]

\[\tilde{\Phi} \propto e^{i(\omega t - k \cdot x)}. \hspace{1cm} (C.43)\]

As a result, the integrated conductivity can be written as a constant term and perturbation term:
The integrated electron density, \( N \), does not have a perturbation because we assume that the plasma content in a flux tube cannot change in the absence of ionization and recombination, both of which are neglected in the analysis.

Plugging in the perturbed values of \( \Sigma_P \) and \( \Phi \) into Equation C.38:

\[
\begin{align*}
\frac{\partial (\Sigma_{P,0} + \tilde{\Sigma}_P)}{\partial t} + \nabla_\perp (\Sigma_{P,0} + \tilde{\Sigma}_P) \cdot \left( \frac{g \times \hat{z}}{\Omega_i} - \frac{\nabla \Phi \times \hat{c}}{B} \right) &= \frac{ec \sin^2 DgN_0}{\Omega_i BH_n} + (\Sigma_{P,0} + \tilde{\Sigma}_P) \left( \frac{\partial (\Phi_0 + \tilde{\Phi})}{\partial y} c \cos D \right) BH_n \\
\gamma \tilde{\Sigma}_P &= \frac{ec \sin^2 DgN_0}{\Omega_i BH_n} + (\Sigma_{P,0} + \tilde{\Sigma}_P) \left( -E_{0y} + ik_y \tilde{\Phi} \right) c \cos D BH_n \\
&= \frac{ec \sin^2 DgN_0}{\Omega_i BH_n} + \left( -\Sigma_{P,0}E_{0y} + \Sigma_{P,0}ik_y \tilde{\Phi} - \tilde{\Sigma}_P E_{0y} + \Sigma_{P,0}ik_y \tilde{\Phi} \right) c \cos D BH_n \\
&= 0 \text{ by Equation C.39} \\
\gamma \tilde{\Sigma}_P &= -\frac{c \cos D}{BH_n} (\tilde{\Sigma}_P E_{0y} - ik_y \tilde{\Phi} \Sigma_{P,0}).
\end{align*}
\]

(C.45)

Next, we follow a similar analysis by perturbing Equation C.24:

\[
\begin{align*}
\nabla_\perp \cdot (\Sigma_{P,0} + \tilde{\Sigma}_P) \nabla_\perp (\Phi_0 + \tilde{\Phi}) - \frac{\partial N}{\partial y} \frac{ge \cos D}{\Omega_i} - \frac{2T}{e} \nabla_\perp^2 (\Sigma_{P,0} + \tilde{\Sigma}_P) + \frac{\partial (\Sigma_{P,0} + \tilde{\Sigma}_P)}{\partial x} \cos D \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) &= 0.
\end{align*}
\]

(C.46)

Noting that:
\[ \nabla_\perp \cdot (\Sigma_{P,0} + \bar{\Sigma}_P) \nabla_\perp (\Phi_0 + \bar{\Phi}) \]
\[ = \nabla_\perp \cdot (\Sigma_{P,0} + \bar{\Sigma}_P) (\mathbf{E}_0 + i\mathbf{k}\bar{\Phi}) \]
\[ = \nabla_\perp \cdot \left[ \begin{array}{l}
\nabla_\perp () = 0 \\
-\Sigma_{P,0} \mathbf{E}_0 + \Sigma_{P,0} i\mathbf{k}\bar{\Phi} - \bar{\Sigma}_P \mathbf{E}_0 + \bar{\Sigma}_P i\mathbf{k}\bar{\Phi}
\end{array} \right] \]
\[ = -\Sigma_{P,0} k^2 \bar{\Phi} - i\mathbf{k} \cdot \bar{\Sigma}_P \mathbf{E}_0, \quad (C.47) \]

we have:

\[ -\Sigma_{P,0} k^2 \bar{\Phi} - i\mathbf{k} \cdot \bar{\Sigma}_P \mathbf{E}_0 - \frac{2T}{e} k^2 \bar{\Sigma}_P + ik_x \bar{\Sigma}_P \cos D \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) = 0. \quad (C.48) \]

