Experiments with Programs for the Simulation of Large Scale Automata on a Digital Computer

Report R-59

November 1954

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Contract DA-36-039-SC-56695
Project 8-103A, D/A Project 3-99-10-101
"The research reported in this document was made possible by support extended to the University of Illinois, Control Systems Laboratory, jointly by the Department of the Army (Signal Corps and Ordnance Corps), Department of the Navy (Office of Naval Research), and the Department of the Air Force (Office of Scientific Research, Air Research and Development Command), under Signal Corps Contract DA-36-039-SC-56695, Project 8-103A, D/A Project 3-99-10-101."
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EXPERIMENTS WITH THE PROGRAMS
FOR THE SIMULATION OF LARGE SCALE AUTOMATA
ON A DIGITAL COMPUTER

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Contract DA-36-039-SC-56695
Project 8-103A, D/A Project 3-99-10-101

Numbered Pages: 92
Acknowledgement

The authors wish to express their thanks to Miss Carol Stewart and Mrs. Jean Linden who performed many of the calculations in the data analysis. They also wish to thank Professor Ralph Meagher and Professor John Nash of the Digital Computer Laboratory for their cooperation in making Illiac available to us, and Mr. William Huffman, Miss Ramona Russell, and Mrs. Caroline Brown also of the Digital Computer Laboratory for their assistance in preparation of tapes and running the programs.
I. Introduction

This report presents some of the more interesting results of experiments with the simulation code written for Illiac, described in the CSL report R-58,¹ which provides a 1/5-scale model of the tracking and sorting operations of the air traffic control system proposed in the CSL report R-35. These experiments are only the first of a series which are being conducted for the purpose of investigating the properties of the air traffic control system. Although only a few of the many possible parametric configurations have been studied it appeared worthwhile to report some preliminary results at this time to indicate the course of the experimental program and to help guide construction of the actual hardware for the system.

A strong attempt has been made to avoid value judgements concerning the quality of tracking which are subject to many qualifications and which, within certain limits, are usually a matter of taste. A number of quantities which are capable of precise definitions on which such judgements usually seem to be based have been measured and are reported here. Because of the large mass of data which must be analyzed our choice of these quantities was governed partly by their ease of measurement, however it is felt that the quantities so chosen represent a useful description of important properties of the system.

¹Hereafter this report is simply referred to as R-58.
II. Preset Parameters, Smoothing and Sorting Parameters, and some Logical Operations which have been used

The values of the preset parameters have remained practically constant for all experiments, the ones most commonly used are listed in Table 1. Unless specifically stated otherwise the preset parameters in an experiment have the values listed in this table.

Attention has been confined to targets in a 256 mi. x 256 mi. area. The x, y coordinate system and radar coverage is indicated in Figure 1. Radar locations are indicated by + .

![Figure 1](image)

Area in which the Targets are Located and the Radar Coverage

Track position coordinates will be represented by X and Y, corresponding coordinates for a radar report will be represented by $S'$ and $\gamma$. The necessary and sufficient condition for association of a radar report with a track is

$$|S' - X| \leq \varepsilon \quad \text{and} \quad |\gamma - Y| \leq \varepsilon$$  \hspace{1cm} (1)
| Preset Parameters |  
|-------------------|---|
| Location of the radar, \((X_1, Y_1)\), angular velocity of the antenna, \(\Omega_1\), (a.u.=angular unit=1/128 of a circle), and the range, \(R_1\). Subscript refers to number of radar. | \(X_1=64\) mi, \(Y_1=192\) mi, \(\Omega_1=10.5\) a.u./sec, \(R_1=48\) mi. |
| \(Q_i\), the number of radar buffer store locations. | \(Q_1 = 8, Q_2 = 8, Q_3 = 8, Q_4 = 8, Q_5 = 8\) |
| \(N_i\), the number of priority noise reports placed in the buffer store every second. | \(N_1 = 6, N_2 = 2, N_3 = 2, N_4 = 4, N_5 = 2\) |
| \(\sigma_i\), the threshold strength. (Strength of report, \(\sigma_i\), can take values 0, 2, 4, 6). | \(\sigma_1 = 4, \sigma_2 = 2, \sigma_3 = 2, \sigma_4 = 2, \sigma_5 = 4\) |
| \(\sigma_0\), the minimum initiation strength. (In order for a report to be initiated as a track it must have \(\sigma > \sigma_0\)). | \(\sigma_0 = 4\) |
| \(f_i\), the amount by which the firmness is increased on every association. | \(f_i = 2\) |
| \(f_m\), the maximum possible firmness. (\(f \leq f_m\) always). | \(f_m = 6\) |
| \(f_0\), the firmness of a freshly initiated track. | \(f_0 = 1\) |
| \(f_t\), the maximum scratching firmness. (\(f_s\), the firmness such that tracks are scratched if \(f \leq f_s\) must satisfy \(f_s \leq f_t\)). | \(f_t = 5\) |
| \(f_D\), the amount by which the firmness is decreased after each interval \(t_s\). | \(f_D = 1\) |
| \(n_0\); scratch tracks if number of free tracks on drum = \(n < n_0\). | \(n_0 = 5\) |
| \(t_s\); the interval after which the firmness will be decreased by \(f_0\) if not associated | \(t_s = 15\) sec. |
The smoothing equations for $x$ and $y$ are

\begin{align}
X^* &= X + \alpha \left( S - X \right), \\
y^* &= y + \alpha \left( \eta - y \right),
\end{align}

where $*$ denotes the smoothed quantity. The smoothing equations for $u$ and $v$, the $x$- and $y$-components of velocity, are

\begin{align}
u^* &= u + \frac{\beta}{t} \left( S - x \right), \\
v^* &= v + \frac{\beta}{t} \left( \eta - y \right).
\end{align}

Five sets of sorting and smoothing parameters $\alpha, \beta, \gamma$ have been used in the experiments reported here. They will be referred to as SS-1, SS-2, ..., SS-5 and they are given in Table 2. (In order to simplify certain computer operations the simulation code uses a firmness function which differs by one from that used in CSL R-35 and R-45: $f(R-35) = f(\text{simulation code}) + 1$. Throughout this report the simulation code firmness function will be used.)

The parameters SS-1 were determined by the $K, L, M$ theory, described in CSL report R-45. The remaining sets of parameters essentially represent educated guesses. Parameters SS-4 and SS-5 were arrived at after a number of "pencil and paper experiments". In these experiments tracking is performed by a human being going through the association, smoothing and bringing-up-to-date operations for a plane flying in the cover of a single radar. The blip scan ratio was simulated by throwing a die and a gaussian distribution was used to simulate the radar error.

In one experiment reported here double bin sorting has been used. A description of this sorting technique is given in CSL report R-45; briefly
### Table 2

**Smoothing and Sorting Parameters**

| SS-1: | $\alpha = 0.8 + 0.0083 (t-8f-8)$ if $t - 8f < 26$  
|       | $\alpha = 0.95$ if $t - 8f \geq 26$  
|       | $\beta = 0.058$ if $f = 0$ or $1$ and $t \leq 15$  
|       | $\beta = 0.029$ otherwise  
|       | $\epsilon = 1.10 + \frac{1}{T+1} + (t/32)^2$  

| SS-2: | $\alpha =$ (same as for SS-1)  
|       | $\beta = 2/t$ if $|x-x'| > 2$ or $|x-\eta| > 2$  
|       | $\beta =$ (same as for SS-1) otherwise  
|       | $\epsilon =$ (same for SS-1)  

| SS-3: | $\alpha =$ (same as for SS-1)  
|       | $\beta =$ (same as for SS-1)  
|       | $\epsilon = 1.6 + \frac{1}{T+1} + (t/32)^2$  

| SS-4: | $\alpha =$ (same as for SS-1)  
|       | $\beta = 0.1$ if $f = 1$ and $t \leq 15$  
|       | $\beta = 0.04$ otherwise  
|       | $\epsilon = 1.3 + \frac{1}{T+1/2} + (t/32)^2$  

| SS-5: | $\alpha = 0.498$ if $t - 8f \leq -2h$  
|       | $\alpha = 0.417$ if $-23 \leq t - 8f \leq 14$  
|       | $\alpha = 0.996$ if $15 \leq t - 8f$  
|       | $\beta = 0.03$ if $t > 60$  
|       | $\beta = 0.1$ if $t \leq 15$ and $f = 1$  
|       | $\beta = 0.04$ otherwise  
|       | $\epsilon = 2.3 + 0.18t$ if $t < 30$ and $f = 1$  
|       | $\epsilon = 3.3 + 0.18t$ if $t > 15$  
|       | $\epsilon = 1.1 + \frac{1.3}{T+4}$ if $t < 15$  

it is this—if the association criterion, Eq. (1), is not satisfied but is satisfied if $\xi$ is replaced by $E$ (a constant) then initiation of a track by the report is inhibited. In our experiments $E = \frac{1}{4}$ mi.

The least significant digit in the radar reports entering the system is the 1 mile digit. The error which results from truncating the position computed by the orbit preparator to the 1 mile digit is the only error in these reports.

