ANALYSIS AND EXPERIMENTAL INVESTIGATION OF THREE SYNTHETIC APERTURE RADAR FORMATS

DAVID AARON SCHWARTZ

UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS
ANALYSIS AND EXPERIMENTAL INVESTIGATION OF
THREE SYNTHETIC APERTURE RADAR FORMATS

by

DAVID AARON SCHWARTZ

This work was supported in part by the Air Force Office of Scientific Research under contract no. AFOSR-79-0029.
ACKNOWLEDGEMENT

The author would like to thank Prof. W. K. Jenkins for his assistance, technical guidance and patience. He also would like to acknowledge the Air Force Office of Scientific Research whose financial support, under Grant No. AFOSR-79-0029, made this research possible.
# TABLE OF CONTENTS

**1 INTRODUCTION TO BASIC PRINCIPLES OF SAR**
- 1.1 Relationship of SAR to Linear Phased Array Antenna ........ 1
- 1.2 Approximate Analysis of 2-D System ......................... 2
- 1.3 Implementation of SAR System ............................... 5

**2 IMPLEMENTING SAR AS A 2-D SPECTRAL ANALYSIS USING THE FFT**
- 2.1 Stretch Processing ........................................ 7
- 2.2 Ideal Response of a Point Target .......................... 10

**3 ANALYSIS AND DESCRIPTION OF POLAR FORMAT**
- 3.1 Actual Response of a Point Target ........................ 13
- 3.2 Approximate Phase Function ................................. 16
- 3.3 Exact Phase Function ....................................... 19
- 3.4 How to Record the Data in the p,u Plane ................ 20
- 3.5 Polar/Keystone to Rectangular Interpolation ............. 24

**4 RECTANGULAR FORMAT**
- 4.1 Rectangular format as an Approximation to Polar Format 33
- 4.2 Source of Image Degradation ............................... 34
- 4.3 Analysis of Spotlight Parameters .......................... 35
- 4.4 Multipatch Format ....................................... 38

**5 COMPUTER SIMULATION RESULTS**
- 5.1 Image Degradation ........................................ 41
- 5.1 First Order Interpolation of Polar Format ............... 58

**APPENDIX** .............................................................. 63

**REFERENCES** ............................................................ 67
1.1 RELATIONSHIP OF SAR TO LINEAR PHASED ARRAY ANTENNA

Synthetic aperture radar (SAR) is a technique for achieving high resolution map images (i.e. terrain images) with a physically small antenna (aperture). In a conventional radar system azimuth resolution is limited by the beamwidth of the antenna. In a SAR system the radar is mounted on a moving platform (i.e. airplane, satellite) which flies along a path. By using a coherent system and signal processing techniques it is possible to achieve an effective aperture approximately proportional to the length of the path flown.

To understand the basic principle, consider a linear phased array antenna of length L consisting of N elements. A target is illuminated with a pulse and the reflection is received at the N elements where the returns are coherently summed together. Now consider the case where one antenna element is initially placed at the same position as the first element of the array. The target is illuminated with the same pulse as in the first system and the return data is stored. Next the antenna is moved to the same position as the second element of the array and the procedure is repeated. This procedure is continued until the return for the antenna in the Nth position is stored. If the stored returns are now coherently summed together the effective azimuth resolution of the
synthesized aperture will be twice that of the linear array of the same length. The factor of two arises from having a two way signal path. Thus fine azimuth resolution can be achieved by flying long aperture paths.

However in the synthetic aperture case the same antenna is used for receiving and transmitting the pulse. Thus as the receiver moves along the aperture the pulses are transmitted from an antenna with a phase center at the same position as the synthetic element and not from an antenna with a phase center at the center of the aperture. It will be shown that this can be approximately compensated for by using 'stretch processing' to simulate a constant phase contour along the synthetic aperture.

1.2 APPROXIMATE ANALYSIS OF 2-D SYSTEM

To better understand the basic principles involved and the historical basis of SAR consider the two dimensional system shown in Fig. 1-1. At time t the radar platform (platform) is at position \( x(t) \) and is moving with velocity \( v \) along the x axis. Assume the transmitted pulse is CW of frequency \( \omega_0 \), then the received signal is:

\[
S(t) = \exp \left[ j \omega_0 \left( t - \frac{2R(t)}{c} \right) \right]
\]  

(1-1)

The instantaneous range to the target then is:

\[
R(t) = \sqrt{R_0^2 + (vt - x_0)^2}
\]

(1-2)

The time derivative of the phase is the instantaneous received frequency:
Fig. 1-1 2-D SAR coordinate system.
The second term is the doppler shift. Taking the the first two terms of the Taylor series expansion of $R^{-1}(t)$ the received signal can be approximated as:

$$R(t) = R_0 + \frac{1}{2R_0} (vt - x_0)^2$$

$$S(t) = \exp \left[ -j \frac{\omega_0}{c R_0} (vt - x_0)^2 \right] \cdot \exp \left[ j \omega_0 (t - \frac{2R_0}{c}) \right]$$

The first exponential term is recognized as a linear frequency modulation (LFM) of the received signal along the x, or azimuth coordinate. Thus the FM rate is:

$$\gamma = \frac{2\omega_0 v^2}{c R_0} \text{ rad/sec}$$

The time duration of the LFM pulse is the time it takes to fly the aperture

$$T = \frac{L_s}{v}$$

The bandwidth is the product of the FM rate and the duration. The time bandwidth product is the product of the bandwidth and the duration.

$$\text{BW} = \gamma \cdot T = \frac{2vL_s}{\lambda R_0}$$

$$\text{TBP} = T \cdot \text{BW} = \frac{2vL_s T}{\lambda R_0}$$

Therefore if the received LFM pulse is passed through a matched filter the effective azimuth resolution is the spatial dwell distance divided by the TBP (see [1] and [2]).
\[ \Delta a = \frac{V_{aT}}{T_{BW}} = \frac{\lambda R_0}{2L_s} \quad (1-10) \]

As noted before the resolution of a normal physical antenna is:

\[ \Delta a = \frac{\lambda R_0}{L_s} \quad (1-11) \]

Therefore if the transmitted pulse is an LFM chirp, of FM rate \( \gamma \), instead of CW, and the returns are processed through a range matched filter, range resolution can be achieved in a manner similar to azimuth.

\[ \Delta r = \frac{PC}{2} \quad (1-12) \]

\[ p = \frac{1}{\gamma T} \quad (1-13) \]

(\( T \) is the length of the pulse and \( p \) is the effective pulse width)

### 1.3 IMPLEMENTATION OF SAR SYSTEM

This leads to a possible system implemented with a two-dimensional array of matched filters, one two-dimensional matched filter for each range azimuth cell. This is of course a two-dimensional correlator. Referring to it in terms of an array of matched filters will be shown to be a useful conceptual tool in understanding multipatch format.

