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ILLINOIS RIVER FLOW SYSTEM MODEL

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INTRODUCTION

Diversion of water from Lake Michigan to the Illinois River drainage system is presently limited to 3200 cfs (cubic feet per second) by a Supreme Court decree. This amount includes surface runoff and groundwater pumpage in the lake watershed. The court decree stipulated that the State of Illinois should allocate the lake diversion water among various users. The state has entrusted the Illinois Department of Transportation, IDOT, with the responsibility and authority to make the allocations.

In the past, about 1700 cfs of Lake Michigan water was used by municipalities for water supplies. After deductions for storm runoff, pumpage, and lockage and leakage, the remainder allowable diversion or discretionary diversion has been used by the Metropolitan Sanitary District of Greater Chicago, MSDGC, to dilute wastewater effluents in the Chicago Sanitary and Ship Canal and Calumet Sag Channel. At the request of IDOT, the court has further agreed to a revised method for measuring and accounting for the allowed diversion. A 40-year accounting period is used now instead of the previous 5-year period and this will aid in better management of the diversion waters. The Division of Water Resources of the IDOT has recently outlined allocation of lake water through 2020.

A few years ago, an increase in levels of the Great Lakes caused significant damage to shoreline and shore properties. Increased lake diversion can reduce such damages but it may result in some hydroelectric power revenue loss in other states. Increased diversion can improve the water quality in the Illinois River but it can cause some damages to agriculture in the lower reaches if the enhanced diversion increases the flood flows. The U.S. Army Corps of Engineers, Chicago District Office, has been conducting studies on the environmental effects of increasing lake diversion. The optimum lake diversion, and its management and scheduling, can be derived with the use of system operation models, suitable benefit and damage functions, historical as well as stochastic flows for the Illinois River and its tributaries, and a multiobjective function.

The goal of the study discussed in this report was to construct a flow system simulation and prediction model for the Illinois River, using upstream and significant tributary inflows as inputs. Such a model can be used to assess the flows downstream resulting from changes in flow diverted from the lake. It can help in deciding an optimal sequence of increased diversion from Lake Michigan.

Acknowledgments

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THE ILLINOIS RIVER BASIN

The Illinois River Basin covers 28,906 square miles in northern and central Illinois. The river, major tributaries, and gaging stations are shown in figure 1. Flow in the Illinois River is regulated through a series of locks and dams for navigation purposes. Figure 2 shows the bed profile, the longitudinal water surface profile during low flows, the locations of the seven locks and dams near Lockport, Brandon Road, Dresden Island, Marseilles, Starved Rock, Peoria, and LaGrange; and the size of the pools they create. These pools have very little surface slope during low flows.

The major tributaries, their USGS gaging stations, their drainage areas at the station as well as at the confluence with the Illinois River, and their periods of available daily flow record together with mean and maximum discharges are given in table 1. The daily flow data from October 1960 to September 1965 for these tributaries and the Illinois River were used in developing suitable structures for the flow models, and the data from October 1965 to September 1970 were utilized for testing the performance of flow predictions from the models.

From Lockport (Chicago Sanitary and Ship Canal) to Meredosia, there are four gaging stations on the river, at Lockport, Marseilles, Kingston Mines, and Meredosia. The river length is divided into 3 reaches: reach I from Lockport to Marseilles, reach II from Marseilles to Kingston Mines, and reach III from Kingston Mines to Meredosia. Flow simulation models were developed for each reach separately. The tributaries and gaging stations in each reach are shown schematically in figures 3, 4, and 5, with mileage from Grafton for the Illinois River and from the gaging

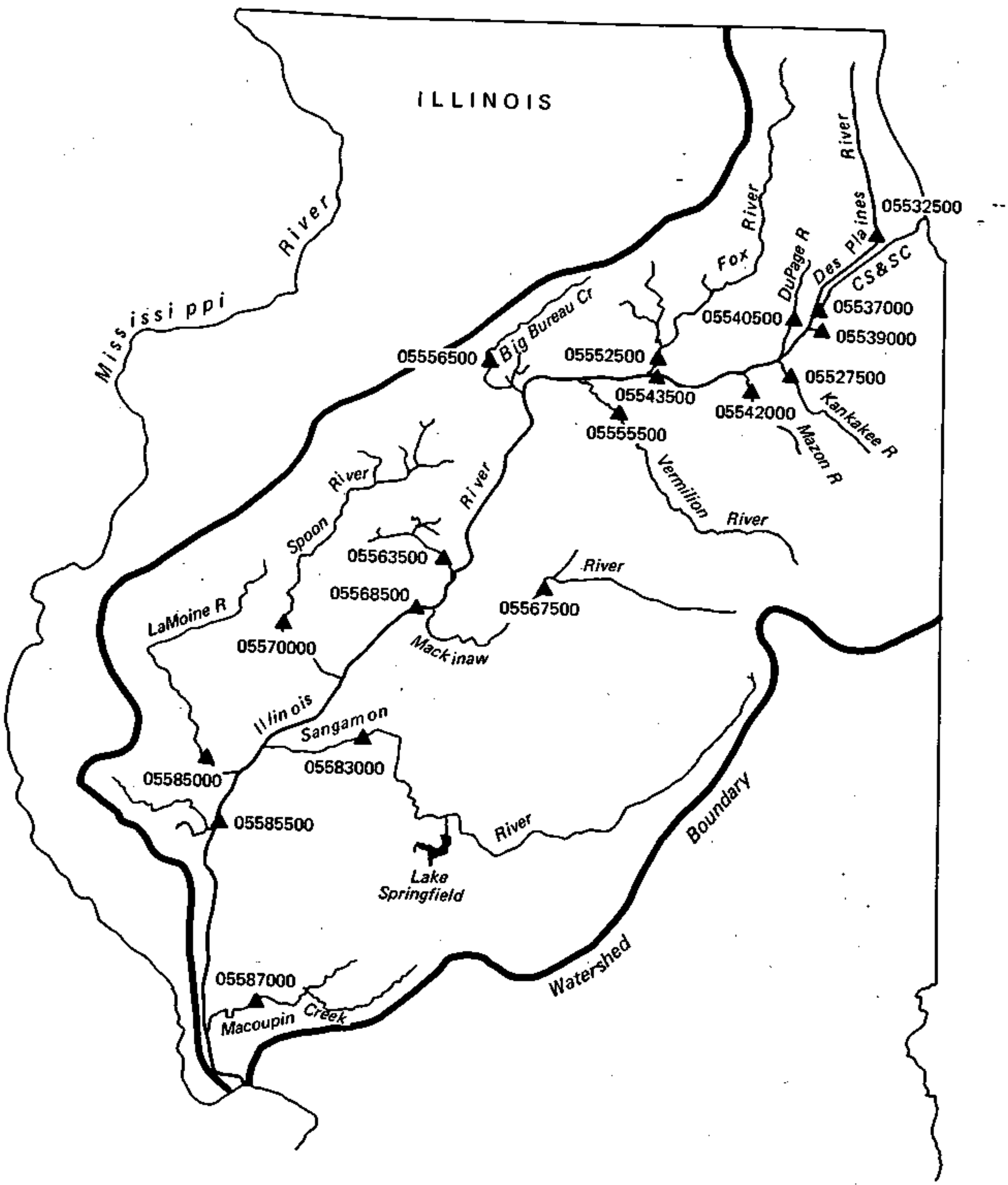


Figure 1. Illinois River Basin

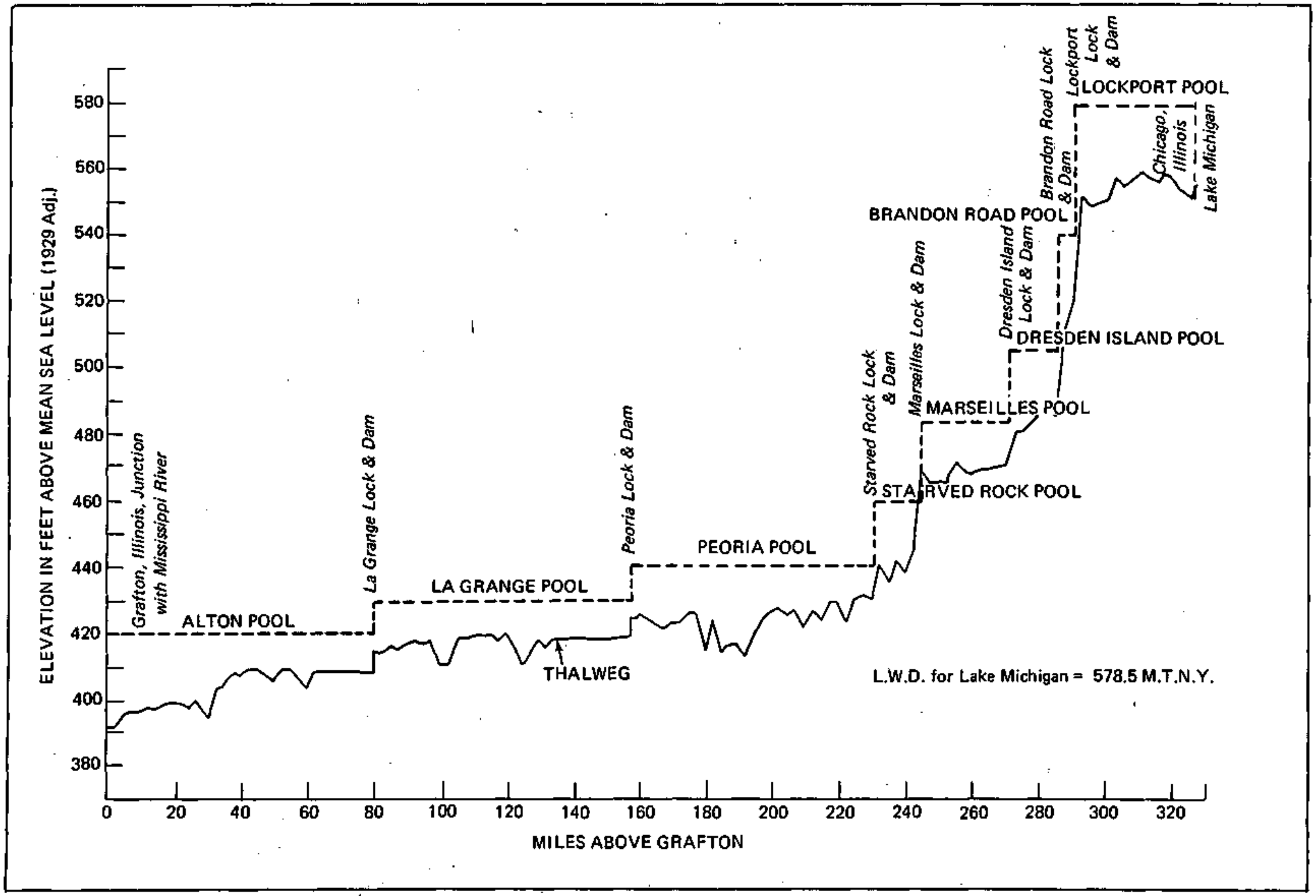
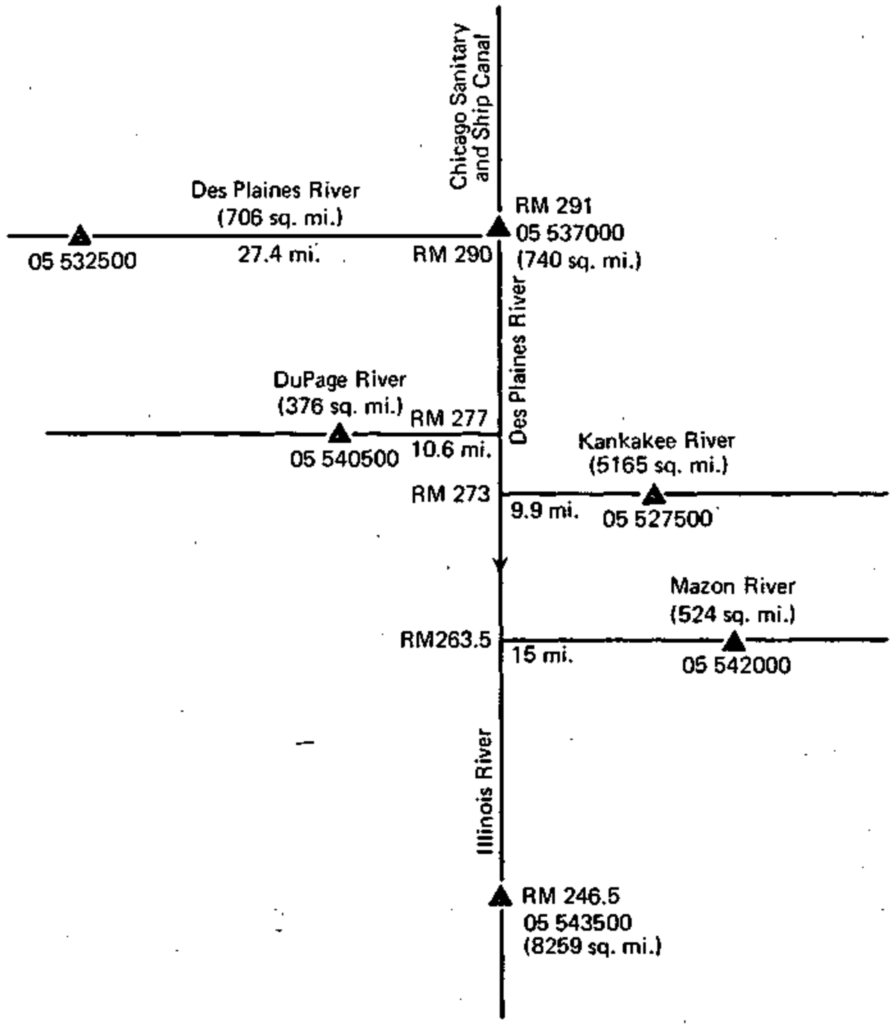


