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ON SIGNALS IMBEDDED IN NOISE

Report R-2  
(Second Running)

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A detector scans a series of channels. On one of them is a true signal which is being searched for. In all of them are noise signals. These will be idealized as  $n_0$  (on the average) noise pulses per channel, of equal height. The statistical distribution of noise pulses in any channel is assumed given by the Poisson distribution

$$P_0(n) = \frac{n_0^n e^{-n_0}}{n!} \approx \frac{1}{\sqrt{2\pi n_0}} e^{-\frac{(n-n_0)^2}{2n_0}} \quad \text{if } n_0 \gg 1$$

The Gaussian approximation holds well if the mean number  $n_0$  is large compared to one.

The height of the true signal is  $\epsilon n_0$  where it is anticipated that  $\epsilon$  will be of order one or less. In the channel containing the true signal, the distribution of pulses (noise plus signal) is

$$P_1(n) = \frac{1}{\sqrt{2\pi n_0(1+\epsilon)}} e^{-\frac{(n-n_0-\epsilon n_0)^2}{2n_0(1+\epsilon)}}$$

where we have assumed that true signal and noise add linearly.

If now the receiver has bias  $B$ , then the probability for exceeding the bias in scanning a channel with noise only is

$$R_0 = \int_B^{\infty} dn P_0(n)$$

and correspondingly, for the channel with the true signal

$$R_1 = \int_B^{\infty} dn P_1(n)$$

Both  $R_0$  and  $R_1$  approach one for  $B$  going to zero. We will in fact treat the case where  $B$  is small so that both  $R_0$  and  $R_1$  are nearly one. Using the asymptotic expansion of the error integral we get, for this case

$$R_0 = 1 - \frac{1}{2\sqrt{\pi}} \frac{e^{-\left\{\frac{B-n_0}{\sqrt{2n_0}}\right\}^2}}{\left\{\frac{B-n_0}{\sqrt{2n_0}}\right\}} = 1 - \theta_0$$

$$R_1 = 1 - \frac{1}{2\sqrt{\pi}} \frac{e^{-\left\{\frac{B-n_0-\epsilon n_0}{\sqrt{2n_0(1+\epsilon)}}\right\}^2}}{\left\{\frac{B-n_0-\epsilon n_0}{\sqrt{2n_0(1+\epsilon)}}\right\}} = 1 - \theta$$

Putting in the following illustrative numbers

$$n_0 = 18$$

$$B = 6$$

$$\epsilon = 1/3 \text{ (signal to noise ratio)}$$

Then

$$R_0 = (1 - 2.6 \times 10^{-3})$$

$$R_1 = (1 - 1.16 \times 10^{-5})$$

It is practically certain that the bias will be exceeded in both bands of channels.

However, operate now in the following way. Scan through the entire series again and again. Whenever by chance one channel fails to produce a response over the bias delete that channel from then on. The only channels that remain after many scannings are those which have produced a response every time without exception.

After K scannings the probability for a channel without true signal to survive is

$$R_0^k = (1-\theta_0)^k \approx e^{-k\theta_0}$$

and, similarly, for the channel with the true signal

$$R_1^k = (1-\theta_1)^k \approx e^{-k\theta_1}$$

With the numbers used above, and with  $K = 5000$ , we get

$$R_0^K = 2 \times 10^{-6}$$

$$R_1^K = 0.942$$

Thus by this time the channel with signal is most likely still in operation, while those without signal have almost certainly been cut out at some point along the way. If represented on a scope screen, the screen would start all white, would darken gradually, and would finally have just a single spot covering the channel be lost. The sequence of scannings should therefore be interrupted at some number which is optimum. Then, if no true signal has appeared, the whole procedure is begun again.

The suitability of this method clearly depends on  $N_0$  being a large number, that is on there being so much noise that the fractional width of the distribution  $P_0(n)$  is very small. If  $n_0$  were increased over the value 18 used here, even more striking effects could be obtained and even lower signal to noise ratios could be detected.

In taking the noise to follow a Poisson distribution it is implicit however, that all the noise pulses are independent of each other. To increase  $n_0$  can mean only to increase the number of detectors or to lengthen the time for scanning each channel.