Traditionally SAR processing has been performed with a two-dimensional coherent optical correlator. In an optical system the data is recorded onto film. After the film is developed it is illuminated by coherent light which then passes through an optical correlator, finally exposing a second film plate. The second film plate is then developed to produce the final image. This is a time consuming
process and is clearly not suited to a real time system.

With the discovery of the FFT in 1965 the basis of performing real time processing was possible. The computational complexity of a 2-D correlation is $O(N^4)$. Implemented with an FFT the complexity is reduced to $O(2N^3\log_2 N)$. With $N$ typically between 512 and 4096 this represents too large a computational load for today's processors for real time results. However the problem can be approximated as a 2-D spectral analysis problem, which using the FFT reduces the computational complexity to $O(2N\log N)$. It is this type of realization that leads to real time processing rates.
2.1 STRETCH PROCESSING

A technique to map range and azimuth into a 2-D spectrum is stretch processing. This leads to an algorithm that processes range and azimuth identically. The basis of this method is to mix the incoming returns with a synthetic reference return that simulates the return of a reference target at the center of the ground patch and to obtain the resulting difference signal from the mixer. The reference return is a time delayed version of the transmitted LFM chirp with the delay proportional to the range to the reference point. Similarly the return of an actual target is also a delayed version of the LFM chirp, with delay proportional to the range to the target as illustrated in Fig. 2-1. Therefore if the two returns are mixed, the difference signal is a constant frequency proportional to the range difference.

Again consider the transmitted signal to be CW. The doppler shift will result in an approximate LFM chirp with delay proportional to the azimuth of the target as illustrated in Fig. 2-2. Therefore the difference signal from the mixer is a constant frequency proportional to the azimuth of the target.
Fig. 2-1

a) Return from target P during one IPP

b) Lowpassed result after mixing with return from the reference.
Δf = Doppler Shift

a) Doppler shift from target at P

Target P - Reference Return

b) Lowpassed output after mixing target P return with reference

Fig. 2-2
It should be kept in mind that this is only an approximation to the actual system. The doppler LFM approximation comes from only the first three terms of an infinite series expansion. It will be shown that in the polar format case the 2-D DFT is still valid.

To formulate the SAR problem in such a way as to map range and azimuth time domain returns to a 2-D spectrum that can be resolved by a 2-D FFT, the time domain response for the ideal case is derived. This will provide a the basis for determining the form into which the received returns must be manipulated. This will also determine the necessary sampling conditions to achieve a desired resolution.

The problem can be characterized in three data planes; 1) the ground plane, 2) the data recording plane and 3) the image plane. Ideally the image plane is the radar reflectivity of the ground plane. The data plane and the image plane form a Fourier transform pair.

2.2 IDEAL RESPONSE OF A POINT TARGET

Let there be a single point target reflector at \((x_t, y_t)\) in the \(x, y\) ground plane. The \(p, u\) data plane should contain data such that the 2-D FFT will result in a digital impulse at a point corresponding (by some scale factor) to \((x_t, y_t)\).

The following properties of the DFT, where \(\Leftrightarrow\) indicates a transform pair, are needed:
\[
\begin{align*}
X(0,0) &= a \\
X(k,l) &= 0 : k, l \neq 0 \\
\end{align*}
\]

(2-1)

\[
W_m^p w_n^q x(m,n) \leftrightarrow X((k+p, l+q)_{MxN}) = W M = \exp(-j2\pi/M)
\]

(2-2)

Therefore the function in the p, u plane must be of the form \( e^{j\psi(p,u)} \), such that:

\[
\frac{\partial \psi}{\partial p} = Bx_t \text{ (rad/m)} \\
\frac{\partial \psi}{\partial u} = By_t \text{ (rad/m)}
\]

(2-3a) (2-3b)

Where B is an arbitrary scaling factor. The only function that satisfies equations (2-3a) and (2-3b) is:

\[
\psi(p,u) = B(x_t p + y_t u) + C
\]

(2-4)

The constant c is arbitrary as it introduces only a constant phase which has no effect on the magnitude of the resulting image.

Converting to the polar coordinates R, \( \theta \), the phase function is given by:

\[
\psi(R, \theta) = BR (x_t \sin \theta + y_t \cos \theta) + C
\]

(2-5)

Equations (2-4) and (2-5) should be kept in mind, for discussion in the next section will attempt to transform the radar returns into something approximating this functional form.

To find the the resolution of ideal data in the image plane assume an MxN array of discrete data in the p, u plane. Let the data have sample spacings of \( \Delta p, \Delta u \) respectively. The definition of the 2-D DFT is:

\[
\begin{align*}
W_m^p w_n^q x(m,n) \leftrightarrow X((k+p, l+q)_{MxN}) = W M = \exp(-j2\pi/M)
\end{align*}
\]
\[
X(k, \ell) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) W_m^k W_n^\ell
\]

(2-6)

Since there is only \(2\pi\) of unambiguous phase, equating phase terms

\[
B\Delta x \Delta p = 2\pi/M
\]

\[
B\Delta y \Delta u = 2\pi/N
\]

or

\[
\Delta x = \frac{2\pi}{B\Delta p} \quad (2-7a)
\]

\[
\Delta y = \frac{2\pi}{B\Delta u} \quad (2-7b)
\]

Therefore for equal range and azimuth resolution \(\Delta x=\Delta y=\Delta\). The data in the \(p, u\) plane is a square of dimensions \(2\pi/B\Delta\) by \(2\pi/B\Delta\). This corresponds to a ground patch of \(M\Delta\) by \(N\Delta\).

From equation (2-3) the change in spatial frequency from 0 to \(M\Delta\) is \(2\pi\), that is \(B\Delta M = 2\pi\) (rad/m). Converting to cycles, \(B\Delta M / 2\pi = 1\) (cycle/m). Multiplying by \(p\) yields one cycle of bandwidth/sample. Therefore the system is sampled at the Nyquist rate.

In a real system the sampling rate will be greater than the Nyquist rate to prevent aliasing from returns outside of the image patch due to the limitations on realizable low pass filters. The returns from the oversampled system is doppler prefiltered and decimated. The doppler prefiltering and decimation is typically implemented for an oversampling factor of \(K\) by taking the weighted sum of \(K\) returns. This is also referred to as presumming. The weighting function is a window such as the Kaiser-Bessel or Dolph-Chebychev window.
CHAPTER 3

ANALYSIS AND DESCRIPTION OF POLAR FORMAT

3.1 ACTUAL RESPONSE OF A POINT TARGET

This section derives the analytic basis of polar format. This derivation is based on [3], using their notation. Their derivation is based on the work of Dr. G.W. Zeoli of Hughes Aircraft Co.