Figure 2. Illinois Waterway Profile

Table 1. Availability of Streamflow Data for Illinois River Basin

No.	USGS No.	Stream and Station Name	D.A., sq mi Gage	D.A., sq mi Mouth	Available Record* From	Available Record* To	Discharge, cfs. Mean	Discharge, cfs. Max
1	05 537000	Chicago Sanitary and Ship Canal at Lockport	740		Oct. '36		3,428	34,948
2	05 532500	DesPlaines River at Riverside	630	706	Oct. '43		451	6,510
3	05 539000	Hickory Creek at Joliet	107	109	Oct. '44		83	15,200
4	05 540500	DuPage River at Shorewood	324	376	Oct. '40		250	12,000
5	05 527500	Kankakee River near Wilmington	5,150	5,165	Oct. '33		4,089	75,900
6	05 542000	Mazon River near Coal City	455	524	Oct. '39		320	17,600
7	05 543500	Illinois River at Marseilles	8,259		Oct. '19		10,680	93,900
8	05 552500	Fox River at Dayton	2,642	2,658	Nov. '14		1,660	47,100
9	05 555500	Vermilion River at Lowell	1,278	1,331	May '31	Sep. '71	734	33,500
10	05 556500	Big Bureau Creek at Princeton	196	486	Mar. '36		130	12,500
11	05 563500	Kickapoo Creek at Peoria	297	306	Mar. '42	Sep. '71	168	18,600
12	05 567500	Mackinaw River near Congerville	767	1,136	Oct. '44		484	36,000
13	05 568500	Illinois River at Kingston Mines	15,819		Oct. '39		14,639	83,100
14	05 570000	Spoon River at Seville	1,636	1,855	Jul. '14		1,026	37,300
15	05 583000	Sangamon River near Oakford	5,093	5,418	Oct. '39		3,230	123,000
16	05 585000	LaMoine River at Ripley	1,293	1,350	Mar. '21		774	24,100
17	05 585500	Illinois River at Meredosia	26,028		Oct. '38		21,311	123,000
18	05 587000	Macoupin Creek near Kane	868	961	Oct. '40		525	40,000
19	-	Illinois River (at mouth) at Grafton	-	28,906				

* Available record shown up to September 1980, unless specified otherwise.
No. 1 Mean and maximum discharge for a 20 year period (1961-1980)
No.11 Partial record station since October 1971
No.15 Mean discharge determined for 53 years, discontinuous period (1910-1980)
No.18 Mean discharge determined for 52 years, discontinuous period (1922-1980)



RM denotes Illinois River Mile

Figure 3. Illinois River from Lockport to Marseilles (Reach I)

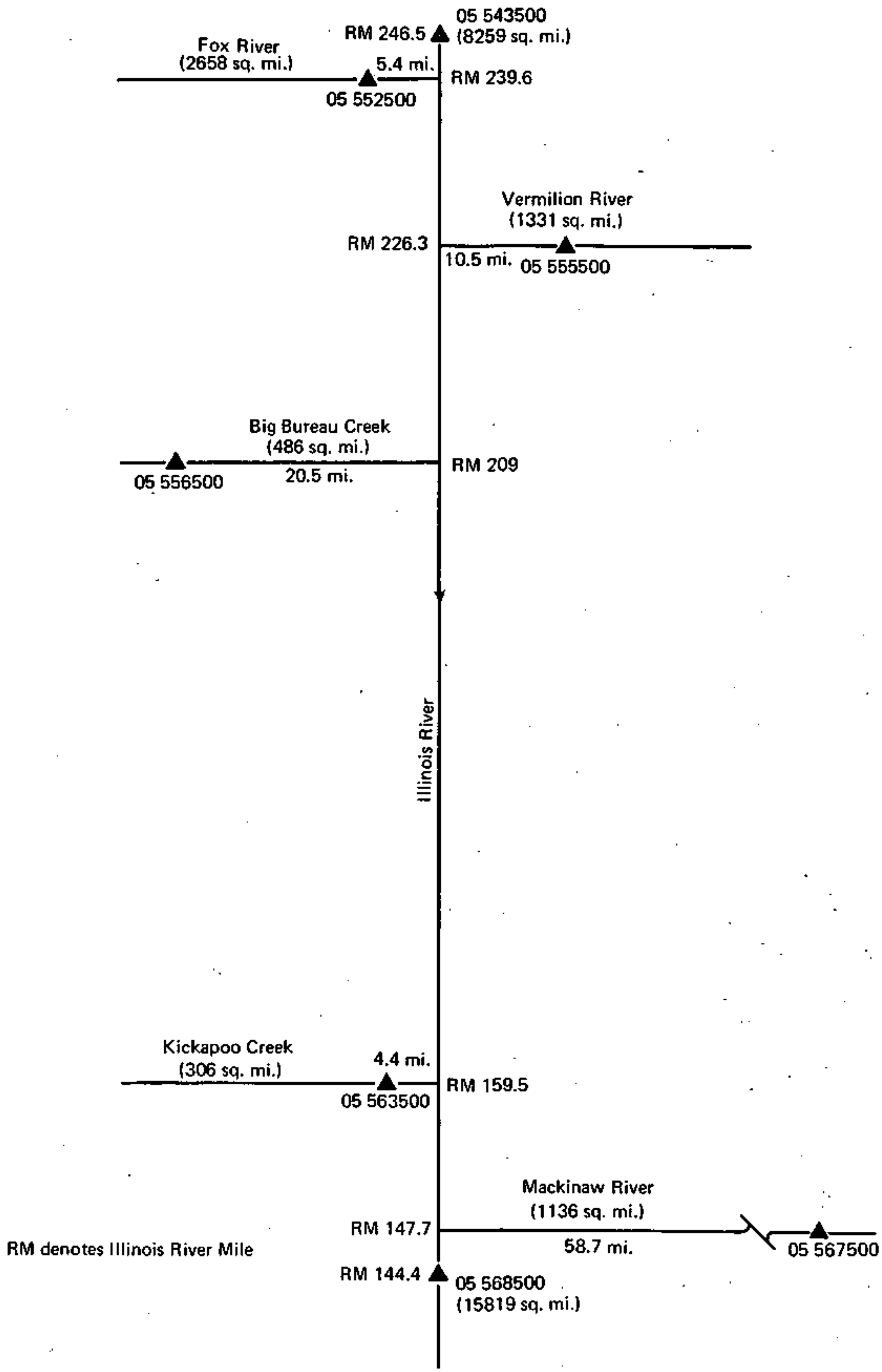
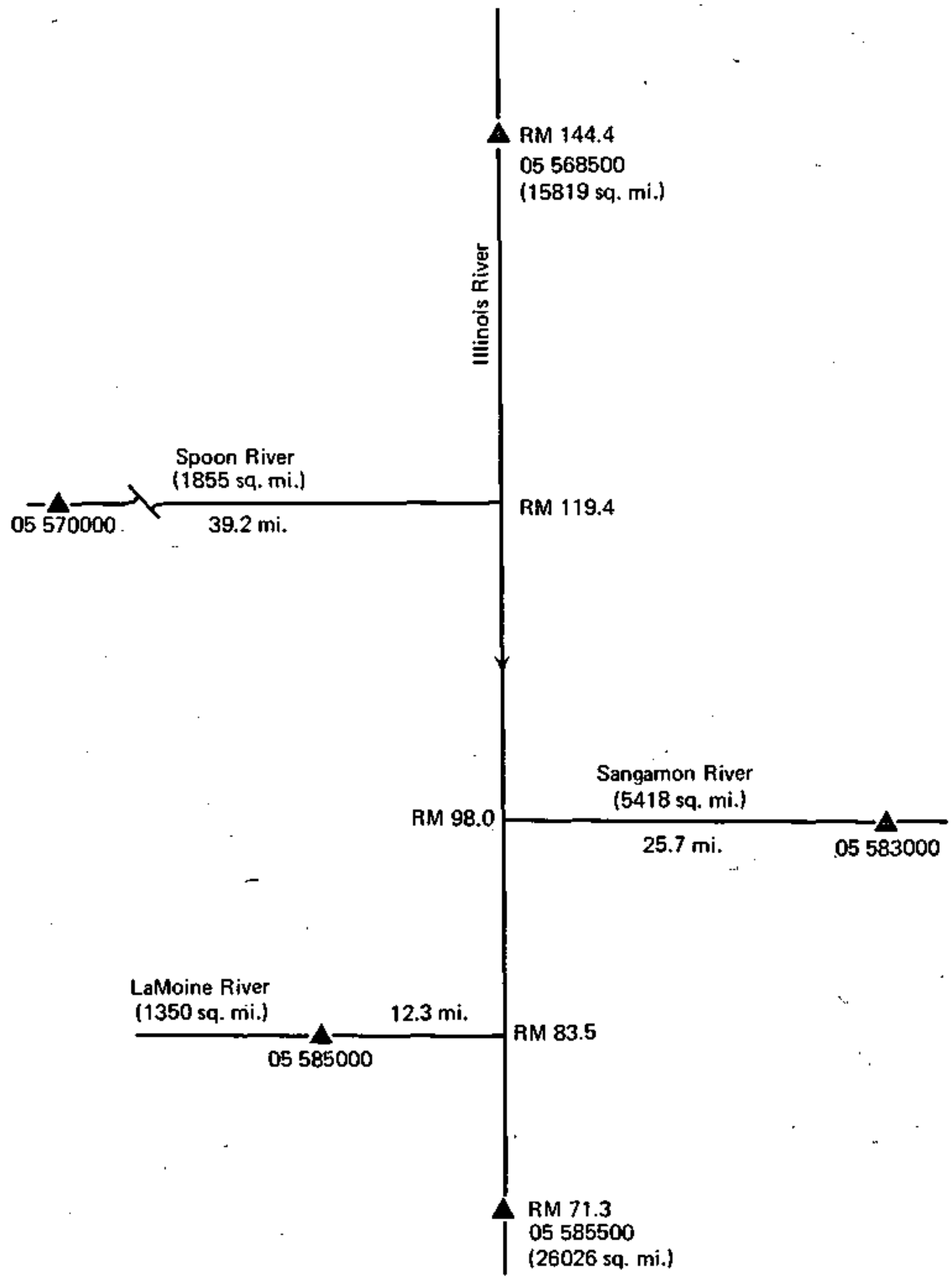


Figure 4. Illinois River from Marseilles to Kingston Mines (Reach II)



RM denotes Illinois River Mile

Figure 5- Illinois River from Kingston Mines to Meredosia (Reach III)

station to the confluence with the Illinois River for the tributaries. The total drainage areas at the mouths of the tributaries as well as at the gaging stations on the Illinois River are shown in parentheses.