Noise was introduced in all experiments, it was either distributed randomly over the entire area ($0 < X < 256$, $0 < Y < 256$) or one-half the area ($0 < X < 128$, $0 < Y < 256$). There are two ways for noise to enter the system. The first way is in the form of priority noise; the number of priority noise reports placed in the buffer stores every second is given in Table 1. Whenever the system is operated at the full report rate (20 reports per second) 20 - $n$ additional noise reports are sent to the drum sorter and tracker when the total number of priority noise reports and plane reports sent to the drum during a second is $n$, thus keeping the report rate constant; this is the second way noise can enter the system. In certain experiments reported here this source of noise is omitted—then the number of reports entering the drum in a second fluctuates; this is called operation at a reduced report rate. In all of the experiments reported here where the system operated at a reduced report rate the priority noise preset parameter $N_1$ was set equal to one for all radars.

The amount of noise sent to the drum sorter is considerably greater than would be expected on the average in a real situation. In setting up these experiments we have been guided by the philosophy that the system should be operable even when conditions are unfavorable to good tracking.
The high speeds of the aircraft and destructive power of their weapons make it vitally important that the system be operable under extreme conditions, such as high noise density, even though their probability of occurrence is small.

In a recent CSL report, R-49, Wax has presented the results of some theoretical studies of "Birth and Death Processes in Certain Signal and Noise Populations". Some of the quantities which he has computed have also been measured experimentally and are reported on here. The number of tracks per report, with firmness $f$, when the system is at equilibrium, represented by Wax as $w_f$, is in this report given by $N_t/N$ where $N$ is the report rate. No attempt has been made to compare our experimental results with this theoretical work since the experimental conditions are quite different from the conditions assumed in the theoretical work.

In all of the experiments described here the problem of saturation of the radar buffer stores was handled by sorting the reports according to strength. Specifically, if a buffer store becomes completely filled with reports and another report comes to it from the radar, then the strength, $\sigma$, of all of the reports is examined and that one having the smallest $\sigma$ is discarded. Another method, called "serial overwrite", for handling this saturation problem has not yet been studied.

Saturation of the type described above is called saturation of the second kind. Another saturation problem arises if, at any time, the total number of reports in all of the buffer stores exceeds twenty. Since the drum can only process twenty reports a second, this type of saturation, called saturation of the first kind, results in some information being older than one second. In all but one experiment "old" information...
resulting from saturation of the first kind was not tampered with, it was handled as if it were up-to-date information. However, in one experiment, Exp. I₁₉,¹ all information older than one second was discarded.

III. Orbits of the Targets

At the beginning of the experimental program it was felt that the orbits used should form a fairly complex flight pattern in order to obtain information about a number of different tracking situations (crossing planes, turning planes, etc.) from a single long experiment.

The flight pattern for Exp. I₁ is shown in Figure 2; the course of each plane is indicated by a continuous straight line with the heading indicated by an arrow. Radar coverage is indicated by the dashed circles. This pattern was generated by the straight line and parabola orbit preparer described in R-58. The southbound planes all started at time t = 0 (the start of the experiment) and traveled at a constant speed of \( \frac{5}{32} \) mi/sec. The northbound planes all started at t = 0, and had a constant y-component of velocity \( v = \frac{1}{8} \) mi/sec with their speeds ranging from \( \frac{1}{8} \) to \( \sqrt{2}/8 \) mi/sec. The eastbound planes traveled at speeds ranging from \( \frac{23}{128} \) mi/sec to \( \frac{1}{16} \) mi/sec; starting at the lower left corner of Figure 2 and going up, the first two planes had a speed of \( \frac{23}{128} \) mi/sec, the next two \( \frac{22}{128} \) mi/sec, and so on in decreasing steps of \( \frac{1}{128} \) mi/sec. The eastbound planes started at various times so as to line up vertically at \( x = 6\frac{1}{4} \) mi and \( t = 102\frac{1}{4} \) sec. Points at which the planes collided fall along the dashed line, called the "collision line" in Figure 2.

¹This notation or the alternate Exp. I-19 is used throughout for labelling experiments. The I refers to the fact that the radars and data processors are simulated by the Illiac.
FIGURE 2. FLIGHT PATTERN FOR EXPERIMENT I
In Exp. I2 the polygonal orbit preparer1 was used. Ninety-five planes were in the "sky" and they traveled between certain subsets of the 15 turning points, indicated by • in Figure 3. A few of the orbits are shown in this figure. One plane had a speed of 1/16 mi/sec, two had a speed of 1/4 mi/sec, forty-six had a speed of 1/8 mi/sec and forty-six had a speed of 3/16 mi/sec. Each plane collided approximately 5 to 10 times with other planes along its path.

The complexity of the flight patterns in these experiments led to considerable difficulties in analyzing the results. For this reason a series of experiments was made with relatively simple flight patterns where the turns or collisions could be considered as isolated events. Two simple flight patterns which are reported on here are generated by the fishhook and scissors orbit preparers described in R-58.

Figure 4 shows a sample fishhook pattern generated by the fishhook orbit preparer. The numbers shown are the identification numbers of the planes. (These are sexadecimal numbers (base 16), K, S, N, J, F, L representing the decimal numbers 10, 11, 12, 13, 14, 15, respectively.) The orbit for plane number 1 is called the basic orbit, the orbit for any other plane, say plane P, is derived from the basic orbit by the transformation

\[
X \text{(plane P)} = X \text{(plane 1)} + n \Delta X \\
Y \text{(plane P)} = Y \text{(plane 1)} + m \Delta Y
\]

In all experiments reported here n and m are integers ranging from 0 to 9 and \(\Delta X = \Delta Y = 25\) miles.

Figure 5 shows a sample pattern generated by the scissors orbit preparer, and the identification numbers of the planes. The orbit for plane

1See R-58.
FIGURE 3. A PORTION OF THE FLIGHT PATTERN FOR EXPERIMENT I₂
**FIGURE 4. A SAMPLE FISHHOOK PATTERN**

**FIGURE 5. A SAMPLE SCISSORS PATTERN**
number 1 is the basic orbit for the odd numbered planes and the orbit for plane number 2 is the basic orbit for the even numbered planes. The orbit for any odd numbered plane is derived from the basic orbit by the above transformation, the orbit for any even numbered plane is derived from the same transformation with X (plane 1) and Y (plane 1) replaced by X (plane 2) and Y (plane 2). In all experiments reported here n is an integer ranging from 0 to 9 and m is an integer ranging from 0 to 5, and \( \Delta X = 25 \text{ miles} \), \( \Delta Y = 50 \text{ miles} \).

IV. Collection of Data

The "association print" output program is the only output program which has been used. The data printed out by this program is the "raw data" for all of the data analysis which has been performed. It is recalled that this program prints out information on a track only at the times at which the track associates with some report; information on a track which is initiated but never associates with a report is not printed. Unless explicitly referred to, these "one-hit tracks" are ignored in the following results.

V. Results of Experiments

A. Exp. I\(_1\)

After all parts of the simulation program were checked separately, Exp. I\(_1\) was set up in order to test the operation of the program as a single unit. Sorting and smoothing parameters SS-1 were used. Examination of the results of this experiment showed that all of the logical operations were being performed correctly.
This experiment also furnished information on the ability of the system to track planes flying with constant velocity through different kinds of radar cover. We will describe radars 2, 3, and 4, which have a blip-scan ratio of 0.75, as good radars, and radars 1 and 5, which have a blip-scan ratio of 0.50, as poor radars. For those planes flying in the cover of at least one good radar the following results were obtained.

1. Isolated planes with speeds of less than about 560 mph can be tracked for periods of at least 10 to 15 minutes with negligible probability of losing the track. For these tracks the average error in position is about 1 mile in each coordinate.

2. Tracks on planes flying at speeds greater than about 560 mph showed considerable stuttering; that is, the track is lost and reinitiated several times within a short period of time. The period between successive initiations was in the neighborhood of 1-2 minutes.

3. There are just eight collisions between eastbound and southbound planes in which tracks are being carried on both planes involved at the time of the collision. Out of these eight collisions there is only one in which both tracks are subsequently lost, there is one in which only one track is lost and in the other six both planes are successfully tracked through the collision. In each collision, at least one track had an incorrect association—that is, an association with a report from the other plane involved in the collision.

4. There are five collisions between eastbound and southbound planes in which only one of the planes is being tracked at the time of the collision. All of these tracks are successfully continued through the collision. However, only three had an incorrect association at the time of the collision.
(5) Eleven of the twelve southbound planes colliding with northbound planes were being tracked at the time of the collision. All of these tracks successfully pass through the collision. Only four of these had an incorrect association at the time of the collision. The northbound planes are so close together at the time of the collision that it is difficult to say whether loss of their tracks is due to collisions with southbound planes or their mutual interference.

For planes in poor radar coverage only the tracking was very unsatisfactory because of stuttering. No collisions occurred in areas of poor coverage.

B. Exp. I

In this experiment the planes make instantaneous changes in heading at various times. The changes in heading appearing in this experiment ranged from $26^\circ$ to $90^\circ$, and the speeds ranged from 450 to 900 mph. These instantaneous turns can be approximated by turns through the same angle having constant speed and acceleration when a few simplifying assumptions are made; the turns approximated in this experiment range from about $1/2g$ to about $1g$.

As mentioned earlier each plane made a number of collisions with other aircraft during its flight. These collisions involved from two to four planes and the angle between the orbits of the colliding planes ranged from 0 to $180^\circ$.

Smoothing and sorting parameters SS-1 were used. From the preliminary data processor\(^1\) the distribution of the number of tracks initiated on a plane was found and is shown in Figure 6; there are, for example, 5 tracks initiated on each of 12 planes. The average number of tracks initiated per plane is about $5\frac{1}{2}$.