To arrive at the growth rate equation, Equations C.45 and C.48 are combined. First, \( \bar{\Sigma}_P \) is isolated from Equation C.45:

\[ \gamma \bar{\Sigma}_P + \frac{c \cos D}{BH_n} E_{0y} \bar{\Sigma}_P = \frac{c \cos D}{BH_n} ik_y \bar{\Phi} \Sigma_{P,0} \]
\[ \bar{\Sigma}_P = \frac{c \cos D}{BH_n} ik_y \bar{\Phi} \Sigma_{P,0} \gamma + \frac{c \cos D}{BH_n} E_{0y}. \quad (C.49) \]

Next, Equation C.48 is divided by \( \bar{\Sigma}_P \) and Equation C.49 is substituted in. Here we use \( \theta \) as the angle between \( \mathbf{E}_0 \) and magnetic east, and \( \alpha \) as the angle between \( \mathbf{k} \) and magnetic east. Also, the neutral scale height is given as \( H_n = T/m_n g \), and an expression for gravity is found from Equation C.39. The derivation is as follows:

\[ -k^2 \gamma BH_n + \frac{c \cos D E_{0y}}{c \cos D ik_y} - \frac{2T}{e} k^2 + ik_x \cos D \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right) = 0 \]
\[ -k^2 \gamma BH_n + \frac{c \cos D \mathbf{E}_{0y}}{c \cos D ik_y} = \mathbf{k} \cdot \mathbf{E}_0 + \frac{2T}{e} k^2 - ik_x \cos D \left( \frac{2T}{eH_n} + \frac{m_i g}{e} \right). \quad (C.50) \]
Looking at the right-hand side (RHS), and plugging in \( T = H_n m_n g \),

\[
\text{RHS} = i k E_0 \cos(\theta - \alpha) + \frac{2 H_n m_n g k^2 - i k_x \cos D}{e} \left( \frac{2 m_n g}{e} + \frac{m_i g}{e} \right)
\]

\[
= i k E_0 \cos(\theta - \alpha) + \frac{m_i g}{e} \left[ \frac{2 H_n m_n k^2}{m_i} - i k_x \cos D \left( \frac{2 m_n}{m_i} + 1 \right) \right]
\]

\[
= i k E_0 \cos(\theta - \alpha) + \frac{c}{B} \sum_{P,0} E_0 k \cos D m_i \left[ \cdot \right]
\]

(Using \( \langle \nu_i \rangle = \sum_{P,0} \Omega_i B/e \cos D )

\[
= i k E_0 \cos(\theta - \alpha) + \frac{c}{B} \langle \nu_i \rangle E_0 \cos \theta \cos D m_i \left[ \cdot \right]
\]

\[
= i k E_0 \cos(\theta - \alpha) + \frac{1}{\Omega_i} \langle \nu_i \rangle E_0 \cos \theta \cos D \left[ \cdot \right]. \quad (C.51)
\]

As a result, we have:

\[
\gamma B H_n + c \cos D E_0 \cos \theta
\]

\[
= - \frac{c \cos D k y}{k^2} \left[ i k E_0 \cos(\theta - \alpha) + \frac{\langle \nu_i \rangle E_0 \cos \theta \cos D}{\Omega_i \sin^2 D} \left[ \cdot \right] \right]
\]

\[
= c E_0 \cos D \cos \alpha \cos(\theta - \alpha) - \frac{i c}{k^2 k y} \cos D \langle \nu_i \rangle \frac{E_0 \cos \theta \cos D}{\Omega_i \sin^2 D} \left[ \cdot \right]
\]

\[
= c E_0 \cos D \cos \alpha \cos(\theta - \alpha) - \frac{E_0 \cos D \cos \langle \nu_i \rangle}{\sin^2 D \Omega_i} \times
\]

\[
\left[ \frac{i c}{k^2 k y} \cos D \frac{2 H_n m_n k^2}{m_i} - i c \frac{k y}{k^2} \cos D k x \cos D \left( \frac{2 m_n}{m_i} + 1 \right) \right]
\]

\[
= c E_0 \cos D \cos \alpha \cos(\theta - \alpha) - \frac{E_0 \cos D \cos \langle \nu_i \rangle}{\sin^2 D \Omega_i} \times
\]