The coordinate system is referenced to the platform which moves in the negative x direction with velocity v (Fig. 3-1). Thus the platform is always located at (0,0,h). The reference point Q is at \((x_n,Y_0,0)\). The ideal point target is at \((x_n+x_t,Y_0+y_t,0)\). The quantity \(r_{0n}\) is the range to Q, \(r_n\) is the range to P, \(d\) is the distance between P and Q, and \(\gamma\) is the forward ground squint angle. Subscripts of n indicate a time varying quantity. However for analytic purposes it is assumed to be constant over an interpulse period (IPP). The functional form to indicate the two time coordinates is \(f_n(t)\), where \(t\) is the time index during one IPP and \(n\) is the discretised time index indicating the nth interpulse time period.

The nth transmitted pulse is:

\[
S_n(t) = \cos \left( w_0 t + \frac{\gamma}{2} t^2 \right) \quad |t| \leq \frac{T}{2}
\]  

(3-1)

Where \(w_0\) (rad/sec) is the center frequency, \(\gamma\) (rad/sec/sec) is the FM rate and \(T\) (sec) is the pulse width. The nth received pulse from a
Fig. 3-1 Coordinate System for Analysis
point target is:

\[ g_n(t) = \cos (\omega_o(t - \tau_n) + \frac{\gamma}{2} (t - \tau_n)^2) \]  \hspace{1cm} (3-2) \]

\( \tau_n \) is the two way path delay to the target.

Reformulating in analytic form let us mix the return from the reference Q with the return from an arbitrary point target at P. The resulting difference frequency has a phase function \( \Phi_n(t) \)

\[ \Phi_n(t) = \omega_o(t - \tau_{on}) + \frac{\gamma}{2} (t - \tau_{on})^2 - \omega_o(t - \tau_n) - \frac{\gamma}{2} (t - \tau_n)^2 \]

\[ \Phi_n(t) = (\tau_n - \tau_{on}) (\omega_o + \gamma t) - \frac{\gamma}{2} (\tau_n^2 - \tau_{on}^2) \]  \hspace{1cm} (3-3) \]

The two way path delay for a distance \( r \) is \( \tau = 2r/c \) where \( c \) is the propagation velocity in free space. Expressing \( \tau_n \), \( \tau_{on} \) in terms of \( r_n \), \( r_{on} \) equ. (3-3) can be rewritten as:

\[ \Phi_n(t) = \frac{2}{c} (r_n - r_{on}) (\omega_o + \gamma t) - \frac{2\gamma}{c^2} (r_n^2 - r_{on}^2) \]  \hspace{1cm} (3-4) \]

Dr. Zeoli suggested that the following change of variable be used.

\[ Z_n = \left( \frac{r_n}{r_{on}} \right)^2 - 1 \]  \hspace{1cm} (3-5) \]

In typical system geometries the magnitude of \( Z_n \) is much less than one.

Applying some elementary geometry to Fig. 3-1

\[ \left( \frac{r_n}{r_{on}} \right)^2 = \frac{(x_n + x_t)^2 + (y_o + y_t)^2 + h^2}{x_n^2 + Y_o^2 + h^2} \]  \hspace{1cm} (3-6) \]

\[ \left( \frac{r_n}{r_{on}} \right)^2 = 1 + \frac{2x_n x_t + 2Y_o y_t + x_t^2 + y_t^2}{r_{on}^2} \]

Replacing \( x_t^2 + y_t^2 \) with \( \rho^2 \) and \( x_n \) with \( Y_o \tan \theta_n \)
The needed terms in equ. (3-4) in terms of \( z_n \) are:

\[
\begin{align*}
  r_n - r_{on} &= r_{on} (\sqrt{1 + z_n} - 1) \\
  r^2_n - r^2_{on} &= r^2_{on} z_n
\end{align*}
\]

\( (3-8) \) \( (3-9) \)

3.2 APPROXIMATE PHASE FUNCTION

For now it is instructive to approximate equ. (3-8) by taking only the first two terms of the binomial expansion of \((1 + z_n)^{1/2}\)

\[
\begin{align*}
  r_n - r_{on} &= \frac{r_{on} z_n}{2} \\
  (3-10)
\end{align*}
\]

Substituting equ. (3-9) and (3-10) into equ. (3-4) \( \phi_n(t) \) is approximated as:

\[
\phi_n(t) = \frac{r_{on} Z_n}{c} [ w_o + \gamma t - \frac{2\gamma r_{on}}{c} ]
\]

\( (3-11) \)

It is now possible to determine how to write the data in polar format. It was suggested by ref. [4] that the data be written in the polar format of an annular sector, where the polar angle is equal to the ground squint angle. The sampled data spacing along the nth radius is constant. The polar angle increment is constant. The result is that the data is uniformly sampled in \( R \) and \( \theta \).
Let the polar radius during the nth interpulse period be defined as;

\[ R_n(t) = K_{vn} t + K_{rn} \]  \hspace{1cm} (3-12)

Where \( K_{vn} \) is the writing velocity and \( K_{rn} \) is the radial offset during the nth IPP. This terminology arises from the optical processing approach where the I and Q channels modulate the writing intensity of a light source moving linearly along the radius of a circular sheet of film. At the beginning of each IPP the film is rotated by the squint angle increment.

Solving equ. (3-12) for \( t \) and substituting into equ. (3-11) an equation is obtained for expressing the phase in terms of the recorded data in the data plane.

\[ \phi_n(R) = \frac{r_{on} Z_n}{c} \left[ \omega_o + \gamma \left( \frac{R-K_{rn}}{K_{vn}} \right) - \frac{2\gamma r_{on}}{c} \right] \]  \hspace{1cm} (3-13)

With respect to equ. (2-5) \( K_{vn} \), and \( K_{rn} \) should be choosen such that the phase function equ. (3-13) is in as close to the ideal form as possible. An obvious choice for \( \frac{K_{rn}}{K_{vn}} \) is to cancel out the first and fourth term of equ. (3-13),

\[ \frac{K_{rn}}{K_{vn}} = \frac{\omega_o}{\gamma} - \frac{2r_{on}}{c} \]  \hspace{1cm} (3-14)

which simplifies \( \phi_n(R) \) to:

\[ \phi_n(R) = \frac{\gamma R}{cK_{vn}} r_{on} Z_n \]  \hspace{1cm} (3-15)