\(^1\)See R-58.
Distribution of number of tracks on planes

FIG. 6
Unfortunately, it turned out to be an extremely difficult task to determine why tracks were lost. Turns and collisions frequently occurred so close together that the loss of a plane might be considered as due to either one of these events or both. Approximately ten such events occurred for each plane. Excluding initiation stuttering it appears that in about half of these events a new track is initiated. Since many planes were tracked successfully through collisions in Exp I, it seems that most of the trouble is due to the turns. Because of the difficulties encountered in trying to analyze the results of this experiment it was decided to try simpler flight patterns with isolated turns and collisions.

C. Exps. I, I', I, and I

In these experiments the fishhook flight pattern was used. The orbit parameters for the basic course are the same for all of them and are given below:

\[
\begin{align*}
x_1 &= 3 \text{ mi}, & y_1 &= 28 \text{ mi}, \\
u_1 &= 0, & v_1 &= 1/8 \text{ mi/sec}, \\
t_1 &= 80 \text{ sec}, & t_2-t_1 &= 126 \text{ sec}, \\
u_2 &= 1/8 \text{ rad/sec}, & v_2 &= 0
\end{align*}
\]

where \(x_1\) and \(y_1\) are the initial position coordinates, \(u_1\) and \(v_1\) are the initial \(x\)- and \(y\)-components of velocity, \(t_1\) is the time of the start of the turn, \(t_2\) is the time the turn is completed, \(\omega\) is the angular velocity of the turn, \(R\) is the radius of the turn \(u_2\) and \(v_2\) are the \(x\)- and \(y\)-components of velocity after completion of the turn. This is a \(1/4\) turn through 90° at constant speed \(V = 450\) mph. After completion of the turn the flight continued for about 50 seconds.

Smoothing and sorting parameters SS-1 were used in Exps. I, I', and I, SS-2 in Exp. I', and SS-3 in Exp. I. Noise was confined to the region \(0 \leq x < 128, 0 \leq y < 256\).
In Exps. I3-I6 the quality of the tracking was very poor compared to that obtained in subsequent experiments. The primary sources of trouble in these four experiments were track stuttering and the so-called northeast wind which were eliminated or at least partially eliminated in later experiments. For this reason the experimental results will only be discussed insofar as they relate to these two phenomena.

In order to observe the general character of the tracking, photographs of the output of the "association print", data processing program\(^\text{1}\) were made. (This data processing code displays on a cathode ray tube the coordinates of a track whenever it associates with a radar report provided the firmness, \(f\), immediately preceding the association is greater than or equal to an adjustable parameter \(f_0\).) Photographs for Exp. I3 are typical for these four experiments and are shown in Figure 7, where \(f_0 = 0\), and Figure 8, where \(f_0 = 4\). Radar coverage is indicated by the circles. Comparison with the true courses, Figure 4, shows that tracking in poor coverage is very unsatisfactory. The discontinuous character of the tracks at the turns indicates that they are being lost and reinitiated even in good radar cover.

It is interesting to compare these photographs with photographs of the raw data in which there is a spot corresponding to every report received by the drum. Figure 9 is a photograph of the raw data (we call this a \(\xi, \eta\) print) for \(1/4g\) fishook experiments described in this report in which the full report rate (20 reports per second) is used. (This includes Exps. I3, I3', I5, I6, I7, I8, I12, I17.) This comparison gives a good qualitative picture of the noise discriminating properties of the system, though

\(^{1}\text{See R-58.}\)
FIG. 7 ASSOCIATION PRINT FOR EXP. I₃ WITH f₀ = 0

FIG. 8 ASSOCIATION PRINT FOR EXP. I₈ WITH f₀ = 4
FIG. 9 $\xi, \eta$ PRINT FOR EXPS. $I_3, I_4, I_5, I_6, I_7, I_8, I_{12}$, AND $I_{17}$
some of the effect is lost because of the integration provided by the photo-
graphs. Quantitative data on noise will be presented later.

The photographs do not furnish an unambiguous track history. It is
not possible to tell definitely when tracks are lost and reinitiated, which
dots actually belong to the same track, and the order of the events in
time. As an attempt to understand the reasons why tracks were lost and
reinitiated, more precise track histories were plotted by human beings.
This procedure is very time consuming, but fortunately most of the useful
information contained in these histories can be extracted by considering
only the most ill-behaved or "interesting" tracks which appear in the
photographs. Several of these track histories are shown in Figures 10, 11,
12, and 13.

In each figure the true course of the aircraft is shown in an x, y
coordinate system in 1/4 mile units, with the time of arrival at points
along the course marked in seconds. The x, y coordinates are in the
sexadecimal number system. Each track is indicated by a continuous line
drawn from the point of the first association to that of the last associ-
ation. The track is identified by the number (sexadecimal) of its drum
location which appears in a small box alongside the corresponding track.
Whenever an association occurs a dot appears at that point on the track
and the time (in seconds) of the association is indicated; if the dot is
enclosed by a small circle the association was made with a noise report,
if there is no circle the association was a correct one—that is, the track
associated with a report from the plane it represents. The planes are
numbered sexadecimally according to the scheme shown in Figure 4. The
quantity \( t_f \) gives the time of the end of the experiment.

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Exp. 1-3
Plane #22
$\tau_F = 289$ sec.
Exp. 1-5
Plane # OS
\( t_F = 257 \text{ sec.} \)
FIGURE 13

Exp. I-5
Plane #19
\( t_F = 257 \) sec.
For illustration we consider Figure 10 in detail. This is a track history on plane 19; Figure 4 shows the course of this plane in relation to the entire area. The starting coordinates of this plane are \( x = 19 \text{N} \) (mi/li), \( y = 138 \text{ (mi/li)} \) which when converted to the decimal number system, and one mile units are \( x = 103 \text{ mi}, y = 78 \text{ mi} \). Six tracks are initiated on this plane. They are in drum locations 2NN, 30N, 3Kl, 2S0, and 338. Tracks will be referred to by drum location. They have their first association after initiation at 13, 32, 50, 136, 186, and 236 seconds, and their last associations at 88, 36, 123, 171, 195, and 257 seconds, respectively. Track 316 flies out of the area and is lost. Track 2NN associates with a report from plane 19 at \( t = 13 \text{ sec} \), and with noise reports at \( t = 24 \text{ sec} \), and \( t = 88 \text{ sec} \).

After examination of a number of these detailed track histories it appeared that there was a general tendency for the tracks to be displaced southwest of the plane's course; this was called the northeast wind effect because of its appearance. This effect can be seen in Figures 10-14. The source of the northeast wind was found to be a truncation error which arose in the smoothing calculation for the track velocity. In the computer seven digits, of which one is a sign digit, are allowed for the velocity; the decimal point falls immediately after the sign digit so that the smallest unit of velocity is \( 2^{-6} \text{ mi/sec} \). However, in computing the velocity correction in the smoothing calculation all 140 digits of the accumulator are used and the result is chopped off at the seventh digit to give a velocity correction truncated to \( 2^{-6} \text{ mi/sec} \). The result is that any negative correction less than \( 2^{-6} \text{ mi/sec} \) is equal to \( -2^{-6} \text{ mi/sec} \) after truncation, while positive corrections less than \( 2^{-6} \text{ mi/sec} \) are always equal to zero after truncation.
The velocity correction is therefore biased in the negative direction. In addition to this there is a tendency for planes flying directly south to have a small, less than $-2^{-6}$, velocity correction because of the way the numbers are handled in the Illiac. The northeast wind effect was eliminated from Exp. I6 and the following experiments by rounding the velocity correction before truncating. The resulting improvement in tracking is indicated by comparing Figure 10 with Figure 14 where a detailed track history on plane 19 for Exp. I6 is plotted. (Exp. I6 differs from Exp. I3 only in the removal of the northeast wind.) In Figure 14 the track is on the average closer to the true course and is not lost on the turn.

It was frequently observed in these experiments that when a plane first appears, or when a track is lost, several short-lived tracks are initiated on the plane before a "good" track on the plane is established; this is particularly true if the plane is executing a turn at the time of initiation. Track stuttering, as this phenomenon is called, can be caused by missing reports (no radar report on the plane during a scan) in combination with too small an $\epsilon$, and $\Delta / t$ which does not allow the correct velocity to be picked up quickly. Let us consider the $\epsilon$ in SS-1 for a newly initiated track in single radar cover. After one scan time, approximately 12 seconds, $f = 1$ and $\epsilon = 1.74$ mi. At this time the $x, y$ coordinates of the track are integral since $\xi, \eta$, the coordinates of the radar report are the coordinates of the track on initiation. Therefore, when comparing radar reports with this track for association the position error will always be integral. Since $\epsilon = 1.74$ mi the position error in $x$ and $y$ must be either 1 or 0 miles for an association. In 12 seconds a 450 mph plane will travel 1.5 mi and after the new position has been truncated to the
nearest mile this may appear as a 2 mile change in position, thus the second report can easily fall outside the association bin. The stuttering on initiation was in part due to this effect. This, however, was not the only source of trouble since stuttering was observed in Exp. I5 where parameters SS-3 were used; here $\varepsilon = 2.24$ mi 12 seconds after initiation. It also appeared that the correct velocity was not picked up quickly enough, indicating that $\beta/t$ was too small during the birth of a track. In the new parameters SS-4 and SS-5, which were used in later experiments, the values of $\varepsilon$ and $\beta/t$ during initiation of a track were increased in an attempt to overcome the stuttering. The velocity smoothing parameter in SS-4 and SS-5 is 0.1 if $f = 1$ and $t \leq 15$, a condition which can only be satisfied by a newly initiated track. With this value of $\beta/t$ the position errors $(\xi - X), (\gamma - Y)$ on the first scan after initiation are treated as if they are almost entirely due to the velocity of the plane; in particular, if the first association occurs 10 seconds after initiation $\beta$ itself is equal to 1.0.