LINEAR DRAINAGE NETWORK MODELS

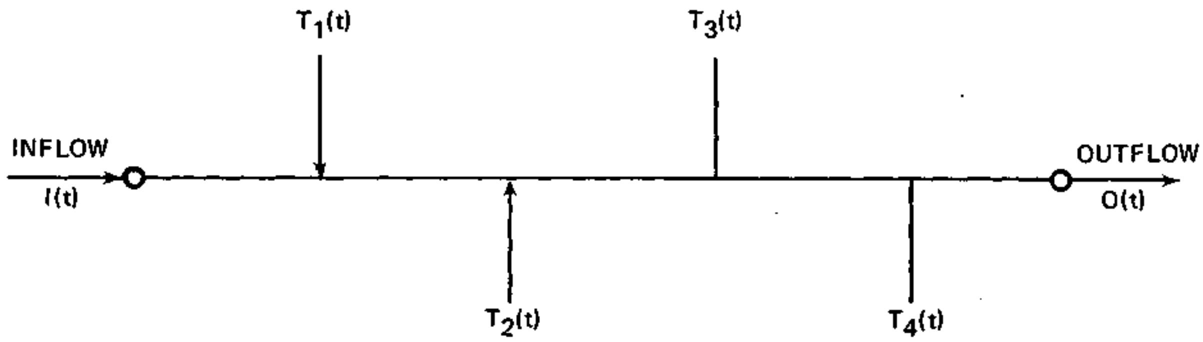
A reach of a river can be conceptualized as a system, as shown in figure 6a, where the reach inflow I(t) and tributary inflows $T_r(t)$ are the system inputs and the reach outflow 0(t) is the system output. The systems approach is concerned with the way in which the system transfers the inputs to an output and not with the physical laws themselves that govern the system (Dooge, 1973). The system operation itself is treated as a black box as shown in figure 6b. A linear drainage network, LDN, model for a river reach with one output and N inputs can be expressed as a system (Natale and Todini, 1977):

$$Q_d(t) = \int_0^t U_1(\tau)\, Q_{U_1}(t-\tau)\, d\tau + \int_0^t U_2(\tau)\, Q_{U_2}(t-\tau)\, d\tau + \int_0^t U_N(\tau)\, Q_{U_N}(t-\tau)\, d\tau \qquad (1)$$

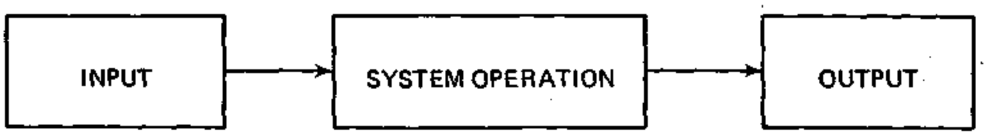
in which Q_d is the outflow from the system, Q_{U_1}, Q_{U_2}, etc. are the inflows to the system, and U_1, U_2, etc. are the impulse responses of the system. The inputs may exist for only a finite time and in that case the limits should be modified accordingly. Through use of the notation in figure 6a for inputs and output, the LDN model can be written in a discrete form for a system of finite memory:

$$O(t) = \sum_{j=k_0}^{l_0} U_I(j)\, I(t-j) + \sum_{r=1}^{s} \sum_{j=k_r}^{l_r} U_{T_r}(j)\, T_r(t-j) \qquad (2)$$

in which U (j) and $U_{T_r}(j)$ are the impulse response functions corresponding to the reach inflow and tributary inflows, s is the number of



a. Hydrologic Flow Network



b. The Concept of System Approach

Figure 6. A hydrologic flow network and the concept of system approach

gaged tributaries, j is the time lag, and k and ℓ define the range of this lag.

The linear drainage network model of equation 2 can be written as follows (Ψ = total number of days):

$$\underset{\sim}{q} = \underset{\sim}{H}_o \underset{\sim}{U}_I + \underset{\sim}{H}_1 \underset{\sim}{U}_{T_1} + \underset{\sim}{H}_2 \underset{\sim}{U}_{T_2} + \dots + \underset{\sim}{H}_s \underset{\sim}{U}_{T_s} \tag{3}$$

in which

$$q = \begin{bmatrix} O(0) \\ O(1) \\ \cdot \\ \cdot \\ O(\Psi) \end{bmatrix} \tag{4}$$

$$\underset{\sim}{H}_i = \begin{bmatrix} I(0) & & & \\ I(1) & I(0) & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot\,\cdot & \\ I(\Psi-1) & I(\Psi-2) & \dots & I(\Psi-\ell+k-1) \\ I(\Psi) & I(\Psi-1) & \dots & I(\Psi-\ell+k) \end{bmatrix};\ i=0,\ 1,\dots,\ s \tag{5}$$

$$\underset{\sim}{U}_I = \begin{bmatrix} U_I(k_0) \\ U_I(k_0+1) \\ \cdot \\ \cdot \\ U_I(\ell_0) \end{bmatrix} \tag{6}$$

and

$$\underset{\sim}{U}_{T_r} = \begin{bmatrix} U_{T_r}(k_r) \\ U_{T_r}(k_r+1) \\ \cdot \\ \cdot \\ U_{T_r}(\ell_r) \end{bmatrix} \tag{7}$$

Or, it can be written in a more compact form:

$$\underset{\sim}{q} = \underset{\sim}{H}\ \underset{\sim}{U} \tag{8}$$

in which

$$\underset{\sim}{H} = \left[\underset{\sim}{H}_0 \; \underset{\sim}{H}_1 \; \underset{\sim}{H}_2 \; \ldots \underset{\sim}{H}_s\right] \quad (9)$$

and

$$\underset{\sim}{U} = \begin{bmatrix} \underset{\sim}{U}_I \\ \underset{\sim}{U}_{T_1} \\ \cdot \\ \cdot \\ \underset{\sim}{U}_{T_s} \end{bmatrix} \quad (10)$$

1. <u>LDN Model with an Error Term</u>

The LDN model is an approximation; therefore, it is more realistic to express it with an error term of the form:

$$\varepsilon = \begin{bmatrix} \varepsilon(0) \\ \varepsilon(1) \\ \cdot \\ \cdot \\ \varepsilon(\dot{V}) \end{bmatrix} \quad (11)$$

Then, equation 8 becomes

$$q = H\,U + \varepsilon \quad (12)$$

If the error series in equation 12 has an autocorrelation structure, different from a white noise process, it can be modeled as an autoregressive moving average (ARMA) process by extracting the information from the error series. An iterative procedure for fitting the model, following the procedure of Box and Jenkins (1976), was used to identify the structure in the error series. It was found that the error series could be modeled satisfactorily by an autoregressive (AR) process represented by

$$\varepsilon_t = \sum_{i=1}^{p} \phi_i \, \varepsilon_{t-i} + \eta_t \quad (13)$$

in which ε_t is an observed error for the t-th day, p is the order of the AR model, ϕ_i are the AR (p) coefficients, and η_t is a component from a white noise process. The LDN model with an error term can be written following the expression of equation 2 as shown below.

$$O(t) = \sum_{j=k_0}^{l_0} U_I(j)\ I(t-j) + \sum_{r=1}^{s} \sum_{j=k_r}^{l_r} U_{T_r}(j)\ T_r\ (t-j) + \sum_{i=1}^{p} \phi_i\ \varepsilon_{t-i} + \eta_t \tag{14}$$

2. <u>LDN Model with an Autoregressive Term</u>

The LDN model can be modified by incorporating an autoregressive term with regard to the output in the right-hand side of equation 2:

$$O(t) = \sum_{j=k_0}^{l_0} U_I(j)\ I(t-j) + \sum_{r=1}^{s} \sum_{j=k_r}^{l_r} U_{T_r}(j)\ T_r(t-j) + U_0(1)\ O(t-1) + \varepsilon_t \tag{15}$$

The error term ε_t, in equation 15 was not modeled because the error series did not have a strong systematic autocorrelation structure. The error series can be modeled using an auto regressive moving average process if a strong autocorrelation structure is indicated.

The two flow forecasting models—the LDN model with an error term and the LDN model with an autoregressive term of output—require the same information. These two models are considered as alternative ways to forecast the river flows.

PARAMETER ESTIMATION AND SYSTEM IDENTIFICATION

The purpose of system identification is to characterize the system response from a given record of input and output. The components of the vector $\underset{\sim}{U}$ of the response function are called parameters, and these must be estimated. The methods of estimating parameters can be grouped into two general categories: transform methods and correlation methods (Dooge, 1973). The least-squares method was used as a correlation method in this study for estimating the parameter values of the $\underset{\sim}{U}$.

Parameter Estimation of $\underset{\sim}{U}$

The $\underset{\sim}{U}$ can be estimated with the least-squares method by minimizing the following quadratic form (Natale and Todini, 1977) for equation 12:

$$J(\underset{\sim}{\varepsilon}) = \frac{1}{2} \underset{\sim}{\varepsilon}^T R^{-1} \underset{\sim}{\varepsilon} \qquad (16)$$

in which $\underset{\sim}{R}^{-1}$ must be a symmetric positive definite matrix to ensure the existence of a minimum. Equation 16 can be written in terms of U:

$$J(\underset{\sim}{U}) = \frac{1}{2} (\underset{\sim}{q} - \underset{\sim}{H}\underset{\sim}{U})^T \underset{\sim}{R}^{-1} (\underset{\sim}{q} - \underset{\sim}{H}\underset{\sim}{U}) \qquad (17)$$

The necessary condition for the existence of an extremum is:

$$\frac{\partial J(\underset{\sim}{U})}{\partial \underset{\sim}{U}} = (\underset{\sim}{H}^T \underset{\sim}{R}^{-1} \underset{\sim}{H})\, \underset{\sim}{U} - \underset{\sim}{H}^T \underset{\sim}{R}^{-1} \underset{\sim}{q} = 0 \qquad (18)$$

and the sufficient condition for a minimum is satisfied by

$$\frac{\partial^2 J(U)}{\partial \underset{\sim}{U}^2} = \underset{\sim}{H}^T \underset{\sim}{R}^{-1} \underset{\sim}{H} \qquad (19)$$

being positive definite. The least-squares estimate of the parameters $\underset{\sim}{U}$ can be obtained from equation 18:

$$\underset{\sim}{U} = (\underset{\sim}{H}^T \underset{\sim}{R}^{-1} \underset{\sim}{H})^{-1} \underset{\sim}{H}^T \underset{\sim}{R}^{-1} \underset{\sim}{q} \tag{20}$$

Computation to estimate the parameter values of the $\underset{\sim}{U}$ were performed using a computer program of the International Mathematical Scientific Library.

The values of k_r and l_r in equation 2 are related to the length of system memory or, physically, the flow travel time required between two river gaging stations. The flow travel time between two stations can be estimated from the cross-correlation functions of two time series or by the observation of flow hydrographs of the stations plotted together. Based on the estimated flow travel time, the structure of the model, i.e., the values of k_r and l_r, are determined by trial and error using the values of variance of the error series as a guide. The equation 12 models the original flow time series better than equation 8 and is thus preferred.

When it is difficult to identify the model structure, the most important input, usually I(t), can first be included in the input matrix, and then the structure of this model input can be determined. The second most important input can then be added to the input matrix and the structure of this model input determined with a fixed structure of the first input. This procedure can be continued until all significant model inputs are included in the input matrix.

It is possible to determine parameter values by constraining response functions to be positive and the system to conserve mass:

$$\sum_{j=k_0}^{l_0} U_I(j) = 1 \tag{21}$$

and

$$\sum_{j=k_r}^{l_r} U_{T_r}(j) = 1 \qquad (22)$$

This approach was developed by Natale and Todini (1977) and applied by Yazicigil, Rao, and Toebes (1979) to the Green River Basin in Kentucky. However, the inputs to the model do not include all the actual inputs to the Illinois River because there are many ungaged small streams and groundwater inflows along the Illinois River. This fact leads to the idea that the sum of parameter values of a tributary can be greater than unity. Moreover, the constraints imposed on the parameter may reduce the fitting ability of the least squares method. Therefore, the parameter values were not constrained in the identification process.