\[
\left[ \frac{c \cos D k y H_n}{m_i} + \frac{c \cos D \sin \alpha \cos \alpha \cos D}{\sin^2 D \Omega_i} \left( \frac{2 m_n}{m_i} + 1 \right) \right]
\]

\[
= c \cos D E_0 \left\{ \cos \alpha \cos(\theta - \alpha) - \frac{E_0 \cos D \cos \langle \nu_i \rangle}{\sin^2 D \Omega_i} \times
\]

\[
\left[ i k y H_n \frac{2 m_n}{m_i} + \sin \alpha \cos \alpha \cos D \left( \frac{2 m_n}{m_i} + 1 \right) \right] \right\}. \quad (C.52)
\]
Next, we need the trigonometric relation:

\[-\cos \theta + \cos \alpha \cos(\theta - \alpha) = \]
\[= -\cos \theta + \cos \alpha \cos \theta \cos^2 \alpha + \cos \alpha \sin \theta \sin \alpha \]
\[= -\cos \theta + \cos \theta (1 - \sin^2 \alpha) + \cos \alpha \sin \theta \sin \alpha \]
\[= -\cos \theta \sin^2 \alpha + \cos \alpha \sin \theta \sin \alpha \]
\[= \sin \alpha [-\cos \theta \sin \alpha + \cos \alpha \sin \theta] \]
\[= \sin \alpha \sin(\theta - \alpha). \quad \text{(C.54)}\]

Finally, we arrive at the growth rate equation:

\[\gamma = \frac{cE_0 \cos D}{BH_n} \left\{ \sin \alpha \sin(\theta - \alpha) - \frac{\cos D \cos \theta \langle \nu_{in} \rangle}{\sin^2 D\Omega_i} \times \right.\]
\[\left. \left[ i(k_y H_n) \left( \frac{2 m_n}{m_i} \right) + \sin \alpha \cos \alpha \cos D \left( \frac{2 m_n}{m_i} + 1 \right) \right] \right\}. \quad \text{(C.55)}\]

Equation C.55 matches Equation 20 of Perkins (1973). As mentioned in Perkins (1973), the first term in the brackets is significant due to \(\langle \nu_{in} \rangle \ll \Omega_i\), and as a result Equation C.55 can be simplified to:

\[\gamma = \frac{cE_0 \cos D}{BH_n} \sin \alpha \sin(\theta - \alpha). \quad \text{(C.56)}\]
APPENDIX D

DERIVATION OF POTENTIAL EQUATION
IN SAMI3

In this Appendix, we will derive the equation that is commonly used in the SAMI3 numerical model to solve for the electrostatic potential self-consistently. Variants of this potential equation are found in the work by Huba et al. (2008); Krall et al. (2009); Huba et al. (2009); Huba and Joyce (2010). The potential equation is derived by assuming divergence free current density in the ionosphere, $\nabla \cdot \mathbf{J} = 0$.

To begin, the curvilinear factors are provided, which are needed to take divergences and gradients on the SAMI3 grid. Then, the current density equation is given, and with $\nabla \cdot \mathbf{J} = 0$, the equation is split up into the unknown potential terms and various source terms consisting of the neutral wind and gravity. The two-dimensional, scalar elliptical partial differential equation is then derived.

D.1 Curvilinear Factors

To begin, within the SAMI3 curvilinear coordinates, the divergence and gradient operators are given as (Orens et al., 1979; Huba et al., 2000; Swisdak, 2006):

$$\nabla \cdot \mathbf{A} = \frac{1}{h_p h_q h_\phi} \left[ \frac{\partial}{\partial p} h_q h_p A_p + \frac{\partial}{\partial q} h_p h_\phi A_q + \frac{\partial}{\partial \phi} h_p h_q A_\phi \right]. \quad (D.1)$$

$$\nabla f = \frac{1}{h_p} \frac{\partial}{\partial p} f e_p + \frac{1}{h_q} \frac{\partial}{\partial q} f e_q + \frac{1}{h_\phi} \frac{\partial}{\partial \phi} f e_\phi. \quad (D.2)$$