D. Exp. I7

Experiment I7 was a fishhook experiment with orbit parameters for the basic course given by Eqs. (5). The system operated at the full report rate and noise was confined to the left half of the area, $0 \leq x < 128$ and $0 \leq y < 256$. Smoothing and sorting parameters SS-4 were used.

Association prints were made at $f_0 = 0$ and $f_0 = 4$. They are shown in Figures 15 and 16. Comparison with Figures 7 and 8 indicates that a qualitative improvement in the tracking has been achieved. This is entirely due to the new smoothing and sorting parameters and the removal of the northeast wind since Exps. I3 and I7 are otherwise identical. Comparison of
FIG. 15 ASSOCIATION PRINT FOR EXP. I₇ WITH fₒ = 0

FIG. 16 ASSOCIATION PRINT FOR EXP. I₇ WITH fₒ = 4
Figures 15 and 16 show that a track display at \( t \geq 1 \) reduces the noise considerably without seriously deteriorating the tracks.

Two important questions to be answered in a fishhook experiment are:

"Can the system successfully track a turning plane and how does this depend on the radar cover and noise density?" In order to answer these questions the following rules were set up for classifying the tracking of a turn in one of five categories, A, B, C, D, or E:

The tracking of the turn is type A if:
1. A track is initiated at \( t \leq 0.05 \).
2. No secondary tracks are initiated in the interval \( 0.05 < t \leq 1 \).
3. There is at least one correct association with this track in the interval \( ON8 < t \leq t_f \).

The tracking of the turn is type B if:
1. A track, called the primary track, is initiated at \( t \leq 0.05 \).
2. Secondary tracks are initiated in the interval \( 0.05 < t \leq 1 \).
3. There is at least one correct association with the primary track in the interval \( ON8 < t \leq t_f \).

The tracking of the turn is type C if:
1. A track (primary) is initiated at \( t \leq 0.05 \).
2. At least one secondary track is initiated in the interval \( 0.05 < t \leq 1 \).
3. There is no correct association with the primary track in the interval \( ON8 < t \leq t_f \).

The tracking of the turn is type D if:
1. A track is initiated at \( t \leq 0.05 \).
2. No secondary tracks are initiated in the interval \( 0.05 < t \leq 1 \).
3. There is no correct association in the interval \( ON8 < t \leq t_f \).
The tracking of the turn is type E if:

1. A track is initiated at $t > 0.50$, but not at $t \leq 0.50$.

The quantity $t$ is the time in sexadecimal seconds, and $t_f$ is the time of the end of the experiment; $t_f \approx 100$ sexadecimal seconds for the fishhook experiments reported here. In all fishhook experiments for which the parameters of the basic course are given by Eqs. (5) the plane begins the turn at $t = 0.50$ and comes out of the turn at $t = ONF$. In order to illustrate the significance of these rules consider those for type A tracking. Rule 1 requires that a track is initiated before the plane begins the turn. The possibility of more than one track being initiated, for example stuttering on initiation, before the turn begins is not precluded. A secondary track is a track initiated by a report from a plane on which a track has already been initiated. Thus, rule 2 demands that no new tracks on the plane are initiated after the turn is begun and before $t = 100$, which is approximately equal to $t_f$. A correct association is one in which a report from a plane associates with a track on that plane. Thus rule 3 demands that the track be sufficiently close to the plane at some time between completion of the turn and before the end of the experiment that it associates with a report from the plane. The interpretation of the rules for the other types of tracking is clear from the explanation of these rules for type A tracking.

This classification of the tracking of turning aircraft is arbitrary but seems to represent a simple and reasonable separation of the interesting events which can occur. Simplicity is important here because of the large amount of data that must be handled. Rule 3 above is somewhat simplified criterion for determining whether the plane is still being tracked properly after the turn. An alternate rule for type A tracking might be the following: The primary track must be within $\varepsilon$ (the sorting parameter) of the plane at
t = 100—that is, the track must be sufficiently close to the plane to be able to associate with a report from it a short time after completion of the turn. Use of this rule, and its equivalent for the other types of tracking, was found to have a negligible effect on the distribution of types of tracking obtained in this experiment and it was abandoned in favor of the simpler rule 3 which requires only that there be a correct association after the turn.

Type A tracking and type B tracking will be called successful tracking of a turn. The existence of secondaries here will alter but little a clear picture display since the fact that the primary track continues around the turn usually implies that the secondaries have a low average firmness and are scratched a short time after initiation. Type C tracking and type D tracking will be called unsuccessful tracking since no track initiated before the start of the turn is in the neighborhood of the plane after completion of the turn. Type E tracking is a result of faulty initiation; a label of successful or unsuccessful tracking of the turn is not particularly meaningful since the track is initiated after the beginning of the turn.

In Table 3 the number of turns in each tracking category in this experiment is displayed according to radar cover and the presence or absence of noise. In this experiment and in the other fishhook experiments reported here there are \( \frac{1}{6} \) turns in radar cover in the noisy area and \( 3\frac{1}{4} \) turns in radar cover where noise is absent. The remaining 20 turns are either partly covered by the radars or completely out of cover.
Although a number of turns occur in the noisy area many are never directly perturbed by noise. It is of some interest to consider only those turns in which a noise report associated with the track during the turn in order to study the effect of perturbation by noise. Lines 1 and 2 in Table 3 list the number of successful turns in each type of cover having one or more noise associations during the turn. Line 3 lists the number of unsuccessful turns in which there was a noise association during the turn and the track was subsequently lost—that is, there were no more correct associations on this track following association with the noise report.

Table 3
Classification of turns for Exp. I7

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>Number of A</td>
<td>7</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>&quot; B</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>&quot; C</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>&quot; D</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>&quot; E</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4
Effect of noise on turns for Exp. I7

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of successful turns with one noise association during the turn.</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Number of successful turns with more than one noise association during the turn.</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Number of unsuccessful turns with a noise association and subsequent loss of the track.</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
The lifetimes of noise tracks on a clear picture display are described in Figures 17a, 17b, 17c, 17d, 17e, and 17f which show distributions of the time interval $\Delta t_{f_0}$ that the firmness of a noise track is greater than or equal to $f_0$, for $f_0 = 0, 1, 2, 3, 4,$ and 5, respectively. The ordinate, $n$, is the number of noise tracks and the abcissa, $\Delta t_{f_0}$, is the time interval, quantized in five second intervals, during which the firmness was greater than $f_0$; e.g., in Figure 17a, 11 tracks had a firmness greater than or equal to zero for 65 to 70 seconds, 3 tracks had a firmness greater than or equal to zero for 100 to 105 seconds, etc. These distributions are for noise tracks initiated during the first minute of the experiment only, in order to allow for a complete history on the long-lived noise tracks. In this experiment two noise tracks which were initiated in the first minute were still alive at $t = t_f$; data on these tracks does not appear in the above figures. Noise tracks which become confused with plane tracks are not included in these distributions; confusion is said to arise when a noise track has two successive associations with reports from the same plane. The number of these cases is very small. It has been assumed that all of these tracks die of $f$-death (i.e. the firmness becomes equal to -1) and not from operation of the track scratcher. However, since the track scratcher only operates at $f = 0$ and $f = 1$ in this experiment, this will only cause a slight shift to the left in the distribution of Figure 17a, and Figure 17b and will leave the others unaffected. Finally, it should be remembered that noise tracks which are initiated but never associate with anything are necessarily omitted since the association print output prints out information on a track only on associations. Inclusion of these one-hit noise tracks would cause the distributions in Figures 17a
Lifetimes of noise tracks on a clear picture display - Exp. I

FIG. 17f $f_0 = 5$
AVG. $\Delta t_5 = 15$ sec.

FIG. 17e $f_0 = 4$
AVG. $\Delta t_4 = 33$ sec.

FIG. 17d $f_0 = 3$
AVG. $\Delta t_3 = 24.2$ sec.

FIG. 17c $f_0 = 2$
AVG. $\Delta t_2 = 37.3$ sec.

FIG. 17b $f_0 = 1$
AVG. $\Delta t_1 = 61.7$ sec.

FIG. 17a $f_0 = 0$
AVG. $\Delta t_0 = 76.9$ sec.
and 17b to be more sharply peaked around $\Delta t_f = 15$ seconds, the others would be unaffected. The average number of one-hit noise tracks on the drum at any time is roughly 95. Since tracks die when $f = -1$ it can be seen that Figure 17a presents a distribution of lifetimes of noise tracks on the drum.

The average number, $\bar{N}_f$, of noise tracks (excluding one-hit noise tracks) on the drum at any time with firmness $f$ has been computed as a function of $f$. This is a time average taken at 20 second intervals from $t = 60$ seconds to $t_f$. The results are given in Table 5. From this table one easily obtains the average number of noise tracks which will appear on a clear picture display for any display firmness, $f_0$. Thus, for $f_0 = 4$ there will, on the average, be 7.0 noise tracks displayed at any time.