Substituting equ. (3-7) for \( z_n \)

\[ \phi_n(R) = \frac{2\gamma Y_o R}{cK_{vn} r_{on}} \left( y_t + x_t \tan \theta_n + \rho^2/2Y_o \right) \]  \hspace{1cm} (3-16)
Recognizing that dividing the $x_t$ and $y_t$ term by $\cos \theta_n$ leads to the ideal form an obvious choice for $K_{vn}$ is:

$$K_{vn} = \frac{K}{\tau_{on} \cos \theta_n}$$  \hspace{1cm} (3-17)

where $K$ is a scaling factor of dimension $m^2/sec$. Thus $\xi_n(R)$ is:

$$\xi_n(R) = \frac{2\gamma Y_o R}{cK} \left( y_t \cos \theta_n + x_t \sin \theta_n + \frac{\rho^2 \cos \theta_n}{2Y_o} \right)$$  \hspace{1cm} (3-18)

Under the assumption that the phase function can be treated as being a continuous function of time, the phase function of the recorded data can be written as a function $R$ and $\theta$:

$$\xi_n(R, \theta) = \frac{2\gamma Y_o R}{cK} \left( y_t \cos \theta \right) x_t \sin \theta + \frac{\rho^2 \cos \theta_n}{2Y_o}$$  \hspace{1cm} (3-19)

Therefore the phase function in rectangular coordinates is given by:

$$\xi(p, u) = \frac{2\gamma Y_o}{cK} \left( y_t u + x_t p + \frac{\rho^2 u}{2Y_o} \right)$$  \hspace{1cm} (3-20)

Differentiating the phase function with respect to $p$, and $u$:

$$\frac{\partial \xi}{\partial p} = \frac{2\gamma Y_o}{cK} \ y_t$$  \hspace{1cm} (3-21)

$$\frac{\partial \xi}{\partial u} = \frac{2\gamma Y_o}{cK} \left( y_t + \frac{\rho^2}{2Y_o} \right)$$  \hspace{1cm} (3-22)

Let $B = 2\gamma Y_o/cK$ (rad/m$^2$) Then equ. (3-21) has the ideal form of equ. (2-3a) and equ. (3-21b) has the ideal form of equ. (2-3b) plus an error term. The error is seen to vary with the square of the distance from the reference. Since the spatial frequency in the $u$ direction is linear in $y_t$ and independent of $x_t$ the 2-D DFT relationship between the data and image plane still holds. However in using the approximation of equ. (3-10) all the higher order terms were dropped in $z_n$. Therefore
this only holds when the diagonal length of the ground patch is much smaller than the slant range to the target. This error can be interpreted as the $p^2/2Y_0$ term introducing the dominant linear misregistration in the $u$ direction and the neglected terms introducing a linear misregistration and spectral spreading (defocusing) in the $p$, and $u$ directions.

From equ. (3-12) the nature of the constant $K$ can be seen. If $K$ is chosen to equal $Y_0 c/2$ (m$^2$/sec) then the spatial frequency can be interpreted directly in meters, any other choice of $K$ represents only a scaling factor. In an optical data recording system $K$ is chosen to match the film size in the recorder. In a digital system $K$ is arbitrary and can be chosen to perform scaling if the processor is an integer processor.

3.3 EXACT PHASE FUNCTION

Now that $K_{vn}$, and $K_{rn}$ have been determined equations (3-7), (3-8), (3-9), (3-12), (3-14) and (3-15) can be substituted into equ. (3-4) to determine the exact phase function. This exact phase function will be the basis for the computer simulation, described in Chapter 5, to investigate the image distortion as a function of resolution for the polar format case.

$$\Phi(R, \theta) = \frac{2Y_{on}^2}{c^2} \left( \frac{cR \cos \theta}{K} + 2 \right) \left( \sqrt{1 + Z_n} - 1 \right) - Z_n$$  (3-23)

It can be seen in the geometry of Fig. 3-1 that
$$r_{on}(\theta) = Y_o \sec^2 \theta + h^2$$  \hspace{1cm} (3-24)

$$Z_n(\theta) = \frac{2Y_o(x_t \tan \theta + y_t)}{r_{on}(\theta)} + x_t^2 + y_t^2$$  \hspace{1cm} (3-25)

Converting to the $p$, $u$ rectangular coordinates results in

$$Z(p,u) = \frac{2Y_0^2}{c} [(\frac{pu}{K} + 2)(\sqrt{1+Z_n} - 1) - Z_n]$$  \hspace{1cm} (3-26)

The identity $\tan \theta = \frac{p}{u}$ gives

$$r_{on}^2(p,u) = Y_o^2 (1 + \frac{p^2}{u^2}) + h^2$$  \hspace{1cm} (3-27)

$$Z_n(p,u) = \frac{2Y_o(x_t(p/u) + y_t)}{r_{on}(p,u)} + (x_t^2 + y_t^2)$$  \hspace{1cm} (3-28)

### 3.4 HOW TO RECORD THE DATA IN THE P, U PLANE

Before simulating the SAR system, using equations (3-23) - (3-28), it is necessary to determine where in the data plane there must be data. This also solves the question of the proper form for writing incoming data returns in the recording plane.

Good data starts at the time of the reception of the leading edge of the return from the farthest point of the ground patch and ends at the time of the reception of the trailing edge of the return from the nearest point of the ground patch. Let $t_B$ be the time good data starts and $t_E$ be the time when good data ends for the nth IPP.
\[ t_B = -\frac{T}{2} + \frac{2r_{2n}}{c} \] (3-29)

\[ t_E = \frac{T}{2} + \frac{2r_{1n}}{c} \] (3-30)

where \( r_{2n} \) is the range to the far edge of the ground patch and \( r_{1n} \) is the range to the near edge of the ground patch for the nth IPP.

Substituting equations (3-14), (3-17) and (3-29) into equation (3-12) yields the beginning value of \( R \) for the nth IPP.

\[ R_{Bn} = \frac{K}{r_{on} \cos \theta_n} \left[ \omega \gamma - \frac{T}{2} + \frac{2}{c} (r_{2n} - r_{on}) \right] \] (3-31)

Similarly substituting equations (3-14), (3-17), (3-30) into equation (3-12) yields the ending value of \( R \) for the nth IPP.

\[ R_{En} = \frac{K}{r_{on} \cos \theta_n} \left[ \omega \gamma + \frac{T}{2} + \frac{2}{c} (r_{1n} - r_{on}) \right] \] (3-32)

Since \( r_{on}, r_{1n} \) and \( r_{2n} \) are not constant but vary as a function of \( n \) and the system geometry the shape of the region of received data is very complex, but describing it as an annular sector is a reasonable approximation (Fig. 3-2). The annular sector has an inner radius of \( R_{B0} \) and an outer radius of \( R_{E0} \). It sweeps out a polar angle of \( \theta_{max} \), where \( \theta_{max} \) is sufficiently large to achieve the desired ideal resolution.