Following Huba et al. (2000), the following curvilinear factors are used:
Table D.1: Table describing terms commonly used for the derivation of the potential equation in SAMI3.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_E$</td>
<td>Radius of Earth ($\approx 6378.1$ km)</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial distance from center of eccentric coordinate system</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Magnetic latitude [°] (in eccentric coordinates)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Eccentric coordinate in longitudinal direction describing the longitude</td>
</tr>
<tr>
<td>$q$</td>
<td>Eccentric coordinate along magnetic field line, $q = \frac{R_E^2}{r^2} \cos \theta$</td>
</tr>
<tr>
<td>$p$</td>
<td>Eccentric coordinate across magnetic field line (i.e., a field line), $p = \frac{r}{R_E \sin \theta}$</td>
</tr>
<tr>
<td>$s_i$</td>
<td>$s_i = R_E q_i$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>$= \sqrt{1 + 3 \cos^2 \theta}$</td>
</tr>
<tr>
<td>$b_s$</td>
<td>$= \frac{R_E^2}{r^2} \Delta$</td>
</tr>
</tbody>
</table>

\[
h_q = \frac{r^3}{R_E^2 \left[1 + 3 \cos^2 \theta \right]^{1/2}} = \frac{1}{R_E/b_s}, \quad \text{(D.3)}
\]

\[
h_p = \frac{R_E \sin^3 \theta}{\left[1 + 3 \cos^2 \theta \right]^{1/2}} = \frac{R_E^3 \sin^3 \theta}{\Delta} = \frac{\Delta}{\Delta} = \frac{r \sin \theta \cdot R_E \sin^2 \theta}{r} = \frac{\Delta}{\Delta} \cdot \frac{r}{p} = \frac{r \sin \theta}{p \Delta}. \quad \text{(D.4)}
\]

\[
h_\phi = r \sin \theta. \quad \text{(D.5)}
\]

Table D.1 provides a description for the terms used in this derivation, and also Section 5.3 discusses the dipole coordinate system in SAMI3.
D.2 Divergence Free Current Density

The current density perpendicular to the magnetic field line, $J_\perp$, can be written as:

$$
J_\perp = \sigma_p \left( E_\perp + U \times B + \frac{g \times B}{\nu_{in}} \right) + \sigma_H \hat{q} \times \left( E_\perp + U \times B + \frac{g \times B}{\nu_{in}} \right).
$$

Now, break apart each term, first for $U$:

$$
U \times B = B \left( \frac{U_p \hat{p} + U_\phi \hat{\phi}}{c} \right) \times \hat{q}
= \frac{B}{c} U_p \hat{p} \times \hat{q} + \frac{B}{c} U_\phi \hat{\phi} \times \hat{q}
= -\frac{B}{c} U_p \hat{\phi} + \frac{B}{c} U_\phi \hat{p},
$$

and also for $g$:

$$
\nu_{in}^{-1} g \times B = \nu_{in}^{-1} \left( g_p \hat{p} \times \frac{B}{c} \hat{q} \right)
= -\nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi},
$$

and also for $\hat{b}$:

$$
\hat{b} \times \nu_{in}^{-1} g \times B = \hat{q} \times -\nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi}
= \nu_{in}^{-1} g_p \frac{B}{c} \hat{p}.
$$

With this, and using the relation $E_\perp = -\nabla_\perp \Phi$, we can write the current density as:

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\[ J_\perp = \sigma_P(-\nabla_\perp \Phi) + \sigma_H \hat{q} \times (-\nabla_\perp \Phi) \]
\[ = -\sigma_P \frac{B}{c} U_p \hat{\phi} + \sigma_P \frac{B}{c} U_\phi \hat{p} - \sigma_P \nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi} \]
\[ + \sigma_H \frac{B}{c} U_p \hat{p} + \sigma_H \frac{B}{c} U_\phi \hat{\phi} + \sigma_H \nu_{in}^{-1} g_p \frac{B}{c} \hat{p}. \]  

(D.11)

Following Section 1.4, we take the divergence of \( J_\perp \) and set it equal to 0:

\[ \nabla \cdot J_\perp = 0 \]