The alertness of the system has been investigated. In normal operation it is expected that the clear picture display will be biased so as to display only those tracks having a firmness equal to some value greater than the initiating firmness in order to suppress the majority of the noise tracks. In this case a track must have at least one association before it can be displayed on the clear picture. The time interval between the first appearance of the plane in the sky and the first association with a track on that plane is therefore a measure of the alertness of the system;

<table>
<thead>
<tr>
<th>$f$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}_f$</td>
<td>4.2</td>
<td>5.0</td>
<td>5.8</td>
<td>7.6</td>
<td>3.6</td>
<td>2.0</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 5

$\bar{N}_f$ as a function of $f$ for Exp. 17
this quantity will be denoted by TD, for time delay. The average value of TD, $\bar{TD}$, and the RMS deviation, $\sqrt{(\Delta TD)^2}$, for the different types of radar cover is given in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TD$ (sec)</td>
<td>42.8</td>
<td>30.7</td>
<td>18.0</td>
</tr>
<tr>
<td>$\sqrt{(\Delta TD)^2}$ (sec)</td>
<td>30.3</td>
<td>23.7</td>
<td>11.0</td>
</tr>
</tbody>
</table>

Table 6

Average time delay for first association in Exp. I7

During this experiment the drum occasionally became overloaded and the track scratcher was brought into operation. The drum parameter $n_0$ was equal to five so that the track scratcher operated when there were less than five empty drum locations. Figure 18 illustrates the times at which the scratching occurred and the level of firmness at which the track scratcher operated during the first portion of the experiment. The horizontal line is the time axis ($t$ is given in sexadecimal seconds), a vertical line appears each time the scratcher operates and the number above the vertical line is the level at which scratching occurred. The track scratcher is first called into operation at $t = 16$ sec, as time proceeds the track scratcher operates with increasing frequency scratching tracks at $f = 0$. Finally scratching at $f = 0$ does not clear enough drum locations and the scratcher operates at $f = 1$, scratching all tracks with $f = 1$ and $f = 0$. Enough drum locations are apparently cleared then to keep the scratcher quiet for about 11 seconds when it starts to operate again at $f = 0$ and then at $f = 1$. This general pattern continues throughout the experiment. It is easy to see that frequent operation of the track scratcher will cause initiation time delays and

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Times at which the track scratcher operated in Exp. I

FIG. 18
stuttering on initiation since a newly born track, having a low firmness, has a relatively good probability of being killed by the track scratcher. In addition to saturation of the drum there was evidence of saturation of the first and second kind. (This was detected by watching the memory slave tube on the Iliac.) It appears from these results that a report rate of 20 reports per second overloads the system. The importance of the fact that the system does not go completely haywire, and in fact does not do too badly in tracking these turns, even when overloaded should not be overlooked. The complete break-down under saturation of similar systems in which the tracking and sorting operations are performed by human beings is not uncommon.

Though one should not attach too much significance to the behavior of a single turn it is interesting to compare the tracking of plane 19 in this experiment with that in earlier experiments (Figs. 10, 13, and 14). A detailed track history on the flight of plane 19 in Exp. I7 is shown in Figure 19. The improved tracking is due to the new sorting and smoothing parameters and the elimination of the northeast wind.

E. Exp. I8

Experiment I8 is a fishook type experiment with orbit parameters for the basic course given by Eqs. (5). The system operated at the full report rate and noise was confined to the left half of the area, \(0 \leq x \leq 128, 0 \leq y \leq 256\). Smoothing and sorting parameters SS-4 were used. Double bin sorting was used; this is the only point on which Exps. I7 and I8 differ.

The association prints do not differ significantly from those obtained for Exp. I7.
FIGURE 19

Exp. I - 7
Plane 19

$\tau_F = 265$ sec.
The number of turns in each tracking category is displayed in Table 7. Data on the effect of noise on turns is presented in Table 8. There appears to be no significant difference between these results and those for Exp. I7. It is noted here that two planes for which tracks appear in Exp. I7 are never tracked in this experiment.

<table>
<thead>
<tr>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
</tr>
<tr>
<td>Number of A</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>&quot;           B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&quot;           C</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>&quot;           D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&quot;           E</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 7
Classification of turns for Exp. I8

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of successful turns with one noise association during the turn.</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Number of successful turns with more than one noise association during the turn.</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of unsuccessful turns with a noise association and subsequent loss of the track.</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8
Effect of noise on turns for Exp. I8

Such a result is not surprising when double bin sorting is used since there will be certain areas in the "sky" where initiation is suppressed due
to the presence of noise tracks or plane tracks in that region. The real value of double bin sorting unfortunately is not apparent in this experiment because of the relatively small radar errors. In some early experiments with a real time tracking code designed for a general purpose computer working with real radar data, double bin sorting gave a significant improvement in the tracking. It is expected that double bin sorting will result in a similar improvement when real radar data is fed into the drum simulation code.

The distribution of $\Delta t_{f_0}$ of noise tracks for $f_0 = 0$ is given in Figure 20. Comparison with Figure 17a shows little difference between the two distributions.

Table 9 shows $N_f$ as a function of $f$. Here again the results differ but little from Exp. I7.

<table>
<thead>
<tr>
<th>$f$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f$</td>
<td>7.0</td>
<td>7.5</td>
<td>10.2</td>
<td>11.8</td>
<td>3.6</td>
<td>2.1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 9

$N_f$ as a function of $f$ for Exp. I8

Table 10 shows $\overline{T_D}$ and $\sqrt{(\overline{\Delta T_D})^2}$ for different types of cover. The average time delay is somewhat greater than that for Exp. I7 because of the presence of areas in which initiation is suppressed.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_D$ (sec)</td>
<td>52.2</td>
<td>40.4</td>
<td>18.3</td>
</tr>
<tr>
<td>$\sqrt{(\overline{\Delta T_D})^2}$ (sec)</td>
<td>50.4</td>
<td>42.1</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Table 10

Average time delay for first association in Exp. I8

1 To be described in a forthcoming report.

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Lifetimes of noise tracks in Exp. $I_0$

FIG. 20 - $t_0 = 0$

AVERAGE $\Delta t_0 = 8.6$ sec.
The operation of the track scratcher was similar to that in Exp. I7.

F. Exp. I9

Experiment I9 was a fishhook experiment with the following orbit parameters for the basic course:

\[ x_1 = 3 \text{ mi} \quad y_1 = 28 \text{ mi} \]
\[ u_1 = 0 \quad v_1 = \frac{3}{16} \text{ mi/sec} \]
\[ t_1 = 80 \text{ sec} \quad t_{2-t_1} = 84 \text{ sec} \]
\[ \omega = 0.01875 \text{ rad/sec} \quad R = 10 \text{ mi} \]
\[ u_2 = \frac{3}{16} \text{ mi/sec} \quad v_2 = 0 \]

This is a 1 g turn through 90° at constant speed \( V = 675 \text{ mph} \). Otherwise this experiment was identical to Exp. I7.

A photograph of the association print at \( f_0 = 0 \) is shown in Figure 21. It appears from this picture and the results of the preliminary data processor that very few turns are successfully tracked.

Smoothing and sorting parameters SS-4 seem to be inadequate for turns of this type, though some of the difficulties may be due to the effects of saturation.

G. Exp. I12

This experiment was identical to Exp. I7 except that \( \sigma_o = 6 \). Since the strength of report, \( \sigma \), has four equally probable values, \( \sigma = 0, 2, 4, 6 \), the probability that a report, which does not associate with any track, can initiate a track is here equal to \( 1/4 \).

Photographs of the association print indicate that the quality of the tracking is not very different from that in Exp. I7, except that the time delay on initiation was naturally longer here because of the high value of \( \sigma_o \); \( \overline{TD} \) and \( \sqrt{(\Delta TD)^2} \) are shown in Table 11.

The effect of varying \( \sigma_o \) is not very realistic here because the strength of noise reports and plane reports have the same distributions
FIG. 21 ASSOCIATION PRINT FOR EXP. I_9 WITH f_o = 0
in the simulation code. Experiments in which the strength distributions for noise reports and plane reports are more realistic are being considered.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TD \text{ (sec)}$</td>
<td>67.6</td>
<td>74.6</td>
<td>34.2</td>
</tr>
<tr>
<td>$\sqrt{\text{(TD)}^2} \text{ (sec)}$</td>
<td>41.0</td>
<td>64.5</td>
<td>36.2</td>
</tr>
</tbody>
</table>

Table 11
Average time delay for first association in Exp. I_{12}

H. Exp. I_{15}

This experiment was identical to Exp. I_7 except that it was run at a reduced report rate (only one priority noise report and plane reports enter a buffer store in each second).

Association prints at $f_0 = 0$ and $f_0 = 1$ are shown in Figs. 22 and 23. Comparison with Figs. 15 and 16 (Exp. I_7) indicates that the quality of the tracking has been improved somewhat.

The number of turns in each tracking category is displayed in Table 12. Data on the effect of noise on turns is presented in Table 13. In Exp. I_{15} the probability of a successful turn in multiple coverage appears to be somewhat greater than in Exp. I_7. The differences in single coverage are not significant.

The distribution of $\Delta t_{f_0}$ for noise tracks with $f_0 = 0, 1, 2, 3, 4, 5$ is shown in Figs. 24a, 24b, 24c, 24d, 24e, 24f, respectively. Table 14 shows $N_f$ as a function of $f$.