The gaging stations for the tributaries do not exist near the confluence with the Illinois River; the flows at these gaging stations were multiplied by the ratio of the drainage area at the mouth of a tributary to that at the gaging station. This is not an essential procedure for the parameter identification; however, such a procedure is considered to help tributary inflows represent actual inputs to the Illinois River.

Estimation of the $\underset{\sim}{\phi}$ Parameters

Identifications of the autoregressive model order, p, and the parameter values, ϕ_i, are performed iteratively according to the processes of model identification, estimation, and diagnostic checking for fitting the Box-Jenkins type of models. An error series, $\underset{\sim}{\varepsilon}$, after fitting the $\underset{\sim}{q} = \underset{\sim}{H}\underset{\sim}{U}$ model as in equation 12, was identified as a second order autoregressive model for each reach. Verification is made by the residual series obtained after fitting the AR model. The residual series from the second-order model gave autocorrelation functions resembling white noise. Reductions in variances were found to be insignificant for higher order models. An interactive computational method developed by Kline and Devor (1979) at the University of Illinois was utilized to perform these tasks. Some streams, which are gaged but do not have significant contribution to the Illinois River, were ignored. These streams are Hickory Creek, Big Bureau Creek, and Kickapoo Creek.

Results of Parameter Estimation

The parameter values for the LDN model with an error term and the LDN model with an autoregressive term were determined for each of the three reaches. These are given in table 2. The table includes reach number (I,II, or III), model number (1 for LDN with an error term and 2 for LDN with an autoregressive term), streams which constitute inputs to the model, response function terms, e.g., U (1) or U_{T_1} (1), corresponding to those in equations 14 and 15, and parameter values for the response function and AR(2) error term as well as the autoregressive term $U_0(1)$. The first stream for a model is the Illinois River itself at the upper end of the reach, followed by the tributaries to the reach of the river. The numbers in the parentheses with the terms indicate orders or lags; for example 1 and 2 are 1-day time lag and 2-day time lag, respectively, between input and output. The parameters of the response function and error terms correspond to equation 14 for model 1 (e.g., model I-1A), and the parameters for the response function terms and autoregressive term correspond to equation 15 for model 2 (e.g., model I-2A).

In this study, the $\tilde{\varepsilon}$ series is called an error series and η series is called a residual series. The variances of observed flows, error series,

Table 2. LDN Models and Parameters

Model No.	Illinois River		Tributary 1		Tributary 2		Tributary 3		Tributary 4		Error model		Autoregressive model
Reach I													
	CS&SC at Lockport		DesPlaines		DuPage		Kankakee		Mazon				
	$U_I(0)$	$U_I(1)$	$U_{T_1}(0)$	$U_{T_1}(1)$	$U_{T_2}(0)$	$U_{T_2}(1)$	$U_{T_3}(0)$	$U_{T_3}(1)$	$U_{T_4}(0)$	$U_{T_4}(1)$	ϕ_1	ϕ_2	$0^{(1)}$
I-1A	0.7049	0.3579	1.0766	-0.2034	1.3958	0.9240	0.6691	0.3108	1.0212	0.7403	0.3396	0.2435	
I-2A	0.5961	0.0862	1.2301	-0.6761	1.5553	-0.1869	0.8149	-0.2046	0.8762	0.3954			0.3657
I-1B		1.1218		1.0279		1.6961		0.9357		1.8099	0.3097	0.0007	
I-2B		0.9599		0.9187		1.2239		0.7608		1.6548			0.1617
Reach II													
	Ill. at Marseilles		Fox		Vermilion		Mackinaw						
	$U_I(3)$	$U_I(4)$	$U_{T_1}(2)$	$U_{T_1}(3)$	$U_{T_2}(2)$	$U_{T_2}(3)$	$U_{T_3}(1)$	$U_{T_3}(2)$					
II-1	0.3365	0.6395	1.6375	0.5275	0.7746	-0.6572	1.4633	-0.8789			0.9441	-0.2009	
II-2	0.1686	-0.0949	0.5192	-0.2979	0.2891	-0.2185	0.4959	-0.3518					0.9061
Reach III													
	Ill. at Kingston Mines		Spoon		Sangamon		LaMoine						
	$U_I(3)$	$U_I(4)$	$U_{T_1}(2)$	$U_{T_1}(3)$	$U_{T_2}(2)$	$U_{T_2}(3)$	$U_{T_3}(1)$						
III-1	0.3556	0.8030	1.1066	-0.8319	0.4065	0.4286	1.0913				1.0683	-0.2759	
III-2	0.2044	-0.1422	0.1207	-0.1242	0.3321	-0.2410	0.2949						0.9276

and residual series are given in table 3 for all types of models; however, the variance of residual series is not given for the LDN model with an autoregressive term because its error series was not modeled. The correlograms of the error series and residual series for the LDN model with an error term and of the error series for the LDN model with an autoregressive term are shown in figures 7 through 10.

Reach I. This reach is about 44 miles long, from Lockport to Marseilles, and has steeper bed slope than other reaches. The major tributaries to this reach are fairly well distributed and consist of the DesPlaines, DuPage, Kankakee, and Mazon Rivers. The reach inflow at Lockport comes mainly from Lake Michigan water diversion and wastewater effluents from Metropolitan Sanitary District of Greater Chicago (MSDGC) plants, and it is quite stable within a year and throughout years. About one-half of the reach outflow comes from the reach inflow at Lockport, and the remaining half comes mainly from the Kankakee River.

Two model structures A and B (with 0- and 1-day lag, and 1-day lag) were investigated. The error series variance with model I-1A and I-2A is very small (table 3), indicating that the fitting is very good. The residual series variance is 75 percent of the error series variance and thus the AR(2) error term model does not explain much variance of the error series because the LDN model itself explained most (approximately 98 percent) of the original variance. In comparing the A and B model structures, it is evident that the term of 0-day lag between input and output significantly reduces variances of the residual series.

Reach II. The second reach is about 102 miles long, from Marseilles to Kingston Mines, and the bed slopes are much milder than in the first reach. Two gaged creeks, Big Bureau and Kickapoo, were ignored in the

Table 3. Variances of Observed Flow, Error Series, and Residual Series

Reach No.	Illinois River at	Model No.	Variances of: Observed Flow Series	Variances of: Error Series	Variances of: Residual Series	% $\frac{Col.5}{Col.4}$	% $\frac{Col.6}{Col.5}$	% $\frac{Col.6}{Col.4}$
1	2	3	4	5	6	7	8	9
I	Marseilles	I-1A	2.736×10^7	5.371×10^5	4.037×10^5	1.96	75.16	1.48
		I-2A	"	4.466×10^5		1.63		
		I-1B	"	1.962×10^6	1.773×10^6	7.17	90.37	6.48
		I-2B	"	1.941×10^6		7.09		
II	Kingston Mines	II-1	7.817×10^7	8.338×10^6	3.056×10^6	10.67	36.65	3.91
		II-2	"	1.368×10^6		1.75		
III	Meredosia	III-1	1.900×10^8	1.489×10^7	2.972×10^6	7.84	19.96	1.56
		III-2	"	2.031×10^6		1.07		

Observed flows are in cfs.

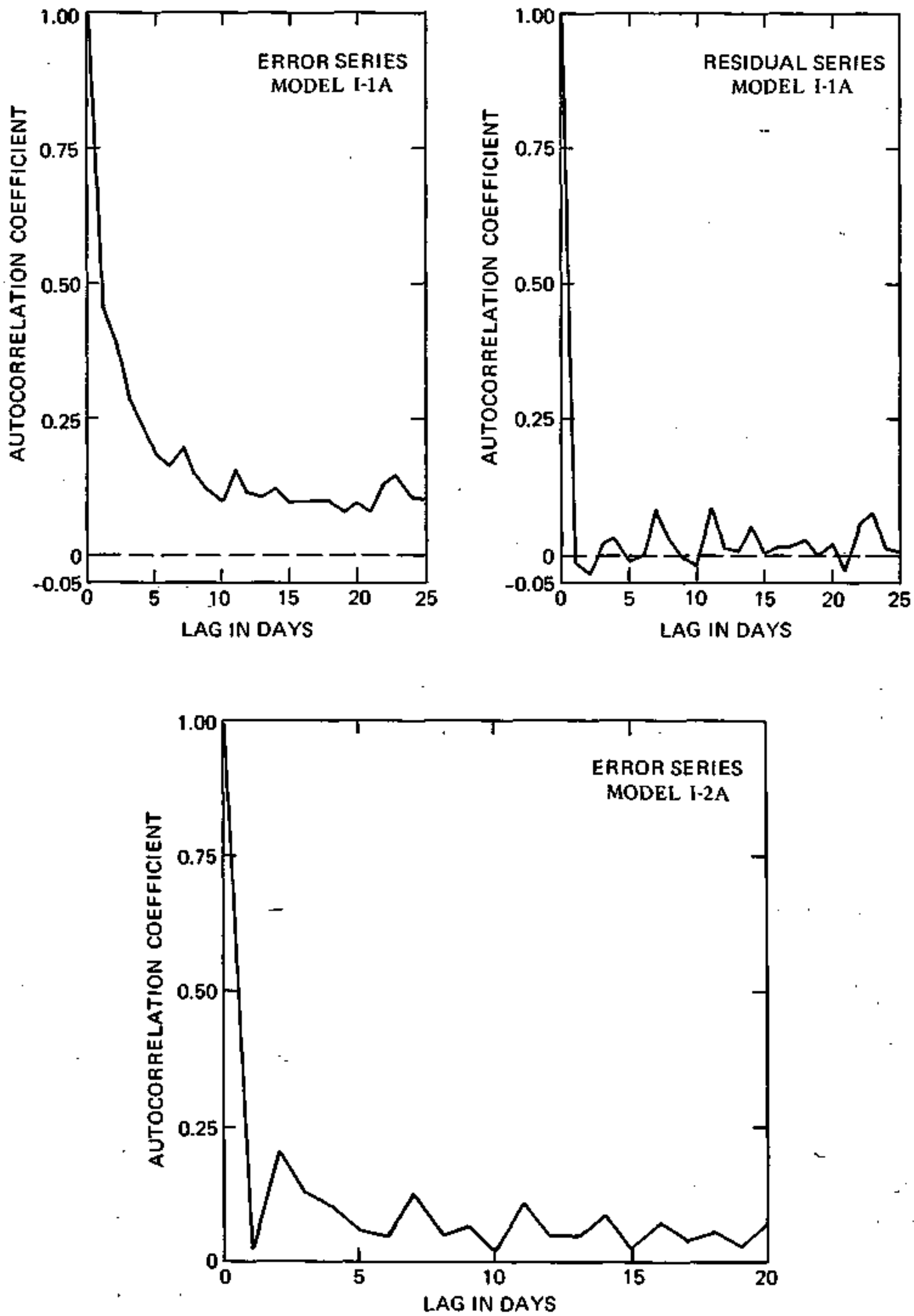


Figure 7. Autocorrelation coefficients of error and residual series for LDN model with error term (I-1A) and of error series for LDN model with autoregressive term (I-2A), first reach