Unknown Terms (U) + Source Terms (S) = 0

\[
\begin{align*}
\nabla \cdot \sigma_P(-\nabla_\perp \Phi) &+ \nabla \cdot \sigma_H \hat{b} \times (-\nabla_\perp \Phi) \\
= U1 &+ U2 \\
-\nabla \cdot \sigma_P \frac{B}{c} U_p \hat{\phi} &+ \nabla \cdot \sigma_P \frac{B}{c} U_\phi \hat{p} - \nabla \cdot \nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi} \\
= S3 &+ S1 &+ S6 \\
+ \nabla \cdot \sigma_H \frac{B}{c} U_p \hat{p} &+ \nabla \cdot \sigma_H \frac{B}{c} U_\phi \hat{\phi} + \nabla \cdot \sigma_H \nu_{in}^{-1} g_p \frac{B}{c} \hat{p} \\
= S2 &+ S4 &+ S7
\end{align*}
\]

\[ = 0. \]  

(D.12)

In this equation, the unknown variable to be solved for is \( \Phi \), hence the grouping of terms with \( \Phi \) as “Unknown Terms”. In the next sections, we will analyze each term individually, beginning with the source terms, and then with the unknown terms. The following relations will be useful, which define the divergence on the curvilinear grid:

\[
\nabla \cdot \alpha \hat{p} = \frac{1}{h_p h_q h_\phi} \frac{\partial}{\partial p} h_q h_\phi \alpha \\
= \frac{1}{h_p h_q h_\phi} R_E \frac{\partial}{\partial r} r \sin \theta \frac{1}{b_s} \alpha. \]  

(D.13)
\[ \nabla \cdot \beta \hat{\phi} = \frac{1}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} h_p h_q \beta \]
\[ = \frac{1}{h_p h_q h_\phi} R_E \frac{\partial}{\partial \phi} \frac{R_E \sin^3 \theta}{\Delta b_s} \beta. \] \hfill (D.14)

The next sections will include substituting \( \alpha \) and \( \beta \) for the unknown and source terms listed in Equation D.12.

### D.2.1 Source Terms

The source terms can be written as:

**S1**

With \( \alpha = \sigma_p \frac{B_c}{c} U_\phi \),

\[ \nabla \cdot \alpha \hat{p} = \nabla \cdot \sigma_p \frac{B_c}{c} U_\phi \hat{p} \]
\[ = \frac{\partial}{\partial p} R_E \frac{1}{b_s} \sigma_p \frac{B_c}{c} U_\phi \]
\[ = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \int r \sin \theta \frac{B_0}{c} \sigma_p U_\phi \, ds. \] \hfill (D.15)

**S2**

With \( \alpha = \sigma_H \frac{B_c}{c} U_p \),

\[ \nabla \cdot \alpha \hat{p} = \nabla \cdot \sigma_H \frac{B_c}{c} U_p \hat{p} \]
\[ = \frac{\partial}{\partial p} R_E \frac{1}{b_s} \sigma_H \frac{B_c}{c} U_p \]
\[ = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \int r \sin \theta \frac{B_0}{c} \sigma_H U_p \, ds. \] \hfill (D.16)
Combining the two previous terms, $S_1$ and $S_2$:

$$\nabla \cdot \sigma P \frac{B}{c} U_p \hat{p} + \nabla \cdot \sigma H \frac{B}{c} U_p \hat{p} = \left( \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \int r \sin \theta \frac{B_0}{c} (\sigma P U_\phi + \sigma H U_p) \right) ds = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} F_{pV}. \quad \text{(D.17)}$$

$S_3$

With $\beta = -\sigma P \frac{B}{c} U_p$,

$$\nabla \cdot \beta \hat{\phi} = \nabla \cdot -\sigma P \frac{B}{c} U_p \hat{\phi} = \frac{\partial}{\partial \phi} \frac{R_E}{h_p h_q h_\phi} \frac{R_E \sin^3 \theta}{\Delta b_s} \left( -\sigma P \frac{B}{c} U_p \right)$$

(Integrate along field line, $ds$, and use $B = B_0 b_s$)

$$= \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \int \frac{R_E \sin^3 \theta}{\Delta} \frac{B_0}{c} (-\sigma P U_p) \right) ds. \quad \text{(D.18)}$$

$S_4$

With $\beta = \sigma H \frac{B}{c} U_\phi$,

$$\nabla \cdot \beta \hat{\phi} = \nabla \cdot \sigma H \frac{B}{c} U_\phi \hat{\phi}$$

$$= \frac{\partial}{\partial \phi} \frac{R_E}{h_p h_q h_\phi} \frac{R_E \sin^3 \theta}{\Delta b_s} \sigma H U_\phi$$