Since the DFT only exists in cartesian Coordinate systems, the 2-D DFT must be of a rectangular region within the annular sector of recorded data. It was shown previously that in the ideal case, to achieve a ground resolution of \( \Delta \), the data region must be a square of
Fig. 3-2 Annular sector indicates collected 'good data', where square region indicates data needed for the DFT
side $\pi c K \sqrt{Y_0}$. Since $M$ and $N$ must be powers of two the possible aspect ratios for the image plane are powers of two. This usually leads to a choice of $N=M$, which implies that $4\theta = \Delta u$.

Assume that the center of the square is at $(0,u_0)$, this being the case for the ground squint angle symmetrically ranging about the broadside angle. From Fig. 3-2 it is now simple to find $\theta_{\text{max}}$, which is also the maximum ground squint angle.

\[
\theta_{\text{max}} = \tan^{-1}\left(\frac{\pi c K / 2 \gamma Y_0}{\frac{u_0 - \pi c K / 2 \gamma Y_0}{\Delta}}\right) = \tan^{-1}\left(\frac{1}{2 \gamma Y_0 \Delta u} \frac{1}{\pi c} \left(\frac{o}{K}\right) - 1\right)
\]  

(3-33)

$u_0$ is located approximately at the midpoint between $R_{B0}$ and $R_{E0}$. Adding equations (3-31) and (3-32) together and dividing by two, for $n$ equals zero, $u_0$ can be approximated as:

\[
u_0 = \frac{K}{r_{oo}} \left[\frac{u_0}{\gamma} + \frac{1}{C} (r_{10} + r_{20} - 2r_{oo})\right]
\]

(3-34)

This can be further simplified by dropping the second term, as it tends to be negligible in typical geometries. A small error in $u_0$ results in a small error in $\theta_{\text{max}}$, so this approximation is sufficiently accurate.

It is now possible to solve for the minimum resolution achievable. This occurs when $\theta_{\text{max}}$ equals $\pi/2$. This implies that the denominator of equ. (3-33) equals zero, or

\[
u_0 = \frac{K u_0}{r_{oo} \gamma}
\]

(3-35)

The distance that must be flown, or the synthetic aperture length $L_s$, is a distance that sweeps out $\theta_{\text{max}}$.  

\[ L_s = 2Y_o \sin \theta_{\text{max}} \] (3-36)

The final equation that is needed characterizes the required length of the transmitted pulse. Looking at Fig. 3-1 it is seen that \( R_{E0} - R_{B0} \) is approximately equal to \( \tau qk / Y_0 \Delta \). Therefore subtracting equ. (3-31) from (3-32) with \( n \) equal to zero, setting it equal to \( \tau qk / Y_0 \Delta \), and solving for the effective pulse width \( T_e \) gives

\[ T_e = \frac{\pi c r_{\infty}}{\gamma Y_0 \Delta} + \frac{2}{c} (r_{10 - r_{20}}) \frac{\pi c r_{\infty}}{\gamma Y_0 \Delta} \] (3-37)

The transmitted pulse must be longer than the effective pulse by the two way time delay across the projection of the ground patch in the slant (range) plane.

\[ T = \frac{\pi c r_{\infty}}{\gamma Y_0 \Delta} + \frac{2N \Delta}{c} \left( \frac{Y_0}{r_{\infty}} \right) \] (3-38)

The recorded data is in polar format with uniform sample spacing in \( R \) and \( \theta \). It is necessary to interpolate the data in the inscribed square region to a uniform sample spacing in \( \rho \) and \( u \). The exact method used will be the result of a tradeoff between hardware and software, where the hardware is affected in both the front end A/D converter and the digital processor.

3.5 POLAR/KEYSTONE TO RECTANGULAR INTERPOLATION

In all of these methods the timing along the IPP's is the same. Let \( t_n \) be the time of the \( n \)th IPP, and \( t_n \) equal \( t_n - t_{n-1} \). Given that the platform moves in a straight line with velocity \( v \), a simple algorithm to compute the flight time to the next IPP (\( t_n \)) is
With the simplest A/D converter scheme, for the nth IPP, the first data point is sampled at $t_{BO}$ and the Nth data point is sampled at $t_{EO}$.

$$t_{BO} = -\frac{T}{2} + \frac{2r_{20}}{C}$$  \hspace{1cm} (3-41)$$

$$t_{EO} = \frac{T}{2} + \frac{2r_{10}}{C}$$ \hspace{1cm} (3-42)

For N data samples the time between samples, $\Delta t$, is given by:

$$\Delta t = \frac{t_{EO} - t_{BO}}{N-1} = \frac{T}{N-1} - \frac{2}{C} \left(\frac{r_{20} - r_{10}}{N-1}\right)$$ \hspace{1cm} (3-43)

Using this method results in uniform polar sampling with all samples lying on equally spaced circles and at equal polar angle increments, as illustrated in Fig. 3-3. This will be referred to as polar sampling.

The second A/D converter scheme requires a faster and more agile converter. In this method the samples lie on equally spaced, constant $u$ contours and at equal polar angle increments as illustrated in Fig. 3.4. This will be referred to as keystone sampling.

The first sample, for the nth IPP, must be at $u$ equal $R_{BO}$. This implies that $R_{Bn}$, the radius at the first sample of the nth IPP, equals $R_{BO}/\cos\theta_n$, or in terms of time

\[ R_{Bn} = \frac{R_{BO}}{\cos\theta_n} \]
Fig. 3-3 Uniform polar sampling grid
Fig. 3-4 Keystone format
Equally spaced, constant u contours require that ΔR equal Δu/cosθ. \(NΔu\) is length of the square of required data divided by \(N-1\), or

\[ΔR_n = \frac{πcK}{\gamma Y_o Δ(N-1)cosθ_n}\]

Substituting equations (3-14) and (3-17) into equ. (3-12) gives

\[R_n(t) = \frac{K}{r_{on} cosθ_n} (t + \frac{w_o}{γ} - \frac{2r_{on}}{c})\]

Since \(R_n(t)\) is linear in \(t\), \(ΔR_n\) is given by:

\[ΔR_n = \frac{K Δt}{r_{on} cosθ_n}\]

Substituting equ. (3-47) into equ. (3-45) and solving for \(Δt\), results in an expression for the time between samples.

\[Δt = \frac{πcr_{on}}{\gamma Y_o Δ(N-1)}\]

The advantage of the keystone format is that the interpolation to rectangular format requires interpolation in only one dimension, i.e. along the p coordinate as shown in Fig. 3-5. In essence the equivalent of optimal bandlimited interpolation along the range returns was performed by sampling the D/A converter at the proper times.