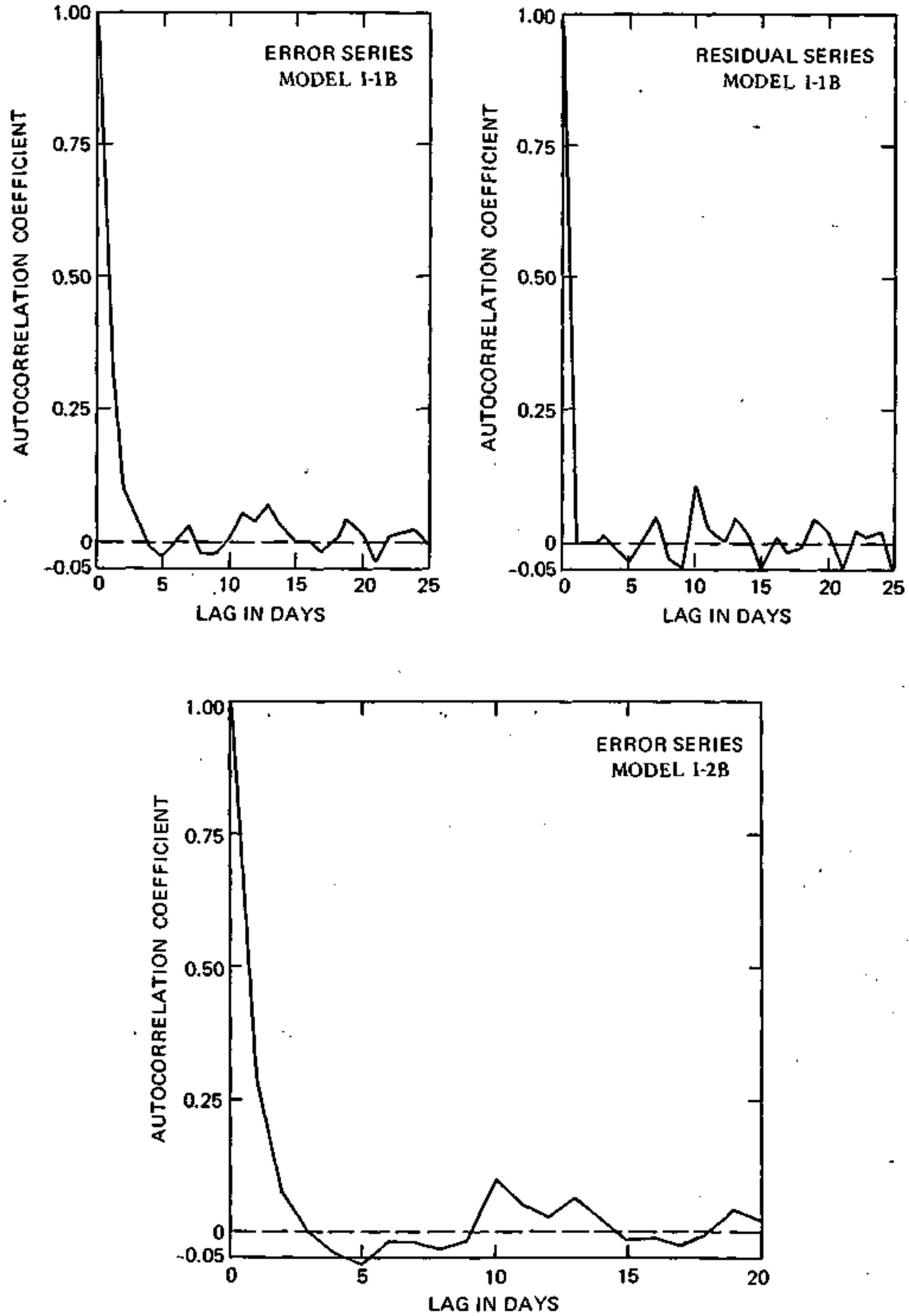


Figure 8. Autocorrelation coefficients of error and residual series for LDN model with error term (I-lB) and of error series for LDN model with autoregressive term (I-2B), first reach

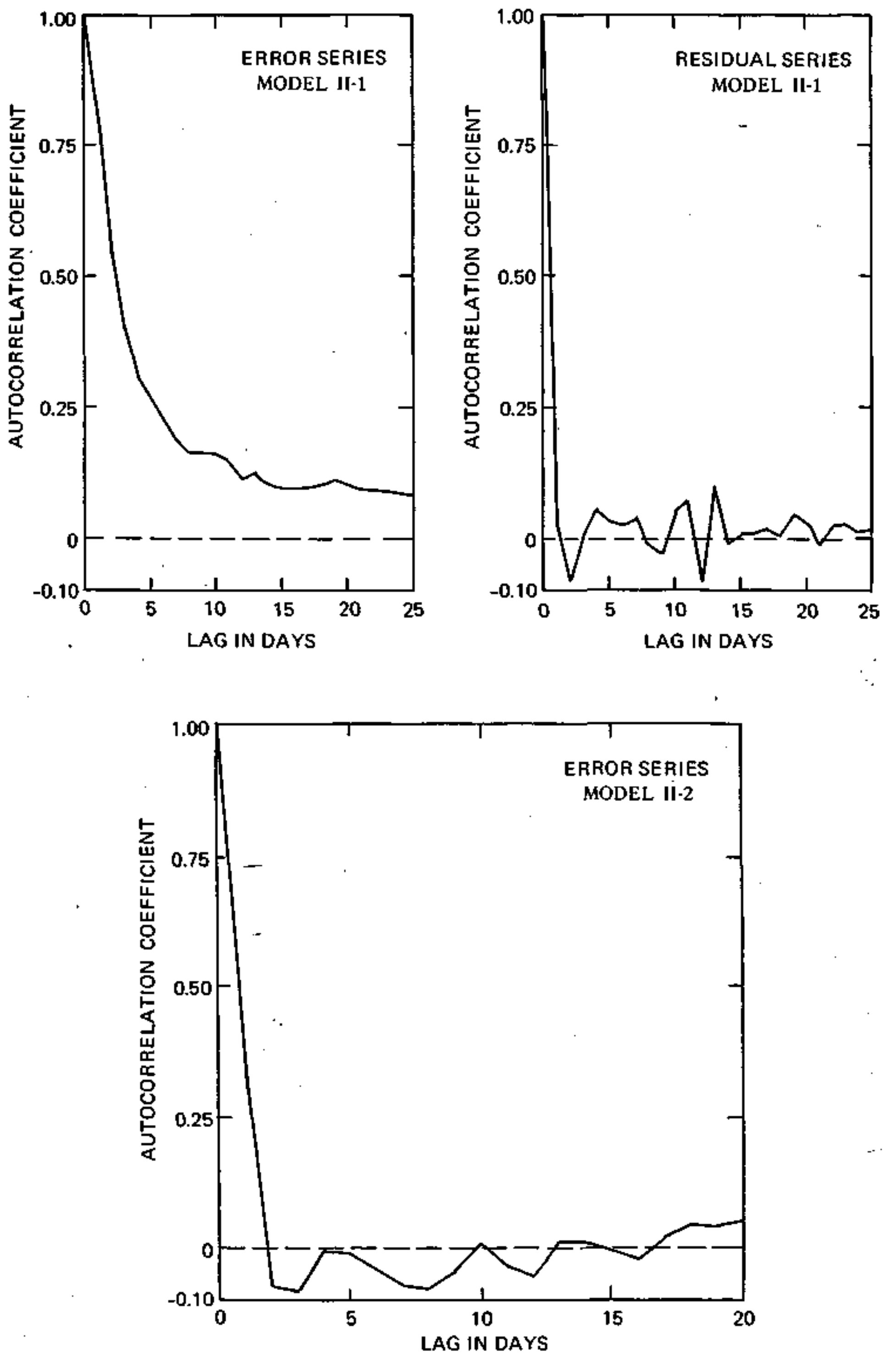


Figure 9. Autocorrelation coefficients of error and residual series for LDN model with error term (II-1) and of error series for LDN model with autoregressive term (II-2), second reach

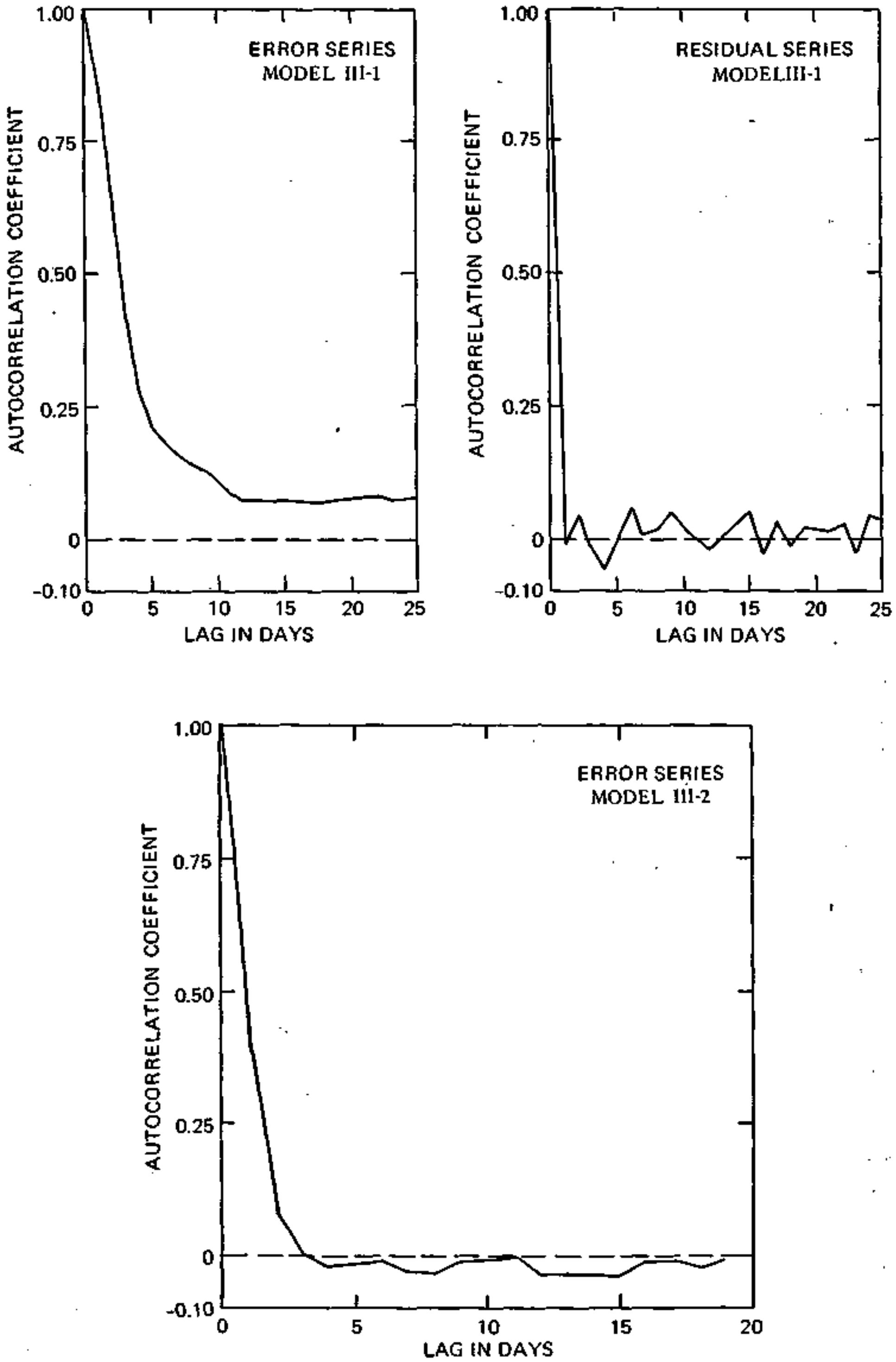


Figure 10. Autocorrelation coefficients of error and residual series for LDN model with error term (III-1), and of error series for LDN model with autoregressive term (III-2), third reach

parameter identification process because their effects are considered to be negligible for modeling flows in the main river.

The tributaries are not well distributed; the largest tributary in the reach, Fox River, is located only 7 miles downstream of Marseilles and the next-largest tributary, Vermilion River, is about 30 miles downstream of the Fox. The Mackinaw River joins the Illinois River about.3 miles upstream of the Kingston gage. The reach contains a series of large lakes such as Senachwine Lake, Goose Lake, and Peoria Lake, implying that storage effects will be dominant. These facts make the parameter identification process difficult because effects of the reach inflow and tributary inflow are masked by the storage effects to some extent and because the two major model inputs are located too close to each other to distinguish between their effects.

The percentage of variance for the error series is larger than for the first reach; however, the percentage of variance for the residual series is smaller. The correlogram of the error series for the second reach (figure 9) shows that the autoregressive process is stronger than for the first reach (figures 7 and 8); therefore, the error model can explain more of the variance of the error series for the second reach than for the first reach. The correlograms of residual series are virtually a random process and no systematic structure can be observed. The LDN model with an autoregressive term gives much smaller variance for the error series than the LDN model with an error term because outflow at Kingston Mines is practically an autoregressive process due to storage effects of the lakes and channel.