Saturation of the first and second kind was not observed, and the track
FIG. 22 ASSOCIATION PRINT FOR EXP. $I_{18}$ WITH $f_o = 0$

FIG. 23 ASSOCIATION PRINT FOR EXP. $I_{15}$ WITH $f_o = 4$
| FIG. 24b - $f_0 = 1$ | AVERAGE $\Delta t_1 = 69.6$ Sec. |
| FIG. 24c - $f_0 = 2$ | AVERAGE $\Delta t_2 = 36.5$ Sec. |
| FIG. 24d - $f_0 = 3$ | AVERAGE $\Delta t_3 = 36.4$ Sec. |
| FIG. 24e - $f_0 = 4$ | AVERAGE $\Delta t_4 = 30.0$ Sec. |
| FIG. 24f - $f_0 = 5$ | AVERAGE $\Delta t_5 = 15.0$ Sec. |

**Lifetimes of noise tracks on a clear picture display - Exp. $I_n$**

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scratcher was never called into operation indicating that the drum was never saturated.

Table 15 shows $\overline{TD}$ and $\overline{(\Delta TD)^2}$ for different types of cover.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>Number of A</td>
<td>9</td>
<td>14</td>
<td>7</td>
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<td>&quot;</td>
<td>0</td>
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<tr>
<td>&quot;</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>&quot;</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 12

Classification of turns for Exp. $I_{15}$

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of successful turns with one noise association during the turn.</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Number of successful turns with more than one noise association during the turn.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of unsuccessful turns with a noise association and subsequent loss of the track.</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 13

Effect of noise on turns for Exp. $I_{15}$

<table>
<thead>
<tr>
<th>$f$ = 0</th>
<th>$f$ = 1</th>
<th>$f$ = 2</th>
<th>$f$ = 3</th>
<th>$f$ = 4</th>
<th>$f$ = 5</th>
<th>$f$ = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{N}_f$</td>
<td>5.1</td>
<td>6.7</td>
<td>8.6</td>
<td>2.9</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 14

$\overline{N}_f$ as a function of $f$ for Exp. $I_{15}$
Table 15

Average time delay for first association in Exp. I \(_{15}^{15}\)

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD (sec)</td>
<td>42.1</td>
<td>44.3</td>
<td>23.9</td>
</tr>
<tr>
<td>(\sqrt{(\Delta TD)^2}) (sec)</td>
<td>29.7</td>
<td>40.0</td>
<td>33.0</td>
</tr>
</tbody>
</table>

I. Exp. I \(_{17}^{17}\)

Experiment I \(_{17}^{17}\) was identical to Exp. I \(_{7}^{7}\) except that smoothing and sorting parameters SS-5 were used.

Photographs of the association print with \(f_0 = 0\) and \(f_0 = 4\) are shown in Figs. 25 and 26. These prints are noisier than those for Exp. I \(_{7}^{7}\) though there does not appear to be a significant difference in the tracking of the turns.

The number of turns in each tracking category is displayed in Table 16. Comparison with the results for Exp. I \(_{7}^{7}\) (Table 3) indicates that there has been a general increase in the number of type E turns and a decrease in the number of type A turns in the noisy area. The effect of noise perturbations is shown in Table 17. Comparison with the results for Exp. I \(_{7}^{7}\) (Table 4) indicates that tracks are more easily perturbed by noise in the cover of one good radar here than in Exp. I \(_{7}^{7}\).

The distribution of \(\Delta t_{f_0}\) of noise tracks for \(f_0 = 0, 1, 2, 3, 4, 5, 6\) is shown in Figs. 27a, 27b, 27c, 27d, 27e, 27f, and 27g. Eight noise tracks, initiated in the first minute, lived beyond the end of this experiment and are not shown in these figures. The noisy appearance of the association prints is described quantitatively in these figures. An increase in the lifetimes of noise tracks is not too surprising here since \(\xi\) increased.
FIG. 25 ASSOCIATION PRINT FOR EXP. $I_{17}$ WITH $f_o = 0$

FIG. 26 ASSOCIATION PRINT FOR EXP. $I_{17}$ WITH $f_o = 4$
<table>
<thead>
<tr>
<th>FIG.</th>
<th>( f_a )</th>
<th>( \Delta t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>27g</td>
<td>6</td>
<td>19.33 sec.</td>
</tr>
<tr>
<td>27f</td>
<td>5</td>
<td>36.6 sec.</td>
</tr>
<tr>
<td>27e</td>
<td>4</td>
<td>46.6 sec.</td>
</tr>
<tr>
<td>27d</td>
<td>3</td>
<td>46.0 sec.</td>
</tr>
<tr>
<td>27c</td>
<td>2</td>
<td>55.0 sec.</td>
</tr>
<tr>
<td>27b</td>
<td>1</td>
<td>67.6 sec.</td>
</tr>
<tr>
<td>27a</td>
<td>0</td>
<td>102.7 sec.</td>
</tr>
</tbody>
</table>
more rapidly just after initiation than in Exp. I_7. Table 18 shows $N_f$ as a function of $f$.

<table>
<thead>
<tr>
<th>Number of A</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>$n$ B</td>
<td>7</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>$n$ C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n$ D</td>
<td>0</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>$n$ E</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 16
Classification of turns for Exp. I_7

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of successful turns with one noise association during the turn.</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of successful turns with more than one noise association during the turn.</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of unsuccessful turns with a noise association and subsequent loss of the track.</td>
<td>10</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 17
Effect of noise on turns for Exp. I_7

<table>
<thead>
<tr>
<th>$f$</th>
<th>$f = 0$</th>
<th>$f = 1$</th>
<th>$f = 2$</th>
<th>$f = 3$</th>
<th>$f = 4$</th>
<th>$f = 5$</th>
<th>$f = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f$</td>
<td>6.3</td>
<td>6.0</td>
<td>11.6</td>
<td>15.4</td>
<td>10.6</td>
<td>5.7</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 18
$N_f$ as a function of $f$ for Exp. I_7

Table 19 shows $\overline{TD}$ and $\sqrt{(\Delta TD)^2}$ for different types of cover.
Table 19

<table>
<thead>
<tr>
<th>TD (sec)</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.3</td>
<td>41.7</td>
<td>17.6</td>
<td></td>
</tr>
<tr>
<td>√(ΔTD)² (sec)</td>
<td>24.8</td>
<td>55.1</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Average time delay for first association in Exp. I17

The operation of the track scratcher was similar to that in Exp. I7.

J. Exp. I18

Experiment I18 was identical to Exp. I17 except that it was run at a reduced report rate. It is identical to Exp. I15 except that smoothing and sorting parameters SS-5 rather than SS-4 were used.

Photographs of the association print with f₀ = 0 and f₀ = 1 are shown in Figs. 28 and 29. Less noise is displayed than in Exp. I17, as is to be expected with a reduced report rate, and the tracking of the turns appears to be improved. There does not appear to be a significant difference with the association prints for Exp. I15 (Figs. 22 and 23).

The number of turns in each tracking category is displayed in Table 20. Comparison with the results for Exp. I17 shows that a significant increase in the number of successful turns has resulted from reducing the report rate. Table 21 contains data on the effect of noise on turns.

The distribution of δf₀ for noise tracks with f₀ = 0, 1, 2, 3, 4, 5 is shown in Figs. 30a, 30b, 30c, 30d, 30e, and 30f. No noise tracks reached a firmness of 6. The lifetimes of noise tracks are less than for Exp. I17 and they do not differ significantly from those for Exp. I15. Table 22 shows Nf as a function of f.
FIG. 28 ASSOCIATION PRINT FOR EXP. I_{18} WITH f_o = 0

FIG. 29 ASSOCIATION PRINT FOR EXP. I_{18} WITH f_o = 4
Lifetimes of noise tracks on a clear picture display - Exp. $I_{in}$

FIG. 30f - $f_a = 5$
AVERAGE $\Delta t_5 = 22.5$ sec.

FIG. 30e - $f_a = 4$
AVERAGE $\Delta t_4 = 30.0$ sec.

FIG. 30d - $f_a = 3$
AVERAGE $\Delta t_3 = 46.0$ sec.

FIG. 30c - $f_a = 2$
AVERAGE $\Delta t_2 = 40.1$ sec.

FIG. 30b - $f_a = 1$
AVERAGE $\Delta t_1 = 74.3$ sec.

FIG. 30a - $f_a = 0$
AVERAGE $\Delta t_0 = 97.7$ sec.
Table 20

Classification of turns for Exp. I18

<table>
<thead>
<tr>
<th>Number of A</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>&quot;A&quot; B</td>
<td>9 14</td>
<td>8 4</td>
<td>11 13</td>
</tr>
<tr>
<td>&quot;C&quot; D</td>
<td>0 1</td>
<td>1 0</td>
<td>0 0</td>
</tr>
<tr>
<td>&quot;E&quot;</td>
<td>0 6</td>
<td>1 1</td>
<td>0 2</td>
</tr>
</tbody>
</table>

Table 21

Effect of noise on turns for Exp. I18

<table>
<thead>
<tr>
<th>Number of successful turns with one noise association during the turn.</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of successful turns with more than one noise association during the turn.</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of unsuccessful turns with a noise association and subsequent loss of the track.</th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 22

$N_f$ as a function of $f$ for Exp. I18

<table>
<thead>
<tr>
<th>$f$ = 0</th>
<th>$f$ = 1</th>
<th>$f$ = 2</th>
<th>$f$ = 3</th>
<th>$f$ = 4</th>
<th>$f$ = 5</th>
<th>$f$ = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f$</td>
<td>4.9</td>
<td>6.9</td>
<td>9.2</td>
<td>5.1</td>
<td>1.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 23 shows $TD$ and $\sqrt{(\Delta TD)^2}$ for different types of radar cover.
In the preceding experiments it was found that operation of the system at the full report rate results in saturation of the buffer stores and drum. Saturation of the first kind in the buffer stores will cause certain reports to be older than one second when they are finally sent to the drum. There is also the possibility that some new reports may not find an empty space in the buffer store because of the presence of these older-than-one-second reports. It might be thought that in such a situation preference should always be given to the most up-to-date information. Exp. I_{18} was run to test this idea. This experiment was identical to Exp. I_{17} except that the buffer stores were cleared at the end of each second, thus no information was older than one second.