In the polar format it is necessary to perform this range interpolation, as shown in Fig. 3-6. The range interpolation is ideally performed by a bandlimited interpolator (linear filter). The interpolation in the range case is a sample rate reduction of \(cosθ_n\). For background on bandlimited interpolation see [6] and [7].
Fig. 3-5 Interpolation of Keystone to rectangular format
Fig. 3-6 Interpolation of polar to Keystone format
The interpolation in azimuth is more complex due to the non-uniform data sample spacing. In ref. [4] it is indicated that the azimuth case can also be handled by linear filtering since the sample spacing has a constant increment, however the complete solution is not presented. In ref. [8] azimuth interpolation was performed with a third order polynomial interpolator. Polynomial interpolation is very similar to bandlimited interpolation, but the bandlimited interpolation is always superior in signal processing applications ([8]).

Since the R and \( \theta \) are not orthogonal the 2-D data is not separable. Therefore a 2-D interpolator would have better performance with respect to interpolation error. However, due to the nonuniform sample spacing, formulation of a 2-D interpolator is very complex. This presents a computational burden possibly exceeding all the rest of the processing task, depending on the order of the interpolator.

There are two very simple 2-D interpolators, the zero order and the first order interpolator. The zero order interpolator consists of choosing the value of the closest known data point. The first order interpolator takes the weighted average of the four nearest polar neighbors see (Fig. 3-7). The weighting varying inversely with the distance between the point and the neighbor.

\[
f(c) = \sum_{i=0}^{3} \frac{f(p_i)}{d_i} / \sum_{i=0}^{3} \frac{1}{d_i}
\]

(3-49)

In ref. [9] this first order interpolator had fair results in polar to rectangular interpolation for 2-D reconstruction from projections.
The major advantage of performing the range and azimuth interpolation separately, in the polar format case, is that the range interpolation can take place in real time as each individual return is recorded. Since most of the processing takes place after all the returns are collected the processor probably has a large fraction of idle time that can be used for interpolation. This may result in a cost savings by allowing a simpler A/D converter.

Fig. 3-7 2-D linear interpolation.
CHAPTER 4

RECTANGULAR FORMAT

4.1 RECTANGULAR FORMAT AS AN APPROXIMATION TO POLAR FORMAT

Where lower resolution is required a simpler mode can be used, rectangular format. Rectangular format predates polar format but can be interpreted in terms of the analysis of polar format. In rectangular format the recorded data plane is treated as being Cartesian instead of polar by assuming that the range returns along an IPP are orthogonal to the range returns across N IPP's. This is equivalent to approximating $u$ as $R$ and $p$ as $u_0 \theta_n$. This arises from the approximations made in formulating stretch processing as described in chapter 1. Making these substitutions to equation (3-23) we get the following phase function:

$$\mathcal{G}(p,u) = \frac{2 \nu_t^2}{c^2} \left[ \left( \frac{uc\cos p}{K} + 2 \right)(\sqrt{1+Z_n} - 1) - Z_n \right]$$

$$Z_n = \frac{2x_n x_t + 2y_n y_t + \rho^2}{r_{on}^2}$$

$$r_{on}^2 = x_n^2 + y_n^2 + h^2$$

In rectangular format samples are taken at equal spacings along the aperture. This implies that $x_n$ is proportional to $p$. This also implies that the sample density is at a minimum in the center of the $p$, $u$ plane, with most of the samples concentrated near the edges.
Although it is difficult to interpret equ's (4-1) - (4-3), this is not an unreasonable approximation for the case where the slant range $r_{00}$ is much greater than the distance across the patch and the aperture length $L_s$. This is the low to medium resolution case.

For low resolution the maximum squint angle is small, so making small angle approximations for $\sin x$, $\cos x$ and the approximation of equ. (3-10) we arrive at the following approximate phase function:

$$\bar{\gamma}(p,u) = \frac{2\gamma_0}{cK} (y_{tu} + u u_0 x_t p + \frac{\rho^2 u}{2Y_0}) \quad (4-4)$$

Since data is only recorded in the neighborhood of $u_0$, $u/u_0$ is approximately unity, thus we can further approximate the phase function as:

$$\bar{\gamma}(p,u) = \frac{2\gamma_0}{cK} (y_{tu} + x_t p + \frac{\rho^2 u}{2Y_0}) \quad (4-5)$$

This should be recognized immediately as equ. (3-20) which is of near ideal form.

4.2 SOURCE OF IMAGE DEGRADATION

Rectangular format results in images that are perfectly focused at the center (reference point) but with defocusing and misregistration that monotonically increases with the distance from the reference ($\rho$). Recall that the 2-D correlator can be thought of as a 2-D array of matched filters. Rectangular format approximates the 2-D correlator as an array of matched filters where each matched filter is matched to the range and azimuth of the reference instead of the actual range and azimuth of the pixel. Therefore as the distance from the reference...
increases the mismatch between the filter and the ideal matched filter increases.

One interpretation of why the image degradation increases with \( \rho \) is explained by the term rangewalking. Refering to Fig. 4-1, a constant phase contour is generated by mixing the returns with a synthetic reference from point Q. This synthesizes an circular reference path of radius \( r_{00} \) about point Q. The distance from P to the reference path is not constant, i.e. \( r_{2} \) is greater then \( r_{1} \). For \( r_{1} \) to equal \( r_{2} \) implies that P moved to \( P' \) when \( r_{2} \) is measured. Thus P appears to walk through several range and azimuth cells, moving in a curve towards Q. The rangewalk error is clearly zero for a point at Q, and maximized for a point lying along the back diagonal at T or T'.

### 4.3 ANALYSIS OF SPOTLIGHT PARAMETERS

Solving equ. (1-10) for the synthetic aperture length, \( L_{s} \)

\[
L_{s} = \frac{\lambda r_{00}}{2\Delta} \quad (4-6)
\]

Note that for small angle approximations equ. (3-36) reduces to the same value.

For a platform moving at velocity \( v \) the aperture integration interval is:

\[
T_{\text{int}} = \frac{L_{s}}{v} \quad (4-7)
\]
Fig. 4-1 Rangewalk based on constant phase contour model
Solving equ. (1-12) for the effective pulse width of the transmitted pulse we get:

\[ T_e = \frac{c}{2\gamma \Delta} \]  \hspace{1cm} (4-8)

As in the polar case, the transmitted pulse width is given by:

\[ T = T_e + \frac{2(\gamma r_{20} - \gamma r_{10})}{c} + \frac{2(\gamma r_{20} - \gamma r_{10})^2}{c} \]  \hspace{1cm} (4-9)

Sampling begins at \( t_{B0} \), which is given by:

\[ t_{B0} = - \frac{T}{2} + \frac{2r_{20}}{c} \hspace{1cm} (4-10) \]

To solve for the sample spacing the phase function given by equ. (3-4) is repeated here for convenience.

\[ \phi_n(t) = \frac{2}{c} (r_n - r_{on})(\omega_o + \gamma t) - \frac{2\gamma}{c^2} (r_n^2 - r_{on}^2) \]  \hspace{1cm} (3-4)