Reach III. The third reach is about 73 miles long from Kingston Mines to Meredosia, and the bed slopes are as mild as those in the second reach. The tributaries are well distributed along the reach: Spoon River, Sangamon River, and LaMoine River. The Sangamon River has the largest drainage area among all the tributaries in the three reaches.

The 10-year flow record from October 1960 to September 1970 was used for the parameter identification of the U because the first five-year flow record did not yield reasonable values. This reach seems to have fewer storage effects than the second reach; therefore, the parameter identification was easier. The percentage of error variance is smaller than for the second reach, indicating that the LDN model can explain the flow system better for this reach (but not as well as for the first reach). Moreover, the correlogram of the error series (figure 10) shows the strongest autoregressive process. Consequently, the AR(2) error model can explain the error series well and the percentage of residual variance is the smallest of the three reaches. The correlogram of the residual series indicates no systematic structure. The LDN model with an autoregressive term gives a very small variance but the parameter value for the autoregressive term is quite high, 0.9276, and parameter values for reach inflow and tributaries are quite low.

FLOW FORECASTING'

With the model parameters presented in table 2, flow forecasts for the Illinois River at Marseilles, Kingston Mines, and Meredosia were made for each reach with given historical (October 1965 to September 1970) inflows and tributary flows as the model inputs. The model output from the upstream reach was not utilized as reach input for the downstream reach because the predictive performance for the second reach, particularly with the LDN model with an error term, was not good enough to use the predicted outflows of the second reach as an input to the third reach. However, predicted model output (i.e., reach outflows) should be used as an input (i.e., reach inflows) for the next reach in order to estimate the effects of increased diversion from Lake Michigan and to derive a schedule of optimal diversions from the lake. This can be done successfully in a later study after the model for the second reach and its predictive performance have been improved.

Three kinds of models were used for the flow forecasting: the LDN model itself, the LDN model with an error term, and the LDN model with an autoregressive term. Flow forecasting with the LDN model can be performed easily using equation 2. Observed reach inflows and observed tributary inflows are necessary to implement the LDN model, and observed outflows are not needed. Flow forecasts for the third reach can be made several days ahead with the LDN model. However, the LDN models with an error term and an autoregressive term forecast flows only one day ahead because these models require, for example, today's observed flow to predict tomorrow's. For the LDN model with an autoregressive term, this requirement is indicated by the third term in the right-hand side of equation 15.

The LDN model with an error term for a reach provides flow forecasts through the use of the following equation (Yazicigil, Rao, and Toebes, 1979):

$$\hat{O}^f(t+1 \ t) = O^f(t+1|t) - \left\{ \sum_{i=1}^{p} \phi_i \ [O^f(t-i+1|t-i) - O^h(t-i+1)] \right\} \quad (23)$$

in which $\hat{O}^f(t+1|t)$ is the modified one-day-ahead flow forecast, $O^f(t+1|t)$ is the unmodified one-day-ahead flow forecast given by the LDN model, $O^h(t-i+l)$ is the historic flow, and p is the number of autoregressive parameters in the error model as in equation 13. The notation $O(t+1|t)$ signifies a (t+l)th-day flow forecast based on the observed data up to and including day t. The above equation indicates that a reach outflow can be modified by the observed past reach outflows.

Flow forecasts were made for the period of October 1965 through September 1970 with the three methods (the LDN model itself, the LDN model with an error term, and the LDN model with an autoregressive term) for each of the three reaches. Forecasted flows were plotted together with the observed flows for comparison. The year 1970 indicates wide variations (low to very high) of flows. A 200-day period, beginning January 9, 1970, was selected for illustrative purposes. In many cases, particularly for the first reach, predicted flows with the LDN model and the LDN model with an error term are close to each other. Therefore, the flows predicted with the LDN model with error terms and the LDN model with an autoregressive term are compared with the observed flows for the three reaches in figures 11 through 18.

First Reach. The predicted and observed flows for the first reach, derived using the LDN model with an error term, are shown in figure 11 for I-1A and in figure 12 for I-1B. The fitting is better with I-1A than with

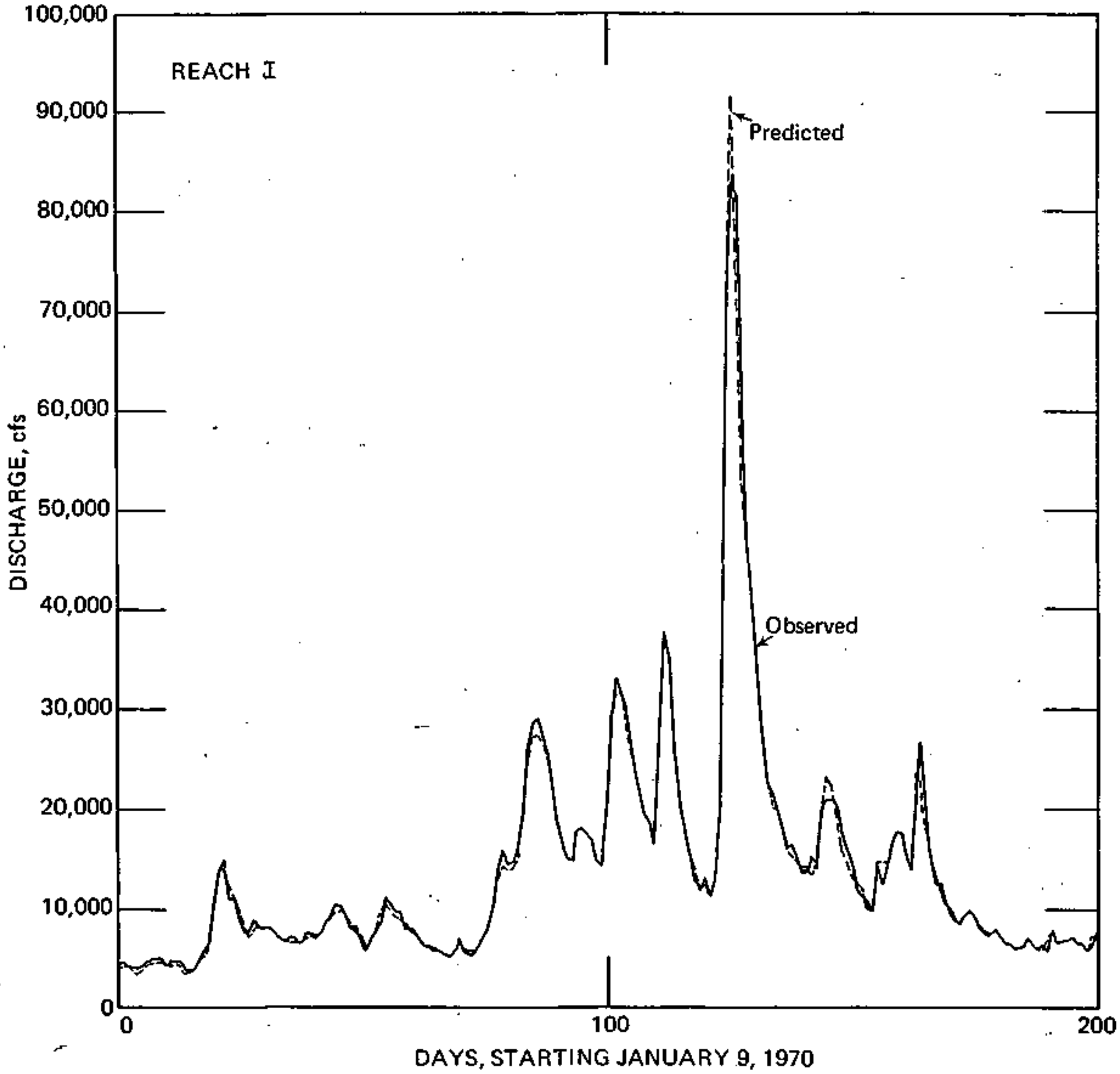


Figure 11. Predicted and observed flows at Marseilles (LDN with error term, model I-1A)

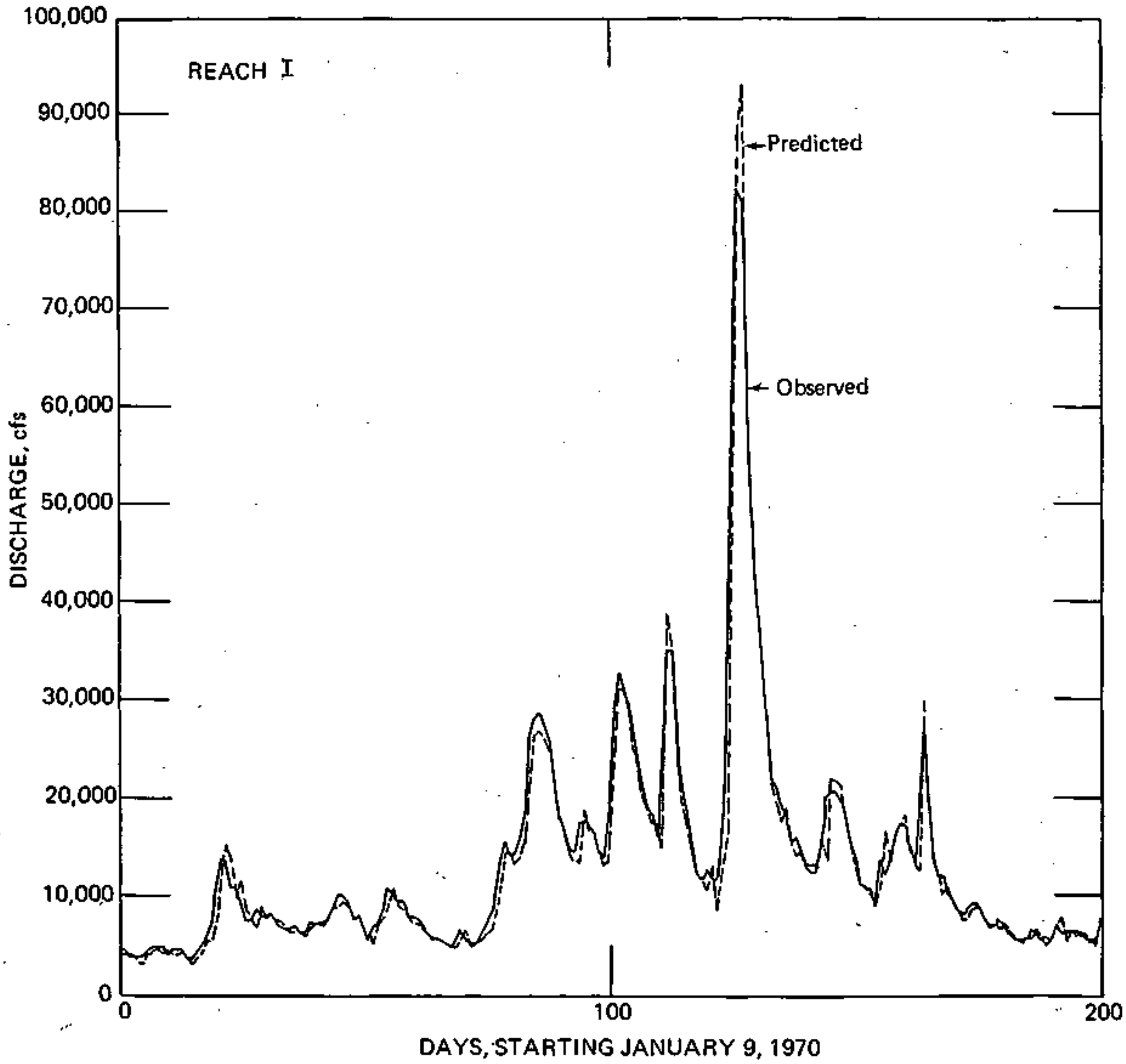


Figure 12. Predicted and observed flows at Marseilles
(LDN with error term, model I-1B)