(integrate along field line and use $B = B_0 b_s$)

$$= \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \int \frac{R_E \sin^3 \theta}{\Delta} \frac{B_0}{c} \sigma H U_\phi \right) ds. \quad \text{(D.19)}$$

Combining the two previous terms, $S_3$ and $S_4$.
\[
- \nabla \cdot \frac{B}{c} U_p \hat{\phi} + \nabla \cdot \sigma_H \frac{B}{c} U_\phi \hat{\phi} = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \left( \int \frac{R_E \sin^3 \theta B_0}{\Delta} \frac{c}{c} (\sigma_H U_\phi - \sigma_P U_p) ds \right) \\
= \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} F_{\phi V}. \tag{D.20}
\]

S6

With \( \beta = -\sigma_P \nu_{in}^{-1} g_p \frac{B}{c} \),

\[
\nabla \cdot \beta \hat{\phi} = \nabla \cdot -\sigma_P \nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi} \\
= -\frac{\partial}{\partial \phi} \frac{R_E}{h_p h_q h_\phi} \frac{R_E \sin^3 \theta B_0}{\Delta b_s} \sigma_P \nu_{in}^{-1} g_p \frac{B}{c} \hat{\phi} \\
= -\frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \left( \int \frac{R_E \sin^3 \theta B_0}{\Delta} \frac{c}{c} \sigma_P \nu_{in}^{-1} g_p \hat{\phi} \right) \\
= -\frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} F_{\phi g}. \tag{D.21}
\]

S7

With \( \alpha = \sigma_H \nu_{in}^{-1} g_p \frac{B}{c} \),

\[
\nabla \cdot \alpha \hat{p} = \nabla \cdot \sigma_H \nu_{in}^{-1} g_p \frac{B}{c} \hat{p} \\
= \frac{\partial}{\partial p} \frac{R_E}{h_p h_q h_\phi} r \sin \frac{1}{b_s} \sigma_H \nu_{in}^{-1} g_p \frac{B}{c} \\
= \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \left( \int r \sin \frac{B_0 \nu_{in}^{-1} g_p}{c} ds \right) \\
= \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} F_{pg}. \tag{D.22}
\]
D.2.2 Unknown Terms

First compute $\nabla \cdot \psi \nabla \Phi$. Here, the $q$ component is removed, due to the equipotential field line assumption (i.e., there are no gradients of $\Phi$ along the field line direction).

\[
\nabla \cdot \psi \nabla \Phi = \nabla \cdot \left\{ \frac{1}{h_p} \frac{\partial}{\partial p} \Phi e_p + \frac{1}{h_\phi} \frac{\partial}{\partial \phi} \Phi e_\phi \right\} \\
= \frac{1}{h_p h_q h_\phi} \left[ \frac{\partial}{\partial p} \psi h_q h_\phi \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial \phi} \psi h_p h_q \frac{\partial}{\partial \phi} \Phi \right]. \quad (D.23)
\]

U1

With this relation, we have:

\[
\nabla \cdot \sigma_p \mathbf{E}_\perp = -\nabla \cdot \sigma_p \nabla \Phi \\
= \frac{1}{h_p h_q h_\phi} \left[ -\frac{\partial}{\partial p} \sigma_p \frac{p \Delta}{b_s} \frac{\partial}{\partial p} \Phi - \frac{\partial}{\partial \phi} \sigma_p \frac{1}{p \Delta b_s} \frac{\partial}{\partial \phi} \Phi \right] \\
(\text{Integrate along field line, } ds, \text{ and use } B = B_0 b_s) \\
= \frac{1}{h_p h_q h_\phi} \left[ -\frac{\partial}{\partial p} \int \left( \frac{\Sigma_{pp \phi}}{p b_s} \frac{\partial}{\partial p} \Phi + \Sigma_{p \phi \phi} \frac{\partial}{\partial \phi} \Phi \right) \right] \\
= \frac{1}{h_p h_q h_\phi} \left[ -\frac{\partial}{\partial p} \Sigma_{pp \phi} \frac{\partial}{\partial p} \Phi - \frac{\partial}{\partial \phi} \Sigma_{p \phi \phi} \frac{\partial}{\partial \phi} \Phi \right]. \quad (D.24)
\]