Photographs of the association print with \( f_0 = 0 \) and \( f_0 = 1 \) are shown in Figs. 31 and 32. It is clear from these figures that tossing out "old" reports degenerates the tracking performance considerably.

L. Discussion of the Fishook Experiments

The more significant results of the series of fishook experiments conducted up to the present time have been shown in the preceding pages, and it appears worthwhile here to summarize these observations. It is difficult to make very accurate quantitative statements about the statistical behavior
FIG. 31 ASSOCIATION PRINT FOR EXP. $I_{19}$ WITH $f_o = 0$

FIG. 32 ASSOCIATION PRINT FOR EXP. $I_{19}$ WITH $f_o = 4$
of some properties of the system because of the small populations of the samples. This is simply a result of the large amount of time required to gather data. Some of these experiments will be repeated with different initial entries to the random number routine in order to increase the size of the sample populations.

There is saturation of the first and second kinds in the buffer stores and saturation of the drum in fishhook experiments when the system operates at the full report rate. It is important to remember that saturation depends on the target pattern and radar characteristics. In an extreme example where all reports are position reports on a single plane, it is clear that there would be no saturation of the drum, though there might be saturation in the radar buffer stores, depending on the radar configuration, blip-scan ratios and size of the buffer stores. It is perhaps worthwhile to here emphasize the difference between saturation of the first and second kinds. The latter, which occurs when a radar buffer store becomes completely filled with reports, depends only on the size of the buffer store, its associated radar, and the targets covered by that radar. It therefore does not depend on the characteristics of the system as a whole. Saturation of the first kind, which arises when the drum cannot process all of the reports coming into the system in one second, is a property of the system considered as a whole, the target configuration over the entire area, the characteristics of all of the radars, the rate at which the drum can process reports, etc. The two kinds of saturation can occur independently or together.

With a few simple assumptions we will derive an equation which gives an upper bound on the report rate which the drum can handle before it
becomes saturated. Let

\[ R = \text{number of reports/second entering computer.} \]
\[ A = \text{number of associations/second taking place.} \]
\[ D = \text{number of drum addresses.} \]
\[ L = \text{lifetime of report which has no further associations.} \]
\[ T = \text{radar scan time.} \]
\[ K = \text{the redundant coverage factor.} \]

Obviously, if all reports are nonsense, and there are no associations, the equilibrium condition is that the report rate just fill the drum in the time "L". Then after L seconds, there appear just enough addresses to handle the R reports which arrive at L+1 seconds; or:

\[ R = \frac{D}{L} \text{ reports/sec - for no associations.} \]

Let us now assume that there are some tracks on the drum, some reports will then associate and we no longer need new addresses for them. For simplicity we assume that these tracks are never lost—that is, they occupy a fixed set of drum addresses—that there is one report in every plane during the time T, and that the remaining addresses on the drum, \( D' \), are available for incoming reports which never associate with anything. Thus,

\[ R - A = \frac{D'}{L} \]

But we know that,

\[ D' = D - AT \] (for single coverage)

where AT is simply the number of associations/second times the scan time, or in other words, the number of tracks. Now some tracks will in general be seen by more than 1 radar - so that more than 1 report/second time will associate with the same address. Therefore,

\[ D' = D - KAT, \quad K < 1 \]
so \[ R - A = \frac{D - KAT}{L} \]
and
\[ R = \frac{D}{L} + A\left(1 - \frac{kT}{L}\right). \]
This is the maximum reporting rate which will keep the "track-scratcher" quiet. However, we contemplate inhibiting the entry on the drum of reports whose \( \sigma \) is less than some predetermined value. Thus only a fraction, \( P \), of the reports will ever need drum addresses. Hence for the nonsense case:
\[ PR = \frac{D}{L} \]
\[ R = \frac{D}{LP} \]
and for the general case - because the \( \sigma \) will not affect associations:
\[ P(R - A) = D - \frac{KAT}{L} \]
\[ PR = \frac{D}{L} - \frac{KAT}{L} + AP \]
\[ R = \frac{1}{P}\left\{\frac{D}{L} + A\left(P - \frac{KT}{L}\right)\right\} \tag{7} \]

Using this result it is found that in a fishhook experiment \( R \) is approximately equal to 16 reports per second. This agrees with the experiments; when the system operated at the reduced report rate, which is slightly less than 16 reports per second, the drum did not become saturated, but when operated at the full report rate, 20 reports per second, the drum was saturated.

One of the most important results here is that the quality of the tracking appears to decrease continuously as the system becomes saturated and does not display a sharp cut-off.

A comparison of the tables for \( N_f \) and the distributions of \( \Delta t_{fo} \) indicates that parameters SS-4 will present a more noise-free display than parameters SS-5 when the system is operating at the full report rate. When operating at a reduced report rate the difference is not nearly so great though parameters SS-4 still appear to give a more noise-free picture.
Smoothing and sorting parameters SS-4 and SS-5 are superior to the others in tracking 450 mph targets through a 1/4 g turn. The differences in the quality of the tracking for parameters SS-4 and SS-5 though not very great indicate that parameters SS-4 are to be preferred. Comparison of the results for Exps. I7 and I17 shows that there are a significantly greater number of successful turns in Exp. I7 for coverage by one good radar and there are no significant differences in other types of cover; thus, parameters SS-4 give slightly better results under saturation conditions. Comparison of the results of Exps. I5 and I8 indicates that the two sets of parameters give about the same results on the tracking of turns when the system is operated at the reduced report rate.

The differences in the first association time delay, TD, resulting from the use of parameters SS-4 or SS-5 are not important.

The results indicate that with the system parameters given in Table 1, and parameters SS-4 or SS-5, 450 mph targets can almost always be successfully tracked around a 90°, 1/4 g turn when in double cover. In the coverage of one good radar the probability of a successful turn is approximately 0.75 for the same target. In the coverage of one poor radar this probability is approximately 0.6. These results refer to operation at the reduced report rate and to targets located in the noisy half of the area.

The present parameters do not give successful tracking of 675 mph targets executing 90°, 1/4 g turns.

M. Exp. I22

Experiment I22 is a scissors experiment. The parameters for the basic course for the odd numbered planes are
and for the even numbered planes

\[ x_2 = 3 \text{ mi} \quad y_2 = 28 \text{ mi} \]
\[ u_2 = 0.0879 \text{ mi/sec} \quad v_2 = 0.0879 \text{ mi/sec} \] (9)

Each plane's speed is 1/8 mi/sec, the courses of the two planes intersect at an angle of 90°, and \( t_f = 257 \) seconds.

Smoothing and sorting parameters SS-4 were used and the system was operated at the reduced report rate. Noise was confined to the left half of the area, \( 0 \leq x \leq 128 \) and \( 0 \leq y \leq 256 \).

Photographs of the association print at \( f_0 = 0 \) and \( f_0 = 4 \) are shown in Figs. 33 and 34. The \( \xi, \gamma \) print (raw data) for this experiment and Exp. I23 is shown in Fig. 35.

The tracking of intersecting planes has been classified in an analogous fashion to the classification of turning planes. Six categories have been set up and they are defined as follows.

The tracking of the intersection is type A if:

1. Tracks are initiated on both planes at \( t \leq 0.050 \).
2. There is at least one correct association with each of these tracks at \( \text{ONK} \leq t \leq t_f \).

The tracking of the intersection is type B if:

1. A track is initiated on only one plane at \( t \leq 0.050 \).
2. There is at least one correct association with this track at \( \text{ONK} \leq t \leq t_f \).

The tracking of the intersection is type C if:

1. A track is initiated on only one plane at \( t \leq 0.050 \).
2. There is no correct association with this track at \( \text{ONK} \leq t \leq t_f \).
FIG. 33 ASSOCIATION PRINT FOR EXP. I_{22} WITH f_0 = 0

FIG. 34 ASSOCIATION PRINT FOR EXP. I_{22} WITH f_0 = 4
FIG. 35 $\xi, \eta$ PRINT FOR EXPS. $I_{22}$ AND $I_{23}$
The tracking of the intersection is type D if:

1. Tracks on both planes are initiated at $t \leq 050$.
2. There is a correct association with only one of these tracks at $ONK \leq t \leq t_f$.

The tracking of the intersection is type E if:

1. Tracks are initiated on both planes at $t \leq 050$.
2. There is no correct association with either of these tracks at $ONK \leq t \leq t_f$.

The tracking of the intersection is type F if:

1. No tracks are initiated on either plane at $t \leq 050$.

The times are given in sexadecimal seconds and $t_f$ is the time of the end of the experiment. For $t \leq 050$ and $t > ONK$ the planes are sufficiently well separated to make the probability of a track associating with a report from the wrong plane negligible.