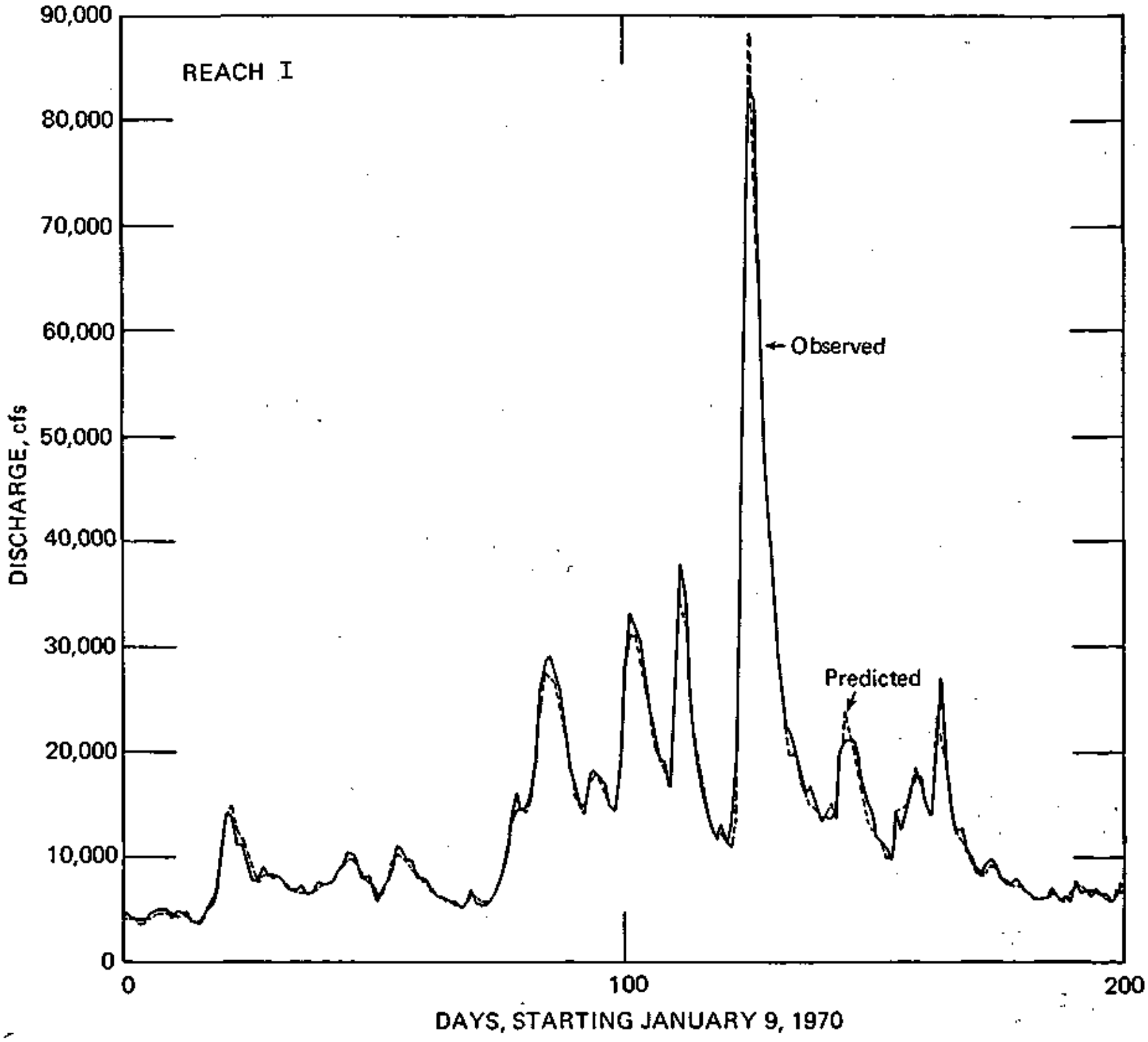


Figure 13. Predicted and observed flows at Marseilles
(LDN with autoregressive term, model I-2A)

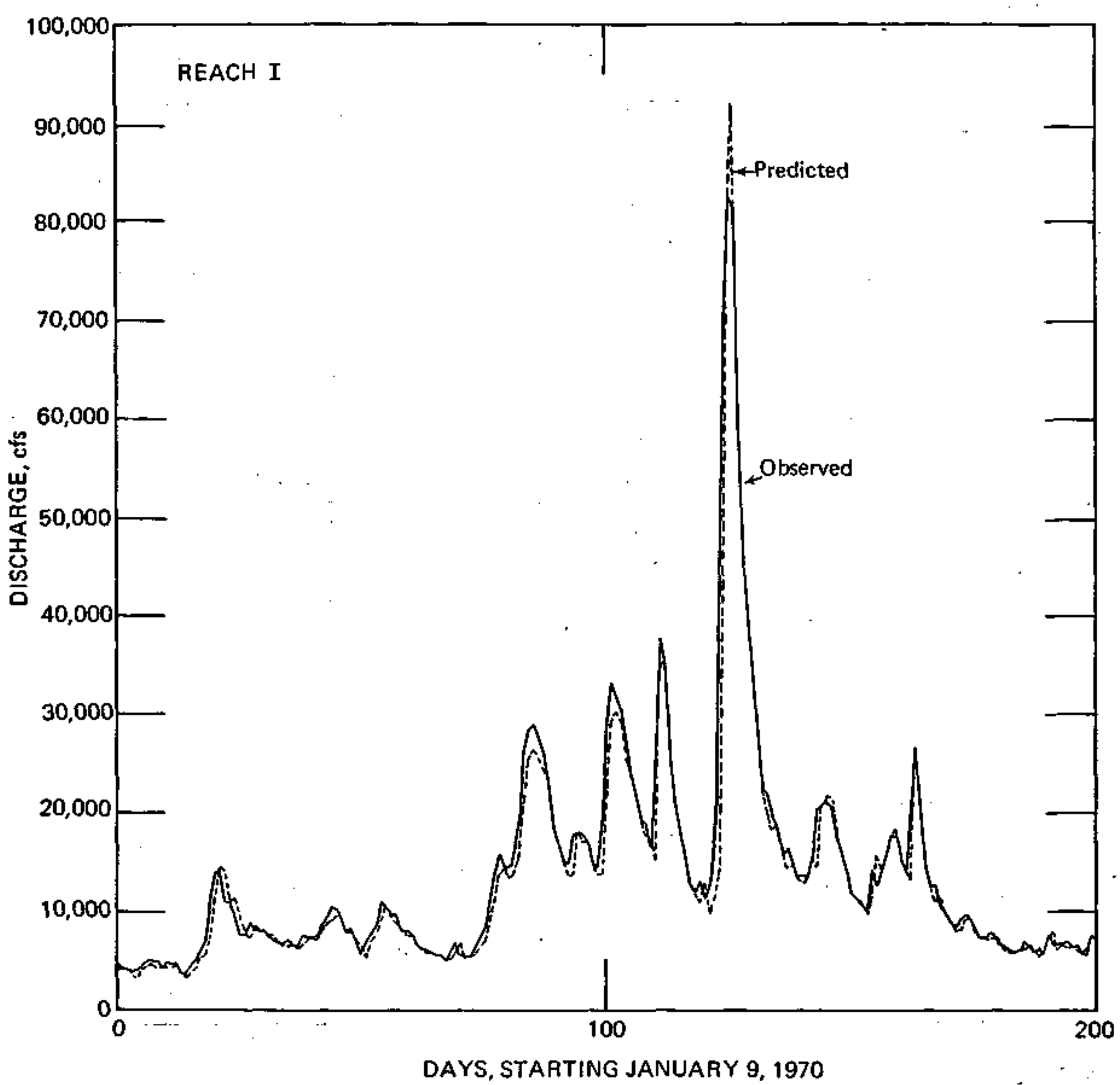


Figure 14. Predicted and observed flows at Marseilles (LDN with autoregressive term, model I-2B)

I-1B. A similar conclusion is drawn from figures 13 and 14, in which the predicted flows are obtained with the LDN model with an autoregressive term according to I-2A and I-2B, respectively. Both LDN models, with an error term and with an autoregressive term, yield equally good predictions. Even predictions with I-lB and I-2B are satisfactory for evaluating the effects of changes in flow at Lockport.

Second Reach. The flow forecasts with the LDN model with an error term (figure 15) do not fit the observed flows as well as for the other two reaches. This is attributed to rather poor parameter identification because of strong storage effects. The LDN model with an error term generally underestimates low flows and overestimates high flows for this reach. The hydrograph generated with the model shows more frequent oscillations than the observed hydrograph. The flow forecast with the LDN model with an autoregressive term (figure 16) fit the observed flows very well. However, a 1-day lag between the predicted and observed flows is found in the hydrographs.

Third Reach. The flow forecasts for the third reach based on the LDN model with an error term (figure 17) are much better than those for the second reach (figure 15). This reach has fewer storage effects, and reach inflows from the Kingston Mines gage have already passed a storage filter effect through the second reach. The flow forecasts using the LDN model with an autoregressive term (figure 18) fit the observed flows much better, but they do exhibit the lag phenomenon mentioned earlier with regard to the second reach. The value of the autoregressive term parameter is close to 0.93.

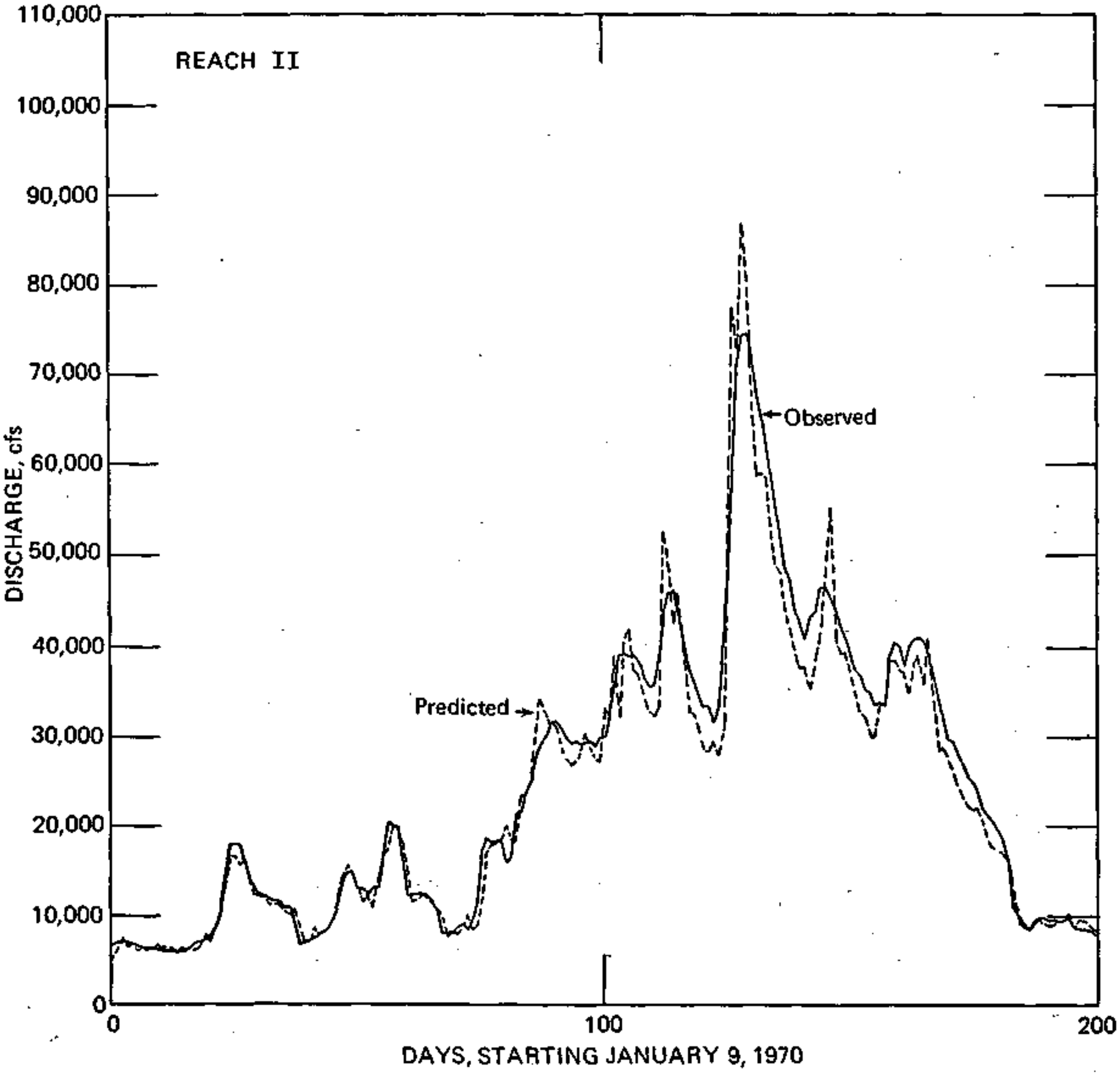


Figure 15. Predicted and observed flows at Kingston Mines (LDN with error term, model II-l)