Demodulating the received signal to baseband and neglecting the \( r_n^2 - r_{on}^2 \) term, as it only contributes a constant phase term within one IPP, we arrive at the following form of the received baseband signal.

\[ e(t) = \exp \{ j \frac{2\gamma t}{c} (r_n - r_{on})t \} \hspace{1cm} (4-11) \]

Equating the the phase function with the kernel phase function of the DFT, in order to prevent aliasing:

\[ \frac{2\gamma \Delta t}{c} \Delta r = \frac{2\pi}{N} \hspace{1cm} (4-12) \]

or

\[ \Delta t = \frac{\pi c}{N \gamma \Delta r} \hspace{1cm} (4-13) \]
The bandwidth is the product of the FM rate and the pulse width.

\[ BW = \frac{\sqrt{T}}{2\pi} \approx \frac{c}{2\pi \Delta} \quad (4-14) \]

To avoid azimuth ambiguity the pulse repetition frequency (PRF) must greater than, or equal to, the bandwidth of the azimuth LFM. It must also be less than, or equal to, the two way path delay to the furthest range point.

\[ \frac{v}{\Delta} \leq PRF \leq \frac{c}{2r_{oo}} \quad (4-15) \]

A PRF of \( v \Delta \) is the case of no oversampling. In practice PRF's near the maximum are used with doppler prefiltering.

4.4 MULTIPATCH FORMAT

In the short discussion on the degradation inherent in rectangular format it was pointed out that the degradation of the image increased with increasing distance from the reference (focus) point. An obvious solution to increase the resolution obtainable is to divide the image up into multiple patches. The image within each subpatch is the result of correlating the returns from points in the subpatch with the return from a reference at the center of the subpatch. Thus the maximum distance from a focus point is the farthest distance in a subpatch from it's focus point. Dividing an \( NxN \) pixel image into \( LxL \) subpatches results in a reduction in \( \theta_{\text{max}} \) by a factor of \( L \).

An alternate explanation of how it improves the image can be understood by recalling that the optimal solution was to process each (range, azimuth) cell with the matched filter for that cell. In the
rectangular format case each range, azimuth cell was processed with a filter matched for the reference point Q. Dividing the NxN pixel image into NxN subpatches results in the optimal solution, the 2-D correlator. Choosing L less then N results in a system realization somewhere between the performance of spotlight and a true 2-D correlator.

Since the degradation is maximized along the back diagonals of the subpatch choosing the reference point closer to the back side may result in better performance then locating the reference in the center of the patch.

There are three basic approaches to implementing multipatch. The first approach is to separate the system into L^2 channels. In each channel the return is mixed with it's associated reference. The L^2 channels are then processed identically to one channel of rectangular format. The final image is constructed by generating a mosaic made of the image pixels in the subpatch of each associated channel (see [10]). This results in a processing load increase greater then a factor of L^2, as well as a memory requirement increase of L^2. If N is of order 1024 only small values of L are practical in today's memory systems. Aside from the memory size problem, trying to access parts of 2-D arrays from a 1-D mass memory will probably result in a processing bottleneck.

The second approach is a slight variation of the first. Since the image plane is the DFT of a NxN pixels and only N/L x N/L points are needed the DFT of just the needed pixels is more efficient if the following condition is satisfied.
For \( N \) equals 1024, \( L \) must be greater than 229. This results in subpatches of a size of 5x5 pixels, not a very likely implementation.

There probably exists an FFT like algorithm that could compute the \( N/L \times N/L \) pixels of order \( O[(2N^2/L^2)\log_2 N] \). This assumption is based on the fact that the order of computation of an FIR filter of order \( N \), decimated by a factor \( L \), is \( O(N/L) \). No existing work on this problem is known to the author.

The third method is to mix the return with its associated reference in each of the \( L^2 \) channels. Next the output of each filter is low pass filtered with a filter whose passband lies in the desired subpatch. After filtering all the channels are summed together and then 2-D FFT transformed, the advantage being that only one 2-D FFT need be performed on the data plane. The drawback is that there is no currently known algorithm for 2-D lowpass complex filtering that is extremely efficient (see [11]).
5.1 IMAGE DEGRADATION

A computer model was developed to experimentally investigate the nature of misregistration and defocusing present in polar, spotlight and multipatch formats. The system parameters were as follows.

\[ Y_0 = 10,000 \text{ m} \]
\[ h = 5000. \text{ m} \]
\[ v = 200 \text{ m/sec} \]
\[ f_0 = 1.00 \text{ GHz} \]
\[ T_e = 100. \mu\text{sec} \]

There was no oversampling and all other parameters are a function of resolution.

Equations (3-26) - (3-28) formed the basis for the polar format program. This bypasses the effects of the interpolator by generating the data directly in Cartesian coordinates, and therefore simulates processing with ideal interpolation. Due to constraints in funding and achieving reasonable execution times \( N \) was constrained to be 64. In a 64x64 pixel image with resolutions between 20 m and 0.01 m misregistration was always less than one pixel and loss of resolution due to defocusing was dominated by the effects of the window. Fig. 5-1 - 5-4 clearly show almost no loss in resolution in resolutions at least
Fig. 5-1 Polar format, resolution equals 1.0m.
Fig. 5-2 Polar format, resolution equals 0.5m.
Fig. 5-3 Polar format, resolution equals 0.5m.
Fig. 5-4  Polar format, resolution equals 0.01m.
as fine as .01 m. Recalling that the wavelength of the center frequency of the transmitted pulse is .3 m, this clearly demonstrates subwavelength resolution! The target on the back diagonal is located at (51,51) in terms of pixels, and the target in the center of the plane, at (31,31), is at the location of the reference point.

It is suggested that larger values of N be investigated in the future so that misregistration and defocusing effects will not be masked by limitations in resolution imposed by the relatively large spatial bandwidth associated with each pixel.

Equation (3-4) formed the basis for the rectangular format program. In the rectangular format defocusing and misregistration became significant for resolutions greater than 1. m. In Fig. 5-5 - 5-8 the range walk phenomenon is very obvious at resolutions greater than .5 m. As in the polar format case, the targets are at (51,51) and (31,31). Note that the target at the reference point stays in perfect focus as expected since the filter is exactly matched to a return from that point. The target on the back diagonal increasingly defocuses as the resolution gets finer. Note the clear demonstration of rangewalking. This defocusing is characterized by the center of the target moving towards the reference and distributing its energy in a curve related to the rangewalking path.