U2

\[
\sigma_H \hat{b} \times \mathbf{E}_\perp = \sigma_H \hat{q} \times \nabla \Phi \\
= \sigma_H \hat{q} \times \left[ \frac{1}{h_p} \frac{\partial}{\partial p} \Phi \hat{p} + \frac{1}{h_q} \frac{\partial}{\partial q} \Phi \hat{q} + \frac{1}{h_\phi} \frac{\partial}{\partial \phi} \Phi \hat{\phi} \right] \\
= \sigma_H \left[ \frac{1}{h_p} \frac{\partial}{\partial p} \Phi \hat{\phi} - \frac{1}{h_\phi} \frac{\partial}{\partial \phi} \Phi \hat{\phi} \right]. \quad (D.25)
\]
\[ \nabla \cdot \sigma_H \left[ \frac{1}{h_p} \frac{\partial}{\partial p} \Phi \hat{\phi} - \frac{1}{h_\phi} \frac{\partial}{\partial \phi} \Phi \hat{p} \right] \]

\[ = \frac{1}{h_p h_q h_\phi} \left[ \frac{\partial}{\partial \phi} h_p h_q \sigma_H \frac{1}{h_p} \frac{\partial}{\partial p} \Phi - \frac{\partial}{\partial p} h_q h_\phi \sigma_H \frac{1}{h_\phi} \frac{\partial}{\partial \phi} \Phi \right] \]

\[ = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \frac{1}{\sigma_H} \frac{\partial}{\partial p} \Phi - \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \frac{1}{\sigma_H} \frac{\partial}{\partial \phi} \Phi \]

(Integrate along field line, \( ds \), and use \( B = B_0 b_s \))

\[ = \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial \phi} \int_{b_s}^{1} \sigma_H \, ds \frac{\partial}{\partial p} \Phi - \frac{R_E}{h_p h_q h_\phi} \frac{\partial}{\partial p} \int_{b_s}^{1} \sigma_H \, ds \frac{\partial}{\partial \phi} \Phi. \]

(D.26)

### D.3 Potential Equation

Given the relations derived in the previous sections, we simply combine the terms to derive the potential equation. Substituting each relation into Equation D.12:

\[ U_1 + U_2 + S_3 + S_1 + S_6 + S_2 + S_4 + S_7 = 0 \]

\[ - \frac{\partial}{\partial p} \Sigma_{pp} \frac{\partial}{\partial p} \Phi - \frac{\partial}{\partial \phi} \Sigma_{p\phi} \frac{\partial}{\partial \phi} \Phi + \frac{\partial}{\partial \phi} \Sigma_H \frac{\partial}{\partial p} \Phi - \frac{\partial}{\partial p} \Sigma_H \frac{\partial}{\partial \phi} \Phi + \frac{\partial}{\partial p} F_{PV} + \frac{\partial}{\partial \phi} F_{PV} - \frac{\partial}{\partial \phi} F_{\phi g} + \frac{\partial}{\partial p} F_{\phi g} = 0. \]

(D.27)

Rearranging, this gives the final form for the potential equation:

\[ \frac{\partial}{\partial p} \Sigma_{pp} \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial \phi} \Sigma_{p\phi} \frac{\partial}{\partial \phi} \Phi - \frac{\partial}{\partial \phi} \Sigma_H \frac{\partial}{\partial p} \Phi + \frac{\partial}{\partial p} \Sigma_H \frac{\partial}{\partial \phi} \Phi \]

\[ = \frac{\partial}{\partial p} F_{PV} + \frac{\partial}{\partial \phi} F_{PV} - \frac{\partial}{\partial \phi} F_{\phi g} + \frac{\partial}{\partial p} F_{\phi g}. \]

(D.28)

Equation D.28 is a second-order, elliptical, scalar partial differential equation and can be solved using the numerical techniques described in Section 5.6.3.
REFERENCES


Kudeki, E. (2010), Applications of Radiowave Propagation, University of Illinois at Urbana-Champaign: ECE 458 Lecture Notes.