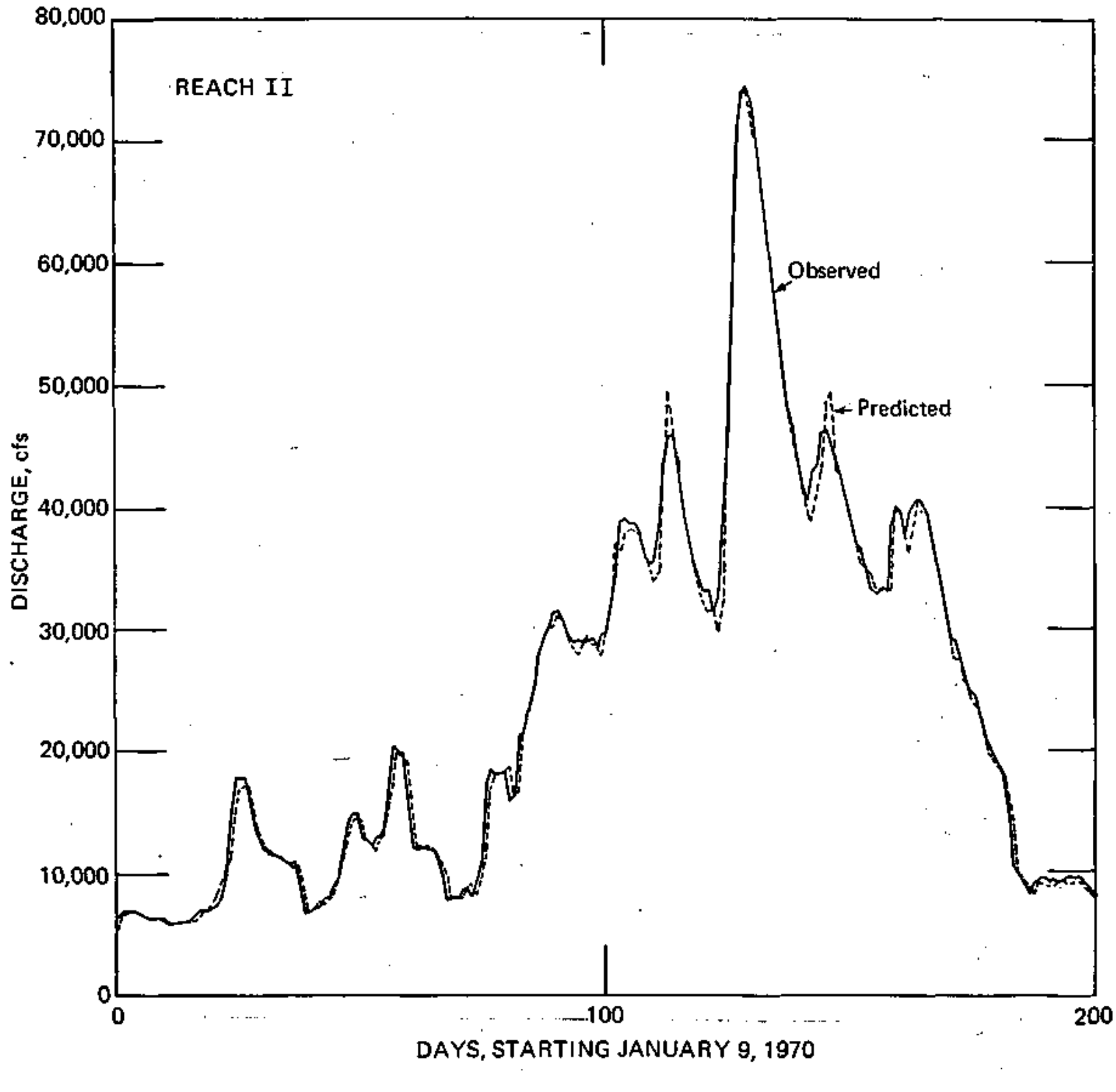


Figure 16. Predicted and observed flows at Kingston Mines
(LDN with autoregressive term, model II-2)

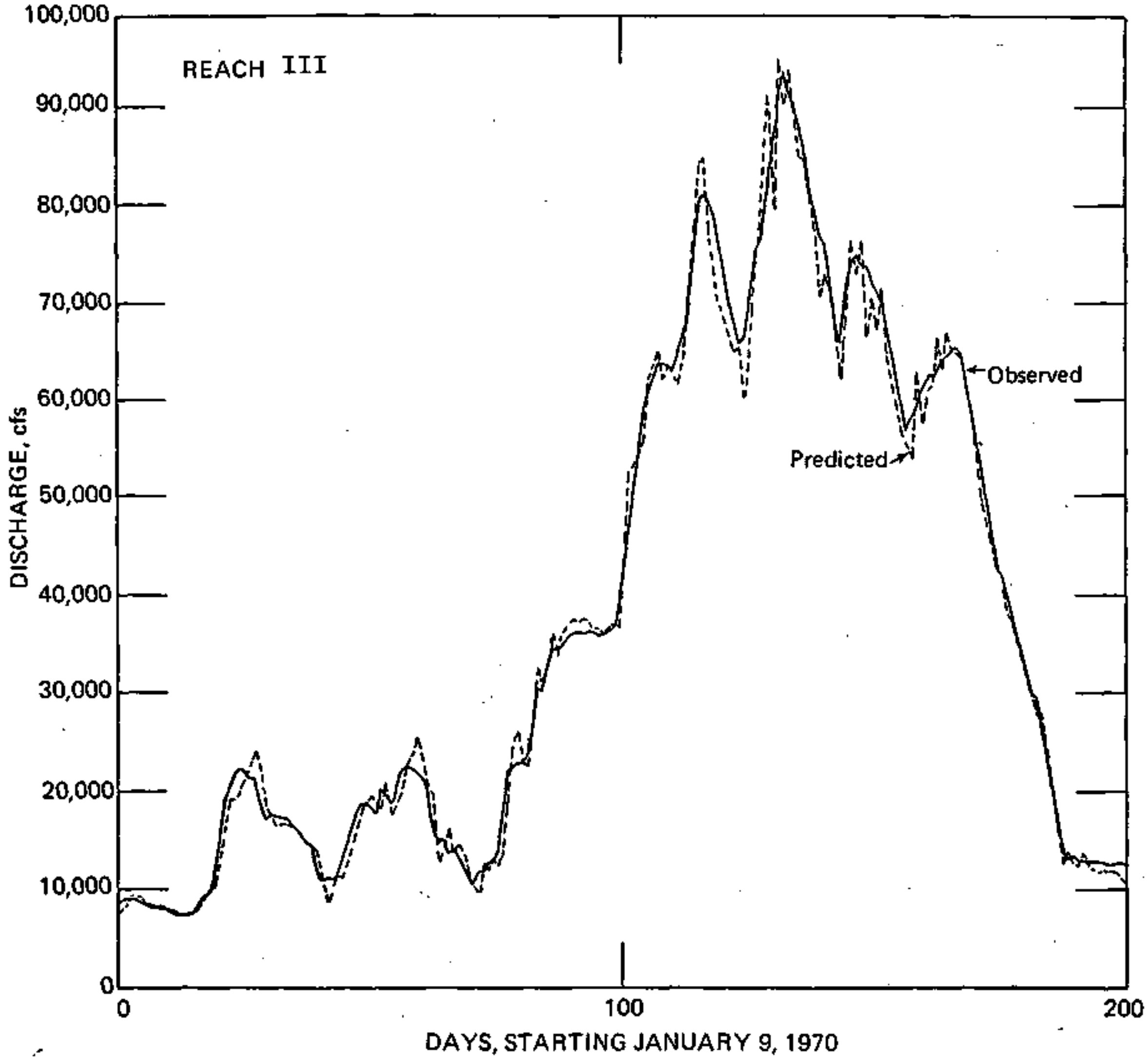


Figure 17. Predicted and observed flows at Meredosia
(LDN with error term, model III-l)

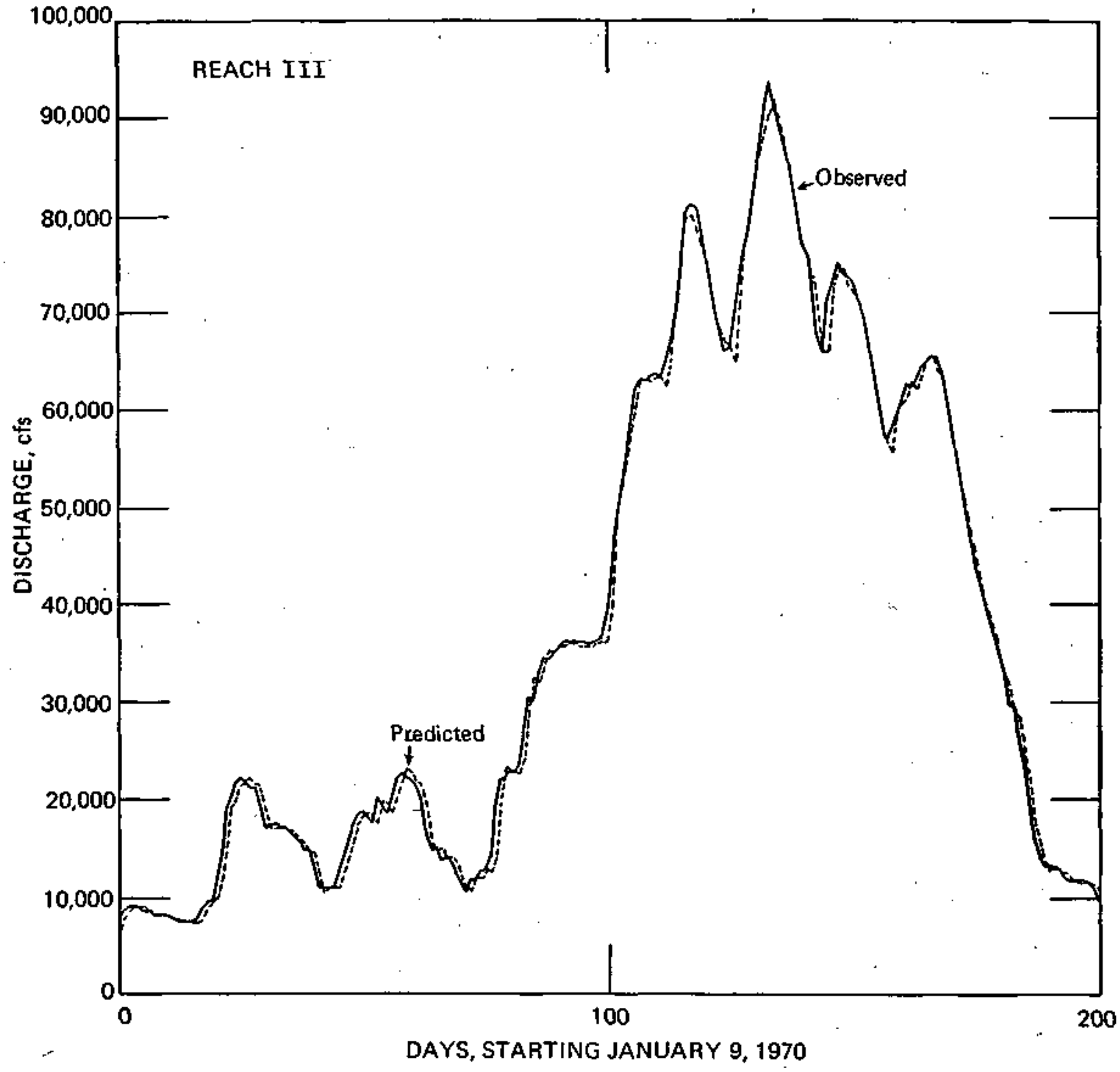


Figure 18. Predicted and observed flows at Meredosia
(LDN with autoregressive term, model III-2)

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