Fig. 5-9 - 5-12 show how defocusing varies with distance from the reference point. All plots in this series are at 0.2 m resolution. The targets are all located on the back diagonal as this maximizes the defocusing as a function of distance. Fig. 5-12 illustrates wraparound in both range and azimuth of the defocused target. This is due to
Fig. 5-5 Rectangular format, resolution equals 20.0m.
Fig. 5-6 Rectangular format, resolution equals 0.5m.
Fig. 5-7 Rectangular format, resolution equals 0.2m.
Fig. 5-8 Rectangular format, resolution equals 0.1m.
Fig. 5-9 Rectangular format, targets @ (31,31) and @ (39,39)
Fig. 5-10 Rectangular format, targets @ (31,31) and @ (47,47)
Fig. 5-11 Rectangular format, targets @ (31,31) and @ (57,57)
Fig. 5-12 Rectangular format, targets @ (31,31) and @ (63,63)
aliasing of the sampled time domain data. Some of the energy of the
defocused target lies in frequencies above half the sampling frequency.
This is normally eliminated by oversampling and doppler prefiltering.
To increase efficiency of the computer model it was decided not to
oversample and prefilter since the data was generated in the baseband
and only targets within the ground patch to be mapped were to be
simulated.

An efficient measured, called spread, was developed to evaluate the
defocusing. Spread is a measure of the width of the mainlobe, defined
as the square root of the number of pixels exceeding a given threshold.
This gives an approximate measure of the average width of the mainlobe
in an arbitrary direction. It has the dimension of pixels. Experimentally it was determined that a threshold of -40 dB gave the
most meaningful results.

Referring to Fig. 5-13, it was found experimentally that spread is
approximately a linear function of the distance from the reference
point. A spread of three pixels is an artifact of the window
(Appendix). Thus an ideal curve would be a line parallel to the axis
at three pixels.

As expected plotting spread as a function of resolution results in
a curve that asymptotically approaches infinity as approaches zero,
and asymptotically approaches three as approaches infinity, as shown
in Fig. 5-14.
Fig. 5-13 Spread as a function of target distance from reference

Resolution = 0.2 m
Threshold = -40.08
Fig. 5-14 Spread as a function of resolution
Work needs to be done to develop a normalized version of these graphs in terms of the system geometry and the center frequency. This would allow the system designer to immediately determine the achievable resolution for a given center frequency and geometry for rectangular format.

The real value of these graphs is in choosing the subpatch size for a multipatch format. Consider the following example. It is desired to obtain 0.20 m resolution with a worst case spread of nine pixels. Refering to Fig. 5-13 it is seen that if the subpatch is 8x8 pixels the desired spread is met. The number of subpatches being given by:

$$N_{SUB} = \left(\frac{NA}{8}\right)^2 = \frac{N^2}{1600} \tag{5-1}$$

where N equals the desired length of the side of the image.

5.2 FIRST ORDER INTERPOLATION OF POLAR FORMAT

The first order interpolator given in equ. (3-49) was evaluated. The first order interpolator was choosen because of its computational speed. Although it was realized that it's performance would be far from ideal it was invstigated as the beginning of a study of 'how good is good enough.' Refering to Fig's 5-15 - 5-17, and comparing them to Fig's 5-1 - 5-3, it can be observed that the first order interpolator eliminated the rangewalking. The mainlobe is precisely where it ideally should be. This is a very significant result, for it clearly demonstrates the need and the power of the polar format in high resolution cases. The simplest possible interpolator eliminated rangewalk! There is however so much interpolator noise that if a full
Fig. 5-15 Polor format with first order interpolation resolution equals 1.0m.
Fig. 5-16 Polar format with first order interpolation, resolution equals 0.5m.
Fig. 5-17 Polar format with first order interpolation, resolution equals 0.5m.
image were processed it would probably be buried in noise, since the sidelobes are only 30 dB down. As expected the noise is highly correlated near the mainlobe and weakly correlated farther away.

Much work remains to be done investigating the tradeoffs involved in different interpolation schemes.
When the ideal response of a point target was derived it was shown to be an impulse in the spatial frequency domain, which has a spread of unity. It is not possible to actually achieve a spread of unity because the recorded time domain data is of finite extent. Collecting only an \(N \times N\) block of data is equivalent to multiplying an infinite plane of data by a rectangular window which is unity over the \(N \times N\) region and zero elsewhere. This is equivalent to convolving the transformed data with the transform of the window in the frequency domain. The rectangular window in the frequency domain being:

\[
\overline{W}(e^{ju}, e^{jv}) = \frac{\sin(uN/2)}{\sin(u/2)} \cdot \frac{\sin(vN/2)}{\sin(v/2)} \quad (A-1)
\]

This results in a mainlobe \(\pi/N\) wide with the first sidelobes down only 13dB. Since the convolution of an impulse with any function yields the original function this implies that to transform the \(N \times N\) block of data directly would result in a worst case spread of \(N\) pixels (the entire image plane could have sidelobes less than 40DB down).

To improve the resolution and decrease the spectral leakage the time domain data can be multiplied by a window chosen to minimize the mainlobe width and to minimize the sidelobes. To better understand the nature of an ideal window the problem of spectral leakage can also be understood in terms of the periodic extension of the time domain.
sequence. The DFT treats the data sequence as being circularly periodic, the next data point after the Nth data point being the first data point of the sequence. Thus if the sequence is not periodic in N or N/I, where I is an integer, at the point where the sequence is periodically extended a periodic discontinuity arises. This periodic discontinuity generates the unwanted spectral leakage.

Therefore an ideal window in a general sense is a sequence that when multiplied by the data yields a sequence where at the boundary of the periodic extension as many orders of derivatives of the sequence to the right of the boundary match the derivatives to the right of the sequence. Commonly the derivatives are set to zero at the boundary.

The window used in all the simulation work was the Kaiser-Bessel window, which is optimum in the sense that for a finite sequence it minimizes the spectral energy beyond a specified frequency [12]. It also has the advantage that of several windows that are optimal in various ways it is relatively easy to compute. The window being given by:

$$W_B(n,m) = \frac{I_0[\pi \alpha \sqrt{1 - \left(\frac{n}{N/2}\right)^2}] \cdot I_0[\pi \alpha \sqrt{1 - \left(\frac{m}{N/2}\right)^2}]}{I_0(\pi \alpha)} ; \quad 0 \leq |n| \leq \frac{N}{2}$$

(A-2)

Where $I_0(x)$ is the modified zero order Bessel function

$$I_0(x) = \sum_{k=0}^{\infty} \left(\frac{x}{2}\right)^k \frac{1}{k!}$$

(A-3)

The spatial frequency domain response of the window with $\alpha=1.8$ is shown in Fig. A-1 and A-2.
Fig. A-1  Spacial frequency domain response of Kaiser-Bessel window.
Fig. A-2 Spacial frequency response of Kaiser-Bessel window.
REFERENCES