Categories A, D, and E describe the tracking of the intersection of two planes on which there are well established tracks before the intersection. Categories B and C describe the tracking of one plane which intersects with another plane not being tracked just before the intersection. Categories B, C and F contain all situations in which there has been some difficulty in track initiation.

Table 24 shows the number of intersections in each tracking category listed according to type of cover and the presence or absence of noise.

In a scissors experiment a wrong association can occur in two ways, by association with a noise report, and by association with a report from the wrong plane (i.e. the other plane involved in the intersection). The information in Table 25 helps to describe the effects of these wrong associations. In this table the number of intersections in categories A, D, and E in which one or both tracks associated with a noise report and one
or both tracks associated with a report from the wrong plane are listed according to cover and the presence or absence of noise.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>Number of A</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>&quot; B</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>&quot; C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&quot; D</td>
<td>0</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>&quot; E</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>&quot; F</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2h

Classification of intersections for Exp. I22

Detailed track histories of four intersections are shown in Figs. 36, 37, 38 and 39. Points of association with the track on the odd numbered plane are marked by x and by • for the even numbered plane. A small square around one of these marks □ or □ indicates an association with the wrong plane, a small circle x or o indicates an association with noise. Otherwise the notation is like that in the detailed track histories already shown. All tracks on the even numbered planes show an approximate displacement of 1 mile south of the true course, this results from the position error introduced by truncating the reports at the 1 mile digit.

N. Exp. I23

In this experiment smoothing and sorting parameters SS-5 were used, otherwise it was identical to Exp. I22.

Photographs of the association print at f_o = 0 and f_o = 4 are shown.
### NOISY AREA

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Noise Assn</td>
<td>Assn with wrong plane</td>
<td>Noise Assn</td>
</tr>
<tr>
<td></td>
<td>one track</td>
<td>both tracks</td>
<td>one track</td>
</tr>
<tr>
<td>No. of A</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>No. of D</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>No. of E</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### NOISE-FREE AREA

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Assn with wrong plane</td>
<td>Assn with wrong plane</td>
<td>Assn with wrong plane</td>
</tr>
<tr>
<td></td>
<td>one track</td>
<td>both tracks</td>
<td>one track</td>
</tr>
<tr>
<td>No. of A</td>
<td>4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>No. of D</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>No. of E</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 25

Description of the effects of wrong associations for Exp. I22
EXP. I-22
PLANES 3 AND 4
$t_f = 257$ SEC.

FIGURE 36
EXP. 1-22
PLANES 37 AND 38
$t_f = 257$ SEC.

FIGURE 38

CONFIDENTIAL

CONFIDENTIAL
EXP. I-22
PLANE 49 AND 4K
$t_f = 257$ SEC.

FIGURE 39
in Figs. 40 and 41. (These pictures do not include the last 43 seconds of the flight.)

Table 26 shows the number of intersections in each tracking category listed according to type of cover and the presence or absence of noise.

<table>
<thead>
<tr>
<th></th>
<th>1 Good Radar</th>
<th>1 Poor Radar</th>
<th>2 or 3 Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Noise</td>
<td>Noise</td>
<td>No Noise</td>
</tr>
<tr>
<td>Number of A</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>&quot; B</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>&quot; C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&quot; D</td>
<td>0</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>&quot; E</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>&quot; F</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 26

Classification of intersections for Exp. 123

Table 27 describes the effects of wrong associations.

Figures 42, 43, 44, 45 show detailed track histories on the same intersections for which detailed track histories have been presented in Exp. 122.

0. Discussion of Scissors Experiments

It is obvious that there is an important difference in the requirements for tracking turning planes and tracking planes through intersections. The former case requires that acceleration be picked up quickly, the latter case requires that the system be fairly insensitive to apparent changes in velocity since there exists a good probability for a wrong association. The conditions under which these requirements imply significant changes
Table 27

Description of the effects of wrong associations for Exp. I23
FIG. 40 ASSOCIATION PRINT FOR EXP. I_{23} WITH f_o = 0

FIG. 41 ASSOCIATION PRINT FOR EXP. I_{23} WITH f_o = 4
FIGURE 43

EXP. I-23
PLANES 35 AND 36
$t_f = 260$ SEC.

CONFIDENTIAL
EXP. 1:23
PLANES 49 AND 4K
$t_f = 260$ SEC.

FIGURE 45
in the sorting and smoothing parameters need to be studied. It may be possible to find a single set of parameters which are suitable for a wide variety of turns and intersections. The two scissors experiments presented here are of particular interest since they have been performed under the same conditions which, so far, have given the best results in fishhook experiments.

The only significant difference, and this is very small, between the results of the two scissors experiments is in the classifications of intersections in the cover of 1 good radar in the noisy area; a comparison indicates that parameters SS-4 are to be preferred slightly. The results in multiple cover are clearly superior to those in single cover for both experiments and if we call type A tracking of an intersection successful, then it appears that in multiple cover the probability of success is nearly equal to the probability of success in tracking a turn. However, more data must be collected before this can be said with reasonable certainty.

P. Exps. I28 and I29

In these two experiments there were no planes in the "sky," the only reports entering the system were noise reports. We are here trying to simulate the real situation in which the system is on the alert but the air traffic is very low. Under these conditions the system must be able to detect the presence of a real target as quickly as possible while not creating too many false alarms from noise tracks. A study of the characteristics of noise tracks under these conditions is, therefore, of considerable importance.

In both of these experiments the system operated at the full report rate of 20 reports per second. Smoothing and sorting parameters SS-4 were
used in Exp. I_{28} and parameters SS-5 were used in Exp. I_{29}. Noise was confined to the left half of the area \((0 \leq x < 128 \text{ and } 0 \leq y < 256)\) in both experiments. The running time, \(t_f\), in the experiments was approximately 212 seconds.

Plots of \(t_{f_0}\) for noise tracks initiated in the first minute of the experiment, for the various \(f_0\), in Exps. I_{28} and I_{29} are shown in Figs. H6a, H6b, H6c, H6d, H6e, H6f and H7a, H7b, H7c, H7d, H7e, H7f, H7g, respectively. Differences in the results produced by the two sets of sorting and smoothing parameters are more striking here than anywhere else. Not all of this difference is apparent from these figures. The histories on 135 noise tracks in Exp. I_{29} have been omitted because they are still alive at the end of the experiment. We can only say that these tracks lived longer than 212 seconds. In Exp. I_{28} none of the noise tracks initiated in the first minute lived beyond the end of the experiment.

It is apparent that parameters SS-1 produce noise tracks with shorter lives than parameters SS-5 when the system is operating at a rate of 20 noise reports per second.

Tables 28 and 29 show \(\bar{N}_f\) as a function of \(f\) for Exps. I_{28} and I_{29}, respectively. The differences noted above are reflected in these tables.

<table>
<thead>
<tr>
<th>(f)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{N}_f)</td>
<td>12.6</td>
<td>12.8</td>
<td>20.0</td>
<td>19.9</td>
<td>5.5</td>
<td>3.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 28

\(\bar{N}_f\) as a function of \(f\) for Exp. I_{28}
<table>
<thead>
<tr>
<th>$f$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f$</td>
<td>9.9</td>
<td>12.2</td>
<td>17.5</td>
<td>27.1</td>
<td>19.2</td>
<td>10.9</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Table 29

$N_f$ as a function of $f$ for Exp I$_{29}$
Lifetimes of noise tracks on a clear picture display - Exp. I

**FIG. 46c - \( f_o = 2 \)**

Average \( \Delta t_2 = 44.47 \text{ sec.} \)

**FIG. 46b - \( f_o = 1 \)**

Average \( \Delta t_1 = 69.73 \text{ sec.} \)

**FIG. 46a - \( f_o = 0 \)**

Average \( \Delta t_0 = 85.30 \text{ sec.} \)
Lifetimes of noise tracks on a clear picture display - Exp. I_{28}

FIG. 46 f - \( f_o = 5 \)
AVERAGE \( \Delta t_5 = 25.35 \text{ sec.} \)

FIG. 46 e - \( f_o = 4 \)
AVERAGE \( \Delta t_4 = 25.74 \text{ sec.} \)

FIG. 46 d - \( f_o = 3 \)
AVERAGE \( \Delta t_3 = 27.26 \text{ sec.} \)
Lifetimes of noise tracks on a clear picture display - Exp. I

FIG. 47c - \( f_0 = 2 \)  
AVERAGE \( \Delta t_c \) = 54.5 sec.

FIG. 47b - \( f_0 = 1 \)  
AVERAGE \( \Delta t_b \) = 77.5 sec.

FIG. 47a - \( f_0 = 0 \)  
AVERAGE \( \Delta t_0 \) = 92.7 sec.
Lifetimes of noise tracks on a clear picture display - Exp. I

FIG. 47g - f<sub>o</sub> = 6
AVERAGE \( \Delta t <sub>f_o</sub> \) = 22.5 sec.

FIG. 47f - f<sub>o</sub> = 5
AVERAGE \( \Delta t <sub>f_o</sub> \) = 29.2 sec.

FIG. 47e - f<sub>o</sub> = 4
AVERAGE \( \Delta t <sub>f_o</sub> \) = 29.7 sec.

FIG. 47d - f<sub>o</sub> = 3
AVERAGE \( \Delta t <sub>f_o</sub> \) = 39.4 sec.